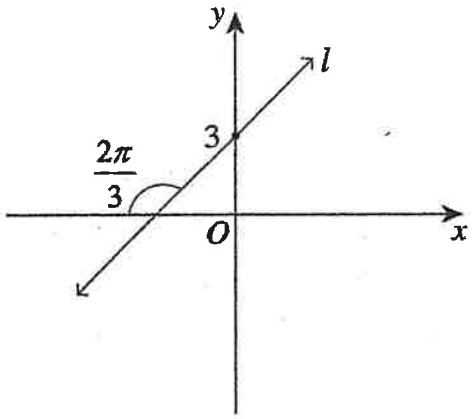
Name: SOLUTIONS

Year 12 Advanced Half Yearly MOCK Exam

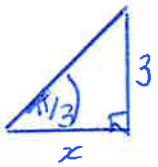
Section 1 – 10 Multiple Choice

Circle the correct answer.

1. Line l is shown in the graph.



What is the gradient of line l?



$\frac{3}{x} = \tan \pi/3$

$x = \frac{3}{\tan \pi/3}$

$= \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

A. $-\frac{1}{\sqrt{3}}$

B. $-\sqrt{3}$

C. $\frac{1}{\sqrt{3}}$

(D.) $\sqrt{3}$

2. What are the solution(s) to $|2x-1|=5$?

$2x-1=5$ $\quad$ $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$

A. $x=-2$

B. $x=3$

(C.) $x=-2$ and $x=3$

D. $x=-3$ and $x=2$

3. Which of the following is the derivative of $y=\ln\sqrt{\frac{x+1}{x-1}}$?

$y = \ln f(x)$

$y' = \frac{f'(x)}{f(x)}$

$y = \ln\left(\frac{x+1}{x-1}\right)^{1/2}$

$= \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$= \frac{1}{2}\{\ln(x+1) - \ln(x-1)\}$

$\frac{dy}{dx} = \frac{1}{2}\left\{\frac{1}{x+1} - \frac{1}{x-1}\right\}$

A. $\frac{dy}{dx}=\frac{1}{2}\left(\frac{x+1}{x-1}\right)$

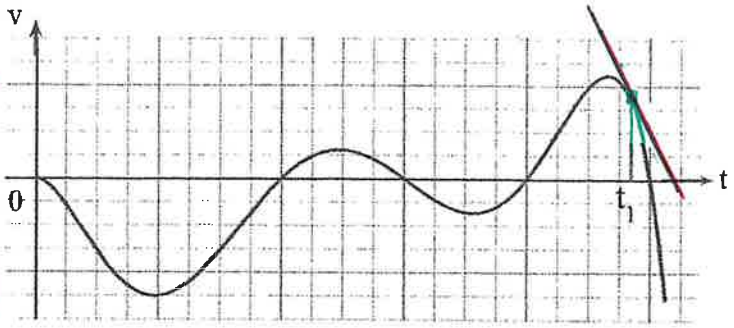
(B.) $\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x+1}-\frac{1}{x-1}\right)$

C. $\frac{dy}{dx}=2\left(\frac{1}{x+1}-\frac{1}{x-1}\right)$

D. $\frac{dy}{dx}=\frac{1}{2}(\ln(x+1)-\ln(x-1))$

(*) 4. The graph shows the velocity of a particle that starts to move from the origin along the t axis.

Note to Advanced Students - this is content not yet covered! (Sorry)



At t_1 $v > 0$

$a < 0$

Not in current exam scope

Which statement describes the motion of the particle at the point where $t = t_1$?

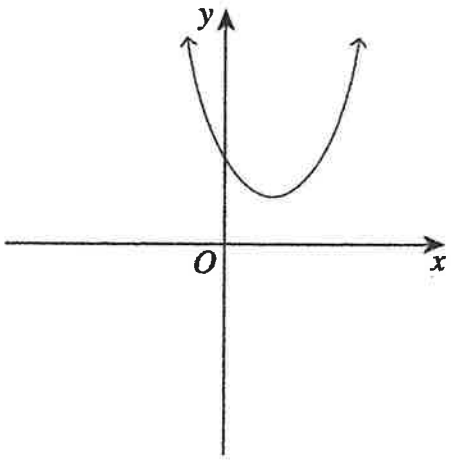
A) The displacement is positive and the acceleration is negative.

B) The displacement is negative and the acceleration is positive.

C) The displacement is positive and the acceleration is positive.

D) The displacement is negative and the acceleration is negative.

5. The graph shows the quadratic function $y = ax^2 + bx + c$.



Which of the following statements is correct?

