Answers

Answers are not provided for certain questions of the type ‘show that’ or ‘prove that’. Please see worked solutions in these cases for a model.

Chapter 1

Exercise 1A

1 a 850, 1000, 1150, 1300, 1450, 1600, 1750, 1900, 2050, 2200, 2350, 2500, . . .
b 9 months

2 a 36, 46, 56, 66 **b** 24, 48, 96, 192
c 26, 22, 18, 14 **d** 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$
e 1, −1, 1, −1 **f** 16, 25, 36, 49
g $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$ **h** −2, 1 $-\frac{1}{2}$, $\frac{1}{4}$

3 a 3, 8, 13, 18 **b** 5, 25, 125, 625
c 4, 2, 0, −2 **d** 70, 700, 7000, 70 000
e 1, 8, 27, 64 **f** 2, 6, 12, 20
g −1, 1, 1, −1 **h** −3, 9, −27, 81

4 a 11, 61, 111, 161 **b** 15, 12, 9, 6
c 5, 10, 20, 40 **d** −100, −20, −4, $-\frac{4}{5}$

5 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62
a 5 **b** 4 **c** 52 **d** 7th term
e Yes; 17th term.
f No; they all end in 2 or 7.

6 $\frac{3}{4}$, $1\frac{1}{2}$, 3, 6, 12, 24, 48, 96, 192, 384, 768, 1536
a 10 **b** 3
c 384 **d** 9th term
e Yes; 8th term. **f** No

7 a 13, 14, 15, 16, 17. Add 1.
b 9, 14, 19, 24, 29. Add 5.
c 10, 5, 0, −5, −10. Subtract 5.
d 6, 12, 24, 48, 96. Multiply by 2.
e −7, 7, −7, 7, −7. Multiply by −1.
f 40, 20, 10, 5, $2\frac{1}{2}$. Divide by 2.

8 a $40 = T_{13}$
c $100 = T_{33}$, 200 is not a term, $1000 = T_{333}$.

9 a $44 = T_5$, 200 and 306 are not terms.
b 40 is not a term, $72 = T_6$, $200 = T_{10}$.
c $8 = T_3$, 96 is not a term, $128 = T_7$.

10 a The 9 terms T_1 to T_9 are less than 100.
b $T_6 = 64$

11 a 52 **b** $T_{21} = 103$

12 a 5, 17, 29, 41 **b** 12, 2, −8, −18
c 20, 10, 5, $2\frac{1}{2}$ **d** −1, −1, 1, −1

13 a $T_n = T_{n-1} + 5$ **b** $T_n = 2T_{n-1}$
c $T_n = T_{n-1} - 7$ **d** $T_n = -T_{n-1}$

14 a 1, 0, −1, 0, T_n where n is even.
b 0, −1, 0, 1, T_n where n is odd.
c −1, 1, −1, 1. No terms are zero.
d 0, 0, 0, 0. All terms are zero.

15 a $28 = T_7$, $70 = T_{10}$ **b** 5 terms

16 a $1\dfrac{1}{2} = T_4$, $96 = T_{10}$ **b** $T_7 = 12$

17 a $y = 10x - 4$ **b** $y = 2^{x-1} \times 3$
c $y = 42 - 4x$ **d** $y = 48 \times 2^{-x}$
e Here $T_n = (-1)^n$, but there is no curve and no real-valued function.
f $y = x^2$ **g** $y = \dfrac{x}{x + 1}$
h Here $T_n = (-2)^{5-n}$, but there is no curve and no real-valued function.

18 a $\frac{4}{5}$, $\frac{n}{n + 1}$ **b** $\frac{1}{30} = T_5$

19 a $0.9 = T_{10}$, $0.99 = T_{100}$
b $n^2 : (n^2 - 1)$ **c** $\dfrac{1}{n}$

20 a 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .
b 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, . . .
c The sum of two odd integers is even, and the sum of an even and an odd integer is odd.
d The first is 2, 4, 6, 10, 16, . . . , which is $2F_{n+1}$. The second is 0, 2, 2, 4, 6, . . . , which is $2F_{n-1}$.

22 a The 20th number is 10, and −20 is the 41st number on the list.
b Start by writing down the successive diagonals 1, 2, $\frac{1}{2}$, 3, $\frac{2}{2}$, $\frac{1}{3}$, 4, $\frac{3}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, . . . Then remove every fraction that can be cancelled because it has previously been listed.
c The number x is not on the list because it differs from the nth number on the list at the nth decimal place.

Exercise 1B

1 a 18, 23, 28 **b** 5, −5, −15 **c** 9, $10\frac{1}{2}$, 12

2 a 3, 5, 7, 9 **b** 7, 3, −1, −5
c 30, 19, 8, −3 **d** −9, −5, −1, 3
e $3\frac{1}{2}$, $1\frac{1}{2}$, $-\frac{1}{2}$, $-2\frac{1}{2}$ **f** 0.9, 1.6, 2.3, 3

3 a AP: $a = 3, d = 4$ **b** AP: $a = 11, d = -4$
c AP: $a = 23, d = 11$ **d** AP: $a = -12, d = 5$
e not an AP **f** not an AP
g AP: $a = 8, d = -10$ **h** AP: $a = -17, d = 17$
i AP: $a = 10, d = -2\frac{1}{2}$

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4 a 67 **b** -55 **c** $50\frac{1}{2}$

5 a $T_n = 4n - 3$
b $T_n = 107 - 7n$
c $T_n = -19 + 6n$

6 a $a = 6$ $d = 10$
b 86, 206, 996
c $T_n = 10n - 4$

7 a $d = 3, T_n = 5 + 3n$
b $d = -6, T_n = 27 - 6n$
c not an AP
d $d = 4, T_n = 4n - 7$
e $d = 1\frac{1}{4}, T_n = \frac{1}{4}(2 + 5n)$
f $d = -17, T_n = 29 - 17n$
g $d = \sqrt{2}, T_n = n\sqrt{2}$
h not an AP
i $d = 3\frac{1}{2}, T_n = \frac{1}{2}(7n - 12) = \frac{7}{2}n - 6$

8 a $T_n = 170 - 5n$ **b** 26 terms
c $T_{35} = -5$

9 a $T_n = 23 - 3n, T_8 = -1$
b $T_n = 85 - 3n, T_{29} = -2$
c $T_n = 25 - \frac{1}{2}n, T_{51} = -\frac{1}{2}$

10 a 11 terms **b** 34 terms **c** 16 terms
d 13 terms **e** 9 terms **f** 667 terms

11 a 11, 15, 19, 23, $a = 11, d = 4$
b $T_{50} + T_{25} = 314, T_{50} - T_{25} = 100$
d $815 = T_{202}$
e $T_{248} = 999, T_{249} = 1003$
f $T_{49} = 203, \ldots, T_{73} = 299$ lie between 200 and 300, making 25 terms.

12 a i $T_n = 8n$
ii $T_{63} = 504, T_{106} = 848$
iii 44 terms
b $T_{91} = 1001, T_{181} = 1991$, 91 terms
c $T_{115} = 805, T_{285} = 1995$, 171 terms

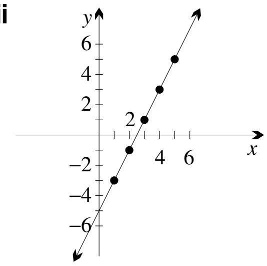
13 a $d = 3$; 7, 10, 13, 16
b $d = 8; T_{20} = 180$
c $d = -2; T_{100} = -166$

14 a \$500, \$800, \$1100, \$1400, . . .
b \$4700
c cost $= 200 + 300n$
d 32

15 a 180, 200, 220, . . . **b** 400 km
c length $= 160 + 20n$ **d** 19 months

16 a 2120, 2240, 2360, 2480
b $A_n = 2000 + 120n, A_{12} = 3440$
c 34 years

17 a 9, 6, 3, 0, -3, . . . and $T_n = 12 - 3n$
b i $T_n = 2n - 5, f(x) = 2x - 5$
ii



18 a $d = 4, x = 1$ **b** $d = 6x, x = \frac{1}{3}$

19 a $d = \log_3 2, T_n = n\log_3 2$
b $d = -\log_a 3, T_n = \log_a 2 + (4 - n)\log_a 3$
c $d = x + 4y, T_n = nx + (4n - 7)y$
d $d = -4 + 7\sqrt{5}, T_n = 9 - 4n + (7n - 13)\sqrt{5}$
e $d = -1.88, T_n = 3.24 - 1.88n$
f $d = -\log_a x, T_n = \log_a 3 + (3 - n)\log_a x$

20 The 13 terms $T_{28} = 19, \ldots, T_{40} = -17$ have squares less than 400.

21 a $a = m + b, d = m$
b gradient $= d$, y-intercept $= m - a$

22 a $a = \lambda a_1 + \mu a_2, d = \lambda d_1 + \mu d_2$
b $A(1,0)$ is 1, 1, 1, . . ., $A(0,1)$ is 0, 1, 2 . . ., $A(a,d) = aA(1,0) + dA(0,1)$.

Exercise 1C

1 a 8, 16, 32 **b** 3, 1, $\frac{1}{3}$
c $-56, -112, -224$ **d** $-20, -4, -\frac{4}{5}$
e $-24, 48, -96$ **f** $200, -400, 800$
g $-5, 5, -5$ **h** $1, -\frac{1}{10}, \frac{1}{100}$
i 40, 400, 4000

2 a 12, 24, 48, 96 **b** $5, -10, 20, -40$
c $18, 6, 2, \frac{2}{3}$ **d** $18, -6, 2, -\frac{2}{3}$
e $6, -3, 1\frac{1}{2}, -\frac{3}{4}$ **f** $-7, 7, -7, 7$

3 a GP: $a = 4, r = 2$
b GP: $a = 16, r = \frac{1}{2}$
c not a GP
d GP: $a = -1000, r = \frac{1}{10}$
e GP: $a = -80, r = -\frac{1}{2}$
f GP: $a = 29, r = 1$
g not a GP
h GP: $a = -14, r = -1$
i GP: $a = 6, r = \frac{1}{6}$

4 a 40 **b** $\frac{3}{10}$ **c** -56
d -8 **e** -88 **f** 120

5 a 3^{n-1} **b** $5 \times 7^{n-1}$ **c** $8 \times \left(-\frac{1}{3}\right)^{n-1}$

6 a $a = 7, r = 2$
b $T_6 = 224, T_{50} = 7 \times 2^{49}$
c $T_n = 7 \times 2^{n-1}$

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7 a $a = 10, r = -3$
  b $T_6 = -2430, T_{25} = 10 \times (-3)^{24} = 10 \times 3^{24}$
  c $T_n = 10 \times (-3)^{n-1}$

8 a $T_n = 10 \times 2^{n-1}, T_6 = 320$
  b $T_n = 180 \times \left(\frac{1}{3}\right)^{n-1}, T_6 = \frac{20}{27}$
  c not a GP
  d not a GP
  e $T_n = \frac{3}{4} \times 4^{n-1}, T_6 = 768$
  f $T_n = -48 \times \left(\frac{1}{2}\right)^{n-1}, T_6 = -1\frac{1}{2}$

9 a $r = -1, T_n = (-1)^{n-1}, T_6 = -1$
  b $r = -2, T_n = -2 \times (-2)^{n-1} = (-2)^n, T_6 = 64$
  c $r = -3, T_n = -8 \times (-3)^{n-1}, T_6 = 1944$
  d $r = -\frac{1}{2}, T_n = 60 \times \left(-\frac{1}{2}\right)^{n-1}, T_6 = -\frac{15}{8}$
  e $r = -\frac{1}{2}, T_n = -1024 \times \left(-\frac{1}{2}\right)^{n-1}, T_6 = 32$
  f $r = -6, T_n = \frac{1}{16} \times (-6)^{n-1}, T_6 = -486$

10 a $T_n = 2^{n-1}$, 7 terms
  b $T_n = -3^{n-1}$, 5 terms
  c $T_n = 8 \times 5^{n-1}$, 7 terms
  d $T_n = 7 \times 2^{n-1}$, 6 terms
  e $T_n = 2 \times 7^{n-1}$, 5 terms
  f $T_n = 5^{n-3}$, 7 terms

11 a $r = 2$; 25, 50, 100, 200, 400
  b i $r = 2$ **ii** $r = 0.1$ or -0.1
  iii $r = -\frac{3}{2}$ **iv** $r = \sqrt{2}$ or $-\sqrt{2}$

12 a 50, 100, 200, 400, 800, 1600, $a = 50, r = 2$
  b $6400 = T_8$
  c $T_{50} \times T_{25} = 5^4 \times 2^{75}, T_{50} \div T_{25} = 2^{25}$
  e The six terms $T_6 = 1600, \ldots, T_{11} = 51\,200$ lie between 1000 and 100 000.

13 The successive thicknesses form a GP with 101 terms, and with $a = 0.1$ mm and $r = 2$. Hence thickness $= T_{101} = \frac{2^{100}}{10}$ mm $\doteqdot 1.27 \times 10^{23}$ km $\doteqdot 1.34 \times 10^{10}$ light years, which is close to the present estimate of the distance to the Big Bang.

14 a $P \times 1.07, P \times (1.07)^2, P \times (1.07)^3$
  b $A_n = P \times (1.07)^n$
  c 11 full years to double, 35 full years to increase tenfold.

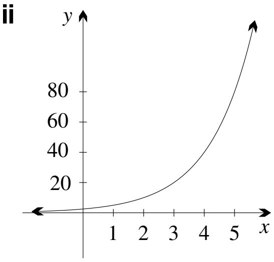
15 a $W_1 = 20\,000 \times 0.8, W_2 = 20\,000 \times (0.8)^2$, $W_3 = 20\,000 \times (0.8)^3, W_n = 20\,000 \times (0.8)^n$
  b 11 years

16 a $r = \sqrt{2}, T_n = \sqrt{6} \times \left(\sqrt{2}\right)^{n-1} = \sqrt{3} \times \left(\sqrt{2}\right)^n$
  b $r = ax^2, T_n = a^n x^{2n-1}$
  c $r = \frac{y}{x}, T_n = -x^{2-n}y^{n-2}$

17 a $T_n = 2x^n, x = 1$ or -1
  b $T_n = x^{6-2n}, x = \frac{1}{3}$ or $-\frac{1}{3}$
  c $T_n = 2^{-16} \times 2^{4n-4}x = 2^{4n-20}x, x = 6$

18 a $T_n = 2^{8-3n}$

19 a $\frac{4}{5}$, 4, 20, 100, 500, . . . and $T_n = \frac{4}{25} \times 5^n$
  b i $T_n = \frac{5}{2} \times 2^n, f(x) = \frac{5}{2} \times 2^x$
  ii



20 a $a = kb, r = b$ **b** $f(x) = ar^{x-1}$

21 a $a = cb, r = b$ **b** $f(x) = \frac{a}{r} \times r^x$

22 a first term $= aA$, ratio $= rR$
  b $W_n = (A + a)r^{n-1}$

Exercise 1D

1 a 11 **b** 23 **c** -31
  d -8 **e** 12 **f** 10

2 a 6 or -6 **b** 12 or -12 **c** 30 or -30
  d 14 or -14 **e** 5 **f** -16

3 a 10; 8 or -8 **b** 25; 7 or -7
  c $20\frac{1}{2}$; 20 or -20 **d** $-12\frac{1}{2}$; 10 or -10
  e -30; 2 **f** 0; 6
  g -3; 1 **h** 24; -3
  i 40; 45 **j** 84; -16
  k $-5\frac{3}{4}$; -36 **l** -21; 7

4 a 7, 14, 21, 28, 35, 42
  b 27, 18, 12, 8
  c 48, $36\frac{3}{4}$, $25\frac{1}{2}$, $14\frac{1}{4}$, 3
  d 48, 24, 12, 6, 3 or 48, -24, 12, -6, 3

5 a $d = 3, a = -9$
  b $d = -9, a = 60$
  c $d = 3\frac{1}{2}, a = -4\frac{1}{2}$

6 a $r = 2, a = 4$ $r = 4, a = \frac{1}{16}$
  b $r = 3$ and $a = \frac{1}{9}$, or $r = -3$ and $a = -\frac{1}{9}$
  c $r = \sqrt{2}$ and $a = \frac{3}{2}$, or $r = -\sqrt{2}$ and $a = \frac{3}{2}$

7 a $T_8 = 37$ **b** $T_2 = 59$ **c** $T_2 = \frac{3}{8}$

8 a $n = 13$ **b** $n = 8$
  c $n = 11$ **d** $n = 8$

9 b $n = 19$ **c** $n = 29$
  d $n = 66$ **e** 10 terms
  f 37 terms

10 a $T_n = 98 \times \left(\frac{1}{7}\right)^{n-1}$, 10 terms
  b $T_n = 25 \times \left(\frac{1}{5}\right)^{n-1} = \left(\frac{1}{5}\right)^{n-3}$, 11 terms
  c $T_n = (0.9)^{n-1}$, 132 terms

11 a about 78% **b** 152 sheets

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12 a $a = 28, d = -1$
b $a = \frac{1}{3}$ and $r = 3$, or $a = \frac{2}{3}$ and $r = -3$
c $T_6 = -2$
13 a $x = 10; 9, 17, 25$
b $x = -2; -2, -6, -10$
c $x = 2; -1, 5, 11$
d $x = -4; -14, -4, 6$
14 a $x = -\frac{1}{2}; -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$
b $x = 1; 1, 2, 4$ or $x = 6; -4, 2, -1$
15 a **i** $x = -48$ **ii** $x = 6$
b **i** $x = 0.100\,01$
ii $x = 0.002$ or $x = -0.002$
c **i** $x = 0.398$ **ii** $x = 20$
d **i** They can't form an AP. **ii** $x = 9$
e **i** $x = 2$ **ii** $x = 4$ or $x = 0$
f **i** $x = \sqrt{5}$ **ii** $x = 2$ or $x = -2$
g **i** $x = \frac{3}{2}\sqrt{2}$ **ii** $x = 2$ or $x = -2$
h **i** $x = 40$ **ii** 2^5 or -2^5
i **i** $x = 0$ **ii** They can't form a GP.
16 a $a = 6\frac{1}{4}$ and $b = 2\frac{1}{2}$, or $a = 4$ and $b = -2$
b $a = 1, b = 0$
18 c $r = 1, \frac{1}{2} + \frac{1}{2}\sqrt{5}$ or $\frac{1}{2} - \frac{1}{2}\sqrt{5}$
d 1, 2, 4, 8, . . .
19 a Squares can't be negative.
c 1, 1, 1 is an AP and a GP. 1, 5, 9 is an AP and 1, 3, 9 is a GP.
20 b $\frac{T_8}{T_1} = \left(\frac{1}{2}\right)^{\frac{7}{12}} \doteqdot 0.6674 \doteqdot \frac{2}{3}$
c $\frac{T_5}{T_1} = \left(\frac{1}{2}\right)^{\frac{4}{12}} \doteqdot 0.7937 \doteqdot \frac{4}{5}$
d $\frac{T_6}{T_1} = \left(\frac{1}{2}\right)^{\frac{5}{12}} \doteqdot 0.7491 \doteqdot \frac{3}{4}$,
$\frac{T_4}{T_1} = \left(\frac{1}{2}\right)^{\frac{3}{12}} \doteqdot 0.8409 \doteqdot \frac{5}{6}$
e $\frac{T_3}{T_1} = \left(\frac{1}{2}\right)^{\frac{2}{12}} \doteqdot 0.8908 \doteqdot \frac{8}{9}$,
$\frac{T_2}{T_1} = \left(\frac{1}{2}\right)^{\frac{1}{12}} \doteqdot 0.9439 \doteqdot \frac{17}{18}$
21 e $\angle OTM = 90°$ because it is an angle in a semi-circle, so OT is a tangent. Now use similar triangles to prove that $OT^2 = OA \times OB$.

Exercise 1E

1 a 24 **b** 80 **c** $3\frac{3}{4}$ **d** 20
2 a $-2, 3, -3$ **b** $120, 121, 121\frac{1}{3}$ **c** 60, 50, 30
d 0.1111, 0.111 11, 0.111 111
3 a S_n: 2, 7, 15, 26, 40, 57, 77
b S_n: 40, 78, 114, 148, 180, 210, 238
c S_n: 2, −2, 4, −4, 6, −6, 8
d S_n: 7, 0, 7, 0, 7, 0, 7
4 a 42 **b** 75 **c** 15 **d** 174
e 100 **f** 63 **g** 117 **h** −1
i 0 **j** 404 **k** 7 **l** −7
5 c 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, . . .
6 a T_n: 1, 3, 5, 7, 9, 11, 13
b T_n: 2, 4, 8, 16, 32, 64, 128
c T_n: −3, −5, −7, −9, −11, −13, −15
d T_n: 8, −8, 8, −8, 8, −8, 8
7 a T_n: 1, 1, 1, 2, 3, 5, 8, 13
b T_n: 3, 1, 3, 4, 7, 11, 18, 29
8 a 2, 8, 26, 80, 242
b 2, 6, 18, 54, 162
c $T_n = 2 \times 3^{n-1}$
9 a $T_n = 5 \times 2^n$
b $T_n = 16 \times 5^{n-1}$
c $T_n = 3 \times 4^{n-2}$
10 a $T_n = 6n$, 6, 12, 18
b $T_n = 6 - 2n$, 4, 2, 0
c $T_n = 4$, 4, 4, 4
d $T_n = 3n^2 - 3n + 1$, 1, 7, 19
e $T_n = 2 \times 3^{-n}, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$,
f $T_n = -6 \times 7^{-n}, -\frac{6}{7}, -\frac{6}{49}, -\frac{6}{343}$
11 a $\sum_{n=1}^{40} n^3$ **b** $\sum_{n=1}^{40} \frac{1}{n}$ **c** $\sum_{n=1}^{20} (n+2)$ **d** $\sum_{n=1}^{12} 2^n$
e $\sum_{n=1}^{10} (-1)^n n$ **f** $\sum_{n=1}^{10} (-1)^{n+1} n$ or $\sum_{n=1}^{10} (-1)^{n-1} n$
12 a $T_1 = 8, T_n = 2n + 3$ for $n \geq 2$
b $T_1 = -7, T_n = 14 \times 3^{n-1}$ for $n \geq 2$
c $T_1 = 1, T_n = \dfrac{-1}{n(n-1)}$ for $n \geq 2$
d $T_n = 3n^2 - n + 1$ for $n \geq 1$
The formula holds for $n = 1$ when $S_0 = 0$.
13 a $T_1 = 2, T_n = 2^{n-1}$ for $n \geq 2$
b 2, 2, 4, 8, 16, . . .
c The derivative of e^x is the original function e^x. Remove the initial term 2 from the sequence in part **b**, and the successive differences are the original sequence.
14 b $T_1 = 1$ and $T_n = 3n^2 - 3n + 1$ for $n \geq 2$
c $U_1 = 1$ and $U_n = 6n$ for $n \geq 2$
d 1, 7, 19, 37, 61, 91, . . . and 1, 6, 12, 18, 24, 30, . . .
e The derivative of x^3 is the quadratic $3x^2$, and its derivative is the linear function $6x$. Taking successive differences once gives a quadratic, and taking them twice gives a linear function.
15 b 3 **c** 1000

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Exercise 1F

1 77

2 a $n = 100, 5050$ **b** $n = 50, 2500$
c $n = 50, 2550$ **d** $n = 100, 15\,150$
e $n = 50, 7500$ **f** $n = 9000, 49\,504\,500$

3 a 180 **b** 78 **c** -153 **d** -222

4 a 222 **b** -630 **c** 78 400
d 0 **e** 65 **f** 30

5 a 101 terms, 10 100 **b** 13 terms, 650
c 11 terms, 275 **d** 100 terms, 15 250
e 11 terms, 319 **f** 10 terms, $61\frac{2}{3}$

6 a 500 terms, 250 500 **b** 2001 terms, 4 002 000
c 3160 **d** 1440

7 a $S_n = \frac{1}{2}n(5 + 5n)$ **b** $S_n = \frac{1}{2}n(17 + 3n)$
c $S_n = n(1 + 2n)$ **d** $\frac{1}{2}n(5n - 23)$
e $S_n = \frac{1}{4}n(21 - n)$ **f** $\frac{1}{2}n\left(2 + n\sqrt{2} - 3\sqrt{2}\right)$

8 a $\frac{1}{2}n(n + 1)$ **b** n^2
c $\frac{3}{2}n(n + 1)$ **d** $100n^2$

9 a 450 legs. No creatures have the mean number of 5 legs.
b 16 860 years **c** \$352 000

10 a $a = 598, \ell = 200, 79\,800$
b $a = 90, \ell = -90, 0$
c $a = -47, \ell = 70, 460$
d $a = 53, \ell = 153, 2163$

11 a 10 terms, $55 \log_a 2$
b 11 terms, 0
c 6 terms, $3(4 \log_b 3 - \log_b 2)$
d $15(\log_x 2 - \log_x 3)$

12 a $\ell = 22$ **b** $a = -7.1$
c $d = 11$ **d** $a = -3$

13 b i 16 terms **ii** more than 16 terms
c 5 terms or 11 terms
d $n = 18$ or $n = -2$, but n must be a positive integer.
e $n = 4, 5, 6, \ldots, 12$
f Solving $S_n > 256$ gives $(n - 8)^2 < 0$, which has no solutions.

14 a $S_n = n(43 - n)$, 43 terms
b $S_n = \frac{3}{2}n(41 - n)$, 41 term
c $S_n = 3n(n + 14)$, 3 terms
d $\frac{1}{4}n(n + 9)$, 6 terms

15 a $n = 17, a = -32$
b $n = 11, a = 20$

16 a 20 rows, 29 logs on bottom row
b $S_n = 5n^2$, 7 seconds
c 11 trips, deposits are 1 km apart.

