EXERCISE 5.1 GRADIENT OF A STRAIGHT LINE

2 **(a)** $\tan\theta = \dfrac{6-(-2)}{4-(-4)}$

$= 1$

$\theta = 45°$

(b) $\tan\theta = \dfrac{4-5}{(-2)-0}$

$= \dfrac{1}{2}$

$\theta \approx 26°34'$

(c) $\tan\theta = \dfrac{3-6}{3-(-5)}$

$= -\dfrac{3}{8}$

$\theta \approx 180° - 20°33'$

$\approx 159°27'$

(d) $\tan\theta = \dfrac{(-4)-5}{(-2)-4}$

$= \dfrac{3}{2}$

$\theta \approx 56°19'$

(e) $\tan\theta = \dfrac{a-b}{2b-2a}$

$= \dfrac{-(b-a)}{2(b-a)}$

$= -\dfrac{1}{2}$

$\theta \approx 180° - 26°34'$

$\approx 153°26'$

(f) $\tan\theta = \dfrac{b-c}{c-b}$

$= \dfrac{b-c}{-(b-c)}$

$= -1$

$\theta = 180° - 45°$

$= 135°$

4 **(a)** $m_{AB} = \dfrac{0-0}{3-0}$

$= 0$

$m_{CD} = \dfrac{5-5}{2-5}$

$= 0$

$m_{BC} = \dfrac{5-0}{5-3}$

$= \dfrac{5}{2}$

$m_{AD} = \dfrac{5-0}{2-0}$

$= \dfrac{5}{2}$

Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, two pairs of opposite sides are parallel, hence $ABCD$ is a parallelogram.

(b) $m_{AB} = \dfrac{1-(-1)}{4-(-3)}$

$= \dfrac{2}{7}$

$m_{CD} = \dfrac{3-5}{1-8}$

$= \dfrac{2}{7}$

$m_{BC} = \dfrac{5-1}{8-4}$

$= 1$

$m_{AD} = \dfrac{3-(-1)}{1-(-3)}$

$= 1$

Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, two pairs of opposite sides are parallel, hence $ABCD$ is a parallelogram.

(c) $m_{AB} = \dfrac{6-4}{4-(-1)}$

$= \dfrac{2}{5}$

$m_{CD} = \dfrac{5-7}{(-3)-2}$

$= \dfrac{2}{5}$

$m_{BC} = \dfrac{7-6}{2-4}$

$= -\dfrac{1}{2}$

$m_{AD} = \dfrac{5-4}{(-3)-(-1)}$

$= -\dfrac{1}{2}$

Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, two pairs of opposite sides are parallel, hence $ABCD$ is a parallelogram.

(d) $m_{AB} = \dfrac{2-(-3)}{6-(-2)}$

$= \dfrac{5}{8}$

$m_{CD} = \dfrac{2-7}{0-8}$

$= \dfrac{5}{8}$

$m_{BC} = \dfrac{7-2}{8-6}$

$= \dfrac{5}{2}$

$m_{AD} = \dfrac{2-(-3)}{0-(-2)}$

$= \dfrac{5}{2}$

Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, two pairs of opposite sides are parallel, hence $ABCD$ is a parallelogram.

6 Find the gradient of the line joining the first two points (m_1) and the gradient of the line joining the last two points (m_2).

$m_1 = \dfrac{12-0}{2-(-2)}$

$= 3$

$m_2 = \dfrac{-9-12}{-5-2}$

$= 3$

Since $m_1 = m_2$, the points are collinear.

8 **(a)** $m_{AB}=\dfrac{2-(-3)}{5-2}$

$=\dfrac{5}{3}$

$m_{BC}=\dfrac{0-2}{(-3)-5}$

$=\dfrac{1}{4}$

$m_{AC}=\dfrac{0-(-3)}{(-3)-2}$

$=-\dfrac{3}{5}$

$m_{AB}\times m_{AC}=\dfrac{5}{3}\times\left(-\dfrac{3}{5}\right)$

$=-1$

Hence ABC is a right-angled triangle with $\angle BAC=90^\circ$.

(b) $m_{AB}=\dfrac{4-2}{3-(-1)}$

$=\dfrac{1}{2}$

$m_{BC}=\dfrac{(-4)-4}{7-3}$

$=-2$

$m_{AB}\times m_{BC}=\dfrac{1}{2}\times(-2)$

$=-1$

Hence ABC is a right-angled triangle with $\angle ABC=90^\circ$.

10 The diagonals are AC and BD.

$m_{AC}=\dfrac{(-7)-6}{5-(-8)}$

$=-1$

$m_{BD}=\dfrac{(-3)-4}{(-5)-2}$

$=1$

$m_{AC}\times m_{BD}=(-1)\times 1$

$=-1$

Hence the diagonals of the quadrilateral are perpendicular.

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EXERCISE 5.2 EQUATION OF A STRAIGHT LINE

2 **(a)** $m=\dfrac{(-5)-3}{(-4)-3}$

$=\dfrac{8}{7}$

$$y-3=\frac{8}{7}(x-3)$$
$$7y-21=8x-24$$
$$8x-7y-3=0$$

(b) $m=\dfrac{2-(-8)}{7-2}$

$=2$

$$y-2=2(x-7)$$
$$y-2=2x-14$$
$$2x-y-12=0$$

4 It is parallel to x-axis so it is a horizontal straight line passing through $(5,\ 2)$.

$\therefore y=2$

6 **B**

The x- and y-intercepts are $(2,\ 0)$ and $(0,\ \text{-}5)$

$m=\dfrac{(-5)-0}{0-2}$

$=\dfrac{5}{2}$

$$y-5=\frac{5}{2}(x-0)$$
$$2y-10=5x$$
$$5x-2y-10=0$$

8 **(a)** $2x+3y=4$

$$3y=-2x+4$$
$$y=-\frac{2}{3}x+\frac{4}{3}$$
$$m=-\frac{2}{3}$$

(b) $3x-2y=7$

$$2y=3x-7$$
$$y=\frac{3}{2}x-\frac{7}{2}$$
$$m=\frac{3}{2}$$

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(c) $2y = 6 - 3x$

$$\begin{aligned} 2y &= -3x + 6 \\ y &= -\frac{3}{2}x + 3 \\ m &= -\frac{3}{2} \end{aligned}$$

(d) $5y - 2x = 8$

$$\begin{aligned} 5y &= 2x + 8 \\ y &= \frac{2}{5}x + \frac{8}{5} \\ m &= \frac{2}{5} \end{aligned}$$

10 (a) Substitute $(2,\ 3)$ into $2x + 3y - 13 = 0$.

$$\begin{aligned} LHS &= 2\times 2 + 3\times 3 - 13 \\ &= 0 \\ &= RHS \end{aligned}$$

$\therefore (2,\ 3)$ lies on the line $2x + 3y - 13 = 0$.

(b) Substitute $(-1,\ 2)$ into $2x + 3y - 13 = 0$.

$$\begin{aligned} a\times(-1) - 4\times 2 + 11 &= 0 \\ -a - 8 + 11 &= 0 \\ -a &= -3 \\ a &= 3 \end{aligned}$$

12 (a) To be parallel to $4x - 5y + 3 = 0$, the gradient must be the same.

$$\begin{aligned} 5y &= 4x + 3 \\ y &= \frac{4}{5}x + \frac{3}{5} \\ m &= \frac{4}{5} \end{aligned}$$

The line passes through the origin $(0,\ 0)$.

$$\begin{aligned} y - 0 &= \frac{4}{5}(x - 0) \\ y &= \frac{4}{5}x \\ 5y &= 4x \\ 4x - 5y &= 0 \end{aligned}$$

(b) To be perpendicular to $4x - 5y + 3 = 0$, gradient is the negative reciprocal.

$$\begin{aligned} 4x - 5y + 3 &= 0 \\ -5y &= -4x - 3 \\ y &= \frac{4}{5}x + \frac{3}{5} \\ m_1 &= \frac{4}{5} \end{aligned}$$

$$m_2 = -\frac{5}{4}$$

$$\begin{aligned} y - 0 &= \frac{4}{5}(x - 0) \\ y &= \frac{4}{5}x \\ 5y &= 4x \\ 4x - 5y &= 0 \end{aligned}$$

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14 $2x - y = 5$

$$y = 2x - 5$$
$$m_1 = 2$$

$$m_2 = \frac{9-5}{1-(-1)}$$
$$= 2$$

Since $m_1 = m_2$, the lines are parallel.

