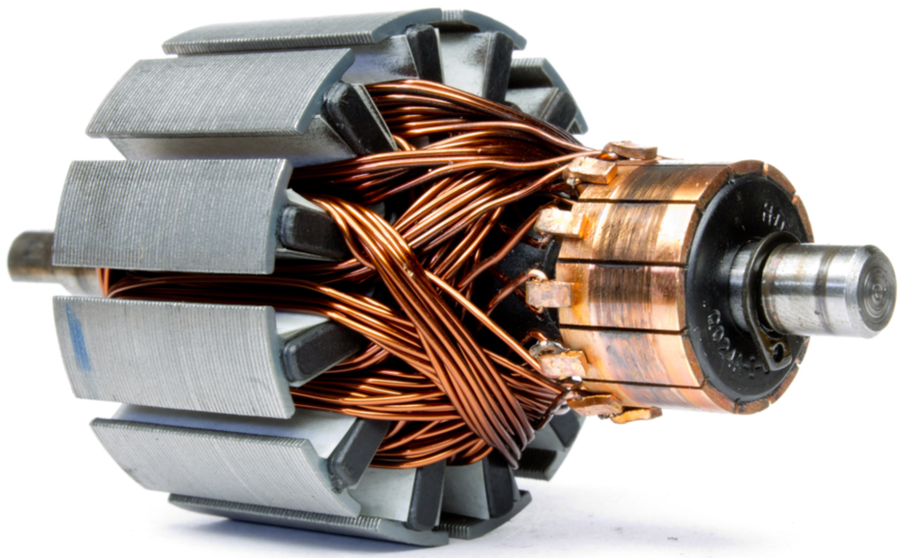
Module 6: Electromagnetism

Part 4: Applications of the Motor Effect



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Contents

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Syllabus content: Electromagnetism

Applications of the Motor Effect

Inquiry question: How has knowledge about the Motor Effect been applied to technological advances?

Students:

- investigate the operation of a simple DC motor to analyse:
  - the functions of its components
  - production of a torque $\tau = nIA_{\perp}B = nIAB\sin\theta$
  - effects of back emf (ACSPH108)
- analyse the operation of simple DC and AC generators and AC induction motors (ACSPH110)
- relate Lenz's Law to the law of conservation of energy and apply the law of conservation of energy to:
  - DC motors and
  - magnetic braking

Revision

The new physics we have introduced in this module is:

- That a charge moving relative to a magnetic field will experience a force $F = qvB \sin\theta$ (together with $F = qE$ this is the Lorentz force)
- That a changing magnetic field will produce an emf equal to $\varepsilon = -\frac{d\Phi}{dt}$ (Faraday's law)

Starting with $F = qvB \sin\theta$, we have shown that:

- A current in a conductor in a magnetic field experiences a force of $F = IlB \sin\theta$. We have called this "the motor effect".
- Parallel current-carrying wires experience a force towards (for parallel currents) or away from each other (for antiparallel currents)
- Free charges inside a conductor that moving through a magnetic field experience a force to one side of the conductor, producing a separation of charge that results in a motional emf appearing across the conductor (you should assume that, although not mentioned explicitly in the syllabus, the *concept* of motional emf will be tested, as this was the situation in the previous syllabus).

Starting with Farday's law of electromagnetic induction, $\varepsilon = -\frac{d\Phi}{dt}$, we have explored how:

- Induced currents are arise in complete circuits in which a changing magnetic field produces an emf, producing their own magnetic field
- Lenz's law, which follows from conservation of energy, can be used to predict the direction of the induced emf
- Eddy currents appear in conductors subject to a changing magnetic flux
- Transformers utilise Faraday's law to step-up and step-down AC voltages, and that this is useful to reduce power losses when transmitting electricity

In this part of the module, we will continue to look at applications of the these two new ideas in the context of electric motors and generators.

Anatomy of a DC motor

DC motors utilise the force on a current-carrying conductor in a magnetic field (the "motor effect") to convert electrical energy to rotational mechanical energy.

