Waves

(yeet.)

Chapter 2 of PC5131 is Waves... I. This is effectively singular waves, rather than interfering waves. This includes normal wave properties and equations, wave intensity, string waves, polarisation, sound waves (longitudinal waves) and the Doppler Effect. This is relatively easier than the next topic (Waves II) so uh yay thank god only this much is tested in the Common Test on Monday. Anyways yeah good luck, and let's see what comes out of this, right? Aight. See yah.

Basics

Wave Quantities

$$\begin{aligned}
\omega &= 2\pi f = \frac{2\pi}{T} \\
k &= \frac{2\pi}{\lambda} \\
v &= f\lambda = \frac{f}{T} = \frac{\omega}{k}
\end{aligned}$$

Wave Equation

$$\begin{aligned}
\psi(x,t) &= A\sin(\phi_x - \phi_t + \phi_0) \\
&= A\sin(kx - \omega t + \phi_0)
\end{aligned}$$

For wave moving backwards:

$$\psi(x,t) = A\sin(kx + \omega t + \phi_0)$$

Phase Difference

$$\begin{aligned}
\Delta\phi &= \phi_2 - \phi_1 \\
&= (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0) \\
&= k(x_2 - x_1) \\
&= k\Delta x \\
&= 2\pi\frac{\Delta x}{\lambda} \\
&= -\omega\Delta t \\
&= -2\pi\frac{\Delta t}{T}
\end{aligned}$$

This applies to both Δt and Δx, which is great. If the wave is moving backwards, the phase difference in terms of Δt is positive, not negative.

Intensity of Wave

$$
\begin{aligned}
I &= \frac{P}{A} \\
&= \frac{P}{4\pi r^2} \\
I &\propto \frac{1}{r^2} \\
\frac{I_1}{I_2} &= \left(\frac{r_2}{r_1}\right)^2
\end{aligned}
$$

String Wave

$$
v = \sqrt{\frac{T}{\mu}}
$$

Polarisation

Polarising Filter

$$
\begin{aligned}
I_1 &= \frac{I_0}{2} \\
A_{n+1} &= A_n \cos\phi_{n+1,n} \\
I_{n+1} &= I_n \cos^2\phi_{n+1,n}
\end{aligned}
$$

Brewster's Angle

$$
\begin{aligned}
n_a \sin\theta_i &= n_a \sin\theta_{reflect} \\
n_a \sin\theta_i &= n_b \sin\theta_{refract} \\
n_a \sin\theta_{reflect} &= n_b \sin\theta_{refract}
\end{aligned}
$$

From here, we note the following for **Brewster's Angle**, which is defined as θ_b.

$$
\begin{aligned}
\theta_{reflect} &= \theta_{brewster} = \theta_b \\
\theta_{refract} &= \theta_{polarised} = \theta_p \\
\theta_b + \theta_p &= \frac{\pi}{2} \\
\sin\theta_p &= \cos\theta_b \\
n_a \sin\theta_b &= n_b \sin\theta_p = n_b \cos\theta_b \\
\frac{\sin\theta_b}{\cos\theta_b} &= \frac{n_b}{n_a} \\
\tan\theta_b &= \frac{n_b}{n_a} \\
\theta_b &= tan^{-1}\left(\frac{n_b}{n_a}\right)
\end{aligned}
$$

Sound

Speeds of Sound

Condition	Speed
0°C (Standard Temperature and Pressure)	330 m/s
30°C (Room Temperature and Pressure)	343 m/s
T°C (Any Temperature)	$331.5 \times \sqrt{1 + \frac{T}{273.15}}$

Doppler Effect

$$f' = f\frac{c \pm v_o}{c \mp v_s}$$

Source	Observer	Numerator	Denominator	Remarks
stationary	stationary	c	c	-
approaching	stationary	c	$c - v_s$	-
moving away	stationary	c	$c + v_s$	-
stationary	approaching	$c + v_o$	c	-
stationary,	moving away	$c - v_o$	c	-
approaching	approaching	$c + v_o$	$c - v_s$	maximum
moving away	approaching	$c + v_o$	$c + v_s$	-
approaching	moving away	$c - v_o$	$c - v_s$	-
moving away	moving away	$c - v_o$	$c + v_s$	minimum

Just draw a diagram, for god's sake.