16 (a) $m_{AB} = \frac{2-4}{5-1}$

$$= -\frac{1}{2}$$

Equation of AB:

$$y - 2 = -\frac{1}{2}(x-5)$$
$$2y - 4 = -x + 5$$
$$x + 2y - 9 = 0$$

(b) Since $ABCD$ is a rectangle, $AB \perp AD$

$$m_{AB} \times m_{AD} = -1$$
$$-\frac{1}{2} \times m_{AD} = -1$$
$$m_{AD} = 2$$

Equation of AD:

$$y - 4 = 2(x-1)$$
$$y + 4 = 2x - 2$$
$$2x - y + 2 = 0$$

(c) Substitute $x = -2$ into $2x - y + 2 = 0$

$$2 \times (-2) - y + 2 = 0$$
$$-4 - y + 2 = 0$$
$$-y - 2 = 0$$
$$y = -2$$

The y-coordinate of D is -2.

(d) BC is parallel to AD, so their gradient is the same.

$$m_{BC} = 2$$

Equation of BC is:

$$y - 2 = 2(x-5)$$
$$y - 2 = 2x - 10$$
$$2x - y - 8 = 0$$

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(e) DC is parallel to AB, so their gradient is the same.

$$m_{DC} = -\frac{1}{2}$$

Equation of DC is:

$$\begin{aligned} y+2 &= -\frac{1}{2}(x+2) \\ 2y+4 &= -x-2 \\ x+2y+6 &= 0 \end{aligned}$$

(f) BC and DC intersect at C

$$\begin{aligned} 2x-y-8 &= 0 \ldots [1] \\ x+2y+6 &= 0 \ldots [2] \end{aligned}$$

$[2]\times 2$

$$2x+4y+12 = 0 \ldots [3]$$

$[3]-[1]$

$$\begin{aligned} 5y+20 &= 0 \\ y &= -4 \end{aligned}$$

Substitute $y=-4$ into $[2]$

$$\begin{aligned} x+2\times(-4)+6 &= 0 \\ x-2 &= 0 \\ x &= 2 \end{aligned}$$

$\therefore C$ is the point $A(1,\ 5)$.

EXERCISE 5.3 INTERSECTION OF TWO LINES

2

$$\begin{aligned} 2x+5y-19 &= 0 \ldots [1] \\ 3x-4y+6 &= 0 \ldots [2] \end{aligned}$$

$[1]\times 3$

$$6x+15y-57 = 0 \ldots [3]$$

$[2]\times 2$

$$6x-8y+12 = 0 \ldots [4]$$

$[3]-[4]$

$$\begin{aligned} 23y-69 &= 0 \\ y &= 3 \end{aligned}$$

Substitute $y=3$ into $[1]$

$$\begin{aligned} 2x+5\times 3-19 &= 0 \\ 2x+15-19 &= 0 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

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Point of intersection is $(2,\ 3)$

The line is parallel to $4x-y-8=0$, so its gradient is the same.

$$\begin{aligned}4x-y-8&=0\\ y&=4x-8\\ m&=4\end{aligned}$$

Equation of the line:

$$\begin{aligned}y-3&=4(x-2)\\ y-3&=4x-8\\ 4x-y-5&=0\end{aligned}$$

4 (a) $x+y+1=0$

$$\begin{aligned}y&=-x-1\\ m_1&=-1\end{aligned}$$

$$\begin{aligned}y&=2-x\\ y&=-x+2\\ m_2&=-1\end{aligned}$$

$$\begin{aligned}3y&=2x+1\\ y&=\frac{2}{3}x+\frac{1}{3}\\ m_3&=\frac{2}{3}\end{aligned}$$

$$\begin{aligned}2x-3y+6&=0\\ 3y&=2x+6\\ y&=\frac{2}{3}x+2\\ m_4&=\frac{2}{3}\end{aligned}$$

$m_1=m_2$ and $m_3=m_4$

The first two lines are parallel and the last two lines are parallel, so these lines are sides of a parallelogram.

(b) $x+y+1=0\ldots[1]$

$$\begin{aligned}3y&=2x+1\\ y&=\frac{2}{3}x+\frac{1}{3}\ldots[2]\end{aligned}$$

Substitute $[2]$ into $[1]$

$$\begin{aligned}x+\frac{2}{3}x+\frac{1}{3}+1&=0\\ \frac{5}{3}x&=-\frac{4}{3}\\ x&=-\frac{4}{5}\end{aligned}$$

Substitute into $[1]$

$$\begin{aligned}-\frac{4}{5}+y+1&=0\\ y&=-\frac{1}{5}\end{aligned}$$

$$\left(-\frac{4}{5},\ -\frac{1}{5}\right)$$

$$\begin{aligned}y&=2-x\ldots[1]\\ 2x-3y+6&=0\ldots[2]\end{aligned}$$

Substitute $[1]$ into $[2]$

$$\begin{aligned}2x-3(2-x)+6&=0\\ 2x-6+3x+6&=0\\ 5x&=0\\ x&=0\end{aligned}$$

Substitute into $[1]$

$$\begin{aligned}y&=2-0\\ &=2\end{aligned}$$

$$(0,\ 2)$$

$$\begin{aligned}x+y+1&=0\ldots[1]\\ 2x-3y+6&=0\ldots[2]\end{aligned}$$

$[1]\times 2$

$$2x+2y+2=0\ldots[3]$$

$[3]-[2]$

$$5y-4=0$$
$$y=\frac{4}{5}$$

Substitute into $[1]$

$$x+\frac{4}{5}+1=0$$
$$x=-\frac{9}{5}$$

$$\left(-\frac{9}{5},\ \frac{4}{5}\right)$$

$$y=2-x\ldots[1]$$
$$3y=2x+1\ldots[2]$$

Substitute $[1]$ into $[2]$

$$3(2-x)=2x+1$$
$$6-3x=2x+1$$
$$-5x=-5$$
$$x=1$$

Substitute into $[1]$

$$y=2-1$$
$$=1$$

$$(1,\ 1)$$

The four vertices of the parallelogram are: $\left(-\frac{4}{5},\ -\frac{1}{5}\right), \left(-\frac{9}{5},\ \frac{4}{5}\right), (0,\ 2), (1,\ 1)$

(c) $m_1=\dfrac{2-\left(-\frac{1}{5}\right)}{0-\left(-\frac{4}{5}\right)}$

$$=\frac{11}{4}$$

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$$y-2=\frac{11}{4}(x-0)$$
$$4y-8=11x$$
$$11x-4y+8=0$$

$$m_2=\frac{1-\frac{4}{5}}{1-\left(-\frac{9}{5}\right)}$$
$$=\frac{1}{14}$$

$$y-1=\frac{1}{14}(x-1)$$
$$14y-14=x-1$$
$$x-14y+13=0$$

The equations of the diagonals of the parallelograms are: $11x-4y+8=0$ and $x-14y+13=0$.

6 $m_{AB}=\frac{6-4}{4-(-1)}$

$$=\frac{2}{5}$$

$$m_{BC}=\frac{7-6}{2-4}$$
$$=-\frac{1}{2}$$

Since $ABCD$ is a parallelogram, the opposite sides are parallel to each other.

$$m_{CD}=m_{AB}$$
$$m_{AD}=m_{BC}$$

Equation of AD:

$$y-4=-\frac{1}{2}(x-(-1))$$
$$y-4=-\frac{1}{2}x-\frac{1}{2}$$
$$2y-8=-x-1$$
$$x+2y-7=0$$

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Equation of CD:

$$\begin{aligned} y-7&=\frac{2}{5}(x-2) \\ 5y-35&=2x-4 \\ 2x-5y+31&=0 \end{aligned}$$

AD and CD intersect at D:

$$\begin{aligned} x+2y-7&=0\ldots[1] \\ 2x-5y+31&=0\ldots[2] \end{aligned}$$

$[1]\times 2$

$$2x+4y-14=0\ldots[3]$$

$[3]-[2]$

$$\begin{aligned} 9y-45&=0 \\ y&=5 \end{aligned}$$

Substitute into $[1]$

$$\begin{aligned} x+2\times 5-7&=0 \\ x&=-3 \end{aligned}$$

$\therefore D(-3,\ 5)$

(a) correct

(b) correct

(c) incorrect

(d) correct

8 **(a)** Find the gradients of AB and AC.

$$\begin{aligned} 5x+y-10&=0 \\ y&=-5x+10 \\ m_{AB}&=-5 \end{aligned}$$

$$\begin{aligned} 3x-2y-6&=0 \\ 2y&=3x-6 \\ y&=\frac{3}{2}x-3 \\ m_{BC}&=\frac{3}{2} \end{aligned}$$

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$$m_{AB} \times m_{CA} = -5 \times \frac{1}{5}$$
$$= -1$$

$AB \perp CA$

$\therefore \angle BAC$ is a right angle.

