keep it simple science

KEEP IT SIMPLE SCIENCE

*PhotoMaster Format*

*Physics Module 5*

# *Advanced Mechanics*

## *Topic Outline*

### 1. Projectiles

- Features of projectile motion
- Analysing projectile motion
- Max.height, time of flight, range, etc.
- Projectiles launched horizontally

### 2. Circular Motion

- Features of circular motion
- Orbital speed & velocity
- Centripital acceleration & force
- Causes of centripital force
- "Centrifugal" force, inertia & banked curves
- Angular velocity
- Work & energy in circular motion
- The concept of torque

### 3. Motion in Gravitational Fields

- Newton's Universal Gravitation
- Grav. field strength, "g"
- Gravity & weight on other planets
- Concept of an orbit & escape velocity
- Launching a spacecraft
- Types of orbits & their uses
- Orbital speed & radius of satellites
- Brief history of Astronomy
- Kepler's Laws & Newton's proof
- Re-entry of spacecraft
- Energy of a satellite

## What is this topic about?

**To keep it as simple as possible, (K.I.S.S. Principle) this topic covers:**

### 1. Projectile Motion

**Characteristics of projectile motion. Analysis of projectile motion.
How to calculate max.height, time of flight, range, position or velocity at any instant.
Projectiles launched horizontally. Prac.work.**

### 2. Circular Motion

**Characteristics of uniform circular motion. Orbital speed and tangential velocity.
Centripital acceleration and force. Where centripital force comes from in different cases.
"Centrifugal" pseudo-force. Tangential inertia and banked curves.
Angular velocity. Work & energy in circular motion. The concept of "torque".**

### 3. Motion in Gravitational Fields

**Newton's Law of Universal Gravitation. Gravitational field strength measured by "g".
Gravity& weight on other planets. Concept of an orbit and escape velocity. Launching spacecraft.
Why rockets? Different orbit types & uses. Orbital velocity & radius.
Brief history of Astronomy. Kepler's Laws & Newton's proof.
Re-entry of spacecraft... shedding energy. Energy of satellites.**

keep it simple science

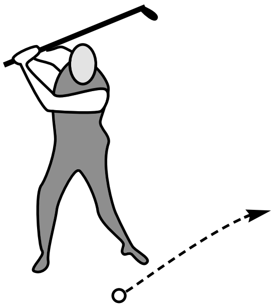
# 1. Projectile Motion

## What is a Projectile?

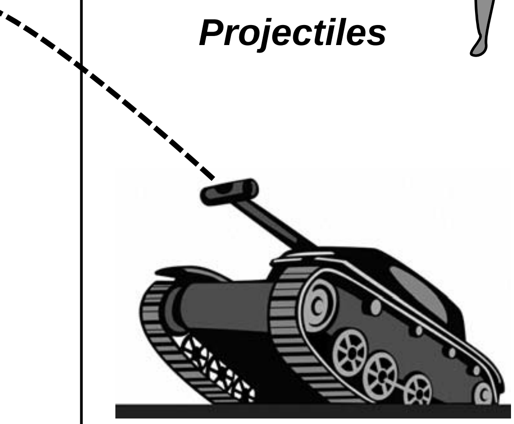
**A projectile is any object that is launched, and then <u>moves only under the influence of gravity</u>.**

**<u>Examples</u>:**

**Once struck, kicked or thrown, a ball in any sport becomes a projectile.**

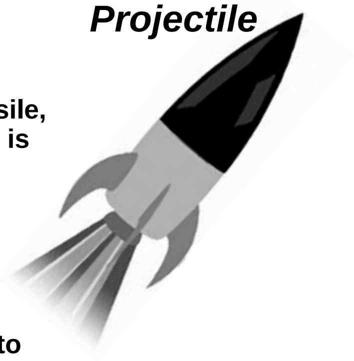


***Projectiles***



**Any bullet, shell or bomb is a projectile once it is fired, launched or dropped.**

***Not a Projectile***



**An example which is <u>NOT a Projectile</u>:**

**A rocket or guided missile, while still under power, is NOT a projectile.**

**Once the engine stops firing it becomes a projectile.**

**Projectiles are subject to only one force... Gravity!**

**When a projectile is travelling through air, there is, of course, an air-resistance force acting as well. For simplicity, (K.I.S.S. Principle) <u>air-resistance is assumed to be negligible</u> throughout this topic.**

**In reality, a projectile in air, does not behave the way described here because of the effects of air-resistance.**

**The exact motion depends on many factors and the Physics becomes very complex, and beyond the scope of this course.**

## Projectile Motion

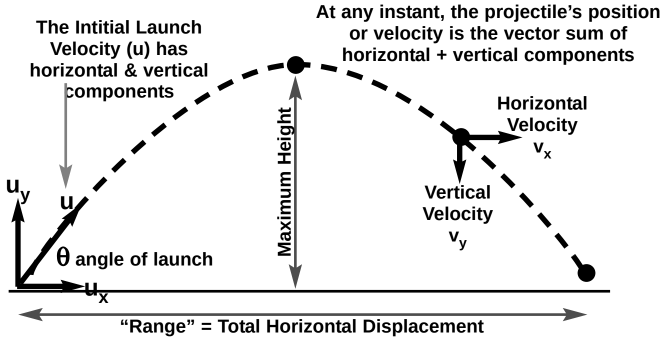
**By simple observation of a golf ball trajectory, or a thrown cricket ball, the motion of any projectile can be seen to follow a curve. It is in fact a <u>parabola</u>, and you might think the Physics of this is going to be difficult. NOT SO... it is really very simple.**

**<u>Horizontal Motion</u> is CONSTANT VELOCITY**

**<u>Vertical Motion</u> is CONSTANT ACCELERATION at "g", DOWNWARDS**

**Just remember the following:**
**You must analyse projectile motion as 2 separate motions; horizontal (x-axis) and vertical (y-axis) must be dealt with separately, and combined as vectors if necessary.**

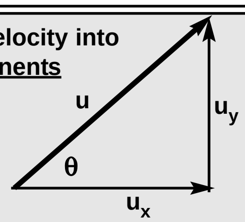
## The Trajectory (Path) of a Projectile




## Equations for Projectile Motion

**To enable analysis, all you need is the initial "launch velocity" (U) & angle of launch (θ)**

**1. Resolve the Initial Launch Velocity into <u>Vertical & Horizontal Components</u>**




$$\sin\theta = \frac{u_y}{u} \quad \& \quad \cos\theta = \frac{u_x}{u}$$

$$\therefore u_y = u.\sin\theta, \quad u_x = u.\cos\theta$$

**2. <u>Horizontal Motion</u> is constant velocity, so**

$$v_x = \frac{S_x}{t}$$ **is all you need**

**3. Vertical Motion is constant acceleration at "g"**

**To find vertical velocity:**

$$v_y = u_y + g.t \quad \text{(from } v = u+at\text{)}$$

**To find vertical displacement:**

$$S_y = u_y.t + \frac{1}{2}.g.t^2 \quad \text{(from } S = ut + \frac{1}{2}at^2\text{)}$$

**It is wise to always consider "up" as positive, "down" as negative vector directions.**

keep it simple science ®

# Analysing Projectile Motion

Example 1

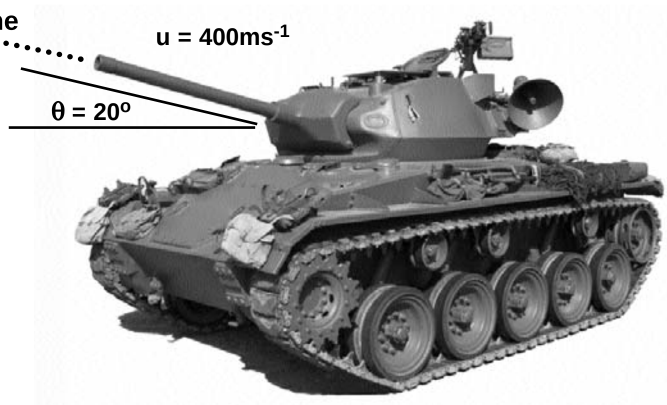
**The cannon shown fires a shell at an initial velocity of $400ms^{-1}$.**
**If it fires at an angle of $20^o$, calculate:**

**a) the vertical and horizontal components of the initial velocity.**

**b) the time of flight. (assuming the shell lands at the same horizontal level)**

**c) the range. (same assumption)**

**d) the maximum height it reaches.**




**a)**

$u_y = u.\text{Sin}\theta$
$= 400.\text{Sin}20$
$= 136.8ms^{-1}$ (upwards)

$u_x = u.\text{Cos}\theta$
$= 400.\text{Cos}20$
$= 375.9ms^{-1}$ (horizontal)

**Point to Note:**
**The mass of the projectile does NOT enter into any calculation. The trajectory is determined by launch velocity & angle, plus gravity. Mass is irrelevant!**
(Remember that all masses accelerate the same, at "g")

**b) The shell is fired upwards, but acceleration due to gravity is downwards.**
**You must assign up = (+ve), down = ( -ve).**

**Key Point of analysis:**
**At the top of its arc, the shell will have an instantaneous vertical velocity = zero.**

$v_y = u_y + g.t$
$0 = 136.8 + (-9.81)xt$ (at the highest point)
$\therefore t = -136.8/-9.81$
$= 13.95$ s

**This means it takes 13.95s to reach the top of its arc. Since the motion is symmetrical, it must take twice as long for the total flight.**
$\therefore$ **time of flight = 27.9s**

**c) Range is horizontal displacement**

**Remember**
$v_x = u_x =$ constant velocity

$v_x = \frac{S_x}{t}$

$\therefore S_x = v_x.t$ (use time of flight)

$= 375.9 \times 27.9$
$= 10{,}488m$

**Range = $1.05 \times 10^4 m$ (i.e. 10.5 km)**

**d) Vertical Height (up = (+ve), down = ( -ve))**

$S_y = v_y.t + \frac{1}{2}.g.t^2$

$= 136.8 \times 13.95 + 0.5 \times (-9.81) \times (13.95)^2$
$= 1901.5 + (-947.7)$
$= 953.8m = 9.54 \times 10^2 m.$ (almost 1 km high)

