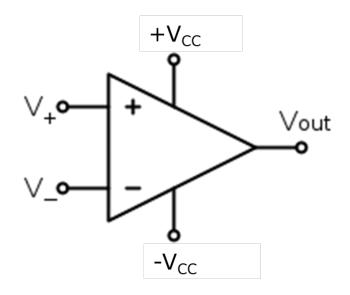
CG1111A: Engineering Principles & Practice I

Tutorial 4: Reflections & Problem Solutions (12/13 Oct 2022)



Comparator

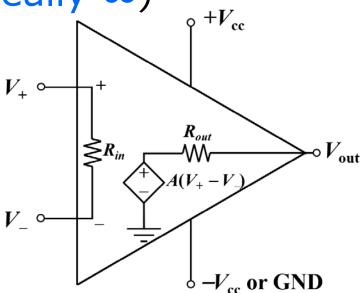


The comparator is an electronic decisionmaking circuit that makes use of an opamp's very high gain in its open-loop state (i.e., there is no feedback resistor)

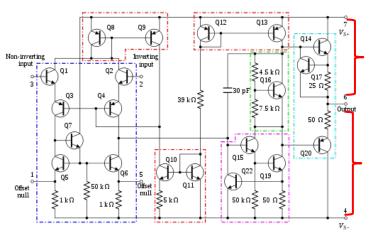
Op-Amp as a Comparator – How It Works

Recall that for op-amp:

- The difference between the two inputs is amplified as $(A(V_{+} V_{-}))'$ at the output
- The open-loop voltage gain ('A') of the op-amp is very high (ideally ∞)
- Even if there is a very small difference between the inputs, the high 'A' will pull the output to "saturation"

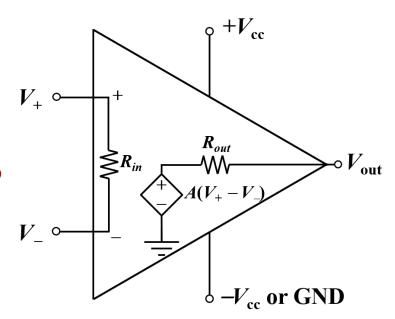


What Are Saturation Voltages?



V_{ovh1} needed for NPN transistor to turn on

V_{ovh2} needed for PNP transistor to turn on



If V₊ > V_−:

$$V_{\text{out}} = V_{CC} - V_{\text{ovh1}}$$

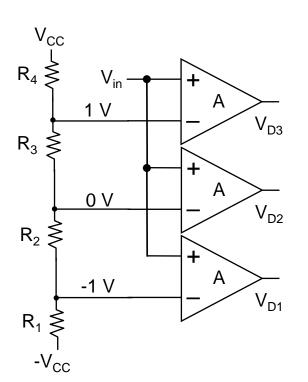
If V_− > V₊:

$$V_{\text{out}} = \begin{cases} -V_{CC} + V_{\text{ovh2}} & \text{if dual power supply} \\ V_{\text{ovh2}} & \text{if single power supply} \end{cases}$$

V_{ovh1} & V_{ovh2} are the voltage headrooms from the supply rails (V_{CC}, -V_{CC}, or GND) needed to sustain proper output transistor turn-on voltage, and it varies between 0.05 to 1.5 V depending on the output current

Common Application of Comparator

 The comparator is ideal for converting analog signals to digital signals at certain threshold values

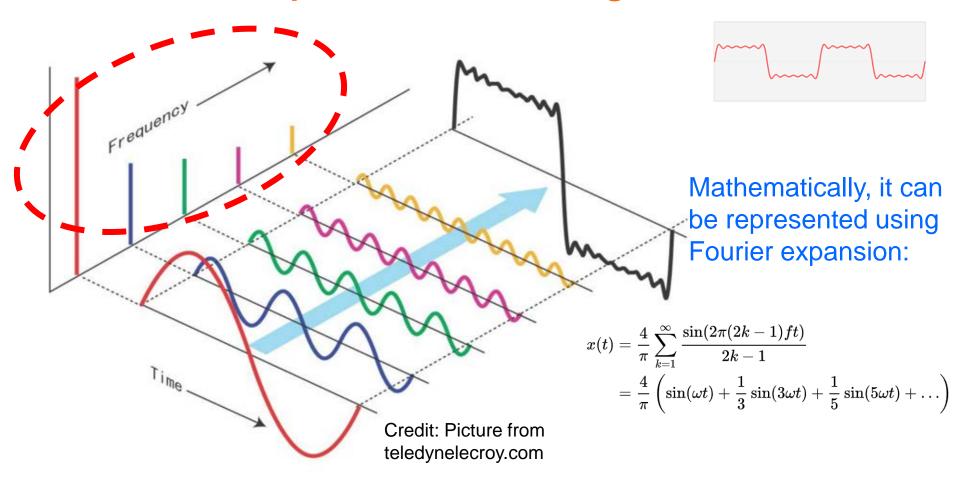


V _{in}	V_{D3}	V_{D2}	V_{D1}	ADC
$V_{in} < -1 V$	L	L	L	00
$-1 V < V_{in} < 0 V$	L	L	Н	01
$0 V < V_{in} < 1 V$	L	Н	Н	10
$V_{in} > 1 V$	Н	Н	Н	11

