Name: _____

Task 1 – Advanced Mathematics Mock Test

Section I – Multiple Choice (5 Marks)

1. If $3^x = 7$, what is the value of x?

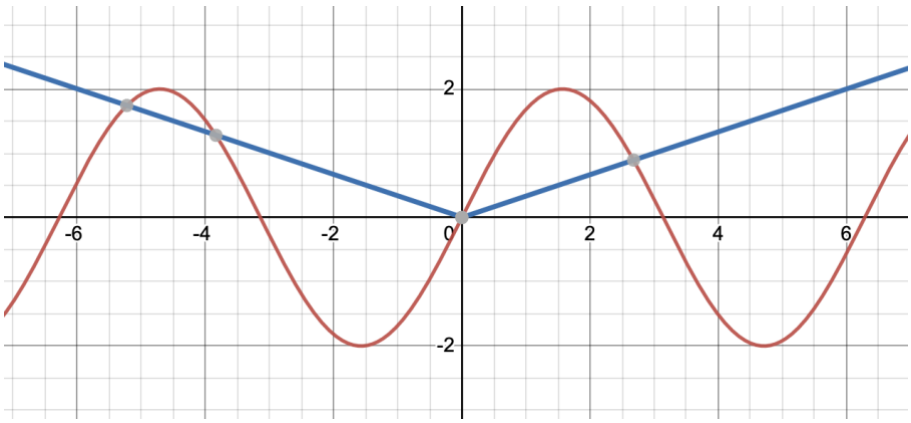
   (A) $\frac{7}{3}$

   (B) $\log_3 \frac{7}{3}$

   (C) $\log_3 4$

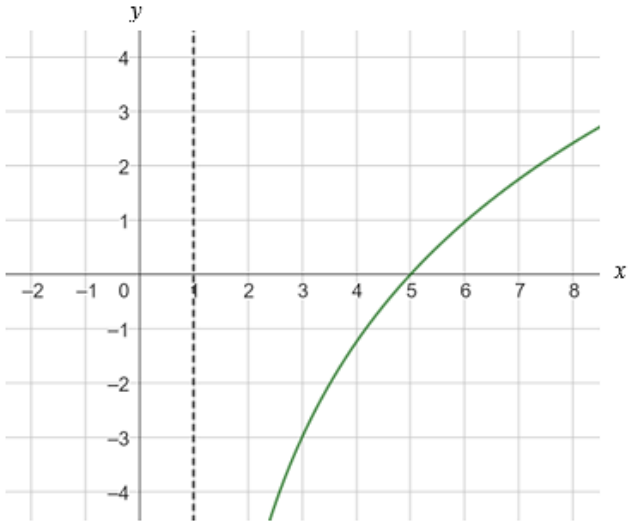
   (D) $\frac{\log_e 7}{\log_e 3}$

2. How many solutions are there to the equation $2\sin x = \left|\frac{x}{3}\right|$ in the domain $(-\infty, \infty)$.



   (A) 3 (B) 2 (C) 4 (D) 5

3. What could be the equation of the graph below?



   (A) $y = 3\log_2(x-1) - 6$ (C) $y = 3\log_2(x+1) - 3$

   (B) $y = 2\log_3(x-1) + 1$ (D) $y = 2\log_4(x-1) - 2$

4. The domain of the function $y = f(x)$ is $[-2, \infty)$.

What is the domain of the function $y = 3f(-2x) - 4$?

