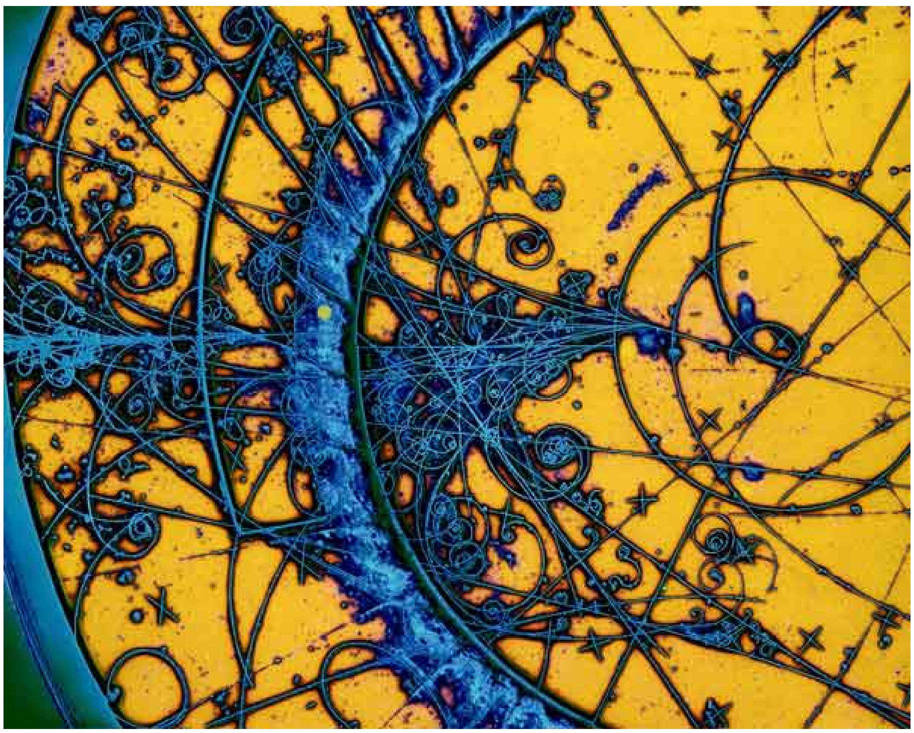
Module 6: Electromagnetism

Part 1: Charges in Electric and Magnetic fields



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Cover photo credit: This artistically enhanced image was produced by the Big European Bubble Chamber (BEBC), which started up at CERN in 1973. Charged particles in a magnetic field passing through a chamber filled with hydrogen-neon liquid leave bubbles along their paths. `https://home.cern/about/updates/2015/06/seeing-invisible-event-displays-particle-physics`

Syllabus content: Electromagnetism

Charged particles in Electric and Magnetic fields

Inquiry question: What happens to stationary and moving charged particles when they interact with an electric or magnetic field?

Students:

- investigate and quantitatively derive and analyse the interaction between charged particles and uniform electric fields, including:
  - a electric field between parallel charged plates, $E = \frac{V}{d}$
  - acceleration of charged particles by the electric field $\vec{F}_{net} = m\vec{a}$, $\vec{F} = q\vec{E}$
  - work done on the charge, $W = qV$, $W = qEd$, $K = \frac{1}{2}mv^2$
- model qualitatively and quantitatively the trajectories of charged particles in electric fields and compare them with the trajectories of projectiles in a gravitational field
- analyse the interaction between charged particles and uniform magnetic fields, including:
  - acceleration, perpendicular to the field, of charged particles
  - the force on the charge $F = qv \perp B = qvB \sin\theta$
- compare the interaction of charged particles moving in magnetic fields to:
  - the interaction of charged particles with electric fields
  - other examples of uniform circular motion

Overview of electromagnetism

A Qualitative summary (emphasising the symmetry of EM)

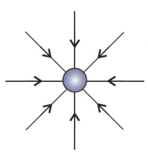
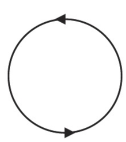
- Charges produce electric fields, and electric fields exert forces on charges.
- Moving charges produce magnetic fields and magnetic fields exert forces on moving charges.
- Changing magnetic fields produce electric fields and changing electric fields produce magnetic fields.

$$\oiint E \cdot dA = \frac{q}{\epsilon_0}$$
$$\oiint B \cdot dA = 0$$
$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$
$$\oint B \cdot ds = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Figure 1: (Left) James Clerk Maxwell and (right) his equations of electromagnetism (in integral form) - just so you've seen them!.

