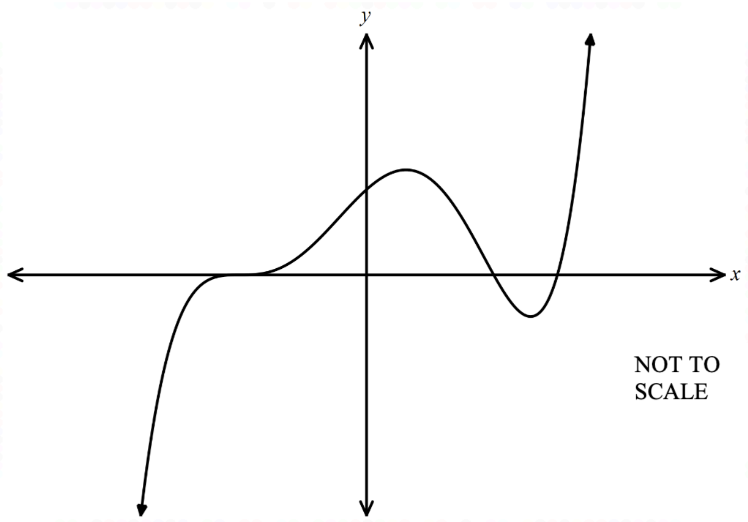
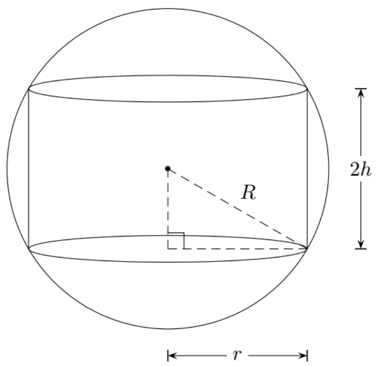
10. The graph of $y = f'(x)$ is shown below.



How many inflection points does $y = f(x)$ have?

Question 17 (7 marks)

The diagram below shows a cylinder of height $2h$ and radius r inscribed within a sphere of fixed radius R.



Let the volume of the cylinder be V.

(a) Show that $V = 2\pi(R^2h - h^3)$. **2**

....

(b) Show that V is maximised when the height of the cylinder is $\dfrac{2R}{\sqrt{3}}$. **3**

....

(c) Find the ratio of the radius of the cylinder to the radius of the sphere when the volume is maximised. **2**

3 Which expression is the derivative of $cos^2 3x$ when differentiated with respect to x ?

A $-6\sin 3x\cos 3x$

B $-2\sin 3x\cos 3x$

C $2\sin 3x\cos 3x$

D $6\sin 3x\cos 3x$

4 The amount M of a drug present in the blood after t hours is given by $M = 9t^2 - t^3$ for $0 \le t \le 9$.

When is the amount of drug in the blood increasing most rapidly?

A $t = 0$

B $t = 9$

C $t = 6$

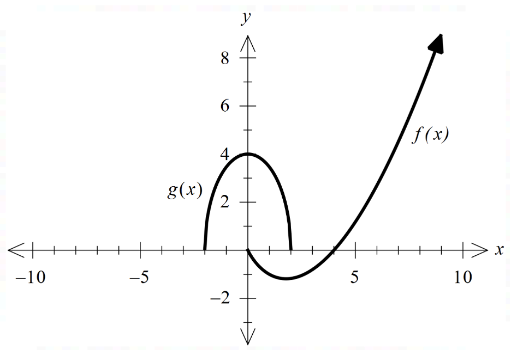
D $t = 3$

Question 27 (2 marks)

Edward plays a game in which he has a probability p of winning, probability q of losing, and probability r of moving to the next round $(p + q + r = 1)$.

What is his probability of eventually winning, in terms of p and q ? **2**

10 The diagram below shows the graph of the functions $f(x)$ and $g(x)$.



What is the domain for which $f(g(x))$ is defined?

A. $x \ge -2$

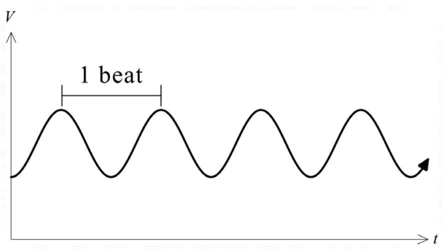
B. $x \ge 0$

C. $0 \le x \le 2$

D. $-2 \le x \le 2$

Question 29 (5 marks)

Blood is pumped into and out of a human heart as it beats. The maximum capacity of the average human heart is 140 millilitres and drops to a minimum of 70 millilitres, before refilling again to the maximum of 140 millilitres. The capacity $V(t)$ in millilitres of blood in the heart as a function of time, t seconds, can be modelled by the equation $V(t) = A - B\cos(nt)$. Some of this information is shown in the diagram below.



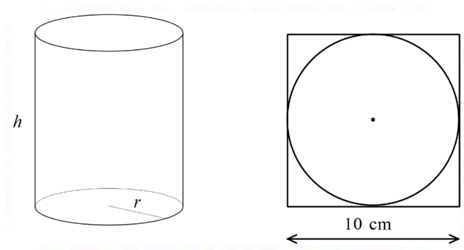
A person's heart is measured as 60 beats/minute. Initially, their heart has 70 millilitres of blood in it.

When is the first time the volume of the heart will be increasing at 200 millilitres/second? **5**

Question 33 (5 marks)

A large cylindrical can with volume 840 cm^3 is to be constructed from 3 sheets of metal. One of the circular ends is cut from a square sheet of metal of side length 10 cm. The other circular end is cut from a second square sheet of metal of side length 90 cm.

To be able to fit items in the cylinder, the radius of the circular ends must be at least 3 cm.



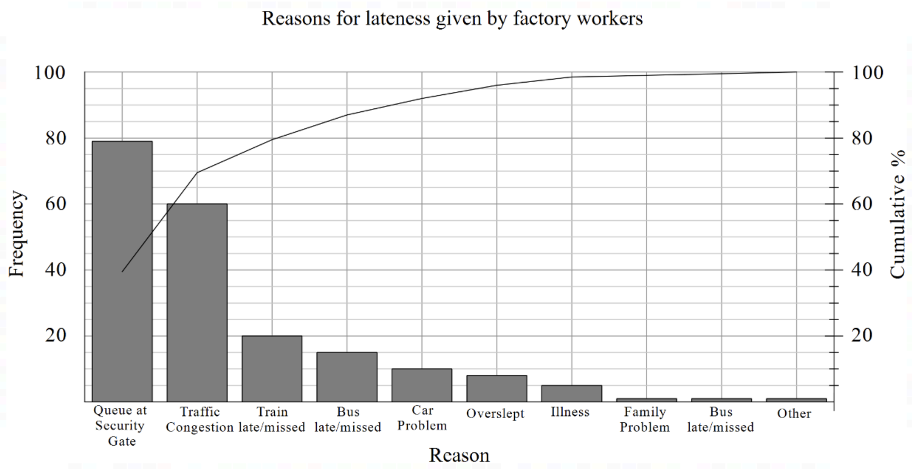
Let the height of the cylinder be h and the radius of each of the circular ends be r.

a) Show that the surface area (A) of the cylinder is given by $A = 2\pi r^2 + \dfrac{1680}{r}$. **1**

b) Hence find the minimum surface area of the cylinder to the nearest whole number. **4**

Spacer 1

1 In a survey to find the main causes of lateness in a factory's work force, a random sample of 200 employees who were late for work were asked the reason why. The Pareto chart below shows the results.



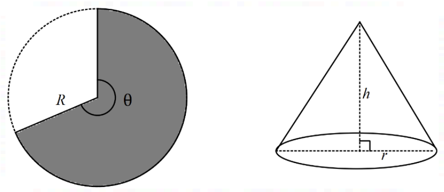
What is the minimum number of causes that accounts for the top 80% of the problems?

A. 1
B. 2
C. 3
D. 4

(This may be wrong → See explanation in solution)

Question 35 (9 marks)

From a circular disc of metal whose area is 100m^2, a sector is cut out and used to make a right cone. The radius of the disc is R metres. The perpendicular height of the cone with radius r metres, is h metres.



a) Show that the height of the cone is given by $h = \sqrt{\dfrac{100}{\pi} - r^2}$. **2**

b) Show that the volume of the cone is given by $V = \dfrac{r^2\sqrt{100\pi - \pi^2 r^2}}{3}$. **1**

c) Show that the volume is maximised when $r = \sqrt{\dfrac{200}{3\pi}}$. **4**

d) Given that the area of the sector used to make the cone is $A = \pi R r$, show that the angle in this sector θ, which gives a maximum volume for the cone, is $\dfrac{2\pi\sqrt{6}}{3}$ radians. **2**

9. Given a function $f(x) = \dfrac{x}{x^2 - 5}$

Which of the following statements is true?

(A) $f(x)$ is even and one-to-one.

(B) $f(x)$ is even and many-to-one.

(C) $f(x)$ is odd and one-to-one.

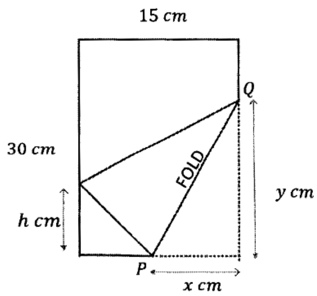
(D) $f(x)$ is odd and many-to-one.

Question 12 (13 Marks)

a) Find the value(s) of m such that $y = 2x + m$ is a tangent to the parabola 2

$$y = 2x^2 + 6x - 5.$$

(d) A rectangular metal sheet is 30 cm high and 15 cm wide. The lower right-hand corner of the sheet is folded over along PQ so as to reach the leftmost edge of the sheet. Let x be the horizontal distance and y be the vertical distance folded as shown in the diagram.



NOT TO SCALE

(i) Show that $y = \frac{hx}{2x-15}$ where $h = \sqrt{x^2 - (15 - x)^2}$. 2

(ii) Show that L, the length of the FOLD, is given by $L^2 = \frac{2x^3}{2x-15}$. 2

(iii) Hence, find the minimum length of L. 2

Spacer 2

10. If A and B are two **independent events**, then the probability of occurrence of at least one of A and B is given by:

A. $1 + P(\bar{A})P(\bar{B})$

B. $1 - P(\bar{A})P(\bar{B})$

C. $P(A) + P(B) - P(\bar{A})P(\bar{B})$

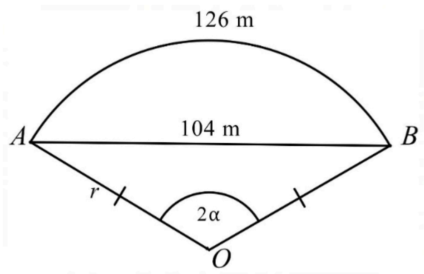
D. $1 - P(A)P(B)$

Question 26 (4 marks) **MA11.3**

A sector of a circle, centre O, is shown below.

The points A and B lie on the circle, such that the length of the chord AB is 104 metres, and the length of the arc AB is 126 metres.

The radius is r metres and the angle subtended at the centre by the arc is 2α radians.



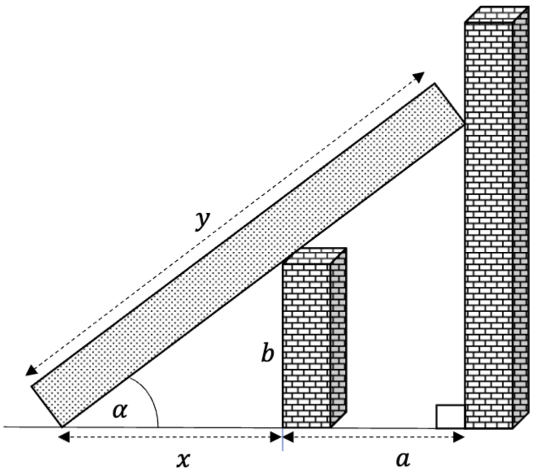
(a) Show that $\sin\alpha = \frac{52\alpha}{63}$. **2**

(b) If $\alpha = \frac{\pi}{3}$, find the exact radius of the circle. **2**

Express your answer with a rational denominator.

Question 32 (6 marks) **MA12.3**

A vertical wall in danger of collapse is to be braced by a beam, which must pass over a second lower wall b metres high and located a metres from the first wall. Let the length of the beam be y metres, the angle the beam makes with the horizontal be α and x is the distance from the foot of the beam to the smaller wall.



2

i. Show that $y = a\sec\alpha + b\operatorname{cosec}\alpha$

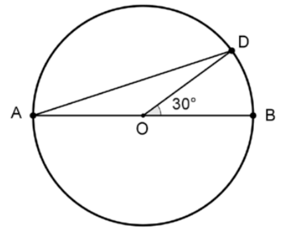
ii. By finding the stationary points on the curve $y = a\sec\alpha + b\operatorname{cosec}\alpha$,

Prove that $\tan\alpha = \sqrt[3]{\frac{b}{a}}$ **2**

iii. Hence show that the shortest beam that can be used is given by **2**

$$y = a\sqrt{1 + \left(\frac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1 + \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

(d) The diagram below shows a circle radius 1 unit, diameter AB and $\angle DOB = 30°$



i) Explain why $\angle DAO = 15°$. *1*

ii) Show that $AD^2 = 2 + \sqrt{3}$. *2*

iii) Hence show that $\sin 15° = \frac{1}{2}\sqrt{2 - \sqrt{3}}$ *2*

Spacer 3

(d) You are asked to provide an 8-letter password which may only be comprised of the letters A, B and C.

(i) How many 8-letter passwords are there to choose from? **1**

(ii) How many of the 8-letter passwords contain only B's and C's? **1**

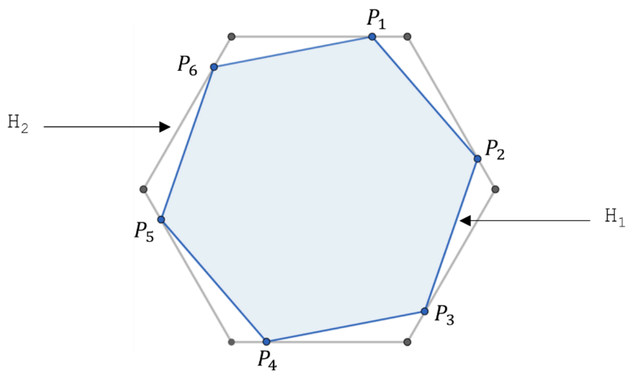
(iii) If you decide that your 8-letter password must use each of the letters A, B and C at least once, how many possible passwords are there to choose from? **2**

(a) A student is given the chance to win 6^X dollars on two rolls of an unbiased, six-sided die, where X is the number of times a '3' is rolled.

(i) What is the most the student may win? **1**

(ii) How much is the student expected to win? Give your answer correct to two decimal places. **2**

(b) Six points P_1, P_2, P_3, P_4, P_5 and P_6 form the vertices of a regular hexagon H_1. An animated program is rotating these six points at a constant speed of 3 cm/s around the perimeter of another regular hexagon, H_2, which has side length of 20 cm. **4**



Initially, the six vertices of H_1 lie on top of the six vertices of H_2. Determine the rate of change of the area of H_1 ten seconds after the animation begins.

