

Q2-1

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MA105 : Final Exam (17:30 - 19:30 on 16/11/2023)

Total Marks: 40

Evaluate $\int \int_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = (Axe^y, Bze^x - Ae^y, x^2 + y^2)$.

$\phi(u, v) = (4\cos v, 4\sin v)$

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(13) (3 marks) Let \mathbf{F} be a C^2 vector field on \mathbb{R}^2 . For any point $P \in \mathbb{R}^2$, suppose that $\int_{\gamma_1} \mathbf{F} \cdot d\mathbf{s} = \int_{\gamma_2} \mathbf{F} \cdot d\mathbf{s}$, where γ_1 and γ_2 are paths with initial point (A, B) and end point P . Which of the following statements is/are TRUE?

- (A) $\nabla \times \mathbf{F}$ is zero except the point (A, B) .
- (B) $\nabla \times \mathbf{F} = 0$.
- (C) $\nabla \cdot \mathbf{F} = 0$.
- (D) $\nabla \cdot \mathbf{F}$ is zero at (A, B) .
- (E) $\nabla \cdot \mathbf{F}$ can be non-zero at every point.

B, C, D

Division 22/ Tutorial Batch T9
Name: Puyanshu Sharma

Roll Number: 2380624
A = 9 B = 7

• Fill in the numbers "A" and "B" above as follows:
If the last digit a of your roll number satisfies $0 \leq a \leq 4$, let $A = a + 5$. If $5 \leq a \leq 9$, let $A = a$.
If the second-last digit b of your roll number satisfies $0 \leq b \leq 4$, let $B = b + 5$. If $5 \leq b \leq 9$, let $B = b$. Thus $5 \leq A, B \leq 9$.
Example: Your Roll number is 238092. Then $A = 7$ and $B = 9$.
You must use these values of A and B below. Using the wrong value of A or B in even one question may lead to the loss of full marks in this exam.

Write the answers of the appropriate Questions only in the box provided below the questions.

You will get full (respectively, partial) marks in the multiple choice/selection questions below only if you select all (respectively, some but not all) of the TRUE statements and only the TRUE statements (that is, if you select a FALSE statement, you will get ZERO mark in that question).

(1) (1 + 1 = 2 marks) Write down the area between the circles $x^2 + y^2 = Ax$ and $x^2 + y^2 = Bx$ as a double integral.

$\int_0^A \int_0^B x^2 dx dy$ where $B = Ax^2$

The area is

2080πb

(2) (2 marks) Find the flow line for the vector field $(Ax, By, (A + B)z)$ passing through the origin.

$9Ax + 7By + 16z = 0$

(3) (2 marks) Rewrite the integral

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y^2} f(x, y, z) dz dy dx$$

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as an iterated integral with order of integration $dx dy dz$.

- (4) (3 marks) Let W be the set defined by $x^2 + y^2 + z^2 \leq A^2$ and $y \leq x$. Find the flux of $(x^3 - Bxz, y^3 + Bxy, z^3 - Bzx)$ out of W .

$\int_0^A \int_0^x \int_0^{\sqrt{A^2-x^2-y^2}} (x^3 - Bxz, y^3 + Bxy, z^3 - Bzx) \cdot d\mathbf{r} \, dy \, dz$

- (5) (2 marks) If C is a simple closed curve in \mathbb{R}^3 , and \mathbf{v} is the constant vector $(A, B, 0)$, what is the value of $\int_C \mathbf{v} \cdot d\mathbf{s}$?

0

- (6) (3 marks) Let S be the surface of the tetrahedron whose vertices are $(A, 0, 0)$, $(0, A, 0)$, $(0, 0, 2A)$ and the origin. Evaluate $\int_S f \, dS$, where $f(x, y, z) = Bxz$.

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- (7) (2 + 1 + 2 = 5 marks) Let \mathbf{n} be the outer unit normal (normal away from the origin) of the parabolic shell

$S : Ax^2 + y + Az^2 = B, y \geq 0,$

and let

$\mathbf{F} = \left(-z + \frac{1}{2+x}, \tan^{-1} y, x + \frac{1}{4+z} \right).$

Then \mathbf{n} is

$\frac{1}{\sqrt{2}}(1, 1, 1)$

Curl \mathbf{F} is

-2j

The value of

$\iint_S \text{Curl } \mathbf{F} \cdot d\mathbf{S}$

-14π

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- (8) (1 + 2 = 3 marks) Let S be the surface $z = \sin x \sin y$ with upward pointing normals and where $0 \leq x \leq \pi$ and $0 \leq y \leq \pi$. Find $\text{Curl } \mathbf{F}$, where $\mathbf{F} = (5z - y^2, x \cos z, xy)$.

$\text{Curl } \mathbf{F} = (2 + 2\sin z)\mathbf{i} + (5 - y)\mathbf{j} + (\cos z + y)\mathbf{k}$

Find the surface integral of $\text{Curl } \mathbf{F}$ over S .

0

- (9) (2 marks) Consider the vector field \mathbf{F} on \mathbb{R}^2 , given by $\mathbf{F}(x, y) = (Ae^x(x \cos x + \sin x) - By, Be^y(y \sin y - \cos y) + Ax)$. Evaluate $\int_\gamma \mathbf{F} \cdot d\mathbf{s}$, where γ is the unit circle with anti-clockwise orientation.

0

- (10) (5 marks) Let \mathbf{F} be a vector field defined by

$\mathbf{F}(x, y) = \begin{cases} \frac{x^2(x \cos y + y \sin x) - x^2}{x^2 + y^2} \mathbf{i} + \frac{x^2(x \sin y - y \cos x) + y^2}{x^2 + y^2} \mathbf{j} \end{cases}$
 $(x, y) \neq (0, 0)$
 $(x, y) = (0, 0)$

Which of the following statements is/are TRUE?

- (A) \mathbf{F} is a conservative vector field.
(B) \mathbf{F} is not a conservative vector field.
(C) The line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{s}$ is non-zero for any simple closed curve containing the origin.
(D) The line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{s}$ is non-zero for any simple closed curve in \mathbb{R}^2 .

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- (11) (3 marks) Find an equation for the plane through the origin such that the circulation of the flow field $\mathbf{F} = Az\mathbf{i} + Bz\mathbf{j} + y\mathbf{k}$ around the circle of intersection of the plane with the sphere $x^2 + y^2 + z^2 = 4$ is maximum.

$\text{div } \mathbf{F} = 0$

- (12) (2 + 3 = 5 marks) Let S be the paraboloid $z = x^2 + y^2, z \leq 4$. Let the orientation on S be such that it induces anti-clockwise orientation on the boundary circle when viewed from a high point upon the positive z -axis. Find an orientation preserving parametrization of S .

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