CG1111A Tutorial 1

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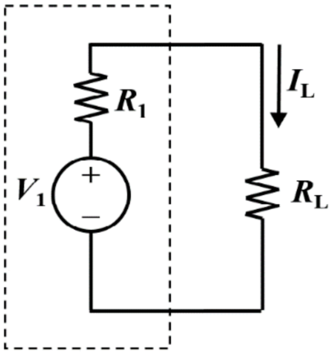
1. Consider the following battery with open-circuit voltage $V_1 = 12$ V, and internal resistance $R_1 = 0.15\ \Omega$. Find the load current I_L and the corresponding power efficiency η_L for the following load:



(a) $R_L = 10\ \Omega$

Solution:

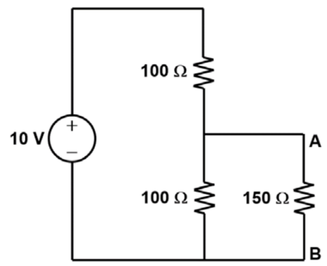
$$\begin{aligned} I_L &= \frac{\varepsilon}{R_T} \\ &= \frac{V_1}{R_1 + R_L} \\ &= \frac{12}{0.15 + 10} \\ &= 1.18227 \\ &\approx \mathbf{1.18\ A} \\ \eta_L &= \frac{P_L}{P_S} \\ &= \frac{1.18227^2 \times 10}{1.18227 \times 12} \times 100\% \\ &= 98.712\% \\ &\approx \mathbf{98.7\%} \end{aligned}$$

(b) $R_L = 1\ \Omega$

Solution:

$$\begin{aligned}
I_L &= \frac{\varepsilon}{R_T} \\
&= \frac{V_1}{R_1 + R_L} \\
&= \frac{12}{0.15 + 1} \\
&= 10.43478 \\
&\approx \mathbf{1.18\ A} \\
\eta_L &= \frac{P_L}{P_S} \\
&= \frac{10.43478^2 \times 1}{10.43478 \times 12} \times 100\% \\
&= 86.957\% \\
&\approx \mathbf{87.0\%}
\end{aligned}$$

2. The figure below shows a **loaded** voltage divider circuit. Calculate the voltage difference V_{AB} (given by $V_A - V_B$):

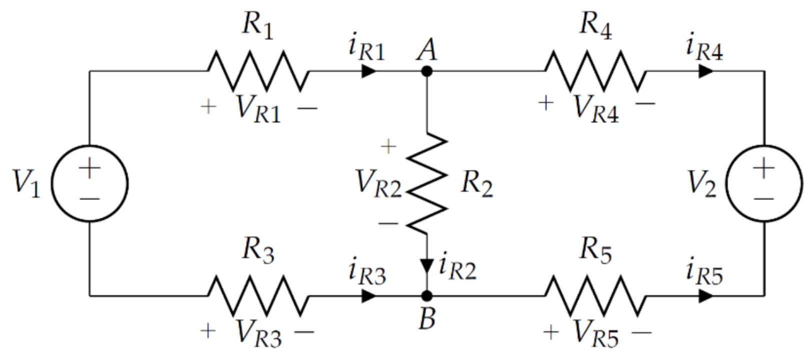


Solution:

$$\begin{aligned}
R_{\text{AB}} &= (\frac{1}{100} + \frac{1}{150})^{-1} \\
&= 60\ \Omega \\
R_{\text{eff}} &= 100 + 60 = 160\ \Omega \\
I_g &= \frac{\varepsilon}{R_{\text{eff}}} \\
&= \frac{10}{160} \\
&= 0.0625\ \text{A} \\
V_{\text{AB}} &= I_g \times R_{\text{AB}} \\
&= 0.0625 \times 60 \\
&= \mathbf{3.75\ V}
\end{aligned}$$