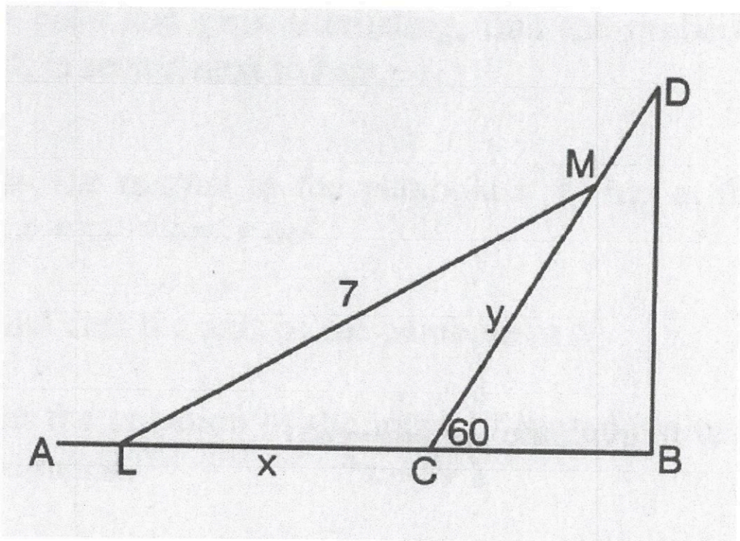
(e) The function $y = f(x)$ is odd, and $f(x) \geq 0$ for $x \geq 0$.

$\int_0^a f(x)\,dx = e^a + \frac{1}{e^a} - 2$ if $a > 0$.

Find the area bounded by $y = f(x+4)$ and the straight lines $x = 5, x = -5$,

and the x-axis. (Leave answer in exact form). **2**

(c) The diagram shows a wall CD inclined at an angle of 60° to the horizontal floor AB. A plank LM of length 7 metres is slipping down the wall with end L moving along CA at a constant speed of 11m/s.



(i) If $LC = x$ and $MC = y$, show that $x^2 + xy + y^2 = 49$ 1

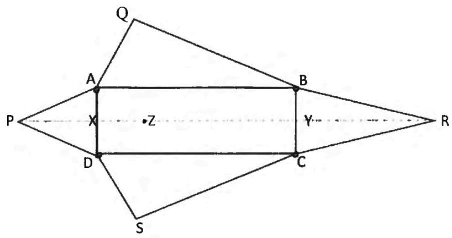
(ii) Given $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$, find the speed at which end M moves along the incline at the instant when $x = 5$ metres. 2

(iii) Find an expression for $\sin\theta$ where $\angle MLC = \theta$ 1

(iv) Calculate the value of $\frac{d\theta}{dt}$ in degrees per second at the instant when $x = 5$ metres. 2

) w. sol#v

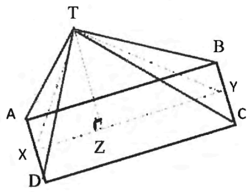
(d)



The figure shows the net of an oblique pyramid with a rectangular base. In this figure, PXZYR is a straight line, $PX = 15cm$, $RY = 20cm$, $AB = 25cm$, and $BC = 10cm$.

Further, $AP = PD$ and $BR = RC$.

When the net is folded, points P, Q, R and S all meet at the apex T, which lies vertically above the point Z in the horizontal base, as shown below.



(i)Show that ΔTXY is right angled. 1

(ii) Hence show that T is 12cm above the base. 2

(iii) Hence find the angle that the face DCT makes with the base. 1

2019 w. sol#v

(c) Find $\int_0^2 3^{-x}\, dx$ 2

Spacer 4

(a) Express n in terms of x and y, given $(\sqrt{2})^n = \frac{4^x}{32^y}$ 1

(a) Express $5cot^2x - 2cosecx + 2$ in terms of $cosecx$

If $y = \frac{\log_e x}{x}$

(a) Find $\frac{dy}{dx}$ 1

(b) Hence show that $\int_e^{e^2} \frac{1 - \log_e x}{x \log_e x}\, dx = \log_e 2 - 1$ 2

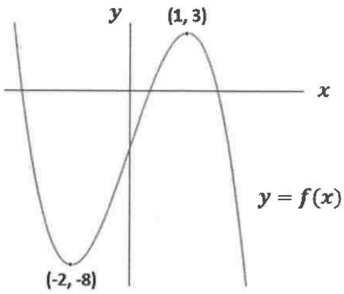
Question 29 (3 marks)

Given that the function $f(x)$ has a derivative $y' = 4e^{4x} + 3$ and the equation of the tangent to this curve is $y = 7x + 2$. Find the exact value of $f(3)$. 3

MA12.6

Question 28 (4 marks)

Consider the graph of $y = f(x)$ shown:



(a) Use the space below to sketch the graph of $y = f'(x)$ **2**

(b) Find the area bounded by $y = f'(x)$ and the x-axis. **2**

Question 22 (5 marks)

(a) By sketching the graph of $y = \sqrt{4 - x^2}$, show that **3**

$$\int_0^1 \sqrt{4 - x^2}\, dx = \frac{\sqrt{3}}{2\cancel{4}} + \frac{\pi}{\cancel{12}\,3}$$

(b) Hence or otherwise, find $\displaystyle\int_2^{\cancel{4}\,3} \sqrt{16x - 4x^2}\, dx$. **2**

Question 17 (6 Marks)

The acceleration of a moving body is given by $a = \sqrt{2t + 1}$ ms^{-2}.

(a) If the body starts from rest, find its velocity after 4 seconds. **3**

(b) Find, in exact form, the average velocity of the body during the fourth second. **3**

Question 25 (6 Marks)

A game consists of a player tossing a fair coin three times. You lose \$3.00 if three heads appear and lose \$2.00 if two heads appear. You win \$1.00 if one head appears and win \$3.00 if no heads appear. Let X be the amount you win or lose per game.

(a) Complete the probability distribution table below. **2**

x	-3	-2	1	3
$P(X = x)$				

....

....

(b) Calculate $E(X)$ and hence determine the expected profit or loss if you played the game 1000 times. **2**

....

....

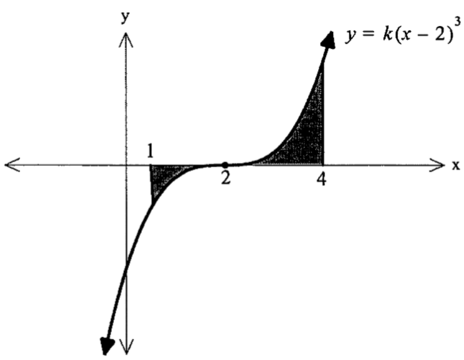
....

....

(c) Evaluate $P(X > -2 | X \geq -2)$. **2**

Spacer 5

6. The graph with the equation $y = k(x-2)^3$ is shown below, for some positive constant k.



If the area of the shaded region is 34, what is the value of k?

(A) $\dfrac{136}{15}$

(B) 8

(C) 4

(D) $\dfrac{34}{9}$

Question 27 (3 marks)

Consider the function $f(x)=\frac{1}{2}-\frac{1}{2^x+1}$

(a) Show that $\frac{1}{2}-\frac{1}{2^x+1}=\frac{2^x-1}{2(2^x+1)}$ **1**

....

....

....

....

....

....

....

....

(b) Hence determine whether $f(x)$ is even, odd or neither. Show all working. **2**

(b) Find the exact area bounded by the curve $y=x^2-2$, the x axis and the line $y=4$. **4**

Question 2 (6 marks)

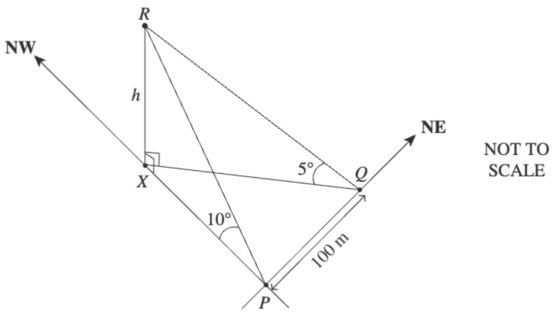
Differentiate the following expressions. Simplify your answers, if possible.

(a) $\frac{e^{3x}}{x-1}$

(c) $\log_4\left(x^2+3\right)$

Question 25 (4 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q.
A peak is located at point R, which is h metres directly above point X.
From point P the angle of elevation to peak R is 10°. At point Q, the angle of elevation to the peak is 5°.



(a) Show that $XP = h \cot 10°$, and write down a similar expression for XQ. **1**

....

....

....

....

(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

....

(e) A particle is moving such that, at time, t seconds, its displacement, x metres, satisfies the equation $t = 2 - \frac{1}{e^{3x}}$. Find the acceleration of the particle after 1 second. **3**

(e) A particle is moving such that, at time, t seconds, its displacement, x metres, satisfies the equation $t = 2 - \dfrac{1}{e^{3x}}$.

i) Show that the velocity of the particle can be represented by

$\dfrac{dx}{dt} = \dfrac{e^{3x}}{3}\,\text{ms}^{-1}$ **2**

ii) Using the formula $a = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$, or otherwise, find the acceleration of the particle after 1 second. **2**

10 Three fair dice of different colours are rolled together. What is the probability that the product of the three scores is a perfect square?

(A) $\dfrac{6}{216}$

(B) $\dfrac{13}{216}$

(C) $\dfrac{32}{216}$

(D) $\dfrac{38}{216}$

9 A geometric series $1 - \dfrac{1}{x} + \dfrac{1}{x^2} - \dfrac{1}{x^3} + \ldots$ has limiting sum S.

For what values of x is $S < 1$?

(A) $x > -1$

(B) $x > -1,\ x \neq 0$

(C) $x > 0$

(D) $x > 1$

Question 34 (3 marks)

If $y = \tan^2 x$, find the values of the constants a and b, such that $\dfrac{d^2y}{dx^2} = ay^2 + by + 2$. **3**

Question 28 (5 marks)

The length of time, in minutes, that a customer queues to buy toilet paper is a random variable, X, with probability density function

$$f(x) = \begin{cases} k(64 - x^2) & \text{for } 0 \leq x \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

a) Show that the value of k is $\dfrac{3}{1024}$. **2**

Question 28 (continued)

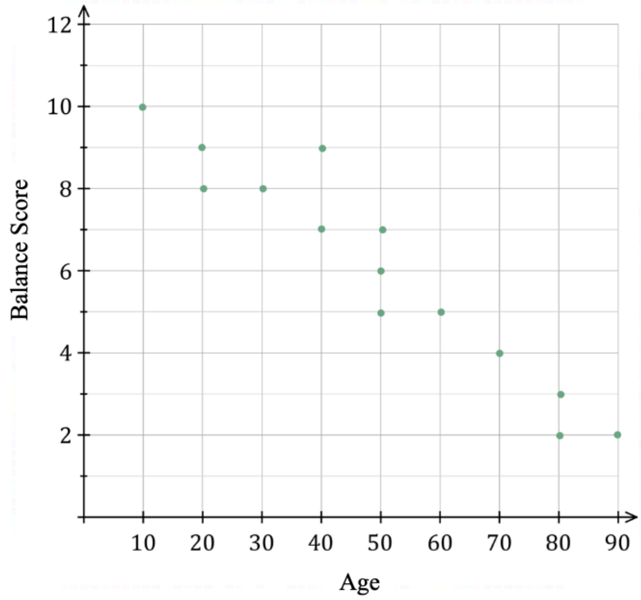
b) Find the cumulative distribution function. **2**

....

Differentiate

a) $y = \tan^3\left(\dfrac{x}{4}\right)$ **2**

(d) A scatter plot below shows the relationship between Age and Balance Score.



(i) The correlation coefficient is -0.955. Describe the association between Age and Balance Score with reference to the correlation. **1**

....

....

(ii) The least squares regression line for this data is $y = 11.1249 - 0.1025x$. Using this regression line, predict the Balance Score of a 65 year old. **1**

....

....

(iii) Comment on whether your answer in part (ii) is reliable. **1**

(e) Let A and B be two events such that $P(A) = 0.4$, $P(B) = 0.55$ and $P(B \mid A) = 0.6$.

(i) Determine whether A and B are independent events. **1**

....

....

....

(ii) Find $P(A \cup B)$. **2**

Spacer 6

9. Which transformations listed are required to obtain the graph of $y = x^2 + \frac{1}{2}x - 3$ from the graph of $y = 4x^2 + x$?

A. Horizontal dilation by a factor of 2; vertical translation of 3 units upwards

B. Horizontal dilation by a factor of 2; vertical translation of 3 units downwards

C. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units upwards

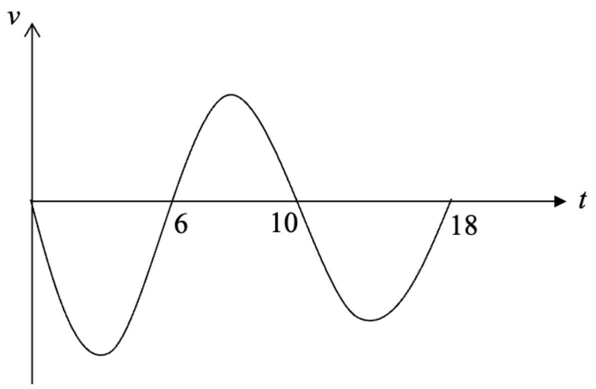
D. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units downwards

(a) An artist posted a song online. Each day there were $2^n + 3n$ downloads, where n is the number of days after the song was posted.

(i) Find the number of downloads on the third day after the song was posted. **1**

(ii) What is the total number of times the song was downloaded in the first 20 days after it was posted? **3**

7. A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



For what value of t is the displacement of the particle a maximum?

A. 18

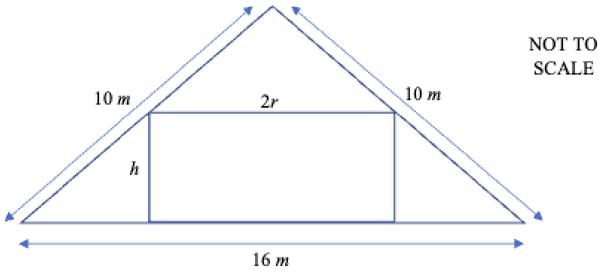
B. 10

C. 8

D. 6

Question 16 (15 marks) **Student Number:**

(a) In some rural areas hot water tanks are installed in the roofs of houses. The diagram below shows a cross-section of a cylindrical tank in a roof. The cylindrical tank snugly fits exactly into the roof with diameter $2r$ metres and height h metres. The cross-section of the roof is an isosceles triangle with dimensions show.



(i) Show that the height of the roof is 6 metres. **1**

....

....

....

(ii) Hence show that $h = \frac{3}{4}(8 - r)$. **2**

....

....

....

....

....

Question 16(a) continues on page 40

Question 16(a) (continued)

(iii) Show that the volume of the cylindrical tank can be expressed by **1**

$$V = \frac{3\pi}{4}\left(8r^2 - r^3\right).$$

....

