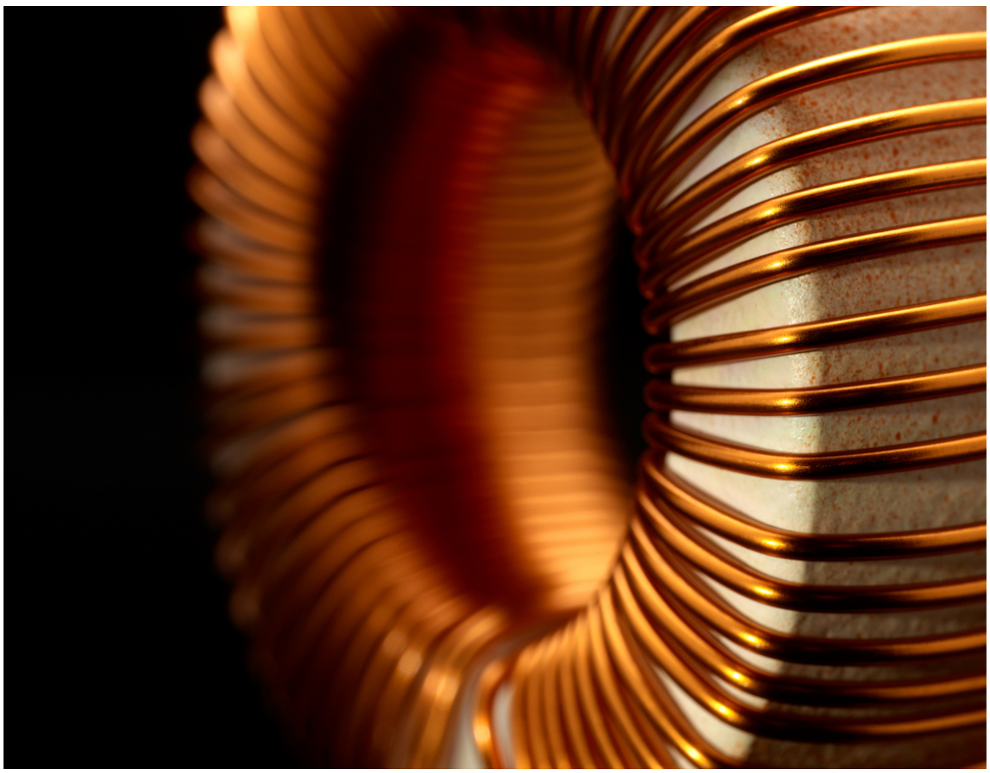
Module 6: Electromagnetism

Part 3: Electromagnetic Induction



Tammy Humphrey

Contents

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Syllabus content: Electromagnetism

Electromagnetic Induction

Inquiry question: How are electric and magnetic fields related?

Students:

- describe how magnetic flux can change, with reference to the relationship $\Phi = B_{\parallel}A = BA\cos\theta$
- analyse qualitatively and quantitatively, with reference to energy transfers and transformations, examples of Faraday's Law and Lenz's Law $\epsilon = -N\frac{\Delta\Phi}{\Delta t}$ including but not limited to:
  - the generation of an electromotive force (emf) and evidence for Lenz's Law produced by the relative movement between a magnet, straight conductors, metal plates and solenoids
  - the generation of an emf produced by the relative movement or changes in current in one solenoid in the vicinity of another solenoid
- analyse quantitatively the operation of ideal transformers through the application of:
  - $\frac{V_p}{V_s} = \frac{N_p}{N_s}$
  - $V_pI_p = V_sI_s$
- evaluate qualitatively the limitations of the ideal transformer model and the strategies used to improve transformer efficiency, including but not limited to:
  - incomplete flux linkage
  - resistive heat production and eddy currents
- analyse applications of step-up and step-down transformers, including but not limited to:
  - the distribution of energy using high-voltage transmission lines

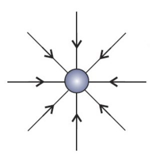
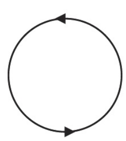
Revision - overview of electromagnetism

We will start this section by looking again at the big picture in electromagnetism. We have already examined how:

- isolated electric charges produce an electric field that begins on positive charges and ends on negative charges
- magnetic fields with circulation/curl (i.e. magnetic field loops) are produced by a current (a line of moving charges)
- electric charges experience a force in an electric field equal to $\vec{F} = q\vec{E}$
- moving electric charges experience a force in magnetic fields equal to $|\vec{F}| = q|\vec{v}||\vec{B}|\sin\theta$ (alternatively, if the moving charges are a current in a wire, then the wire experiences a force of $F = IlB\sin\theta$)

$$\oiint E \cdot dA = \frac{q}{\epsilon_0}$$
$$\oiint B \cdot dA = 0$$
$$\oint E \cdot ds = -\frac{d\Phi_B}{dt}$$
$$\oint B \cdot ds = \mu_0 i + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Figure 1: Maxwell and his equations of electromagnetism

Type of field			How it is made
Fields with divergence		**Electric fields**	**1. Electric charges**
		Magnetic fields	**2. Do not exist** (There are no magnetic charges/monopoles)
Fields with circulation		**Electric fields**	**3. Changing magnetic fields**
		Magnetic fields	**4. Changing electric fields & moving electric charges**

Force laws: $F = qE$ & $F = qvB\sin\theta$

Figure 2: A qualitative summary of Maxwell's equations

Electromotive force (EMF)

In this section we will focus on the third of Maxwell's equations, that tells us that a changing magnetic field will produce a looped electric field (i.e. an electric field with circulation).

This type of electric field is a bit different to the diverging electric fields we have encountered so far, as it is not *conservative*. You cannot define an electric potential energy associated with position in this electric field, as the work done on a charge by this non-conservative field depends on the path taken by the charge.

