ADVANCED MATHEMATICS
Logs and Exponentials (Adv), E1 Logs and Exponentials (Adv)

Log/Index Laws and Equations (Y11)

Teacher: Cathyanne Horvat
Exam Equivalent Time: 30 minutes *(based on allocation of 1.5 minutes per mark)*

HISTORICAL CONTRIBUTION

- *E1 Logs and Exponentials* is a Year 11 topic covering significant amounts of gateway content that is applicable to multiple other Year 12 topic areas.
- It's contribution to other sub-topics is highly significant, including *Tangents, Areas Under Curves, Probability Density Functions, Rates of Change and L&E Calculus* just to name a few. Importantly, these are not covered in this analysis as they are included in other topic areas.
- *E1 Logs and Exponentials* has been split into two sub-categories for the purposes of analysis which are: *1-Index Laws and Equations and 2-L&E Graphs and Applications.*
- This analysis looks at *Index Laws and Equations.*

HSC ANALYSIS - What to expect and common pitfalls

- *Index Laws and Other Equations* has not been examined with a dedicated question in new syllabus exams to date, although they consistently appeared before 2020 within the Adv multiple choice section (including twice in 2019).
- The database, despite the absence of new syllabus HSC exam questions, provides important practice for algebraic manipulation of logs and exponential equations, which is a critical skill for later *Calculus* topics.
- Notwithstanding the above, the database does include many pre-2020 dedicated questions. While generally well answered, students had notable difficulties with *2019 Adv 15a, 2016 Adv 14e*, and *2008 Adv 7a,* which deserve attention.

Questions

1. Functions, 2ADV F1 SM-Bank 53

   i. If $\frac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^x$, find x. ***(1 mark)***

   ii. Calculate the value of $\frac{1}{\sqrt[3]{7+\pi}}$ to 3 significant figures. ***(1 mark)***

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2. L&E, 2ADV E1 SM-Bank 8

   Solve the equation $3^{-4x} = 9^{6-x}$ for x. ***(2 marks)***

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3. L&E, 2ADV E1 SM-Bank 12

   Solve the equation $\log_e(3x+5) + \log_e(2) = 2$, for x. ***(2 marks)***

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4. L&E, 2ADV E1 2011 HSC 1c

   Solve $2^{2x+1} = 32$. ***(2 marks)***

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5. L&E, 2ADV E1 2005 HSC 5a

   Use the change of base formula to evaluate $\log_3 7$, correct to two decimal places. ***(1 mark)***

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6. L&E, 2ADV E1 SM-Bank 7

   Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. ***(2 marks)***

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7. L&E, 2ADV E1 SM-Bank 9

   Solve $\log_2(6-x) - \log_2(4-x) = 2$ for x, where $x < 4$. ***(2 marks)***

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8. L&E, 2ADV E1 SM-Bank 10

   Solve the equation $2^{3x-3} = 8^{2-x}$ for x. ***(2 marks)***

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9. L&E, 2ADV E1 SM-Bank 13

   Find x given $100^{x-2} = 1000^x$. ***(2 marks)***

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10. L&E, 2ADV E1 2019 MET1-N 4

    Solve $\log_3(t) - \log_3(t^2-4) = -1$ for t. ***(3 marks)***

Worked Solutions

1. Functions, 2ADV F1 SM-Bank 53

i. $\dfrac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^{-\frac{1}{3}}$

ii. $\dfrac{1}{\sqrt[3]{7+\pi}} = 0.4619\ldots$

$= 0.462 \quad \text{(to 3 sig. fig.)}$

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2. L&E, 2ADV E1 SM-Bank 8

$$3^{-4x} = \left(3^2\right)^{6-x}$$
$$3^{-4x} = 3^{12-2x}$$
$$-4x = 12-2x$$
$$2x = -12$$
$$\therefore x = -6$$

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3. L&E, 2ADV E1 SM-Bank 12

Simplify using log laws:

$$\log_e(6x+10) = 2$$
$$6x+10 = e^2$$
$$\therefore x = \frac{e^2-10}{6}$$

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4. L&E, 2ADV E1 2011 HSC 1c

$$2^{2x+1} = 32$$
$$2^{2x+1} = 2^5$$
$$2x+1 = 5$$
$$\therefore x = 2$$

MARKER'S COMMENT: Many students also correctly solved this by taking the logarithms of both sides.

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5. L&E, 2ADV E1 2005 HSC 5a

$$\log_3 7 = \frac{\log_e 7}{\log_e 3}$$
$$= 1.771\ldots$$
$$= 1.77 \quad \text{(to 2 d.p.)}$$

Worked Solutions

6. L&E, 2ADV E1 SM-Bank 7

$$\log_3(5)^2 - \log_3(2) + \log_3(x) = 2$$
$$\log_3(25x) - \log_3(2) = 2$$
$$\log_3\left(\frac{25x}{2}\right) = 2$$
$$\frac{25x}{2} = 3^2$$
$$\therefore x = \frac{18}{25}$$

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7. L&E, 2ADV E1 SM-Bank 9

Simplify using log laws:

$$\log_2\left(\frac{6-x}{4-x}\right) = 2$$
$$2^2 = \frac{6-x}{4-x}$$
$$16-4x = 6-x$$
$$3x = 10$$
$$\therefore x = \frac{10}{3}$$

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8. L&E, 2ADV E1 SM-Bank 10

$$2^{3x-3} = 2^{3(2-x)}$$
$$3x-3 = 6-3x$$
$$6x = 9$$
$$\therefore x = \frac{3}{2}$$

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9. L&E, 2ADV E1 SM-Bank 13

$$100^{x-2} = 1000^x$$
$$\left(10^2\right)^{x-2} = \left(10^3\right)^x$$
$$10^{2x-4} = (10)^{3x}$$
$$2x-4 = 3x$$
$$\therefore x = -4$$

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10. L&E, 2ADV E1 2019 MET1-N 4

$$\begin{aligned}
\log_3(t) - \log_3\left(t^2 - 4\right) &= -1 \\
\log_3\left(\frac{t}{t^2-4}\right) &= -1 \\
\frac{t}{t^2-4} &= \frac{1}{3} \\
t^2 - 4 &= 3t \\
t^2 - 3t - 4 &= 0 \\
(t-4)(t+1) &= 0
\end{aligned}$$

$\therefore t = 4 \quad (t > 0,\ t \neq -1)$