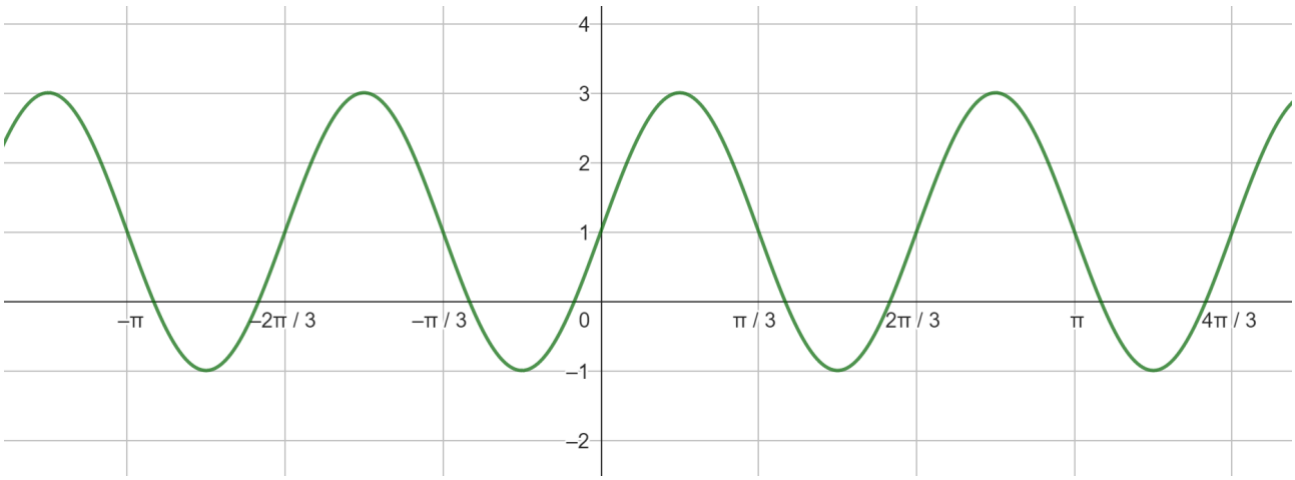
(A) $[1, \infty)$

(B) $(-\infty, -4]$

(C) $(-\infty, 1]$

(D) $[-4, \infty)$

5. In the diagram, a graph of a trigonometric function is given.



Which of the following could be the equation of the given graph?

(A) $y = 2\sin\left(3x - \frac{\pi}{2}\right) + 1$

(B) $y = 2\sin(2x - \pi) + 1$

(C) $y = 2\cos\left(3x - \frac{\pi}{2}\right) + 1$

(D) $y = 2\cos\left(2x + \frac{\pi}{2}\right) + 1$

End of Section 1

Section II – Show all necessary working (30 marks)

6. Differentiate $y = 2xe^{7x-3}$ **2**

7. Find the domain of $y = \ln(x - 4)$ **1**

8. Solve for θ, where $0 \leq \theta \leq 2\pi$ **3**

$$2\sin\left(\theta - \frac{\pi}{3}\right) = -1$$

9. It is given that $\log_3 6 = a$ and $\log_3 5 = b$. **1**

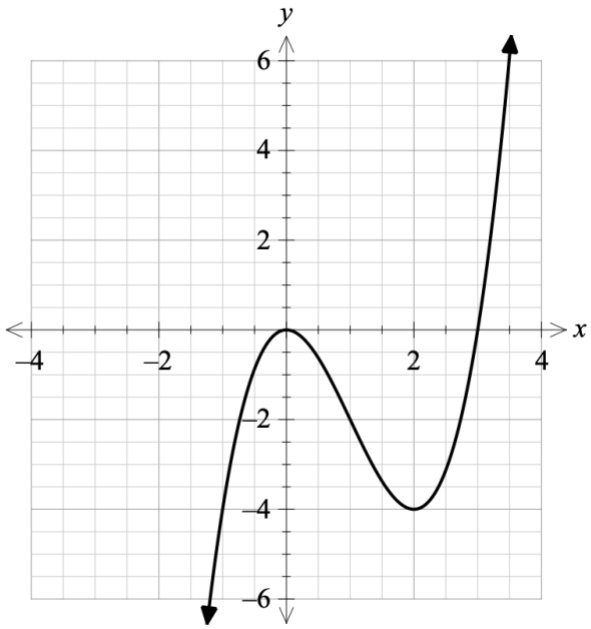
Express $\log_3 150$ in terms of a and b.

10. Describe two transformations which, when applied in succession, change the graph of $y = x^2$ to the graph with equation $y = \left(\frac{x+1}{2}\right)^2$. **2**

