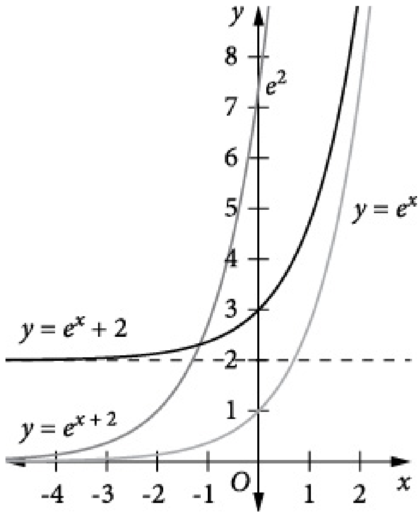
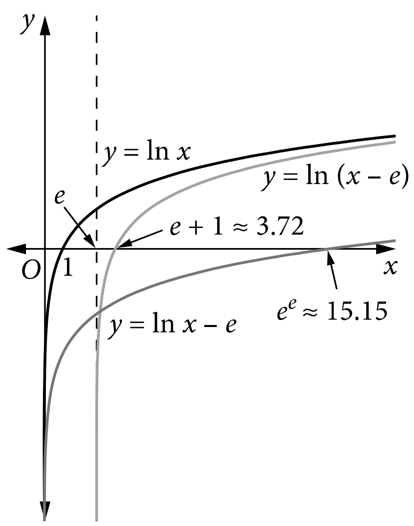
EXERCISE 15.1 TRANSFORMATION OF GRAPHS USING $y = f(x + b)$ AND $y = f(x)+c$

2 (a)



$y = e^x$ cuts the y-axis at 1. Where $x \to +\infty$, $y \to +\infty$. Where $x \to -\infty$, $y \to 0^+$.
$y = e^{x+2}$ is the graph of $y = e^x$ translated to the left by 2 units. $(0,\ 1)$ on the graph $y = e^x$ becomes $(-2,\ 1)$ on the graph of $y = e^{x+2}$. Where $x = 0$, $y = e^2$, so the graph cuts the y-axis at e^2. Where $x \to +\infty$, $y \to +\infty$. Where $x \to -\infty$, $y \to 0^+$.
$y = e^x + 2$ is the graph of $y = e^x$ translated up by 2 units. $(0,\ 1)$ on the graph $y = e^x$ becomes $(0,\ 3)$ on the graph of $y = e^x + 2$. Where $x \to +\infty$, $y \to +\infty$. Where $x \to -\infty$, $y \to 2^+$.

(b)



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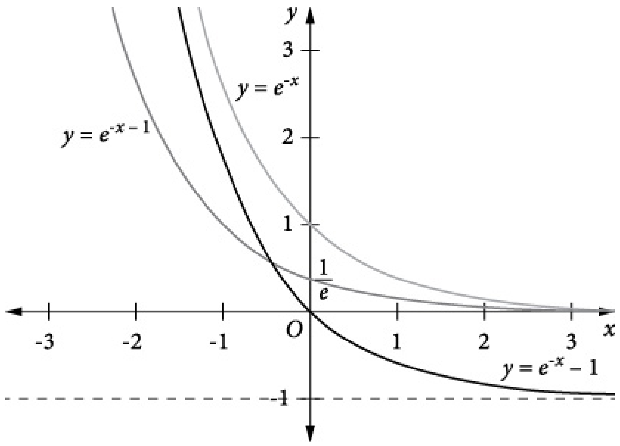
$y = \ln x$ cuts the x-axis at 1. Where $x \to +\infty$, $y \to +\infty$. Where $x \to 0^+$, $y \to -\infty$.
$y = \ln(x-e)$ is the graph $y = \ln x$ translated to the right by e units. Where $x \to +\infty$, $y \to +\infty$. Where $x \to e^+$, $y \to -\infty$. $y = \ln(x-e)$ cuts the x-axis at $1+e$.

$$\begin{aligned} \ln(x-e) &= 0 \\ x-e &= 1 \\ x &= 1+e \end{aligned}$$

$y = \ln x + e$ is the graph $y = \ln x$ translated up by e units. Where $x \to +\infty$, $y \to +\infty$. Where $x \to 0^+$, $y \to -\infty$. $y = \ln(x-e)$ cuts the x-axis at $\frac{1}{e^e}$.

$$\begin{aligned} \ln x + e &= 0 \\ \ln x &= -e \\ x &= e^{-e} \\ x &= \frac{1}{e^e} \end{aligned}$$

(c)



$y = e^{-x}$ is the graph $y = e^x$ reflected along the y-axis. $y = e^{-x}$ cuts the y-axis at 1. Where $x \to +\infty$, $y \to 0^+$. Where $x \to -\infty$, $y \to +\infty$.
$y = e^{-x-1}$ is the graph $y = e^{-x}$ translated to the left by 1 unit. $(0,\ 1)$ on the graph $y = e^{-x}$ becomes $(-1,\ 1)$ on the graph of $y = e^{-x-1}$. Where $x = 0$, $y = e^{-1}$, so the graph cuts the y-axis at $\frac{1}{e}$.Where $x \to +\infty$, $y \to 0^+$. Where $x \to -\infty$, $y \to +\infty$.
$y = e^{-x} - 1$ is the graph $y = e^{-x}$ translated down by 1 unit. $y = e^{-x} - 1$ crosses the origin $(0,\ 0)$. Where $x \to +\infty$, $y \to -1^+$. Where $x \to -\infty$, $y \to +\infty$.

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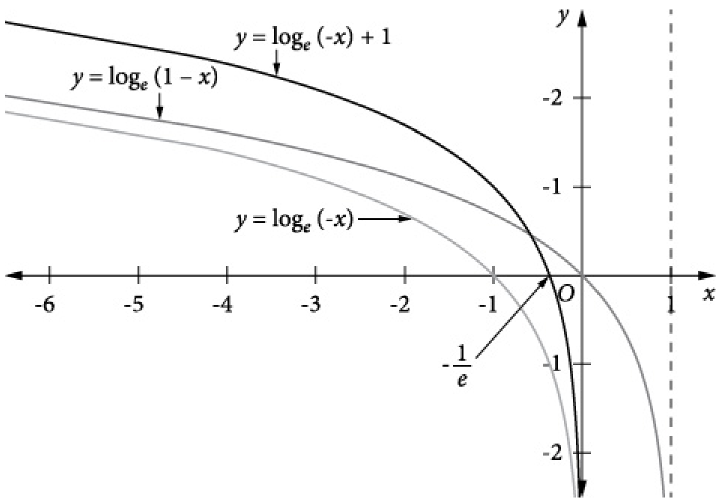
(d)



$y=\ln(-x)$ is the graph $y=\ln x$ reflected along the y-axis. $y=\ln(-x)$ cuts the x-axis at -1. Where $x\to-\infty$, $y\to+\infty$. Where $x\to0^-$, $y\to-\infty$.

$y=\ln(1-x)$ is the graph $y=\ln(-x)$ translated to the right by 1 unit. Where $x\to-\infty$, $y\to+\infty$. Where $x\to1^-$, $y\to-\infty$. $y=\ln(1-x)$ crosses the origin $(0,\ 0)$.

$y=\ln(-x)+1$ is the graph $y=\ln(-x)$ translated up by 1 unit. $(-1,\ 0)$ on the graph $y=\ln(-x)$ becomes $(-1,\ 1)$ on the graph of $y=\ln(-x)+1$. Where $x\to-\infty$, $y\to+\infty$. Where $x\to0^-$, $y\to-\infty$. $y=\ln(-x)+1$ cuts the x-axis at $-\frac{1}{e}$.

$$\begin{aligned}\ln(-x)+1&=0\\ \ln(-x)&=-1\\ -x&=e^{-1}\\ x&=-\frac{1}{e}\end{aligned}$$

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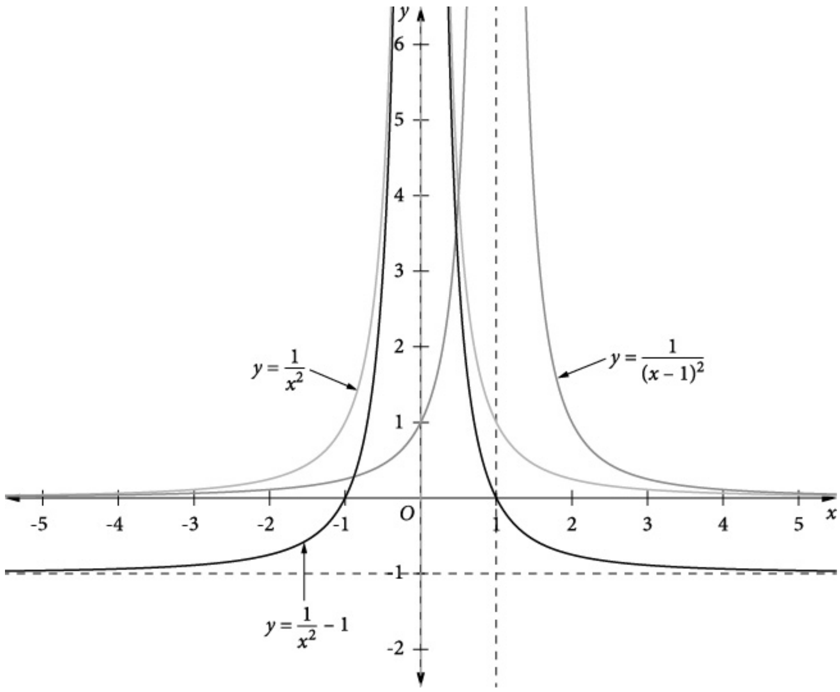
4 (a)

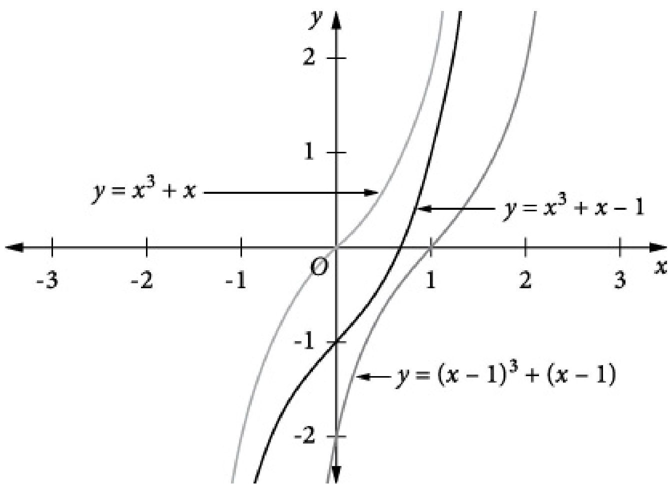


$y = \dfrac{1}{x^2}$ has horizontal asymptote at $y = 0$ and vertical asymptote at $x = 0$. Where $x \to \pm\infty$, $y \to 0$. Where $x \to 0$, $y \to +\infty$.

$y = \dfrac{1}{(x-1)^2}$ is the graph $y = \dfrac{1}{x^2}$ translated to the right by 1 unit. $y = \dfrac{1}{(x-1)^2}$ has horizontal asymptote at $y = 0$ and vertical asymptote at $x = 1$. Where $x \to \pm\infty$, $y \to 0$. Where $x \to 1$, $y \to +\infty$.

$y = \dfrac{1}{x^2} - 1$ is the graph $y = \dfrac{1}{x^2}$ translated down by 1 unit. $y = \dfrac{1}{x^2} - 1$ has horizontal asymptote at $y = -1$ and vertical asymptote at $x = 0$. Where $x \to \pm\infty$, $y \to -1$. Where $x \to 0$, $y \to +\infty$.

(b)



$$\begin{aligned} y &= x^3 + x \\ &= x\left(x^2+1\right) \end{aligned}$$

$$\begin{aligned} x\left(x^2+1\right) &= 0 \\ x &= 0 \end{aligned}$$

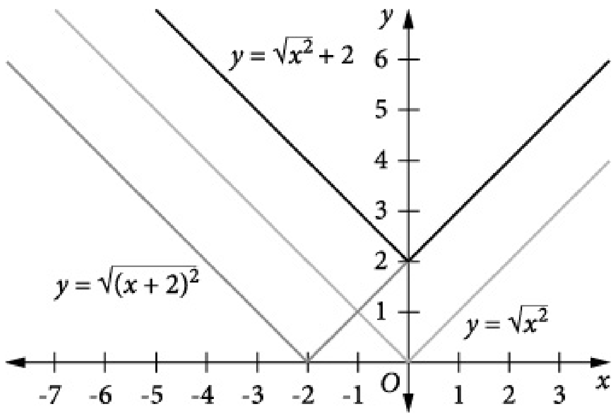
$y = x^3 + x$ crosses the origin at $(0,\ 0)$. Where $x \to +\infty$, $y \to +\infty$. Where $x \to -\infty$, $y \to -\infty$.

$y = (x-1)^3 + (x-1)$ is the graph $y = x^3 + x$ translated to the right by 1 unit. Where $x \to +\infty$, $y \to +\infty$. Where $x \to -\infty$, $y \to -\infty$. $y = (x-1)^3 + (x-1)$ cuts the x-axis at 1 and the y-axis at -2.

$$\begin{aligned} y &= (0-1)^3 + (0-1) \\ &= 2 \end{aligned}$$

$y = x^3 + x - 1$ is the graph $y = x^3 + x$ translated down by 1 unit. Where $x \to +\infty$, $y \to +\infty$. When $x \to -\infty$, $y \to -\infty$. $y = x^3 + x - 1$ cuts the y-axis at -1.

(c)



$y = \sqrt{x^2} = |x|$.

Where $x \geq 0$, $y = x$, and where $x < 0$, $y = -x$. There is a sharp turning point at $(0,\ 0)$.

$y = \sqrt{(x+2)^2}$ is the graph $y = \sqrt{x^2}$ translated to the left by 2 units. Sharp turning point at $(-2,\ 0)$. It cuts the y-axis at $\sqrt{(0+2)^2} = 2$.

$y = \sqrt{x^2} + 2$ is the graph $y = \sqrt{x^2}$ translated up by 2 units. Sharp turning point at $(0,\ -2)$.

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EXERCISE 15.2 TRANSFORMATION OF GRAPHS USING $y = kf(x)$ AND $y = kf(x+b)$

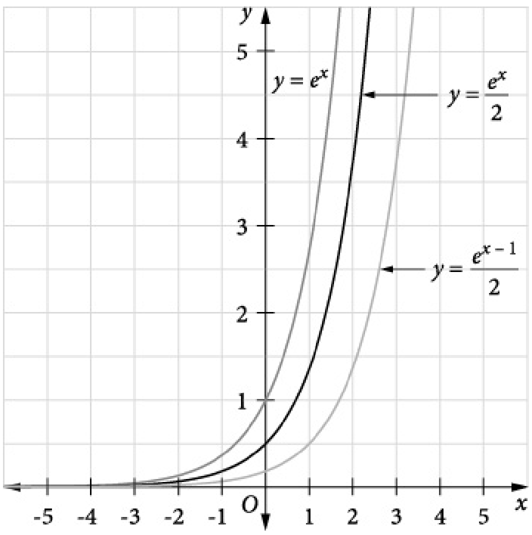
2 (a) $y = e^x$ is the exponential function with y-intercept $(0,\ 1)$ and the x-axis as the asymptote.

$y = \frac{e^x}{2}$ is the function $y = e^x$ dilated from the x-axis with factor $\frac{1}{2}$. The y-intercept will be $\left(0,\ \frac{1}{2}\right)$.

$y = \frac{e^{x-1}}{2}$ is the function $y = e^x$ translated 1 unit to the right, then dilated from the x-axis with factor $\frac{1}{2}$.

The y-intercept will be $\left(0,\ \frac{1}{2e}\right)$ i.e. about $(0,\ 0.18)$.

The dilation from the x-axis for the second and third graphs has factor $\frac{1}{2}$.



(b) $y = \sin x$ is the basic sine graph with key points

$(-\pi,\ 0),\ \left(-\frac{\pi}{2},\ -1\right),\ (0,\ 0),\ \left(\frac{\pi}{2},\ 1\right),\ (\pi,\ 0)$.

$y = 2\sin x$ is the function $y = \sin x$ dilated from the x-axis with factor 2.

