EXERCISE 1.1 SIMPLIFYING ALGEBRAIC EXPRESSIONS

2 $7x-3+3x-2=7x+3x-3-2$

$=10x-5$

4 $6ab+3ab+5a+4a=9ab+9a$

6 $3a^2b$ and $2a^2b$ are like terms and can be added, but $3ab^2$ is not.

$3a^2b-3ab^2+2a^2b=5a^2b-3ab^2$

8 $3abc+5bca-2cba=3abc+5abc-2abc$

$=6abc$

10 $x^2-3x+2x+4x^2=x^2+4x^2-3x+2x$

$=5x^2-x$

12 $5a-3(a+b)=5a-3a-3b$

$=2a-3b$

14 $8m-5(2m-3n)=8m-10m+15n$

$=15n-2m$

16 $5(2x+3)-5(x+7)=10x+15-5x-35$

$=5x-20$

18 $5a(a+2)-3a(a+1)=5a^2+10a-3a^2-3a$

$=2a^2+7a$

20 $2a+3b-(a-b)=2a+3b-a+b$

$=a+4b$

22 $5x(2x+1)-(x^2+x)=10x^2+5x-x^2-x$

$=9x^2+4x$

24 $3x(x-2)-4(x-1)=3x^2-6x-4x+4$

$=3x^2-10x+4$

26 $5x+2y-3-(x-7y+9)=5x+2y-3-x+7y-9$

$=4x+9y-12$

28 B

$3(m^2-m)-2(m^2+2m+5)=3m^2-3m-2m^2-4m-10$

$=m^2-7m-10$

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EXERCISE 1.2 SUBSTITUTION IN FORMULAE

2 $E = 2.4 \times 40$

$= 96$

4 **(a)** $F = \dfrac{9 \times 60}{5} + 32$

$= 140$

(b) $41 = \dfrac{9 \times C}{5} + 32$

$41 - 32 = \dfrac{9 \times C}{5}$

$\dfrac{9 \times C}{5} = 9$

$9 \times C = 9 \times 5$

$C = 5$

6 $V = \pi \times 4.2^2 \times 10$

$= 554.2$

8 $v = 20 + 1.8 \times 10$

$= 38$

10 $v^2 = 12^2 + 2 \times 2 \times 20.25$

$v^2 = 225$

$v = \pm\sqrt{225}$

$v = \pm 15$

12 $S = 2 \times \pi \times 2.5 \times 3.5$

$= 17.5\pi$

≈ 55.0

14 $E = \dfrac{4}{2}\left(4^2 - 2^2\right)$

$= 24$

16 $F = \dfrac{20\left(4 - 2\right)}{6}$

$= \dfrac{20}{3}$

$= 6.7$

18 $S = \dfrac{5\left(3^3 - 1\right)}{3 - 1}$

$= 65$

20 $V = \pi \times \left(0.9^2 - 0.2^2\right) \times 1.5$

$= 3.6$

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22 (a) $P=\sqrt{\dfrac{2\times 50-20}{5}}$

$=\sqrt{16}$
$=4$

22 (b) $0.2=\sqrt{\dfrac{2\times 20-V}{5}}$

$$\begin{aligned}0.2^2&=\frac{40-V}{5}\\40-V&=0.2\\V&=39.8\end{aligned}$$

24 (a) $f=\dfrac{20\times 25}{20+25}$

$$\begin{aligned}&=\frac{20\times 25}{45}\\&=\frac{100}{9}\quad\text{(after cancelling)}\\&=11.1\end{aligned}$$

24 (b) $20=\dfrac{v\times 25}{v+25}$

$$\begin{aligned}20(v+25)&=v\times 25\\20v+500&=25v\\25v-20v&=500\\5v&=500\\v&=100\end{aligned}$$

EXERCISE 1.3 BASIC POLYNOMIALS

2 $(x-2)(x-3)=x^2-2x-3x+6$

$=x^2-5x+6$

4 $(x-2)^2=x^2-2\times 2x+2^2$

$=x^2-4x+4$

6 $(2x+3)(x+5)=2x^2+10x+3x+15$

$=2x^2+13x+15$

8 $(3m+7)(2m-1)=6m^2+14m-3m-7$

$=6m^2+11m-7$

10 $(2p-9)(2p+9)=(2p)^2-9^2$

$=4p^2-81$

12 $(4p-5)^2=(4p)^2-2\times 4p\times 5+5^2$

$=16p^2-40p+25$

14 $(x-3)(2x^2+3x+1)=2x^3+3x^2+x-6x^2-9x-3$

$=2x^3-3x^2-8x-3$

16 $x(x-2)(x+2)=x(x^2-4)$

$=x^3-4x$

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18 $2(x-1)(x-2)(x-3)=2(x^2-x-2x+2)(x-3)$

$$\begin{aligned}&=2(x^2-3x+2)(x-3)\\&=2(x^3-3x^2-3x^2+9x+2x-6)\\&=2(x^3-6x^2+11x-6)\\&=2x^3-12x^2+22x-12\end{aligned}$$

20 $(x-2)(x+2)(x+2)=(x^2-4)(x+2)$

$$=x^3+2x^2-4x-8$$

22 C

$$\begin{aligned}\left(\sqrt{x}+\sqrt{y}\right)^2&=\left(\sqrt{x}\right)^2+2\sqrt{xy}+\left(\sqrt{y}\right)^2\\&=x+2\sqrt{xy}+y\\&=x+y+2\sqrt{xy}\end{aligned}$$

EXERCISE 1.4 FACTORISING BY GROUPING IN PAIRS

2 The common factor is $(2b-3c)$.

$3a(2b-3c)-m(2b-3c)=(3a-m)(2b-3c)$

4 The common factor is $(2x-1)$.

$x^2(2x-1)+4(2x-1)=(x^2+4)(2x-1)$

6 $x^2-xy+xz-yz=x(x-y)+z(x-y)$

$$=(x+z)(x-y)$$

8 $a^2-ab-ac+bc=a(a-b)-c(a-b)$

$$=(a-c)(a-b)$$

10 $a^3+3a^2b+ab+3b^2=a^2(a+3b)+b(a+3b)$

$$=(a+3b)(a^2+b)$$

12 $3xy-6y+7x-14=3y(x-2)+7(x-2)$

$$=(3y+7)(x-2)$$

14 $a^3-a^2b-ab+b^2=a^2(a-b)-b(a-b)$

$$=(a^2-b)(a-b)$$

16 $x^3+3x^2+4x+12=x^2(x+3)+4(x+3)$

$$=(x^2+4)(x+3)$$

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18 $m^2p+m^2+np+n=m^2(p+1)+n(p+1)$

$$=(m^2+n)(p+1)$$

20 $ab-3a-4b+12=a(b-3)-4(b-3)$

$$=(a-4)(b-3)$$

22 **A**

$$3m^2-3mn-m+n=3m(m-n)-(m-n)$$
$$=(3m-1)(m-n)$$

EXERCISE 1.5 STANDARD FACTORISATIONS

2 $x^2-16=x^2-4^2$

$$=(x-4)(x+4)$$

4 $9a^2-25=(3a)^2-5^2$

$$=(3a-5)(3a+5)$$

6 $a^2b^2-c^2=(ab)^2-c^2$

$$=(ab-c)(ab+c)$$

8 $(x+1)^2-9=(x+1)^2-3^2$

$$=((x+1)-3)((x+1)+3)$$
$$=(x+1-3)(x+1+3)$$
$$=(x-2)(x+4)$$

18 $12a^3-3ab^2=3a(4a^2-b^2)$

$$=3a((2a)^2-b^2)$$
$$=3a(2a-b)(2a+b)$$

10 $\frac{a^2}{25}-1=\left(\frac{a}{5}\right)^2-1^2$

$$=\left(\frac{a}{5}-1\right)\left(\frac{a}{5}+1\right)$$

12 $\frac{x^2}{4}-\frac{1}{9}=\left(\frac{x}{2}\right)^2-\left(\frac{1}{3}\right)^2$

$$=\left(\frac{x}{2}-\frac{1}{3}\right)\left(\frac{x}{2}+\frac{1}{3}\right)$$

14 $x^2-(y+z)^2=[x-(y+z)][x+(y+z)]$

$$=(x-y-z)(x+y+z)$$

16 $523^2-477^2=(523-477)(523+477)$

$$=46\times1000$$
$$=46\ 000$$

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20 $$\begin{aligned}(x+y)^2-4&=(x+y)^2-2^2\\&=(x+y-2)(x+y+2)\end{aligned}$$

22 $$\begin{aligned}x^3-x^2y-9x+9y&=x^2(x-y)-9(x-y)\\&=(x^2-9)(x-y)\\&=(x-3)(x+3)(x-y)\end{aligned}$$

24 $$\begin{aligned}p^2q-p^2-16q+16&=p^2(q-1)-16(q-1)\\&=(p^2-16)(q-1)\\&=(p-4)(p+4)(q-1)\end{aligned}$$

26 $$\begin{aligned}48a^2-75b^2&=3(16a^2-25b^2)\\&=3\left((4a)^2-(5b)^2\right)\\&=3(4a-5b)(4a+5b)\end{aligned}$$

28 $$\begin{aligned}\frac{x^2}{25}-y&=\left(\frac{x}{5}\right)^2-y^2\\&=\left(\frac{x}{5}-y\right)\left(\frac{x}{5}+y\right)\end{aligned}$$

30 $$\begin{aligned}\frac{a^2}{b^2}-\frac{b^2}{a^2}&=\left(\frac{a}{b}\right)^2-\left(\frac{b}{a}\right)^2\\&=\left(\frac{a}{b}-\frac{b}{a}\right)\left(\frac{a}{b}+\frac{b}{a}\right)\\&=\left(\frac{a^2-b^2}{ab}\right)\left(\frac{a^2+b^2}{ab}\right)\\&=\frac{(a-b)(a+b)(a^2+b^2)}{a^2b^2}\end{aligned}$$

(a) correct

(b) correct

(c) incorrect

(d) correct

32 $$\begin{aligned}z^3+1&=z^3+1^3\\&=(z+1)(z^2-z+1)\end{aligned}$$

34 $$\begin{aligned}216-a^3&=6^3-a^3\\&=(6-a)(36+6a+a^2)\end{aligned}$$

36 $$\begin{aligned}(2x+3)^3-(x-4)^3&=\left[(2x+3)-(x-4)\right]\left[(2x+3)^2+(2x+3)(x-4)+(x-4)^2\right]\\&=\left[2x+3-x+4\right]\left[(4x^2+12x+9)+(2x^2-8x+3x-12)+(x^2-8x+16)\right]\\&=(x+7)(4x^2+12x+9+2x^2-8x+3x-12+x^2-8x+16)\\&=(x+7)(7x^2-x+13)\end{aligned}$$

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38 $64a^3+8b^3=8\left(8a^3+b^3\right)$

$$=8\left((2a)^3+b^3\right)$$
$$=8(2a+b)\left(4a^2-2ab+b^2\right)$$

42 $\frac{8}{a^3}-\frac{27}{b^3}=\left(\frac{2}{a}\right)^3-\left(\frac{3}{b}\right)^3$

$$=\left(\frac{2}{a}-\frac{3}{b}\right)\left(\frac{4}{a^2}+\frac{6}{ab}+\frac{9}{b^2}\right)$$

40 $p^7x^4-p^4x^7=p^4x^4\left(p^3-x^3\right)$

$$=p^4x^4(p-x)\left(p^2+px+x^2\right)$$

44 $4x^5-9x^3-4x^2+9=x^3\left(4x^2-9\right)-\left(4x^2-9\right)$

$$=\left(4x^2-9\right)\left(x^3-1\right)$$
$$=\left((2x)^2-3^2\right)\left(x^3-1^3\right)$$
$$=(2x-3)(2x+3)(x-1)\left(x^2+x+1\right)$$