As a result, we will define and use a new quantity in this section, called *electromotive force* (EMF). EMF is not a force, but has units of volts, and is a measure of the work done per unit charge by source.[1]

[1] For olympiad students: Emf is the line integral of the force acting on a charge around a complete circuit $\varepsilon = \oint \vec{F} \cdot d\vec{s}$

Motional emf

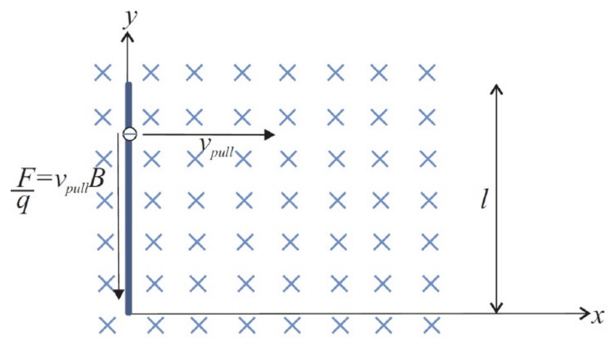


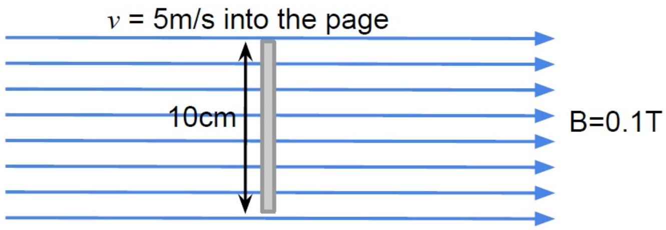
Figure 3: A motional emf appears over a conductor moving through a magnetic field.

Consider a conductor moving through a magnetic field. Charges (protons and electrons) inside the conductor experience a force due to their motion through the field. Protons are unable to move (the force keeping them in the nucleus is much stronger!), however electrons are free to move. We can use the right-hand rule to determine that the electrons experience a force towards the bottom of the rod, causing this end of the rod to become negatively charged so that a "motional emf" appears across the rod.

In the situation shown, the work done on the electrons is $W = Fl$, where $F = qvB$, so we can obtain an expression for motional emf as

$$\varepsilon = \frac{W}{q} = \frac{Fl}{q} = \frac{qv_{pull}Bl}{q} = v_{pull}Bl$$

Example 1.



What is the magnitude and polarity of the motional emf induced across a conducting rod of length 10cm moving into the page with a velocity of $v = 5\text{ms}^{-1}$ in a uniform magnetic field of strength 0.1T?

A. 5V, top of rod positive

B. 0.05V, bottom of rod positive

C. 5V, bottom of rod positive

D. 0.05V, top of rod positive

Example 2.

The earth's magnetic field is $\approx 5 \times 10^{-5}$T.

Calculate the motional emf generated across the wings of a airplane with a wingspan of 60m which is flying across the north pole at a velocity of $v = 800\text{km/h}$.

Which wing is positively charged?

Magnetic Flux

Loosely speaking, magnetic flux is a measure of *how much* magnetic field passes through a particular area, for example, through a loop of wire. We could speak in an analogous way about how much light enters a room through a window (see figure 4) - this depends on:

- how big the window is
- how bright (intense) the light is
- and the angle between the plane of the window and the direction of the source of the light (more light enters through a west facing window in the late afternoon than in the middle of the day).

Figure 4: The flux of light through a window depends on the size of the window, intensity (brightness) of the light, and the angle between the plane of the window and the direction of the light.

Definition of magnetic flux

Magnetic flux can be defined as

$$\Phi = BA\cos\theta$$

where

- Φ is the magnetic flux, measured in Webers (Wb).
- B is the magnetic field strength (sometimes called the magnetic 'flux density') and is measured in Teslas
- A is the area we are considering (often, but not necessarily, the area of a conducting loop)
- θ is the angle between the **normal** to the plane of the loop and the direction of the magnetic field

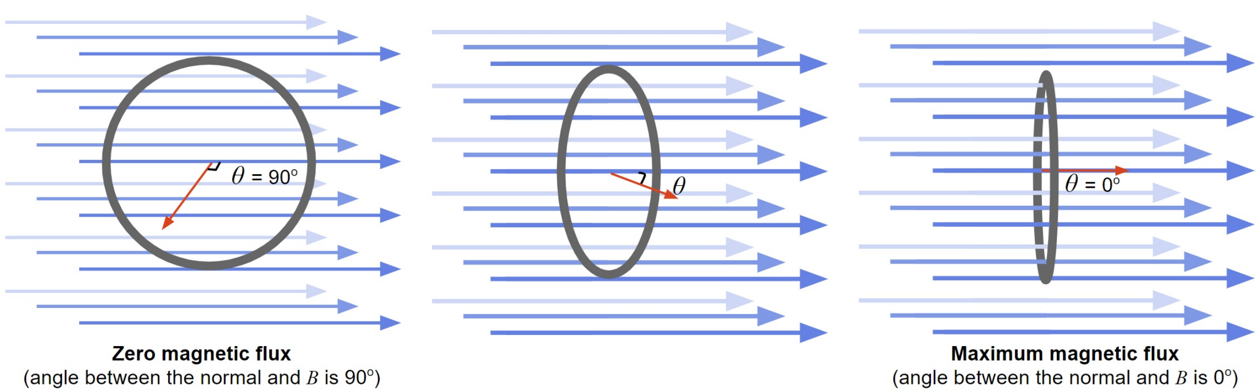


Figure 5: Magnetic flux is maximum when the angle between the normal to the loop and the magnetic field is zero.

Example 3.

A square conducting loop of side length 0.1m shown in figure 11, is originally fully inside a magnetic field of strength $B = 1\text{mT}$. It is then pulled out of the field.

(a) What is the magnetic flux through the ring when it is fully inside the magnetic field?

(b) What is the magnetic flux through the ring after it has been completely pulled out of the magnetic field?

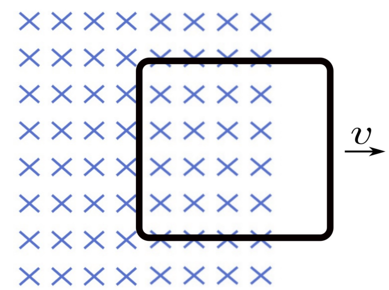


Figure 6: A square ring is pulled out of a magnetic field

Faraday and electromagnetic induction

Michael Faraday was an extremely talented experimentalist who worked at the Royal Institution in the 1800s. Faraday was inspired by Oested's discovery that electric currents produce magnetic fields to search for a way to produce an electric current from a magnetic field. Eventually he made the discovery that a magnetic field *can* make an electric current, but only if it is *changing* with time.

