Mock Test - Sol^{n}s

(V2)

① $3^x = 7 \implies \log_3 7 = x$

OR $\dfrac{\log_e 7}{\log_e 3} = x \implies$ D

② 4 solutions $\implies$ C

③ Here $x \neq 1$, $(5,0)$ lies on graph

A $y = 3\log_2(5-1) - 6 = 3\log_2 4 - 6$
$= 3\log_2 2^2 - 6$
$= 6\log_2 2 - 6$
$= 0$ ✓

B $y = 2\log_3 4 + 1$ ✗

C $y = 3\log_2 6 - 3$ ✗

D $y = 2\log_4 4 - 2 = 2 - 2 = 0$ ✓

A or D

$(3,-3)$ lies on curve: A $y = 3\log_2 2 - 6 = -3$

D $y = 2\log_4 2 - 2 \neq -3$

$\implies$ A

(4) $y = f(x)$ has domain $[-2, \infty)$

$x \geqslant -2$

$y = 3f(-2x) - 4$

VD sf 3

(doesn't affect domain)

VT 4 down

(doesn't affect domain

$y = f(-2x) = f(-2(x))$

HD sf of $-\frac{1}{2}$

$\Rightarrow x \leqslant 2 \Leftarrow$ negative sf $\frac{1}{2}$

$x \leqslant 2(\frac{1}{2})$

$x \leqslant 1 \Leftarrow$ sf of $\frac{1}{2}$

or $(-\infty, 1]$ $\Rightarrow$ (C)

(5) In diagram, amp = 2

V translation 1 up.

period $= \frac{2\pi}{6} = \frac{2\pi}{3}$

graph is sine curve.

$y = 2\sin 3x + 1$, however, this is not an option.

An equivalent option is

$y = 2\cos 3(x - \frac{\pi}{6}) + 1$ OR

$y = 2\cos(3x - \frac{\pi}{2}) + 1$ $\Rightarrow$ (C)

(6) $y = 2x e^{7x-3}$

$\frac{dy}{dx} = 2x(7)e^{7x-3} + e^{7x-3}(2)$

OR

$14x e^{7x-3} + 2e^{7x-3} = 2e^{7x-3}(7x+1)$

(or equivalent)

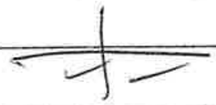
(7) $y = \ln(x-4)$

$x - 4 > 0$

$\underline{\underline{x > 4}}$ or $\underline{\underline{(4, \infty)}}$

(8) $2\sin\left(\theta - \frac{\pi}{3}\right) = -1$ $\qquad 0 \leq \theta \leq 2\pi$

$\sin\left(\theta - \frac{\pi}{3}\right) = -\frac{1}{2}$ $\qquad -\frac{\pi}{3} \leq \theta - \frac{\pi}{3} \leq \frac{5\pi}{3}$



$\left[\theta - \frac{\pi}{3} = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}\right]$

$\theta - \frac{\pi}{3} = \frac{7\pi}{6}, \frac{11\pi}{6}, -\frac{\pi}{6}$

$\theta = \frac{3\pi}{2}, \cancel{\frac{13\pi}{6}}, \frac{\pi}{6}$

$\theta = \underline{\underline{\frac{3\pi}{2}, \frac{\pi}{6}}}$ ✓ ✓

(9) $\log_3 6 = a \qquad \log_3 5 = b$

$\log_3 150 = \log_3(3 \times 5 \times 2 \times 5)$

$= \log_3(6 \times 5^2)$

$= \log_3 6 + 2\log_3 5$

$= \underline{\underline{a + 2b}}$

4

(10) $y = x^2$

$\Rightarrow y = \left(\frac{x}{2}\right)^2$

$y = \left(\frac{x+1}{2}\right)^2$

(1) H Dilation s f of 2

(2) H translation 1 unit (L)

