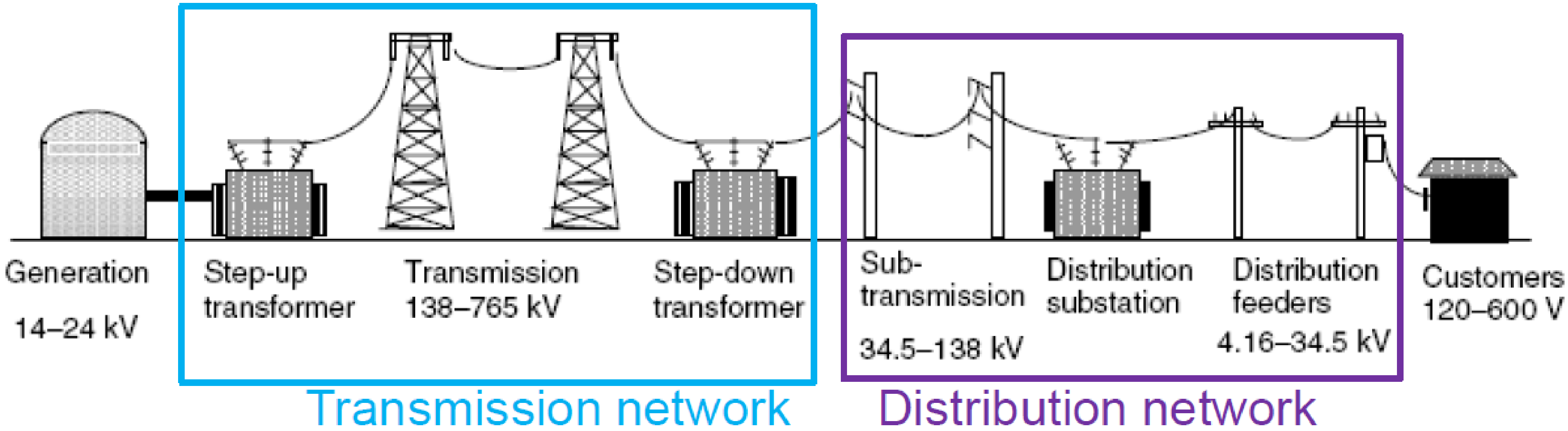
CG1111A: Engineering Principles & Practice I

Tutorial 3: Reflections & Problem Solutions
(28/29 Sep 2022)

NUS National University of Singapore

Why Transformers?



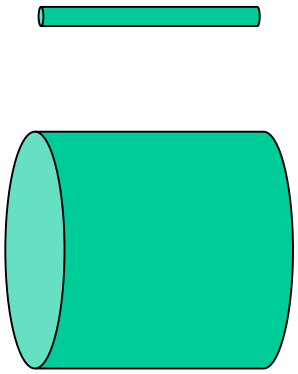
- More efficient if electricity transmitted at very high voltage:
  - –For the **same power *P***, higher *V* → smaller *I*
  - –Smaller *I* → smaller power loss (Recall that $P = I^2R$)
  - –Smaller *I* → can use thinner transmission lines

Example

Transmitted Power	Source Voltage	Current	Transmission Line's Resistance	Power Loss in Transmission Line (I^2R)
1 MW	100 kV	10 A	1 Ω	100 W
1 MW	10 kV	100 A	1 Ω	10000 W = 10 kW
1 MW	10 kV	100 A	0.01 Ω	100 W

Recall that $R = \frac{\rho l}{A}$

If V decreases by 10x, you need to increase wire thickness by 100x to maintain the same low power loss!

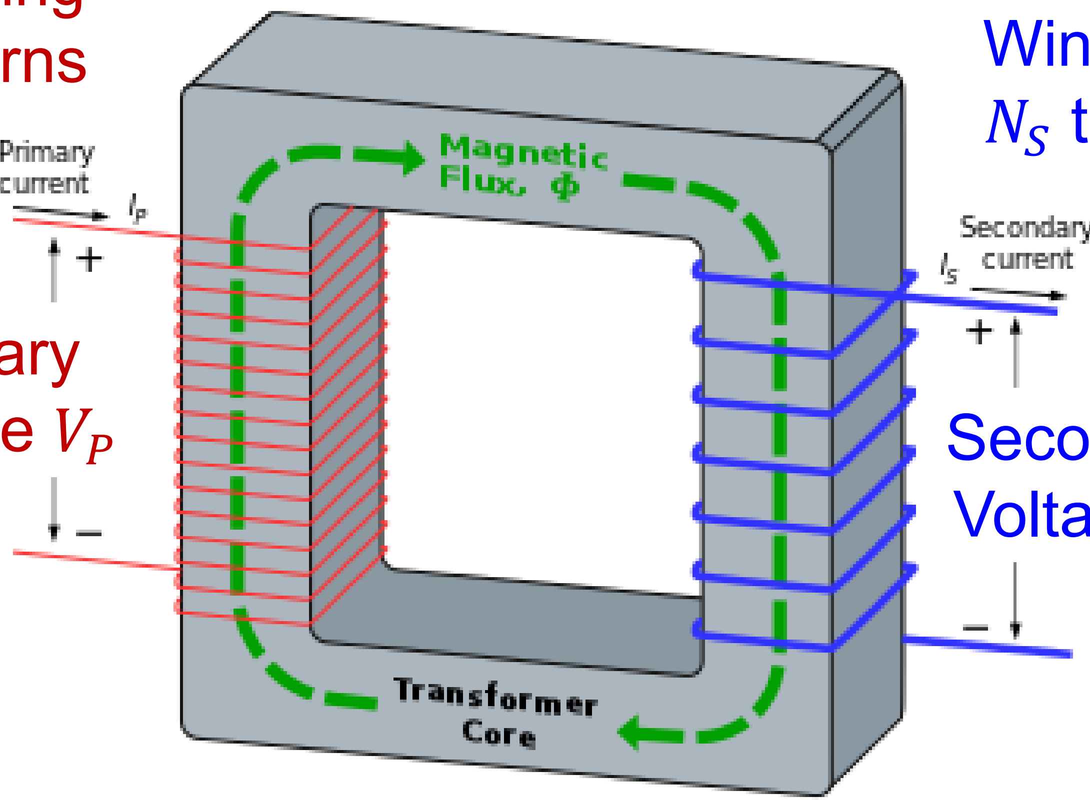


Step-Up/Down Transformer

Primary Winding N_P turns

Primary Current I_P

Primary Voltage V_P



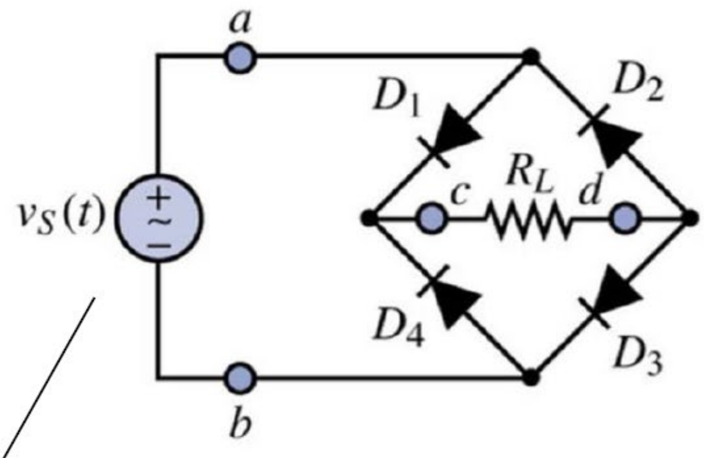
Secondary Winding N_S turns

Secondary Current I_S

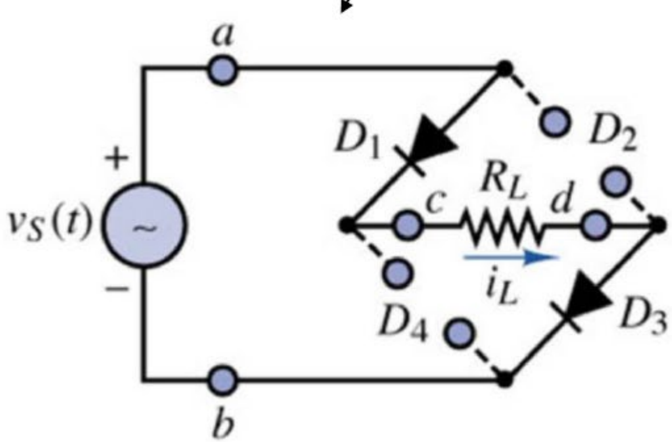
Secondary Voltage V_S

$$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P}} \qquad \boxed{\frac{I_S}{I_P} = \frac{N_P}{N_S}}$$

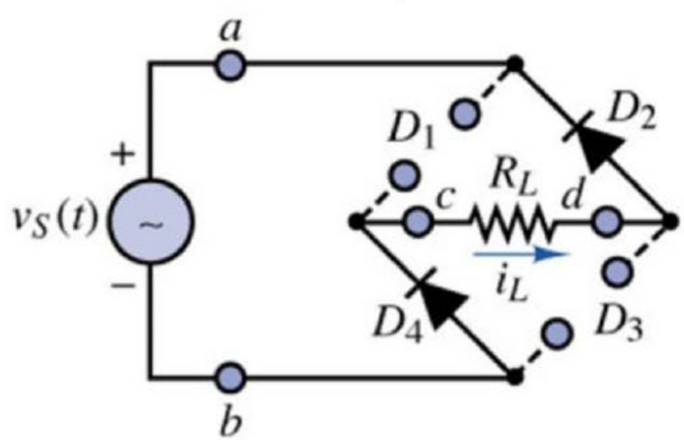
Why Rectifier?