The essential components of a DC motor (and their function):

- **Magnets** - These can be permanent magnets (for very small motors) but are most commonly electromagnets.
- **Armature** - This is also called the rotor. Usually has many coils of wire arranged symmetrically around the axle (as in the image on the front page of this booklet).
- **Brushes** - These are usually made from graphite in real motors. These provide electrical contact between the coils on the armature (rotating part of the motor) and the source of electrical power (which is stationary).
- **Power source** - To produce a current, an source of (DC) electrical power is required.
- **Split-ring commutator** - Reverses the direction of the current every half cycle, to ensure that the motor continues to rotate in a constant direction.
- **Laminated iron core** - The coils in real DC motors are wound onto a laminated iron core. This serves the same purpose as the laminated iron core in a transformer, amplifying the magnetic field inside the motor while minimising the eddy currents that arise as the core rotates in the external B-field.

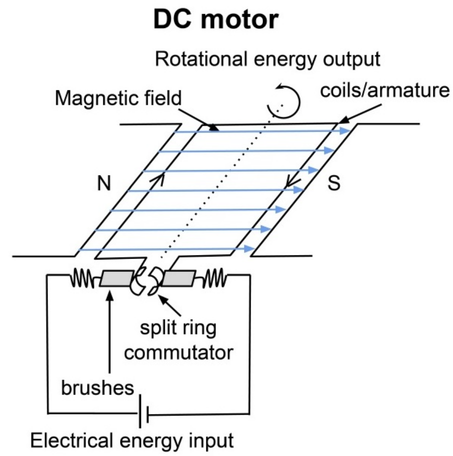

Figure 1: Line diagram of a simple DC motor.

Torque

In module 5 we introduced the concept of torque as the rotational analogue of force. Torques act to produce rotation of an object, and add as vectors. A *net* torque produces angular acceleration.

In the syllabus, torque is defined as:

$$\tau = r_{\perp}F = rF\sin\theta$$

where r is the magnitude of the displacement of the point of application of the force from the axis of rotation (no, there is no simpler definition of r!) and F is the force applied to the object.

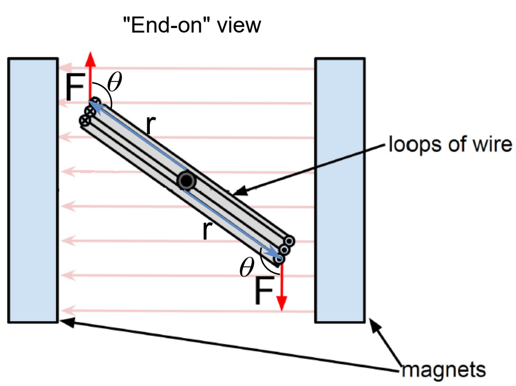

Figure 2: A schematic of how torque is produced in a DC motor as a result of the force acting on the coils due to the motor effect. An "end-on" view of the coils on the armature is shown. Current is flowing into the page in the wires at the top left, experiencing a force upwards, and out of the page in the wires at the bottom right, experiencing a force downwards. These two forces are applied at a distance r from the axis and so can produce a torque.

Figure 2 illustrates how torque is produced in a DC motor due to the motor effect. The torque acting on the coil is maximum when the angle between r and F are perpendicular, and zero when they are parallel (or anti-parallel). The force acting on a length l of the coils that are perpendicular to the field be expressed using $F = nBIl\sin\theta = nBIl$, where n is the number of windings. Note that the angle between the direction of the current and the magnetic field is *always* $90°$ in a DC motor, so $\sin\theta = 1$. The torque acting on the motor can then be expressed as

$$\tau = rF\sin\theta = 2nrBIl\sin\theta$$

where the factor of 2 appears as there is a force on the top left and bottom right lengths of wire. Note (very!) carefully that the θ in the equation for torque is the angle between F and r [1].

The factor $2rl$ in the above equation can be replaced by the area of the loop A, so that we obtain:

> Torque acting on a DC motor
>
> $$\tau = nIA_{\perp}B = nIAB\sin\theta$$

[1] Somewhat unfortunately, the same variable θ has been used in the syllabus for the angle between F and r in the equation for torque, and for the angle between I and B in the "motor effect" equation.