(11) $\log_2 x + \log_2 (x-3) = 2$

$\log_2 x(x-3) = 2$

$2^2 = x(x-3)$

$4 = x(x-3)$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or -1

but $x > 0$ so $x \neq -1$

and $x - 3 > 0$

$x > 3$ so $x = 4$ ✓

(12) $y = 3e^{-2x}$

$\frac{dy}{dx} = -6e^{-2x}$ $\frac{d^2y}{dx^2} = 12e^{-2x}$

$LHS = 12e^{-2x} + 3 \times -6e^{-2x} + 2(3e^{-2x})$

$= 12e^{-2x} - 18e^{-2x} + 6e^{-2x}$

$= 0$

$= RHS$

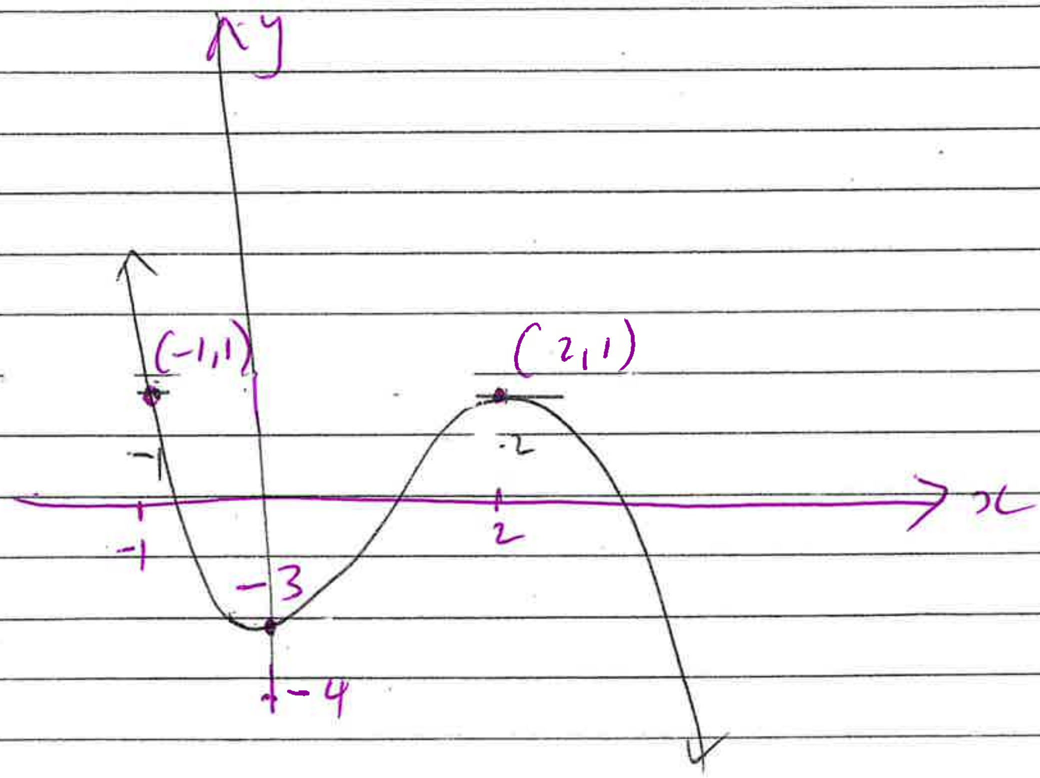
(13) $y = f(2-x) + 1$

$y = f(-(x-2)) + 1$

(1) H Dilation s f of -1

(2) H Translation 2 units (R)

(3) V translation 1 unit up



(13) (a) when $t=0$, $T = 20e^{0} + 15 = 35$

(b) $\frac{dT}{dt} = 0.4(20e^{0.4t})$

when $t=5$, $\frac{dT}{dt} = 0.4(20)e^{0.4 \times 5}$

$\doteq 59.1$

$\therefore$ temp is increasing at a rate of

59° C after 5 min

(c) $100 = 20e^{0.4t} + 15$

$\frac{85}{20} = e^{0.4t}$

$\ln \frac{85}{20} = 0.4t$

$t = \frac{\ln \frac{85}{20}}{0.4}$

$t \doteqdot \underline{3.6 \text{ min}}$ (1 dp)