The key points become $(-\pi,0),\left(-\frac{\pi}{2},-2\right),(0,0),\left(\frac{\pi}{2},2\right),(\pi,0)$.

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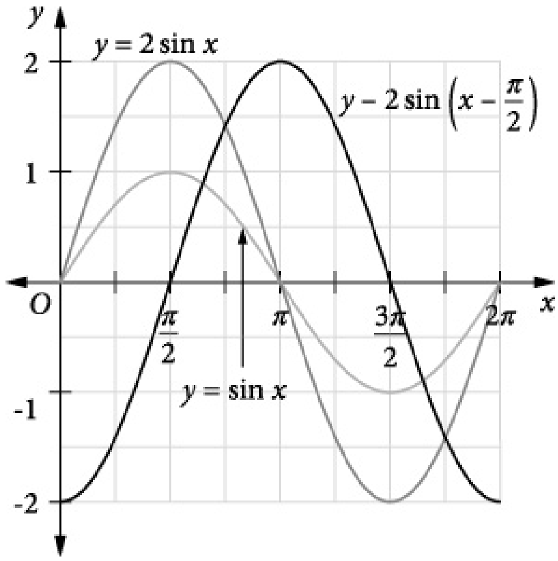
$y = 2\sin\left(x - \frac{\pi}{2}\right)$ is the function $y = \sin x$ translated $\frac{\pi}{2}$ to the right, then dilated from the x-axis with factor 2.

The key points are $(-\pi,\ 2)$, $\left(-\frac{\pi}{2},\ 0\right)$, $(0,\ -2)$, $\left(\frac{\pi}{2},\ 0\right)$, $(\pi,\ 2)$.

The dilation from the x-axis for the second and third graphs has factor 2.



(c) $y = \sec x$ is the basic secant graph with maximums at $(-\pi,\ -1)$, $(\pi,\ -1)$, a minimum at $(0,\ 1)$, and asymptotes $x = -\frac{\pi}{2},\ x = \frac{\pi}{2}$.

$y = \frac{\sec x}{2}$ is the function $y = \sec x$ dilated from the x-axis with factor $\frac{1}{2}$, so maximums are at $\left(-\pi,\ -\frac{1}{2}\right)$ and $\left(\pi,\ -\frac{1}{2}\right)$, and there is a minimum at $\left(0,\ \frac{1}{2}\right)$.

$y = \frac{\sec\left(x - \frac{\pi}{6}\right)}{2}$ is the function $y = \sec x$ translated $\frac{\pi}{6}$ units to the right, then dilated from the x-axis with factor $\frac{1}{2}$ so there is a maximum at $\left(-\frac{5\pi}{6},\ -\frac{1}{2}\right)$ and a minimum at $\left(\frac{\pi}{6},\ \frac{1}{2}\right)$.

The asymptotes are $x = -\frac{\pi}{3},\ x = \frac{2\pi}{3}$.

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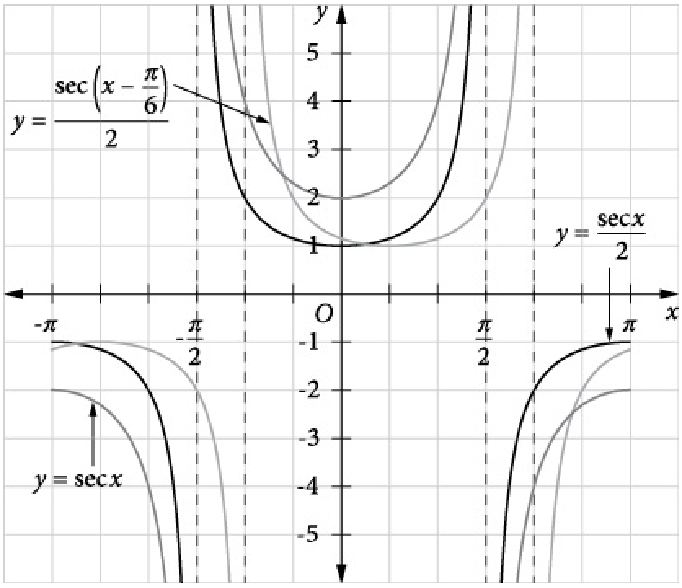
The endpoints are $\left(-\pi,\ -\frac{1}{\sqrt{3}}\right),\ \left(\pi,\ -\frac{1}{\sqrt{3}}\right)$, i.e. about $(-\pi,\ -0.58),\ (\pi,\ -0.58)$

The dilation from the *x*-axis for the second and third graphs has factor $\frac{1}{2}$.



4 B

The graph of $y = f(x)$ has vertex $(0,\ 2)$ and x-interceptss $(-\sqrt{2},\ 0),\ (\sqrt{2},\ 0)$.

The graph of $y = 3f(x)$ will have vertex $(0,\ 6)$, due to the dilation of factor 3 from the x-axis, and the x-axis points will be unchanged. The only option fitting this is **B**.

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EXERCISE 15.3 TRANSFORMATION OF GRAPHS USING $y = f(ax)$ AND $y = f(a(x + b))$

2 **(a)** $f(x) = x^3$

Replace x with $2x$.

$$f(2x) = (2x)^3 = 8x^3$$

(b) $f(x) = x^3$

Replace x with $x-1$.

$$f(x-1) = (x-1)^3$$

(c) $f(x) = x^3$

$$f(x) + 3 = x^3 + 3$$

(d) $f(x) = x^3$

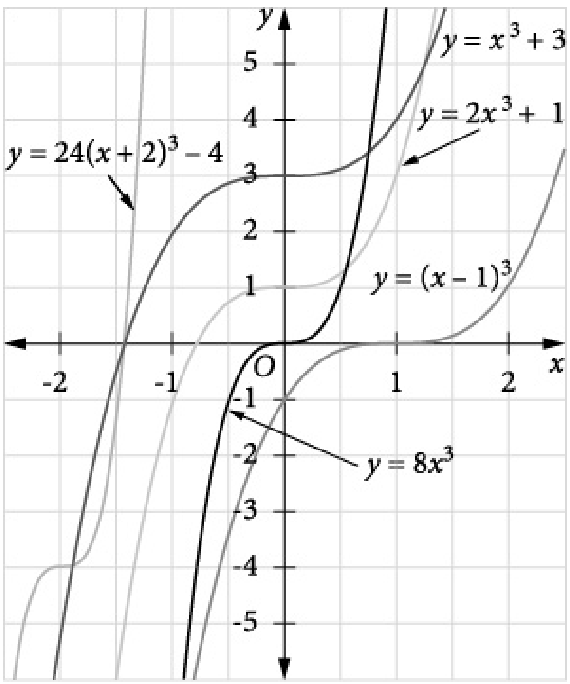
$$\begin{aligned} 2f(x) + 1 &= 2 \times x^3 + 1 \\ &= 2x^3 + 1 \end{aligned}$$

(e) $f(x) = x^3$

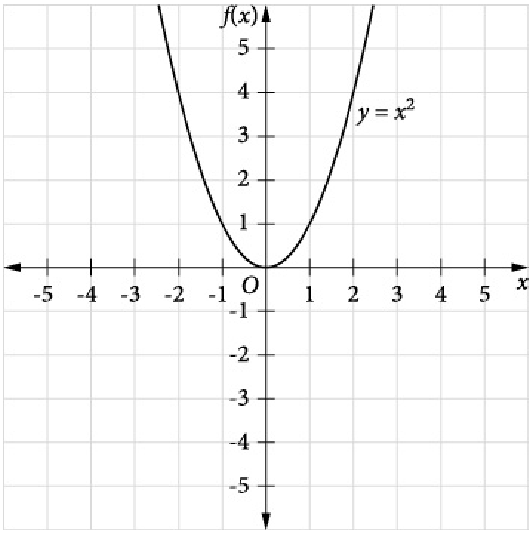
Replace x with $2(x+2)$.

$$\begin{aligned} 3f(2(x+2)) - 4 &= 3(2(x+2))^3 - 4 \\ &= 3 \times 2^3 (x+2)^3 - 4 \\ &= 24(x+2)^3 - 4 \end{aligned}$$

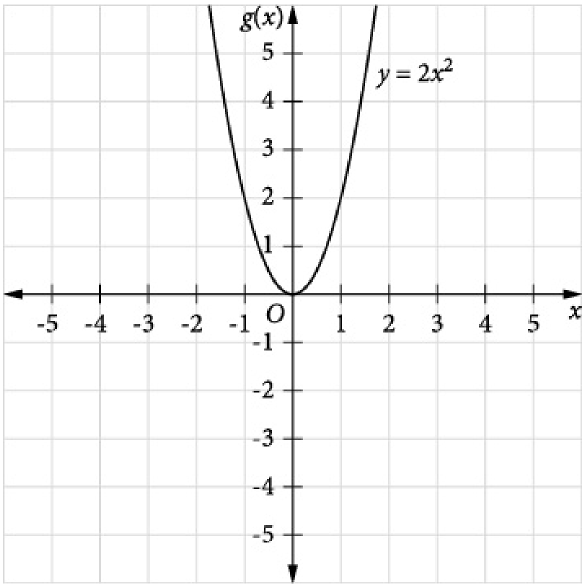
4 All of the resultant functions have been drawn on the following set of axes.



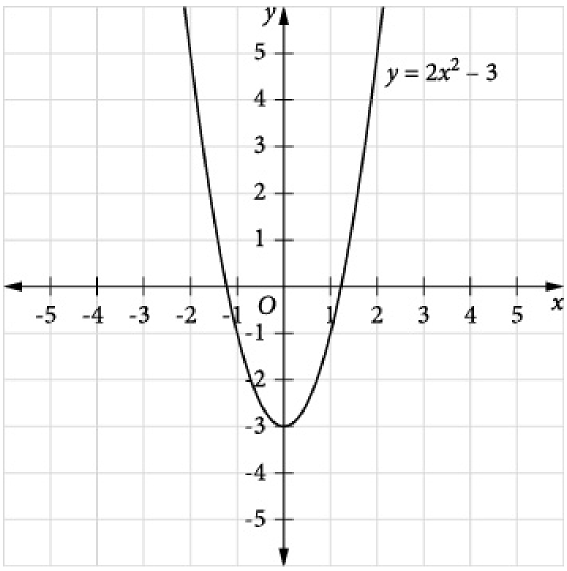
6 (a)



$y = 2f(x)$ is the graph of $f(x) = x^2$ dilated a factor of 2 from the x-axis.



$y = g(x) - 3$ is the graph of $y = g(x)$ translated 3 units down.



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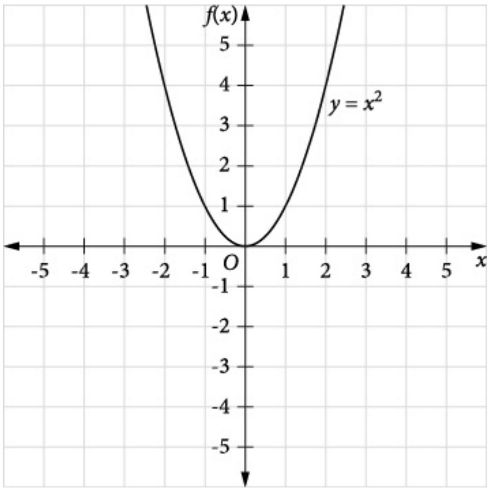
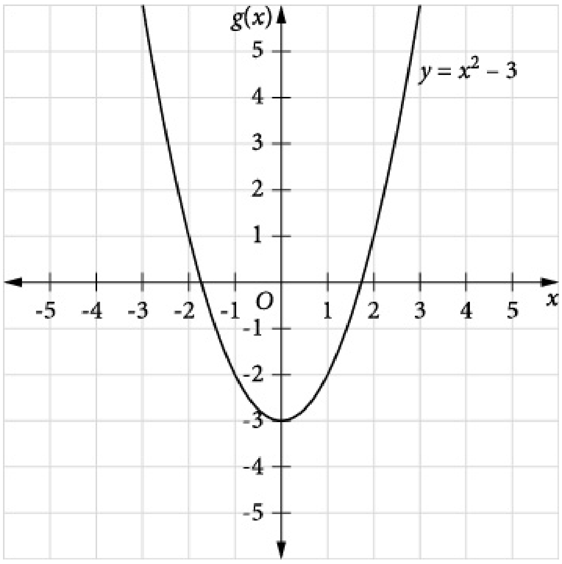
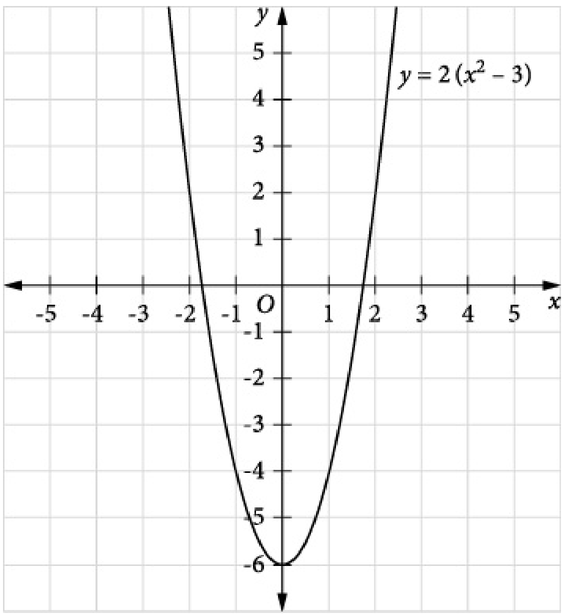
(b)



$g(x) = f(x) - 3$ is the graph of $f(x)$ translated 3 units down.

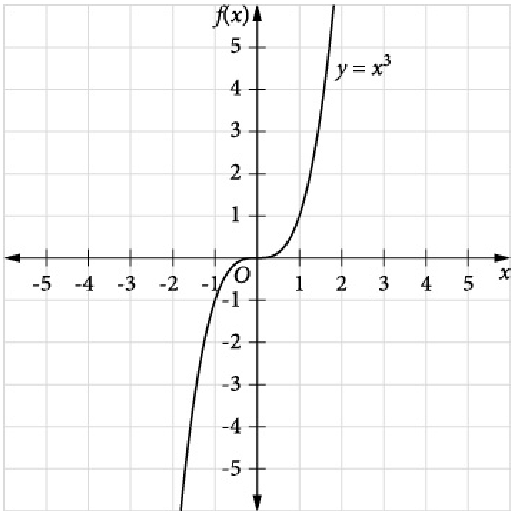


$y = 2g\left(x\right)$ is the graph of $g(x)$ dilated a factor of 2 from the x-axis.

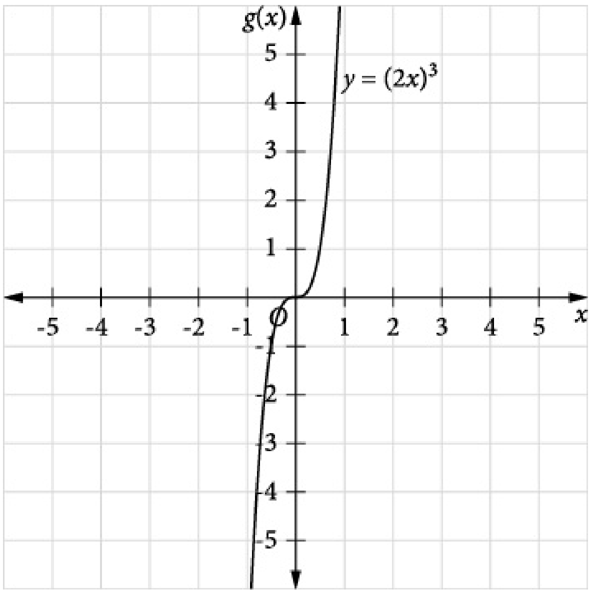


(c) The dilation is a factor of 2 in each case but in the **(b)** graph the vertical translation is double that of the **(a)** graph. This is because the multiplication by 2 came after the translation.