An alternate qualitative summary (aligned with Maxwell's 4 equations)

The difference between the previous summary and the summary below is that here the emphasis is on describing the content of each of Maxwell's four equations in terms of how charges make electric and magnetic fields. The equations (called the Lorentz force law) which describe how these electric and magnetic fields exert forces on charges are given separately below.

Type of field			How it is made
Fields with divergence		**Electric fields**	**1. Electric charges**
		Magnetic fields	**2. Do not exist** (There are no magnetic charges/monopoles)
Fields with circulation		**Electric fields**	**3. Changing magnetic fields**
		Magnetic fields	**4. Changing electric fields & moving electric charges**

Force laws: $F = qE$ & $F = qvB\sin\theta$

Figure 2: Qualitative summary of Maxwell's equations (with the Lorentz force laws below)

In this part of Module 6, our focus is on the two force laws (together known as the **Lorentz force**), that govern the force that acts on a charge in an electric or magnetic field, $\vec{F} = q\vec{E}$ and $F = qvB\sin\theta$, respectively.

Charged particles in electric fields

Revision of the work-energy theorem

Work is the mechanism by which energy is transferred to or from an object. In module 1 we derived the work-energy theorem from Newton's 2nd law for the case of a constant force as:

$$\Delta KE = W_{net} = |\Sigma \vec{F}||\vec{d}|\cos\theta$$

where d is the displacement, and θ the angle between the direction of the net force and the displacement.

The electric field between parallel charged plates

In module 4 we used superposition to argue that the electric field between parallel charged plates is uniform.

Space for revising this:

The work done on a charge by a uniform electric field

The force on a charge in an electric field is given by $\vec{F} = q\vec{E}$.

If the electric field is **uniform**, such as the electric field between two parallel charged plates, then the work done by the electric field when the charge undergoes a displacement is

$$W = |\vec{F}||\vec{d}|\cos\theta = q|\vec{E}||\vec{d}|\cos\theta$$

where θ is the angle between the direction of the electric field and the displacement. If $\cos\theta > 0$, then the field has done positive work on positive charges (meaning the charge gains kinetic energy) and negative work if the charge is negative (the charge loses kinetic energy). If $\cos\theta < 0$, then the converse is true.

The voltage between parallel charged plates

In module 4 we defined the change in electric potential energy as the negative of the work done by an electric field on a charge.

$$\Delta U = -W = -q|\vec{E}||\vec{d}|\cos\theta$$

We also defined the change in electrical *potential* as the change in electrical potential energy *per unit charge*. That is

$$V = \frac{\Delta U}{q} = -|\vec{E}||\vec{d}|\cos\theta$$

This means that if a positive charge undergoes a displacement in the *opposite* direction to the electric field, then its electrical potential energy increases, and we would say that it is now at a higher *voltage* (or a higher electrical *potential*, this is exactly the same thing) than it was before.

It is worth spending time to convince yourself that this makes sense.

Parallel plates: Relating the magnitude of the voltage, E-field and separation of the plates

In the syllabus, the **magnitude** of the potential difference between two charged plates is defined as:

$$E = \frac{V}{d} \tag{1}$$

where in this new context (where we are only using in the magnitude of E and V), d is simply the *distance* or separation of the plates (not the displacement of the charge).

Most generally, the electric field is the spatial derivative of the potential. If the potential is the same at two positions, then there is no electric field with a component in the direction along a line joining those two positions.