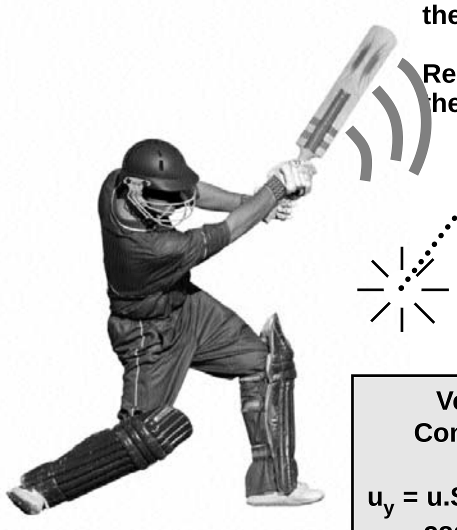
**Note: the time used is the time to reach the top of the arc... the time at the highest point, NOT total time of flight.**

keep it simple science ®

# Analysing Projectile Motion *cont.*

Example 2

**The batsman has just hit the ball upwards at an angle of 55°, with an intial velocity of 28.0ms$^{-1}$. The boundary of the field is 62.0m away from the batsman.**



**Resolve the velocity into vertical and horizontal components, then use these to find:**

**a) the time of flight of the ball.**

**b) the maximum height reached.**

**c) whether or not he has "hit a 6" by clearing the boundary.**

**d) the velocity of the ball (including direction) at the instant t = 3.50s.**

**Remember to let UP = (+ve)**
**DOWN = ( -ve)**
**acceleration = "g" = -9.81ms$^{-2}$**

**Vertical & Horizontal Components of Velocity**

$u_y = u.\sin\theta$, $u_x = u.\cos\theta$
$= 28\sin55$ $= 28\cos55$
$= 22.9ms^{-1}$ $= 16.1ms^{-1}$

**a) Time of Flight**

At highest point $v_y = 0$, so

$v_y = u_y + g.t$
$0 = 22.9 + (-9.81)xt$
$\therefore t = -22.9/-9.81$
$= 2.33s$

This is the mid-point of the arc, so time of flight = 4.66s

**b) Maximum Height**
is achieved at t = 2.33s, so

$S_y = u_y.t + \frac{1}{2}.g.t^2$
$= 22.9x2.33 + 0.5x(-9.81)x(2.33)^2$
$= 53.5 + (-26.6)$
$= 26.9m$

**c) Range will determine if he's "hit a 6".**

$v_x = u_x$ = constant velocity
$S_x = v_x.t$ (use total time of flight)
$= 16.1 \times 4.66$
$= 75.0m$ That'll be 6 !

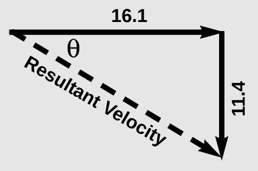
**d) Velocity at t = 3.50s ?**

**Vertical**

$v_y = u_y + g.t$
$= 22.9+(-9.81)x3.50$
$= -11.4ms^{-1}$
(this means it is downwards)

**Horizontal**

$v_x = u_x$ = constant
$= 16.1ms^{-1}$




By Pythagorus,
$v^2 = v_y^2 + v_x^2$
$= (-11.4)^2 + 16.1^2$
$\therefore v = \sqrt{389.17} = 19.7ms^{-1}$

Tan θ = 11.4/16.1
$\therefore \theta \cong 35°$
at an angle 35° below horizontal

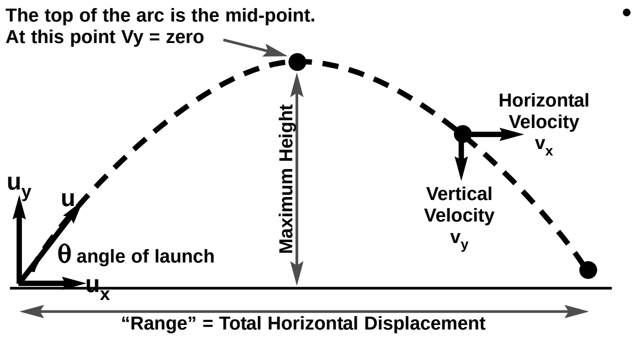
keep it simple science

# Analysing Projectile Motion *cont.*

If you find solving Projectile Motion problems is difficult, try to learn these basic "rules":

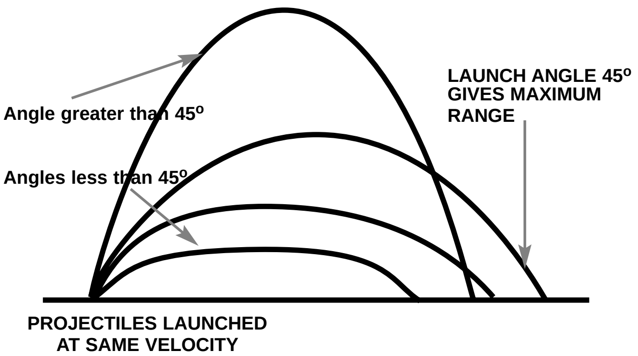
• The "launch velocity" must be <u>resolved</u> into a horizontal velocity ($u_x$) and a vertical velocity ($u_y$).
Once you have these, you can deal with vertical and horizontal motion as 2 separate things.

• The motion is symmetrical, so at the highest point, the elapsed time is exactly half the total time of flight.




Also, at the highest point, $v_y$ = zero.
The projectile has been rising to this point.
After this point it begins falling.
For an instant $v_y = 0$. Very useful knowledge!

• <u>Maximum Range</u> is achieved at launch angle 45°.




• <u>Horizontal Motion</u> is constant velocity... easy.
Use $v_x = u_x$ and $S_x = u_x.t$

• <u>Vertical Motion</u> is constant acceleration at $g = -9.81\text{ms}^{-2}$, so use $v_y = u_y + g.t$ to find "t" at the max.height (when $v_y=0$) or, find $v_y$ at a known time.

Try Worksheet 1

Use $S_y = u_y.t + \frac{1}{2}.g.t^2$
to find vertical displacement ($S_y$) at a known time, or find the time to fall through a known height (if $u_y=0$)

## Projectiles Launched Horizontally

A common situation with projectile motion is when a projectile is launched horizontally, as in the following example. This involves half the normal trajectory.

Plane flying horizontally, at constant $50.0\text{ms}^{-1}$

Releases a bomb from altitude = 700m

<u>Questions</u>
a) How long does it take for the bomb to hit the ground?
b) At what velocity does it hit?
c) If the plane continues flying straight and level, where is it when the bomb hits?

### Solution

Because the plane is flying horizontally, the intitial velocity vectors of the bomb are:
Horizontal, $u_x = 50.0\text{ms}^{-1}$,
Vertical, $u_y$ = zero

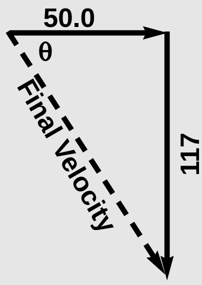
a) Time to hit the ground
We know the vertical distance to fall (-700m (down)), the acceleration rate (g= $-9.81\text{ms}^{-2}$) and that $u_y=0$.
(Initially, its vertical velocity = zero)

$$S_y = u_y.t + \frac{1}{2}.g.t^2$$
$$-700 = 0xt + 0.5\ x(-9.81)x\ t^2$$
$$-700 = -4.905xt^2$$
$$\therefore t^2 = -700/-4.905$$
$$t = 11.9s$$

b) Final Velocity at impact

| <u>Vertical</u> | <u>Horizontal</u> |
|---|---|
| $v_y = u_y + g.t$ | $v_x = u_x$ |
| $= 0 + (-9.81)x11.9$ | $v_x = 50.0\text{ms}^{-1}$. |
| $v_y = -117\text{ms}^{-1}$. (down) | |

$$v^2 = v_y^2 + v_x^2$$
$$= 117^2 + 50.0^2$$
$$\therefore v = \sqrt{16,189}$$
$$= 127\text{ms}^{-1}.$$




Tan θ = 117/50
$\therefore \theta \cong 67°$.

Bomb hits the ground at $127\text{ms}^{-1}$, at angle 67° below horizontal.

c) Where is the Plane?
Since both plane and bomb travel at the same horizontal velocity, it follows that they have both travelled exactly the same horizontal distance when the bomb hits. i.e. the plane is directly above the bomb at impact.

(In warfare, this is a problem for low-level bombers... the bombs must have delayed-action fuses) keep it simple science®

Prac Work: *Projectile Motion*

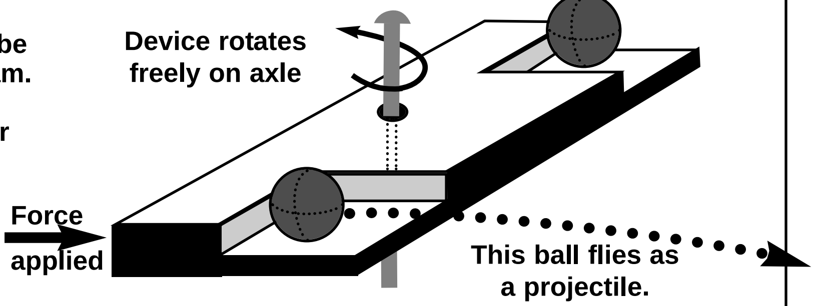
## *Activity 1* *A Simple Observation*

Someone with some basic woodwork skills may be able to make a small device similar to this diagram.

It can be "loaded" with a pair of identical coins or other small objects as shown.

If struck sharply, it spins on its axle.

This launches one coin/ball horizontally as a projectile. The other is dropped vertically.




The observation to make is whether or not the 2 objects hit the ground at the same time. Discuss the result.

## *Activity 2* *Measuring a Projectile Trajectory*

The diagram shows a simple laboratory set up to make some basic measurements of the trajectory of a projectile.

You need to practice dropping a heavy ball bearing into the top end and making sure it lands consistently at the same point on the floor. You can then place a flat pad of putty (blu-tack) at this point. The ball will leave a mark which gives an accurate point for measuring distance.

The plastic tubing can be adjusted (with a clamp stand) to different heights for the ball to accelerate before launching <u>horizontally</u> as a projectile.