Spectral Analysis

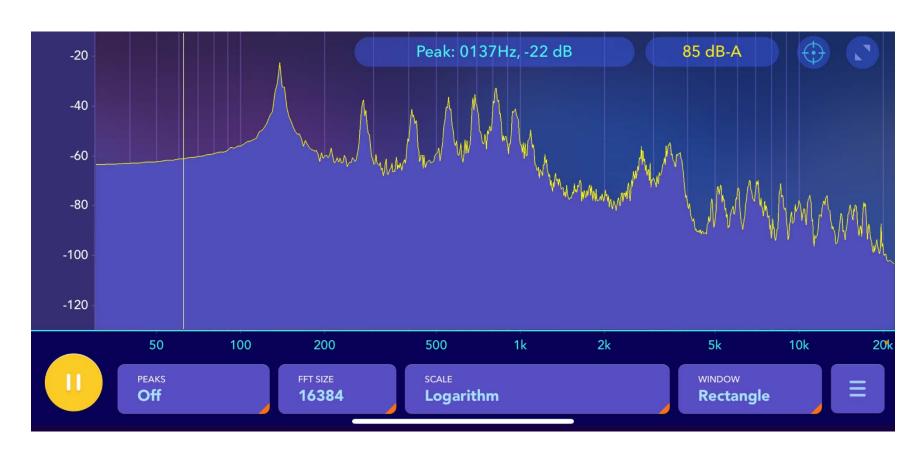
- Any function of time can be described as a sum of sinusoidal waves, each with different amplitudes and frequencies
- Spectral analysis studies the distribution of a signal's frequency components
- The plot of a signal's frequency components and their corresponding magnitudes is called "frequency spectrum"

Spectral Analysis



- We are analysing a signal in the frequency domain
- A square wave can be decomposed into an infinite sum of sinusoidal waves

Example of Frequency Spectrum



The frequency spectrum of a particular audio tone

Filter

 A filter is a device or process that removes some unwanted components or features from a signal

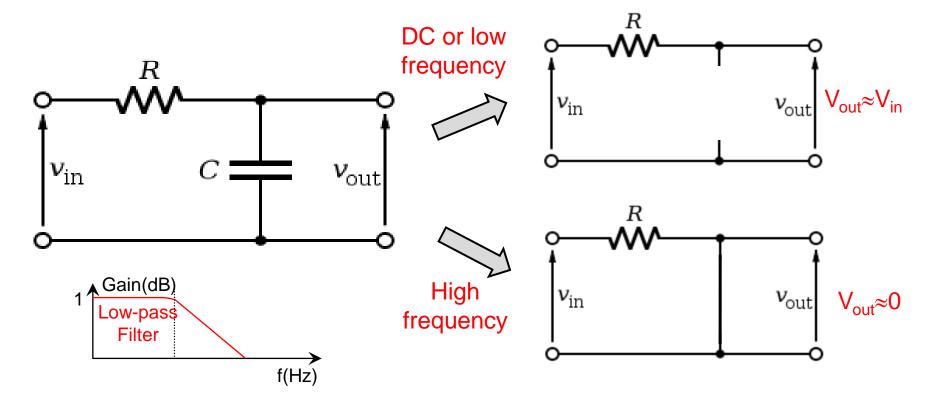
• Examples:

 Removing the noise from measured ECG signal using a filter to help a doctor understand the heart better



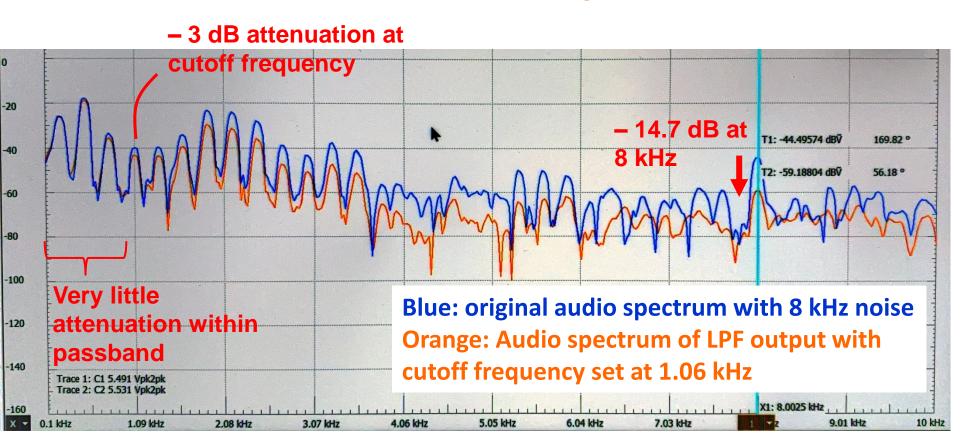
Removing some frequencies or frequency bands from an audio signal

Passive Low-Pass Filter



- Capacitor impedance is given as 1/(jωC)
- At low frequency or DC, capacitor behaves like open circuit
- At high frequency, capacitor behaves like short circuit
- Allow low frequency signal to pass through and reject high frequency signal ⇒ Low-pass filter

Filters Can Help Suppress (Attenuate) Undesirable Frequencies



Note:

The above spectrum is plotted in linear scale; usual practice is to plot in logarithmic scale if plotted over wide frequency range

Power Gain in decibels (dB)

• The Voltage Amplification (A_v) or Gain of a voltage amplifier/filter is given by:

$$A_{v} = \frac{V_{\text{out}}}{V_{\text{in}}}$$

The voltage gain is commonly expressed in terms of the resulting power gain in dB:

Power Gain (dB) =
$$10 \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) dB$$

(Under certain conditions, to be explained in Q1) = $10 \log_{10} \left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 dB = 20 \log_{10} \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| dB$

Frequency Response

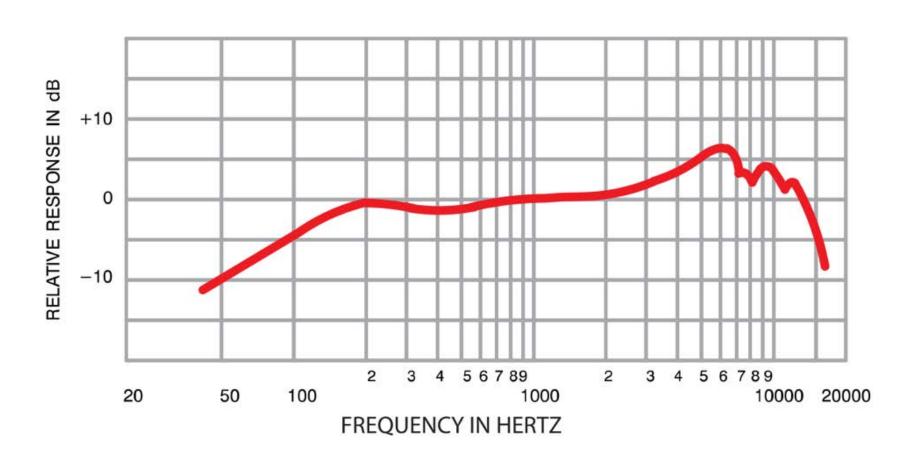
- It is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system
 - -Frequency in logarithmic scale: horizontal x-axis