3. Considering the circuit diagram shown in the figure below, which of the following

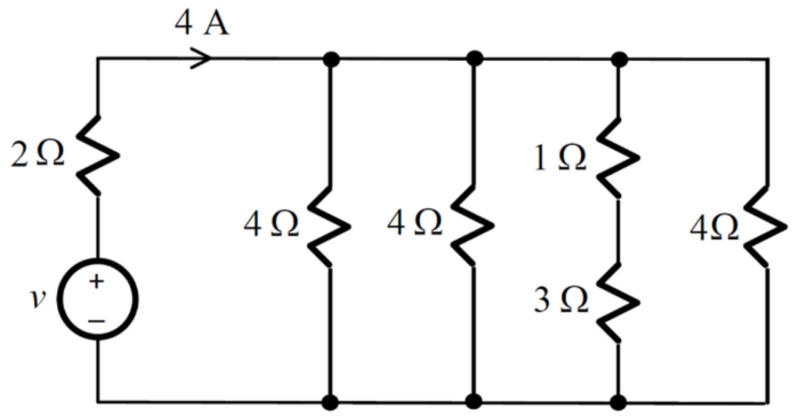
correctly applies **both** KVL and KCL?



A. $V_1 - V_{R1} - V_{R2} - V_{R3} = 0;\quad i_{R1} - i_{R2} - i_{R4} = 0$
B. $V_1 + V_{R3} - V_{R1} - V_{R2} = 0;\quad i_{R1} + i_{R3} = 0$
C. $V_2 + V_{R4} + V_{R2} + V_{R5} = 0;\quad i_{R4} + i_{R5} = 0$
D. $V_2 + V_{R4} - V_{R2} - V_{R5} = 0;\quad i_{R3} - i_{R2} - i_{R5} = 0$

Solution: D

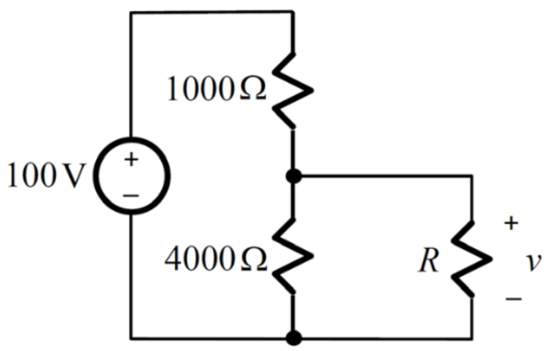
4. Determine the source voltage v and the voltage across the 3 Ω resistor in the following circuit.



Solution:

$$
\begin{aligned}
R_{//} &= (3 \times \frac{1}{4} + \frac{1}{3+1})^{-1} \\
&= 1\ \Omega \\
v &= I_g R_{\text{eff}} \\
&= (4) \times (2+1) \\
&= \mathbf{12\ V} \\
V_{//} &= I_g \times R_{//} \\
&= 4 \times 1 = 4\ \text{V} \\
V_{3\ \Omega} &= V_{//} \times \frac{3}{3+1} \\
&= \mathbf{3\ V}
\end{aligned}
$$

5. THe following circuit shows a common voltage divider for obtaining a certain voltage v across a load resistor R.



A novice may forget to include the loading effects of R. To understand these effects, determine v and the current in R, I_R when

(a) $R = \infty$ (open circuit)

Solution:

$$
\begin{aligned}
R_{//} &= (\frac{1}{4000} + \frac{1}{\infty})^{-1} \\
&= 4000\ \Omega \\
V_{//} &= \varepsilon \times \frac{4000}{4000+1000} \\
v &= 100 \times 0.8 \\
&= \mathbf{80\ V} \\
I_R &= \mathbf{0\ A}
\end{aligned}
$$

(b) $R = 8000\ \Omega$

Solution:

$$
\begin{aligned}
R_{//} &= (\frac{1}{4000} + \frac{1}{8000})^{-1} \\
&= \frac{8000}{3}\ \Omega \\
V_{//} &= \varepsilon \times \frac{\frac{8000}{3}}{\frac{8000}{3} + 1000} \\
v &= 100 \times \frac{8}{11} \\
&= 72.72727 \\
&\approx \mathbf{72.7\ V} \\
I_R &= \frac{v}{R} \\
&= \frac{72.72727}{8000} \\
&= 9.0909 \times 10^{-3}\ \text{A} \\
&\approx \mathbf{9.09\ mA}
\end{aligned}
$$

