Solutions.

TASK 1 – REVIEW TEST SCORE: ___/30

NAME:____________________

1. What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \le x \le 2\pi$? (1)

$\tan x = -\frac{1}{\sqrt{3}}$

$\pi - \pi/6 = 5\pi/6$ ✓

$\pi/6$

S | A
T | C

$2\pi - \pi/6 = 11\pi/6$ ✓

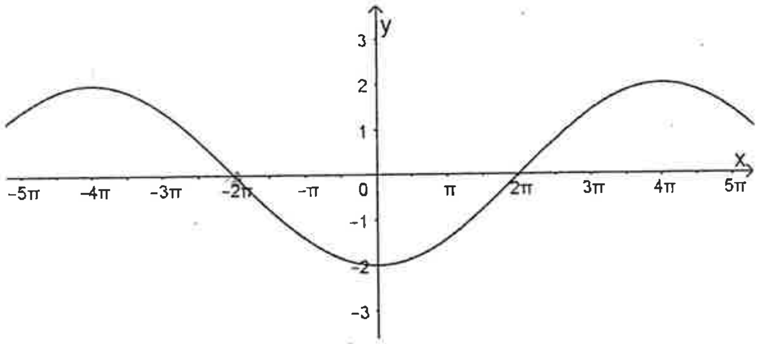
A $x = \frac{\pi}{3}$ or $x = \frac{2\pi}{3}$

B $x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$

C $x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{3}$

D $x = \frac{5\pi}{6}$ or $x = \frac{11\pi}{6}$

2. Which of the following is the correct equation for the graph shown? (1)



$\frac{1}{2}$ period $= 4\pi$

period $= 8\pi$

$\frac{2\pi}{b} = 8\pi$

$b = \frac{1}{4}$

A. $y = -2\sin\left(\frac{x}{4} + \frac{\pi}{2}\right)$ $\frac{1}{4}(x + 2\pi)$ ✓

B. $y = 2\sin\left(\frac{x}{4} - \frac{\pi}{4}\right)$ $\frac{1}{4}(x - \pi)$ ✗

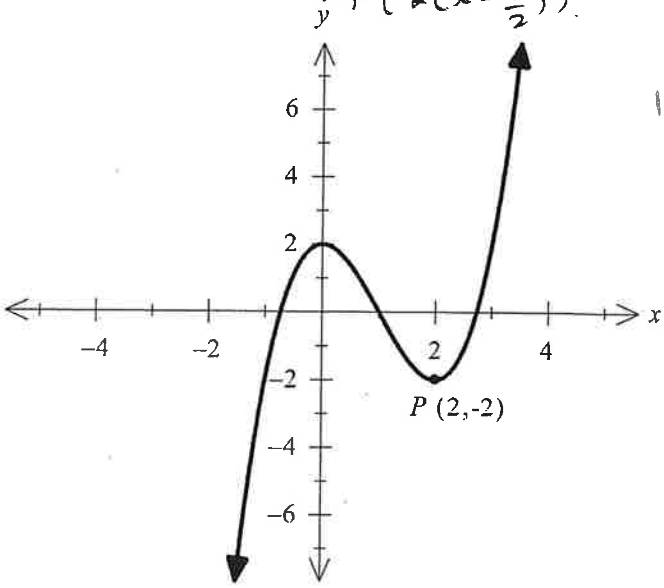
C. $y = -2\sin\left(\frac{x}{2} + \frac{\pi}{2}\right)$

D. $y = 2\sin\left(\frac{x}{4} + \frac{\pi}{4}\right)$

$\frac{1}{4}(x + \pi)$ ✗

3. P lies on the graph of $y = f(x)$ as shown on the diagram below. A transformation maps the graph of $f(x)$ to $g(x)$ such that $g(x) = 2f(1-2x)$. The same transformation maps the point P to P'. (1)

$= 2f\left(-2\left(x - \frac{1}{2}\right)\right)$



$1 - 2x = 2$

$-2x = 2 - 1 = 1$

$x = -\frac{1}{2}$

$y \to 2y$

$-2 \times 2 = -4$

$\left(-\frac{1}{2}, -4\right)$

What are the coordinates of P'?

A. $(2, -6)$

B. $\left(\frac{-1}{2}, -4\right)$

C. $\left(\frac{1}{2}, -4\right)$

D. $(-2, -4)$

4. What is the derivative of e^{x^6}? (1)

$e^{x^6} \times 6x^5$

(A) $6x^5 e^{x^6}$

(B) $6xe^{x^6}$

(C) $6x^5 e^{6x^5}$

(D) $x^6 e^{x^6 - 1}$

5. (3)

Find the gradient of the normal to the curve $y = \frac{(4 - x^2)}{e^{3x}}$ at the point $(0, 4)$.

