5.3 Motion in Gravitational Fields

Multiple-choice questions: 1 mark each

1. An object on Earth has a weight of 490 N and experiences an acceleration due to gravity of 9.8 m s^{-2}. On Mars, this object would experience an acceleration due to gravity of 3.7 m s^{-2}.

On Mars, what would be the weight of this object?

(A) 490 N

(B) $\frac{490}{9.8}$ N

(C) $\frac{490}{9.8} \times 3.7$ N

(D) $\frac{490}{3.7} \times 9.8$ N

2008 HSC Q1

2. During a lunar eclipse, Earth moves between the Sun and the Moon.

What happens to the force exerted by the Sun on the Moon?

(A) It increases.

(B) It decreases.

(C) It remains unchanged.

(D) It depends on the closeness of Earth to the Moon.

2009 HSC Q5

3. Two masses have a gravitational force of 12 N between them.

If the distance between the masses is doubled, what would be the new gravitational force between them?

(A) 3 N

(B) 6 N

(C) 12 N

(D) 24 N

2014 HSC Q15

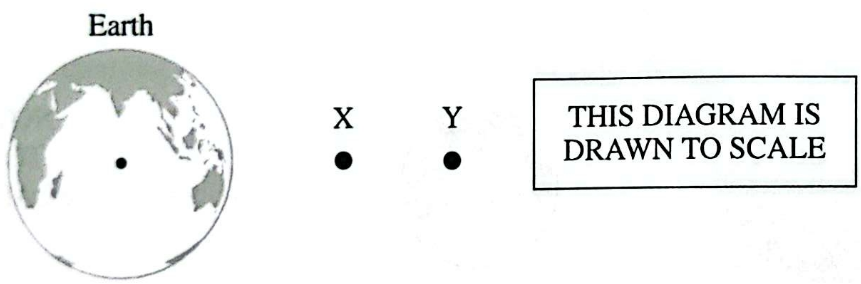
4. The acceleration due to gravity on the surface of Mercury is 3.7 m s^{-2}.

How much does a 2.0 kg brick weigh on Earth and on Mercury?

	Weight of brick on Earth	*Weight of brick on Mercury*
(A)	2.0 kg	2.0 kg
(B)	19.6 kg	7.4 kg
(C)	19.6 N	19.6 N
(D)	19.6 N	7.4 N

2010 HSC Q7 (adapted)

5. The gravitational force, due to Earth, on a mass positioned at X is F_x and on the same mass positioned at Y is F_y. The diagram is drawn to scale.



What is the value of $\frac{F_x}{F_y}$?

(A) 1.5

(B) 2.0

(C) 2.25

(D) 4.0

2012 HSC Q18

6. A spaceship at a distance r metres from the centre of a star experiences a gravitational force of x newtons. The spaceship moves a distance $\frac{r}{2}$ towards the star.

What is the gravitational force acting on the spaceship when it is at this new location?

(A) $\frac{x}{2}$ newtons

(B) x newtons

(C) $2x$ newtons

(D) $4x$ newtons

2004 HSC Q3

7. The table shows the value of the acceleration due to gravity on the surface of Earth and on the surface of Mercury.

	Acceleration due to gravity ($m s^{-2}$)
Earth	9.8
Mercury	3.7

A person has a weight of 550 N on the surface of Earth.

What would be the person's weight on the surface of Mercury?

(A) 56.1 N

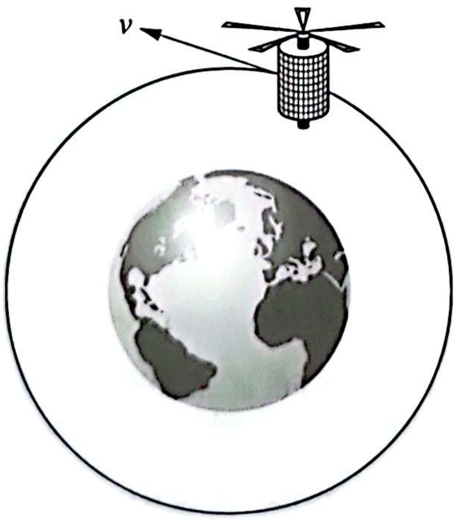
(B) 208 N

(C) 550 N

(D) 1457 N

2002 HSC Q3 (adapted)

8. A satellite is in orbit around Earth with tangential velocity v as shown.



Which of the following describes the direction of the centripetal force acting on the satellite?

(A) Same direction as the gravitational force

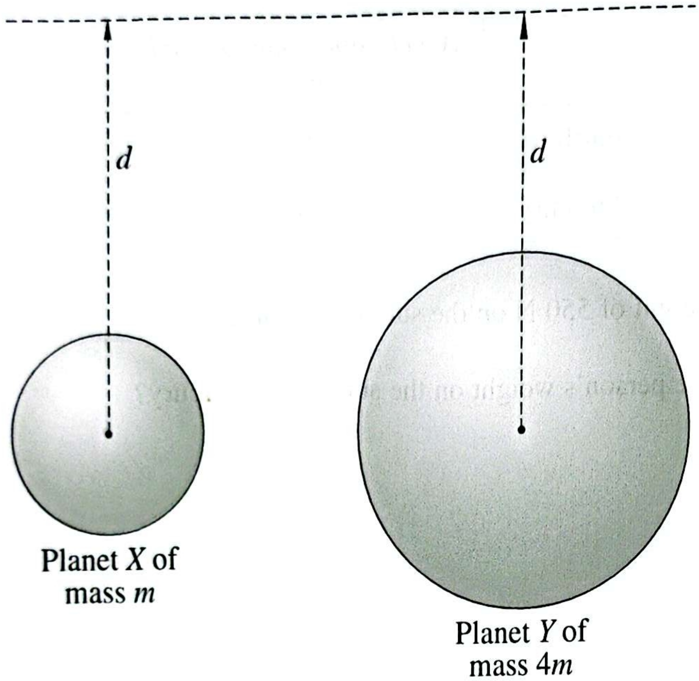
(B) Opposite direction to the gravitational force

(C) Same direction as the tangential velocity

(D) Opposite direction to the tangential velocity

2007 HSC Q1

9. The diagram shows two planets X and Y of mass m and $4m$ respectively.



At the distance d from the centre of planet Y the acceleration due to gravity is 4.0 m s^{-2}.

What is the acceleration due to gravity at distance d from the centre of planet X?

(A) 1.0 m s^{-2}

(B) 2.0 m s^{-2}

(C) 2.8 m s^{-2}

(D) 4.0 m s^{-2}

2004 HSC Q2

10. The International Space Station orbits Earth at an altitude of approximately 330 km. Another satellite, Meteosat, is in geostationary orbit at an altitude of 36 000 km.

Which of the following correctly compares the orbital velocity and orbital period of these satellites?

	International Space Station	*Meteosat*
(A)	Greater orbital velocity	Shorter orbital period
(B)	Lesser orbital velocity	Shorter orbital period
(C)	Greater orbital velocity	Longer orbital period
(D)	Lesser orbital velocity	Longer orbital period

2010 HSC Q1

11. The acceleration due to gravity on Earth's surface is g. Suppose the radius of Earth was reduced to a quarter of its present value while its mass remained the same.

What would be the new value of the acceleration due to gravity on the surface?

(A) $\frac{1}{16}g$

(B) $\frac{1}{4}g$

(C) $4g$

(D) $16g$

2007 HSC Q4

12. Given that G is the universal gravitational constant, and g is the magnitude of the acceleration due to gravity, which statement is true?

(A) The values of G and g depend on location.

(B) The values of G and g are independent of location.

(C) G is the same everywhere in the universe, but g is not.

(D) g is the same everywhere in the universe, but G is not.

2006 HSC Q1

13. Why would a satellite in low orbit around Earth eventually fall to Earth?

(A) It is not in a geostationary orbit.

(B) Gravity is too strong at low orbits.

(C) The sun's solar wind pushes it out of orbit.

(D) The upper atmosphere gradually slows it down.

2005 HSC Q2

14. A satellite is in a high orbit around the Earth. A particle of dust is in the same orbit.

Which row of the table correctly compares their potential energy and orbital speed?

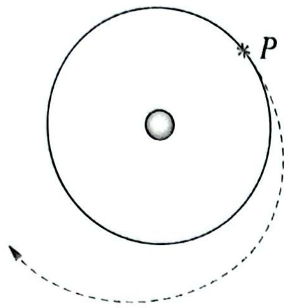
	Potential energy	*Orbital speed*
(A)	Different	Same
(B)	Different	Different
(C)	Same	Same
(D)	Same	Different

2014 HSC Q6

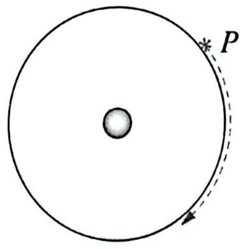
15. A space probe, P, is in a stable orbit around a small, distant planet. The probe fires a forward-facing rocket that reduces its orbital speed by half.

Which of the following best illustrates the subsequent motion of the probe?

