EXERCISE 10.1 DISCRETE RANDOM VARIABLES

2 Rethink the question using complementary events.

$$P(\text{scoring at least one goal}) = 1 - P(\text{scoring no goals})$$

You need to find:

$$P(\text{scoring no goals}) < 0.05$$

Let M be misses goal.

Use trial and error to see how many shots at goal he needs to take so the probability is less than 0.05

$$\begin{aligned} P(M) &= 1 - 60\% \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} P(MM) &= 0.4 \times 0.4 \\ &= 0.16 \end{aligned}$$

$$\begin{aligned} P(MMM) &= 0.4 \times 0.4 \times 0.4 \\ &= 0.064 \end{aligned}$$

$$\begin{aligned} P(MMMM) &= 0.4 \times 0.4 \times 0.4 \times 0.4 \\ &= 0.0256 \\ &< 0.05 \end{aligned}$$

Wayne would need to take at least 4 shots at goal to be at least 95% certain of scoring at least 1 goal.

4 **(a)** Add the probabilities. A discrete probability distribution has the sum of probabilities equal to 1, and all values are between zero and one, inclusive.

$$\begin{aligned} 1 &= \frac{1}{18} + \frac{1}{6} + \frac{1}{9} + \frac{5}{18} + k + \frac{3}{18} \\ k &= 1 - \left(\frac{1}{18} + \frac{1}{6} + \frac{1}{9} + \frac{5}{18} + \frac{3}{18} \right) \\ &= \frac{2}{9} \end{aligned}$$

(b) Add the probabilities. A discrete probability distribution has the sum of probabilities equal to 1, and all values are between zero and one, inclusive.

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$$1=\frac{1}{8}+\frac{5}{24}+k+\frac{7}{24}+\frac{1}{24}+\frac{1}{6}$$
$$k=1-\left(\frac{1}{8}+\frac{5}{24}+\frac{7}{24}+\frac{1}{24}+\frac{1}{6}\right)$$
$$=\frac{1}{6}$$

(c) Add the probabilities. A discrete probability distribution has the sum of probabilities equal to 1, and all values are between zero and one, inclusive.

$$1=\frac{1}{9}+k+\frac{k}{2}+\frac{4}{9}+\frac{1}{9}$$
$$k+\frac{k}{2}=1-\left(\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right)$$
$$\frac{3k}{2}=\frac{1}{3}$$
$$k=\frac{1}{3}\times\frac{2}{3}$$
$$=\frac{2}{9}$$

(d) Add the probabilities. A discrete probability distribution has the sum of probabilities equal to 1, and all values are between zero and one, inclusive.

$$1=\frac{1}{6}+3k+\frac{1}{3}+k+\frac{1}{6}$$
$$3k+k=1-\left(\frac{1}{6}+\frac{1}{3}+\frac{1}{6}\right)$$
$$4k=\frac{1}{3}$$
$$k=\frac{1}{12}$$

6 (a) C

Let X be 'the number of diamonds drawn'.

When 2 cards are drawn, with replacement, you may get 0, 1 or 2 of any of the three suits.

Hence 0, 1 or 2 diamonds are possible.

(b) $P(\text{diamond})=\frac{3}{10}$

$=0.3$

Let X be 'number of diamonds drawn'.

The sample space is: $\{\text{DD, DH, DS, HH, HD, HS, SS, SD, SH}\}$

$$\begin{aligned}P(X=0)&=P(HH)+P(HS)+P(SS)+P(SH)\\&=\frac{5}{10}\times\frac{5}{10}+\frac{5}{10}\times\frac{2}{10}+\frac{2}{10}\times\frac{2}{10}+\frac{2}{10}\times\frac{5}{10}\\&=0.49\end{aligned}$$

$$\begin{aligned}P(X=1)&=P(DS)+P(SD)+P(DH)+P(HD)\\&=\frac{3}{10}\times\frac{2}{10}+\frac{2}{10}\times\frac{3}{10}+\frac{3}{10}\times\frac{5}{10}+\frac{5}{10}\times\frac{3}{10}\\&=0.42\end{aligned}$$

$$\begin{aligned}P(X=2)&=P(DD)\\&=\frac{3}{10}\times\frac{3}{10}\\&=0.09\end{aligned}$$

x	0	1	2
$P(X=x)$	0.49	0.42	0.09

(c) The one red card can be the first card or the second card.

$$P(\text{red})=\frac{8}{10}=\frac{4}{5}=0.8$$

$$P(\text{not red})=\frac{2}{10}=\frac{1}{5}=0.2$$

Let X be ‘number of red cards drawn’.

$$P(X=1)=\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}\times\frac{4}{5}=\frac{8}{25}=0.32$$

The probability of drawing exactly one red card from the pack is 0.32

(d) $P(\text{at least 1 red})=1-P(\text{not red first, not red second})$

$$=1-\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}=0.96$$

The probability of drawing at least one red card from the pack is 0.96.

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8 B

$$\begin{aligned} 1 &= (k+0.1)+(k-0.1)+(k+0.55)+(k-0.15) \\ 1 &= 4k+0.4 \\ 4k &= 1-0.4 \\ 4k &= 0.6 \\ k &= 0.15 \end{aligned}$$

The value of k that makes the table a discrete probability distribution table is 0.15

10 (a) Total number of parts in the ratio is $3+6+1=10$

Let X be 'number of bedrooms'.

$$\begin{aligned} P(X=2) &= \frac{3}{10} \\ &= 0.3 \end{aligned}$$

$$\begin{aligned} P(X=3) &= \frac{6}{10} \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} P(X=4) &= \frac{1}{10} \\ &= 0.1 \end{aligned}$$

x	2	3	4
$P(X=x)$	0.3	0.6	0.1

(b) A discrete probability distribution can be graphed using a bar graph or a dot graph because there are only three distinct outcomes.

