EXERCISE 3.1 LINEAR EQUATIONS IN ONE VARIABLE

2 $4(2x-7)=3x-5$

$$\begin{aligned}8x-28&=3x-5\\8x-3x&=-5+28\\5x&=23\\x&=\frac{23}{5}\\x&=4.6\end{aligned}$$

4 $x-7=3-x$

$$\begin{aligned}x+x&=3+7\\2x&=10\\x&=5\end{aligned}$$

6 $3(5x-1)-2(4-x)=6$

$$\begin{aligned}15x-3-8+2x&=6\\17x&=6+3+8\\17x&=17\\x&=1\end{aligned}$$

8 $3x-2=5x$

$$\begin{aligned}3x-5x&=2\\-2x&=2\\x&=\frac{2}{-2}\\x&=-1\end{aligned}$$

10 $4(3a+2)-6(3-a)=8$

$$\begin{aligned}12a+8-18+6a&=8\\18a&=8-8+18\\18a&=18\\a&=1\end{aligned}$$

12 $4(3m-1)=11+2m$

$$\begin{aligned}12m-4&=11+2m\\12m-2m&=11+4\\10m&=15\\m&=\frac{15}{10}\\m&=1.5\end{aligned}$$

14 $m+8=5(m-1)-2m$

$$\begin{aligned}m+8&=5m-5-2m\\m-5m+2m&=-5-8\\-2m&=-13\\m&=\frac{-13}{-2}\\m&=6.5\end{aligned}$$

16 $18-3(a-2)=2(a+2)$

$$\begin{aligned}18-3a+6&=2a+4\\-3a-2a&=4-6-18\\-5a&=-20\\a&=\frac{-20}{-5}\\a&=4\end{aligned}$$

18 $5(c-7)=3(3c+8)$

$$\begin{aligned}5c-35&=9c+24\\5c-9c&=24+35\\-4c&=59\\c&=-\frac{59}{4}\\c&=-14.75\end{aligned}$$

20 $a+6=3-a$

$$\begin{aligned}a+a&=3-6\\2a&=-3\\a&=-\frac{3}{2}\\a&=-1.5\end{aligned}$$

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EXERCISE 3.2 LINEAR EQUATIONS INVOLVING FRACTIONS

2 $\frac{3x}{4}-\frac{2x}{5}=14$

$$\frac{15x}{20}-\frac{8x}{20}=14$$
$$\frac{7x}{20}=14$$
$$x=14\times\frac{20}{7}$$
$$x=40$$

4 $\frac{2x+5}{3}=\frac{x+2}{7}+4$

$$\frac{7(2x+5)}{21}=\frac{3(x+2)}{21}+\frac{84}{21}$$
$$14x+35=3x+6+84$$
$$11x=6+84-35$$
$$11x=55$$
$$x=5$$

6 $\frac{x+3}{3}-\frac{2x-1}{6}=x+1$

$$\frac{2(x+3)}{6}-\frac{2x-1}{6}=x+1$$
$$\frac{2x+6-2x+1}{6}=x+1$$
$$\frac{7}{6}=x+1$$
$$x=\frac{7}{6}-1$$
$$x=\frac{1}{6}$$

8 $\frac{2x-1}{3}-5=\frac{x}{6}$

$$\frac{2x-1}{3}-\frac{x}{6}=5$$
$$\frac{2(2x-1)}{6}-\frac{x}{6}=5$$
$$\frac{4x-2-x}{6}=5$$
$$3x-2=30$$
$$3x=32$$
$$x=\frac{32}{3}$$
$$x=10\frac{2}{3}$$

10 $\frac{5y+1}{4}=6-\frac{2y}{3}$

$$\frac{5y+1}{4}+\frac{2y}{3}=6$$
$$\frac{3(5y+1)}{12}+\frac{8y}{12}=6$$
$$15y+3+8y=72$$
$$23y=69$$
$$y=3$$

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12 $\frac{x+4}{2}-\frac{3-4x}{4}=\frac{5-x}{8}$

$$\frac{4(x+4)}{8}-\frac{2(3-4x)}{8}=\frac{5-x}{8}$$
$$4x+16-6+8x=5-x$$
$$4x+8x+x=5-16+6$$
$$13x=-5$$
$$x=-\frac{5}{13}$$

14 $\frac{x-4}{x+2}=5$

$$x-4=5(x+2)$$
$$x-4=5x+10$$
$$x-5x=10+4$$
$$-4x=14$$
$$x=\frac{14}{-4}$$
$$x=-\frac{7}{2}$$
$$x=-3.5$$

16 $\frac{7}{x}+2=\frac{3}{x}$

$$\frac{7}{x}-\frac{3}{x}=-2$$
$$\frac{4}{x}=-2$$
$$4=-2x$$
$$x=\frac{4}{-2}$$
$$x=-2$$

18 $\frac{3}{x+2}=\frac{5}{2x-1}$

$$3(2x-1)=5(x+2)$$
$$6x-3=5x+10$$
$$6x-5x=10+3$$
$$x=13$$

20 $\frac{y+3}{y+2}=\frac{y+1}{y+4}$

$$(y+3)(y+4)=(y+1)(y+2)$$
$$y^2+7y+12=y^2+3y+2$$
$$7y-3y=2-12$$
$$4y=-10$$
$$y=-\frac{10}{4}$$
$$y=-\frac{5}{2}$$
$$y=-2.5$$