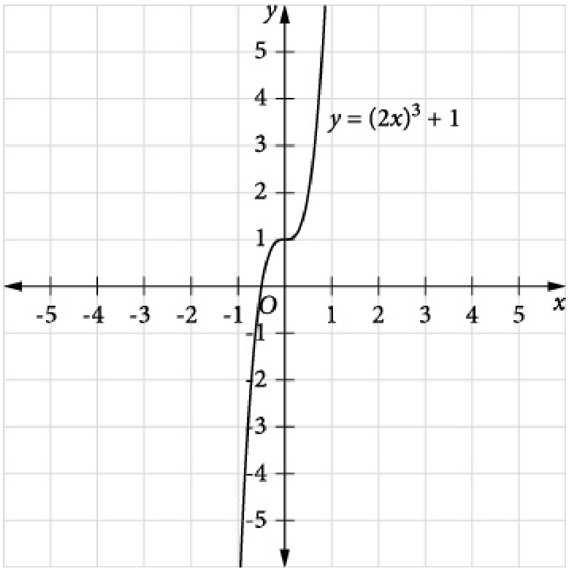
8 (a)



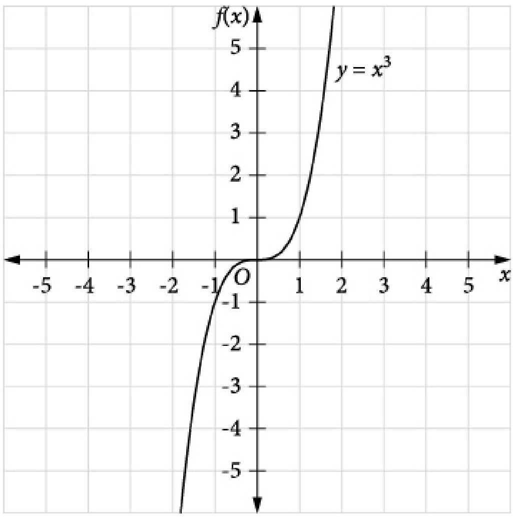
$y = f\left(2x\right)$ is the graph of $f(x) = x^3$ dilated a factor of $\frac{1}{2}$ from the y-axis.



$y = g(x) + 1$ is the graph of $g(x)$ translated 1 unit up.



(b)



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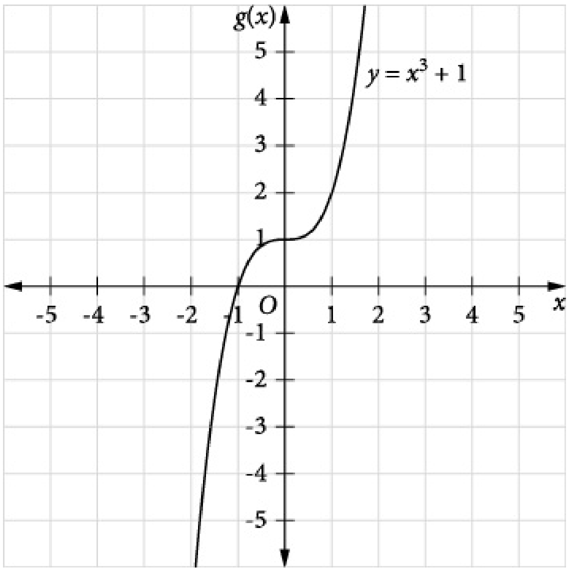
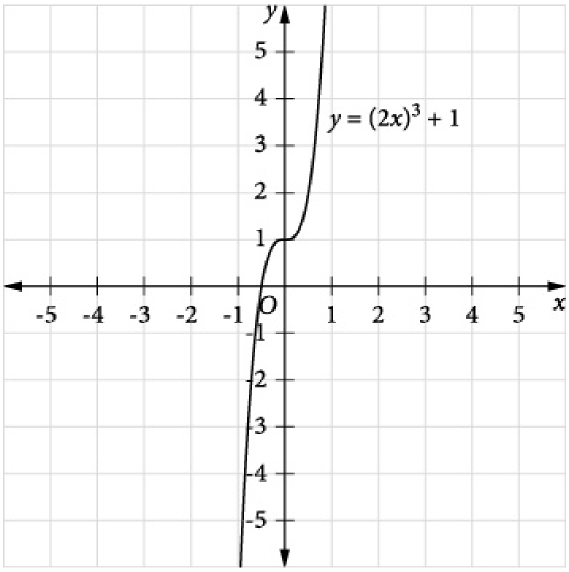
$y = f(x) + 1$ is the graph of $f(x) = x^3$ translated 1 unit up.



$y = g(2x)$ is the graph of $g(x)$ dilated a factor of $\frac{1}{2}$ from the y-axis.



(c) In this case the final graphs in both parts **(a)** and **(b)** are the same.

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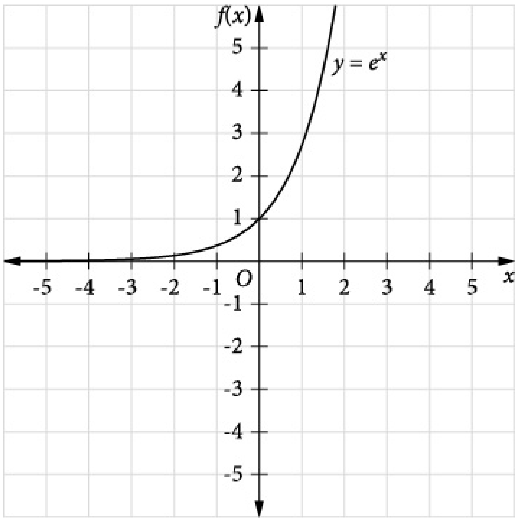
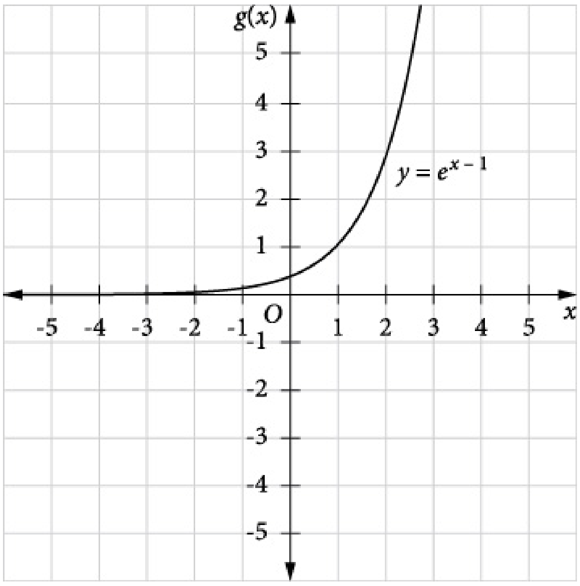
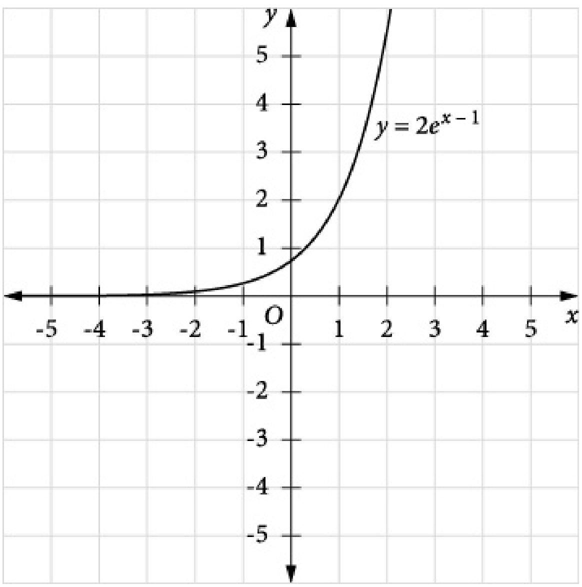
10 (a)



$g(x) = f(x-1)$ is the graph of $f(x) = e^x$ translated 1 unit right.



$y = 2g(x)$ is the graph of $g(x)$ dilated a factor of 2 from the x-axis.



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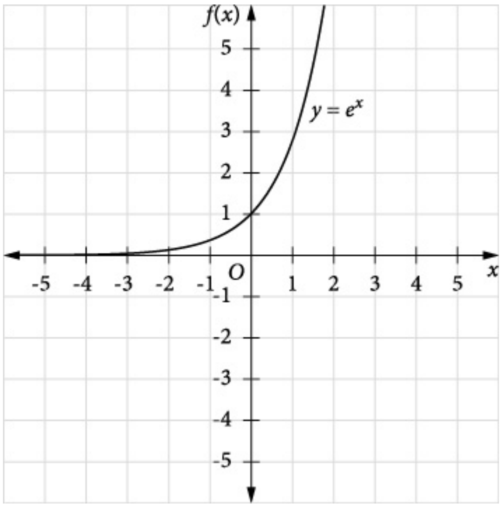
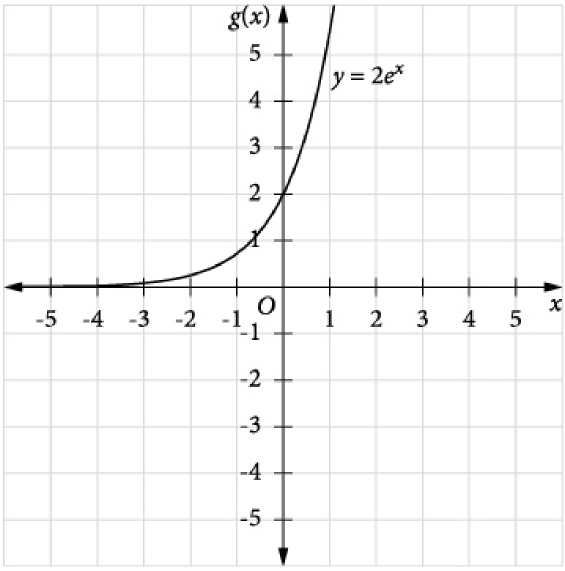
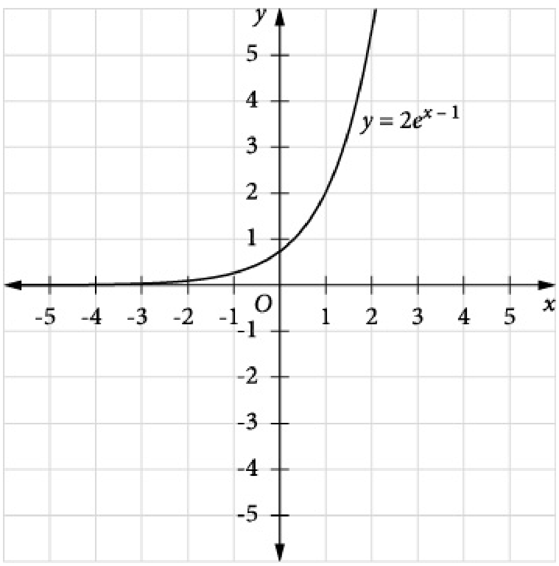
(b)



$g(x) = 2f(x)$ is the graph of $f(x)$ dilated a factor of 2 from the x-axis.



$y = g(x-1)$ is the graph of $g(x)$ translated 1 unit right.



(c) In this case the final graphs are the same.

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EXERCISE 15.4 GRAPHING RATIONAL ALGEBRAIC FUNCTIONS

2 (a)

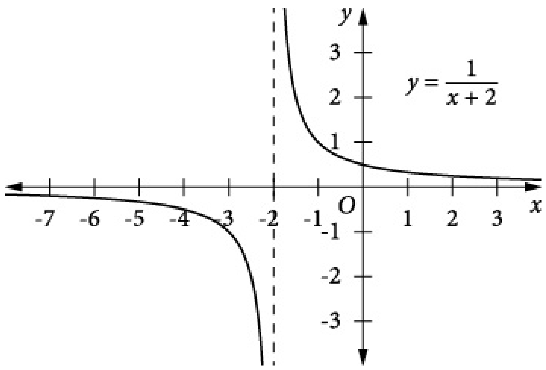
$x \neq -2$, so $x = -2$ is a vertical asymptote.

For $x > -2$, $x + 2 > 0$, so $y > 0$. As $x \to -2^+$, $y \to +\infty$.

For $x < -2$, $x + 2 < 0$, so $y < 0$. As $x \to -2^-$, $y \to -\infty$.

The numerator of $y = \dfrac{1}{x+2}$ is not zero, so the curve does not cut the x-axis.

At $x = 0$, $y = \dfrac{1}{2}$, so the y-intercept is $\dfrac{1}{2}$.



(b) $y = \dfrac{1}{x-1}$

$x \neq 1$, so $x = 1$ is a vertical asymptote.

For $x < 1$, $x - 1 < 0$, so $y < 0$. As $x \to 1^-$, $y \to -\infty$.

The numerator of $y = \dfrac{1}{x-1}$ is not zero, so the curve does not cut the x-axis.

At $x = 0$, $y = -1$, so the y-intercept is -1.

As $x \to +\infty$, $y \to 0^+$. As $x \to -\infty$, $y \to 0^-$. So $y = 0$ is a horizontal asymptote.

$$\begin{aligned}\frac{dy}{dx} &= -\left(x-1\right)^{-2} \\ &= -\frac{1}{\left(x-1\right)^2}\end{aligned}$$

Since $\left(x-1\right)^2 > 0$, then $\dfrac{dy}{dx} < 0$ for all x in the domain. Thus $y = \dfrac{1}{x-1}$ is a decreasing function in each part of its domain.

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Also $\frac{dy}{dx} \neq 0$ in the domain, so there are no turning points.

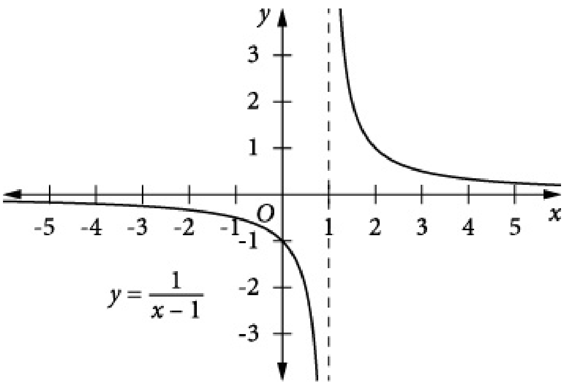
$$\frac{d^2y}{dx^2} = -1 \times (-2)(x-1)^{-3}$$
$$= \frac{2}{(x-1)^3}$$

For $x > 1$, $x - 1 > 0$, so $\frac{d^2y}{dx^2} > 0$.

For $x < 1$, $x - 1 < 0$, so $\frac{d^2y}{dx^2} < 0$.

The curve is concave down for $x < 1$ and concave up for $x > 1$.

The range: real y, $y \neq 0$.



(c) $y = \frac{1}{2-x}$

$x \neq 2$, so $x = 2$ is a vertical asymptote.

For $x > 2$, $2 - x < 0$, so $y < 0$. As $x \to 2^+$, $y \to -\infty$.

For $x < 2$, $2 - x < 0$, so $y > 0$. As $x \to 2^-$, $y \to +\infty$.

The numerator of $y = \frac{1}{2-x}$ is not zero, so the curve does not cut the x-axis.

At $x = 0$, $y = \frac{1}{2}$, so the y-intercept is $\frac{1}{2}$.

As $x \to +\infty$, $y \to 0^-$. As $x \to -\infty$, $y \to 0^+$. So $y = 0$ is a horizontal asymptote.

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$$\frac{dy}{dx} = -(2-x)^{-2}$$
$$= -\frac{1}{(2-x)^2}$$

Since $(2-x)^2 > 0$, then $\frac{dy}{dx} < 0$ for all x in the domain. Thus $y = \frac{1}{2-x}$ is a decreasing function in each part of its domain.

Also $\frac{dy}{dx} \neq 0$ in the domain, so there are no turning points.

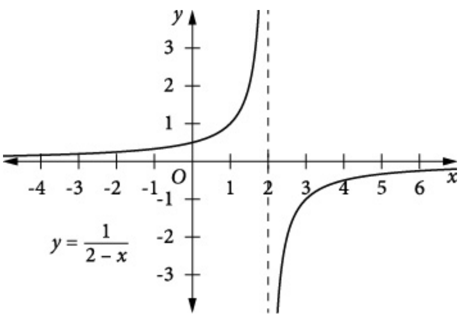
$$\frac{d^2y}{dx^2} = -1 \times (-2)(2-x)^{-3}$$
$$= \frac{2}{(2-x)^3}$$

For $x > 2$, $2 - x < 0$, so $\frac{d^2y}{dx^2} < 0$.

For $x < 2$, $2 - x > 0$, so $\frac{d^2y}{dx^2} > 0$.

The curve is concave down for $x > 2$ and concave up for $x < 2$.

The range: real y, $y \neq 0$.