Example 1.

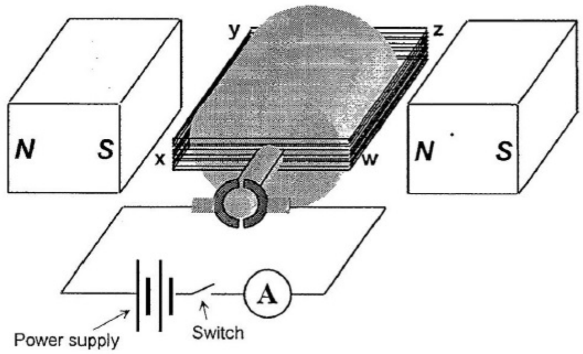

Figure 3: Image credit - 2006 STANSW trial paper Q6.

In the motor shown above, the dimensions of sides $XY = WZ =$ 3cm and sides $YZ = XW =$ 2cm, $n = 40$ turns and the magnetic field strength due to the magnets is $B =$ 25mT.

(a) Which side of the rotor will move upwards when the switch is closed, XY or WZ?

Using the right-hand rule, side XY

(b) A current of 2A flows through the ammeter when the switch is closed.

What is the maximum torque due to the motor effect which acts on the motor?

Back emf

Back emf if an effect that occurs in a DC motor when it is rotating. The change in magnetic flux through the turns of wire on the rotor produces an induced emf due to Faraday's law.

Back emf opposes the supply emf

As we discussed in part 3 of this module, we can reason about the direction of this induced emf in a number of ways.

Firstly, we can see that as the wire moves upward through the magnetic field, a motional-emf is generated, in the direction given by the right hand rule. Consider the motor shown in figure 3 on page 7. When the switch is closed and side XY begins to move upwards, it is now a conductor moving upwards through a magnetic field. Using the right-hand rule with our thumb pointing upwards (as this is the direction of motion of the charges in this conductor as it moves upwards), we see that there is a force on positive charges *towards* X, which is the opposite direction to the emf applied by the DC source. From this argument, we see that the "back emf" opposes the supply emf.

The second way we can reason is to use conservation of energy. If the induced emf was to appear in the *same* direction as the supply emf, then as the motor began to rotate, the overall emf would increase, producing a larger current, an associated increase in torque, an increase rotational mechanical energy, which in turn produces an even larger induced emf due to the larger rate of change of flux, *with no additional input of energy*. As this would violate conservation of energy, the induced emf must appear in a direction which opposes the supply emf.

Finally, we can reason using Lenz's law. Recall that Lenz's law merely tells us the direction of the induced emf that is consistent with conservation of energy, so it is not actually different to our previous line of reasoning. It simply gives us alternate language to use to phrase our argument. Lenz's law tells us that:

Consider again the motor in figure 3. At the position shown there is zero flux through the coils. As side XY moves upwards there is an increase in the flux passing through the coil from right to left. Lenz's law tells us that an induced emf appears in a direction so that the current it would induce would produce a magnetic field to oppose this change. Using the right-hand grip rule we can picture

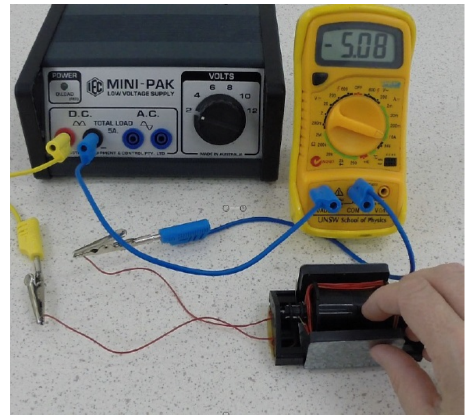

Figure 4: The current flowing through a small DC motor when the coils are held to prevent rotation can be very high - here it is 5A, enough to melt the insulation on the wire if held for more than a few moments.

Note that the negative value is not important here - it just indicates the direction of the current.