(c) $R = 200\ \Omega$

Solution:

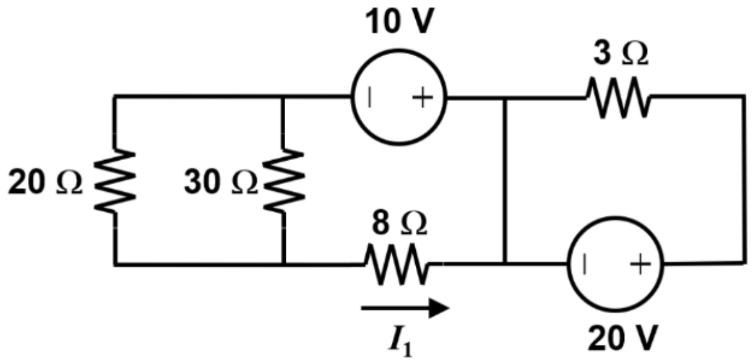
$$
\begin{aligned}
R_{//} &= (\frac{1}{4000} + \frac{1}{200})^{-1} \\
&= \frac{4000}{21}\ \Omega \\
V_{//} &= \varepsilon \times \frac{\frac{4000}{21}}{\frac{4000}{21} + 1000} \\
v &= 100 \times \frac{4}{25} \\
&= \mathbf{16\ V} \\
I_R &= \frac{v}{R} \\
&= \frac{16}{200} \\
&= 0.08\ \text{A} \\
&= \mathbf{80\ mA}
\end{aligned}
$$

(d) $R = 0$ (short circuit)

Solution:

$$
\begin{aligned}
R_{//} &= (\frac{1}{4000} + \frac{1}{0})^{-1} \\
&= 0\ \Omega \\
V_{//} &= \varepsilon \times \frac{0}{0+1000} \\
&= \mathbf{0\ V} \\
I_R &= \frac{\varepsilon - V_{//}}{1000} \\
&= \frac{100}{1000} \\
&= 0.1\ \text{A} \\
&= \mathbf{100\ mA}
\end{aligned}
$$

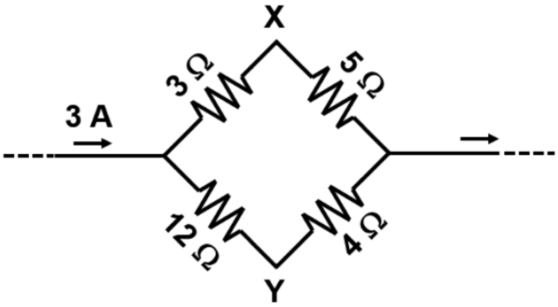
6. For the circuit shown in the figure below, what is the value of current I_1?



Solution:
Notice that due to the single wire, the 20 V source and the 3 Ω resistor are in a separate circuit altogether. Thus, we only need to simply apply Ohm's Law as follows:

$$
\begin{aligned}
R_{//} &= (\frac{1}{20} + \frac{1}{30})^{-1} = 12\ \Omega \\
I &= \frac{\varepsilon}{R_{\text{eff}}} \\
-I_1 &= \frac{10}{12+8} \\
I_1 &= \textbf{-0.5 A}
\end{aligned}
$$

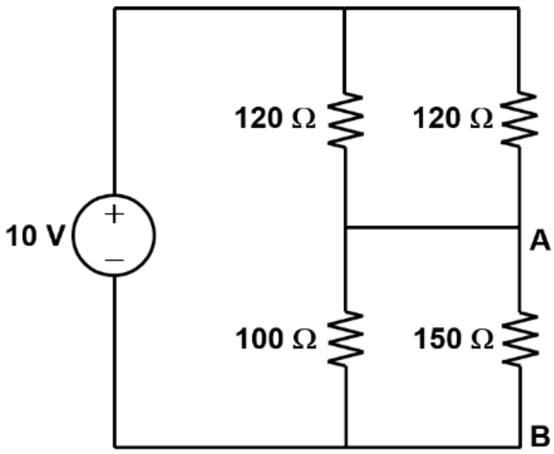
7. A current of 3 A flows through a resistor network as shown in the figure below. What is the voltage difference V_{XY} (given by $V_X - V_Y$)?