Faraday performed an experiment in which one coil wound around an iron ring was connected via a switch to a battery, shown on the left in figure 9. A second coil, shown on the right in figure 9, was wound on the same ring but electrically isolated from the first coil, and was connected to a sensitive galvanometer (device to measure small currents).

Faraday discovered that:

- When the switch was connected, current flowed briefly through the second ring, and then stopped once the current was established and flowing continuously.
- When the switch was disconnected, current again flowed briefly through the second ring, but in the opposite direction.

Figure 7: Michael Faraday by Thomas Phillips oil on canvas, 1841-1842.

Figure 8: Faraday's induction ring (1831). Image credit: Royal Society/Science and Society Picture Library.

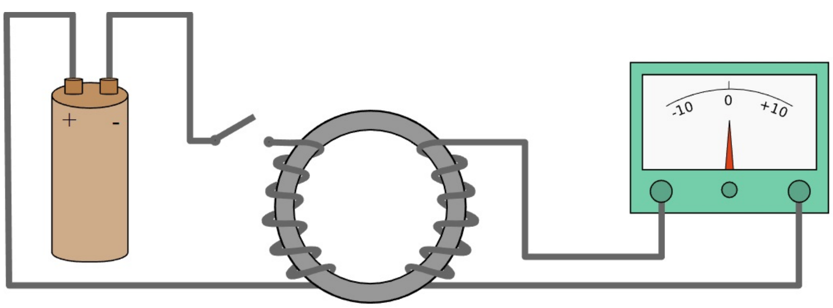


Figure 9: A diagram of Faraday's electromagnetic induction experiment. Image credit: `https://en.wikipedia.org/wiki/File:Faraday_emf_experiment.svg`

Faraday's law

Faraday's law is one of Maxwell's four equations of electromagnetism, and describes how a changing magnetic flux produces an electric field.

Faraday's law:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

where:

- ε is the emf induced around the perimeter of the area through which there is a change in the magnetic flux
- $\Phi = BA\cos\theta$ is the magnetic flux, and
- $\frac{\Delta\Phi_B}{\Delta t}$ is the rate of change in the magnetic flux
- N is the number of turns (if there is more than one turn of wire in the loop)

The magnetic flux through a given area (often a loop) can change in the following ways (illustrated in figure 10):

- the loop moves out of the magnetic field
- the area of the loop changes (e.g. it is squashed or stretched)
- the strength (magnetic flux density, B) of the magnetic field changes
- the angle between the magnetic field lines and the normal to the loop changes due to rotation of the loop (or field).

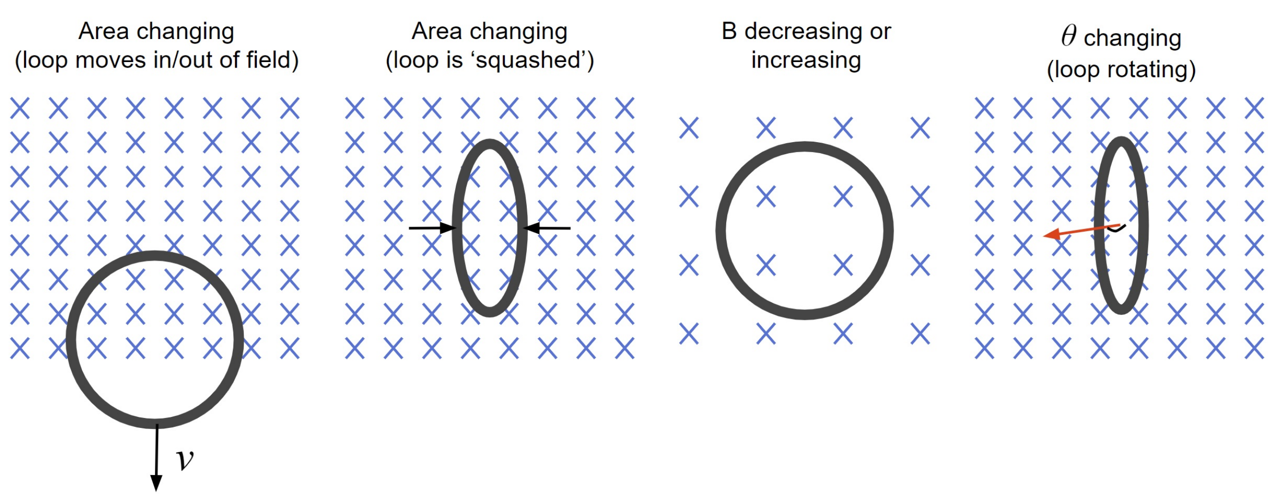


Figure 10: A range of ways the magnetic flux through a loop can change over time.

Example 4.

(a) The conducting loop considered in the previous example (a square with side length 0.1m in a field of $B = 1\text{mT}$) is pulled from the field with a velocity of $v = 5\text{ms}^{-1}$. Calculate the EMF induced in the loop during the time it is being pulled at constant speed.

(b) If the resistance of the loop is $R = 2\Omega$, calculate the induced current that flows in the loop.

