Singapore Junior Physics Olympiad 2017

Answers

Question 1 - A

Using basic kinematics with downwards displacement $s = 0.05\,\mathrm{m}$, initial velocity $u = 0$ and acceleration g, the time taken is $t = \sqrt{2s/g} = 0.1\,\mathrm{s}$.

Question 2 - B

The electron experiences an additional downwards electric force. The resultant force is directed downwards, with greater magnitude than in question 1. By Newton's 2nd Law, the acceleration of the electron is increased. As $t \propto 1/\sqrt{\text{acceleration}}$, the time taken is reduced.

Question 3 - C

Newton's 3rd Law states that *when object* A *exerts a force on object* B*, object* B *exerts an equal but opposite force (of the same type) on object* A. The apparatus exerts an electrostatic force on the electron, so the reaction force is an electrostatic force exerted by the electron on the apparatus.

Question 4 - D

After the electric field is reduced to zero, the electron experiences a net resultant force downwards due to gravity. The electron experiences a downwards acceleration g from an initial upwards velocity of $u = 15\,\mathrm{m\,s^{-1}}$ to a velocity of $v = 5\,\mathrm{m\,s^{-1}}$. Using kinematics, the distance travelled is $s = \frac{(v^2-u^2)}{2(-g)} = 10\,\mathrm{m}$.

Question 5 - D

Both D and P have the same charge, $+e$. The work done by the electric field on each particle is eE_0x_0. By the work-energy theorem, change in kinetic energy $= \frac{p_P^2}{2m_p} - \frac{p_0^2}{2m_p} = \frac{p_D^2}{4m_p} - \frac{p_0^2}{4m_p} = eE_0x_0$ for the proton and the deuteron respectively. (p_0 is the initial momentum of the deuteron/proton.) Therefore, $p_D^2 - p_0^2 = 2(p_P^2 - p_0^2) \implies p_D^2 - p_P^2 = p_P^2 - p_0^2 = 2m_peE_0x_0$. The relative size of p_D and p_P depends on the direction of $\mathbf{E_0}$, i.e. the sign of E_0.

Question 6 - C

Let the initial kinetic energy of proton 1 be K_0. Then $K_1 = K_0 + eE_0x_0$ and $K_2 = 2K_0 + eE_0x_0$. (Note that $E_0 > 0$, as defined in the question.) Clearly $2K_1 - K_2 = eE_0x_0 > 0$. Momentum and kinetic energy are related by $p = \sqrt{2mK}$. Hence $\sqrt{2}p_1 - p_2 > 0$ as well.

Question 7 - E

Firstly note that the initial momenta of deuteron 1 and proton 2 are the same. As the impulses $F\Delta t = eE_0t_0$ experienced by the particles are the same, the final momenta of the two particles are the same as well, i.e. $p_1 = p_2$. This then implies $2K_1 = K_2$.

Question 8 - E

We consider each statement individually.

A. The electric field strength need not be a constant in the time intervals 0 to t and t to $2t$. The instantaneous electric field strength at a certain time between t and $2t$ could exceed that at another time between 0 and t.

B. The velocity could have changed direction between time t and $3t$, such that the magnitude of velocity satisfied the values given in the question. The electric field vector could hence have remained in the same direction. In fact, we cannot make a statement about the time at which the electric field vector changed direction even if it did.

C. The electric field strength could possibly remain constant (as in projectile motion), but we cannot say so with certainty.

D. The electric field strength need not change monotonically from 0 to $2t$ and from $2t$ to $4t$.

Option E is hence the correct option. (Providing velocities at fixed time intervals provides insufficient information to make a suitable conclusion.)

Question 9 - D

By Newton's 2nd Law, $evB = m_p v^2/r$, where v is the speed of the proton, B is the magnetic flux density and r is the radius of gyration. Hence, linear momentum $m_p v = eBr = 1.6 \times 10^{-23}\,\mathrm{kg\,m\,s^{-1}}$.

Question 10 - D

As before, $m_e v = eBr$. Hence, kinetic energy $= \frac{(m_e v)^2}{2m_e} = \frac{(eBr)^2}{2m_e} = 1.41 \times 10^{-16}\,\mathrm{J} = 880\,\mathrm{eV}$.

Question 11 - B

The proton gyrates in a clockwise direction. The proton has a lower speed in the upper half of the orbit compared to the bottom half of the orbit by conservation of energy. Hence, the guiding center of the proton moves to the left.