Solution:
Denoting the one above as branch 1, and the one below as branch 2,

$$
\begin{aligned}
R_{//} &= (\frac{1}{3+5} + \frac{1}{12+4})^{-1} = \frac{16}{3}\ \Omega \\
I_1 &= I \times \frac{R_{//}}{R_1} = 3 \times \frac{\frac{16}{3}}{3+5} = 2\ \text{A} \\
I_2 &= I \times \frac{R_{//}}{R_2} = 3 \times \frac{\frac{16}{3}}{12+4} = 1\text{A} \\
V_{3\ \Omega} &= I_1 \times 2 = \text{-6 V} \\
V_{12\ \Omega} &= I_2 \times 1 = \text{-12 V} \\
V_{\text{XY}} &= -6 - (-12) \\
&= \mathbf{6\ V}
\end{aligned}
$$

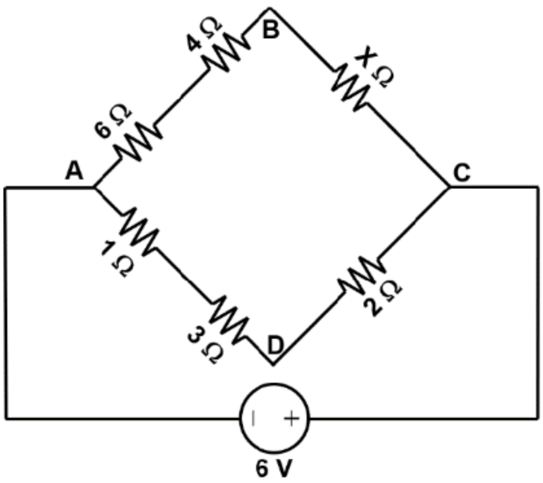
8. What is the voltage difference V_{AB} (given $V_A - V_B$)?



Solution:
This is effectively two parallel sets in series, which can be denoted as //1 and //2:

$$\begin{aligned}
R_{//1} &= (2 \times \frac{1}{120})^{-1} = 60\ \Omega \\
R_{//2} &= (\frac{1}{100} + \frac{1}{150})^{-1} = 60\ \Omega \\
V_{//2} &= \varepsilon \times \frac{R_{//2}}{R_{//2} + R_{//1}} \\
V_{\text{AB}} &= 10 \times \frac{1}{2} \\
&= \mathbf{5\ V}
\end{aligned}$$

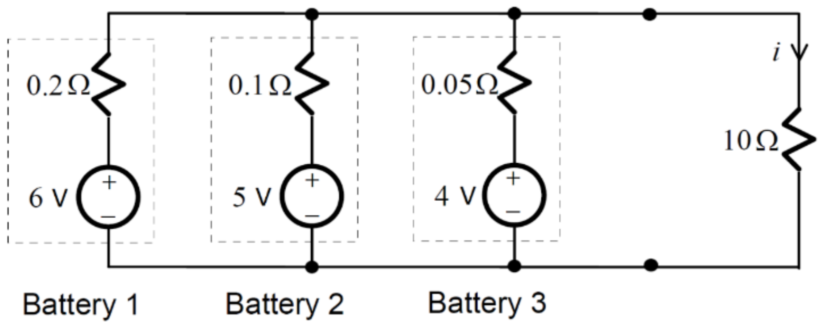
9. For the circuit shown in the figure below, if the voltage difference V_{BD} (given by $V_B - V_D$) is 1 V, what is the value of resistance X?