(b) AB and CA intersect at A

$$5x + y - 10 = 0 \ldots[1]$$
$$x - 5y + 24 = 0 \ldots[2]$$

$[1] \times 5$

$$25x + 5y - 50 = 0 \ldots[3]$$

$[3] + [2]$

$$26x - 26 = 0$$
$$x = 1$$

Substitute $x = 1$ into $[2]$

$$1 - 5y + 24 = 0$$
$$-5y = -25$$
$$y = 5$$

A is the point $(1,\ 5)$.

Find the gradient of BC.

$$3x - 2y - 6 = 0$$
$$2y = 3x - 6$$
$$y = \frac{3}{2}x - 3$$
$$m_{BC} = \frac{3}{2}$$

The gradient of the line perpendicular to BC is the negative reciprocal.

$$m = -\frac{2}{3}$$

The equation of line perpendicular to BC going through A is

$$\begin{aligned} y-5&=-\frac{2}{3}(x-1) \\ 3y-15&=-2x+2 \\ 2x+3y-17&=0 \end{aligned}$$

Find the intersection of this line and BC.

$$\begin{aligned} 2x+3y-17&=0...[1] \\ 3x-2y-6&=0...[2] \end{aligned}$$

$[1]\times 2$

$$4x+6y-34=0...[3]$$

$[2]\times 3$

$$9x-6y-18=0...[4]$$

$[3]+[4]$

$$\begin{aligned} 13x-52&=0 \\ x&=4 \end{aligned}$$

Substitute $x=4$ into [1]

$$\begin{aligned} 2\times 4+3y-17&=0 \\ 8+3y-17&=0 \\ 3y&=9 \\ y&=3 \end{aligned}$$

The coordinates of the foot of the perpendicular from A to BC are $(4,\ 3)$.

EXERCISE 5.4 SIMULTANEOUS EQUATIONS

2 $x+5y=34...[1]$

$x-5y=-6...[2]$

$[1]+[2]$

$$\begin{aligned} 2x&=28 \\ x&=14 \end{aligned}$$

Substitute into [1]

$$
\begin{aligned}
14+5y &= 34 \\
5y &= 20 \\
y &= 4
\end{aligned}
$$

$\therefore x = 14,\ y = 4$

4 $3x - y = 5 \ldots [1]$

$5x + 3y = -8 \ldots [2]$

$[1] \times 3$

$9x - 3y = 15 \ldots [3]$

$[3] + [2]$

$$
\begin{aligned}
14x &= 7 \\
x &= \frac{1}{2}
\end{aligned}
$$

Substitute into $[1]$

$$
\begin{aligned}
3 \times \frac{1}{2} - y &= 5 \\
-y &= \frac{7}{2} \\
y &= -\frac{7}{2}
\end{aligned}
$$

$\therefore x = \frac{1}{2},\ y = -\frac{7}{2}$

6 $-2x + 7y = 4 \ldots [1]$

$-3x + 5y = -5 \ldots [2]$

$[1] \times 3$

$-6x + 21y = 12 \ldots [3]$

$[2] \times 2$

$-6x + 10y = -10 \ldots [4]$

$[3] - [4]$

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$$\begin{aligned} 11y &= 22 \\ y &= 2 \end{aligned}$$

Substitute into $[1]$

$$\begin{aligned} -2x + 7 \times 2 &= 4 \\ -2x &= -10 \\ x &= 5 \end{aligned}$$

$\therefore x = 5,\ y = 2$

8 $5x + 2y = 9 \ldots [1]$

$9x - 7y = -5 \ldots [2]$

$[1] \times 7$

$35x + 14y = 63 \ldots [3]$

$[2] \times 2$

$18x - 14y = -10 \ldots [4]$

$[3] + [4]$

$$\begin{aligned} 53x &= 53 \\ x &= 1 \end{aligned}$$

Substitute into $[1]$

$$\begin{aligned} 5 \times 1 + 2y &= 9 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

$\therefore x = 1,\ y = 2$

10 $2x + 5y = 16 \ldots [1]$

$10x - 3y = -4 \ldots [2]$

$[1] \times 5$

$10x + 25y = 80 \ldots [3]$

$[3] - [2]$

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$$\begin{aligned} 28y &= 84 \\ y &= 3 \end{aligned}$$

Substitute into [1]

$$\begin{aligned} 2x + 5\times 3 &= 16 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

$$\therefore x = \frac{1}{2},\ y = 3$$

12 $5m - 6n = 12 \ldots [1]$

$2m + 9n = 20 \ldots [2]$

$[1]\times 2$

$10m - 12n = 24 \ldots [3]$

$[2]\times 5$

$10m + 45n = 100 \ldots [4]$

$[4]-[3]$

$$\begin{aligned} 57n &= 76 \\ y &= \frac{76}{57} \\ &= \frac{4}{3} \end{aligned}$$

Substitute into [1]

$$\begin{aligned} 5m - 6\times\frac{4}{3} &= 12 \\ 5m &= 20 \\ m &= 4 \end{aligned}$$

$$\therefore m = 4,\ n = \frac{4}{3}$$

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14 $x-4=4(y+2)$

$$\begin{aligned} x-4&=4y+8 \\ x-4y&=12\ldots[1] \\ 3(x-2)&=2y+20 \\ 3x-6&=2y+20 \\ 3x-2y&=26\ldots[2] \end{aligned}$$

$[1]\times 3$

$3x-12y=36\ldots[3]$

$[2]-[3]$

$$\begin{aligned} 10y&=-10 \\ y&=-1 \end{aligned}$$

Substitute into $[1]$

$$\begin{aligned} x-4\times(-1)&=12 \\ x+4&=12 \\ x&=8 \end{aligned}$$

$\therefore x=8,\ y=-1$

16 $2x-\dfrac{y}{4}=5\ldots[1]$

$x+\dfrac{3y}{4}=-1\ldots[2]$

$[2]\times 2$

$2x+\dfrac{3y}{2}=-2\ldots[3]$

$[3]-[1]$

$$\begin{aligned} \frac{7y}{4}&=-7 \\ y&=-4 \end{aligned}$$

Substitute into $[2]$

$$x + \frac{3 \times (-4)}{4} = -1$$
$$x = 2$$

$$\therefore x = 2,\ y = -4$$

18 $$\frac{x-3}{2} = \frac{2y+1}{3}$$

$$3x - 9 = 4y + 2$$
$$3x - 4y = 11 \ldots [1]$$
$$\frac{3x-1}{5} - \frac{2y+1}{2} = 1$$
$$6x - 2 - 10y - 5 = 10$$
$$6x - 10y = 17 \ldots [2]$$

$[1] \times 2$

$$6x - 8y = 22 \ldots [3]$$

$[3] - [2]$

$$2y = 5$$
$$y = \frac{5}{2}$$

Substitute into $[1]$

$$3x - 4 \times \frac{5}{2} = 11$$
$$3x = 21$$
$$x = 7$$

$$\therefore x = 7,\ y = \frac{5}{2}$$

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20 $2(3a-b)=3(a+b)$

$$\begin{aligned}6a-2b&=3a+3b\\3a-5b&=0\ldots[1]\end{aligned}$$

$$\begin{aligned}3(a-4b)+46&=5a\\3a-12b+46&=5a\\-2a-12b&=-46\\a+6b&=23\ldots[2]\end{aligned}$$

$[2]\times 3$

$3a+18b=69\ldots[3]$

$[3]-[1]$

$$\begin{aligned}23b&=69\\b&=3\end{aligned}$$

Substitute into $[2]$

$$\begin{aligned}a+6\times 3&=23\\a&=5\end{aligned}$$

$\therefore a=5,\ b=3$

EXERCISE 5.5 PROBLEM SOLVING WITH SIMULTANEOUS EQUATIONS

2 Let the number of trucks carry a load of 10 tonnes be x, and the number of trucks carry a load of 5 tonnes be y. The total number of trucks is $x+y$, which is 8, and the load carried by the 10 tonne trucks is $x\times 10$, and the load carried by the 5 tonne trucks is $y\times 5$.

$$\begin{aligned}x+y&=8\ldots[1]\\10x+5y&=70\ldots[2]\end{aligned}$$

$[1]\times 10$

$10x+10y=80\ldots[3]$

$[3]-[2]$

$$\begin{aligned}5y&=10\\y&=2\end{aligned}$$

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Substitute into [1]

$$\begin{aligned} x+2&=8 \\ x&=6 \end{aligned}$$

$\therefore x=6,\ y=2$

The contractor owns six 10 tonne trucks and two 5 tonne trucks

4 Let John's age now be x, and John's mother's age now be y.

This means $y=5x$...[1]

Three years ago, John's age was $x-3$, and John's mother's age was $y-3$, which was nine times John's age then.

$$\begin{aligned} y-3&=9(x-3) \\ y-3&=9x-27 \\ 9x-y&=24\text{...[2]} \end{aligned}$$

Substitute [1] into [2]

$$\begin{aligned} 9x-5x&=24 \\ 4x&=24 \\ x&=6 \end{aligned}$$

Substitute into [1]

$$\begin{aligned} y&=5\times 6 \\ &=30 \end{aligned}$$

John is now 6 and his mother is 30.