4 **(a)** $y = x + \frac{4}{x}$

$x \neq 0$, so the curve does not cut the y-axis and the y-axis $(x = 0)$ is a vertical asymptote.

$$x + \frac{4}{x} = 0$$
$$\frac{x^2+4}{x} = 0$$

Since $x^2 + 4 \neq 0$ for all real x, the curve does not cut the x-axis.

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As $x \to +\infty$, $y \to x^+$

As $x \to -\infty$, $y \to x^-$

$y = x$ is an asymptote.

$$\frac{dy}{dx} = 1 - 4x^{-2} = 1 - \frac{4}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{8}{x^3}$$

For stationary points, $\frac{dy}{dx} = 0$.

$$1 - \frac{4}{x^2} = 0$$
$$\frac{x^2 - 4}{x^2} = 0$$
$$x^2 - 4 = 0$$
$$x = \pm 2$$

At $x = 2$, $y = 2 + \frac{4}{2} = 4$

$$\frac{d^2y}{dx^2} = \frac{8}{2^3} = 1 > 0$$

$(2,\ 4)$ is a minimum turning point.

At $x = -2$, $y = -2 + \frac{4}{-2} = -4$

$$\frac{d^2y}{dx^2} = \frac{8}{(-2)^3} = -1 < 0$$

$(-2,\ -4)$ is a maximum turning point.

For points of inflections: $\frac{d^2y}{dx^2} = 0$

$\frac{8}{x^3} \neq 0$, so there are no points of inflections.

$\frac{d^2y}{dx^2} > 0$ when $x > 0$ and $\frac{d^2y}{dx^2} < 0$ when $x < 0$.

The curve is concave up for $x > 0$ and concave down for $x < 0$.

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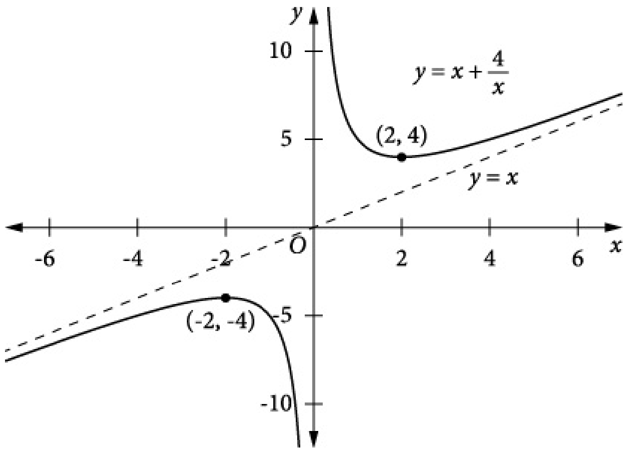

Range: All values of y except between the turning points, i.e. except for $-2 < y < 2.$, or $|y| < 2.$

Range: $|y| \geq 2$.

(b) $y = x - \dfrac{1}{x}$

$x \neq 0$, so the curve does not cut y-axis and the y-axis $(x = 0)$ is a vertical asymptote.

$$x - \frac{1}{x} = 0$$
$$\frac{x^2 - 1}{x} = 0$$
$$x^2 - 1 = 0$$
$$x = \pm 1$$

So the curve cut x-axis at $x = \pm 1$.

As $x \to +\infty$, $y \to x^+$

As $x \to -\infty$, $y \to x^-$

$y = x$ is an asymptote.

$$\frac{dy}{dx} = 1 + x^{-2}$$
$$= 1 + \frac{1}{x^2}$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x^3}$$

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For stationary points: $\frac{dy}{dx}=0$

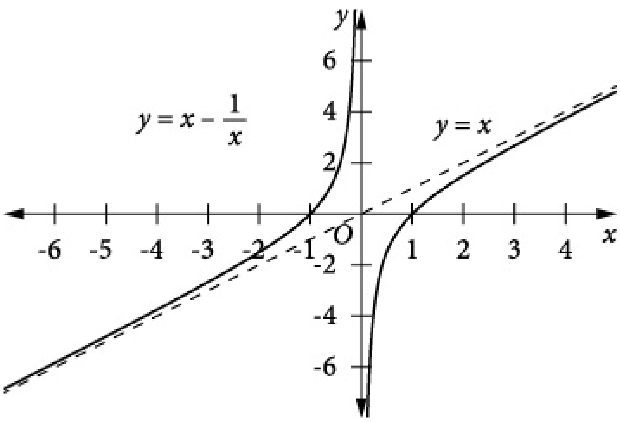
$$1+\frac{1}{x^2}=0$$
$$\frac{x^2+1}{x^2}=0$$

$x^2+1\neq 0$ for all real x

Therefore, there are no turning points.

For points of inflection: $\frac{d^2y}{dx^2}=0$

$-\frac{2}{x^3}\neq 0$, there are no points of inflections.



Range: real y

(c) $y=2x+\frac{8}{x}$

$x\neq 0$, so the curve does not cut y-axis and the y-axis $(x=0)$ is a vertical asymptote.

$$2x+\frac{8}{x}=0$$
$$\frac{2x^2+8}{x}=0$$

Since $2x^2+8\neq 0$ for all real x, so the curve does not cut the x-axis.

As $x\to+\infty$, $y\to 2x^+$

As $x\to-\infty$, $y\to 2x^-$

$y=2x$ is an asymptote.

$$\frac{dy}{dx} = 2 - 8x^{-2}$$
$$= 2 - \frac{8}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{16}{x^3}$$

For stationary points, $\frac{dy}{dx} = 0$.

$$2 - \frac{8}{x^2} = 0$$
$$\frac{2x^2 - 8}{x^2} = 0$$
$$2x^2 - 8 = 0$$
$$2\left(x^2 - 4\right) = 0$$
$$x = \pm 2$$

At $x = 2$, $y = 2 \times 2 + \frac{8}{2} = 8$

$$\frac{d^2y}{dx^2} = \frac{16}{2^3} = 2 > 0$$

$(2,\ 8)$ is a minimum turning point.

At $x = -2$, $y = 2 \times (-2) + \frac{8}{(-2)} = -8$

$$\frac{d^2y}{dx^2} = \frac{16}{(-2)^3} = -2 < 0$$

$(-2,\ -8)$ is a maximum turning point.

For points of inflection, $\frac{d^2y}{dx^2} = 0$

$\frac{d^2y}{dx^2} = \frac{16}{x^3} \neq 0$, so there are no points of inflection.

$\frac{d^2y}{dx^2} > 0$ when $x > 0$ and $\frac{d^2y}{dx^2} < 0$ when $x < 0$.

The curve is concave up for $x > 0$ and concave down for $x < 0$.

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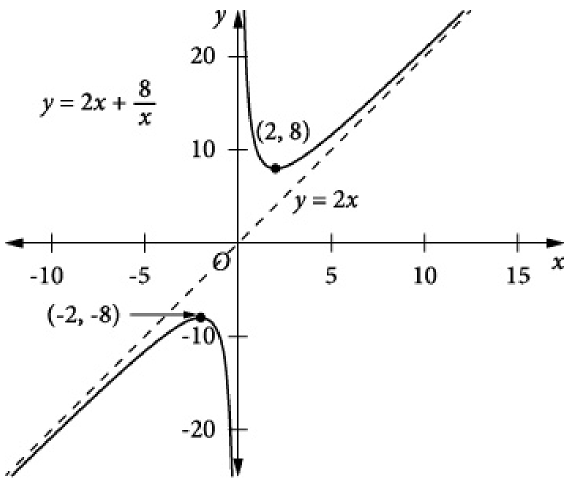

Range: All values of y except between the turning points, i.e. except for $-8 < y < 8.$, or $|y| < 8.$

Range: $|y| \geq 8$.

6 **(a)** $y = 1 + \frac{1}{x+2}$

$x \neq -2$, so $x = -2$ is a vertical asymptote.

For $x > -2$, $x + 2 > 0$, so $y > 1$. As $x \to -2^+$, $y \to +\infty$.

For $x < -2$, $x + 2 < 0$, so $y < 1$. As $x \to -2^-$, $y \to -\infty$.

$$1 + \frac{1}{x+2} = 0$$
$$\frac{x+2+1}{x+2} = 0$$
$$\frac{x+3}{x+2} = 0$$
$$x + 3 = 0$$
$$x = -3$$

The curve cuts the x-axis at $x = -3$.

At $x = 0$, $y = 1 + \frac{1}{2} = 1\frac{1}{2}$, so the y-intercept is $1\frac{1}{2}$.

As $x \to +\infty$, $y \to 1^+$. As $x \to -\infty$, $y \to 1^-$. So $y = 1$ is a horizontal asymptote.

$$\frac{dy}{dx} = -(x+2)^{-2} = -\frac{1}{(x+2)^2}$$

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Since $(x+2)^2 \geq 0$, and the graph is undefined when $(x+2)^2 = 0$, then $\frac{dy}{dx} < 0$ for all x in the domain. Thus $y = 1 + \frac{1}{x+2}$ is a decreasing function in each part of its domain.

Also $\frac{dy}{dx} \neq 0$ in the domain, so there are no turning points.

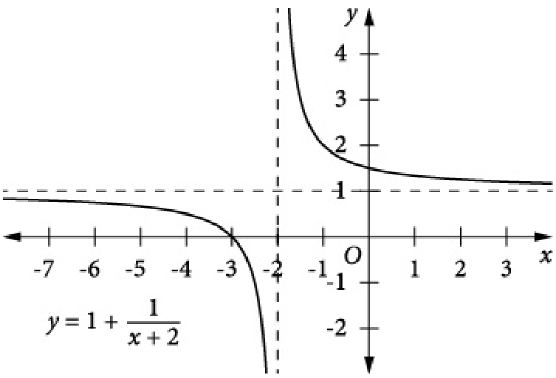
$$\frac{d^2y}{dx^2} = -1 \times (-2)(x+2)^{-3} = \frac{2}{(x+2)^3}$$

For $x > -2$, $x + 2 > 0$, so $\frac{d^2y}{dx^2} > 0$.

For $x < -2$, $x + 2 < 0$, so $\frac{d^2y}{dx^2} < 0$.

The curve is concave down for $x < -2$ and concave up for $x > -2$.

The range: real y, $y \neq 1$.



(b) $y = \frac{x-1+1}{x-1}$

$= 1 + \frac{1}{x-1}$

$x \neq 1$, $x = 1$ is a vertical asymptote.

For $x > 1$, $x - 1 > 0$, so $y > 1$. As $x \to 1^+$, $y \to +\infty$.

For $x < 1$, $x - 1 < 0$, so $y < 1$. As $x \to 1^-$, $y \to -\infty$.

$$\frac{x-1+1}{x-1} = 0$$

$$\frac{x}{x-1} = 0$$

$$x = 0$$

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The curve passes through the origin $(0,\ 0)$.

As $x \to +\infty$, $y \to 1^+$. As $x \to -\infty$, $y \to 1^-$. So $y = 1$ is a horizontal asymptote.

$$\frac{dy}{dx} = -(x-1)^{-2}$$
$$= -\frac{1}{(x-1)^2}$$

Since $(x-1)^2 > 0$, then $\frac{dy}{dx} < 0$ for all x in the domain. Thus $y = 1 + \frac{1}{x-1}$ is a decreasing function in each part of its domain.

Also $\frac{dy}{dx} \neq 0$ in the domain, so there are no turning points.

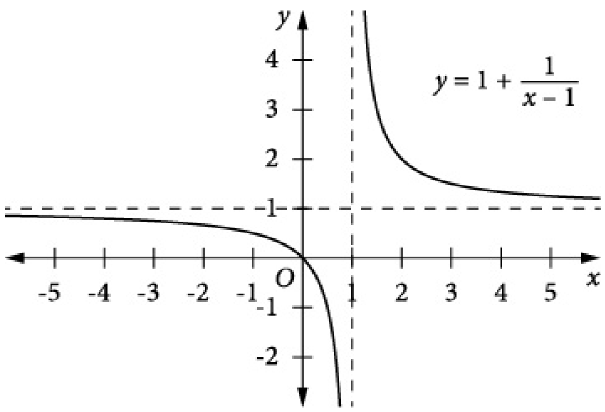
$$\frac{d^2y}{dx^2} = -1 \times (-2)(x-1)^{-3}$$
$$= \frac{2}{(x-1)^3}$$

For $x > 1$, $x - 1 > 0$, so $\frac{d^2y}{dx^2} > 0$.

For $x < 1$, $x - 1 < 0$, so $\frac{d^2y}{dx^2} < 0$.

The curve is concave down for $x < 1$ and concave up for $x > 1$.

The range: real y, $y \neq 1$.



(c) $y = \frac{x-2}{x-3}$

$$= \frac{x-3+1}{x-3}$$
$$= 1 + \frac{1}{x-3}$$

$x \neq 3$, so $x = 3$ is a vertical asymptote.

For $x > 3$, $x - 3 > 0$, so $y > 1$. As $x \to 3^+$, $y \to +\infty$.

For $x < 3$, $x - 3 < 0$, so $y < 1$. As $x \to 3^-$, $y \to -\infty$.

$$\frac{x-2}{x-3} = 0$$
$$x - 2 = 0$$
$$x = 2$$

The curve cuts the x-axis at 2.

As $x \to +\infty$, $y \to 1^+$.

As $x \to -\infty$, $y \to 1^-$.

$y = 1$ is a horizontal asymptote.

$$\frac{dy}{dx} = -(x-3)^{-2} = -\frac{1}{(x-3)^2}$$

Since $(x-3)^2 > 0$, $\frac{dy}{dx} < 0$ for all x in the domain. Thus $y = 1 + \frac{1}{x-3}$ is a decreasing function in each part of its domain.

Also $\frac{dy}{dx} \neq 0$ in the domain, so there are no turning points.

$$\frac{d^2y}{dx^2} = -1 \times (-2)(x-3)^{-3} = \frac{2}{(x-3)^3}$$

For $x > 3$, $x - 3 > 0$, so $\frac{d^2y}{dx^2} > 0$.

For $x < 3$, $x - 3 < 0$, so $\frac{d^2y}{dx^2} < 0$.

The curve is concave down for $x < 3$ and concave up for $x > 3$.

The range: real y, $y \neq 1$.

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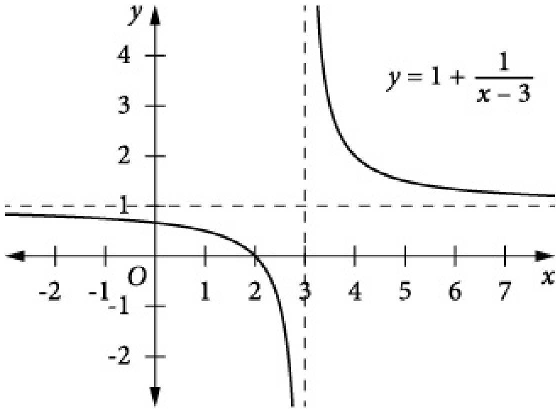
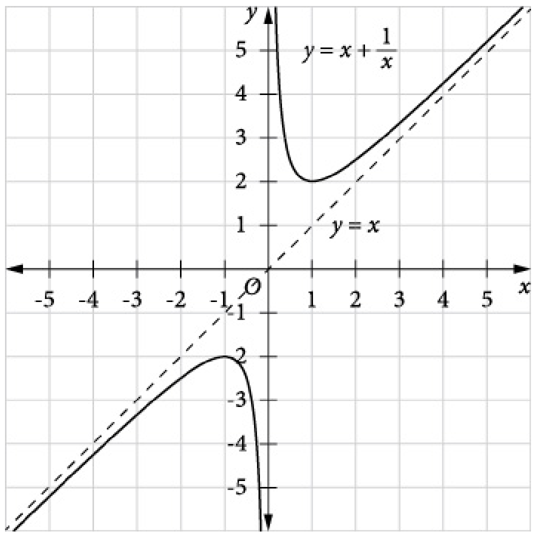


EXERCISE 15.5 APPLICATIONS INVOLVING GRAPHING FUNCTIONS

2 (a) The lines $y = x$ and $x = 0$ are asymptotes for this graph. There is no y-intercept.

$y = x + \frac{1}{x} = \frac{x^2+1}{x} \neq 0$ so there is no x-intercept.