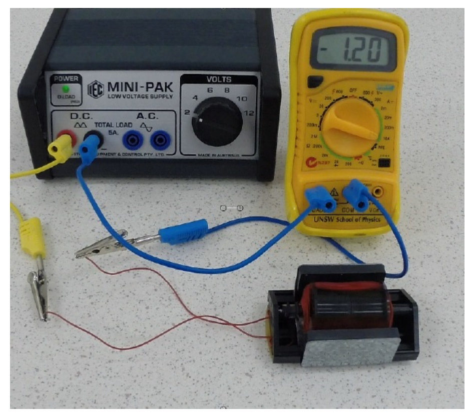

Figure 5: The current flowing in the same DC motor when it is allowed to rotate freely is much less (1.2A than when it is prevented from rotating - due to **back emf**.

holding the coil so that our fingers point left to right through the coil (to oppose the change in flux) and then our thumb points in the direction that the current due to this induced emf would flow in the opposite direction to the current produced by the external supply.

Again we have obtained the same answer, that the induced emf, which we will call **back emf** in this situation, appears in a direction which opposes the supply emf.

How current varies as a DC motor speeds up

When the motor is held stationary (i.e. there is no *net* torque)

$$I_{stat} = \frac{\varepsilon_{supply}}{R_{coils}}$$

where ε_{supply} is the supply voltage and R_{coils} is the resistance of the coil (the windings on the rotor). If the angular velocity of the motor increases, there is a change in the magnetic flux through the coil, inducing a _back emf $\varepsilon_{back} = -\frac{d\Phi}{dt}$ in the opposite direction to the supply emf.

The current flowing through the motor when it is rotating is

$$I_{rot} = \frac{\varepsilon_{supply} - \varepsilon_{back}}{R_{coils}}$$

which is *lower* than the current when the motor is held stationary.

Torque and back emf

The net torque acting on the motor is the difference between the torque due to the motor effect and any opposing torque due to friction or due an applied load.

The torque due to the motor effect is proportional to the current flowing, and is given by

$$\tau = nIAB\sin\theta$$

If the net torque is non-zero then the angular velocity of the motor increases, and so the current flowing through the coils *decreases* (due to increasing back emf). This means that the torque due to the motor effect decreases in proportion to the current.

As the motor rotates faster, the torque due to the motor effect decreases *until it exactly balances any opposing torque*. At this point the motor rotates at a constant angular velocity (as zero net torque produces zero angular acceleration).

If the opposing torque increases (for example, the motor is driving a drill bit which begins to drill a hole), then the net torque and angular acceleration is negative, slowing the rotation of the motor, so increasing the current and the torque due to the motor effect until the net torque is again zero and the motor again rotates with a constant (but slower) angular velocity.

Example 2.

(a) Sketch qualitatively the current flowing through the coils of a DC motor as a function of *time* as it is switched on and speeds up to eventually reach a constant angular velocity. Assume there is no friction or applied load. How would the graph differ for a real motor in which there is an opposing torque due to friction in the motor.

(b) Sketch qualitatively the current flowing through the coils of a DC motor as a function of *angular velocity*.

DC and AC generators

A generator is a device for converting rotational mechanical energy to electric energy. All generators share the same basic design components, consisting of

- a rotor and a stator
- coils and a magnetic field (which may be located on either the stator or the rotor)
- produce either direct current (DC) or alternating current (AC) electricity.

Generators operate on the principle that changing the magnetic flux through a coil induces an emf which drives a current around the loop according to Faraday's law.

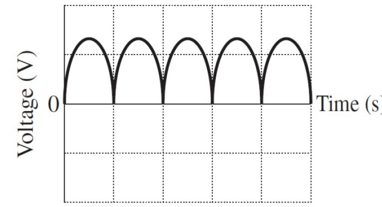

Figure 6: Voltage produced by a (single loop) DC generator

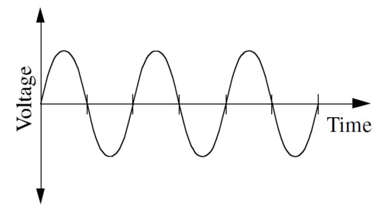

Figure 7: Voltage produced by an AC generator

DC generators

A DC generator has all the same components as a DC motor, except that the energy inputs and outputs are reversed, as shown in figure 8.