12 (a) The sample space is: $\{\text{TT, TH, HT, HH}\}$

$$\begin{aligned} P(T=0) &= P(HH) \\ &= \frac{1}{2}\times\frac{1}{2} \\ &= 0.25 \end{aligned}$$

$$\begin{aligned} P(T=1) &= P(TH)+P(HT) \\ &= \frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2} \\ &= 0.5 \end{aligned}$$

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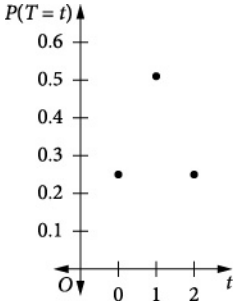
$$\begin{aligned} P(T=2) &= P(TT) \\ &= \frac{1}{2}\times\frac{1}{2} \\ &= 0.25 \end{aligned}$$

t	0	1	2
$P(T=t)$	0.25	0.5	0.25

(b) You could draw a bar graph or a dot graph.



14 (a) The sample space is: $\{$H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6$\}$

H1 $\rightarrow$ $X=1$; H2 $\rightarrow$ $X=4$; H3 $\rightarrow$ $X=9$; H4 $\rightarrow$ $X=16$; H5 $\rightarrow$ $X=25$; H6 $\rightarrow$ $X=36$

T1 $\rightarrow$ $X=1$; T2 $\rightarrow$ $X=2$; T3 $\rightarrow$ $X=3$; T4 $\rightarrow$ $X=4$; T5 $\rightarrow$ $X=5$; T6 $\rightarrow$ $X=6$

$$P(X=1)=P(X=4)=\frac{2}{12}=\frac{1}{6}$$

$$\begin{aligned} P(X=2) &= P(X=3) \\ &= P(X=5) \\ &= P(X=6) \\ &= P(X=9) \\ &= P(X=16) \\ &= \frac{1}{12} \end{aligned}$$

The probability distribution of X is shown in the table.

x	1	2	3	4	5	6	9	16	25	36
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

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(b) $P(X \le 12) = 1 - P(X > 12)$

$$= 1 - [P(X = 16) + P(X = 25) + P(X = 36)]$$
$$= 1 - \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right)$$
$$= \frac{9}{12}$$
$$= \frac{3}{4}$$

(c) $P(X \ge 16) = P(X = 16) + P(X = 25) + P(X = 36)$

$$= \left(\frac{1}{12} + \frac{1}{12} + \frac{1}{12}\right)$$
$$= \frac{3}{12}$$
$$= \frac{1}{4}$$

(d) The new sample space (given coin shows heads) is the six equally likely outcomes:

$\{1,\ 4,\ 9,\ 16,\ 25,\ 36\}$

$$P(X \le 9 \mid H) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$
$$= \frac{1}{2}$$

16 (a) The sample space is:

$$\left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix}\right\}$$

$$P(Y = 2) = P(1,1) = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$$

$$
\begin{aligned}
P(Y=3) &= P(1,2)+P(2,1)\\
&= \frac{1}{9}\times\frac{2}{9}+\frac{2}{9}+\frac{1}{9}\\
&= \frac{4}{81}
\end{aligned}
$$

$$
\begin{aligned}
P(Y=4) &= P(1,3)+P(2,2)+P(3,1)\\
&= \frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}\\
&= \frac{6}{81}\\
&= \frac{2}{27}
\end{aligned}
$$

$$
\begin{aligned}
P(Y=5) &= P(1,4)+P(2,3)+P(3,2)+P(4,1)\\
&= \frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}+\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}\\
&= \frac{8}{81}
\end{aligned}
$$

$$
\begin{aligned}
P(Y=6) &= P(1,5)+P(2,4)+P(3,3)+P(4,2)+P(5,1)\\
&= \frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}\\
&= \frac{11}{81}
\end{aligned}
$$

$$
\begin{aligned}
P(Y=7) &= P(1,6)+P(2,5)+P(3,4)+P(4,3)+P(5,2)+P(6,1)\\
&= \frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}+\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}+\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}\\
&= \frac{12}{81}\\
&= \frac{4}{27}
\end{aligned}
$$

$$
\begin{aligned}
P(Y=8) &= P(2,6)+P(3,5)+P(4,4)+P(5,3)+P(6,2)\\
&= \frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}\\
&= \frac{14}{81}
\end{aligned}
$$

$$\begin{aligned}P(Y=9)&=P(3,6)+P(4,5)+P(5,4)+P(6,3)\\&=\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}+\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}\\&=\frac{8}{81}\end{aligned}$$

$$\begin{aligned}P(Y=10)&=P(4,6)+P(5,5)+P(6,4)\\&=\frac{2}{9}\times\frac{2}{9}+\frac{1}{9}\times\frac{1}{9}+\frac{2}{9}\times\frac{2}{9}\\&=\frac{9}{81}\\&=\frac{1}{9}\end{aligned}$$

$$\begin{aligned}P(Y=11)&=P(5,6)+P(6,5)\\&=\frac{1}{9}\times\frac{2}{9}+\frac{2}{9}\times\frac{1}{9}\\&=\frac{4}{81}\end{aligned}$$

$$\begin{aligned}P(Y=12)&=P(6,6)\\&=\frac{2}{9}\times\frac{2}{9}\\&=\frac{4}{81}\end{aligned}$$

The probability distribution of Y is shown in the table.

y	2	3	4	5	6	7	8	9	10	11	12
$P(Y=y)$	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{6}{81}$	$\frac{8}{81}$	$\frac{11}{81}$	$\frac{12}{81}$	$\frac{14}{81}$	$\frac{8}{81}$	$\frac{9}{81}$	$\frac{4}{81}$	$\frac{4}{81}$

(b) $P(Y\geq 4)=1-P(Y<4)$

$$\begin{aligned}&=1-\left[P(2)+P(3)\right]\\&=1-\left(\frac{1}{81}+\frac{4}{81}\right)\\&=1-\frac{5}{81}\\&=\frac{76}{81}\end{aligned}$$

(c) $P(Y<7)=P(2)+P(3)+P(4)+P(5)+P(6)$

$$=\frac{1}{81}+\frac{4}{81}+\frac{6}{81}+\frac{8}{81}+\frac{11}{81}$$
$$=\frac{30}{81}$$
$$=\frac{10}{27}$$

(d) $P(3\le Y\le 9)=P(3)+P(4)+P(5)+P(6)+P(7)+P(8)+P(9)$

$$=\frac{4}{81}+\frac{6}{81}+\frac{8}{81}+\frac{11}{81}+\frac{12}{81}+\frac{14}{81}+\frac{8}{81}$$
$$=\frac{63}{81}$$
$$=\frac{7}{9}$$

