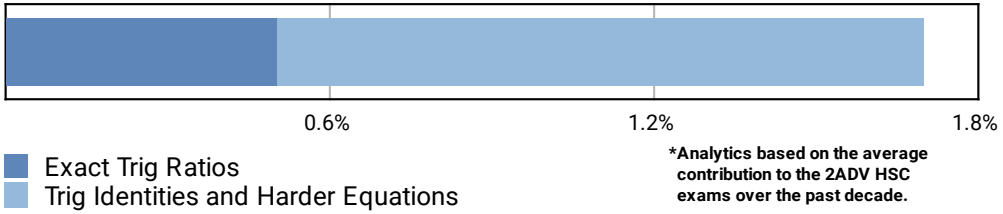
ADV: Trigonometry (Adv), T2 Trig Functions and Identities (Adv)

Trig Identities and Harder Equations (Y11)
Exact Trig Ratios (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 108 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

T2 Trig Functions and Identities



HISTORICAL CONTRIBUTION

- *T2 Trig Functions and Identities* is a small contributor to past Mathematics exams, accounting for an average of 1.7% of past papers.
- This small contribution belies the importance of this area which must be well understood to answer a wide range of questions in other topics such as series, maxima and minima and trig calculus.
- This topic has been split into two sub-topics for analysis purposes: *1-Exact Trig Ratios (0.5%), and 2-Trig Identities and Harder Equations (1.2%).*
- This analysis looks at the sub-topic *Trig Identities and Harder Equations.*

HSC ANALYSIS - What to expect and common pitfalls

- *Trig Identities and Harder Equations* has been examined in a dedicated question in 6 of the last 8 years (most recently in 2021).
- Dedicated HSC questions on this topic area have produced sub-50% mean marks on a majority of occasions.
- While we are cognisant of the small contribution of "dedicated" questions in this area, note that examiners have shown a tendency to test trig identities in a multitude of cross-topic ways.
- Good skills in identifying and manipulating trig identities will reap benefits in tackling some of the harder questions that might appear in the later sections of the advanced HSC paper.

Questions

1. Trigonometry, 2ADV T2 2021 HSC 1 MC

   Which of the following is equivalent to $\sin^2 5x$?

   A. $1+\cos^2 5x$

   B. $1-\cos^2 5x$

   C. $-1+\cos^2 5x$

   D. $-1-\cos^2 5x$

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2. Trigonometry, 2ADV T2 2012 HSC 6 MC

   What are the solutions of $\sqrt{3}\tan x = -1$ for $0 \leq x \leq 2\pi$?

   (A) $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$

   (B) $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

   (C) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

   (D) $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

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3. Trigonometry, 2ADV T2 2017 HSC 7 MC

   Which expression is equivalent to $\tan\theta + \cot\theta$?

   (A) $\operatorname{cosec}\theta + \sec\theta$

   (B) $\sec\theta \operatorname{cosec}\theta$

   (C) 2

   (D) 1

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4. Trigonometry, 2ADV T2 SM-Bank 44 MC

The domain of the function with rule $f(x) = 1 - \sec\left(x + \frac{\pi}{4}\right)$ is

A. $\text{all real } x$

B. $\left\{\frac{(4k-1)\pi}{4}\right\}, (\text{for } k \text{ integer})$

C. $\left\{\frac{(4k+1)\pi}{4}\right\}, (\text{for } k \text{ integer})$

D. $\left\{\frac{(2k-1)\pi}{4}\right\}, (\text{for } k \text{ integer})$

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5. Trigonometry, 2ADV T2 2014 HSC 7 MC

How many solutions of the equation $(\sin x - 1)(\tan x + 2) = 0$ lie between 0 and 2π?

(A) 1

(B) 2

(C) 3

(D) 4

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6. Trigonometry, 2ADV T2 2016 HSC 8 MC

How many solutions does the equation $|\cos(2x)| = 1$ have for $0 \leq x \leq 2\pi$?