Solution:
Denoting the one above as branch 1, and the one below as branch 2,

$$
\begin{aligned}
I_1 &= \frac{\varepsilon}{6+4+X} = \frac{6}{10+X} \\
I_2 &= \frac{\varepsilon}{1+3+2} = 1 \\
V_X &= -I_1 \times X = -\frac{6X}{10+X} \\
V_{2\,\Omega} &= -I_2 \times 2 = -2 \\
V_{\mathrm{BD}} &= 2 - \frac{6X}{10+X} \\
1\text{ V} &= 2 - \frac{6X}{10+X} \\
\frac{6X}{10+X} &= 1 \\
6X &= 10 + X \\
5X &= 10 \\
X &= \mathbf{2\ \Omega}
\end{aligned}
$$

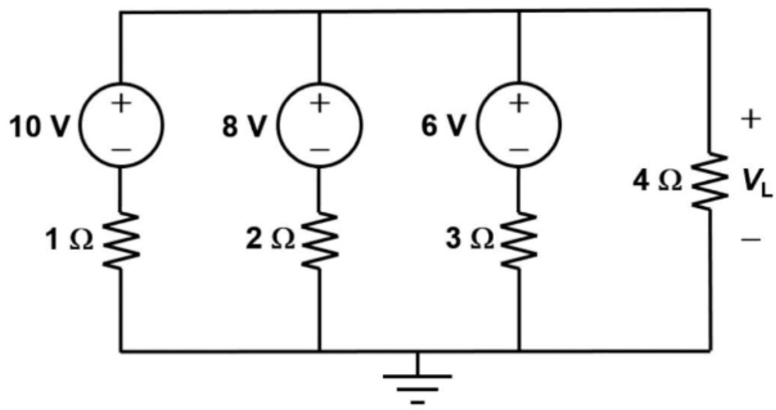
10. The circuit below shows a 10 Ω load connected to three batteries in parallel. Using node voltage analysis method, determine the voltage across the 10 Ω load, V, and its current i.



Solution:
Ground the bottom wire. Denote Voltage of top wire as V.

$$
\begin{aligned}
\frac{6-V}{0.2} + \frac{5-V}{0.1} + \frac{4-V}{0.05} &= \frac{V}{10} \\
300 - 50V + 500 - 100V + 800 - 200V &= V \\
351V &= 1600 \\
V &= 4.55840 \approx \mathbf{4.56\ V} \\
i &= \frac{V}{10} \approx \mathbf{0.456\ A}
\end{aligned}
$$

11. For the circuit shown in the figure below, what is the voltage V_L? (*Hint: Use Node Voltage Analysis method*) How much power is the 6V source supplying/consuming?

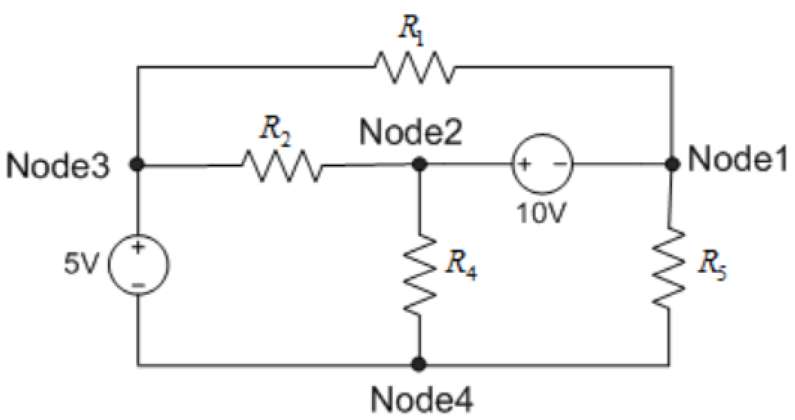


Solution:

$$
\begin{aligned}
\frac{10-V_L}{1}+\frac{8-V_L}{2}+\frac{6-V_L}{3} &= \frac{V_L}{4} \\
120-12V_L+48-6V_L+24-4V_L &= 3V_L \\
25V_L &= 192 \\
V_L &= \mathbf{7.68\ V} \\
I_{6\text{ V}} &= \frac{6-V_L}{3} = -0.56\text{ A} \\
P = IV &= -0.56\times 6 \\
&= \mathbf{-3.36\ W}
\end{aligned}
$$

Thus it is consuming **3.36 W** of power.

12. Consider the circuit given below. Suppose R_5 is the load resistance, derive and draw the Thevenin equivalent circuit as seen by R_5. clearly labeling Node 1 and Node 4 in the equivalent circuit. (Assume that $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_4 = 1\ \Omega$)



Solution: NO.