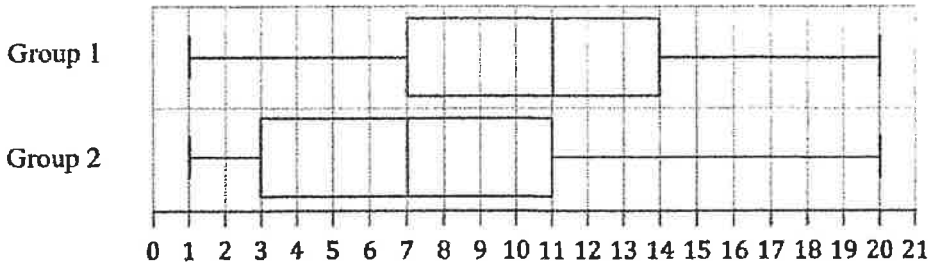
A. $a > 0, c > 0$ and $b^2 - 4ac = 0$

B. $a > 0, c > 0$ and $b^2 - 4ac > 0$

C. $a > 0, c < 0$ and $b^2 - 4ac < 0$

D. $a > 0, c > 0$ and $b^2 - 4ac < 0$

6. Consider the parallel box plots below.



Which of the following statements is CORRECT?

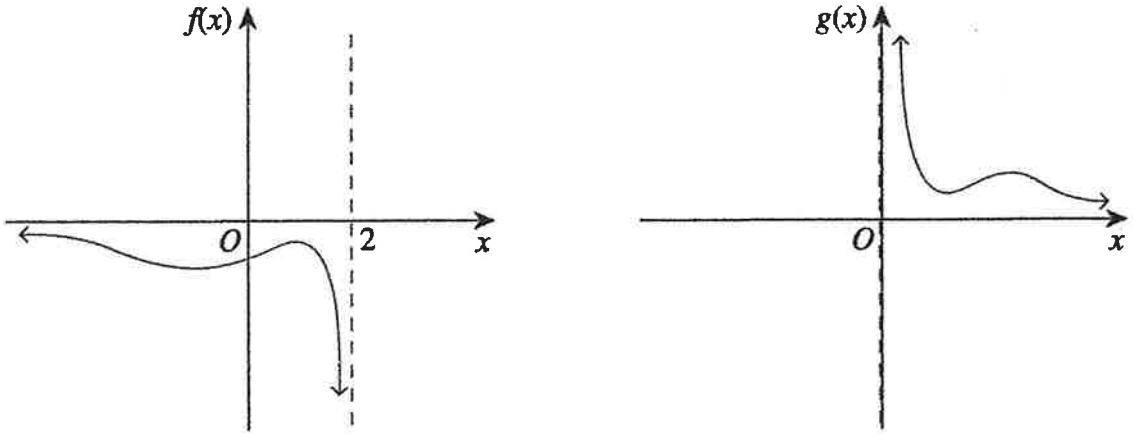
A. Group 1 is positively skewed.

B. Group 2 is negatively skewed.

C. The difference between the median and Q_1 of Group 1 is the same as the difference between the median and Q_3 of Group 2.

D. The range and IQR are equal for both sets of data.

7. The function $y = f(x)$ is transformed to $y = g(x)$, as shown in the diagram.



Which of the following equations best represents the transformed function?

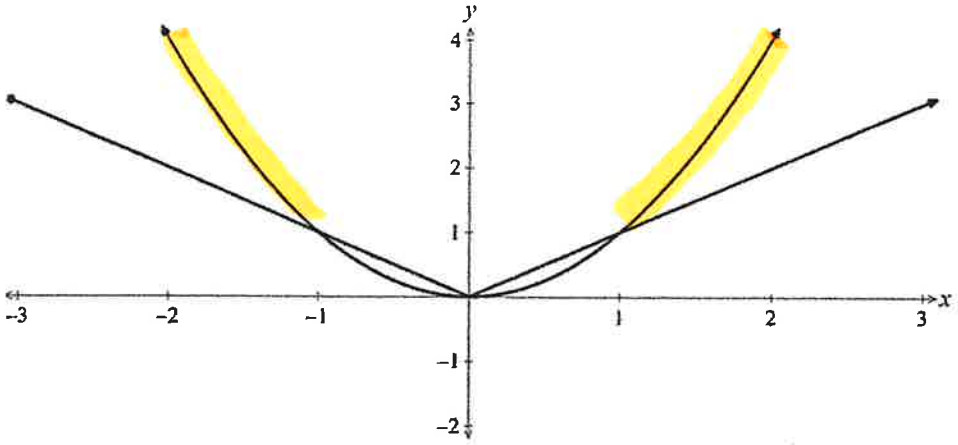
A. $g(x) = -f(2 - x)$

B. $g(x) = -f(x - 2)$

C. $g(x) = f(2 - x)$

D. $g(x) = f(x - 2)$

8. The graph shows $y = x^2$ and $y = |x|$.



What is the solution to the inequality $x^2 - |x| > 0$? $x^2 > |x|$

A. $\{x: (-\infty, -1) \cup (1, \infty)\}$

B. $\{x: (-\infty, -1] \cup [1, \infty)\}$

C. $\{x: (-1,1)\}$

D. $\{x: [-1,1]\}$

9. Consider the information about events A and B.

- $P(B) = 0.6$
- $P(A|B) = 0.4$
- $P(A|\bar{B}) = 0.8$

What is the value of $P(B|A)$?