17 a $d = -2, a = 11, S_{10} = 20$
b $a = 9, d = -2, T_2 = 7$
c $d = -3, a = 28\frac{1}{2}, T_4 = 19\frac{1}{2}$

18 a 300 **b** 162

19 a $S_n = \frac{1}{2}n(n + 1)$
b i n ends in 4, 5, 9 or 0.
ii n has remainder 3 or 0 after division by 4.
c i $n = 28$ **ii** $n = 14$ **iii** $n = 12$
iv $n = 19$ **v** $n = 3$ **vi** $n = 11$
vii $n = 20$

Exercise 1G

1 728

2 2801 kits, cats, sacks, wives and man

3 a 1093 **b** 547

4 a $1023, 2^n - 1$ **b** $242, 3^n - 1$
c $-11\,111, -\frac{1}{9}\left(10^n - 1\right)$ **d** $-781, -\frac{1}{4}\left(5^n - 1\right)$
e $-341, \frac{1}{3}\left(1 - (-2)^n\right)$ **f** $122, \frac{1}{2}\left(1 - (-3)^n\right)$
g $-9091, -\frac{1}{11}\left(1 - (-10)^n\right)$
h $-521, -\frac{1}{6} = \left(1 - (-5)^n\right)$

5 a $\frac{1023}{64}, 16\left(1 - \left(\frac{1}{2}\right)^n\right)$ **b** $\frac{364}{27}, \frac{27}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$
c $\frac{605}{9}, \frac{135}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$ **d** $\frac{211}{24}, \frac{4}{3}\left(\left(\frac{3}{2}\right)^n - 1\right)$
e $\frac{341}{64}, \frac{16}{3}\left(1 - \left(-\frac{1}{2}\right)^n\right)$ **f** $\frac{182}{27}, \frac{27}{4}\left(1 - \left(-\frac{1}{3}\right)^n\right)$
g $-\frac{305}{9}, -\frac{135}{4}\left(1 - \left(-\frac{1}{3}\right)^n\right)$ **h** $\frac{55}{24}, \frac{4}{15}\left(1 - \left(-\frac{3}{2}\right)^n\right)$

6 a $5\left((1.2)^n - 1\right), 25.96$
b $20\left(1 - (0.95)^n\right), 8.025$
c $100\left((1.01)^n - 1\right), 10.46$
d $100\left(1 - (0.99)^n\right), 9.562$

7 a i 2^{63} **ii** $2^{64} - 1$
b 615 km^3

8 a $S_n = \left(\left(\sqrt{2}\right)^n - 1\right)\left(\sqrt{2} + 1\right)$,
$S_{10} = 31\left(\sqrt{2} + 1\right)$
b $S_n = \frac{1}{2}\left(1 - \left(-\sqrt{5}\right)^n\right)\left(\sqrt{5} - 1\right)$,
$S_{10} = -1562\left(\sqrt{5} - 1\right)$

9 a $a = 6, r = 2, 762$
b $a = 9, r = 3, 3276$
c $a = 12, r = \frac{1}{2}, \frac{765}{32}$

10 a $\frac{1}{8} + \frac{3}{4} + \frac{9}{2} + 27 + 162 = 194\frac{3}{8}$ or
$\frac{1}{8} - \frac{3}{4} + \frac{9}{2} - 27 + 162 = 138\frac{7}{8}$
b $15\frac{3}{4}$ **c** 1562.496 **d** 7 **e** 640

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11 a **i** 0.01172 tonnes
ii 11.99 tonnes
b 4.9×10^{-3}g
c **i** $S_n = 10P(1.1^{10} - 1)$
ii \$56.47
12 a 34 010 and 26 491 **c** 3.30
13 b $n = 8$ **c** 14 terms
d $S_{14} = 114\,681$
14 a 41 powers of 3 **b** 42 terms
15 a 6 terms **b** 8 terms **c** 5 terms **d** 7 terms
16 a $\dfrac{n+1}{n}$ **b** $\dfrac{3}{2^n + 1}$
17 a $r = 2$ or $r = -2$ **c** $r = 3^{-\frac{1}{2}}$ or $-3^{-\frac{1}{2}}$
18 112
19 a **i** 2097151 **ii** $\frac{6560}{4374}$
b $r = 4$ and $n = 4$
c $n = 6$ and $\ell = -1215$
20 a $3 \times 3^n + 6 \times 2^n - 9$ **b** $2 \times 2^n + n^2 + 4n - 2$
c $a = 1, d = 3, b = 3,$
$S_n = \frac{3}{2}n^2 + \frac{5}{2}n - 6 + 6 \times 2^n$

Exercise 1H

1 a 18, 24, 26, $26\frac{2}{3}$, $26\frac{8}{9}$, $26\frac{26}{27}$
b $S_\infty = 27$
c $S_\infty - S_6 = 27 - 26\frac{26}{27} = \frac{1}{27}$
2 a 24, 12, 18, 15, $16\frac{1}{2}$, $15\frac{3}{4}$
b $S_\infty = 16$
c $S_\infty - S_6 = 16 - 15\frac{3}{4} = \frac{1}{4}$
3 a $a = 8, r = \frac{1}{2}, S_\infty = 16$
b $a = -4, r = \frac{1}{2}, S_\infty = -8$
c $a = 1, r = -\frac{1}{3}, S_\infty = \frac{3}{4}$
d $a = 36, r = -\frac{1}{3}, S_\infty = 27$
e $r = -\frac{1}{2}, r = -\frac{1}{2}, S_\infty = 40$
f $r = -\frac{1}{5}, r = -\frac{1}{5}, S_\infty = 50$
4 a $r = -\frac{1}{2}, S_\infty = \frac{2}{3}$
b $r = -\frac{3}{2}$, no limiting sum
c $r = \frac{1}{3}, S_\infty = 18$
d $r = \frac{1}{10}, S_\infty = 1111\frac{1}{9}$
e $r = -\frac{1}{5}, S_\infty = -\frac{5}{3}$
f $r = \frac{1}{5}, S_\infty = -\frac{5}{6}$
5 a The successive down-and-up distances form a GP with $a = 12$ and $r = \frac{1}{2}$.
b $S_\infty = 24$ metres
6 a T_n: 10, 10, 10, 10, 10, 10. S_n: 10, 20, 30, 40, 50, 60. $S_n \to \infty$ as $n \to \infty$.
b T_n: 10, −10, 10, −10, 10, −10.
S_n: 10, 0, 10, 0, 10, 0. S_n oscillates between 10 and 0 as $n \to \infty$.
c T_n: 10, 20, 40, 80, 160, 320.
S_n: 10, 30, 70, 150, 310, 630. $S_n \to \infty$ as $n \to \infty$.
d T_n: 10, −20, 40, −80, 160, −320.
S_n: 10, −10, 30, −50, 110, −210. S_n oscillates between larger and larger positive and negative numbers as $n \to \infty$.
7 a $S_\infty - S_4 = 160 - 150 = 10$
b $S_\infty - S_4 = 111\frac{1}{9} - 111\frac{1}{10} = \frac{1}{90}$
c $S_\infty - S_4 = 55\frac{5}{9} - 32\frac{4}{5} = 22\frac{34}{45}$
8 a $a = 2000$ and $r = \frac{1}{5}$
b $S_\infty = 2500$
c $S_\infty - S_4 = 4$
9 a $S_\infty = 10000$ **b** $S_\infty - S_{10} \doteqdot 3487$
10 a $r = 1.01$, no limiting sum
b $r = (1.01)^{-1}, S_\infty = 101$
c $r = \frac{1}{4}, S_\infty = \frac{64}{3}\sqrt{5}$
d $\frac{7}{6}\left(7 + \sqrt{7}\right)$
e $4\left(2 - \sqrt{2}\right)$
f $5\left(5 - 2\sqrt{5}\right)$
g $r = \frac{1}{3}\sqrt{10} > 1$, so there is no limiting sum.
h $\frac{1}{3}\sqrt{3}$
11 a $a = \frac{1}{3}, r = \frac{1}{3}, S_\infty = \frac{1}{2}$ **b** $a = \frac{7}{2}, r = \frac{1}{2}, S_\infty = 7$
c $a = -24, r = -\frac{3}{5}, S_\infty = -15$
12 a $S_\infty = \frac{5}{1 - x}, x = \frac{1}{2}$ **b** $S_\infty = \frac{5}{1 + x}, x = -\frac{2}{3}$
c $S_\infty = \frac{3x}{2}, x = \frac{4}{3}$ **d** $S_\infty = \frac{3x}{4}, x = \frac{8}{3}$
13 b **i** 96 **ii** 32 **iii** 64 **iv** 32
14 a $-1 < x < 1, \frac{7}{1 - x}$ **b** $-\frac{1}{3} < x < \frac{1}{3}, \frac{2x}{1 - 3x}$
c $0 < x < 2, \frac{1}{2 - x}$ **d** $-2 < x < 0, -\frac{1}{x}$
15 a $-\sqrt{2} < x < \sqrt{2}$ and $x \neq 0, S_\infty = \frac{1}{2 - x^2}$
b $x \neq 0, S_\infty = \frac{1 + x^2}{x^2}$
16 b $r = -3$, which is impossible.
d **i** $S_\infty > 3$ **ii** $S_\infty < -4$
iii $S_\infty > \frac{1}{2}a$ **iv** $S_\infty < \frac{1}{2}a$
17 a $w = \frac{1}{1 - v}$ **b** $v = \frac{w}{1 + w}$ **c** v
18 a $r = \frac{4}{5}$
b $18 + 6 + 2 + \cdots$ or $9 + 6 + 4 + \cdots$
c $r = \frac{5}{6}$
d **i** $r = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$ ($r = -\frac{1}{2} - \sqrt{5} < -1$, so it is not a possible solution.)
ii $r = \frac{1}{2}$ **iii** $r = \frac{1}{2}\sqrt{2}$ or $-\frac{1}{2}\sqrt{2}$

Exercise 1I

1 a $0.3 + 0.03 + 0.003 + \cdots = \frac{1}{3}$
b $0.1 + 0.01 + 0.001 + \cdots = \frac{1}{9}$
c $0.7 + 0.07 + 0.007 + \cdots = \frac{7}{9}$
d $0.6 + 0.06 + 0.006 + \cdots = \frac{2}{3}$

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2 a $0.27 + 0.0027 + 0.000027 + \cdots = \frac{3}{11}$

b $\frac{81}{99} = \frac{9}{11}$ **c** $\frac{1}{11}$ **d** $\frac{4}{33}$ **e** $\frac{26}{33}$

f $\frac{1}{37}$ **g** $\frac{5}{37}$ **h** $\frac{5}{27}$

3 a $12 + (0.4 + 0.04 + \cdots) = 12\frac{4}{9}$

b $7 + 0.81 + 0.0081 + \cdots = 7\frac{9}{11}$

c $8.4 + (0.06 + 0.006 + \cdots) = 8\frac{7}{15}$

d $0.2 + (0.036 + 0.00036 + \cdots) = \frac{13}{55}$

4 a $0.\dot{9} = 0.9 + 0.09 + 0.009 + \cdots = \frac{0.9}{1 - 0.1} = 1$

b $2.7\dot{9} = 2.7 + (0.09 + 0.009 + 0.0009 + \cdots)$

$= 2.7 + \frac{0.09}{1 - 0.1} = 2.7 + 0.1 = 2.8$

5 a $\frac{29}{303}$ **b** $\frac{25}{101}$ **c** $\frac{3}{13}$ **d** $\frac{3}{7}$

e $0.25 + (0.0057 + 0.000057 + \cdots) = \frac{211}{825}$

f $1\frac{14}{135}$ **g** $\frac{1}{3690}$ **h** $7\frac{27}{35}$

6 If $\sqrt{2}$ were a recurring decimal, then we could use the methods of this section to write it as a fraction.

7 a Notice that $\frac{1}{9} = 0.\dot{1}$, $\frac{1}{99} = 0.\dot{0}\dot{1}$, $\frac{1}{999} = 0.\dot{0}0\dot{1}$, and so on. If the denominator of a fraction can be made a string of nines, then the fraction will be a multiple of one of these recurring decimals.

b Periods: 1, 6, 1, 2, 6, 3, 3, 5, 4, 5

8 d The fourth sentence should be changed to, 'Imagine that each real number T_n in the sequence is written as an infinite decimal string of digits 0.dddddd . . . , where each d represents a digit. Add an infinite string of zeroes to every terminating decimal, and if there is an infinite string of 9s, rewrite the decimal as a terminating decimal.'

Chapter 1 review exercise

1 14, 5, −4, −13, −22, −31, −40, −49

a 6 **b** 4 **c** −31

d T_8 **e** No **f** $T_{11} = -40$

2 a 52, −62, −542, −5 999 942

b 20 no, $10 = T_8$, $-56 = T_{19}$, −100 no

c $T_{44} = -206$

d $T_{109} = -596$

3 a 4, 7, 7, 7, 7, 7, . . .

b 0, 1, 2, 3, 4, 5, 6, . . .

c $T_1 = 5$, $T_n = 2n - 1$ for $n > 1$

d $T_1 = 3$, $T_n = 2^{3n-1}$ for $n > 1$

4 a 82 **b** −15 **c** 1 **d** $\frac{63}{64}$

5 a −5, 5, −5, 5, −5, 5, −5, 5 **b** −5, 0

c Take the opposite. **d** 5, −5, −5

6 a AP, $d = 7$ **b** AP, $d = -121$

c neither **d** GP, $r = 3$

e neither **f** GP, $r = -\frac{1}{2}$

7 a $a = 23, d = 12$

b $T_{20} = 251$, $T_{600} = 7211$

d $143 = T_{11}$, 173 is not a term.

e $T_{83} = 1007$, $T_{165} = 1991$

f 83 (Count both T_{83} and T_{165}.)

8 a $a = 20, d = 16$ **b** $T_n = 4 + 16n$

c 12 cases, $4 change **d** 18

9 a $a = 50, r = 2$

b $T_n = 50 \times 2^{n-1}$ (or 25×2^n)

c $T_8 = 6400$, $T_{12} = 102\,400$

d $1600 = T_6$, 4800 is not a term.

e 320 000 **f** 18 terms

10 a $a = 486, r = \frac{1}{3}$

b 486, 162, 54, 18, 6, 2 (no fractions)

c 4 **d** $S_6 = 728$ **e** 729

11 a 75 **b** 45 or −45

12 a 11 111 **b** −16 400 **c** 1025

13 a $n = 45$, $S_{45} = 4995$ **b** $n = 101$, $S_{101} = 5050$

c $n = 77$, $S_{77} = 2387$

14 a 189 **b** −1092 **c** $-157\frac{1}{2}$

15 a 300

b $r = -\frac{3}{2} < -1$, so there is no limiting sum.

c $-303\frac{3}{4}$

16 a $-3 < x < -1$ **b** $S_\infty = -\frac{2 + x}{1 + x}$

17 a $\frac{13}{33}$ **b** $\frac{52}{111}$ **c** $12\frac{335}{1100} = 12\frac{67}{220}$

18 a $d = 5$, 511 **b** −1450 **c** $r = -2$, −24

d $d = -5$ **e** $n = 2$ or $n = 8$

f $r = -\frac{1}{3}$ **g** 16

Chapter 2

Exercise 2A

1 A When $n = 1$, RHS $= 1^2$
$= $ LHS,

so the statement is true for $n = 1$.

B Suppose that $k \geq 1$ is a positive integer for which the statement is true.

That is, suppose

$$1 + 3 + 5 + \cdots + (2k - 1) = k^2. \qquad (**)$$

We prove the statement for $n = k + 1$.

That is, we prove

$$1 + 3 + 5 + \cdots + (2k + 1) = (k + 1)^2.$$

$$\begin{aligned} \text{LHS} &= 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) \\ &= k^2 + (2k + 1), \\ &\qquad \text{by the induction hypothesis } (**), \\ &= (k + 1)^2 \\ &= \text{RHS}. \end{aligned}$$

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C It follows from parts A and B by mathematical induction that the statement is true for all positive integers n.

3 1 and $\frac{1}{2}$

6 b It is not true for $n = 1$.

7 b If it is true for $n = k$, it does not follow that it is true for $n = k + 1$.

8 a $P(n) = (n + 1)(4n^2 + 14n + 9)$

Exercise 2B

1 A When $n = 1$, $7^n - 1 = 6$, which is divisible by 6, so the statement is true for $n = 1$.

B Suppose that $k \geq 1$ is a positive integer for which the statement is true.
That is, suppose $7^k - 1 = 6m$, for some integer m. $\quad (**)$
We prove the statement for $n = k + 1$.
That is, we prove $7^{k+1} - 1$ is divisible by 6.

$$\begin{aligned} 7^{k+1} - 1 &= 7 \times 7^k - 1 \\ &= 7 \times (6m + 1) - 1, \\ &\quad \text{by the induction hypothesis } (**), \\ &= 42m + 6 \\ &= 6(7m + 1), \\ &\quad \text{which is divisible by 6, as required.} \end{aligned}$$

C It follows from parts A and B by mathematical induction that the statement is true for all positive integers n.

3 a 0, 10, 120, 1330, 14 640, ...
The expression is always divisible by 10.

Chapter 2 review exercise

3 a 0, 5, 55, 485. . . .
The expression is always divisible by 5.

4 b The limiting sum is 1

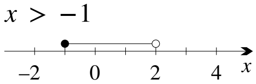
Chapter 3

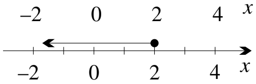
Exercise 3A

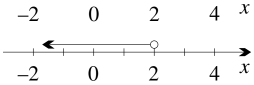
1 a i $-1 \leq x \leq 2$ **ii** $[-1, 2]$

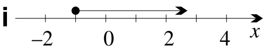
b i $-1 < x \leq 2$ **ii** $(-1, 2]$

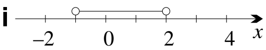
c i $x > -1$ **ii** $(-1, \infty)$

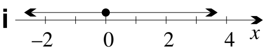
2 a i  **ii** $[-1, 2)$

b i  **ii** $(-\infty, 2]$

c i  **ii** $(-\infty, 2)$

3 a i  **ii** $x \geq -1$

b i  **ii** $-1 < x < 2$

c i 

ii R (There is no way of writing the interval using inequalities.)

4 a i $2^4 = 16$ **ii** $8 + 1 = 9$

iii $2^8 = 256$ **iv** $4 + 1 = 5$

b i 2^{x+1} **ii** $2^x + 1$

iii 2^{2^x} **iv** $x + 2$

5 a $(-\infty, 1)$ **b** $(0, 2)$ **c** $(0, 1)$ **d** $(4, \infty)$

6 a $-1 \leq x \leq 0$ or $x \geq 1$

b $-5 \leq x \leq -2$ or $x \geq 1$

c $x < -2$ or $x > 4$

d $-2 \leq x \leq 2$

e $x < -2$ or $0 < x < 2$

f $-1 \leq x < 0$ or $2 < x \leq 3$

7 a $x \neq -\dfrac{3}{2}$ **b** $x \leq 2$ **c** all real x

d $x > -1$ **e** $x > -3$ **f** all real x

8 a i $-1 < x < 1$ or $2 \leq x \leq 3$

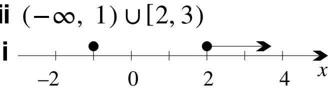
ii $(-1, 1) \cup [2, 3]$

b i $x < 1$ or $x \geq 2$

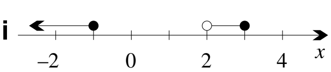
ii $(-\infty, 1) \cup [2, \infty)$

c i $x < 1$ or $2 \leq x < 3$

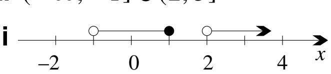
ii $(-\infty, 1) \cup [2, 3)$

9 a i 

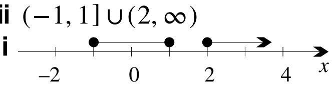
ii $[-1, -1] \cup [2, \infty)$

b i 

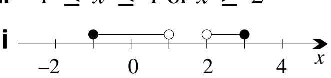
ii $(-\infty, -1] \cup (2, 3]$

c i 

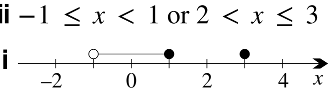
ii $(-1, 1] \cup (2, \infty)$

10 a i 

ii $-1 \leq x \leq 1$ or $x \geq 2$

b i 

ii $-1 \leq x < 1$ or $2 < x \leq 3$

c i 

ii $-1 < x \leq 1$ or $x = 3$

11 a $[-1, 0] \cup [1, \infty)$ **b** $[-5, -2] \cup [1, \infty)$

c $(-\infty, -2) \cup (4, \infty)$ **d** $[-2, 2]$

e $(-\infty, -2) \cup (0, 2)$ **f** $[-1, 0) \cup (2, 3]$

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12 a $x < -1$ or $x \geq 2$

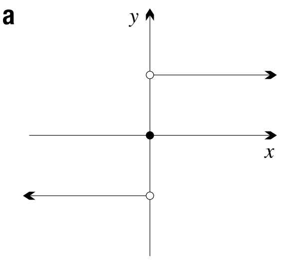
b $-1 < x \leq 1$ or $x > 3$

c $x < -1$ or $-1 < x < 2$

13 a $[0,\ 1) \cup (1,\ \infty)$ **b** $(-\infty, -1) \cup (6,\ \infty)$

c $(-1,\ 3)$ **d** $(-\infty,\ \infty)$

14 a



b $x \geq 0$

15 a $x \neq 0$

b $h'(x) = \dfrac{-4}{(e^x - e^{-x})^2}$

The denominator is a square so is positive.

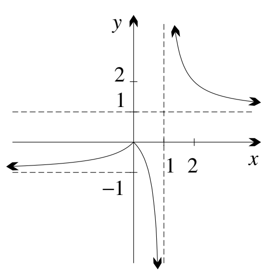
Thus $h'(x)$ is negative for $x \neq 0$.

16 a i $x \neq 1$

ii $x = 0$

iii $[0, 0] \cup (1, \infty)$

iv

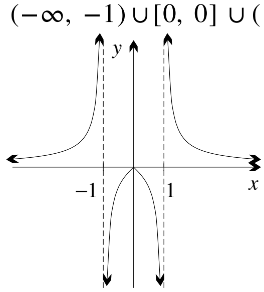


b i $x = 0$

ii $x \neq -1$ or 1

iii $(-\infty,\ -1) \cup [0,\ 0] \cup (1, \infty)$

iv



17 a Both sides equal $\sin\left(e^{1-x^2} + \dfrac{\pi}{3}\right)$

b LHS $= (f \circ g)h(x) = f\Big(g\big(h(x)\big)\Big)$,

RHS $= f\big(g \circ h(x)\big) = f\Big(g\big(h(x)\big)\Big) =$ LHS.

18 a It has one endpoint at 5 which it contains.

b It does not contain any end points.

c It contains all its endpoints. (There are none!)

Exercise 3B

1 a $f(x) \to 0 \quad x \to \infty \qquad x \to -\infty$

b $f(x) \to 1$ as $x \to \infty$ and $x \to -\infty$

c $f(x) \to -2$ as $x \to \infty$ and $x \to -\infty$

d $f(x) \to \dfrac{1}{2}$ as $x \to \infty$ and $x \to -\infty$

e $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

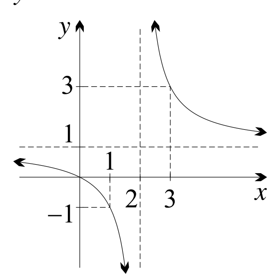
f $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

2 a $x \neq 2$

b $x = 0$ and $y = 0$

c $y \to 1$ as $x \to \infty$ and as $x \to -\infty$.

d $x = 2$ is a vertical asymptote, $y \to \infty$ as $x \to 2^+$, $y \to -\infty$ as $x \to 2^-$.

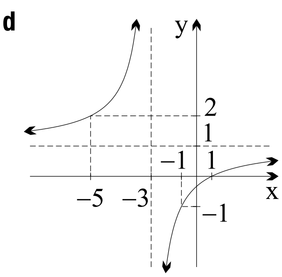


3 a $x = -3$

b $x = 1$ and $y = -\dfrac{1}{3}$

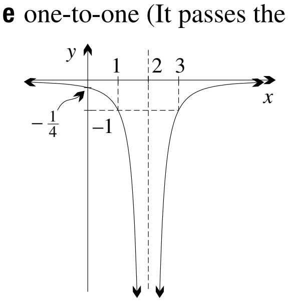
c $y \to 1$ as $x \to \infty$ and as $x \to -\infty$, $y \to -\infty$ as $x \to -3^+$, $y \to \infty$ as $x \to -3^-$.

d



e one-to-one (It passes the horizontal line test.)

4



domain: $x \neq 2$,

$y \to 0$ as $x \to \infty$ and as $x \to -\infty$

vert'l asymptote $x = 2$,

as $x \to 2^+$, $y < 0$ so $y \to -\infty$,

as $x \to 2^-$, $y < 0$ so $y \to -\infty$

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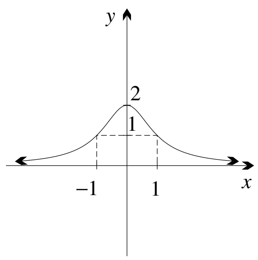
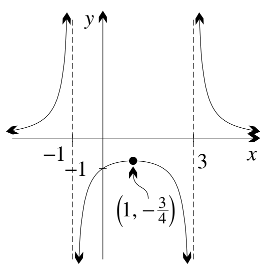
5



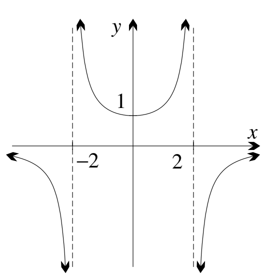
a $y \to 0$ as $x \to \infty$ and $x \to -\infty$.
b $x^2 + 1$ is never 0.
c $y' = -4x(x^2 + 1)^{-2}$
e $0 < y \le 2$
f many-to-one (It fails the horizontal line test.)

6



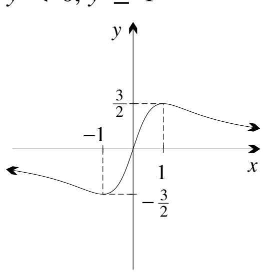
a $x \ne -1, 3$
b $(0, -1)$
c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.
e as $x \to 3^+$, $y > 0$ so $y \to \infty$, as $x \to 3^-$, $y < 0$ so $y \to -\infty$, as $x \to 1^+$, $y < 0$ so $y \to -\infty$, and as $x \to 1^-$, $y > 0$ so $y \to \infty$
f $y \le -\frac{3}{4}, y > 0$

7 a



b $y < 0, y \ge 1$

8



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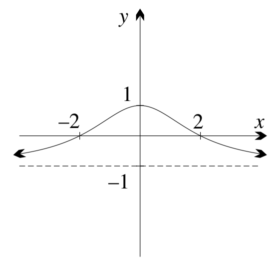


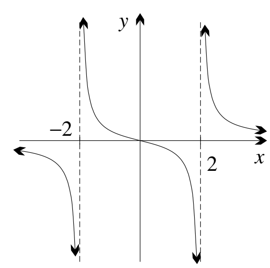
10 a $y = \dfrac{(x + 2)(x + 3)}{(x - 1)(x - 3)}$, $x = 1$, $x = 3$ and $y = 1$

b $y = \dfrac{(x - 1)^2}{(x + 1)(x + 4)}$, $x = -1$, $x = -4$ and $y = 1$

c $y = \dfrac{x - 5}{(x - 2)(x + 5)}$, $x = -5$, $x = 2$ and $y = 0$

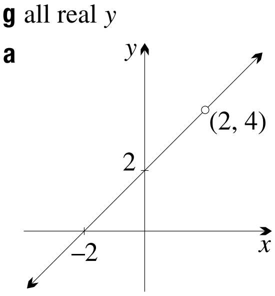
d $y = \dfrac{(1 - 2x)(1 + 2x)}{(1 - 3x)(1 + 3x)}$, $x = \dfrac{1}{3}$, $x = -\dfrac{1}{3}$ and $y = \dfrac{4}{9}$

11

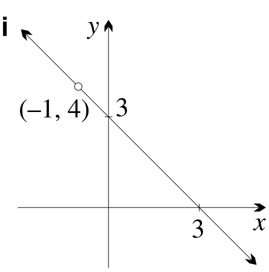


a odd
b domain: $x \ne 2$ and $x \ne -2$,
asymptotes: $x = 2$ and $x = -2$
d $y = 0$
e $f'(x) = -\dfrac{x^2 + 4}{(x^2 - 4)^2}$ $f'(x) < 0$ for $x \ne 2$ & $x \ne -2$
g all real y

12 a



b i



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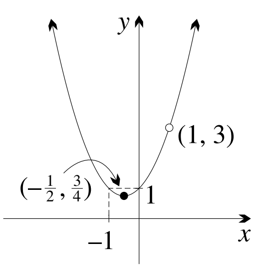

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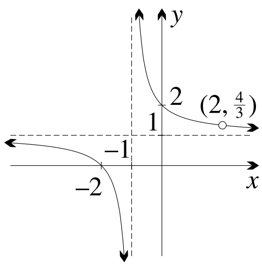

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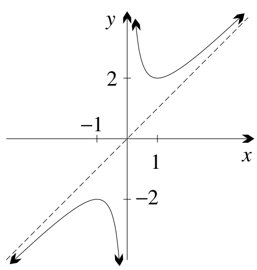


a point symmetry in the origin

b domain: $x \neq 0$, asymptote: $x = 0$

e $(-1, -2)$ and $(1, 2)$

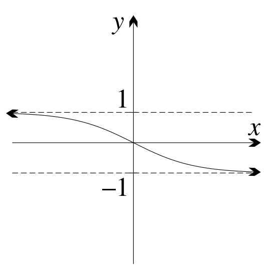
h $y \geq 2$ or $y \leq -2$

14 a 1

b -1

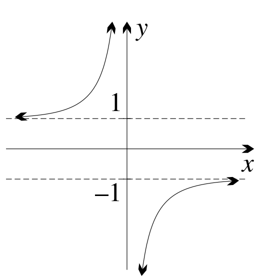
c $(0, 0)$

d

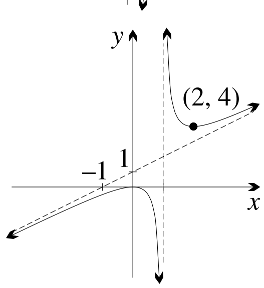


e odd

f



15 a



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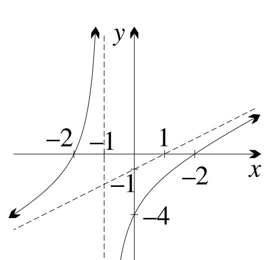
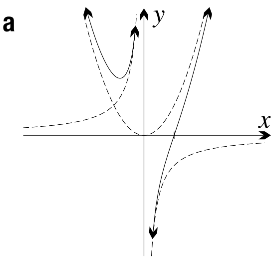


16 a



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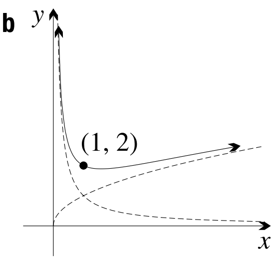

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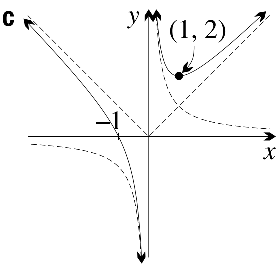


Exercise 3C

1 a $y = \dfrac{9}{(x - 3)(x + 3)}$

b $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

c symmetry in the y-axis

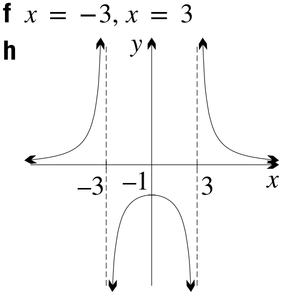
d $(0, -1)$

e $-3 < x < 3$

f $x = -3, x = 3$

g $y = 0$

h



i $y' = \dfrac{-18x}{(x^2 - 9)^2}$ so $y'(0) = 0$

2 a $y = \dfrac{x}{(2 - x)(2 + x)}$

b $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

c point symmetry in the origin

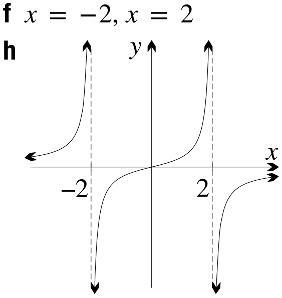
d $(0, 0)$

e $x < -2$ or $0 \leq x < 2$

f $x = -2, x = 2$

g $y = 0$

h



i $y' > 0$ in the domain.

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3 a $\dfrac{2\left(x - 2\frac{1}{2}\right)}{(x - 1)(x - 4)}$

b $x \neq 1$ and $x \neq 4$

c The domain is not symmetric about $x = 0$.

