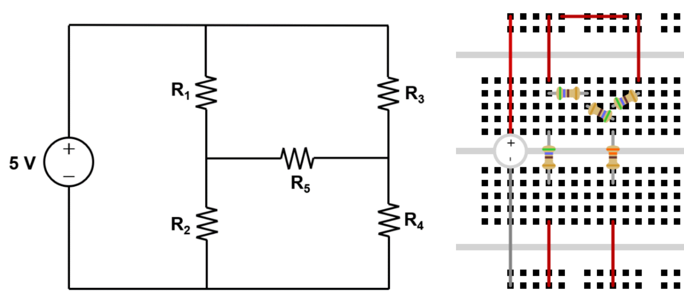
Nodal Analysis Studio Report
CG1111A Studio 3

Prannaya Gupta (B02)

August 17, 2022

1 Activity 1



(a) The Given Set-Up (b) Breadboard Used

Figure 1: Set-Up Planned

In this experiment, to utilise a Breadboard to its best usage under this system, we have designed it as shown in Figure **??**.

We know the individual resistances of each of these resistors (both Nominal and Measured via DMM), as shown in Table **??**.

Resistor	Nominal Resistance (Ω)	Measured Resistance (Ω)
R_1	560	545
R_2	560	546
R_3	560	549
R_4	330	324
R_5	560	549

Table 1: Table of Resistance Values

We use nodal analysis to determine the relevant equations. We note that we denote I_n as the current flowing through the resistor R_n.

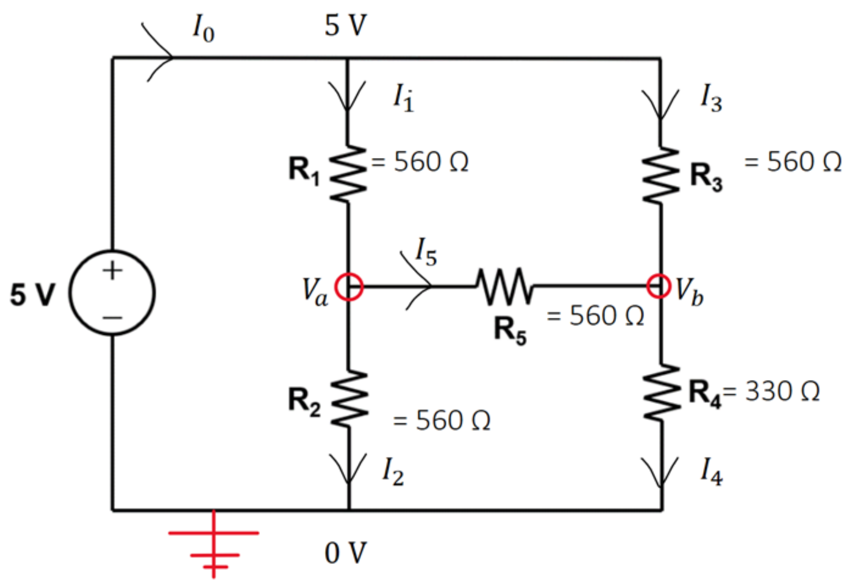


Figure 2: Nodal Analysis performed on the Set-Up.

From here, we can use Kirchoff's Junction Law (KCL) to get the following equations:

$$\frac{5-V_a}{R_1} + \frac{V_b - V_a}{R_5} + \frac{0-V_a}{R_2} = 0 \tag{1}$$

$$\frac{5-V_b}{R_3} + \frac{V_a - V_b}{R_5} + \frac{0-V_b}{R_4} = 0 \tag{2}$$

We can simplify this to get the following equations:

$$\left(\frac{1}{R_1} + \frac{1}{R_5} + \frac{1}{R_2}\right) V_a - \frac{1}{R_5} V_b = \frac{5}{R_1} \tag{3}$$

$$\left(\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_4}\right) V_b - \frac{1}{R_5} V_a = \frac{5}{R_3} \tag{4}$$

From here, we have two simultaneous equations which we can solve via Python as shown with the following code.

```
import numpy as np
```

```python
R1 = 545
R2 = 546
R3 = 549
R4 = 324
R5 = 549

A = np.array([
    [1/R1 + 1/R5 + 1/R2, -1/R5],
    [-1/R5, 1/R3 + 1/R5 + 1/R4]
])

Ainv = np.linalg.pinv(A)

v = np.array([5/R1, 5/R3])[:, np.newaxis]


Va, Vb = ((Ainv @ v).T)[0]

print("Va =", Va, "V")
print("Vb =", Vb, "V")
print()

V = np.array([5-Va, Va, 5-Vb, Vb, Va-Vb])
R = np.array([R1, R2, R3, R4, R5])
I = V/R

for i in range(5):
    print(f"V{i+1} =", round(V[i],3), "V")
    print(f"I{i+1} =", round(1000*I[i],3), "mA")
    print()
```

The output of this code is as follows:

```
Va = 2.3303115367817906 V
Vb = 1.9841444761213873 V
V1 = 2.67 V
I1 = 4.899 mA
V2 = 2.33 V
I2 = 4.268 mA
V3 = 3.016 V
I3 = 5.493 mA
V4 = 1.984 V
I4 = 6.124 mA
V5 = 0.346 V
I5 = 0.631 mA
```