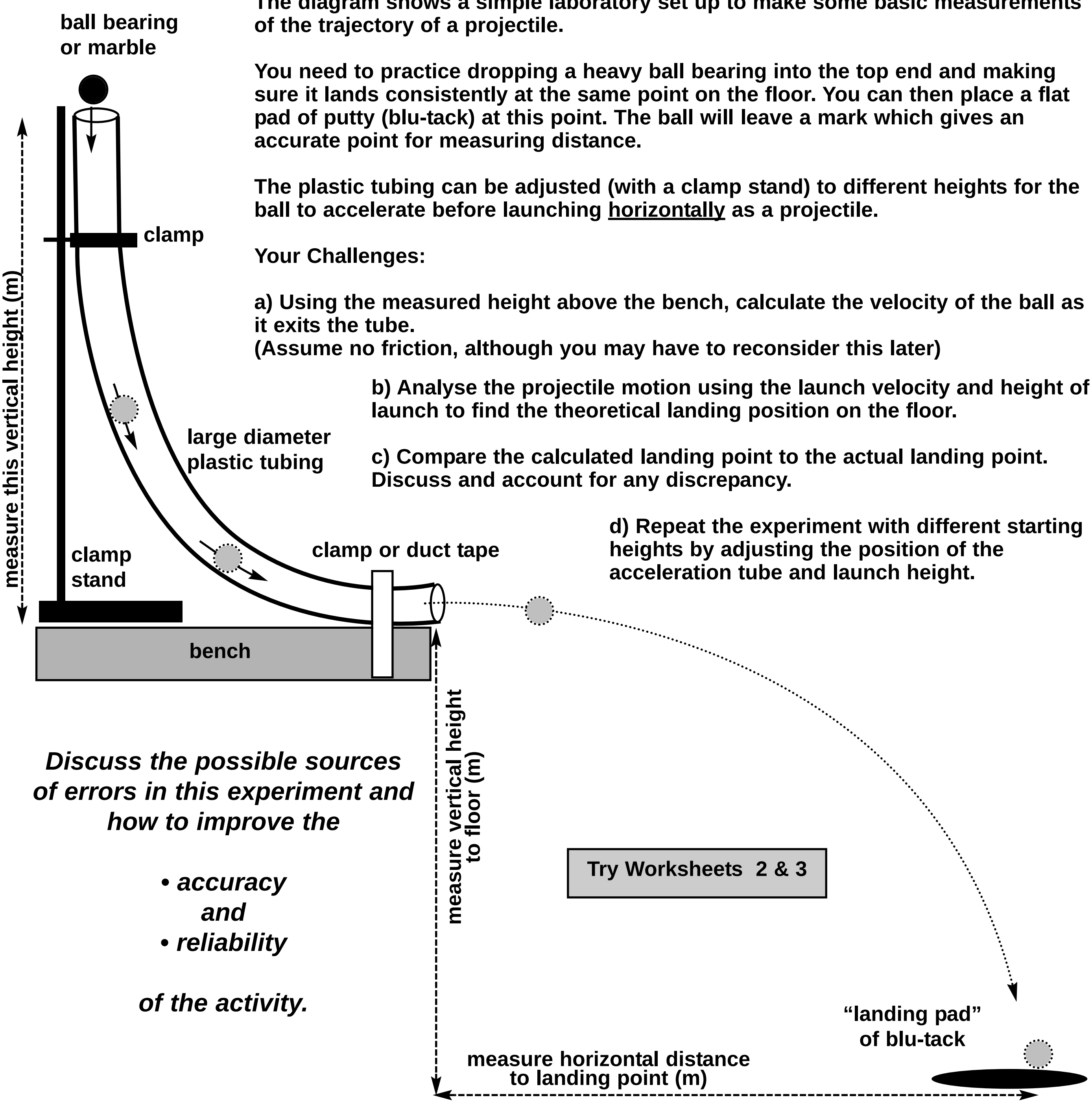
Your Challenges:

a) Using the measured height above the bench, calculate the velocity of the ball as it exits the tube.
(Assume no friction, although you may have to reconsider this later)

b) Analyse the projectile motion using the launch velocity and height of launch to find the theoretical landing position on the floor.

c) Compare the calculated landing point to the actual landing point. Discuss and account for any discrepancy.

d) Repeat the experiment with different starting heights by adjusting the position of the acceleration tube and launch height.




***Discuss the possible sources of errors in this experiment and how to improve the***

- ***accuracy*** ***and***
- ***reliability***

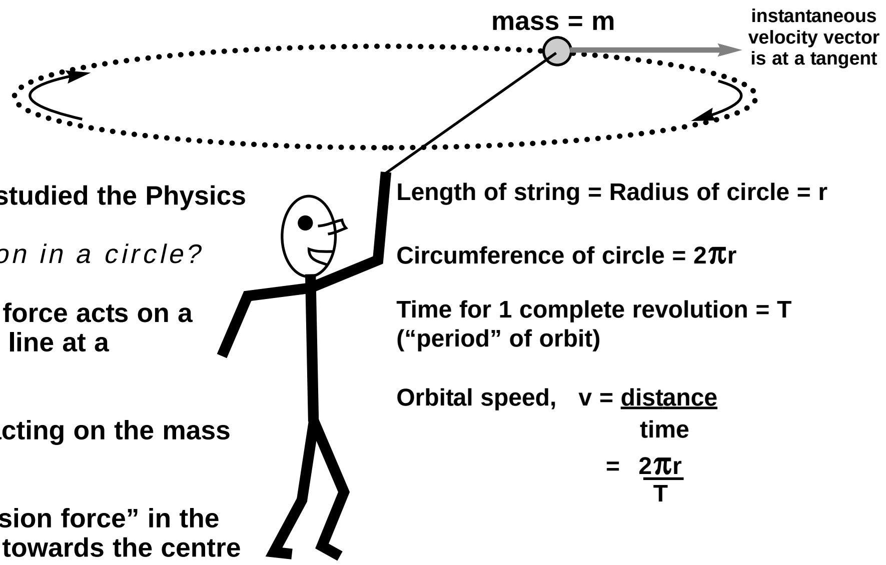
***of the activity.***

**Try Worksheets 2 & 3**

keep it simple science

# 2. Circular Motion

**Imagine swinging a mass around on the end of a string. The mass moves in a circle, but if you let go of the string, it flies off at a tangent to the circle. (and it becomes a projectile)**




**Length of string = Radius of circle = r**

**Circumference of circle = 2πr**

**Time for 1 complete revolution = T („period" of orbit)**

**Orbital speed,** $v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$

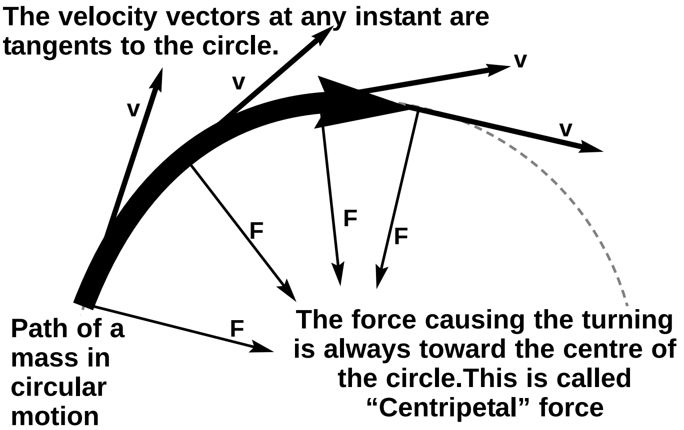
**In the earlier topic on "Dynamics" you studied the Physics of motion in a straight line.**
*How can we understand this motion in a circle?*

**Newton's 1st Law tells us that unless a force acts on a moving object, it will move in a straight line at a constant velocity.**

**Obviously then, there must be a force acting on the mass so that it moves in a circle.**

**This force, of course, is due to the "tension force" in the string. It is constantly pulling the mass towards the centre of the circle. If the string breaks there is no more force, so the mass obeys Newton's 1st Law and flies off (initially) in a straight line at constant velocity.**

**The diagram below shows the circular motion as seen from above. The mass is travelling at a constant speed around the circle.**




**Orbital Speed (& instantaneous tangential velocity)**

$$v = \frac{2\pi r}{T}$$

**Centripetal Acceleration**

$$a_c = \frac{v^2}{r}$$

**Centripetal Force** $$F_c = \frac{mv^2}{r}$$

**r = radius of the circle (in metres)**
**T= period of rotation. (s)**
**v = instantaneous velocity ($ms^{-1}$) ("orbital speed")**
**m = mass of object in motion (in kg)**

**Even though the speed may be constant, the object is constantly accelerating because its direction is constantly changing. This acceleration is towards the centre of the circle and is called "<u>centripetal acceleration</u>".**

**The force causing this acceleration is called "<u>centripetal force</u>" and is always directed to the centre of the circle.**

**In the case of our mass on a string, the centripital force is the tension in the string. In the case of a car turning a circular corner, the centripital force is due to friction between the tyres and the road. In the case of a satellite in circular orbit, the force is due to gravity.**

**The <u>velocity vector</u> is constantly changing, but at any instant it is a tangent to the circle, and therefore, at right angles to the acceleration and force vectors.**

*Example Problem*

**A 100g ball is being swung in a circle on a string of length 1.50m. It completes one "orbit" every 0.25s. Calculate:**
**a) its orbital speed.**
**b) the centripetal acceleration.**
**c) the tension force in the string.**

*Solution*

**a)** $v = \frac{2\pi r}{T} = 2 \times \pi \times 1.50 / 0.25 = 37.7\ ms^{-1}$.

**b)** $a_c = \frac{v^2}{r} = 37.7^2 / 1.50 = 948\ ms^{-2}$.

**c)** $F_c = \frac{m v^2}{r} = 0.100 \times 37.7^2 / 1.50 = 94.8$ N

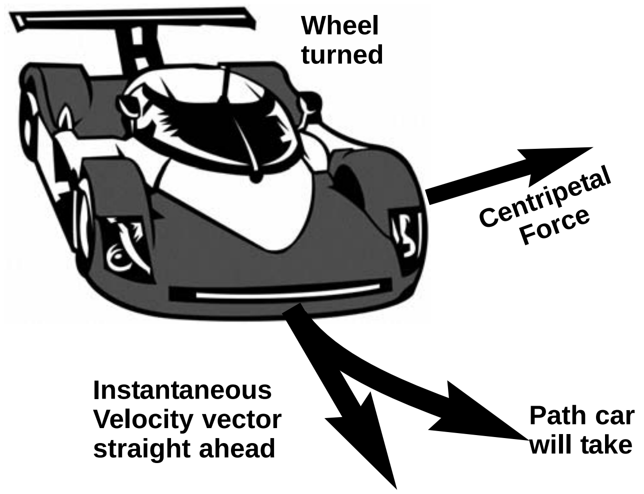
**(It had better be a strong string... $F_c$ is about 10g !)**

keep it simple science

# Circular Motion of Vehicles Turning Corners

**So, where does the centripetal force come from to push a moving vehicle, such as a car, around a corner?**

**The centripetal force comes from the frictional "grip" of the tyres on the road. Turning the steering wheel creates new friction forces which are directed to the centre of an imaginary circle.**




**So long as the frictional forces are strong enough, the vehicle will follow a circular path around the corner.**

**If the centripetal force <u>required</u> for a particular corner exceeds the friction "grip" of the tyres, then the vehicle will not make it, and may "spin out" and crash.**

**This can happen because:**

- **speed is too high for the radius of the curve. (i.e. the radius is too small compared to velocity)**
- **loss of friction between tyres and road. (e.g. road is wet, or tyres are worn smooth)**