(e) $P(Y\ge 6\cap Y\le 10)=P(6)+P(7)+P(8)+P(9)+P(10)$

$$=\frac{11}{81}+\frac{12}{81}+\frac{14}{81}+\frac{8}{81}+\frac{9}{81}$$
$$=\frac{54}{81}$$
$$=\frac{2}{3}$$

$$P(Y\le 10)=1-\left[P(11)+P(12)\right]$$
$$=1-\left(\frac{4}{81}+\frac{4}{81}\right)$$
$$=\frac{73}{81}$$

$$P(Y\ge 6\,|\,Y\le 10)=\frac{P(Y\ge 6\cap Y\le 10)}{P(Y\le 10)}$$
$$=\frac{2}{3}\div\frac{73}{81}$$
$$=\frac{54}{73}$$

18 (a) $P(\text{scoring at least one goal})=1-P(\text{scoring no goals})$

$P(\text{scoring no goals})<0.05$

Let M be 'misses a goal'.

Use trial and error to see how many goals Lou needs to take so the probability is less than 0.05

$P(M) = 1 - 0.71 = 0.29$

$P(MM) = 0.29 \times 0.29 = 0.0841$

$P(MMM) = 0.29 \times 0.29 \times 0.29 = 0.024\ 38 < 0.05$

Lou would need to take at least 3 shots at goal to be at least 95% certain of scoring at least 1 goal.

(b) $P(\text{scoring at least one goal}) = 1 - P(\text{scoring no goals})$

$P(\text{scoring no goals}) < 0.05$

Let M be 'misses a goal'.

Use trial and error to see how many goals Buddy needs to take so the probability is less than 0.05

$P(M) = 1 - 0.68 = 0.32$

$P(MM) = 0.32 \times 0.32 = 0.1024$

$P(MMM) = 0.32 \times 0.32 \times 0.32 = 0.032\ 768 < 0.05$

Buddy would need to take at least 3 shots at goal to be at least 95% certain of scoring at least 1 goal.

(c) $P(\text{scoring at least one goal}) = 1 - P(\text{scoring no goals})$

$P(\text{scoring no goals}) < 0.05$

Let M be 'misses a goal'.

Use trial and error to see how many goals Angela needs to take so the probability is less than 0.05

$P(M) = 0.32$

$$\begin{aligned} P(MM) &= 0.32 \times 0.32 \\ &= 0.1024 \end{aligned}$$

$$\begin{aligned} P(MMM) &= 0.32 \times 0.32 \times 0.32 \\ &= 0.032768 \end{aligned}$$

Buddy would need to take at least 3 shots at goal to be at least 95% certain of scoring at least 1 goal.

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EXERCISE 10.2 EXPECTED VALUE, VARIANCE AND STANDARD DEVIATION OF DISCRETE PROBABILITY DISTRIBUTIONS

2 **(a)** $0.4+0.1+a+0.1+b=1$

$a+b=0.4 \quad [1]$

$$2\times0.4+3\times0.1+4\times a+5\times0.1+6\times b=3.8$$
$$4a+6b=2.2$$
$$2a+3b=1.1 \quad [2]$$

$[1]\times 2$

$2a+2b=0.8 \quad [3]$

$[2]-[3]$

$b=0.3$

Substitute $b=0.3$ into $[1]$

$$a+0.3=0.4$$
$$a=0.1$$

$\therefore a=0.1,\ b=0.3$

(b) $0.05+i+0.05+j+0.55=1$

$i+j=0.35 \quad [1]$

$$(-3)\times0.05+(-2)\times i+(-1)\times0.05+0\times j+1\times0.55=0.15$$
$$-2i=-0.2$$
$$i=0.1$$

Substitute $i=0.1$ into $[1]$

$$0.1+j=0.35$$
$$j=0.25$$

$\therefore i=0.1,\ j=0.25$

(c) $\frac{3}{20}+\frac{5}{20}+i+\frac{7}{20}+j=1$

$i+j=\frac{1}{4} \quad [1]$

$$2\times\frac{3}{20}+4\times\frac{5}{20}+6\times i+8\times\frac{7}{20}+10\times j=6\frac{2}{5}$$

$$6i+10j=\frac{23}{10}$$

$$3i+5j=\frac{23}{20} \quad [2]$$

$[1]\times 3$

$$3i+3j=\frac{3}{4} \quad [3]$$

$[2]-[3]$

$$2j=\frac{23}{20}-\frac{3}{4}=\frac{23}{20}-\frac{15}{20}=\frac{8}{20}=\frac{2}{5}$$

$$j=\frac{1}{5}$$

Substitute $j=\frac{1}{5}$ into $[1]$

$$i+\frac{1}{5}=\frac{1}{4}$$

$$i=\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$$

$$\therefore i=\frac{1}{20},\ j=\frac{1}{5}$$

(d) $a+\frac{1}{5}+\frac{1}{15}+b+\frac{1}{5}=1$

$$a+b=\frac{8}{15} \quad [1]$$

$$(-3)\times a+(-1)\times\frac{1}{5}+1\times\frac{1}{15}+3\times b+5\times\frac{1}{5}=1\frac{4}{15}$$

$$-3a+3b=\frac{2}{5}$$

$$-a+b=\frac{2}{15} \quad [2]$$

$[1]-[2]$

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$$2a=\frac{6}{15}=\frac{2}{5}$$
$$a=\frac{1}{5}$$

$$[1]+[2]$$

$$2b=\frac{10}{15}=\frac{2}{3}$$
$$b=\frac{1}{3}$$

$$\therefore a=\frac{1}{5},\ b=\frac{1}{3}$$

4 **(a)** $E(X)=\sum x_i p_i$

$$=1\times0.1+2\times0.3+3\times0.25+4\times0.05+5\times0.15+6\times0.15$$
$$=3.3$$

$$Var(X)=E(X^2)-[E(X)]^2$$
$$=1\times0.1+2^2\times0.3+3^2\times0.25+4^2\times0.05+5^2\times0.15+6^2\times0.15-3.3^2$$
$$=2.61$$