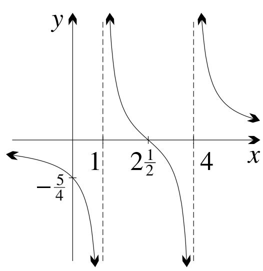
d $\left(0, -\frac{5}{4}\right)$ and $\left(2\frac{1}{2}, 0\right)$

e $1 < x < 2\frac{1}{2}$ or $x > 4$

f $x = 1$ and $x = 4$

g $y = 0$

h



4 a $y = x(x - 2)(x + 2)$

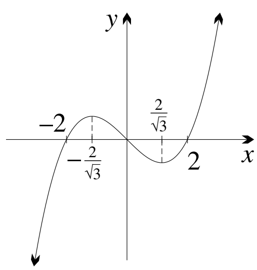
b $(-\infty, \infty)$

c $(-2, 0)$, $(0, 0)$, $(2, 0)$

d point symmetry in the origin

e no

g



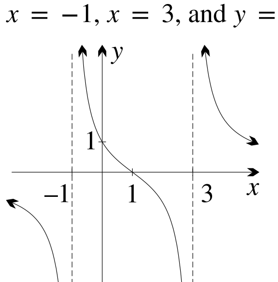
5 $y = \dfrac{3(x - 1)}{(x - 3)(x + 1)}$

a domain: $x \neq -1$ and $x \neq 3$ intercepts: $(1, 0)$ and $(0, 1)$

b The domain is not symmetric about $x = 0$.

c $x = -1$, $x = 3$, and $y = 0$

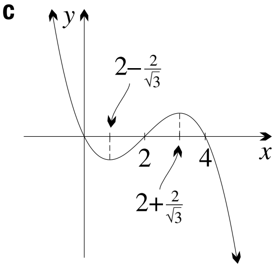
d



6 $y = -x(x - 2)(x - 4)$

a $-\infty < x < \infty$ $(0, 0)$, $(2, 0)$, $(4, 0)$

c



7 $y = \dfrac{(x + 1)^2}{(x - 1)(x + 3)}$

a domain:

$x \neq -3$ and $x \neq 1$

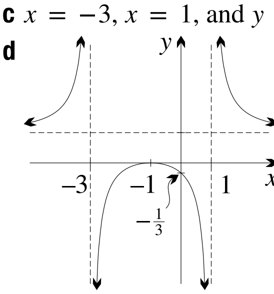
intercepts:

$(-1, 0)$ and $\left(0, -\frac{1}{3}\right)$

b The domain is not symmetric about $x = 0$.

c $x = -3$, $x = 1$, and $y = 1$

d



e $y \leq 0$ or $y > 1$

8 $f(x) = \dfrac{(x - 2)(x + 2)}{x(x - 4)}$

a domain:

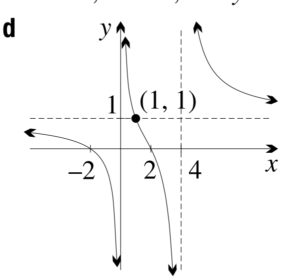
$x \neq 0$ and $x \neq 4$

intercepts:

$(-2, 0)$ and $(2, 0)$

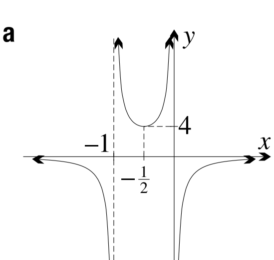
b $x = 0$, $x = 4$, and $y = 1$

d



e all real y

9 a



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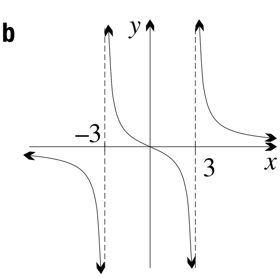

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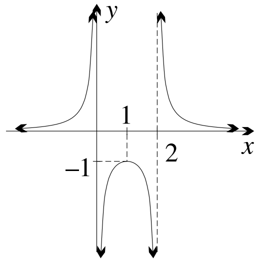
10 a



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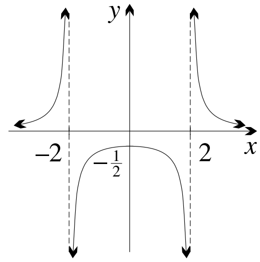
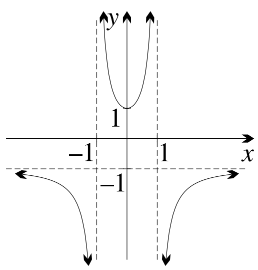


11 a



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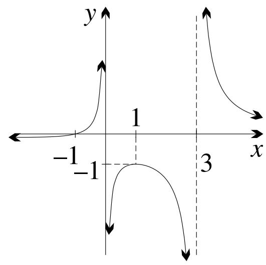

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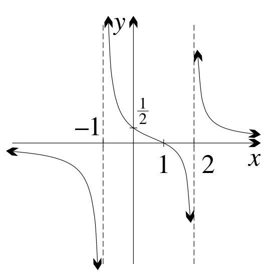

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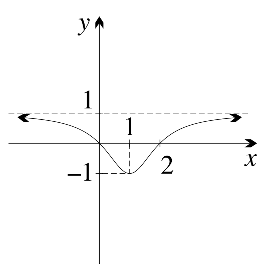

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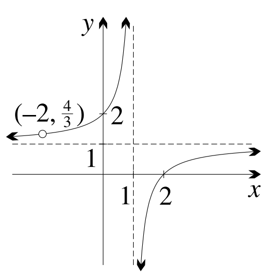

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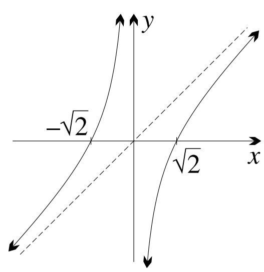


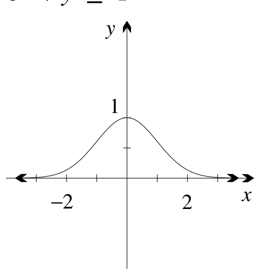
12 a domain: all real x intercept: $(0, 1)$

b It is an even function with asymptote $y = 0$.

c $(0, 1)$

d $y' = -xe^{-x^2/2}$, so $y' = 0$ at $x = 0$

e $0 < y \leq 1$



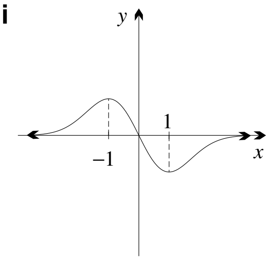
f $e^{-\frac{1}{2}} < 2^{-\frac{1}{2}}$ so $y = 2^{-\frac{1}{2}x^2}$ is higher, except at $x = 0$ where they are equal.

13 a **i** $e^x > x$ for $x \geq 0$

ii Replace x with $\frac{1}{2}x^2$ in part **i** and take reciprocals.

iii 0

b **i**

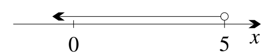


ii $f' = e^{-\frac{1}{2}x^2}(x^2 - 1)$ so $f'(x) = 0$ at $x = -1$ or 1.

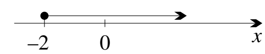
The graph shows that $f(x)$ is greatest at $x = -1$ and least at $x = 1$.

Exercise 3D

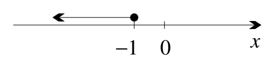
1 a $x < 5$



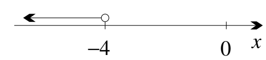
b $x \geq -2$



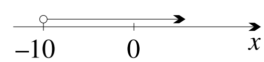
c $x \leq -1$



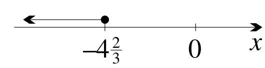
d $x < -4$



e $x > -10$



f $x \leq -4\frac{2}{3}$



2 a $x < -2$, $(-\infty, -2)$

b $x \geq -4$, $[-4, \infty)$

c $x < 6$, $(-\infty, 6)$

3 $5x - 4 < 7 - \frac{1}{2}x$, with solution $x < 2$

4 a $-\frac{1}{2} \leq x \leq 1\frac{1}{2}$, $\left[-\frac{1}{2}, 1\frac{1}{2}\right]$

b $-4 < x < 2$, $(-4, 2)$

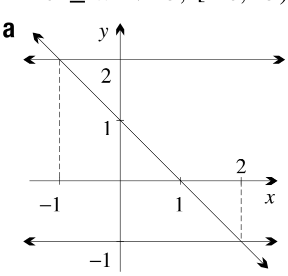
c $\frac{1}{3} < x \leq 4$, $\left(-\frac{1}{3}, 4\right]$

d $-2 < x \leq 7$, $(-2, 7]$

e $-2 \leq x < 0$, $[-2, 0)$

f $-6 \leq x < 15$, $[-6, 15)$

5 a



b $-1 \leq x < 2$. The solution of the inequation is where the diagonal line lies between the horizontal lines.

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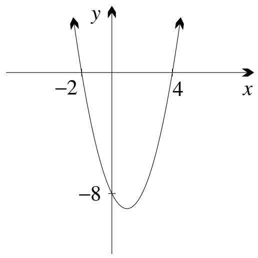
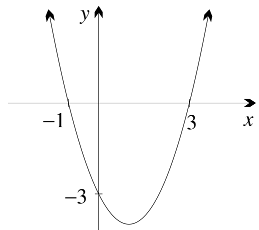
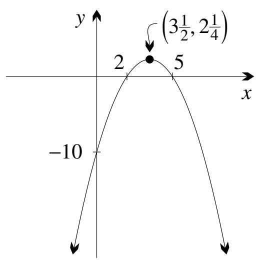
6 a $-2 < x < 4$



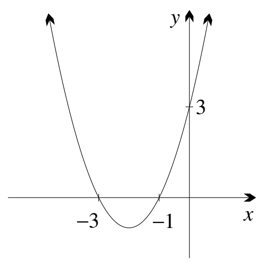
b $x < -1$ or $x > 3$



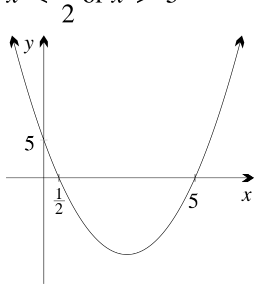
c $2 \le x \le 5$



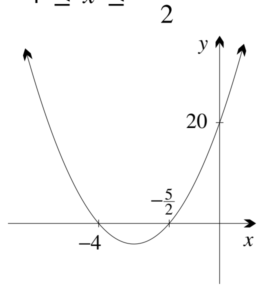
d $x \le -3$ or $x \ge -1$



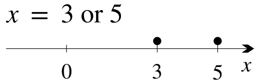
e $x < \dfrac{1}{2}$ or $x > 5$



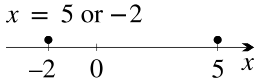
f $-4 \le x \le -\dfrac{5}{2}$



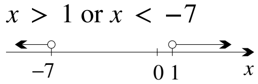
7 a $x = 3$ or 5



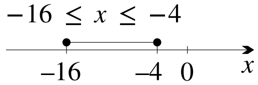
b $x = 5$ or -2



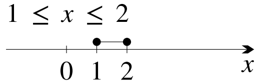
c $x > 1$ or $x < -7$



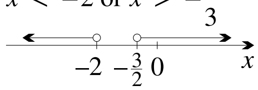
d $-16 \le x \le -4$



e $1 \le x \le 2$



f $x < -2$ or $x > -\dfrac{2}{3}$



8 a $x < 0$ or $x \ge 2$

b $-1 < x < 2$

c $x > \frac{3}{2}$ or $x < -\frac{1}{2}$

d $x \le \frac{1}{8}$ or $x > \frac{3}{4}$

9 a $y = \begin{cases} 2x - 1 & \text{for } x \ge 2 \\ 3 & \text{for } x < 2 \end{cases}$

b $y = \begin{cases} x + 9 & \text{for } x \ge -2 \\ -3x + 1 & \text{for } x < -2 \end{cases}$

c $y = \begin{cases} 4x - 4 & \text{for } x \ge -1 \\ -2x + 2 & \text{for } x < -1 \end{cases}$

10 a $x \ge 3$ **b** $0 < x \le 3$

c $-4 \le x \le 4$ **d** $x < -4$

e $0 < x < 8$ **f** $\dfrac{1}{25} \le x \le 625$

11 a $0 < x < 3$ **b** $x < -3$ or $x > 2$

c $x < 0$ **d** $x < 0$ or $1 < x < 5$

e $-4 < x < 1$

12 a i $y = \begin{cases} 3x & \text{for } x \ge 0 \\ -x & \text{for } x < 0 \end{cases}$ **ii** $x < -1$ or $x > \frac{1}{3}$

b i $y = \begin{cases} 4x - 8 & \text{for } x \ge 2 \\ 4 - 2x & \text{for } x < 2 \end{cases}$ so $1 \le x \le 2\frac{1}{2}$

ii $y = \begin{cases} \frac{1}{2}x + 1 & \text{for } x \ge -1 \\ -\frac{3}{2}x - 1 & \text{for } x < -1 \end{cases}$ so $-\frac{8}{3} < x < 4$

13 a $x < -4$ or $x > 3$

b $x \le -3$ or $x > -1$

c $\frac{1}{2} < x \le \frac{4}{5}$

14 a $0 < x < \frac{\pi}{2}$ or $\frac{3\pi}{2} < x < 2\pi$

b $-\frac{\pi}{2} < x \le 0$ or $\frac{\pi}{4} \le x < \frac{\pi}{2}$

15 a $-5 \le x < -3$ or $-1 < x \le 1$

b $-\frac{1}{2} < x \le 1$ or $2 \le x < 3\frac{1}{2}$

16 a false: $x = 2$ and $y = -2$

b true

c false: $x = 2$ and $y = -2$

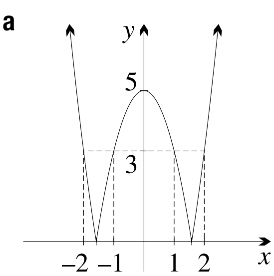
d true

e true

f false: $x = -2$

17 a It assumes $x + 1 = |x + 1|$. **b** $x < 1$

18 a



Intercepts at $x = -\sqrt{\frac{5}{2}}$ and $x = \sqrt{\frac{5}{2}}$

b $x \le -2$ or $-1 \le x \le 1$ or $x \ge 2$

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19 $x^2 + xy + y^2 = \frac{1}{2}(x^2 + y^2) + \frac{1}{2}(x + y)^2$ or otherwise.

20 a Start with $(x - y)^2 \ge 0$.

b Replace x with x^2 and y with y^2, and divide.

21 a $2(a^2 + b^2 + c^2 - ab - bc - ac)$

b $2(a^3 + b^3 + c^3 - 3abc)$

Exercise 3E

1 a 1 **b** 2 **c** 3 **d** 2 **e** 2 **f** 3

2 a $x = \frac{1}{2}$

b $x = -\frac{3\pi}{4}$ or $\frac{\pi}{4}$

c $x \doteqdot -2.1,\ 0.3,\ 1.9$

d $x = 1$ or $x \doteqdot 3.5$

e $x = 1$ or $x \doteqdot -1.9$

f $x = 0, x \doteqdot -1.9$ or 1.9

3 a $x \le -3$ **b** $0 \le x \le 2$ **c** $x = 1$

4 a $x < -2$ or $x > 1$

b $0 \le x \le 1$

c $-1 < x < 0$ or $x > 1$

5 a $\sqrt{2} \doteqdot 1.4, \sqrt{3} \doteqdot 1.7$

b $x = -1$ or $x = 2$

c $x < -1$ or $x > 2$

d $x = -2$ or $x = 1, -2 \le x \le 1$

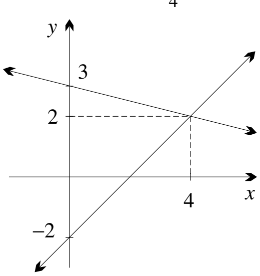
e $x \doteqdot 1.62$ or $x \doteqdot -0.62$

f i Draw $y = -x$; $x = 0$ or $x = -1$.

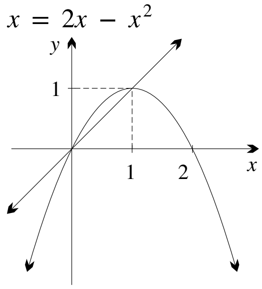
ii Draw $y = x + \frac{1}{2}$; $x \doteqdot 1.37$ or $x \doteqdot -0.37$.

iii Draw $y = \frac{1}{2}x + \frac{1}{2}$; $x = 1$ or $x = -\frac{1}{2}$.

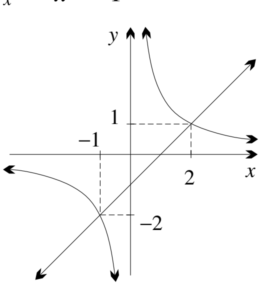
6 a $(4, 2)$, $x - 2 = 3 - \frac{1}{4}x$



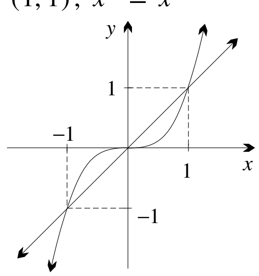
b $(0, 0)$ and $(1, 1)$, $x = 2x - x^2$



c $(-1, -2)$ and $(2, 1)$, $\frac{2}{x} = x - 1$



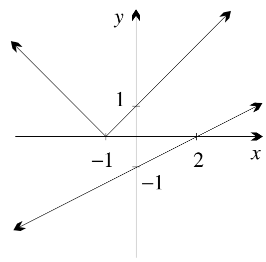
d $(-1, -1)$, $(0, 0)$ and $(1, 1)$, $x^3 = x$



7 a $x \ge 4$ **b** $0 < x < 1$

c $x < -1$ or $0 < x < 2$ **d** $-1 < x < 0$ or $x > 1$

8 a i

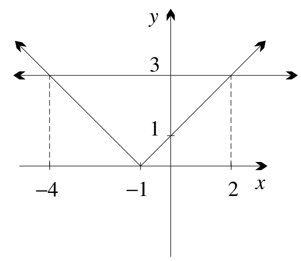


ii The graph of $y = |x + 1|$ is always above the graph of $y = \frac{1}{2}x - 1$.

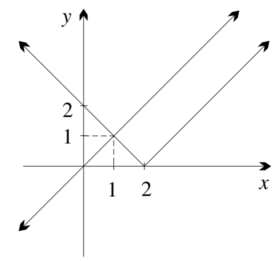
b The curve is always above the line.

c The two lines are parallel and thus the first is always below the second.

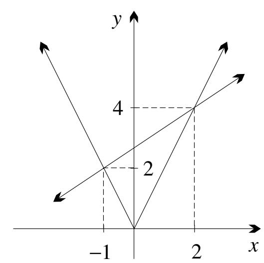
9 a $(-4, 3)$, $(2, 3)$



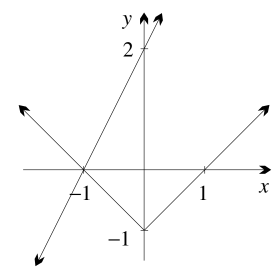
b $(1, 1)$



c $(-1, 2)$, $(2, 4)$



d $(-1, 0)$



10 a $-4 \le x \le 2$ **b** $x < 1$

c $x \le -1$ or $x \ge 2$ **d** $x < -1$

11 a Divide by e^x to get $e^x = e^{1-x}$

b Multiply by $\cos x$ to get $\sin x = \cos x$

c Subtract 1 then divide by x to get $x^2 - 4 = -\frac{1}{x}$

12 a The table below traps the solution between $x = -1.690$ and $x = -1.6905$, so it is $x = -1.690$, correct to three decimal places.

x	-2	-1.7	-1.6	-1.68
2^x	0.25	0.3078	0.3299	0.3121
$x + 2$	0	0.3	0.4	0.32

x	-1.69	-1.691	-1.6905
2^x	0.3099	0.3097	0.3098
$x + 2$	0.31	0.309	0.3095

b Part (c): $x \doteqdot -2.115$. Part (e): $x \doteqdot -1.872$.

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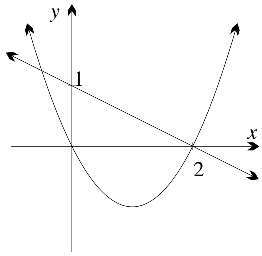
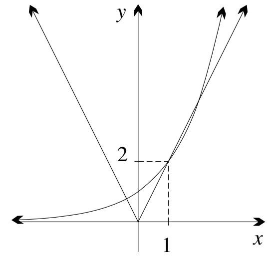
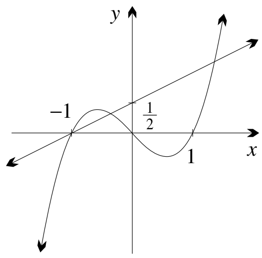
13 a 2 solutions



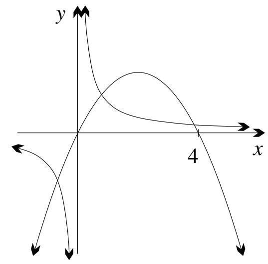
b 3 solutions



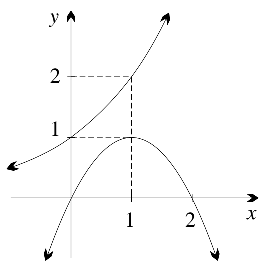
c 3 solutions



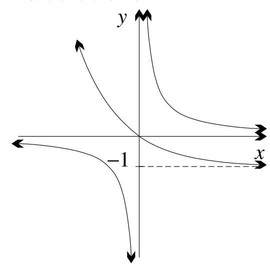
d 3 solutions



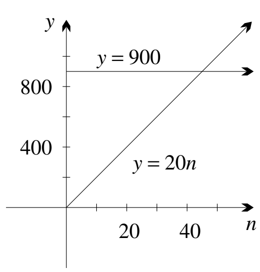
e no solutions



f no solutions



14 a



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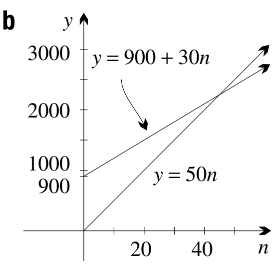
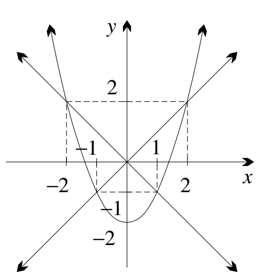


In both cases the break-even point is $n = 45$.
Total sales are \$2250 at that point.

15 a

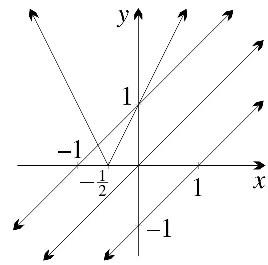


b $x = 2$ or -2

c $x < -2$ or $x > 2$

d $-1 \le x \le 1$

16



c $c > \frac{1}{2}$

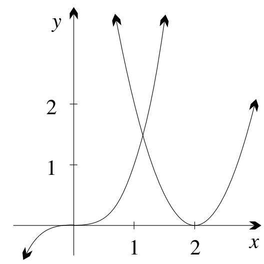
17 b $-\dfrac{3\sqrt{2}}{2} < b < \dfrac{3\sqrt{2}}{2}$

18 a 2

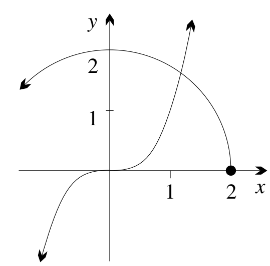
b The solutions are not integers.

c $x = \frac{1}{11}$ or $\frac{7}{3}$

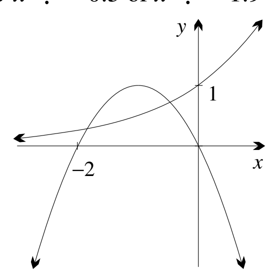
19 a $x \doteqdot 1.1$



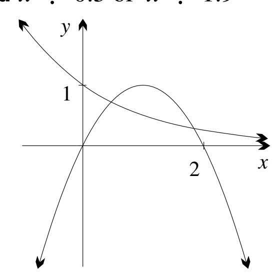
b $x \doteqdot 1.2$



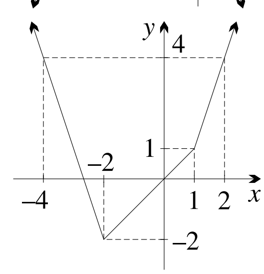
c $x \doteqdot -0.5$ or $x \doteqdot -1.9$



d $x \doteqdot 0.5$ or $x \doteqdot 1.9$



20



a $y = \begin{cases} -3x - 8, & \text{for } x < -2, \\ x, & \text{for } -2 \le x < 1, \\ 3x - 2, & \text{for } x \ge 1. \end{cases}$

b $-3\frac{1}{3} \le x \le -2\frac{1}{3}$ or $-1 \le x \le 1\frac{1}{3}$

21 b $b < m$

c $-p \le m \le p$ and $b < -\frac{qm}{p}$

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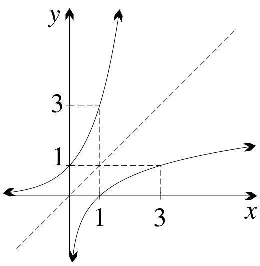
22 a i



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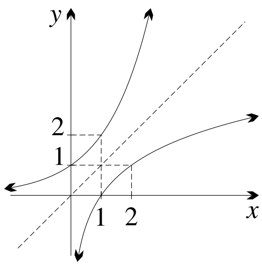
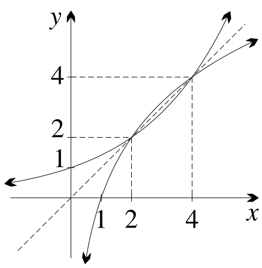


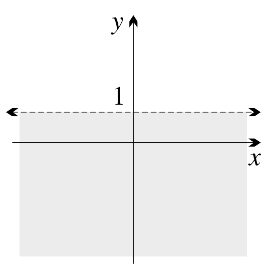
iii



b 0, 1 or 2

Exercise 3F

1 a



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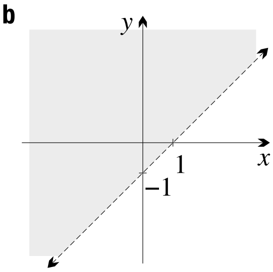
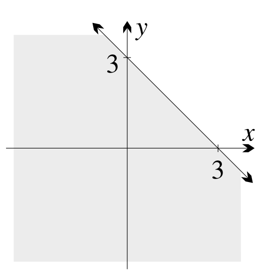


c



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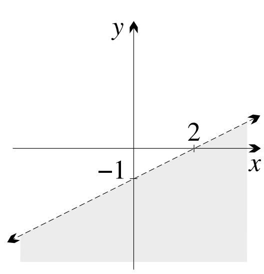
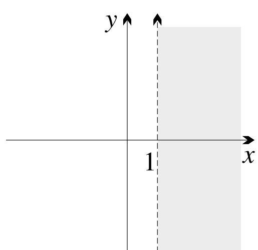


2 a



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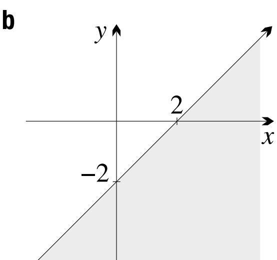

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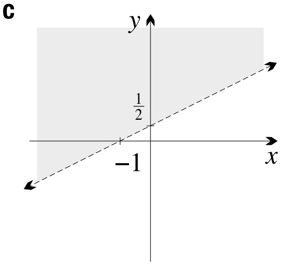

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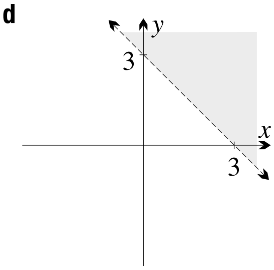
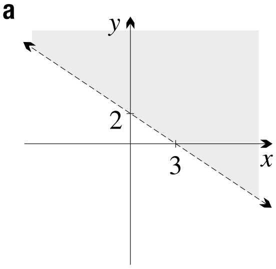


3 a



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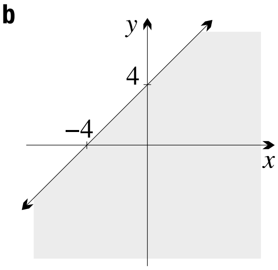
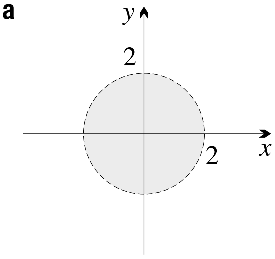
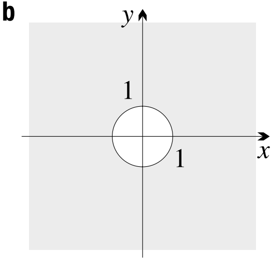


4 a



b



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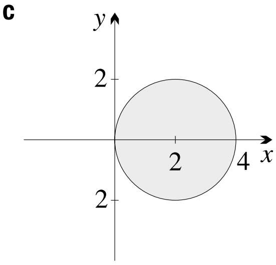

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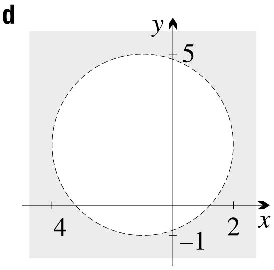
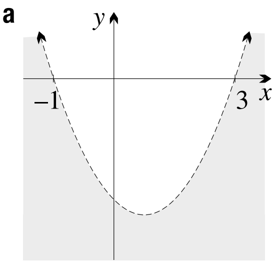


5 a



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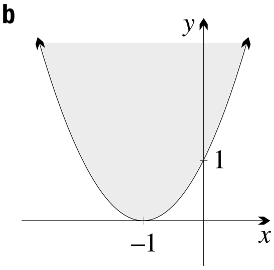

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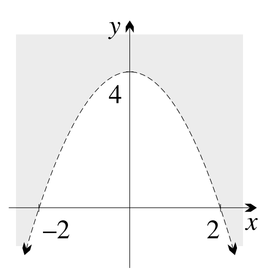

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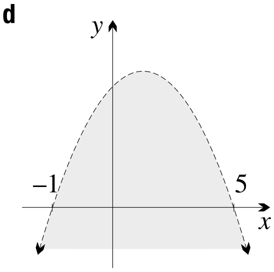