-Power Gain in decibels (dB): <u>vertical</u> y-axis To describe a change in output power over the whole frequency range

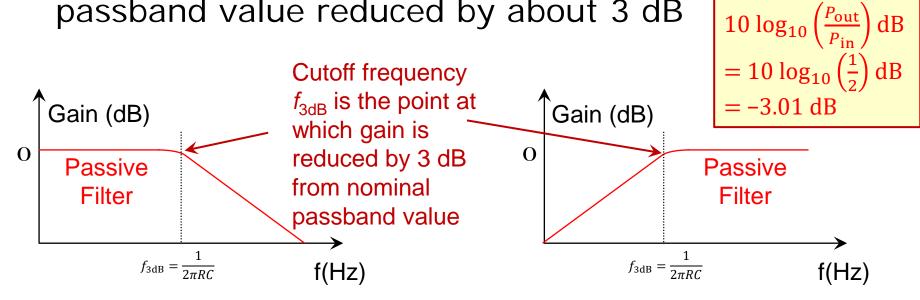
Power Gain in dB $(f) = 20 \log_{10} |A_v(f)|$

Example of a Microphone's Frequency Response Graph



Cut-off Frequency

- In filters, the cut-off frequency characterizes a "boundary" between a passband and a stopband
- It is defined as the frequency at which the output power is reduced by half compared to the nominal passband value
- In dB scale, this is equivalent to the nominal passband value reduced by about 3 dB



Cut-off Frequency: -3 dB Point (i.e., Half-power Point)

• Graphical approach:

-Find the passband gain from the magnitude vs frequency plot

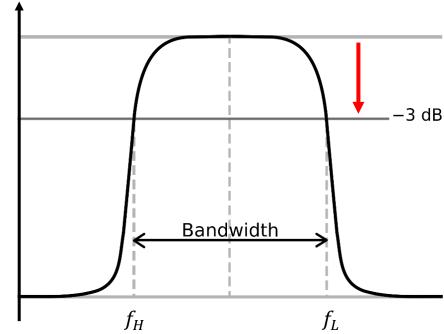
-Subtract 3 dB from the passband gain and draw a horizontal line on the plot

10 dB

7 dB

-Identify the pointswhere this horizontalline intercepts the plot

-The frequencies corresponds to these intercepts are the cut-off frequencies (f_H and f_I)



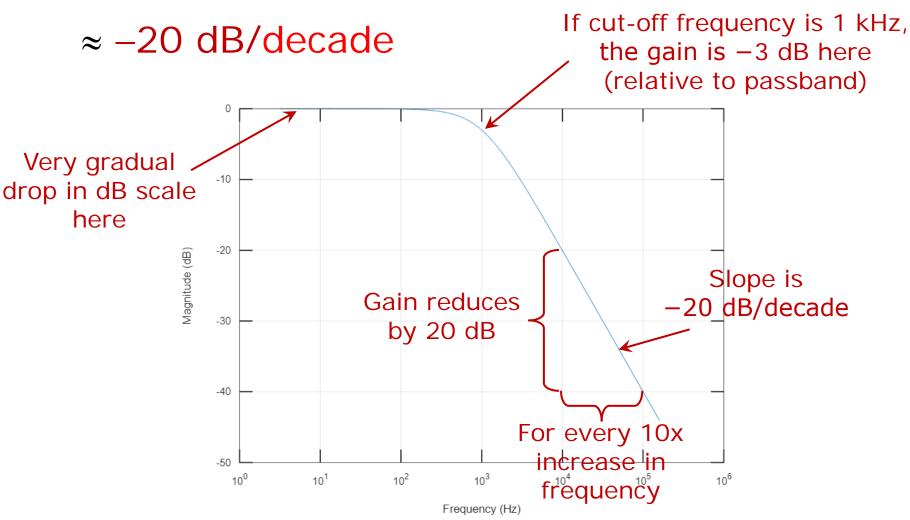
Cut-off Frequency: -3 dB Point (i.e., Half-power Point)

• Quantitative approach (for <u>first</u>-order filters):

$$f_H = \frac{1}{2\pi R_H C_H}, \quad f_L = \frac{1}{2\pi R_L C_L}$$

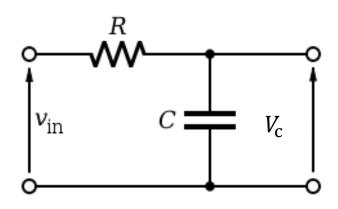
First-order Low-Pass Filter

Slope after cut-off frequency



Passive Low-Pass Filter

Using voltage divider rule to find the voltage gain:



$$\frac{V_{\rm c}}{V_{\rm in}} = \left[\frac{1}{\sqrt{1 + (R\omega C)^2}} \right]$$

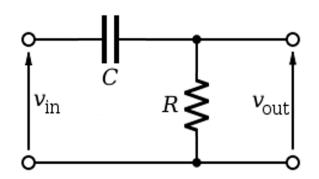
Gain in dB =
$$\frac{20 \log_{10} 1}{-20 \log_{10} \sqrt{1 + (\omega CR)^2}} dB$$

Passband gain Change in gain with ω
(= 0 dB) (-3 dB occurs at $f_L = \frac{1}{2\pi RC}$)

Passive High-Pass Filter

 Using voltage divider rule to find the voltage gain:

$$ightharpoonup V_{\text{out}} = \left[\frac{R}{R + \frac{1}{j\omega C}}\right] V_{\text{in}}$$



$$\frac{V_{\text{out}}}{V_{\text{in}}} = \left[\frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}} \right]$$