## *Example Problem*

**The maximum frictional force possible from <u>each tyre</u> of this 750kg car is 5,000 N.**

**What is the maximum speed that the car can go around a circular curve with a radius of of 40m?**

## *Solution*

**Max. Force possible from 4 tyres = 4x 5,000**
**= 20,000N**
**Centripetal Force cannot exceed this value.**

$F_c = m v^2 / r$, so $v^2 = F_c r / m$

$= 20{,}000 \times 40 / 750$

$v^2 = 1067$

$\therefore$ max. $v = \sqrt{1067} \cong 33\ ms^{-1}$

**(This is almost 120 km/hr, so for a tight 40m radius curve, these must be VERY good tyres!)**

# Centrifugal Forces & Banked Corners

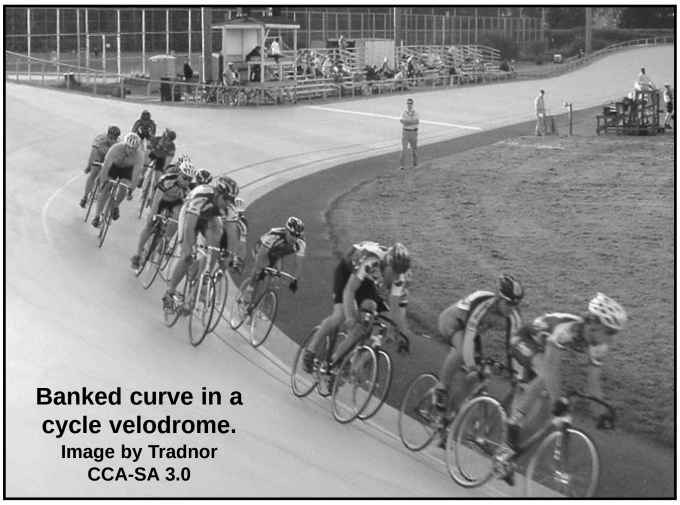
***"Centrifugal Force"*** **is the <u>apparent</u> force that objects <u>seem to</u> experience when travelling around a curve, such as happens in circular motion. For example, we are all familiar with loose objects on the dashboard sliding to the left as our car turns a curve to the right. We would say that "centrifugal force pushed them towards the outside of the curve".**

**In fact, centrifugal force is a "pseudo-force" which arises due to inertia. It seems real when you are inside the turning car, but when analysed from a non-accelerating position** (eg measuring the Physics from a stationary over-pass as the car travels under you) **the only <u>real force</u> acting on the car (and contents) is <u>centripetal force</u> causing acceleration towards the centre of the circle.**

**The loose object on the dashboard has insufficient friction to stay attached to the car, so its inertia** (Newton's 1st Law... tendency to continue travelling in a straight line) **takes over. It actually tries to fly off at a tangent to the curve, but from within the car this seems to be an outwards push from a force we call "centrifugal".**

**The force is fake, but the inertia is real. On a bicycle at speed, the rider always "leans into the curve" to conteract the outward (actually tangential) inertia.**

**On a curve on a horizontal track, this inward lean**



**Banked curve in a cycle velodrome.**
Image by Tradnor CCA-SA 3.0

**can cause the rider to fall because the side-wall of the tyres comes into contact with the road surface and it may lack the friction to hold on at high speed.**

**Banking the curve up at an angle allows the rider to lean into a curve at high speed while still keeping the tyre tread in full frictional contact. Not only are cycle velodromes "banked", but so are well-made roadways and railways. Aircraft bank to turn, too. It's all about maintaining friction (or aircraft "lift") so centripetal force "pulls" you around the corner.**

keep it simple science

# Angular Velocity

When analysing circular motion, sometimes it is useful to measure the angle through which an object moves (measured from the centre of the circle) per unit of time, rather than measuring the distance travelled per unit of time.

For example, if you are observing a satellite pass overhead in the night sky, it is not easy to measure its velocity because (for instance) you don't know how high up it is and therefore you cannot measure distance travelled per second. BUT you can measure the angle it moves through per second.

For convenience when working with circles, the angles are always measured in radians, not degrees.
(If not familiar with radians, note that

$$1 \text{ rad.} \cong 57.3^o$$

because, by definition, there are $2\pi$ radians in a full circle $360^o$.) The convenience of using radians should become clear to you soon.

## Angular Velocity

The angular velocity of an object in circular motion can be defined as the "angular change of position (displacement) per unit of time" when viewed from the centre of the circle. The symbol used for angular velocity is the (lower case) Greek letter omega ( $\omega$ ).

$$\omega = \frac{\Delta\phi}{t}$$

where $\Delta\phi$ is the change in the angle (in radians) and "t" is the elapsed time in seconds. The units for angular velocity are radians per second ($rad.s^{-1}$)

This is the formula presented in the syllabus and your HSC Data Sheet. However, it may be more informative to consider this concept as follows:

Imagine an object in circular motion. To travel one complete revolution, its angular displacement is $2\pi$ radians ($360^o$). The time it takes for one revolution ("period") is "T" seconds.

Therefore, $$\omega = \frac{\Delta\phi}{t} = \frac{2\pi}{T}$$

This allows us to derive an alternate set of equations to apply to problems on circular motion.

### Orbital Speed & Angular Velocity

$$v = \frac{2\pi r}{T} \quad \text{but} \quad \omega = \frac{2\pi}{T} \quad \text{so} \quad v = \omega r$$

### Centripetal Acceleration

$$a_c = \frac{v^2}{r} \quad \text{but} \quad v = \omega r$$

$$\text{so} \quad a_c = \frac{\omega^2 r^{\cancel{2}}}{\cancel{r}} = \omega^2 r$$

### Centripetal Force

$$F_c = \frac{mv^2}{r}$$

Substituting as above gives $F_c = m\omega^2 r$

### Example Problem

A 250g ball is being swung around on a string which is 2.5m long. Its period of rotation is 1.25s.

a) Find its angular velocity.

b) What is the centripetal force in the string?

c) What is the orbital speed?

### Solution

a) $\omega = \frac{2\pi}{T} = 2\pi \ / \ 1.25 = 5.03 \ rad.s^{-1}$.

b) $F_c = m\omega^2 r = 0.250 \times (5.03)^2 \times 2.5$
$= 15.8 \text{ N}$

c) $v = \omega r = 5.03 \times 2.5 = 12.6 \ ms^{-1}$.

Try Worksheet 4

## Work Done in Circular Motion

How much work is done by the centripetal force during circular motion?

You will recall that "work" is done when a force acts over a distance and that the amount of work is equivalent to the energy applied or used.

$$W = F.S$$

However, you may also recall that the displacement in this equation must be in the direction of the force.

In circular motion, the centripetal force acts towards the centre of the circle. Since the revolving object always stays the same distance (= the radius) from the centre, then there is NO DISPLACEMENT in the direction of the force.

Therefore, the work done is zero!

In circular motion, no work is done by the centripetal force and no energy change occurs. This is why a planet in orbit can remain that way for ever, without running out of energy... no work is being done.

keep it simple science

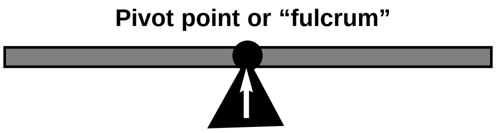
# The Concept of Torque

## ...and now for something completely different!

**At this point the syllabus specifies that you must learn about another situation where an object may rotate in a circle. However, this rotation has nothing to do with "circular motion" involving centripetal force.**

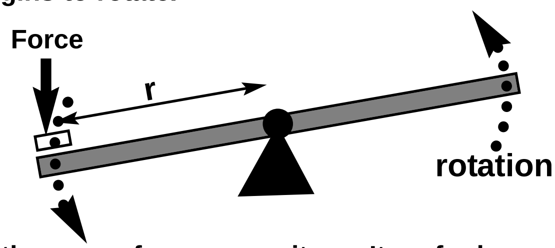
**Instead, here we will consider the motion which occurs when one or more forces act, not towards the centre, but (usually) on the line of a tangent to the circle.**

**Let's begin with a simple example: a see-saw.**




**Basically, this is a rigid beam which is able to rotate around its pivot point, or "fulcrum". To begin with, it is perfectly balanced and motionless.**

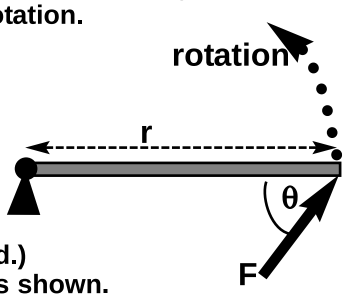
**If you place a weight on one end the see-saw experiences a "turning moment" or "torque" and begins to rotate.**




**In the case of a see-saw, it won't go far because it will hit the ground, but in the case of many mechanical devices, the bar can continue to rotate in a circle if the force continues to act along the line of a tangent to the circle of rotation.**

### *Calculating Torque*

**A force acts on the end of a rod or bar at distance "r" from its pivot point. (We use "r" because that distance is the radius of the circle it will turn around.) The force acts at angle θ as shown. Then:**




$$\tau = r.F.\sin\theta$$

**τ = torque on the system, in newton-metres (Nm).**
**r = distance from pivot to point where force is applied, in metres (m).**
**F = force, in newtons (N).**

**Note that when θ = 90°, sinθ = 1. This means that maximum torque occurs when the force acts at right angles to the bar. If θ = 0°, torque is zero.**

**The symbol for torque is the Greek letter "tau", τ.**

### *Example Problems*

**1. A rigid bar has a pivot point at one end and is 3.0m long. It can rotate in a horizontal plane and for simplicity, friction and the weight of the bar itself are taken to be zero.**

**Find the magnitude of the torque if:**
**a) a force of 10N is applied at right angles, at a point halfway along the bar.**
**b) The same force is applied at the end of the bar.**
**c) The same force is applied just 10cm from the pivot.**
**d) A force of 20N is applied to the end of the bar at an angle of 30°.**

**Try Worksheet 5**

### *Solutions*

**a)** $\tau = r.F.\sin\theta$ **= 1.5 x 10 x sin90 = 15 Nm.**

**b)** $\tau = r.F.\sin\theta$ **= 3.0 x 10 x sin90 = 30 Nm.**

**c)** $\tau = r.F.\sin\theta$ **= 0.1 x 10 x sin90 = 1.0 Nm.**
**(note the effects of distance from the pivot)**

**d)** $\tau = r.F.\sin\theta$ **= 3.0 x 20 x sin30 = 30 Nm.**

**(Compare (d) with (b) to note the effect of angle)**

**Everyday examples of applying torque include:**

**• pushing a door open on its hinges. If you push on the door at a point close to the hinges, you need much more force to get the same torque as pushing at the outside edge.**