46 $a^3+(a-b)^3=(a+(a-b))\left[a^2-a(a-b)+(a-b)^2\right]$

$$=(2a-b)\left(a^2-a^2+ab+a^2-2ab+b^2\right)$$
$$=(2a-b)\left(a^2-ab+b^2\right)$$

48 $(2x+1)^3-(2x-1)^3=\left[(2x+1)-(2x-1)\right]\left[(2x+1)^2+(2x+1)(2x-1)+(2x-1)^2\right]$

$$=(2x+1-2x+1)\left(4x^2+4x+1+4x^2-1+4x^2-4x+1\right)$$
$$=2\left(4x^2+4x+1+4x^2-1+4x^2-4x+1\right)$$
$$=2\left(12x^2+1\right)$$

50 $a^5b^4-a^2b=a^2b\left(a^3b^3-1\right)$

$$=a^2b\left((ab)^3-1^3\right)$$
$$=a^2b(ab-1)\left(a^2b^2+ab+1\right)$$

52 B

$1000p^3-q^6=(10p)^3-\left(q^2\right)^3$

$$=\left(10p-q^2\right)\left(100p^2+10pq^2+q^4\right)$$

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EXERCISE 1.6 FACTORISING QUADRATIC TRINOMIALS

2 Two numbers which add to $+10$ and multiply to make $+21$ are $+3$ and $+7$.

$x^2+10x+21=(x+3)(x+7)$

4 Two numbers which add to $+12$ and multiply to make $+32$ are $+4$ and $+8$.

$a^2+12a+32=(a+4)(a+8)$

6 Two numbers which add to $+13$ and multiply to make $+12$ are $+1$ and $+12$.

$x^2+13x+12=(x+1)(x+12)$

8 Two numbers which add to -7 and multiply to make $+12$ are -3 and -4.

$x^2-7x+12=(x-3)(x-4)$

10 Two numbers which add to -8 and multiply to make $+12$ are -2 and -6.

$x^2-8x+12=(x-2)(x-6)$

12 Two numbers which add to $+14$ and multiply to make -15 are -1 and $+15$.

$p^2+14p-15=(p+15)(p-1)$

14 Two numbers which add to -14 and multiply to make -15 are -15 and $+1$.

$p^2-14p-15=(p-15)(p+1)$

16 Two numbers which add to -3 and multiply to make -10 are -5 and $+2$.

$x^2-3x-10=(x-5)(x+2)$

18 Two numbers which add to -4 and multiply to make -12 are -6 and $+2$.

$a^2-4a-12=(a-6)(a+2)$

20 Two numbers which add to -1 (the coefficient of x) and multiply to make -72 are -9 and $+8$.

$x^2-x-72=(x-9)(x+8)$

22 Two numbers which add to -21 and multiply to make -72 are -24 and $+3$.

$x^2-21x-72=(x-24)(x+3)$

24 Two numbers which add to -1 (the coefficient of x) and multiply to make -42 are -7 and $+6$.

$x^2-x-42=(x-7)(x+6)$

26 Two numbers which add to -19 and multiply to make -42 are $+21$ and -2.

$x^2+19x-42=(x+21)(x-2)$

28 Two numbers which add to -8 and multiply to make $+7$ are -1 and -7.

$x^2-8x+7=(x-1)(x-7)$

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Since $-1\times-1=1$, if we multiply both factors by -1, we are multiplying by 1, leaving the result unchanged.

$$\begin{aligned}x^2-8x+7&=(x-1)(x-7)\times1\\&=(x-1)(x-7)\times-1\times-1\\&=-1(x-1)\times-1(x-7)\\&=(-x+1)(-x+7)\\&=(1-x)(7-x)\end{aligned}$$

(a) incorrect

(b) correct

(c) incorrect

(d) correct

EXERCISE 1.7 FACTORISING NON-MONIC TRINOMIALS

2 The factors of $3x^2$ are $3x$ and x.

The factors of -4 are -4 and 1; 4 and -1; or -2 and 2.

Set up the cross method.

$$\begin{array}{cccccccc}3x & \diagdown\!\!\!\diagup & -4 & 1 & 4 & -1 & -2 & 2\\ x & & 1 & -4 & -1 & 4 & 2 & -2\end{array}$$

Calculate the sums, looking for a sum of $11x$.

$3x\times-4+x\times1=-12x+x=-11x$

$3x\times1+x\times-4=3x-4x=-x$

$3x\times4+x\times-1=12x-x=11x$

$3x\times-1+x\times4=-3x+4x=x$

$3x\times-2+x\times2=-6x+2x=4x$

$3x\times2+x\times-2=6x-2x=4x$

You can ignore the factors -2 and 2 as they must produce an even multiple of x and hence cannot produce $11x$.

The only combination that gives $11x$ is: $\begin{array}{ccc}3x & \diagdown\!\!\!\diagup & -1\\ x & & 4\end{array}$.

$3x^2+11x-4=(3x-1)(x+4)$

4 The factors of $4a^2$ are $4a$ and a; or $2a$ and $2a$.

The only factors of 3 are 3 and 1.

We can ignore any two negative factors of 3 as these cannot make the middle term positive.

Set up the cross method.

$$\begin{matrix} 4a & 2a \\ a & 2a \end{matrix} \times \begin{matrix} 3 & 1 \\ 1 & 3 \end{matrix}$$

We can also ignore $2a$ and $2a$ as they cannot produce $13a$, an odd multiple of a.

Calculate the sums, looking for a sum of $13a$.

$4a\times1+a\times3=4a+3a=7a$

$4a\times3+a\times1=12a+a=13a$

The only combination that gives $13a$ is: $\begin{matrix} 4a & & 1 \\ & \times & \\ a & & 3 \end{matrix}$.

$4a^2+13a+3=(4a+1)(a+3)$

6 The factors of $8x^2$ are $8x$ and x; or $4x$ and $2x$.

The only possible factors of 3 (that could work) are -3 and -1.

We can ignore any two positive factors of 3 as these cannot make the middle term negative.

Set up the cross method.

$$\begin{matrix} 8x & 4x \\ x & 2x \end{matrix} \times \begin{matrix} -3 & -1 \\ -1 & -3 \end{matrix}$$

We can also ignore $8x$ and x because as both factors are odd, they must produce an odd (negative) number and an even number, which must add to an odd number, and so cannot produce $-14x,$ an even multiple of x.

Calculate the sums, looking for a sum of $-14x$.

$4x\times-1+2x\times-3=-4x-6x=-10x$

$4x\times-3+2x\times-1=-12x-2x=-14x$

The only combination that gives $-14x$ is: $\begin{matrix} 4x & & -1 \\ & \times & \\ 2x & & -3 \end{matrix}$.

$8x^2-14x+3=(4x-1)(2x-3)$

8 The factors of $8x^2$ are $8x$ and x; or $4x$ and $2x$.

The only possible factors of 5 (that could work) are 5 and 1.

We can ignore any two negative factors of 5 as these cannot make the middle term positive.

Set up the cross method.

$$\begin{matrix} 8x & 4x \\ x & 2x \end{matrix} \;\times\; \begin{matrix} -5 & -1 \\ -1 & -5 \end{matrix}$$

We can also ignore $8x$ and x because as both factors are odd, they must produce an odd (positive) number and an even number, which must add to an odd number, and so cannot produce $14x$, an even multiple of x.

Calculate the sums, looking for a sum of $14x$.

$$4x\times1+2x\times5=4x+10x=14x$$

$$4x\times-5+2x\times-1=-20x-2x=-22x$$

The only combination that gives $14x$ is: $\begin{matrix} 4x \\ 2x \end{matrix} \times \begin{matrix} 5 \\ 1 \end{matrix}$.

$$8x^2+14x+5=(4x+5)(2x+1)$$

10 The factors of $6a^2$ are $6a$ and a; or $3a$ and $2a$.

The factors of 63 are 63 and 1; 21 and 3; and 9 and 7.

We can ignore any two negative factors of 3 as these cannot make the middle term positive.

Set up the cross method.

$$\begin{matrix} 6a & 3a \\ a & 2a \end{matrix} \;\times\; \begin{matrix} 63 & -1 & -63 & 1 & 21 & -3 & -21 & 3 & 9 & -7 & -9 & 7 \\ -1 & 63 & 1 & -63 & -3 & 21 & 3 & -21 & -7 & 9 & 7 & -9 \end{matrix}$$

There are a lot of options, so it would be good if we could rule some out.

The middle four are both multiples of 3 and cannot produce $-13a$.

If we look at the first four, we will soon see that the size of the difference must be much greater than $13a$, so they cannot produce $-13a$. This leaves only 9 and 7 to consider.

Calculate the sums, looking for a sum of $-13a$.

You may realise that the closest possibility using $6a$ and a is $6a\times-7+a\times9=-42a+9a\neq-13a$.

The differences will be too large.

$3a\times9+a\times-7$ is positive.

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$3a \times -7 + 2a \times 9 = -21a + 18a = -3a$

$3a \times -9 + 2a \times 7 = -27a + 14a = -13a$

$3a \times 9 + 2a \times -7 = 27a - 14a = 13a$

The only combination that gives $-13a$ is: $\begin{matrix} 4a & \times & 1 \\ a & & 3 \end{matrix}$.

$6a^2 - 13a - 63 = (3a + 7)(2a - 9)$

You may observe that if one combination of factors gives a certain answer, when the signs are both changed, the resulting answer will be the opposite sign of the first. This will halve the number of calculations you need to make. Also, if there is no common factor, there will be only one solution, so once you have found factors that work, you need not go any further.

12 The factors of $10x^2$ are $10x$ and x; or $5x$ and $2x$.

The only possible factors of 8 (that could work) are 8 and 1 or 4 and 2.

We can ignore any two negative factors of 5 as these cannot make the middle term positive.

Set up the cross method.

$$\begin{matrix} 10x & 5x & \times & 8 & -1 & -8 & 1 & 4 & -2 & -4 & 2 \\ x & 2x & & -1 & 8 & 1 & -8 & -2 & 4 & 2 & -4 \end{matrix}$$

We can also ignore 4 and 2 because they must produce an even number, and cannot produce $-11x$.

Calculate the sums, looking for a sum of $-11x$, and avoiding combinations with an even result.

$10x \times -1 + x \times 8$ is even.

$10x \times 8 + x \times -1 = 4x + 10x = 14x$ is obviously too big.

Forget $10x$ and x.

$5x \times 8 + 2x \times -1$ is obviously too big.

$5x \times -1 + 2x \times 8 = -5x + 16x = 11x$

We are looking for $-11x$, so use the same factors with opposite sign.

$5x \times 1 + 2x \times -8 = 5x - 16x = -11x$

The only combination that gives $-11x$ is: $\begin{matrix} 5x & \times & -8 \\ 2x & & 1 \end{matrix}$.

$10x^2 - 11x - 8 = (5x - 8)(2x + 1)$

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14 You may notice that the first and last terms are both perfect squares. Could this be a perfect square?

If it was, it would be $(2x)^2 - 12x + 3^2 = (2x)^2 - 2\times 3\times 2x + 3^2 = (2x-3)^2$

$4x^2 - 12x + 9 = (2x-3)^2$

Using the cross method, you get $\begin{matrix} 4x & 2x \\ x & 2x \end{matrix} \times \begin{matrix} -9 & -1 & -3 \\ -1 & -9 & -3 \end{matrix}$.