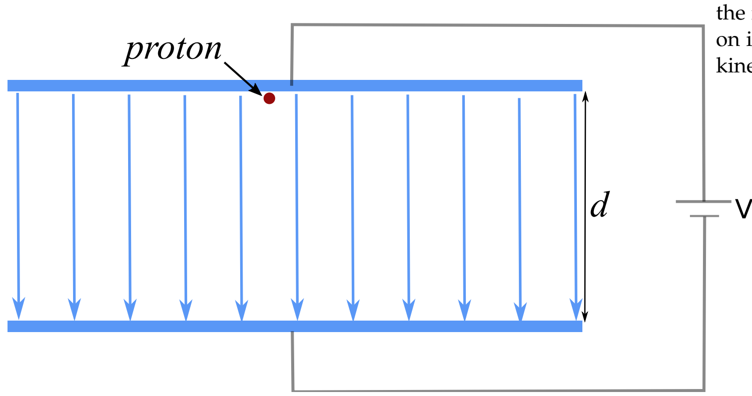


Figure 3: A proton released from rest at the negative plate will have work done on it by the electric field to change its kinetic energy

Example 1. Figure 3 shows a proton positioned at the positive plate of a pair of parallel plates. A voltage of $V = 1500\text{V}$ is applied between the plates, with are separated by a distance of 5mm. The mass of a proton is $m_p = 1.67 \times 10^{-27}\text{kg}$ and the charge on a proton is $q = 1.60 \times 10^{-19}\text{C}$.

The proton is released from rest. How fast is it going when it collides with the negative plate?

Charges in uniform electric fields move in a parabola

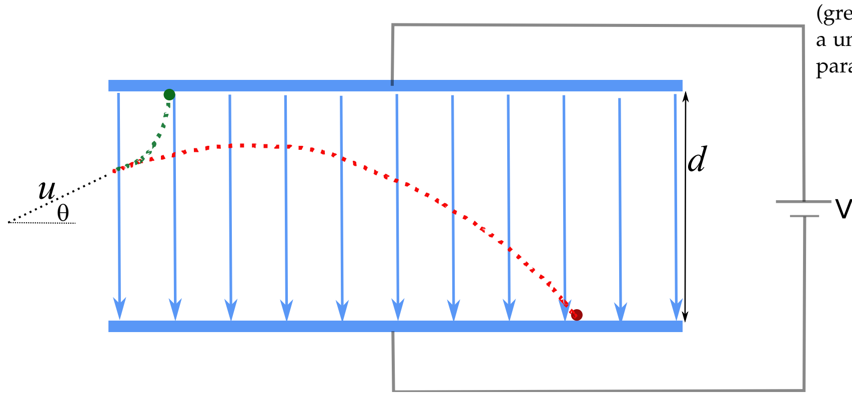


Figure 4: A positive (red) and negative (green) charge fired at an angle into a uniform electric field move in a parabolic path.

So far, we have focused on discussing the motion of charges in an electric field in terms of energy: work, electrical potential energy and kinetic energy. In this section we will use an approach based on force and acceleration.

In module 5, we discussed motion under constant acceleration, and determined that the trajectory of a particle subject to a constant force would be parabolic. We can apply this understanding here to the motion of charges subject to a constant force, and so a constant acceleration, in a uniform electric field.