Question 12 - A

The electron gyrates in an anti-clockwise direction. The electron has a higher speed in the upper half of the orbit compared to the bottom half of the orbit by conservation of energy. Hence, the guiding center of the electron moves to the right.

Question 13 - B

By the nature of the set-up, the speed of the guiding centre should be independent of time. Out of options A, B and C (which are independent of t), only the expression in object B has the dimensions of speed.

Remark 1 (Mathematical analysis for questions 11 to 13)**.** *Using Newton's Laws of Motion in the conventional notation,*

$$\begin{aligned} a_x &= \frac{qB_0}{m} v_y \\ a_y &= -\frac{qB_0}{m} v_x - \frac{q}{m} E_0. \end{aligned}$$

Now we consider the S' frame, moving at a velocity E_0/B_0 in the negative x-direction relative to the S (lab) frame. (Assume that $E_0/B_0 \ll c$.) Then

$$\begin{aligned} v_x' &= v_x + E_0/B_0 \\ v_y' &= v_y, \end{aligned}$$

which means that

$$a'_x = a_x = \frac{qB_0}{m} v'_y$$
$$a'_y = a_y = -\frac{qB_0}{m} v'_x.$$

These are the equations of uniform circular motion.
Hence, the charged particle moves in a circle with a fixed centre in the S' frame, i.e. the particle moves in a circle whose centre moves to the left with velocity E_0/B_0 in the S frame.

Question 14 - E

Sketch the full magnetic field. Using the fact that the magnetic force vector is perpendicular to the magnetic field lines, observe that the magnetic force vector has a component directed towards the region of lower magnetic flux density. Hence the proton experiences a net force directed towards the region of lower magnetic flux density, i.e. along the magnetic field lines (as given in the question).

Remark 2 (Mathematical analysis for question 14)**.** *We can decompose the magnetic flux density* $\mathbf{B}$ *into its components relative to the plane of gyration:* $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_\parallel$*. The resultant force on the proton can then be written as* $\mathbf{F} = q\mathbf{v} \times \mathbf{B} = q\mathbf{v} \times \mathbf{B}_\perp + q\mathbf{v} \times \mathbf{B}_\parallel$ *in the usual notation. We now consider the average force* $\langle \mathbf{F} \rangle$ *over time (e.g. over multiple periods of gyration). As the component* $q\mathbf{v} \times \mathbf{B}_\perp$ *provides the centripetal force for the proton's gyration, its time average vanishes, i.e.* $\langle q\mathbf{v} \times \mathbf{B}_\perp \rangle = 0$*.*
Hence, $\langle \mathbf{F} \rangle$ *is perpendicular to the plane of gyration, and such an average force would indeed cause a corresponding acceleration (parallel to* $\langle \mathbf{F} \rangle$*) of the guiding centre.*
As the magnetic flux density decreases in the direction of the magnetic field, we can conclude that the magnetic field $\mathbf{B}_\parallel$ *in the plane of gyration is directed outwards through the proton's circle of gyration by Gauss' Law (*$\oint \mathbf{B} \cdot d\mathbf{A} = 0$*).*
This means that $\mathbf{v} \times \mathbf{B}_\parallel \parallel -\mathbf{\Omega}$*, where* $\mathbf{\Omega}$ *is the angular velocity vector of the proton's gyration. (To aid in visualisation, note that as* $q > 0$*,* $\mathbf{\Omega}$ *is opposite in direction to the magnetic field, i.e.* $\mathbf{\Omega} \cdot \mathbf{B} < 0$*.)*
We can hence write

$$\langle \mathbf{F} \rangle = \langle q\mathbf{v} \times \mathbf{B} \rangle = qv \left\langle \mathbf{B}_\parallel \cdot \frac{\mathbf{A}}{A} \right\rangle \frac{-\mathbf{\Omega}}{\Omega} \equiv qv \left\langle \mathbf{B}_\parallel \cdot \hat{\mathbf{A}} \right\rangle (-\hat{\mathbf{\Omega}}).$$

Using Gauss' Law, the average force is $\langle \mathbf{F} \rangle = (qvr/2)(d\mathbf{B}_\perp/dz)$*, where* z *is the spatial coordinate perpendicular to the plane of gyration.*
The acceleration of the guiding centre is therefore $(qvr/2m)(dB_\perp/dz)$*.*

Remark 3 (Extension). *A curious student may question what the result would be when protons are replaced with electrons in question 14.*
In the case of electrons, the guiding centre will still move in the direction of the magnetic field. A simple way to understand why is to observe that $q\hat{\boldsymbol{\Omega}}$ *is unchanged regardless of the sign of the particle.* $\langle \mathbf{F} \rangle$ *is hence in the same direction as before.*

Question 15 - E

The proton gyrates in a clockwise direction. The radius of curvature is smaller at the top and bigger at the bottom, causing the guiding centre to move to the right.