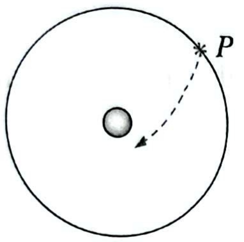
(A)



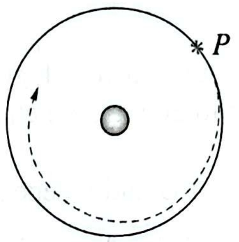
(B)



(C)



(D)



2005 HSC Q4 (adapted)

16. Which of these statements best describes the forces acting on a satellite in orbit around Earth?

(A) Although gravity has no effect, there is still an outward force.

(B) The satellite is kept up by an outward force that balances the force due to gravity.

(C) Gravity is the only force acting on the satellite and this results in an inward acceleration.

(D) The effect of gravity is negligible, the satellite is kept in orbit by its momentum and the net force on it is zero.

2008 HSC Q2

17. A fast-moving space probe passes close to a planet.

During its journey, how does the gravitational field of the planet affect the speed and direction of the probe?

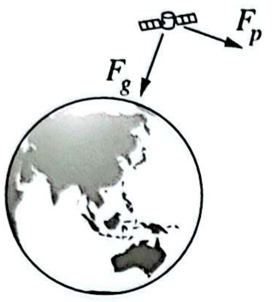
	Speed	*Direction*
(A)	Remains constant	Remains constant
(B)	Remains constant	Changes
(C)	Changes	Changes
(D)	Changes	Remains constant

2009 HSC Q1

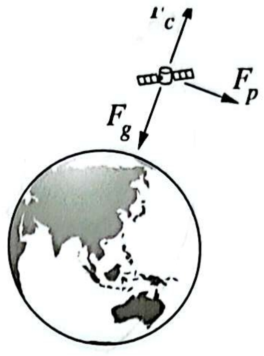
18. Which of the following diagrams correctly represents the force(s) acting on a satellite in a stable circular orbit around Earth?

F_g = gravitational force	F_p = propulsive force
F_c = centripetal force	F_r = reaction force

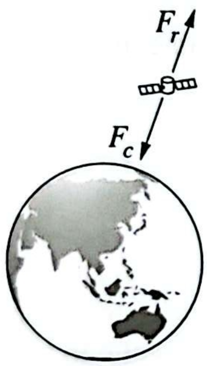
(A)



(B)



(C)



(D)

2015 HSC Q11

19. The table contains information related to two planets orbiting a distant star.

Planets	*Mass* (kg)	*Orbital radius* (m)	*Radius of planet* (m)	*Length of day* (s)	*Orbital period* (s)
Alif	1.21×10^{25}	4.00×10^{11}	8.0×10^{6}	9.5×10^{4}	8.75×10^{7}
Ba	1.50×10^{24}	8.00×10^{11}	4.0×10^{6}	4.7×10^{4}	—

The orbital period of the planet Ba can be determined by using data selected from this table.

What is the orbital period of the planet Ba?

(A) 3.10×10^{7} s

(B) 5.51×10^{7} s

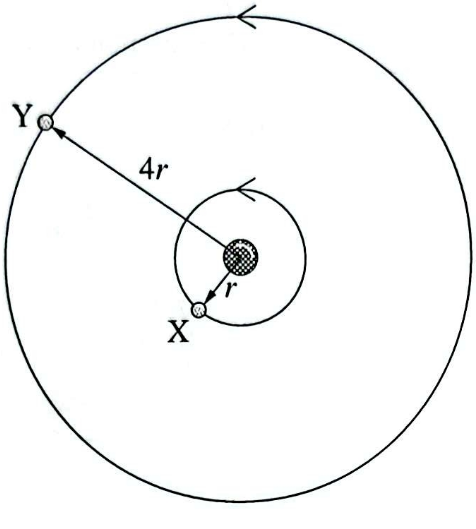
(C) 1.39×10^{8} s

(D) 2.47×10^{8} s

2002 HSC Q5

20. Two planets, X and Y, travel around a star in the same direction, in circular orbits.

Planet X completes one revolution about the star in time T. The radii of the orbits are in the ratio 1 : 4.



How many revolutions does planet Y make about the star in the same time T?

(A) $\frac{1}{8}$ revolution

(B) $\frac{1}{2}$ revolution

(C) 2 revolutions

(D) 8 revolutions

2003 HSC Q4

21. Two satellites, X and Y, are in circular orbits around Earth. Their masses are identical and their orbital radii are R and 16R, respectively.

What is the ratio of their orbital periods, $T_X : T_Y$?

(A) 1 : 4

(B) 1 : 16

(C) 1 : 32

(D) 1 : 64

2006 HSC Q5

22. What is the main cause of orbital decay of a satellite in low Earth orbit?

(A) Tidal effects of the Moon

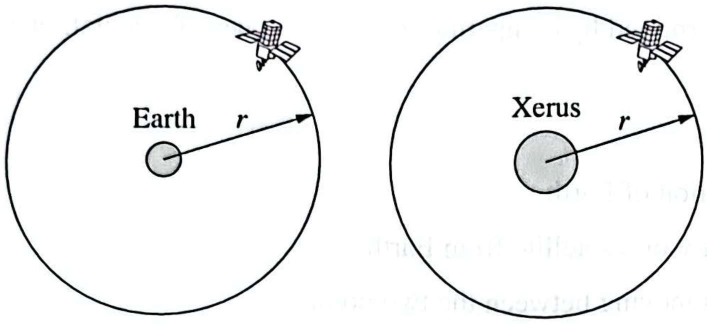
(B) The Sun's gravitational field

(C) Friction between the atmosphere and the satellite

(D) The interaction of the solar wind with the satellite

2011 HSC Q1

23. A satellite orbits Earth with period T. An identical satellite orbits the planet Xerus which has a mass four times that of Earth. Both satellites have the same orbital radius r.



What is the period of the satellite orbiting Xerus?

(A) $\frac{T}{4}$

(B) $\frac{T}{2}$

(C) $2T$

(D) $4T$

2016 HSC Q14

24. Compared to a geostationary orbit, which row of the table correctly describes the relative properties of a low Earth orbit?

	Orbital velocity	*Orbital period*
(A)	Higher	Higher
(B)	Higher	Lower
(C)	Lower	Higher
(D)	Lower	Lower

2012 HSC Q9

25. The initial velocity required by a space probe to just escape the gravitational pull of a planet is called *escape velocity.*

Which of the following quantities does NOT affect the magnitude of the escape velocity?

(A) Mass of the planet

(B) Mass of the space probe

(C) Radius of the planet

(D) Universal gravitational constant

2005 HSC Q3

26. The gravitational potential energy of a given mass is known at both Earth's surface and at a fixed distance above Earth.

What CANNOT be determined by comparing these two values of gravitational potential energy?

(A) The mass of Earth

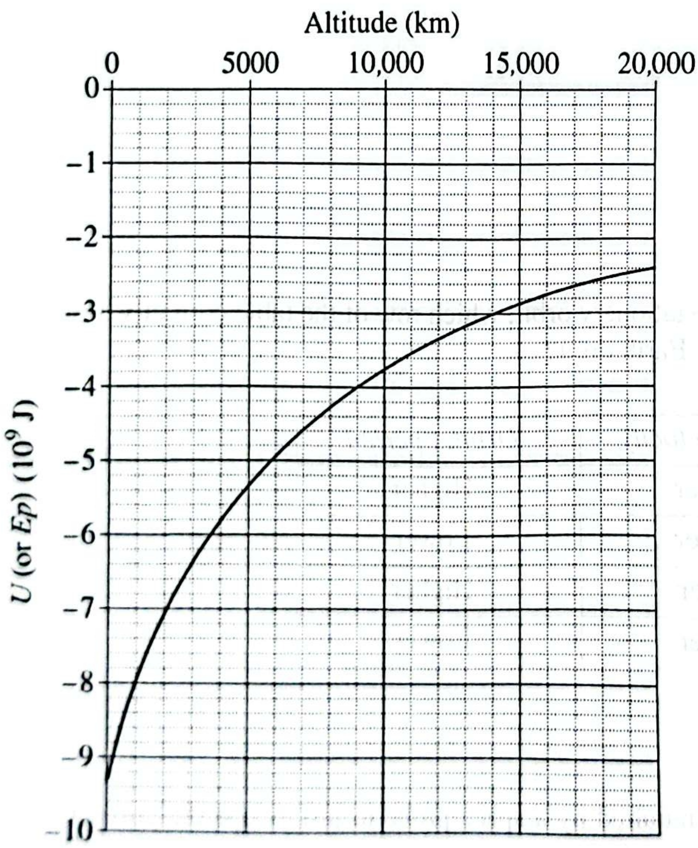
(B) The speed of rotation of Earth

(C) The escape velocity of a satellite from Earth

(D) The work done in moving between the two points

2007 HSC Q3

27. The graph shows how the gravitational potential energy, *U (or* E_p), of a satellite changes with its altitude.



What is the change in gravitational potential energy of the satellite when its altitude is reduced from 14,000 km to 4000 km?