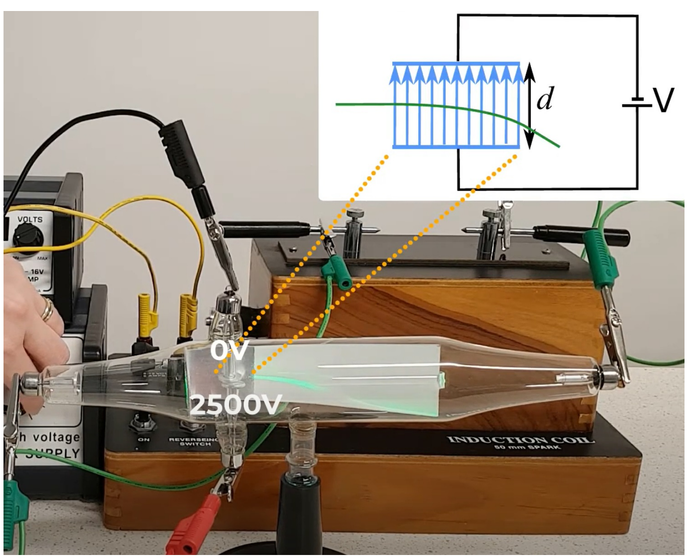


Figure 5: Electrons in a cathode ray tube travel with an initially horizontal velocity equal to almost 0.2 times the speed of light. Here, the electrons in the beam are deflected in a parabolic path by parallel plates with a voltage difference of 2500V. The path of the electrons is visible due to a fluorescent screen (see https://www.youtube.com/watch?v=dNouwkpNrqw)

Example 2.

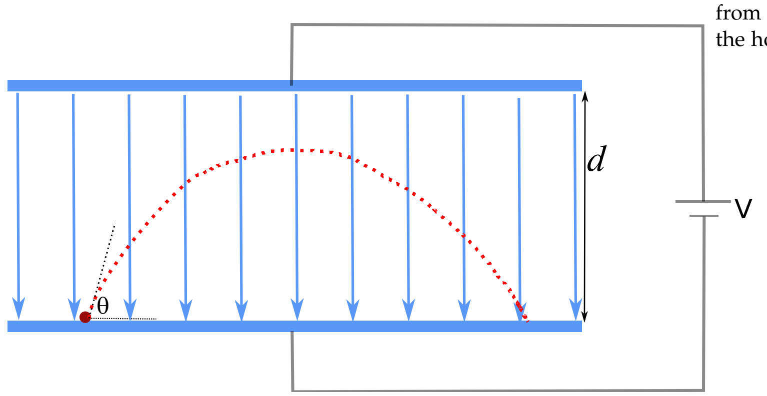


Figure 6: An proton is fired upwards from the negative plate at an angle to the horizontal of θ

A proton (mass $m_p = 1.27 \times 10^{-27}$kg) is fired upwards from the negative plate of a pair of parallel charged plates with an initial velocity of $u = 2.0 \times 10^5 \text{ms}^{-1}$ at an angle of $60°$ above the horizontal. The voltage on the plates is $V = 3000$V and the separation of the plates is 6cm.

If the parallel plates are 1cm in length, will the proton will hit the negative plate again? (i.e. determine if the range if larger or smaller than 1cm).

Charges in magnetic fields

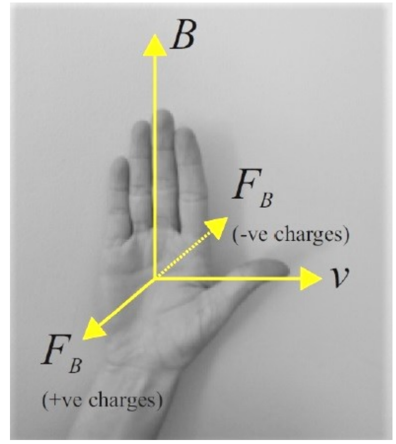


Figure 7: The right hand rule for the force on a charge moving in a magnetic field.

Experimentally, it is found that the force acting on a charge moving in a magnetic field is given by

$$F = qvB\sin\theta \tag{2}$$

where F is the magnitude of the force, q is the charge, v is the magnitude of the velocity of the charge, B is the magnitude of the magnetic field, and θ is the angle between the direction of the velocity and the magnetic field.

Figure 7 illustrates the "Right hand rule" which can be used to work out the direction of the force on a charge due to a magnetic field. You take your right hand and point your fingers in the direction of the magnetic field (picture them a bit wiggly, like field lines...). Your thumb points in the direction of the component of the velocity perpendicular to the field. Your palm then points in the direction of the force on a positive charge (the back of your hand points in the direction of the force on a negative charge).

Remember this as **P**alm **P**ositive.

Note that if the charge is moving parallel to the field, then there is no component of its motion perpendicular to the field and so no force.

An aside

Note that this is very a peculiar force. It acts perpendicular to the direction of the field, and, most disturbingly, it depends on the *velocity* of the charge.