22 C

$$\frac{2x+3}{3x-1}=\frac{2x-5}{3x+4}$$
$$(2x+3)(3x+4)=(2x-5)(3x-1)$$
$$6x^2+8x+9x+12=6x^2-2x-15x+5$$
$$6x^2+17x+12=6x^2-17x+5$$
$$17x+17x=5-12$$
$$34x=-7$$
$$x=-\frac{7}{34}$$

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24 $\dfrac{2}{x+1}+\dfrac{3}{x+9}=0$

$$\frac{2}{x+1}=-\frac{3}{x+9}$$
$$2(x+9)=-3(x+1)$$
$$2x+18=-3x-3$$
$$2x+3x=-3-18$$
$$5x=-21$$
$$x=-\frac{21}{5}$$
$$x=-4.2$$

26 $\dfrac{3}{x-2}-\dfrac{2}{x+2}=\dfrac{1}{x^2-4}$

$$\frac{3(x+2)}{(x-2)(x+2)}-\frac{2(x-2)}{(x-2)(x+2)}=\frac{1}{x^2-4}$$
$$\frac{3x+6-2x+4}{x^2-4}=\frac{1}{x^2-4}$$
$$x+10=1$$
$$x=-9$$

28 $\dfrac{1}{x-1}+\dfrac{1}{2x-1}=0$

$$\frac{1}{x-1}=-\frac{1}{2x-1}$$
$$2x-1=-(x-1)$$
$$2x-1=-x+1$$
$$2x+x=2$$
$$3x=2$$
$$x=\frac{2}{3}$$

30 $\dfrac{1}{x+1}+\dfrac{1}{x+2}=\dfrac{1}{x^2+3x+2}$

$$\frac{x+2}{(x+1)(x+2)}+\frac{x+1}{(x+1)(x+2)}=\frac{1}{(x+1)(x+2)}$$
$$x+2+x+1=1$$
$$2x=1-1-2$$
$$2x=-2$$
$$x=-1$$

However, the initial equation is only defined if $x\neq -1, x\neq -2$. Therefore, there is no solution.

32 $\dfrac{1}{1-t}+\dfrac{1}{1+t}=\dfrac{2t}{1-t^2}$

$$\frac{1+t}{(1-t)(1+t)}+\frac{1-t}{(1-t)(1+t)}=\frac{2t}{(1-t)(1+t)}$$
$$1+t+1-t=2t$$
$$2t=2$$
$$t=1$$

However, the initial equation is only defined if $t\neq \pm 1$. Therefore, there is no solution.

(a) incorrect

(b) incorrect

(c) incorrect

(d) incorrect

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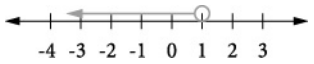
EXERCISE 3.3 SIMPLE LINEAR INEQUALITIES

2 $2x-2<0$

$2x<2$

$x<1$

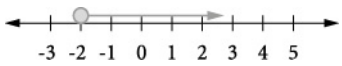


4 $6x+2\geq -10$

$6x\geq -10-2$

$6x\geq -12$

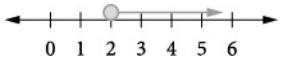
$x\geq -2$



6 $3(x+1)\geq 9$

$x+1\geq 3$

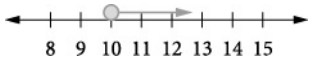
$x\geq 2$



8 $2(x-6)\geq 8$

$x-6\geq 4$

$x\geq 10$



10 $\frac{3x}{2}-\frac{x}{3}<\frac{3}{2}$

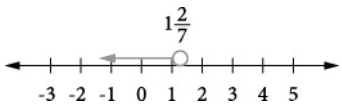
$\frac{9x}{6}-\frac{2x}{6}<\frac{3}{2}$

$\frac{7x}{6}<\frac{3}{2}$

$x<\frac{3}{2}\times\frac{6}{7}$

$x<\frac{9}{7}$

$x<1\frac{2}{7}$

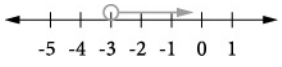


12 $2x+7<3x+10$

$2x-3x<10-7$

$-x<3$

$x>-3$



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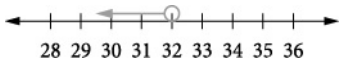
14 $\dfrac{x+3}{5} < 7$

$x+3 < 35$

$x < 35-3$

$x < 32$



16 D

$4x < x+15$

$4x - x < 15$

$3x < 15$

$x < 5$

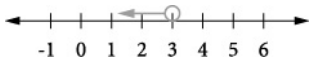
Since x is a positive integer, $x = 1, 2, 3, 4$

18 $\dfrac{7x}{3} < 3 + \dfrac{4x}{3}$

$\dfrac{7x}{3} - \dfrac{4x}{3} < 3$

$\dfrac{3x}{3} < 3$

$x < 3$



20 $\dfrac{5x-3}{2} < x+2$

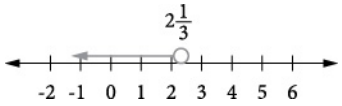
$5x - 3 < 2x + 4$

$5x - 2x < 4 + 3$

$3x < 7$

$x < \dfrac{7}{3}$

$x < 2\dfrac{1}{3}$



22 $22 \le 5x - 3 \le 32$

$22 + 3 \le 5x \le 32 + 3$

$25 \le 5x \le 35$

$\dfrac{25}{5} \le x \le \dfrac{35}{5}$

$5 \le x \le 7$

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24 $x-2>-2$

$$x>-2+2$$
$$x>0$$

$$x-3\le 0$$
$$x\le 3$$

This requires $x>0$ *and* $x\le 3$ to both be true, so (d) is incorrect.