....

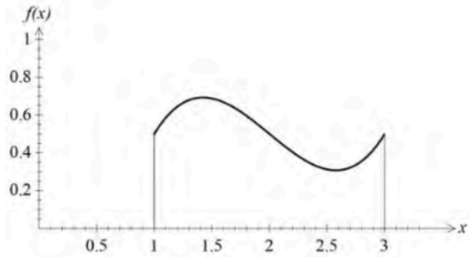
....

(iv) Find the value of r which gives the tank its greatest volume and calculate that volume, correct to the nearest litre. **4**

....

Question 31 (5 marks)
A continuous probability distribution is graphed below and is defined by:

$$f(x)=\begin{cases}\frac{1}{2}(x^3-6x^2+11x-5), & [1,3]\\ 0, & (-\infty,1)\cup(3,\infty)\end{cases}$$



i) Use the graph of $y=f(x)$ to **estimate** the mode of the distribution. **1**

i) Find the cumulative distribution function. **2**

ii) Find $P(X>2)$. **2**

10. Consider the cumulative probability density function

$$F(x)=\begin{cases}0, & x<\frac{\pi}{2}\\ \cos^2 x, & \frac{\pi}{2}\le x\le\pi\\ 1, & x>\pi\end{cases}$$

What is the 3rd quartile of the distribution?

(A) $x=\frac{\pi}{6}$ (B) $x=\frac{5\pi}{8}$ (C) $x=\frac{3\pi}{4}$ (D) $x=\frac{5\pi}{6}$

14 The home loan from Building Society N is advertised with an interest rate of 5.4% compounded monthly. Credit Union T advertises their home loan with an interest rate of 5.35% compounded daily. If there are no other fees involved, which financial institution offers the better deal?

2 Write a recursion relation for each of the following.

(a) An investment of $1000 at an interest of 3.1% with the interest compounded annually.
(b) A loan of $5000 at an interest rate of 6.2% with the interest compounded six-monthly.
(c) An investment of $10 000 at an interest of 2.7% with the interest monthly.

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5 A principal of $1000 is invested for three years at an interest rate of 5.6% pa compounded half-yearly.
Determine how much needs to be invested to achieve the same interest if the interest rate was 3.5% pa compounded monthly. **Solution**

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics
NESA Mathematics Standard 2 Year 12 Topic Guide: Financial mathematics

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16 The table shows the future value of an annuity of $1 for varying interest rates and time periods. The contribution is made at the beginning of each period. **Solution**

n	1%	2%	3%	4%	5%
1	1.0100	1.0200	1.0300	1.0400	1.0500
2	2.0301	2.0604	2.0909	2.1216	2.1525
3	3.0604	3.1216	3.1836	3.2465	3.3101
4	4.1010	4.2040	4.3091	4.4163	4.5256
5	5.1520	5.3081	5.4684	5.6330	5.8019
6	6.2135	6.4343	6.6625	6.8983	7.1420
7	7.2857	7.5830	7.8923	8.2142	8.5491
8	8.3685	8.7546	9.1591	9.5828	10.0265
9	9.4622	9.9497	10.4639	11.0061	11.5779
10	10.5668	11.1687	11.8078	12.4864	13.2068
11	11.6825	12.4121	13.1920	14.0258	14.9171
12	12.8093	13.6803	14.6178	15.6268	16.7130

(a) Ken invests $200 at the start of each year for 8 years at an interest rate of 5% per annum. Calculate the future value of Ken's investment. **2**

(b) Holly is planning to take a holiday in 3 years. She needs $5000 for this holiday. She is going to make regular quarterly payments into an account that earns interest at the rate of 4% pa. compounded quarterly. **2**
What is the minimum investment, to the nearest $100, Holly needs to make each quarter in order to take this holiday? Support your answer with an explanation.

NESA Mathematics Standard 2 Sample examination materials

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21 MA **30** The number of hours for which light bulbs will work before failing can be modelled by the random variable X with cumulative distribution function. **2** **Solution**

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs. Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal place

NESA 2021 Mathematics Advanced HSC Examination

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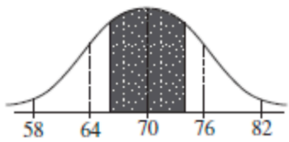
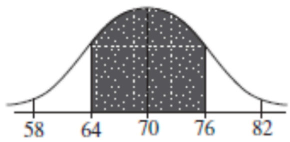
9 **Band 5-6** The scores on an examination are normally distributed with a mean of 70 and a standard deviation of 6. Michael received a score on the examination between the lower quartile and the upper quartile of he scores. **1** **Solution**
Which shaded region most accurately represents where Michael's score lies?

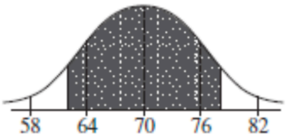
15 A.



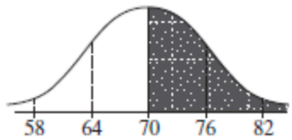
B.



C.

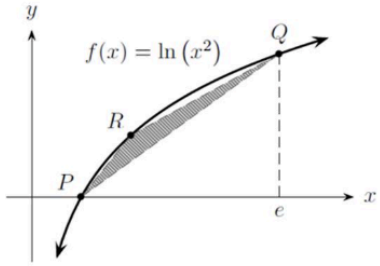


D.



NESA Mathematics Advanced Sample Examination Paper (2020)
NESA 2019 Mathematics Standard 2 HSC Examination

39. The diagram shows the graph of the function $y = \ln(x^2)$, where $x > 0$.
The points $P(1, 0)$, $Q(e, 2)$ and $R(t, \ln t^2)$ all lie on the curve.
The area of ΔPQR is maximum when the tangent at R is parallel to the line through P and Q.



a) Show that R has coordinates $(e - 1, \ln[(e - 1)^2])$ for ΔPQR to have maximum area. **2**

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Question continues the next page

b) Hence find the size of $\angle RPQ$ correct to the nearest degree. **2**

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In a music class of 28, 17 students play the violin (V) and 13 students play the clarinet (C). Five students play neither of these musical instruments.

(a) Draw a Venn diagram to represent these events using the labels V and C. **1**

(b) A student is chosen at random from the class.
Find the probability that the student plays:

(i) Both the violin and the clarinet **1**

(ii) Either the violin or the clarinet **1**

(c) A duet is a performance of two students, one playing the violin and the other playing the clarinet. **3**

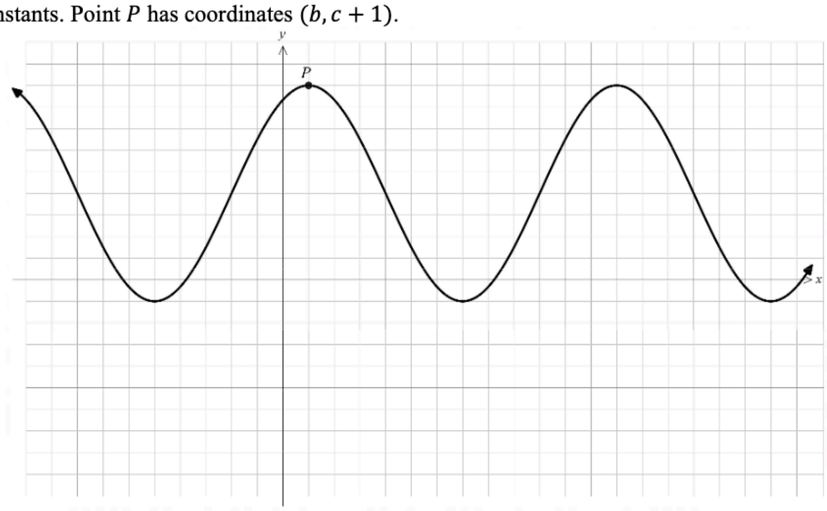
If two students are chosen at random from the class, what is the probability that they can play a duet together?

Question 24 (2 marks)

Given that $\sin\theta = \frac{6+\sqrt{2}}{2\sqrt{19}}$ and θ is obtuse, find the exact value of $\tan\theta$ in simplest form. **2**

Question 32 (4 marks)

The graph of the function $y = \cos\big(2a(x-b)\big) + c$ is shown below, where a, b and c are constants. Point P has coordinates $(b, c+1)$. **4**



On the same set of axes, sketch:

$$y = -c\sin(3ax - 3ab)$$

You may use the space below for working.

The hours of daylight in Sydney can be modelled by

$$y(t) = 2.24 \sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$

Where y is the number of hours of daylight and t is the months of the year, each month being represented with a whole number, e.g. t =1 represents January 1st.

(a) In this model what is the maximum number of daylight hours in one day. **1**

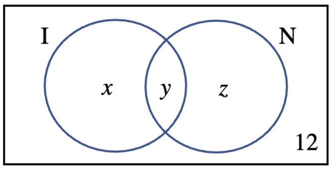
(b) Find the month that has the longest day in terms of number hours of daylight. **2**

(c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should she be looking to run her experiment? **3**

(c) Eighty people were asked if they had holidayed interstate (I) or within NSW (N) in the past five years.

The results showed that,

- 12 people had not holidayed at all.
- 42 people had holidayed interstate (I).
- 36 people had holidayed within NSW (N).



(i) Copy the Venn diagram above onto you answer sheet and complete it by finding values for x, y and z. **1**

(ii) One of the surveyed people is selected at random. Find the probability that the person selected has holidayed both interstate and within NSW. **1**

(iii) A randomly selected person is known to have travelled. Find the probability that this person holidayed within NSW. **1**

(iv) Two people are chosen, in turn, at random. Find the probability that exactly one of those chosen holidayed interstate. **2**

Question 23 (3 marks)

A geometric progression has 5^{th} term 9 and 13^{th} term 59 049.

(a) Find the first term and the common ratio. **2**

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(b) Find the 19^{th} term. **1**

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Spacer 8

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a)

(i) Find the volume of the solid generated when the area bounded by the curve $y = \tan x$, the x axis and $x = \frac{\pi}{6}$ is rotated about the x axis. **2**

(ii) By considering the volume of a cylinder , prove that **3**

$$\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}.$$

(b) If $(ax)^{\ln a} = (bx)^{\ln b}$, show that $x = \frac{1}{ab}$, where $a \neq b$. **3**

(c) A person deposits $\$M$ into their superannuation fund at the beginning of each month. The money is invested at r % per month, compounded monthly. Let P be the amount of money in the account after n months.

Let $R = 1 + \frac{r}{100}$ and let $S = \frac{R^n - 1}{R - 1}$.

(i) Show that $P = MRS$. **2**

After retiring, $\$kM$ is withdrawn from the account at the end of each month, without making any further deposits, until there is no money left in the account. The account continues to earn the same amount of interest per month.

(ii) Let m be the number of months that pass after retirement until there is no money expected to remain in the account. **3**

Prove that $m = \frac{1}{\ln R} \times \ln\left(\frac{k}{k - R(R^n - 1)}\right)$.

(iii) Describe what happens to the amount in the person's account over time if **1**

$$k \leq R(R^n - 1).$$

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Rationalise the denominator of **1**

$$\frac{1}{\sqrt[3]{2}-1}.$$

(d) Prove that if x, y and z are in geometric progression and **3**

$$x^{1/a} = y^{1/b} = z^{1/c},$$

then a, b and c are in arithmetic progression.

(d) Alissa receives an income of A at the beginning of every year which is deposited into a bank account. The bank account earns an interest rate of $100r_1\%$ per annum. **3**

However, just before she deposits her income into the bank account each year, she withdraws $100r_2\%$ of her balance for everyday expenses.

Show that if

$$\frac{1}{r_2}-\frac{1}{r_1}<1,$$

then her total savings in the bank amount will never exceed $\dfrac{A}{r_2+r_1r_2-r_1}$.

(d)

(i) Find the value of a such that **1**

$$n(n+1)(n+2)(n+3)=(n^2+an)(n^2+an+2).$$

(ii) Let n be an integer. **2**

Deduce that $2011\times2012\times2013\times2014+1$ is a perfect square.

7 Consider the region $\mathcal{R}$ bounded by the curves $y=x^2$, $y=(x-2)^2$ and the y axis.

Which of the following regions defined below have the same area as $\mathcal{R}$?

(A) $y\le -x^2$, $y\ge -(x-2)^2$ and $y\le 0$.

(B) $y\ge x^2$, $y\le (x-2)^2$ and $y\ge 0$.

(C) $y\le 4-4x$, $0\le x\le 1$ and $y\ge 0$.

(D) $y\le 4x-4$, $0\le x\le 1$ and $y\le 0$.

(a) Solve the following equation for x. **2**

$$\log_2 x\log_3 x=\log_4 x$$

(b) Show that **2**

$$\frac{\sin 0^\circ+\sin 1^\circ+\sin 2^\circ+\sin 3^\circ+...+\sin 90^\circ}{\cos 0^\circ+\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+...+\cos 90^\circ}=1.$$

(f) Let $x^2+xy+y^2=0$, where $x+y\ne 0$. **3**

Simplify

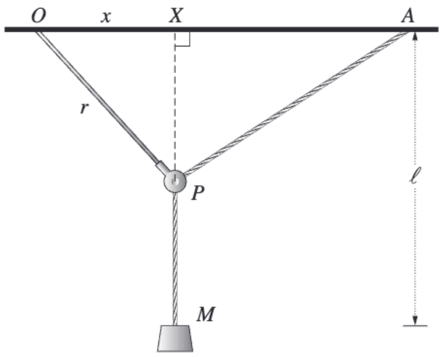
$$\left(\frac{x}{x+y}\right)^{2015}+\left(\frac{y}{x+y}\right)^{2015}.$$

(a) Show that $a^{\log_b x}=x^{\log_b a}$. **2**

(b) Show that for integer values of $n\ge 2$, **3**

$$\frac{1}{1+2}+\frac{1}{1+2+3}+\frac{1}{1+2+3+4}+...+\frac{1}{1+2+3+...+n}=\frac{n-1}{n+1}.$$

(b) A pulley P is attached to the ceiling at O by a piece of metal that can swing freely. One end of a rope is attached to the ceiling at A. The rope is passed through the pulley P and a weight is attached to the other end of the rope at M, as shown in the diagram.



The distance OA is 1 m, the length of the rope is 2 m, and the length of the piece of metal $OP = r$ metres, where $0 < r < 1$. Let X be the point where the line MP produced meets OA. Let $OX = x$ metres and $XM = \ell$ metres.

(i) By considering triangles OXP and AXP, show that **1**

$$\ell = 2 + \sqrt{r^2 - x^2} - \sqrt{1 - 2x + r^2}\,.$$

(ii) Show that $\dfrac{d\ell}{dx} = \dfrac{\left(r^2 - x^2\right) - x^2\left(1 - 2x + r^2\right)}{\sqrt{r^2 - x^2}\,\sqrt{1 - 2x + r^2}\left(\sqrt{r^2 - x^2} + x\sqrt{1 - 2x + r^2}\right)}$. **2**

(iii) You are given the factorisation **2**

$$\left(r^2 - x^2\right) - x^2\left(1 - 2x + r^2\right) = (x - 1)\left(2x^2 - r^2x - r^2\right).$$

(Do NOT prove this.)