15 $y = \frac{4-3x}{x+2}$

HA $\Rightarrow$ $y = \frac{\frac{4}{x} - 3}{1 + \frac{2}{x}}$

as $x \to \infty$, $y \to \frac{-3}{1} = -3$

$\therefore$ HA at $\underline{y = -3}$

VA $\Rightarrow$ $x = -2$

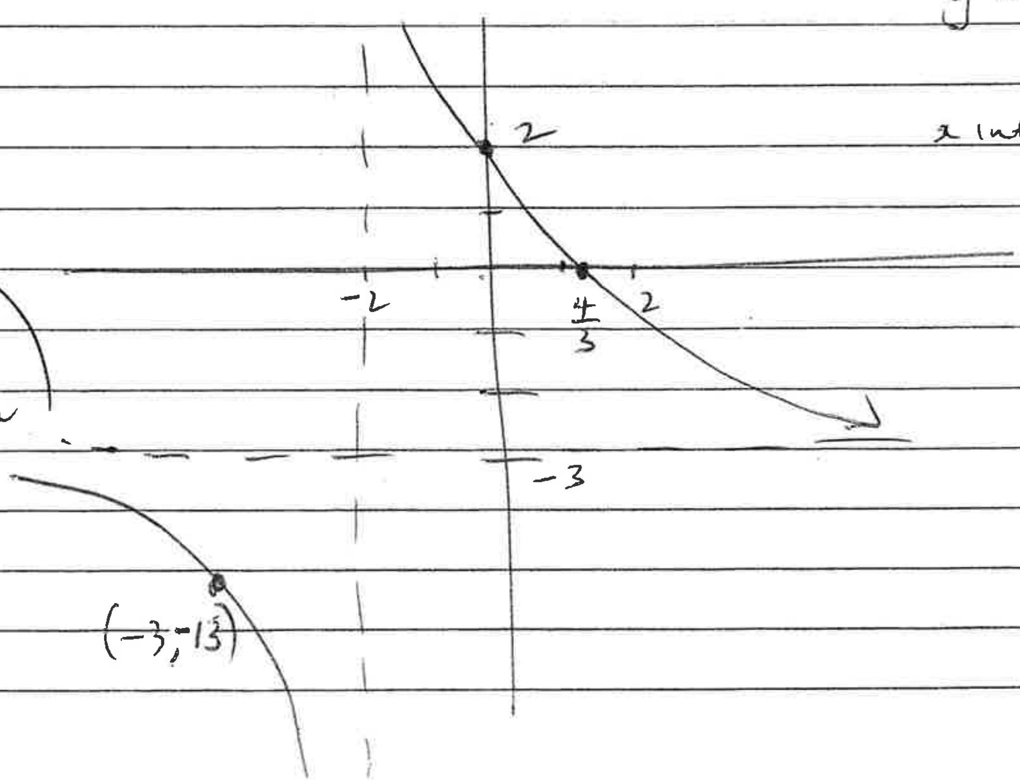
y int ($x=0$)

$y = \frac{4}{2} = 2$

x int ($y=0$)

$0 = 4 - 3x$

$x = \frac{4}{3}$



* (Not drawn to scale)

(16) $x = 10 + 3e^{-0.2t}$

$v = -0.2 \times 3 e^{-0.2t}$

$= -0.6 e^{-0.2t}$

as $t \to \infty$, $e^{-0.2t} > 0$

$\therefore -0.6 e^{-0.2t} < 0$

$\therefore$ Motion is to left but is never zero.