Gain in dB =
$$\frac{20 \log_{10} 1 - 20 \log_{10} \sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}}{1 + \left(\frac{1}{\omega CR}\right)^2} dB$$

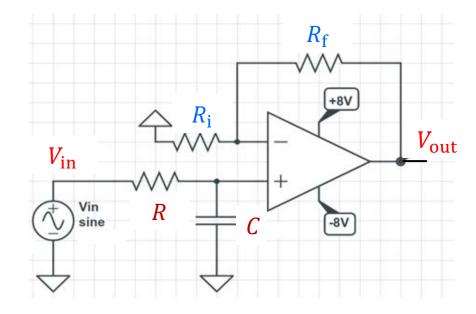
Passband gain Change in gain with ω
(= 0 dB) (-3 dB occurs at $f_H = \frac{1}{2\pi RC}$)

The Need for Amplifying Signals

- Voltage output from sensors may be in the order of mV, e.g., microphone signals
- The sensor voltage output would need to be scaled before A-to-D conversion for more accurate measurements (e.g., using Arduino Uno)

Active Low-Pass Filter

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_{\text{out}}}{V_{+}} \times \frac{V_{+}}{V_{\text{in}}} = \left(1 + \frac{R_{\text{f}}}{R_{\text{i}}}\right) \frac{V_{+}}{V_{\text{in}}}$$



$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \left(1 + \frac{R_{\text{f}}}{R_{\text{i}}}\right) \frac{1}{\sqrt{1 + (R\omega C)^2}}$$

Gain in dB =
$$20 \log_{10} \left(1 + \frac{R_{\rm f}}{R_{\rm i}}\right) - 20 \log_{10} \sqrt{1 + (\omega CR)^2} \, dB$$

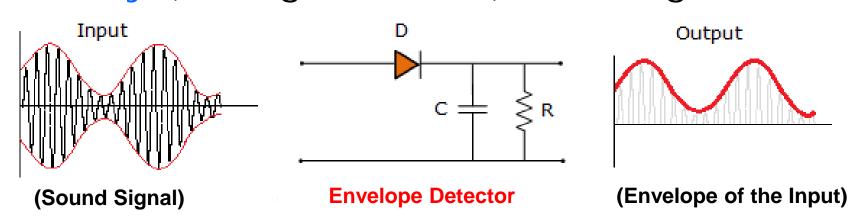
Passband gain
(= XX dB)

Change in gain with ω
(= XX dB)

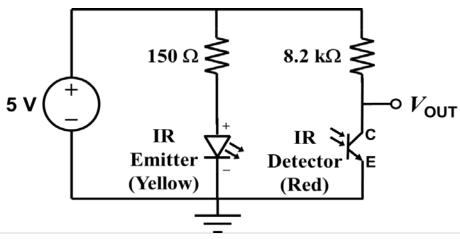
(-3 dB from XX dB occurs at $f_{\rm L} = \frac{1}{2\pi RC}$)

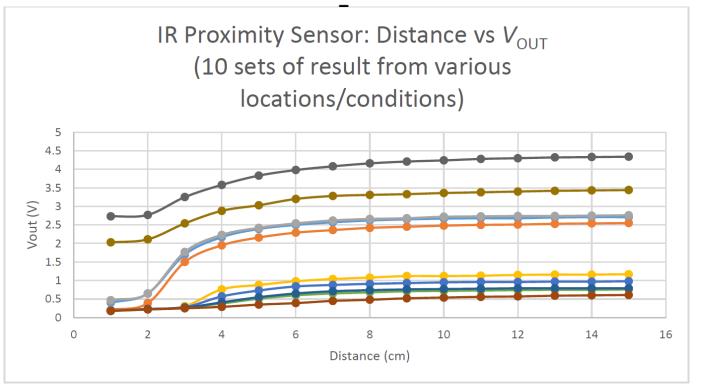
Envelope Detector

- An envelope detector is an electronic circuit that takes a high-frequency signal as input (sound) and provides an output which is the envelope of the original signal
- The capacitor in the circuit stores up charge on the rising edge, and releases it slowly (through the load) when signal falls

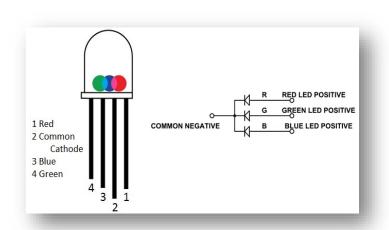


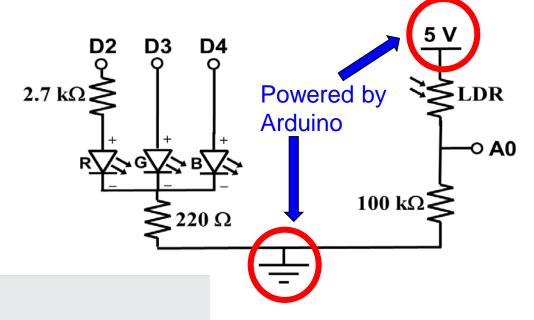
IR Proximity Sensor





LDR Colour Sensor





255

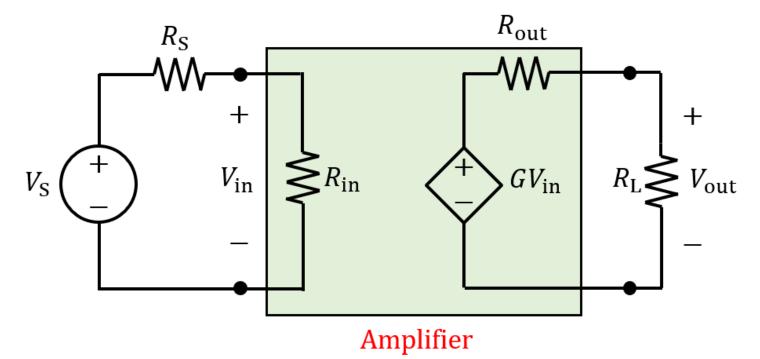
RGB Calculator



- An RGB colour value is specified with: rgb(red, green, blue)
 - Each parameter (red, green, and blue) defines the intensity of the colour as an integer between 0 and 255