We have already discussed *relativity* (or more precisely, *Galilean relativity*), which contends that it should not be possible to detect absolute motion. Yet, here we have a force which appears to depend on the reference frame of the observer! In module 7 (time permitting) we will revisit this issue when we discuss Einstein's theory of special relativity [1].

[1] Spoiler: if you change reference frames so that the charge has a different velocity a force due to an electric field appears which exactly balances any change in the force due to the magnetic field... ...the universe is truly an amazing place!

The trajectory of a charge in a magnetic field

As the force on a charged particle in a magnetic field always acts perpendicular to its velocity, this force acts a **centripetal force**, causing the charge to undergo uniform circular motion. The circular motion is *uniform* as the magnetic field can never change the speed of the particle, unlike an electric field, as there is no component of the force in the direction of motion.

$$\Sigma|\vec{F}| = m|\vec{a}|$$

From our work on uniform circular motion in module 5, we know that the magnitude of centripetal acceleration is

$$a_c = \frac{v^2}{r}$$

We need to be careful to note that in this context v is taken as the component of the velocity perpendicular to the field (i.e. $|\vec{v}|\sin\theta$ in our previous equation for magnetic force becomes v here).

We can then write down a relationship between the magnitude of the force due to the magnetic field, the component of the velocity perpendicular to the magnetic field and the radius of the circular path taken by the charge as:

$$qvB = m\frac{v^2}{r}$$

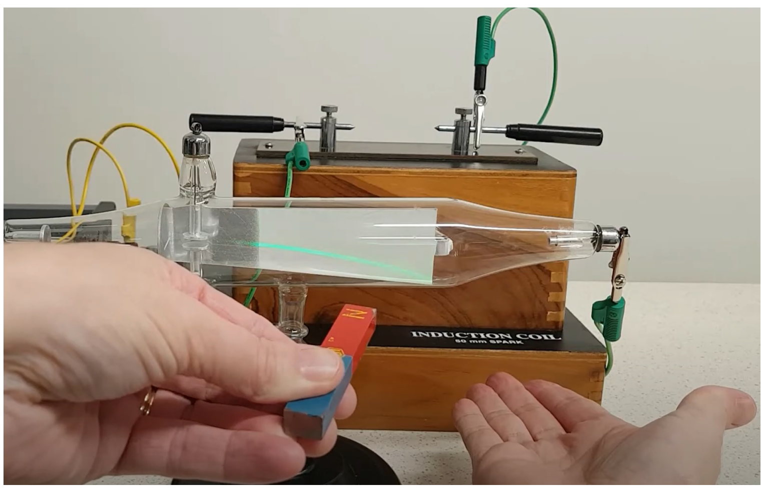


Figure 8: A beam of electrons in a cathode ray tube are deflected by a magnetic field in the direction of the right hand rule.

Example 3.

Electrons in a cathode ray tube travel at $0.2c$ (that is, 0.2 times the speed of light) (see `https://www.youtube.com/watch?v=9yF-7I-ThbI`). Calculate their radius of curvature if they are subjected to a magnetic field of 1mT in a direction perpendicular to their velocity.

Worked answers

Worked Example 1. Figure 3 shows a proton positioned at the positive plate of a pair of parallel plates. A voltage of $V = 1500\text{V}$ is applied between the plates, with are separated by a distance of 5mm. The mass of a proton is $m_p = 1.67 \times 10^{-27}\text{kg}$ and the charge on a proton is $q = 1.60 \times 10^{-19}\text{C}$.

The proton is released from rest. How fast is it going when it collides with the negative plate?

The best way to do this question is to conserve mechanical energy, and say that the change in the electrical potential energy is equal and opposite to the change in kinetic energy.

$$\Delta KE + \Delta U = 0$$

so

$$\frac{1}{2}mv^2 = -q\Delta V$$

In this case the change in electric potential is negative (as the displacement is in the same direction as the electric field, work done by the field is positive), so the negative signs cancel out and

$$v = \sqrt{\left(\frac{2qV}{m}\right)} = \sqrt{\left(\frac{2 \times 1.60 \times 10^{-19}\text{C} \times 1500\text{V}}{1.67 \times 10^{-27}\text{kg}}\right)} = 5.4 \times 10^5 \text{ms}^{-1}$$

Worked Example 2.