Find the value of x for which M is closest to the floor. Justify your answer.

iii) is pretty hard

Question 28 (4 marks) **Marks**

A new artist releases a song on a music streaming platform. The number of 'listens' each hour for the first 5 hours is recorded in the table below.

Hour (H)	1	2	3	4	5
Listens (L)	13	36	62	94	138

(a) Show that the predicted number of listens in the 6[th] hour is 206. **2**

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(b) How many predicted 'listens' will the song have had at the conclusion of one day? **2**

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Answers

10. (**2 points of inflection**)

Question 17 –

(c) Find the ratio of the radius of the cylinder to the radius of the sphere when the volume is maximised. **2**

$r^2 = R^2 - h^2$

$= R^2 - \frac{R^2}{3} \quad (h^2 = \frac{R^2}{3} \text{ from b)})$ 1

$= \frac{2R^2}{3}$

$\therefore r : R = \frac{\sqrt{2}}{\sqrt{3}}R : R$

$= \sqrt{2} : \sqrt{3}$ 1

Question 3 – **A**

Question 4 – **D**

Question 27 –

Question 27 (2 marks)

Edward plays a game in which he has a probability p of winning, probability q of losing, and probability r of moving to the next round $(p + q + r = 1)$.

What is his probability of eventually winning, in terms of p and q? **2**

Let W = 'win round 1', L = 'lose round 1',

C = 'continue after round 1'.

The $P(\text{eventually wins}) = P(\text{eventually wins} \mid W) \cdot P(W)$

$+ P(\text{eventually wins} \mid L) \cdot P(L)$

$+ P(\text{eventually wins} \mid C) \cdot P(C)$

$= 1 \times p + 0 \times q + r \times p(\text{eventually wins})$,

since $p(\text{eventually wins} \mid C) = p(\text{eventually wins})$.

$\therefore P(\text{eventually wins}) = p + r \times p(\text{eventually wins})$

$\therefore p(\text{eventually wins}) = \frac{p}{1-r}$

$= \frac{p}{p+q}$ ✓✓

Question 28 (4 marks)

10	When $-2 \le g(x) \le 2$, then $f(g(x))$ $\therefore$ the domain is $-2 \le x \le 2$	D

29	$140mL$ is the maximum amount of blood in the heart & 70mL is the minimum amount of blood at any one time. $\therefore$ Amplitude is 35 ie $B = 35$ $Mean\,Value = \frac{140+70}{2}$ $= 105$ $\therefore A = 105$ Since athlete's heart beats at 60 beats/minute $\rightarrow$ 1 beat/second or 1 second/beat *ie* Period $= 1$ $\frac{2\pi}{n} = 1$ $n = 2\pi$ $V(t) = 105 - 35\cos(2\pi t)$ $V'(t) = 35 \times 2\pi \sin(2\pi t)$ $200 = 70\pi \sin(2\pi t)$ $\sin(2\pi t) = \frac{200}{70\pi}$ $2\pi t = 1.14197583$ $t = 0.1817510982$ *ie* after 0.18 seconds (2 dp) 5 – correct solution 4 – obtains $\sin(2\pi t) = \frac{200}{70\pi}$ 3 – correctly differentiates $V(t)$ 2 – Finds n, A and B 1 – Finds A and B or n or obtains $V'(t) = Bn\sin(nt)$

33b	$A = 2\pi r^2 + \frac{1680}{r}$ $= 2\pi r^2 + 1680r^{-1}$ $\frac{dA}{dr} = 4\pi r - 1680r^{-2}$ Stationery points occur when $\frac{dA}{dr} = 0$ $4\pi r - 1680r^{-2} = 0$ $4\pi r - \frac{1680}{r^2} = 0$ $4\pi r = \frac{1680}{r^2}$ $4\pi r^3 = 1680$ $r^3 = \frac{420}{\pi}$ $r = \sqrt[3]{\frac{420}{\pi}}$ $r = 5.113282718$ but $r > 5$ $\frac{d^2A}{dr^2} = 4\pi + 3360r^{-3}$ $= 4\pi + \frac{3360}{r^3}$ Since $r > 0, \frac{d^2A}{dr^2} > 0$ ie A is always concave up $\therefore r = 5.11...$ is a minimum turning point But $3 \le r \le 5$ $\therefore$ Minimum must occur when $r = 5$ When $r = 5, A = 2 \times \pi \times 5^2 + \frac{1680}{5}$ $= 493.0796327\text{cm}$ $\therefore$ *Minimum Area* $= 493\text{cm}$ (*nearest* cm) 4 –correct solution 3 – shows r is a minimum at $r = \sqrt[3]{\frac{420}{\pi}}$ **or** correctly finds the value of r which obtains max/min **and** calculates the area when $r = 5$ 2-finds the value of r which obtains max/min 1 – correctly differentiates and equates to 0

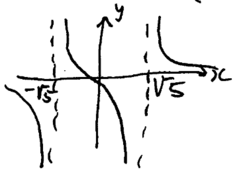
Spacer 1

1	Line up 80% on the cumulative percentage to the ogive. This will show that 4 of the main causes takes up 80% of the problem.	D

35a	Area: $\pi R^2 = 100$ $\therefore R^2 = \frac{100}{\pi}$ Using pythagoras theorem $R^2 = h^2 + r^2$ $h = \sqrt{R^2 - r^2}$ $\therefore h = \sqrt{\frac{100}{\pi} - r^2}$	2- Correct solution 1- correctly substituting $R = \sqrt{\frac{100}{\pi}}$
35b	$V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi r^2 \sqrt{\frac{100}{\pi} - r^2}$ $= \frac{1}{3}\pi r^2 \sqrt{\frac{100 - \pi r^2}{\pi}}$ $= \frac{1}{3}r^2 \sqrt{\pi^2 \times \frac{100 - \pi r^2}{\pi}}$ $= \frac{r^2\sqrt{100\pi - \pi^2 r^2}}{3}$	1- correct solution

35c	$v=\sqrt{\dfrac{r^4(100\pi-\pi^2r^2)}{9}}$ $=\left(\dfrac{100\pi r^4}{9}-\dfrac{\pi^2r^6}{9}\right)^{\frac{1}{2}}$ $v'=\dfrac{1}{2}\left(\dfrac{100\pi r^4}{9}-\dfrac{\pi^2r^6}{9}\right)^{-\frac{1}{2}}\times\left(\dfrac{400\pi r^3}{9}-\dfrac{6\pi^2r^5}{9}\right)$ $=\dfrac{\left(\dfrac{400\pi r^3}{9}-\dfrac{6\pi^2 r}{9}\right)}{2\sqrt{\dfrac{100\pi r^4}{9}-\dfrac{\pi^2r^6}{9}}}$ $v'=0$ when $\dfrac{400\pi r^3}{9}-\dfrac{6\pi^2r^5}{9}=0$ $\pi r^3(400-6\pi r^2)=0$ $400-6\pi r^2=0$ $r=0,\quad r=\sqrt{\dfrac{200}{3\pi}}$ Test for max when $r=\sqrt{\dfrac{200}{3\pi}}$ $v'=\dfrac{\left(\dfrac{400\pi r^3}{9}-\dfrac{6\pi^2 r}{9}\right)}{2\sqrt{\dfrac{100\pi r^4}{9}-\dfrac{\pi^2r^6}{9}}}$	4- correct solution 3- correctly obtained $r=\sqrt{\frac{200}{3\pi}}$ but did not check it was a maximum 2- some significant progress towards obtaining r 1- correctly finds v' in any form

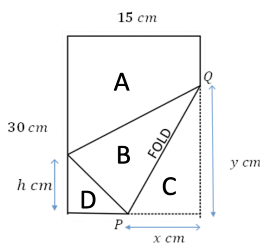
9 $f(-x)=\dfrac{-x}{(-x)^2-5}=\dfrac{-x}{x^2-5}=-f(x)$ ∴ odd



by horiz. line test ∴ many to one

D

	2 - correct soln.
$y = 2x + m$ $2 \Rightarrow$ gradient of tangent	1 - finds point of contact by using calculus
$\therefore y' = 4x + 6$ where $m = 2$ $\therefore 2 = 4x + 6$ $-1 = x$ $\quad \therefore y = 2(-1)^2 + 6(-1) - 5$ $y = -9$ $\therefore$ pt. of contact $(-1, -9)$	1 - finds Δ correctly by using simult. eqns.
sub. $(-1, -9)$ into $y = 2x + m$ $-9 = 2(-1) + m$ $\therefore m = -7$	1 - finds gradient function correctly & x-coord. of pt. of contact
OR by simultaneous eqn. $2x + m = 2x^2 + 6x - 5$ $2x^2 + 4x - 5 - m = 0$ $\Delta = 0$ (since tangent $\therefore$ only one solution) $0 = b^2 - 4ac$ $0 = 4^2 - 4(2)(-5 - m)$ $0 = 16 + 40 + 8m$ $m = -7$	1 - creates quadr. eqn. correctly by solving simult. eqns, & attempts to solve $\Delta = 0$

16(d)(i)	 Area of the rectangle = Area of A + B + C + D $B = C = \frac{1}{2}xy$ $D = \frac{1}{2}(15 - x)h$ where $h = \sqrt{x^2 - (15 - x)^2}$ $A = \frac{1}{2} \times 15[(30 - h) + (30 - y)]$ $30 \times 15 = xy + \frac{1}{2}(15 - x)h + \frac{1}{2} \times 15[(30 - h) + (30 - y)]$ $0 = 2xy - hx - 15y$ $y(2x - 15) = hx$ $\therefore \ \boldsymbol{y = \frac{hx}{2x - 15}}$	2- correct proof 1- formed an equation in terms of x, y and h.

<table>
<tr>
<td>16(d)(ii)</td>
<td>

$$L^2 = x^2 + y^2 \text{ ------ (A)}$$

$$L^2 = x^2 + \left(\frac{hx}{2x-15}\right)^2$$

$By\ substituting\ for\ h$

$$= \frac{x^2[(2x-15)^2 + (2x-15)15]}{(2x-15)^2}$$

$$\boldsymbol{L^2 = \frac{2x^3}{2x-15}}$$

</td>
<td>2- correct proof
1- equation (A) to find L</td>
</tr>
<tr>
<td>16(d)(iii)</td>
<td>

$$\frac{d(L^2)}{dx} = \frac{(2x-15)\times 6x^2 - 2x^3 \times 2}{(2x-15)^2}$$

$$\frac{d(L^2)}{dx} = 0 \rightarrow (2x-15)\times 6x^2 - 2x^3 \times 2 = 0$$

$$2x^2(4x-45) = 0$$

$$As\ x \neq 0, \quad x = \frac{45}{4} = 11.25$$

<table>
<tr><th>x</th><th>11.20</th><th>11.25</th><th>11.30</th></tr>
<tr><td>$\frac{d(L^2)}{dx}$</td><td>-50.176</td><td>0</td><td>$+51.076$</td></tr>
</table>

Hence for $minimum\ L, x = 11.25$

$$L = \sqrt{\frac{2\times 11.25^3}{2\times 11.25 - 15}} = 19.485..$$

$\boldsymbol{minimum\ length\ of\ the\ FOLD\ is\ 19.5\ cm\ (1dp)}$

</td>
<td>2- correct answer

1- valid value for x</td>
</tr>
</table>

Spacer 2

10. If A and B are two **independent events**, then the probability of occurrence of at least one of A and B is given by:

A. $1 + P(\bar{A})P(\bar{B})$
B. $1 - P(\bar{A})P(\bar{B})$
C. $P(A) + P(B) - P(\bar{A})P(\bar{B})$
D. $1 - P(A)P(B)$

at least one of A and B is equivalent to $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

(For independent events, $P(A \cap B) = P(A)\cdot P(B)$

$= P(A) + P(B)(1 - P(A))$

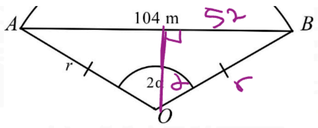
$= P(A) + P(B)\cdot P(\bar{A})$

$= 1 - P(\bar{A}) + P(B)\cdot P(\bar{A})$

$= 1 - P(\bar{A})[1 - P(B)]$

$= 1 - P(\bar{A})P(\bar{B})$

B



(a) Show that $\sin\alpha = \frac{52\alpha}{63}$.

$l = r\theta$

$126 = r\theta \qquad r = \frac{126}{2\alpha} = \frac{63}{\alpha}$

$\sin\alpha = \frac{52}{r}$

$\sin\alpha = \frac{52}{\frac{63}{\alpha}} = \frac{52\alpha}{63}$

(b) If $\alpha = \frac{\pi}{3}$, find the exact radius of the circle.

Express your answer with a rational denominator.

$\sin\alpha = \frac{52}{r}$

$r = \frac{52}{\sin\frac{\pi}{3}} = \frac{52}{\frac{\sqrt{3}}{2}}$

$r = \frac{104}{\sqrt{3}} = \frac{104\sqrt{3}}{3}$ m

iii. Hence show that the shortest beam that can be used is given by **2**

$$y = a\sqrt{1+\left(\frac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1+\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

$\tan^3\alpha = \frac{b}{a} \qquad \tan\alpha = \frac{\sqrt[3]{b}}{\sqrt[3]{a}}$

$y' = \frac{a\sin\alpha}{\cos^2\alpha} - \frac{b\cos\alpha}{\sin^2\alpha}$

$u = a\sin\alpha \qquad u = b\cos\alpha$

$u' = a\cos\alpha \qquad u' = -b\sin\alpha$

$v = \cos^2\alpha \qquad v = \sin^2\alpha$

$v' = -2\cos\alpha\sin\alpha \qquad v' = 2\sin\alpha\cos\alpha$

$y'' = \frac{a\cos^3\alpha + 2a\sin^2\alpha\cos\alpha}{\cos^4\alpha} - \frac{-b\sin^3\alpha - 2b\sin\alpha\cos^2\alpha}{\sin^4\alpha}$

$= \frac{a\cos^3\alpha + 2a\sin^2\alpha\cos\alpha}{\cos^4\alpha} + \frac{b\sin^3\alpha + 2b\sin\alpha\cos^2\alpha}{\sin^4\alpha} > 0$

$\therefore$ min point

$\tan\alpha = \sqrt[3]{\frac{b}{a}} \qquad 1+\tan^2\alpha = \sec^2\alpha$

$1+\left(\frac{b}{a}\right)^{\frac{2}{3}} = \sec^2\alpha$

$\cot\alpha = \sqrt[3]{\frac{a}{b}} \qquad 1+\cot^2\alpha = \operatorname{cosec}^2\alpha$

$1+\left(\frac{a}{b}\right)^{2/3} = \operatorname{cosec}^2\alpha$

$y = a\sec\alpha + b\operatorname{cosec}\alpha$

$y = a\sqrt{1+\left(\frac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1+\left(\frac{a}{b}\right)^{2/3}}$