$x>0$ is the same as $0<x$, so these can be combined into $0<x\le 3$.

(a) incorrect

(b) correct

(c) incorrect

(d) incorrect

26 Let this number be x.

$$4<\frac{x}{2}<8$$
$$4\times 2<x<8\times 2$$
$$8<x<16$$

28 Let the number of women be x.

The number of men must be $x-3$.

$$7\le x+x-3\le 15$$
$$7+3\le 2x\le 15+3$$
$$10\le 2x\le 18$$
$$5\le x\le 9$$

The number of women on the committee could be 5, 6, 7, 8, or 9.

30 Let x and the length of the equal sides be y.

$$x=2y-4$$
$$2y=x+4$$

$$x+2y<80$$
$$x+x+4<80$$
$$2x<80-4$$
$$2x<76$$
$$x<38$$

Since x is a length, it is greater than 0.

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Therefore $0 < x < 38,$ where x is an integer.

32 Any side of a triangle must be less than the sum of the remaining two sides.

$x < 8 + 10 \Rightarrow x < 18$
$10 < x + 8 \Rightarrow 2 < x$
$\therefore 2 < x < 18$

There is no need to consider $8 < x + 10,$ since it must be true.

EXERCISE 3.4 QUADRATIC EQUATIONS

2 $(x-2)(x-3) = 0$

$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ or $x = 3$

4 $(x-7)(2x+5) = 0$

$x - 7 = 0$ or $2x + 5 = 0$

$x = 7$ or $x = -\frac{5}{2}$

6 $-5x(x+1) = 0$

$-5x = 0$ or $x + 1 = 0$

$x = 0$ or $x = -1$

8 $(x-3a)(x+2b) = 0$

$x - 3a = 0$ or $x + 2b = 0$

$x = 3a$ or $x = -2b$

10 $(2x-11)(2x+11) = 0$

$2x - 11 = 0$ or $2x + 11 = 0$

$x = \frac{11}{2}$ or $x = -\frac{11}{2}$

$x = 5.5$ or $x = -5.5$

12 $(2x+3)^2 = 0$

$2x + 3 = 0$

$x = -\frac{3}{2}$

$x = -1.5$

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EXERCISE 3.5 QUADRATIC EQUATIONS WITHOUT A LINEAR TERM

2 $x^2 - 25 = 0$

$(x-5)(x+5) = 0$

$x = 5$ or $x = -5$

OR

$x^2 - 25 = 0$

$x^2 = 25$

$x = \pm 5$

4 $x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$x = 4$ or $x = -4$

OR

$x^2 = 16$

$x = \pm 4$

6 $4x^2 - 9 = 0$

$(2x-3)(2x+3) = 0$

$x = \frac{3}{2}$ or $x = -\frac{3}{2}$

$x = 1.5$ or $x = -1.5$

OR

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \pm\frac{3}{2}$

$x = \pm 1.5$

8 $4x^2 - 25 = 0$

$(2x-5)(2x+5) = 0$

$x = \frac{5}{2}$ or $x = -\frac{5}{2}$

$x = 2.5$ or $x = -2.5$

OR

$4x^2 - 25 = 0$

$4x^2 = 25$

$x^2 = \frac{25}{4}$

$x = \pm\frac{5}{2}$

$x = \pm 2.5$

10 $5x^2 - 5 = 0$

$5(x^2 - 1) = 0$

$5(x-1)(x+1) = 0$

$x = 1$ or $x = -1$

OR

$5x^2 - 5 = 0$

$5x^2 = 5$

$x^2 = \frac{5}{5}$

$x^2 = 1$

$x = \pm 1$

12 $16 - x^2 = 0$

$(4-x)(4+x) = 0$

$x = 4$ or $x = -4$

OR

$16 - x^2 = 0$

$16 = x^2$

$x = \pm 4$

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14 $9(x-1)^2-36=0$

$$(x-1)^2-4=0$$
$$(x-1-2)(x-1+2)=0$$
$$(x-3)(x+1)=0$$

$x=3$ or $x=-1$

OR

$$9(x-1)^2-36=0$$
$$9(x-1)^2=36$$
$$(x-1)^2=\frac{36}{9}$$
$$(x-1)^2=4$$
$$x-1=\pm 2$$
$$x=1\pm 2$$