Question 16 - B

The electron gyrates in an anti-clockwise direction. The radius of curvature is bigger at the top and smaller at the bottom, causing the guiding centre to move to the left.

Question 17 - A

As the electron is negatively charged, $\mathbf{v}$ will be in a direction opposite to that of $\mathbf{E}$. The force experienced by the electron would then be $\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$, which is always into the paper.
(Alternatively, use Fleming's Left Hand Rule.)

Question 18 - C

Use a first-order Taylor expansion of $\frac{Q}{4\pi\varepsilon_0 r}$ about the sphere's surface. Alternatively, potential difference $= \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 (r+d)} = \frac{Q}{4\pi\varepsilon_0 r}(1-(1+\frac{d}{r})^{-1}) \approx \frac{Qd}{4\pi\varepsilon_0 r^2}$.

Question 19 - A

Consider the maximum distance d the outer shell of electrons can move in a radial direction. Each electron has kinetic energy on the order of $E = kT_e$. The energy needed to move the electrons outwards by a distance d is given by $U = \frac{eQd}{4\pi\varepsilon_0 r^2}$, where $Q = 4\pi r^2 d n_e e$ is the charge of the outer shell of fixed protons. (The electrons in the outer shell do not exert a net electric force on any individual electron in the same shell.) By conservation of energy, change in total energy $= U - E = 0$. Ignoring the constants, we obtain $T_e \propto d^2 n_e$, i.e. $d \propto \sqrt{T_e/n_e}$.

Question 20 - E

From question 19, $U = Cn_e d^2$ for some constant C. This is a simple harmonic potential, with angular frequency $\omega = \sqrt{2Cn_e/m}$. Hence, frequency $= \omega/2\pi \propto \sqrt{n_e}$.

Question 21 - C

By conservation of energy, decrease in kinetic energy = increase in potential energy. When the electron reaches its maximum height, it has zero speed & zero kinetic energy, so the decrease in kinetic energy = 0.5 eV. As the increase in potential energy $= mg_s r_s(1 - r_s/r) = 0.5\,\mathrm{eV}$, we have $r/r_s = 1.85$. The electron's height about the sun's surface is $r - r_s = 0.85r_s$.

Question 22 - B

There is no net external torque acting on the system, so angular momentum about any axis is conserved. The initial angular momentum of the system is $L_0 = m_p vr$ where the mass of a hydrogen atom is approximated to m_p and $r = 1.5\,\mathrm{nm}$ is the initial separation of the two hydrogen atoms. Let I be the moment of inertia of the hydrogen molecule about its centre of mass, and let ω be the angular frequency of the molecule's rotation. As $I = 2m_p(l/2)^2$, $\omega = L_0/I = 1.4 \times 10^{15}\,\mathrm{rad\,s^{-1}}$.

Question 23 - C

For the He nucleus to pass through Au in a head-on collision, it just needs to reach the Au nucleus with a non-zero velocity. By conservation of energy, decrease in kinetic energy = increase in potential energy. Hence,
initial kinetic energy > electric potential energy at the centre of Au
= electric potential energy at the surface of the Au nucleus + potential difference between surface and centre of Au nucleus
$= (79e)(2e)/(4\pi\varepsilon_0 r) + (\text{maximum electric field strength})(r)/2$
$= (79e)(2e)/(4\pi\varepsilon_0 r) + (79e)(2e)/(4\pi\varepsilon_0 r^2) \times r/2$
$= 474e^2/(8\pi\varepsilon_0 r) = 2.4\,\mathrm{keV}$.

Question 24 - C

By the Stefan-Boltzmann law, $P = A\sigma T^4$, where A is the surface area of the star and T is its temperature. As $A \propto r^2$, where r is the radius of the star, $r^2 \propto P/T^4$, i.e. $r \propto \sqrt{P}/T^2$. Hence $r/r_s = \sqrt{1000}/4 \approx 8$.