Show that if $R_{in} = R_L$, then the power gain in dB for an amplifier circuit is given by

Power gain (dB) =
$$20 \log_{10} \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right|$$
 dB



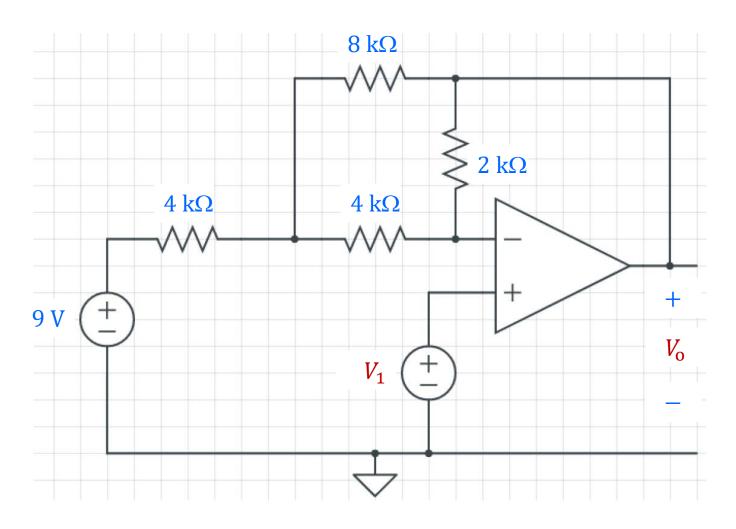
- Power at amplifier's input: $P_{\text{in}} = \frac{V_{\text{in}}^2}{R_{\text{in}}}$
- Power delivered to load: $P_{\text{out}} = \frac{V_{\text{out}}^2}{R_{\text{L}}}$
- Power Gain in dB

$$= 10 \log_{10} \left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \qquad \text{Equals 0 if} \\ R_{\text{in}} = R_{\text{L}}$$

$$= 10 \log_{10} \left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^{2} + 10 \log_{10} \left(\frac{R_{\text{in}}}{R_{\text{L}}}\right)$$

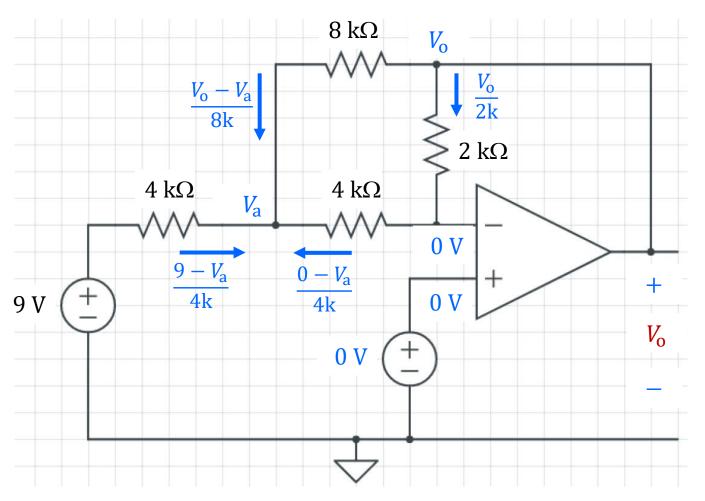
$$= 20 \log_{10} \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right|$$

• Calculate V_0 in the circuit if $V_1 = 0$



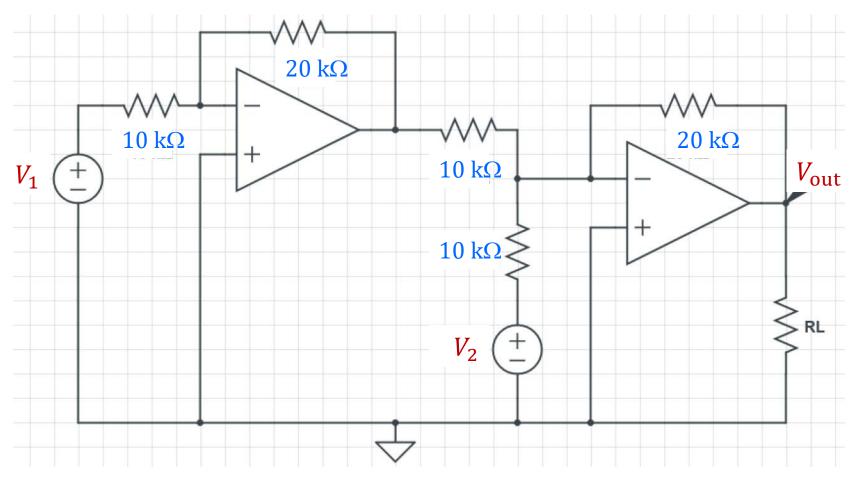
$$\frac{9 - V_{a}}{4k} + \frac{V_{o} - V_{a}}{8k} + \frac{0 - V_{a}}{4k} = 0 \qquad \qquad \frac{V_{o}}{2k} + \frac{V_{a} - 0}{4k} = 0 \rightarrow V_{a} = -2V_{o}$$