The expected value is 3.3 and the variance is 2.61

(b) $E(X)=\sum x_i p_i$

$$=5\times0.15+6\times0.35+7\times0.1+8\times0.25+9\times0.15$$
$$=6.9$$

$$Var(X)=E(X^2)-[E(X)]^2$$
$$=5^2\times0.15+6^2\times0.35+7^2\times0.1+8^2\times0.25+9^2\times0.15-6.9^2$$
$$=1.79$$

The expected value is 6.9 and the variance is 1.79

6 **(a)** $E(Y)=\sum y_i p_i$

$$=(-3)\times0.02+(-2)\times0.03+(-1)\times0.25+0\times0.35+1\times0.3+2\times0.05$$
$$=0.03$$

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$$\begin{aligned}Var(Y) &= E(Y^2)-[E(Y)]^2\\ &= (-3)^2\times 0.02+(-2)^2\times 0.03+(-1)^2\times 0.25+0^2\times 0.35+1^2\times 0.3+2^2\times 0.05-0.03^2\\ &= 1.0491\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{Var(Y)}\\ &= \sqrt{1.0491}\\ &\approx 1.024\end{aligned}$$

(b) $\mu = 0.03$

$\sigma = 1.024$

$$\begin{aligned}\mu-2\sigma &= 0.03-2\times 1.024\\ &= -2.018\end{aligned}$$

$$\begin{aligned}\mu+2\sigma &= 0.03+2\times 1.024\\ &= 2.078\end{aligned}$$

$$P(\mu-2\sigma\le Y\le \mu+2\sigma) = P(Y=-2)+P(Y=-1)+P(Y=0)+P(Y=1)+P(Y=2)$$

$$\begin{aligned}P(-2.018\le Y\le 2.078) &= 0.03+0.25+0.35+0.3+0.05\\ &= 0.98\end{aligned}$$

8 **(a)** $E(W) = 2\times\frac{5}{16}+4\times\frac{1}{8}+6\times\frac{3}{8}+8\times\frac{3}{16}$

$= 4\frac{7}{8}$

(b) $E(3W-4) = 3E(W)-4$

$= 3\times 4\frac{7}{8}-4$

$= 10\frac{5}{8}$

(c) $E(2W+5) = 2E(W)+5$

$= 2\times 4\frac{7}{8}+5$

$= 14\frac{3}{4}$

(d) $E(W^2-7) = E(W^2)-7$

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$$= 2^2 \times \frac{5}{16} + 4^2 \times \frac{1}{8} + 6^2 \times \frac{3}{8} + 8^2 \times \frac{3}{16} - 7$$
$$= 21\frac{3}{4}$$

10 C

$$E(Y) = 0\times\frac{1}{9} + 1\times\frac{7}{18} + 2\times\frac{1}{3} + 3\times\frac{1}{6} = \frac{28}{18} = \frac{14}{9}$$
$$E(Y^2) = 0^2\times\frac{1}{9} + 1^2\times\frac{7}{18} + 2^2\times\frac{1}{3} + 3^2\times\frac{1}{6} = \frac{58}{18} = \frac{29}{9}$$
$$Var(Y) = E(Y^2) - \left[E(Y)\right]^2$$
$$= \left(\frac{29}{9}\right) - \left(\frac{14}{9}\right)^2$$
$$= \frac{29\times 9}{81} - \frac{196}{81}$$
$$= \frac{65}{81}$$

12 (a) $E(D) = 1\times\frac{1}{12} + 2\times\frac{1}{6} + 3\times\frac{1}{4} + 4\times\frac{1}{4} + 5\times\frac{1}{6} + 6\times\frac{1}{12}$

$$= 3\frac{1}{2}$$

(b) $E(D^2) = 1^2\times\frac{1}{12} + 2^2\times\frac{1}{6} + 3^2\times\frac{1}{4} + 4^2\times\frac{1}{4} + 5^2\times\frac{1}{6} + 6^2\times\frac{1}{12}$

$$= 14\frac{1}{6}$$

$$\sigma^2 = 14\frac{1}{6} - \left(3\frac{1}{2}\right)^2$$
$$= \frac{23}{12}$$
$$\sigma = \sqrt{\frac{23}{12}}$$
$$= 1.3844...$$

$$\mu + 2\sigma = 3\frac{1}{2} + 2\times 1.3844...$$
$$\approx 6.27$$

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$$\begin{aligned}\mu - 2\sigma &= 3\tfrac{1}{2} - 2\times 1.3844... \\ &\approx 0.73\end{aligned}$$

The range is 0.73 to 6.27

(c) As the range of $\mu \pm 2\sigma$ does cover all of the possible results $\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ when rolling the die, none of the values should be considered unusual.

14 (a)

g	0	1	2	3	4
$P(G=g)$	$6k$	$5k$	$4k$	$3k$	$2k$

$$\begin{aligned}1 &= 6k + 5k + 4k + 3k + 2k \\ 1 &= 20k \\ k &= \frac{1}{20}\end{aligned}$$

(b) $E(G) = 0\times\frac{6}{20} + 1\times\frac{5}{20} + 2\times\frac{4}{20} + 3\times\frac{3}{20} + 4\times\frac{2}{20}$

$$= 1\frac{1}{2}$$

(c) $E(G^2) = 0^2\times\frac{6}{20} + 1^2\times\frac{5}{20} + 2^2\times\frac{4}{20} + 3^2\times\frac{3}{20} + 4^2\times\frac{2}{20}$

$$\begin{aligned}&= \frac{5}{20} + \frac{16}{20} + \frac{27}{20} + \frac{32}{20} \\ &= \frac{80}{20} \\ &= 4\end{aligned}$$

$$\begin{aligned}Var(G) &= E(G^2) - [E(G)]^2 \\ &= 4 - \left(\frac{3}{2}\right)^2 \\ &= 1\frac{3}{4}\end{aligned}$$

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(d) $\sigma = \sqrt{Var(G)}$

$= \sqrt{\frac{7}{4}}$

$= \frac{\sqrt{7}}{2}$

$= 1.3228...$

≈ 1.32

16 (a) (i) If the player selects a black card, they gain \$1. So Enrico loses \$1.

(ii) If the player selects a diamond, they gain \$5. So Enrico loses \$5.

(b) If the player selects a heart, Enrico collects the game fee.