A. $\frac{1}{5}$

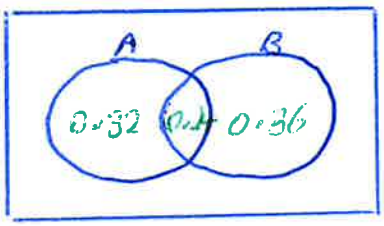
B. $\frac{2}{5}$

C. $\frac{3}{7}$

D. $\frac{4}{5}$

$\frac{P(A \cap B)}{P(B)} = 0.4$

$\frac{P(A \cap B)}{0.6} = 0.4$ $P(A \cap B) = 0.24$



$\frac{P(A \cap \bar{B})}{P(\bar{B})} = 0.8$ $P(\bar{B}) = 1 - 0.6 = 0.4$

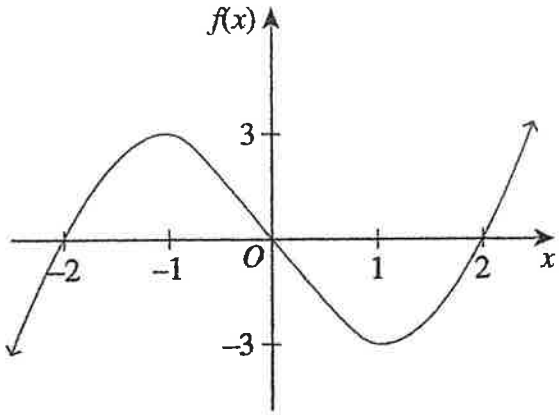
$P(A \cap \bar{B}) = 0.8 \times 0.4 = 0.32$

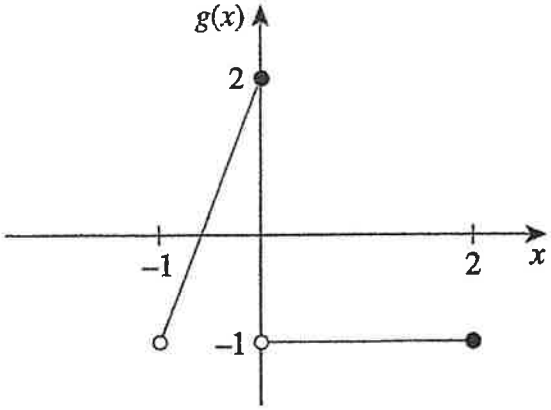
$P(B|A) = \frac{P(B \cap A)}{P(A)}$ $P(A) = 0.32 + 0.24 = 0.56$

$= \frac{0.24}{0.56}$

$= 3/7$

10. The graphs of $y = f(x)$ and $y = g(x)$ are shown.





What is the domain and range for $y = f(g(x))$?

	Domain	*Range*
A.	$(-1, 2]$	$[-3, 3]$
B.	$(-1, 2]$	$(-1, 2]$
C.	$(-\infty, \infty)$	$[-3, 3)$
D.	$(-\infty, \infty)$	$(-1, 2]$

End of Section 1

Section 2 – 60 Marks

Show your working in the space provided.

11. Solve $9^{2x-3} = 27^{x}$. 2

$(3^2)^{2x-3} = (3^3)^x$

$3^{4x-6} = 3^{3x}$

$4x - 6 = 3x$

$x = 6$

12. A triangular park is shown below. 2

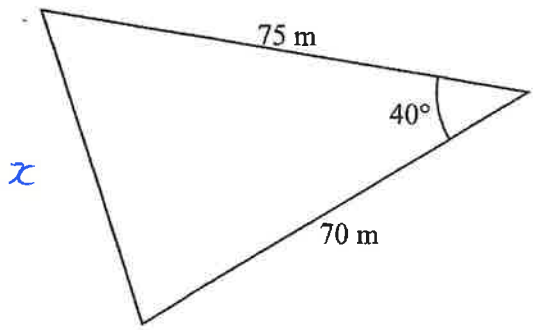


DIAGRAM NOT TO SCALE

Calculate the length of the unknown side, correct to the nearest metre.

$x^2 = 75^2 + 70^2 - 2 \times 75 \times 70 \times \cos 40°$

$x^2 = 2481.533347$

$x = 49.81499119$

$x = 50\text{m}$ (nearest metre)

13. **Solve the equation $2\ln(x+2) - \ln x = \ln(2x+1)$ where $x > 0$, for x.** 2