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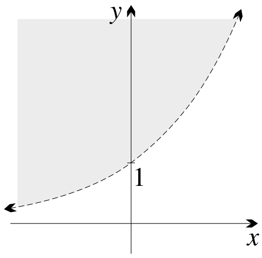
6 a



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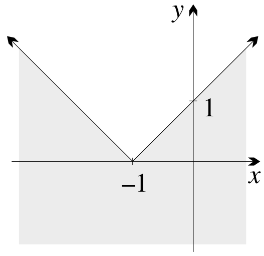

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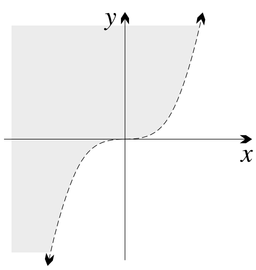

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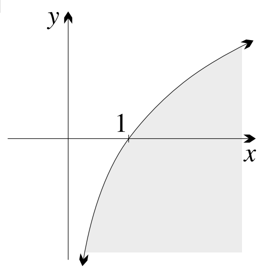
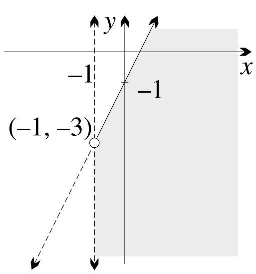


7 b



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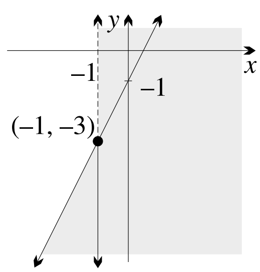
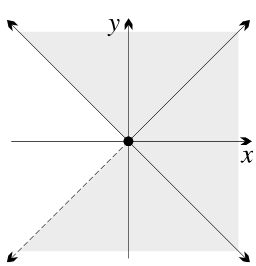


8 a i



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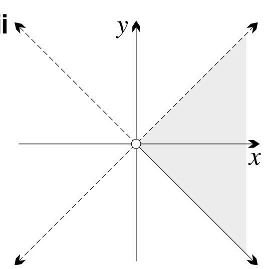
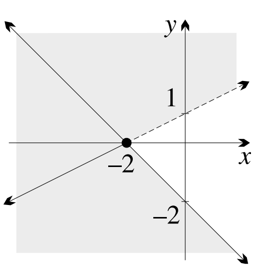


b i



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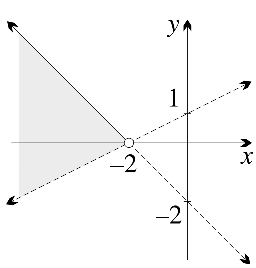


9 a $x \geq 0$ and $y \geq 0$ **b** $x \leq 0$ and $y \geq 0$

c $x \leq 0$ and $y \leq 0$ **d** $x \geq 0$ and $y \leq 0$

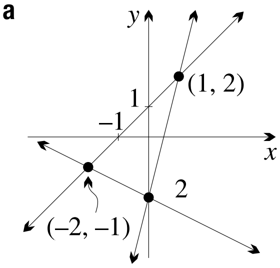
e $x \geq 0$ or $y \geq 0$ **f** $x \geq 0$ or $y \leq 0$

10 a $y < x$ and $y \leq 2 - x$

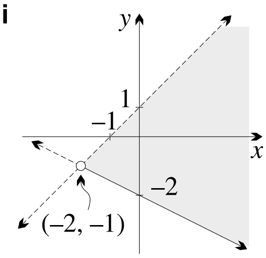
b $y \leq -\frac{1}{2}x - 1$ or $y \geq 2 - 2x$

c $y < x + 2$ or $y > 4x - 1$

11 a



b i



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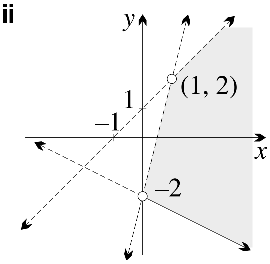
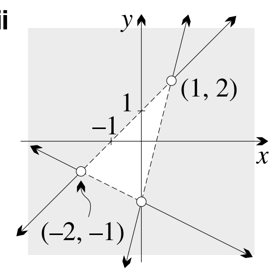
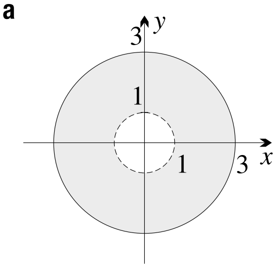


iii

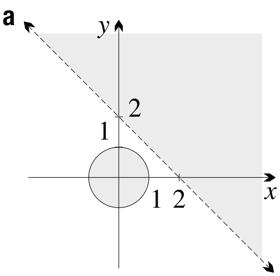


12 a



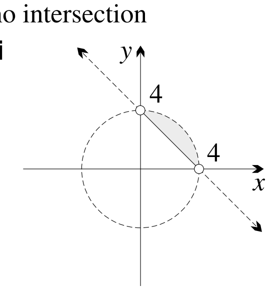
b whole plane

13 a



b no intersection

14 b i



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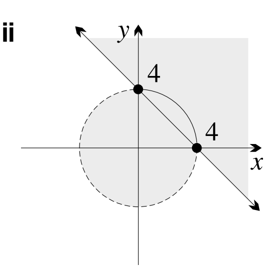

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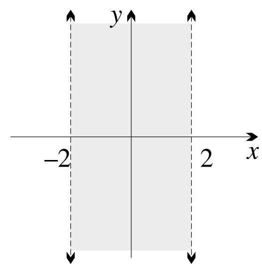
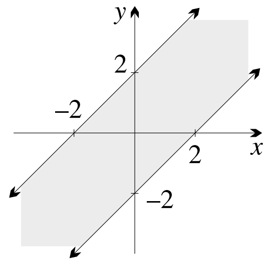
15 a $x < 2$ and $x > -2$



b $x - y \le 2$ and $x - y \ge -2$



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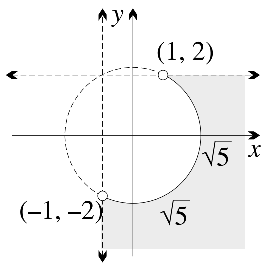
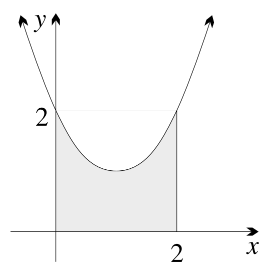
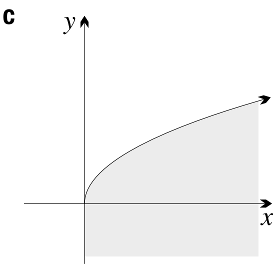


17



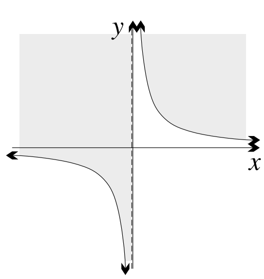
18 b The curve is undefined for $x < 0$.

c



19 a The curve is undefined when $x = 0$.

b



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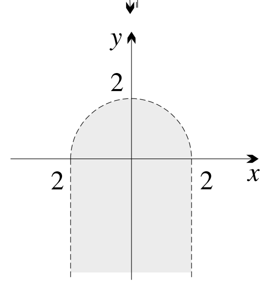

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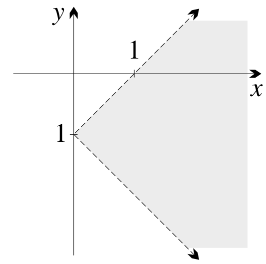
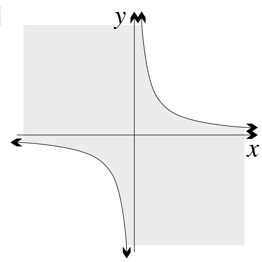


22 a 6

b i



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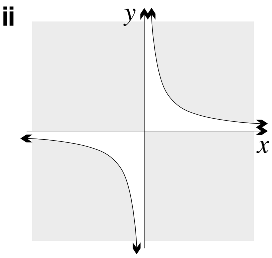
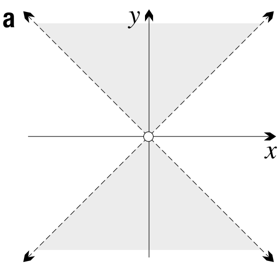
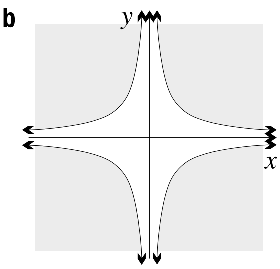


23 a



b



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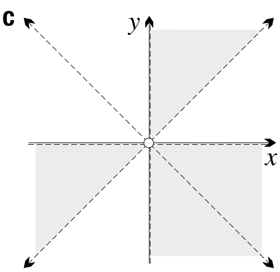
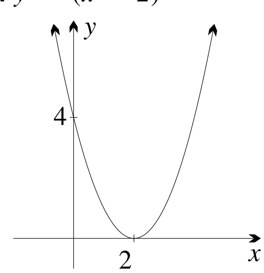
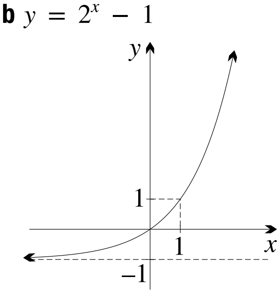


Exercise 3G

1 a $y = (x - 2)^2$



b $y = 2^x - 1$



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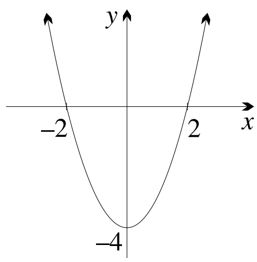
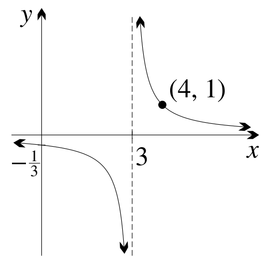
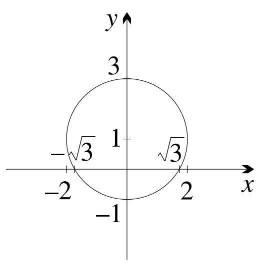
c $y = x^2 - 4$



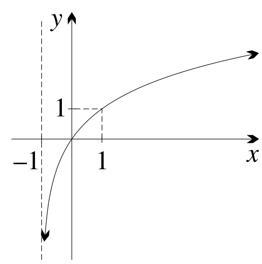
d $y = \frac{1}{x-3}$



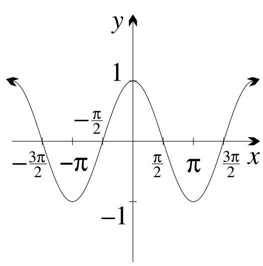
e $x^2 + (y - 1)^2 = 4$



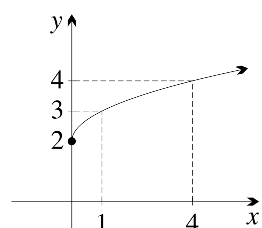
f $y = \log_2(x + 1)$



g $y = \sin(x + \frac{\pi}{2})$

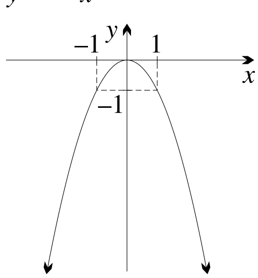


h $y = \sqrt{x} + 2$

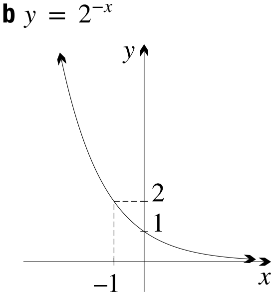


This is also $y = \cos x$.

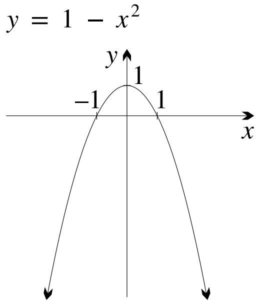
2 a $y = -x^2$



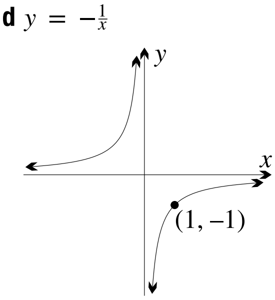
b $y = 2^{-x}$



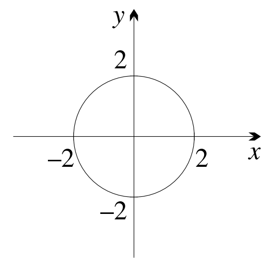
c $y = 1 - x^2$



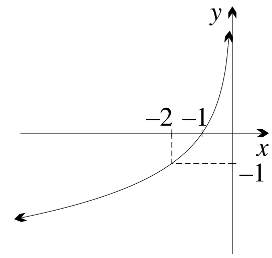
d $y = -\frac{1}{x}$



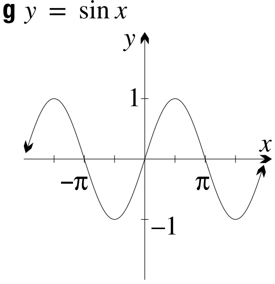
e $x^2 + y^2 = 4$



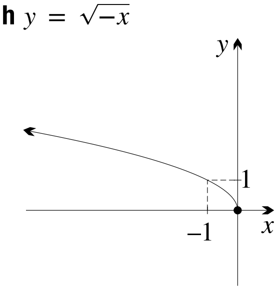
f $y = -\log_2(-x)$



g $y = \sin x$



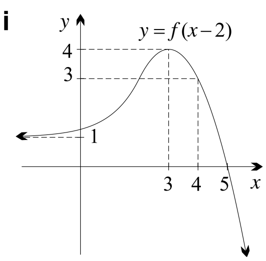
h $y = \sqrt{-x}$



3 In part **e** the circle is symmetric in the y-axis.
In part **g** $y = \sin x$ is an odd function, and so is unchanged by a rotation of 180°.

4 a $r = 2,\ (-1, 0)$ **b** $r = 1,\ (1, 2)$
c $r = 2,\ (2, 0)$ **d** $r = 5,\ (0, 3)$

5 a i



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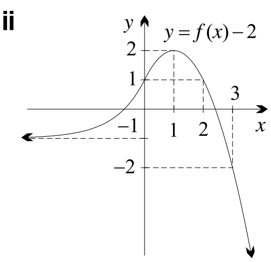
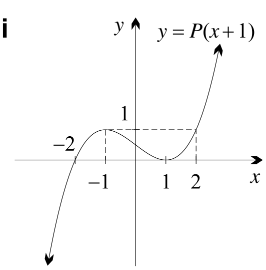


b i



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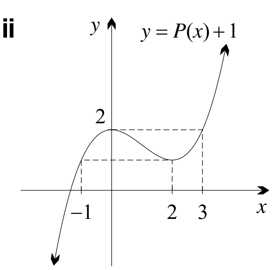
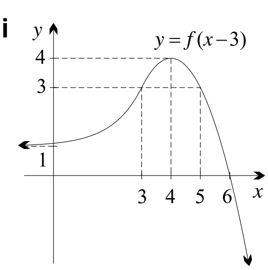


6 a i



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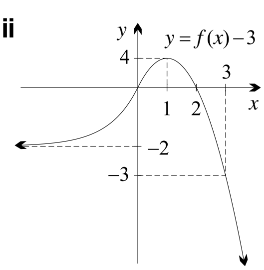

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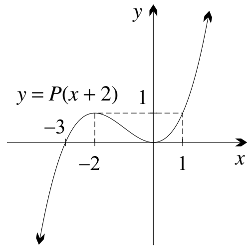
b i



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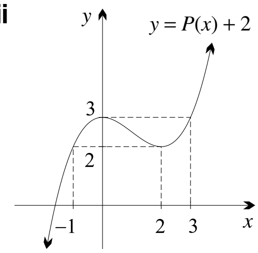


7 a $y = (x + 1)^2 + 2$ **b** $y = \frac{1}{x - 2} + 3$

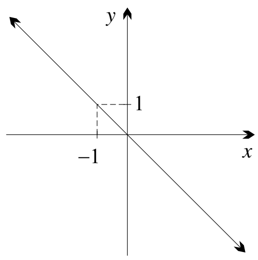
c $y = \cos(x - \frac{\pi}{3}) - 2$ **d** $y = e^{x+2} - 1$

8 a From $y = -x$:

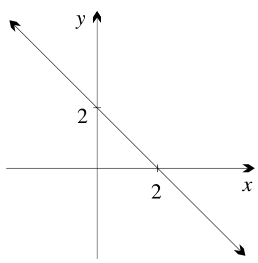
i shift up 2 (or right 2)

ii shift down 2 (or left 2)

iii reflect in x-axis (or y-axis) and shift up 4 (or left 4)



i



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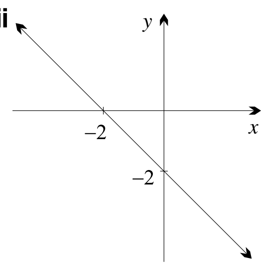

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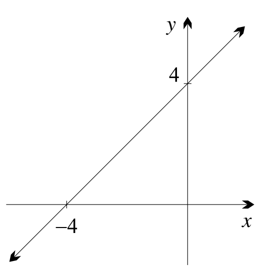
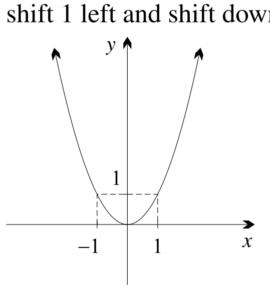


b From $y = x^2$:

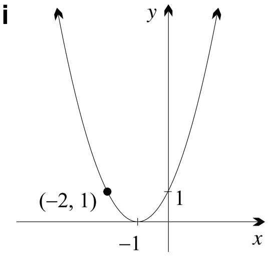
i shift 1 left

ii shift 1 left and reflect in x-axis

iii shift 1 left and shift down 1



i



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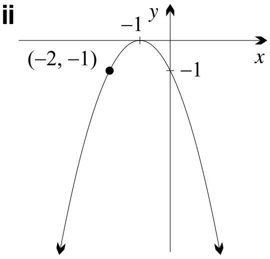

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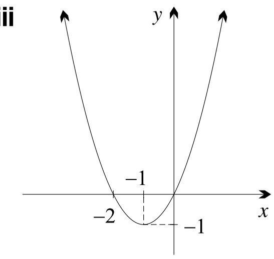
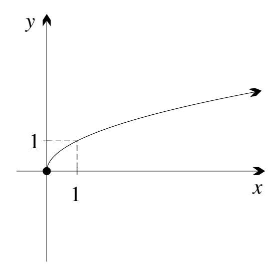


c From $y = \sqrt{x}$:

i shift 4 left

ii reflect in x-axis

iii shift 4 left and reflect in x-axis



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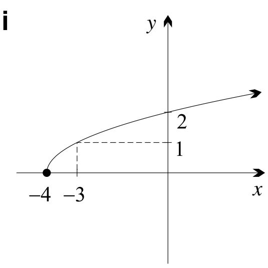

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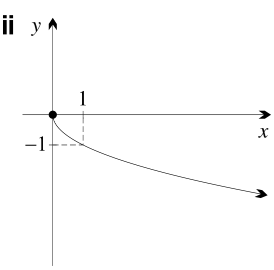

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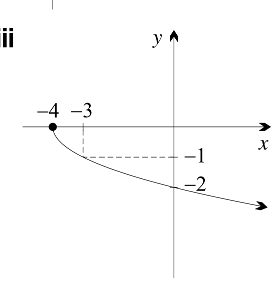

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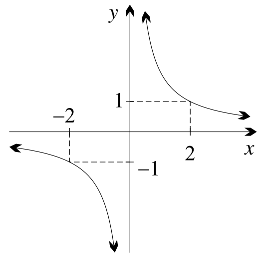
d From $y = \dfrac{2}{x}$:

i shift down 1

ii shift down 1, left 2

iii reflect in the x-axis or in the y-axis



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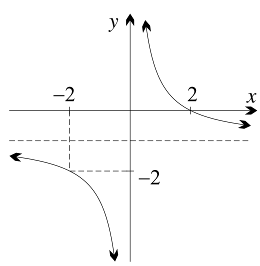

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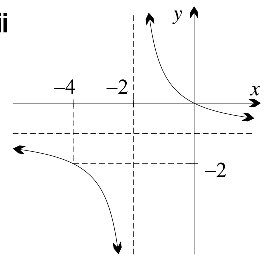

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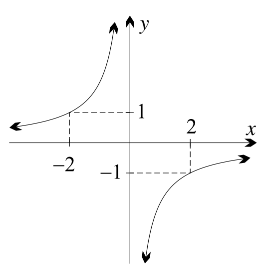
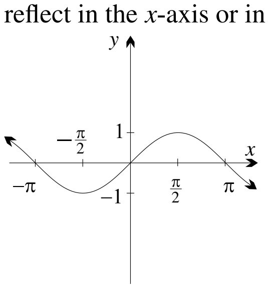


e From $y = \sin x$:

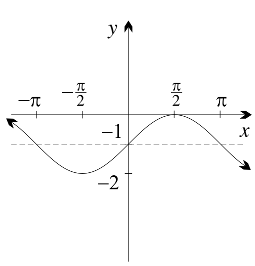
i shift down 1

ii shift down 1, left 2

iii reflect in the x-axis or in the y-axis



i



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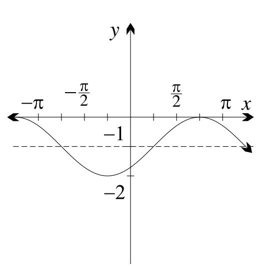

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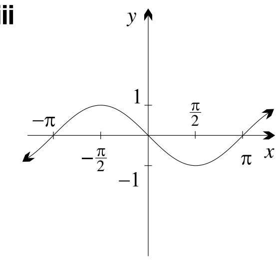


9 a $(1, -2)$ and $(-1, 2)$

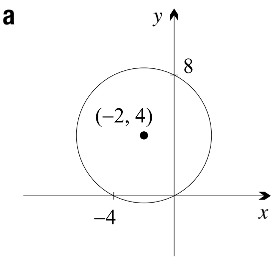
b i $y = x^3 - 3x + 1$

ii $(1, -1)$ and $(-1, 3)$

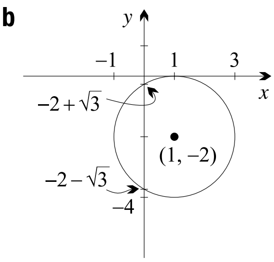
c i $y = x^3 + 3x^2 - 2$

ii $(0, -2)$ and $(-2, 2)$

10 a



b



11 a The parabola $y = x^2$ shifted left 2, down 1.
$y + 1 = (x + 2)^2$

b The hyperbola $xy = 1$ shifted right 2, down 1.
$y + 1 = \frac{1}{x - 2}$

c The exponential $y = 2^x$ reflected in the x-axis, shifted 1 up. $y = 1 - 2^x$

d The curve $y = \cos x$ reflected in the x-axis and shifted 1 up. $y = 1 - \cos x$

12 a The parabola $y = x^2$ reflected in the x-axis, then shifted 3 right and 1 up. $y - 1 = -(x - 3)^2$

b The curve $y = \log_2 x$ reflected in the y-axis, then shifted right 2, down 1. $y + 1 = -\log_2(x - 2)$

c The half parabola $y = \sqrt{x}$ reflected in the x-axis , then shifted left 4 and 2 up.
$y - 2 = -\sqrt{x + 4}$

13 a i The result is a rotation of 180°, so odd functions are unchanged.

ii $\mathcal{I}$ and $\mathcal{H}$ do not commute.

b i $y = f(2a - x)$

ii $x = a$

iii $g(a + t) = g(a - t)$ so $g(x)$ is symmetric in $x = a$.

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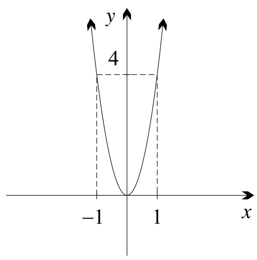
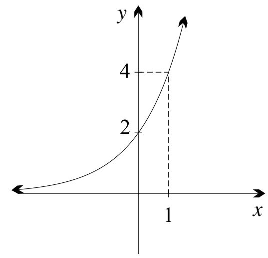
Exercise 3H

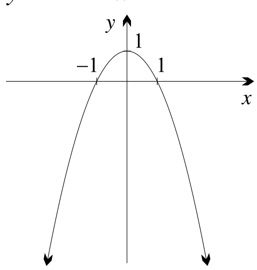
1 a $y = 4x^2$



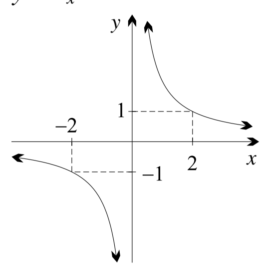
b $y = 2 \times 2^x = 2^{x+1}$



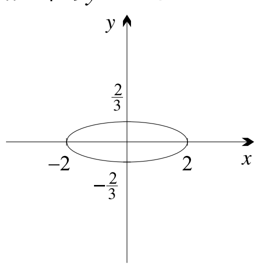
c $y = 1 - x^2$



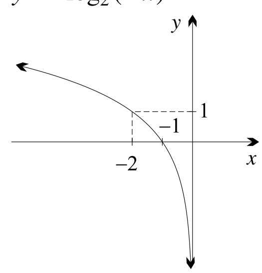
d $y = \frac{2}{x}$



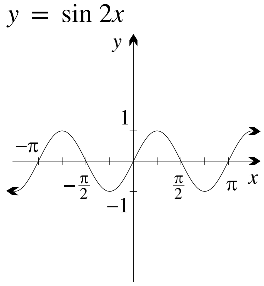
e $x^2 + 9y^2 = 4$



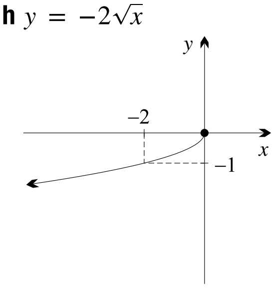
f $y = \log_2(-x)$



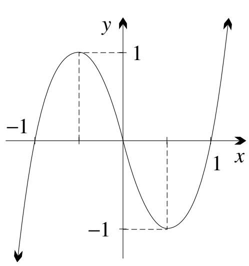
g $y = \sin 2x$



h $y = -2\sqrt{x}$



2 a i



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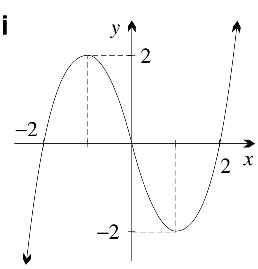
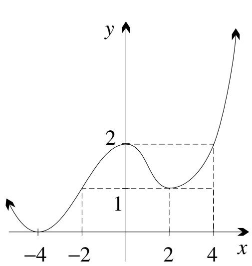


b i



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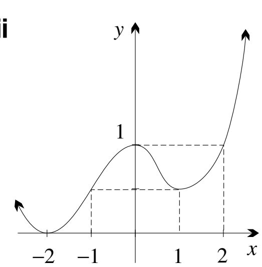
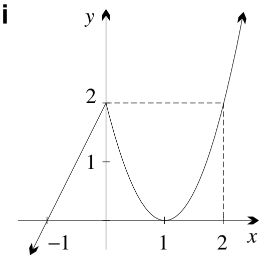


c i



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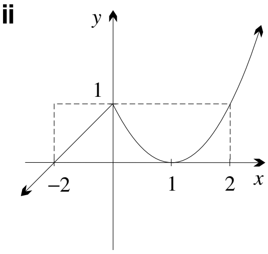
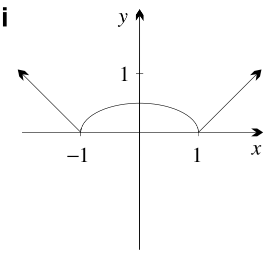


d i



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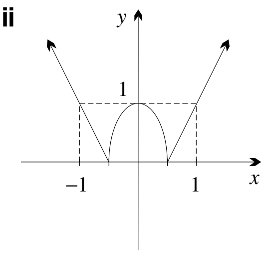
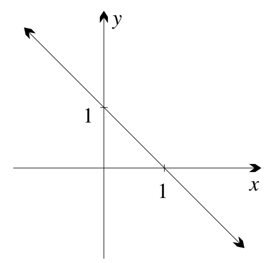
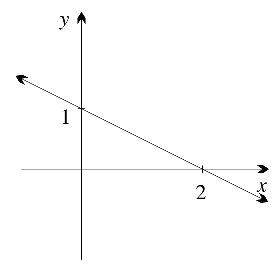


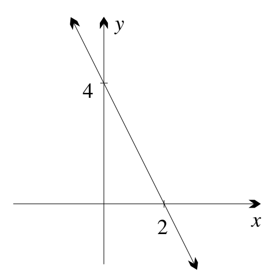
3 a stretch horizontally by factor 2

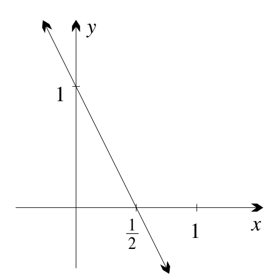
b stretch horizontally by factor 2, vertically by factor 4

c stretch horizontally by factor $\frac{1}{2}$

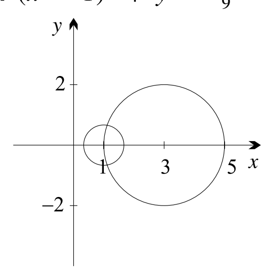




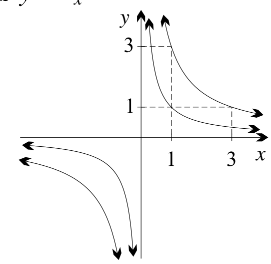




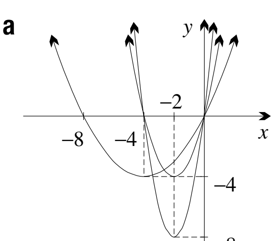
4 a $(x - 1)^2 + y^2 = \frac{4}{9}$



b $y = \frac{3}{x}$



5 a



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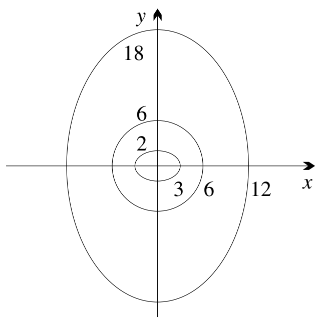
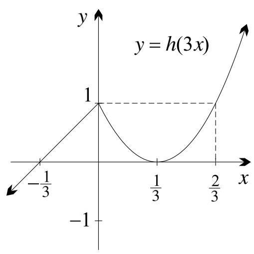
b