Question 25 - A

After a long time t, the RC-circuit will reach a steady-state, where current in the circuit is zero. The potential difference across the resistor will hence tend towards zero, implying that the potential differences across the capacitors are the same. Let the charge in capacitor C be q. As charge is conserved, the charge in capacitor $2C$ is $Q - q$. Using potential differences, $q/C = (Q-q)/2C$, i.e. $q = Q/3$. Then the total energy stored in the two capacitors is $\frac{(Q/3)^2}{2C} + \frac{(2Q/3)^2}{2C} = \frac{Q^2}{6C} = E/3$ as $E = Q^2/C$.

Question 26 - D

Pressure is a scalar quantity, defined as *the normal force per unit surface area*. Mathematically, $p = \mathbf{F} \cdot \mathbf{A}/|\mathbf{A}|^2$ for force $\mathbf{F}$ and surface area $\mathbf{A}$. Hence, it is incorrect to say that pressure has a direction (upwards or downwards).

Question 27 - E

Conceptually, the forces in options A to D are physical forces, i.e. they are forces that act on an object and cause its acceleration. The centripetal force, on the other hand, is not a physical force; it refers to the resultant force acting on an object moving in circular motion.
This means that we can draw the forces in options A to D on a free-body diagram. However, we do not draw the centripetal force as a separate force on a free-body diagram.

Question 28 - D

Each electron experiences a constant acceleration of $a = eE/m_e = e\Delta V/m_e d$ between each pair of electrodes, where E is the electric field strength, $\Delta V = 125\,\mathrm{V}$ is the potential difference and $d = 1\,\mathrm{cm}$ is the distance between adjacent electrodes. The time taken for each electron to travel between adjacent electrodes is hence $t = \sqrt{2d/a} = \sqrt{2m_e/e\Delta V}d$. The total time taken to travel from the cathode to the anode (across 11 spaces) is $11t = 33\,\mathrm{ns}$.

Question 29 - A

Let $l = 1\,\mathrm{cm}$ be the length of the lens, $n_0 = 1.67$ be the value of n at $r = 0$ and $n_1 = 1.66$ be the value of n at $r = 1\,\mathrm{cm}$.
The focal length f is the distance between the edge of the lens and the focal point, where light rays passing through the lens converge. We can find the

focal point by considering the central ($r = 0$) and outermost ($r = 1\,\text{cm}$) rays. The difference in optical path length between the central and outermost rays is $\Delta = (n_0 l + f) - (n_1 l + \sqrt{f^2 + r^2}) = (n_0 - n_1)l - (\sqrt{f^2 + r^2} - f)$. As different points on the same wavefront must reach the focal point at the same time, the optical path difference Δ must vanish. Hence, $\sqrt{f^2 + r^2} - f = 0.01l \implies f = (r^2 - 0.0001l^2)/0.02l$. This means $f = 50\,\text{cm}$.

Question 30 - E

There is an upper limit on the intensity at the wood as the sun is not a perfect point source, i.e. it has a non-zero size. Therefore, the reflected image of the sun at the focal point of the curved mirror has non-zero size.
The intensity of sunlight incident on the earth is given by $I_0 = \sigma T^4 (r_s/r_{SE})^2$. The apparent power of the sun, i.e. the total power reflected by the curved mirror, is $P = \pi(d/2)^2 I_0$, where $d = 20\,\text{cm}$ is the diameter of the curved mirror. The area of the sun's image is $\alpha = \pi(r_s f/r_{SE})^2$, where f is the focal length and f/r_{SE} is the magnification factor of the curved mirror. The maximum intensity is hence given by $I_{\text{max}} = P/\alpha = \sigma T^4 (d/2f)^2 = 2.5 \times 10^6\,\text{W}\,\text{m}^{-2}$.

Question 31 - A

By analogy to simple harmonic motion, the maximum velocity of the electron v_0 is related to the maximum acceleration a_0 by $a_0 = 2\pi f v_0$. The maximum acceleration of the electron is $a_0 = eE_0/m_e$. Hence, the maximum kinetic energy of the electron is $m_e v_0^2/2 = \frac{1}{8m_e}(\frac{eE_0}{\pi f})^2$.