$2\ln(x+2) - \ln x = \ln(2x+1)$

$\ln(x+2)^2 - \ln x = \ln(2x+1)$

$\ln\left(\frac{(x+2)^2}{x}\right) = \ln(2x+1)$

$\frac{(x+2)^2}{x} = 2x+1$

But $x > 0$

$\therefore x = 4$

$x^2 + 4x + 4 = 2x^2 + x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

14. Differentiate with respect to x:

(a) $x\sin^2 x$ **2**

$u = x \qquad v = (\sin x)^2$

$u' = 1 \qquad v' = 2\cos x \sin x$

$vu' + uv' = (\sin x)^2 + 2x\cos x \sin x$

$= \sin x(\sin x + 2x\cos x)$

(b) $\ln\sqrt{4x^2 - 1}$ **2**

$y = \ln(4x^2-1)^{1/2} = \frac{1}{2}\ln(4x^2-1)$

$\frac{dy}{dx} = \frac{1}{2}\left(\frac{8x}{4x^2-1}\right)$

$= \frac{4x}{4x^2-1}$

(c) $\dfrac{3x+1}{x+4}$ **2**

$u = 3x+1 \qquad v = x+4$

$u' = 3 \qquad v' = 1$

$\frac{vu' - uv'}{v^2} = \frac{3(x+4) - 1(3x+1)}{(x+4)^2} = \frac{11}{(x+4)^2}$

15. The probability distribution table for the discrete random variable X is shown. **3**

x	0	1	2	3	4	5
$P(X=x)$	0.35	a	b	0.15	0.05	0.01

If $E(X) = 1.5$, find the values of a and b.

$0.35 + a + b + 0.15 + 0.05 + 0.01 = 1$

$a + b = 0.44$ —— ①

$E(X) = 0\times0.35 + 1\times a + 2\times b + 3\times0.15 + 4\times0.05 + 5\times0.01$

$1.5 = a + 2b + 0.7$

$a + 2b = 0.8$ —— ②

$a + b = 0.44$

$a + 2b = 0.8$

$b = 0.36$

$a = 0.08$

16. A survey contained a question asking eight households to estimate their weekly income.

The results of the survey were as follows.

\$600 \$1200 \$1600 \$1800 \$2000 \$2400 \$2600 \$4200

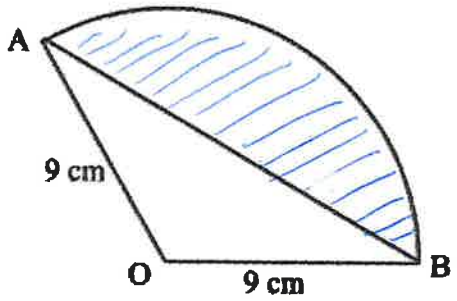
Identify any outlier(s) in this dataset. Justify your answer. 2

median = 1900 $Q_1 - 1.5\,IQR = -250$

UQ = 2500 } IQR = 1100 $Q_3 + 1.5\,IQR = 4150$

LQ = 1400 } $\therefore$ \$4200 is an outlier

17. In the diagram, AOB is a sector of a circle with centre at O and radius 9 cm.
AB is a chord, and the length of arc AB is 6π *cm*.



a) Show that the size of $\angle$ AOB $= \frac{2\pi}{3}$. 1

$l = r\theta$

$6\pi = 9 \times \theta$

$\theta = 6\pi/9 = 2\pi/3$ as required.

b) Find the perimeter of the shaded minor segment. 2

$AB^2 = 9^2 + 9^2 - 2 \times 9 \times 9 \times \cos 2\pi/3$

$AB^2 = 243$

$AB = \sqrt{243}$

Perimeter $= 6\pi + \sqrt{243}$

$= 34.43801319$

$= 34.4$ cm (1dp)

18. The function $y = \sin(x)$ undergoes a series of graphical transformations and becomes $y = 5\sin\left(2x + \frac{\pi}{3}\right)$. **3**

Outline the transformations that were applied to $y = \sin(x)$ in the correct order.

$y = 5\sin(2(x+\pi/6))$

$f(x) = y = \sin(x)$

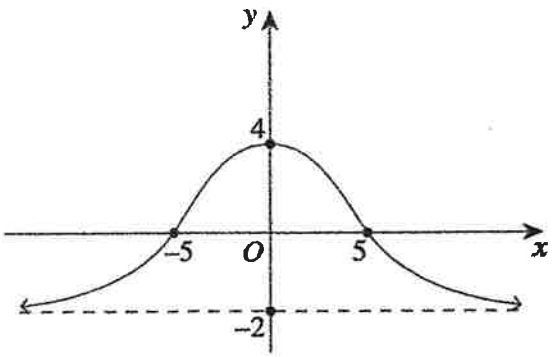
Vertically: $a(x) = 5f(x) = 5\sin(x)$ → vertical dilation with scale factor 5.

Horizontally: $b(x) = a(2x) = 5\sin 2x$ → horizontal dilation with scale factor 2