$x=3$ or $x=-1$

(a) correct

(b) correct

(c) incorrect

(d) correct

16 $5x^2-245=0$

$$5(x^2-49)=0$$
$$5(x-7)(x+7)=0$$

$x=7$ or $x=-7$

OR

$$5x^2-245=0$$
$$5x^2=245$$
$$x^2=49$$
$$x=\pm 7$$

18 $(5x-1)^2=16$

$$(5x-1)^2-16=0$$
$$(5x-1-4)(5x-1+4)=0$$
$$(5x-5)(5x+3)=0$$
$$5(x-1)(5x+3)=0$$

$x=1$ or $x=-\frac{3}{5}$

$x=1$ or $x=-0.6$

OR

$$(5x-1)^2=16$$
$$(5x-1)=\pm 4$$
$$5x=1\pm 4$$

$5x=5$ or $5x=-3$

$x=1$ or $x=-\frac{3}{5}$

$x=1$ or $x=-0.6$

20 $x^2-5=0$

$$(x-\sqrt{5})(x+\sqrt{5})=0$$

$x=\sqrt{5}$ or $x=-\sqrt{5}$

OR

$$x^2-5=0$$
$$x^2=5$$
$$x=\pm\sqrt{5}$$

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22 $(x+1)^2 = 8$

$(x+1)^2 - 8 = 0$

$\left(x+1-\sqrt{8}\right)\left(x+1+\sqrt{8}\right) = 0$

$\left(x+1-\sqrt{4\times 2}\right)\left(x+1+\sqrt{4\times 2}\right) = 0$

$\left(x+1-2\sqrt{2}\right)\left(x+1+2\sqrt{2}\right) = 0$

$x = -1+2\sqrt{2}$ or $x = -1-2\sqrt{2}$

OR

$(x+1)^2 = 8$

$(x+1) = \pm\sqrt{8}$

$x+1 = \pm 2\sqrt{2}$

$x = -1 \pm 2\sqrt{2}$

EXERCISE 3.6 QUADRATIC EQUATIONS WITHOUT A CONSTANT TERM

2 $x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or $x = 5$

4 $x^2 + 10x = 0$

$x(x+10) = 0$

$x = 0$ or $x = -10$

6 $2x^2 - 5x = 0$

$x(2x-5) = 0$.

$x = 0$ or $x = \frac{5}{2}$

$x = 0$ or $x = 2.5$

8 $3x^2 - 21x = 0$

$3x(x-7) = 0$

$x = 0$ or $x = 7$

10 $6x^2 = 24x$

$6x^2 - 24x = 0$

$6x(x-4) = 0$

$x = 0$ or $x = 4$

12 $5x^2 + x = 0$

$x(5x+1) = 0$

$x = 0$ or $x = -\frac{1}{5}$

$x = 0$ or $x = -0.2$

14 $12x^2 - 5x = 0$

$x(12x-5) = 0$

$x = 0$ or $x = \frac{5}{12}$

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EXERCISE 3.7 GENERAL QUADRATIC EQUATIONS

2 $x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1$ or $x = 5$

4 $x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$

6 $x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1$ or $x = 4$

8 You will need the cross method or another suitable method to first factorise this quadratic.

$9x^2 + 4x - 5 = 0$

$(x+1)(9x-5) = 0$

$x = \frac{5}{9}$ or $x = -1$

10 $x^2 + 4x - 12 = 0$

$(x-2)(x+6) = 0$

$x = 2$ or $x = -6$

12 $4x^2 - 12x - 7 = 0$

$(2x-7)(2x+1) = 0$

$x = \frac{7}{2}$ or $x = -\frac{1}{2}$

$x = 3.5$ or $x = -0.5$

14 $x^2 + 10x + 25 = 0$

$(x+5)^2 = 0$

$x = -5$

16 $4x^2 - 8x - 21 = 0$

$(2x-7)(2x+3) = 0$

$x = \frac{7}{2}$ or $x = -\frac{3}{2}$

$x = 3.5$ or $x = -1.5$

18 $x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$

$x = 4$

20 $3x^2 - 41x + 60 = 0$

$(3x-5)(x-12) = 0$

$x = \frac{5}{3}$ or $x = 12$

$x = 1\frac{2}{3}$ or $x = 12$

22 $5x^2 = 8x - 3$

$5x^2 - 8x + 3 = 0$

$(5x-3)(x-1) = 0$

$x = \frac{3}{5}$ or $x = 1$

$x = 0.6$ or $x = 1$

24 $x(x+5) = 6$

$x^2 + 5x = 6$

$x^2 + 5x - 6 = 0$

$(x-1)(x+6) = 0$

$x = 1$ or $x = -6$

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26 $x^2+15=8x$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3$ or $x=5$

28 Note: There is no need to take out a common factor of -1 as it is easier to just divide (or multiply) both sides of the equation by -1.

$12-4x-x^2=0$

$x^2+4x-12=0$

$(x-2)(x+6)=0$

$x=2$ or $x=-6$

30 $(x+1)^2=4x$

$x^2+2x+1-4x=0$

$x^2-2x+1=0$

$(x-1)^2=0$

$x=1$

32 $6x^2=10-11x$

$6x^2+11x-10=0$

$(3x-2)(2x+5)=0$

$x=\frac{2}{3}$ or $x=-\frac{5}{2}$

EXERCISE 3.8 COMPLETING THE SQUARE

2 Half of -6 is -3 and the square of -3 is 9.

Hence 9 must be added.

4 Half of 2 is 1 and the square of 1 is 1.

Hence 1 must be added.

6 Half of -1 is $-\frac{1}{2}$ and the square of $-\frac{1}{2}$ is $\frac{1}{4}$.

Hence $\frac{1}{4}$ must be added.

8 Half of 3 is $\frac{3}{2}$ and the square of $\frac{3}{2}$ is $\frac{9}{4}$.

Hence $\frac{9}{4}$ must be added.

10 Half of 1 is $\frac{1}{2}$ and the square of $\frac{1}{2}$ is $\frac{1}{4}$.

Hence $\frac{1}{4}$ must be added.

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12 Half of $-2b$ is b and the square of b is b^2.

Hence b^2 must be added.

14 Half of $-a$ is $\frac{-a}{2}$ and the square of $\frac{-a}{2}$ is $\frac{a^2}{4}$.

Hence $\frac{a^2}{4}$ must be added.