6 From $(2,\ 2)$, we get $2a+2b=12\text{...[1]}$.

From $(-4,\ 5)$, we get $-4a+5b=12\text{...[2]}$.

$[1]\times -2$

$-4a-4b=-24\text{...[3]}$

$[2]-[3]$

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$$\begin{aligned}9b &= 36\\ b &= 4\end{aligned}$$

Substitute into $[1]$

$$\begin{aligned}2a+2\times 4 &= 12\\ 2a &= 4\\ a &= 2\end{aligned}$$

$\therefore a=2, b=4$

The equation of the line is $2x+4y=12$, which can be simplified to $x+2y=6$.

8 Let the first number be x and the second number be y.

$$\begin{aligned}x+18 &= 2y\\ x-2y &= -18...[1]\\ 3x &= y+6\\ 3x-y &= 6...[2]\end{aligned}$$

$[2]\times 2$

$6x-2y=12...[3]$

$[3]-[1]$

$$\begin{aligned}5x &= 30\\ x &= 6\end{aligned}$$

Substitute into $[1]$

$$\begin{aligned}6-2y &= -18\\ -2y &= -24\\ y &= 12\end{aligned}$$

$\therefore x=6, y=12$

The first number is 6 and the second number is 12.

10 (a) It passes through $(4,\ 1)$, so $1=4m+c...[1]$

It passes through $(-1,\ -9)$, so $-9=-m+c...[2]$

$[1]-[2]$

$10 = 5m$
$m = 2$

Substitute into $[1]$

$1 = 4 \times 2 + c$
$c = -7$

$\therefore y = 2x - 7$

(b) It passes through $(0,\ 4)$, so $4 = 0 + c ... [1]$

$c = 4$

It passes through $[1,\ 0]$, so $0 = m + c ... [2]$

Substitute into $[2]$

$0 = m + 4$
$m = -4$

$\therefore y = -4x + 4$

(c) It passes through $(2,\ -1.5)$, so $-1.5 = 2m + c ... [1]$

It passes through $(-4,\ -6)$, so $-6 = -4m + c ... [2]$

$[1] - [2]$

$\frac{9}{2} = 6m$

$m = \frac{3}{4}$

Substitute into $[1]$

$-1.5 = 2 \times \frac{3}{4} + c$
$c = -3$

$\therefore y = \frac{3}{4}x - 3$

(d) It passes through $(2,\ 4)$, so $4 = 2m + c ... [1]$

It passes through $(-6,\ 8)$, so $8 = -6m + c ... [2]$

$[1] - [2]$

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$-4 = 8m$

$m = -\frac{1}{2}$

Substitute into [1]

$4 = 2 \times \left(-\frac{1}{2}\right) + c$

$c = 5$

$\therefore y = -\frac{1}{2}x + 5$

12 (a) It will cost him $\$2$ per cup each day, as well as $\$80$.

$C = 2x + 80$

His revenue is $\$4$ per cup multiplied by the number of cups he sells.

$R = 4x$

(b) The break-even point occurs when cost and revenue are equal.

$$\begin{aligned} 2x + 80 &= 4x \\ 80 &= 2x \\ x &= 40 \end{aligned}$$

40 cups per day

(c) His profit is found by subtracting his costs from his revenue.

$$\begin{aligned} P &= R - C \\ &= 4x - (2x + 80) \\ &= 4x - 2x - 80 \\ &= 2x - 80 \end{aligned}$$

(d) $x = 100$

$$\begin{aligned} P &= 2x - 80 \\ &= 2 \times 100 - 80 \\ &= 200 - 80 \\ &= 120 \end{aligned}$$

$\$120$ profit

EXERCISE 5.6 SOLVING SIMULTANEOUS EQUATIONS—LINEAR AND SECOND DEGREE

2 $y = 3x - 2 ...[1]$

$y = x^2 ...[2]$

Equating $[1]$ and $[2]$

$$\begin{aligned} 3x - 2 &= x^2 \\ x^2 - 3x + 2 &= 0 \\ (x-1)(x-2) &= 0 \\ x &= 1,\ x = 2 \end{aligned}$$

Substitute $x = 1$ into $[2]$

$$\begin{aligned} y &= 1^2 \\ &= 1 \end{aligned}$$

Substitute $x = 2$ into $[2]$

$$\begin{aligned} y &= 2^2 \\ &= 4 \end{aligned}$$

$\therefore x = 1,\ y = 1;\ x = 2,\ y = 4$

4 $x + y = 15 ...[1]$

$y = x^2 - 6x + 1 ...[2]$

Substitute $[2]$ into $[1]$

$$\begin{aligned} x + x^2 - 6x + 1 &= 15 \\ x^2 - 5x - 14 &= 0 \\ (x+2)(x-7) &= 0 \\ x &= -2,\ x = 7 \end{aligned}$$

Substitute $x = -2$ into [1]

$$\begin{aligned} -2 + y &= 15 \\ y &= 17 \end{aligned}$$

Substitute $x = 7$ into [1]

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$$7+y=15$$
$$y=8$$

$$\therefore x=-2,\ y=17;\ x=7,\ y=8$$

6 $y-2x=1$

$$y=2x+1\ldots[1]$$
$$x^2+y^2=10\ldots[2]$$

Substitute [1] into [2]

$$x^2+(2x+1)^2=10$$
$$x^2+4x^2+4x+1=10$$
$$5x^2+4x-9=0$$
$$(5x+9)(x-1)=0$$
$$x=-\frac{9}{5},\ x=1$$

Substitute $x=-\frac{9}{5}$ into [1]

$$y=2\times\left(-\frac{9}{5}\right)+1$$
$$=-\frac{13}{5}$$

Substitute $x=1$ into [1]

$$y=2\times1+1$$
$$=3$$

$$\therefore x=-\frac{9}{5},\ y=\frac{13}{5};\ x=1,\ y=3$$

8 $2x-y=2\ldots[1]$

$$y=x^2-x-2\ldots[2]$$

Substitute [1] into [2]

$$\begin{aligned} 2x-\left(x^2-x-2\right)&=2 \\ 2x-x^2+x+2&=2 \\ x^2-3x&=0 \\ x(x-3)&=0 \\ x&=0,\ x=3 \end{aligned}$$

Substitute $x=0$ into [1]

$$\begin{aligned} 2\times0-y&=2 \\ 0-y&=2 \\ y&=-2 \end{aligned}$$

Substitute $x=3$ into [1]

$$\begin{aligned} 2\times3-y&=2 \\ 6-y&=2 \\ y&=4 \end{aligned}$$

$\therefore x=0,\ y=-2;\ x=3,\ y=4$

10 $y=2x-5 \ldots [1]$

$y=x^2-4x+4 \ldots [2]$

Equating [1] and [2]

$$\begin{aligned} 2x-5&=x^2-4x+4 \\ x^2-6x+9&=0 \\ (x-3)^2&=0 \\ x&=3 \end{aligned}$$

Substitute $x=3$ into [1]

$$\begin{aligned} y&=2\times3-5 \\ &=1 \end{aligned}$$

$\therefore x=3,\ y=1$

12 $x+y=5$

$$\begin{aligned} y&=5-x \ldots [1] \\ 3x^2+xy-y^2&=29 \ldots [2] \end{aligned}$$

Substitute [1] into [2]

$$\begin{aligned}3x^2+x(5-x)-(5-x)^2&=29\\3x^2+5x-x^2-\left(25-10x+x^2\right)&=29\\3x^2+5x-x^2-25+10x-x^2&=29\\x^2+15x-54&=0\\(x+18)(x-3)&=0\\x&=-18,\ x=3\end{aligned}$$