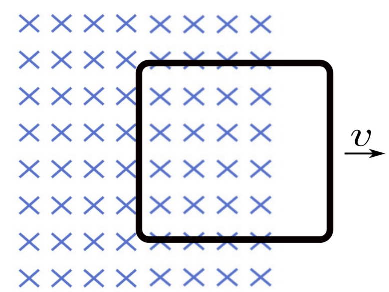


Figure 11: A square ring is pulled out of a magnetic field

Experiment: Electromagnetic Induction

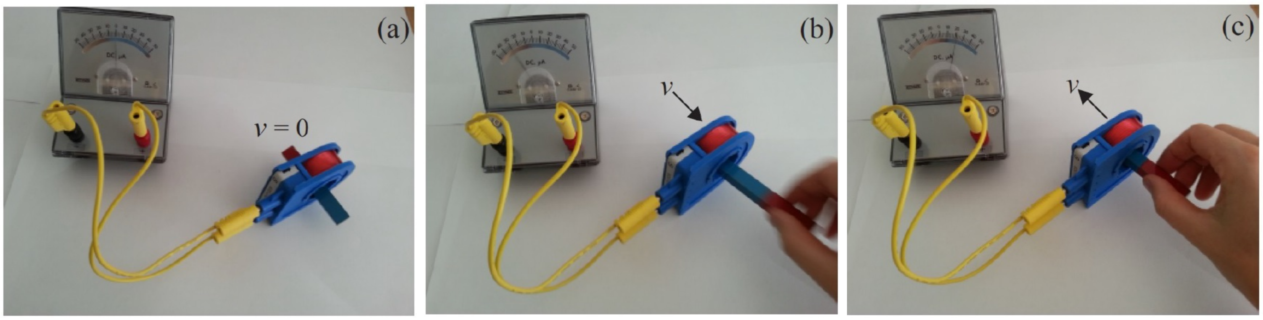


Figure 12: A bar magnet used to change the magnetic flux through a coil of wire, inducing a current which is detected with a galvanometer.

We will perform a similar experiment to Michael Faraday, using a bar magnet to produce a changing magnetic flux through a solenoid connected to a galvanometer.

In your experiment, you will need to determine what happens when:

- The north side of the magnet is pushed into the coil
- The north side is pulled out of the coil
- The south side is pushed into the coil
- The south side is pulled out of the coil
- The magnet is pushed faster or slower into the coil
- More than one magnet (aligned so both north ends are pointing the same way) is pushed into the coil
- The magnet is left stationary inside the coil

Write up your experiment on the next page in a form that would be suitable for you to use as an exam answer. Generally (unless told otherwise) you don't need to write in the form of an 'aim', 'equipment', 'method' and 'results'. Instead, when answering questions about practicals in an exam, it's often useful to consider three questions:

- What did you do? (often best communicated with a labelled diagram)
- What did you measure? or What results did you obtain?
- How did you analyse your results? (This might take the form of describing what you graphed and what information you obtained from the graph, or you might discuss the physical significance of your results - how did they demonstrate the phenomenon you set out to investigate?)

Space for your write up of your experiment to demonstrate electromagnetic induction.

Relationship between motional emf and Faraday's law

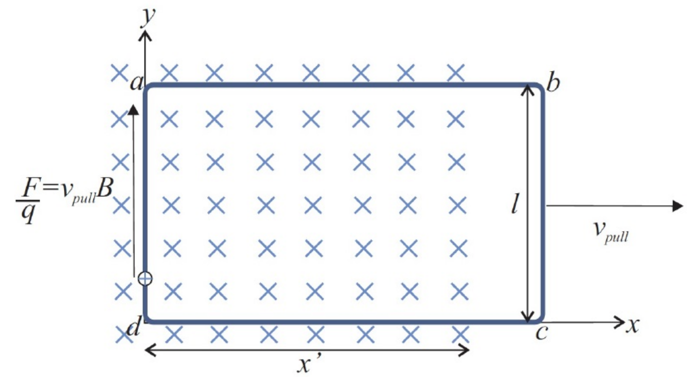


Figure 13: A motional emf appears over a conductor moving through a magnetic field. Here the ends of the conductor have been connected to allow a current to flow around the loop

Earlier, we calculated the motional emf which appears across a conductor moving through a magnetic field, using $F = qvB\sin\theta$ we found this to be:

$$\varepsilon = v_{pull}Bl$$

In this section, we consider the situation when we connect the ends of the moving conductor with a wire. In this case, there is a magnetic flux through the loop consisting of the conductor and connecting wires, equal to:

$$\Phi = BA\cos\theta = Bx'l$$

This flux is changing with time as the loop is pulled out of the field, and the rate of change of flux is equal to:

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{\Delta(Bx'l)}{\Delta t} = v_{pull}Bl$$

Using Faraday's law to calculating the emf generated around the loop, we obtain:

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = v_{pull}Bl$$

This is exactly the same result we obtained earlier, treating the system using using $F = qvB\sin\theta$, and illustrates the equivalence of these two approaches to calculating the emf in reference frames in which there is motion of the conductor relative to the magnetic field.

The value of this observation is that in cases like this we have the option of using the right-hand rule when reasoning about the **direction** in which an induced current flows.

The direction of the emf and induced current

Argument using the right-hand rule

We can use the right hand rule to reason about the direction of the induced current in situations in which the conductor is moving relative to a magnetic field.

If we consider pulling the ring shown in figure 14 out of the magnetic field, then we can use the right-hand rule to argue that conventional current must flow clockwise in the loop as shown.

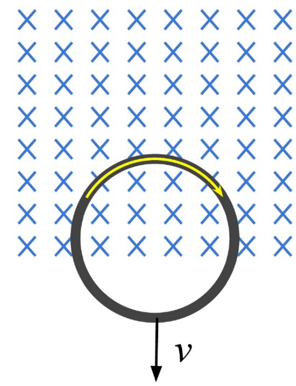


Figure 14: Using the right-hand rule to reason about the direction of the induced current when a ring is pulled from a magnetic field

- electrons in top of loop are moving downwards
- they experience a force to the left using the RH rule
- This generates a clockwise induced current

Argument using conservation of energy

We can make the same argument, but using conservation of energy:

- If there is a current in the loop there is an electrical power $P = \varepsilon I$ produced by the motion of the loop
- This power must be dissipated in the loop as heat
- Energy is conserved, so this power must be provided by the agent pulling the loop
- If the induced current due to the motional emf is flowing clockwise, then there is now a force upwards on the loop (by the RH rule)
- This upward force is in the opposite direction to the motion of the loop, so the agent pulling the loop must do work to continue to pull the loop down
- Energy is therefore conserved as the work done by the agent pulling the loop supplies the electrical energy dissipated in the loop as heat

We can also reason in in reverse as follows:

- If the induced current was instead flowing anticlockwise then it would exert a force downwards on the loop (by the RH rule).
- If this were to occur then loop could be used to *supply* mechanical *and* electrical energy with no external source of energy.
- This would violate the law of conservation of energy, therefore, current *must* flow clockwise so that energy is conserved.