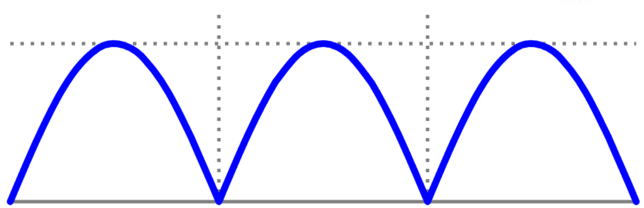


Positive
half-cycle

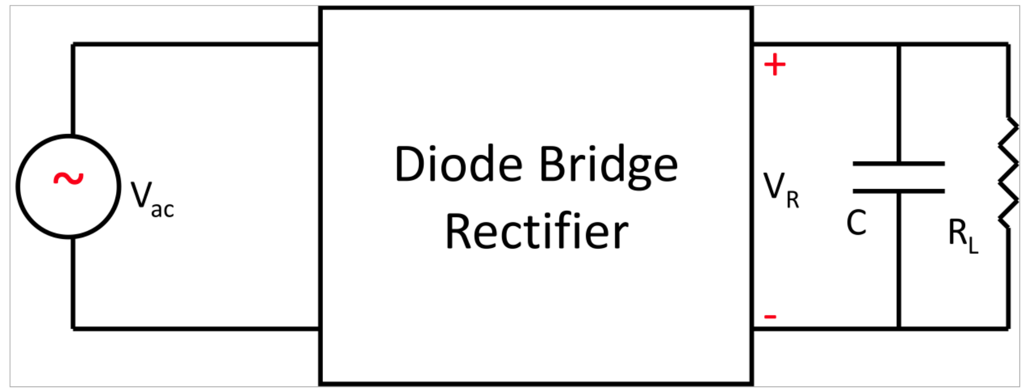


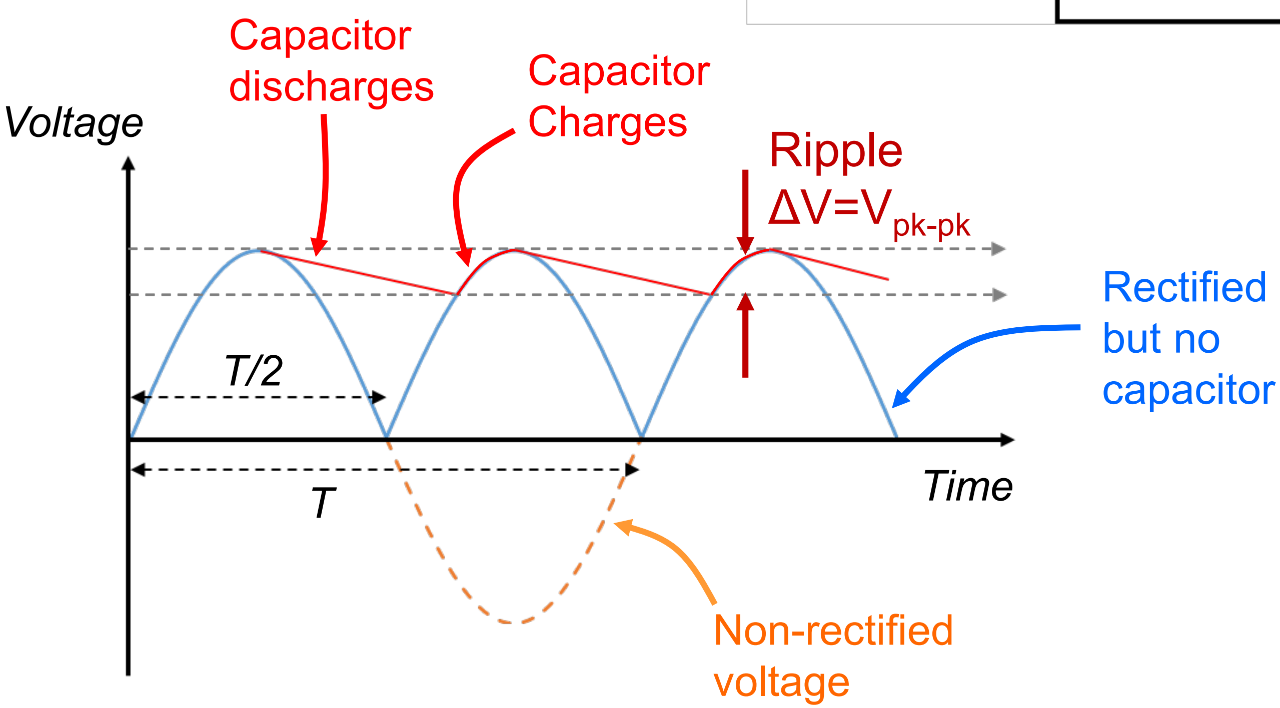
Negative
half-cycle





Why Filter Capacitor?





Characteristics of DC Motors

Torque of shaft: $$\boxed{T_{\text{shaft}} = K_t\, I_m \;\; [\text{N.m}]}$$

Back emf: $$\boxed{E_b = K_e\, \omega \;\; [\text{V}]}$$

For PMDC motor: $$\boxed{K_t = K_e}$$

Note: $$\boxed{\omega = 2\pi \times \frac{RPM}{60} \;\; [\text{rad/s}]}$$

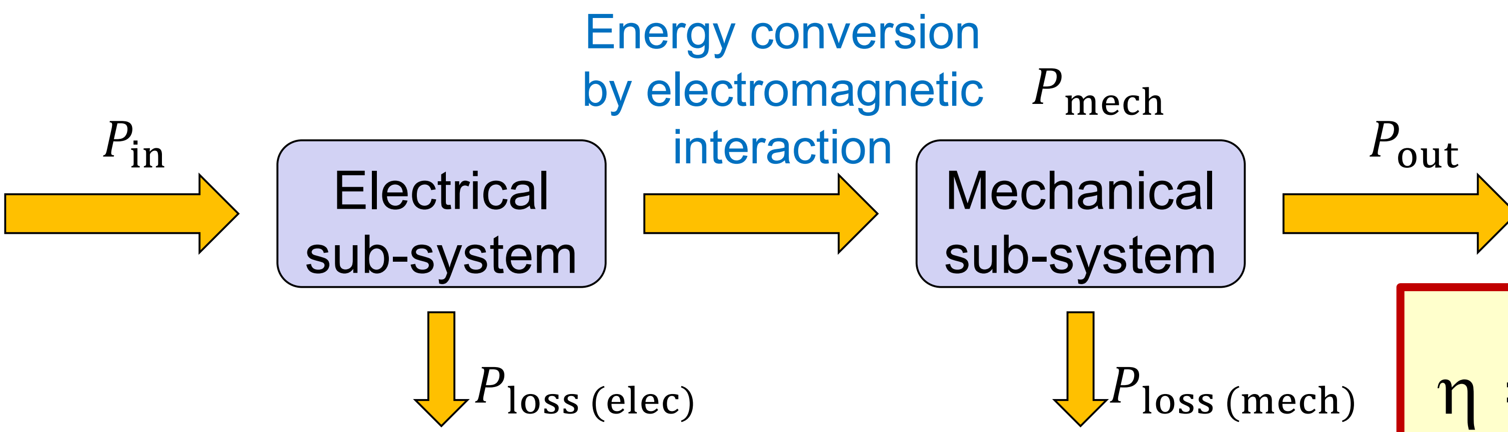
Power Conversion in Motors

- Mechanical power at motor shaft:

$$\boxed{P_{\text{mech}} = T_{\text{shaft}}\,\omega \quad [\mathrm{W}]}$$

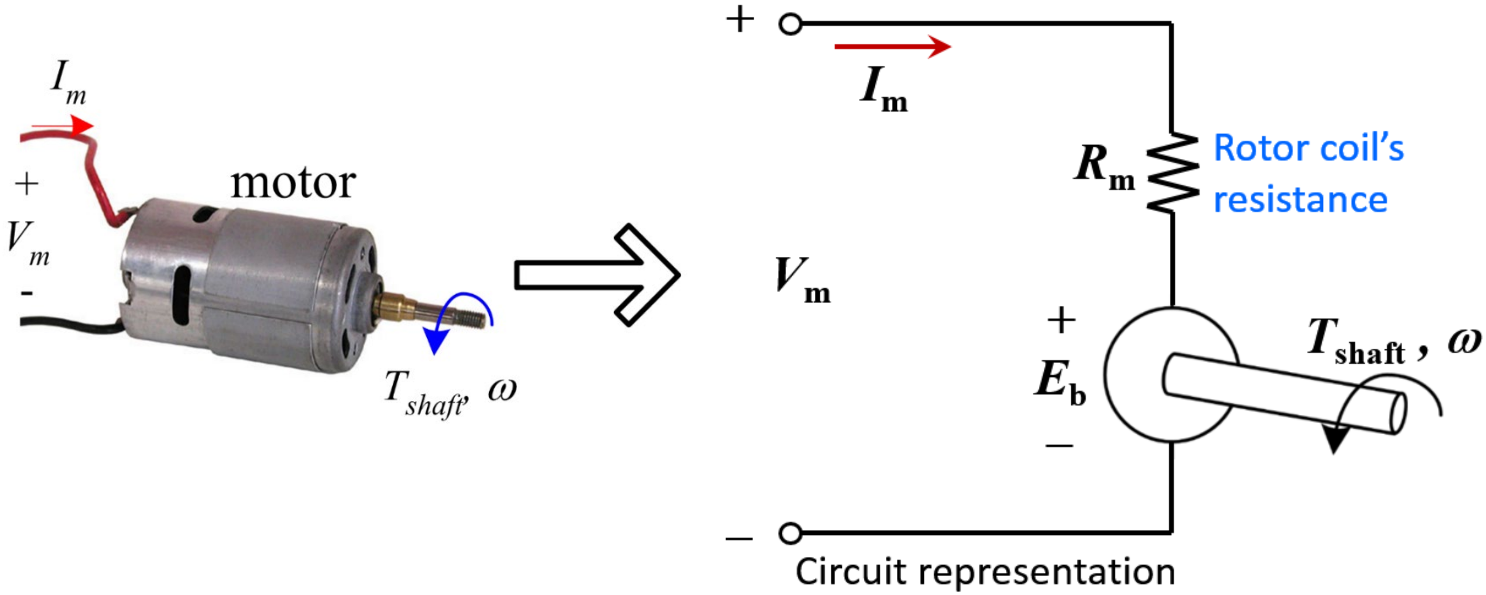
- Electrical power supplied to motor:

$$\boxed{P_{\text{in}} = V_m\, I_m \quad [\mathrm{W}]}$$



$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}}}$$

Circuit Representation: PMDC Motor



- From the circuit: $\boxed{I_m = \dfrac{V_m - E_b}{R_m}}$

- Since $E_b = K_e\,\omega$, we have: $\boxed{I_m = \dfrac{V_m}{R_m} - \dfrac{K_e\,\omega}{R_m}}$

Basic Properties of PMDC Motor

Rearranging: $$\boxed{\omega = \frac{V_m}{K_e} - \frac{R_m\, I_m}{K_e}}$$

- For a fixed load (i.e., fixed T_{shaft}, which implies fixed I_m since $T_{\text{shaft}} = K_t\, I_m$):

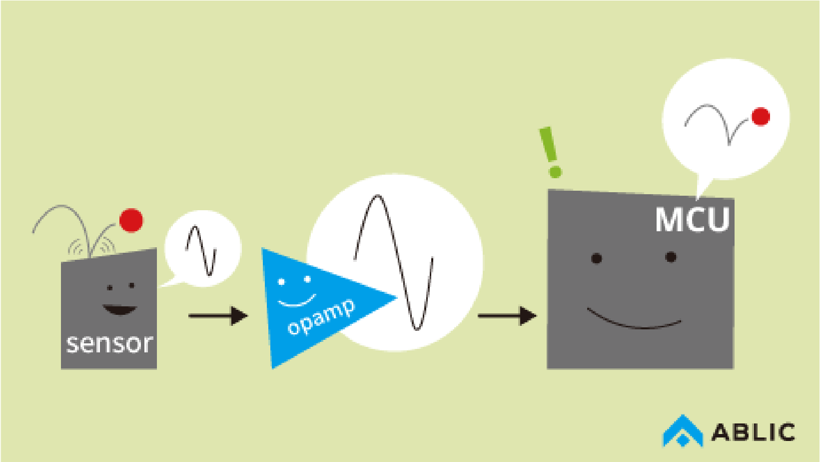
> Shaft speed ω can be increased by increasing motor voltage V_m

- For a fixed voltage, if T_{shaft} increases, I_m increases, and hence ω decreases:

> Shaft speed ω decreases with increasing load T_{shaft}

What is an "Operational Amplifier"?

- An "operational amplifier (**op-amp**)" is an integrated circuit that can amplify weak electric signals



Credit: Image from ablic.com

- It has **two input** signal terminals and **one output** signal terminal

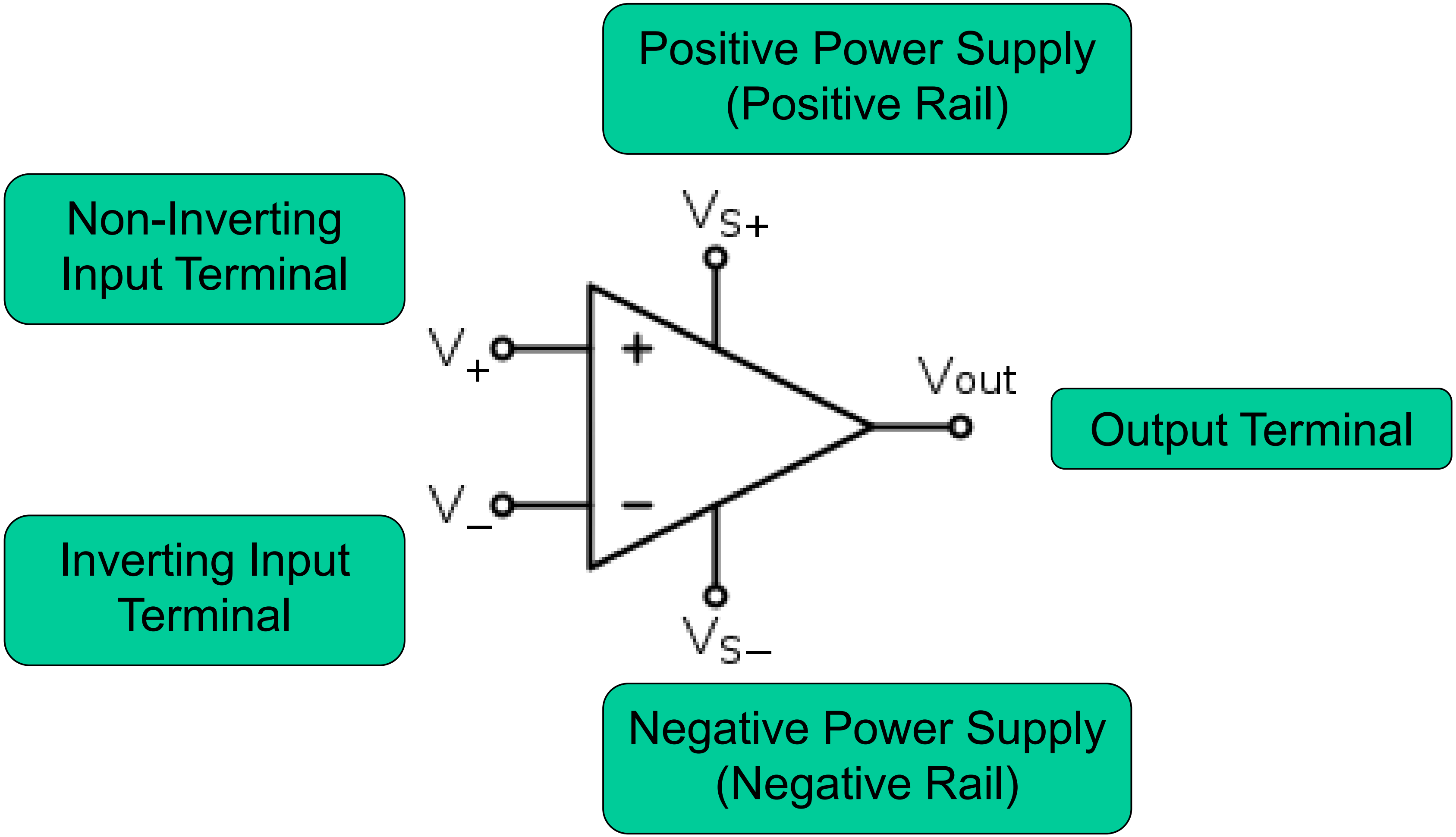
- It amplifies and outputs the **voltage difference** between the two input terminals