This will result in $x^2 - ax + \frac{a^2}{4} = \left(x - \frac{a}{2}\right)^2$

(a) incorrect

(b) correct

(c) correct

(d) incorrect

EXERCISE 3.9
SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

2 $x^2 - 2x - 8 = 0$

$x^2 - 2x = 8$

$x^2 - 2x + 1 = 8 + 1$

$(x-1)^2 = 9$

$x - 1 = \pm 3$

$x - 1 = 3$ or $x - 1 = -3$

$x = 4$ or $x = -2$

4 $x^2 + 4x = 12$

$x^2 + 4x + 4 = 12 + 4$

$(x+2)^2 = 16$

$x + 2 = \pm 4$

$x + 2 = 4$ or $x + 2 = -4$

$x = 2$ or $x = -6$

6 $x^2 - 4x = 21$

$x^2 - 4x + 4 = 21 + 4$

$(x-2)^2 = 25$

$x - 2 = \pm 5$

$x - 2 = 5$ or $x - 2 = -5$

$x = 7$ or $x = -3$

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8 $x^2 - 3x + 2 = 0$

$$x^2 - 3x = -2$$
$$x^2 - 3x + \frac{9}{4} = -2 + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$$
$$x - \frac{3}{2} = \pm\frac{1}{2}$$
$$x - \frac{3}{2} = \frac{1}{2} \text{ or } x - \frac{3}{2} = -\frac{1}{2}$$
$$x = 2 \text{ or } x = 1$$

10 $x^2 - 5x + 4 = 0$

$$x^2 - 5x = -4$$
$$x^2 - 5x + \frac{25}{4} = -4 + \frac{25}{4}$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{9}{4}$$
$$x - \frac{5}{2} = \pm\frac{3}{2}$$
$$x - \frac{5}{2} = \frac{3}{2} \text{ or } x - \frac{5}{2} = -\frac{3}{2}$$
$$x = 4 \text{ or } x = 1$$

12 $x^2 - 11x = 12$

$$x^2 - 11x + \frac{121}{4} = 12 + \frac{121}{4}$$
$$\left(x - \frac{11}{2}\right)^2 = \frac{169}{4}$$
$$x - \frac{11}{2} = \pm\frac{13}{2}$$
$$x - \frac{11}{2} = \frac{13}{2} \text{ or } x - \frac{11}{2} = -\frac{13}{2}$$
$$x = 12 \text{ or } x = -1$$

14 $x^2 = 7x - 10$

$$x^2 - 7x = -10$$
$$x^2 - 7x + \frac{49}{4} = -10 + \frac{49}{4}$$
$$\left(x - \frac{7}{2}\right)^2 = \frac{9}{4}$$
$$x - \frac{7}{2} = \pm\frac{3}{2}$$
$$x - \frac{7}{2} = \frac{3}{2} \text{ or } x - \frac{7}{2} = -\frac{3}{2}$$
$$x = 5 \text{ or } x = 2$$

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16 $x^2 - 10x - 11 = 0$

$$x^2 - 10x = 11$$
$$x^2 - 10x + 25 = 11 + 25$$
$$(x-5)^2 = 36$$
$$x - 5 = \pm 6$$
$$x - 5 = 6 \text{ or } x - 5 = -6$$
$$x = 11 \text{ or } x = -1$$

18 $x^2 = 3x$

$$x^2 - 3x = 0$$
$$x^2 - 3x + \frac{9}{4} = 0 + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{9}{4}$$
$$x - \frac{3}{2} = \pm\frac{3}{2}$$
$$x - \frac{3}{2} = \frac{3}{2} \text{ or } x - \frac{3}{2} = -\frac{3}{2}$$
$$x = 3 \text{ or } x = 0$$

(a) correct **(b)** incorrect

(c) correct **(d)** correct

EXERCISE 3.10 QUADRATIC EQUATIONS WITH NON-RATIONAL SOLUTIONS

2 $x^2 + 4x - 4 = 0$

$$x^2 + 4x = 4$$
$$x^2 + 4x + 4 = 4 + 4$$
$$(x+2)^2 = 8$$
$$x + 2 = \pm 2\sqrt{2}$$
$$x = -2 \pm 2\sqrt{2}$$

4 $x^2 - 6x + 2 = 0$

$$x^2 - 6x = -2$$
$$x^2 - 6x + 9 = -2 + 9$$
$$(x-3)^2 = 7$$
$$x - 3 = \pm\sqrt{7}$$
$$x = 3 \pm \sqrt{7}$$

6 $x^2 + 2x - 2 = 0$

$$x^2 + 2x = 2$$
$$x^2 + 2x + 1 = 2 + 1$$
$$(x+1)^2 = 3$$
$$x + 1 = \pm\sqrt{3}$$
$$x = -1 \pm \sqrt{3}$$

8 $x^2 + x - 1 = 0$

10 $x^2 + 4x = 1$

$$x^2 + 4x = 1$$
$$x^2 + 4x + 4 = 1 + 4$$
$$(x+2)^2 = 5$$
$$x + 2 = \pm\sqrt{5}$$
$$x = -2 \pm \sqrt{5}$$

12 $x^2 + 3x - 6 = 0$

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$$x^2 + x = 1$$
$$x^2 + x + \frac{1}{4} = 1 + \frac{1}{4}$$
$$\left(x + \frac{1}{2}\right)^2 = \frac{5}{4}$$
$$x + \frac{1}{2} = \pm\frac{\sqrt{5}}{2}$$
$$x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

$$x^2 + 3x = 6$$
$$x^2 + 3x + \frac{9}{4} = 6 + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{33}{4}$$
$$x + \frac{3}{2} = \pm\frac{\sqrt{33}}{2}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{33}}{2}$$