(b) $y = x + \frac{1}{x} = x + x^{-1}$

$$\frac{dy}{dx} = 1 - 1 \times x^{-2}$$
$$= 1 - \frac{1}{x^2}$$
$$= \frac{x^2-1}{x^2}$$
$$= \frac{(x+1)(x-1)}{x^2}$$

$\frac{dy}{dx} = 0$ when $x = -1,\ 1$

When $x = -1,\ y = -1 - \frac{1}{1} = -2$

There is a stationary point at $(-1, -2)$.

When $x = 1,\ y = 1 + \frac{1}{1} = 2$

There is a stationary point at $(1, 2)$.

$\frac{d^2y}{dx^2} = 2 \times x^{-3} = \frac{2}{x^3}$

When $x = -1,\ \frac{d^2y}{dx^2} = -2 < 0$

$(-1, -2)$ is a local maximum.

When $x = 1,\ \frac{d^2y}{dx^2} = 2 > 0$

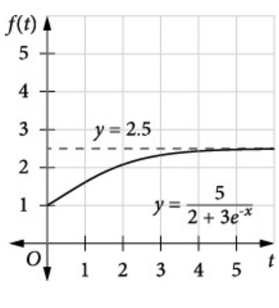
$(1, 2)$ is a local minimum.

(c) The least value in this domain is at the local minimum turning point 2.

4 (a) As $t \to +\infty,\ f(t) \to \frac{5}{2 + 3 \times 0} = \frac{5}{2}$, and when $t > 0,\ f(t) < \frac{5}{2}$, so $f(t)$ approaches from below. $y = \frac{5}{2}$ is an asymptote.

$f(t)$ cannot be zero, so there is no x-intercept.

When $t = 0,\ f(t) = \frac{5}{2 + 3e^0} = \frac{5}{2 + 3 \times 1} = 1$ so the x-intercept is 1.



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(b) Use the chain rule where $u = 2+3e^{-t}$ so $f(t) = \frac{5}{u}$ and $\frac{du}{dt} = -3e^{-t}$.

$$\begin{aligned} f'(t) &= \frac{df(t)}{du} \times \frac{du}{dt} \\ &= -\frac{5}{u^2} \times -3e^{-t} \\ &= \frac{15e^{-t}}{\left(3+2e^{-t}\right)^2} \end{aligned}$$

Since $e^{-t} > 0$, $\frac{15e^{-t}}{\left(3+2e^{-t}\right)^2} > 0$ for all values of t.

(c) Use $f(t) = \frac{5}{2+3e^{-t}}$

As $t \to \infty$, $e^{-t} \to 0$

Hence,

$$\lim_{t\to\infty} \frac{5}{2+3e^{-t}} = \frac{5}{2} = 2.5$$

(d) Since $f(t)$ is continually increasing, the minimum value of $f(t)$ occurs when $t = 0$.

$$f(0) = \frac{5}{2+3e^0} = \frac{5}{2+3} = 1$$

Range: $1 \le f(t) < 2.5$

6 **(a)** $f(x) = \log_e(\sin x)$

The logarithm function requires a positive input, so $f(x)$ is defined when $\sin x > 0$, i.e. $0 < x < \pi$.

(b) Use $\frac{d}{dx}\log\left(f(x)\right) = \frac{f'(x)}{f(x)}$ where $f(x) = \sin x$ and $f'(x) = \cos x$.

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)} = \frac{\cos x}{\sin x} = \cot x$$

Alternatively, use the chain rule (from which this is derived).

Let $u = \sin x$ so $f(x) = \log_e u$ and $\frac{du}{dx} = \cos x$.

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$$\frac{df(x)}{du}=\frac{1}{u}$$

$$\begin{aligned}\frac{df(x)}{dx}&=\frac{df(x)}{du}\times\frac{du}{dx}\\&=\frac{1}{u}\times\cos x\\&=\frac{1}{\sin x}\times\cos x\\&=\cot x\end{aligned}$$

(c) $f'(x)=0$ when $\cot x=0$, so $\cos x=0$.

Hence $x=\frac{\pi}{2}$

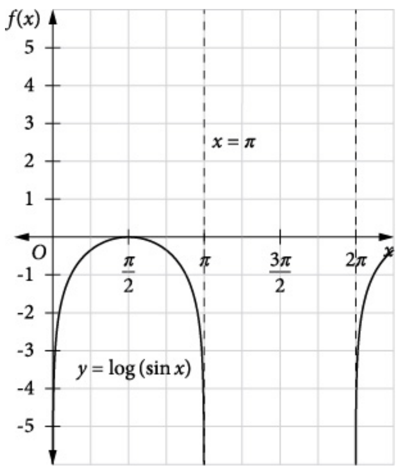
$$\begin{aligned}f'(x)&=\cot x\\f''(x)&=-\text{cosec}^2 x\\f''\left(\frac{\pi}{2}\right)&=-\text{cosec}^2\left(\frac{\pi}{2}\right)=-(1)^2<0\end{aligned}$$

This will be a local maximum.

$$f\left(\frac{\pi}{2}\right)=\log_e\left(\sin\frac{\pi}{2}\right)=\log_e 1=0$$

So $\left(\frac{\pi}{2},\ 0\right)$ is the maximum value of $f(x)$ in the stated domain.

(d)



8 (a) Use the chain rule to differentiate the function.

Let $u=\cos\left(\frac{\pi(t-8)}{2}\right)+2$ so $C=1000u^2-1000$ and $\frac{du}{dt}=-\frac{\pi}{2}\sin\left(\frac{\pi(t-8)}{2}\right)$.

$$\frac{dC}{du}=2000u=2000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]$$

$$\begin{aligned}\frac{dC}{dt}&=\frac{dC}{du}\times\frac{du}{dt}\\&=2000\left[\cos\left(\frac{\pi(t-8}{2}\right)+2\right]\times-\frac{\pi}{2}\sin\left(\frac{\pi(t-8)}{2}\right)\\&=-1000\pi\sin\left(\frac{\pi(t-8}{2}\right)\left[\cos\left(\frac{\pi(t-8}{2}\right)+2\right]\end{aligned}$$

$\frac{dC}{dt}=0\Rightarrow\sin\left(\frac{\pi(t-8)}{2}\right)=0$ since $\cos\left(\frac{\pi(t-8}{2}\right)+2>0$.

$$\begin{aligned}&8\le t\le 16\\&0\le t-8\le 8\\&0\le\frac{\pi\left(t-8\right)}{2}\le 8\times\frac{\pi}{2}\\&0\le\frac{\pi\left(t-8\right)}{2}\le 4\pi\end{aligned}$$

$$\begin{aligned}\sin\left(\frac{\pi(t-8)}{2}\right)=0\Rightarrow\frac{\pi(t-8)}{2}&=0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi\\t-8&=0,\ 2,\ 4,\ 6,\ 8\\t&=8,\ 10,\ 12,\ 14,\ 16\end{aligned}$$

$$\begin{aligned}t=8:\ C(t)&=1000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]^2-1000\\&=1000\left[\cos 0+2\right]^2-1000\\&=1000\times3^2-1000\\&=8000\end{aligned}$$

$$\begin{aligned}t=10:\ C(t)&=1000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]^2-1000\\&=1000\left[\cos\pi+2\right]^2-1000\\&=1000\times1^2-1000\\&=0\end{aligned}$$

$$\begin{aligned} t=12:\ C(t) &= 1000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]^2-1000 \\ &= 1000\left[\cos 2\pi+2\right]^2-1000 \\ &= 1000\times 3^2-1000 \\ &= 8000 \end{aligned}$$

$$\begin{aligned} t=14:\ C(t) &= 1000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]^2-1000 \\ &= 1000\left[\cos 3\pi+2\right]^2-1000 \\ &= 1000\times 1^2-1000 \\ &= 0 \end{aligned}$$

$$\begin{aligned} t=16:\ C(t) &= 1000\left[\cos\left(\frac{\pi(t-8)}{2}\right)+2\right]^2-1000 \\ &= 1000\left[\cos 4\pi+2\right]^2-1000 \\ &= 1000\times 3^2-1000 \\ &= 8000 \end{aligned}$$

The least value of $C(t)$ will occur when $t=10,\ 14$ i.e. after 10 hours and after 14 hours.

$$C(10)=1000\left[\cos\pi+2\right]^2-1000=0$$

$$C(14)=1000\left[\cos 3\pi+2\right]^2-1000=0$$

The minimum concentration of insects is zero and occurs after 10 hours and after 14 hours.

(b) Mark in the maximum and minimum points already calculated. You should add the middle ones. When $t=9,\ 11,\ 13,\ 15$; $\cos\left(\dfrac{\pi(t-8)}{2}\right)=0$, so $C(t)=1000\left[0+2\right]^2-1000=3000$.

This tells you that while this curve is periodic it is not a transformation of the sine curve and you will need to plot individual points to work out its shape.

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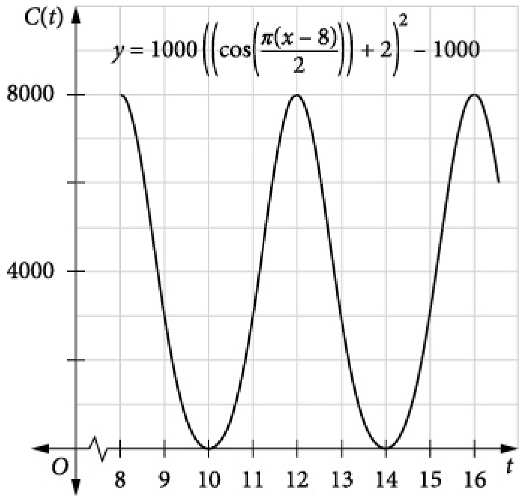


10 (a) $I(t)=100\left(1-e^{-5t}\right)$

$I(0)=100\left(1-e^{0}\right)=0$ amps

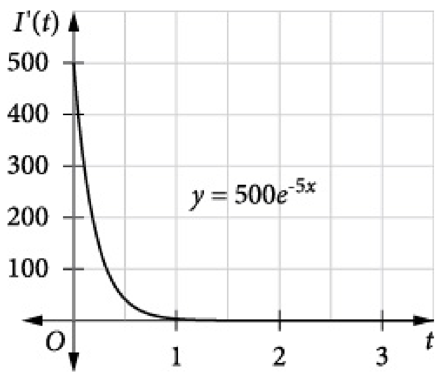
$I(0.2)=100\left(1-e^{-1}\right)\approx 63.2$ amps

$I(1)=100\left(1-e^{-5}\right)\approx 99.3$ amps

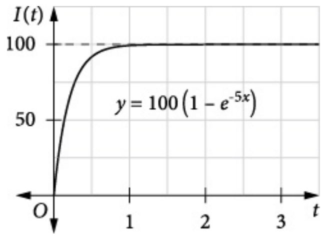
(b) As $t\to\infty, e^{-5t}\to 0$ so the current approaches $100(1-0)=100$ amps.

(c) $I(t)=100\left(1-e^{-5t}\right)=100-100e^{-5t}$

$I'(t)=-100\times(-5)e^{-5t}=500e^{-5t}$



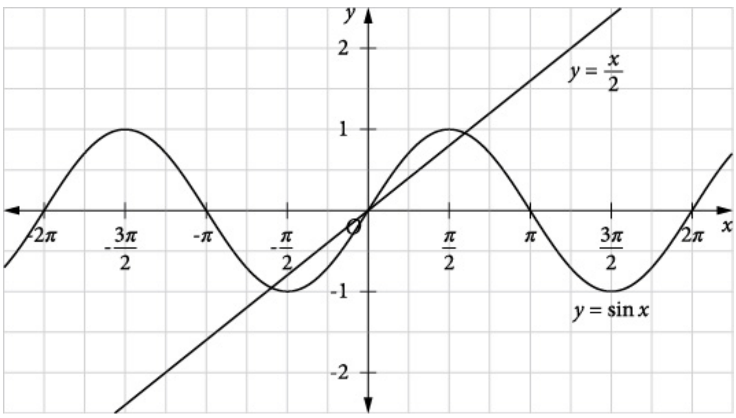
(d) From part **(a)**, $I(t)=100\left(1-e^{-5t}\right)$ starts at the origin and has a horizontal asymptote at $I=100$.



(e) The graph of $y = I'(t)$ shows the gradient of $y = I(t)$ for $t \geq 0$. The gradient shows the current increasing rapidly at the start, then increasing at a progressively slower rate until it is staying practically the same.

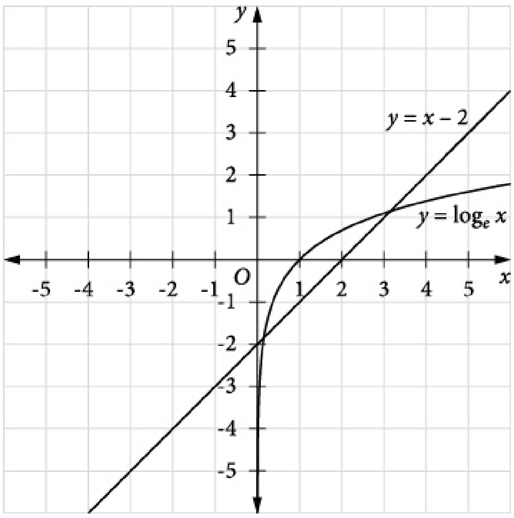
EXERCISE 15.6 GRAPHICAL SOLUTION OF EQUATIONS

2 (a)



There are three solutions within the given domain for the stated equation.

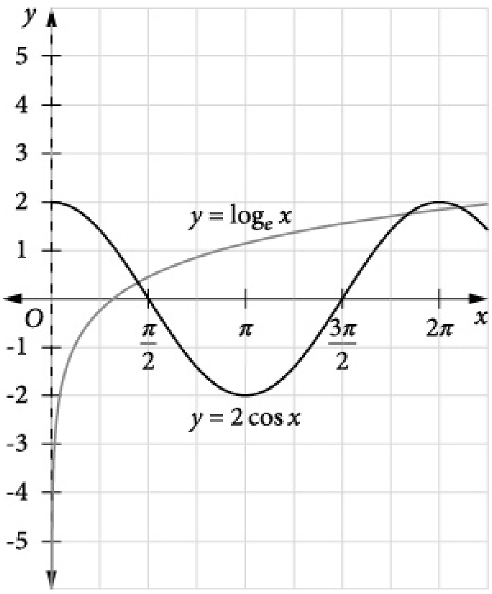
(b) $\log_e x - x + 2 = 0$ is equivalent to $\log_e x = x - 2$.



There are two solutions for the stated equation.

(c)

$2\cos x - \log_e x = 0$ is equivalent to $2\cos x = \log_e x$.



Within the given domain there are two solutions for the stated equation. (The third point shown on the graph occurs just outside the given domain.)

(d) $e^x - \sin x = 0$ is equivalent to $e^x = \sin x$.

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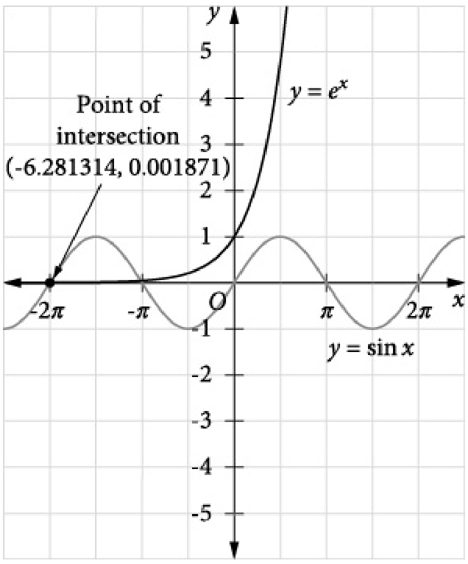
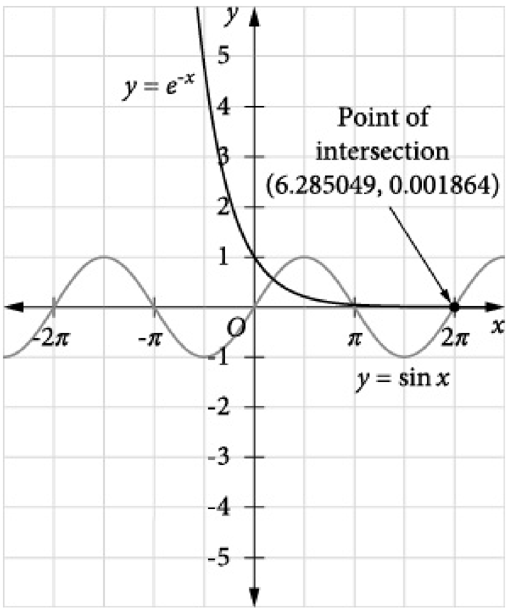


Within the given domain there are two solutions for the stated equation. The left-hand point occurs just inside the beginning of the domain.