Substitute $x=-18$ into [1]

$$\begin{aligned}y&=5-(-18)\\&=23\end{aligned}$$

Substitute $x=3$ into [1]

$$\begin{aligned}y&=5-3\\&=2\end{aligned}$$

$\therefore x=-18, y=23; x=3, y=2$

14 $y-4x-8=0\ldots[1]$

$$y=4-x^2\ldots[2]$$

Substitute [2] into [1]

$$\begin{aligned}\left(4-x^2\right)-4x-8&=0\\-x^2-4x-4&=0\\x^2+4x+4&=0\\(x+2)^2&=0\\x&=-2\end{aligned}$$

Substitute $x=-2$ into [2]

$$\begin{aligned}y&=4-(-2)^2\\&=0\end{aligned}$$

$\therefore x=-2,\ y=0$

16 $3x+y=11$

$$y=11-3x\ldots[1]$$

$$2x^2-xy-y=10\ldots[2]$$

Substitute [1] into [2]

$$\begin{aligned}2x^2-x(11-3x)-(11-3x)&=10\\2x^2-11x+3x^2-11+3x&=10\\5x^2-8x-21&=0\\(5x+7)(x-3)&=0\\x&=-\frac{7}{5},\ x=3\end{aligned}$$

Substitute $x=-\frac{7}{5}$ into [1]

$$\begin{aligned}y&=11-3\times\left(-\frac{7}{5}\right)\\&=\frac{76}{5}\end{aligned}$$

Substitute $x=3$ into [1]

$$\begin{aligned}y&=11-3\times3\\&=2\end{aligned}$$

$$\therefore x=-\frac{7}{5},\ y=\frac{76}{5};\ x=3,\ y=2$$

18 $x=2y-1\ldots[1]$

$$3x^2=x+2y^2\ldots[2]$$

Substitute [1] into [2]

$$
\begin{aligned}
3(2y-1)^2 &= (2y-1)+2y^2 \\
3(4y^2-4y+1) &= 2y-1+2y^2 \\
12y^2-12y+3-2y+1-2y^2 &= 0 \\
10y^2-14y+4 &= 0 \\
5y^2-7y+2 &= 0 \\
(5y-2)(y-1) &= 0 \\
y &= \frac{2}{5},\ y=1
\end{aligned}
$$

Substitute $y=\frac{2}{5}$ into [1]

$$
\begin{aligned}
x &= 2\times\frac{2}{5}-1 \\
&= -\frac{1}{5}
\end{aligned}
$$

Substitute $y=1$ into [1]

$$
\begin{aligned}
x &= 2\times1-1 \\
&= 1
\end{aligned}
$$

$$\therefore x=-\frac{1}{5},\ y=\frac{2}{5};\ x=1,\ y=1$$

20 $y=x+9\ldots[1]$

$y=x^2-x-6\ldots[2]$

Equating [1] and [2]

$$
\begin{aligned}
x+9 &= x^2-x-6 \\
x^2-2x-15 &= 0 \\
(x+3)(x-5) &= 0 \\
x &= -3,\ x=5
\end{aligned}
$$

Substitute $x=-3$ into [1]

$$
\begin{aligned}
y &= -3+9 \\
&= 6
\end{aligned}
$$

Substitute $x=5$ into [1]

$$y = 5 + 9$$
$$= 14$$

$$\therefore x = -3,\ y = 6;\ x = 5,\ y = 14$$

EXERCISE 5.7 SOLVING SIMULTANEOUS EQUATIONS—LINEAR AND SECOND DEGREE IN THE GENERAL FORM

2 $3y - 4x = 0$

$$3y = 4x$$
$$y = \frac{4x}{3} \text{...[1]}$$
$$x^2 + y^2 = 25\text{...[2]}$$

Substitute [1] into [2]

$$x^2 + \left(\frac{4x}{3}\right)^2 = 25$$
$$x^2 + \frac{16x^2}{9} = 25$$
$$9x^2 + 16x^2 = 225$$
$$25x^2 = 225$$
$$x^2 = 9$$
$$x = \pm 3$$

Substitute $x = -3$ into [1]

$$y = \frac{4 \times (-3)}{3}$$
$$= -4$$

Substitute $x = 3$ into [1]

$$y = \frac{4 \times 3}{3}$$
$$= 4$$

$$\therefore x = -3,\ y = -4;\ x = 3,\ y = 4$$

4 $3x-2y=2$

$$2y=3x-2$$
$$y=\frac{3x-2}{2}...[1]$$
$$x^2-xy+y^2=21...[2]$$

Substitute [1]into [2]

$$x^2-x\left(\frac{3x-2}{2}\right)+\left(\frac{3x-2}{2}\right)^2=21$$
$$x^2-\frac{3x^2-2x}{2}+\frac{9x^2-12x+4}{4}=21$$
$$4x^2-2\left(3x^2-2x\right)+9x^2-12x+4=84$$
$$4x^2-6x^2+4x+9x^2-12x+4=84$$
$$7x^2-8x-80=0$$
$$(7x+20)(x-4)=0$$
$$x=-\frac{20}{7},\ x=4$$

Substitute $x=-\frac{20}{7}$ into [1]

$$y=\frac{3\times\left(-\frac{20}{7}\right)-2}{2}$$
$$=-\frac{37}{7}$$

Substitute $x=4$ into [1]

$$y=\frac{3\times4-2}{2}$$
$$=5$$
$$\therefore x=-\frac{20}{7},\ y=-\frac{37}{7};\ x=4,\ y=5$$

6 $2x=3y+1$

$$x=\frac{3y+1}{2}...[1]$$
$$xy+x+y=23...[2]$$

Substitute [1] into [2]

$$y\left(\frac{3y+1}{2}\right)+\frac{3y+1}{2}+y=23$$
$$3y^2+y+3y+1+2y=46$$
$$3y^2+6y-45=0$$
$$y^2+2y-15=0$$
$$(y+5)(y-3)=0$$
$$y=-5,\ y=3$$

Substitute $y=-5$ into [1]

$$x=\frac{3\times(-5)+1}{2}$$
$$=-7$$

Substitute $y=3$ into [1]

$$x=\frac{3\times3+1}{2}$$
$$=5$$

$\therefore x=-7,\ y=-5;\ x=3,\ y=5$

EXERCISE 5.8 QUADRATIC FUNCTIONS

2 **(a)** $y=2x^2-4x$

$$=2\left(x^2-2x\right)$$
$$=2\left(x^2-2x+1\right)-2$$
$$=2\left(x-1\right)^2-2$$

$a>0$, so this parabola is concave up, with a minimum value of -2.

Range: $y\geq-2$

(b) $y=-2x^2+8x-3$

$$=-2\left(x^2-4x\right)-3$$
$$=-2\left(x^2-4x+4\right)-3+8$$
$$=-2\left(x-2\right)^2+5$$

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$a < 0$, so this parabola is concave down, with a maximum value of 5.

Range: $y \le 5$

(c) $y = 7 + 16x - 4x^2$

$$
\begin{aligned}
&= -4\left(x^2 - 4x\right) + 7 \\
&= -4\left(x^2 - 4x + 4\right) + 7 + 16 \\
&= -4\left(x - 2\right)^2 + 23
\end{aligned}
$$

$a < 0$, so this parabola is concave down, with a maximum value of 23.

Range: $y \le 23$

(d) $y = 4x^2 + 8x - 7$

$$
\begin{aligned}
&= 4\left(x^2 + 2x\right) - 7 \\
&= 4\left(x^2 + 2x + 1\right) - 7 - 4 \\
&= 4\left(x - 1\right)^2 - 11
\end{aligned}
$$

$a > 0$, so this parabola is concave up, with a minimum value of -11.

Range: $y \ge -11$

(e) $y = 8 - 2x^2$

$$= -2x^2 + 8$$

$a < 0$, so this parabola is concave down, with a maximum value of 8.

Range: $y \le 8$

(f) $y = 7 - 2x - x^2$

$$
\begin{aligned}
&= -\left(x^2 + 2x\right) + 7 \\
&= -\left(x^2 + 2x + 1\right) + 7 + 1 \\
&= -\left(x + 1\right)^2 + 8
\end{aligned}
$$

$a < 0$, so this parabola is concave down, with a maximum value of 8.

Range: $y \le 8$

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(g) $y = 2x^2 - 6x$

$$
\begin{aligned}
&= 2\left(x^2 - 3x\right) \\
&= 2\left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{2} \\
&= 2\left(x - \frac{3}{2}\right)^2 - \frac{9}{2}
\end{aligned}
$$

$a > 0$, so this parabola is concave up, with a minimum value of $-\frac{9}{2}$.