The Need for Amplifying Signals

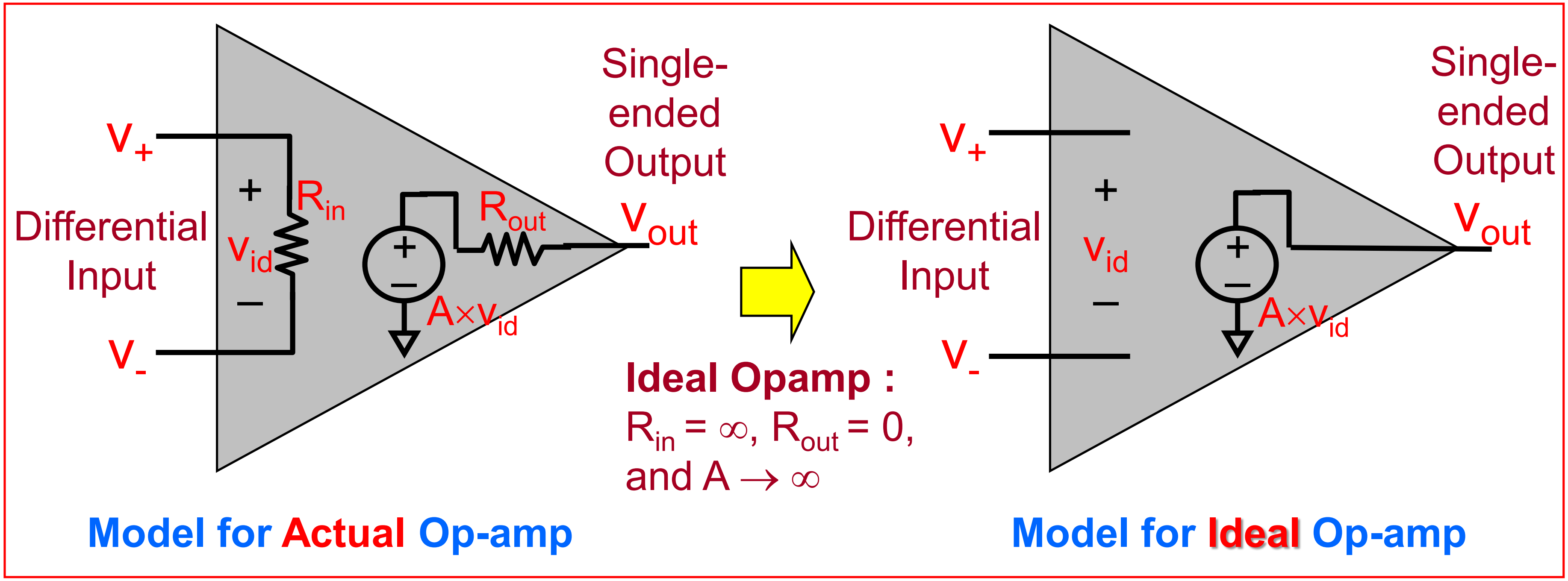
- Voltage output from sensors may be in the order of mV, e.g., microphone signals

- The sensor voltage output would need to be scaled before A-to-D conversion for more accurate measurements (e.g., using Arduino Uno)

Op-Amp Terminals

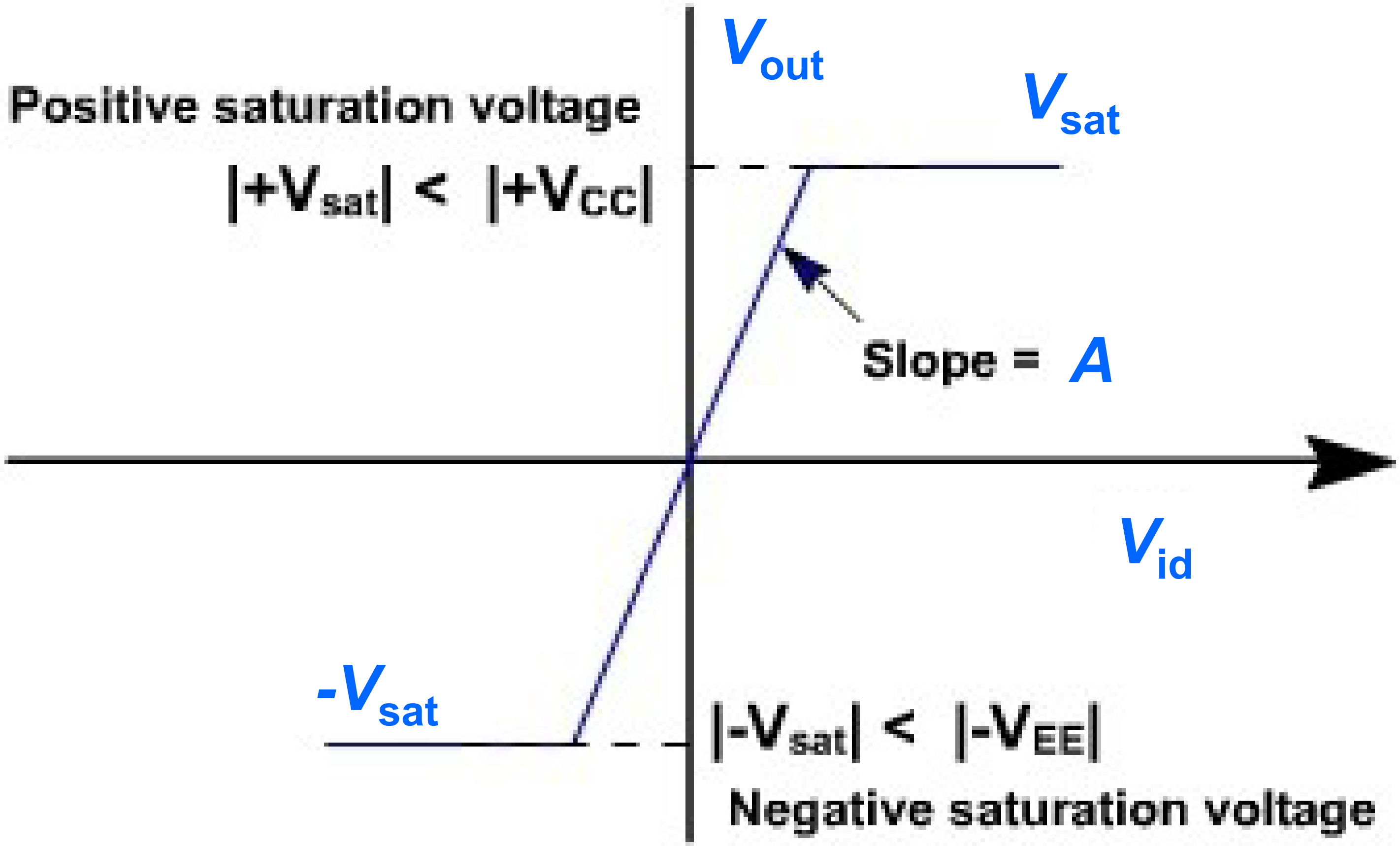


Op-Amp Equivalent Circuit



- ***A*** is the open-loop voltage gain
  - It is very large, approaching infinity
- ***R*$_{in}$** is the input impedance (very large) & ***R*$_{out}$** is the output impedance (very small)
- To **simplify** analysis, we always assume **infinite** ***R*$_{in}$** and ***A***, and **zero** ***R*$_{out}$**

Op-amp's Saturation Voltage

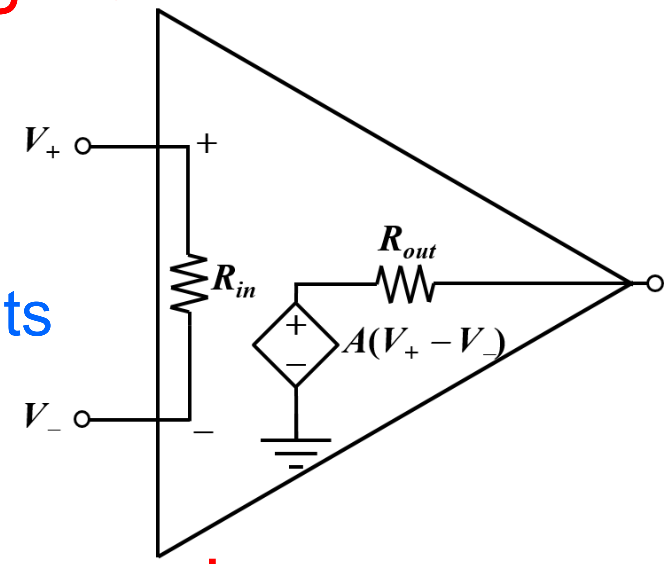


Typical Op Amp Parameters

Parameter	Variable	Typical Ranges	Ideal Values
Voltage Gain	A	10^5 to 10^8	∞
Input Impedance	R_{in}	10^5 to 10^8 Ω	∞ Ω
Output Impedance	R_{out}	10s to 100s Ω	0 Ω
Supply Voltage	V_S $-V_S$	3 to 30 V –30 to 0 V	N/A N/A

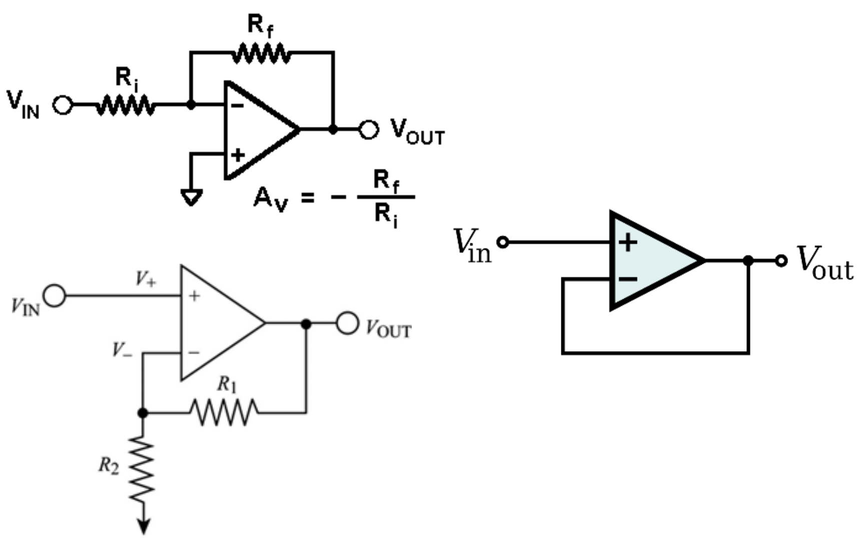
Op-amp Golden Rules

- Rule 1: In a closed loop with –ve feedback, the output attempts to do whatever is necessary to make the voltage difference between the inputs zero
  - The voltage gain of a real op-amp is so high that a fraction of a mV difference between the V_+ & V_- inputs will achieve the desired finite output
  - **"Virtual short"**, i.e., $V_+ \approx V_-$

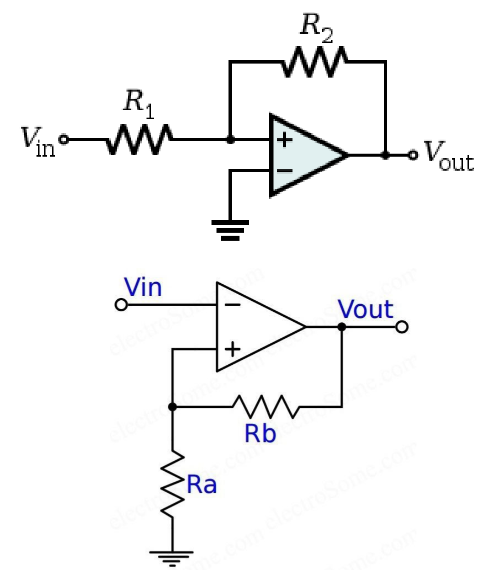


- Rule 2: The inputs draw no current
  - The ideal op-amp has very large input impedance (R_{in}). Thus, the current drawn at the two input terminals ~0.