(A) -8.8×10^9 J

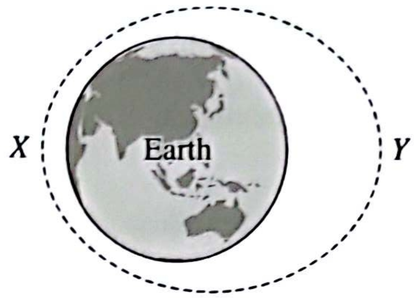
(B) -2.8×10^9 J

(C) 2.8×10^9 J

(D) 8.8×10^9 J

2012 HSC Q4 (amended)

28. A satellite orbits Earth with an elliptical orbit that passes through positions X and Y.



Which row of the table correctly identifies the position at which the satellite has greater kinetic energy and the position at which it has greater potential energy?

	Greater kinetic energy	*Greater potential energy*
A.	X	X
B.	X	Y
C.	Y	X
D.	Y	Y

2017 HSC Q12

29. A satellite is moved from a geostationary orbit to a higher orbit.

Which statement about the orbit change is correct?

(A) During the move the gravitational potential energy decreases.

(B) The change in gravitational potential energy is independent of the mass of the satellite.

(C) The work done is the difference between the gravitational potential energy of the higher orbit and that of the geostationary orbit.

(D) The work done is the energy required to move the satellite, which is in the gravitational field, from a very large distance away, to the higher orbit.

2009 HSC Q3

30. A 60 kg object has a weight of 240 N on the surface of Planet X.

What is the acceleration due to gravity on the surface of Planet X?

(A) 0.25 m s^{-2}

(B) 4 m s^{-2}

(C) 250 m s^{-2}

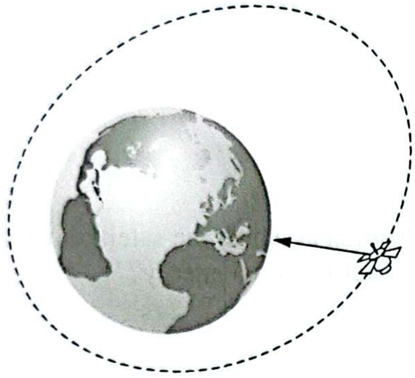
(D) $14\,400 \text{ m s}^{-2}$

2011 HSC Q2

31. A satellite orbits Earth as shown.

Which diagram correctly shows the direction of the satellite's acceleration?

A.



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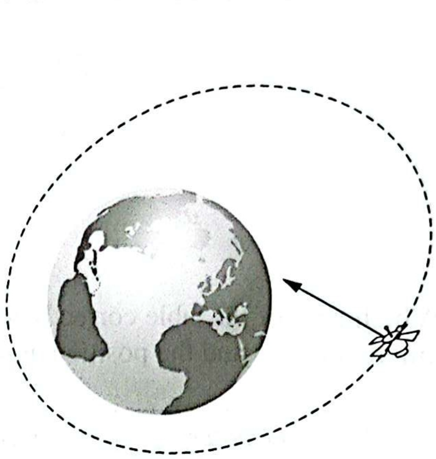

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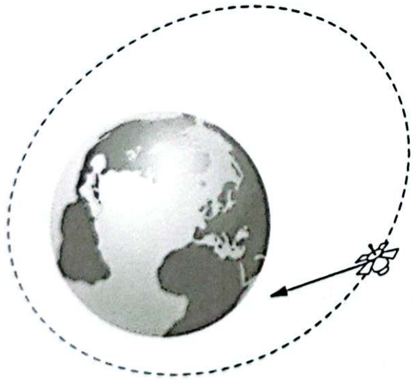
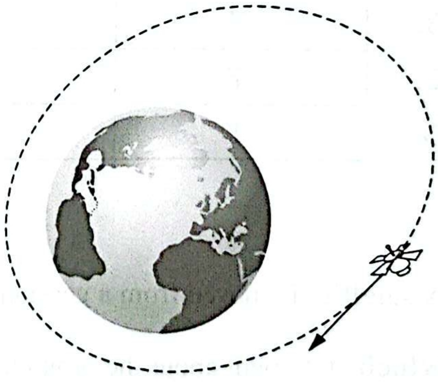


D.



2018 HSC Q1

32. A planet X has twice the mass and twice the radius of Earth.

What is the magnitude of the gravitational acceleration close to the surface of planet X?

A. $\frac{1}{2}g$

B. $1g$

C. $2g$

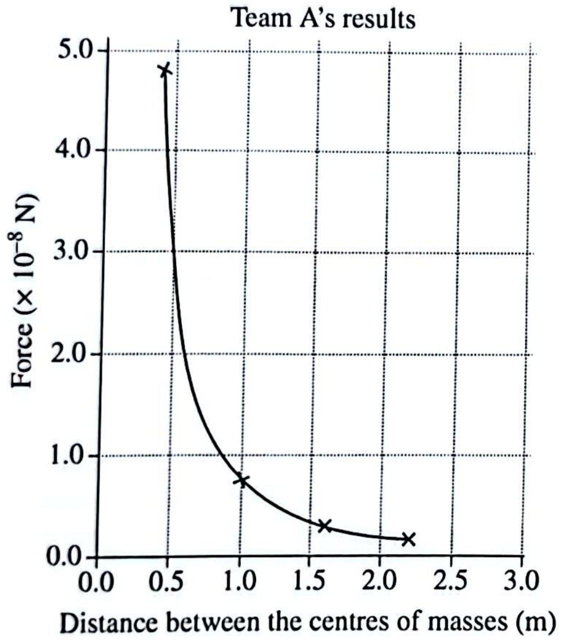
D. $4g$

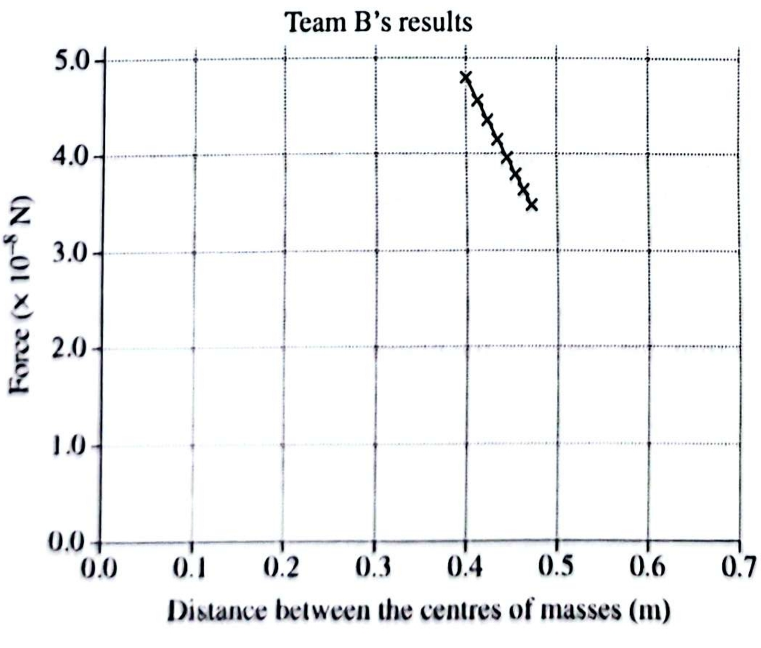
2018 HSC Q7

Short-answer questions

33. Two teams carried out independent experiments with the purpose of investigating Newton's Law of Universal Gravitation. Each team used the same procedure to accurately measure the gravitational force acting between two spherical masses over a range of distances.

The following graphs show the data collected by each team.





Question 33 continues

Question 33 (continued)

(a) Compare qualitatively the relationship between force and distance in the graphs.

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(b) Assess the appropriateness of Team A's data and Team B's data in achieving the purpose of the experiments.

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End of Question 33

2016 HSC Q25 – 2 + 3 = 5 marks

34. An object is stationary in space and located at a distance 10 000 km from the centre of a certain planet. It is found that 1.0 MJ of work needs to be done to move the object to a stationary point 20 000 km from the centre of the planet.

Calculate how much more work needs to be done to move the object to a stationary point 80 000 km from the centre of the planet.

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2006 HSC Q18 – 3 marks

35. The following table contains information about the planet Uranus and the dwarf planet Pluto, that were discovered after Kepler died.

Planet	*Orbital Period (years)*	*Average distance from the Sun (km)*
Uranus	84.0	2.87×10^9
Pluto	248.4	5.91×10^9

(a) Show that the information above is consistent with Kepler's third law (the law of periods).

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(b) If the mass of the dwarf planet Pluto is 1.3×10^{22} kg and its radius is 2.3×10^6 m, determine the acceleration due to gravity on the surface of Pluto.

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(c) Explain the significance of the word *universal* in Newton's Law of Universal Gravitation.