(A) 1

(B) 3

(C) 4

(D) 5

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7. Trigonometry, 2ADV T2 2005 HSC 2a

Solve $\cos\theta = \frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 2\pi$. *(2 marks)*

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8. Trigonometry, 2ADV T2 2007 HSC 4a

Solve $\sqrt{2}\sin x = 1$ for $0 \leq x \leq 2\pi$. *(2 marks)*

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9. Trigonometry, 2ADV T2 2009 HSC 1e

Find the exact value of θ such that $2\cos\theta = 1$, where $0 \leq \theta \leq \frac{\pi}{2}$. *(2 marks)*

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10. Trigonometry, 2ADV T2 2015 HSC 12a

Find the solutions of $2\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$. *(2 marks)*

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11. Trigonometry, 2ADV T2 SM-Bank 41

Prove that

$(\sec x + \tan x)(\sec x - \tan x) = 1$. *(2 marks)*

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12. Trigonometry, 2ADV T2 SM-Bank 43

Find the exact value of

$\cot\left(-\frac{5\pi}{6}\right)$. *(2 marks)*

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13. Trigonometry, 2ADV T2 SM-Bank 31

Given $\cot\theta = -\frac{24}{7}$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, find the exact value of

i. $\sec\theta$ *(2 marks)*

ii. $\sin\theta$ *(1 mark)*

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14. Trigonometry, 2ADV T2 2011 HSC 2b

Find the exact values of x such that $2\sin x = -\sqrt{3}$, where $0 \leq x \leq 2\pi$. *(2 marks)*

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15. Trigonometry, 2ADV T2 2016 HSC 11g

Solve $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$. *(2 marks)*

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16. Trigonometry, 2ADV T2 2020 HSC 19

Prove that $\sec\theta - \cos\theta = \sin\theta\tan\theta$. *(2 marks)*

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17. Trigonometry, 2ADV T2 SM-Bank 2

Find all solutions of the equation $2\cos\theta = \sqrt{3}\cot\theta$, for $0 \leq \theta \leq 2\pi$ *(3 marks)*

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18. Trigonometry, 2ADV T2 SM-Bank 32

Express $5\cot^2 x - 2\text{cosec}\, x + 2$ in terms of $\text{cosec}\, x$ and hence solve

$5\cot^2 x - 2\text{cosec}\, x + 2 = 0$ for $0 < x < 2\pi$. *(3 marks)*

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19. Trigonometry, 2ADV T2 SM-Bank 33

Given $\sec\theta = -\frac{37}{12}$ for $0 < \theta < \pi$,

find the exact value of $\operatorname{cosec}\theta$. *(2 marks)*

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20. Trigonometry, 2ADV T2 SM-Bank 34

Solve the equation $\sqrt{3}\sin x = \cos x$ for $-\pi \leq x \leq \pi$. *(2 marks)*

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21. Trigonometry, 2ADV T2 SM-Bank 35

Solve the equation

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } 0 \leq x \leq \pi \text{ } \textit{(2 marks)}$$

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22. Trigonometry, 2ADV T2 SM-Bank 36

Solve the equation $\sin\left(\frac{x}{2}\right) = -\frac{1}{2}$ for $2\pi \leq x \leq 4\pi$ *(2 marks)*

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23. Trigonometry, 2ADV T2 SM-Bank 37

Solve the equation $\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. *(2 marks)*

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24. Trigonometry, 2ADV T2 SM-Bank 38

Solve $2\cos(2x) = -\sqrt{3}$ for x, where $0 \leq x \leq \pi$. *(2 marks)*

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25. Trigonometry, 2ADV T2 SM-Bank 39

Let $f(x) = \sin\left(\frac{2\pi x}{3}\right)$.

Solve the equation $\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$ for $0 \leq x \leq 3$ *(2 marks)*

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26. Trigonometry, 2ADV T2 SM-Bank 42

Prove that

$$\frac{1 - \sin^2 x \cos^2 x}{\sin^2 x} = \cot^2 x + \sin^2 x. \text{ } \textit{(2 marks)}$$

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27. Calculus, 2ADV C4 2010 HSC 5b

i. Prove that $\sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$. *(1 mark)*

ii. Hence prove that $\sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$. *(1 mark)*

iii. Hence, use the identity $\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x} \, dx. \text{ } \textit{(2 marks)}$$

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28. Trigonometry, 2ADV T2 2004 HSC 8a

i. Show that $\cos\theta \tan\theta = \sin\theta$. *(1 mark)*

ii. Hence solve $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$ for $0 \leq \theta \leq 2\pi$. *(2 marks)*