6 a i



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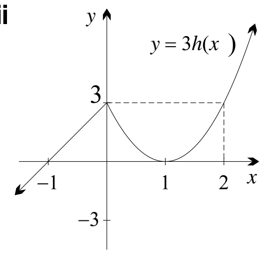
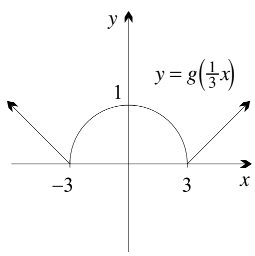


b i



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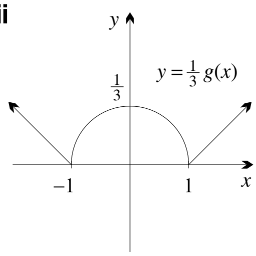
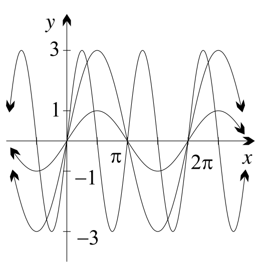


7 a



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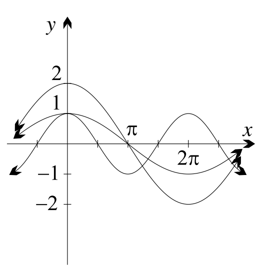


8 a $(1, -2)$ and $(-1, 2)$

b i $y = 2x^3 - 6x$

ii $(1, -4)$ and $(-1, 4)$

c i $y = \frac{1}{27}x^3 - x$

ii $(3, -2)$ and $(-3, 2)$

9 a vertical factor 3 **b** horizontal factor $\frac{1}{2}$

c horizontal factor 4 **d** vertical factor 2

10 a $y = \frac{2}{x}$ **b** $y = \frac{2}{x}$

c Both dilations give the same graph.

d yes: by factor $\sqrt{2}$

11 a $y = 4x^2$ **b** $y = 4x^2$

c Both dilations give the same graph.

d no

12 a $M(0) = 3$ **b** 53 years

c i The mass has been dilated by factor 2, so $M = 6 \times 2^{-\frac{1}{53}t}$

14 a The unit circle $x^2 + y^2 = 1$, horizontally by 3, vertically by 2. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b The exponential $y = 3^x$, vertically by -2. $y = -2 \times 3^x$

c The curve $y = \tan x$, horizontally by 3, vertically by 2. $y = 2\tan\frac{x}{3}$

15 a i stretch vertically by factor 2, $\frac{y}{2} = 2^x$, or translate left by 1, $y = 2^{(x+1)}$

ii stretch along both axes by k, $\frac{y}{k} = \frac{1}{\frac{x}{k}}$, or stretch horizontally by k^2, $y = \frac{1}{\frac{x}{k^2}}$

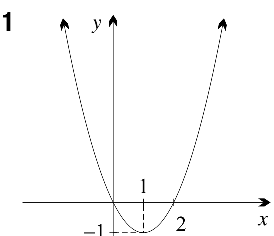
iii reciprocal, $y = \frac{1}{3^x}$, or reflect in the y-axis , $y = 3^{-x}$

16 vertically by factor a^2

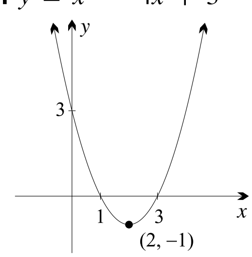
17 stretch horizontally by factor $\sqrt{3}$ and vertically by factor $3\sqrt{3}$

Exercise 3I

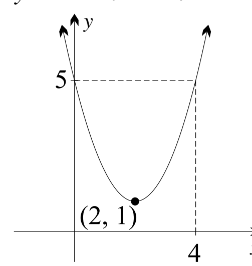
1



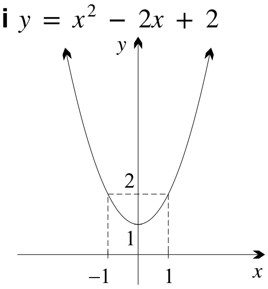
a i $y = x^2 - 4x + 3$



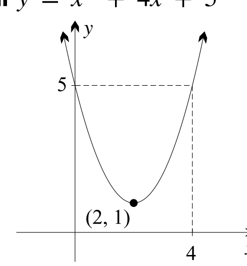
ii $y = x^2 + 4x + 5$



b i $y = x^2 - 2x + 2$



ii $y = x^2 + 4x + 5$



c yes

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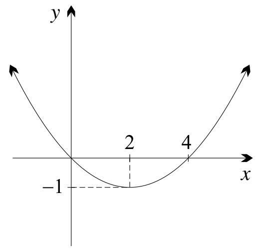
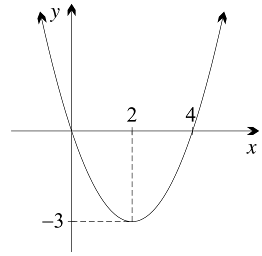
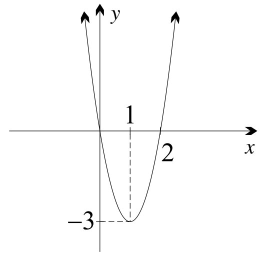
2 a i $y = \frac{1}{4}x^2 - x$



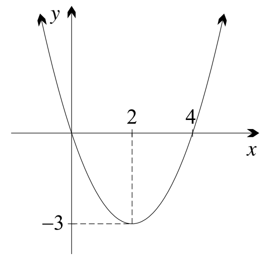
ii $y = \frac{3}{4}x^2 - 3x$



b i $y = 3x^2 - 6x$

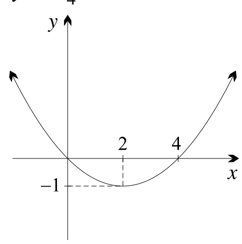


ii $y = \frac{3}{4}x^2 - 3x$

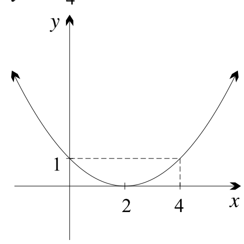


c yes

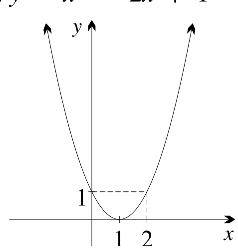
3 a i $y = \frac{1}{4}x^2 - x$



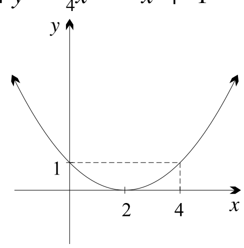
ii $y = \frac{1}{4}x^2 - x + 1$



b i $y = x^2 - 2x + 1$

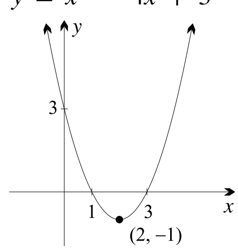


ii $y = \frac{1}{4}x^2 - x + 1$

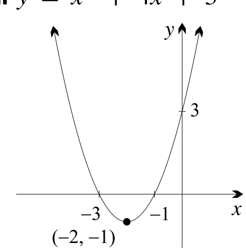


c yes

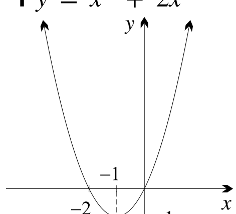
4 a i $y = x^2 - 4x + 3$



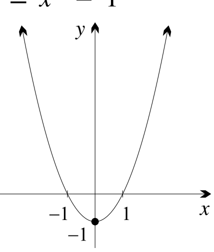
ii $y = x^2 + 4x + 3$



b i $y = x^2 + 2x$



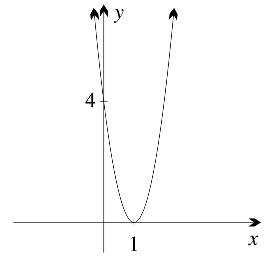
ii $y = x^2 - 1$



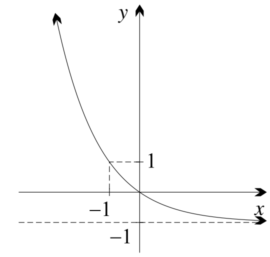
c no: order matters

5 a no **b** no **c** yes **d** yes **e** no **f** yes

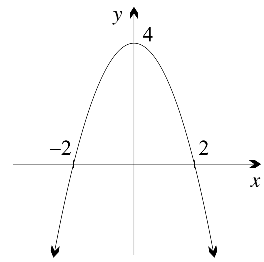
6 a $y = 4(x - 1)^2$



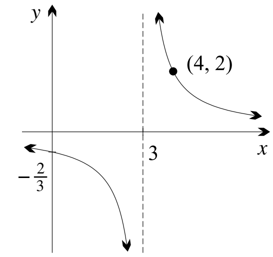
b $y = 2^{-x} - 1$



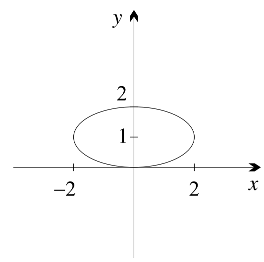
c $y = 4 - x^2$



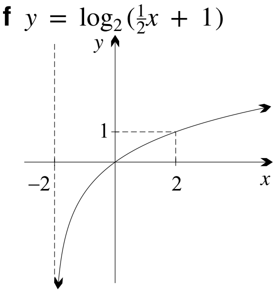
d $y = \frac{2}{x - 3}$



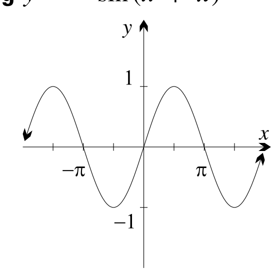
e $x^2 + 4(y - 1)^2 = 4$



f $y = \log_2(\frac{1}{2}x + 1)$

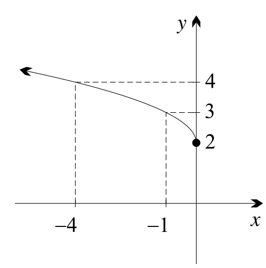


g $y = -\sin(x + \pi)$

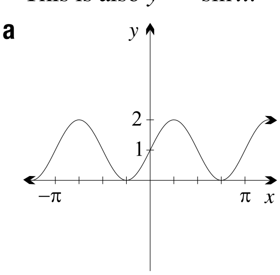


This is also $y = \sin x$.

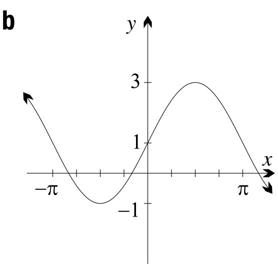
h $y = -\sqrt{x} + 2$



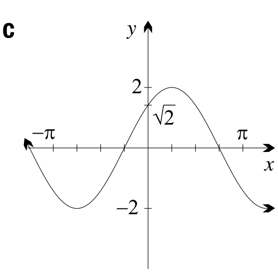
7 a



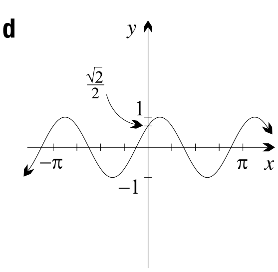
b



c



d



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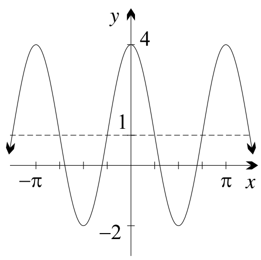
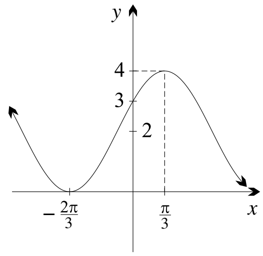
8 a $y = \frac{1}{4}(x + 2)^2 - 4$ **b** $y = \frac{1}{4}(x + 1)^2 - 4$

c $y = 2 - 2^x$ **d** $y = \frac{2}{x - 2} + 1$

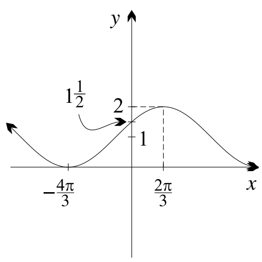
10 a



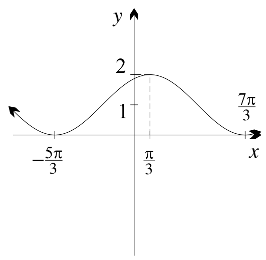
b



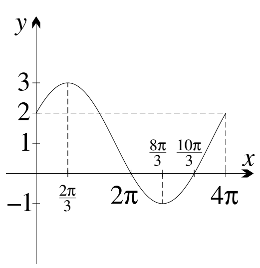
c



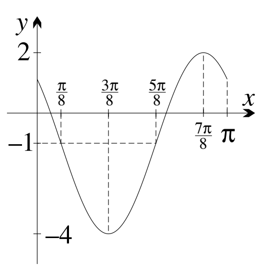
d



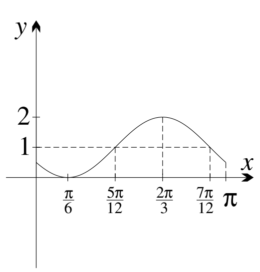
11 a



b



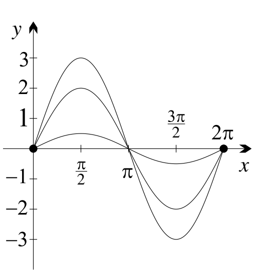
c



12 d If the transformations are dilations or reflections then they commute. A reflection is a special type of dilation, and any pair of dilations commute. If one of the transformations is a translation in a different direction to the other transformation, for example $\mathcal{H}$ and $\mathcal{V}$, then they commute.

Exercise 3J

1 a

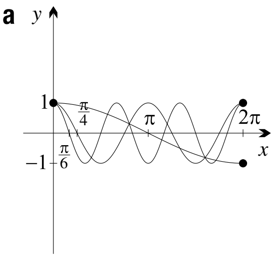


i $\frac{1}{2}$ **ii** 2 **iii** 3

b The graph $y = \sin x$ is stretched vertically by a factor of k.

c The amplitude increases. The bigger the amplitude, the steeper the wave.

2 a

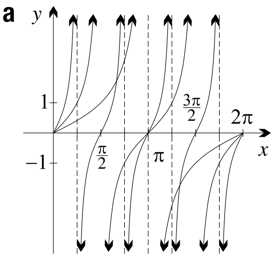


i 4π **ii** π **iii** $\frac{2\pi}{3}$

b The graph $y = \cos x$ is stretched horizontally by a factor of $\dfrac{1}{n}$.

c The period decreases.

3 a

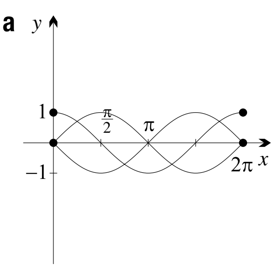


i π **ii** 2π **iii** $\frac{\pi}{2}$

b The graph $y = \tan x$ is stretched horizontally by a factor of $\frac{1}{a}$.

c The period decreases.

4 a

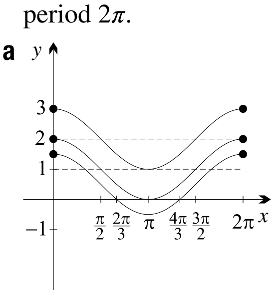


i $\frac{\pi}{2}$ **ii** π **iii** 2π or 0

b The graph $y = \sin x$ is shifted α units to the left.

c The graph stays the same, because $y = \sin x$ has period 2π.

5 a



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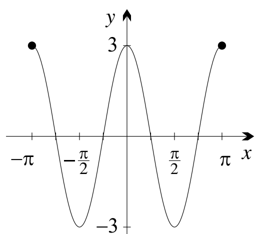
i Range: $0 \le y \le 2$ or $[0, 2]$, mean value: 1
ii Range: $[1, 3]$, mean value: 2
iii Range: $\left[\frac{1}{2}, \frac{3}{2}\right]$, mean value: $\frac{1}{2}$

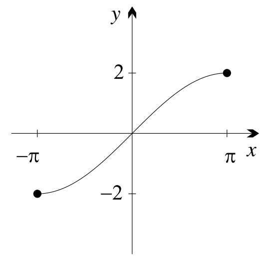
b The graph $y = \cos x$ is shifted c units up, and the mean value is c.

c It moves up.

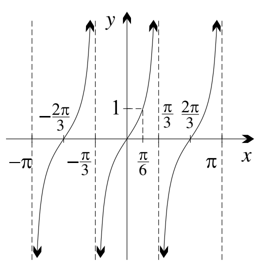
6 a period $= \pi$, amplitude $= 3$



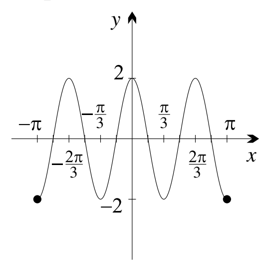
b period $= 4\pi$, amplitude $= 2$



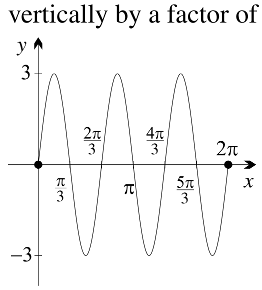
c period $= \frac{2\pi}{3}$, no amplitude



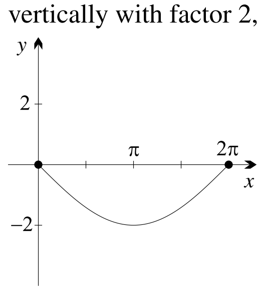
d period $= \frac{2\pi}{3}$, amplitude $= 2$



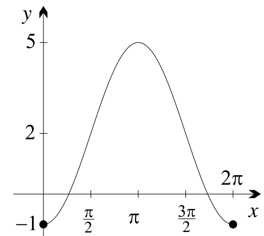
7 a Stretch horizontally by a factor of $\frac{1}{3}$, then stretch vertically by a factor of 3.



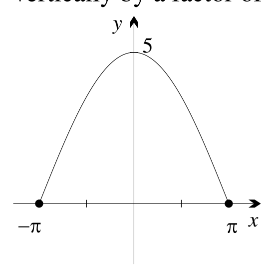
b Stretch horizontally with factor 2, then stretch vertically with factor 2, then reflect in the x-axis.



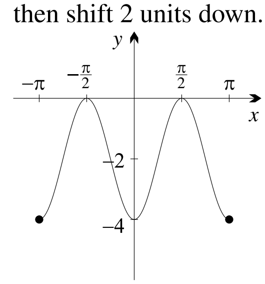
c Shift $\frac{\pi}{2}$ units right, then stretch vertically by a factor of 3, then shift 2 units up.



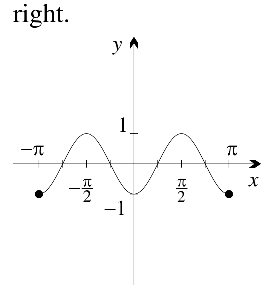
8 a Stretch horizontally by a factor of 2, then stretch vertically by a factor of 5.



b Stretch horizontally by a factor of $\frac{1}{2}$, then stretch vertically by a factor of 2, then reflect in the x-axis, then shift 2 units down.



c Stretch horizontally by a factor of $\frac{1}{2}$, then shift $\frac{\pi}{2}$ units right.



9 a Stretch horizontally by a factor of $\frac{1}{3}$, then shift $\frac{\pi}{6}$ units left.

b Stretch horizontally by a factor of $\frac{1}{4}$, then shift $\frac{\pi}{2}$ units right, then stretch vertically by a factor of $\frac{1}{4}$, then shift 4 units down.

c Stretch horizontally by a factor of 2, then shift $\frac{\pi}{2}$ units left, then stretch vertically by a factor of 6, then reflect in the x-axis.

10 a Part **a**: period $= \frac{2\pi}{3}$, phase $= 0 + \frac{\pi}{2} = \frac{\pi}{2}$.
Part **b**: period $= \frac{2\pi}{4} = \frac{\pi}{2}$, phase $= -\pi$ (but this is twice the period, so we can also say that phase $= 0$).
Part **c**: period $= 4\pi$, phase $= \frac{\pi}{4}$.

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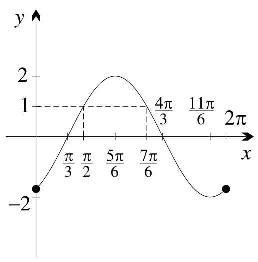
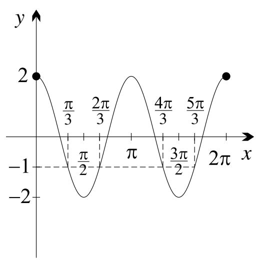
b **i** period $= \pi$, phase $= 2\left(0 - \frac{\pi}{3}\right) = -\frac{2\pi}{3}$
ii period $= 6\pi$, phase $= \frac{\pi}{3}$
iii period $= \frac{\pi}{3}$, phase $= \frac{3\pi}{8}$

11 a $x = \frac{\pi}{2}$ or $\frac{7\pi}{6}$



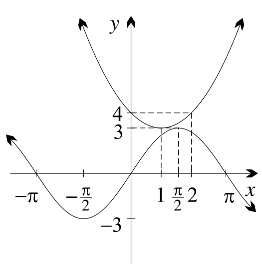
b $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$



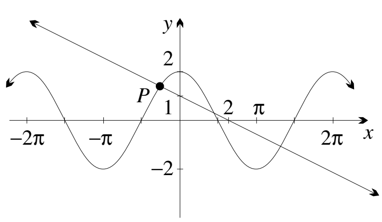
12 a $x \doteqdot 1.675$
b $x \doteqdot 0.232$ or 1.803

13 a $(1, 3)$

b

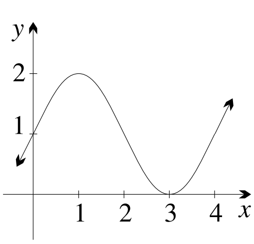


14



c 3
d P is in the second quadrant.

15



a 4
c the origin
d $m > \frac{1}{4}$ or $m = 0$

16 a 10 metres
b 6 metres
c 2 pm
d 9.20 am

17 c amplitude $= 5$

18 a **i** 1
ii $0 < k < 1$
b **ii** 1.3
iii $\angle AOB = 2\theta \doteqdot 2.6$ radians
c **ii** $\ell > 300$

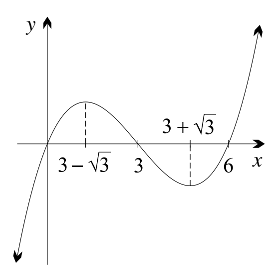
Chapter 3 review exercise

1 a **i** $-1 < x < 2$
ii $(-1, 2)$
b **i** $-1 \le x < 2$
ii $[-1, 2)$
c **i** $x \le 2$
ii $(-\infty, 2]$

2 a **i** 0
ii 4
iii 8
iv 0
b **i** $x^2 + 2x$
ii x^2
iii $x^4 - 2x^2$
iv $x + 2$

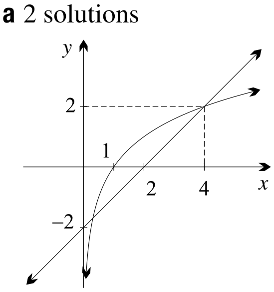
3 a $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$
b $f(x) \to \frac{1}{2}$ as $x \to \infty$ and $x \to -\infty$
c $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

4 $y = x(x - 3)(x - 6)$
a $-\infty < x < \infty$
b $(0, 0), (3, 0), (6, 0)$
c no asymptotes
e $y' = 3(x^2 - 6x + 6)$ so $y' = 0$ at $x = 3 - \sqrt{3}$ or $x = 3 + \sqrt{3}$



5 a $-4 \le x < 2$, $[-4, 2)$
b $-\frac{3}{2} < x < 0$, $\left(-\frac{3}{2}, 0\right)$
c $-3 \le x < \frac{1}{2}$, $[-3, \frac{1}{2})$
d $-4 \le x \le 10$, $[-6, 10]$

6 a 2 solutions



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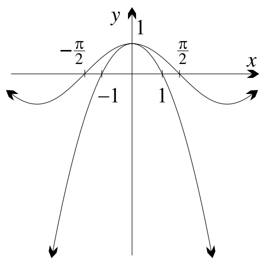
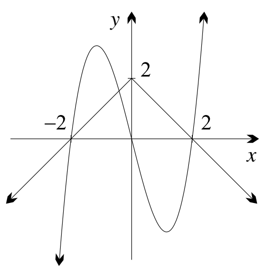
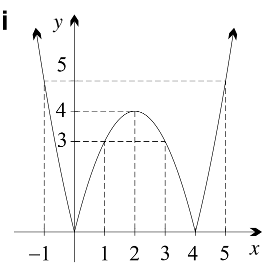
b 1 solution



c 3 solutions



7 a i



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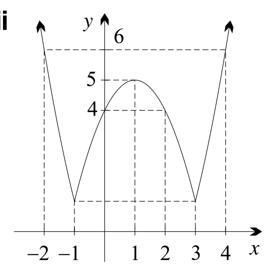
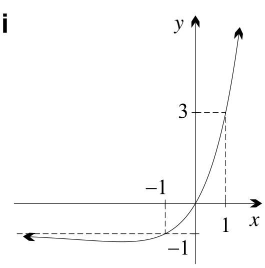
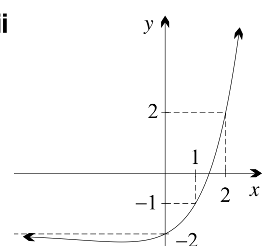


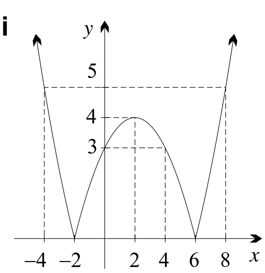
b i



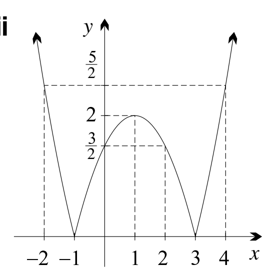
ii



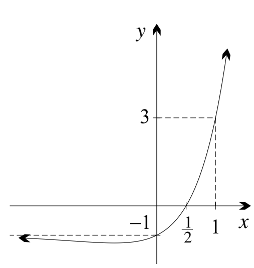
8 a i



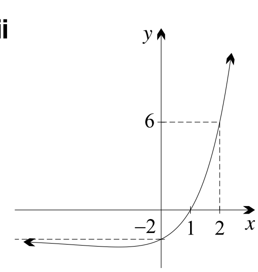
ii



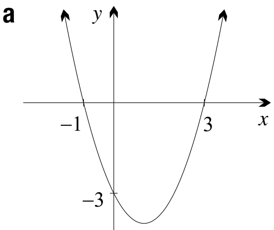
b i



ii

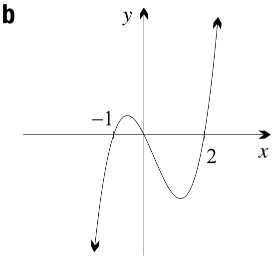


9 a



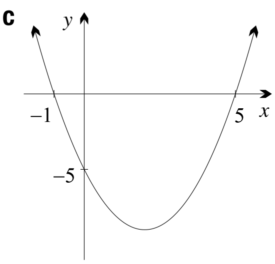
$-1 \leq x \leq 3$

b



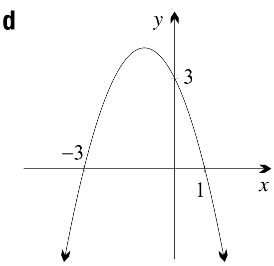
$x \leq -1$ or $0 \leq x \leq 2$

c



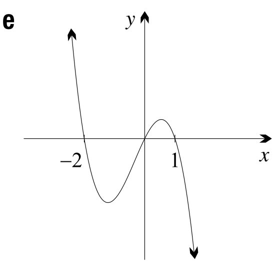
$-1 \leq x \leq 5$

d



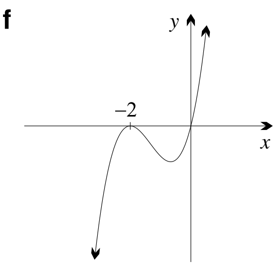
$x \leq -3$ or $x \geq 1$

e



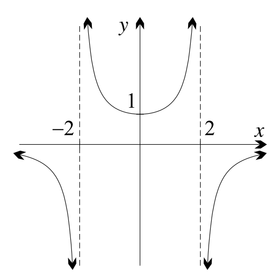
$x \leq -2$ or $0 \leq x \leq 1$

f



$x \leq 0$

10



a $x \neq -2, 2$

b $(0, 1)$

c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$

e as $x \to 2^+$, $y < 0$ so $y \to -\infty$, as $x \to 2^-$, $y > 0$ so $y \to \infty$, as $x \to -2^+$, $y > 0$ so $y \to \infty$, and as $x \to -2^-$, $y < 0$ so $y \to -\infty$

f $(-\infty, 0) \cup [1, \infty)$

11 a $y = \frac{3(x+1)}{(x+3)(x-1)}$

b domain: $x \neq 1$ and $x \neq -3$ intercepts: $(-1, 0)$ and $(0, -1)$

c The domain is not symmetric about $x = 0$.