(d)	(i)	$\angle DAO$, $\angle ADO$ and $\angle DOB$ all refer to the acute angles $\angle DAO + \angle ADO = \angle DOB$ (exterior angle sum of ΔAOD) but as ΔAOD is isosceles $\angle DAO = \angle ADO$ $\therefore$ $2\angle DAO = \angle DOB$ $2\angle DAO = 30°$ $\angle DAO = 15°$	✓ Finding $\angle DAO$ with geometrical reasons stated
	(ii)	$\angle AOD$ and $\angle DOB$ refer to the obtuse and acute angles respectively $\angle AOD = 180° - \angle DOB$ (angles on a straight line are supplementary) $\angle AOD = 180° - 30°$ $= 150°$ $AD^2 = AO^2 + DO^2 - 2 \times AO \times DO \times \cos(\angle AOD)$ $= 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos(150°)$ $= 2 - \cancel{2} \times \left(\frac{-\sqrt{3}}{\cancel{2}}\right)$ $= 2 - (-\sqrt{3})$ $= 2 + \sqrt{3}$	✓ Using the cosine rule with appropriate sides and angles ✓ Clear substitutions and steps to arrive at the answer
	(iii)	$\angle DAO$ and $\angle AOD$ refer to the acute and obtuse angles respectively $\frac{\sin(\angle DAO)}{DO} = \frac{\sin(\angle AOD)}{AD}$ $\frac{\sin(15°)}{1} = \frac{\sin(150°)}{\sqrt{2+\sqrt{3}}}$ $\sin(15°) = \frac{\left(\frac{1}{2}\right)}{\sqrt{2+\sqrt{3}}}$ $= \frac{1}{2} \times \frac{1}{\sqrt{2+\sqrt{3}}} \times \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$ $= \frac{1}{2} \times \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2^2-(\sqrt{3})^2}}$ $= \frac{1}{2} \times \frac{\sqrt{2-\sqrt{3}}}{\sqrt{4-3}}$ $= \frac{1}{2} \times \frac{\sqrt{2-\sqrt{3}}}{1}$ $= \frac{1}{2}\sqrt{2-\sqrt{3}}$	✓ Using the sine rule with correct values ✓ Clearly show the steps (including rationalising the denominato r) leading to the answer

Spacer 3

d) i) $3^8 = 6561$ ① mark for correct answer

ii) $2^8 = 256$ ① mark for correct answer

Also accepted interpretation of $2^8 - 2 = 254$ which contain only B's & C's but excludes all B's & all C's

iii) $3^8 - 3 \times 256 + 3 = 5796$

or

$3^8 - 3 \times 254 = 5796$

② marks for full correct answer

① for $3^8 - 3 \times 256 = 5793$

Suggested Solutions	Awarded	
a) i) $6^2 = 36$	1	well done
(ii) $p = \frac{1}{6}$ $E(6^x)$		
$= 6^0 \binom{2}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2 + 6^1\binom{2}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^1 + 6^2\binom{2}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0$	1	1 for binomial 2 for correct answer
$= \frac{1}{36}(25 + 60 + 36)$		• poorly done most students used 0, 1, 2 not $6^0, 6^1, 6^2$ \$1.82 common incorrect answer.
$= \$3.36$	1	• students who showed binomial gained 1 mark

b)

Let x be the distance each vertex on H_1 travels after t seconds

$\frac{dx}{dt} = 3$ cm/s

Let A = area of H_1

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

Let l be the side length of H_1

$l^2 = x^2 + (20-x)^2 - 2 \times x(20-x)\cos 120°$

$= x^2 + 400 - 40x + x^2 - 2x(20-x)(-\frac{1}{2})$

$= x^2 + 400 - 40x + x^2 + 20x - x^2$

$= x^2 - 20x + 400$ ① for finding an expression for the side length of H_1

$A = 6 \times \frac{1}{2} \times l \times l \times \sin 60°$

$= \frac{3\sqrt{3}}{2} l^2$

$= \frac{3\sqrt{3}}{2}(x^2 - 20x + 400)$ ① for finding an expression for the area of H_1

$\frac{dA}{dx} = \frac{3\sqrt{3}}{2}(2x - 20)$

$= 3\sqrt{3}(x-10)$

$\frac{dA}{dt} = 3\sqrt{3}(x-10) \times 3$

$= 9\sqrt{3}(x-10)$

After 10s, the point has travelled $3 \times 10 = 30$cm, $30 - 20 = 10$ so it is 10cm from the vertex

ie need to sub in $x = 10$ ① for finding correct value of x to sub in

$\frac{dA}{dt} = 9\sqrt{3}(10-10)$

$= 0$ cm^2/s ① for finding the correct expression for $\frac{dA}{dt}$ and evaluating correctly when $x = 10$.

Q14 - (e) $y = f(x)$ is odd, $f(x) \geq 0$ for $x \geq 0$.

$$\text{Area} = \int_{-5}^{5} f(x+4)\,dx$$

Let $u = x+4$
$du = dx$
$x = 5 \rightarrow u = 9$
$x = -5 \rightarrow u = -1$

$$= \int_{-1}^{9} f(u)\,du$$

$$= \int_{-1}^{0} f(u)\,du + \int_{0}^{9} f(u)\,du$$

$$= \int_{0}^{1} f(u)\,du + \int_{0}^{9} f(u)\,du$$

as $f(x)$ is odd

$$= \left(e^1 + \frac{1}{e^1} - 2\right) + \left(e^9 + \frac{1}{e^9} - 2\right)$$

$$\therefore \text{Area} = e + \frac{1}{e} + e^9 + \frac{1}{e^9} - 4 \text{ units}^2 \ \#$$

✓ ✓

(i) $\angle MCL = 120$.
$\therefore$ In $\triangle LCM$, using cosine rule.
$7^2 = x^2 + y^2 - 2xy\cos 120$
$49 = x^2 + y^2 - 2xy(-\cos 60)$
$49 = x^2 + y^2 + 2xy \cdot \frac{1}{2}$
$49 = x^2 + y^2 + xy$.

(ii) Find $\frac{dy}{dt}$ when $x = 5$; $y = 3$.

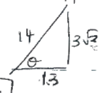
$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$
$$= -\frac{(2x+y)}{x+2y} \times 11$$
$$= -\left(\frac{10+3}{5+6}\right) \times 11$$
$$= \boxed{-13 \text{ m/s}}.$$

When $x = 5$.
$49 = 25 + y^2 + 5y$.
$24 = y^2 + 5y$
$y^2 + 5y - 24 = 0$
$(y+8)(y-3) = 0$
$\therefore y = 3$ as $y > 0$

(iii) $\frac{\sin\theta}{y} = \frac{\sin 120}{7}$
$\sin\theta = \frac{y\sin 60}{7}$
$\boxed{\sin\theta = \frac{y\sqrt{3}}{14}}$

$y = \frac{14\sin\theta}{\sqrt{3}}$
$\frac{dy}{d\theta} = \frac{14}{\sqrt{3}}\cos\theta$.

When $x = 5, y = 3$.
$\sin\theta = \frac{\sqrt{3} \cdot 3}{14}$



(iv) $\frac{d\theta}{dt} = \frac{d\theta}{dy} \times \frac{dy}{dt}$
$= \frac{\sqrt{3}}{14\cos\theta} \times -13$.

? $\frac{d\theta}{dt} = \frac{\sqrt{3}}{14 \times \frac{13}{14}} \times -13 = \boxed{99^\circ/s}$.

d) i) If right angled then from converse of Pyth. Thm.
$XY^2 = XT^2 + TY^2$
$LHS = 25^2$
$= 625$
$RHS = 15^2 + 20^2$
$= 225 + 400$
$= 625$
$LHS = RHS$

1

(ii) $\tan\theta = \frac{20}{15} = \frac{4}{3}$
$\theta = \tan^{-1}\frac{4}{3}$
$\therefore \sin\theta = \frac{h}{15}$
$h = 15\sin(\tan^{-1}\frac{4}{3})$
$h = 12$
OR using similar Δ's.

2

(iii) Let the angle be θ
$\tan\theta = \frac{12}{5}$
$\theta = \tan^{-1}\frac{12}{5}$
$= 67°23'$

(c) Find $\int_0^2 3^{-x}\,dx$ **2**

$= -\int_0^2 -3^{-x}\,dx$

$= \left[\ln\left(\frac{1}{3}\right)\cdot\left(\frac{1}{3}\right)^x\right]_0^2$

$= \frac{1}{\log_e 3}\cdot\left(3^{-2} - 1\right)$

$= \frac{1}{\log_e 3}\cdot\left(1 - \frac{1}{9}\right)$

$= \frac{8}{9\log_e 3}$

Spacer 4

Question 11 (7 marks)

(a) Express n in terms of x and y, given $(\sqrt{2})^n = \frac{4^x}{32^y}$ **1**

MA12.1

$2^{\frac{n}{2}} = \frac{2^{2x}}{2^{5y}}$

$2^{\frac{n}{2}} = 2^{2x-5y}$

$\frac{n}{2} = 2x - 5y$

$n = 4x - 10y$

1

(a) Express $5\cot^2 x - 2\operatorname{cosec} x + 2$ in terms of $\operatorname{cosec} x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$5(\operatorname{cosec}^2 x - 1) - 2\operatorname{cosec} x + 2$

$= 5\operatorname{cosec}^2 x - 5 - 2\operatorname{cosec} x + 2$

$= 5\operatorname{cosec}^2 x - 2\operatorname{cosec} x - 3$

(b) Hence show that $\int_e^{e^2} \frac{1-\log_e x}{x\log_e x}\,dx = \log_e 2 - 1$ **2**

$$\frac{1-\log_e x}{x\log_e x} = \frac{\frac{1-\log_e x}{x^2}}{\frac{\log_e x}{x}}$$

$$\int_e^{e^2} \frac{\left(\frac{1-\log_e x}{x^2}\right) f'(x)}{\left(\frac{\log_e x}{x}\right) f(x)} = \ln\left|\left[\frac{\log_e x}{x}\right]_e^{e^2}\right|$$

$$= \ln\left[\frac{\log_e e^2}{e^2}\right] - \ln\left[\frac{\log_e e}{e}\right]$$

$$= \ln\left(\frac{2}{e^2}\right) - \ln\frac{1}{e}$$

$$= \ln 2 - \ln e^2 + \ln e$$

$$= \ln 2 - 2 + 1 = \ln 2 - 1$$

$$= \log_e 2 - 1$$

Question 29 (3 marks)

Given that the function $f(x)$ has a derivative $y' = 4e^{4x} + 3$ and the equation of the tangent to this curve is $y = 7x + 2$. Find the exact value of $f(3)$. **3**

MA12.6

$m_{tan} = 7$

$7 = 4e^{4x} + 3$

$4 = 4e^{4x}$

$1 = e^{4x}$

$e^0 = e^{4x}$

$x = 0$

$y = 7(0) + 2$

$(0, 2)$

$y = \int 4e^{4x} + 3\,dx$

$y = e^{4x} + 3x + C$

$2 = e^0 + 3(0) + C$

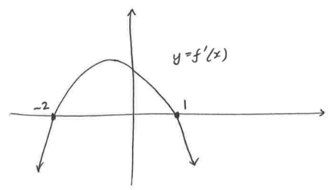
$C = 1$

$y = e^{4x} + 3x + 1$

$f(3) = e^{12} + 12 + 1$

$f(3) = e^{12} + 13$

(a) Use the space below to sketch the graph of $y = f'(x)$ **2**



✓ x-ints,
✓ shape.

(b) Find the area bounded by $y = f'(x)$ and the x-axis. **2**

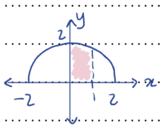
Area $= \int_{-2}^{1} f'(x)\,dx$

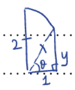
$= \left[f(x)\right]_{-2}^{1}$

$= 3 - (-8) = 11\,u^2$. ✓✓

(a) By sketching the graph of $y = \sqrt{4 - x^2}$, show that **3**

$$\int_0^1 \sqrt{4 - x^2}\,dx = \frac{\sqrt{3}}{\cancel{4}\,2} + \frac{\pi}{\cancel{12}\,3}$$





$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{y}{2}$

$\therefore y = 2\sin\frac{\pi}{3}$

$= \sqrt{3}$

$\therefore A_\Delta = \frac{1}{2} \times 1 \times \sqrt{3}$ (1)

$= \frac{\sqrt{3}}{2}$

$A_{sector} = \frac{1}{2} \times 2^2 \times \frac{\pi}{6}$ (1)

$= \frac{\pi}{3}$

$\therefore \int_0^1 \sqrt{4 - x^2}\,dx = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$ (1)

(b) Hence or otherwise, find $\int_2^{\cancel{4}\,3} \sqrt{16x - 4x^2}\,dx$. **2**

$\int_2^4 \sqrt{4(4x - x^2)}\,dx = 2\int_2^4 \sqrt{4x - x^2}$

$f(x) = \sqrt{4 - x^2}$

$f(x-2) = \sqrt{4 - (x-2)^2}$

$= \sqrt{4 - (x^2 - 4x + 4)}$

$= \sqrt{4x - x^2}$ (1)

$\int_2^3 \sqrt{16 - 4x^2}\,dx = 2 \times \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) = \sqrt{3} + \frac{2\pi}{3}$ (1)

sol#v

Question 17 (Ho)

(a) (3 marks)

- ✓ [1] for obtaining $v = \dfrac{(2t+1)^{\frac{3}{2}}}{3} + C$
- ✓ [1] for finding the value of C
- ✓ [1] for finding v when $t = 4$

$$\begin{aligned} a &= (2t+1)^{\frac{1}{2}} \\ \therefore v &= \int (2t+1)^{\frac{1}{2}} dt \\ &= \frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + C \\ &= \frac{(2t+1)^{\frac{3}{2}}}{3} + C \end{aligned}$$

When $t = 0$, $v = 0$

$$\begin{aligned} \therefore 0 &= \frac{1}{3} + C \\ \therefore C &= -\frac{1}{3} \\ \therefore v &= \frac{(2t+1)^{\frac{3}{2}}}{3} - \frac{1}{3} \end{aligned}$$

When $t = 4$,

$$\begin{aligned} v &= \frac{9^{\frac{3}{2}}}{3} - \frac{1}{3} \\ &= \frac{27}{3} - \frac{1}{3} \\ &= \frac{26}{3} \text{ m/s} \end{aligned}$$

(b) (3 marks)

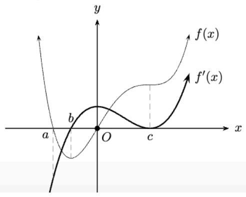
- ✓ [1] for correct interpretation of 'during the fourth second'
- ✓ [1] for obtaining $v_{\text{avg}} = \int_3^4 v(t)\,dt$
- ✓ [1] for finding the exact average velocity

During the fourth second means between $t = 3$ to $t = 4$.