Question 32 - C

We ignore all constants as we only need an expression that is correct to a proportionality constant.
First note that the magnetic field $\mathbf{B}$ does not result in any magnetic pressure as $\mathbf{j} \parallel \mathbf{B} \implies \mathbf{j} \times \mathbf{B} = 0$. (This immediately rules out options A, B and D. A simple qualitative analysis should show that the magnetic pressure is non-zero, leaving option C as the only viable option.)
The magnetic field $\mathbf{B}'$ induced by the current density $\mathbf{j}$ does cause a resultant force as $\mathbf{j} \times \mathbf{B}' \neq 0$. The magnetic pressure p would hence be due to the

induced field $\mathbf{B}'$. Using the following relations,

$$\begin{aligned} B' &\propto jr \\ F = B'Il &\propto (jr)(jr^2)l = rl(jr)^2 \\ p = F/A &\propto F/rl \end{aligned}$$

where $B' \equiv |\mathbf{B}'|$ and $j \equiv |\mathbf{j}|$.
Hence, $p \propto (jr)^2$.

Question 33 - A

For a $E = 12.4\,\mathrm{keV}$ photon, the angular frequency is given by $\omega = E/\hbar = 2\pi E/h = 1.88 \times 10^{19}\,\mathrm{rad\,s^{-1}}$.
Note that the given angle is the angle between the critical ray and the plasma interface, so by Snell's Law, $n = \sin(\pi/2 - \theta_{\text{crit}}) = \cos\theta_{\text{crit}} = 1 - 1.88 \times 10^{-5}$.
Hence,

$$n_e = \frac{\varepsilon_0 m_e \omega^2}{e^2}(1 - n^2) = 4.18 \times 10^{30}\,\mathrm{m^{-3}}.$$

The total number density of electrons in gold (including conduction/non-conduction electrons) is $n_0 = 79\rho N_A/m_r = 4.66 \times 10^{30}\,\mathrm{m^{-3}}$, where $\rho = 19300\,\mathrm{kg\,m^{-3}}$ is the density of gold and $m_r = 0.197\,\mathrm{kg\,mol^{-1}}$ is the molar mass of gold.
The required fraction is $n_e/n_0 = 0.9$.
(This is different from the case where only the conduction electrons are treated as "free" electrons, such that $n_e/n_0 = 1/79$.)

Question 34 - C

At thermal equilibrium, heat lost from the house = heat gained by the house. Heat is lost via conduction through the floor to the external environment, while heat is gained via radiation from the hot water radiator.
Let $A = 100\,\mathrm{m^2}$ be the area of the floor and $A' = 220\,\mathrm{m^2}$ be the total area of the walls and the roof. T is the temperature of the floor, T_i is the inside temperature and T_o is the outside temperature.
Thus, by the Stefan-Boltzmann Law,

$$\text{heat gained} = A\sigma(T^4 - T_i^4).$$

On the other hand,

$$\text{heat lost} = kA'\frac{T_i - T_o}{\Delta x} + kA\frac{T - T_o}{\Delta x}$$

where $k = 0.14\,\mathrm{W\,m^{-1}\,K^{-1}}$ is the conductivity of wood and $\Delta x = 0.05\,\mathrm{m}$ is the thickness of the floor, roof and walls.
To keep the inside temperature at $T_i = 293.15\,\mathrm{K}$ in thermal equilibrium, we need

$$\sigma T^4 - \frac{k}{\Delta x}T + \left(\frac{k}{\Delta x}T_o - k\frac{A'}{A}\frac{\Delta T}{\Delta x} - \sigma T_i^4\right) = 0.$$

We can find the solution to the above quartic equation by substituting the values provided in the options. Clearly, the temperatures given in options A, B, D and E do not satisfy the equation. Hence, $T = 335\,\mathrm{K}$. (The same answer is obtained via analytical solution.)

Question 35 - B

A potential difference is only produced when cylinder B rotates within a magnetic field. If cylinder A rotates but cylinder B does not, no potential difference results as there is no change in magnetic flux linkage nor any motional electromotive force. (Note that by rotational symmetry, the magnetic field due to cylinder A remains constant over time even if cylinder A rotates.) Therefore, only the rotation of cylinder B determines the potential difference produced.
Cases I, II and III hence produce the same potential difference, while case IV produces no potential difference. Only option B is true.