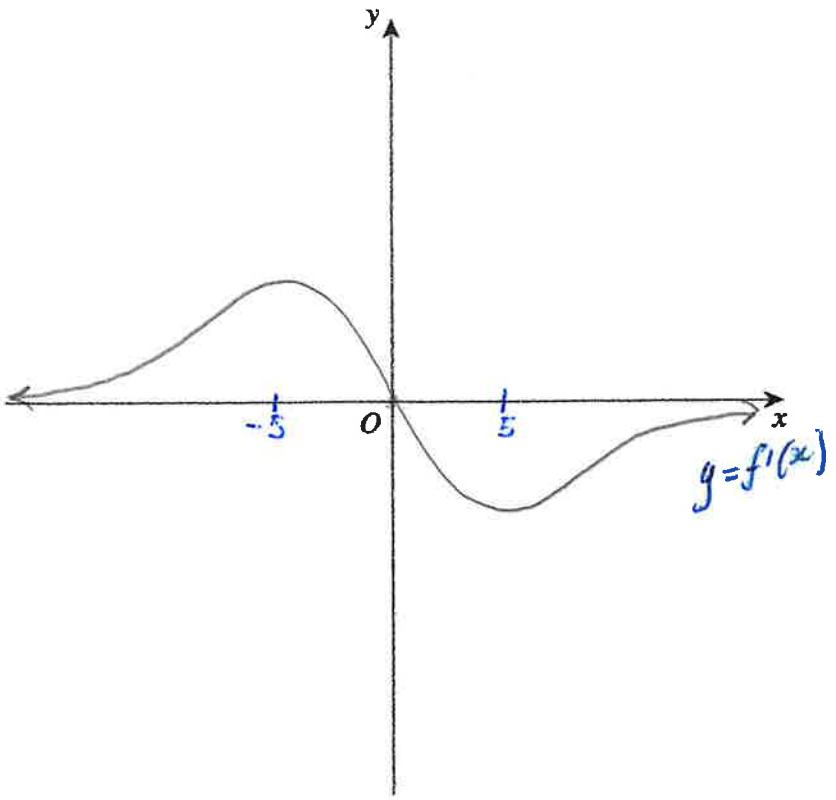
$c(x) = b(x+\pi/6)$

$= 5\sin 2(x+\pi/6)$ → horizontal translation $\pi/6$ to the left

19. **The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = -2$.** **3**



On the axes below, sketch the graph of $y = f'(x)$, clearly showing the behaviour at the intercepts and any asymptotes.



20. Prove that $(1-\sin x)(\sec x+\tan x)\equiv\cos x$. 2

LHS $= (1-\sin x)(\sec x+\tan x)$

$= (1-\sin x)\left(\dfrac{1+\sin x}{\cos x}\right)$

$= \dfrac{1-\sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x}$

$= \cos x$ $= RHS$

21. **Daniel has a bag that contains 5 red, and 8 black balls. Two balls are drawn randomly from the bag.**

(a) **Find the probability that the first ball drawn is red.** **1**

$P(R) = \frac{5}{13}$

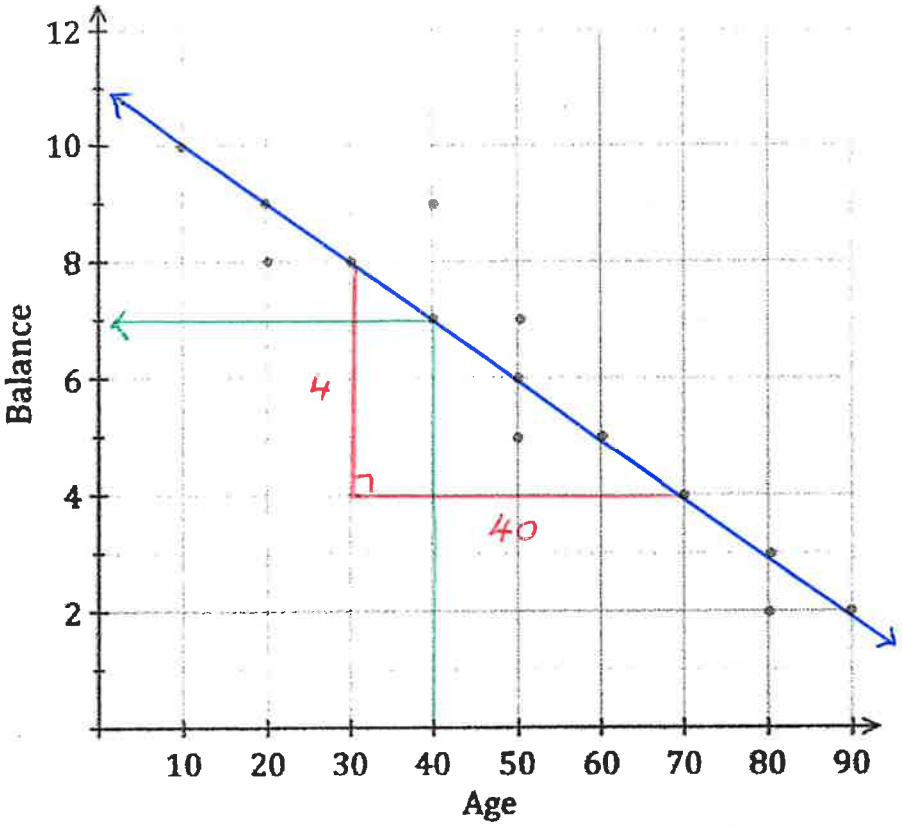
(b) **Find the probability that both balls drawn are the same colour.** **1**