Lenz's law

Lenz's law allows us to use conservation of energy to determine the direction of an induced current.

You will find a variety of wordings for Lenz's law (generally all awkwardly phrased and difficult to remember!).

I choose to use the following definition:

> Lenz's law:
> The induced emf gives rise to a current whose magnetic field opposes the change in the magnetic flux which produced it.

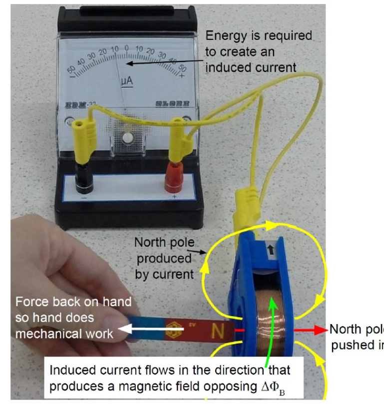


Figure 15: We can use Lenz's law to reason about the direction of the induced current produced by a change in magnetic flux in our earlier experiment on electromagnetic induction.

I suggest a "four-step" approach to finding the direction of an induced current using Lenz's law:

1. Which way is the magnetic field causing the magnetic flux pointing? (e.g. left, right, into the page, out of the page, up, down?)
2. Is the flux in that direction increasing or decreasing with time?
3. Which way would the magnetic field due to the current have to point to oppose this *change*?
4. Use the right-hand grip rule to determine the direction of the induced current.

Let's check that Lenz's law correctly predicts the direction of the induced current in the example we considered on the previous page (figure 16).

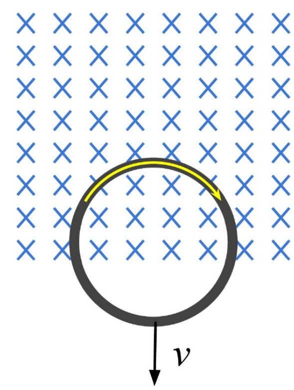


Figure 16: Lenz's law can be used to determine the direction of the induced current in our previous example of a loop pulled out of a magnetic field.

1. Firstly, we note that the direction of the magnetic field is into the page (in other problems this may be more difficult to determine).
2. The magnetic flux into the page through the loop is decreasing as it is pulled out of the field.
3. To oppose this change, the magnetic field due to the induced current will need to point into the page as well.
4. Using the right-hand grip rule, we orientate our curled fingers to point into the loop, and our thumb then points in a clockwise direction around the loop, in agreement with the direction we found using conservation of energy and the right-hand rule on the previous page.

Example 5.

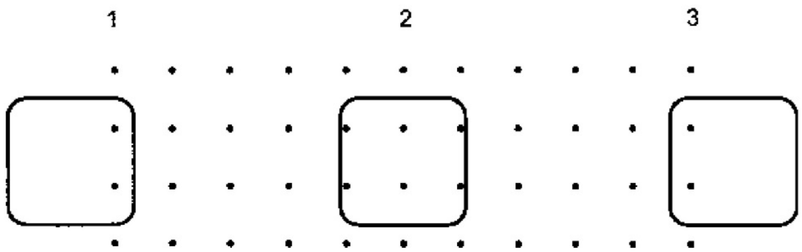


Figure 17: A loop is pulled (1) into a field, (2) through the field, then (3) out of the field.

A loop is pulled through a uniform magnetic field directed out of the page, as shown in figure 17.

In each of the following positions, determine if an induced current flows, and if so, in what direction.

(a) Position 1

(b) Position 2

(c) Position 3

Eddy currents

Eddy currents are simply induced currents due to a changing magnetic flux in a *solid conductor* (rather than a coil or loop of wire). We will look at several practical examples together, such as a magnet falling on a copper bar, relative movement between a magnet and a copper bar, and magnets falling through copper and aluminium tubes. We will also introduce the concept of magnetic braking, and revisit this at the end of the module (where it is specifically referred to in the syllabus).

Figure 18: Disk eddy current brake on 700 Series Shinkansen, a Japanese bullet train. Image credit: By Take-y at the Japanese language Wikipedia, CC BY-SA 3.0, `https://commons.wikimedia.org/w/index.php?curid=5188379`

Magnetic braking

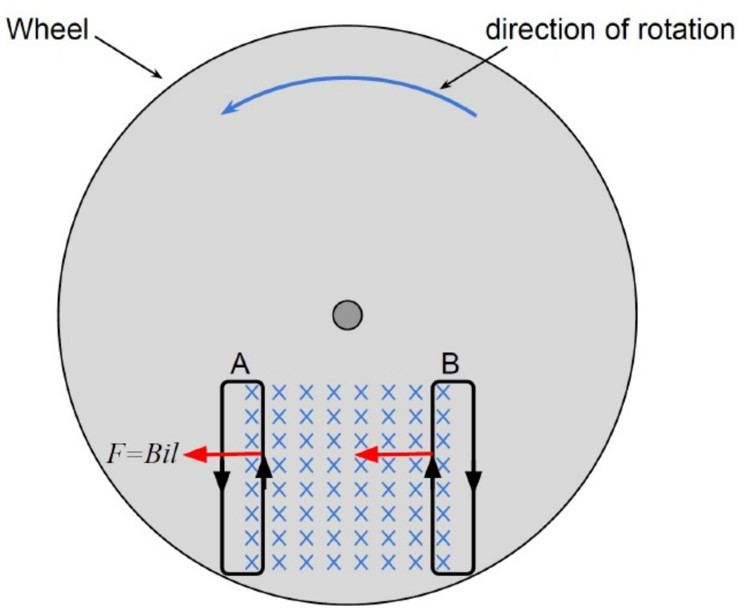


Figure 19: Diagram of the eddy currents (black loops) and forces that act on this currents due to the motor effect (red arrows).

