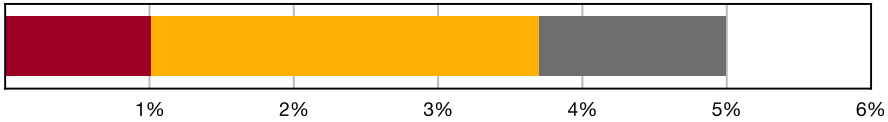
ADVANCED MATHEMATICS
Statistics (Adv), S1 Probability & Probability Distributions (Adv)

Multi-Stage Events (Y11)
Relative Frequency (Y11)
Conditional Probability and Venn Diagrams (Y11)
Discrete Probability Distributions (Y11)

Teacher: Cathyanne Horvat
Exam Equivalent Time: 58.5 minutes *(based on allocation of 1.5 minutes per mark)*

S1 Probability and Probability Distributions



Multi-Stage Events
Conditional Probability and Venn Diagrams
Discrete Probability Distributions

***SmarterMaths analytics based on the average contribution to new syllabus Advanced Maths exams since 2020.**

HISTORICAL CONTRIBUTION

- *S1 Probability* has contributed an average of 5.0% per new syllabus Advanced exam since it was introduced in 2020.
- *S1 Probability* has been split into four sub-categories for the purposes of this analysis: *1-Multi-Stage Events (1.0%), 2-Relative Frequency (0%), 3-Conditional Probability and Venn Diagrams (2.7%) and 4-Discrete Probability Distributions (1.3%).*
- This analysis looks at *Relative Frequency.*

HSC ANALYSIS - What to expect and common pitfalls

- *Relative Frequency* is a sub-topic that has previously only been examined in the *Standard* course but now represents common content with the new *Advanced* syllabus.
- *Relative Frequency* was not examined in the first three *Advanced exams* in 2020-22.
- It has been a substantial contributor to past *Std2* HSC exams, accounting for a very healthy average of 2.7% per year.
- Our database is made up of an extensive history of past HSC questions from the *Std2* course, which can be identified by the "STD2 S2" in their title.
- Although generally well answered, note the sub-50% mean marks in *2018 Adv 26a, 2017 Adv 29c* and in *2016 Adv 23 MC* that deserve attention (note that all mean marks and comments are taken from *Std2* results).

Questions

1. Probability, 2ADV S1 2014 HSC 12c

A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.

i. Draw a tree diagram to show the possible outcomes. Include the probability on each branch. ***(2 marks)***

ii. What is the probability that the two lollies are of different colours? ***(1 mark)***

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2. Probability, 2ADV S1 2019 HSC 11f

A bag contains 5 green beads and 7 purple beads. Two beads are selected at random, without replacement.

What is the probability that the two beads are the same colour? ***(2 marks)***

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3. Probability, STD2 S2 2011 HSC 25c

At another school, students who use mobile phones were surveyed. The set of data is shown in the table.

	Pre-paid	*Plan*	TOTAL
Female students	172	147	319
Male students	158	103	261
TOTAL	330	250	

i. How many students were surveyed at this school? ***(1 mark)***

ii. Of the female students surveyed, one is chosen at random. What is the probability that she uses pre-paid? ***(1 mark)***

Ten new male students are surveyed and all ten are on a plan. The set of data is updated to include this information.

iii. What percentage of the male students surveyed are now on a plan? Give your answer to the nearest per cent. ***(1 mark)***

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4. Probability, 2ADV S1 2007 HSC 4b

Two ordinary dice are rolled. The score is the sum of the numbers on the top faces.

i. What is the probability that the score is 10? ***(2 marks)***

ii. What is the probability that the score is not 10? ***(1 mark)***

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5. Probability, 2ADV S1 SM-Bank 3

In a workplace of 25 employees, each employee speaks either French or German, or both.