$P(\text{same}) = P(R \text{ and } R) \text{ or } (B \text{ and } B)$

$= \frac{5}{13} \times \frac{4}{12} + \frac{8}{13} \times \frac{7}{12}$

$= \frac{19}{39}$

22. The scatterplot below shows the relationship between age and balance.



(a) Draw a line of best fit on the scatterplot. Find the gradient of this line. **2**

$m = -4/40$

$m = -1/10$

(b) Hannah is 40 years old. What is her expected balance ? **1**

7

(c) Calculate the value of the Pearson's correlation coefficient. Answer correct to two decimal places. **2**

$r = -0.957814821$

$r = -0.96$ (2dp)

23. (a) Find the turning points and points of inflection on $y = x^4 - 2x^3 + 1$. **3**

$y' = 4x^3 - 6x^2$

$y' = 0 \rightarrow 4x^3 - 6x^2 = 0$

$2x^2(2x - 3) = 0$

$x = 0 \quad x = 3/2$

At $x = 0$ $y = 1$

$x = 3/2$ $y = -11/16$

$y'' = 12x^2 - 12x$

At $x = 0$ $y'' = 0$

At $x = 3/2$ $y'' = 9 > 0$ $\therefore$ minimum at $(3/2, -11/16)$

x	-1	0	$1/2$
y''	24	0	-3

$\therefore$ Change in concavity

$\therefore$ Point of horizontal inflection at $(0, 1)$

$y'' = 0 \rightarrow 12x^2 - 12x = 0$

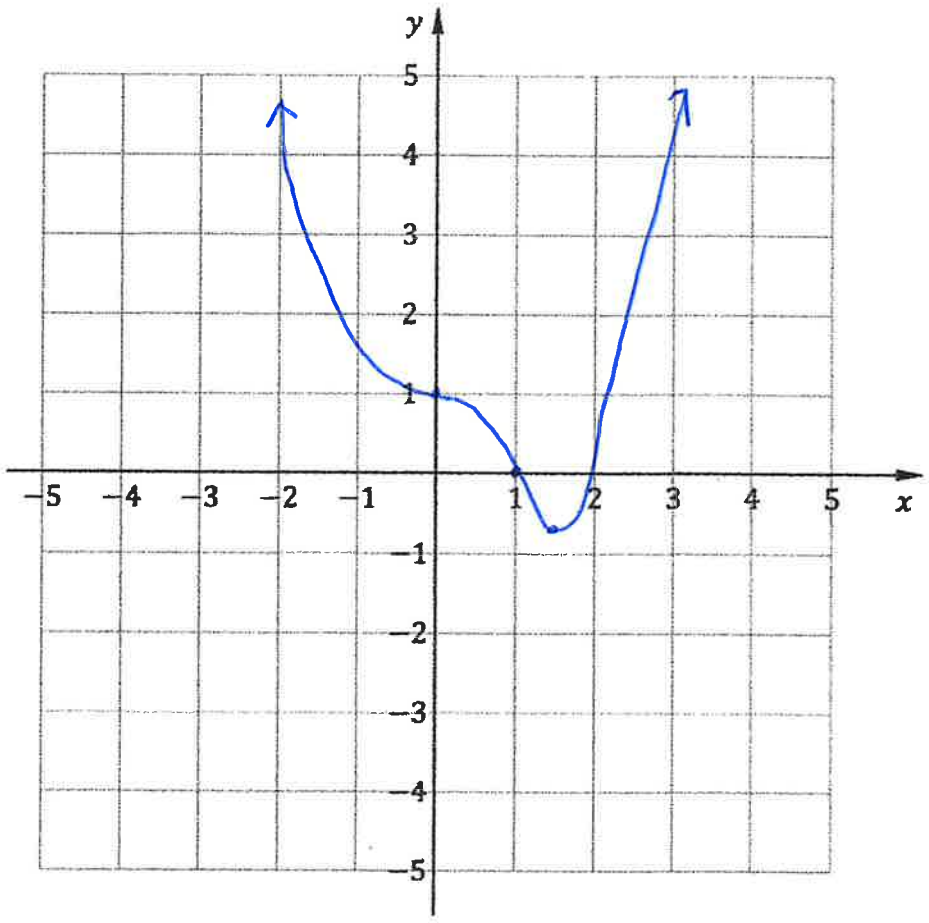
$12x(x - 1) = 0$

$x = 0 \checkmark \quad x = 1$

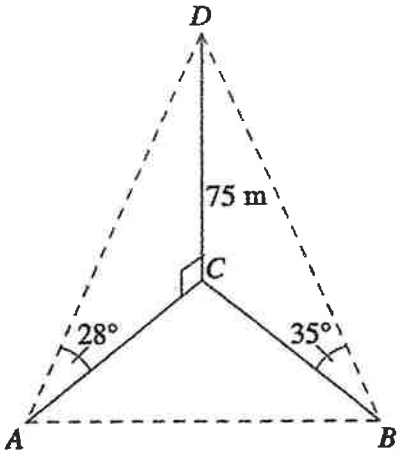
x	$1/2$	1	2
y''	-3	0	24

$\therefore$ point of inflection at $(1, 0)$

(b) Sketch the graph $y = x^4 - 2x^3 + 1$ on the axes below, clearly showing the turning points and points of inflection. It is not necessary to find all x-intercepts. **1**