$$\begin{aligned} \therefore v_{\text{avg}} &= \frac{x(4) - x(3)}{4 - 3} \\ &= x(4) - x(3) \\ &= \int_3^4 v(t)\,dt \\ &= \int_3^4 \frac{(2t+1)^{\frac{3}{2}}}{3} - \frac{1}{3}\,dt \\ &= \frac{1}{3}\int_3^4 (2t+1)^{\frac{3}{2}} - 1\,dt \\ &= \frac{1}{3}\left[\frac{(2t+1)^{\frac{5}{2}}}{\frac{5}{2} \times 2} - t\right]_3^4 \\ &= \frac{1}{3}\left[\frac{(2t+1)^{\frac{5}{2}}}{5} - t\right]_3^4 \\ &= \frac{1}{3}\left(\frac{223}{5} - \left(\frac{7^{\frac{5}{2}}}{5} - 3\right)\right) \\ &= \frac{1}{3}\left(\frac{238}{5} - \frac{7^{\frac{5}{2}}}{5}\right) \text{ m/s} \end{aligned}$$

Question 18 (Ho) (3 marks)

- ✓ [1] for correct overall shape of $f'(x)$
- ✓ [1] for correct location of $f'(x)$ at either O, $x = b$, OR $x = c$
- ✓ [1] for correct locations of $f'(x)$ at O, $x = a$, $x = b$, AND $x = c$



(b) (2 marks)

- ✓ [1] for calculating $E(X)$ correctly
- ✓ [1] for final answer

$$\begin{aligned} E(X) &= -3 \times \frac{1}{8} - 2 \times \frac{3}{8} + 1 \times \frac{3}{8} + 3 \times \frac{1}{8} \\ &= -\frac{3}{8} \end{aligned}$$

$$\begin{aligned} \therefore \text{Expected loss after 1000 games} &= 1000 \times \frac{3}{8} \\ &= \$375 \end{aligned}$$

(c) (2 marks)

- ✓ [1] for simplification to $\dfrac{P(X > -2)}{P(X \geq -2)}$
- ✓ [1] for final answer

$$\begin{aligned} P(X > -2 | X \geq -2) &= \frac{P(X > -2 \cap X \geq -2)}{P(X \geq -2)} \\ &= \frac{P(X > -2)}{P(X \geq -2)} \\ &= \frac{\frac{4}{8}}{\frac{7}{8}} \\ &= \frac{4}{7} \end{aligned}$$

Spacer 5

$$⑥\ \text{Area} = 34 = \left|\int_1^2 k(x-2)^3\,dx\right| + \int_2^4 k(x-2)^3\,dx$$

$$34 = k\left|\int_1^2 (x-2)^3\,dx\right| + k\int_2^4 (x-2)^3\,dx$$

$$34 = k\left|\left[\frac{(x-2)^4}{4}\right]_1^2\right| + k\left[\frac{(x-2)^4}{4}\right]_2^4$$

$$34 = k\left|0 - \frac{1}{4}\right| + k\left[\frac{16}{4} - 0\right]$$

$$34 = k\left(\frac{1}{4} + 4\right)$$

$$\therefore k = 8$$

B

(a) Show that $\frac{1}{2} - \frac{1}{2^x+1} = \frac{2^x-1}{2(2^x+1)}$ **1**

$$\text{L.H.S.} = \frac{1}{2} - \frac{1}{2^x+1}$$

$$= \frac{2^x+1-2}{2(2^x+1)}$$

$$= \frac{2^x-1}{2(2^x+1)}$$

(b) Hence determine whether $f(x)$ is even, odd or neither. Show all working. **2**

$$f(-x) = \frac{2^{-x}-1}{2(2^{-x}+1)}$$

$$= \frac{\frac{1}{2^x} - 1}{2\left(\frac{1}{2^x}+1\right)}$$

$$= \frac{1-2^x}{2(1+2^x)}$$

$$= -\frac{(2^x-1)}{2(2^x+1)}$$

$$= -f(x)$$

Alternative Soln:

$$f(-x) = \frac{2^{-x}-1}{2(2^{-x}+1)} \times \frac{2^x}{2^x}$$

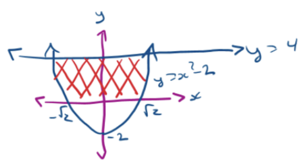
$$= \frac{1-2^x}{2(1+2^x)}$$

$$= \frac{-(2^x-1)}{2(2^x+1)}$$

$$= -f(x)$$

$\therefore f(x)$ is an odd fn.

(b) Find the exact area bounded by the curve $y = x^2 - 2$, the x axis and the line $y = 4$. **4**



$$y = x^2 - 2$$

$$x = \pm\sqrt{y+2}$$

$$A = 2\int_0^4 \sqrt{y+2}\,dy$$

$$= 2\int_0^4 (y+2)^{1/2}\,dy$$

$$= 2\left[\frac{(y+2)^{3/2}}{\left(\frac{3}{2}\right)}\right]_0^4$$

$$= \frac{4}{3}\left[(y+2)^{3/2}\right]_0^4$$

$$= \frac{4}{3}\left[(6)^{3/2} - 2^{3/2}\right]$$

$$= \frac{4}{3}\left[6\sqrt{6} - 2\sqrt{2}\right] = \frac{8}{3}\left(3\sqrt{6} - \sqrt{2}\right)\ u^2$$

4	Correct response
3	One error in correct method or Found area between curves but find not remove the excess area between $-\sqrt{2} \le x \le \sqrt{2}$
2	Two errors with correct procedure or or error or incomplete solution with the method of area between two curves
1	Finding the 2 lots of the area under the curve $y = x^2 - 2$ between $\sqrt{2}$ and $\sqrt{6}$

Markers comments:

Criteria	Marks
Provides correct solution in simplified form (Factorisation of numerator is not required)	2
Attempts to use the quotient rule	1

let $y = \dfrac{e^{3x}}{x-1}$

$\dfrac{dy}{dx} = \dfrac{(x-1)3e^{3x} - e^{3x}(1)}{(x-1)^2}$

$= \dfrac{3xe^{3x} - 3e^{3x} - e^{3x}}{(x-1)^2}$

$= \dfrac{3xe^{3x} - 4e^{3x}}{(x-1)^2} = \dfrac{e^{3x}(3x-4)}{(x-1)^2}$

Marker's Comments:

Well done.

Some students needed to take care in simplifying the numerator.

(b) $\tan x \cos x$

Method 2

let $y = \tan x \cos x$

$= \dfrac{\sin x}{\cancel{\cos x}} \times \cancel{\cos x}$

$y = \sin x$

$\therefore \dfrac{dy}{dx} = \cos x$

(c) $\log_4\left(x^2+3\right)$

Criteria	Marks
Provides correct solution	1

let $y = \log_4(x^2+3)$

$= \dfrac{\ln(x^2+3)}{\ln 4}$

$\therefore \dfrac{dy}{dx} = \dfrac{1}{\ln 4} \cdot \dfrac{2x}{x^2+3}$

$= \dfrac{2x}{(\ln 4)(x^2+3)}$

$= \dfrac{x}{(\ln 2)(x^2+3)}$

Marker's Comments:

Overall, well done.

(a) Show that $XP = h \cot 10°$, and write down a similar expression for XQ. **1**

In ΔRXP, $\tan 10° = \dfrac{h}{XP}$

$XP = \dfrac{h}{\tan 10°}$

$= h\cot 10°$ $\frac{1}{2}$ mark

Similarly, $XQ = h\cot 5°$ $\frac{1}{2}$ mark

Many students did not answer the second part of the question.

Make sure you read the question!!!!!

(b) Hence, find the value of h. Give your answer correct to the nearest metre. **3**

$\angle XPQ = 90°$ (NW $\perp$ NE)

$XP^2 + PQ^2 = XQ^2$ 1 mark

$(h\cot 10°)^2 + 100^2 = (h\cot 5°)^2$

$h^2 \cot^2 10° + 100^2 = h^2 \cot^2 5°$

$h^2(\cot^2 5° - \cot^2 10°) = 100^2$

$h^2 = \dfrac{100^2}{\cot^2 5° - \cot^2 10°}$

$h = \dfrac{100}{\sqrt{\cot^2 5° - \cot^2 10°}}$ 1 mark

$= 10$ m (nearest metre) 1 mark

Some students ignored the diagram and did not see NE and NW and so $\angle XPQ = 90°$. Not realising this meant you could not answer the question successfully.

Answer

$$t = 2 - \frac{1}{e^{3x}}$$
$$= 2 - e^{-3x}$$
$$\frac{dt}{dx} = 3e^{-3x}$$
$$= \frac{3}{e^{3x}}$$
$$\therefore \frac{dx}{dt} = \frac{e^{3x}}{3}$$
ie $v = \dfrac{e^{3x}}{3}$

$$x = \frac{1}{3}\ln(\frac{1}{2-t})$$
$$v = \frac{e^{3\times\frac{1}{3}\ln\left(\frac{1}{2-t}\right)}}{3}$$
$$v = \frac{1}{3(2-t)}$$
$$\frac{dx}{dt} = \frac{1}{3(2-t)}$$
$$a = \frac{d^2x}{dt^2} = \frac{1}{3}\left(\frac{1}{2-t}\right)$$
when $t = 1$
$$a = \frac{1}{3} \times \frac{1}{2-1}$$
$$= \frac{1}{3}\,\text{ms}^{-2}$$

	Solution		Marks	Allocation of marks
(e)	i) $t = 2 - \frac{1}{e^{3x}}$ $= 2 - e^{-3x}$ $\frac{dt}{dx} = 3e^{-3x}$ $= \frac{3}{e^{3x}}$ $\therefore \frac{dx}{dt} = \frac{e^{3x}}{3}$ ie $v = \frac{e^{3x}}{3}$	OR $t = 2 - \frac{1}{e^{3x}}$ $t - 2 = -\frac{1}{e^{3x}}$ $e^{3x} = -\frac{1}{t-2}$ $= \frac{1}{2-t}$ $3x = \ln\left(\frac{1}{2-t}\right)$ $x = \frac{1}{3}\ln\left(\frac{1}{2-t}\right)$ $\frac{dx}{dt} = \frac{1}{3}\left(\frac{\frac{1}{(2-t)^2}}{\frac{1}{2-t}}\right)$ $= \frac{1}{3}\left(\frac{1}{2-t}\right)$ $= \frac{e^{3x}}{3}$	2	2 marks- correctly finds $\frac{dt}{dx}$ or rearranges to make t the subject and finds $\frac{dx}{dt}$ 1 mark - attempts to differentiate t in terms of x or correctly expresses x in terms of t
	ii) $a = \frac{d}{dx}\left(\frac{v^2}{2}\right)$ $= \frac{d}{dx}\left(\frac{e^{6x}}{18}\right)$ $= \frac{e^{6x}}{3}$ when $t = 1$ $1 = 2 - \frac{1}{e^{3x}}$ $-1 = -\frac{1}{e^{3x}}$ $e^{3x} = 1$ $3x = \ln 1$ $x = 0$ $\therefore a = \frac{e^0}{3}$ $= \frac{1}{3}\text{ms}^{-2}$	OR $\frac{dx}{dt} = \frac{1}{3(2-t)} = \frac{1}{3}(2-t)^{-1}$ $a = \frac{d^2x}{dt^2} = \frac{-1}{3}(2-t)^{-2} \times -1$ $= \frac{1}{3}\left(\frac{1}{(2-t)^2}\right)$ When t = 1 $a = \frac{1}{3} \times \frac{1}{(2-1)^2} = \frac{1}{3}\text{ms}^{-2}$	2	2 marks - correct answer after obtaining correct expression for acceleration in terms of t or x, or equivalent 1 mark - attempt to use acceleration formula $\frac{d}{dx}\frac{v^2}{2}$ or to find acceleration in terms of t.

10) D. 9) D

Question 34 (3 marks)

If $y = \tan^2 x$, find the values of the constants a and b, such that $\frac{d^2y}{dx^2} = ay^2 + by + 2$. **3**

$\frac{dy}{dx} = 2\tan x \sec^2 x$
$= 2\tan x(1+\tan^2 x)$
$= 2\tan x + 2\tan^3 x$
$\frac{d^2y}{dx^2} = 2\sec^2 x + 6\tan^2 x \sec^2 x$
$= 2(1+\tan^2 x) + 6\tan^2 x(1+\tan^2 x)$
$= 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x$
$= 2 + 8\tan^2 x + 6\tan^4 x$
sub $y = \tan^2 x$,
$\frac{d^2y}{dx^2} = 2 + 8y + 6y^2$
$\therefore a = 6, b = 8$

Q28a	$\int_0^8 k(64-x^2)=1$ $k\left[64x-\frac{x^3}{3}\right]_0^8=1$ $k\left[64\times 8-\frac{8^3}{3}-0\right]=1$ $k\times\frac{1024}{3}=1$ $k=\frac{3}{1024}$	2 Marks Correct solution 1 Mark Correct primitive function
Q28b	$f(x)=\begin{cases}\frac{3}{1024}(64-x^2) & \text{for } 0\le x\le 8\\ 0 & \text{otherwise}\end{cases}$ $F(x)=\int_0^x \frac{3}{1024}(64-x^2)dx$ $F(x)=\frac{3}{1024}\left[64x-\frac{x^3}{3}\right]_0^x$ $F(x)=\frac{3}{1024}\left[64x-\frac{x^3}{3}\right]_0^x$ $F(x)=\frac{3}{1024}\left(64x-\frac{x^3}{3}\right)$ $F(x)=\frac{3x}{16}-\frac{x^3}{1024}$ $\therefore F(x)=\begin{cases}0 & x<0\\ \frac{3x}{16}-\frac{x^3}{1024} & 0\le x\le 8\\ 1 & x>8\end{cases}$	2 Marks Correct solution 1 Mark Correct primitive function

Q14a	$y=\tan^3\left(\frac{x}{4}\right)$ $\frac{dy}{dx}=3\times\frac{1}{4}\sec^2\left(\frac{x}{4}\right)\times\tan^2\left(\frac{x}{4}\right)$ $\frac{dy}{dx}=\frac{3}{4}\sec^2\left(\frac{x}{4}\right)\tan^2\left(\frac{x}{4}\right)$	2 Marks Correct solution 1 Mark Correct differentiation of $\tan\frac{x}{4}$

Age

(i) The correlation coefficient is -0.955. Describe the association between Age and Balance Score with reference to the correlation. **1**

Strong negative correlation 1

(ii) The least squares regression line for this data is $y = 11.1249 - 0.1025x$. **1**

Using this regression line, predict the Balance Score of a 65 year old.

$y = 11.1249 - 0.1025 \times 65$

$= 4.4624$ 1

(iii) Comment on whether your answer in part (ii) is reliable. **1**

Yes as strong correlation & interpolation. 1

Question 14 continues on next page

Question 12 (continued)

(e) Let A and B be two events such that $P(A)=0.4$, $P(B)=0.55$ and $P(B|A)=0.6$.

(i) Determine whether A and B are independent events. **1**

$P(B|A) \neq P(B)$

$\therefore$ not independent 1

(ii) Find $P(A \cup B)$. **2**

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.4 + 0.55 -$

$P(A\cap B) = P(B|A) \times P(A)$

$= 0.6 \times 0.4$

$= 0.24$ 1

$P(A\cup B) = 0.4 + 0.55 - 0.24$ 1

$= 0.71$

Spacer 6

9. Which transformations listed are required to obtain the graph of $y = x^2 + \frac{1}{2}x - 3$ from the graph of $y = 4x^2 + x$?