What is Closed Loop with Negative Feedback?



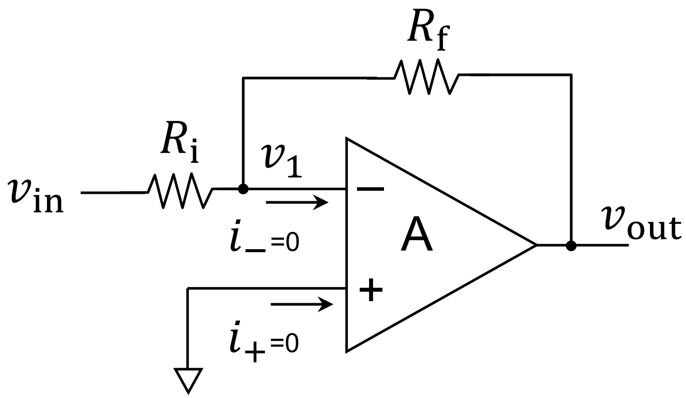
Negative Feedback



Positive Feedback

- Closed loop: There is connection between output and input
- Negative feedback: The output is fed back to the input in such a way to **reduce** the **output fluctuations**

Inverting Amplifier



$$\boxed{\frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i}}$$

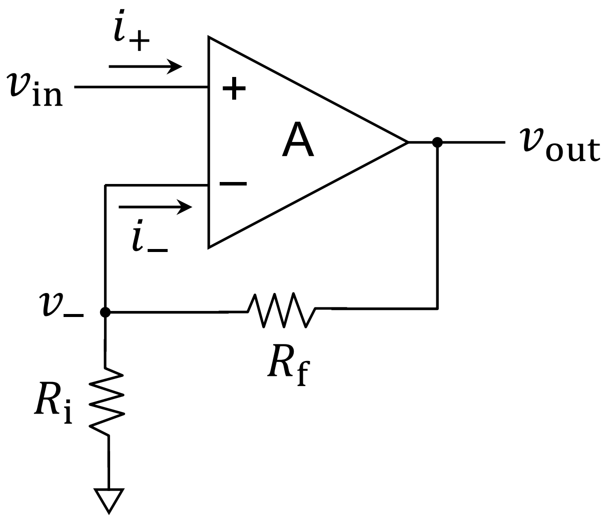
Derivation based on Golden Rules

$$v_1 = v_- \approx v_+ = 0 \quad [\because \text{ Virtual Short}]$$

$$\frac{v_{in} - v_1}{R_i} = \frac{v_1 - v_{out}}{R_f} \quad [\because \; i_- \approx 0]$$

$$\Rightarrow \frac{v_{in}}{R_i} = \frac{-v_{out}}{R_f} \quad \Rightarrow \quad \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_i}$$

Non-Inverting Amplifier



$$\boxed{\frac{v_{out}}{v_{in}} = 1 + \frac{R_f}{R_i}}$$

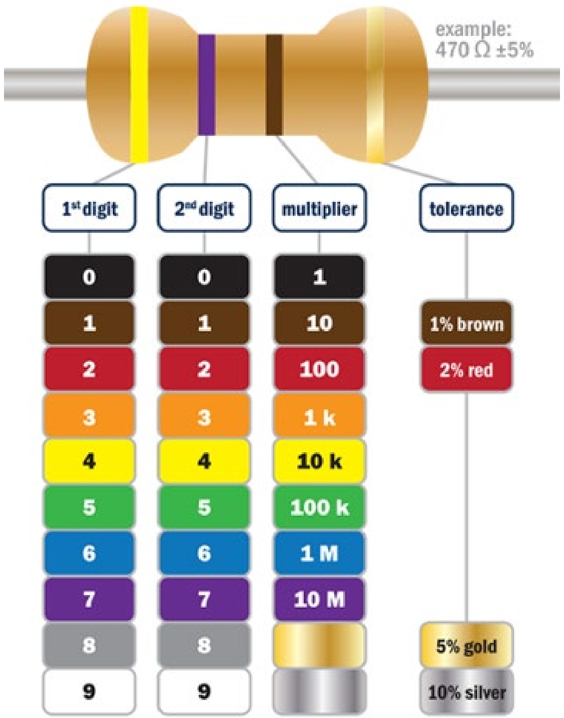
Derivation based on Golden Rules

$$v_- \approx v_+ = v_{in} \quad [\because \quad \text{Virtual Short}]$$

$$v_- = v_{out} \times \frac{R_i}{R_i + R_f} \approx v_{in} \quad [\because \quad i_- \approx 0]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \left(1 + \frac{R_f}{R_i}\right)$$

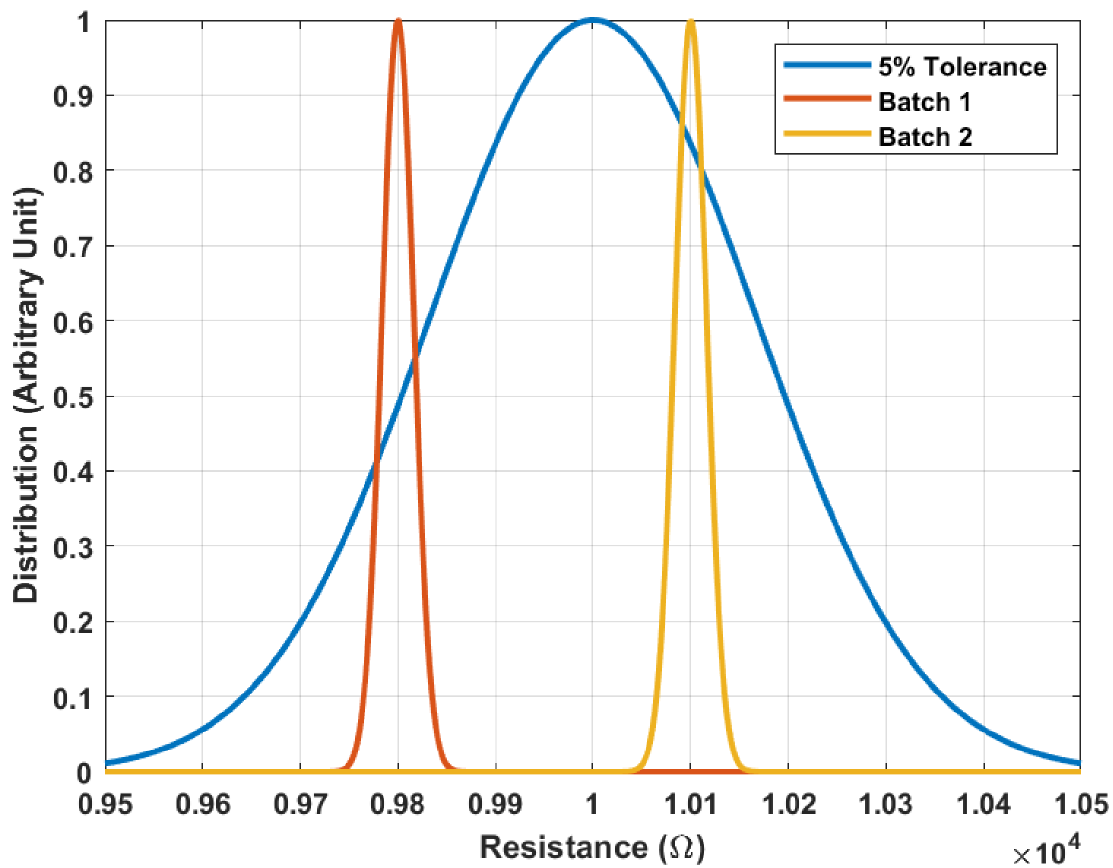
Significance of Ratio



$$\boxed{\frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{R_{\text{f}}}{R_{\text{i}}}}$$

$$\boxed{\frac{v_{\text{out}}}{v_{\text{in}}} = 1 + \frac{R_{\text{f}}}{R_{\text{i}}}}$$

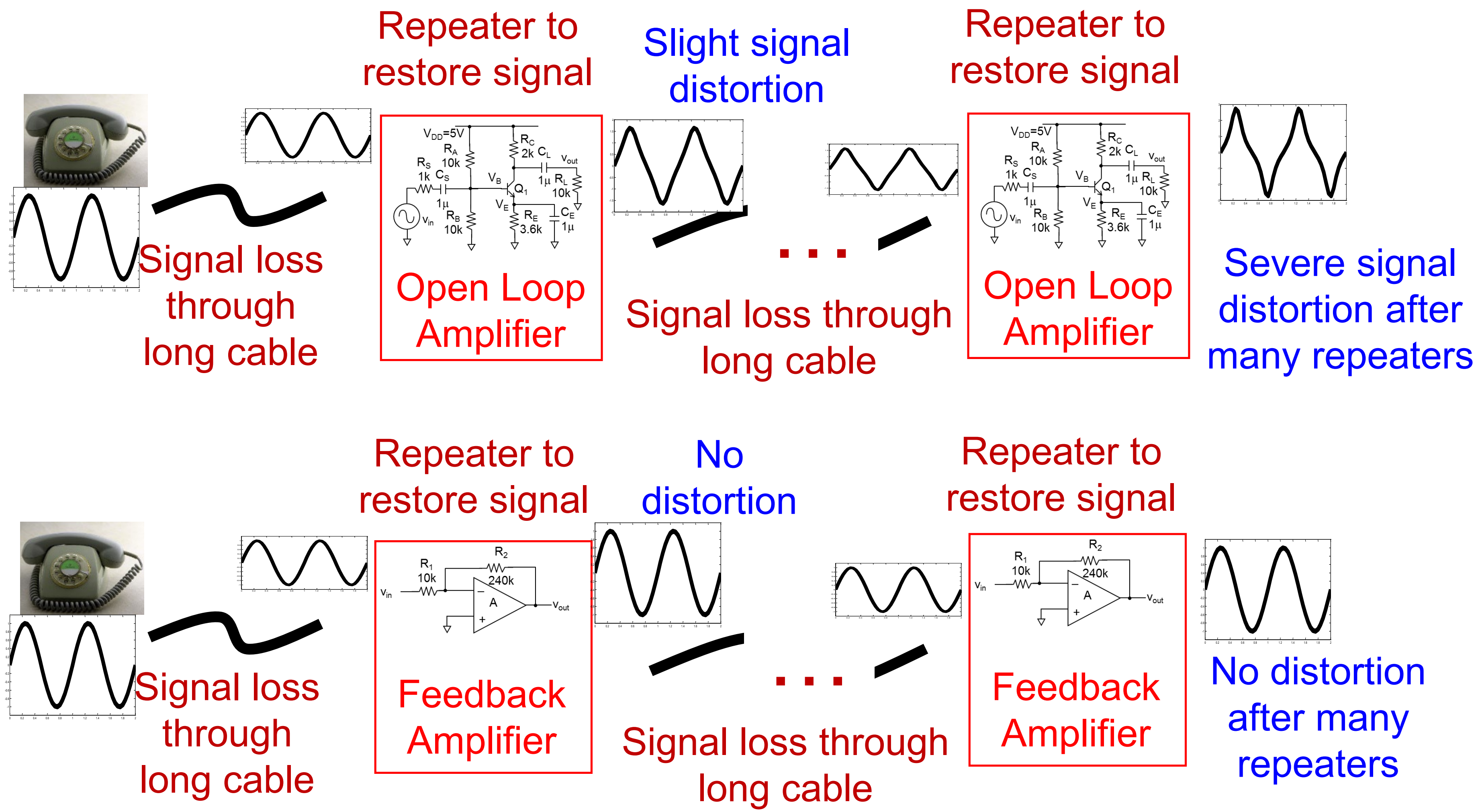
If R_f and R_i varies by 5% each, the resulting gain could vary by as much as 10% from desired value!