$2x\times -3 + 2x\times -3 = -6x - 6x = -12x$

The only combination which works is $\begin{matrix} 2x \\ 2x \end{matrix} \times \begin{matrix} -3 \\ -3 \end{matrix}$.

$$\begin{aligned} 4x^2 - 12x + 9 &= (2x-3)(2x-3) \\ &= (2x-3)^2 \end{aligned}$$

16 The only factors of $2x^2$ are $2x$ and x.

The only possible factors of 10 (that could work) are -10 and -1 or -5 and -2.

We can ignore any two positive factors of 10 as these cannot make the middle term negative.

Set up the cross method.

$\begin{matrix} 2x \\ x \end{matrix} \times \begin{matrix} -10 & -1 & -5 & -2 \\ -1 & -10 & -2 & -5 \end{matrix}$

Calculate the sums, looking for a sum of $-9x$.

$2x\times -10 + x\times -1 = -20x - x = -21x$

$2x\times -1 + x\times -10 = -2x - 10x = -12x$

$2x\times -2 + x\times -5 = -4x - 5x = -9x$

$2x\times -5 + x\times -2 = -10x - 2x = -12x$

The only combination that gives $-9x$ is: $\begin{matrix} 2x \\ x \end{matrix} \times \begin{matrix} -5 \\ -2 \end{matrix}$.

$2x^2 - 9x + 10 = (2x-5)(x-2)$

18 Look for two numbers which add to -2 and multiply to -3.

One must be positive and one negative.

$-3 + 1 = -2$ and $-3\times 1 = -3$.

$y^2 - 2y - 3 = (y-3)(y+1)$

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20 The factors of $6x^2$ are $6x$ and x or $3x$ and $2x$.

The only possible factors of 14 (that could work) are -14 and -1 or -7 and -2.

We can ignore any two positive factors of 14 as these cannot make the middle term negative.

Set up the cross method.

$$\begin{matrix} 6x & 2x \\ x & 3x \end{matrix} \times \begin{matrix} -14 & -1 & -7 & -2 \\ -1 & -14 & -2 & -7 \end{matrix}$$

Calculate the sums, looking for a sum of $-25x$. Ignore sums with an obviously even answer, or obviously too large or too small a difference.

$$2x\times -14+3x\times -1=-28x-3x=-31x$$

$$2x\times -2+3x\times -7=-4x-21x=-25x$$

There is no need to look any further.

The only combination that gives $-25x$ is: $\begin{matrix} 3x \\ 2x \end{matrix} \times \begin{matrix} -2 \\ -7 \end{matrix}$.

$$6x^2-25x+14=(3x-2)(2x-7)$$

22 The factors of $6x^2$ are $6x$ and x or $3x$ and $2x$.

The only possible factors of 14 (that could work) are -14 and -1 or -7 and -2.

We can ignore any two positive factors of 14 as these cannot make the middle term negative.

Set up the cross method.

$$\begin{matrix} 6x & 2x \\ x & 3x \end{matrix} \times \begin{matrix} -14 & -1 & -7 & -2 \\ -1 & -14 & -2 & -7 \end{matrix}$$

Calculate the sums, looking for a sum of $-19x$. Ignore sums with an obviously even answer, or obviously too large or too small a difference.

$$6x\times -2+x\times -7=-12x-7x=-19x$$

There is no need to look any further.

The only combination that gives $-19x$ is: $\begin{matrix} 6x \\ x \end{matrix} \times \begin{matrix} -7 \\ -2 \end{matrix}$.

$$6x^2-19x+14=(6x-7)(x-2)$$

24 The factors of $8x^2$ are $8x$ and x or $4x$ and $2x$.

The only factors of 3 are 3 and 1.

Use the cross method.

$$\begin{array}{cc} 8x & 4x \\ x & 2x \end{array} \times \begin{array}{cccc} -3 & 1 & 3 & -1 \\ 1 & -3 & -1 & 3 \end{array}$$

Calculate the sums, looking for a sum of $2x$. Ignore $8x$ and x as this can only produce an odd sum.

$4x \times 1 + 2x \times -3 = 4x - 6x = -2x$

Swap the signs of the factors of 3.

$4x \times -1 + 2x \times 3 = -4x + 6x = 2x$

The combination that gives $2x$ is: $\begin{array}{c} 4x \\ 2x \end{array} \times \begin{array}{c} 3 \\ -1 \end{array}$.

$8x^2 + 2x - 3 = (4x+3)(2x-1)$

26 The factors of $10a^2$ are $10a$ and a; or $5a$ and $2a$.

The factors of 6 are 6 and 1; or 3 and 2.

Set up the cross method.

$$\begin{array}{cc} 10a & 5a \\ a & 2a \end{array} \times \begin{array}{cccccccc} 6 & -1 & -6 & 1 & 3 & -2 & 2 & -3 \\ -1 & 6 & 1 & -6 & -2 & 3 & -3 & 2 \end{array}$$

There are a lot of options, so it would be good if we could rule some out. Ignore options which give an even result, or are obviously too large, like $\begin{array}{c} 10a \\ a \end{array} \times \begin{array}{c} -1 \\ 6 \end{array}$.

Calculate the sums, looking for a sum of $-11a$.

$3a \times 9 + a \times -7$ is positive.

$10a \times -2 + a \times 3 = -20a + 3a = -17a$

$10a \times 2 + a \times -3 = 20a - 3a = 17a$

$5a \times -1 + 2a \times 6 = -5a + 12a = 7a$

$5a \times 1 + 2a \times -6 = 5a - 12a = -7a$

$5a \times -2 + 2a \times 3 = -10a + 6a = -4a$

$5a \times 3 + 2a \times -2 = 15a - 4a = 11a$

$5a \times -3 + 2a \times 2 = -15a + 4a = -11a$

The combination that gives $-11a$ is: $\begin{array}{c} 5a \\ 2a \end{array} \times \begin{array}{c} 2 \\ -3 \end{array}$.

$10a^2 - 11a - 6 = (5a+2)(2a-3)$

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28 The factors of $24x^2$ are $24x$ and x; $12x$ and $2x$; $8x$ and $3x$; or $6x$ and $4x$.

Rule out $12x$ and $2x$; and also $6x$ and $4x$ as these must produce an even middle term.

The factors of 36 are -36 and -1; or -18 and -2; or -12 and -3; or -9 and -4; or -6 and -6.

We can ignore all except -36 and -1; or -9 and -4; as these will make the middle term a multiple of 2 or 3 or both.

Set up the cross method.

$$\begin{array}{cc} 36x & 8x \\ x & 3x \end{array} \times \begin{array}{cccc} -36 & -1 & -9 & -4 \\ -1 & -36 & -4 & -9 \end{array}$$

Calculate the sums, looking for a sum of $-59x$. Start with the smaller numbers as these are more likely to give a reasonable middle term. Ignore sums which must give a multiple of 2 or 3, or an obviously too large or too small a difference.

$$8x\times-4+3x\times-9=-32x-27x=-59x$$

There is no need to look any further. It's amazing how a little thought (and a little luck) can give such a quick result from such a large number of options.

The combination that gives $-59x$ is: $\begin{array}{c} 8x \\ 3x \end{array} \times \begin{array}{c} -9 \\ -4 \end{array}$.

$$24x^2-59x+36=(8x-9)(3x-4)$$

30 The factors of $3x^2$ are $3x$ and x.

The only factors of 1 are 1 and 1.

Use the cross method.

$$\begin{array}{c} 3x \\ x \end{array} \times \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array}$$

It is easy to find a sum of $-2x$. Ignore $8x$ and x as this can only produce an odd sum.

$$3x\times-1+x\times1=-3x+x=-2x$$

The combination that gives $-2x$ is: $\begin{array}{c} 3x \\ x \end{array} \times \begin{array}{c} 1 \\ -1 \end{array}$.

$$3x^2-2x-1=(3x+1)(x-1)$$

32 The factors of $2x^2$ are $2x$.

The only possible factors of 4 (that could work) are -4 and -1 or -2 and -2.

We can ignore any two positive factors of 14 as these cannot make the middle term negative.

Set up the cross method.

$$\begin{array}{ccccc} 2x & \multirow{2}{*}{$\times$} & -4 & -1 & -2 \\ x & & -1 & -4 & -2 \end{array}$$

-2 and -2 will make the middle term even, so we can ignore these. So will the first -4 and -1.

Calculate the sums, looking for a sum of $-9x$.

$$2x\times -4 + x\times -1 = -8x - x = -9x$$

The combination that gives $-9x$ is: $\begin{array}{ccc} 2x & \times & -1 \\ x & & -4 \end{array}$.

$$2x^2 - 9x + 4 = (2x-1)(x-4)$$

34 You may notice that the first and last terms are both perfect squares. Could this be a perfect square?

If it was, it would be $(2x)^2 + 12x + 3^2 = (2x)^2 + 2\times 3\times 2x + 3^2 = (2x+3)^2$

$$4x^2 + 12x + 9 = (2x+3)^2$$

Using the cross method, you get $\begin{array}{cccccc} 4x & 2x & \times & 9 & 1 & 3 \\ x & 2x & & 1 & 9 & 3 \end{array}$.

$2x\times 3 + 2x\times 3 = 6x + 6x = 12x$ (The only option that gives a multiple of 3)

The only combination which works is $\begin{array}{ccc} 2x & \times & 3 \\ 2x & & 3 \end{array}$.

$$\begin{aligned} 4x^2 + 12x + 9 &= (2x+3)(2x+3) \\ &= (2x+3)^2 \end{aligned}$$

Note that $2x+3$ is the same as $3+2x$.

(a) correct

(b) correct

(c) correct

(d) incorrect

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EXERCISE 1.8 MIXED FACTORISATIONS

2 $$\begin{aligned} 2a^3-8a &= 2a\left(a^2-4\right) \\ &= 2a\left(a^2-2^2\right) \\ &= 2a\left(a-2\right)\left(a+2\right) \end{aligned}$$

4 Two numbers which add to -8 and multiply to make -9 are -9 and -1.

$$x^2-8x-9=\left(x-9\right)\left(x+1\right)$$

6 $$\begin{aligned} 5x^3y-20xy^3 &= 5xy\left(x^2-4y^2\right) \\ &= 5xy\left(x^2-\left(2y\right)^2\right) \\ &= 5xy\left(x-2y\right)\left(x+2y\right) \end{aligned}$$

8 The factors of $10x^2$ are $10x$ and x; or $5x$ and $2x$.

The only factors of -1 are -1 and 1.

Set up the cross method.

$$\begin{array}{cccc} 10x & 5x & -1 & 1 \\ x & 2x & 1 & -1 \end{array}$$

Calculate the sums, looking for a sum of $9x$.

$$10x\times1+x\times-1=10x-x=9x$$

The combination that gives $9x$ is: $\begin{array}{cc} 10x & -1 \\ x & 1 \end{array}$.

$$10x^2+9x-1=\left(10x-1\right)\left(x+1\right)$$

10 $$\begin{aligned} 6x^2-24 &= 6\left(x^2-4\right) \\ &= 6\left(x^2-2^2\right) \\ &= 6\left(x-2\right)\left(x+2\right) \end{aligned}$$

12 The common factor is $\left(m+n\right)$.

$$a\left(m+n\right)-b\left(m+n\right)=\left(a-b\right)\left(m+n\right)$$

14 $$\begin{aligned} 3a^3+24a^2+21a &= 3a\left(a^2+8a+7\right) \\ &= 3a\left(a+7\right)\left(a+1\right) \end{aligned}$$

Two numbers which add to 8 and multiply to make 7 are 7 and 1.

$$3a^3+24a^2+21a=3a\left(a+7\right)\left(a+1\right)$$

16 The highest common factor is ab.

$$ab^2+abc+abd=ab\left(b+c+d\right)$$

18 The common factor is $\left(y-z\right)$.

$$x\left(y-z\right)+y\left(y-z\right)=\left(x+y\right)\left(y-z\right)$$

20 $$\begin{aligned} bx^2-14bxy+49by^2 &= b\left(x^2-14xy+49y^2\right) \\ &= b\left(x-7y\right)^2 \end{aligned}$$

Two numbers which add to -14 and multiply to make 49 are -7 and -7.

$$bx^2-14bxy+49by^2=b\left(x-7y\right)^2$$

You may also recognise that

$$\begin{aligned} x^2-14xy+49y^2 &= x^2-2\times x\times7y+\left(7y\right)^2 \\ &= \left(x-7y\right)^2 \end{aligned}$$

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22 $6y^3+26y^2+8y=2y\left(3y^2+13y+4\right)$

The factors of $3y^2$ are $3y$ and y.