Current on the coil reverses direction every half cycle. You can think of this either in terms of Faraday's law (the flux through the loop varies as $\sin\theta$, and as $\varepsilon = -\frac{d\Phi}{dt}$, the induced emf changes sign) or in terms of the motional emf generated as the coils are alternately moving upwards and then downwards through the magnetic field so the direction of the emf reverses. The split-ring commutator reverses the connection between the external circuit and the coil every half-cycle as well, so that the current flows through the external circuit in a constant direction as the motor rotates, as shown in figure 6. Note (as exam questions regularly test this idea) that if the frequency of rotation of the motor increases, both the frequency *and* the amplitude of the induced voltage will increase, as the rate of change of magnetic flux will increase.

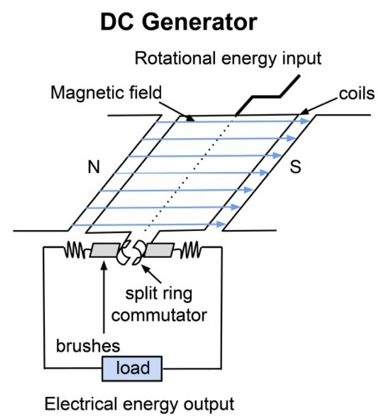

Figure 8: Line diagram of a simple DC generator.

AC generators

AC generators which have the magnet on the stator and the coils on the rotor utilise a "slip ring" commutator to transfer generated current off the rotor to an external circuit, as shown in figure 9. The slip-ring commutator maintains a constant connection from one side of the coil to one end of the external circuit, unlike the *split-ring commutator* on a DC generator. The direction of the current induced in the coil changes direction every half cycle (as discussed in the section on DC generators), so this means that the direction of the

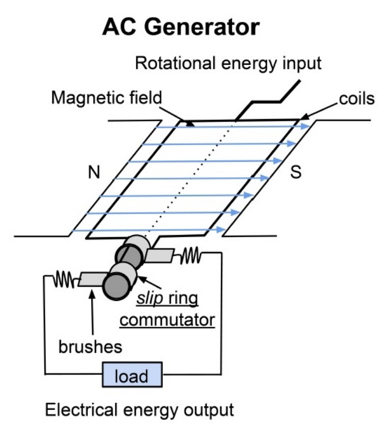

Figure 9: Line diagram of a simple (slip ring) AC generator.

current through the external circuit also reverses direction every half cycle, as shown in figure 7.

Back torque

In a motor, the torque produced by a current carrying coil is used to convert electrical energy to rotational mechanical energy. The emf induced when this coil begins to rotate in the field (the "back emf") opposes the supply emf.

In a generator, the analogous effect occurs when the induced emf due to the rotation of the coil in the magnetic field drives a current through the coils. The force exerted on this induced current by the magnetic field produces a "back torque" in a direction opposite to the direction that the generator is rotating, so that mechanical work must be done by the external agent to continue to rotate the generator coils.

AC motors

AC induction motors

In an AC induction motor stationary coils on the outside of the rotor produce a magnetic field which rotates. This is done by running AC current through two (or three) pairs of coils on the stator (see figure 11), with each coil out of phase with the other, so that the north and south pole progressively move around the rotor.

The rotor is made of multiple bars connected at the ends (see figure 15). The changing magnetic flux through the rotor induces currents due to Faraday's law, which in turn generate magnetic fields which cause the rotor to rotate in the same direction as the field (or, alternatively, you can speak of the induced current experiencing a force in the external magnetic field in a direction given by the right-hand rule. This force produces a torque on the motor causing rotation in the same direction as the rotating magnetic field. The rotor always lags behind the field (called 'slip') so that there is a continuous change in flux and a resulting torque.

An important advantage of AC motors is the absence of mechanical contact between the rotor and the stator - the motor is 'brushless'. This ensures reliability and means that it is often used in situations where maintenance would be difficult. A drawback is the difficulty in adjusting the speed of the motors.