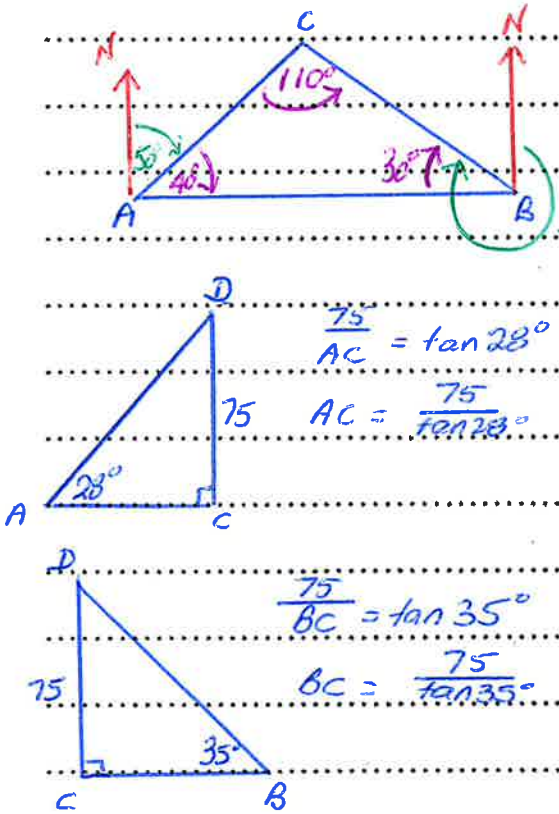
24. The diagram shows a 75 m vertical tower, represented by line DC. Points A and B are in the same horizontal plane as the base of the tower, point C, and point A is west of point B. The angle of elevation from point A to point D is 28°, and the angle of elevation from point B to point D is 35°. **3**



NOT TO SCALE

The bearing of point C from point A is 050°T, and the bearing of point C from point B is 300°T.

Find the distance between points A and B, correct to the nearest metre.



$\frac{75}{AC} = \tan 28°$

$AC = \frac{75}{\tan 28°}$

$\frac{75}{BC} = \tan 35°$

$BC = \frac{75}{\tan 35°}$

$AB^2 = AC^2 + BC^2 - 2.AC.BC.\cos 110°$

$AB^2 = \left(\frac{75}{\tan 28°}\right)^2 + \left(\frac{75}{\tan 35°}\right)^2 - 2\left(\frac{75}{\tan 28°}\right)\left(\frac{75}{\tan 35°}\right) \times \cos 110°$

$AB^2 = 11791.31477$

$AB = 108.5878206$

The distance between A and B is 109 m (nearest metre)

25. (a) Show that $\frac{d}{dx}(x\ln x - x) = \ln x$. **2**

$x \ln x$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = 1/x$

$\frac{d}{dx}(x\ln x - x) = 1 \times \ln x + x \times 1/x - 1$

$= \ln x + 1 - 1$

$= \ln x$ as required.

(b) Show that $y = \frac{1}{e}x$ is the equation of the tangent to the curve $y = \ln x$ at the point $(e, 1)$. **2**

$y = \ln x \longrightarrow y' = 1/x$

At $x = e$, $m_T = 1/e$

$y - y_1 = m(x - x_1)$

$y - 1 = 1/e\,(x - e)$

$y - 1 = 1/e \cdot x - 1$

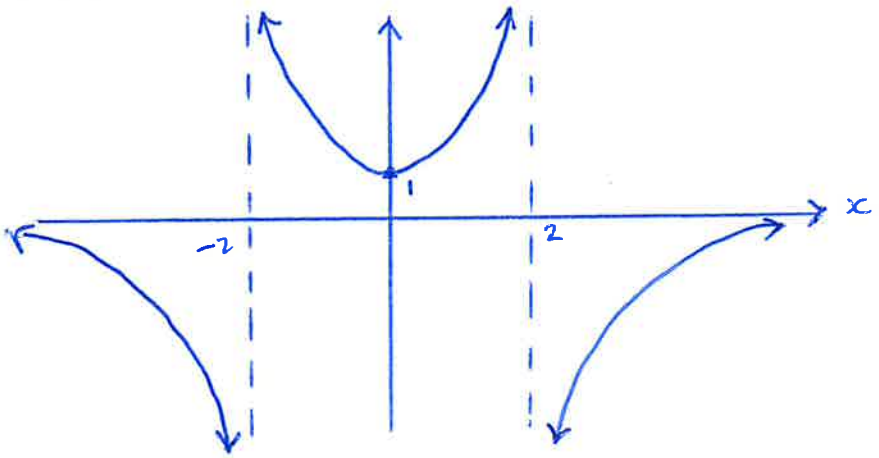
$y = 1/e\, x$ as required

26. A rational function $f(x)$ has the following properties, **3**

- The horizontal asymptote of its graph is $y = 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivates at various points.