A. Horizontal dilation by a factor of 2; vertical translation of 3 units upwards

B. Horizontal dilation by a factor of 2; vertical translation of 3 units downwards

C. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units upwards

D. Horizontal dilation by a factor of $\frac{1}{2}$; vertical translation of 3 units downwards

$y = 4x^2 + x \xRightarrow{x \to \frac{x}{2}} y = 4\left(\frac{x}{2}\right)^2 + \frac{x}{2} \xRightarrow{y \to y+3} y + 3 = x^2 + \frac{1}{2}x$

$x^2 + \frac{1}{2}x$

$y = x^2 + \frac{1}{2}x - 3$

16.	(a)	(i)	$T_3 = 2^3 + 3(3) = 17$ songs ☑
		(ii)	$S_n = 2^n \quad a=2 \quad r=2 \quad \therefore S_{20} = \dfrac{2\left(2^{20}-1\right)}{2-1}$ $= 2\ 097\ 150$ songs ☑ $S_n = 3n \quad a=3 \quad d=3 \quad \therefore S_{20} = \dfrac{20}{2}(2\times 3 + (20-1)\times 3)$ $= 630$ *songs* ☑ $\therefore$ *Total songs* $= 20\ 971\ 150 + 630 = 2\,097\,780$ songs ☑

7.	Max when moves from v > 0 to v < 0 Therefore at $x = 10$ $\therefore B$

(iii) Show that the volume of the cylindrical tank can be expressed by **1**

$$V = \frac{3\pi}{4}\left(8r^2 - r^3\right).$$

$V = \pi r^2 h$

$= \pi r^2 \times \frac{3}{4}(8-r)$

$= \frac{3\pi}{4}\left[r^2(8-r)\right] = \frac{3\pi}{4}(8r^2 - r^3)$ as req'd

(iv) Find the value of r which gives the tank its greatest volume and calculate that volume, correct to the nearest litre. **4**

$\frac{dV}{dr} = \frac{3\pi}{4}\left[16r - 3r^2\right]$

$\frac{d^2V}{dr^2} = \frac{3\pi}{4}\left[16 - 6r\right]$

$\frac{dV}{dr} = 0 \quad 0 = \frac{3\pi}{4} r(16-3r)$

$r = 0, \quad 16 - 3r = 0$

$r = \frac{16}{3}$ ($r > 0$)

when $r = \frac{16}{3}$ $\frac{d^2V}{dr^2} = \frac{3\pi}{4}\left(16 - 6 \times \frac{16}{3}\right)$

$= \frac{3\pi}{4}(16 - 32) = \frac{3\pi}{4} \times -16 < 0$

$\therefore$ maximum (greatest) volume

when $r = \frac{16}{3}$ $V = \frac{3\pi}{4}\left(8 \times \left(\frac{16}{3}\right)^2 - \left(\frac{16}{3}\right)^3\right)$

$= \frac{3\pi}{4} \times \frac{\cancel{2048}\ 512}{\cancel{729}\ 9} = \frac{512\pi}{9}$

$= 178.7217154$

$\times 1000$

$= 178\,722$ L

or 178.722 kL

$\boxed{1m^3 = 1000L}$

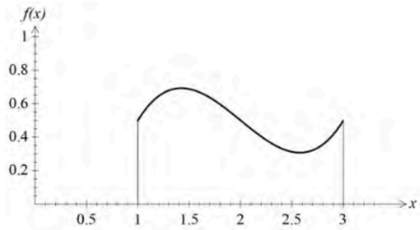
$1m^3 = 1kL$

Question 16 continues on page 41

Question 31 (5 marks)

A continuous probability distribution is graphed below and is defined by:

$$f(x)=\begin{cases}\frac{1}{2}(x^3-6x^2+11x-5), & [1,3]\\ 0, & (-\infty,1)\cup(3,\infty)\end{cases}$$



i) Use the graph of $y=f(x)$ to **estimate** the mode of the distribution. **1**

Mode: $x=1\cdot4$

i) Find the cumulative distribution function. **2**

$$F(x)=\int_1^x \frac{1}{2}(x^3-6x^2+11x-5)\,dx$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x\right]_1^x$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x-\left(\frac{1}{4}-2+\frac{11}{2}-5\right)\right]$$

$$=\frac{1}{2}\left[\frac{x^4}{4}-2x^3+\frac{11x^2}{2}-5x-\left(-\frac{5}{4}\right)\right]$$

$$=\frac{1}{8}\left[x^4-8x^3+22x^2-20x+5\right]$$

ii) Find $P(X>2)$. **2**

$$P(X>2)=1-P(X<2)$$

$$=1-F(2)$$

$$=1-\left[\frac{1}{8}\left(2^4-8(2)^3+22(2)^2-20(2)+5\right)\right]$$

$$=\frac{3}{8}$$

10. Consider the cumulative probability density function

$$F(x)=\begin{cases}0, & x<\frac{\pi}{2}\\ \cos^2 x, & \frac{\pi}{2}\le x\le\pi\\ 1, & x>\pi\end{cases}$$

$\cos^2 x=\frac{3}{4}$

$\cos x=\pm\frac{\sqrt{3}}{4}$

rel $\angle=\frac{\pi}{6}$

$\therefore x=\frac{5\pi}{6}$.

What is the 3rd quartile of the distribution?

(A) $x=\frac{\pi}{6}$ (B) $x=\frac{5\pi}{8}$ (C) $x=\frac{3\pi}{4}$ (D) $x=\frac{5\pi}{6}$

14 N: $R = 5.536\%$. T: $R = 5.495\%$. T is best.

2 (a) $A_0 = 1000$, $A_n = 1.031 \times A_{n-1}$

(b) $A_0 = 5000$, $r = \frac{6.1}{2} = 3.1\%$ per 6 months. $R = 1.031$.

$A_n = 1.031 \times A_{n-1}$

(c) $A_0 = 10\,000$, $r = \frac{2.7}{12} = 0.225\%$ p.m. $R = 1.002\,25$.

$A_n = 1.002\,25 \times A_{n-1}$

TG **5** A principal of $1000 is invested for three years at an interest rate of 5.6% pa compounded half-yearly.

Determine how much needs to be invested to achieve the same interest if the interest rate was 3.5% pa compounded monthly.

5.6% per annum = 2.8% per half-year, 3 years = 6 half-years

$$\begin{aligned} FV &= PV(1+r)^n \\ &= 1000(1+0.028)^6 \\ &= 1000(1.028)^6 \\ &= 1180.208364\ldots \\ &= 1180.21 \text{ (2 dec pl)} \end{aligned}$$

∴ the compounded amount is $1180.21.

∴ the interest is $180.21.

3.5% per annum = 0.2916…% per month, 3 years = 36 months

$$\begin{aligned} A + 180.21 &= A(1+0.002917)^{36} \\ A + 180.21 &= 1.002917^{36}A \\ A(1.002917^{36} - 1) &= 180.21 \\ A &= \frac{180.21}{1.002917^{36} - 1} \\ &= 1630.060725\ldots \\ &= 1630 \text{ (nearest whole)} \end{aligned}$$

∴ the investment is $1630.

(a) FV = 200 × 10.0265

= 2005.3

The investment will be worth $2005.30.

(b) 3 years = 12 quarters, 4% pa = 1% per quarter

Let investment = A

5000 = A × 12.8093

$$\begin{aligned} A &= \frac{5000}{12.8093} \\ &= 390.3413926\ldots \\ &= 400 \text{ (nearest 100)} \end{aligned}$$

Holly should invest $400 per quarter.

21 MA	**30**	The number of hours for which light bulbs will work before failing can be modelled by the random variable X with cumulative distribution function. $$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$ Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs. Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal places.	**2**

$F(x) = 1 - e^{-0.01x}$

Substitute $F(x) = 0.99$:

$$0.99 = 1 - e^{-0.01x}$$

$$e^{-0.01x} = 0.01$$

$$-0.01x = \ln 0.01$$

$$x = \frac{\ln 0.01}{-0.01}$$

$$= 460.5170186\ \ldots$$

$$= 460.52 \text{ (2 dec. pl.)}$$

$\therefore$ the light bulb should work for 460.52 hours.

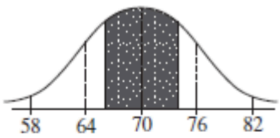
State Mean:
0.44/2

A

As $\mu = 70$ and $\sigma = 6$, then between $\mu - \sigma = 64$ and $\mu + \sigma = 76$.
This means 68% of scores are between 64 and 76.

But, between the lower and upper quartile is 50% of the scores.

Hence the correct graph is



a) Show that R has coordinates $(e-1, \ln[(e-1)^2])$ for ΔPQR to have maximum area. 2

$f(x) = \ln(x^2)$

$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$ 1m

$m(PQ) = \frac{2-0}{e-1} = \frac{2}{e-1}$

$\therefore \frac{2}{x} = \frac{2}{e-1}$ $\therefore x = e-1$

$y = f(x) = \ln(e-1)^2$

$\therefore R = ((e-1), \ln(e-1)^2)$ 1m

Question continues the next page

b) Hence find the size of $\angle RPQ$ correct to the nearest degree. 2

Let $m(PQ) = \frac{2}{e-1} = \tan\alpha$ 1m for $m(PQ)$, $m(RP)$

1m for correct angle $7°$ with working

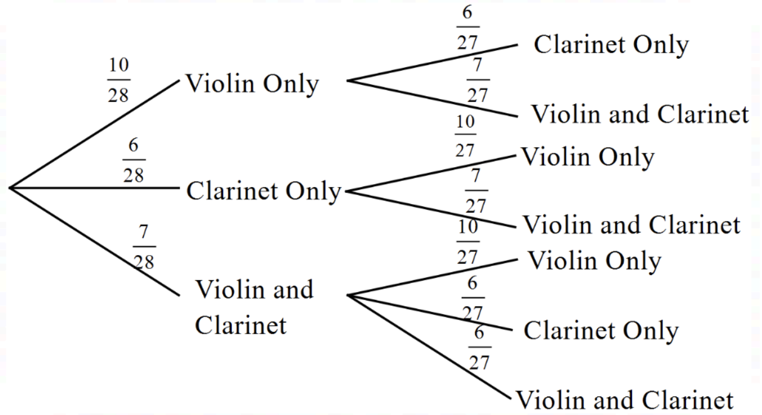
$\alpha = 49°20'$

$m(RP) = \frac{\ln(e-1)^2 - 0}{e-1-1} = \frac{\ln(e-1)^2}{e-2}$

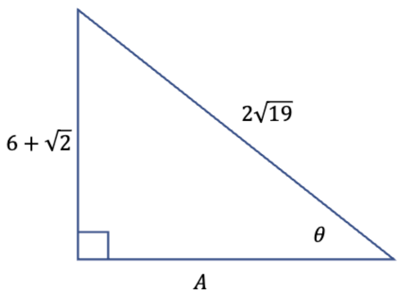
Let $m(RP) = \tan\beta$ $\beta = 56°26'$

$\angle RPQ = 7°6' \doteqdot 7°$ (nearest degree)

See below for alternative ☆

c	 $P(duet) = P(V,C) + P(V,both) + P(C,V) + P(C,both) + P(both,V) + P(both,C) + P(both,both)$ $= (2 \times (P(V,C) + P(V,both) + P(C,both)) + P(both,both)$ $= 2 \times \left(\frac{10}{28} \times \frac{6}{27} + \frac{10}{28} \times \frac{7}{27} + \frac{6}{28} \times \frac{7}{27}\right) + (\frac{7}{28} \times \frac{6}{27})$ $= \frac{193}{378}$ $= 51.06\%$	3 marks for correct answer 2 marks for applying addition and multiplication rules with minor errors 1 mark for finding one possible duet or partially correct tree diagram

Spacer 7

24	Constructing a right-angled triangle in quadrant 2.  	1 mark for $A^2 = \left(2\sqrt{19}\right)^2 - \left(6+\sqrt{2}\right)^2$ 1 mark for $\tan\theta = -\frac{6+\sqrt{2}}{6-\sqrt{2}}$ OR $\tan\theta = \frac{6+\sqrt{2}}{\sqrt{2}-6}$

	$A^2 = \left(2\sqrt{19}\right)^2 - \left(6+\sqrt{2}\right)^2$ $= 76 - \left(36 + 12\sqrt{2} + 2\right)$ $= 76 - \left(38 + 12\sqrt{2}\right)$ $= 38 - 12\sqrt{2}$ $= 36 - 12\sqrt{2} + 2$ $= \left(6-\sqrt{2}\right)^2$ $A = 6 - \sqrt{2}$ $\therefore \tan\theta = -\frac{6+\sqrt{2}}{6-\sqrt{2}}$ OR $\tan\theta = \frac{6+\sqrt{2}}{\sqrt{2}-6}$	

32	**Original function:** $y = \cos\big(2a(x-b)\big) + c$ New function: $y = -c\sin(3a(x-b))$ - Centre of motion at c corresponds to the amplitude of the new function - Both functions have been translated horizontally by b - Ratio of periods of the original function to the new function: $2a: 3a = 2: 3$. Therefore, the new function will sketch 3 periods within the same time that the original function sketched 2 periods. (i.e. 3 periods in 24 boxes or 1 period in 8 boxes) - The new function will be a negative sine curve.	1 mark for each of the following: - Recognises the functions have the same horizontal translation and/or starts the centre of motion of the new function below point P at $x = b$ - Sketches a negative sine curve from $x = b$ or $x = 0$ - Matches amplitude of the new function to the centre of motion of the original function - New function has period of 8 boxes.

(c) A scientist needs 13 hours of continuous daylight for an experiment. Between which dates should she be looking to run her experiment? **3**

Let $y = 13$

$$13 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right) + 12.19$$
$$0.81 = 2.24\sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$\frac{81}{224} = \sin\left(\frac{\pi t}{6} + 1.34\right)$$
$$\sin^{-1}\left(\frac{81}{224}\right) = \frac{\pi t}{6} + 1.34$$
$$\frac{\pi t}{6} + 1.34 = \sin^{-1}\left(\frac{81}{224}\right), \pi - \sin^{-1}\left(\frac{81}{224}\right), 2\pi + \sin^{-1}\left(\frac{81}{224}\right)$$
$$t = \frac{\sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}, \frac{\pi - \sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}, \frac{2\pi + \sin^{-1}\left(\frac{81}{224}\right) - 1.34}{\frac{\pi}{6}}$$
$$t = -1.817\ldots, 2.73\ldots, 10.14742\ldots$$

February has either 28/29 days
$0.73 \times 28 = 20.44$ or $0.69 \times 27 = 19.71$

October has 31 days
0.14*31=4.34

Therefore, the scientist should perform her experiment between October 5th and February 19th or 20th.

1 mark - finds one correct solutions, or translates two incorrect solutions into correct dates
2 marks – finds two correct solutions and attempts, or finds one correct solution and correctly translates to a date.
3 marks – finds two correct solutions and translates to the dates, specifying between October and February, rather than between February and October.