The factors of 4 are 4 and 1; or 2 and 2.

Set up the cross method.

$$\begin{array}{cccccc} 3y & \multirow{2}{*}{$\times$} & 4 & 1 & 2 \\ y & & 1 & 4 & 2 \end{array}$$

Calculate the sums, looking for a sum of $13y$.

$3y\times1+y\times4=3y+4y=7y$

$3y\times4+y\times1=12y+y=13y$

The only combination that gives $13y$ is: $\begin{array}{ccc} 3y & \times & 1 \\ y & & 4 \end{array}$.

$6y^3+26y^2+8y=2y\left(3y+1\right)\left(y+4\right)$

24 The highest common factor is mn.

$$\begin{aligned} 9mn-25m^3n^3 &= mn\left(9-25m^2n^2\right) \\ &= mn\left(3^2-\left(5mn\right)^2\right) \\ &= mn\left(3-5mn\right)\left(3+5mn\right) \end{aligned}$$

26 $\left(x+y\right)^2-\left(x-y\right)^2=\left[\left(x+y\right)-\left(x-y\right)\right]\left[\left(x+y\right)+\left(x-y\right)\right]$

$$\begin{aligned} &=\left(x+y-x+y\right)\left(x+y+x-y\right) \\ &=2y\times2x \\ &=4xy \end{aligned}$$

28 Factorise the terms in pairs.

$$\begin{aligned} m^2-mn+6m-6n &= m\left(m-n\right)+6\left(m-n\right) \\ &= \left(m+6\right)\left(m-n\right) \end{aligned}$$

30 Factorise the terms in pairs.

$$\begin{aligned} mx^2-xy+ly-mlx &= x\left(mx-y\right)-l\left(-y+mx\right) \\ &= \left(x-l\right)\left(mx-y\right) \end{aligned}$$

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32 The side length of the square is $2r$.

The area of the square is $2r \times 2r = 4r^2$.

The area of the circle is πr^2.

Shaded area $= 4r^2 - \pi r^2 = r^2(4-\pi)$

34 The area of the outer circle is πR^2.

The area of the inner circle is πr^2.

Shaded area $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
$= \pi(R-r)(R+r)$

36 The area of the outer circle is πa^2.

The area of the inner ellipse is πab.

The shaded area $= \pi a^2 - \pi ab$

The highest common factor is πa.

The shaded area $= \pi a(a-b)$.

38 The side length of the square is $4r$.

The area of the square is $4r \times 4r = 16r^2$.

The area of the four circles is $4 \times \pi r^2 = 4\pi r^2$.

The shaded area is $16r^2 - 4\pi r^2$

The highest common factor is $4r^2$.

The shaded area is $4r^2(4-\pi)$

40 The area can be the difference of a rectangle with sides $a+b$ and $a+c$, and a rectangle with sides b and c.

Area $= (a+b)(a+c) - bc$

The area can be the sum of a square and two rectangles.

Area $= a \times a + a \times b + c \times a$ and $a+c$.
$= a^2 + ab + ac$

This expression has a common factor of a.

Area $= a(a+b+c)$

(a) correct

(b) correct

(c) incorrect

(d) correct

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EXERCISE 1.9 ALGEBRAIC FRACTIONS

2 $\frac{15x+10y}{15}=\frac{5(3x+2y)}{15}$

$=\frac{3x+2y}{3}$

4 $\frac{8x^2-4xy}{8xy}=\frac{4x(2x-y)}{8xy}$

$=\frac{2x-y}{2y}$

6 $\frac{8x+2}{4x+1}=\frac{2(4x+1)}{4x+1}$

$=2$

8 $\frac{m+m^2}{m}=\frac{m(1+m)}{m}$

$=1+m$

10 $\frac{p^2q-pq^2}{pq}=\frac{pq(p-q)}{pq}$

$=p-q$

12 $\frac{2rs-12r}{r^2+rs}=\frac{2r(s-6)}{r(r+s)}$

$=\frac{2(s-6)}{r+s}$

14 $\frac{k^2+k}{k+1}=\frac{k(k+1)}{k+1}$

$=k$

16 $\frac{15a^2-5ab}{3ab-b^2}=\frac{5a(3a-b)}{b(3a-b)}$

$=\frac{5a}{b}$

18 $\frac{x^2-7x+6}{x^2-36}=\frac{(x-6)(x+1)}{(x-6)(x+6)}$

$=\frac{x+1}{x+6}$

20 $\frac{a^2-b^2}{a^2+ab}=\frac{(a-b)(a+b)}{a(a+b)}$

$=\frac{a-b}{a}$

22 $\frac{x^2-6x+8}{x^2-x-2}=\frac{(x-2)(x-4)}{(x-2)(x+1)}$

$=\frac{x-4}{x+1}$

24 $\frac{x^2-5x+6}{x^2+x-12}=\frac{(x-2)(x-3)}{(x+4)(x-3)}$

$=\frac{x-2}{x+4}$

26 $\frac{4x^3y-16xy}{x^2+2x-8}=\frac{4xy(x^2-4)}{(x+4)(x-2)}$

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$$= \frac{4xy(x-2)(x+2)}{(x+4)(x-2)}$$

$$= \frac{4xy(x+2)}{x+4}$$

28 $\dfrac{3x^2 - xy}{\cancel{x}\cancel{y}} \times \dfrac{x^{\cancel{2}}\cancel{y}}{3xy - y^2} = \dfrac{x(3x-y)}{1} \times \dfrac{x}{y(3x-y)}$

$$= \frac{x^2}{y}$$

30 $\dfrac{2a^2 - 3ab}{ab - b^2} \times \dfrac{2a^2 - 2ab}{4a - 6b} = \dfrac{a(2a-3b)}{b(a-b)} \times \dfrac{2a(a-b)}{2(2a-3b)}$

$$= \frac{a^2}{b}$$

32 $\dfrac{12x^2 - 4x}{3x^2 - x} \div \dfrac{10x^2 y}{5x^2 y^2} = \dfrac{4x(3x-1)}{x(3x-1)} \times \dfrac{5x^2 y^2}{10x^2 y}$

$$= 4 \times \frac{y}{2}$$
$$= 2y$$

34 $\dfrac{(a+2b)(a-b)}{a^2 - 4b^2} \times \dfrac{a^2 - 3ab + 2b^2}{ab - b^2} = \dfrac{(a+2b)(a-b)}{(a-2b)(a+2b)} \times \dfrac{(a-2b)(a-b)}{b(a-b)}$

$$= \frac{a-b}{a-2b} \times \frac{a-2b}{b}$$
$$= \frac{a-b}{b}$$

36 $\dfrac{x^3 + y^3}{x^2 - y^2} = \dfrac{(x+y)(x^2 - xy + y^2)}{(x-y)(x+y)}$

$$= \frac{x^2 - xy + y^2}{x+y}$$

38 $\dfrac{2x + 2y}{x^3 - y^3} \times \dfrac{x^2 - 2xy + y^2}{x^2 - y^2} = \dfrac{2(x+y)}{(x-y)(x^2 + xy + y^2)} \times \dfrac{(x-y)^2}{(x-y)(x+y)}$

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$$= \frac{2(x+y)}{(x-y)(x^2+xy+y^2)} \times \frac{(x-y)}{(x+y)}$$
$$= \frac{2}{x^2+xy+y^2}$$

40 $\dfrac{x^3-(x-y)^3}{x^2-(x-y)^2} = \dfrac{[x-(x-y)][x^2+x(x-y)+(x-y)^2]}{[x-(x-y)][x+(x-y)]}$

$$= \frac{(x-x+y)(x^2+x^2-xy+x^2-2xy+y^2)}{(x-x+y)(x+x-y)}$$
$$= \frac{y(3x^2-3xy+y^2)}{y(2x-y)}$$
$$= \frac{3x^2-3xy+y^2}{2x-y}$$

EXERCISE 1.10 ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

2 $\dfrac{3x}{8}+\dfrac{x}{2}=\dfrac{3x}{8}+\dfrac{4x}{8}$

$$=\frac{7x}{8}$$

4 $\dfrac{y}{2}+\dfrac{2y}{3}-\dfrac{y}{4}=\dfrac{6y}{12}+\dfrac{8y}{12}-\dfrac{3y}{12}$

$$=\frac{11y}{12}$$

6 $\dfrac{2x-y}{3}-\dfrac{x-3y}{6}=\dfrac{2(2x-y)}{6}-\dfrac{x-3y}{6}$

$$=\frac{4x-2y-x+3y}{6}$$
$$=\frac{3x+y}{6}$$

8 $\dfrac{3m-2n}{5}+\dfrac{m+n}{10}=\dfrac{2(3m-2)}{10}+\dfrac{m+n}{10}$

$$=\frac{6m-4+m+n}{10}$$
$$=\frac{7m-3n}{10}$$

10 $\dfrac{a-2b}{6}-\dfrac{2a+b}{9}=\dfrac{6(a-2b)}{36}-\dfrac{4(2a+b)}{36}$

12 $\dfrac{1}{x}-\dfrac{2}{3x}=\dfrac{3}{3x}-\dfrac{2}{3x}$

$$=\frac{1}{3x}$$

$$= \frac{6a - 12b - 8a - 4b}{36}$$
$$= \frac{-2a - 16b}{36}$$
$$= \frac{-2(a + 8b)}{36}$$
$$= -\frac{(a + 8b)}{18}$$

14 $\frac{1}{ab} - \frac{2}{b} = \frac{1}{ab} - \frac{2a}{ab}$

$$= \frac{1 - 2a}{ab}$$

16 $\frac{4}{xy} + \frac{3}{yz} = \frac{4z}{xyz} + \frac{3x}{xyz}$

$$= \frac{4z + 3x}{xyz}$$

18 $\frac{a+1}{6a} + \frac{a-4}{2a} = \frac{a+1}{6a} + \frac{3(a-4)}{6a}$

$$= \frac{a + 1 + 3a - 12}{6a}$$
$$= \frac{4a - 11}{6a}$$

20 $\frac{1}{x} + \frac{2}{x} - \frac{1}{x^2} = \frac{x}{x^2} + \frac{2x}{x^2} - \frac{1}{x^2}$

$$= \frac{3x - 1}{x^2}$$

22 $(x - 3)(x + 3)$

24 $6(x - 2)$

26 $x^2 - 4x = x(x - 4)$

$x^2 - 16 = (x - 4)(x + 4)$

The LCM is $x(x - 4)(x + 4)$.