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1996 HSC Q32A(d) (adapted) – 5 marks

36. If the mass of Jupiter is 1.87×10^{27} kg and its radius is 7.18×10^7, what is the value of the acceleration due to gravity at the surface of Jupiter?

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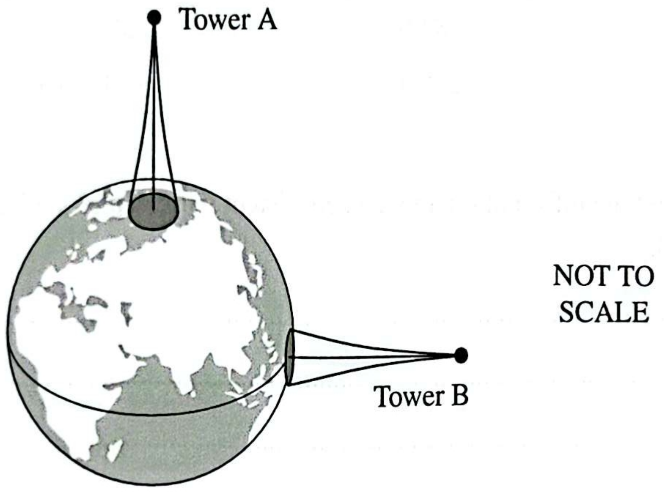
1995 HSC Q32A(b)(ii) – 2 marks

37. Consider the following thought experiment.

Two towers are built on Earth's surface. The height of each of the towers is equal to the altitude of a satellite in geostationary orbit about Earth. Tower A is built at Earth's North Pole and Tower B is built at the equator.



Identical masses are simultaneously released from rest from the top of each tower. Explain the motion of each of the masses after their release.

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2012 HSC Q23 – 4 marks

38. In July 1969 the Apollo 11 Command Module with Michael Collins on board orbited the Moon waiting for the Ascent Module to return from the Moon's surface. The mass of the Command Module was 9.98×10^3 kg, its period was 119 minutes, and the radius of its orbit from the Moon's centre was 1.85×10^6 metres.

(a) Assuming the Command Module was in circular orbit, calculate

(i) the mass of the Moon;

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(ii) the magnitude of the orbital velocity of the Command Module.

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(b) The docking of the Ascent Module with the Command Module resulted in an increase in mass of the orbiting spacecraft. The spacecraft remained at the same altitude.

This docking procedure made no difference to the orbital speed. Justify this statement.

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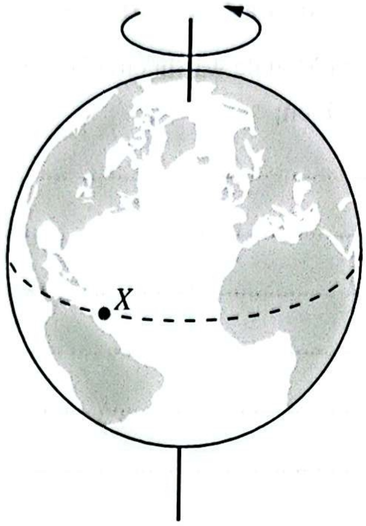
2004 HSC Q17 – 2 + 2 + 2 = 6 marks

39. The diagram shows the position X on Earth's surface from which a satellite is to be launched into a geostationary orbit.



Given that the radius of Earth is 6.38×10^6 m, calculate the height of the satellite above Earth's surface.

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2007 HSC Q17(b) – 3 marks

40. The acceleration due to gravity at the surface of Earth and at the surface of Saturn are approximately the same. The mass of Saturn is approximately 100 times that of the Earth.

What is the ratio of the radius of Saturn to the radius of the Earth?

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1999 HSC Q32A(e)(ii) 2 marks

41. (a) Why does orbital decay occur more rapidly for satellites in a low-Earth orbit than for satellites in other orbits?

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(b) Calculate the magnitude of the gravitational force that acts on a 50 kg satellite when it is 8000 km from Earth's centre.

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2016 HSC Q21 – 2 + 3 = 5 marks

42. NASA recently landed a space probe on an asteroid found between the orbits of Earth and Mars. The 500 kg space probe had a weight of 2.5 N when it landed on the asteroid.

(a) What would be the weight of this space probe on the surface of Earth?

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(b) Before landing on the asteroid, the space probe was placed in an orbit with radius 50 km. The orbital period was 5.9×10^4 s.

What was the mass of the asteroid?

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2009 HSC Q16 – 1 + 2 = 3 marks

43. From nearest to furthest, the four satellite moons of Jupiter first observed by Galileo in the year 1610 are called Io, Europa, Ganymede and Callisto. For the first three moons, the orbital period T of each is exactly twice the period of the one orbiting immediately inside it. That is,

$$T_{Europa} = 2 \times T_{Io}$$

$$T_{Ganymede} = 2 \times T_{Europa}$$

The mass of Jupiter is 1.90×10^{27} kg, and the orbital radius of Io is 421 600 km.

(a) Use Kepler's Law of Periods to calculate Ganymede's orbital radius.

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(b) Calculate Ganymede's orbital speed.

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2005 HSC Q16 – 2 + 3 = 5 marks

44. The escape velocity from a planet is given by $v = \sqrt{\dfrac{2GM}{r}}$.

(a) The radius of Mars is 3.39×10^6 m and its mass is 6.39×10^{23} kg.

Calculate the escape velocity from the surface of Mars.

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Question 44 continues

Question 44 (continued)

(b) Using the law of conservation of energy, show that the escape velocity of an object is independent of its mass.

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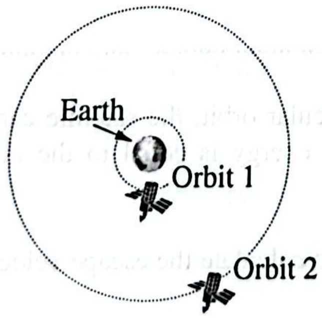
End of Question 44

2017 HSC Q24 2 + 3 = 5 marks

45. (a) A satellite is propelled from Orbit 1 to Orbit 2 as shown in the diagram.



Orbit 2 has a radius of 27 000 km. What is the satellite's speed in this orbit?

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Question 45 continues

Question 45 (continued)

(b) The radius of Orbit 2 is four times that of Orbit 1. What is the ratio of the new orbital period to the original period?

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End of Question 45

2008 HSC Q19(b), (c) 3 + 2 = 5 marks

46. A satellite of mass 150 kg is launched from Earth's surface into a uniform circular orbit of radius 7.5×10^6 m.

(a) Calculate the magnitude of the gravitational potential energy E_p of the satellite.

[where $U = E_p$]

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(b) From this uniform circular orbit, the satellite can escape Earth's gravitational field when its kinetic energy is equal to the magnitude of the gravitational potential energy.

Use this relationship to calculate the escape velocity of the satellite.

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(c) Discuss the effect of Earth's rotational motion on the launch of this satellite.

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2003 HSC Q17(a), (b) – 1 + 3 + 2 = 6 marks

47. The radius of the moon is 1740 km. The moon's mass is 7.35×10^{22} kg. In this question, ignore the moon's rotational and orbital motion.

A 20 kg mass is launched vertically from the moon's surface at a velocity of 1200 m s^{-1}.

(a) Show that the change in potential energy of the mass in moving from the surface to an altitude of 500 km is 1.26×10^7 J.

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(b) Calculate the velocity of the 20 kg mass at an altitude of 500 km.

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2018 HSC Q28 2 + 3 = 5 marks

48. A planet orbits the star, Pollux, at a distance of 1.64 astronomical units (AU). It takes 590 Earth days to complete one orbit.

(a) Why does the mass of the planet play NO role in determining its orbital speed around Pollux?

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(b) A satellite orbits Pollux with a period of 365 Earth days.

How far is the satellite from Pollux in astronomical units (A U)?