How to Achieve Accurate Gain?

- Use resistors from **same batch** (they usually match very well)!
- For example, if the desired $R_\mathrm{f}/R_\mathrm{i}$ ratio is 4, we use 5 identical resistors:
  - R_f uses 4 resistors, and
  - R_i uses 1 resistor.
  - An accurate gain of 4 can be obtained!
- $\Rightarrow$ Principle exploited in IC design

Significance of Closed Loop Negative Feedback



- Harold Stephen Black invented negative feedback amplifier in 1928

Question 1

Given:

- Primary voltage V_P of transformer is 65 V RMS
- Number of turns:
  - Primary windings N_P: 60
  - Secondary windings N_S: 90

Calculate RMS voltage across secondary winding.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{90}{60} = 1.5$$

Therefore, $V_S = 65 \times 1.5 = 97.5 \text{ V RMS}$

Question 2

A given transformer has:

- Primary voltage V_P : 25 V RMS
- Secondary voltage V_S : 90 V RMS
- Secondary windings N_S: 36

Calculate no. of turns in primary winding, N_P.

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{25}{90}$$

Therefore, $N_P = \frac{25}{90} \times 36 = 10$

Question 3

A given transformer has:

- Secondary voltage V_S : 100 V RMS
- Primary voltage V_P : 30 V RMS

What is the primary voltage needed to obtain a secondary voltage of 225 V RMS?

$$\frac{V_P}{V_S} = \frac{30}{100} = 0.3$$

Therefore, to obtain $V_S = 225$, we need

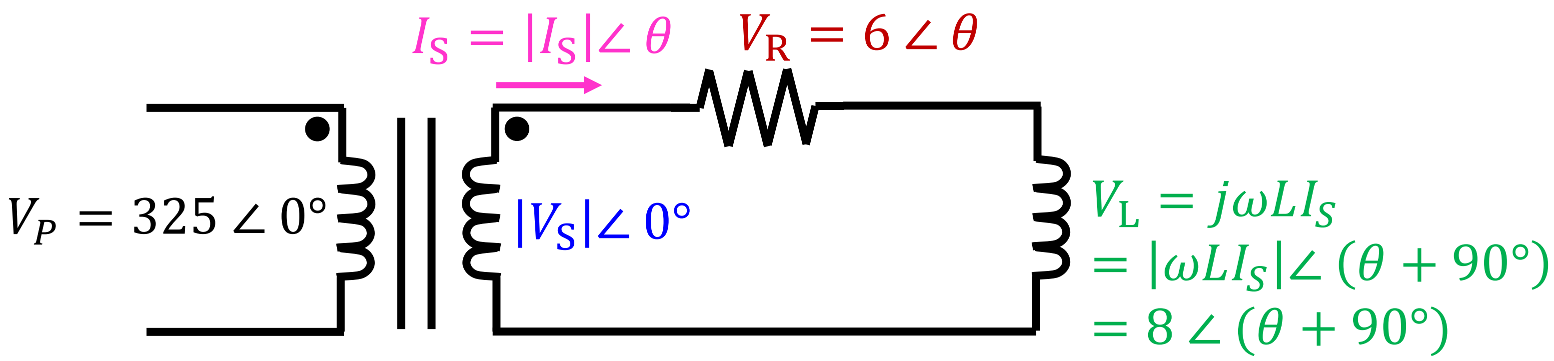
$$V_P = 0.3 \times 225 = 67.5 \text{ V RMS}$$

Question 4

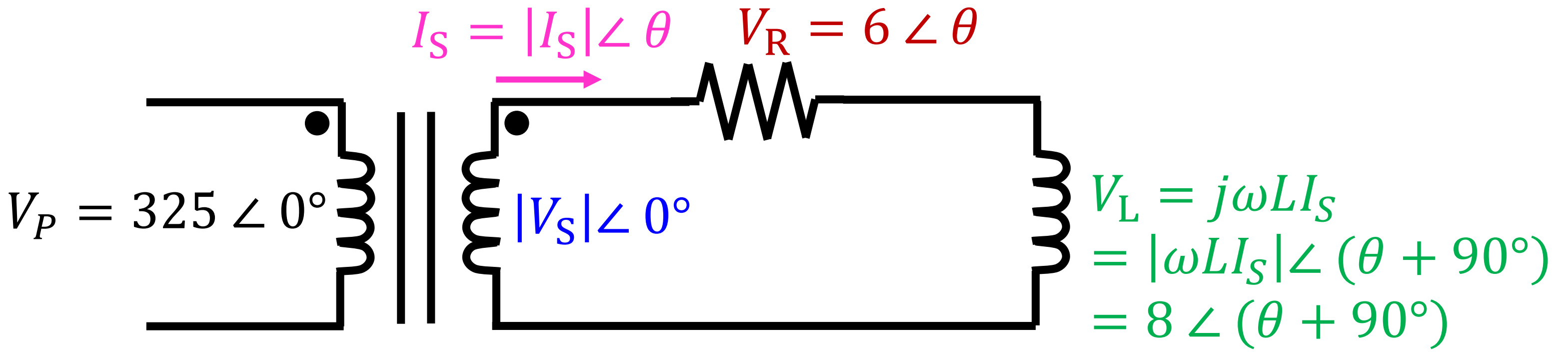
A given transformer has:

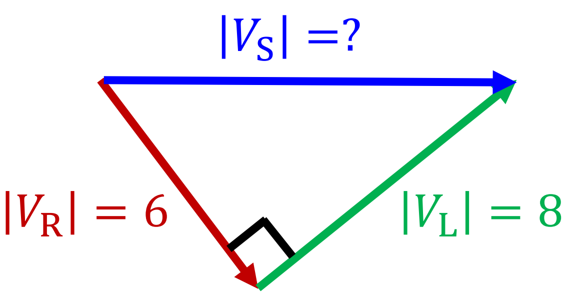
- Primary voltage V_P : $325 \angle 0°$
- Secondary side has series RL load:
  - $|V_\mathrm{R}| = 6\ \mathrm{V}, \quad |V_\mathrm{L}| = 8\ \mathrm{V}$

Calculate the turns ratio $\left(\frac{N_P}{N_S}\right)$ of the transformer.



Question 4





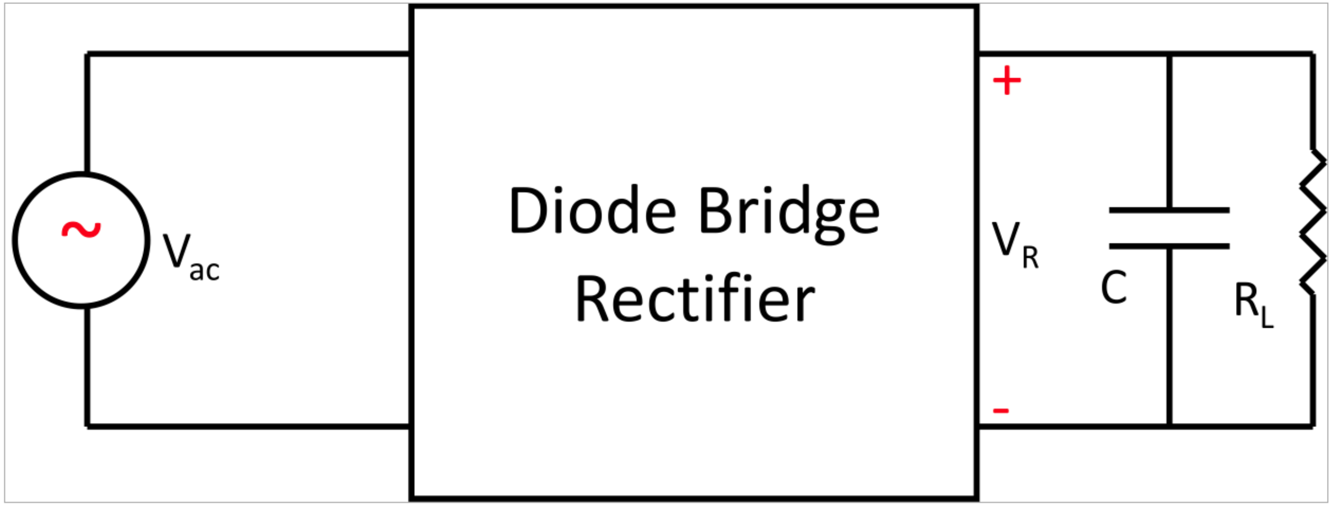
$$|V_S| = \sqrt{6^2 + 8^2} = 10$$

Therefore, $\dfrac{N_P}{N_S} = \dfrac{|V_P|}{|V_S|} = \dfrac{325}{10} = 32.5$

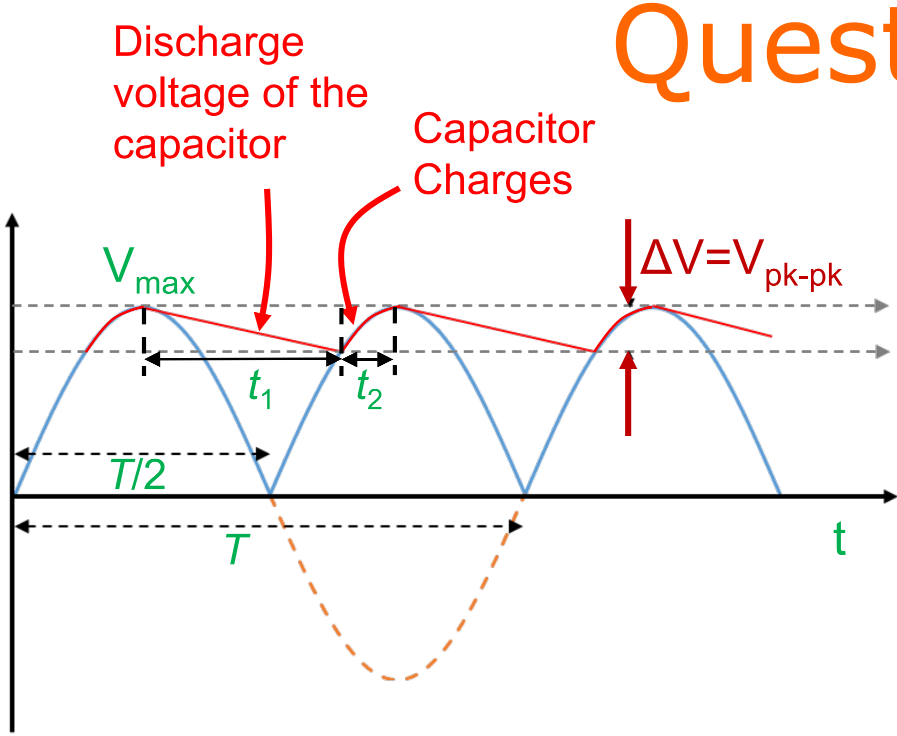
Question 5

Derive an expression for the voltage ripple at load R_L, given that:

- Average load voltage is V_{Load}
- AC power supply's frequency is f_s.