Space to practice drawing an induction motor:

Figure 10: A deconstructed AC induction motor. The rotor, called a "squirrel cage" is on the left, and the stator coils, which produce the rotating magnetic field are on the right.

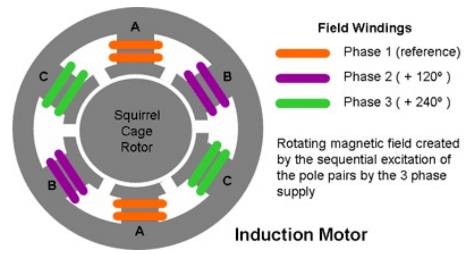

Figure 11: A diagram of an AC induction motor. Image credit: Barrie Lawson, "Electropaedia" (`www.mpoweruk.com`).

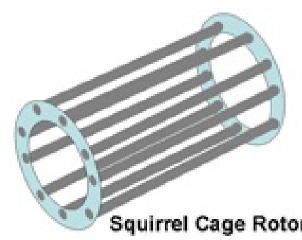

Figure 12: A squirrel cage rotor. Image credit: Barrie Lawson, "Electropaedia" (`www.mpoweruk.com`).

Universal motors

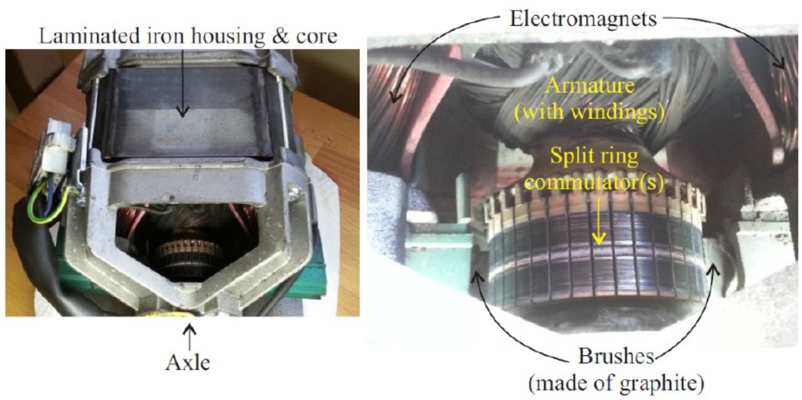

Figure 13: Labelled universal motor from a washing machine.

Universal motors have split ring commutators like DC motors however the magnetic field is created by running current through stator coils that are in series with the rotor. This means that if the motor is powered by an AC current, then every time the current changes direction, the direction of the magnetic field changes, allowing the motor to continue to rotate in the same direction.

From experience (in pulling apart broken appliances!) most motors in your home that need to be powerful (e.g. washing machines, blenders, vacuum cleaners) are *universal motors*. I have found *induction motors* in a fan and powering a microwave turntable.

Space for you to practice drawing a universal motor:

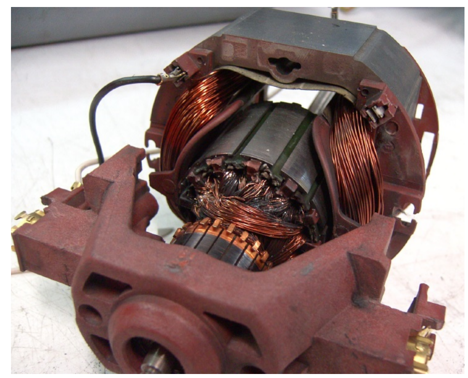

Figure 14: A "Universal" motor, that runs on either AC or DC electricity.

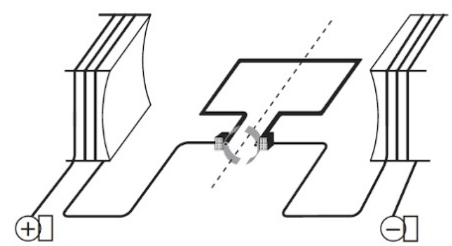

Figure 15: A line diagram of a universal motor (from 2010 HSC Q5)

Lenz's law and conservation of energy

DC Motors

We have covered this in our earlier discussion of back-emf.