(c) Let $X =$ amount won by Enrico (\$)

x	-5	-1	p
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(d) $E(X) = (-5)\times\frac{1}{4} + (-1)\times\frac{1}{2} + p\times\frac{1}{4}$

$0 = -\frac{7}{4} + \frac{p}{4}$

$\frac{p}{4} = \frac{7}{4}$

$p = 7$

When $p = 7$, the expected value for the distribution is 0.

(e) Enrico must charge more than \$7 if he wishes to make a profit. Hence, the minimum whole dollar amount he should charge to play the game would be \$8.

18 (a)

Colour	Yellow	Blue	Red
$P(X = x)$	$\frac{5}{9}$	$\frac{3}{9}$	$\frac{1}{9}$
$x =$ payment	\$1	\$3	\$5

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(b) $E(X)=\frac{5}{9}\times1+\frac{3}{9}\times3+\frac{1}{9}\times5$

$=2\frac{1}{9}$

(c) Being charged an amount equivalent to the expected gain would result in a long term net gain of zero.

Charge $\$2\frac{1}{9}\approx 2.11$.

If the game is to be fair, you should pay \$2.11 to play.

(d) It is unlikely that this would be the cost of any game in real life, so players would likely be charged \$2.50 or \$3.00 to play the game in which case the operator will make a profit.

20 (a) $E(Y)=1\times0.1+2\times0.2+3\times0.3+4\times0.4$

$=3$

(b) $E(X)=0\times0.6+1\times0.1+2\times0.1+3\times0.1+4\times0.1$

$=1$

(c) (i) The sample space is:

$$\begin{Bmatrix}(1,0) & (1,1) & (1,2) & (1,3) & (1,4)\\(2,0) & (2,1) & (2,2) & (2,3) & (2,4)\\(3,0) & (3,1) & (3,2) & (3,3) & (3,4)\\(4,0) & (4,1) & (4,2) & (4,3) & (4,4)\end{Bmatrix}$$

$$\begin{aligned}P(M=1)&=P(1,0)\\&=0.1\times0.6\\&=0.06\end{aligned}$$

$$\begin{aligned}P(M=2)&=P(2,0)+P(1,1)\\&=0.2\times0.6+0.1\times0.1\\&=0.13\end{aligned}$$

$$\begin{aligned}P(M=3)&=P(3,0)+P(2,1)+P(1,2)\\&=0.3\times0.6+0.2\times0.1+0.1\times0.1\\&=0.21\end{aligned}$$

$$\begin{aligned}P(M=4)&=P(4,0)+P(3,1)+P(2,2)+P(1,3)\\&=0.4\times0.6+0.3\times0.1+0.2\times0.1+0.1\times0.1\\&=0.3\end{aligned}$$

$$\begin{aligned}P(M=5)&=P(4,1)+P(3,2)+P(2,3)+P(1,4)\\&=0.4\times0.1+0.3\times0.1+0.2\times0.1+0.1\times0.1\\&=0.1\end{aligned}$$

$$\begin{aligned}P(M=6)&=P(4,2)+P(3,3)+P(2,4)\\&=0.4\times0.1+0.3\times0.1+0.2\times0.1\\&=0.09\end{aligned}$$

$$\begin{aligned}P(M=7)&=P(4,3)+P(3,4)\\&=0.4\times0.1+0.3\times0.1\\&=0.07\end{aligned}$$

$$\begin{aligned}P(M=8)&=P(4,4)\\&=0.4\times0.1\\&=0.04\end{aligned}$$

The new probability distribution table is:

m	1	2	3	4	5	6	7	8
$P(M=m)$	0.06	0.13	0.21	0.3	0.1	0.09	0.07	0.04

(ii) $E(M)=1\times0.06+2\times0.13+3\times0.21+4\times0.3+5\times0.1+6\times0.09+7\times0.07+8\times0.04$

$=4$

(d) $E(X)+E(Y)=3+1=E(M)$

$\therefore E(M)=E(X)+E(Y)$

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EXERCISE 10.3 THE UNIFORM DISTRIBUTION

2 (a) Since all keys are equally likely, each number of attempts is equally likely, and so this is a uniform distribution.

Since there are nine keys, $n = 9$.

$$\begin{aligned} E(X) &= \frac{n+1}{2} \\ &= \frac{9+1}{2} \\ &= 5 \end{aligned}$$

The expected number of attempts is 5

(b) $$\begin{aligned} Var(X) &= \frac{n^2-1}{12} \\ &= \frac{9^2-1}{12} \\ &= 6\frac{2}{3} \end{aligned}$$

4 (a) C

$$\begin{aligned} E(X) &= \frac{n+1}{2} \\ 11.5 &= \frac{n+1}{2} \\ 23 &= n+1 \\ n &= 22 \end{aligned}$$

(b) D

$$\begin{aligned} Var(X) &= \frac{n^2-1}{12} \\ 14 &= \frac{n^2-1}{12} \\ 168 &= n^2-1 \\ n^2 &= 169 \\ n &= 13 \end{aligned}$$

6 (a) The minutes possible are: $00,\ 01,\ 02, \ldots, 58,\ 59$.

The numbers are evenly distributed between 00 and 59.

Since the probabilities are all equal, the distribution is symmetric, and so the mean will be the middle of 00 and 59.

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$E(T) = \frac{0+59}{2} = 29.5$

(b) $Var(T) = \frac{n^2 - 1}{12}$

$= \frac{60^2 - 1}{12}$
$= 299.91\overline{6}$

$\sigma(T) = \sqrt{Var(T)}$
$= \sqrt{299.91\overline{6}}$
$= 17.3181...$

$Var(T) = 299.917$ and $\sigma(T) = 17.318$, correct to 3 decimal places

(c) $Var(T)$ and $\sigma(T)$ won't change because they are measures of spread, and the new set of data has the same range or spread as the original set of data.

If the numbers shown were 1 to 60, instead of 0 to 59, then these values would be the same.

EXERCISE 10.4 DISCRETE DISTRIBUTIONS IN PRACTICAL SITUATIONS

2 (a)

Number	1	2	3	4	5	6
z	1	-2	-3	4	-5	6
$P(Z=z)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(b) All the probabilities are equal, so this is a uniform distribution.