Question 5



- Let V_{Load}: average load voltage
- Average load current is given by

$$I_L = \frac{V_{Load}}{R_L}$$

- Since it is a full-wave diode bridge rectifier, the pattern repeats every $T/2$

- Let ΔV be the peak-to-peak ripple voltage
- $\Delta V = \frac{\Delta Q}{C}$ (from capacitance's definition)
- For small ΔV, $t_1 \approx T/2$. Since $i(t) = \frac{dQ}{dt}$, average current $I_L \approx \frac{\Delta Q}{T/2}$.
- Hence

$$\boxed{\Delta V \approx \frac{I_L * T/2}{C} = \frac{V_{Load}}{R_L} * \frac{1}{2f_S} * \frac{1}{C}}$$

Question 6

Full-wave rectifier with the following:

- Resistive load R_L: 100 Ω
- Average voltage V_{Load} : 9 V
- AC source's frequency f_S : 100π rad/s
- Filter capacitor C: 1.5 mF
- Assume: Ideal diodes with no voltage drop
- Find the voltage ripple

$$\Delta V = \frac{V_{Load}}{2 f_s R_L C} = \frac{9\ \mathrm{V}}{2 \times \frac{100\pi}{2\pi}\,\mathrm{Hz} \times 100\ \Omega \times 1.5\ \mathrm{mF}} = 0.6\ \mathrm{V}$$

Question 7

Full-wave rectifier with the following:

- Average current I_{L} : 0.2 A
- Average voltage V_{Load} : 15 V
- AC source's frequency f_{S} : 50 Hz
- Required peak-to-peak ripple $\Delta V \leq 0.5$ V
- Assume: Ideal diodes with no voltage drop
- Find the minimum value of the filter capacitor needed

Question 7

- $\Delta V = \frac{V_{Load}}{2f_S R_L C} = \frac{I_L}{2f_S C}$

- $I_L = 0.2\ \text{A},\ f_s = 50\ \text{Hz},$ and we need $\Delta V \leq 0.5\ V$

Therefore,

$$0.5 \geq \frac{I_L}{2f_s C}$$

$$C \geq \frac{I_L}{2f_s(0.5)} = \frac{0.2}{2 * 50 * 0.5} = 4\ \text{mF}$$

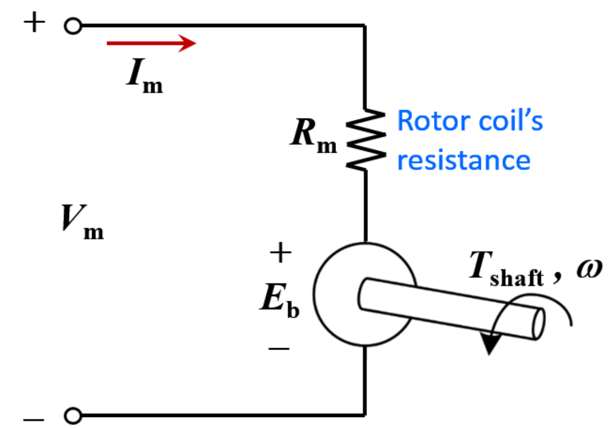
Question 8

- PMDC motor, 12 V source
- No-load speed = 3800 RPM
- Stall torque = 30 mNm

- If now running at 2500 RPM, find
  a) I_m
  b) T_{shaft}
  c) E_b
  d) Electrical power consumed
  e) Shaft power
  f) Power loss in rotor coil

Question 8

- First, find all unknown motor parameters



Circuit representation

- 3800 RPM $\rightarrow$ $\omega_{\text{no-load}} = 398\ \text{rad/s}$
- Recall that: $\boxed{I_m = \dfrac{V_m}{R_m} - \dfrac{K_e\ \omega}{R_m}}$

- At no-load, $I_m \approx 0$ since $T_{\text{shaft}} \approx 0$
  - Hence, $V_m \approx K_e\, \omega_{\text{no-load}} \rightarrow K_e \approx \dfrac{12}{398} = 30\ \dfrac{\text{mV}}{\text{rad/s}}$
  - For PMDC motor, $K_t = K_e = 30\ \dfrac{\text{mNm}}{\text{A}}$

- At stall condition, $T_{\text{stall}} = 30\ \text{mNm}$
  - $I_{\text{stall}} = \dfrac{T_{\text{stall}}}{K_t} = 1\ \text{A}$
  - Since $\omega_{\text{stall}} = 0,\ I_{\text{stall}} = \dfrac{V_m}{R_m} \rightarrow R_m = \dfrac{12\ \text{V}}{1\ \text{A}} = 12\ \Omega$

Question 8

At 2500 RPM:

$$I_m = \frac{V_m}{R_m} - \frac{K_e\,\omega}{R_m}$$

a) Find $I_{2500\ \mathrm{RPM}}$ and b) $T_{2500\ \mathrm{RPM}}$

$$\omega_{2500\ \mathrm{RPM}} = 261.8\ \mathrm{rad/s}$$

a) $$I_{2500\ \mathrm{RPM}} = \frac{V_m}{R_m} - \frac{K_e \omega_{2500\ \mathrm{RPM}}}{R_m}$$

$$= \frac{12}{12} - \frac{0.03 \times 261.8}{12} = 0.346\ \mathrm{A}$$

b) $$T_{2500\ \mathrm{RPM}} = K_t I_{2500\ \mathrm{RPM}}$$

$$= 0.03\ \times 0.346 = 10.4\ \mathrm{mNm}$$

Question 8

c) Find back emf $E_{2500\ \mathrm{RPM}}$

$$E_{2500\ \mathrm{RPM}} = K_e \omega_{2500\ \mathrm{RPM}} = 7.85\ \mathrm{V}$$

d) Find total electrical power consumed P_{e}

$$P_{\mathrm{e}} = V_{\mathrm{m}} \times I_{2500\ \mathrm{RPM}} = 4.15\ \mathrm{W}$$

Question 8

e) Find shaft power P_{shaft}

$$P_{\text{shaft}} = T_{2500\text{ RPM}} \times \omega_{2500\text{ RPM}} = 2.72\text{ W}$$

f) Find power loss in rotor coil

$$P_{\text{loss}} = R_{\text{m}} \times (I_{2500\text{ RPM}})^2 = 1.44\text{ W}$$

Question 9

- Same motor as Q8
- Load condition unchanged
  - (i.e., same torque as $T_{2500\ \mathrm{RPM}}$ previously)
- New desired speed = 1500 RPM

- Find:
  - a) Rotor current I_m and the required V_m
  - b) PWM duty cycle if DC source still 12 V
  - c) T_on and T_off if PWM frequency = 5 kHz
  - d) Electrical power consumed
  - e) Power loss in rotor coil

Question 9

a) Find current I_m and the required V_m

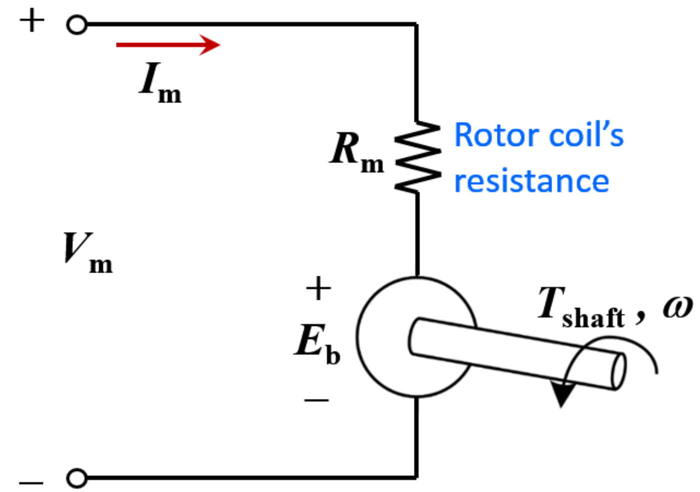
Same load means same torque as $T_{2500\ \mathrm{RPM}}$ previously, hence current remains unchanged at

$$I_\mathrm{m} = 0.346\ \mathrm{A}$$

$$\omega_{1500\ \mathrm{RPM}} = 157.1\ \mathrm{rad/s}$$

$$E_b = K_e\omega = 4.71\ \mathrm{V}$$

$$V_\mathrm{m} = R_\mathrm{m}I_\mathrm{m} + E_b = 8.86\ \mathrm{V}$$



Circuit representation

Question 9

b) Find PWM duty cycle if DC source is 12 V

$$\text{Duty Cycle} = \frac{8.86}{12} = 73.8\%$$

c) T_{on} and T_{off} if PWM frequency = 5 kHz

$$T_{\text{p}} = \frac{1}{5000} = 200\ \mu\text{s}$$

$$T_{\text{on}} = 0.738 \times 200\ \mu\text{s} = 147.6\ \mu\text{s}$$

$$T_{\text{off}} = T_{\text{p}} - T_{\text{on}} = 52.4\ \mu\text{s}$$

Question 9

d) Find total electrical power consumed

$$P_e = V_m \times I_m = 8.86 \times 0.346 = 3.07\ \mathrm{W}$$

e) Find power loss in rotor coil

$$P_{loss} = R_m \times (I_m)^2 = 1.44\ \mathrm{W}$$

(same as Q8 because load torque and I_m unchanged!)

Question 10

- PMDC motor with fan
- Fan's load torque $T_L = 0.05\omega + 0.001\omega^2$
- $K_\mathrm{t} = 2.42\ \mathrm{Nm/A}$
- $R_\mathrm{m} = 0.2\ \Omega$
- 50 V DC power supply

- Find:

  Speed of fan

Question 10

Recall that:

$$\boxed{T_{\text{shaft}} = K_t I_m}$$

$$\boxed{I_m = \frac{V_m}{R_m} - \frac{K_e \omega}{R_m}}$$

For PMDC motor: $\boxed{K_t = K_e}$

Therefore:

$$T_{\text{shaft}} = 2.42 \times \left(\frac{50}{0.2} - \frac{2.42}{0.2}\omega\right) = 605 - 29.3\omega$$

Question 10

Since $T_{\text{shaft}} = T_L = 0.05\omega + 0.001\omega^2$

$$605 - 29.3\omega = 0.05\omega + 0.001\omega^2$$
$$\omega^2 + 29350\omega - 605000 = 0$$

Solution of quadratic equation:

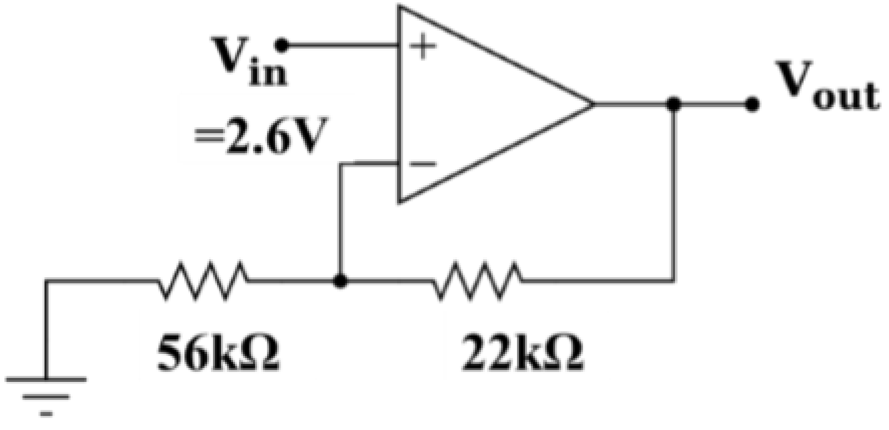
$$\omega = +20.6 \text{ or } -29371$$

Keeping the positive value, we have

$$\omega = 20.6 \text{ rad/s} \rightarrow 197 \text{ RPM}$$

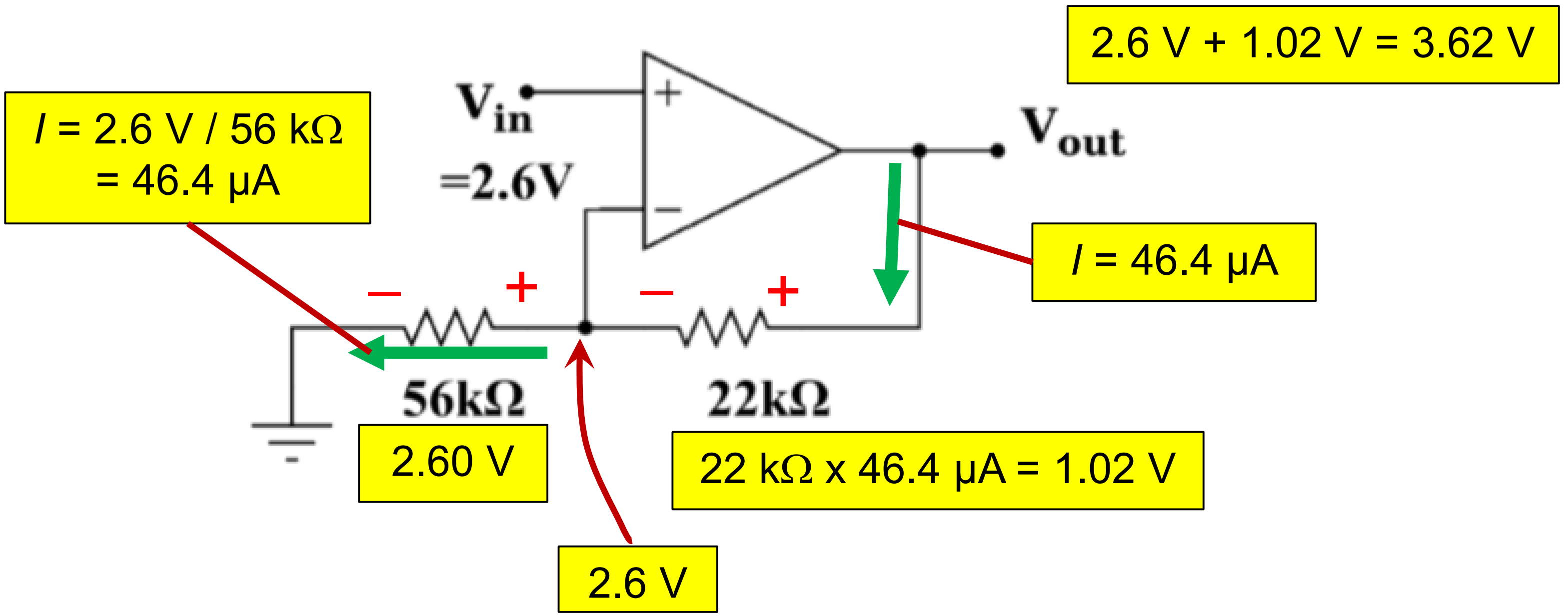
Question 11

- Calculate all voltage drops and currents in this circuit, and label the currents' directions & voltage polarities
- Calculate the overall voltage gain of this amplifier circuit (A_V), both (in V/V) and in units of decibels (dB)