d $x = -3$, $x = 1$, and $y = 0$

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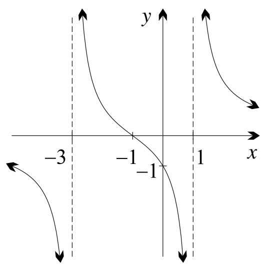
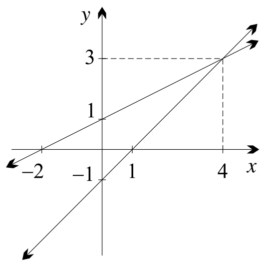
e



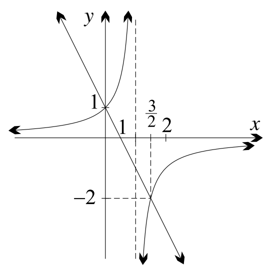
12 a $x = -3\frac{1}{2}$ or $3\frac{1}{2}$ **b** $x = \frac{1}{3}$ or 1
c $-3 \le x \le \frac{1}{3}$ **d** $x < -2$ or $x > -\frac{1}{3}$

13 a



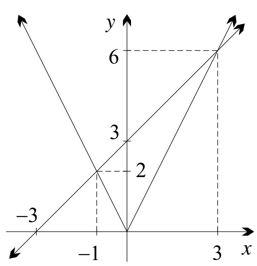
$x \ge 4$

b



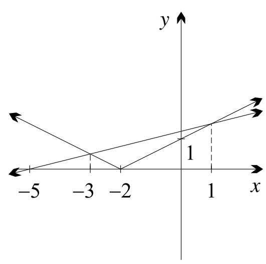
$0 < x < 1$ or $x > 1\frac{1}{2}$

c



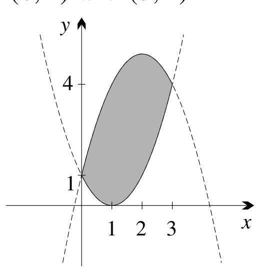
$-1 \le x \le 3$

d



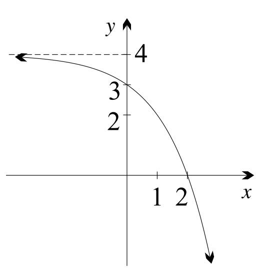
$x < -3$ or $x > 1$

14 a $(0, 1)$ and $(3, 4)$
b

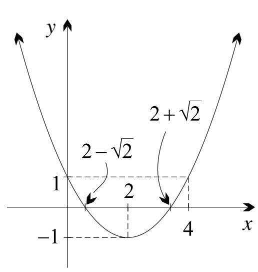


15 a $y = (x - 2)^2 + 1$ **b** $y = \frac{1}{x + 2} - 3$
c $y = \sin(x + \frac{\pi}{6}) - 1$ **d** $y = e^{x-2} + 1$
16 a horizontal factor 2 **b** vertical factor $\frac{1}{2}$
c vertical factor $\frac{1}{3}$ **d** horizontal factor 2
17 a yes **b** no **c** no **d** yes

18 a



b



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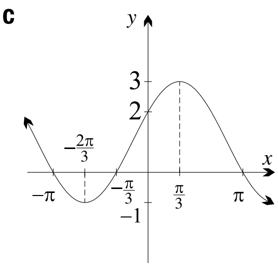
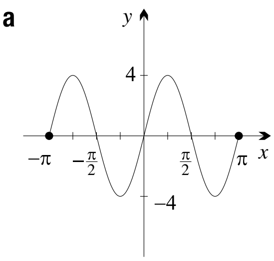
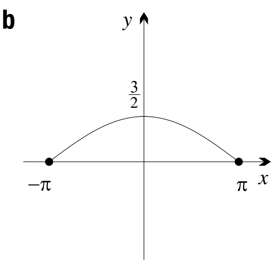


19 a



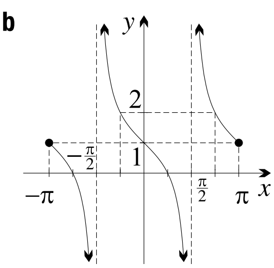
amplitude is 4, period is π

b



amplitude is $\frac{3}{2}$, period is 4π

20 a Reflect in the x-axis , then shift up 1 unit.
b



21 a Reflect in the y-axis, then stretch vertically with factor 3, then shift down 2 units. Actually, the first transformation, reflect in the y-axis , is unnecessary because $y = \cos x$ is even.
b Stretch horizontally with factor $\frac{1}{4}$, and vertically with factor 4. There is no need to shift left $\frac{\pi}{2}$ units because the period is $\frac{\pi}{2}$.
c Stretch horizontally with factor $\frac{1}{2}$, then shift right $\frac{\pi}{6}$ units.

22 a 0
b $4(0 + \frac{\pi}{2}) = 2\pi$, or more simply 0
c $0 - \frac{\pi}{3} = -\frac{\pi}{3}$

23 a 3
b 3 solutions, 1 positive solution
c Outside this domain the line is beyond the range of the sine curve.

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Chapter 4

Exercise 4A

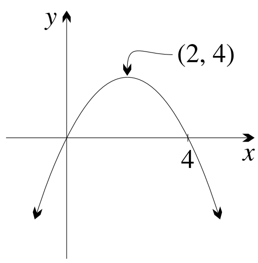
1 a A, G and I **b** C and E

c B, D, F and H

3 a $4 - 2x$

b i $x < 2$ **ii** $x > 2$ **iii** $x = 2$

c

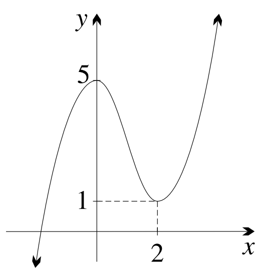


4 a $3x^2 - 6x$

b i $x < 0$ or $x > 2$ **ii** $0 < x < 2$

iii $x = 0$ or $x = 2$

c



5 a $\dfrac{3}{x^2}$

b The function is not continuous at $x = 0$.

6 a $x > 2$ **b** $x < -3$

c $x > 1$ or $x < -1$ **d** $x < 0$ or $x > 2$

7 a $-\frac{1}{3} < x < 1$ **b** $x < -2$ or $x > 4$

8 a $f'(x) = x^2 + 2x + 5$

b $f'(x) = (x + 1)^2 + 4 > 0$ for all x.

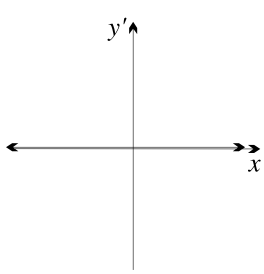
c The graph of $f(x)$ is increasing for all x, so it has exactly one x-intercept.

9 a $f'(x) = -\dfrac{6}{(x - 3)^2} < 0$ for $x \neq 3$.

b $f'(x) = \dfrac{x^2(x^2 + 3)}{(x^2 + 1)^2} > 0$ for $x \neq 0$.

11 a III **b** I **c** IV **d** II

12 a



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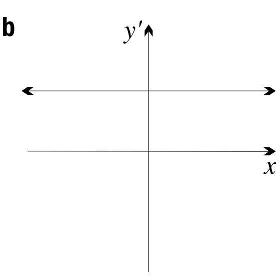

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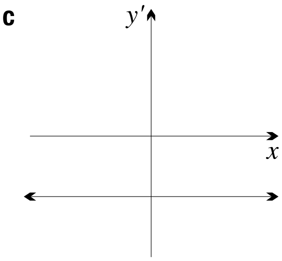

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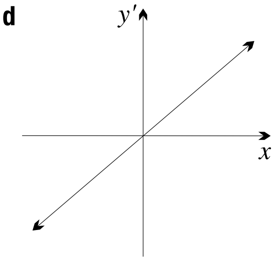

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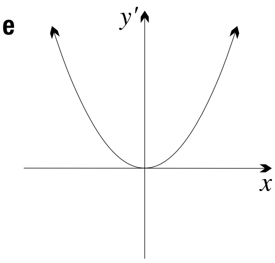

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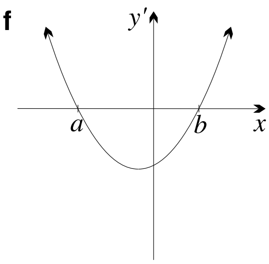

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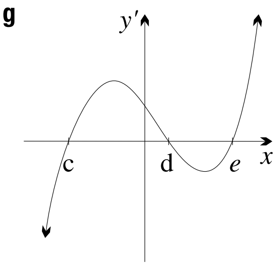
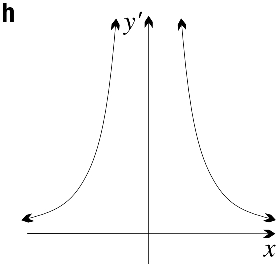


h

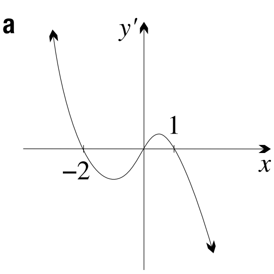


13 $-2 < x < 0$

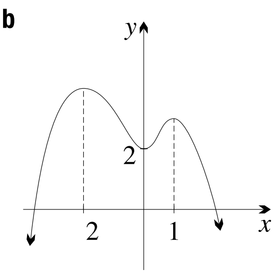
14 a i $f'(x) = \dfrac{-4x}{(x^2 + 1)^2}$ **ii** $f(0) = 1$

b $f(x)$ is a continuous function that is increasing for $x < 0$, decreasing for $x > 0$ and stationary at $x = 0$. So it reaches its maximum value at $x = 0$.

15 a



b



Exercise 4B

1 a $x = 3$ **b** $x = -2$ **c** $x = 1$ or -1

2 a $(2, 3)$ **b** $(4, 0)$

c $(1, -2)$ **d** $(1, 0)$

e $(0, 0)$ and $(2, -4)$ **f** $(1, -2)$

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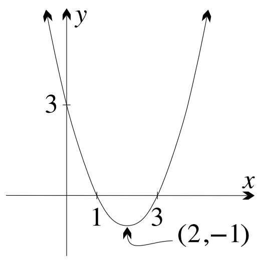
3 a



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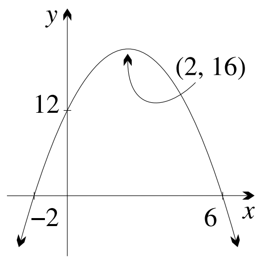

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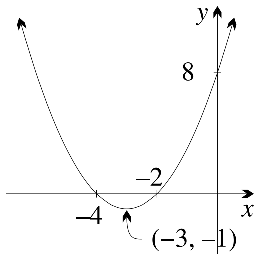

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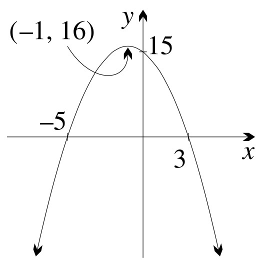
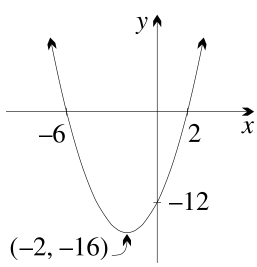


4 a minimum **b** maximum **c** minimum

d horizontal (or stationary) point of inflection

5 a



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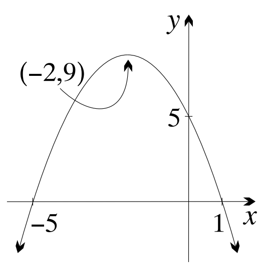
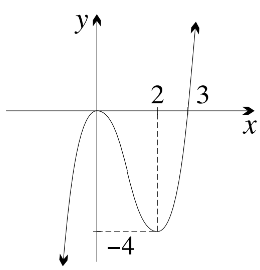


6 c



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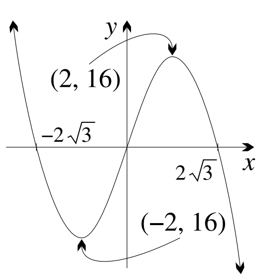
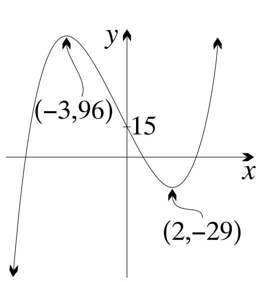


8 a



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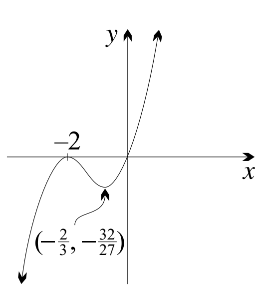

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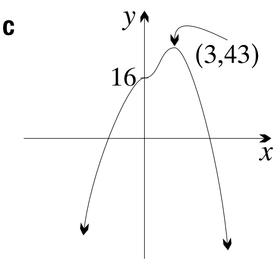

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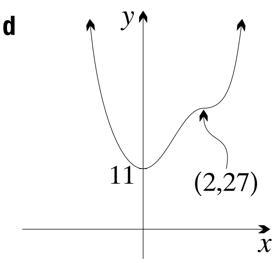

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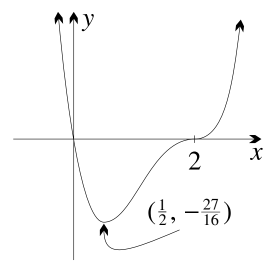

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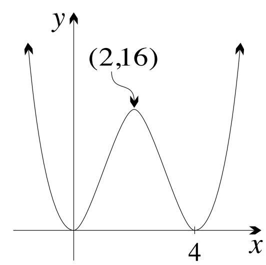

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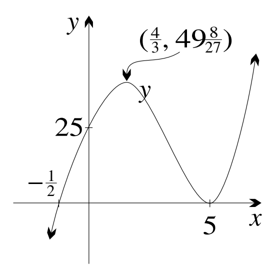


12 a $a = -8$ **b** $a = 2$

13 a $a = 2$ and $c = 3$ **b** $b = -3$ and $c = -24$

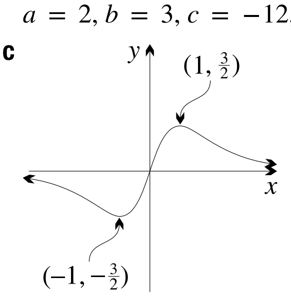
14 b $a = b = -1, c = 6$

15 a The curve passes through the origin.

c $a = -1$

16 $a = 2, b = 3, c = -12, d = 7$

17 c



d **i** no roots **ii** 1 root

iii 2 roots **iv** 1 root

19 c Hint: Consider the equation $P'(x) = 0$.

Answers 4B

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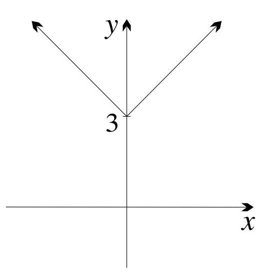
Exercise 4C

1 a A maximum turning point, B minimum turning point
b C minimum
c D horizontal point of inflection, E maximum turning point
d F minimum turning point, G maximum, H minimum turning point
e I minimum
f J horizontal point of inflection, K minimum turning point, L maximum turning point

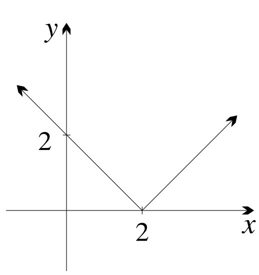
2 a $x = 0$ turning point, $x = 3$ horizontal point of inflection
b $x = -2$ turning point, $x = 4$ turning point
c $x = 0$ turning point, $x = 1$ discontinuity of y'
d $x = 0$ horizontal point of inflection, $x = 1$ discontinuity of y'
e $x = 0$ turning point, $x = 1$ discontinuity of y'
f $x = 0$ horizontal point of inflection, $x = 1$ discontinuity of y'
g $x = -1$ turning point, $x = 1$ turning point, $x = 0$ discontinuity of y'
h $x = 1$ turning point, $x = 0$ discontinuity of y'
i $x = 2$ turning point, $x = -2$ discontinuity of y', $x = 1$ discontinuity of y'

3 a



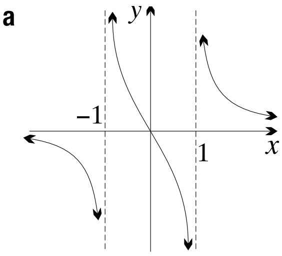
b When $x > 0$, $y' = 1$. When $x < 0$, $y = -1$.
c y' is not defined at $(0, 3)$.

4 a



b When $x > 2$, $y' = 1$. When $x < 2$, $y' = -1$
c y' is not defined at $(2, 0)$.

5 a



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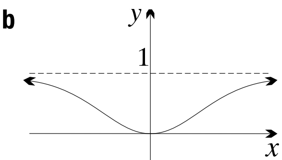

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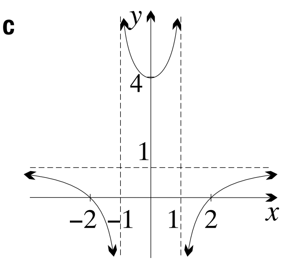

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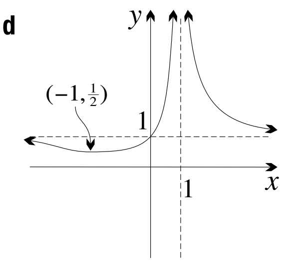

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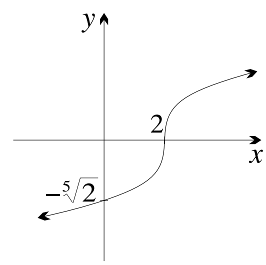
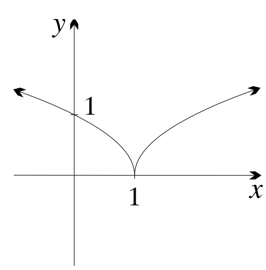


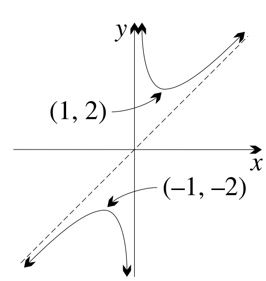
a $f'(x) = \frac{1}{5}(x - 2)^{-\frac{4}{5}}$
b There is a vertical tangent at $(2, 0)$.

7



b There is a cusp at $(1, 0)$.
c $f(x) = (x - 1)^{\frac{2}{3}}$ in Question 7 has a global minimum at $(1, 0)$.

8



a $x \neq 0$
b Zeroes at $x = 1$ and $x = -1$, discontinuity at $x = 0$.

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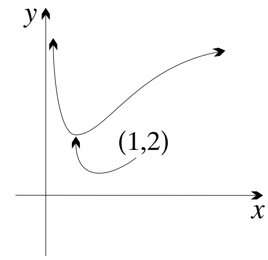
d The oblique asymptote is $y = x$ because $y - x \to 0$ as $|x| \to \infty$. The y-axis is a vertical asymptote.

9



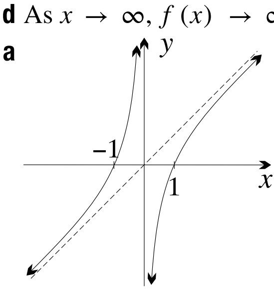
a $x > 0$

b $x = 1$

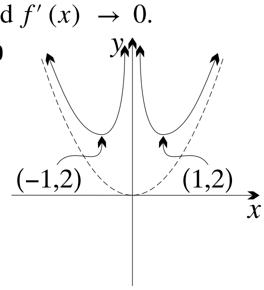
c $(1, 2)$ is a minimum turning point.

d As $x \to \infty$, $f(x) \to \infty$ and $f'(x) \to 0$.

10 a



b



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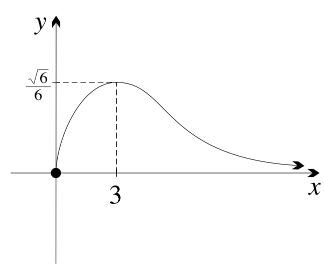
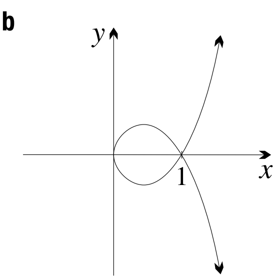


a domain: $x \geq 0$. horizontal asymptotes: $x = 0$

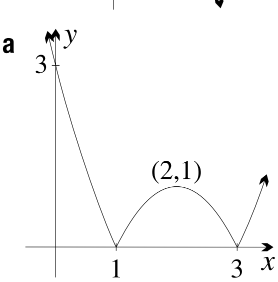
c $\left(3, \frac{1}{6}\sqrt{6}\right)$ is a maximum turning point.

d As $x \to 0^+$, $y \to 0$ and $y' \to \infty$, so the curve emerges vertically from the origin. (Notice that $y(0) = 0$, so the origin lies on the curve.)

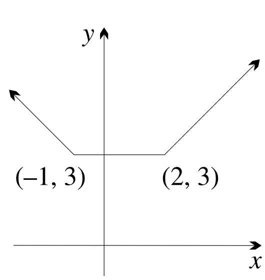
12 b



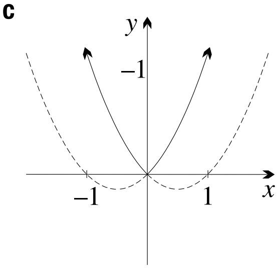
13 a



b



c



Exercise 4D

1 a $3x^2, 6x, 6$ **b** $10x^9, 90x^8, 720x^7$

c $7x^6, 42x^5, 210x^4$ **d** $2x, 2, 0$

e $8x^3, 24x^2, 48x$ **f** $15x^4, 60x^3, 180x^2$

g $-3, 0, 0$ **h** $2x - 3,\ 2,\ 0$

i $12x^2 - 2x, 24x - 2, 24$

j $20x^4 + 6x^2, 80x^3 + 12x, 240x^2 + 12$

2 a $2x + 3, 2$ **b** $3x^2 - 8x, 6x - 8$

c $2x - 1, 2$ **d** $6x - 13, 6$

e $30x^4 - 36x^3, 120x^3 - 108x^2$

f $32x^7 + 40x^4, 224x^6 + 160x^3$

3 a $0.3x^{-0.7}, -0.21x^{-1.7}, 0.357x^{-2.7}$

b $-\dfrac{1}{x^2}, \dfrac{2}{x^3}, -\dfrac{6}{x^4}$ **c** $-\dfrac{2}{x^3}, \dfrac{6}{x^4}, -\dfrac{24}{x^5}$

d $-\dfrac{15}{x^4}, \dfrac{60}{x^5}, -\dfrac{300}{x^6}$ **e** $2x - \dfrac{1}{x^2}, 2 + \dfrac{2}{x^3}, -\dfrac{6}{x^4}$

4 a $-\dfrac{3}{x^4}, \dfrac{12}{x^5}$ **b** $-\dfrac{4}{x^5}, \dfrac{20}{x^6}$

c $-\dfrac{6}{x^3}, \dfrac{18}{x^4}$ **d** $-\dfrac{6}{x^4}, \dfrac{24}{x^5}$

5 a $2(x + 1), 2$

b $9(3x - 5)^2, 54(3x - 5)$

c $8(4x - 1), 32$

d $-11(8 - x)^{10}, 110(8 - x)^9$

6 a $\dfrac{-1}{(x + 2)^2}, \dfrac{2}{(x + 2)^3}$ **b** $\dfrac{2}{(3 - x)^3}, \dfrac{6}{(3 - x)^4}$

c $\dfrac{-15}{(5x + 4)^4}, \dfrac{300}{(5x + 4)^5}$ **d** $\dfrac{12}{(4 - 3x)^3}, \dfrac{108}{(4 - 3x)^4}$

7 a $\dfrac{1}{2\sqrt{x}}, \dfrac{-1}{4x\sqrt{x}}$ **b** $\frac{1}{3}x^{-\frac{2}{3}}, -\frac{2}{9}x^{-\frac{5}{3}}$

c $\frac{3}{2}\sqrt{x}, \dfrac{3}{4\sqrt{x}}$ **d** $-\frac{1}{2}x^{-\frac{3}{2}}, \frac{3}{4}x^{-\frac{5}{2}}$

e $\dfrac{1}{2\sqrt{x + 2}}, \dfrac{-1}{4(x + 2)^{\frac{3}{2}}}$ **f** $\dfrac{-2}{\sqrt{1 - 4x}}, \dfrac{-4}{(1 - 4x)^{\frac{3}{2}}}$

8 a $f'(x) = 3x^2 + 6x + 5, f''(x) = 6x + 6$

b i 5 **ii** 14 **iii** 6 **iv** 12

9 a i 15 **ii** 12 **iii** 6 **iv** 0

b i -8 **ii** 48 **iii** -192 **iv** 384

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10 a $\dfrac{1}{(x+1)^2}, \dfrac{-2}{(x+1)^3}$ **b** $\dfrac{7}{(2x+5)^2}, \dfrac{-28}{(2x+5)^3}$

11 $(x-1)^3(5x-1), 4(x-1)^2(5x-2)$

12 a $1, -1$ **b** $-\frac{1}{3}$

13 a $nx^{n-1}, n(n-1)x^{n-2}, n(n-1)(n-2)x^{n-3}$

b $n(n-1)(n-2)\ldots 1, 0$

15 $a = 3, b = 4$

Exercise 4E

1

Point	A	B	C	D	E	F	G	H	I
y'	0	+	0	−	0	−	0	+	0
y''	+	0	−	0	0	0	+	0	0

2 a concave down **b** concave up

c concave up **d** concave down

3 a minimum **b** maximum

c minimum **d** minimum

4 a $y'' = 2$, so $y'' > 0$ for all values of x.

b $y'' = -6$, so $y'' < 0$ for all values of x.

5 a $y'' = 6x - 6$

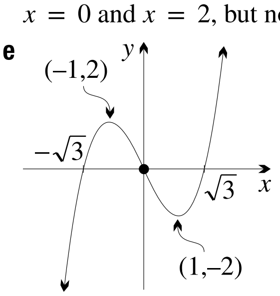
b i $x > 1$ **ii** $x < 1$

6 a $y'' = 6x - 2$

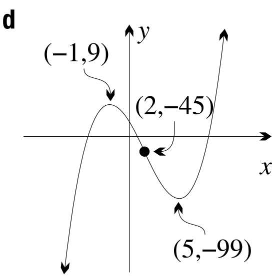
b i $x > \frac{1}{3}$ **ii** $x < \frac{1}{3}$

7 $x = 0$ and $x = 2$, but not $x = -3$.

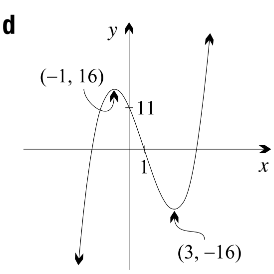
8 e



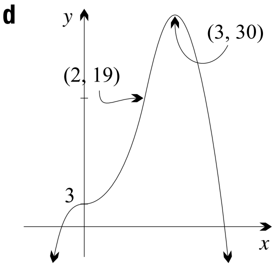
9 d



10 d



11 d



12 a $x > 2$ or $x < -1$ **b** $-1 < x < 2$

c $x > \frac{1}{2}$ **d** $x < \frac{1}{2}$

13 a $y' = 3x^2 + 6x - 72, y'' = 6x + 6$

d $75x + y - 13 = 0$

14 b $f''(x) = g''(x) = 0$, no

c $f(x)$ has a horizontal (or stationary) point of inflection, $g(x)$ has a minimum turning point.

15 a $y'' = 6x - 2a, a = 6$

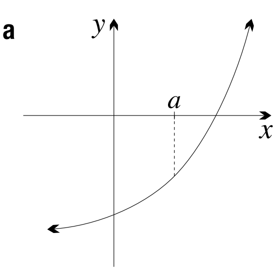
b $y'' = 6x + 4a, a > 1\frac{1}{2}$

c $y'' = 12x^2 + 6ax + 2b, a = -5, b = 6$

d $a > -\frac{2}{3}$

16 a Increasing. **b** Concave down.

17 a



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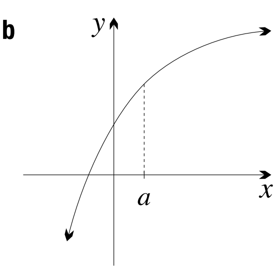

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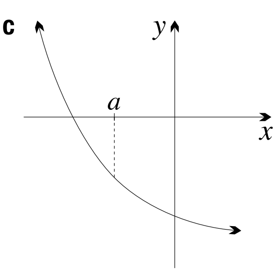
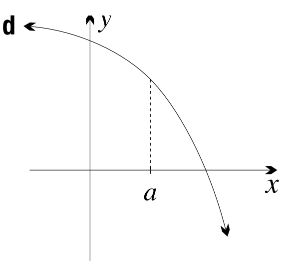


d



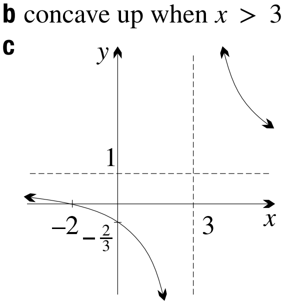
18 a $y' = (x-3)^2 + 2 \geq 2$ for all real x.

b There is a point of inflection at $x = 3$.

c One, because the function is continuous and increasing for all real x.