$x = 0$

$m_{tan} = y'(x=0)$

$y' = ?$ $\frac{u'v - v'u}{v^2}$

$u = 4 - x^2$ $v = e^{3x}$

$u' = -2x$ $v' = 3e^{3x}$

$y' = \frac{-2xe^{3x} - (4-x^2)3e^{3x}}{e^{6x}}$

$= \frac{-e^{3x}(2x - 12 + 3x^2)}{e^{6x}}$

$= -\frac{1}{e^{3x}}(3x^2 + 2x - 12)$

$y'(x=0) = -\frac{1}{e^0}(0 - 12) = 12$

$m_{normal} = \frac{-1}{m_{tan}}$

$= -\frac{1}{12}$

6. A bottle of vintage wine cost \$375 when first released. After t years its value, $\$V$, is given by $V = 375e^{0.05t}$.

(a) Find the value of the bottle of wine after 7 years, correct to the nearest dollar. (1)

$V(7) = 375e^{0.05 \times 7} = 532.15$

$\doteqdot \$532$ (nearest \$)

(b) Find how many years it takes for the value of the wine to increase to \$1200 per bottle. Round your answer to 1 decimal place. (2)

$V(?) = 1200$

$375e^{0.05t} = 1200$

$e^{0.05t} = \frac{1200}{375}$

$\ln e^{0.05t} = \ln\left(\frac{1200}{375}\right)$

$0.05t = \ln\left(\frac{1200}{375}\right)$

$t = \frac{1}{0.05} \times \ln\left(\frac{1200}{375}\right)$

$= 23.263$ years

$\doteqdot 23.3$ years (1 dp)

(c) What is the rate of increase in the value of the wine 7 years after it was first released? Round your answer to 1 decimal place. (2)

$\left.\frac{dV}{dt}\right|_{t=7} = ?$

$\frac{dV}{dt} = 375 \times 0.05e^{0.05t}$

$\frac{dV}{dt} = 18.75e^{0.05t}$

$\left.\frac{dV}{dt}\right|_{t=7} = 18.75e^{0.05 \times 7}$

$= 26.608$

$\doteqdot 26.6$ \$/year

∴ the value is increasing at a rate of \$26.6/year after 7 years

7.

The water level in an estuary is cyclical, with a maximum depth of 5 metres, a minimum depth of 2 metres, and the cycle repeats every 12 hours.

$c = \frac{5+2}{2} = 3.5$

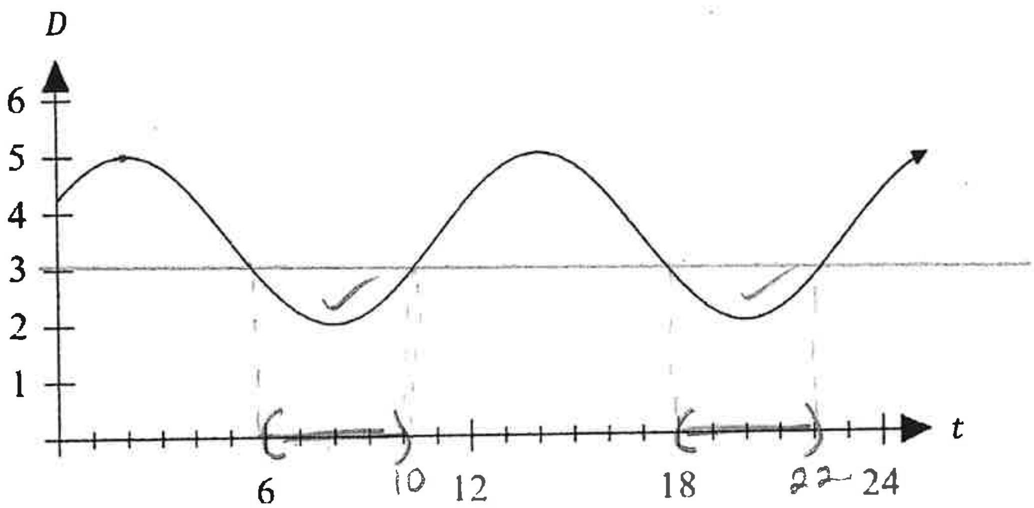
$A = 1.5$

$P = 12$ hrs.

$\therefore HSF = \frac{6}{\pi}$

The last high tide was at 2:00 am. $\rightarrow t = 2 \quad \therefore D(2) = 5$

The function of the form $D = k\cos\frac{\pi}{6}(t+b)+c$ models the water depth, where D is the water depth (in metres), t is the hours since 12:00 am (midnight) and k, b and c are constants.