(c) $E(Z) = 1\times\frac{1}{6} + (-2)\times\frac{1}{6} + (-3)\times\frac{1}{6} + 4\times\frac{1}{6} + (-5)\times\frac{1}{6} + 6\times\frac{1}{6}$

$= \frac{1}{6}$

(d) The game is not fair as it is biased in favour of the player.

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4 (a) $1 = k + 2k + 3k + 4k + k(9-5) + k(9-6) + k(9-7) + k(9-8)$

$$1 = 20k$$
$$k = \frac{1}{20}$$
$$k = 0.05$$

(b)

x	1	2	3	4	5	6	7	8
$P(X=x)$	k	$2k$	$3k$	$4k$	$4k$	$3k$	$2k$	k
$P(X=x)$	0.05	0.1	0.15	0.2	0.2	0.15	0.1	0.05

(c) $E(X) = 1\times 0.05 + 2\times 0.1 + 3\times 0.15 + 4\times 0.2 + 5\times 0.2 + 6\times 0.15 + 7\times 0.1 + 8\times 0.05$

$$= 4.5$$

(d)

$$E(X^2) = 1^2\times 0.05 + 2^2\times 0.1 + 3^2\times 0.15 + 4^2\times 0.2 + 5^2\times 0.2 + 6^2\times 0.15 + 7^2\times 0.1 + 8^2\times 0.05$$

$$= 23.5$$
$$Var(X) = E(X^2) - E(X)^2$$
$$= 23.5 - 4.5^2$$
$$= 3.25$$

6 $P(5) = \dfrac{4-5}{5}$

$$= -\frac{1}{5}$$

Tomino's probability distribution is incorrect because there is a negative probability for $x = 5$ and probabilities cannot be negative.

8 (a) The sample space is:

$$\begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

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There are 36 ordered pairs.

(b) Count the number of ordered pairs where each number is a maximum. For example, there is only one ordered pair with 1 the maximum, $(1,1)$, there are 3 ordered pairs with 2 the maximum, and so on.

z	1	2	3	4	5	6
$P(Z=z)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

(c) Count the number of ordered pairs where each number is a minimum. For example, there are eleven ordered pairs with 1 the minimum, there are nine ordered pairs with 2 the minimum, and so on.

y	1	2	3	4	5	6
$P(Y=y)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(d) The two distributions are mirror images of each other.

(e) (i) $E(Z)=1\times\frac{1}{36}+2\times\frac{3}{36}+3\times\frac{5}{36}+4\times\frac{7}{36}+5\times\frac{9}{36}+6\times\frac{11}{36}$

$=4\frac{17}{36}$

(ii) $E(Y)=1\times\frac{11}{36}+2\times\frac{9}{36}+3\times\frac{7}{36}+4\times\frac{5}{36}+5\times\frac{3}{36}+6\times\frac{1}{36}$

$=2\frac{19}{36}$

(f) $6-4\frac{17}{36}=1\frac{19}{36}$

The difference between the greatest value of Z and $E(Z)$ is $1\frac{19}{36}$

(g) $2\frac{19}{36}-1=1\frac{19}{36}$

The difference between the least value of Y and $E(Y)$ is $1\frac{19}{36}$

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(h) The difference between the greatest value of Z and $E(Z)$ is $1\frac{19}{36}$, which is equal to the difference between the least value of Y and $E(Y)$ is $1\frac{19}{36}$. This is because the two distributions are mirror images of each other.

(i) $Var(Z) = Var(Y)$ as the mirror images have the same spread, so any measure of spread, like variance or standard deviation, would be the same.

$$Var(Y) = 1\frac{926}{1296}$$

10 (a) $E(X) = 1\times 0.25 + 2\times 0.1 + 3\times 0.45 + 4\times 0.2$

$= 2.6$

(b) $$\begin{aligned} E(2X) &= 2\times 0.25 + 4\times 0.1 + 6\times 0.45 + 8\times 0.2 \\ &= 5.2 \end{aligned}$$

$$\begin{aligned} 2E(X) &= 2.6\times 2 \\ &= 5.2 \end{aligned}$$

$$E(2X) = 2E(X)$$

(c) $$\begin{aligned} E(X^2) &= 1^2\times 0.25 + 2^2\times 0.1 + 3^2\times 0.45 + 4^2\times 0.2 \\ &= 7.9 \end{aligned}$$

$$\begin{aligned} Var(X) &= E(X^2) - E(X)^2 \\ &= 7.9 - 2.6^2 \\ &= 1.14 \end{aligned}$$

(d) (i) $$\begin{aligned} E\left((2X)^2\right) &= 2^2\times 0.25 + 4^2\times 0.1 + 6^2\times 0.45 + 8^2\times 0.2 \\ &= 31.6 \end{aligned}$$

$$\begin{aligned} Var(2X) &= E\left((2X)^2\right) - E(2X)^2 \\ &= 31.6 - 5.2^2 \\ &= 4.56 \end{aligned}$$

(ii) $$\begin{aligned} E(3X) &= 3E(X) \\ &= 3\times 2.6 \\ &= 7.8 \end{aligned}$$

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$$E\left((3X)^2\right) = 3^2 \times 0.25 + 6^2 \times 0.1 + 9^2 \times 0.45 + 12^2 \times 0.2$$
$$= 71.1$$
$$Var(3X) = E\left((3X)^2\right) - E(3X)^2$$
$$Var(3X) = 71.1 - 7.8^2$$
$$= 10.26$$

(e) $4Var(X) = 4 \times 1.14$

$$= 4.56$$
$$2^2 Var(X) = Var(2X)$$
$$9Var(X) = 9 \times 1.14$$
$$= 10.26$$
$$3^2 Var(X) = Var(3X)$$
$$\therefore Var(kX) = k^2 Var(X)$$

CHAPTER REVIEW 10

2 (a)

$$\begin{Bmatrix} (3,3) & (3,4) & (3,5) & (3,6) & (3,7) & (3,8) \\ (4,3) & (4,4) & (4,5) & (4,6) & (4,7) & (4,8) \\ (5,3) & (5,4) & (5,5) & (5,6) & (5,7) & (5,8) \\ (6,3) & (6,4) & (6,5) & (6,6) & (6,7) & (6,8) \\ (7,3) & (7,4) & (7,5) & (7,6) & (7,7) & (7,8) \\ (8,3) & (8,4) & (8,5) & (8,6) & (8,7) & (8,8) \end{Bmatrix}$$