$$\frac{V_0}{2k} + \frac{V_a - 0}{4k} = 0 \rightarrow V_a = -2V_0$$

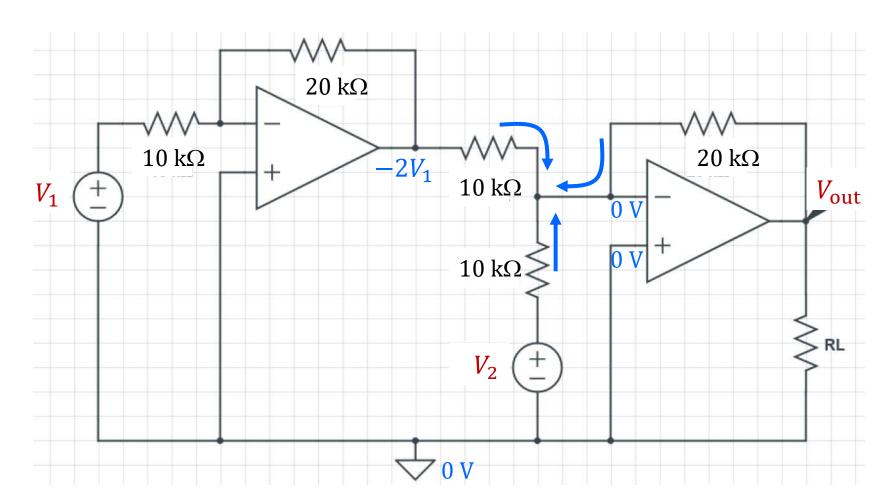


Solving, $V_{\rm o}$ $V_{\rm o} = -1.64 \, \mathrm{V}$

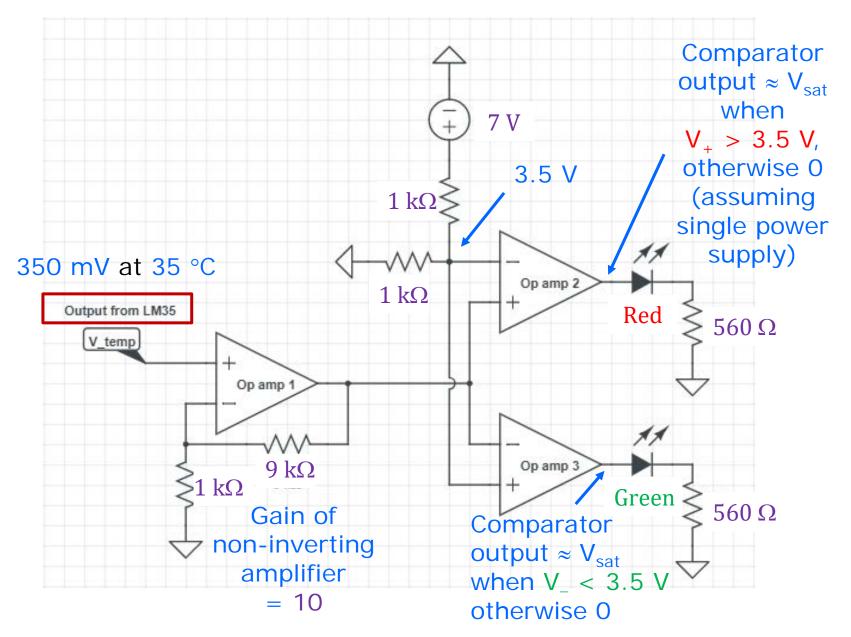
• Find an expression for V_{out} (in terms of V_1 and V_2)



$$\frac{-2V_1 - 0}{10k} + \frac{V_2 - 0}{10k} + \frac{V_{\text{out}} - 0}{20k} = 0 \quad \to \quad V_{\text{out}} = 4V_1 - 2V_2$$



- Design a temperature sensing circuit using LM35 (temperature sensor IC), Op amps, resistors, and LEDs
- LM35 output = 250 mV at 25 °C
- LM35 output varies as 10 mV/°C
- Temperature below 35 °C, GREEN LED on
- Temperature above 35 °C, RED LED on



- Op Amp 1 closed loop gain of 10
- Op Amps 2 & 3 are comparators with reference voltage of 3.5 V
 - -Op Amp 2 non-inverting comparator
 - -Op Amp 3 inverting comparator
- When output of op-amp 1 is more than 3.5 V (i.e., when temperature > 35 °C), Red LED is turned on
- When output of op-amp 1 is less than 3.5 V (i.e., when temperature < 35 °C), Green LED is turned on

- Suppose:
 - -An audio clip: 100-3000 Hz
 - -Corrupted with 10 kHz noise
 - -Signal very soft
- Desired outcome:
 - Active LPF with passband gain of 6 dB
 - -Suppress the 10 kHz noise by 20 dB <u>relative</u> to the <u>passband gain</u>
- What is the cut-off frequency of the low-pass filter?

A Note About -20 dB in Power

- Suppressing the noise by 20 dB is equivalent to reducing its power to just 1% compared to no filtering
- Also equivalent to reducing its voltage to just 10% compared to no filtering

■ 10
$$\log_{10} \left(\frac{P_{\text{noise(filtered)}}}{P_{\text{noise(no filter)}}} \right) = 10 \log_{10} (0.01) = -20 \text{ dB}$$

$$20 \log_{10} \left(\frac{V_{\text{noise(filtered)}}}{V_{\text{noise(no filter)}}} \right) = 20 \log_{10} (0.1) = -20 \text{ dB}$$

Vin sine

Active low-pass filter (i.e., <u>amplifies entire</u> signal while suppressing high frequency signals' power **relative** to signals in passband)

Blue:

Active gain due to noninverting amplifier

Red:

Passive filter's (= 2, i.e., 6 dB) gain due to RC potential divider (ω -dependent)

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_{\text{out}}}{V_{+}} \times \frac{V_{+}}{V_{\text{in}}} = \left(1 + \frac{R_f}{R_1}\right) \frac{V_{+}}{V_{\text{in}}} = 2 \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right] = \frac{2}{1 + j\omega CR}$$

Hence, gain's magnitude w.r.t.
$$\omega = \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \frac{2}{\sqrt{1 + (\omega CR)^2}}$$

Vout

Rf

10 kΩ

R1 10 kΩ

Gain in dB =
$$\frac{20 \log_{10} 2}{20 \log_{10} 2} - \frac{20 \log_{10} \sqrt{1 + (\omega CR)^2}}{20 \log_{10} \sqrt{1 + (\omega CR)^2}} dB$$

A gain reduction of 20 dB at f = 10 kHz means:

$$-20 \log_{10} \sqrt{1 + (\omega CR)^2} \Big|_{f = 10 \text{ kHz}} = -20 \text{ dB}$$