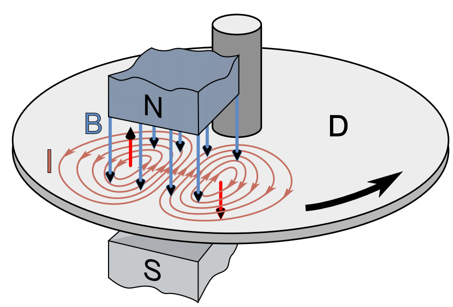


Figure 20: A three-dimensional view of the eddy currents created in EM braking. This diagram illustrated how the magnetic field produced by the eddy currents acts to oppose the rotation of the wheel. This is an alternative way to understand the braking effect (the explanation in the text emphasises the force on the eddy currents due to the "motor effect"). Figure credit: By Chetvorno - Own work, CC0, `https://commons.wikimedia.org/w/index.php?curid=40937881`

Eddy currents are utilised in electromagnetic braking as follows: Consider the part of the wheel (marked 'A') in figure 20 that is about to enter to the field. As the flux into the wheel in that area is increasing with time, an eddy current flows in an anticlockwise direction to oppose the change in magnetic flux (Lenz's law).

An eddy current flows in the opposite direction in the part of the wheel moving out of the field (marked B), as the flux into the wheel in area B is decreasing with time.

The right (upward moving) side of the eddy current A experiences a force F = Bil in the direction opposite to the rotation of the wheel (by the right hand rule), which exerts a torque on the wheel in the opposite direction to its rotation, slowing it down The left (upward moving) side of eddy current B similarly produces a torque in the opposite direction to the rotation of the wheel.

Conservation of energy considerations in EM braking

Another way to reason about EM braking is to note that if eddy currents are induced, then electrical energy is dissipated in the wheel due to I^2R heating. This energy *must* come from somewhere (so that energy is conserved). The kinetic energy of the wheel is transformed first into electrical energy and then into heat in the wheel, and this loss of kinetic energy slows the wheel down.

Transformers

In this section we begin to look at practical applications of electromagnetic induction.

> Transformers are devices which utilise electromagnetic induction to convert AC electricity from one voltage to another.

Electricity is generated at power stations at voltages between 2000V and 27000V[2]. Transformers play an important role in the power transmission as they allow this voltage to increased via "step-up" transformers to around 500kV for transmission, then decreased to lower, safer voltages via "step-down" transformers for use by consumers.

[2] For typical generator technical specifications see, for example, `https://new.siemens.com/global/en/products/energy/power-generation/generators.html`

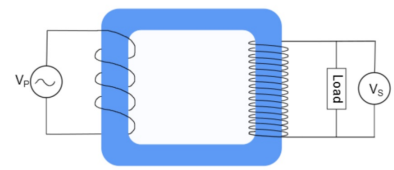


Figure 21: A transformer. The primary coil is connected to an AC voltage and the secondary coil (with a different number of turns to the primary) is connected to a load. Both coils are wound onto the same iron core.

The physics of transformers

Transformers consist of two coils wound around an iron core. The "primary" coil is connected to a source of AC voltage, and the "secondary" coil is connected to a load, as shown in figure 21.

The AC source of emf produces a changing current through the primary coil, producing a changing magnetic field. As both coils are wound on the same iron core, the changing magnetic field due to the primary coil produces a changing magnetic flux through the secondary coils. This changing magnetic flux will induce an emf and, as a result, a current in the secondary coil.

The role of the iron core

The role of the iron core is to greatly increase the magnetic field, and so the magnetic flux, through the coils, compared to that which would be present with an air core.

Iron is used as it is a ferromagnetic material, in which the magnetic moment of individual atoms are locally aligned in regions called "domains". In the absence of an external field, the magnetic field in each small domain is randomly aligned compared to the magnetic field of adjacent domains. When a ferromagnetic material is placed in an external magnetic field, the magnetic field of all the domains tends to align in the same direction as the external field, to an extent that depends on the strength of the external field.

The net effect is that the magnetic field produced by the iron core adds to the external field, producing a much stronger field than would be obtained without the core.

Ideal transformers and the transformer equation

An **ideal** transformer is one in which the flux through each turn on both the primary and the secondary coil is the same (there is **complete flux linkage**). In this case, each coil has the same *rate of change of flux* through each turn, which means that the *emf per turn is the same* in the primary and secondary coils.

The emf per turn over the primary is determined by the source of emf. This means that in an **ideal** transformer

$$\varepsilon_P = \varepsilon_S = -\frac{d\Phi}{dt}$$

where ε_P is the emf per turn in the primary and ε_S is the emf per turn in the secondary. However the emf per turn in the primary is equal to $\varepsilon_P = \frac{V_P}{N_P}$, and similarly the emf in the secondary is $\varepsilon_S = \frac{V_P}{N_P}$, so

> The (ideal) transformer equation is:
>
> $$\frac{V_P}{N_P} = \frac{V_S}{N_S}$$

The syllabus expresses this as $\frac{V_P}{V_S} = \frac{N_P}{N_S}$. Personally I prefer the above definition, as it can be remembered simply as "the emf per turn in the primary equals the emf per turn in the secondary".

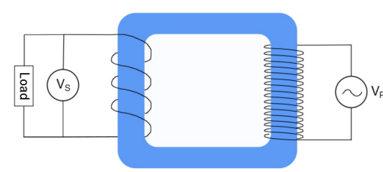


Figure 22: A step-down transformer. The primary coil is connected to an AC voltage and the secondary coil (with less turns than the primary) is connected to a load. Both coils are wound onto the same iron core.

Step-up and Step-down transformers

From the transformer equation, it can be seen that if $N_S > N_P$, then $V_S > V_P$ and we call the transformer a "**step-up**" transformer. This type of transformer is illustrated in figure 21. As mentioned in the introduction it is important to increase the voltage before electricity is transmitted long distances in order to minimise the power lost due to I^2R heating in the transmission lines.

Conversely, if $N_S < N_P$, then $V_S < V_P$ and we call the transformer a "**step-down**" transformer, as shown in figure 22. These are important in reducing the voltage to safe levels for use by consumers. These types of transformers may also be used for other applications which require a reduction in voltage, such as converting 240V to 12V for some types of lighting.

Example 6.

An ikea light contains a transformer that converts 240V to 12V.

(a) Is this a step-up or step-down transformer?