**• winding the handle on a fishing reel, or winch. If the shaft of the handle is longer, you get more torque and the job is easier.**

**The concept of torque is especially important with motors. It will be re-visited in a later topic when electric motors are covered.**

### *Is Torque the Same as "Work"?*

**The unit of torque is a newton-metre. This is the same unit as Work. (W = F.S) Work is equivalent to energy, so a newton-metre of work is equal to a joule of energy.**

**But wait! This does NOT work. Torque is NOT the same as energy and it is simply a co-incidence that the units of measurement are the same.**

**Torque is a measure of the rate of change of angular momentum and is not equivalent to energy until multiplied by the rotation rate.**

**We are NOT going there, but for the petrol-heads, the torque of a car engine IS related to how powerful it is. An engine's power is described as "(number of) kilowatts at (number of) RPM". RPM is "revolutions per minute". Engine power (power is rate of energy change) is actually given by**

$$P = \tau.\omega \quad \text{(Power = torque x angular velocity)}$$

keep it simple science

# 3. Motion in Gravitational Fields

## Newton's Universal Gravitation

### Gravitational Fields

**Every mass acts as if surrounded by an invisible "force field" which attracts any other mass within the field. Theoretically, the field extends to infinity, and therefore every mass in the universe is exerting some force on every other mass in the universe... that's why it's called Universal Gravitation.**

### Newton's Gravitation Equation

**It was Isaac Newton who showed that the strength of the gravitational force between 2 masses:**

- **is proportional to the product of the masses, and**
- **inversely proportional to the square of the distance between them.**

**He came up with this idea in 1666 as a way to solve a long-standing problem in Astronomy to do with "Kepler's Laws of Planetary Motion". More later!**

$$F_G = \frac{GMm}{r^2}$$

**$F_G$ = Gravitational Force, in N.**
**G = "Universal Gravitational Constant" = $6.67 \times 10^{-11}$**
**M & m = the 2 masses involved, in kg.**
**r = distance between M & m (centre to centre) in m.**

### Example Calculation 1

**Find the gravitational force acting between the Earth and a 750kg satellite in orbit 42,000km from the Earth's centre.**

*Solution*

$$F_G = \frac{GMm}{r^2} = \frac{6.67\times10^{-11}\times6.0\times10^{24}\times750}{(4.2\times10^7)^2} = 170\text{ N.}$$

### Example 2

**Find the gravitational force acting between the Earth, and an 80kg person standing on the surface, 6,370km from Earth's centre (d = $6.37 \times 10^6$m).**

**Solution**

$$F_G = \frac{GMm}{r^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times 80}{(6.37\times10^6)^2} = 785\text{ N.}$$

**This is, of course, the person's weight!**
**... and sure enough**

$$F = mg = 80 \times 9.81 = 785\text{N.}$$

$$\therefore \text{Gravitational Force = Weight Force}$$

**Try Worksheet 6**

### Effects of Mass & Distance

**How does the Gravitational Force change for different masses, and different distances?**

**Imagine 2 masses, each 1kg, separated by a distance of 1 metre.**

$$F_G = \frac{GMm}{r^2} = \frac{G \times 1 \times 1}{1^2} = G$$

**_Effect of masses_**

**Now imagine doubling the mass of one object:**

$$F_G = \frac{GMm}{r^2} = \frac{G \times 2 \times 1}{1^2} = 2G \quad \text{(Twice the force)}$$

**What if both masses are doubled?**

$$F_G = \frac{GMm}{r^2} = \frac{G \times 2 \times 2}{1^2} = 4G \quad \text{(4X the force)}$$

**_Effect of Distance_**

**Go back to the original masses, and double the distance:**

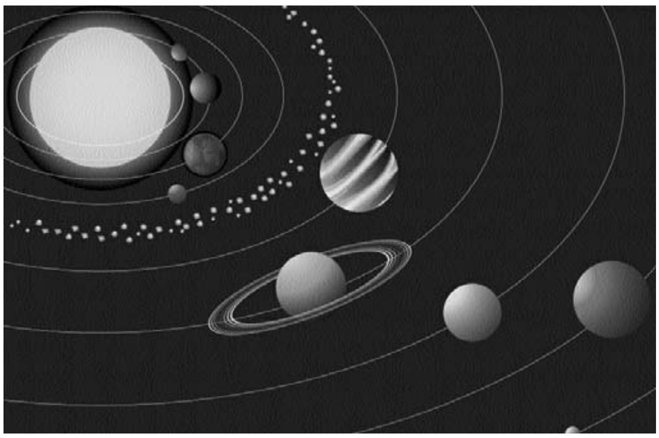
$$F_G = \frac{GMm}{r^2} = \frac{G \times 1 \times 1}{2^2} = \frac{G}{4} \quad \text{( 1/4 the force)}$$

**Gravitational Force shows the "Inverse Square" relationship...**
**triple the distance = one ninth the force**
**10 x the distance = 1/100 the force, etc.**

### Universal Gravitation & Orbiting Satellites

**It should be obvious by now that it is $F_G$ which provides the centripetal force to hold any satellite in its orbit.**

**Not only does this apply to artificial satellites launched into Earth orbit, but for the orbiting of the Moon around the Earth, and of all the planets around the Sun.**



**Our entire Solar System is orbiting the Galaxy because of gravity, and whole galaxies orbit each other. Ultimately, gravity holds the entire universe together, and its strength, compared to the expansion of the Big Bang, will determine the final fate of the Universe.**

keep it simple science

# *The Gravitational Field Strength, "g"*

**In earlier topics you learnt that "g" is the acceleration due to gravity. That's fine, but there is another way to think of it. Look again at problem 2 on the previous page.**

From that problem we can say that:

<u>weight force</u> = <u>gravitational force</u>

$$mg = GMm / r^2$$

Now imagine placing a 1kg mass at a point within the gravity field of a planet with mass "M".
If m = 1 in the equation above, then:

$$g = GM / r^2$$

This can be interpreted as the <u>strength of the gravitational field</u> at that point, because it defines the effect of the field on a "<u>test mass</u>" of 1 unit (kg).
(Compare this idea to how the Electric Field strength was defined in a previous module.)

So, "g" is both the strength of the grav. field (measured in $N.kg^{-1}$) at a given point AND it is the acceleration due to gravity ($ms^{-2}$) at that point.

We tend to think of gravity as being the same everywhere, but that is only because we are always on, or very close to, the Earth's surface where $g = 9.8\ N.kg^{-1}$ or $ms^{-2}$.

However, the equation at left means that the value of "g" depends on the mass of the planet you are on AND how far you are from its centre.

Here on the surface of Earth we are $r_E = 6.371 \times 10^6$ m from the centre of a mass of $M_E = 6.0 \times 10^{24}$ kg.

Substituting this gives

$$g = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} / (6.371 \times 10^6)^2$$
$$= 9.86\ ms^{-2}.$$

(<u>Discussion</u>: why doesn't this agree exactly with standard data?)

Go up 1,000 km above the surface and $g = 7.4\ ms^{-2}$.

If the Earth became denser and shrank so that the surface was 1,000km lower, (but same mass) then surface gravity would be $g = 13.9\ ms^{-2}$.

However, if you go lower by digging a hole, "g" actually <u>decreases</u>. As you go deep into the Earth, some of its mass is above you, attracting you upwards.

At the centre of the Earth g = zero!

**Try Worksheet 7 parts a-h only**

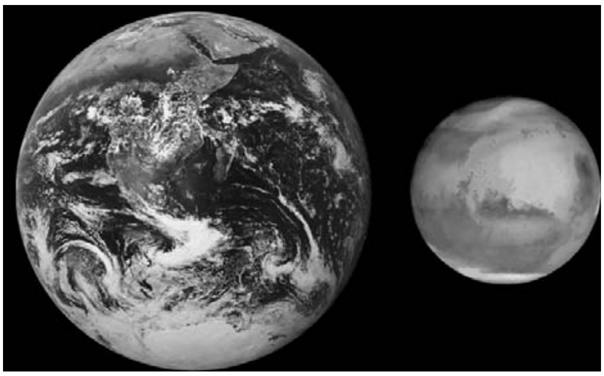
# *Gravity and Weight on Other Planets*

We are so used to the gravity effects on Earth that we need to be reminded that "g" is different elsewhere, such as on another planet in our Solar System.

Since "g" is different, and weight force F = mg it follows that things have a different weight if taken to another planet.

Here are values of "g" on the surface of some other planets. (Good luck finding the surface of Jupiter if you go there!)

| <u>Planet</u> | g ($ms^{-2}$) | g (as multiple of Earth's) |
|---|---|---|
| Earth | 9.8 | 1.00 |
| Mars | 3.8 | 0.39 |
| Jupiter | 25.8 | 2.63 |
| Neptune | 10.4 | 1.06 |
| Moon | 1.6 | 0.17 |



Composite photo of Earth and Mars to the same scale. Mars is much smaller and is lower in both density and total mass.
Its surface gravity is only 39% of Earth's.
Photo by NASA

# *Calculating Your Weight on another Planet*

<u>Example</u>
If an astronaut in his space suit weighs 1,350N on Earth, what will he weigh on Mars where $g = 3.84ms^{-2}$?

<u>Solution</u> F = mg
On Earth, 1,350 = m x 9.81
∴ mass = 1,350/9.81
= 137.6 kg

So on Mars,

W = mg
= 137.6 x 3.84
= 528kg.