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2013 HSC Q23 2 + 3 = 5 marks

49. Compare the force of gravity exerted on the moon by Earth with the force of gravity exerted on Earth by the moon.

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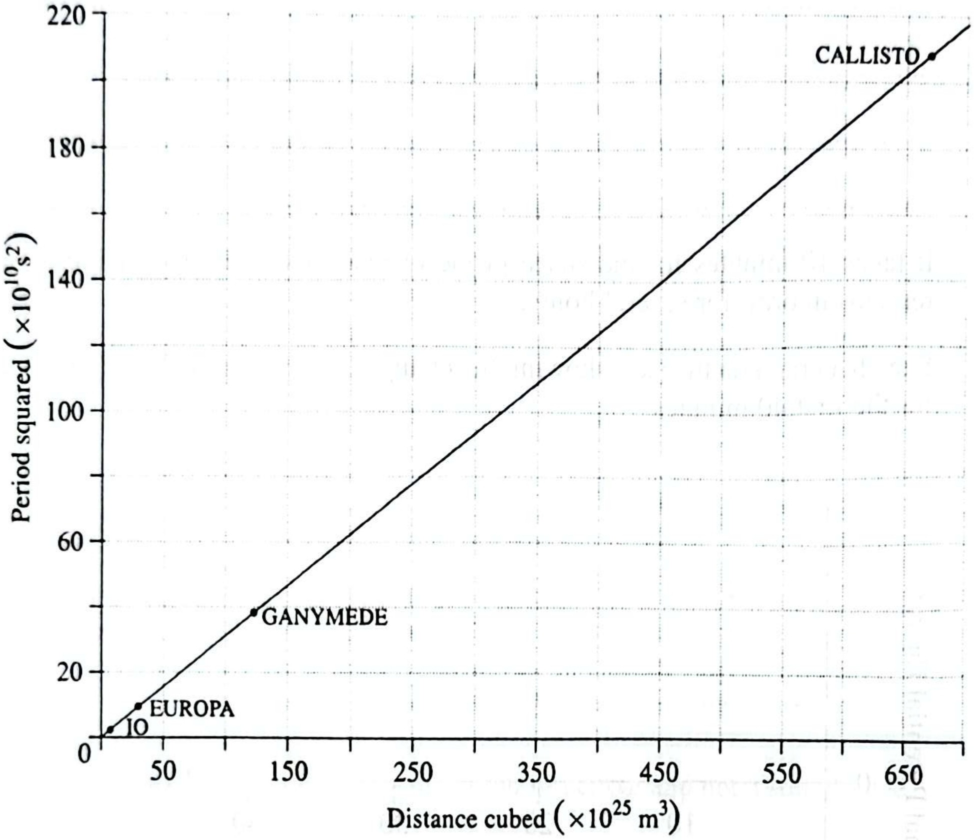
2018 HSC Q21(a) 2 marks

50. The graph below relates to four moons of Jupiter. Each moon has a period denoted by T and a distance from Jupiter's centre denoted by R. The graph is a plot of the period squared against distance cubed.



(a) What TWO features of the graph show that Kepler's third law applies to the moons of Jupiter?

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(b) When Newton's law of gravitation is combined with Kepler's third law, the equation becomes:

$$T^2 = \frac{4\pi^2}{GM} R^3.$$

From the graph, determine the mass M of Jupiter.

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1998 HSC Q32A(c) – 5 marks

51. (a) A space probe is placed in an orbit at an altitude of 188 km above Earth.

Given Earth has a radius of 6380 km, calculate the period of this orbit.

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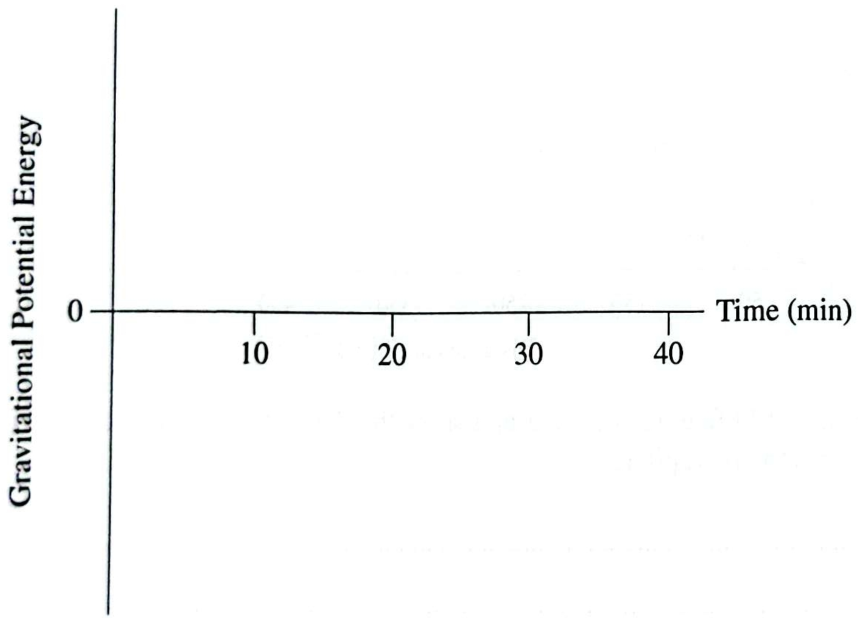
(b) It takes 10 minutes for the space probe to reach its orbit around Earth and it remains in orbit for several hours.

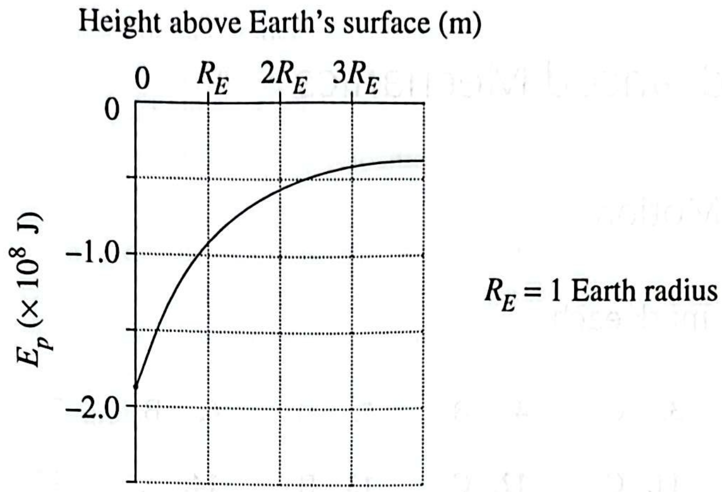
Sketch a graph on the axes showing the changes in gravitational potential energy for the first 40 minutes.



2014 HSC Q27(b), (c) – 2 + 2 = 4 marks

52. The graph below represents the gravitational potential energy (E_p) of a mass as it is raised above Earth's surface.

[where $E_p = U$
$E_k = K$]



(a) From the graph, what is the gravitational potential energy of the mass when it is one Earth radius above Earth's surface?

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(b) Use an equation to explain why the graph is a curve and not a straight line.

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(c) Explain what happens to a rocket's chemical energy, kinetic energy and gravitational potential energy when it is being launched from the surface of Earth.

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2008 HSC Q17 – 1 + 1 + 3 = 5 marks (adapted)

5.3 Motion in Gravitational Fields

Multiple choice: 1 mark each

1. C	**2.** C	**3.** A	**4.** D	**5.** C	**6.** D	**7.** B	**8.** A
9. A	**10.** C	**11.** D	**12.** C	**13.** D	**14.** A	**15.** C	**16.** C
17. C	**18.** D	**19.** D	**20.** A	**21.** D	**22.** C	**23.** B	**24.** B
25. B	**26.** B	**27.** B	**28.** B	**29.** C	**30.** B	**31.** A	**32.** A

Explanations:

1. **C** $F_{Wt} = mg$, and hence the mass of the object, $m = \frac{F_{Wt}}{g} = \frac{490}{9.8}$. Therefore, the weight force on Mars is given by $F_{Wt\,Mars} = mg_{Mars} = \frac{490}{9.8} \times 3.7$ N, as in (C).

2. **C** Newton's Law of Universal Gravitation can be expressed as $F = \frac{Gm_{Sun}m_{Moon}}{d^2}$.

The force exerted by the Sun on the Moon depends ONLY on the masses of the Sun and the Moon and the distance between the Sun and Moon. During an eclipse of the Moon, it will neither increase nor decrease but will remain constant, so (A) and (B) are incorrect and (C) is the answer. The independent gravitational force between the Earth and the Moon depends on their closeness, so (D) is irrelevant to the size of the force between the Sun and the Moon and is therefore incorrect.

3. **A** Force is inversely proportional to the square of the distance between the masses, $F \propto \frac{1}{d^2}$.

So if distance is doubled, force is divided by 4 to become 3 N, as in (A).

4. **D** $F_{weight} = mg$

$Weight_{Earth} = 2.0 \times 9.8 = 19.6$ N

$Weight_{Mercury} = 2.0 \times 3.7 = 7.4$ N ... so only (D) can be the answer.

5. **C** $F = G\frac{m_{Earth}m_{Mass}}{r^2}$ where G, m_{Earth} and m_{mass} are all constant.

So $\frac{F_X}{F_Y} = \frac{d_Y^2}{d_X^2} = \frac{4.2^2}{2.8^2} = 2.25$ ... as in (C)

[Note: Remember to measure distances between centres of masses.]

6. D $F = G\frac{m_1 m_2}{d^2}$ The gravitational force is inversely proportional to d^2, so in moving from r to $\frac{r}{2}$, the distance is halved. Therefore, the force (F) will be $2^2 = 4$ times the force. So, the force in the new location will be $4x$ newtons, as in (D).

7. B Weight = mg, where m = mass and g = acceleration due to gravity

On Earth: $550 \text{ N} = m \times 9.8 \text{ m s}^{-2}$ $\therefore$ $m = \frac{550}{9.8} = 56.1 \text{ kg}$

On Mercury: weight = $56.1 \text{ kg} \times 3.7 \text{ m s}^{-2} = 208 \text{ N}$ ... as in (B).

8. A A satellite in orbit is in circular motion. The centripetal acceleration is inwards towards the centre of the motion, i.e. the centre of the Earth. It is the result of the gravitational force acting on the satellite. Therefore it is in the same direction as the Earth's gravitational force, as in (A).