28 $x^2 - y^2 = (x - y)(x + y)$

The LCM is $(x - y)(x + y)$

30 A

$x^2 - 9 = (x - 3)(x + 3)$

The LCM is $x(x - 3)(x + 3)$

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32 $\frac{3}{x-y}-\frac{2}{x+y}=\frac{3(x+y)}{(x-y)(x+y)}-\frac{2(x-y)}{(x-y)(x+y)}$

$$=\frac{3x+3y-2x+2y}{(x-y)(x+y)}$$
$$=\frac{x+5y}{(x-y)(x+y)}$$

34 $\frac{x}{x-y}+\frac{y}{x+y}=\frac{x(x+y)}{(x-y)(x+y)}+\frac{y(x-y)}{(x-y)(x+y)}$

$$=\frac{x^2+xy+xy-y^2}{(x-y)(x+y)}$$
$$=\frac{x^2+2xy-y^2}{(x-y)(x+y)}$$

36 $\frac{1}{x^2-4}-\frac{1}{x+2}=\frac{1}{(x-2)(x+2)}-\frac{x-2}{(x-2)(x+2)}$

$$=\frac{1-x+2}{(x-2)(x+2)}$$
$$=\frac{3-x}{(x-2)(x+2)}$$

38 $\frac{3}{(x-2)^2}+\frac{2}{x-2}=\frac{3}{(x-2)^2}+\frac{2(x-2)}{(x-2)^2}$

$$=\frac{3+2x-4}{(x-2)^2}$$
$$=\frac{2x-1}{(x-2)^2}$$

40 $\frac{3}{x-2}+\frac{1}{x+3}=\frac{3(x+3)}{(x-2)(x+3)}+\frac{x-2}{(x-2)(x+3)}$

$$=\frac{3x+9+x-2}{(x-2)(x+3)}$$
$$=\frac{4x+7}{(x-2)(x+3)}$$

42 $\dfrac{3}{x-2}+\dfrac{1}{x+3}=\dfrac{3(x+3)}{(x-2)(x+3)}+\dfrac{x-2}{(x-2)(x+3)}$

$$=\frac{3x+9+x-2}{(x-2)(x+3)}$$
$$=\frac{4x+7}{(x-2)(x+3)}$$

44 $\dfrac{6}{3x-2}-\dfrac{8}{4x+1}=\dfrac{6(4x+1)}{(3x-2)(4x+1)}-\dfrac{8(3x-2)}{(3x-2)(4x+1)}$

$$=\frac{24x+6-24x+16}{(3x-2)(4x+1)}$$
$$=\frac{22}{(3x-2)(4x+1)}$$

46 $\dfrac{y}{x^2-xy}+\dfrac{1}{x}=\dfrac{y}{x(x-y)}+\dfrac{1}{x}$

$$=\frac{y}{x(x-y)}+\frac{x-y}{x(x-y)}$$
$$=\frac{y+x-y}{x(x-y)}$$
$$=\frac{x}{x(x-y)}$$
$$=\frac{1}{x-y}$$

48 $\dfrac{1}{x+2}+\dfrac{1}{x-2}+\dfrac{4}{x^2-4}=\dfrac{x-2}{(x-2)(x+2)}+\dfrac{x+2}{(x-2)(x+2)}+\dfrac{4}{(x-2)(x+2)}$

$$=\frac{x-2+x+2+4}{(x-2)(x+2)}$$
$$=\frac{2x+4}{(x-2)(x+2)}$$
$$=\frac{2(x+2)}{(x-2)(x+2)}$$
$$=\frac{2}{x-2}$$

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50 $\dfrac{3}{x^2-4}-\dfrac{2}{x^2-3x+2}=\dfrac{3}{(x-2)(x+2)}-\dfrac{2}{(x-2)(x-1)}$

$$=\frac{3(x-1)}{(x-2)(x+2)(x-1)}-\frac{2(x+2)}{(x-2)(x+2)(x-1)}$$
$$=\frac{3x-3-2x-4}{(x-2)(x+2)(x-1)}$$
$$=\frac{x-7}{(x-2)(x+2)(x-1)}$$

52 $\dfrac{5}{x-2}+\dfrac{3}{x^2-4}=\dfrac{5}{x-2}+\dfrac{3}{(x-2)(x+2)}$

$$=\frac{5(x+2)}{(x-2)(x+2)}+\frac{3}{(x-2)(x+2)}$$
$$=\frac{5x+10+3}{(x-2)(x+2)}$$
$$=\frac{5x+13}{(x-2)(x+2)}$$

54 $\dfrac{5}{3x}-\dfrac{2}{x^2-5x}=\dfrac{5}{3x}-\dfrac{2}{x(x-5)}$

$$=\frac{5(x-5)}{3x(x-5)}-\frac{2\times 3}{3x(x-5)}$$
$$=\frac{5x-25-6}{3x(x-5)}$$
$$=\frac{5x-31}{3x(x-5)}$$

EXERCISE 1.11 REAL NUMBERS AND SURDS

2 $\sqrt{20}=\sqrt{4}\times\sqrt{5}$
$=2\sqrt{5}$

4 $\sqrt{32}=\sqrt{16}\times\sqrt{2}$
$=4\sqrt{2}$

6 $\sqrt{45}=\sqrt{9}\times\sqrt{5}$
$=3\sqrt{5}$

8 $\sqrt{84}=\sqrt{4}\times\sqrt{21}$
$=2\sqrt{21}$

10 $\sqrt{108}=\sqrt{36}\times\sqrt{3}$
$=6\sqrt{3}$

12 $\sqrt{162}=\sqrt{81}\times\sqrt{2}$
$=9\sqrt{2}$

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14 $5\sqrt{128} = 5\sqrt{64} \times \sqrt{2}$

$= 5 \times 8 \times \sqrt{2}$

$= 40\sqrt{2}$

16 $2\sqrt{150} = 2\sqrt{25} \times \sqrt{6}$

$= 2 \times 5 \times \sqrt{6}$

$= 10\sqrt{6}$

18 D

$7\sqrt{245} = 7\sqrt{49} \times \sqrt{5}$

$= 7 \times 7 \times \sqrt{5}$

$= 49\sqrt{5}$

20 $\dfrac{\sqrt{175}}{5} = \dfrac{\sqrt{25} \times \sqrt{7}}{5}$

$= \dfrac{5\sqrt{7}}{5}$

$= \sqrt{7}$

22 $\dfrac{\sqrt{90}}{9} = \dfrac{\sqrt{9} \times \sqrt{10}}{9}$

$= \dfrac{3\sqrt{10}}{9}$

$= \dfrac{\sqrt{10}}{3}$

24 $\dfrac{2\sqrt{7}}{\sqrt{35}} = \dfrac{2\sqrt{7}}{\sqrt{7} \times \sqrt{5}}$

$= \dfrac{2}{\sqrt{5}}$

$= \dfrac{2}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{2\sqrt{5}}{5}$

(a) correct

(b) incorrect

(c) correct

(d) incorrect

26 $\sqrt{8} \times \sqrt{2} = \sqrt{4} \times \sqrt{2} \times \sqrt{2}$

$= 2 \times 2$

$= 4$

Alternatively, $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$

28 $\sqrt{6} \times \sqrt{10} = \sqrt{60}$

$= \sqrt{4} \times \sqrt{15}$

$= 2\sqrt{15}$

30 $\sqrt{8} \times 2\sqrt{2} = 2\sqrt{16} = 2 \times 4 = 8$

32 $4\sqrt{6} \times 2\sqrt{3} = 8\sqrt{18}$

$= 8 \times \sqrt{9} \times \sqrt{2}$

$= 8 \times 3 \times \sqrt{2}$

$= 24\sqrt{2}$

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34 $4\sqrt{3}\times\sqrt{18}=4\sqrt{54}$

$$=4\times\sqrt{9}\times\sqrt{6}$$
$$=4\times3\times\sqrt{6}$$
$$=12\sqrt{6}$$

36 $4\sqrt{5}\times\sqrt{20}=4\sqrt{100}=4\times10=40$

38 $\frac{2}{\sqrt{3}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$$=\frac{2\sqrt{3}}{3}$$

40 $\frac{7}{\sqrt{7}}=\frac{7}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$

$$=\frac{7\sqrt{7}}{7}$$
$$=\sqrt{7}$$

Alternatively, $\frac{7}{\sqrt{7}}=\frac{\sqrt{49}}{\sqrt{7}}=\sqrt{7}$.

42 $\frac{3\sqrt{2}}{\sqrt{6}}=\frac{3\sqrt{2}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$

$$=\frac{3\sqrt{12}}{6}$$
$$=\frac{3\times\sqrt{4}\times\sqrt{3}}{6}$$
$$=\frac{6\sqrt{3}}{6}$$
$$=\sqrt{3}$$

Alternatively, $\frac{3\sqrt{2}}{\sqrt{6}}=\frac{3}{\sqrt{3}}=\frac{\sqrt{9}}{\sqrt{3}}=\sqrt{3}$

44 $\frac{2\sqrt{18}}{\sqrt{8}}=\frac{2\sqrt{18}}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}}$

$$=\frac{2\sqrt{144}}{8}$$
$$=\frac{2\times12}{8}$$
$$=3$$

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EXERCISE 1.12 ADDING AND SUBTRACTING SURDS

2 $5\sqrt{7}-2\sqrt{7}+4\sqrt{7}=(5-2+4)\sqrt{7}=7\sqrt{7}$

4 $4\sqrt{2}-\sqrt{3}+4\sqrt{3}-\sqrt{2}=4\sqrt{2}-\sqrt{2}-\sqrt{3}+4\sqrt{3}$

$$=(4-1)\sqrt{2}+(-1+4)\sqrt{3}$$
$$=3\sqrt{2}+3\sqrt{3}$$

6 $\sqrt{8}-\sqrt{2}=\sqrt{4}\times\sqrt{2}-\sqrt{2}$

$$=2\sqrt{2}-\sqrt{2}$$
$$=\sqrt{2}$$

8 $\sqrt{20}+\sqrt{5}=\sqrt{4}\times\sqrt{5}+\sqrt{5}$

$$=2\sqrt{5}+\sqrt{5}$$
$$=3\sqrt{5}$$

10 $\sqrt{27}+2\sqrt{48}-4\sqrt{3}=\sqrt{9}\times\sqrt{3}+2\times\sqrt{16}\times\sqrt{3}-4\sqrt{3}$

$$=3\sqrt{3}+2\times4\times\sqrt{3}-4\sqrt{3}$$
$$=3\sqrt{3}+8\sqrt{3}-4\sqrt{3}$$
$$=7\sqrt{3}$$

12 $2\sqrt{50}-3\sqrt{18}=2\times\sqrt{25}\times\sqrt{2}-3\times\sqrt{9}\times\sqrt{2}$

$$=2\times5\times\sqrt{2}-3\times3\times\sqrt{2}$$
$$=10\sqrt{2}-9\sqrt{2}$$
$$=\sqrt{2}$$