(h) $y = 6 - 10x - 5x^2$

$$
\begin{aligned}
&= -5\left(x^2 + 2x\right) + 6 \\
&= -5\left(x^2 + 2x + 1\right) + 6 + 5 \\
&= -5\left(x + 1\right)^2 + 11
\end{aligned}
$$

$a < 0$, so this parabola is concave down, with a maximum value of 11.

Range: $y \le 11$

4 **(a)** $t \ge 0$ and $t = 0$ when the stone is thrown.

The equation of motion only applies while the stone is in the air.

$$
\begin{aligned}
&20t - 5t^2 \ge 0 \\
&5t\left(4 - t\right) \ge 0 \\
&0 \le t \le 4
\end{aligned}
$$

(b) $h\left(t\right) = 20t - 5t^2$

$$
\begin{aligned}
&= -5\left(t^2 - 4t + 4\right) + 20 \\
&= -5\left(t - 2\right)^2 + 20
\end{aligned}
$$

$a < 0$, so this parabola is concave down, with a maximum value of 20.

Hence the greatest height reached is 20 m.

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6 Let the two numbers be x and y

$$\begin{aligned} x + y &= 20 \\ y &= 20 - x \\ P &= xy \\ &= x(20 - x) \\ &= -x^2 + 20x \\ &= -\left(x^2 - 20x + 100\right) + 100 \\ &= -(x - 10)^2 + 100 \end{aligned}$$

$a < 0$, so this parabola is concave down, with a maximum value of 100.

$$\begin{aligned} -(x-10)^2 + 100 &= 100 \\ -(x-10)^2 &= 0 \\ x &= 10 \\ x + y &= 20 \\ y &= 10 \end{aligned}$$

Hence 100 is the maximum product and the two numbers are 10 and 10.

8 Let the two sides of the rectangle be x and y.

$$\begin{aligned} 2x + y &= 20 \\ y &= 20 - 2x \\ A &= xy \\ &= x(20 - 2x) \\ &= -2x^2 + 20x \\ &= -2\left(x^2 - 10x + 25\right) + 50 \\ &= -2(x - 5)^2 + 50 \end{aligned}$$

$a < 0$, so this parabola is concave down, with a maximum value of 50.

$$\begin{aligned} -2(x-5)^2 + 50 &= 50 \\ -2(x-5)^2 &= 0 \\ x &= 5 \\ y &= 20 - 2\times 5 \\ &= 10 \end{aligned}$$

Hence the maximum area is $50\ \text{m}^2$ and the two sides are 5 m and 10 m.

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10 Let the length of the rectangle be x and the breadth of the rectangle be y.

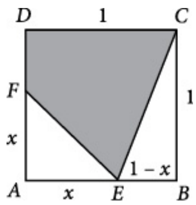
There will be three fences parallel to its length, each of length x m and there will be four fences parallel to its width, each of length y m.

$$\begin{aligned}
3x+4y&=1200\\
y&=\frac{1200-3x}{4}\\
A&=xy\\
&=x\left(\frac{1200-3x}{4}\right)\\
&=300x-\frac{3}{4}x^2\\
&=-\frac{3}{4}\left(x^2-400x\right)\\
&=-\frac{3}{4}\left(x^2-400x+40\ 000\right)+30\ 000
\end{aligned}$$

$a<0$, so this parabola is concave down, with a maximum value of $30\ 000$.

The maximum possible area is $30\ 000\ \text{m}^2$.

12



Area of $CDFE$ is the area of the square $ABCD$ minus the area of the triangles AFE and EBC.

$$\begin{aligned}
y&=1\times1-\frac{1}{2}\times x\times x-\frac{1}{2}\times(1-x)\times1\\
&=1-\frac{x^2}{2}-\frac{1-x}{2}\\
&=\frac{2-x^2-1+x}{2}\\
&=\frac{1+x-x^2}{2}\\
&=\frac{1}{2}\left(1+x-x^2\right)
\end{aligned}$$

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$$\begin{aligned} y &= \frac{1}{2}\left(1+x-x^2\right) \\ &= -\frac{1}{2}\left(x^2-x-1\right) \\ &= -\frac{1}{2}\left(x^2-x\right)+\frac{1}{2} \\ &= -\frac{1}{2}\left(x^2-x+\frac{1}{4}\right)+\frac{1}{2}+\frac{1}{8} \\ &= -\frac{1}{2}\left(x^2-x+\frac{1}{4}\right)+\frac{5}{8} \end{aligned}$$

$a<0$, so this parabola is concave down, with a maximum value of $\frac{5}{8}$.

The quadrilateral's greatest possible area is $\frac{5}{8}$.

EXERCISE 5.9 PARABOLAS AND DISCRIMINANTS

2 (a) $x^2+6x+2=0$

$$\begin{aligned} \Delta &= 6^2-4\times1\times2 \\ &= 28 \end{aligned}$$

$\Delta>0$, there are two real roots.

(b) $2x^2+3x+4=0$

$$\begin{aligned} \Delta &= 3^2-4\times2\times4 \\ &= -23 \end{aligned}$$

$\Delta<0$, there are no real roots.

(c) $4x^2-12x+9=0$

$$\begin{aligned} \Delta &= (-12)^2-4\times4\times9 \\ &= 0 \end{aligned}$$

$\Delta=0$, there is one real root.

(d) $-3x^2+2x-1=0$

$$\begin{aligned} \Delta &= 2^2-4\times(-3)\times(-1) \\ &= -8 \end{aligned}$$

$\Delta<0$, there are no real roots.

(e) $2x^2=3x+7$

$2x^2-3x-7=0$

$$\begin{aligned} \Delta &= (-3)^2-4\times2\times(-7) \\ &= 65 \end{aligned}$$

$\Delta>0$, there are two real roots.

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4 (a) $y = x^2 - 5x + 2$

It crosses the x-axis when $y = 0$

$x^2 - 5x + 2 = 0$

$$\begin{aligned}\Delta &= (-5)^2 - 4 \times 1 \times 2 \\ &= 17\end{aligned}$$

$\Delta > 0$, so there are two real roots. Hence it does cross the x-axis (twice).

(b) $y = -4x^2 + 2x - 1$

It crosses the x-axis when $y = 0$

$-4x^2 + 2x - 1 = 0$

$$\begin{aligned}\Delta &= 2^2 - 4 \times (-4) \times (-1) \\ &= -12\end{aligned}$$

$\Delta < 0$, so there are no real roots. Hence it does not cross the x-axis.

(c) $y = x^2 - 6x + 9$

It crosses the x-axis when $y = 0$

$x^2 - 6x + 9 = 0$

$$\begin{aligned}\Delta &= (-6)^2 - 4 \times 1 \times 9 \\ &= 0\end{aligned}$$

$\Delta = 0$, so there is one real root. Hence it touches the x-axis.

(d) $y = 8 - 3x - 2x^2$

It crosses the x-axis when $y = 0$

$8 - 3x - 2x^2 = 0$
$-2x^2 - 3x + 8 = 0$

$$\begin{aligned}\Delta &= (-3)^2 - 4 \times (-2) \times 8 \\ &= 73\end{aligned}$$

$\Delta > 0$, so there are two real roots. Hence it does cross the x-axis.

(e) $y = 3x^2 + 2x + 5$

It crosses the x-axis when $y = 0$

$3x^2 + 2x + 5 = 0$

$$\begin{aligned}\Delta &= 2^2 - 4 \times 3 \times 5 \\ &= -56\end{aligned}$$

$\Delta < 0$, so there are no real roots. Hence it does not cross the x-axis.

(f) $y = -x^2 - x - 1$

It crosses the x-axis when $y = 0$

$-x^2 - x - 1 = 0$

$$\begin{aligned}\Delta &= (-1)^2 - 4 \times (-1) \times (-1) \\ &= -3\end{aligned}$$

$\Delta < 0$, so there are no real roots. Hence it does not cross the x-axis.