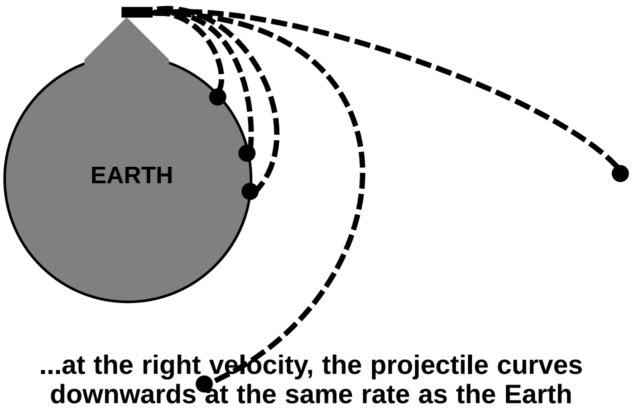
**Try Worksheet 8**

keep it simple science

# Isaac Newton and Orbiting

**Once Isaac Newton had developed the Maths and discovered the laws of motion and gravity, he thought about Projectile Motion.**

**Newton imagined a cannon on a very high mountain, firing projectiles horizontally with ever-increasing launch velocities. The faster each ball is launched, the further it flies before hitting the ground. But then...**




**...at the right velocity, the projectile curves downwards at the same rate as the Earth curves... it will circle the Earth in orbit!**

**...if the launch velocity is high enough, the projectile can escape from the Earth's gravity completely.**

**Newton had discovered the concepts of a gravitational orbit, and of "escape velocity".**

## *Escape Velocity*

**is defined as the launch velocity needed for a projectile to escape from the Earth's gravitational field.**

**Mathematically, it can be shown that (for any planet)**

**Escape Velocity,** $v_{esc} = \sqrt{2GM / r}$

**G= Gravitational Constant**
**M = Mass of the planet (kg)**
**r = Radius of planet (m)**

**Complete Worksheet 7 parts p-w**

**Note that:**

- **The mass of the projectile is not a factor. Therefore, all projectiles, regardless of mass, need the same velocity to escape from Earth, about 11km per second!**
- **The Escape Velocity depends only on the mass and radius of the planet.**

**It follows that different planets have different escape velocities. Here are a few examples...**

| PLANET | ESCAPE VELOCITY in km/sec | ($ms^{-1}$) |
|---|---|---|
| Earth | 11.2 | $1.12 \times 10^4$ |
| Moon | 2.3 | $2.3 \times 10^3$ |
| Mars | 5.0 | $5.0 \times 10^3$ |
| Jupiter | 60.0 | $6.0 \times 10^4$ |

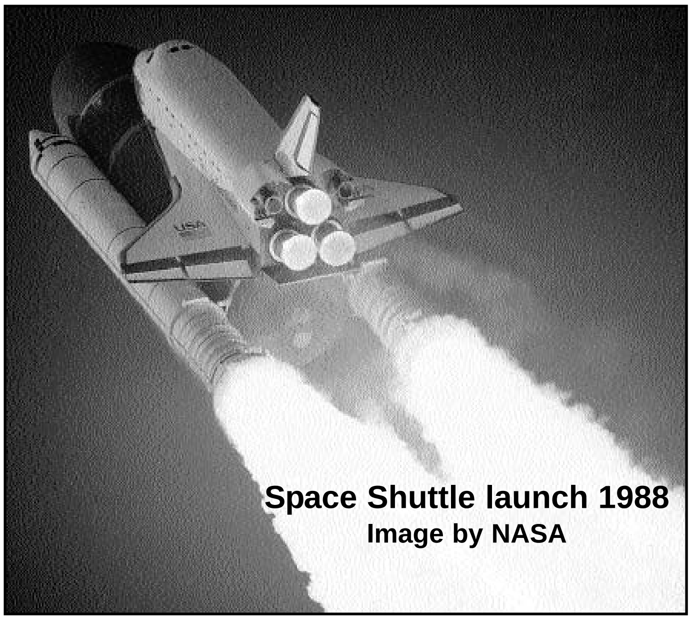
# Placing a Spacecraft into Earth Orbit

**A projectile needs an enormous velocity to escape from the Earth's gravitational field... about 11 km per second. Think of a place 11 km away from you, and imagine getting there in 1 second flat!**

**What about Newton's idea of an orbiting projectile? If it is travelling at the right velocity, a projectile's down-curving trajectory will match the curvature of the Earth, so it keeps falling down, but can never reach the surface. A projectile "in orbit" becomes a "satellite".**

**It can be shown that to achieve orbit, the launch velocity required is less than escape velocity, but still very high... about 8 km per second. How is this velocity possible?**

**In a 19th century novel, author Jules Verne proposed using a huge cannon to fire a space capsule (including human passengers) into space.**



**Space Shuttle launch 1988**
**Image by NASA**

**The problem with this idea is the rate of acceleration to go from zero to orbital velocity in the very short time it takes to fire a cannon. A fit, trained astronaut can tolerate sustained forces of "5g", but anything above about "10g" is life-threatening. Jules Verne's cannon astronauts would have suffered forces of about 200g... instantly fatal.**

**That's why we use rockets, not cannons. A large rocket can produce a steady acceleration at a "g-force" which is acceptable for trained astronauts, over the 10 minutes (or so) that it takes to reach orbit. If you're wondering why we don't use jet engines, or similar, you need to remember that there is no air (with oxygen to burn the fuel) in space. Rockets carry their own oxidisers plus fuel and so do not need air. In fact, of course, they work better in space where there is no air resistance.**

keep it simple science

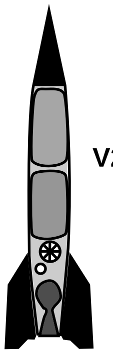
# A Brief History of Rocketry

**Simple solid-fuel (e.g. gunpowder) rockets have been used as fireworks and weapons for over 500 years.**

**Over 100 years ago, the Russian Tsiolkovsky (1857-1935) was the first to seriously propose rockets as vehicles to reach outer space. He developed the theory of multi-stage, liquid-fuel rockets as being the only practical means of achieving space flight.**

**Goddard and his first liquid-fuel rocket.**

**The American Robert Goddard (1882-1945) developed rocketry theory futher, but also carried out practical experiments including the first liquid-fuel rocket engine.**

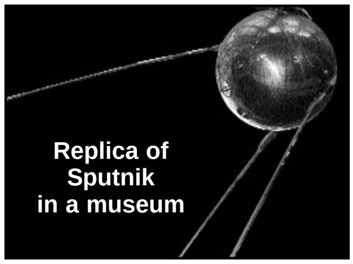


**V2**

**Goddard's experiments were the basis of new weapons research during World War II, especially by Nazi Germany.**

**Wernher von Braun (1912-1977) and others developed the liquid-fuel "V2" rocket to deliver explosive warheads at supersonic speeds from hundreds of kilometers away.**

**At the end of the war many V2's, and the German scientists who developed them, were captured by either the Russians or the Americans. They continued their research in their "new" countries, firstly to develop rockets to carry nuclear weapons (during the "Cold War") and later for space research.**



**Replica of Sputnik in a museum**

**The Russians achieved the first satellite ("Sputnik" 1957) and the first human in orbit, and the Americans the first manned missions to the Moon (1969).**

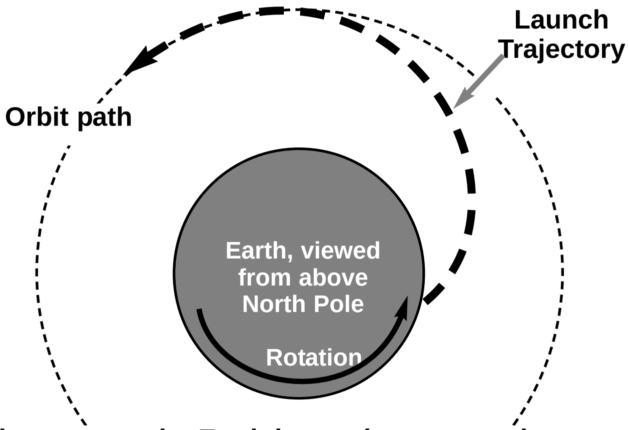
**Since the 1970's, the use of satellites has become routine and essential to our communications & research, while (unmanned) probes have visited every other planet in the Solar System, plus an asteroid and a comet.**

# Physics of a Rocket Launch

## Direction of Launch

**Straight upwards, right? Wrong!**

**To reach Earth orbit, rockets are aimed toward the EAST to take advantage of the Earth's rotation. The rocket will climb vertically to clear the launch pad, then be turned eastward.**

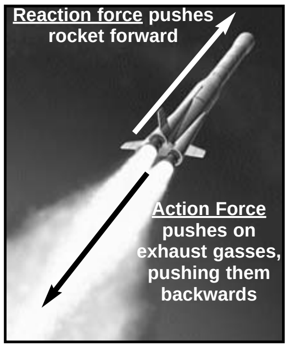



**At the equator, the Earth is rotating eastwards at about 1,700km/hr (almost 0.5km/sec) so the rocket already has that much velocity towards its orbital speed.**

**Rocket launch facilities are always sited as close to the equator as possible, and usually near the east coast of a continent so the launch is outwards over the ocean.**

## Conservation of Momentum

**Why a rocket moves is yet another phenomenon explained by Newtonian Physics.**




### Newton's 3rd Law

**Force on Exhaust Gases = Force on Rocket**

**It can also be shown that**

**Change of Momentum of Exhaust Gases = Change of Momentum of Rocket**

backwards ( -ve) ← → forwards (+ve)

**Mass x velocity = Mass x velocity**

**The mass x velocity of the exhaust gases stays fairly constant during the launch. However, the mass of the rocket decreases as its fuel is burnt. Therefore, the rocket's velocity must keep increasing in order to maintain the Conservation of Momemtum.**

**This increasing acceleration (and g-forces) used to make early astronauts very uncomfortable, but modern space vehicles are "throttled-back" during launch to keep the g-force more tolerable.**

# Satellites and Orbits

There are 2 main types of satellite orbits:

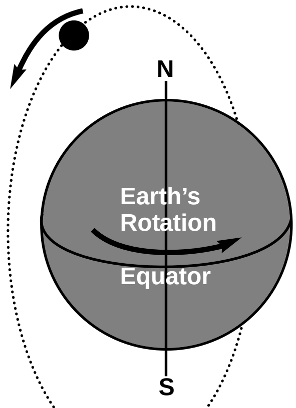
## Low-Earth Orbit

**As the name suggests, this type of orbit is relatively close to the Earth, generally from about 200km, out to about 1,000km above the surface.**

**For any satellite, the closer it is, the faster it must travel to stay in orbit. Therefore, in a Low-Earth Orbit a satellite is travelling quickly and will complete an orbit in only a few hours.**

**A common low orbit is a "Polar Orbit" in which the satellite tracks over the north and south poles while the Earth rotates underneath it.**

**Polar Orbit**




Satellites of this type can be seen as "moving stars" in the night sky. They always move on a N-S path.