Markers feedback
These questions were very rarely seriously attempted. Part (a) and (b) were generally well done when attempted. Many students forgot that they would need to use radians, getting answers that were far too large. Some students found the correct dates, but set the region to be between February and October, rather than between October and February.

12(c)(iv)	$P(\text{exactly 1 interstate traveller}) = P(\mathrm{I},\tilde{\mathrm{I}}) + P(\tilde{\mathrm{I}},\mathrm{I})$ $= \left(\frac{42}{80} \times \frac{38}{79}\right) + \left(\frac{36}{80} \times \frac{42}{79}\right)$ $= \frac{399}{790}$ ✓ ✓	Aw 1 for one of the brackets. Aw 2 for answer.

Question 23 (3 marks)

A geometric progression has 5th term 9 and 13th term 59 049.

(a) Find the first term and the common ratio. **2**

$T_5 = ar^4 = 9$

$T_{13} = ar^{12} = 59049$

$\therefore \dfrac{ar^{12}}{ar^4} = \dfrac{59\,049}{9}$ $\quad \therefore r^8 = 6561$

$\therefore r = 3$ or -3

$\therefore a(81) = 9 \quad \therefore a = \dfrac{1}{9}$

Spacer 8

$$= \pi\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) \quad \text{cubic units}$$

(a)(ii) Let the volume of the cylinder which has radius $\frac{1}{\sqrt{3}}$ and height $\frac{\pi}{6}$ be denoted V_C. We can see that

$$V_C > V$$

$$\pi \times \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{\pi}{6} > \pi\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right)$$

$$\frac{\pi}{18} + \frac{\pi}{6} > \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\pi > \frac{3\sqrt{3}}{2}$$

Also, we know that $V > 0$

$$\pi\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6}\right) > 0$$

$$\frac{\pi}{6} < \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\pi < 2\sqrt{3}$$

$$\therefore \frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$$

(b)

$$(ax)^{\ln a} = (bx)^{\ln b}$$

$$\ln\left((ax)^{\ln a}\right) = \ln\left((bx)^{\ln b}\right)$$

$$\ln a \ln ax = \ln b \ln bx$$

$$\ln a(\ln a + \ln x) = \ln b(\ln b + \ln x)$$

$$\ln x(\ln a - \ln b) = (\ln b)^2 - (\ln a)^2$$

$$\ln x(\ln a - \ln b) = (\ln b - \ln a)(\ln b + \ln a) \quad \text{but } a \neq b \Rightarrow \ln a \neq \ln b$$

$$-\ln x = \ln b + \ln a$$

$$\ln\left(\frac{1}{x}\right) = \ln ab$$

$$\frac{1}{x} = ab$$

$$\therefore x = \frac{1}{ab}$$

(c)(ii) Let B_j be the amount remaining in the account j months after retirement.

$B_1 = RP - kM$

$B_2 = RB_1 - kM$

$= R^2P - kMR - kM$

$B_3 = RB_2 - kM$

$= R^3P - kMR^2 - kMR - kM$

......

......

$B_m = R^mP - kMR^{m-1} - kMR^{m-2} - ... - kMR - kM$

$B_m = R^mP - kM(1 + R + R^2 + ... + R^{m-1})$

$$= R^mP - \frac{kM(R^m - 1)}{R - 1}$$

We expect $B_m = 0$ where the account is exhausted

$$R^mP - \frac{kM(R^m - 1)}{R - 1} = 0$$

$$R^mP = \frac{kM(R^m - 1)}{R - 1}$$

$$R^mP(R - 1) = kMR^m - kM$$

$$R^m(kM - P(R - 1)) = kM$$

$$R^m = \frac{kM}{(kM - P(R - 1))}$$

$$\ln(R^m) = \ln\left(\frac{kM}{(kM - P(R - 1))}\right) \quad \text{but } P = MRS$$

$$m\ln R = \ln\left(\frac{kM}{(kM - MRS(R - 1))}\right) \quad \text{but } S = \frac{R^n - 1}{R - 1}$$

$$m = \frac{1}{\ln R} \times \ln\left(\frac{k}{k - R(R^n - 1)}\right)$$

(c)(iii) When $k \leq R(R^n - 1)$ then m is undefined which means there does not exist an m such that account gets reduced to zero. In other words, the account actually grows over time where the rate of interest gained exceeds the rate of monthly withdrawals (as an aside, one can formally show that $B_j > B_{j-1}$).

(a)

$$\frac{1}{\sqrt[3]{2}-1} = \frac{1}{\sqrt[3]{2}-1} \times \frac{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}$$

$$= \frac{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}{(\sqrt[3]{2})^3-1^3}$$

$$= 2^{\frac{2}{3}}+2^{\frac{1}{3}}+1$$

(d) If x, y and z are in geometric progression then $\frac{y}{x} = \frac{z}{y} \Rightarrow xz = y^2$. From the property provided

$$x^{\frac{1}{a}} = y^{\frac{1}{b}}$$

$$x^b = y^a$$

similarly $y^c = z^b$

$$\Rightarrow x^b \times z^b = y^a \times y^c$$

$$(xz)^b = y^{a+c}$$

$y^2b = y^{a+c}$ since x, y, z are in geometric progression

$$\Rightarrow 2b = a + c$$

$$b - a = c - b$$

Hence a, b and c are in arithmetic progression

(d) Let B_t be the balance in the account after t years. After each year, the balance grows by $100r_1\%$ and at the same time declines by $100r_2\%$ before the next payment comes in. Let the first payment occur at $t = 0$. Hence we have

$$B_1 = A(1+r_1)(1-r_2) + A$$

$$B_2 = B_1(1+r_1)(1-r_2) + A$$

$$= A(1+r_1)^2(1-r_2)^2 + A(1+r_1)(1+r_2) + A$$

$$B_3 = B_2(1+r_1)(1-r_2) + A$$

$$= A(1+r_1)^3(1-r_2)^3 + A(1+r_1)^2(1-r_2)^2 + A(1+r_1)(1+r_2) + A$$

and so on.

Note that each B_t forms a geometric series.

If $\dfrac{1}{r_2} - \dfrac{1}{r_1} < 1$ this implies that

$$r_1 - r_2 < r_1 r_2$$

$$1 + r_1 - r_2 - r_1 r_2 < 1$$

$$(1+r_1) - r_2(1+r_1) < 1$$

$$(1+r_1)(1-r_2) < 1$$

With r_1 and r_2 being positive, this condition suggests if we let $t \to \infty$ then B_t will actually converge to a finite value as it becomes a limiting sum. Hence

$$B_\infty = A + A(1+r_1)(1-r_2) + A(1+r_1)^2(1-r_2)^2 + A(1+r_1)^3(1+r_2)^3 + ...$$

$$= \frac{A}{1-(1+r_1)(1-r_2)}$$

$$= \frac{A}{r_2 + r_1 r_2 - r_1}$$

Since each term in the geometric series is positive, then when t is finite the value of B_t in the bank account will never exceed $\dfrac{A}{r_2 + r_1 r_2 - r_1}$

(d) (i) Note that

$$n(n+1)(n+2)(n+3) = [n(n+3)][(n+1)(n+2)]$$

$$= (n^2+3n)(n^2+3n+2)$$

Comparing this to $(n^2+an)(n^2+an+2)$ it is clear that $a=3$

(ii) Let $x = n^2+3n$ it follows that

$$n(n+1)(n+2)(n+3) = x(x+2)$$

$$= x^2+2x+1-1$$

$$= (x+1)^2-1$$

$$n(n+1)(n+2)(n+3)+1 = (x+1)^2$$

Thus, when $n = 2011$ the LHS becomes $2011 \times 2012 \times 2013 \times 2014 + 1$. Since x is an integer (as it is a polynomial function of integer n) then $x+1$ is an integer hence $2011 \times 2012 \times 2013 \times 2014 + 1$ is a perfect square.

Alternatively, let $u = n^2+3n+1$ which leads to

$$n(n+1)(n+2)(n+3) = (u-1)(u+1)$$

$$= u^2-1$$

$$n(n+1)(n+2)(n+3)+1 = u^2$$

Thus, when $n = 2011$ the LHS becomes $2011 \times 2012 \times 2013 \times 2014 + 1$. Since u is an integer (as it is a polynomial function of integer n) then $2011 \times 2012 \times 2013 \times 2014 + 1$ is a perfect square.

Question 7

From sketching the region defined, the area of $\mathcal{R}$ is $\displaystyle\int_0^1 \left((x-2)^2 - x^2\right)\,dx$, which then simplifies to the integral $\displaystyle\int_0^1 (-4x+4)\,dx$. From sketching each of the choices, it can be seen that (A) and (B) have no resemblance to the region $\mathcal{R}$. From the remaining choices (C) and (D) we see that (C) is the correct answer because the area of the region is given by $\displaystyle\int_0^1 (-4x+4)\,dx$.

(a) Using the change of base law to common base 4

$$\log_2 x \log_3 x = \log_4 x$$

$$\frac{\log_4 x}{\log_4 2} \times \frac{\log_4 x}{\log_4 3} = \log_4 x$$

$$\log_4 x \left(\frac{\log_4 x}{\frac{1}{2}\log_4 3} - 1 \right) = 0$$

$$\log_4 x = 0 \quad \text{or} \quad \log_4 x = \log_4 \sqrt{3}$$

$$x = 1, \sqrt{3}$$

(b) Using the fact that $\cos(90° - x) = \sin x$

$$\frac{\sin 0° + \sin 1° + \sin 2° + ... + \sin 90°}{\cos 0° + \cos 1° + \cos 2° + ... + \cos 90°} = \frac{\sin 0° + \sin 1° + \sin 2° + ... + \sin 90°}{\sin 90° + \sin 89° + \sin 88° + ... + \sin 0°}$$

$$= \frac{\sin 0° + \sin 1° + \sin 2° + ... + \sin 90°}{\sin 0° + \sin 1° + \sin 2° + ... + \sin 90°}$$

$$= 1$$

(f) If $x^2 + xy + y^2 = 0$ then $(x-y)(x^2 - xy - y^2) = 0$ or equivalently $x^3 = y^3$ (1). Also note that

$$\frac{x^2}{y^2} + \frac{x}{y} + 1 = 0$$

$$\frac{x}{y} + 1 = -\frac{x^2}{y^2} \quad (2)$$

$$\begin{aligned}
\left(\frac{x}{x+y}\right)^{2015} + \left(\frac{y}{x+y}\right)^{2015} &= \frac{x^{2015} + y^{2015}}{(x+y)^{2015}} \\
&= \frac{\frac{x^{2015}+y^{2015}}{y^{2015}}}{\frac{(x+y)^{2015}}{y^{2015}}} \\
&= \frac{\left(\frac{x}{y}\right)^{2013} \times \frac{x^2}{y^2} + 1}{\left(\frac{x}{y}+1\right)^{2015}} \\
&= \frac{\left(\frac{x^3}{y^3}\right)^{671} \times \frac{x^2}{y^2} + 1}{\left(\frac{x}{y}+1\right)^{2015}} \quad \text{substitute results (1) and (2)} \\
&= \frac{\frac{x^2}{y^2} + 1}{\left(-\frac{x^2}{y^2}\right)^{2015}} \\
&= \frac{\frac{x^2}{y^2} + 1}{-\frac{x^{4030}}{y^{4030}}} \\
&= \frac{\frac{x^2}{y^2} + 1}{-\left(\frac{x^3}{y^3}\right)^{1343} \times \frac{x}{y}} \\
&= \frac{\frac{x^2}{y^2} + 1}{-\frac{x}{y}} \\
&= 1 \quad \text{using (2)}
\end{aligned}$$

(a)

$$a^{\log_b x}$$

$$\begin{aligned}
&= a^{\frac{\log_a x}{\log_a b}} \quad \text{by change of base} \\
&= a^{\log_a x^{\frac{1}{\log_a b}}} \\
&= x^{\frac{1}{\log_a b}} \\
&= x^{\frac{\log_a a}{\log_a b}} \\
&= x^{\log_b a}
\end{aligned}$$

(b) Note in each term, the denominator is an arithmetic series, so

$$\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+3+...+n}=\frac{1}{\frac{2}{2}(1+2)}+\frac{1}{\frac{3}{2}(1+3)}+...+\frac{1}{\frac{n}{2}(1+n)}$$

$$=\sum_{k=2}^{n}\frac{1}{\frac{k}{2}(1+k)}$$

$$=\sum_{k=2}^{n}\frac{2}{k(1+k)}$$

$$=2\sum_{k=2}^{n}\frac{1+k-k}{k(1+k)}$$

$$=2\sum_{k=2}^{n}\left(\frac{1+k}{k(1+k)}-\frac{k}{k(1+k)}\right)$$

$$=2\sum_{k=2}^{n}\left(\frac{1}{k}-\frac{1}{1+k}\right)$$

But note that

$$2\sum_{k=2}^{n}\left(\frac{1}{k}-\frac{1}{1+k}\right)=2\left(\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+...+\left(\frac{1}{n}-\frac{1}{1+n}\right)\right)$$

$$=2\left(\frac{1}{2}-\frac{1}{1+n}\right)$$

$$=1-\frac{2}{n+1}$$

$$=\frac{n-1}{n+1}$$

$$\therefore \frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+3+...+n}=\frac{n-1}{n+1}$$

Pulley Question

The Show parts are pretty simple, the last part isn't

- To do the last part, understand that in the obtained dl/dx equation, the denominator is >0. This means that the sign of the expression (x-1)(2x^2-r^2x-r^2) determines the slope. Understand that x-1<0 for all valid x. Then say that when the parabola (concave up) is just before the valid x stationary point, the expression 2x^2-r^2x-r^2 is negative, making (x-1)(2x^2-r^2x-r^2) positive and thus, dl/dx positive. Then repeat for values of x just after this. As a result, a positive, then negative slope will be before and after this stationary point, indicating it is a local maximum

Question 28 (4 marks) Poorly done. **Marks**

A new artist releases a song on a music streaming platform. The number of 'listens' each hour for the first 5 hours is recorded in the table below.

Hour (H)	1	2	3	4	5
Listens (L)	13	36	62	94	138

(a) Show that the predicted number of listens in the 6th hour is 206. **2**

$L_2 - L_1 = 23 = 20+3$

$L_3 - L_2 = 26 = 20+6$ AP + GP

$L_4 - L_3 = 32 = 20+12$ (1) pattern building

$L_5 - L_4 = 44 = 20+24$

$\therefore L_6 - L_5 = 20+48$

$= 68$

$L_6 = 138+68$ (1) answer

$= 206$

(b) How many predicted 'listens' will the song have had at the conclusion of one day? **2**

$L_{24} = 10 + 20(23) + 3(2)^{23}$ GP

$S_{24} = \frac{24}{2}(10 + 10+20(23)) + \frac{3(2^{24}-1)}{2-1}$ (1) for showing correct either AP or GP

AP

$= 12(480) + 3(2^{24}-1)$

$= 50\ 337\ 405$ (2) Correct answer.