Question 36 - B

Let N_0 be the initial number of ^{238}U. Also let N_{238} be the number of ^{238}U and N_{234} be the number of ^{234}U at any given time t. $T_{238} = 4.468 \times 10^9\,\mathrm{y}$ is the half-life of ^{238}U, while $T_{234} = 2.445 \times 10^5\,\mathrm{y}$ is the half-life of ^{234}U.
As the half-lives of the decays from ^{234}Th to ^{234}Pa and to ^{234}U are extremely short compared to the relevant time-scales, i.e. $7\,\mathrm{h} < 24.1\,\mathrm{d} \ll 10^5\,\mathrm{y}$, we can assume that these decays occur so rapidly that any ^{234}Th or ^{234}Pa produced decays immediately to give their daughter products.
Then we have $N_{238} = N_0(2^{-t/T_{238}})$.
To find N_{234}, consider the following facts:

1. If we assume that ^{234}U does not decay further, we will overestimate the value of N_{234}. This means $N_{234} \leq 1 - N_{238}$.

2. If we assume that all the decay of ^{234}U occurs suddenly at the relevant time t, we will underestimate the value of N_{234}. This means $N_{234} \geq (1 - N_{238})(2^{-t/T_{234}})$.

Applying this to $t = 10^5\,\text{y}$, we have $1.17 \times 10^{-5} \leq N_{234}/N_{238} \leq 1.55 \times 10^{-5}$. (We can rule out options C to E.)
This method, however, is inconclusive for $t = 2 \times 10^6\,\text{y}$. Instead, consider the fact that $t \gg T_{234}$. We can hence assume that the value of N_{234} has reached equilibrium, such that the rate of formation of ^{234}U is the same as its rate of decay. Thus, $-dN_{238}/dt = (\ln 2/T_{234})N_{234} \implies N_{234} = (T_{234}/T_{238})N_{238}$.
Hence, at large t (e.g. $t = 2 \times 10^6\,\text{y}$), $N_{234}/N_{238} = T_{234}/T_{238} = 5.5 \times 10^{-5}$.

Remark 4 (Calculus-based analysis for question 36)**.** *For the exact values of N_{234}/N_{238}, consider the system of differential equations:*

$$\frac{dN_{238}}{dt} = -\frac{\ln 2}{T_{238}}N_{238}$$
$$\frac{dN_{234}}{dt} = +\frac{\ln 2}{T_{238}}N_{238} - \frac{\ln 2}{T_{234}}N_{234}$$

with initial conditions $N_{238} = N_0$ and $N_{234} = 0$ at $t = 0$.
The solution of the differential equations is:

$$N_{238} = N_0(2^{-t/T_{238}})$$
$$N_{234} = N_0\frac{T_{234}}{T_{238} - T_{234}}(2^{-t/T_{238}} - 2^{-t/T_{234}}).$$

Hence,

$$\frac{N_{234}}{N_{238}} = \frac{T_{234}}{T_{238} - T_{234}}\left(1 - 2^{(1/T_{238} - 1/T_{234})t}\right).$$

When $t = 10^5\,\text{y}$, $N_{234}/N_{238} = 1.4 \times 10^{-5}$.
When $t = 2 \times 10^6\,\text{y}$, $N_{234}/N_{238} = 5.5 \times 10^{-5}$.

Question 37 - A

Using the typical notation for direct-current circuits, $P = V^2/R \implies R = V^2/P$. Hence, the first bulb has resistance $14.4\,\Omega$ and the second bulb has resistance $28.8\,\Omega$. The power supplied when these bulbs are connected in series is $P = V^2/R_{\text{total}} = 3.33\,\text{W} < 5\,\text{W}$.

Question 38 - A

The power output of the helicopter is $P = Tv$, where T is the downwards thrust exerted by the helicopter blades on the air and v is the mean velocity of the downward-moving column of air. By Newtons 3rd Law, $T = T$ where T is the upwards force exerted on the helicopter. Then, by Newtons 2nd Law,

$T = W$ where W is the weight of the helicopter, i.e. $P = Wv$. We can write $T = \Delta(mv)/\Delta t = v\Delta m/\Delta t = v\rho A v$ where A is the cross-sectional area of the helicopter blades and ρ is the density of air. From above, $W = T = \rho A v^2$. Let a characteristic linear dimension of the helicopter be L. Then $W \propto L^3$ and $A \propto L^2$. Thus $v^2 \propto W/A \propto L$. Hence $P \propto L^{3.5}$. The required power when all linear dimensions are reduced by half is $P' = 2^{-3.5}P$.

Question 39 - B

1. The difference in temperature arises as the centre of gravity of the first block of iron rises when the iron expands, but that of the second block of iron falls. (A hint is provided when the coefficient of thermal expansion is given.)