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EXERCISE 5.10 FURTHER EXAMPLES INVOLVING DISCRIMINANTS

2 A

$(2k-3)x^2+(k+1)x-1=0$ has two real unique roots when $\Delta > 0$

$$\begin{aligned}\Delta &= (k+1)^2-4\times(2k-3)\times(-1)\\ &= k^2+2k+1+8k-12\\ &= k^2+10k-11\\ &= (k+11)(k-1)\end{aligned}$$

$(k+11)(k-1)>0$
$k<-11$ or $k>1$

4 (a) $x^2+2x+m^2-1=0$ has real roots when $\Delta \geq 0$

$$\begin{aligned}\Delta &= 2^2-4\times1\times(m^2-1)\\ &= 4-4m^2+4\\ &= 8-4m^2\end{aligned}$$

$$\begin{aligned}8-4m^2 &\geq 0\\ 4(2-m^2) &\geq 0\\ -\sqrt{2} \leq m &\leq \sqrt{2}\end{aligned}$$

(b) $(m-1)x^2+(m+1)x+m-1=0$ has real roots when $\Delta \geq 0$

$$\begin{aligned}\Delta &= (m+1)^2-4\times(m-1)\times(m-1)\\ &= m^2+2m+1-4(m^2-2m+1)\\ &= -3m^2+10m-3\end{aligned}$$

$$\begin{aligned}-3m^2+10m-3 &\geq 0\\ 3m^2-10m+3 &\leq 0\\ (3m-1)(m-3) &\leq 0\\ \frac{1}{3} \leq m &\leq 3\end{aligned}$$

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(c) $x^2+2mx+2(m+12)=0$ has real roots when $\Delta \geq 0$

$$\begin{aligned}\Delta &= (2m)^2-4\times1\times2(m+12)\\ &= 4m^2-8m-96\end{aligned}$$

$$\begin{aligned}4m^2-8m-96 &\geq 0\\ 4(m^2-2m-24) &\geq 0\\ 4(m+4)(m-6) &\geq 0\end{aligned}$$

$m \leq -4$ or $m \geq 6$

6 $4(m+1)x^2-4(m-1)x-3=0$ has real roots when $\Delta \geq 0$

$$\begin{aligned}\Delta &= \left[-4(m-1)\right]^2-4\times4(m+1)\times(-3)\\ &= 16(m-1)^2+48m+48\\ &= 16(m^2-2m+1)+48m+48\\ &= 16m^2-32m+16+48m+48\\ &= 16m^2+16m+64\\ &= 16\left(m^2+m+\frac{1}{4}\right)+64-4\\ &= 16\left(m+\frac{1}{2}\right)^2+60\end{aligned}$$

$\therefore \Delta > 0$ for all values of m.

So $4(m+1)x^2-4(m-1)x-3=0$ has real roots for all real m, although it is not a quadratic when $m=-1$ (but still has one root).

8 $x^2-(2a+b)x+ab=0$

$$\begin{aligned}\Delta &= \left[-(2a+b)\right]^2-4\times1\times ab\\ &= 4a^2+4ab+b^2-4ab\\ &= 4a^2+b^2\end{aligned}$$

$a^2 \geq 0$ and $b^2 \geq 0$ for all real values of a and b, so $4a^2+b^2 \geq 0$

Since $\Delta \geq 0$, $x^2-(2a+b)x+ab=0$ has real roots for all real values of a and b.

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10 (a) $x^2 - 2mx + 8m - 15 = 0$

$$\begin{aligned}\Delta &= (-2m)^2 - 4\times 1\times (8m-15)\\ &= 4m^2 - 32m + 60\\ &= 4(m^2 - 8m + 15)\\ &= 4(m-3)(m-5)\end{aligned}$$

For one unique root, $\Delta = 0$

$$4(m-3)(m-5) = 0$$
$$m = 3 \text{ or } m = 5$$

(b) For two roots, $\Delta > 0$

$$4(m-3)(m-5) > 0$$
$$m < 3 \text{ or } m > 5$$

12 $x^2 + mx + (m+1)^2 = 0$

$$\begin{aligned}\Delta &= m^2 - 4\times 1\times (m+1)^2\\ &= m^2 - 4(m^2 + 2m + 1)\\ &= m^2 - 4m^2 - 8m - 4\\ &= -3m^2 - 8m - 4\\ &= -(3m^2 + 8m + 4)\\ &= -(3m+2)(m+2)\end{aligned}$$

For two roots, $\Delta > 0$.

$$-(3m+2)(m+2) > 0$$
$$(3m+2)(m+2) < 0$$

$$-2 < m < -\frac{2}{3}$$

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EXERCISE 5.11 SOLUTION SET OF SIMULTANEOUS EQUATIONS

2 **(a)** $y = x - 3 \ldots [1]$

$x^2 + y^2 = 9 \ldots [2]$

Substitute [1] into [2]

$$\begin{aligned} x^2 + (x-3)^2 &= 9 \\ x^2 + x^2 - 6x + 9 &= 9 \\ 2x^2 - 6x &= 0 \\ 2x(x-3) &= 0 \\ x &= 0, x = 3 \end{aligned}$$

Substitute $x = 0$ into [1]

$$\begin{aligned} y &= 0 - 3 \\ &= -3 \end{aligned}$$

Substitute $x = 3$ into [1]

$$\begin{aligned} y &= 3 - 3 \\ &= 0 \end{aligned}$$

$\therefore (0, -3), (3, 0)$

(b) $y = 2x - 1 \ldots [1]$

$y = x^2 - 3x + 5 \ldots [2]$

Equate

[1] and [2].

$$\begin{aligned} 2x - 1 &= x^2 - 3x + 5 \\ x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \\ x &= 2, x = 3 \end{aligned}$$

Substitute $x = 2$ into [1]

$$\begin{aligned} y &= 2 \times 2 - 1 \\ &= 3 \end{aligned}$$

Substitute $x = 3$ into [1]

$$\begin{aligned} y &= 2 \times 3 - 1 \\ &= 5 \end{aligned}$$

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$\therefore (2,3),(3,5)$

(c) $y = 3 - 2x \ldots [1]$

$y = (x-2)^2 \ldots [2]$

Equate $[1]$ and $[2]$.

$$
\begin{aligned}
3 - 2x &= (x-2)^2 \\
3 - 2x &= x^2 - 4x + 4 \\
x^2 - 2x + 1 &= 0 \\
(x-1)^2 &= 0 \\
x &= 1
\end{aligned}
$$

Substitute $x = 1$ into $[1]$

$$
\begin{aligned}
y &= 3 - 2 \times 1 \\
&= 1
\end{aligned}
$$

$\therefore (1,1)$

4 The line $y = x + c$ is a tangent to the circle $x^2 + y^2 = 4$ if it only touches the circle once.

Because there is only one solution, $\Delta = 0$.

$$
\begin{aligned}
y &= x + c \ldots [1] \\
x^2 + y^2 &= 4 \ldots [2]
\end{aligned}
$$

Substitute $[1]$ into $[2]$

$$
\begin{aligned}
x^2 + (x+c)^2 &= 4 \\
x^2 + x^2 + 2cx + c^2 &= 4 \\
2x^2 + 2cx + c^2 - 4 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\Delta &= (2c)^2 - 4 \times 2 \times (c^2 - 4) \\
&= 4c^2 - 8c^2 + 32 \\
&= -4c^2 + 32
\end{aligned}
$$

$$
\begin{aligned}
-4c^2 + 32 &= 0 \\
c^2 &= 8 \\
c &= \pm\sqrt{8} \\
c &= \pm 2\sqrt{2}
\end{aligned}
$$

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6 The line $y = mx$ is a tangent to the parabola $y = x^2 - 8x + 25$ if it only touches the parabola once. Because there is only one solution, $\Delta = 0$

$$y = mx \ldots [1]$$
$$y = x^2 - 8x + 25 \ldots [2]$$

Equate [1] and [2]

$$\begin{aligned} mx &= x^2 - 8x + 25 \\ x^2 - 8x - mx + 25 &= 0 \\ x^2 - (8+m)x + 25 &= 0 \end{aligned}$$

$$\begin{aligned} \Delta &= [-(8+m)]^2 - 4 \times 1 \times 25 \\ &= 64 + 16m + m^2 - 100 \\ &= m^2 + 16m - 36 \\ &= (m+18)(m-2) \end{aligned}$$

$$(m+18)(m-2) = 0$$
$$m = -18, m = 2$$

8 **(a)** The line $y = mx + 5$ touches the parabola $y = 3 + 5x - 2x^2$. So there is only one solution, $\Delta = 0$

$$y = mx + 5 \ldots [1]$$
$$y = 3 + 5x - 2x^2 \ldots [2]$$

Equate [1] and [2]

$$\begin{aligned} mx + 5 &= 3 + 5x - 2x^2 \\ 2x^2 + (m-5)x + 2 &= 0 \end{aligned}$$

$$\begin{aligned} \Delta &= (m-5)^2 - 4 \times 2 \times 2 \\ &= m^2 - 10m + 25 - 16 \\ &= m^2 - 10m + 9 \\ &= (m-1)(m-9) \end{aligned}$$

$$\begin{aligned} (m-1)(m-9) &= 0 \\ m &= 1, m = 9 \end{aligned}$$

(b) The line $y = mx + 5$ intersects the parabola $y = 3 + 5x - 2x^2$; there are two solutions; $\Delta > 0$