If 36% of the employees speak German, and 20% speak both French and German.

i. Calculate the probability one person chosen could speak German if they could speak French. Give your answer to the nearest percent. ***(1 mark)***

ii. Calculate the probability one person chosen could not speak French if they could speak German. Give your answer to the nearest percent. ***(1 mark)***

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6. Probability, 2ADV S1 2008 MET1 7

Jane drives to work each morning and passes through three intersections with traffic lights. The number $\boldsymbol{X}$ of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.1	0.2	0.3	0.4

i. What is the mode of $\boldsymbol{X}$? ***(1 mark)***

ii. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days? ***(2 marks)***

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7. Probability, 2ADV S1 2009 MET1 7

The random variable $\boldsymbol{X}$ has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

i. $\boldsymbol{P(X > 1 \mid X \leq 3)}$ ***(2 marks)***

ii. $\boldsymbol{P(X)}$, the variance of $\boldsymbol{X}$. ***(3 marks)***

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8. Probability, 2ADV S1 2007 MET1 11

There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island, is 0.8, and the probability of the flight departing on time, given that the weather on the island is not fine, is 0.6.

In March the probability of a day being fine is 0.4.

Find the probability that on a particular day in March

i. the flight from Paradise Island departs on time ***(2 marks)***

ii. the weather is fine on Paradise Island, given that the flight departs on time. ***(2 marks)***

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9. Probability, 2ADV S1 2011 MET1 8

Two events, $\boldsymbol{A}$ and $\boldsymbol{B}$, are such that $\boldsymbol{P = \frac{3}{5}}$ and $\boldsymbol{P = \frac{1}{4}}$.

If $\boldsymbol{A\prime}$ denotes the compliment of $\boldsymbol{A}$, calculate $\boldsymbol{P(A\prime \cap B)}$ when

i. $\boldsymbol{P(A \cup B) = \frac{3}{4}}$ ***(2 marks)***

ii. $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually exclusive. ***(1 mark)***

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10. Probability, 2ADV S1 2014 MET1 9

Sally aims to walk her dog, Mack, most mornings. If the weather is pleasant, the probability that she will walk Mack is $\frac{3}{4}$, and if the weather is unpleasant, the probability that she will walk Mack is $\frac{1}{3}$.

Assume that pleasant weather on any morning is independent of pleasant weather on any other morning.

a. In a particular week, the weather was pleasant on Monday morning and unpleasant on Tuesday morning.

Find the probability that Sally walked Mack on at least one of these two mornings. ***(2 marks)***

b. In the month of April, the probability of pleasant weather in the morning was $\frac{5}{8}$.

i. Find the probability that on a particular morning in April, Sally walked Mack. ***(2 marks)***

ii. Using your answer from **part b.i.**, or otherwise, find the probability that on a particular morning in April, the weather was pleasant, given that Sally walked Mack that morning. ***(2 marks)***

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11. Probability, STD2 S2 2018 HSC 26a

Jeremy rolled a biased 6-sided die a number of times. He recorded the results in a table.

Number	1	2	3	4	5	6
Frequency	23	19	48	20	21	19

What is the relative frequency of rolling a 3? ***(1 mark)***

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12. Probability, 2ADV S1 2018 HSC 16b

A game involves rolling two six-sided dice, followed by rolling a third six-sided die. To win the game, the number rolled on the third die must lie between the two numbers rolled previously. For example, if the first two dice show 1 and 4, the game can only be won by rolling a 2 or 3 with the third die.

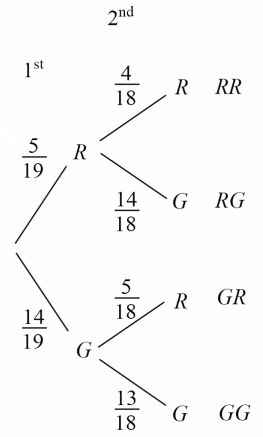
i. What is the probability that a player has no chance of winning before rolling the third die? ***(2 marks)***

ii. What is the probability that a player wins the game? ***(2 marks)***

Worked Solutions

1. Probability, 2ADV S1 2014 HSC 12c

i.