(b) What is the ratio of the number of turns in the primary coil to the number of turns in the secondary coil?

Example 7.

A transformer with 1000 turns on the primary coil must transform 27kV produced by a generator to 500kV for long-distance transmission.

(a) Is this a step-up or step down transformer?

(b) How many turns are required on the secondary coil?

Power transfer in transformers

If a transformer is ideal, then the power delivered to the primary coil by the source of emf is completely transferred to the secondary coil[3], so that

For an ideal transformer:

$$V_P I_P = V_S I_S$$

Example 8.

If the current flowing in the primary coil of an ideal step-up transformer is 200A, calculate the current flowing in the secondary coil if the ratio of turns is $\frac{N_P}{N_S} = \frac{1}{30}$.

[3] Note: In general, AC circuits involve physics that is more complex than we wish to deal with in the HSC. When an AC circuit contains inductors (coils) and capacitors (essentially parallel plates) in addition to resistors, the current will, in general, no longer be "in-phase" with the voltage applied by the AC source of emf, such that the voltage and current are not at their maximum value at the same time.

Energy is only dissipated (or transferred, in the case of transformers) when there is a component of the current that is in-phase with the driving emf. When we deal with energy transfer in transformers in the HSC, we are always assuming that this is true.

Distribution of energy using high-voltage transmission lines

We will consider a numerical example to illustrate why it is more efficient to transmit electricity at high voltages and low currents.// Consider a simplified circuit for electricity distribution where power is produced by a generator at a voltage V_G. It passes through transmission lines, represented by resistors R_T which have a resistance of the order of 2Ω per 1000km. This is very low - but is still a significant source of power loss when large amounts of electricity are transmitted long distances from power stations to the consumer. In particular, we note that the resistance (called the 'impedance' in AC circuits) of the transmission lines is much, much smaller than that of the load, i.e the consumers to whom the electricity is being transmitted , i.e. $R_L >> R_T$.

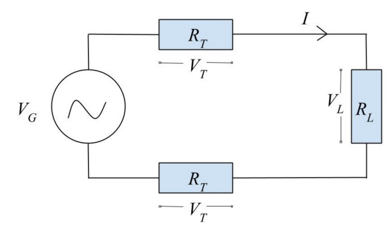


Figure 23: A (very!) simplified circuit diagram of a power distribution network. A source of AC emf from a generator, V_G, transmits electricity through power lines, represented by resistors, to a load, represented by another resistor, R_L where $R_L >> R_T$.

This means that whatever the voltage the power is transmitted at, essentially the entire voltage is dropped over the load. In other words, if we transmit electricity at a voltage of 330kV, for example, the voltage drop across the load is 330kV, but the voltage drop over the transmission line very much smaller than this. We know that the current that passes through the load is the same as that which passes through the transmission line, as these are in series with the load. It therefore makes sense to calculate the power loss in the transmission load using $P = I^2R$, as I and R are known (or can be calculated), rather than using $P = V^2/R$, as V is here the voltage drop over the transmission line (which is *not* 330kV!).

Example 9.

A power station located in the Hunter Valley transmits 100MW of power across 400km of transmission lines (200km there and 200km back), with a resistance of 0.8Ω[4] to a major electricity substation in Sydney.

How much power will be lost (dissipated as heat) in the transmission lines if it is transmitted at a voltage of 330kV?

If the power delivered by the generator is 100MW, the current flowing in the transmission line must be

$$I = \frac{P}{V} = \frac{100 \times 10^6\text{W}}{330 \times 10^3\text{V}} = 303\text{A}$$

The power dissipated in the transmission lines would then be

$$P = I^2R = 303\text{A} \times 0.8\Omega = 73\text{kW}$$

which is a loss of 0.1% of the power originally transmitted.

How much power will be lost if it is transmitted at 20kV?

[4] Value taken from `http://www.egr.unlv.edu/~eebag/TRANSMISSION%20LINES.pdf`

Limitations of the ideal transformer model

Incomplete flux linkage

The ideal transformer model assumes that exactly the same magnetic flux passes through every turn of wire in the primary and secondary coils.

In reality[5], some magnetic field lines do not pass through both coils. The result is that the emf per turn in the secondary coil is not quite equal to the emf per turn in the primary, and so, for a non-ideal transformer, $V_P I_P > V_S I_S$.

5 Information in this section is sourced from the ABB "Transformer Handbook" `https://dotorresg.files.wordpress.com/2011/12/abbtransformerhandbook.pdf`, pg 141-144.

Other losses

- **Eddy currents** induced in the iron core due to the changing magnetic field cause I^2R heating in the core, and so result in a loss of efficiency of the transformer
- **Resistive losses in the winding**s of the coils are simply due to I^2R heating of the coils due to the finite resistance of the wire.
- **Hysteresis** is the "lagging behind" of the magnetisation of the iron core compared to the applied external magnetic field. As a result of hysteresis there is an energy cost to changing the direction of the domains of the core as the direction of the magnetic field changes every half cycle (which produces heat in the core and reduces the efficiency of the transformer)

Strategies to improve transformer efficiency

The transformer core is designed carefully to minimise the I^2R heating due to eddy currents [6].

6 See the ABB transformer handbook, page 159.

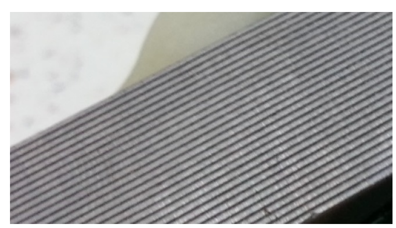


Figure 24: A close up showing a laminated iron core.