(e) $e^{-x} - \sin x = 0$ is equivalent to $e^{-x} = \sin x$.



Within the given domain there are two solutions for the stated equation.

When $x = 2\pi$, $\sin x = 0$ and $e^{-x} > 0$, so this point of intersection will occur when x is just a little greater than 2π.

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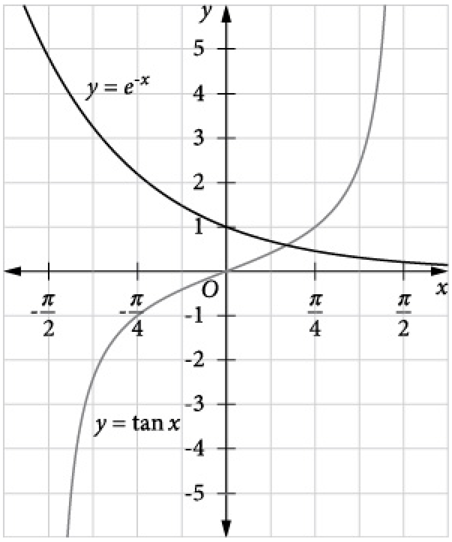
(f) $e^{-x} - \tan x = 0$ is equivalent to $e^{-x} = \tan x$.



Within the given domain there is only one solution for the stated equation.

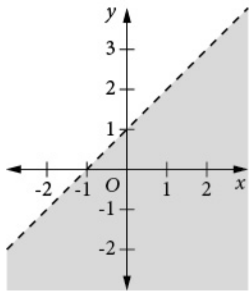
EXERCISE 15.7 REGIONS AND INEQUALITIES

2 Draw a dotted line where $y = x + 1$.

For $y < x + 1$, substitute the non-boundary point $(0,\ 0)$.

$0 < 0 + 1$

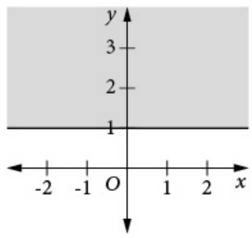
The result is true, so shade below the line.



4 Draw a solid line where $y = 1$.

For $y \geq 1$, substitute the non-boundary point $(0,\ 0)$.

For $y \geq 1$, this is false, so the region is above the line.



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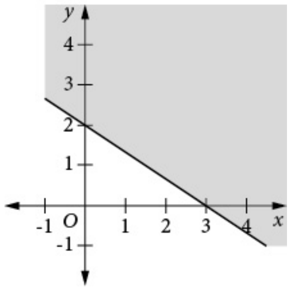
6 $2x+3y \geq 6$



Draw a solid line where $2x+3y=6$.

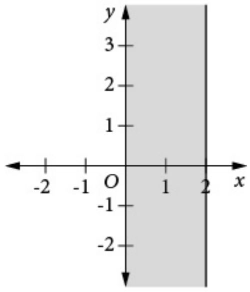
This is best done using the intercepts.

Substitute the non-boundary point $(0,\ 0)$.

$2(0)+3(0) \geq 6$

The result is false, so shade above the line

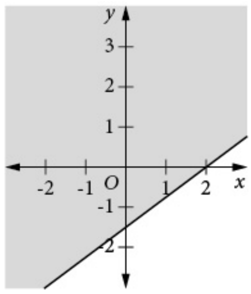
8 $0 \leq x \leq 2$



The region is between $x=0$ and $x=2$.

Draw the solid vertical lines $x=0$ and $x=2$, and shade the area between them.

10 $3x-4y \leq 6$



Draw a solid line where $3x-4y=6$.

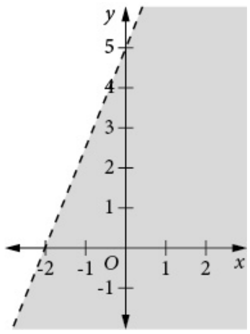
This is best done using the intercepts.

Substitute the non-boundary point $(0,\ 0)$

$3(0)-4(0) \leq 6$

The result is true, so shade above the line

12 $2y-5x<10$



Draw a dotted line where $2y-5x=10$.

This is best done using the intercepts.

Substitute the non-boundary point $(0,\ 0)$

$2(0)-5(0)<10$

The result is true, so shade below the line

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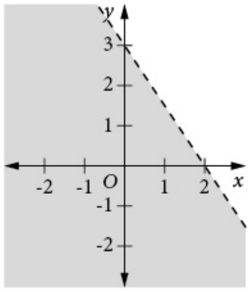
14 $\frac{x}{2}+\frac{y}{3}<1$



Draw a dotted line where $\frac{x}{2}+\frac{y}{3}=1$.

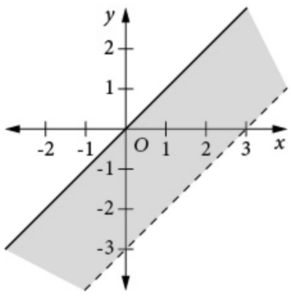
This is best done using the intercepts.

Substitute the non-boundary point $(0,\ 0)$

$\frac{0}{2}+\frac{0}{3}<1$

The result is true, so shade below the line

16 $0\le x-y<3$



Draw a solid line where $x-y=0$, and a dotted line where $x-y=3$.

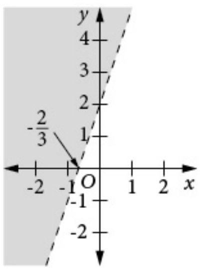
This is best done using the intercepts.

Substitute the non-boundary point $(1,\ 0)$.

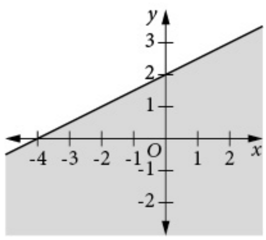
$0\le 1-0<3$

Both inequalities are true, so shade between the lines.

18 **(a)** First draw the dotted (above, not on) line $y=3x+2$ and shade above it.

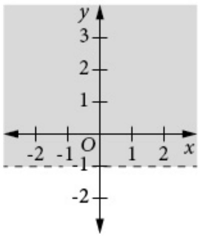


(b) Draw a solid (on or below) line for $x-3y+4=0$ and shade below it.

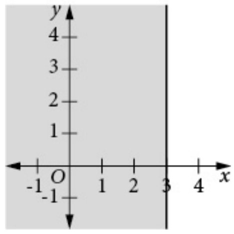


(c) Draw a dotted line $y=-1$ and shade

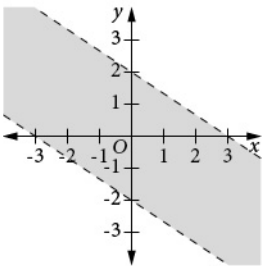
(d) Draw the solid ('on it' is included) vertical above it.



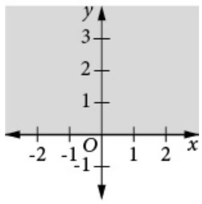
line $x = 3$ and shade left of it.



(e) Draw the (parallel) lines $2x+3y+6=0$ and $2x+3y-6=0$ dotted ('on them' is not included) and shade the region between them.



(f) Draw the solid (on or above) the *x*-axis and shade above it.



20 (a) Check any two points on $x+2y-2=0$.

If $x=0$, $0+2y-2=0 \Rightarrow y=1$.

$(0,\ 1)$ is a point on $x+2y-2=0$ and also on the drawn graph.

If $y=0$, $x+0-2=0 \Rightarrow x=2$.

$(2,\ 0)$ is a point on $x+2y-2=0$ and also on the drawn graph.

Correct

(b) Looking at the intercepts, the rise is -1 and the run is 2.Incorrect

The gradient of the boundary line is $\dfrac{-1}{2}=-\dfrac{1}{2}$.

Incorrect

(c) Substitute the non-boundary point $(0,\ 0)$.

$x+2y-2=0+2\times 0-2=-2<0$

The inequality of the region is $x+2y-2<0$.

Incorrect

(d) The inequality of the region is $x+2y-2<0$. See part **(c)**.

Correct

22 (a) First draw the graphs of $y=3-x^2$ and $y=2x$ with solid lines.

To find where they intersect, equate the values of y.

$$\begin{aligned} 2x &= 3-x^2 \\ x^2+2x-3 &= 0 \\ (x+3)(x-1) &= 0 \\ x &= -3,\ 1 \end{aligned}$$

When $x=-3$, $y=2x=-6$.

When $x=1$, $y=2x=2$.

For $y\le 3-x^2$, substitute the non-boundary point $(0,\ 0)$.

$0\le 3-0^2$

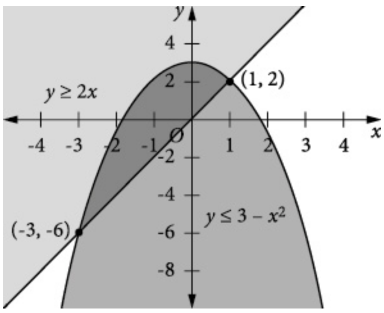
This is true, so lightly shade underneath the graph.

For $y\ge 2x$, substitute the non-boundary point $(0,\ 1)$.

$1\ge 2\times 0$

This is true, so lightly shade above the line.

Finally, shade in the overlapping region.



(b) $x^2 + 2x - 3 \le 0$

$2x \le 3 - x^2$

Look for the values of x where the graph of $y = 2x$ is below the graph of $y = 3 - x^2$.

$-3 \le x \le 1$

24 (a) First draw the graphs of $y = 3x - 2$ with a solid straight line and $y = 2x - x^2$ with a dotted line.

To find where they intersect, equate the values of y.

$$\begin{aligned} 3x - 2 &= 2x - x^2 \\ 3x - 2 - 2x + x^2 &= 0 \\ x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \end{aligned}$$

$x = -2,\ 1$

When $x = -2$, $y = 3x - 2 = 3 \times -2 - 2 = -8$.

When $x = 1$, $y = 3x - 2 = 3 \times 1 - 2 = 1$.

The graphs intersect at $(-2,\ -8)$ and $(1,\ 1)$.

For $y \ge 3x - 2$, substitute the non-boundary point $(0,\ 0)$.

$0 \ge 3 \times 0 - 2$

This is true, so lightly shade above the line.

For $y < 2x - x^2$, substitute the non-boundary point $(0,\ 1)$.

$1 < 2 \times 0 - 0^2$

This is false, so lightly shade below the graph.

Finally, shade in the overlapping region.

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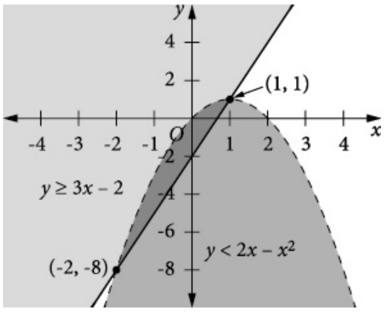


(b) $x^2 + x - 2 < 0$

$x^2 + x - 2 + 2x - x^2 < 2x - x^2$

$3x - 2 < 2x - x^2$

Look for the values of x where the graph of $y = 3x - 2$ is below the graph of $y = 2x - x^2$.

$-2 < x < 1$

(c) $x^2 + x - 2 < 0$

$(x+2)(x-1) = 0$

$x = -2,\ 1$

$-2 < x < 1$

EXERCISE 15.8 SIMULTANEOUS LINEAR INEQUALITIES

2 C

Check each point on the graph.

$(1,\ 1)$ is not in the shaded region.

$(3,\ 1)$ is in the shaded region.

$(1,\ 3)$ is not in the shaded region.

$-(1,\ 3)$ is not in the shaded region.

4 **(a)** Draw the solid lines $x+y=3,\ y=x$.

Test a point not on either boundary, e.g. $(0,\ 1)$.

For $x+y\le 3,\ 0+1\le 3$.

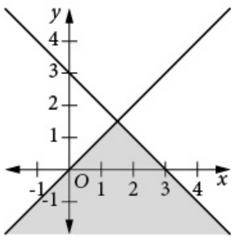
This is true, so this represents the area under $x+y=3$.

For $y\le x,\ 1\le 0$.

This is false, so this represents the area under $y=x$.

Shade the area which is under $x+y=3$ and also under $y=x$.

$x+y\le 3,\ y\le x$



Test each point on the graph above to see whether it lies in the shaded region.

The point $(0,\ 0)$ is on a solid edge of the region, and because both edges are included, it is 'inside' the region.

The point $(2,\ 3)$ is not in the region.

The point $(-1,\ -2)$ is in the region.

(b) Draw the broken lines for $2y=x+2,\ x+y=-1$.

Test a point not on either boundary, e.g. $(0,\ 0)$.

For $2y>x+2,\ 2\times 0>0+2$.

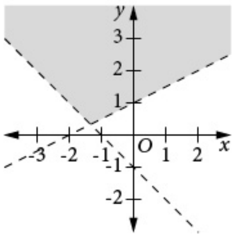
This is false, so this is the area which is above $2y=x+2$.

For $x+y>-1,\ 0+0>-1$.

This is true, so this is the area which is above $x+y=-1$.

Shade the region which is above $2y = x + 2$ and also above $x + y = -1$.

$2y > x + 2,\ x + y > -1$



Test each point on the graph above to see whether it lies in the shaded region.

The point $(0,\ 0)$ is not in the region.

The point $(0,\ 1)$ on the edge of the region, and because neither edge is included, it is not in the region.

The point $(2,\ 5)$ is in the region.

(c) Draw a solid line for $x + 2y = 8,$ and a broken line for $y = 7$.

Test a point not on either boundary, e.g. $(0,\ 0)$.

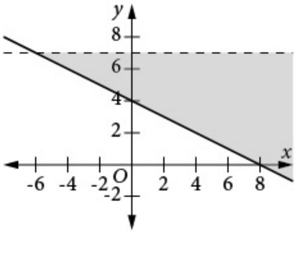
For $x + 2y \geq 8,\ 0 + 2 \times 0 \geq 8$.

This is false, so this is the area which is above $x + 2y = 8$.

$y < 7$ represents the area below $y = 7$.

Shade the region which is above $x + 2y = 8$ and also below $y = 7$.

$x + 2y \geq 8,\ y < 7$



Test each point on the graph above to see whether it lies in the shaded region.

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The point $(0,\ 4)$ is on a solid line, so it is in the region.

The point $(-1,\ -1)$ is not in the region.

The point $(9,\ 2)$ is in the region.

(d) Draw a solid line for $3y = 2x + 6$ and a broken line for $x + y = 2$.

Test a point not on either boundary, e.g. $(0,\ 0)$.

For $3y \le 2x + 6,\ 3\times 0 \le 2\times 0 + 6$.

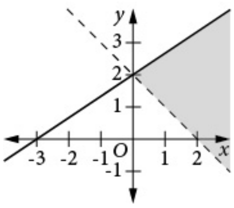
This is true, so this is the area which is below $3y = 2x + 6$.

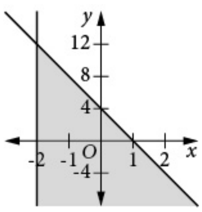
For $x + y > 2,\ 0 + 0 > 2$.

This is false, so this is the area which is above $x + y = 2$.

Shade the region which is below $3y = 2x + 6$ and also above $x + y = 2$.

$3y \le 2x + 6,\ x + y > 2$





Test each point on the graph above to see whether it lies in the shaded region.

The point $(2,\ 0)$ is on a dotted line, so it is not in the region.

The point $(3,\ 3)$ is in the region.

The point $(4,\ -1)$ is in the region.

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(e) Draw a solid line for $4x+y=4$ and for $x=-2$.

Test a point not on either boundary, e.g. $(0,\ 0)$.

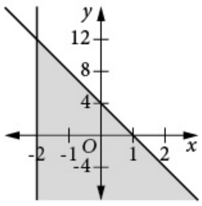
For $4x+y\le 4,\ 4\times 0+0\le 8$.

This is true, so this is the area which is above $4x+y=4$.

$x\ge -2$, represents the area on and to the right of $x=-2$.