EXERCISE 3.11 COMPLETING THE SQUARE FOR NON-MONIC EQUATIONS

2 $2x^2 + 6x - 5 = 0$

$$2x^2 + 6x = 5$$
$$x^2 + 3x = \frac{5}{2}$$
$$x^2 + 3x + \frac{9}{4} = \frac{5}{2} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{19}{4}$$
$$x + \frac{3}{2} = \pm\frac{\sqrt{19}}{2}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{19}}{2}$$
$$x = \frac{-3 \pm \sqrt{19}}{2}$$

4 $2x^2 + 3x - 1 = 0$

$$2x^2 + 3x = 1$$
$$x^2 + \frac{3}{2}x = \frac{1}{2}$$
$$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$$
$$\left(x + \frac{3}{4}\right)^2 = \frac{17}{16}$$
$$x + \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$$
$$x = -\frac{3}{4} \pm \frac{\sqrt{17}}{4}$$
$$x = \frac{-3 \pm \sqrt{17}}{4}$$

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6 $3x^2+4x=5$

$$x^2+\frac{4}{3}x=\frac{5}{3}$$
$$x^2+\frac{4}{3}x+\frac{16}{36}=\frac{5}{3}+\frac{16}{36}$$
$$\left(x+\frac{4}{6}\right)^2=\frac{19}{9}$$
$$x+\frac{2}{3}=\pm\frac{\sqrt{19}}{3}$$
$$x=-\frac{2}{3}\pm\frac{\sqrt{19}}{3}$$
$$x=\frac{-2\pm\sqrt{19}}{3}$$

8 $2x^2-6x+1=0$

$$2x^2-6x=-1$$
$$x^2-3x=-\frac{1}{2}$$
$$x^2-3x+\frac{9}{4}=-\frac{1}{2}+\frac{9}{4}$$
$$\left(x-\frac{3}{2}\right)^2=\frac{7}{4}$$
$$x-\frac{3}{2}=\pm\frac{\sqrt{7}}{2}$$
$$x=\frac{3}{2}\pm\frac{\sqrt{7}}{2}$$
$$x=\frac{3\pm\sqrt{7}}{2}$$

10 $4x^2+4x-5=0$

$$4x^2+4x=5$$
$$x^2+x=\frac{5}{4}$$
$$x^2+x+\frac{1}{4}=\frac{5}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{2}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{\sqrt{2}}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{6}}{2}$$
$$x=-\frac{1}{2}\pm\frac{\sqrt{6}}{2}$$
$$x=\frac{-1\pm\sqrt{6}}{2}$$

12 $3x^2-2x-2=0$

$$3x^2-2x=2$$
$$x^2-\frac{2}{3}x=\frac{2}{3}$$
$$x^2-\frac{2}{3}x+\frac{4}{36}=\frac{2}{3}+\frac{4}{36}$$
$$\left(x-\frac{2}{6}\right)^2=\frac{7}{9}$$
$$x-\frac{1}{3}=\pm\frac{\sqrt{7}}{3}$$
$$x=\frac{1}{3}\pm\frac{\sqrt{7}}{3}$$
$$x=\frac{1\pm\sqrt{7}}{3}$$

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14 $3x^2 - 8x + 3 = 0$

$$3x^2 - 8x = -3$$
$$x^2 - \frac{8}{3}x = -1$$
$$x^2 - \frac{8}{3}x + \frac{16}{9} = -1 + \frac{16}{9}$$
$$\left(x - \frac{4}{3}\right)^2 = \frac{7}{9}$$
$$x - \frac{4}{3} = \pm\frac{\sqrt{7}}{3}$$
$$x = \frac{4}{3} \pm \frac{\sqrt{7}}{3}$$
$$x = \frac{4 \pm \sqrt{7}}{3}$$

EXERCISE 3.12 THE QUADRATIC FORMULA

2 $x^2 + 2x - 8 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1,\ b = 2,\ c = -8$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -8}}{2 \times 1}$

$x = \dfrac{-2 \pm \sqrt{36}}{2}$

$x = \dfrac{-2 \pm 6}{2}$

$x = -1 \pm 3$

$x = 2$ or $x = -4$

4 $x^2 - 7x + 10 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a = 1,\ b = -7,\ c = 10$

$x = \dfrac{7 \pm \sqrt{(-7)^2 - 4 \times 1 \times 10}}{2 \times 1}$

$x = \dfrac{7 \pm \sqrt{9}}{2}$

$x = \dfrac{7 \pm 3}{2}$

$x = 5$ or $x = 2$

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6 $x^2 - 6x + 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = -6,\ c = 4$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 4}}{2 \times 1}$

$x = \dfrac{6 \pm \sqrt{20}}{2}$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$

8 $x^2 + 5x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 5,\ c = -1$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4 \times 1 \times -1}}{2 \times 1}$

$x = \dfrac{-5 \pm \sqrt{29}}{2}$

10 $x^2 + 4x + 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 4,\ c = 2$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 2}}{2 \times 1}$

$x = \dfrac{-4 \pm \sqrt{8}}{2}$

$x = \dfrac{-4 \pm 2\sqrt{2}}{2}$

$x = -2 \pm \sqrt{2}$

12 $x^2 + 2x - 15 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 2,\ c = -15$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -15}}{2 \times 1}$