	$x < -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x > 2$
$f(x)$			1		
$f'(x)$	< 0	< 0	0	> 0	> 0
$f''(x)$	< 0	> 0	> 0	> 0	< 0.

Sketch $y = f(x)$, using the properties in the table above.



27. The height of a giraffe is modelled by $h = 590 - 460\,(1.1)^{-0.5t}$, where h is the height of the giraffe in centimetres (cm) and t is its age in months.

a) What is the height of the giraffe when its age is 10 months? 1

$h = 590 - 460\,(1.1)^{-0.5\times 10}$

$h = 304.3761914$

Height at 10 months is 304.3 cm (1dp)

b) At what rate is the height h increasing when its age is 15 months? 2

$\frac{dh}{dt} = -460 \times \ln 1.1 \times -0.5 \times (1.1)^{-0.5t}$

$= 10.7256963$

Rate of increase at 15 months is 10.7 cm/month (1dp)

$\frac{d}{dx} a^{f(x)} = \ln a \cdot f'(x) a^{f(x)}$

c) At what age will the height of the giraffe become 4 m? 3

$h = 4m = 400cm \longrightarrow 590 - 460(1.1)^{-0.5t} = 400$

$460(1.1)^{-0.5t} = 190$

$(1.1)^{-0.5t} = 190/460$

$\ln(1.1)^{-0.5t} = \ln(19/46)$

$-0.5t \cdot \ln(1.1) = \ln(19/46)$

$t = \frac{\ln(19/46)}{-0.5 \ln 1.1}$

$= 18.55420731$

The giraffe reaches 4m after 18.6 months (1dp)

28. The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA, can be modelled by the function $P(t) = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$ where t is the number of days after 22 March 2023, for $0 \le t \le 366$.

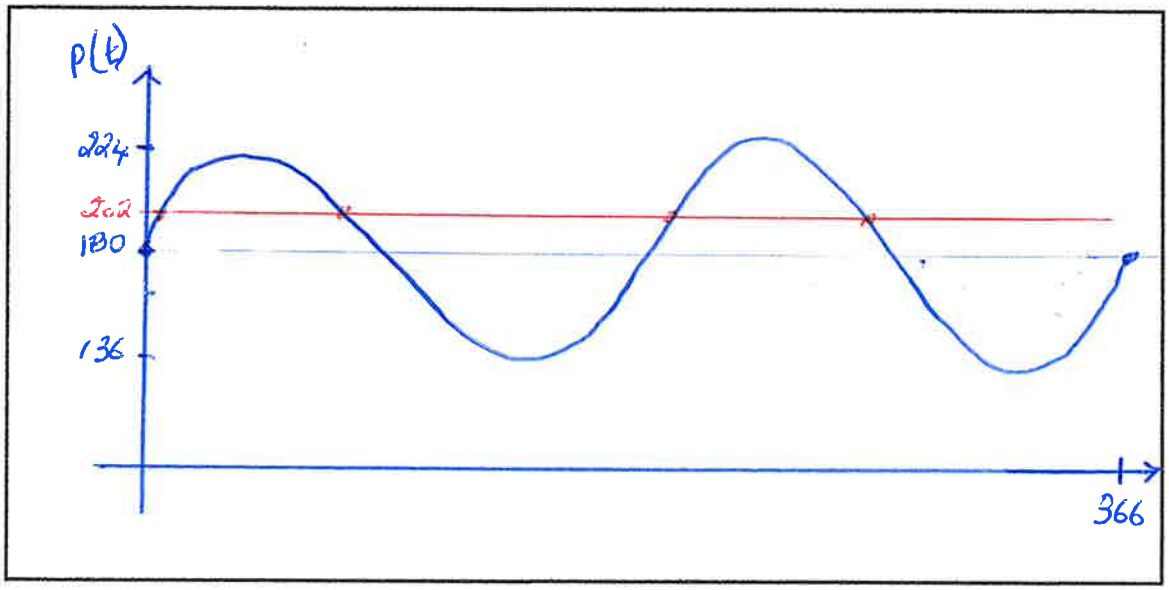
maximum value is 1

(a) What is the maximum price of petrol during the year? 1

$P_{max} = 180 + 44$

$= 224$ c/L

(b) Sketch the function $P(t)$ for $0 \le t \le 366$. 2



(c) What are the values of t for when petrol will cost 202 cents per litre. 2

$202 = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$

$\sin\left(\frac{2\pi t}{183}\right) = 22/44 = 1/2$

$\frac{2\pi t}{183} = \frac{\pi}{6}$ $\qquad$ $\frac{2\pi t}{183} = \frac{5\pi}{6}$

$t = \frac{183\pi}{12\pi}$ $\qquad$ $t = 76.25$

$t = 15.25$

$\frac{2\pi t}{183} = \frac{13\pi}{6}$ $\qquad$ $\frac{2\pi t}{183} = \frac{17\pi}{6}$

$t = 198.25$ $\qquad$ $t = 259.25$

$t = 15.25,\ 76.25,\ 198.25,\ 259.25$