Shade the region which is above $x+2y=8$ and also right of $x=-2$.

$4x+y\le 4,\ x\ge -2$



Test each point on the graph above to see whether it lies in the shaded region.

The point $(0,\ 0)$ is in the region.

The point $(-3,\ 1)$ is not in the region.

The point $(1,\ 0)$ is on a solid line and is therefore in the region.

(f) Draw the broken lines for $y=3x+3,\ x+y=3$.

Test a point not on either boundary, e.g. $(0,\ 0)$.

For $y>3x+3,\ 0>3\times 0+3$.

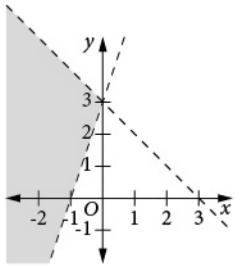
This is false, so this is the area which is above $y=3x+3$.

For $x+y<3,\ 0+0<3$.

This is true, so this is the area which is below $x+y=3$.

Shade the region which is above $y=3x+3$ and also below $x+y=3$.

$y>3x+3,\ x+y<3$



Test each point on the graph above to see whether it lies in the shaded region.

The point $(0,\ 3)$ is on a dotted line and is not in the region.

The point $(2,\ 7)$ is not in the region.

The point $(-1,\ 4)$ is on a dotted line and is not in the region.

6 (a) Draw the solid lines for $y = x + 2,\ 2x + y = 4,\ x + y = 2.$

Test a point not on any boundary, e.g. $(0,\ 0)$.

For $y \le x + 2,\ 0 \le 0 + 2$.

This is true, so this is the area which is below $y = x + 2$.

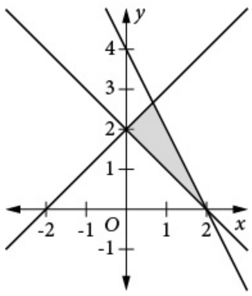
For $2x + y < 4,\ 2 \times 0 + 0 < 4$.

This is true, so this is the area which is below $2x + y = 4$.

For $x + y \ge 2,\ 0 + 0 \ge 2$.

This is false, so this is the area which is above $x + y = 2.$

Shade the region which is below $y = x + 2$ and $2x + y = 4$, and also above $x + y = 2$.



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$y = x + 2$ [1]
$2x + y = 4$ [2]
$x + y = 2$ [3]

Substitute [1] into [2]

$$2x + x + 2 = 4$$
$$3x = 2$$
$$x = \frac{2}{3}$$

Substitute $x = \frac{2}{3}$ into [1]

$$y = \frac{2}{3} + 2 = 2\frac{2}{3}$$

Intersection of [1] and [2]: $\left(\frac{2}{3},\ 2\frac{2}{3}\right)$

Substitute [1] into [3]

$$x + x + 2 = 2$$
$$2x = 0$$
$$x = 0$$

Substitute $x = 0$ into [1]

$$y = 0 + 2 = 2$$

Intersection of [1] and [3]: $(0,\ 2)$

$[2] - [3]$

$$2x + y - (x + y) = 4 - 2$$
$$x = 2$$

Substitute $x = 2$ into [3]

$$2 + y = 2$$
$$y = 0$$

Intersection of [2] and [3]: $(2,\ 0)$

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Vertices: $\left(\frac{2}{3},\ 2\frac{2}{3}\right),\ (0,\ 2),\ (2,\ 0)$

(b) Draw a solid line for $2y-x=4$, and broken lines for $y=3x-6,\ 3x+y=-6.$

Test a point not on any boundary, e.g. $(0,\ 0)$.

For $2y-x\le 4,\ 2\times 0-0\le 4$.

This is true, so this is the area which is below $2y-x=4$.

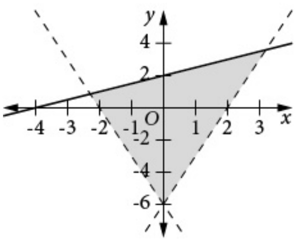
For $y>3x-6,\ 0>3\times 0-6$.

This is true, so this is the area which is above $y=3x-6$.

For $3x+y>-6,\ 3\times 0+0>-6$.

This is true, so this is the area which is above $3x+y=-6.$

Shade the region which is below $2y-x=4$, and also above $y=3x-6$ and $3x+y=-6.$



$2y-x=4$ [1]
$y=3x-6$ [2]
$3x+y=-6$ [3]

Substitute [2] into [1]

$$\begin{aligned} 2(3x-6)-x&=4 \\ 5x&=16 \\ x&=3\frac{1}{5} \end{aligned}$$

Substitute $x=3\frac{1}{5}$ into [2]

$$y=3\left(3\frac{1}{5}\right)-6$$

$$y=3\frac{3}{5}$$

Intersection of [1] and [2]: $\left(3\frac{1}{5},\ 3\frac{3}{5}\right)$

$[1]-2\times[3]$

$$\begin{aligned} 2y-x-2(3x+y)&=4-2(-6) \\ -7x&=16 \\ x&=-2\frac{2}{7} \end{aligned}$$

Substitute $x=-2\frac{2}{7}$ into [1]

$$\begin{aligned} 2y-\left(-2\frac{2}{7}\right)&=4 \\ 2y&=1\frac{5}{7} \\ y&=\frac{6}{7} \end{aligned}$$

Intersection of [1] and [3]: $\left(-2\frac{2}{7},\ \frac{6}{7}\right)$

Substitute [2] into [3]

$$\begin{aligned} 3x+3x-6&=-6 \\ 6x&=0 \\ x&=0 \end{aligned}$$

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Substitute $x = 0$ into [2]

$y = 3(0) - 6$
$y = -6$

Intersection of [2] and [3]: $(0,\ -6)$

Vertices: $\left(3\frac{1}{5},\ 3\frac{3}{5}\right), \left(-2\frac{2}{7},\ \frac{6}{7}\right), (0,\ -6)$

(c) Draw broken lines for $y - 2x = 4,\ y + 2x = 6$ and a solid line for $y = x + 2$.

Test a point not on any boundary, e.g. $(0,\ 0)$.

For $y - 2x < 4,\ 0 - 2 \times 0 < 4$.

This is true, so this is the area which is below $2y - x = 4$.

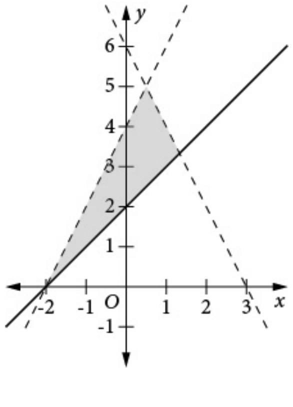
For $y + 2x < 6,\ 0 + 2 \times 0 < 6$.

This is true, so this is the area which is below $y = 3x - 6$.

For $y \geq x + 2,\ 0 \geq 0 + 2$.

This is false, so this is the area which is above $y = x + 2$.

Shade the region which is below $2y - x = 4$, below $y = 3x - 6$ and also above $y = x + 2$.



$y-2x=4 \quad [1]$

$y+2x=6 \quad [2]$

$y=x+2 \quad [3]$

$[1]+[2]$

$$\begin{aligned} y-2x+(y+2x)&=4+6 \\ 2y&=10 \\ y&=5 \end{aligned}$$

Substitute $y=5$ into $[1]$

$$\begin{aligned} 5-2x&=4 \\ x&=\frac{1}{2} \end{aligned}$$

Intersection of $[1]$ and $[2]$: $\left(\frac{1}{2},\ 5\right)$

Substitute $[3]$ into $[1]$

$$\begin{aligned} x+2-2x&=4 \\ x&=-2 \end{aligned}$$

Substitute $x=-2$ into $[3]$

$$\begin{aligned} y&=-2+2 \\ y&=0 \end{aligned}$$

Intersection of $[1]$ and $[3]$: $(2,\ 0)$

Substitute $[3]$ into $[2]$

$$\begin{aligned} x+2+2x&=6 \\ x&=1\frac{1}{3} \end{aligned}$$

Substitute $x=1\frac{1}{3}$ into $[3]$

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$$\begin{aligned} y &= 1\frac{1}{3} + 2 \\ &= 3\frac{1}{3} \end{aligned}$$

Intersection of [2] and [3]: $\left(1\frac{1}{3},\ 3\frac{1}{3}\right)$

Vertices: $\left(\frac{1}{2},\ 5\right)$, $(2,\ 0)$, $\left(1\frac{1}{3},\ 3\frac{1}{3}\right)$

(d) Draw the broken lines for $y-3x=3,\ 3x+4y=12,\ x-2y=4.$

Test a point not on any boundary, e.g. $(0,\ 0)$.

For $y-3x<3,\ 0-3\times 0<3$.

This is true, so this is the area which is below $y-3x=3$.

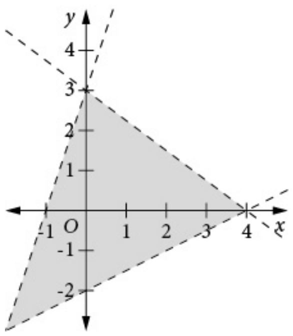
For $3x+4y<12,\ 3\times 0+4\times 0<12$.

This is true, so this is the area which is below $3x+4y=12$.

For $x-2y<4,\ 0-2\times 0<4$.

This is true, so this is the area which is above $x-2y=4$.

Shade the region which is below $y-3x=3$ and $3x+4y=12$, and also above $x-2y=4$.



$y-3x=3 \qquad [1]$
$3x+4y=12 \qquad [2]$
$x-2y=4 \qquad [3]$

$[1]+[2]$

$$\begin{aligned} y-3x+(3x+4y)&=3+12 \\ 5y&=15 \\ y&=3 \end{aligned}$$

Substitute $y=3$ into [1]

$$\begin{aligned} 3-3x&=3 \\ x&=0 \end{aligned}$$

Intersection of [1] and [2]: $(0,\ 3)$

$2\times[1]+[3]$

$$\begin{aligned} 2y-6x+(x-2y)&=6+4 \\ -5x&=10 \\ x&=-2 \end{aligned}$$

Substitute $x=-2$ into [1]

$$\begin{aligned} y-3(-2)&=3 \\ y&=-3 \end{aligned}$$

Intersection of [1] and [3]: $(-2,\ -3)$

$[2]+2\times[3]$

$$\begin{aligned} 3x+4y+2x-4y&=12+8 \\ 5x&=20 \\ x&=4 \end{aligned}$$

Substitute $x=4$ into [3]

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$$4-2y=4$$
$$y=0$$

Intersection of [2] and [3]: $(4,\ 0)$

Vertices: $(0,\ 3)$, $(-2,\ -3)$, $(4,\ 0)$

(e) Draw solid lines for $y=x-1,\ x+y=2,\ x=0,\ y=0.$

Test a point not on any boundary, e.g. $(0,\ 0)$.

For $y \le x+1,\ 0 \le 0+1$.

This is true, so this is the area which is below $y=x+2$.

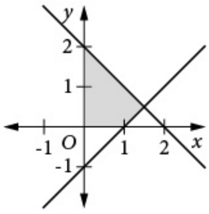
For $x+y \le 2,\ 0+0 \le 2$.

This is true, so this is the area which is below $x+y=2$.

$x \ge 0$ is the region to the right of the y -axis.

$y \ge 0$ is the region above the x -axis.

Shade the region which is above $y=x-1,\ x+y=2,\ x=0$ (the x -axis) and to the right of $y=0$ (the y -axis).



$y=x-1$ [1]
$x+y=2$ [2]
$y=0$ [3]
$x=0$ [4]

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Substitute [1] into [2]

$$x+(x-1)=2$$
$$2x=3$$
$$x=\frac{3}{2}$$

Substitute $x=\dfrac{3}{2}$ into [1]

$$y=\frac{3}{2}-1=\frac{1}{2}$$

Intersection of [1] and [2]: $\left(\dfrac{3}{2},\ \dfrac{1}{2}\right)$

Substitute [3] into [1]

$$0=x-1$$
$$x=1$$

Intersection of [1] and [3]: $(1,\ 0)$

Substitute [4] into [2]

$$0+y=2$$
$$y=2$$

Intersection of [2] and [4]: $(0,\ 2)$

Vertices: $\left(\dfrac{3}{2},\ \dfrac{1}{2}\right)$, $(1,\ 0)$, $(0,\ 2)$, $(0,\ 0)$

(f) Draw the solid lines for $x+y=2,\ x+y=-2,\ x-y=2,\ x-y=-2.$

Test a point not on any boundary, e.g. $(0,\ 0)$.

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For $x+y\le 2,\ 0+0\le 2$.

This is true, so this is the area which is below $x+y=2$.

For $x+y\ge -2,\ 0+0\ge -2$.

This is true, so this is the area which is above $x+y=-2$.

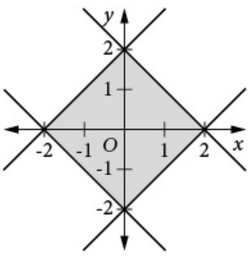
For $x-y\le 2,\ 0-0\le 2$.

This is true, so this is the area which is above $x-y=2$.

For $x-y\ge -2,\ 0-0\ge -2$.

This is true, so this is the area which is below $x-y=-2$.

Shade the region which is below $x+y=2$ and $x-y=-2$, and also above $x+y=-2$ and $x-y=2$.



$x+y=2$ [1]
$x+y=-2$ [2]
$x-y=2$ [3]
$x-y=-2$ [4]

[1]+[3]

$$\begin{aligned} x+y+(x-y) &= 2+2 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

Substitute $x = 2$ into [1]

$$\begin{aligned} 2 + y &= 2 \\ y &= 0 \end{aligned}$$

Intersection of [1] and [3]: $(2,\ 0)$

$[1]+[4]$

$$\begin{aligned} x + y + (x - y) &= 2 + (-2) \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

Substitute $x = 0$ into [1]

$$\begin{aligned} 0 + y &= 2 \\ y &= 2 \end{aligned}$$

Intersection of [1]and [4]: $(0,\ 2)$

$[2]+[3]$

$$\begin{aligned} x + y + (x - y) &= -2 + 2 \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

Substitute $x = 0$ into [2]

$$\begin{aligned} 0 + y &= -2 \\ y &= -2 \end{aligned}$$

Intersection of [2] and [3]: $(0,\ -2)$

$[2]+[4]$

$$\begin{aligned} x + y + (x - y) &= -2 + (-2) \\ 2x &= -4 \\ x &= -2 \end{aligned}$$

Substitute $x = -2$ into [2]

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$$-2+y=-2$$
$$y=0$$

Intersection of [2] and [4]: $(-2,\ 0)$

Vertices: $(2,\ 0)$, $(0,\ 2)$, $(0,\ -2)$, $(-2,\ 0)$

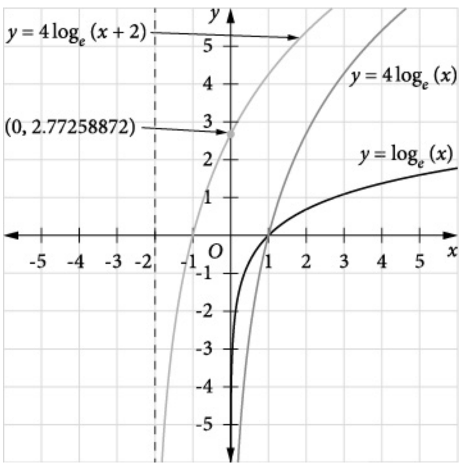
CHAPTER REVIEW 15

2 $y=\ln x$ is the basic logarithm function with x-intercept $(1,0)$ and the y-axis as the asymptote.

$y=4\ln x$ is the first function dilated from the x-axis with factor 4.

$y=4\ln(x+2)$ is the second function translated 2 units to the left.

The y-intercept will be $(0,4\ln 2)$ i.e. about $(0,2.77)$. The x-intercept will be $(-1,0)$.



4 **(a)** $f(x)=e^x$

Replace x with $2x$.

$f(2x)=e^{2x}$

(b) Replace x with $2x$.

$f(x-3)=e^{x-3}$

(c) $f(x)+1=e^x+1$

(d) $2f(x)+4=2e^x+4$

(e) Replace x with $2(x+2)$.