2. The volumetric thermal expansion coefficient is three times that of the linear thermal expansion coefficient. (It is reasonable to assume that the block of iron is homogeneous and isotropic.) Based on the definitions of the thermal expansion coefficients,

$$\begin{aligned} \alpha_v &= \Delta V/V\Delta T \\ \alpha_l &= \Delta L/L\Delta T. \end{aligned}$$

As $V = L^3$, i.e. $\Delta V/V = 3\Delta L/L$,

$$\alpha_l = \alpha_v/3 = 11.8 \times 10^{-6}\,\mathrm{K}^{-1} = 11.8\,\mu\mathrm{m/m\,K}.$$

3. The centre of gravity rises/falls by half the change in length of the block. The vertical coordinate of each point in the block of iron varies proportionally to its vertical distance from the point of suspension.

With these three key ideas, we can write, for the first block,

$$\begin{aligned} Q &= mc\Delta T + mg\Delta h \\ &= mc\Delta T + mg\Delta L/2 \\ &= m\Delta T(c + g\alpha_l s/2), \end{aligned}$$

where s is the side length $0.100\,\mathrm{m}$.
The expression is similar for the second block, but with a minus sign. Thus,

$$\begin{aligned} T - T' &= \frac{Q}{m}\left(\frac{1}{c + g\alpha_l s/2} - \frac{1}{c - g\alpha_l s/2}\right) \\ &= -7.26\,\mathrm{nK}. \end{aligned}$$

Alternatively, we can obtain an approximate answer by treating $\delta c = g\alpha_l s/2$. Then, $Q = m\Delta T(c \pm \delta c) \implies T - T' = Q/(mc) \times (-2\delta c/c) = -7.26\,\mathrm{nK}$.

Question 40 - D

We consider the collisions separately.
The balls both have a velocity of $v = \sqrt{2gh}$ before colliding with the ground. The large ball first bounces up with a velocity v. The small ball is still (momentarily) moving downwards with velocity v. As the large ball has a much larger mass, it continues moving upwards at velocity v even after colliding with the small ball.
By Newton's Law of Restitution,

relative speed of approach = relative speed of separation.

Thus the small ball moves upwards with velocity $3v$ (since $3v - v = v + v$). The initial height of the small ball is D above the ground. The small ball moves to an additional height of $(3v)^2/2g = 9v^2/2g = 9h$. The total height above the ground is thus $D + 9h$.

Question 41 - D

Let v_f be the final speed of the spacecraft. As energy is conserved, we can use Newton's Law of Restitution:

$$\begin{aligned}
\text{relative speed of approach} &= \text{relative speed of separation} \\
v + u &= \sqrt{v_f^2 + u^2} \\
4u^2 &= v_f^2 + u^2 \\
\implies v_f &= v\sqrt{3}.
\end{aligned}$$

Question 42 - C

It is possible to choose the right answer based on the options alone.
To derive the answer without relying on the options, use conservation of energy and conservation of momentum.
Let ϕ be the angle of deflection of mass M, $\mathbf{P}$ be the final momentum of mass M and $\mathbf{p}$ be the final momentum of mass m.
Conservation of energy:

$$P^2 + 2p^2 = M^2V^2$$

Conservation of momentum:

$$\begin{aligned}
\mathbf{p} &= M\mathbf{V} - \mathbf{P} \\
\implies p^2 &= P^2 + M^2V^2 - 2PMV\cos\phi
\end{aligned}$$

If we define $MV/P \equiv \beta$, we have $\cos\phi = (\beta + 3/\beta)/4$. As $\beta > 0$, we use the AM-GM inequality: $\beta + 3/\beta \geq 2\sqrt{(\beta)(3/\beta)} = 2\sqrt{3}$. (Equality holds when $\beta = 3/\beta$, i.e. $\beta = \sqrt{3}$.)
Hence, $\cos\phi \geq \sqrt{3}/2$, i.e. $\phi \leq 30^o$.

Question 43 - A

Using Faraday's Law,

$$V_{\text{in}} = d\Phi_{\text{in}}/dt$$
$$V_{\text{out}} = d\Phi_{\text{out}}/dt.$$

Note that the magnetic flux linkage is $\Phi = N\phi$, where $\phi = BA$ is the magnetic flux. The magnetic flux ϕ is conserved everywhere in an ideal transformer, so $d\phi/dt = V_{\text{in}}/N_{\text{in}} = V_{\text{out}}/N_{\text{out}}$, i.e. the two voltages are in phase.
The explanation is similar for non-ideal transformers.