19 b concave up when $x > 3$, concave down when $x < 3$

c



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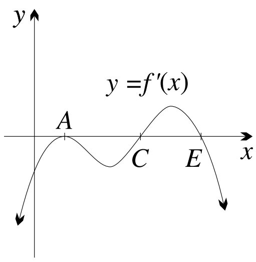
20 a



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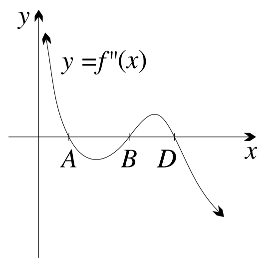
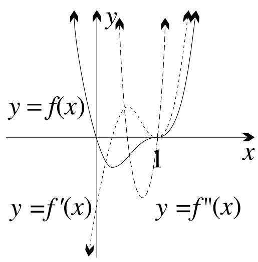
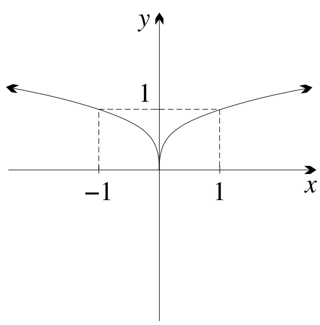


21 b



22 $a = 2, b = -3, c = 0$ and $d = 5$

23 b



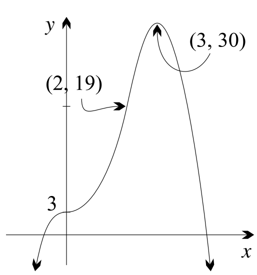
Exercise 4F

1 a $(6, 0)$ **b** $(4, 32)$ **c** $(2, 16)$

2 a $x = -1$ or $x = 2$ **b** $x = 0$

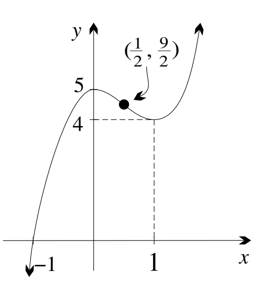
c $-1 < x < 2$ **d** $x < 0$

3 a Show that $f(-x) = -f(x)$. Point symmetry in the origin.



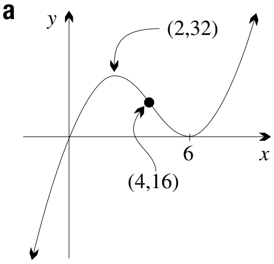
e When $x = 0, y' = 27$.

4

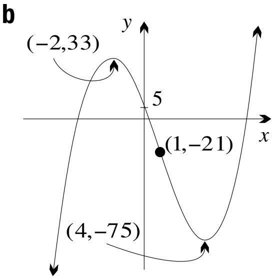


When $x = \frac{1}{2}, y' = 1\frac{1}{2}$.

5 a



b



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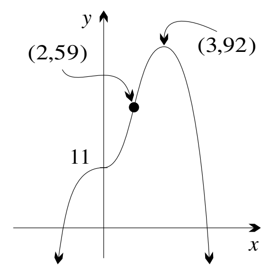

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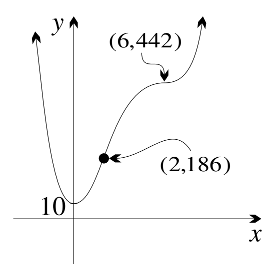

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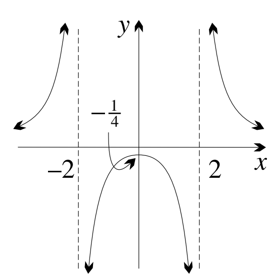


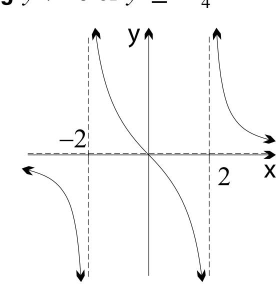
c line symmetry in the y-axis

d domain: $x \neq 2$ and $x \neq -2$, asymptotes: $x = 2$ and $x = -2$

e $y = 0$

g $y > 0$ or $y \leq -\frac{1}{4}$

9



c gradient $= -\frac{1}{4}$

d domain: $x \neq 2$ and $x \neq -2$, asymptotes: $x = 2$ and $x = -2$

e $y = 0$

f point symmetry in the origin

i all real y

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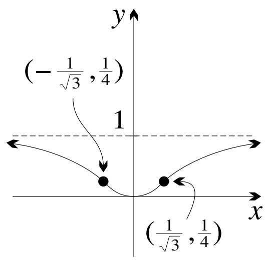
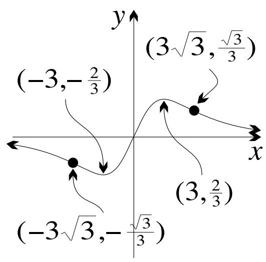
10 e



11 e



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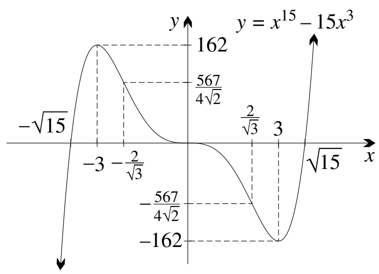
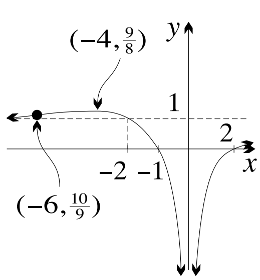
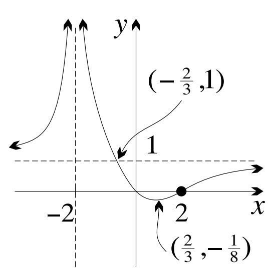


x-intercepts at 0, $\sqrt{15}$ and $-\sqrt{5}$, maximum turning point at $(-3, 162)$, horizontal point of inflection at $(0, 0)$, minimum turning point at $(3, -162)$, points of inflection at $(-\frac{3}{\sqrt{2}}, \frac{567}{4\sqrt{2}})$ and $(\frac{3}{\sqrt{2}}, -\frac{567}{4\sqrt{2}})$.

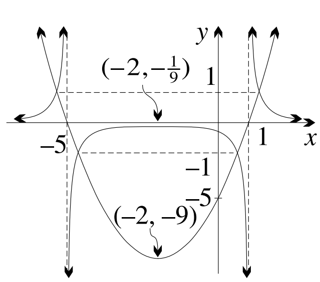
13 a



b



14 a



b $(-2 + 2\sqrt{2}, -1)$, $(-2 - 2\sqrt{2}, -1)$, $(-2 + \sqrt{10}, 1)$, $(-2 - \sqrt{10}, 1)$

Exercise 4G

1 a A local maximum, B local minimum

b C global maximum, D local minimum, E local maximum, F global minimum

c G global maximum, H horizontal point of inflection

d I horizontal point of inflection, J global minimum

2 a $0,\ 4$ **b** $2,\ 5$ **c** $0,\ 4$ **d** $0,\ 5$

e $0, 2\sqrt{2}$ **f** $-1, -\frac{1}{4}$ **g** $-1, 2$

3 a $-1, 8$ **b** $-49, 5$ **c** $0,\ 4$ **d** $0,\ 9$

4 a global minimum -5, global maximum 20

b global minimum -5, local maximum 11, global maximum 139

c global minimum 4, global maximum 11

Exercise 4H

1 a $P = 12x - 2x^2$ **b** 3 **c** 18

2 a $Q = 2x^2 - 16x + 64$

b 4 **c** 32

3 After 2 hours and 40 minutes.

4 c 10 **d** $200\ \text{m}^2$

5 d 24 cm

6 b $x = 30$ m and $y = 20$ m

7 c $h = 2, w = \frac{3}{2}$

8 a $\frac{x}{4}, \frac{10 - x}{4}$ **c** 5 **d** $\frac{25}{8}\ \text{cm}^2$

9 a $R = x(47 - \frac{1}{3}x)$ **b** $-\frac{8}{15}x^2 + 32x - 10$ **c** 30

10 c 4 cm by 4 cm by 2 cm

11 c $\frac{10}{3}$

12 b Width $16\sqrt{3}$ cm and depth $16\sqrt{6}$ cm.

13 b 15 cm by 5 cm by 3.75 cm

14 c $48\ \text{cm}^2$

15 a $c = \pi x^2 a + 2\pi xhb$

16 d $2(\sqrt{10} + 1)$ cm by $4(\sqrt{10} + 1)$ cm

17 b 80 km/h **c** \$400

18 a $I_c = \frac{W}{x^2} + \frac{2w}{(30 - x)^2}$ **b** 13.27 cm

Exercise 4I

1 c $\frac{10}{3\pi}$

d $\frac{1000}{27\pi}\ \text{m}^3$

2 c $20\sqrt{10}\,\pi\ \text{cm}^3$

3 a $S = \pi r_1^{\ 2} + \pi(k - r_1)^2$

5 d $r = 8$

8 b $V = \frac{S}{2}r - \frac{1}{2}\pi r^3$

9 4:3

11 c $2\pi R^2$

12 $r : h = 1:2$

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Exercise 4J

1 a $\frac{1}{7}x^7 + C$ **b** $\frac{1}{4}x^4 + C$
c $\frac{1}{11}x^{11} + C$ **d** $\frac{3}{2}x^2 + C$
e $5x + C$ **f** $\frac{1}{2}x^{10} + C$
g $3x^7 + C$ **h** C

2 a $\frac{1}{3}x^3 + \frac{1}{5}x^5 + C$ **b** $x^4 - x^5 + C$
c $\frac{2}{3}x^3 + \frac{5}{8}x^8 + C$ **d** $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + c$
e $3x - 2x^2 + 2x^8 + C$ **f** $x^3 - x^4 - x^5 + C$

3 a $\frac{1}{3}x^3 - \frac{3}{2}x^2 + C$ **b** $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + C$
c $x^3 + \frac{11}{2}x^2 - 4x + C$ **d** $\frac{5}{6}x^6 - x^4 + C$
e $x^8 + \frac{1}{2}x^4 + C$
f $\frac{1}{2}x^2 + \frac{1}{4}x^4 - 3x - x^3 + C$

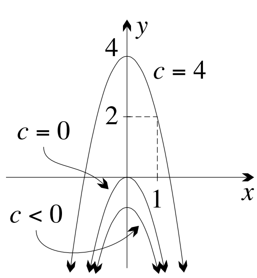
4 a **i** $y = x^2 + 3x + 3$ **ii** $y = x^2 + 3x + 4$
b **i** $y = 3x^3 + 4x + 1$ **ii** $y = 3x^3 + 4x - 2$
c **i** $y = x^3 - 2x^2 + 7x$
ii $y = x^3 - 2x^2 + 7x - 7$

5 a $-\dfrac{1}{x} + C$ **b** $-\dfrac{1}{2x^2} + C$
c $\dfrac{1}{x^2} + C$ **d** $\dfrac{1}{x^3} + C$
e $-\dfrac{1}{x} + \dfrac{1}{2x^2} + C$

6 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $2\sqrt{x} + C$ **c** $\frac{3}{4}x^{\frac{4}{3}} + C$
d $4\sqrt{x} + C$ **e** $\frac{5}{8}x^{\frac{8}{5}} + C$

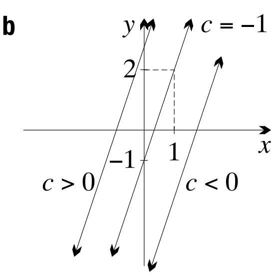
7 a $y = \frac{2}{3}x^{\frac{3}{2}} + 1$ **b** $y = \frac{2}{3}x^{\frac{3}{2}} - 16$

8 a



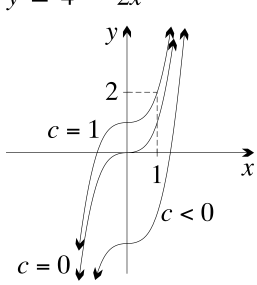
$y = -2x^2 + C$,
$y = 4 - 2x^2$

b



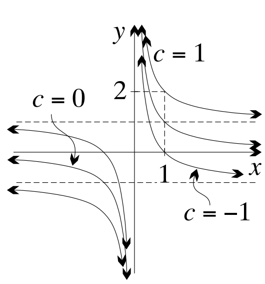
$y = 3x + C$,
$y = 3x - 1$

c



$y = x^3 + C$,
$y = x^3 + 1$

d



$y = \frac{1}{x} + C$,
$y = \frac{1}{x} + 1$

9 a $\frac{1}{4}(x + 1)^4 + C$ **b** $\frac{1}{6}(x - 2)^6 + C$
c $\frac{1}{3}(x + 5)^3 + C$ **d** $\frac{1}{10}(2x + 3)^5 + C$
e $\frac{1}{21}(3x - 4)^7 + C$ **f** $\frac{1}{20}(5x - 1)^4 + C$
g $-\frac{1}{4}(1 - x)^4 + C$ **h** $-\frac{1}{28}(1 - 7x)^4 + C$
i $\dfrac{-1}{3(x - 2)^3} + C$ **j** $\dfrac{1}{9(1 - x)^9} + C$

10 a $\frac{2}{3}(x + 1)^{\frac{3}{2}} + C$ **b** $\frac{2}{3}(x - 5)^{\frac{3}{2}} + C$
c $-\frac{2}{3}(1 - x)^{\frac{3}{2}} + C$ **d** $\frac{1}{3}(2x - 7)^{\frac{3}{2}} + C$
e $\frac{2}{9}(3x - 4)^{\frac{3}{2}} + C$

11 a $y = \frac{1}{5}(x - 1)^5$ **b** $\frac{1}{8}(2x + 1)^4 - \frac{9}{8}$
c $y = \frac{1}{3}(2x + 1)^{\frac{3}{2}}$

12 a $y = \frac{3}{5}x^5 - \frac{1}{4}x^4 + x$
b $y = -\frac{1}{4}x^4 + x^3 + 2x - 2$
c $y = -\frac{1}{20}(2 - 5x)^4 + \frac{21}{20}$

13 30

14 The rule gives the primitive of x^{-1} as $\dfrac{x^0}{0}$, which is undefined. This problem will be addressed in Chapter 6.

15 $y = x^3 + 2x^2 - 5x + 6$

17 $y = -x^3 + 4x^2 + 3$

18 $f(x) = \dfrac{1}{x} + 1$ for $x > 0$, and $f(x) = \dfrac{1}{x} + 3$ for $x < 0$

Chapter 4 review exercise

1 a C and H **b** A and F
c B, D, E and G **d** A, B, G and H
e D **f** C, E and F

2 a $f'(x) = 3x^2 - 2x - 1$
b **i** decreasing **ii** stationary
iii increasing **iv** increasing

3 a $2x - 4$
b **i** $x > 2$ **ii** $x < 2$ **iii** $x = 2$

4 a $y' = 3x^2$, increasing
b $y' = 2x - 1$, increasing
c $y' = 5(x - 1)^4$, stationary
d $y' = -\dfrac{4}{(x - 3)^2}$, decreasing

5 a $7x^6$, $42x^5$
b $3x^2 - 8x$, $6x - 8$
c $5(x - 2)^4$, $20(x - 2)^3$
d $-\dfrac{1}{x^2}$, $\dfrac{2}{x^3}$

6 a concave up **b** concave down

7 a $12x - 6$
b **i** $x > \frac{1}{2}$ **ii** $x < \frac{1}{2}$

8 a $x < 1$ or $x > 3$ **c** $x > 2$
b $1 < x < 3$ **d** $x < 2$

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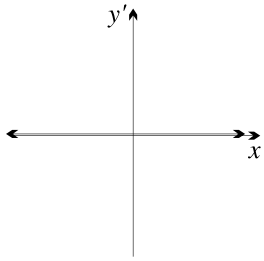
9 a



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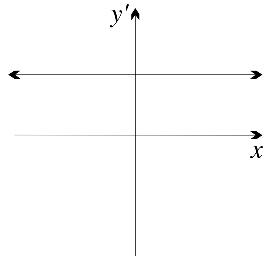

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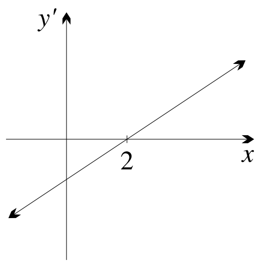

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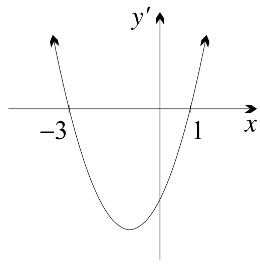
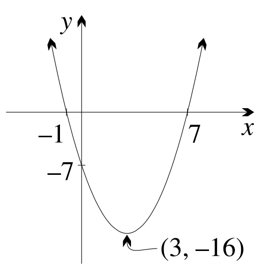


10 a $P(-1, 3)$, $Q(\frac{1}{3}, \frac{49}{27})$ **b** $x > -\frac{1}{3}$

c $\frac{49}{27} < k < 3$

11 a



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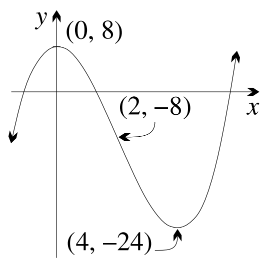

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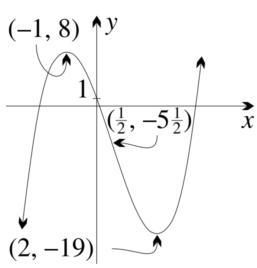
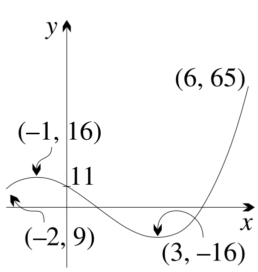


12 a



b 65 and -16

13 a $a = -2$ **b** $a = 3$ and $b = 6$

14 b -16

15 a 175 **b** 0 **c** 256

16 b $\frac{1600}{27}$ cm^3

17 b 30 cm by 40 cm

18 b $r = 8$ m

19 a $\frac{1}{8}x^8 + C$ **b** $x^2 + C$

c $4x + C$ **d** $2x^5 + C$

e $4x^2 + x^3 - x^4 + C$

20 a $x^3 - 3x^2 + C$ **b** $\frac{1}{3}x^3 - 2x^2 - 5x + C$

c $\frac{4}{3}x^3 - 6x^2 + 9x + C$

21 a $\frac{1}{6}(x + 1)^6 + C$ **b** $\frac{1}{8}(x - 4)^8 + C$

c $\frac{1}{8}(2x - 1)^4 + C$

22 a $-\frac{1}{x} + C$ **b** $\frac{2}{3}x^{\frac{3}{2}} + C$

23 $f(x) = x^3 - 2x^2 + x + 3$

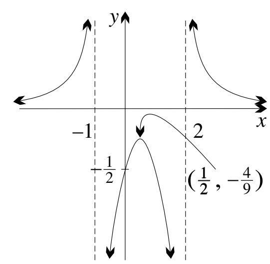
24 25

25 c Maximum turning point

d $x = -1$ and $x = 2$

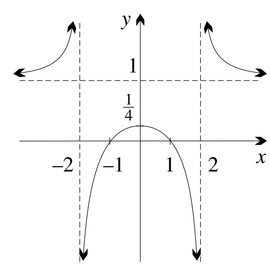
e As $x \longrightarrow \infty$, $f(x) \longrightarrow 0$.

f



26 a $(1, 0)$, $(-1, 0)$ and $(0, \frac{1}{4})$

f



27 a $S = 16x + 4h$ **c** 27 m by 9 m by 18 m

28 b $\dfrac{\pi R^2 h(r - R)}{r}$ **b** $\frac{4}{27}\pi r^2 h$

29 a $4\sqrt{3}$ cm^2

Chapter 5

Exercise 5A

1 a $\frac{1}{2}$ u^2

b The area under the curve is less than the area of the triangle.

2 a $\frac{1}{16}$ u^2 **b** $\frac{5}{16}$ u^2

c The area under the curve is less than the combined area of the triangle and trapezium.

3 b The gaps between the upper line segments and the curve are getting smaller.

4 a 6 **b** 12 **c** 8 **d** 9

e 2 **f** $\frac{25}{2}$ **g** 6 **h** 20

5 a 8 **b** 25 **c** 9 **d** 24

e 36 **f** 24 **g** 9 **h** 8

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6 a 15 **b** 15 **c** 25 **d** 40
e $\frac{25}{2}$ **f** 12 **g** 16 **h** 24
i 8 **j** 18 **k** 4 **l** 16
m 4 **n** 16 **o** $\frac{25}{2}$ **p** $\frac{25}{2}$

7 a 8π **b** $\frac{25}{4}\pi$

8 a $\frac{7}{32}u^2$ **b** $\frac{15}{32}u^2$

c The sum of the areas of the lower rectangles is less than the exact area under the curve which is less than the sum of the areas of the upper rectangles. Note that $\int_0^1 x^2\,dx = \frac{1}{3}$.

9 d As the number of rectangles increases, the interval within which the exact area lies becomes smaller. Note that $\int_0^1 2^x\,dx = \frac{1}{ln\,2} \doteqdot 1.44$.

10 d As the number of rectangles increases, the interval within which the exact area lies becomes smaller. Note that $\int_2^4 \ln x\,dx = 6\ln 2 - 2 \doteqdot 2.16$.

11 e The interval is getting smaller.

f Yes, they appear to be getting closer and closer to the exact value.

13 a You should count approximately 133 squares. $\frac{133}{400} \doteqdot 0.33$. We shall see later that $\int_0^1 x^2\,dx = \frac{1}{3}$.

b The exact values are:

i $\frac{1}{24}$ **ii** $\frac{7}{24}$

14 b 0.79 **c** 3.16

17 a $\frac{1}{3} + \frac{1}{6n^2}$

b The lines P_0P_1, P_1P_2 … lie above the curve. Therefore the combined area of the trapezia is greater than the area under the curve.

Exercise 5B

1 a 1 **b** 15 **c** 16
d 84 **e** 19 **f** 243
g 62 **h** 2 **i** 1

2 a i 4 **ii** 25

iii 1 (Note that $\int_4^5 dx$ means $\int_4^5 1\,dx$.)

b Each function is a horizontal line, so each integral is a rectangle.

3 a 30 **b** 6 **c** 33
d 18 **e** 132 **f** 2
g 23 **h** 44 **i** 60

4 a 2 **b** 2 **c** 9
d 30 **e** 96 **f** 10

5 a $13\frac{1}{2}$ **b** $4\frac{2}{3}$ **c** $29\frac{1}{4}$
d 2 **e** $20\frac{5}{6}$ **f** 98

6 a 24 **b** 18 **c** $2\frac{2}{3}$
d 21 **e** $\frac{1}{4}$ **f** $\frac{8}{15}$

7 a 42 **b** 14 **c** 62
d $8\frac{1}{3}$ **e** $6\frac{2}{3}$ **f** 6

8 a $\frac{1}{24}$ **b** $\frac{20}{27}$ **c** $\frac{7}{8}$

9 a i $\frac{1}{10}$ **ii** $\frac{5}{36}$ **iii** 15
b i $\frac{1}{2}$ **ii** $\frac{15}{32}$ **iii** 7

10 a ii 8 **b ii** 6

11 a $k = 1$ **b** $k = 4$ **c** $k = 8$
d $k = 3$ **e** $k = 3$ **f** $k = 2$

12 a $1 + \frac{\pi}{2}$ **b** $2\frac{1}{2}$

13 a $\frac{3}{2}$ **b** $\frac{5}{8}$ **c** $42\frac{1}{3}$

14 a $13\frac{1}{3}$ **b** $8\frac{59}{120}$ **c** $\frac{1}{24}$

15 a x^2 is never negative.

b The function has an asymptote $x = 0$, which lies in the given interval. Hence the integral is meaningless and the use of the fundamental theorem is invalid.

c Part **ii** is meaningless because it crosses the asymptote at $x = 3$.

16 a i x^2 **ii** $x^3 + 3x$ **iii** $\frac{1}{x}$ **iv** $(x^3 - 3)^4$

17 a $(a - x)u(x)$

Exercise 5C

1 The values are 6 and -6, which differ by a factor of -1.

2 a $LHS = RHS = 2$ **b** $LHS = RHS = 6\frac{3}{4}$

c $LHS = RHS = 0$

3 a The interval has width zero.

b $y = x$ is an odd function.

4 a The area is below the x-axis.

b The area is above the x-axis.

c The areas above and below the x-axis are equal.

d The area below the x-axis is greater than the area above.

5 a The area is above the x-axis.

b The area is below the x-axis.

c $y = 1 - x^2$ is an even function and so is symmetrical about the y-axis.

d The area under the parabola from 0 to $\frac{1}{2}$ is greater than the area from $\frac{1}{2}$ to 1.

6 a -7 **b** 5

7 The area under the line $y = 2x$ from $x = 0$ to $x = 1$ is greater than the area under $y = x$.

8 The area below the x-axis is greater than the area above.

9 a i 6 **ii** -6.

The integrals are opposites because the limits have been reversed.

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b i 5 **ii** 5.
The factor 20 can be taken out of the integral.
c i 45 **ii** 30 **iii** 15.
An integral of a sum is the sum of the integrals.
d i 48 **ii** 3 **iii** 45.
The interval $0 \le x \le 2$ can be dissected into the intervals $0 \le x \le 1$ and $1 \le x \le 2$.
e i 0 **ii** 0.
An integral over an interval of zero width is zero.

10 a 0. The interval has zero width.
b 0. The interval has zero width.
c 0. The integrand is odd.
d 0. The integrand is odd.
e 0. The integrand is odd.
f 0. The integrand is odd.

11 a The curves meet at $(0, 0)$ and at $(1, 1)$.
b In the interval $0 \le x \le 1$, the curve $y = x^3$ is below the curve $y = x^2$.
c $\frac{1}{4}$ and $\frac{1}{3}$

12 a $\frac{\pi}{2} - \frac{1}{2}$ **b** $1 - \frac{\pi}{2}$

13 a i 5 **ii** $\frac{25}{2}$
b i 25 **ii** $17\frac{1}{2}$ **iii** $27\frac{1}{2}$

14 a i $\frac{1}{2}$ **ii** 18 **iii** 8
b i $-\frac{1}{2}$ **ii** -18 **iii** -8

15 a The function is odd, so the integral is zero.
b The function is even, so its graph is symmetrical about the y-axis.

16 a false **b** true **c** false **d** false

17 b i 3 **ii** 4 **iii** $-\frac{7}{3}$
iv $\frac{10}{3}$ **v** $60\frac{2}{3}$ **vi** 33

18 a True, as the function is odd.
b True, as $\sin 4x°$ is odd and $\cos 2x°$ is even.
c False, as $2^{-x^2} > 0$ for all x.
d True, as $2^x < 3^x$ for $0 < x < 1$.
e False, as $2^x > 3^x$ for $-1 < x < 0$.
f True, as $t^n > t^{n+1}$ for $0 \le t \le 1$ and hence $\dfrac{1}{1 + t^n} \le \dfrac{1}{1 + t^{n+1}}$.

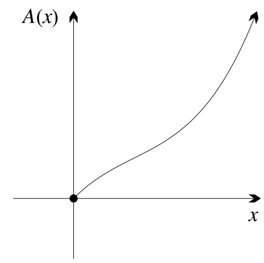
19 a The integral is $-\dfrac{1}{N} + 1$, which converges to 1 as $N \to \infty$.
b The integral is $-1 + \dfrac{1}{\varepsilon}$, which diverges to ∞ as $\varepsilon \to 0^+$.
c The integral is $2\sqrt{N} - 2$, which diverges to ∞ as $N \to \infty$.
d The integral is $2 - 2\sqrt{\varepsilon}$, which converges to 2 as $\varepsilon \to 0^+$.

Exercise 5D

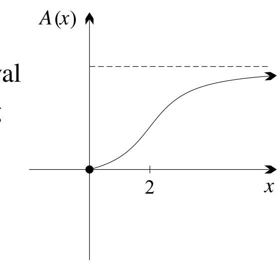
1 a $A(x) = \frac{3}{2}x^2$ **b** $A'(x) = 3x$

2 a $y = 3, A(x) = 3x$
b $y = 2t, A(x) = x^2, A'(x) = 2x$
c $y = 2 + t, A(x) = 2x + \frac{1}{2}x^2, A'(x) = 2 + x$
d $y = 5 - t, A(x) = 5x - \frac{1}{2}x^2, A'(x) = 5 - x$

3 a $A(x)$ is increasing at a decreasing rate in the interval $0 \le x < 2$, and increasing at an increasing rate for $x > 2$. It has an inflection at $x = 2$.



b $A(x)$ is increasing at an increasing rate in the interval $0 \le x \le 2$, and increasing at a decreasing rate for $x \ge 2$. It has a point of inflection at $x = 2$.



4 a $A'(x) = \dfrac{1}{x}$ **b** $A'(x) = \dfrac{1}{1 + t^3}$
c $A'(x) = e^{-\frac{1}{2}x^2}$

5 a $A'(x) = 3x^2 - 12, A(x) = x^3 - 12x + 11$
b $A'(x) = x^3 + 4x, A(x) = \frac{1}{4}x^4 + 2x^2 - 12$
c $A'(x) = \dfrac{1}{x^2}, A(x) = \frac{1}{2} - \frac{1}{x}$

6 a i x^2 **ii** $x^3 + 3x$ **iii** $\frac{1}{x}$ **iv** $(x^3 - 3)^4$

7 a The curve $y = A(x)$ looks like $y = e^x - 1$. The curve is increasing at an increasing rate.
b The curve is zero at $x = 1$, and is increasing at an increasing rate.
c The curve is zero at $x = 1$, and is increasing at a decreasing rate.