$f(2(x+2))=e^{2(x+2)}$

$3f(2(x+2))-1=3e^{2(x+2)}-1$

6 (a) Let $BC=y$ cm

$AB=AE=EB=DC=x$

$BC=BF=FC=AD=y$

Perimeter $=54$ cm so

$$3x+3y=54$$
$$x+y=18$$
$$y=18-x$$

The shape consists of a rectangle and two equilateral triangles of sides x and y respectively.

Rectangle area $=xy=x(18-x)$

Height of triangle ABE $=x\sin 60°=\dfrac{x\sqrt{3}}{2}$

Area of triangle ABE $=\dfrac{1}{2}\times x\times\dfrac{x\sqrt{3}}{2}=\dfrac{x^2\sqrt{3}}{4}$

Height of triangle BCF $=y\sin 60°=\dfrac{y\sqrt{3}}{2}$

Area of triangle BCF $=\dfrac{1}{2}\times y\times\dfrac{y\sqrt{3}}{2}=\dfrac{y^2\sqrt{3}}{4}=\dfrac{(18-x)^2\sqrt{3}}{4}$

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$$\begin{aligned}A(x) &= x(18-x)+\frac{x^2\sqrt{3}}{4}+\frac{(18-x)^2\sqrt{3}}{4}\\ &=18x-x^2+\frac{\sqrt{3}}{4}x^2+\frac{\sqrt{3}}{4}\left(324-36x+x^2\right)\\ &=18x-x^2+\frac{\sqrt{3}}{4}x^2+81\sqrt{3}-9\sqrt{3}x+\frac{\sqrt{3}}{4}x^2\\ &=\left(\frac{\sqrt{3}}{2}-1\right)x^2+\left(18-9\sqrt{3}\right)x+81\sqrt{3}\\ &=\left(\frac{\sqrt{3}-2}{2}\right)x^2+9\left(2-\sqrt{3}\right)x+81\sqrt{3}\end{aligned}$$

(b) $x+y=18$ and $x>0,\ y>0$, so since $y>0$, x must be less than 18 (as well as greater than zero).

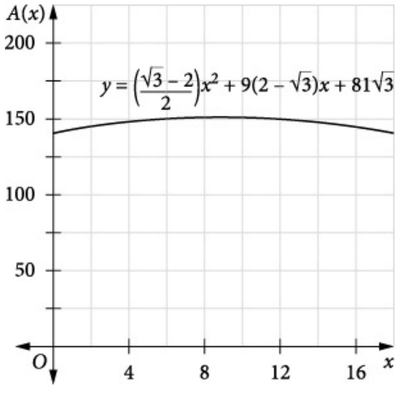
Hence the domain is $0<x<18$.

(c) Since $\sqrt{3}-2<0$, the coefficient of x^2 is negative, and the parabola will be concave down.

When $x=0,\ y=81\sqrt{3}$. This corresponds to the area of one equilateral triangle of perimeter 18 cm. When $x=18,\ y=81\sqrt{3}$ since this also corresponds to the area of an equilateral triangle of perimeter 18 cm. $81\sqrt{3}\approx 140.30$

The maximum will occur at the middle of these two values, when $x=9$. In this case,

$$\begin{aligned}A(x) &=\left(\frac{\sqrt{3}}{2}-1\right)\times 9^2+9\left(2-\sqrt{3}\right)\times 9+81\sqrt{3}\\ &=40.5\sqrt{3}-81+162-81\sqrt{3}+81\sqrt{3}\\ &=40.5\sqrt{3}+81\\ &\approx 151.15\end{aligned}$$



(d) The reasoning in part **(c)** gives $40.5\sqrt{3}+81\approx 151.15$

If calculus is used, the following reasoning will produce the same answer.

$$A(x)=\left(\frac{\sqrt{3}-2}{2}\right)x^2+9\left(2-\sqrt{3}\right)x+81\sqrt{3}$$

$$\frac{dA}{dx}=\left(\sqrt{3}-2\right)x+9\left(2-\sqrt{3}\right)$$

$$\frac{dA}{dx}=0:$$

$$\left(\sqrt{3}-2\right)x+9\left(2-\sqrt{3}\right)=0$$

$$\frac{dA}{dx}=0\Rightarrow x=\frac{-9\left(2-\sqrt{3}\right)}{\sqrt{3}-2}=\frac{9\left(\sqrt{3}-2\right)}{\sqrt{3}-2}=9$$

$\dfrac{d^2A}{dx^2}=\sqrt{3}-2<0$ so the parabola is concave down throughout.

The rectangle becomes a square of side 9 cm when the area is a maximum.

8 (a) (i) $y=\dfrac{4x}{(x-1)^2}$

Use the quotient rule to find the derivative:

$$\begin{aligned}\frac{dy}{dx}&=\frac{(x-1)^2\times 4-4x\times 2(x-1)}{(x-1)^4}\\&=\frac{4(x-1)(x-1-2x)}{(x-1)^4}\\&=\frac{4(-x-1)}{(x-1)^3}\\&=-\frac{4(x+1)}{(x-1)^3}\end{aligned}$$

An interesting alternative which maybe makes life easier is

$$y=\frac{4x-4+4}{(x-1)^2}=\frac{4}{x-1}+\frac{4}{(x-1)^2}$$

$$\frac{dy}{dx}=-\frac{4}{(x-1)^2}-2\times\frac{4}{(x-1)^3}$$

$$=\frac{-4(x-1)-8}{(x-1)^3}$$

$$=-\frac{4x+4}{(x-1)^3}$$

For stationary points, $\frac{dy}{dx}=0\Rightarrow x=-1$

$$x=-1\Rightarrow y=\frac{4\times-1}{\left(-1-1\right)^2}=-1$$

Finding the second derivative is more complex, so compare the gradient before and after.

$$x=-2\Rightarrow\frac{dy}{dx}=-\frac{4\times-2+4}{(-2-1)^3}=-\frac{-4}{-27}=-\frac{4}{27}<0$$

$$x=0\Rightarrow\frac{dy}{dx}=-\frac{4\times0+4}{(0-1)^3}=-\frac{4}{-1}=4>0$$

The gradient changes from negative to positive, so there is a minimum at $\left(-1,\ -1\right)$.

Method using the second derivative:

$$\frac{d^2y}{dx^2}=\frac{-4\left((x-1)^3-(x+1)\times3(x-1)^2\right)}{(x-1)^6}$$

$$=\frac{-4(x-1)^2\left(x-1-(x+1)\times3\right)}{(x-1)^6}$$

$$=\frac{-4(x-1-3x-3)}{(x-1)^4}$$

$$=\frac{-4(-2x-4)}{(x-1)^4}$$

$$=\frac{8(x+2)}{(x-1)^4}$$

$$x=1\Rightarrow\ \frac{d^2y}{dx^2}=\frac{8(-1+2)}{(-1-1)^4}=\frac{8}{16}>0$$

Since this is positive the stationary point will be a minimum.

There is a minimum turning point at $(-1,-1)$.

(ii) $\dfrac{d^2y}{dx^2} = 0 \Rightarrow x = -2$

$x = -2 \Rightarrow y = \dfrac{4 \times -2}{(-2-1)^2} = -\dfrac{8}{9}$

$x = -1 \Rightarrow \dfrac{d^2y}{dx^2} > 0$

$x = -3 \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{-8}{4^4} < 0$

Concavity changes at $x = -2$ so $\left(-2,\ -\dfrac{8}{9}\right)$ is a point of inflection.

(iii) $y = \dfrac{4x}{(x-1)^2}$

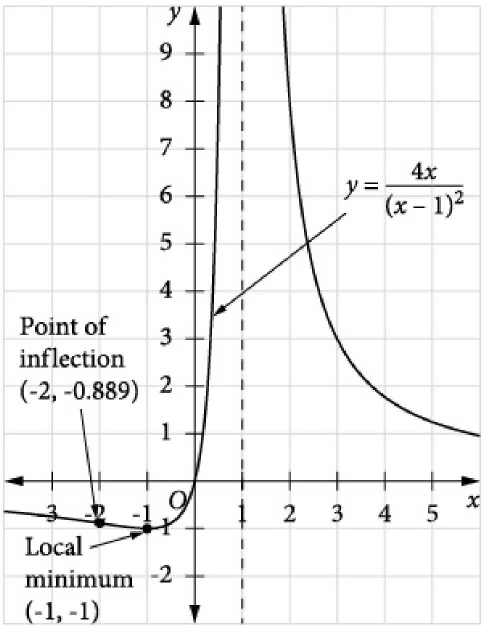
As $x \to -\infty, y \to 0$ from below.

As $x \to \infty, y \to 0$ from above.

As $x \to 1$ from right or left, $y \to \infty$.

The asymptotes are $x = 1, y = 0$

(b) $f(x) = \dfrac{4x}{(x-1)^2}$



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(c) $f'(x) = -\dfrac{4(x+1)}{(x-1)^3}$

For $x = 0$, $y = -\dfrac{4(0+1)}{(0-1)^3} = -\dfrac{4}{-1} = 4$.

The y-intercept is 4.

For $y = 0$, $y = -\dfrac{4(x+1)}{(x-1)^3} = 0 \Rightarrow x = -1$

The x-intercept is -1.

$x = 1$ will be an asymptote.

As $x \to 1$ from the left, $y = -\dfrac{4(x+1)}{(x-1)^3} > 0$ so $y \to +\infty$.

As $x \to 1$ from the right, $y = -\dfrac{4(x+1)}{(x-1)^3} < 0$ so $y \to -\infty$.

As $x \to +\infty$, $y = -\dfrac{4(x+1)}{(x-1)^3} \to 0$ from below.

As $x \to -\infty$, $y = -\dfrac{4(x+1)}{(x-1)^3} \to 0$ from above.

$$\frac{d}{dx}\left(f'(x)\right) = \frac{d^2y}{dx^2} = 0 \Rightarrow x = -2$$

$$x = -2 \Rightarrow f'(x) = -\frac{4(-2+1)}{(-2-1)^3} = -\frac{-4}{-27} = -\frac{4}{27}$$

There is a stationary point at $\left(-2, -\dfrac{4}{27}\right)$. The nature of this stationary point will be obvious from the graph.

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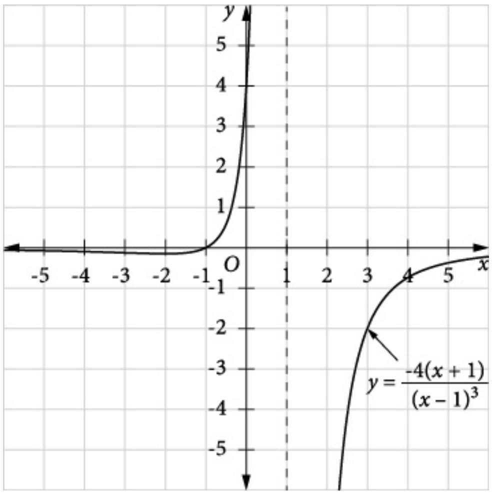
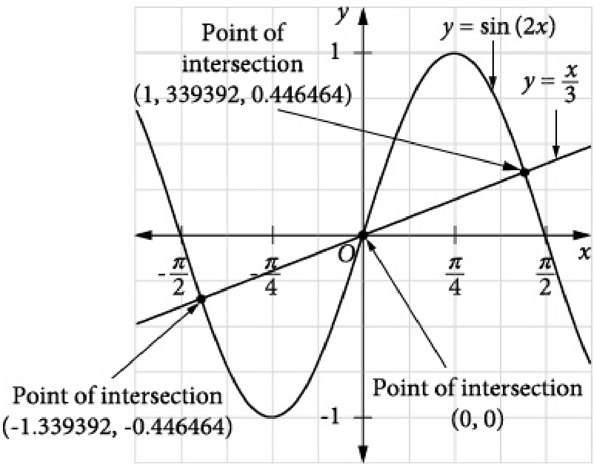


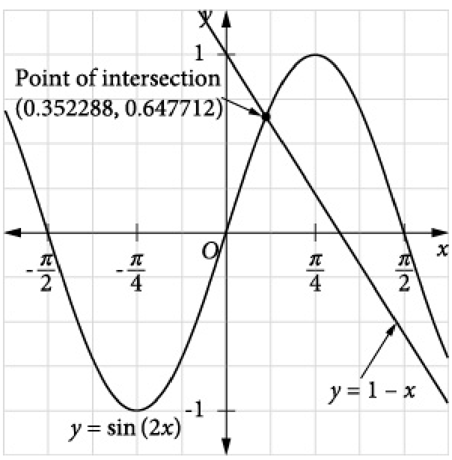
10 (a) Draw the graph of $y = \frac{x}{3}$ on the same axes and find the x values of the intersection points.



The graphs $y = \sin 2x$ and $y = \frac{x}{3}$ intersect at three places: when $x = 0$ and near $x = \pm 0.4\pi$ or ± 1.3.

More accurate solutions using technology: $x = 0,\ \pm 0.43\pi$ i.e. $0,\ \pm 1.34$.

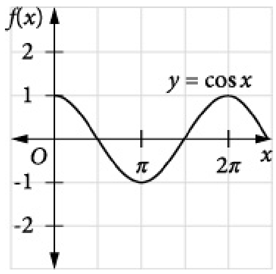
(b) Draw the graph of $y = 1 - x$ on the same axes and find the x values of the intersection points.

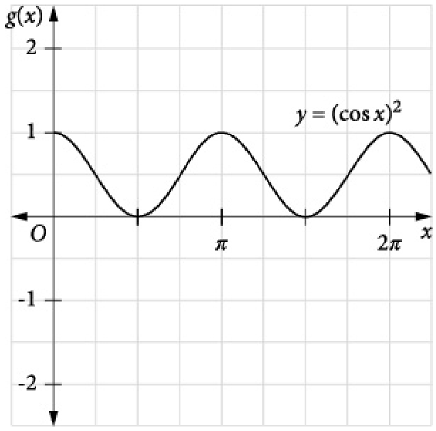


The graphs $y = \sin 2x$ and $y = 1 - x$ intersect at one place: near $x = 0.1\pi$ or 0.3.

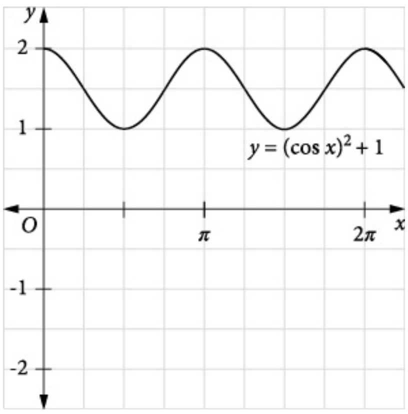
A more accurate solution using technology: $x = 0.11\pi$ i.e. 0.35.

12 (a)





The graph of $y = g(x) + 1$ will be the graph of $y = g(x)$ translated 1 unit up.



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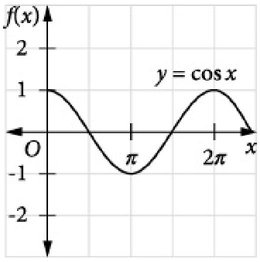
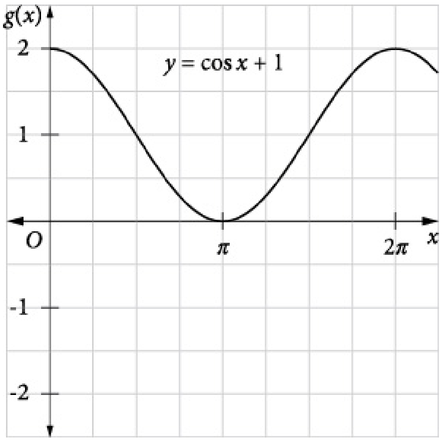
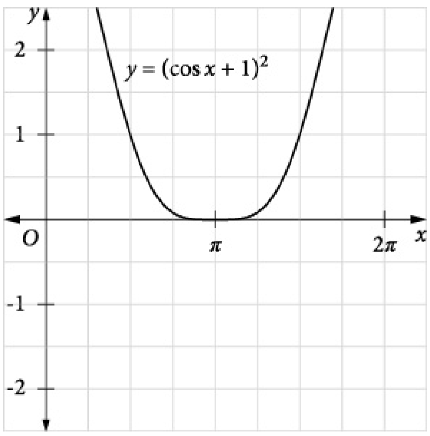
(b)





The graph of $y = f(x)+1$ will be the graph of $y = f(x)$ translated 1 unit up.



(c) The final graphs in each part are completely different due to whether the squaring is done before or after adding 1.

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