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29. Trigonometry, 2ADV T2 2004 HSC 9a

Consider the geometric series $1 - \tan^2\theta + \tan^4\theta - \ldots$

i. When the limiting sum exists, find its value in simplest form. *(2 marks)*

ii. For what values of θ in the interval

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ does the limiting sum of the series exist? *(2 marks)*

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30. Trigonometry, 2ADV T2 2014 HSC 15a

Find all solutions of $2\sin^2 x + \cos x - 2 = 0$, where $0 \leq x \leq 2\pi$. *(3 marks)*

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31. Trigonometry, 2ADV T2 2008 HSC 6a

Solve $2\sin^2\left(\frac{x}{3}\right) = 1$ for $-\pi \leq x \leq \pi$. *(3 marks)*

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32. Trigonometry, 2ADV T2 2019 HSC 13a

Solve $2\sin x \cos x = \sin x$ for $0 \leq x \leq 2\pi$. *(3 marks)*

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33. Trigonometry, 2ADV T2 SM-Bank 40

Let $(\tan\theta - 1)(\sin\theta - \sqrt{3}\cos\theta)(\sin\theta + \sqrt{3}\cos\theta) = 0$.

i. State all possible values of $\tan\theta$. *(1 mark)*

ii. Hence, find all possible solutions for $(\tan\theta - 1)(\sin^2\theta - 3\cos^2\theta) = 0$, where $0 \leq \theta \leq \pi$. *(2 marks)*

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Worked Solutions

1. Trigonometry, 2ADV T2 2021 HSC 1 MC

Using the identity:

$$\sin^2 5x + \cos^2 5x = 1$$
$$\sin^2 5x = 1 - \cos^2 5x$$

$\Rightarrow B$

2. Trigonometry, 2ADV T2 2012 HSC 6 MC

$$\sqrt{3}\tan x = -1$$
$$\tan x = -\frac{1}{\sqrt{3}}$$

When $\tan x = \frac{1}{\sqrt{3}}$, $x = \frac{\pi}{6}$

Since $\tan x$ **is negative in** $2^{\text{nd}}/4^{\text{th}}$ **quadrant**

$$\therefore x = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6},\ \ldots$$
$$= \frac{5\pi}{6},\ \frac{11\pi}{6}$$

$\Rightarrow D$

3. Trigonometry, 2ADV T2 2017 HSC 7 MC

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta\sin\theta}$$
$$= \sec\theta\,\text{cosec}\,\theta$$

$\Rightarrow B$

Worked Solutions

4. Trigonometry, 2ADV T2 SM-Bank 44 MC

$y = \sec(x) = \dfrac{1}{\cos(x)}$ has asymptotes when

$x = -\dfrac{\pi}{2}, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

$\Rightarrow y = \sec\left(x + \dfrac{\pi}{4}\right)$ has asymptotes at when

$x = \dfrac{-3\pi}{4}, \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \ldots$

$\therefore$ Domain: $\left\{\dfrac{(4k+1)\pi}{4}\right\}$, (for k integer)

$\Rightarrow C$

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5. Trigonometry, 2ADV T2 2014 HSC 7 MC

♦♦♦ *Mean mark 25%, making it the toughest MC question in the 2014 exam.*
COMMENT: Note that the "2 solutions" answer relies on the sum of an infinity of zeros not equalling zero. This concept created unintended difficulty in this question.

When $(\sin x - 1)(\tan x + 2) = 0$

$(\sin x - 1) = 0$ or $\tan x + 2 = 0$

If $\sin x - 1 = 0$

$\sin x = 1$

$x = \dfrac{\pi}{2}, \quad 0 < x < 2\pi$

If $\tan x + 2 = 0$

$\tan x = -2$

$\Rightarrow$ Note that since $\tan \dfrac{\pi}{2}$ is undefined, there are only 2 solutions when $\tan x = -2$ (which occurs in the 1st and 4th quadrants).