**This type of orbit is ideal for taking photos or Radar surveys of Earth.**

**The satellite only "sees" a narrow north-south strip of the Earth, but as the Earth rotates, each orbit looks at a new strip.**

**Eventually, the entire Earth can be surveyed.**

**Being a close orbit, fine details can be seen.**

## Geo-Synchronous Orbits

**are those where the period of the satellite (time taken for one orbit) is exactly the same as the Earth itself... 1 day.**

**If the satellite orbit is directly above the equator, the satellite is also "geo-stationary", meaning that it is always directly above the same spot on the Earth, and seems to remain motionless in the same position in the sky. It's not really motionless, of course, but orbiting around at the same angular velocity as the Earth itself.**

**Geostationary orbits are above the equator, and have to be about 36,000km above the surface in order to have the correct orbital speed.**

**Being so far out, these satellites are not much good for photographs or surveys, but are ideal for communications. They stay in the same relative position in the sky and so radio and microwave dishes can be permanently aimed at the satellite, for continuous TV, telephone and internet relays to almost anywhere on Earth.**

Try Worksheet 9

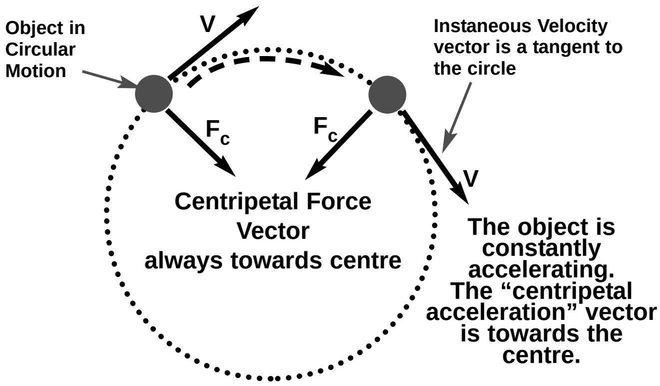
**Three geostationary satellites, spaced evenly around the equator, can cover virtually the whole Earth with their transmissions.**

## Orbits & Centripetal Force

**The orbit of a satellite is often an oval-shape, or an "ellipse". However, in this topic we will always assume the orbits are circular... K.I.S.S. Principle.**

**To maintain motion in a circle an object must be constantly acted upon by "Centripetal Force", which acts towards the centre of the circle.**

**It is (of course) gravity which provides the centripital force which keeps a satellite in orbit.**

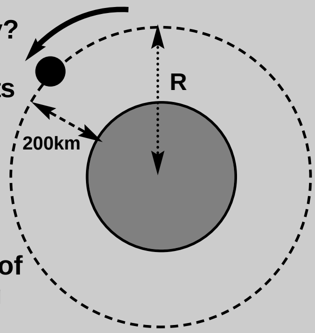



### Example Problem

**A 250kg satellite in a circular orbit 200km above the Earth, has an orbital period of 1.47hours.**

**a) What is its orbital velocity?**

**b) What centripetal force acts on the satellite?**
**(Earth radius = $6.37\times10^6$m)**




### Solution

**a) First, find the true radius of the orbit, and get everything into S.I. units:**

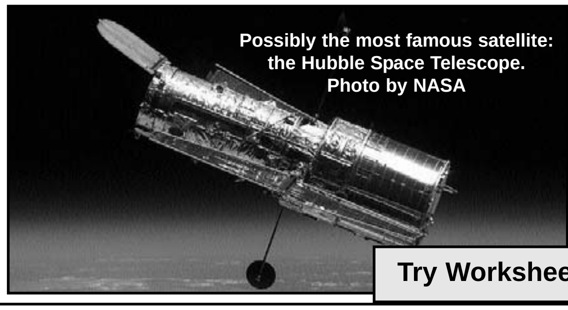
**Radius of orbit = 200,000 + $6.37\times10^6$ = $6.57\times10^6$m**
**Period = 1.47hr = 1.47 x 60 x 60 = $5.29\times10^3$ sec.**

$$v = \frac{2\pi r}{T} = 2 \times \pi \times 6.57\times10^6 / 5.29\times10^3 = 7.80\times10^3 ms^{-1}.$$

b) $$F_c = \frac{mv^2}{r} = 250\times(7.80\times10^3)^2 / 6.57\times10^6$$

$$= 2{,}315 = 2.32\times10^3 \text{ N.}$$

**The satellite is travelling at about 8 km/sec, held in orbit by a gravitational centripetal force of about 2,300N.**



Possibly the most famous satellite: the Hubble Space Telescope. Photo by NASA

Try Worksheet 10

keep it simple science

# Satellites, Planets & Moons

So far, we have used the word “satellite” only for human-made space-craft in Earth orbit.

However, don’t forget that all the planets of the Solar System are satellites of the Sun and that the Moon is a natural satellite of Earth. Most of the other planets have moons (some have dozens) and these are satellites of their “primary”... the body they orbit around.

### Orbital Speed & Radius

Since we know that the centripetal force is due to the gravitational force between the satellite and its “primary”, we can say:

$$F_c = F_G \quad \text{or} \quad \frac{mv^2}{r} = \frac{GMm}{r^2}$$

Simplifying gives:

$$v^2 = \frac{GM}{r} \quad \text{so} \quad v = \sqrt{\frac{GM}{r}}$$

What does this mean?

What this means is, that for any given “primary” object, there is a relationship between the orbital speed of a satellite and the radius of the orbit.

To put it another way, for any given radius of an orbit, there is a certain orbital speed which “fits” that orbit. The relationship is inverse: a larger radius orbit results in a slower orbital speed and vice-versa.

For artificial satellites in Earth orbit, this means that low-level satellites must move very fast in their orbit, while those further out move more slowly.

Same thing for the planets in orbit around the Sun. Those closer to the Sun move faster, those further out move slower. This was first realised and measured by astronomers about 400 years ago and was the stimulus that caused Isaac Newton to get busy.

It will help your understanding if you know some background about what they knew back then, and what was the problem that Newton solved.

# A Brief History of Astronomy *(to the time of Newton)*

### Ancient Beliefs

In ancient times, the Sun, Moon & stars were “The Heavens” and considered the home of Gods. The early Greeks were the first to attempt to explain things based on evidence, observation & calculation... ie scientifically.

Generally, it was believed that the Earth was the centre of the Universe and everything else revolved around us. There were some who realised that movements in the sky could also be explained if the Earth went around the Sun AND rotated on its axis. (if interested, research Aristarchus.) However, no-one could find evidence for movements of the Earth, so the “Geocentric Model” was accepted.

Throughout the Middle Ages, this idea was so prevalent that it was adopted by the Church as religious dogma... any other suggestions were heresy and punishable by torture & death.

### Nicholas Copernicus (Polish. 1473 -1543)

The first modern suggestion that the Earth revolves around the Sun was published by Copernicus in a book released just as he died.

It was a Heliocentric model... Sun centred.

The accuracy of predicted motions of planets, etc. remained much the same as the Geocentric model, but this model was much simpler in its explanations. The Copernicus model was NOT immediately accepted at the time because:

- there was still no evidence that the Earth moved.
- the Church condemned the theory as heresy and banned the book.

### Tycho Brahe (1546-1601 Danish)

Tycho built the most advanced observatory of that time to gather outstandingly accurate data (accurate for measurement without a telescope) of planetary movements. He favoured the geocentric model and hoped his observations would prove Copernicus wrong.

However, this doesn’t mean he accepted the ancient models either. His aim was to develop a new model, but he died before completing it.

He jealously guarded his data from others, but when he died it went to his student Kepler.

### Johannes Kepler (1571-1630 German)

Kepler fitted Brahe's extremely accurate data (especially of the movements of planet Mars) to the Copernicus model. Finally, he found it only fitted if the orbits were ellipses, not circles.

Eventually he proposed 3 "Laws of Planetary Motion" , but could give no explanation of how or why the Earth and planets could orbit around the Sun. The Heliocentric idea was still NOT accepted widely. One of his “Laws” is detailed, next page.

### Galileo Galilei (1564-1642 Italian)

About the same time as Kepler’s Laws, (circa 1610) Galileo was the first to use a TELESCOPE to view the heavens.

His observations of Jupiter’s moons, planet Venus and the Moon conflicted with the Geocentric model and supported the Heliocentric idea of Copernicus.

keep it simple science

# Kepler's "Law of Periods" & Newton's Proof

One of Kepler's "Laws of Planetary Motion" was a mathematical relationship between the Period of the orbit and its Radius:

$r^3 \; \alpha \; T^2$ (Greek letter alpha ($\alpha$) means "proportional to")

This means that

$$\frac{r^3}{T^2} = \text{constant}$$

This means that for every planet, the (Radius)$^3$ divided by (Period)$^2$ has the same value. Similarly, every satellite of the Earth would have the same value of $r^3 / T^2$, but this value would be quite different to the value for the Sun's satellites. The ratio is different for each "primary".

Kepler's Law of Periods was discovered empirically... that is, it was discovered by observing the motion of the planets, calculated from Tycho's data. Kepler had no idea WHY it was so and could not prove mathematically that it was generally true.

For about 50 years, no-one could figure this out. Meanwhile, across Europe the power and control of the Church of Rome was being weakened by the spread of the Protestant movement.

In Protestant countries Science was flourishing and in England, a new generation of brilliant scientists had formed a Scientific Institute called the "Royal Society". They became interested in solving the riddle of Kepler's Laws.

Encouraged by others, Isaac Newton (age 24) solved the problem. There is a legend that he was inspired by seeing an apple fall from a tree, but in fact he developed his famous 3 "Laws of Motion", invented a whole new method of Maths (now called "Calculus") and topped it off with his Law of Gravity... apples have nothing to do with such genius!

With his "Law of Universal Gravitation" he was able to prove the theoretical basis for Kepler's Law, as follows:

The Centripetal Force of orbiting is provided by the Gravitational Force between the satellite and the Earth, so

$$F_c = F_G \quad \text{or} \quad \frac{mv^2}{r} = \frac{GMm}{r^2}$$

Simplifying gives:

$$v^2 = \frac{GM}{r} \quad \text{but} \quad v = \frac{2\pi r}{T}$$

So, $$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

re-arranging: $$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} = \text{constant}$$

Since the right hand side contains all constant values, this proves Kepler's Law and establishes the Force of Gravity as the controlling force for all orbiting satellites, including planets around the Sun.

In the above,
G = Universal Gravitational Constant
M = mass of the "primary" body (body being orbited)
m = mass of satellite... notice that it disappears!

***This proof explained Kepler's Laws and proved mathematically that the Heliocentric idea is correct. Newton's Gravity law fitted precisely with the Astronomical observations to explain how & why the Universe worked. Soon, new telescopic observations followed Galileo's work and eventually found the proof that the Earth was moving. Many consider that this was the beginning of modern Science.***

### Problem 1

In a previous problem, an Earth satellite 200km high had a period of 1.47 hrs. Use this to find the height of a geosynchronous satellite using Kepler's Law.

### Solution

For the satellite above, (units are km & hours) $\frac{R^3}{T^2} = \frac{6.570^3}{1.47^2} = 1.31 \times 10^{11}$

According to the law of periods, ALL satellites of Earth must have the same value for $R^3/T^2$

So, for the geo-stationary satellite: $\frac{R^3}{T^2} = 1.31 \times 10^{11}$

So $R^3 = 1.31\text{x}10^{11} \text{x} (24.0)^2$

$\therefore R = \sqrt[3]{7.55\text{x}10^{13}} = 4.23 \times 10^4$ km

This is approx. 42,000km from Earth's centre, or about 36,000km above the surface.