There are 36 equally likely outcomes.

$$P(X = 6) = P(3,3) = \frac{1}{36}$$

$$P(X = 7) = P(3,4) + P(4,3) = \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$$

$$P(X = 9) = P(3,6) + P(4,5) + P(5,4) + P(6,3) = \frac{4}{36} = \frac{1}{9}$$

$$P(X = 11) = P(3,8) + P(4,7) + P(5,6) + P(6,5) + P(7,4) + P(8,3) = \frac{6}{36} = \frac{1}{6}$$

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$$P(X=12)=P(4,8)+P(5,7)+P(6,6)+P(7,5)+P(8,4)=\frac{5}{36}$$

$$P(X=14)=P(6,8)+P(7,7)+P(8,6)=\frac{3}{36}=\frac{1}{12}$$

$$P(X=15)=P(7,8)+P(8,7)=\frac{2}{36}=\frac{1}{18}$$

The probability distribution of Y is shown in the table.

x	6	7	8	9	10	11	12	13	14	15	16
$P(X=x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

(b)

$$P(X\ge 11)=P(X=11)+P(X=12)+P(X=13)+P(X=14)+P(X=15)+P(X=16)$$
$$=\frac{1}{6}+\frac{5}{36}+\frac{1}{9}+\frac{1}{12}+\frac{1}{18}+\frac{1}{36}$$
$$=\frac{7}{12}$$

(c) $P(X<15)=1-P(X\ge 15)$

$$=1-\left[P(X=15)+P(X=16)\right]$$
$$=1-\left(\frac{1}{18}+\frac{1}{36}\right)$$
$$=\frac{11}{12}$$

(d) $P(7\le X\le 11)=P(X=7)+P(X=8)+P(X=9)+P(X=10)+P(X=11)$

$$=\frac{1}{18}+\frac{1}{12}+\frac{1}{9}+\frac{5}{36}+\frac{1}{6}$$
$$=\frac{5}{9}$$

4 **(a)** $E(X)=(-2)\times 0.15+(-1)\times 0.2+0\times 0.1+1\times 0.35+2\times 0.2$

$$=0.25$$

(b) $E(X^2)=(-2)^2\times 0.15+(-1)^2\times 0.2+0^2\times 0.1+1^2\times 0.35+2^2\times 0.2$

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$$=1.95$$

$$Var(X) = 1.95 - 0.25^2$$

$$= 1.8875$$

(c) $E(3X-2) = 3E(X) - 2$

$$= 3 \times 0.25 - 2$$

$$= -1.25$$

(d) $E(X^2 - 2X) = E(X^2) - 2E(X)$

$$= 1.95 - 2 \times 0.25$$

$$= 1.45$$

(e) $Var(3X-2) = 3^2 Var(X)$

$$= 9 \times 1.8875$$

$$= 16.9875$$

6 Random numbers are equally likely, so this is a uniform distribution with $n = 20$.

(a) $E(R) = \dfrac{n+1}{2}$

$$= \frac{20+1}{2}$$

$$= 10\frac{1}{2}$$

(b) $Var(R) = \dfrac{20^2 - 1}{12}$

$$= 33\frac{1}{4}$$

8 Let $P(T) = x$

$$P(H) = 3 \times P(T)$$

$$= 3x$$

$$P(H) + P(T) = 1$$

$$x + 3x = 1$$

$$4x = 1$$

$$x = \frac{1}{4}$$

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$$E(X) = 100 \times \frac{1}{4}$$
$$= 25$$

10 Let D be defective and N be non-defective.

(a) Zero defective means drawing three non-defective items (DDD).

$$P(0) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}$$
$$= \frac{7}{24}$$

One defective means NND or NDN or DNN.

$$P(1) = \frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} + \frac{7}{10} \times \frac{3}{9} \times \frac{6}{8} + \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8}$$
$$= \frac{21}{40}$$

Two defectives means NDD or DND or DDN.

$$P(2) = \frac{7}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{7}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{7}{8}$$
$$= \frac{7}{40}$$

Three defectives

$$P(3) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$
$$= \frac{1}{120}$$

y	0	1	2	3
$P(Y = y)$	$\frac{7}{24}$	$\frac{21}{40}$	$\frac{7}{40}$	$\frac{1}{120}$

(b) $E(Y) = 0 \times \frac{7}{24} + 1 \times \frac{21}{40} + 2 \times \frac{7}{40} + 3 \times \frac{1}{120}$

$$= \frac{9}{10}$$

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12 C

$$\begin{aligned} P(1W) &= P(W,\ \text{not } W) + P(\text{not } W,\ W) \\ &= \frac{7}{12}\times\frac{5}{12}+\frac{5}{12}\times\frac{7}{12} \\ &= 2\times\frac{7}{12}\times\frac{5}{12} \end{aligned}$$

14 The sample space is:

$$\begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

$$P(Y=2) = P(1,1) = \frac{1}{36}$$

$$P(Y=3) = P(1,2) + P(2,1) = \frac{1}{6}+\frac{1}{6} = \frac{2}{36} = \frac{1}{18}$$

$$P(Y=4) = P(1,3) + P(2,2) + P(3,1) = \frac{3}{36} = \frac{1}{12}$$

$$P(Y=5) = P(1,4) + P(2,3) + P(3,2) + P(4,1) = \frac{4}{36} = \frac{1}{9}$$

$$P(Y=6) = P(1,5) + P(2,4) + P(3,3) + P(4,2) + P(5,1) = \frac{5}{36}$$

$$P(Y=7) = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) = \frac{6}{36} = \frac{1}{6}$$

$$P(Y=8) = P(2,6) + P(3,5) + P(4,4) + P(5,3) + P(6,2) = \frac{5}{36}$$

$$P(Y=9) = P(3,6) + P(4,5) + P(5,4) + P(6,3) = \frac{4}{36} = \frac{1}{9}$$

$$P(Y=10) = P(4,6) + P(5,5) + P(6,4) = \frac{3}{36} = \frac{1}{12}$$

$$P(Y=11) = P(5,6) + P(6,5) = \frac{2}{36} = \frac{1}{18}$$

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$$P(Y=12)=P(6,6)=\frac{1}{36}$$