9. A $F = G\frac{m_1 m_2}{d^2}$ Since the distance from the centre of mass of the planets is the same, the gravitational force (F) and therefore the acceleration due to gravity (g) is proportional to the mass of the planet. Planet X has $\frac{1}{4}$ the mass of planet Y, so g for planet $X = 1.0 \text{ m s}^{-2}$ as in (A).

10. C Orbital velocity decreases with increasing height above the Earth so the International Space Station (ISS) will have a greater orbital velocity as in (A) or (C) than the Meteosat, which has a greater height than the ISS. The orbital period increases with height above the Earth. A geostationary satellite, such as Meteosat, at 36,000 km has greater altitude than the ISS and so will have a much longer orbital period, so (C) is correct and (A) is incorrect.

[Note: 1. Meteosat is not a single satellite but rather is a series of several geostationary weather satellites. The first generation satellites were Meteosat-1 to Meteosat-7 while Meteosat-8 and Meteosat-9 are second generation satellites. Meteosat-6 to Meteosat-9 were still in orbit in 2010. 2. A geostationery satellite will have an orbital period of ~23 h 56 m (as it is one sidereal day), while the ISS has an orbital period of ~1.5 h.]

11. D $F = G\frac{m_{Earth} m_2}{r^2}$ and $F = m_2 g$. The force of gravity is proportional to the product of the masses and that remains unchanged. It is inversely proportional to the square of the distance between the centres of mass. If this distance is decreased to ¼ its value, the gravitational force and therefore g will increase by a factor of 16, so (D) is the answer.

12. C The universal gravitational constant, G, always equals $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ (as on the Data Sheet). Constant means that it is always the same. However, g is gravitational acceleration. The value of g depends on location and depends on mass and radius, e.g. on Earth, $g = 9.8 \text{ m s}^{-2}$ but on the Moon, $g = 1.6 \text{ m s}^{-2}$.
So (C) is the only possible answer.

13. D The atmosphere thins out with increasing altitude, reaching pressure equivalent to a good vacuum at 300 km altitude, although traces extend to altitudes over 1000 km. Even at this low atmospheric density, low orbit satellites experience frictional drag that slows them down, as in (D). A low orbit satellite cannot be in a geostationary orbit, so (A) is not relevant. While gravity is stronger at lower altitudes, this is not the reason for their fall, so (B) is incorrect. Effects of the solar wind are much less than effects of the upper atmosphere, so (C) is incorrect.

14. A Orbital speed is a characteristic of the particular orbit and is independent of mass. So the dust particle and satellite will have the *same orbital speed*. Potential energy is dependent on the position in the Earth's gravitation field and also on the mass, so objects of different mass in the same orbit will have *different potential energy*. This is given in (A).

15. C* In the probe's orbit, which is about 7–8 times the planet's radius, it will be far above any atmosphere the planet might have. A retro-rocket that reduces the probe's orbital speed by half, will reduce its K (or E_k) to a quarter its previous value. The retro-rocket would only fire over a small fraction of an orbit.

This loss in orbital velocity at P will be followed by the probe accelerating more towards the planet as gravitational U (or E_P) is converted into K (or E_k) The satellite will move in a new, highly elliptical orbit, with the planet at one focus and P being the farthest point from the planet. If the satellite does not hit the planet, nor encounter any significant atmosphere, it will continue the ellipse back to P. Therefore path (C) is the most likely path, rather than the paths shown in (A), (B) or (D).

[* Note: Answer (D) was wrongly accepted for the 2005 HSC exam. It is incorrect as explained above, even though it appears to be the 'commonsense answer'. (D) appears to match the path of a *low* orbit decay, but the orbit as shown is not low. In (A), (B) and (D), the orbit followed only indicates a small change in orbital velocity, if any change at all, and not a sudden decrease in velocity to half its previous value.]

16. C A gravitational force acts between Earth and an orbiting satellite around Earth. This is the only force acting on the satellite and acts inwards towards Earth providing the acceleration to maintain the satellite's circular motion, as in (C). Gravity still has an effect and there is no outward force, so (A) is incorrect. Since there is no outward force and gravity alone provides the force necessary for circular motion, (B) is incorrect. Since gravity is not negligible, (D) is incorrect.

17. C The gravitational force of a planet on a space probe will act towards the centre of the planet at each instant as the space probe approaches, passes and departs the vicinity of the planet. The space probe will therefore experience an increasing acceleration on its approach and a decreasing acceleration as its leaves and thus will change both its speed and direction as it swings past the planet, as in (C).

18. D Any satellites in a stable orbit have a constant instantaneous velocity. So they have no net force in the direction of their motion. Once a satellite has been placed in orbit, the only force acting is Earth's gravity. This provides the satellite's centripetal force that keeps it in orbit around Earth. So (D) is the answer. (A), (B) and (C) are all incorrect, as they show additional non-existent forces.

19. **D** Using Kepler's Law: $\frac{r^3}{T^2} = \frac{GM}{4\pi^2} = k$ for a distant star with mass M.

All planets revolving around a star have the same value of k. So $k_{\text{Alif}} = k_{\text{Ba}}$

$$\frac{(4.00 \times 10^{11})^3}{(8.75 \times 10^7)^2} = \frac{(8.00 \times 10^{11})^3}{(T_{\text{Ba}})^2}$$

So $(T_{\text{Ba}})^2 = 8.00 \times (8.75 \times 10^7)^2 \text{ s}^2$

$\therefore$ $T_{\text{Ba}} = 2.47 \times 10^8$ s ... as in (D).

20. **A** Kepler's Law as explained by Newton applies to the star. The relationship between the orbital period (time) and the radius of the orbits about a star or a planet is given by:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

Since both planets orbit the same star: $\frac{r^3}{T^2} = \text{a constant}$.

$\therefore \frac{r_X^3}{T_X^2} = \frac{r_Y^3}{T_Y^2}$. And so: $\frac{r_Y^3}{r_X^3} = \frac{T_Y^2}{T_X^2}$

$\therefore \frac{r_Y^3}{r_X^3} = \frac{(4r_X)^3}{r_X^3} = 4^3 = 64 = \frac{T_Y^2}{T_X^2}$. $\therefore \frac{T_Y}{T_X} = \sqrt{64} = 8$ $\therefore T_Y = 8T_X$

So planet Y makes $\frac{1}{8}$ of a revolution while planet X makes one revolution.

$\therefore$ (A) is the answer.

21. **D** This question requires you to use Kepler's law of periods (from Formulae Sheet).

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \qquad \frac{\text{radius}^3}{\text{period}^2} = \frac{6.67 \times 10^{-11} \times \text{mass of central body}}{4\pi^2}$$

$\frac{r^3}{T^2} = \text{a constant}$ (as G is the Universal Gravitation Constant, $4\pi^2$ is a constant, and since both satellites orbit the Earth, M is a constant.)

$\frac{R^3}{T_x^2} = \frac{(16R)^3}{T_y^2}$ (since the radii of X and Y are R and $R16$ respectively

$R^3 \times T_y^2 = (16R)^3 \times T_x^2$ So, $T_y^2 = 4096 \times T_x^2$

$\therefore T_y = 64T_x$ ... so (D) is the answer.

22. **C** A low Earth orbit is about 200–2000 km above the Earth's surface and so some of Earth's atmosphere extends into this range. The drag force that a satellite in low Earth orbit experiences is due to its interaction with the few air molecules that are present at these altitudes, so (C) is the answer. The only other major force acting on the satellite is Earth's gravity. Therefore (A), (B) and (D) are all incorrect.

23. **B** Using Kepler's Law: $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

Since r and G are constants, $\therefore \frac{1}{T^2} \propto M$ i.e. $T \propto \sqrt{\frac{1}{M}}$

$M_{Xerus} = 4 \times M_{Earth}$ $\therefore$ period of satellite orbit on Xerus $= \sqrt{\frac{1}{4}}\, T = \frac{T}{2}$ ... as in (B).

24. **B** A satellite in orbit moves faster when it is closer to the Earth and slower when it is further away. So a satellite in a low Earth orbit has a much higher velocity than a geostationary satellite. The period of a geostationary satellite matches Earth's rotation period, whereas a satellite in a low Earth orbit has a much shorter period. So (B) is the answer.

25. **B** At the escape velocity, the kinetic energy of a space probe of mass, m is just sufficient to overcome the gravitational force field of the planet.

K (or E_k) $= -U$ (or E_p) $\qquad E_k = \frac{1}{2}mv^2 = -E_p = G\frac{m_1 m_2}{r} = G\frac{mM_P}{r_p} \qquad v^2 = 2G\frac{M_P}{r_p}$

26. **B** The speed of rotation of Earth is totally unrelated to the gravitational U (or E_p) of an object either on the Earth's surface or at a fixed distance above it and therefore cannot be determined from the data for gravitational U (or E_p). So (B) is the answer. The other quantities are all related to gravitational U (or E_p) – and so they can be determined using the data for gravitational U (or E_p).