Magnetic braking

We also covered magnetic braking in our earlier section on eddy currents, but will review it again here (as this is where it appears in the syllabus).

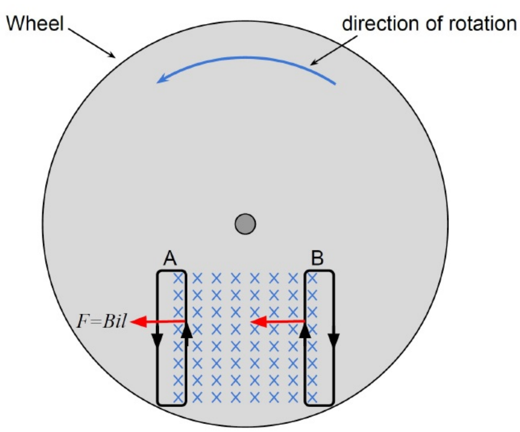

Figure 16: Diagram of the eddy currents (black loops) and forces that act on this currents due to the motor effect (red arrows).

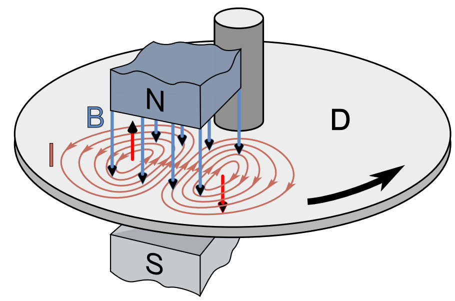

Figure 17: A three-dimensional view of the eddy currents created in EM braking. This diagram illustrated how the magnetic field produced by the eddy currents acts to oppose the rotation of the wheel. This is an alternative way to understand the braking effect (the explanation in the text emphasises the force on the eddy currents due to the "motor effect"). Figure credit: By Chetvorno - Own work, CC0, `https://commons.wikimedia.org/w/index.php?curid=40937881`

Eddy currents are utilised in electromagnetic braking as follows: Consider the part of the wheel (marked 'A') in figure 17 that is about to enter to the field. As the flux into the wheel in that area is increasing with time, an eddy current flows in an anticlockwise direction to oppose the change in magnetic flux (Lenz's law).

An eddy current flows in the opposite direction in the part of the wheel moving out of the field (marked B), as the flux into the wheel in area B is decreasing with time.

The right (upward moving) side of the eddy current A experiences a force F = Bil in the direction opposite to the rotation of the wheel (by the right hand rule), which exerts a torque on the wheel in the opposite direction to its rotation, slowing it down The left (upward moving) side of eddy current B similarly produces a torque in the opposite direction to the rotation of the wheel, also contributing to slowing it down.

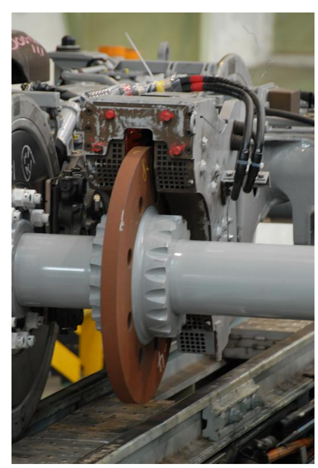

Figure 18: Disk eddy current brake on 700 Series Shinkansen, a Japanese bullet train. Image credit: By Take-y at the Japanese language Wikipedia, CC BY-SA 3.0, `https://commons.wikimedia.org/w/index.php?curid=5188379`

Conservation of energy considerations in EM braking

Another way to reason about EM braking is to note that if eddy currents are induced, then electrical energy is dissipated in the wheel due to I^2R heating. This energy *must* come from somewhere (so that energy is conserved). The kinetic energy of the wheel is transformed first into electrical energy and then into heat in the wheel, and this loss of kinetic energy slows the wheel down.

Answers

Worked Example 1.

(a) Using the right-hand rule, side XY will move upwards.
(b) $\tau = nBIA\sin\theta$ and when torque is maximum $\sin\theta = 1$, so

$$\tau_{max} = 40 \times 25 \times 10^{-3}\text{T} \times 2\text{A} \times (.02 \times 0.03)\text{m}^2 = 1.2 \times 10^{-3}\text{Nm}$$

Worked Example 2.