Question 44 - E

Beta-decay leads to the formation of three particles: a daughter nucleus (^{32}S), a beta-particle (^{0}e^{-}, i.e. an electron) and an antineutrino ($\bar{\nu}$). There are hence three variables (i.e. velocities of the three product particles) but only two equations (i.e. conservation of energy and momentum). There is no unique solution for the velocity of the beta-particle. The kinetic energy of the beta-particle is hence not a distinct value, and varies from 0 to $Q = 1.71\,\text{MeV}$.
If the formula for alpha-decay were applied, option D would be (incorrectly) obtained. It is clear, however, that the daughter nucleus and anti-neutrino cannot possibly have zero energy, as they would need to have some motion via the conservation of momentum.
More information (e.g. kinetic energy & momentum of the antineutrino) is needed to find the kinetic energy of the beta-particle.

Question 45 - A

Intuition gives the correct answer.
Alternatively, the threshold angle for each face of the prism can be calculated using geometry.

Face	Threshold angle	Other angle
A	59.7^o	66.4^o
B	30.3^o	30.3^o
C	48.6^o	59.7^o
D	$< 32.5^o$	not necessary to calculate

Face A is the face with the largest threshold angle.

Question 46 - D

Using the adiabatic equation, $p^{1-\gamma}T^{\gamma}$ = constant, where p is the pressure and T is the temperature of the air. The temperature at the given height is hence $T = (293.15\,\mathrm{K})\left(\frac{100\,\mathrm{kPa}}{84.5\,\mathrm{kPa}}\right)^{(1-\gamma)/\gamma} = 279.4\,\mathrm{K} \approx 6\,^{\circ}\mathrm{C}$.

Question 47 - D

We can analyse the options individually.

A. This would be the case in the absence of diffraction/interference.

B. This is the single-slit diffraction intensity distribution.

C. This is the double-slit interference intensity distribution, with a diffraction envelope.

D. This is the 4-slit interference intensity distribution, with a diffraction envelope.

E. This is the diffraction grating intensity distribution.

Question 48 - C

Two physical quantities are conserved in this collision,

$$\text{momentum } \mathbf{p} = m\mathbf{v}/\sqrt{1-v^2/c^2} + m'\mathbf{v}'/\sqrt{1-v'^2/c^2} \text{ and}$$
$$\text{energy } E = mc^2/\sqrt{1-v^2/c^2} + m'c^2/\sqrt{1-v'^2/c^2}.$$

The expression in option C is E/c^3, which is a conserved quantity.
The expression in option A is non-relativistic momentum. This would be conserved classically, but is not conserved at relativistic speeds.
The expression in option B is similar to that of relativistic momentum, but it is erroneous as the Lorentz (gamma) factor is not the same for both masses.
The expression in option D is also similar to that of relativistic momentum,

but it is erroneous as the Lorentz (gamma) factor is incorrectly written.
The expression in option E refers to the relative speed between the masses in the lab frame. This would be conserved classically, by Newton's Law of Restitution, but is not conserved at relativistic speeds.

Question 49 - B

We can consider the options individually.

A. This would mean acceleration and force are infinite.

B. Between the time τ_1 of first contact and the time τ_2 at which the fly begins to travel with the same velocity as the train, the sign of the velocity changes. By the intermediate value theorem, the magnitude of the velocity must be zero at some time between τ_1 and τ_2.

C. In real life, the train does not stop.

D. By conservation of momentum, the speed of the train/fly is lower than before. Hence, the magnitude of the fly's momentum has decreased.

E. If we doubled the mass of the train, the change in the flys momentum and the time scale of the collision will not vary much, so the force exerted will not double.

Question 50 - B

We can evaluate the options individually.

A. The magnetic field at the position of CC only changes at time $t > x/c$, as changes in the electromagnetic field take time to travel across space.

B. There is already a magnetic field (due to CC) present at the position of PC. When the current is turned on in PC, it immediately experiences a magnetic force due to the perpendicular magnetic flux density and current density.

C. Total momentum is still conserved.

D. Refer to option C.

E. Heisenberg's uncertainty principle relates uncertainty in momentum to uncertainty in position. In fact, assuming that the wires are suitably large, quantum effects will not be evident in the experiment.