27. **B** Using the new symbol for gravitational potential energy, U (rather than Ep):

$\Delta U = U(\text{final}) - U(\text{initial}) = -5.8 \times 10^9 - (-3 \times 10^9) = -2.8 \times 10^9$ J ... as in (B).

[Note: The value is negative because E_p is lost as the satellite and Earth are brought closer together.]

28. **B** A satellite in a circular orbit travels a set distance from Earth with uniform speed and so has constant kinetic and potential energy. In an elliptical orbit, a satellite's kinetic and potential energy are continually interchanged. When the satellite is 'falling' towards the Earth as it nears point X, it speeds up and its kinetic energy approaches a maximum at X. As the satellite swings out towards Y, it slows down towards its minimum speed at Y. As it approaches Y, its kinetic energy is converted to potential energy. This reaches a maximum at Y. So (B) is the answer

29. **C** (C) is the best alternative. The work done involves the difference in gravitational potential energy as in (C), but it also involves the difference between the kinetic energy of the satellite in the two orbits. When a satellite is moved further from the Earth, work must be done against the force of gravity, so the gravitational potential energy increases rather than decreases. So (A) is incorrect. The work done does include a mass term, so (B) is incorrect. The work done to move the satellite from a large distance away to the radius of the higher orbit gives the potential energy of the higher orbit and not the work done, so (D) is incorrect.

30. **B** $F_{\text{Weight}} = mg$

so $g = \frac{F_{\text{Weight}}}{m} = \frac{240 \text{ N}}{60 \text{ kg}} = 4 \text{ m s}^{-2}$ ... as in (B).

31. **A** The satellite shown is in an elliptical orbit with the centre of the Earth at one focus. In such a motion, the acceleration of the satellite is also directed towards that same focal point, as only shown in (A).

32. A $F = mg = \frac{GMm}{r^2}$ ... where m = mass of Earth

M = mass of planet $X = 2\ m$

$r_{\text{planet}} = 2 \times r_{\text{Earth}}$

Since $g = \frac{GM}{r^2}$

$\therefore\ g_{\text{planet}} = \frac{1}{2} \times g_{\text{Earth}}$ ... as in (A).

Short-answer questions

33. (a) Both graphs show a decrease in the gravitational force with an increase in the distance between the two masses. Team A's results show the force decreasing at a decreasing rate (a non-linear relationship), whereas Team B's results show the force decreasing at a constant rate (a linear relationship).

(b) Although Team A had a good range of results, they should have taken more measurements at intermediate points to infer a valid relationship.

Although Team B collected more results than Team A, these were over too narrow a range of values to infer a valid relationship.

34. Work done = change in potential energy (U or E_p)

So $U = E_p = -G\frac{m_1 m_2}{r}$

To move object from 10,000 to 20,000 km: 1.0 MJ = 1.0×10^6 J work was done:

$\therefore\ U = \Delta E_p = 1.0 \times 10^6 = (-G\frac{m_1 m_2}{20{,}000 \times 10^3}) - (-G\frac{m_1 m_2}{10{,}000 \times 10^3})$

$1.0 \times 10^6 \times 20{,}000 \times 10^3 = -Gm_1m_2 + 2\ Gm_1m_2$

$2 \times 10^{13} = Gm_1m_2$

To move object from 20,000 km to 80,000 km:

$U = \Delta E_p = -G\frac{m_1 m_2}{80{,}000 \times 10^3} - (-G\frac{m_1 m_2}{20{,}000 \times 10^3}) = -\frac{2 \times 10^{13}}{80{,}000 \times 10^3} + \frac{2 \times 10^{13}}{20{,}000 \times 10^3}$

$\therefore$ extra work to be done = –250,000 + 1,000,000 = 750,000 J = 0.75 MJ

35. (a) The square of the period (T^2) taken by a planet to orbit the Sun is proportional to the cube of its mean distance (r^3) from the Sun, i.e. $\frac{r^3}{T^2}$ = constant.

For Uranus: $\frac{r^3}{T^2} = \frac{(2.87 \times 10^9)^3}{84.0^2} = 3.35 \times 10^{24}$

For Pluto: $\frac{r^3}{T^2} = \frac{(5.91 \times 10^9)^3}{284.4^2} = 3.35 \times 10^{24}$

$\therefore$ the ratio $\frac{r^3}{T^2}$ is the same.

(b) $F = \frac{Gm_1m_2}{r^2}$ ... but $F = m_1a$

$\therefore a = \frac{Gm_2}{r^2} = \frac{6.7 \times 10^{-11} \times 1.3 \times 10^{22}}{(2.3 \times 10^6)^2} = 0.16 \text{ m s}^{-2}$

(c) It applies to any two objects in the universe.

36. $F = \frac{GMm}{r^2}$ ... but $F = ma$

$\therefore a = \frac{GM}{r^2} = \frac{6.7 \times 10^{-11} \times 1.87 \times 10^{27}}{(7.18 \times 10^7)^2}$

$= 24.3 \text{ m s}^{-2}$

37. Tower A will rotate on the spot as the Earth rotates on its axis and will be carried along with the Earth as it orbits the Sun. A mass released from the top of Tower A will fall vertically to Earth with increasing acceleration, as g increases the closer the mass gets to the Earth's surface.
The mass at the top of Tower B is in a geostationary orbit as it is rotating with the same angular velocity as the Earth and it has the correct height. When a mass is released from the top of Tower B, it will remain where it was released in a geostationary orbit.

38. (a) (i) $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ $\therefore M_{moon} = \frac{r^3 4\pi^2}{T^2 G} = \frac{(1.85 \times 10^6)^3 4\pi^2}{(119 \times 60)^2 \times 6.67 \times 10^{-11}} = 7.351 \times 10^{22}$ kg

(ii) $F_c = \frac{mv^2}{r} = F_g = \frac{GmM}{r^2}$ $\therefore v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 7.351 \times 10^{22}}{1.85 \times 10^6}}$

Orbital velocity, $v = 1627.984 \text{ m s}^{-1} = 1.63 \times 10^3 \text{ m s}^{-1}$

(b) Orbital velocity, $v = \sqrt{\frac{GM}{r}}$ is independent of the mass of the orbiting spacecraft and only depends on the mass of the Moon, the gravitational constant and the radius of the orbit. Provided the docking manoeuvre is gentle, there will be no change in the orbital radius and the spacecraft will therefore continue at the same velocity even though its mass is greater after the ascent module has docked.

39. $\frac{r_{orbit}^3}{T^2} = \frac{GM}{4\pi^2}$

$\therefore\ r_{orbit} = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11} \times 6.0 \times 10^{24} \times 86{,}164^2}{4\pi^2}} = 4.222\times10^7 \text{ m}$

$r_{orbit} - r_{Earth} = 4.222\times10^7 \text{ m} - 6.38\times10^6 \text{ m} = 3.5840\times10^7 \text{ m} = 35{,}840 \text{ km}$

$\therefore$ height of satellite above Earth's surface = 35,840 km

[Note: The period used above, T = one sidereal day = 86,164 seconds (i.e. the time for Earth to rotate 360° which is approx 3 mins 56 seconds less than a solar day). If you calculated this problem using a value for the sidereal day as approximately 4 mins less than a solar day, i.e. T = 86,160 seconds, your answer would be 35,820 km. If you used T = solar day = 86,400 seconds, your answer would be 35,918 km.]

40. Consider an object of mass = m $\qquad M_{\text{Saturn}} = 100 \times M_{\text{Earth}}$

On Saturn: $F = mg_{\text{Saturn}} = \frac{GM_{\text{Saturn}}m}{r_{\text{Saturn}}^2}$ $\qquad$ On Earth: $F = mg_{\text{Earth}} = \frac{GM_{\text{Earth}}m}{r_{\text{Earth}}^2}$

So, $\frac{GM_{\text{Saturn}}m}{r_{\text{Saturn}}^2} = 100 \times \frac{GM_{\text{Earth}}m}{r_{\text{Earth}}^2}$

Since $mg_{\text{Saturn}} = mg_{\text{Earth}}$ $\qquad \frac{1}{r_{Earth}^2} = 100 \times \frac{1}{r_{Saturn}^2}$

$\therefore r_{\text{Saturn}} : r_{\text{Earth}} = 10:1$

41. (a) Satellites in low-Earth orbits experience atmospheric drag, due to traces of atmospheric gases. This decelerates the satellite and leads to orbital decay. The greater the altitude of a satellite's orbit, the less atmospheric drag and so less orbital decay.