Question 11

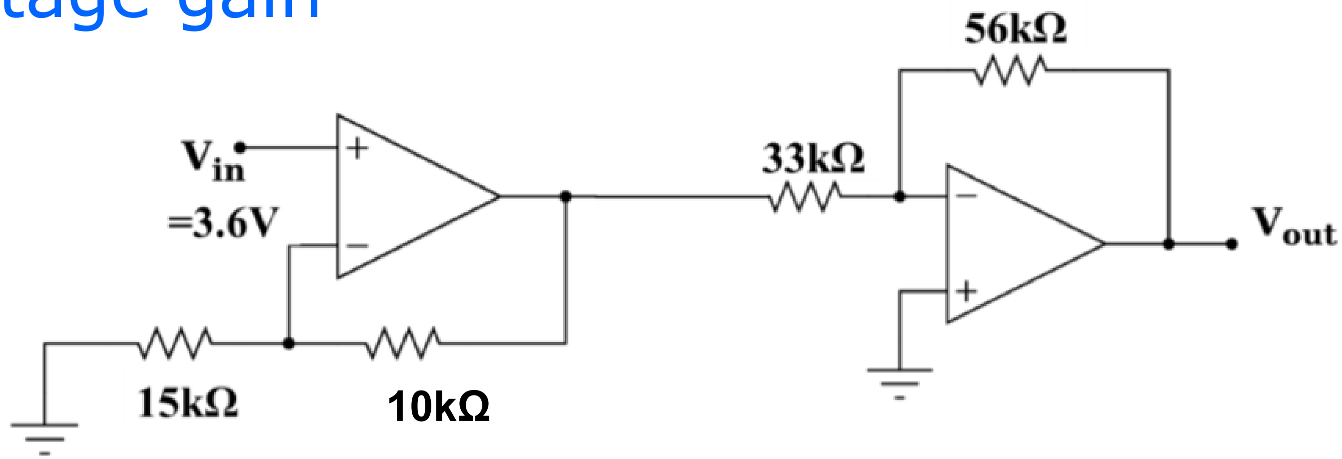
- Opamp's golden rule: V+ ≈ V–
- No current entering '+' & '–' inputs



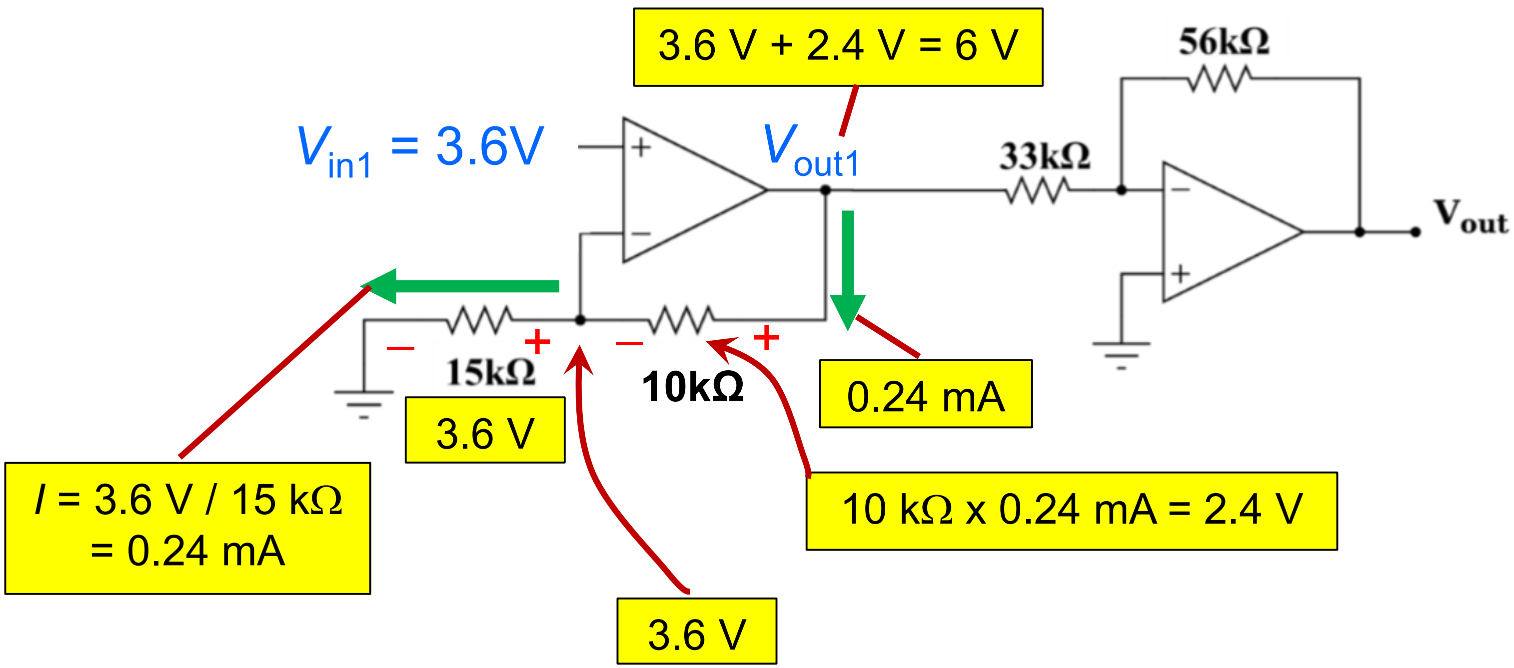
- Gain $= \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{3.62}{2.6} = 1.39\ \text{V/V}$ (Volt per Volt)
- Gain in dB = $20 \log_{10}(1.39)$ = 2.86 dB

Question 12

- Calculate all voltage drops and currents in this circuit, and label the currents' directions & voltage polarities
- Calculate the voltage gain for each stage of this amplifier circuit (both as a ratio and in units of decibels), then calculate the overall voltage gain

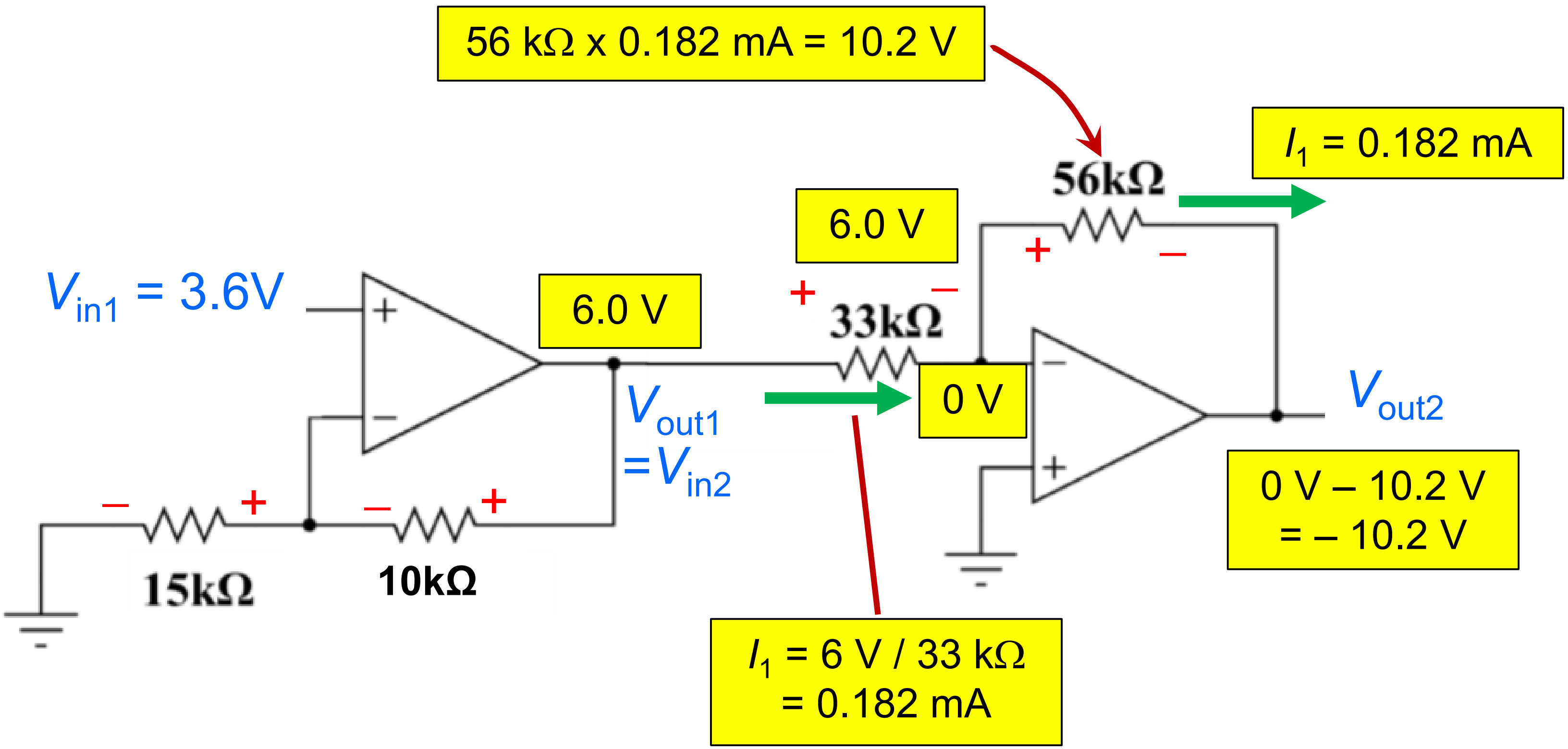


Question 12



- Gain of 1st stage $= \frac{V_{out1}}{V_{in1}} = \frac{6}{3.6} = 1.67 \text{ V/V}$
- Gain in dB = 20 $\log_{10}$ (1.67) = 4.45 dB