Resistor	Nominal Resistance (Ω)	Measured Resistance (Ω)	Calculated Voltage (V)	Calculated Current (mA)	Measured Voltage (V)	Measured Current (A)
R_1	560	545	2.670	4.899	2.56	4.697
R_2	560	546	2.330	4.268	2.96	5.421
R_3	560	549	3.016	5.493	2.89	5.264
R_4	330	324	1.984	6.124	1.75	5.401
R_5	560	549	0.346	0.631	0.34	0.619

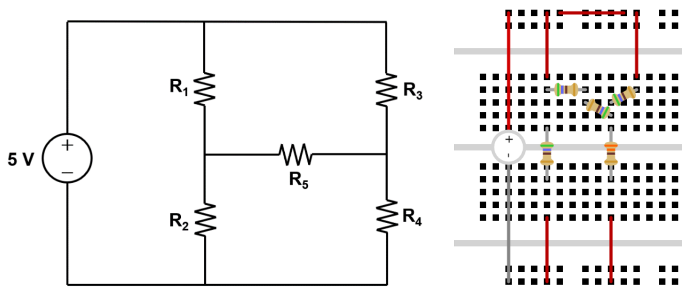
Table 2: Table of Final Values

Of course, with the measured values, there are some inaccuracies, likely brought about due to the resistances of the wires and breadboard connectors, which introduce ineffective values.

It could also be caused by the fact that the DMM is not ideal as a voltmeter/ ammeter/ resistance-meter. The voltmeter may have non-infinite resistance, the ammeter may have non-zero resistance and so on. Hence it is not possible for us to gauge the true value.

2 Activity 2

Using the DMM, we note that the new power supply has a voltage of 5.08 V.



(a) The Given Set-Up (b) Breadboard Used

Figure 3: Set-Up Planned

In this experiment, to utilise a Breadboard to its best usage under this system, we have designed it as shown in Figure ??.

We use nodal analysis to determine the relevant equations. We note that we denote I_n as the current flowing through the resistor R_n.

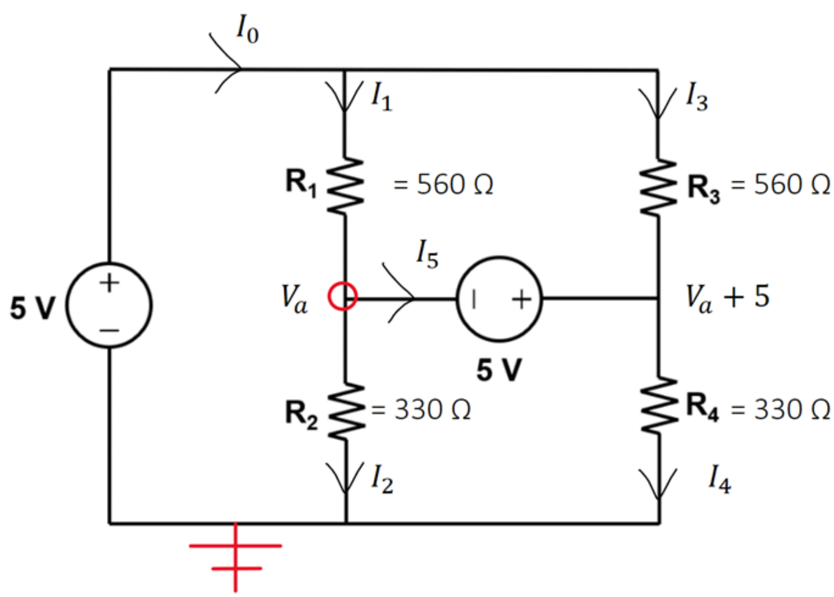


Figure 4: Nodal Analysis performed in the setup.

To imagine the supernode, we go by the following expression:

$$\frac{5-V_a}{R_1}+\frac{5-V_a-5.08}{R_3}+\frac{0-V_a}{R_2}+\frac{0-V_a-5}{R_4}=0 \tag{5}$$

Essentially, we have imagined the strand containing the 5.08 V supply as the node.

We can simply said equation as follows:

$$\frac{5-V_a}{R_1}+\frac{5-V_a-5.08}{R_3}+\frac{0-V_a}{R_2}+\frac{0-V_a-5}{R_4}=0$$
$$\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_2}+\frac{1}{R_4}\right)V_a=\frac{5}{R_1}-\frac{0.08}{R_3}-\frac{5}{R_4}$$

This gives us an expression for V_a, which can be computed as -0.747 V. Thus, we get the following cascade:

$$I_1=\frac{5-V_a}{R_1}=10.545 \text{ mA}$$
$$I_2=\frac{V_a}{R_2}=-1.368 \text{ mA, which means current flows upwards}$$
$$I_3=\frac{5-5.08-V_a}{R_3}=1.215 \text{ mA}$$
$$I_4=\frac{5.08+V_a}{R_4}=13.374 \text{ mA}$$
$$I_5=I_4-I_3=12.159 \text{ mA}$$

These can be plugged in as follows:

Resistor	Nominal Resistance (Ω)	Measured Resistance (Ω)	Calculated Voltage (V)	Calculated Current (mA)	Measured Voltage (V)	Measured Current (A)
R_1	560	545	5.747	10.545	5.64	10.349
R_2	560	546	-0.747	-1.368	-0.79	-1.447
R_3	560	549	0.667	1.215	0.72	1.311
R_4	330	324	4.333	13.374	4.83	14.907
Second 5V Source	–	–		12.159	-5.08	11.12

Table 3: Table of Final Values

Now, notably, since the current flowing through the 5.08 V supply is flowing into the negative terminal and out of the positive terminal, we have that the supply is supplying power.