(a) B

$$\begin{aligned}P(X\geq 10)&=P(X=10)+P(X=11)+P(X=12)\\&=\frac{1}{12}+\frac{1}{18}+\frac{1}{36}\\&=\frac{1}{6}\end{aligned}$$

(b) D

$$\begin{aligned}P(5<X\leq 9)&=P(X=6)+P(X=7)+P(X=8)+P(X=9)\\&=\frac{5}{36}+\frac{1}{6}+\frac{5}{36}+\frac{1}{9}\\&=\frac{5}{9}\end{aligned}$$

(c) A

$$\begin{aligned}E(X)&=2\times\frac{1}{36}+3\times\frac{1}{18}+4\times\frac{1}{12}+5\times\frac{1}{9}+6\times\frac{5}{36}+7\times\frac{1}{6}+8\times\frac{5}{36}+9\times\frac{1}{9}+10\times\frac{1}{12}+11\times\frac{1}{18}+12\times\frac{1}{36}\\&=7\end{aligned}$$

This could also be calculated using symmetry as $\frac{2+12}{36}=7$.

16 C

$$\begin{aligned}Var(X)&=E(X^2)-E(X)^2\\&=6.2-2.3^2\\&=0.91\\\sigma&=\sqrt{0.91}\\&\approx 0.95\end{aligned}$$

18 (a) C

This is a uniform distribution.

(b) D

$$\begin{aligned}18&=\frac{n+1}{2}\\36&=n+1\\n&=35\end{aligned}$$

(c) A

$$\begin{aligned}101&=\frac{n+1}{2}\\202&=n+1\\n&=201\end{aligned}$$

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$$26.5 = \frac{n+1}{2}$$
$$53 = n+1$$
$$n = 52$$

20 (a) $3a + 3b = 1$

Substitute $a = 3b$.

$$3 \times 3b + 3b = 1$$
$$12b = 1$$
$$b = \frac{1}{12}$$

$$a = 3b = \frac{3}{12} = \frac{1}{4}$$

$$a = \frac{1}{4},\ b = \frac{1}{12}$$

(b)

x	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$

(c) $E(X) = 1 \times \frac{1}{4} + 2 \times \frac{1}{12} + 3 \times \frac{1}{4} + 4 \times \frac{1}{12} + 5 \times \frac{1}{4} + 6 \times \frac{1}{12}$

$$= 3\frac{1}{4}$$

(d) $E(X^2) = 1^2 \times \frac{1}{4} + 2^2 \times \frac{1}{12} + 3^2 \times \frac{1}{4} + 4^2 \times \frac{1}{12} + 5^2 \times \frac{1}{4} + 6^2 \times \frac{1}{12}$

$$= \frac{85}{6}$$

$$Var(X) = E(X^2) - E(X)^2$$
$$= \frac{85}{6} - \left(3\frac{1}{4}\right)^2$$
$$= 2\frac{41}{48}$$

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(e) $P(Z>9)=P(Z=10)+P(Z=11)+P(Z=12)$

$$=P(4,6)+P(5,5)+P(6,4)+P(5,6)+P(6,5)+P(6,6)$$
$$=\frac{1}{12}\times\frac{1}{12}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{12}\times\frac{1}{12}+\frac{1}{4}\times\frac{1}{12}+\frac{1}{12}\times\frac{1}{4}+\frac{1}{12}\times\frac{1}{12}$$
$$=\frac{1}{8}$$

22 $P(X=3)=\dfrac{2\times3-3}{15}=\dfrac{3}{15}$

$P(X=4)=\dfrac{2\times4-3}{15}=\dfrac{5}{15}$

$P(X=5)=\dfrac{2\times5-3}{15}=\dfrac{7}{15}$

Calculate $\sum_{x=3}^{5}p(x)$.

$$\sum_{x=3}^{5}p(x)=\frac{3}{15}+\frac{5}{15}+\frac{7}{15}=\frac{15}{15}=1$$

Since each probability is positive and less than 1, and the probabilities add to 1, $P(X=x)$ is a probability distribution for $x\in\{3,\ 4,\ 5\}$.

When $x=1$, $P(X=1)=\dfrac{2\times1-3}{15}=-\dfrac{1}{15}$.

Since probabilities cannot be negative, $P(X=x)$ is not a probability distribution for $x\in\{1,\ 2,\ 3,\ 4,\ 5\}$.

24 (a) There are two possible outcomes with each throw: Odd (O) or Even (E)

The sample space is:

(E), (O, E), (O, O, E), (O, O, O, E), (O, O, O, O, E), (O, O, O, O, O)

Let X be the number of throws

The sample space for X is: $\{1,\ 2,\ 3,\ 4,\ 5\}$

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$$P(X=1)=P(\text{E})=\frac{1}{2}$$
$$P(X=2)=P(\text{O, E})=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$
$$P(X=3)=P(\text{O, O, E})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$$
$$P(X=4)=P(\text{O, O, O, E})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$$
$$\begin{aligned}P(X=5)&=P(\text{O, O, O, O, E})+P(\text{O, O, O, O, O})\\&=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\\&=\frac{1}{16}\end{aligned}$$

x	1	2	3	4	5
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

(b) $E(X)=1\times\frac{1}{2}+2\times\frac{1}{4}+3\times\frac{1}{8}+4\times\frac{1}{16}+5\times\frac{1}{16}$

$=\frac{31}{16}$
$=1.9375$

The expected number of rolls $E(X)=1.9,$ rounded to one decimal place.

(c) This is just the same as the die question as the relevant probabilities are equal. So the expected number of flips is 1.9.

26 (a) $E(X)=1\times\frac{1}{2}+2\times\frac{1}{4}+3\times\frac{1}{8}+4\times\frac{1}{16}+5\times\frac{1}{16}$

$=1.9375$

Karina and Achim should expect to print and sell 3 packs of business cards per hour.

(b) They expect to sell an average of 3.06 packs at an average price of $\$20$, with average running costs of $\$16$ per hour

Profit $=\$20\times3.06-\$16=\$45.20$

Their expected profit is \$45.20 per hour.

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