$\therefore$ 2 solutions

$\Rightarrow B$

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6. Trigonometry, 2ADV T2 2016 HSC 8 MC

♦♦♦ *Mean mark 23%.*

$|\cos(2x)| = 1$

$\cos(2x) = \pm 1$

When $\cos(2x) = 1$

$2x = 0, 2\pi, 4\pi, \ldots$

$\therefore x = 0, \pi, 2\pi, \ldots$

When $\cos(2x) = -1$

$2x = \pi, 3\pi, 5\pi, \ldots$

$\therefore x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \ldots$

$\therefore x = 0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$ for $0 \leq x \leq 2\pi$

$\Rightarrow D$

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7. Trigonometry, 2ADV T2 2005 HSC 2a

$\cos\theta = \dfrac{1}{\sqrt{2}}, \quad 0 \leq \theta \leq 2\pi$

Since $\cos \dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$, and cos is positive in 1st/4th quadrants

$\theta = \dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}$

$= \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

8. Trigonometry, 2ADV T2 2007 HSC 4a

$$\sqrt{2}\sin x = 1 \qquad 0 \le x \le 2\pi$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \text{ and sin is positive in}$$

1st/2nd quadrants,

$$\therefore x = \frac{\pi}{4}\,,\ \pi - \frac{\pi}{4}$$

$$= \frac{\pi}{4}\,,\ \frac{3\pi}{4}$$

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9. Trigonometry, 2ADV T2 2009 HSC 1e

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\therefore\ \theta = \frac{\pi}{3}, \quad 0 \le \theta \le \frac{\pi}{2}$$

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10. Trigonometry, 2ADV T2 2015 HSC 12a

$$2\sin\theta = 1, \quad 0 \le \theta \le 2\pi$$

$$\sin\theta = \frac{1}{2}$$

$$\Rightarrow\ \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \text{ and sin is positive}$$

in the 1st/2nd quadrants

$$\therefore \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

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11. Trigonometry, 2ADV T2 SM-Bank 41

$$\text{LHS} = (\sec x + \tan x)(\sec x - \tan x)$$

$$= \sec^2 x - \tan^2 x$$

$$= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{1 - \sin^2 x}{\cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x}$$

$$= 1$$

$$= \text{RHS}$$

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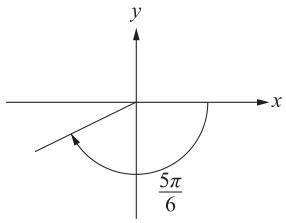
12. Trigonometry, 2ADV T2 SM-Bank 43



$$\cot\left(-\frac{5\pi}{6}\right) = \frac{1}{\tan\left(-\frac{5\pi}{6}\right)}$$

$$= \frac{1}{\tan\left(\frac{\pi}{6}\right)}$$

$$= \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

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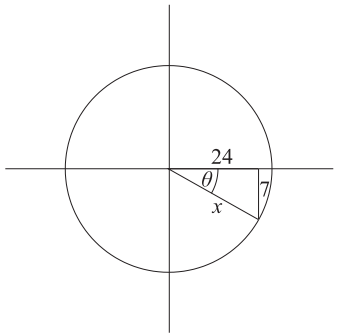
13. Trigonometry, 2ADV T2 SM-Bank 31

i. $\cot\theta = -\frac{24}{7} \Rightarrow \tan\theta = -\frac{7}{24}$

Graphically, given $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$**:**



$x = \sqrt{24^2 + 7^2} = 25$

$$\sec\theta = \frac{1}{\cos\theta}$$
$$= \frac{1}{\frac{24}{25}}$$
$$= \frac{25}{24}$$

ii. $\sin\theta = -\frac{7}{25}$

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14. Trigonometry, 2ADV T2 2011 HSC 2b

$2\sin x = -\sqrt{3}$ **where** $0 \le x \le 2\pi$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Since $\sin x$ **is negative in** 3^{rd} / 4^{th} **quadrants**

$$x = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$$
$$= \frac{4\pi}{3},\ \frac{5\pi}{3} \text{ radians}$$

MARKER'S COMMENT: Better responses found the reference angle and then identified the correct quadrants, as shown clearly in the worked solution.