- **Laminating** the core, as illustrated in figure 24 involves constructing the core from very thin (approximately 0.18mm to 0.3mm) steel sheets, which are insulated from each other via a very thin ($<$ 4¯m) coating of an inorganic material that is corrosion and temperature resistant and compatible with transformer oil.
- Using **iron alloyed with silicon** at around 3% reduces the conductivity, so reducing eddy currents.
- **Hysteresis** losses due to heating caused by changing the direction of the magnetic domains can be reduced by using "soft" iron with a low carbon content (so that the core does not retain its magnetisation after the field is removed)

- **Other alloys** may be used for special applications. For example, for high frequency transformer cores ferrite materials may be used, which are iron-oxide compounds that have been powdered then 'sintered' into the correct shape. These have a low conductivity to minimise eddy currents but maintain suitable magnetic properties.
- Finally, **Preventing overheating** of the transformer using oil as a coolant and heat dissipating fins on the transformer increases efficiency as it minimises the resistance of the metal windings in the transformer (as the resistance of metal increases with temperature), so reducing the I^2R heating which occurs due to the high currents flowing through the coils.

A practical demonstration of these effects

A complete, laminated iron core.

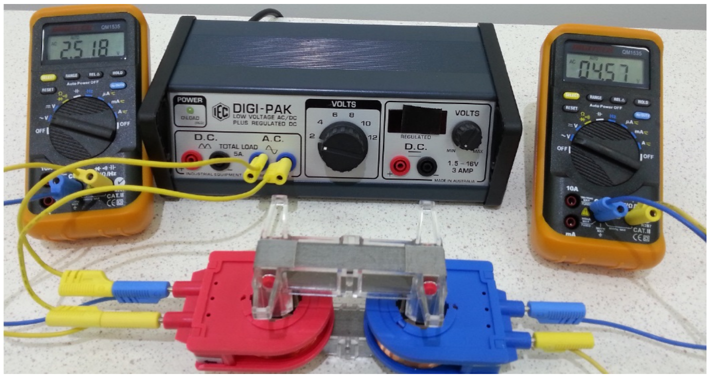


Figure 25: An experimental setup for demonstrating a transformer with a complete, *laminated* iron core (using IEC equipment). The left (primary) coil has 300 turns and has a AC voltage of 2.5V applied across it. The secondary (right) coil has 600 turns an the multimeter is measuring an induced emf of 4.6V across it. This is close to the 5.0V that would be expected from the transformer equation.

Complete but partly un-laminated iron core.

Figure 26: A similar experimental setup as in 25, however part of the iron core is *unlaminated*. The left (primary) coil has 300 turns and has a AC voltage of 2.5V applied across it. The secondary (right) coil has 600 turns an the multimeter is measuring an induced emf of 3.6V across it. This is lower than would be expected from the transformer equation due to increased power loss due to eddy currents.

Incomplete iron core - significant flux leakage.

Figure 27: A similar experimental setup as in 25, however part of the iron core has been removed, so that there is significant flux leakage between the primary and secondary coils. The left (primary) coil has 300 turns and has a AC voltage of 2.5V applied across it. The secondary (right) coil has 600 turns an the multimeter is measuring an induced emf of 1.4V across it. This is much lower than would be expected from the transformer equation as the assumption underlying this equation, that the same magnetic flux passes through both coils, is not met.

Answers

Worked Example 1.

B

Worked Example 2.

$$\varepsilon = vBl = \frac{800}{3.6} \times 5 \times 10^{-5} \times 60 = 0.67\text{V}$$

Worked Example 3.

A ring with radius 0.1m shown in figure 11, is originally fully inside a magnetic field of strength $B = 1\text{mT}$. It is then pulled out with velocity $v = 1\text{ms}^{-1}$.

(a) What is the magnetic flux through the ring when it is fully inside the magnetic field?

The magnetic flux before it is pulled from the field is:

$$\Phi = BA\cos\theta = 1 \times 10^{-3}\text{T} \times 0.1^2 \times \cos 0° = 1.0 \times 10^{-5}\text{Wb}$$

(b) What is the magnetic flux through the ring after it has been completely pulled out of the magnetic field?

The magnetic flux is zero after the loop has been pulled out of the field.

Worked Example 4.

(a) The loop considered in Example 1 is pulled from the field with a velocity of $v = 5\text{ms}^{-1}$. Calculate the magnitude of the EMF induced in the loop.

The time taken for the loop to be pulled from the field is the time taken for the length of the square loop to pass across the edge of the field, given by $t = \frac{d}{v} = \frac{0.1\text{m}}{5\text{ms}^{-1}} = 0.02\text{s}$

$$|\varepsilon| = |\frac{\Delta\Phi_B}{\Delta t}| = |\frac{0 - 1.0 \times 10^{-5}\text{Wb}}{0.02\text{s}}| = 5.0 \times 10^{-4}\text{V}$$

(b) If the resistance of the loop is $R = 2\Omega$, calculate the induced current that flows in the loop.

$$I = \frac{\varepsilon}{R} = \frac{5.0 \times 10^{-4}\text{V}}{2\Omega} = 2.5 \times 10^{-4}\text{A}$$

Worked Example 5.

A loop is pulled through a uniform magnetic field directed out of the page, as shown in figure 17.

In each of the following positions, determine if an induced current flows, and if so, in what direction.

(a) Position 1

Clockwise

(b) Position 2

No induced current

(c) Position 3

Anticlockwise

Worked Example 6.

An ikea light contains a transformer that converts 240V to 12V.

(a) Is this a step-up or step-down transformer?

Step down.

(b) What is the ratio of the number of turns in the primary coil to the number of turns in the secondary coil?

$$\frac{N_P}{N_S} = \frac{240V}{12V} = 20$$

Worked Example 7.

A transformer with 1000 turns on the primary coil must transform 27kV produced by a generator to 500kV for long-distance transmission.

(a) Is this a step-up or step down transformer?

Step up.

(b) How many turns are required on the secondary coil?

$N_S = N_P \frac{V_S}{V_P} = 1000 \times \frac{500}{27} = 18519 \approx 19000$ (2s.f.)

Worked Example 8.

If the current flowing in the primary coil of an ideal step-up transformer is 200A, calculate the current flowing in the secondary coil if the ratio of turns is $\frac{N_P}{N_S} = \frac{1}{30}$.

If $\frac{N_P}{N_S} = \frac{1}{30}$, then $\frac{V_P}{V_S} = \frac{1}{30}$, and $\frac{I_P}{I_S} = 30$. The current is the secondary coil is therefore $I_S = \frac{I_P}{30} = \frac{200\text{A}}{30} = 6.7\text{A}$.