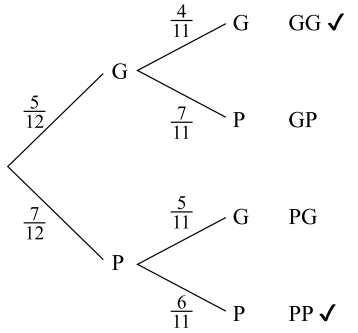


ii. $P(\text{different colours})$

$$
\begin{aligned}
&= P(RG) + P(GR) \\
&= \frac{5}{19} \times \frac{14}{18} + \frac{14}{19} \times \frac{5}{18} \\
&= \frac{70}{342} + \frac{70}{342} \\
&= \frac{140}{342} \\
&= \frac{70}{171}
\end{aligned}
$$

Worked Solutions

2. Probability, 2ADV S1 2019 HSC 11f



$$\begin{aligned} P\text{ (same colour)} &= P(GG) + P(PP) \\ &= \frac{5}{12} \cdot \frac{4}{11} + \frac{7}{12} \cdot \frac{6}{11} \\ &= \frac{62}{132} \\ &= \frac{31}{66} \end{aligned}$$

3. Probability, STD2 S2 2011 HSC 25c

i. $\#\text{ Students surveyed} = 319 + 261 = 580$

ii. $$\begin{aligned} P(\text{Female uses prepaid}) &= \frac{\#\text{ Females on prepaid}}{\text{Total females}} \\ &= \frac{172}{319} \\ &= 0.53918\ldots \\ &= 54\%\ \text{(nearest \%)} \end{aligned}$$

iii. $$\begin{aligned} \%\text{ Males on plan} &= \frac{\#\text{ Males on plan} + 10}{\text{Total males} + 10} \\ &= \frac{103 + 10}{261 + 10} \\ &= \frac{113}{271} \\ &= 0.4169\ldots \\ &= 42\%\ \text{(nearest \%)} \end{aligned}$$

4. Probability, 2ADV S1 2007 HSC 4b

i.

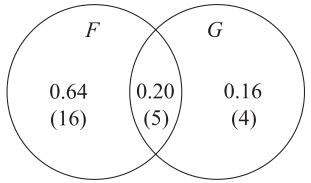
Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	**10**
5	6	7	8	9	**10**	11
6	7	8	9	**10**	11	12

$$\begin{aligned} &\text{P (score} = 10) \\ &= \frac{3}{36} \\ &= \frac{1}{12} \end{aligned}$$

ii. $$\begin{aligned} &\text{P (score is not ten)} \\ &= 1 - \text{P (score is ten)} \\ &= 1 - \frac{1}{12} \\ &= \frac{11}{12} \end{aligned}$$

5. Probability, 2ADV S1 SM-Bank 3

i. Expressing in a Venn diagram:



$$\begin{aligned} P(G \mid F) &= \frac{P(G \cap F)}{P(F)} \\ &= \frac{0.20}{0.84} \\ &= 0.238\ldots \\ &= 24\% \end{aligned}$$

ii. $$\begin{aligned} P(\text{not } F \mid G) &= \frac{P(\text{not } F \cap G)}{P(G)} \\ &= \frac{0.16}{0.36} \\ &= 0.444\ldots \\ &= 44\% \end{aligned}$$

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6. Probability, 2ADV S1 2008 MET1 7

i. 3

ii. $$\begin{aligned} &P(0,0) + P(1,1) + P(2,2) + P(3,3) \\ &= 0.1^2 + 0.2^2 + 0.3^2 + 0.4^2 \\ &= 0.3 \end{aligned}$$

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7. Probability, 2ADV S1 2009 MET1 7

i. $$\begin{aligned} P(X > 1 \mid X \leq 3) &= \frac{P(X=2) + P(X=3)}{1 - P(X=4)} \\ &= \frac{0.4 + 0.2}{1 - 0.1} \\ &= \frac{0.6}{0.9} \\ &= \frac{2}{3} \end{aligned}$$

ii. $$\begin{aligned} E(X) &= 0.1(0) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) \\ &= 0 + 0.2 + 0.8 + 0.6 + 0.4 \\ &= 2 \end{aligned}$$

$$\begin{aligned} E(X^2) &= 0^2(0.1) + 1^2(0.2) + 2^2(0.4) + 3^2(0.2) + 4^2(0.1) \\ &= 0 + 0.2 + 1.6 + 1.8 + 1.6 \\ &= 5.2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 5.2 - (2)^2 \\ &= 1.2 \end{aligned}$$

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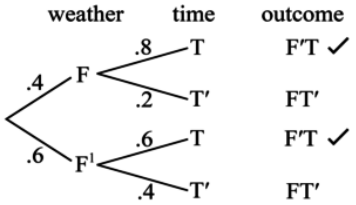
8. Probability, 2ADV S1 2007 MET1 11

i.