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$$(m-1)(m-9) > 0$$
$$m < 1, m > 9$$

(c) The line $y = mx + 5$ does not intersect the parabola $y = 3 + 5x - 2x^2$; there are no solutions;

$$\Delta < 0$$
$$(m-1)(m-9) < 0$$
$$1 < m < 9$$

10 The line $y = ax$ intersects the curve $y = \dfrac{2}{x-3}$; there are two solutions; $\Delta > 0$

$$y = ax \text{...[1]}$$
$$y = \frac{2}{x-3} \text{...[2]}$$

Equate [1] and [2]

$$ax = \frac{2}{x-3}$$
$$ax^2 - 3ax = 2$$
$$ax^2 - 3ax - 2 = 0$$

$$\Delta = (-3a)^2 - 4 \times a \times (-2)$$
$$= 9a^2 + 8a$$

$$9a^2 + 8a > 0$$
$$a(9a+8) > 0$$
$$a < -\frac{8}{9}, a > 0$$

12 The line contains the point $(1, 3)$ is $y - 3 = m(x-1)$

If it is the tangent to the parabola $y = x^2 - 2x + 5$, it touches at one point. So $\Delta = 0$.

$$y - 3 = m(x-1)$$
$$y = mx - m + 3 \text{...[1]}$$
$$y = x^2 - 2x + 5 \text{...[2]}$$

Equate [1] and [2]

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$$mx - m + 3 = x^2 - 2x + 5$$
$$x^2 - (2+m)x + m + 2 = 0$$

$$\begin{aligned}\Delta &= [-(2+m)]^2 - 4\times1\times(m+2)\\ &= m^2 + 4m + 4 - 4m - 8\\ &= m^2 - 4\end{aligned}$$

$$\begin{aligned}m^2 - 4 &= 0\\ m &= \pm2\end{aligned}$$

Substitute $m = -2$ into [1]

$$\begin{aligned}y &= -2x - (-2) + 3\\ &= -2x + 2 + 3\\ &= -2x + 5\end{aligned}$$

Substitute $m = 2$ into [1]

$$\begin{aligned}y &= 2x - 2 + 3\\ &= 2x + 1\end{aligned}$$

The two lines are $y = -2x + 5$ and $y = 2x + 1$

CHAPTER REVIEW 5

2 **(a)** $P(2,3), Q(6,-1)$

Midpoint: $\left(\dfrac{2+6}{2}, \dfrac{3+-1}{2}\right) = (4,1)$

$P(2,3), R(-4,-5)$

Midpoint: $\left(\dfrac{2+-4}{2}, \dfrac{3+-5}{2}\right) = (-1,-1)$

Coordinates: $M(4,1), N(-1,-1)$

(b) Gradient of MN:

$$m_{MN} = \frac{-1-1}{-1-4} = \frac{2}{5}$$

Gradient of QR:

$$m_{QR} = \frac{-5--1}{-4-6} = \frac{-4}{-10} = \frac{2}{5}$$

$$m_{MN} = m_{QR}, \therefore MN \parallel QR$$

(c) $Q(6,-1), R(-4,-5)$

$$\begin{aligned}|QR| &= \sqrt{(-4-6)^2 + (-5--1)^2} \\ &= \sqrt{116} \\ &= 2\sqrt{29}\end{aligned}$$

(d) $M(4,1), N(-1,-1)$

$$|MN| = \sqrt{(-1-1)^2 + (-1-4)^2} = \sqrt{29}$$

$$|QR| = 2\sqrt{29}$$

$$|MN| = \frac{|QR|}{2}$$

4 The midpoint of $(3,-2)$ and $(5,8)$ is $\left(\frac{3+5}{2}, \frac{-2+8}{2}\right) = (4,3)$.

$$\begin{aligned}3x + 4y &= 5 \\ 4y &= 5 - 3x \\ y &= -\frac{3x}{4} + \frac{5}{4}\end{aligned}$$

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The gradient of this line is $-\frac{3}{4}$.

The gradient of a perpendicular line is $\frac{4}{3}$.

This line goes through $(4,3)$.

$$\begin{aligned} y-3&=\frac{4}{3}(x-4) \\ 3(y-3)&=4(x-4) \\ 3y-9&=4x-16 \\ 4x-3y-7&=0 \end{aligned}$$

6 $3(x-3)=2(2y+1)$ [1]

$2(3x-1)=5(2y+1)+10$ [2]

Expand and solve using the elimination method.

$3x-9=4y+2$ [1]

$6x-2=10y+15$ [2]

Multiply [1] by 2.

$6x-18=8y+4$ [3]

Subtract [3] from [2].

$$\begin{aligned} 16&=2y+11 \\ y&=2.5 \end{aligned}$$

Substitute in [1].

$$\begin{aligned} 3x-9&=4\times 2.5+2=12 \\ 3x&=12+9=21 \\ x&=7 \end{aligned}$$

$x=7,\ y=2.5$

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8 $$\begin{aligned} &x+y-9=0 \qquad [1] \\ &y=x^2+4x+3 \qquad [2] \end{aligned}$$

Rewrite [1] and substitute into [2]

$$\begin{aligned} x+y-9&=0 \\ y&=-x+9 \end{aligned}$$

$$\begin{aligned} -x+9&=x^2+4x+3 \\ x^2+5x-6&=0 \\ (x-1)(x+6)&=0 \\ \therefore x&=-6 \text{ or } 1 \end{aligned}$$

Substitute $x=-6$ into [1]

$$\begin{aligned} x+y-9&=0 \\ -6+y-9&=0 \\ y&=15 \end{aligned}$$

Substitute $x=1$ into [1]

$$\begin{aligned} x+y-9&=0 \\ 1+y-9&=0 \\ y&=8 \end{aligned}$$

$\therefore x=-6, y=15$ or $x=1, y=8$

10 (a) $x^2-6x+5=0$

$$\Delta=b^2-4ac=(-6)^2-4(1)(5)=16$$

Because $\Delta>0$, $x^2-6x+5=0$ has two real roots.

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(b) $2x^2 - 3x - 7 = 0$

$\Delta = b^2 - 4ac = (-3)^2 - 4(2)(-7) = 65$

Because $\Delta > 0$, $2x^2 - 3x - 7 = 0$ has two real roots.

(c) $x^2 - 20x + 100 = 0$

$\Delta = b^2 - 4ac = (-20)^2 - 4(1)(100) = 0$

Because $\Delta = 0$, $x^2 - 20x + 100 = 0$ has one real root.

(d) $3x^2 + 4x - 1 = 0$

$\Delta = b^2 - 4ac = (4)^2 - 4(3)(-1) = 28$

Because $\Delta > 0$, $3x^2 + 4x - 1 = 0$ has two real roots.

12 $\Delta = 25 - 4(k-1)$

$$\begin{aligned} &= 25 - 4k + 4 \\ &= 29 - 4k \end{aligned}$$

(a) $\Delta > 0$

$$\begin{aligned} -4k + 29 &> 0 \\ -4k &> -29 \\ k &< 7\frac{1}{4} \end{aligned}$$

(b) $\Delta = 0$

$$\begin{aligned} -4k + 29 &= 0 \\ -4k &= -29 \\ k &= 7\frac{1}{4} \end{aligned}$$

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(c) $\Delta < 0$

$$\begin{aligned} -4k + 29 &< 0 \\ -4k &< -29 \\ k &> 7\frac{1}{4} \end{aligned}$$

14 $y = 2x - 3 \qquad [1]$

$y = x^2 - 4x + 5 \qquad [2]$

Equate the right sides of the two equations.

$$2x - 3 = x^2 - 4x + 5$$
$$x^2 - 6x + 8 = 0$$
$$(x-2)(x-4) = 0$$
$$\therefore x = 2 \text{ or } x = 4$$

Substitute $x = 2$ into [1].

$$y = 2x - 3 = 2(2) - 3 = 1$$

Substitute $x = 4$ into [1].

$$y = 2x - 3 = 2(4) - 3 = 5$$

The points of intersection are $(2,1)$ and $(4,5)$.

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