

Chapter 4 Nuclear Reactions

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Unless otherwise stated:

Elementary Charge, $q_e = 1.60 \times 10^{-19}$ C

Speed of Light, $c = 3.00 \times 10^8$ m/s

Atomic Mass Unit, $u = 1.66 \times 10^{-27}$ kg

Proton Mass, $m_p = 1.007276$ u

Neutron Mass, $m_n = 1.008665$ u

Electron Mass, $m_e = 0.000549$ u

1. By how much does the mass of a heavy nucleus change as it emits a 4.8 MeV gamma ray photon?

Solution:

By the Mass-Energy Equivalence ($\Delta E = \Delta mc^2$), we have that the energy of the photon is equivalent to the change of energy of the heavy nucleus (by the law of conservation of energy), thus we have the following:

$$\Delta E = \Delta mc^2$$

$$4.8 \text{ MeV} = \Delta mc^2$$

$$4.8 \times 10^6 \times q_e = \Delta m(c^2)$$

$$\Delta m = \frac{4.8 \times 10^6 \times 1.6 \times 10^{-19}}{(3.00 \times 10^8)^2}$$

$$= 8.53333 \times 10^{-30} \text{ kg}$$

$$= 5.1406 \times 10^{-3} \text{ u}$$

$$\approx \mathbf{5.2 \times 10^{-3} \text{ u}}$$

2. Find the binding energy of $^{107}_{46}\text{Ag}$, which has an atomic mass of 106.905 u. Express your answer to three significant figures.

Solution:

This is simply the mass-energy equivalence, again.

$$\begin{aligned}
 B.E. &= \Delta mc^2 \\
 &= (46m_p + (107 - 46)m_n + 46m_e - m_{Ag})c^2 \\
 &= (46 \times 1.007276 + 61 \times 1.008665 + 46 \times 0.000549 - 106.905) \text{ u} \times c^2 \\
 &= 0.9826 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 \\
 &= 1.4680044 \times 10^{-10} \text{ J} \\
 &= 917.50275 \text{ MeV} \\
 &\approx \mathbf{918 \text{ MeV}}
 \end{aligned}$$

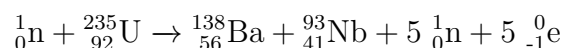
3. The binding energy per nucleon for elements near iron in the periodic table is about 8.90 MeV per nucleon. Estimate the atomic mass (including electrons) of $^{56}_{26}\text{Fe}$.

Solution:

Since $A = 56$ nucleons, we can get the total binding energy to be as follows:

$$\begin{aligned}
 B.E. &= 56 \times 8.90 \text{ MeV} \\
 \Delta mc^2 &= 498.4 \times 10^6 \times 1.60 \times 10^{-19} \text{ J} \\
 (26 \times m_p + 26 \times m_e + 30 \times m_n - m_{\text{Fe}}) &= \frac{7.9744 \times 10^{-11}}{c^2} \\
 m_{\text{Fe}} &= (26 \times 1.007276 + 26 \times 0.000549 + 30 \times 1.008665) \text{ u} - \frac{7.9744 \times 10^{-11}}{c^2} \\
 m_{\text{Fe}} &= (56.4634) - \frac{7.9744 \times 10^{-11}}{(3.00 \times 10^8)^2 \times 1.66 \times 10^{-27}} \\
 &= 55.92964 \text{ u} \approx \mathbf{55.9 \text{ u}}
 \end{aligned}$$

4. Consider the following fission reaction:



The masses of one unit of each component (in terms of u) are given. How much energy is released when:

- (a) 1 atom undergoes this type of fission?