$$P(\text{FT}) + P(\text{F}'\text{T})$$
$$= 0.4 \times 0.8 + 0.6 \times 0.6$$
$$= 0.32 + 0.36$$
$$= 0.68$$

ii. Conditional probability:

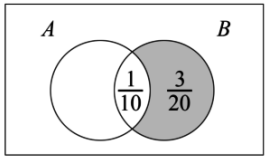
$$P(F \mid T) = \frac{P(F \cap T)}{P(T)}$$
$$= \frac{0.32}{0.68}$$

$$\therefore P(F \mid T) = \frac{8}{17}$$

♦♦ *Mean mark 29%.*
MARKER'S COMMENT: Students continue to struggle with conditional probability. Attention required here.

9. Probability, 2ADV S1 2011 MET1 8

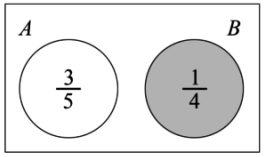
i. Sketch Venn Diagram



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{3}{4} = \frac{3}{5} + \frac{1}{4} - P(A \cap B)$$
$$P(A \cap B) = \frac{1}{10}$$

$$\therefore P(A' \cap B) = \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$$

ii.



$P(A \cap B) = 0$ **(mutually exclusive),**

$$\therefore P(A' \cap B) = P(B) = \frac{1}{4}$$

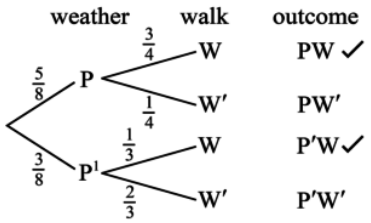
10. Probability, 2ADV S1 2014 MET1 9

a. $P(\text{at least 1 walk}) = 1 - P(\text{no walk})$

$$= 1 - \frac{1}{4} \times \frac{2}{3}$$

$$= \frac{5}{6}$$

b.i. Construct tree diagram:



$$P(PW) + P(P\prime W) = \frac{5}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{3}$$

$$= \frac{19}{32}$$

b.ii. $P(P \mid W) = \dfrac{P(P \cap W)}{P(W)}$

$$= \frac{\frac{5}{8} \times \frac{3}{4}}{\frac{19}{32}}$$

$$= \frac{15}{32} \times \frac{32}{19}$$

$$= \frac{15}{19}$$

♦ *Part (b)(ii) mean mark 38%.*

11. Probability, STD2 S2 2018 HSC 26a

$$\text{Rel Freq} = \frac{\text{number of 3's rolled}}{\text{total rolls}}$$

$$= \frac{48}{150}$$

$$= \frac{8}{25}$$

♦ *Mean mark 40%.*

12. Probability, 2ADV S1 2018 HSC 16b

i. Construct a sample space of the number of possible winning rolls:

♦ *Mean mark 40%.*
COMMENT: Constructing the full sample space is a critical step here..

	1	**2**	**3**	**4**	**5**	**6**
1	–	–	1	2	3	4
2	–	–	–	1	2	3
3	1	–	–	–	1	2
4	2	1	–	–	–	1
5	3	2	1	–	–	–
6	4	3	2	1	–	–

$$\text{P(no chance)} = \frac{\text{number of pairs with no gap}}{\text{total possibilities}}$$

$$= \frac{16}{36}$$

$$= \frac{4}{9}$$

ii. The sample space in the table shows:

♦♦♦ *Mean mark 7%.*

$\rightarrow$ 8 combinations leave a gap for a single winning number,

$\rightarrow$ 6 combinations leave a gap for two winning numbers,

$\vdots$

$$\therefore \text{P(winning)} = \frac{1}{36}\left[8 \times \frac{1}{6} + 6 \times \frac{2}{6} + 4 \times \frac{3}{6} + 2 \times \frac{4}{6}\right]$$

$$= \frac{1}{36}\left(\frac{8}{6} + \frac{12}{6} + \frac{12}{6} + \frac{8}{6}\right)$$

$$= \frac{1}{36}\left(\frac{40}{6}\right)$$

$$= \frac{5}{27}$$