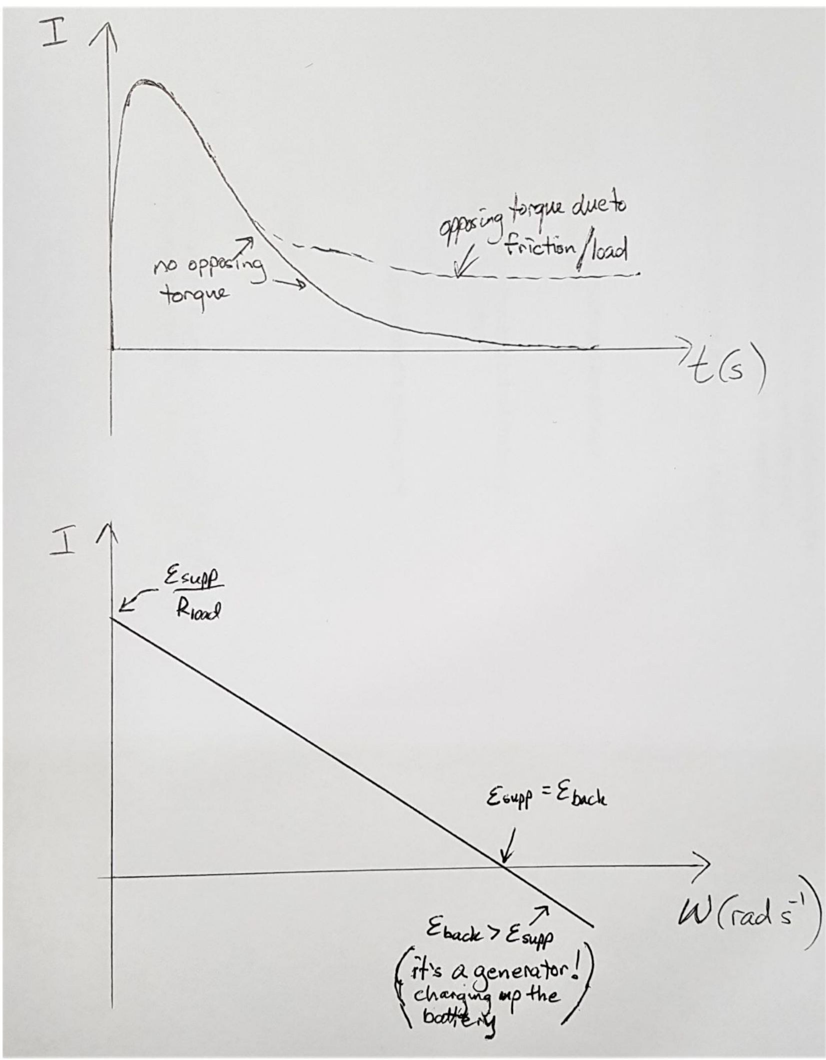

Figure 19: (Top) Current in a DC motor as it is switched on and then speeds up to a constant speed. Solid line: No opposing torque, so the net torque is zero only when the current is zero and so the back emf= supply emf. Dashed line: With an opposing torque the net torque is zero for $\varepsilon_{back} < \varepsilon_{supp}$, and so the current asymptotes to a constant value.

(Bottom) Current versus angular velocity. The current is linearly proportional to the back emf ($I_{rot} = \frac{\varepsilon_{supply} - \varepsilon_{back}}{R_{coils}}$), and the (maximum) back emf is proportional to the (maximum) rate of change of flux, which is proportional to the angular velocity.

Note that there is an angular velocity at which the back emf equals the supply emf, and so the current is zero - this would be the angular velocity of a motor with no opposing torque. The only way that the angular velocity could increase beyond this would be if energy was put in to the motor to actively spin it faster (i.e. it is run as a *generator*. In this case the current flows in the direction of the "back emf", and in the opposite direction to the supply emf, so "charging up" the supply emf.