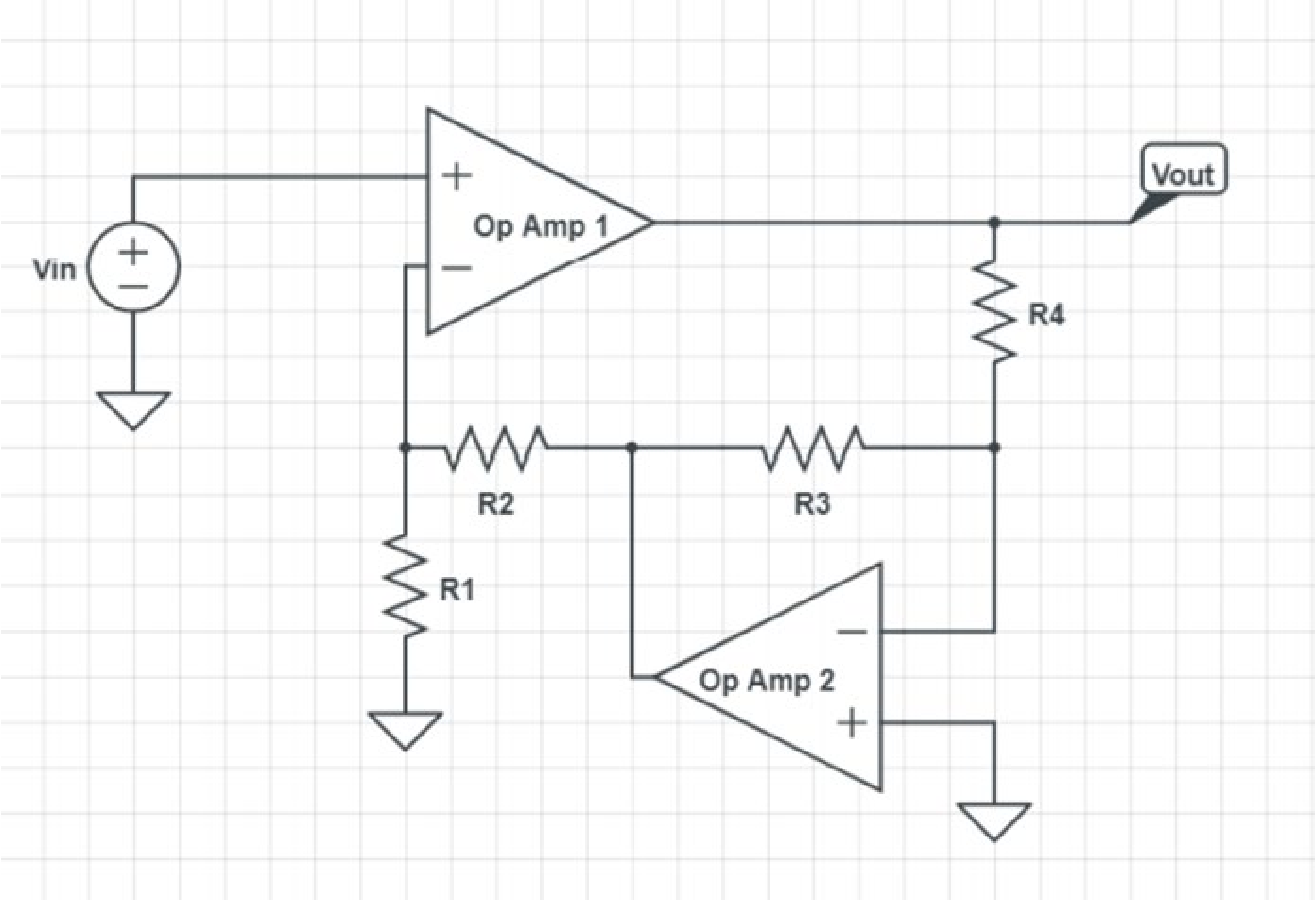
Question 12



- Gain of 2nd stage $= \dfrac{V_{out2}}{V_{in2}} = -\dfrac{10.2}{6} = -1.7$ V/V
- Gain in dB = 20 $\log_{10}(|-1.7|)$ = 4.61 dB
- Overall voltage gain = 1.67 × (–1.7) = –2.84 V/V = 9.07 dB

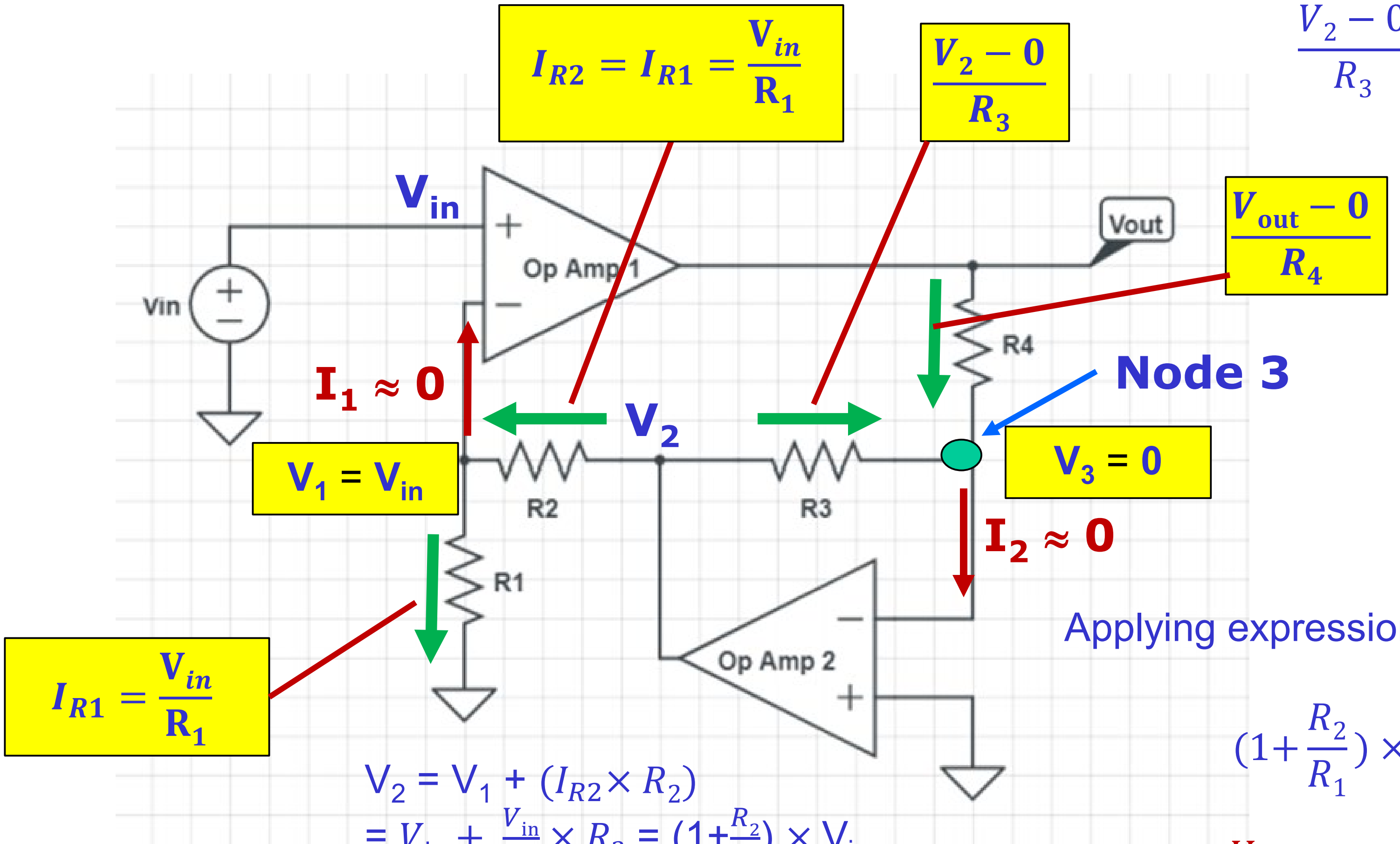
Question 13

- Calculate the voltage gain (V_{out}/V_{in}) of Op Amp 1



Question 13

- Using op amp Golden rules:



Applying KCL at Node 3,

$$\frac{V_2 - 0}{R_3} + \frac{V_{out} - 0}{R_4} = 0$$

Applying expression for V_2 from Eqn 1,

$$(1+\frac{R_2}{R_1}) \times \frac{V_{in}}{R_3} = \frac{-V_{out}}{R_4}$$

$$\frac{V_{out}}{V_{in}} = -(1+\frac{R_2}{R_1}) \times \frac{R_4}{R_3}$$

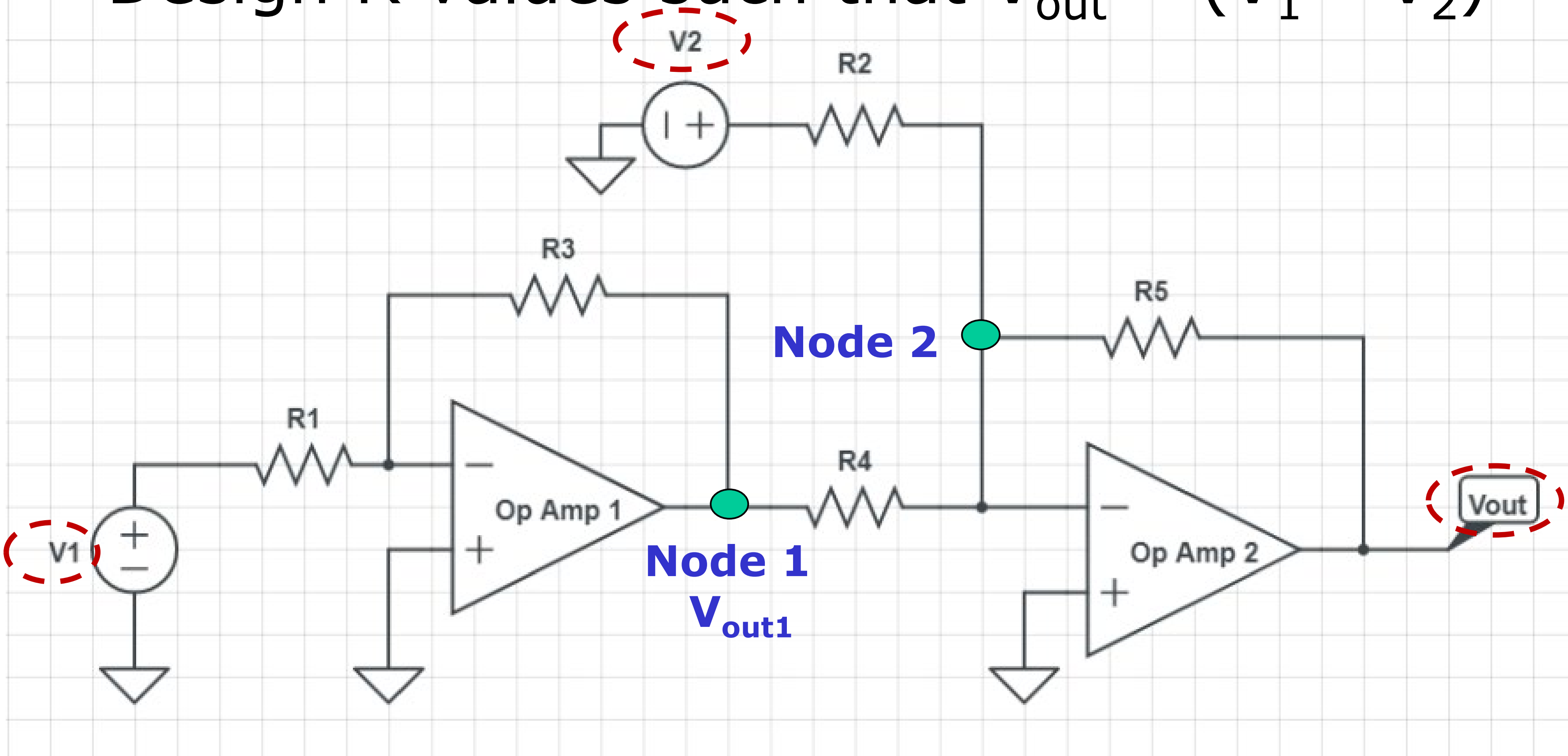
$V_2 = V_1 + (I_{R2} \times R_2)$

$= V_{in} + \frac{V_{in}}{R_1} \times R_2 = (1+\frac{R_2}{R_1}) \times V_{in}$

$\rightarrow V_2 = (1+\frac{R_2}{R_1}) \times V_{in}$ ----->Eqn 1

Question 14

- Derive the expression relating V_{out} and the two inputs, V_1 and V_2
- Design R values such that $V_{out} \propto (V_1 - V_2)$



Question 14

If $\frac{R_5 R_3}{R_4 R_1} = \frac{R_5}{R_2}$, which gives $\frac{R_3}{R_4 R_1} = \frac{1}{R_2}$, then

$$V_{out} = \frac{R_5}{R_2}(V_1 - V_2)$$

$$= K\,(V_1 - V_2)$$

Applying KCL at Node 2,

$$\frac{V_2 - 0}{R_2} + \frac{V_{out1} - 0}{R_4} + \frac{V_{out} - 0}{R_5} = 0$$

$$V_{out} = \left(\frac{R_5}{R_4} * \frac{R_3}{R_1} * V_1\right) - \left(\frac{R_5}{R_2} * V_2\right)$$