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15. Trigonometry, 2ADV T2 2016 HSC 11g

$\sin\frac{x}{2} = \frac{1}{2}$ **for** $0 \le x \le 2\pi$

$$\Rightarrow \text{Base angle} = \frac{\pi}{6},$$
$$\frac{x}{2} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, \ldots$$
$$= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \ldots$$

$\therefore x = \frac{\pi}{3},\ \frac{5\pi}{3}$ **for** $0 \le x \le 2\pi$

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16. Trigonometry, 2ADV T2 2020 HSC 19

$$\text{LHS} = \frac{1}{\cos\theta} - \cos\theta$$
$$= \frac{1 - \cos^2\theta}{\cos\theta}$$
$$= \frac{\sin^2\theta}{\cos\theta}$$
$$= \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$$
$$= \sin\theta\tan\theta \text{ ... as required}$$

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17. Trigonometry, 2ADV T2 SM-Bank 2

$$2\cos\theta = \sqrt{3}\cot\theta$$

$$2\cos\theta - \sqrt{3}\cot\theta = 0$$

$$2\cos\theta - \sqrt{3}\frac{\cos\theta}{\sin\theta} = 0$$

$$\left(2 - \frac{\sqrt{3}}{\sin\theta}\right)\cos\theta = 0$$

If $\cos\theta = 0$,

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

If $2 - \frac{\sqrt{3}}{\sin\theta} = 0 \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

18. Trigonometry, 2ADV T2 SM-Bank 32

$$\begin{aligned}\cot^2 x &= \frac{\cos^2 x}{\sin^2 x} \\ &= \frac{1 - \sin^2 x}{\sin^2 x} \\ &= \operatorname{cosec}^2 x - 1\end{aligned}$$

$$\begin{aligned}5\cot^2 x - 2\operatorname{cosec} x + 2 &= 0 \\ 5(\operatorname{cosec}^2 x - 1) - 2\operatorname{cosec} x + 2 &= 0 \\ 5\operatorname{cosec}^2 x - 2\operatorname{cosec} x - 3 &= 0 \\ (5\operatorname{cosec} x + 3)(\operatorname{cosec} x - 1) &= 0\end{aligned}$$

$$\operatorname{cosec} x = -\frac{3}{5} \qquad \operatorname{cosec} x = 1$$

$$\sin x = -\frac{5}{3} \qquad \sin x = 1$$

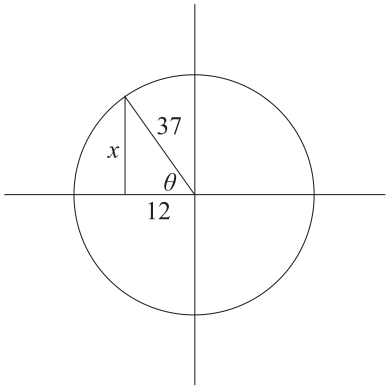
$$\text{(no solution)} \qquad x = \frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{2}$$

19. Trigonometry, 2ADV T2 SM-Bank 33

$$\sec\theta = -\frac{37}{12} \Rightarrow \cos\theta = -\frac{12}{37}$$

Graphically:



$$x = \sqrt{37^2 - 12^2} = 35$$

$$\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$
$$= \frac{1}{\frac{35}{37}}$$
$$= \frac{37}{35}$$

20. Trigonometry, 2ADV T2 SM-Bank 34

Divide both sides by $\cos x$:

$$\sqrt{3}\sin x = \cos x$$
$$\sqrt{3}\tan x = 1$$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \text{Base angle} = \frac{\pi}{6}$$
$$\therefore x = \frac{\pi}{6} \text{ or } -\frac{5\pi}{6}, \quad (-\pi \le x \le \pi)$$

MARKER'S COMMENT: Many students who found the base angle correctly could not solve within the restrictions.

21. Trigonometry, 2ADV T2 SM-Bank 35

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2}$$
$$\Rightarrow \text{Base angle is } \frac{\pi}{6}$$
$$\left(2x + \frac{\pi}{3}\right) = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots$$
$$2x = -\frac{\pi}{6}, \frac{\pi}{2}, \frac{11\pi}{6}, \frac{15\pi}{6}, \ldots$$
$$x = -\frac{\pi}{12}, \frac{\pi}{4}, \frac{11\pi}{12}, \frac{15\pi}{12}, \ldots$$
$$\therefore x = \frac{\pi}{4}, \frac{11\pi}{12} \quad (0 \le x \le \pi)$$