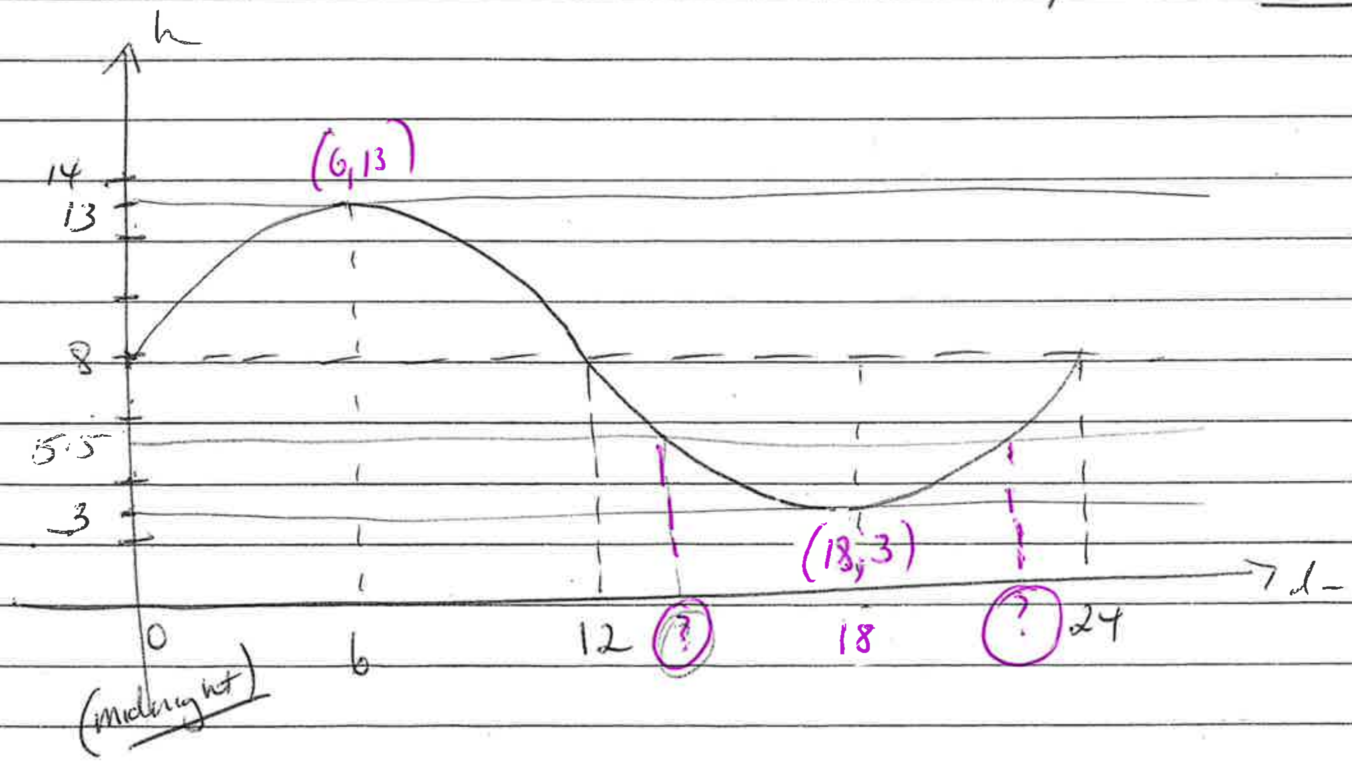
so particle is never at rest

(17) $h(t) = 8 + 5 \sin \frac{\pi}{12} t$

(when $t=0$, time is midnight)

period $= \frac{2\pi}{\frac{\pi}{12}}$

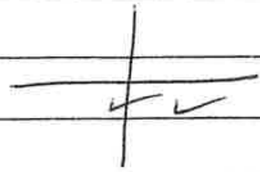
period $= 24$



$$8 + 5\sin\frac{\pi t}{12} = 5.5$$

$$5\sin\frac{\pi t}{12} = -2.5$$

$$\sin\frac{\pi}{12}t = -\frac{1}{2}$$



$$\left[\frac{\pi}{12}t = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}\right] \qquad 0 \le \frac{\pi}{12}t \le 2$$

$$\frac{\pi}{12}t = \pi + \frac{\pi}{6} = \frac{7\pi}{6} \;;\quad 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$t = \frac{7\cancel{\pi}}{6} \times \frac{12}{\cancel{\pi}} \;;\quad \frac{11\cancel{\pi}}{6} \times \frac{12}{\cancel{\pi}}$$

$$t = 14 \;;\quad 22$$

When $t = 14$, time is 2 pm.

$t = 22$, time is 10 pm

$\therefore$ Boat may enter the beach between midnight and 2 pm. and then again from 10pm until midnight.