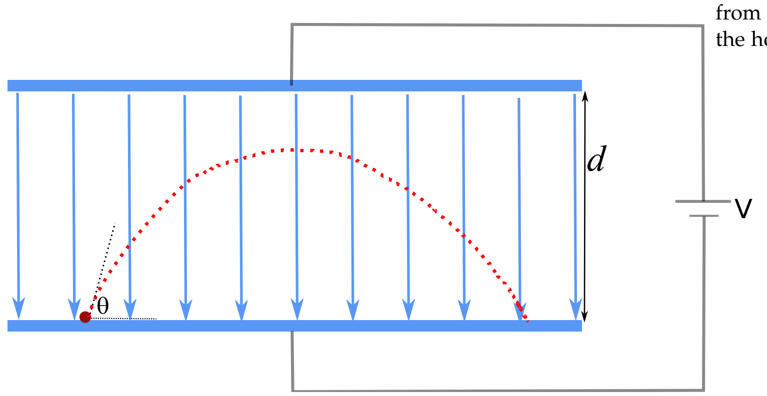


Figure 9: An electron is fired upwards from the negative plate at an angle to the horizontal of θ

A proton (mass $m_p = 1.67 \times 10^{-27}\text{kg}$) is fired upwards from the negative plate of a pair of parallel charged plates with an initial velocity of $u = 2.0 \times 10^5 \text{ms}^{-1}$ at an angle of 60° above the horizontal. The voltage on the plates is $V = 3000\text{V}$ and the separation of the plates is 6cm.

If the parallel plates are 1cm in length, determine if the proton will hit the negative plates again (i.e. determine if the range if larger or smaller than 1cm.

The x- and y- components of the initial velocity are given by:
$u_x = 2.0 \times 10^5 \text{ms}^{-1} \cos(60°) = 1.0 \times 10^5 \text{ms}^{-1}$
$u_y = 2.0 \times 10^5 \text{ms}^{-1} \sin(60°) = 1.73 \times 10^5 \text{ms}^{-1}$
The electric field between the parallel plates is given by:

$$E = \frac{V}{d} = \frac{3000\text{V}}{6 \times 10^{-2}\text{m}} = 5.0 \times 10^4 \text{Vm}^{-1}$$

The force acting on the proton is

$$F = qE = 1.60 \times 10^{-19}\text{C} \times 5.0 \times 10^4 \text{Vm}^{-1} = 8.0 \times 10^{-15}\text{N}$$

and accelerates it towards the negative plate.

$$a = \frac{\Sigma F}{m} = \frac{8.0 \times 10^{-15}\text{N}}{1.27 \times 10^{-27}\text{kg}} = 6.3 \times 10^{12} \text{ms}^{-2}$$

and We now have enough information to calculate the time of flight for each, using

$$\Delta y = u_y t + \frac{1}{2} a t^2$$

Here $\Delta y = 0$, so

$$t = \frac{-2u_y}{a} = \frac{-2 \times 1.73 \times 10^5 \text{ms}^{-1}}{-4.79 \times 10^{12} \text{ms}^{-2}} = 2.70^{-5}\text{s} = 7.2 \times 10^{-8}\text{s}$$

So the horizontal displacement when it hit the negative plate again would be

$$\Delta x = u_x t = 1.0 \times 10^5 \text{ms}^{-1} \times 7.2 \times 10^{-8}\text{s} = 5.5 \times 10^{-3}$$

So the proton would hit the negative plate.

Worked Example 3. Using Newton's 2nd law for the electron travelling in the arc of a circle,

$$qvB = m\frac{v^2}{r}$$

we have

$$r = \frac{v}{qB} = \frac{9.11 \times 10^{-31} \times 0.2 \times 3 \times 10^8}{1.6 \times 10^{-19} \times 1 \times 10^{-3}} = 0.34\text{m}$$