Hence,
$$\sqrt{1 + (\omega CR)^2} = 10$$
 when $f = 10$ kHz

Our low-pass filter needs to have:

$$RC = \frac{\sqrt{10^2 - 1}}{2\pi \times 10000} = 1.584 \times 10^{-4} \text{ s}$$

Choose some *RC* combination that has this value

- Cutoff frequency is the frequency at which the gain decreases by 3 dB from passband gain
- A gain reduction of 3 dB at $f = f_c$ means: $-20 \log_{10} \sqrt{1 + (\omega CR)^2} \Big|_{f = f_c} = -3 \text{ dB}$

$$-20\log_{10}\sqrt{1+(\omega CR)^2}\Big|_{f=f_c} = -3 \text{ dB}$$

• Hence, $\sqrt{1 + (\omega CR)^2} = 10^{3/20} = \sqrt{2}$ at $f = f_c$

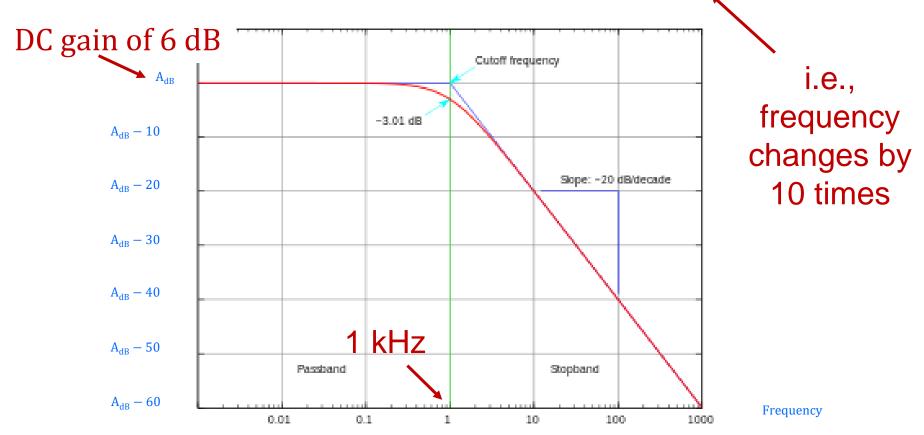
$$\omega CR = 1$$
 at $f = f_c$

Since
$$\omega = 2\pi f$$
, we have $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 1.584 \times 10^{-4}} \approx 1 \text{ kHz}$

Graphical Visualization for Q5

First-order low-pass filter: —20 dB/decade

Each horizontal box is "1 decade"

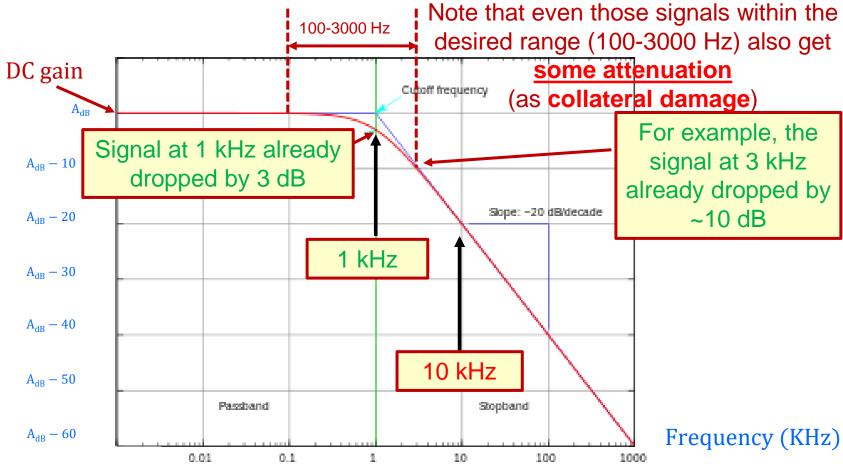


Why -20 dB/decade?

f	$-20\log_{10}\sqrt{1+(\omega CR)^2}$
f_c	≈ -3 dB
$10 \times f_c$	$\approx -20 \text{ dB}$
100 x f _c	$\approx -40 \text{ dB}$
1000 x f_c	$\approx -60 \text{ dB}$

Graphical Visualization for Q5

 If 10 kHz noise has to be reduced by 20 dB, we need to have the cutoff frequency at 1 kHz



Extra Points to Note for Q5

For your curiosity only:

As can be seen, with a first-order filter, we also lose some audio signals that we desire. How do we improve this?

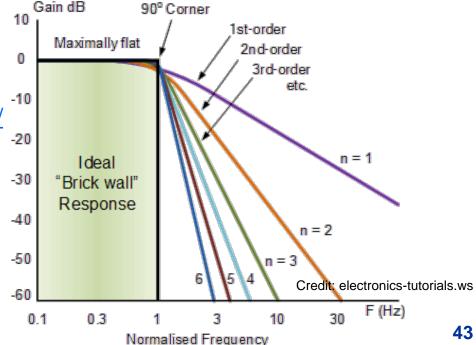
We can use higher-order filters! This allows us to have <u>sharper</u> attenuation slope, so that our desired passband is not

attenuated too much!