Solution:

By Mass-Energy Equivalence,

$$\begin{aligned}
 E_{\text{released}} &= \Delta mc^2 \\
 &= (1.0087 + 235.0439 - 137.9050 - 92.9060 - 5 \\
 &\quad \times 1.0087 - 5 \times 0.00055) \text{ u} \times c^2 \\
 &= 0.19535 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 \\
 &= 2.918529 \times 10^{-11} \text{ J} \\
 &= 182.40806 \text{ MeV} \\
 &\approx \mathbf{182 \text{ MeV}}
 \end{aligned}$$

(b) 1.0 kg of atoms undergoes fission?

Solution:

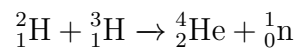
1.0 kg of Uranium atoms of mass 235.0439 u gives the following number of atoms:

$$\begin{aligned}
 n_{\text{atoms}} &= \frac{1.0}{235.0439 \times 1.66 \times 10^{-27}} \\
 &= 2.5630 \times 10^{24} \text{ atoms}
 \end{aligned}$$

By Mass-Energy Equivalence,

$$\begin{aligned}
 E_{\text{released}} &= n_{\text{atoms}} E_{\text{atom}} \\
 &= 2.5630 \times 10^{24} \times 2.918529 \times 10^{-11} \\
 &= 7.48019 \times 10^{13} \text{ J} \\
 &\approx \mathbf{7.5 \times 10^{13} \text{ J}}
 \end{aligned}$$

5. See notes for question, see other page for my answer on the whiteboard.
6. One of the most promising fusion reactions for power generation involves deuterium (H-2) and tritium (H-3):



where the atomic masses (including electrons) are as given. How much energy is produced when 2.0 kg of H-2 fuses with 3.0 kg of H-3 to form He-4?

Solution:

Let's start by getting the amount of energy released in one such reaction (which is also given by the Mass-Energy Equivalence).

$$\begin{aligned}
 E_{\text{reaction}} &= \Delta mc^2 \\
 &= (2.01410 + 3.01605 - 4.00260 - 1.00867) \text{ u} \times c^2 \\
 &= 0.01888 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 \\
 &= 2.82067 \times 10^{-12} \text{ J}.
 \end{aligned}$$

Now we need to find the limiting reagent. To do this, we compute the number of atoms of H-2 and H-3 separately.

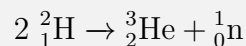
$$\begin{aligned}
 n_{\text{H-2}} &= \frac{2.0}{2.01410 \times 1.66 \times 10^{-27}} \\
 &= 5.98192 \times 10^{26} \text{ atoms} \\
 n_{\text{H-3}} &= \frac{3.0}{3.01605 \times 1.66 \times 10^{-27}} \\
 &= 5.99204 \times 10^{26} \text{ atoms}
 \end{aligned}$$

Thus, deuterium is in the minority, hence we have that $n_{\text{reactions}} = n_{\text{H-2}}$. From here, we can extrapolate the energy released overall, as follows:

$$\begin{aligned}
 E_{\text{released}} &= n_{\text{reactions}} E_{\text{reaction}} \\
 &= 5.98192 \times 10^{26} \times 2.82067 \times 10^{-12} \\
 &= 1.68730 \times 10^{15} \text{ J} \\
 &\approx \mathbf{1.7 \times 10^{15} \text{ J}}
 \end{aligned}$$

7. In a fusion reaction, two deuterons (${}^2_1\text{H}$, atomic mass = 2.01410 u) fuse to form ${}^3_2\text{He}$ (atomic mass = 3.01603 u) with the release of a neutron.

(a) Write the equation for this reaction.

Solution:

- (b) Find the energy released in this fusion reaction. Express your answer to three significant figures.

For the umpteenth time, by Mass-Energy Equivalence, we have the following:

$$\begin{aligned}E_{\text{released}} &= \Delta mc^2 \\&= (2 \times 2.0141 - 3.01603 - 1.008665) \text{ u} \times c^2 \\&= 0.0035050 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 \\&= 5.23647 \times 10^{-13} \text{ J} \\&= 3.2728 \text{ MeV} \\&\approx \mathbf{3.27 \text{ MeV}}\end{aligned}$$