(a) Determine the values of k, b and c. (3)

$c = 3.5$

$k = 1.5$

$1.5\cos\frac{\pi}{6}(2+b) + 3.5 = 5$

$1.5\cos\frac{\pi}{6}(2+b) = 1.5$

$\cos\frac{\pi}{6}(2+b) = 1$

$\frac{\pi}{6}(2+b) = 0$

$\therefore b = -2$

(b) Using the graph or otherwise, when is the water level less than 3m in the 24 hour period? (2)

$\pi - \theta$

S	A
T	C

$\pi + \theta$

$\cos^{-1}(\frac{1}{3})$

$1.5\cos\frac{\pi}{6}(t-2) + 3.5 = 3$

$1.5\cos\frac{\pi}{6}(t-2) = -0.5$

$\cos\frac{\pi}{6}(t-2) = -\frac{0.5}{1.5} = -\frac{1}{3}$

$\therefore \frac{\pi}{6}(t-2) = \pi - \cos^{-1}(\frac{1}{3})$ or

$\pi + \cos^{-1}(\frac{1}{3})$

$= 1.91$ or 4.37

(B) (C)

$t = \frac{1.91 \times 6}{\pi} + 2$ or

$\frac{4.37 \times 6}{\pi} + 2$

$\therefore t = 5.65, \ 10.35$ and in next rev +12

$17.65, \ 22.35$

ANS

$\therefore$ Between 5:38 am and 10:21 am (not inclusive) and between 5:38 pm and 10:21 pm (not inclusive), the water level is less than 3m.

8. $f(x) = x^2$

The graph of $y = x^2$ is transformed into the graph of $y = 3x^2 + 24x + 33$ by the transformations: (3)

- A vertical stretch (dilation) with scale factor k followed by $a(x) = kf(x) = kx^2$
- A horizontal translation of p units followed by $b(x) = a(x-p) = k(x-p)^2$
- A vertical translation of q units $c(x) = a(x-p) + q = k(x-p)^2 + q$

Write down the values of k, p and q.

$k(x^2 - 2xp + p^2) + q$

$= kx^2 - 2kpx + kp^2 + q = 3x^2 + 24x + 33$

$\underline{k = 3}$

$-2kp = -2 \times 3 \times p = 24$

$p = 24 \div -6 = \underline{-4} = p$

$kp^2 + q = 33$

$3 \times 16 + q = 33$

$q = 33 - 48$

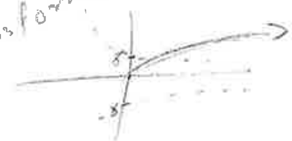
$= \underline{-15}$

$\therefore \boxed{k = 3, \ p = -4, \ q = -15}$

9. The velocity of a particle moving along the x−axis is given by:

$$v(t) = \dot{x} = 8 - 8e^{-2t}$$

transforms →



where t is the time in seconds and x is the displacement in metres.

(a) Show that the particle is initially at rest. $\Rightarrow v(0) = 0$ SHOW (1)

$v(0) = 8 - 8e^{-2 \times 0} = 8 - 8(1) = \underline{0 \text{ m/s}}$

$\therefore$ initially particle is at rest

(b) Show that the acceleration of the particle is always positive. $a(t) = \frac{d}{dt} v(t)$ (1)

$\frac{dv}{dt} = -8 \times -2e^{-2t} = 16e^{-2t}$

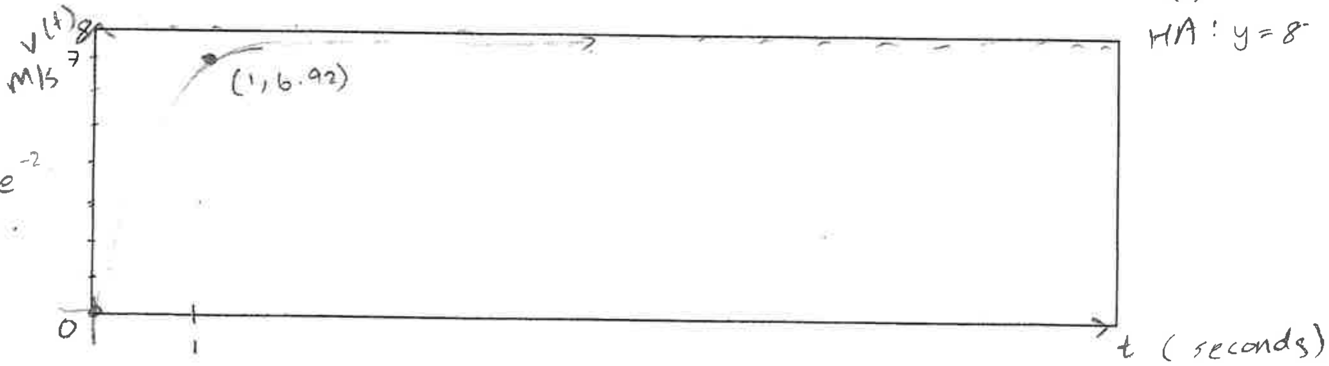
Since e^{-2t} is always positive, $16e^{-2t}$ is always > 0

