Rotational Mechanics

(yeet.)

Translational and Rotational Counterparts

It's just everything we have already learnt, but they added angular in front of it. So let's just walk through a list of equations, and their rotational counterparts.

Translational	Rotational
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$\Delta x = v_i t + \frac{1}{2}at^2 = \frac{v_f+v_i}{2}t$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{\omega_i+\omega_f}{2}t$
$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
$F_{net} = ma$	$\tau_{net} = I\alpha$
$p = mv$	$L = I\omega$
$J = F\Delta t$	$\Delta L = \tau\Delta t$
$W = F \cdot s$	$W = \tau \cdot \Delta\theta$
$E_K = \frac{1}{2}mv^2$	$E_L = \frac{1}{2}I\omega^2$
$P = \frac{W}{t} = F \cdot v$	$P = \tau \cdot \omega$

Moment of Inertia, I

Do note that I is slightly more cursed, and the following table demonstrates the definition of I:

Situation	I	β
Moment of Inertia off from I_{cm}	$I = I_{cm} + Md^2$	-
Moment of Inertia of Multiple Parts	$I = \sum I_{part}$	-
Hoop (from the centre)	$I = MR^2$	1
Circular Disc (from the centre)	$I = \frac{1}{2}MR^2$	$\frac{1}{2}$
Circular Disc (through diameter)	$I = \frac{1}{4}MR^2$	$\frac{1}{4}$

Situation	I	β
Long Rod (from the centre)	$I = \frac{1}{12}ML^2$	$\frac{1}{12}$
Long Rod (from end)	$I = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$	$\frac{1}{3}$
Solid Sphere	$I = \frac{2}{5}MR^2$	$\frac{2}{5}$
Hollow Sphere	$I = \frac{2}{3}MR^2$	$\frac{2}{3}$

Shifting from Translational to Rotational

We also note that we can relate all the quantities to each other via the variable r, as shown below:

Translational	Rotational
Δx	$r\Delta\theta$
v	$r\omega$
a	$r\alpha$
I	βmr^2
$\vec{F} \times \vec{r}$	$\vec{\tau}$
$\vec{r} \times \vec{p}$	$\vec{L}$

Conservation Cases

Cases to be considered for collisions:

Situation	Translational	Rotational
Elastic Collision	COE, COM applies	COE, COAM applies
Inelastic Collison	COM applies, COE does not	COAM applies, COE does not
Completely Inelastic Collision (objects stick together)	COM applies, COE does not	COAM applies, COE does not
Some force acting throughout the motion (e.g. Gravity)	COE applies, COM does not	COE applies, COAM does not

Ramp

Name	Quantity
Rotational Kinetic Energy	$K_{rot} = \frac{\beta}{1+\beta}mgh$
Translational Kinetic Energy	$K_{trans} = \frac{1}{1+\beta}mgh$
Velocity of Centre of Mass	$v_{cm} = \sqrt{\frac{2gh}{1+\beta}}$
Acceleration of Centre of Mass	$a_{cm} = \frac{v_{cm}^2}{2\Delta x} = \frac{g\sin\theta}{1+\beta}$
Velocity at Top of Loop-*de*-Loop (Radius R)	$v_{loop} = \sqrt{g(R-r)}$
Minimum height, h for Loop-*de*-Loop	$h = \frac{5+\beta}{2}(R-r)$

Loop-*de*-Loop

Loop-*de*-Loop for normal circular motion:

$$\begin{aligned} N = \frac{mv^2}{R} - mg = 0 \\ v^2 = Rg \\ \frac{1}{2}mv^2 = mg(h-2R) \\ v^2 = 2g(h-2R) = Rg \\ 2h - 4R = R \\ h = 2.5R \end{aligned}$$

For rigid body of radius r:

$$\begin{aligned} N = \frac{mv^2}{R-r} - mg = 0 \\ v^2 = g(R-r) \\ \frac{1}{2}(1+\beta)mv^2 = mg(h+2r-2R) \\ (1+\beta)g(R-r) = 2g(h+2r-2R) \\ (1+\beta)(R-r) = 2h - 4(R-r) \\ 2h = (5+\beta)(R-r) \\ h = \frac{5+\beta}{2}(R-r) \end{aligned}$$