(b) $F = \frac{Gm_1m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 50}{(8000 \times 10^3)^2}$

$= 312.7 \text{ N}$

$\approx 313 \text{ N}$

42. (a) $\text{Force}_{\text{weight}} = mg$

$= 500 \times 9.8 = 4900 \text{ N}$

(b) $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

$\therefore M = \frac{r^3 4\pi^2}{T^2 G} = \frac{50{,}000^3 \times 4\pi^2}{(5.9 \times 10^4)^2 \times (6.67 \times 10^{-11})} = 2.13\times10^{16} \text{ kg}$

43. (a) $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ $\quad\therefore$ For Jupiter's moons, $\frac{r^3}{T^2} = \frac{GM_{Jupiter}}{4\pi^2}$ = constant

$$\frac{r^3_{Ganymede}}{T^2_{Ganymede}} = \frac{r^3_{Io}}{T^2_{Io}} \qquad \text{So } r^3_{Ganymede} = \frac{r^3_{Io}}{T^2_{Io}} \times T^2_{Ganymede} = \frac{(4.216 \times 10^5 \times 10^3)^3}{T^2_{Io}} \times (4T_{Io})^2$$

$\therefore$ orbital radius, $r_{Ganymede} = \sqrt[3]{(4.216 \times 10^5 \times 10^3)^3 \times 16}$

(b) $\therefore \; v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.90 \times 10^{27}}{1.0624 \times 10^9}}$

$= 10{,}924$ m s^{-1} = 1.09×10^4 m s^{-1}

44. (a) $v_{esc} = \sqrt{\frac{2GM}{r}}$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6.39 \times 10^{23}}{3.39 \times 10^6}} = \sqrt{2.5145 \times 10^7} = 5.01 \times 10^3 \text{ m s}^{-1}$$

(b) The escape velocity is the minimum velocity at the surface that allows an object to reach an infinite displacement, where its E_p is zero. Since energy is conserved, the total energy ($E_p + E_k$) must be zero:

$$\tfrac{1}{2}mv^2_{escape} + \left(-\frac{GM_{planet}m}{r_{planet}}\right) = 0$$

The mass of the object (m) can be cancelled out: $v_{escape} = \sqrt{\frac{2GM_{planet}}{r_{planet}}}$

$\therefore$ escape velocity of an object is independent of its mass.

45. (a) $F = G\frac{m_1 m_2}{d^2}$ $\quad$ so $F_{gravity} = G\frac{m_{Earth}m_{satellite}}{r^2_{orbit}}$

$F = \frac{mv^2}{r}$ $\quad$ so $F_{centripetal} = \frac{m_{satellite}v^2_{satellite}}{r_{orbit}}$ $\quad$ and $\quad F_{gravity} = F_{centripetal}$

So $v_{satellite} = \sqrt{\frac{Gm_{Earth}}{r_{orbit}}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{27\ 000 \times 10^3}} = 3849.96$ m s^{-1}

$\therefore$ satellite's speed, $v_{satellite} = 3850$ m s^{-1} = 3.8×10^3 m s^{-1} or 3.85 km s^{-1}

(b) Since $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ = constant, $\therefore \; \frac{r_1^3}{T_1^2} = \frac{(4 \times r_1)^3}{T_2^2}$ (since $r_2 = 4r_1$)

$\therefore$ 64 $T_1^2 = T_2^2$ $\quad$ so $8T_1 = T_2$, i.e. the ratio of $T_2 : T_1$ = 8:1

46. (a) $U(=E_p) = -\frac{Gm_1m_2}{r} = \frac{-6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 150}{7.5 \times 10^6}$

$= -8.0 \times 10^9$ J

(b) *If* $K(E_K) = -U(E_P)$, then $\frac{1}{2}mv^2 = 8.0 \times 10^9$ J

$\therefore v^2 = \frac{2 \times 8.0 \times 10^9}{150}$ and $v = 1.0328 \times 10^4$ m s^{-1}

$\therefore$ escape velocity for satellite $= 1.0 \times 10^4$ m s^{-1}

(c) The Earth's rotational motion is from west to east. The Earth's spherical shape means that its lineal surface speed towards the east is greatest at the equator.

Therefore, satellite-carrying rockets are usually launched towards the east and from launch sites as close as possible to the equator, to maximise the contribution of the Earth's rotational speed in getting the satellite to reach its orbital velocity. This reduces the amount of fuel and/or the size of the rocket needed to put the satellite into orbit.

47. (a) Initial potential energy (U_i) of mass: $U_i = -\frac{GMm}{r}$

$= -\frac{-6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 20}{1.74 \times 10^6\ (m)}$

$= -5.635 \times 10^7$ J

Final potential energy (U_f) of mass: $U_f = -\frac{GMm}{r}$

$= -\frac{-6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 20}{2.24 \times 10^6\ (m)}$

$= -4.3772 \times 10^7$ J

$\therefore \Delta U = U_f - U_i$

$= 1.2578$

$= 1.26 \times 10^7$ J

(b) Initial kinetic energy (K_i) of mass: $K_i = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 20 \times 1200^2$

$= 1.44 \times 10^7$ J

Final kinetic energy (K_f): $K_f = K_i - \Delta U$

$= 1.44 \times 10^7 - 1.26 \times 10^7$

$= 1.8 \times 10^7$ J

Since $K_f = \frac{1}{2}mv^2$

$\therefore$ velocity, $v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \times 1.8 \times 10^6}{20}}$

$= 424.264$

≈ 424 m s^{-1}

48. (a) Kepler's Law of Periods relates the radius of an orbit to the period of the orbit around a central body, $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ but $T = \frac{2\pi r}{v}$

so substituting for T: $\frac{r^3 v^2}{4\pi^2 r^2} = \frac{GM}{4\pi^2}$ $\therefore v = \sqrt{\frac{GM}{r}}$

The orbital velocity (v) only depends on the mass of the central star (M) and its distance from the planet (r). So the mass of the planet (m) is irrelevant.

(b) Using Kepler's Law: $\frac{r^3_{planet}}{T^2_{planet}} = \frac{r^3_{satellite}}{T^2_{satellite}}$ Substituting: $\frac{1.64^3}{590^2} = \frac{r^3_{satellite}}{365^2}$

$\therefore$ Distance of satellite from Pollux, $r_{satellite} = \sqrt[3]{\frac{1.64^3 \times 365^2}{590^2}} = 1.19$ AU

49. These two forces are equal in magnitude, but opposite in direction.

50. (a)
- When graphed as T^2 versus R^3, the points for each of the moons lie on a straight line
- The line passes through the point (0,0).

(b) $T^2 = \frac{4\pi^2}{GM} R^3$

Slope of graph $= \frac{4\pi^2}{GM} = \frac{140 \times 10^{10} - 0}{450 \times 10^{25} - 0} = \frac{140 \times 10^{10}}{450 \times 10^{25}}$

$\therefore M = \frac{4\pi^2}{G} \times \frac{450 \times 10^{25}}{140 \times 10^{10}} = \frac{4\pi^2}{6.67 \times 10^{-11}} \times \frac{450 \times 10^{25}}{140 \times 10^{10}}$

$= 1.90 \times 10^{27}$ kg

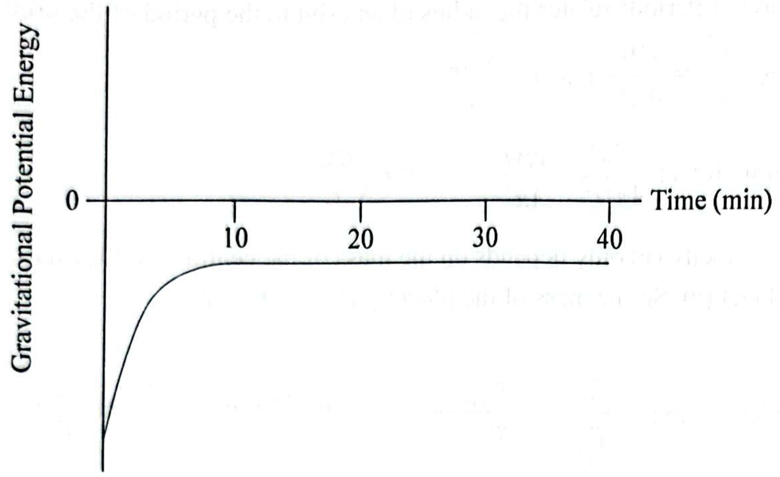
51. (a) $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

So, $T^2 = \frac{4\pi^2 r^3}{GM}$

$= \frac{4\pi^2((6380 + 188) \times 10^3)^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}$

$\therefore$ period, $T = 5287$ s ≈ 88 min

(b)



52. (a) Reading from the graph at R_E: $U = E_p = -0.925 \times 10^8$ J $= -9.25 \times 10^7$ J

(b) $U = E_p = -G\dfrac{m_1 m_2}{r} = -G\dfrac{m_{Earth}m_{Object}}{r_{distance\ from\ Earth's\ centre}}$

Since G, m_{Earth} and m_{Object} are all constants, U (E_p) varies with the inverse of the height above Earth – so the graph will be a curve.

(c) The chemical potential energy in the rocket's fuel is converted into kinetic energy, enabling the rocket to do work against the gravity of the Earth. As the fuel gets used up, its chemical energy is converted to U (potential energy) and K (kinetic energy). K increases as the rocket's velocity increases. U increases as it rises higher.