14 $\sqrt{5}+\sqrt{2}-\sqrt{45}+\sqrt{8}=\sqrt{5}+\sqrt{2}-\sqrt{9}\times\sqrt{5}+\sqrt{4}\times\sqrt{2}$

$$=\sqrt{5}+\sqrt{2}-3\sqrt{5}+2\sqrt{2}$$
$$=3\sqrt{2}-2\sqrt{5}$$

(a) incorrect

(b) incorrect

(c) correct

(d) correct

16 $5\sqrt{3}+\sqrt{27}-\sqrt{45}=5\sqrt{3}+\sqrt{9}\times\sqrt{3}-3\sqrt{5}$

$$=5\sqrt{3}+3\sqrt{3}-3\sqrt{5}$$
$$=8\sqrt{3}-3\sqrt{5}$$

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18 $3\sqrt{15}+\sqrt{60}-\sqrt{40}=3\sqrt{15}+\sqrt{4}\times\sqrt{15}-\sqrt{4}\times\sqrt{10}$

$$=3\sqrt{15}+2\sqrt{15}-2\sqrt{10}$$
$$=5\sqrt{15}-2\sqrt{10}$$

20 $2\sqrt{50}-3\sqrt{18}+\sqrt{3}=2\times\sqrt{25}\times\sqrt{2}-3\times\sqrt{9}\times\sqrt{2}+\sqrt{3}$

$$=10\sqrt{2}-9\sqrt{2}+\sqrt{3}$$
$$=\sqrt{2}+\sqrt{3}$$

22 $\sqrt{150}-\sqrt{200}=\sqrt{25}\times\sqrt{6}-\sqrt{100}\times\sqrt{2}$

$$=5\sqrt{6}-10\sqrt{2}$$

24 $5\sqrt{7}+3\sqrt{5}-2\sqrt{28}=5\sqrt{7}+3\sqrt{5}-2\times\sqrt{4}\times\sqrt{7}$

$$=5\sqrt{7}+3\sqrt{5}-4\sqrt{7}$$
$$=\sqrt{7}+3\sqrt{5}$$

26 $\sqrt{98}-2\sqrt{20}-\sqrt{12}=\sqrt{49}\times\sqrt{2}-2\times\sqrt{4}\times\sqrt{5}-\sqrt{4}\times\sqrt{3}$

$$=7\sqrt{2}-4\sqrt{5}-2\sqrt{3}$$

28 $7\sqrt{3}-2\sqrt{2}+\sqrt{12}+\sqrt{8}=7\sqrt{3}-2\sqrt{2}+\sqrt{4}\times\sqrt{3}+\sqrt{4}\times\sqrt{2}$

$$=7\sqrt{3}-2\sqrt{2}+2\sqrt{3}+2\sqrt{2}$$
$$=9\sqrt{3}$$

30 $6\sqrt{2}+\sqrt{12}-2\sqrt{3}-3\sqrt{8}=6\sqrt{2}+\sqrt{4}\times\sqrt{3}-2\sqrt{3}-3\times\sqrt{4}\times\sqrt{2}$

$$=6\sqrt{2}+2\sqrt{3}-2\sqrt{3}-6\sqrt{2}$$
$$=0$$

EXERCISE 1.13 THE DISTRIBUTIVE LAW

2 $\sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)=\sqrt{5}\times\sqrt{5}+\sqrt{5}\times\sqrt{2}=5+\sqrt{10}$

4 $\sqrt{3}\left(\sqrt{2}-\sqrt{6}\right)=\sqrt{3}\times\sqrt{2}+\sqrt{3}\times\sqrt{6}$

$$=\sqrt{6}-\sqrt{18}$$
$$=\sqrt{6}-\sqrt{9}\times\sqrt{2}$$
$$=\sqrt{6}-3\sqrt{2}$$

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6 $7\left(2\sqrt{5}-1\right)=7\times2\sqrt{5}-7\times1=14\sqrt{5}-7$

8 $3\sqrt{2}\left(2\sqrt{6}-\sqrt{5}\right)=3\sqrt{2}\times2\sqrt{6}-3\sqrt{2}\times\sqrt{5}$

$$\begin{aligned}&=6\sqrt{12}-3\sqrt{10}\\&=6\times\sqrt{4}\times\sqrt{3}-3\sqrt{10}\\&=12\sqrt{3}-3\sqrt{10}\end{aligned}$$

10 $\sqrt{x}\left(\sqrt{x}-\sqrt{y}\right)=\sqrt{x}\times\sqrt{x}-\sqrt{x}\times\sqrt{y}=x-\sqrt{xy}$

12 $\left(\sqrt{2}+\sqrt{7}\right)\left(\sqrt{3}+2\sqrt{2}\right)=\sqrt{2}\times\sqrt{3}+\sqrt{2}\times2\sqrt{2}+\sqrt{7}\times\sqrt{3}+\sqrt{7}\times2\sqrt{2}$

$$\begin{aligned}&=\sqrt{6}+2\times\sqrt{4}+\sqrt{21}+2\sqrt{14}\\&=\sqrt{6}+4+\sqrt{21}+2\sqrt{14}\end{aligned}$$

14 $\left(\sqrt{5}+2\right)\left(2\sqrt{5}+3\right)=2\sqrt{25}+3\sqrt{5}+4\sqrt{5}+6$

$$\begin{aligned}&=10+7\sqrt{5}+6\\&=16+7\sqrt{5}\end{aligned}$$

16 $\left(\sqrt{3}-\sqrt{2}\right)\left(2\sqrt{3}-\sqrt{2}\right)=2\sqrt{9}-\sqrt{6}-2\sqrt{6}+\sqrt{4}$

$$\begin{aligned}&=6-\sqrt{6}-2\sqrt{6}+2\\&=8-3\sqrt{6}\end{aligned}$$

18 $\left(2\sqrt{2}-\sqrt{6}\right)\left(2\sqrt{3}-1\right)=4\sqrt{6}-2\sqrt{2}-2\sqrt{18}+\sqrt{6}$

$$\begin{aligned}&=4\sqrt{6}-2\sqrt{2}-2\sqrt{9}\times\sqrt{2}+\sqrt{6}\\&=5\sqrt{6}-2\sqrt{2}-6\sqrt{2}\\&=5\sqrt{6}-8\sqrt{2}\end{aligned}$$

20 $\left(\sqrt{5}-\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2-2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$

$$\begin{aligned}&=5-2\sqrt{10}+2\\&=7-2\sqrt{10}\end{aligned}$$

22 $\left(2\sqrt{2}-1\right)\left(2\sqrt{2}+1\right)=\left(2\sqrt{2}\right)^2-1^2$

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$$= 2\sqrt{2} \times 2\sqrt{2} - 1$$
$$= 8 - 1$$
$$= 7$$

24 $\left(\sqrt{11} - \sqrt{7}\right)\left(\sqrt{11} + \sqrt{7}\right) = \left(\sqrt{11}\right)^2 - \left(\sqrt{7}\right)^2$

$$= 11 - 7$$
$$= 4$$

26 D

$$\left(3\sqrt{7} - 2\right)^2 = \left(3\sqrt{7}\right)^2 - 2 \times 3\sqrt{7} \times 2 + 2^2$$
$$= 9\sqrt{49} - 12\sqrt{7} + 4$$
$$= 63 - 12\sqrt{7} + 4$$
$$= 67 - 12\sqrt{7}$$

28 $\left(\sqrt{11} + \sqrt{7}\right)^2 = \left(\sqrt{11}\right)^2 - 2 \times \sqrt{11} \times \sqrt{7} + \left(\sqrt{7}\right)^2$

$$= 11 + 2\sqrt{77} + 7$$
$$= 18 + 2\sqrt{77}$$

30 $\left(\sqrt{11} - \sqrt{10}\right)\left(\sqrt{11} + \sqrt{10}\right) = \left(\sqrt{11}\right)^2 - \left(\sqrt{10}\right)^2$

$$= 11 - 10$$
$$= 1$$

32 $\left(2\sqrt{2} + \sqrt{3}\right)^2 = \left(2\sqrt{2}\right)^2 + 2 \times 2\sqrt{2} \times \sqrt{3} + \left(\sqrt{3}\right)^2$

$$= 4\sqrt{4} + 4\sqrt{6} + 3$$
$$= 8 + 4\sqrt{6} + 3$$
$$= 11 + 4\sqrt{6}$$

34 $\left(\sqrt{5} + 2\sqrt{2}\right)\left(\sqrt{6} - 1\right) = \sqrt{30} - \sqrt{5} + 2\sqrt{12} - 2\sqrt{2}$

$$= \sqrt{30} - \sqrt{5} + 2\sqrt{4} \times \sqrt{3} - 2\sqrt{2}$$
$$= \sqrt{30} - \sqrt{5} + 4\sqrt{3} - 2\sqrt{2}$$

36 $\left(4\sqrt{3} + 1\right)\left(2\sqrt{3} - 3\right) = 8\sqrt{9} - 12\sqrt{3} + 2\sqrt{3} - 3$

$$= 24 - 12\sqrt{3} + 2\sqrt{3} - 3$$
$$= 21 - 10\sqrt{3}$$

(a) incorrect

(b) correct

(c) correct

(d) incorrect

38 $\left(2\sqrt{7}+3\sqrt{6}\right)^2=\left(2\sqrt{7}\right)^2+2\times2\sqrt{7}\times3\sqrt{6}+\left(3\sqrt{6}\right)^2$

$$\begin{aligned}&=4\sqrt{49}+12\sqrt{42}+9\sqrt{36}\\&=28+12\sqrt{42}+54\\&=82+12\sqrt{42}\end{aligned}$$

40 $\left(2\sqrt{2}+3\sqrt{3}\right)^2=\left(2\sqrt{2}\right)^2+2\times2\sqrt{2}\times3\sqrt{3}+\left(3\sqrt{3}\right)^2$

$$\begin{aligned}&=4\sqrt{4}+12\sqrt{6}+9\sqrt{9}\\&=8+12\sqrt{6}+27\\&=35+12\sqrt{6}\end{aligned}$$

42 $\left(2\sqrt{3}-1\right)^2=\left(2\sqrt{3}\right)^2-4\sqrt{3}+1$

$$\begin{aligned}&=4\times3-4\sqrt{3}+1\\&=13-4\sqrt{3}\end{aligned}$$

EXERCISE 1.14 RATIONALISING DENOMINATORS

2 $\dfrac{\sqrt{5}}{\sqrt{3}}=\dfrac{\sqrt{5}}{\sqrt{3}}\times\dfrac{\sqrt{3}}{\sqrt{3}}$

$$=\frac{\sqrt{15}}{3}$$

4 $\dfrac{1}{\sqrt{3}-\sqrt{2}}=\dfrac{1}{\sqrt{3}-\sqrt{2}}\times\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\begin{aligned}&=\frac{\sqrt{3}+\sqrt{2}}{3-2}\\&=\sqrt{3}+\sqrt{2}\end{aligned}$$

6 $\dfrac{1}{\sqrt{5}+2}=\dfrac{1}{\sqrt{5}+2}\times\dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

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$$= \frac{\sqrt{5}-2}{5-4}$$
$$= \sqrt{5}-2$$

8 $$\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{3\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{3\sqrt{10}+3\sqrt{6}}{5-3}$$
$$= \frac{3\left(\sqrt{10}+\sqrt{6}\right)}{2}$$

10 $$\frac{2\sqrt{5}}{3\sqrt{11}-2\sqrt{8}} = \frac{2\sqrt{5}}{3\sqrt{11}-2\sqrt{8}} \times \frac{3\sqrt{11}+2\sqrt{8}}{3\sqrt{11}+2\sqrt{8}}$$

$$= \frac{6\sqrt{55}+4\sqrt{40}}{9\times 11-4\times 8}$$
$$= \frac{6\sqrt{55}+4\sqrt{4}\times\sqrt{10}}{9\times 11-4\times 8}$$
$$= \frac{6\sqrt{55}+8\sqrt{10}}{67}$$

12 $$\frac{4\sqrt{2}+3\sqrt{5}}{2\sqrt{5}-\sqrt{2}} = \frac{4\sqrt{2}+3\sqrt{5}}{2\sqrt{5}-\sqrt{2}} \times \frac{2\sqrt{5}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$$

$$= \frac{4\sqrt{2}\times 2\sqrt{5}+4\sqrt{2}\times\sqrt{2}+3\sqrt{5}\times 2\sqrt{5}+3\sqrt{5}\times\sqrt{2}}{4\times 5-2}$$
$$= \frac{8\sqrt{10}+8+30+3\sqrt{10}}{18}$$
$$= \frac{38+11\sqrt{10}}{18}$$