2nd order: 40 dB/decade

 http://www.electronics-tutorials.ws/ filter/second-order-filters.html

- 3rd order: 60 dB/decade
 - http://www.circuitstoday.com/
 higher-order-filters



 Design a passive LPF to suppress 12 kHz noise by at least 15 dB

$$\rightarrow V_{\text{out}} = \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \right] V_{\text{in}}$$

$$\frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \left[\frac{1}{\sqrt{1 + (R\omega C)^2}}\right] \rightarrow 20 \log_{10} 1 - 20 \log_{10} \sqrt{1 + (R\omega C)^2} \, dB$$
Passband gain Change in gain with ω (= 0 dB) (we want -15 dB at 12 kHz)

A gain reduction of 15 dB at f=12 kHz means:

$$-20 \log_{10} \sqrt{1 + (\omega CR)^2} \Big|_{f = 12 \text{ kHz}} = -15 \text{ dB}$$

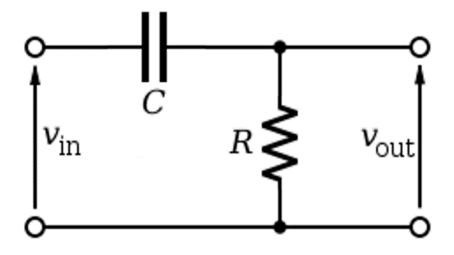
Hence,
$$\sqrt{1 + (\omega CR)^2} = 10^{15/20}$$
 when f = 12 kHz

Our low-pass filter needs to have:

$$RC = \frac{\sqrt{10^{(15/20)2} - 1}}{2\pi \times 12000} = 73.4 \,\mu\text{s}$$

If we choose C = 10 nF, we get $R = 7.34 \text{ k}\Omega$

For the following passive first-order high-pass filter, show that its cutoff frequency is given by $f_{\rm H} = \frac{1}{2\pi CR}$



Using voltage divider rule to find the voltage gain:

$$ightharpoonup V_{\text{out}} = \left[\frac{R}{R + \frac{1}{j\omega C}}\right] V_{\text{in}}$$

$$\frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \left[\frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}}\right] \rightarrow 20 \log_{10} 1 - 20 \log_{10} \sqrt{1 + \left(\frac{1}{\omega CR}\right)^2} dB$$
Magnitude
$$\text{Passband gain} \quad \text{Change in gain with } \omega$$

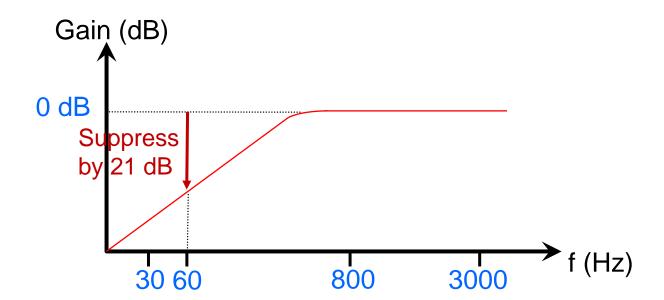
$$\text{(= 0 dB)} \quad \text{(-3 dB occurs at } f_{\text{H}} \text{)}$$

A gain reduction of 3 dB at $f = f_H$ means:

$$-20 \log_{10} \sqrt{1 + \left(\frac{1}{\omega CR}\right)^2} \bigg|_{f=f_{\mathrm{H}}} = -3 \, \mathrm{dB}$$

Hence,
$$\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}=10^{3/20}=\sqrt{2}$$
 at $f=f_{\rm H}$, which gives $\frac{1}{\omega CR}=\frac{1}{2\pi f_{\rm H}CR}=1$ $\rightarrow f_H=\frac{1}{2\pi CR}$

- Low frequency humming from the airplane's engine: 30 – 60 Hz
- Electronic dance music: 800 3000 Hz
- Design a high-pass filter to suppress the humming by at least 21 dB



We will focus on the passive filter. If need be, a non-inverting amplifier can be cascaded to boost the gain.

$$\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \left[\frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}} \right]$$

⇒
$$20 \log_{10} 1 - 20 \log_{10} \sqrt{1 + \left(\frac{1}{\omega CR}\right)^2} dB$$

Passband gain Change in gain with ω
(= 0 dB) (we want –21 dB at 60 Hz)

Hence,

$$-20 \log_{10} \sqrt{1 + \left(\frac{1}{\omega CR}\right)^2} \bigg|_{f=60 \text{ Hz}} = -21 \text{ dB}$$

$$\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2} = 10^{21/20}$$
 when $f = 60$ Hz,

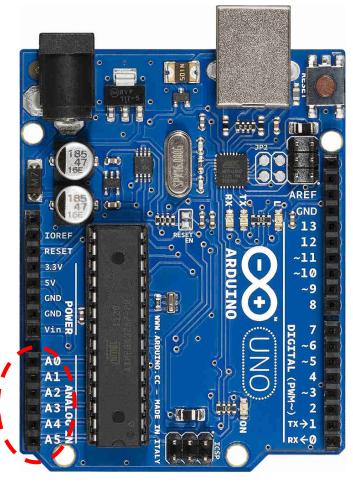
which gives
$$\frac{1}{\omega CR} = \frac{1}{2\pi (60)CR} = 11.176$$

Cutoff frequency of filter =
$$\frac{1}{2\pi CR}$$
 = 670.5 Hz

- Pressure sensor: $p = 250 \left(\frac{V_{\text{out}}}{V_{\text{cc}}} \right) 25$, where $V_{\text{cc}} = 5 \text{ V}$
- Max pressure in setup: 10 PSI
- How to amplify signal before sampling to make good use of Arduino Uno's ADC input range of 0 – 5 V for better accuracy?

ADC input range: 0 – 5 V

ADC Output: 0 – 1023



Since max pressure is 10 PSI, we have

$$10 = 250 \left(\frac{V_{\text{out}}}{5}\right) - 25$$

- Solving, we get max $V_{\text{out}} = 0.7 \text{ V}$
- To make good use of the ADC's input range of 0 5 V, we need a gain of $\frac{5}{0.7} = 7.14$
- We can pick a non-inverting amplifier with

$$\frac{R_{\rm f}}{R_{\rm i}} = 7.14 - 1 = 6.14$$

- HIH-4030 humidity sensor IC chip
- V_{out} varies by 30.68 mV/RH% change
- At 0% RH, $V_{\text{out}} = 0.958 \text{ V}$
- Design a humidity sensing circuit using HIH-4030, op-amp, and Arduino Uno

- At 100% RH, $V_{\text{out}} = 0.958 \text{ V} + 0.03068 \times 100 = 4.026 \text{ V}$
- We can amplify V_{out} to the range of 0 5 V before sampling it using Arduino Uno, using a noninverting amplifier with a gain of $\frac{5}{4.026} = 1.242$