$\therefore$ acceleration is always positi

(c) Sketch the graph of the particle's velocity as a function of time. (2)

$v(1) = 8 - 8e^{-2}$

$= 6.92$.



10. Solve $\log_e x - \dfrac{3}{\log_e x} = 2$ giving your answers in exact form. $x > 0$ (3)

$\ln x - \dfrac{3}{\ln x} = 2$

$\dfrac{(\ln x)^2 - 3}{\ln x} = 2$

$(\ln x)^2 - 3 = 2\ln x$

$(\ln x)^2 - 2\ln x - 3 = 0$ Let $y = \ln x$

$y^2 - 2y - 3 = 0 \Rightarrow (y-3)(y+1) = 0$

$y = 3, -1$

$e^{\ln x} = e^3$ or $e^{\ln x} = e^{-1}$

$x = e^3$ or e^{-1}

11. Sketch $f(x) = \dfrac{1-x}{x-2}$. Clearly label asymptotes, x and y intercepts. (2)

Intercepts:

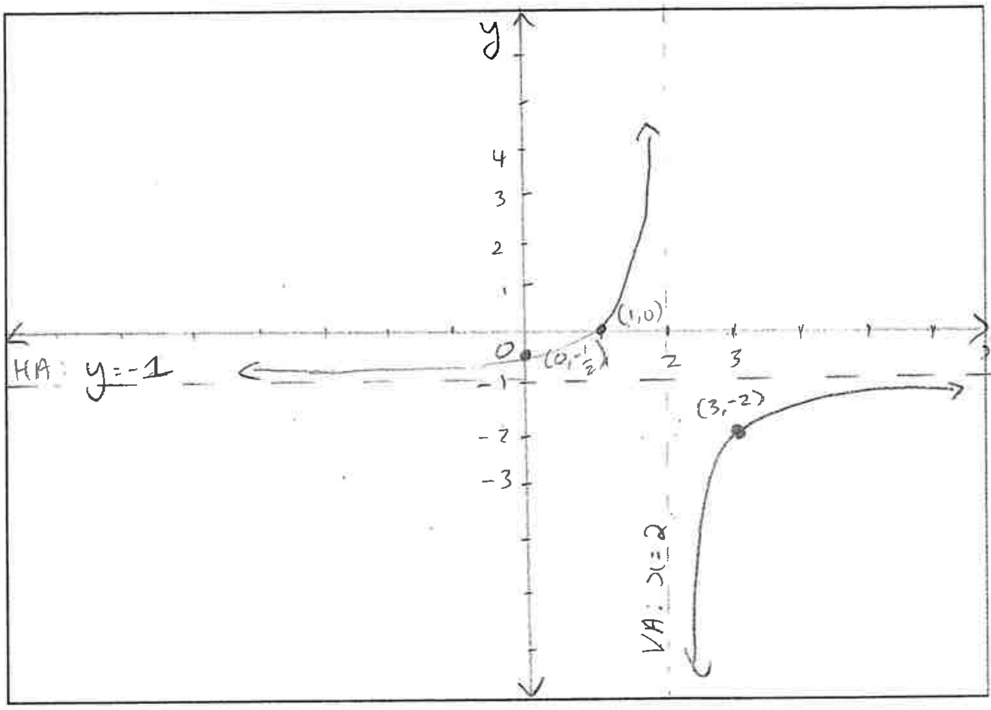
y_{int} $(x=0)$

$y_{int} = \dfrac{1}{-2} = -\dfrac{1}{2}$

x_{int} $[y=0]$

$1 - x = 0$

$x_{int} = 1$



Asymp:

VA: $x = 2$

HA: $\lim_{x\to\infty} \dfrac{1/x - x/x}{x/x - 2/x}$

$= \dfrac{-1}{1} = -1$

[or sub in big value for x]

HA: $y = -1$

point on other arc:

$y(x=3) = \dfrac{1-3}{3-2}$

$= -2$

Hence or otherwise, solve for x, where $\frac{1-x}{x-2} > 0$ (1)

$x \in (1, 2)$