$x = \dfrac{-2 \pm \sqrt{64}}{2}$

$x = \dfrac{-2 \pm 8}{2}$

$x = 3$ or $x = -5$

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14 $2x^2 - 8x + 3 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 2,\ b = -8,\ c = 3$

$x = \dfrac{8 \pm \sqrt{8^2 - 4 \times 2 \times 3}}{2 \times 2}$

$x = \dfrac{8 \pm \sqrt{40}}{4}$

$x = \dfrac{8 \pm 2\sqrt{10}}{4}$

$x = \dfrac{4 \pm \sqrt{10}}{2}$

16 $2x^2 - 3x = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 2,\ b = -3,\ c = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4 \times 2 \times 0}}{2 \times 2}$

$x = \dfrac{3 \pm \sqrt{9}}{4}$

$x = \dfrac{3 \pm 3}{4}$

$x = \dfrac{3}{2}$ or $x = 0$

18 $2x^2 + 3x - 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 2,\ b = 3,\ c = -5$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 2 \times -5}}{2 \times 2}$

$x = \dfrac{-3 \pm \sqrt{49}}{4}$

$x = \dfrac{-3 \pm 7}{4}$

$x = 1$ or $x = -\dfrac{5}{2}$

20 $x^2 - 8x + 16 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = -8,\ c = 16$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4 \times 1 \times 16}}{2 \times 1}$

$x = \dfrac{8 \pm \sqrt{0}}{2}$

$x = 4$

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22 $7x^2 - 7x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 7,\ b = -7,\ c = -2$$
$$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 7 \times -2}}{2 \times 7}$$
$$x = \frac{7 \pm \sqrt{105}}{14}$$

24 $3x^2 - 11x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 3,\ b = -11,\ c = -4$$
$$x = \frac{11 \pm \sqrt{(-11)^2 - 4 \times 3 \times -4}}{2 \times 3}$$
$$x = \frac{11 \pm \sqrt{169}}{6}$$
$$x = \frac{11 \pm 13}{6}$$
$$x = 4 \text{ or } x = -\frac{1}{3}$$

26 $x(x + 3) = 2$

$$x^2 + 3x = 2$$
$$x^2 + 3x - 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 3,\ c = -2$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -2}}{2 \times 1}$$
$$x = \frac{-3 \pm \sqrt{17}}{2}$$

28 $2x^2 - 6x = 3$

$$2x^2 - 6x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 2,\ b = -6,\ c = -3$$
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 2 \times -3}}{2 \times 2}$$
$$x = \frac{6 \pm \sqrt{60}}{4}$$
$$x = \frac{6 \pm 2\sqrt{15}}{4}$$
$$x = \frac{3 \pm \sqrt{15}}{2}$$

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30 $x^2 = 6x + 2$

$x^2 - 6x - 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = -6,\ c = -2$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times -2}}{2 \times 1}$

$x = \dfrac{6 \pm \sqrt{44}}{2}$

$x = \dfrac{6 \pm 2\sqrt{11}}{2}$

$x = 3 \pm \sqrt{11}$

32 $2x^2 + 10x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 2,\ b = 10,\ c = 5$

$x = \dfrac{-10 \pm \sqrt{10^2 - 4 \times 2 \times 5}}{2 \times 2}$

$x = \dfrac{-10 \pm \sqrt{60}}{4}$

$x = \dfrac{-10 \pm 2\sqrt{15}}{4}$

$x = \dfrac{-5 \pm \sqrt{15}}{2}$

34 $3x^2 + 9x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 3,\ b = 9,\ c = 5$

$x = \dfrac{-9 \pm \sqrt{9^2 - 4 \times 3 \times 5}}{2 \times 3}$

$x = \dfrac{-9 \pm \sqrt{21}}{6}$

36 $x(x+1) = 1$

$x^2 + x = 1$

$x^2 + x - 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 1,\ c = -1$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -1}}{2 \times 1}$

$x = \dfrac{-1 \pm \sqrt{5}}{2}$

EXERCISE 3.13 PROBLEMS INVOLVING QUADRATIC EQUATIONS

2 D

Use Pythagoras' rule.

$x^2 + (x-1)^2 = 5^2$

$x^2 + x^2 - 2x + 1 - 25 = 0$

$2x^2 - 2x - 24 = 0$

$x^2 - x - 12 = 0$

Note that:

$x^2 - x - 12 = 0 \Rightarrow x(x-1) = 12$

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4 Let this positive number be x.

$$x + x^2 = 12$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$

$x = -4$ or $x = 3$

Since this number is positive, $x = 3$ is the only solution. The number is 3.

6 Let the smaller number be x. The next consecutive number is $x+1$.

$$x(x+1) = 72$$
$$x^2 + x - 72 = 0$$
$$(x+9)(x-8) = 0$$

$x = -9$ or $x = 8$

Therefore the two consecutive numbers are -9, -8 or 8, 9.

8 Let this positive number be x.

$$x^2 + 4x = 60$$
$$x^2 + 4x - 60 = 0$$
$$(x+10)(x-6) = 0$$

$x = -10$ or $x = 6$

Since this number is positive, $x = 6$ is the only solution. The number is 6.

10 Let the width of the border be x.

The sides of the carpet will be $6-2x$ and $4-2x$.

$$(6-2x)(4-2x) = 8$$
$$24 - 20x + 4x^2 = 8$$
$$4x^2 - 20x + 16 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-1)(x-4) = 0$$

$x = 1$ or $x = 4$

Since the room measures 6 m by 4 m, the width of the border must be less than 4 m. Therefore the width of the border is 1 m.