14 $$\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

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$$=\frac{\left(3\sqrt{2}\right)^2+2\times3\sqrt{2}\times2\sqrt{3}+\left(2\sqrt{3}\right)^2}{9\times2-4\times3}$$

$$=\frac{9\times2+12\sqrt{6}+4\times3}{6}$$

$$=\frac{18+12\sqrt{6}+12}{6}$$

$$=\frac{30+12\sqrt{6}}{6}$$

$$=\frac{6\left(5+2\sqrt{6}\right)}{6}$$

$$=5+2\sqrt{6}$$

16 $\dfrac{2\sqrt{3}}{3\sqrt{3}-2}=\dfrac{2\sqrt{3}}{3\sqrt{3}-2}\times\dfrac{3\sqrt{3}+2}{3\sqrt{3}+2}$

$$=\frac{6\times3+4\sqrt{3}}{9\times3-4}$$

$$=\frac{18+4\sqrt{3}}{23}$$

18 $\dfrac{5\sqrt{11}+3}{3\sqrt{11}-2}=\dfrac{5\sqrt{11}+3}{3\sqrt{11}-2}\times\dfrac{3\sqrt{11}+2}{3\sqrt{11}+2}$

$$=\frac{5\sqrt{11}\times3\sqrt{11}+5\sqrt{11}\times2+3\times3\sqrt{11}+6}{9\times11-4}$$

$$=\frac{15\times11+10\sqrt{11}+9\sqrt{11}+6}{9\times11-4}$$

$$=\frac{165+19\sqrt{11}+6}{95}$$

$$=\frac{171+19\sqrt{11}}{95}$$

$$=\frac{19\left(9+\sqrt{11}\right)}{19\times5}$$

$$=\frac{9+\sqrt{11}}{5}$$

20 $\dfrac{3\sqrt{3}}{2\sqrt{3}+\sqrt{2}}=\dfrac{3\sqrt{3}}{2\sqrt{3}+\sqrt{2}}\times\dfrac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-\sqrt{2}}$

$$= \frac{6\times 3-3\sqrt{6}}{4\times 3-2}$$
$$= \frac{18-3\sqrt{6}}{10}$$
$$= \frac{3\left(6-\sqrt{6}\right)}{10}$$

22 $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}$

$$= \frac{\left(\sqrt{2}\right)^2-2\sqrt{2}+1}{2-1}$$
$$= 2-2\sqrt{2}+1$$
$$= 3-2\sqrt{2}$$

24 $\frac{\sqrt{6}+2\sqrt{3}}{2\sqrt{6}-\sqrt{3}} = \frac{\sqrt{6}+2\sqrt{3}}{2\sqrt{6}-\sqrt{3}}\times\frac{2\sqrt{6}+\sqrt{3}}{2\sqrt{6}+\sqrt{3}}$

$$= \frac{\sqrt{6}\times 2\sqrt{6}+\sqrt{6}\times\sqrt{3}+2\sqrt{3}\times 2\sqrt{6}+2\sqrt{3}\times\sqrt{3}}{4\times 6-3}$$
$$= \frac{2\times 6+\sqrt{18}+4\sqrt{18}+2\times 3}{21}$$
$$= \frac{12+\sqrt{9}\times\sqrt{2}+4\times\sqrt{9}\times\sqrt{2}+6}{21}$$
$$= \frac{18+3\sqrt{2}+12\sqrt{2}}{21}$$
$$= \frac{18+15\sqrt{2}}{21}$$
$$= \frac{3\left(6+5\sqrt{2}\right)}{21}$$
$$= \frac{6+5\sqrt{2}}{7}$$

26 $\frac{\sqrt{5}+\sqrt{3}}{2\sqrt{10}-\sqrt{6}} = \frac{\sqrt{5}+\sqrt{3}}{2\sqrt{10}-\sqrt{6}}\times\frac{2\sqrt{10}+\sqrt{6}}{2\sqrt{10}+\sqrt{6}}$

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$$
\begin{aligned}
&= \frac{\sqrt{5}\times 2\sqrt{10}+\sqrt{5}\times\sqrt{6}+\sqrt{3}\times 2\sqrt{10}+\sqrt{3}\times\sqrt{6}}{4\times 10-6} \\
&= \frac{10\sqrt{2}+\sqrt{30}+2\sqrt{30}+\sqrt{18}}{34} \\
&= \frac{10\sqrt{2}+3\sqrt{30}+\sqrt{9\times 2}}{34} \\
&= \frac{10\sqrt{2}+3\sqrt{30}+3\sqrt{2}}{34} \\
&= \frac{13\sqrt{2}+3\sqrt{30}}{34}
\end{aligned}
$$

28 A

$$
\begin{aligned}
\frac{3-2\sqrt{2}}{3+2\sqrt{2}}+\frac{3+2\sqrt{2}}{3-2\sqrt{2}} &= \frac{\left(3-2\sqrt{2}\right)^2+\left(3+2\sqrt{2}\right)^2}{9-\left(2\sqrt{2}\right)^2} \\
&= \frac{9-2\times 3\times 2\sqrt{2}+\left(2\sqrt{2}\right)^2+9+2\times 3\times 2\sqrt{2}+\left(2\sqrt{2}\right)^2}{9-4\times 2} \\
&= \frac{9+8+9+8}{1} \\
&= 34
\end{aligned}
$$

30

$$
\begin{aligned}
\frac{\left(3\sqrt{2}+1\right)^2-2\left(3\sqrt{2}+1\right)}{3\sqrt{2}+1-1} &= \frac{\left(3\sqrt{2}\right)^2+2\times 3\sqrt{2}+1-6\sqrt{2}-2}{3\sqrt{2}} \\
&= \frac{18+6\sqrt{2}+1-6\sqrt{2}-2}{3\sqrt{2}} \\
&= \frac{17}{3\sqrt{2}} \\
&= \frac{17}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \\
&= \frac{17\sqrt{2}}{6}
\end{aligned}
$$

32 LS $=\left(2\sqrt{2}-3\right)^2+6\times\left(2\sqrt{2}-3\right)+1$

$$
\begin{aligned}
&= \left(2\sqrt{2}\right)^2-12\sqrt{2}+9+12\sqrt{2}-18+1 \\
&= 8+9-18+1 \\
&= 0
\end{aligned}
$$

Therefore, $x=2\sqrt{2}-3$ is one solution of the equation $x^2+6x+1=0$.

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34 $$\begin{aligned}\text{LS} &= \left(\sqrt{5}-1\right)^3 + 3\times\left(\sqrt{5}-1\right)^2 - 2\times\left(\sqrt{5}-1\right) - 4 \\ &= \left(\sqrt{5}\right)^3 - 3\left(\sqrt{5}\right)^2 + 3\sqrt{5} - 1 + 3\left(5 - 2\sqrt{5} + 1\right) - 2\sqrt{5} + 2 - 4 \\ &= 5\sqrt{5} - 15 + 3\sqrt{5} - 1 + 15 - 6\sqrt{5} + 3 - 2\sqrt{5} - 2 \\ &= 0\end{aligned}$$

Therefore, $x = \sqrt{5} - 1$ is one solution of the equation $x^3 + 3x^2 - 2x - 4 = 0$.

36 $$\begin{aligned}\frac{1}{2\sqrt{3}-1} + \frac{3}{\sqrt{3}+1} &= \frac{1}{2\sqrt{3}-1}\times\frac{2\sqrt{3}+1}{2\sqrt{3}+1} + \frac{3}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{2\sqrt{3}+1}{12-1} + \frac{3\sqrt{3}-3}{3-1} \\ &= \frac{2\sqrt{3}+1}{11} + \frac{3\sqrt{3}-3}{2} \\ &= \frac{2\left(2\sqrt{3}+1\right) + 11\left(3\sqrt{3}-3\right)}{22} \\ &= \frac{4\sqrt{3} + 2 + 33\sqrt{3} - 33}{22} \\ &= \frac{37\sqrt{3} - 31}{22}\end{aligned}$$

38 $$\begin{aligned}\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}+\sqrt{3}} &= \frac{2\sqrt{6} - 3 - 4 + \sqrt{6}}{2\sqrt{6} + 3 + 4 + \sqrt{6}} \\ &= \frac{3\sqrt{6}-7}{3\sqrt{6}+7} \\ &= \frac{3\sqrt{6}-7}{3\sqrt{6}+7}\times\frac{3\sqrt{6}-7}{3\sqrt{6}-7} \\ &= \frac{9\times 6 - 21\sqrt{6} - 21\sqrt{6} + 49}{9\times 6 - 49} \\ &= \frac{54 - 42\sqrt{6} + 49}{54 - 49} \\ &= \frac{103 - 42\sqrt{6}}{5}\end{aligned}$$

40 $$\frac{2\sqrt{5}+1}{2\sqrt{5}-1} - \frac{\sqrt{5}-1}{2\sqrt{5}-3} = \frac{2\sqrt{5}+1}{2\sqrt{5}-1}\times\frac{2\sqrt{5}+1}{2\sqrt{5}+1} - \frac{\sqrt{5}-1}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3}$$

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$$=\frac{20+4\sqrt{5}+1}{20-1}-\frac{10+3\sqrt{5}-2\sqrt{5}-3}{20-9}$$
$$=\frac{21+4\sqrt{5}}{19}-\frac{7+\sqrt{5}}{11}$$
$$=\frac{11\left(21+4\sqrt{5}\right)-19\left(7+\sqrt{5}\right)}{209}$$
$$=\frac{231+44\sqrt{5}-133-19\sqrt{5}}{209}$$
$$=\frac{98+25\sqrt{5}}{209}$$

42 $$\frac{\sqrt{3}-1}{\sqrt{3}+2}-\frac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}=\frac{\sqrt{3}-1}{\sqrt{3}+2}\times\frac{\sqrt{3}-2}{\sqrt{3}-2}-\frac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}\times\frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}-\sqrt{3}}$$
$$=\frac{3-2\sqrt{3}-\sqrt{3}+2}{3-4}-\frac{10-\sqrt{15}-2\sqrt{15}+3}{20-3}$$
$$=\frac{5-3\sqrt{3}}{-1}-\frac{13-3\sqrt{15}}{17}$$
$$=\frac{17\left(3\sqrt{3}-5\right)-13+3\sqrt{15}}{17}$$
$$=\frac{51\sqrt{3}-85-13+3\sqrt{15}}{17}$$
$$=\frac{51\sqrt{3}+3\sqrt{15}-98}{17}$$

44 It is easier to simplify the algebraic expression first.

$$\frac{1}{x-1}+\frac{1}{x+1}-\frac{2}{x^2-1}=\frac{x+1}{(x-1)(x+1)}+\frac{x-1}{(x-1)(x+1)}-\frac{2}{(x-1)(x+1)}$$
$$=\frac{2x-2}{x^2-1}$$

Substitute $2\sqrt{3}+1$.

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$$\begin{aligned}\frac{2x-2}{x^2-1}&=\frac{2\left(2\sqrt{3}+1\right)-2}{\left(2\sqrt{3}+1\right)^2-1}\\&=\frac{4\sqrt{3}+2-2}{12+4\sqrt{3}+1-1}\\&=\frac{4\sqrt{3}}{12+4\sqrt{3}}\\&=\frac{\sqrt{3}}{3+\sqrt{3}}\\&=\frac{\sqrt{3}}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}\\&=\frac{3\sqrt{3}-3}{9-3}\\&=\frac{3\left(\sqrt{3}-1\right)}{6}\\&=\frac{\sqrt{3}-1}{2}\end{aligned}$$

CHAPTER REVIEW 1

2 **(a)** $81-4a^2=9^2-(2a)^2$

$$=(9-2a)(9+2a)$$

(b) The highest common factor is $5y$.

$$10xy-5y=5y(2x-1)$$

(c) The highest common factor is $5xy$.

$$5x^2y-10xy^2-5xy=5xy(x-2y-1)$$

(d) Two numbers which add to -18 and multiply to make 56 are -4 and -14.

$$a^2-18a+56=(a-4)(a-14)$$

(e) $16x^2-1=(4x)^2-1^2$

$$=(4x-1)(4x+1)$$

(f) The factors of $3x^2$ are $3x$ and x.