22. Trigonometry, 2ADV T2 SM-Bank 36

$$\frac{x}{2} = \frac{\pi}{6} + \pi, 2\pi - \frac{\pi}{6}, 2\pi + \left(\frac{\pi}{6} + \pi\right), \ldots$$
$$= \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \ldots$$
$$\therefore x = \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{19\pi}{3}, \ldots$$

Given $2\pi \le x \le 4\pi$

$$\therefore x = \frac{7\pi}{3}, \frac{11\pi}{3}$$

23. Trigonometry, 2ADV T2 SM-Bank 37

$$\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$$
$$\Rightarrow \text{Base angle } = \frac{\pi}{3}$$
$$\frac{3x}{2} = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$$
$$\therefore x = \frac{-2\pi}{9}, \frac{2\pi}{9}, \frac{10\pi}{9}$$
$$= \frac{-2\pi}{9}, \frac{2\pi}{9} \quad \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$$

24. Trigonometry, 2ADV T2 SM-Bank 38

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

$$2x = \frac{5\pi}{6}, 2\pi - \frac{5\pi}{6}, 2\pi + \frac{5\pi}{6}$$

$$= \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \ldots$$

$$\therefore x = \frac{5\pi}{12}, \frac{7\pi}{12} \quad (0 \leq x \leq \pi)$$

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25. Trigonometry, 2ADV T2 SM-Bank 39

$$\sin\left(\frac{2\pi x}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \text{Base angle} = \frac{\pi}{3}$$

$$\frac{2\pi x}{3} = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \ldots$$

$$\therefore x = 2 \text{ or } \frac{5}{2}, \quad (0 \leq x \leq 3)$$

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26. Trigonometry, 2ADV T2 SM-Bank 42

$$\text{RHS} = \frac{\cos^2 x}{\sin^2 x} + \sin^2 x$$

$$= \frac{\cos^2 x + \sin^4 x}{\sin^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x(1 - \cos^2 x)}{\sin^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \frac{1 - \sin^2 x \cos^2 x}{\sin^2 x}$$

$$= \text{LHS}$$

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27. Calculus, 2ADV C4 2010 HSC 5b

i. Need to prove

$$\sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$$

$$\text{LHS} = \sec^2 x + \sec x \tan x$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$$

$$= \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$= \frac{1 + \sin x}{\cos^2 x}$$

$$= \text{RHS} \quad \ldots \text{as required}$$

ii. Need to prove

$$\sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$$

♦♦ *Mean mark 31%.*

$$\text{i.e. } \frac{1 + \sin x}{\cos^2 x} = \frac{1}{1 - \sin x} \quad \text{(part (i))}$$

$$\text{LHS} = \frac{1 + \sin x}{\cos^2 x}$$

$$= \frac{1 + \sin x}{1 - \sin^2 x}$$

$$= \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)}$$

$$= \frac{1}{1 - \sin x} \quad \ldots \text{as required}$$

iii. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{1}{1 - \sin x}\, dx$

$$= \int_0^{\frac{\pi}{4}} \left(\sec^2 x + \sec x \tan x\right) dx$$

♦ *Mean mark 37%.*

$$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{4}}$$

$$= \left[\left(\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\right) - (\tan 0 + \sec 0)\right]$$

$$= \left[\left(1+\frac{1}{\cos\left(\frac{\pi}{4}\right)}\right) - \left(0+\frac{1}{\cos 0}\right)\right]$$
$$= 1+\sqrt{2}\ -1$$
$$= \sqrt{2}$$

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28. Trigonometry, 2ADV T2 2004 HSC 8a

i. Prove $\cos\theta\tan\theta = \sin\theta$

$$\text{LHS} = \cos\theta\tan\theta$$
$$= \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$= \sin\theta$$
$$= \text{RHS}$$

ii. $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\theta$

$$\therefore 8\sin\theta(\sin\theta) = \operatorname{cosec}\theta, \quad \text{(part (i))}$$
$$8\sin^2\theta = \frac{1}{\sin\theta}$$
$$8\sin^3\theta = 1$$
$$\sin^3\theta = \frac{1}{8}$$
$$\sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}. \quad (\text{for } 0 \le \theta \le 2\pi)$$