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12 Let the breadth of this rectangle be x, so that its length is $x+8$.

$$x(x+8)=48$$
$$x^2+8x-48=0$$
$$(x+12)(x-4)=0$$

$x=-12$ or $x=4$

Since the length must be positive, this rectangle is 4 cm by 12 cm.

14 (a) Let the length of the rectangle be l.

$$2x+2l=40$$
$$x+l=20$$
$$l=20-x$$

The length is $(20-x)$ cm.

(b) Let the area of the rectangle be A.

$$A=xl$$
$$=x(20-x)$$
$$=20x-x^2$$

(c) $20x-x^2=84$

$$x^2-20x+84=0$$
$$(x-4)(x-16)=0$$

$x=4$ cm or $x=16$ cm

The length is 4 cm and the breadth is 16 cm.

CHAPTER REVIEW 3

2 (a) $\frac{x}{5}=\frac{3}{20}$

$$x=\frac{3}{20}\times 5$$
$$x=\frac{3}{4}$$

(b) $\frac{3x-1}{5}=\frac{x}{20}$

$$\frac{4(3x-1)}{20}=\frac{x}{20}$$
$$12x-4=x$$
$$12x-x=4$$
$$11x=4$$
$$x=\frac{4}{11}$$

(c) $\frac{x-2}{x+3}=\frac{3}{5}$

$$5(x-2)=3(x+3)$$
$$5x-10=3x+9$$
$$5x-3x=9+10$$
$$2x=19$$
$$x=\frac{19}{2}$$
$$x=9.5$$

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4 **(a)** $x^2 = 4$

$x = \pm 2$

OR

$x^2 = 4$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2$ or $x = -2$

(b) $x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$

(c) $x^2 = 4x - 4$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

(d) $(x^2 - 3x)^2 = 16$

$(x^2 - 3x)^2 - 16 = 0$

$(x^2 - 3x - 4)(x^2 - 3x + 4) = 0$

$x^2 - 3x - 4 = 0$ or $x^2 - 3x + 4 = 0$

Solve each equation separately.

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4$ or $x = -1$

$x^2 - 3x + 4 = 0$ cannot be factorised.

You can complete the square or use the quadratic formula.

Competing the square:

$x^2 - 3x + 4 = 0$

$x^2 - 3x = -4$

$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = -4 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = -\frac{7}{4}$

There will be no solutions, as a perfect square cannot be negative.

Using the formula:

$x^2 - 3x + 4 = 0$

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = -3,\ c = 4$$
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times 4}}{2 \times 1}$$
$$x = \frac{3 \pm \sqrt{-7}}{2}$$

There will be no real solutions.

Therefore, the only solutions are $x = 4$ or $x = -1$.

(e) $(x^2 - 3x - 10)(x^2 - 3x - 4) = 0$

$$(x-5)(x+2)(x-4)(x+1) = 0$$
$$x = -2,\ x = -1,\ x = 4 \text{ or } x = 5$$

(f) $6x^2 + 7x - 3 = 0$

$$(3x-1)(2x+3) = 0$$
$$x = \frac{1}{3} \text{ or } x = -\frac{3}{2}$$

6 (a) $2x^2 - x - 5 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-5)}}{4}$$
$$x = \frac{1 \pm \sqrt{41}}{4}$$

(b) $x = \frac{1 - \sqrt{41}}{4} = -1.35$

$$x = \frac{1 + \sqrt{41}}{4} = 1.85$$

$$x = -1.35, 1.85$$

8 $(2x-y)(x^2 - xy + y^2) = x^2(2x-y) - xy(2x-y) + y^2(2x-y)$

$$= 2x^3 - x^2y - 2x^2y + xy^2 + 2xy^2 - y^3$$
$$= 2x^3 - 3x^2y + 3xy^2 - y^3$$

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10 $525\,000 = n(66\,000 + 1500n - 1500)$

$525\,000 = n(64\,500 + 1500n)$

$350 = 43n + n^2$

$n^2 + 43n - 350 = 0$

$(n + 50)(n - 7) = 0$

$n = -50,\ 7$

$n = 7$ since $n > 0$

12 $400 = n\left(6 + 2n - 2\right)$

$400 = n\left(2n + 4\right)$

$400 = 2n^2 + 4n$

$n^2 + 2n = 200$

$n^2 + 2n - 200 = 0$

$n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a = 1,\ b = 2,\ c = -200$

$n = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times -200}}{2a}$

$n = \dfrac{-2 \pm \sqrt{804}}{2}$

Since $n > 0$, $n = \dfrac{-2 \pm \sqrt{804}}{2} = 13.177...$

$n = 13,$ rounded to the nearest whole number.

14 $\dfrac{a - x}{x} = \dfrac{x}{b - x}$

$(a - x)(b - x) = x^2$

$ab - ax - bx + x^2 = x^2$

$x^2 - (a + b)x + ab = x^2$

$(a + b)x = ab$

$x = \dfrac{ab}{a + b}$

16 $x\left(x^2 + 5\right) = 6x^2$

$x^3 + 5x - 6x^2 = 0$

$x\left(x^2 - 6x + 5\right) = 0$

$x(x - 1)(x - 5) = 0$

$x = 0,\ x = 1$ or $x = 5$

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