The factors of 7 are 7 and 1.

Set up the cross method.

$$\begin{array}{ccccc} 3x & 7 & -1 & -7 & 1 \\ x & -1 & 7 & 1 & -7 \end{array}$$

Calculate the sums, looking for a sum of $4x$.

$3x \times -1 + x \times 7 = -3x + 7x = 4x$

The combination that gives $4x$ is: $\begin{array}{cc} 3x & 7 \\ x & -1 \end{array}$.

$3x^2 + 4x - 7 = (3x+7)(x-1)$

(g) Factorise the two pairs of terms.

$$\begin{aligned} a^2 - b^2 + 2a - 2b &= (a-b)(a+b) + 2(a-b) \\ &= (a-b)(a+b+2) \end{aligned}$$

(h) Take out the common factor $-x$ (to make the coefficient of x^2 positive) and rearrange so the terms are in order of descending powers of x.

$3x + 7x^2 - 6x^3 = -x(6x^2 - 7x - 3)$

The factors of $6x^2$ are $6x$ and x; or $3x$ and $2x$.

The factors of 3 are 3 and 1.

Set up the cross method.

$$\begin{array}{cccccc} 6x & 3x & 3 & -1 & -3 & 1 \\ x & 2x & -1 & 3 & 1 & -3 \end{array}$$

Calculate the sums, looking for a sum of $-7x$. Ignore any combinations which will obviously be odd or a multiple of 3.

$6x \times -1 + x \times 3$ is both odd and a multiple of 3.

$6x \times 3 + x \times -1$ is too large.

The other two combinations involving $6x$ and x are similar.

$3x \times -1 + 2x \times 3$ is both odd and a multiple of 3

$3x \times 3 + 2x \times -1 = 9x - 2x = 7x$

$3x \times -3 + 2x \times 1 = -9x + 2x = -7x$

The combination that gives $-7x$ is: $\begin{array}{cc} 3x & 1 \\ 2x & -3 \end{array}$.

$$\begin{aligned}3x+7x^2-6x^3&=-x(3x+1)(2x-3)\\&=x(3x+1)(3-2x)\end{aligned}$$

(i) $8a^3-27=(2a)^3-3^3$

$$\begin{aligned}&=(2a-3)\left((2a)^2+2a\times3+3^2\right)\\&=(2a-3)\left(4a^2+6a+9\right)\end{aligned}$$

(j) $8-(x+h)^3=2^3-(x+h)^3$

$$\begin{aligned}&=\left[2-(x+h)\right]\left[2^2+2(x+h)+(x+h)^2\right]\\&=(2-x-h)\left(4+2x+2h+x^2+2xh+h^2\right)\\&=(2-x-h)\left(x^2+2xh+h^2+2x+2h+4\right)\end{aligned}$$

(k) The factors of $8y^2$ are $8y$ and y; or $4y$ and $2y$.

The factors of 9 are 9 and 1; or 3 and 3.

Set up the cross method.

$$\begin{array}{cccccccccc}8y & 4y & \multirow{2}{*}{$\times$} & 9 & -1 & -9 & 1 & 3 & -3\\ y & 2y & & -1 & 9 & 1 & -9 & -3 & 3\end{array}$$

Calculate the sums, looking for a sum of $-6y$. Since $-6y$ is even, we can ignore $8y$ and y.

$4y\times-1+2y\times9=-4y+18y=14y$

$4y\times9+2y\times-1$ is clearly too large and $4y\times-9+2y\times1$ will be too small (too negative).

$4y\times1+2y\times-9=4y-18y=-14y$

$4y\times-3+2y\times3=-12y+6y=-6y$

The combination that gives $-6y$ is: $\begin{array}{ccc}4y & \multirow{2}{*}{$\times$} & 3\\ 2y & & -3\end{array}$.

$8y^2-6y-9=(4y+3)(2y-3)$

(l) $x^4-8x=x\left(x^3-8\right)$

$$\begin{aligned}&=x\left(x^3-2^3\right)\\&=x(x-2)\left(x^2+2x+4\right)\end{aligned}$$

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4 (a) $$\frac{3x^3}{4a^2}\times\frac{ay-a}{xy^2}\div\frac{3y-3}{4ay^2}=\frac{3x^3}{4a^2}\times\frac{a(y-1)}{xy^2}\times\frac{4ay^2}{3(y-1)}$$

$$=x^2$$

(b) $$\frac{5x}{3}-\frac{2x+3}{4}+\frac{x}{6}=\frac{20x}{12}-\frac{3(2x+3)}{12}+\frac{2x}{12}$$

$$=\frac{20x-6x-9+2x}{12}$$

$$=\frac{16x-9}{12}$$

(c) $$\frac{12m^2-4n}{3m^2-n}\div\frac{10m^2n}{5m^2n^2}=\frac{4(3m^2-n)}{3m^2-n}\times\frac{5m^2n^2}{10m^2n}$$

$$=2n$$

(d) $$\frac{3x+4}{x^2}-\frac{5}{x}=\frac{3x+4}{x^2}-\frac{5x}{x^2}$$

$$=\frac{3x+4-5x}{x^2}$$

$$=\frac{4-2x}{x^2}$$

(e) $$\frac{3}{x+1}+\frac{1}{x^2+2x+1}=\frac{3}{x+1}+\frac{1}{(x+1)^2}$$

$$=\frac{3(x+1)}{(x+1)^2}+\frac{1}{(x+1)^2}$$

$$=\frac{3x+3+1}{(x+1)^2}$$

$$=\frac{3x+4}{(x+1)^2}$$

(f) $$\frac{2x^2-3xy}{xy-y^2}\div\frac{4x-6y}{2x^2-2xy}=\frac{x(2x-3y)}{y(x-y)}\times\frac{2x(x-y)}{2(x-3y)}$$

$$=\frac{x^2}{y}$$

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(g) $\frac{a^3+b^3}{a^2-b^2}=\frac{(a+b)(a^2-ab+b^2)}{(a-b)(a+b)}$

$=\frac{a^2-ab+b^2}{a-b}$

(h) $\frac{2}{m^2-4}-\frac{1}{m^2-3m+2}=\frac{2}{(m-2)(m+2)}-\frac{1}{(m-2)(m-1)}$

$=\frac{2(m-1)}{(m-2)(m+2)(m-1)}-\frac{m+2}{(m-2)(m+2)(m-1)}$

$=\frac{2m-2-m-2}{(m-2)(m+2)(m-1)}$

$=\frac{m-4}{(m-2)(m+2)(m-1)}$

(i) $\frac{1}{x^2-9x+20}+\frac{1}{x^2-11x+30}=\frac{1}{(x-4)(x-5)}+\frac{1}{(x-5)(x-6)}$

$=\frac{x-6}{(x-4)(x-5)(x-6)}+\frac{x-4}{(x-4)(x-5)(x-6)}$

$=\frac{x-6+x-4}{(x-4)(x-5)(x-6)}$

$=\frac{2x-10}{(x-4)(x-5)(x-6)}$

$=\frac{2(x-5)}{(x-4)(x-5)(x-6)}$

$=\frac{2}{(x-4)(x-6)}$

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6 $$\frac{a^4b}{c^4}=\frac{\left[\left(\frac{2}{3}\right)^2\right]^4\times\left(\frac{8}{3}\right)^7}{\left[\left(\frac{4}{3}\right)^4\right]^4}$$

$$\begin{aligned}&=\frac{2^8}{3^8}\times\frac{\left(2^3\right)^7}{3^7}\div\frac{4^{16}}{3^{16}}\\&=\frac{2^8}{3^8}\times\frac{2^{21}}{3^7}\times\frac{3^{16}}{\left(2^2\right)^{16}}\\&=\frac{2^{29}\times3^{16}}{3^{15}\times2^{32}}\\&=\frac{3}{2^3}\\&=\frac{3}{8}\end{aligned}$$

8 **(a)** $$\begin{aligned}\sqrt{12}+4\sqrt{3}+\sqrt{27}-3\sqrt{54}&=\sqrt{4}\times\sqrt{3}+4\sqrt{3}+\sqrt{9}\times\sqrt{3}-3\sqrt{9}\times\sqrt{6}\\&=2\sqrt{3}+4\sqrt{3}+3\sqrt{3}-9\sqrt{6}\\&=9\sqrt{3}-9\sqrt{6}\end{aligned}$$

(b) $$\begin{aligned}\frac{2}{\sqrt{5}}+\sqrt{20}+\frac{8}{\sqrt{80}}&=\frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}+\sqrt{4}\times\sqrt{5}+\frac{8}{\sqrt{16}\times\sqrt{5}}\\&=\frac{2\sqrt{5}}{5}+2\sqrt{5}+\frac{8}{4\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}\\&=\frac{2\sqrt{5}}{5}+\frac{10\sqrt{5}}{5}+\frac{2\sqrt{5}}{5}\\&=\frac{14\sqrt{5}}{5}\end{aligned}$$

10 $$\begin{aligned}\left(3\sqrt{5}-2\sqrt{3}\right)^2&=\left(3\sqrt{5}\right)^2-2\times3\sqrt{5}\times2\sqrt{3}+\left(2\sqrt{3}\right)^2\\&=45-12\sqrt{15}+12\\&=57-12\sqrt{15}\end{aligned}$$

12 $$\begin{aligned}\frac{\sqrt{2}-1}{2\sqrt{2}-1}\times\frac{2\sqrt{2}+1}{2\sqrt{2}+1}&=\frac{4+\sqrt{2}-2\sqrt{2}-1}{8-1}\\&=\frac{3-\sqrt{2}}{7}\end{aligned}$$

14 $x^3+3x^2+x-2=\left(\frac{\sqrt{5}-1}{2}\right)^3+3\left(\frac{\sqrt{5}-1}{2}\right)^2+\frac{\sqrt{5}-1}{2}-2$

$$\begin{aligned}
&=\frac{5\sqrt{5}-3\times5+3\sqrt{5}-1}{8}+3\times\frac{5-2\sqrt{5}+1}{4}+\frac{\sqrt{5}-1}{2}-2\\
&=\frac{8\sqrt{5}-16}{8}+\frac{6\left(6-2\sqrt{5}\right)}{8}+\frac{4\left(\sqrt{5}-1\right)}{8}-\frac{16}{8}\\
&=\frac{8\sqrt{5}-16+36-12\sqrt{5}+4\sqrt{5}-4-16}{8}\\
&=\frac{0}{8}\\
&=0
\end{aligned}$$

Therefore $x=\frac{\sqrt{5}-1}{2}$ is a root of the equation $x^3+3x^2+x-2=0$.

16 (a) $2x^2-8=2\left(x^2-4\right)=2(x-2)(x+2)$

(b) $6x^2-216=6\left(x^2-36\right)=6(x-6)(x+6)$

18 $\frac{H}{H-y}=\frac{R}{x}$

$$\begin{aligned}
\frac{H-y}{H}&=\frac{x}{R}\\
H-y&=\frac{Hx}{R}\\
H&=\frac{Hx}{R}+y\\
y&=\frac{Hx}{R}-H
\end{aligned}$$

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