11. Solve $\log_2 x + \log_2 (x-3) = 2$. **3**

12. Given $y = 3e^{-2x}$, prove that $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ **2**

13. The graph of $y = f(x)$ is shown below. Sketch the curve $y = f(2 - x) + 1$, showing all important features. **2**



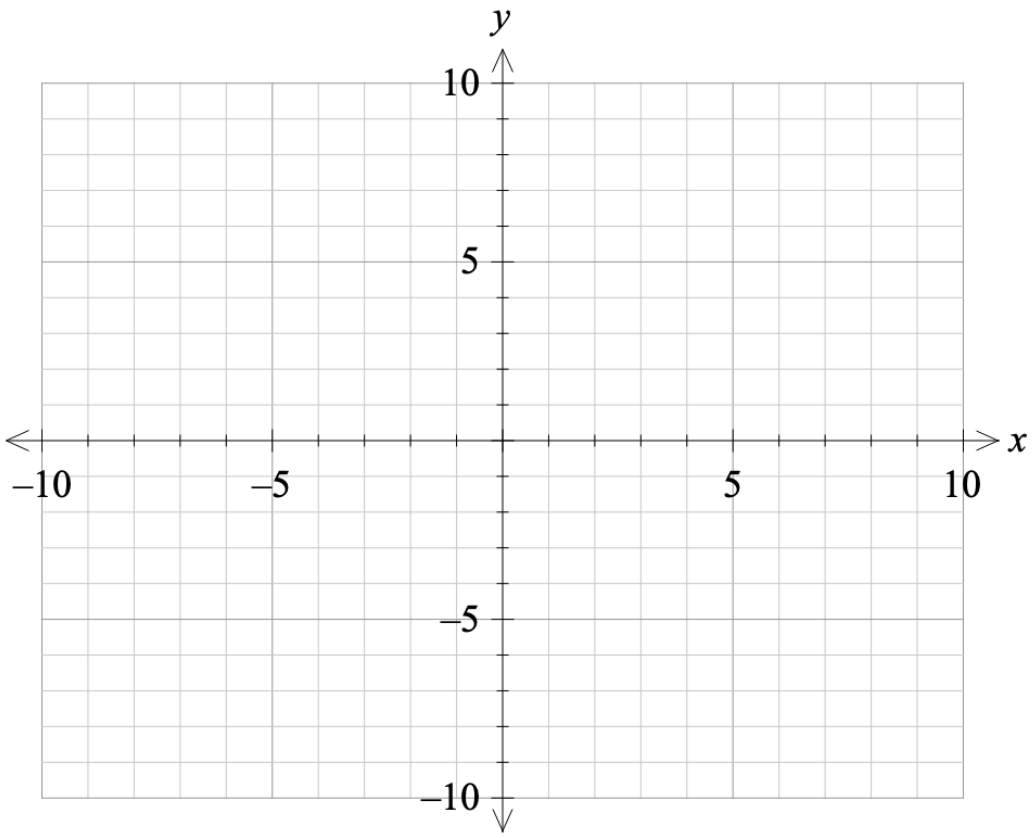
14. The temperature T (°C) of water in a kettle after t minutes is given by $T = 20e^{0.4t} + 15$.

(a) What is the initial temperature of the water? **1**

(b) At what rate is the temperature increasing after 5 minutes? **2**

(c) How many minutes will it take for the temperature to reach 100 °C? Give your answer correct to one decimal place. **2**

15. Sketch $y = \frac{4-3x}{x+2}$ labelling any asymptotes and intercepts. **3**

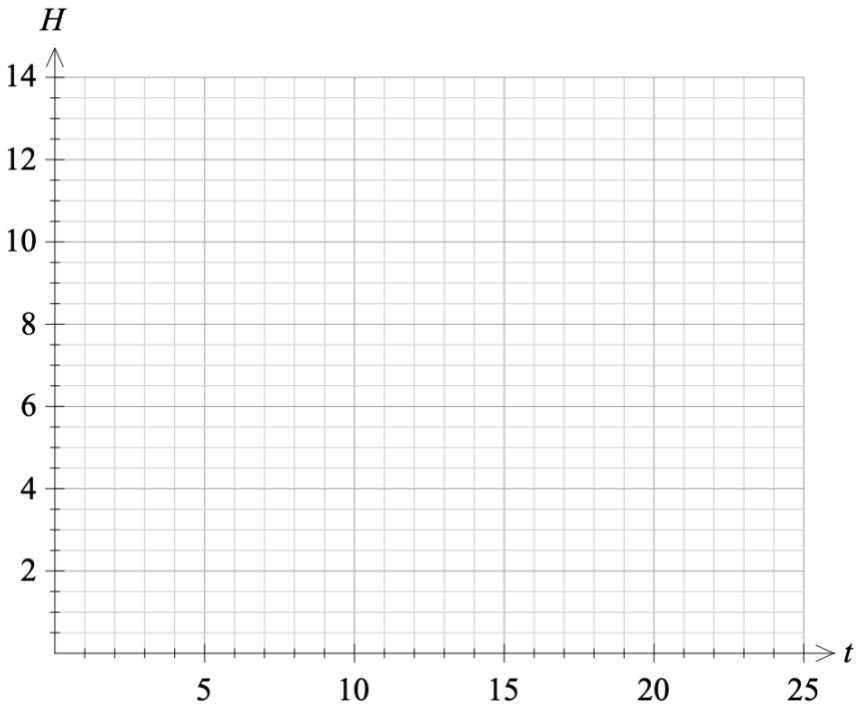


16. The displacement of a particle x metres from the origin, at t seconds is given by $x = 10 + 3e^{-0.2t}$. **2**

Explain why the particle will never be stationary. Include mathematical reasoning.

17. The water at a certain beach rises and falls in a periodic pattern. The height of the water at any time can be modelled by the equation $h(t) = 8 + 5\,sin\left(\frac{\pi t}{12}\right)$, where t is time in hours after midnight.

(a) Sketch the graph of $y = h(t)$ for $0 \leq t \leq 24$. **2**



(b) A boat with a depth of 5.5 metres, is scheduled to arrive at the beach. Between what times will the boat be able to dock? **2**

End of Mock Exam