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29. Trigonometry, 2ADV T2 2004 HSC 9a

i. $1 - \tan^2\theta + \tan^4\theta - \ldots$

$$\Rightarrow \text{GP where } a = 1,\ r = \frac{T_2}{T_1} = -\tan^2\theta$$

$$\therefore S_\infty = \frac{1}{1-\left(-\tan^2\theta\right)}$$
$$= \frac{1}{1+\tan^2\theta}$$
$$= \frac{1}{\sec^2\theta}$$
$$= \cos^2\theta$$

ii. Find θ such that $|r| < 1$

$$\left|-\tan^2\theta\right| < 1$$
$$\tan^2\theta < 1$$
$$-1 < \tan\theta < 1$$
$$\therefore -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

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30. Trigonometry, 2ADV T2 2014 HSC 15a

♦ *Mean mark 42%*

$$2\sin^2 x + \cos x - 2 = 0$$
$$2(1 - \cos^2 x) + \cos x - 2 = 0$$
$$2 - 2\cos^2 x + \cos x - 2 = 0$$
$$-2\cos^2 x + \cos x = 0$$
$$\cos x(-2\cos x + 1) = 0$$

$\therefore -2\cos x + 1 = 0$ or $\cos x = 0$

$2\cos x = 1$ $\qquad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \frac{1}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

Since cos is positive in 1^{st} / 4^{th} quadrants,

$$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$
$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

$\therefore x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$ for $0 \le x \le 2\pi$

31. Trigonometry, 2ADV T2 2008 HSC 6a

♦♦ *Although exact data not available, markers specifically mentioned this question was poorly answered.*

$2\sin^2\left(\frac{x}{3}\right) = 1$ for $-\pi \le x \le \pi$

$\sin^2\left(\frac{x}{3}\right) = \frac{1}{2}$

$\sin\left(\frac{x}{3}\right) = \pm\frac{1}{\sqrt{2}}$

MARKER'S COMMENT: Many students had problems adjusting their answer to the given domain, especially when dealing with negative angles.

When $\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$

$\frac{x}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$

$x = \frac{3\pi}{4}, \frac{9\pi}{4}$

When $\sin\left(\frac{x}{3}\right) = -\frac{1}{\sqrt{2}}$

$\frac{x}{3} = -\frac{\pi}{4}, -\frac{3\pi}{4}$

$x = -\frac{3\pi}{4}, -\frac{9\pi}{4}$

$\therefore x = -\frac{3\pi}{4}$ or $\frac{3\pi}{4}$ for $-\pi \le x \le \pi$

32. Trigonometry, 2ADV T2 2019 HSC 13a

♦ *Mean mark 49%.*

$$2\sin x\cos x - \sin x = 0$$
$$\sin x(2\cos x - 1) = 0$$
$$\sin x = 0$$
$$\Rightarrow x = 0, \pi, 2\pi$$
$$\cos x = \frac{1}{2}$$
$$\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

33. Trigonometry, 2ADV T2 SM-Bank 40

i. $(\tan\theta - 1)(\sin\theta - \sqrt{3}\cos\theta)(\sin\theta + \sqrt{3}\cos\theta) = 0$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = 0$

$\sin\theta = \sqrt{3}\cos\theta$

$\tan\theta = \sqrt{3}$

$\Rightarrow \sin\theta + \sqrt{3}\cos\theta = 0$

$\sin\theta = -\sqrt{3}\cos\theta$

$\tan\theta = -\sqrt{3}$

$\therefore \tan\theta = 1 \text{ or } \tan\theta = \pm\sqrt{3}$

♦ *Mean mark 42%.*

ii. $(\tan\theta - 1)(\sin^2\theta - 3\cos^2\theta) = 0$

Using part a:

$(\tan\theta - 1)(\sin\theta - \sqrt{3}\cos\theta)(\sin\theta + \sqrt{3}\cos\theta) = 0$

$\Rightarrow \tan\theta = 1 \quad \text{or} \quad \tan\theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{4} \qquad \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore \theta = \frac{\pi}{4}, \frac{\pi}{3} \text{ or } \frac{2\pi}{3} \quad (0 \leq \theta \leq \pi)$

♦ *Mean mark 42%.*

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