Note: When using Kepler's Law this way it doesn't matter which units are used, as long as you are consistent. In this example, km & hrs were used. You get the same answer with metres & seconds.

### Problem 2

Find the orbital radius of a geo-stationary satellite, given that its period of orbit is 24.0 hours.
(24.0hr = 24.0x60x60 = 8.64 x $10^4$ sec)
Doing it this way, you MUST use S.I. units!!

(G= Gravitational Constant = 6.67 x $10^{-11}$
M = Mass of Earth = 5.97 x $10^{24}$kg)

$$\frac{R^3}{T^2} = \frac{GM}{4\pi^2}$$

$$R^3 = \frac{6.67\text{x}10^{-11} \text{ x } 5.97\text{x}10^{24} \text{ x } (8.64\text{x}10^4)^2}{4\pi^2}$$

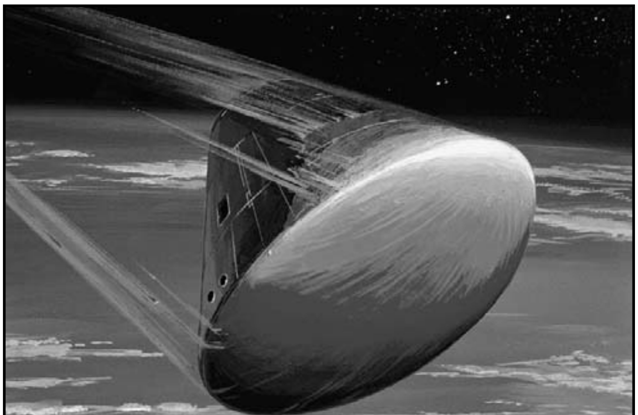
$\therefore R = \sqrt[3]{7.46\text{x}10^{22}} = 4.21 \times 10^7$m

This is about 42,000km, or about 36,000km above the surface... the same answer as before. (It better be!)

Try Worksheet 11

keep it simple science

# Re-Entry From Orbit


Artistic impression of an Apollo spacecraft during Atmospheric Braking

## Decay of Low-Earth Orbits

**Where does "Space" begin?**

It's generally agreed that by 100km above the surface of the Earth the atmosphere has ended, and you're in outer space. However, although this seems to be a vacuum, there are still a few atoms and molecules of gases extending out many hundreds of kilometres.

Therefore, any satellite in a low-Earth orbit will be constantly colliding with this extremely thin "outer atmosphere". The friction or air-resistance this causes is extremely small, but over a period of months or years, it gradually slows the satellite down.

As it slows, its orbit "decays". This means it loses a little altitude and gradually spirals downward. As it gets slightly lower it will encounter even more gas molecules, so the decay process speeds up.

Once the satellite reaches about the 100km level the friction becomes powerful enough to cause heating and rapid loss of speed. At this point the satellite will probably "burn up" and be destroyed as it crashes downward.

Modern satellites are designed to reach their low-Earth orbit with enough fuel still available to carry out short rocket engine "burns" as needed to counteract decay and "boost" themselves back up to the correct orbit. This way they can remain in low-Earth orbits for many years.

Getting a spacecraft into orbit is difficult enough, but the most dangerous process is getting it down again in one piece with any astronauts on board alive and well.

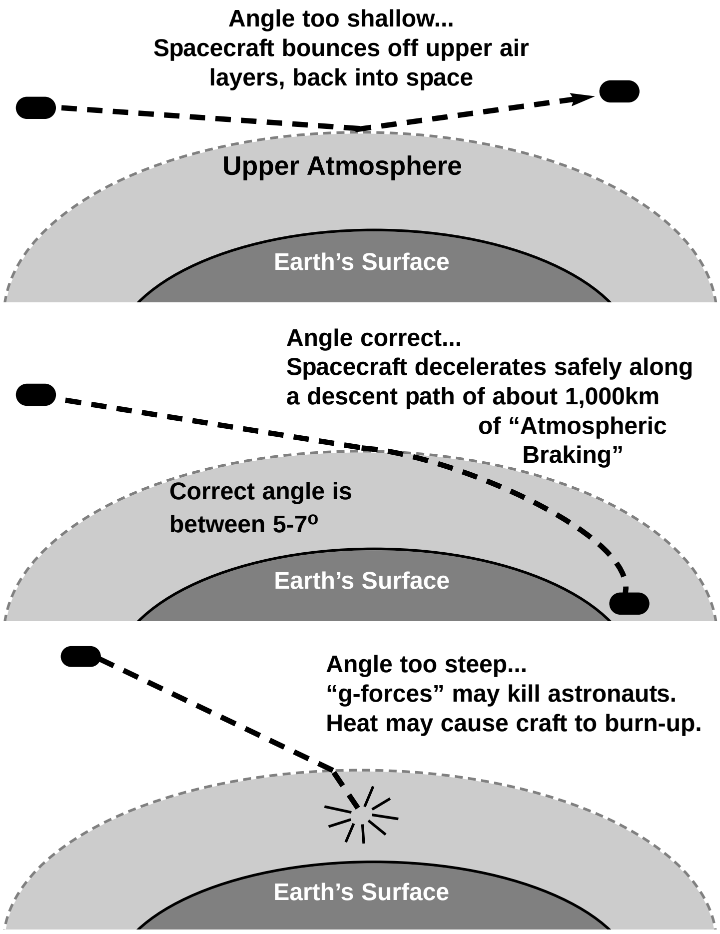
In orbit, the satellite and astronauts have a high velocity (kinetic energy) and a large amount of GPE due to height above the Earth. To get safely back to Earth, the spacecraft must decelerate and shed all that energy.

It is impossible to carry enough fuel to use rocket engines to decelerate downwards in a reverse of the lift-off, riding the rocket back down at the same rate it went up.

Instead, the capsule is slowed by "retro-rockets" just enough to cause it to enter the top of the atmosphere so that friction with the air does 2 things:

- cause deceleration of the capsule at a survivable rate of deceleration not more than (say) "5-g", and
- convert all the $E_k$ and GPE into heat energy.

The trick is to enter the atmosphere at the correct angle:




Early spacecraft used "ablation shields", designed to melt and carry heat away, with the final descent by parachute. The Space Shuttle used high temperature tiles and high-tech insulation for heat protection, and glided in on its wings for final landing like an aircraft.

keep it simple science ®

# Total Energy of a Satellite

In a previous module you studied the relationship for any object on or near the Earth, that the change of Gravitational Potential Energy (GPE) is

$$\Delta U = mg\Delta h$$

If an object is allowed to fall down, it loses some GPE and gains some other form of energy, such as Kinetic or Heat. To raise the object higher, you must "do work" on it, in order to increase the amount of GPE it contains.

Notice that this equation calculates the CHANGE in GPE and assumes a constant value for "g" on or near the Earth's surface. For satellites it is useless.

## Gravitational Potential Energy

For mathematical reasons, the point where an object is defined to have zero GPE is not on Earth, but at a point an infinite distance away. So GPE is defined as follows:

**Gravitational Potential Energy is a measure of the work done to move an object from infinity, to a point within the gravitational field.**

This definition has an important consequence: it defines GPE as the work done to bring an object towards the Earth, but we know that you need to do work to push an object (upwards) away from Earth.

Therefore, GPE is, by definition, a negative quantity! So, if you lift an object upwards against gravity, its GPE increases by becoming less negative. It's value might go from (say) -10 units to -5 units.

$$U = -\frac{GMm}{r}$$

U = GPE in joules (J)
G = Gravitational Constant (= $6.67\times10^{-11}$)
m = mass of object (kg)
M = mass of Earth, or other planet (kg)
r = distance from centre of the "primary" (m).

### Example Problem

Compare the GPE possessed by a geosynchronous ($r = 4.2\times10^7$m) satellite of mass 500kg, with that of a 500kg satellite in low orbit 200km up. ($r = 6.571\times10^6$m)

### Solution

Geosynch. Satellite

$$U = -\frac{GMm}{r} = -6.67\times10^{-11} \times 5.97\times10^{24} \times 500 / 4.2\times10^7$$

$$= -4.74 \times 10^9 \text{ J} \quad (-4{,}740{,}000{,}000)$$

Low Orbit Satellite

$$U = -\frac{GMm}{r} = -6.67\times10^{-11} \times 5.97\times10^{24} \times 500 / 6.571\times10^6$$

$$= -3.03 \times 10^{10} \text{ J} \quad (-30{,}300{,}000{,}000)$$

The higher satellite has more GPE (about 6 times more) by virtue of being less negative in value.

## Kinetic Energy in Orbit

You previously learned that

$$E_k = \frac{1}{2}mv^2$$

but to work it out for a satellite it would be better to express it in terms of G, M, r, etc.

We begin the same way as we have before:

$$F_c = F_G \quad \text{or} \quad \frac{mv^2}{r} = \frac{GMm}{r^2}$$

Multiply both sides by "r" and divide by 2 gives:

$$\frac{mv^2}{2} = \frac{GMm}{2r}$$

The left side is the kinetic energy expression, so:

$$E_k = \frac{GMm}{2r}$$

This means that a satellite in a higher orbit (r is larger) will have less $E_k$. That makes sense, because at higher orbits we know that its velocity is lower.

Total Energy of a Satellite $= E_k + U$

$$= \frac{GMm}{2r} + -\frac{GMm}{r}$$

Express these with a common denominator:

$$= \frac{GMm}{2r} + -\frac{2GMm}{2r} = \frac{GMm - 2GMm}{2r}$$

$$E_k + U = -\frac{GMm}{2r}$$

### Example Problem

How much energy is required to lift a 500kg satellite from a low orbit ($r = 6.571\times10^6$m) up to a geosynchronous orbit? ($r = 4.2\times10^7$m)

### Solution

Total energy in low orbit

$$E_k + U = -\frac{GMm}{2r} = -\frac{6.67\times10^{-11} \times 5.97\times10^{24} \times 500}{2 \times 6.571\times10^6}$$

$$= -1.51 \times 10^{10} \text{ J}$$

Total energy in geosynch. orbit

$$E_k + U = -\frac{GMm}{2r} = -\frac{6.67\times10^{-11} \times 5.97\times10^{24} \times 500}{2 \times 4.2\times10^7}$$

$$= -2.37 \times 10^9 \text{ J}$$

In the higher orbit, its total energy is increased (it is less negative). It has lost $E_k$ (lower velocity) but gained GPE (more height).
The actual answer to the question is the difference between these values:

Energy required = $-2.37\times10^9 - (-1.51\times10^{10}) = 1.27\times10^{10}$ J

Try Worksheet 12

Finish with Worksheet 13