8 a The values are 0, 1, 0, -1, 0. The curve looks like $y = \sin x$, and this suggests that $\sin x$ has derivative $\cos x$.
b The values are 0, 1, 2, 1, 0. The graph looks like $y = 1 - \cos x$, which suggest that the derivative of $\cos x$ is $-\sin x$.

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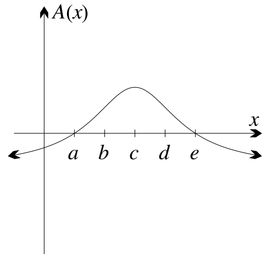
9 a $A(x)$ is increasing when $f(t)$ is positive, that is, for $t < c$, and is decreasing for $t > c$.

b $A(x)$ has a maximum turning point at $x = c$, and no minimum turning points.

c $A(x)$ has inflections when $f'(t)$ changes sign, that is, at $x = b$ and $x = d$.

d Because of the point symmetry of $f(t)$, there are two zeroes of $A(x)$ are $x = a$ and $x = e$.

e $A(x)$ is positive for $a < x < e$ and negative for $x < a$ and for $x > e$.



10 a The function is continuous at every real number, so it is a continuous function.

b The domain is $x \neq 2$, and y is continuous at every value in its domain, so it is a continuous function.

c Zero now lies in the domain, and y is not continuous at $x = 0$, so it is not a continuous function.

d The domain is $x \geq 0$, and y is continuous at every value in its domain, so it is a continuous function.

e The domain is $x > 0$, and y is continuous at every value in its domain, so it is a continuous function.

f The domain is $x \geq 0$, and y is not continuous at $x = 0$, so it is not a continuous function.

Exercise 5E

1 a $4x + C$ **b** $x + C$ **c** C **d** $-2x + C$
e $\frac{x^2}{2} + C$ **f** $\frac{x^3}{3} + C$ **g** $\frac{x^4}{4} + C$ **h** $\frac{x^8}{8} + C$

2 a $x^2 + C$ **b** $2x^2 + C$ **c** $x^3 + C$ **d** $x^4 + C$
e $x^{10} + C$ **f** $\frac{x^4}{2} + C$ **g** $\frac{2x^6}{3} + C$ **h** $\frac{x^9}{3} + C$

3 a $\frac{x^2}{2} + \frac{x^3}{3} + C$ **b** $\frac{x^5}{5} - \frac{x^4}{4} + C$
c $\frac{x^8}{8} + \frac{x^{11}}{11} + C$ **d** $x^2 + x^5 + C$
e $x^9 - 11x + C$ **f** $\frac{x^{14}}{2} + \frac{x^9}{3} + C$
g $4x - \frac{3x^2}{2} + C$ **h** $x - \frac{x^3}{3} + \frac{x^5}{5} + C$
i $x^3 - 2x^4 + \frac{7x^5}{5} + C$

4 a $-x^{-1} + C$ **b** $-\frac{1}{2}x^{-2} + C$ **c** $-\frac{1}{7}x^{-7} + C$
d $-x^{-3} + C$ **e** $-x^{-9} + C$ **f** $-2x^{-5} + C$

5 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $\frac{3}{4}x^{\frac{4}{3}} + C$ **c** $\frac{4}{5}x^{\frac{5}{4}} + C$
d $\frac{3}{5}x^{\frac{5}{3}} + C$ **e** $2x^{\frac{1}{2}} + C$ **f** $\frac{8}{3}x^{\frac{3}{2}} + C$

6 a $\frac{1}{3}x^3 + x^2 + C$ **b** $2x^2 - \frac{1}{4}x^4 + C$
c $\frac{5}{3}x^3 - \frac{3}{4}x^4 + C$ **d** $\frac{1}{5}x^5 - \frac{5}{4}x^4 + C$
e $\frac{1}{3}x^3 - 3x^2 + 9x + C$ **f** $\frac{4}{3}x^3 + 2x^2 + x + C$
g $x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + C$ **h** $4x - 3x^3 + C$
i $\frac{1}{3}x^3 - \frac{1}{2}x^4 - 3x + 3x^2 + C$

7 a $\frac{1}{2}x^2 + 2x + C$ **b** $\frac{1}{2}x^2 + \frac{1}{3}x^3 + C$
c $\frac{1}{6}x^3 - \frac{1}{16}x^4 + C$

8 a $-\frac{1}{x} + C$ **b** $-\frac{1}{2x^2} + C$
c $-\frac{1}{4x^4} + C$ **d** $-\frac{1}{9x^9} + C$
e $-\frac{1}{x^3} + C$ **f** $-\frac{1}{x^5} + C$
g $-\frac{1}{x^7} + C$ **h** $-\frac{1}{3x} + C$
i $-\frac{1}{28x^4} + C$ **j** $\frac{1}{10x^2} + C$
k $\frac{1}{4x^4} - \frac{1}{x} + C$ **l** $-\frac{1}{2x^2} - \frac{1}{3x^3} + C$

9 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $\frac{3}{4}x^{\frac{4}{3}} + C$
c $2\sqrt{x} + C$ **d** $\frac{3}{5}x^{\frac{5}{3}} + C$

10 a 18 **b** 12 **c** 4 **d** $\frac{3}{5}$

11 a $\frac{1}{6}(x + 1)^6 + C$ **b** $\frac{1}{4}(x + 2)^4 + C$
c $-\frac{1}{5}(4 - x)^5 + C$ **d** $-\frac{1}{3}(3 - x)^3 + C$
e $\frac{1}{15}(3x + 1)^5 + C$ **f** $\frac{1}{32}(4x - 3)^8 + C$
g $-\frac{1}{14}(5 - 2x)^7 + C$ **h** $-\frac{1}{40}(1 - 5x)^8 + C$
i $\frac{1}{24}(2x + 9)^{12} + C$ **j** $\frac{3}{22}(2x - 1)^{11} + C$
k $\frac{4}{35}(5x - 4)^7 + C$ **l** $-\frac{7}{8}(3 - 2x)^4 + C$

12 a $\frac{3}{5}\left(\frac{1}{3}x - 7\right)^5 + C$ **b** $\frac{4}{7}\left(\frac{1}{4}x - 7\right)^7 + C$
c $-\frac{5}{4}\left(1 - \frac{1}{4}x\right)^4 + C$

13 a $-\frac{1}{2(x + 1)^2} + C$ **b** $-\frac{1}{3(x - 5)^3} + C$
c $-\frac{1}{3(3x - 4)} + C$ **d** $\frac{1}{4(2 - x)^4} + C$
e $-\frac{3}{5(x - 7)^5} + C$ **f** $-\frac{1}{2(4x + 1)^4} + C$
g $\frac{2}{15(3 - 5x)^3} + C$ **h** $\frac{1}{5 - 20x} + C$
i $-\frac{7}{96(3x + 2)^4} + C$

14 a $\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + C$ **b** $\frac{1}{2}x^2 - 4x + C$
c $2x^2 - \frac{8}{3}x^{\frac{3}{2}} + x + C$

15 a i $\frac{2}{3}$ **ii** 2 **iii** 12
b i $5\frac{1}{3}$ **ii** $96\frac{4}{5}$ **iii** 4

16 a 2 **b** $-\frac{13}{6}$ **c** $12\frac{1}{6}$

17 $\int x^{-1}\,dx = \frac{x^0}{0} + C$ is meaningless. Chapter 6 deals with the resolution of this problem.

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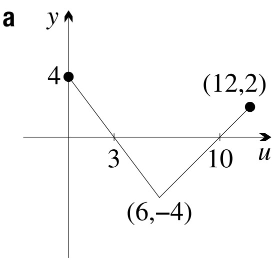
18 a $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + C$ **b** $-\frac{1}{6}(7 - 4x)^{\frac{3}{2}} + C$
c $\frac{3}{16}(4x - 1)^{\frac{4}{3}} + C$ **d** $\frac{2}{3}\sqrt{3x + 5} + C$
19 a $\frac{242}{5}$ **b** 0 **c** $121\frac{1}{3}$
d 1 **e** $\frac{13}{6}$ **f** 2
g 0 **h** $\frac{112}{9}$ **i** $8\frac{2}{5}$
20 b i $\frac{1}{5}x(x - 1)^5 - \frac{1}{30}(x - 1)^6 + C$
ii $\frac{2}{3}x(1 + x)^{\frac{3}{2}} - \frac{4}{15}(1 + x)^{\frac{5}{2}} + C$

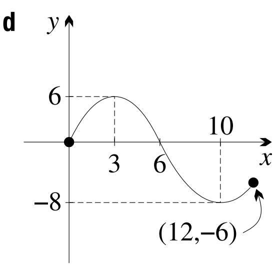
Exercise 5F

1 a 4 u^2 **b** 26 u^2 **c** 81 u^2
d 12 u^2 **e** 9 u^2 **f** $6\frac{2}{3}\text{ u}^2$
g $\frac{128}{3}\text{ u}^2$ **h** 6u^2 **i** $\frac{1}{4}\text{ u}^2$
j $57\frac{1}{6}\text{ u}^2$ **k** 36u^2 **l** 60u^2
2 a 25u^2 **b** 8u^2 **c** 4u^2
d 108u^2 **e** $\frac{9}{2}\text{u}^2$ **f** $34\frac{2}{3}\text{ u}^2$
g 18u^2 **h** 2u^2
3 a $\frac{4}{3}\text{u}^2$ **b** $\frac{27}{2}\text{u}^2$ **c** $\frac{81}{4}\text{u}^2$ **d** $46\frac{2}{5}\text{ u}^2$
4 a $\frac{9}{2}\text{u}^2$ **b** $\frac{4}{3}\text{u}^2$ **c** $\frac{45}{4}\text{u}^2$ **d** 9 u^2
5 b $4\frac{1}{2}\text{u}^2$ **c** 2 u^2 **d** $6\frac{1}{2}\text{u}^2$
e $2\frac{1}{2}$. This is the area above the x-axis minus the area below it.
6 b $10\frac{2}{3}\text{u}^2$ **c** $2\frac{1}{3}\text{u}^2$ **d** 13 u^2
e $-8\frac{1}{3}$. This is the area above the x-axis minus the area below it.
7 b $2\frac{2}{3}\text{u}^2$ **c** $\frac{5}{12}\text{u}^2$ **d** $3\frac{1}{12}\text{u}^2$
e $-2\frac{1}{4}$. This is the area above the x-axis minus the area below it.
8 a $11\frac{2}{3}\text{u}^2$ **b** $128\frac{1}{2}\text{u}^2$ **c** 4 u^2
d $8\frac{1}{2}\text{u}^2$ **e** $32\frac{3}{4}\text{u}^2$ **f** $11\frac{1}{3}\text{u}^2$
9 a 13u^2 **b** $2\frac{1}{2}\text{u}^2$ **c** $9\frac{1}{3}\text{u}^2$ **d** $7\frac{1}{3}\text{u}^2$
10 a i 64 u^2 **ii** 128 u^2 **iii** $64\frac{4}{5}\text{u}^2$
b i 50 u^2 **ii** 18 u^2 **iii** $\frac{32}{3}\text{u}^2$
11 8 u^2
12 a $(2, 0)$, $\left(0, 4\sqrt{2}\right)$, $\left(0, -4\sqrt{2}\right)$
b $\frac{16\sqrt{2}}{3}\text{u}^2$
ii $x = 2 - \frac{y^2}{16}$
13 a $y = \frac{1}{3}x^3 - 2x^2 + 3x$
b The curve passes through the origin, $(1, 1\frac{1}{3})$ is a maximum turning point and $(3, 0)$ is a minimum turning point.
c $\frac{4}{3}\text{u}^2$
15 a $2 : n + 1$ **b** $1 : n + 1$
16 b $a^2 = \frac{1}{2}\left(3 + \sqrt{5}\right), a^4 = \frac{1}{2}\left(7 + 3\sqrt{5}\right),$
$a^5 = \frac{1}{2}\left(11 + 5\sqrt{5}\right)$
c Areas are $\frac{1}{5}\text{u}^2$, $\frac{1}{10}\text{u}^2$ and $\frac{1}{10}\text{u}^2$.

17 a



b maximum at $(3, 6)$, minimum at $(10, -8)$
c 0, 6
d



e 24 u^2

18 a $-\dfrac{1}{n + 1}$ **b** $\dfrac{1}{n + 1}$

Exercise 5G

1 a $\frac{1}{6}\text{u}^2$ **b** $\frac{1}{4}\text{u}^2$ **c** $\frac{3}{10}\text{u}^2$ **d** $\frac{1}{12}\text{u}^2$
e $\frac{2}{35}\text{u}^2$ **f** $20\frac{5}{6}\text{u}^2$ **g** 36 u^2 **h** $20\frac{5}{6}\text{u}^2$
2 a $\frac{4}{3}\text{u}^2$ **b** $\frac{1}{6}\text{u}^2$ **c** $\frac{4}{3}\text{u}^2$ **d** $4\frac{1}{2}\text{u}^2$
3 a $5\frac{1}{3}\text{u}^2$ **b** $\frac{9}{4}\text{u}^2$
4 a $16\frac{2}{3}\text{u}^2$ **b** $9\frac{1}{3}\text{u}^2$
5 a $4\frac{1}{2}\text{ u}^2$
6 a $\frac{4}{3}\text{u}^2$
7 a 36 u^2
8 a $4\frac{1}{2}\text{u}^2$ **b** $20\frac{5}{6}\text{u}^2$ **c** $2\frac{2}{3}\text{u}^2$
9 c 36 u^2
10 c $\frac{4}{3}\text{u}^2$
11 a $4\frac{1}{2}\text{u}^2$ **b** $20\frac{5}{6}\text{u}^2$ **c** $21\frac{1}{3}\text{u}^2$
12 c $\frac{1}{3}\text{u}^2$
13 b $y = x - 2$ **c** $5\frac{1}{3}\text{u}^2$
14 c 108 u^2
15 a The points are $(-4, -67)$, $(1, -2)$, and $(2, 5)$.
c $73\frac{5}{6}\text{u}^2$
16 a $((0, 0), (\frac{1}{2}, \frac{1}{8}), (1, 0)$ **b** $\frac{1}{16}\text{u}^2$
17 a $-1 < x < 1$ or $x > 4$ **b** $21\frac{1}{12}\text{u}^2$
18 b $y = 2x - 7$ **c** $\frac{7}{12}\text{u}^2$
19 $1 - \dfrac{1}{\sqrt[3]{2}}$

Exercise 5H

1 a 40 **b** 22 **c** -26
2 a 164
3 30

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4 a The curve is concave up, so the chord is above the curve, and the area under the chord will be greater than the area under the curve.

b The curve is concave down, so the chord is underneath the curve, and the area under the chord will be less than the area under the curve.

5 b 10

c $10\frac{2}{3}$, the curve is concave down.

d $6\frac{1}{4}\%$

6 b $10\frac{1}{10}$

c y'' is positive in the interval $1 \le x \le 5$, so the curve is concave up.

7 b 24.7

c $24\frac{2}{3}$. y'' is negative in the interval $9 \le x \le 16$, so the curve is concave down.

8 a 0.73 **b** 4.5 **c** 3.4 **d** 37

9 a 1.12 **b** 0.705 **c** 22.9 **d** 0.167

10 9.2 metres

11 $550\,\text{m}^2$

12 5900

13 a 0.7489

b $\pi \doteqdot 3.0$, the approximation is less than the integral, because the curve is concave down.

15 d 876 400

Exercise 5I

1 a $8(2x+3)^3$

b i $(2x+3)^4 + C$ **ii** $2(2x+3)^4 + C$

2 a $9(3x-5)^2$

b i $(3x-5)^3 + C$ **ii** $3(3x-5)^3 + C$

3 a $20(1+4x)^4$

b i $(1+4x)^5 + C$ **ii** $\frac{1}{2}(1+4x)^5 + C$

4 a $-8(1-2x)^3$

b i $(1-2x)^4 + C$ **ii** $\frac{1}{4}(1-2x)^4 + C$

5 a $-4(4x+3)^{-2}$

b i $(4x+3)^{-1} + C$ **ii** $-\frac{1}{4}(4x+3)^{-1} + C$

6 a $(2x-5)^{-\frac{1}{2}}$

b i $(2x-5)^{\frac{1}{2}} + C$ **ii** $\frac{1}{3}(2x-5)^{\frac{1}{2}} + C$

7 a $8x(x^2+3)^3$

b i $(x^2+3)^4 + C$ **ii** $5(x^2+3)^4 + C$

8 a $15x^2(x^3-1)^4$

b i $(x^3-1)^5 + C$ **ii** $\frac{1}{5}(x^3-1)^5 + C$

9 a $\dfrac{2x}{\sqrt{2x^2+3}}$

b i $\sqrt{2x^2+3} + C$

ii $\frac{1}{2}\sqrt{2x^2+3} + C$

10 a $\dfrac{3(\sqrt{x}+1)^2}{2\sqrt{x}}$

b i $(\sqrt{x}+1)^3 + C$ **ii** $\frac{2}{3}(\sqrt{x}+1)^3 + C$

11 a $12(x^2+2x)(x^3+3x^2+5)^3$

b i $(x^3+3x^2+5)^4 + C$

ii $\frac{1}{12}(x^3+3x^2+5)^4 + C$

12 a $-7(2x+1)(5-x^2-x)^6$

b i $(5-x^2-x)^7 + C$

ii $-\frac{1}{7}(5-x^2-x)^7 + C$

13 a $\frac{1}{4}(5x+4)^4 + C$ **b** $\frac{1}{6}(1-3x)^6 + C$

c $\frac{1}{8}(x^2-5)^8 + C$ **d** $\frac{1}{5}(x^3+7)^5 + C$

e $\frac{-1}{3x^2+2} + C$ **f** $2\sqrt{9-2x^3} + C$

14 a $\frac{1}{3}(5x^2+3)^3 + C$ **b** $\frac{1}{4}(x^2+1)^4 + C$

c $\frac{1}{6}(1+4x^3)^6 + C$ **d** $\frac{1}{30}(1+3x^2)^5 + C$

e $-\frac{1}{32}(1-x^4)^8 + C$ **f** $\frac{2}{3}(x^3-1)^{\frac{3}{2}} + C$

g $\dfrac{1}{15}(5x^2+1)^{\frac{3}{2}} + C$ **h** $2\sqrt{x^2+3} + C$

i $\frac{1}{4}\sqrt{4x^2+8x+1} + C$ **j** $-\dfrac{1}{4(x^2+5)^2} + C$

15 a $\frac{32}{15}$ **b** $\frac{7}{144}$ **c** $\frac{1}{12}$ **d** 936

16 a $\frac{1}{6}(1-\frac{1}{x})^6 + C$ **b** $\dfrac{1}{3}$

17 a $x \ge 1$ or $x \le -1$

b $\dfrac{2x^2-1}{\sqrt{x^2-1}}$ **c** $\dfrac{16}{3}\sqrt{2}\,\text{u}^2$

18 a horizontal points of inflection at $\left(\sqrt{7}, 0\right)$ and $\left(-\sqrt{7}, 0\right)$, maximum at $(1, 216)$, minimum at $(-1, -216)$

b $600\frac{1}{4}\,\text{u}^2$

Chapter 5 review exercise

1 a 1 **b** $\frac{3}{2}$ **c** 609

d $\dfrac{2}{5}$ **e** -12 **f** $8\frac{2}{3}$

g 8 **h** -10 **i** $21\frac{1}{3}$

2 a $4\frac{2}{3}$ **b** $-1\frac{2}{3}$ **c** $\dfrac{1}{3}$

3 a $-1\frac{1}{2}$ **b** 15 **c** $-6\frac{1}{6}$

4 a ii $k = 6$ **b ii** $k = 3$

5 a 0. The integral has zero width.

b 0. The integrand is odd.

c 0. The integrand is odd.

6 a 8 **b** $\frac{3}{2}$

7 a i $4x - \frac{1}{2}x^2 + 10$ **ii** $\frac{1}{2} - x^{-1}$

b i $4 - x$ **ii** x^{-2}

c i $x^5 - 5x^3 + 1$

ii $\dfrac{x^2+4}{x^2-1}$

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8 a $\frac{x^2}{2} + 2x + C$
b $\frac{x^4}{4} + x^3 - \frac{5x^2}{2} + x + C$
c $\frac{x^3}{3} - \frac{x^2}{2} + C$
d $-\frac{x^3}{3} + \frac{5x^2}{2} - 6x + C$
e $-x^{-1} + C$
f $-\frac{1}{6x^6} + C$
g $\frac{2x^{\frac{3}{2}}}{3} + c$
h $\frac{1}{5}(x + 1)^5 + C$
i $\frac{1}{12}(2x - 3)^6 + C$

9 a $9\frac{1}{3}\text{u}^2$ **b** $4\,\text{u}^2$ **c** $\frac{4}{3}\text{u}^2$ **d** $1\,\text{u}^2$
e $\frac{1}{6}\text{u}^2$ **f** $\frac{4}{15}\text{u}^2$ **g** $\frac{1}{6}\text{u}^2$ **h** $4\frac{1}{2}\text{u}^2$

10 b $\frac{4}{3}\text{u}^2$

11 a 9 **b** 0.56

12 a $18(3x + 4)^5$
b i $(3x + 4)^6 + C$ **ii** $\frac{1}{2}(3x + 4)^6 + C$

13 a $6x(x^2 - 1)^2$
b i $(x^2 - 1)^3 + C$ **ii** $\frac{1}{6}(x^2 - 1)^3 + C$

14 a $\frac{1}{5}(x^3 + 1)^5 + C$ **b** $-\frac{1}{2(x^2 - 5)^2} + C$

Chapter 6

Exercise 6A

1 a 2^{10} **b** e^7 **c** 2^4
d e^3 **e** 2^{12} **f** e^{30}

2 a e^{7x} **b** e^{2x} **c** e^{10x}
d e^{-5x} **e** e^{-3x} **f** e^{-12x}

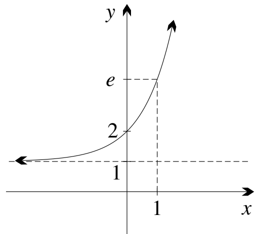
3 a 7.389 **b** 0.04979 **c** $e^1 \doteqdot 2.718$
d $e^{-1} \doteqdot 0.3679$ **e** $e^{\frac{1}{2}} \doteqdot 1.649$ **f** $e^{-\frac{1}{2}} \doteqdot 0.6065$

4 a $y' = e^x$ and $y'' = e^x$
b 'The curve $y = e^x$ is always concave up, and is always increasing at an increasing rate.'

5 a gradient $= e$, $y = ex$.
b $y = x + 1$
c $y = \frac{1}{e}(x + 2)$

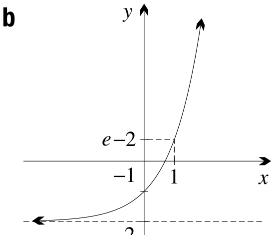
6 a $P = (1, e - 1)$
b $\frac{dy}{dx} = e^x$. When $x = 1$, $\frac{dy}{dx} = e$.
c tangent: $ex - y - 1 = 0$,
normal: $x + ey - e^2 + e - 1 = 0$

7 a



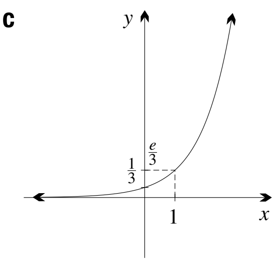
Shift e^x up 1

b



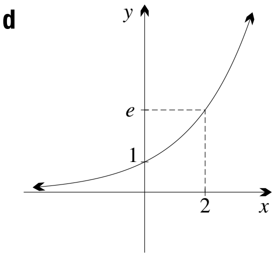
Shift e^x down 2

c



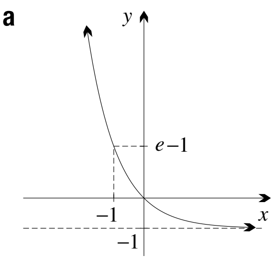
Stretch e^x vertically with factor $\frac{1}{3}$

d



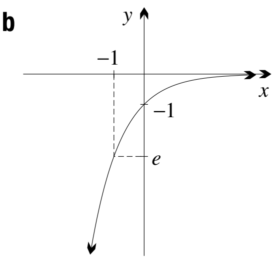
Stretch e^x horizontally with factor 2

8 a



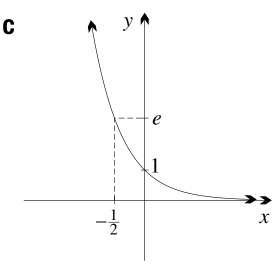
Shift e^{-x} down 1

b



Reflect e^{-x} in x-axis

c



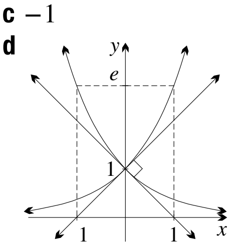
Stretch e^{-x} horizontally with factor $-\frac{1}{2}$

9 It is a vertical dilation of $y = e^x$ with factor $-\frac{1}{3}$. Its equation is $y = -\frac{1}{3}e^x$.

10 a $e^{2x} - 1$ **b** $e^{6x} + 3e^{4x} + 3e^{2x} + 9$
c $1 - 2e^{3x}$ **d** $e^{-4x} + 2 + e^{4x}$

11 a $e^{2x} + e^x$ **b** $e^{-2x} - e^{-x}$
c $e^{20x} + 5e^{30x}$ **d** $2e^{-4x} + 3e^{-5x}$

12 a 1
b Reflection in y-axis
c -1
d



e Horizontal dilation with factor -1

13 a e^x, e^x, e^x, e^x
b $e^x + 3x^2$, $e^x + 6x$, $e^x + 6$, e^x
c $4e^x$, $4e^x$, $4e^x$, $4e^x$
d $5e^x + 10x$, $5e^x + 10$, $5e^x$, $5e^x$. In part **c**, the gradient equals the height.

14 a 1, 45° **b** e, 69°48′
c e^{-2}, 7°42′ **d** e^5, 89°37′

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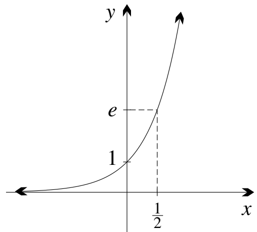
15 a $e - 1$

b $\frac{dy}{dx} = e^x$. When $x = 1$, $\frac{dy}{dx} = e$.

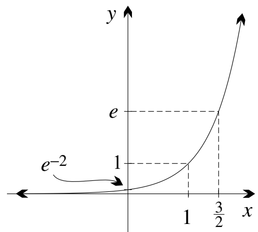
c $y = ex - 1$

16 a



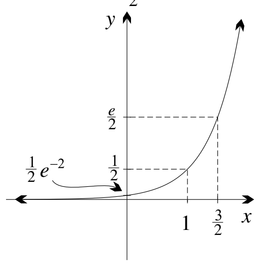
Stretch horizontally with factor $\frac{1}{2}$.

b



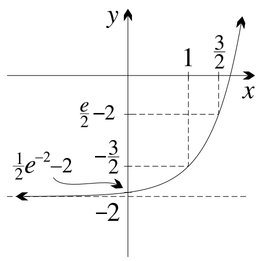
Shift right 1.

c



Stretch vertically with factor $\frac{1}{2}$.

d



Shift down 2.

17 a Shift left 2. Alternatively, $y = e^2 e^x$, so it is a vertical dilation with factor e^2.

b Stretch vertically with factor 2. Alternatively, $y = e^{\log_e 2} e^x = e^{x + \log_e 2}$, so it is a shift left $\log_e 2$.

Exercise 6B

1 a $7e^{7x}$ **b** $12e^{3x}$
c $2e^{\frac{1}{3}x}$ **d** e^{-2x}
e $y' = 3e^{3x+4}$ **f** $y' = 4e^{4x-3}$
g $y' = -3e^{-3x+4}$ **h** $y' = -2e^{-2x-7}$

2 a $e^x - e^{-x}$ **b** $2e^{2x} + 3e^{-3x}$
c $\frac{e^x + e^{-x}}{2}$ **d** $\frac{e^x - e^{-x}}{3}$
e $e^{2x} + e^{3x}$ **f** $e^{4x} + e^{5x}$

3 a $y' = 3e^{3x}$ **b** $y' = 2e^{2x}$
c $y' = 2e^{2x}$ **d** $y' = 6e^{6x}$
e $y' = 3e^{3x}$ **f** $y' = -e^{-x}$
g $y' = -3e^{-3x}$ **h** $y' = -5e^{-5x}$

4 a i $-e^{-x}, e^{-x}, -e^{-x}, e^{-x}$

ii Successive derivatives alternate in sign. More precisely, $f^{(n)}(x) = \begin{cases} e^{-x}, & \text{if } n \text{ is even,} \\ -e^{-x}, & \text{if } n \text{ is odd.} \end{cases}$

b i $2e^{2x}, 4e^{2x}, 8e^{2x}, 16e^{2x}$

ii Each derivative is twice the previous one. More precisely, $f^{(n)}(x) = 2^n e^{2x}$.

5 a $2e^{2x} + e^x$ **b** $e^{-x} - 4e^{-2x}$
c $2e^{2x} + 2e^x$ **d** $2e^{2x} + 6e^x$
e $2e^{2x} - 2e^x$ **f** $2e^{2x} - 4e^x$
g $2(e^{2x} + e^{-2x})$ **h** $10(e^{10x} + e^{-10x})$

6 a $a\,e^{ax+b}$ **b** $2xe^{x^2}$
c $-xe^{-\frac{1}{2}x}$ **d** $2xe^{x^2+1}$
e $-2xe^{1-x^2}$ **f** $2(x + 1)\,e^{x^2+2x}$
g $(1 - 2x)\,e^{6+x-x^2}$ **h** $(3x - 1)\,e^{3x^2-2x+1}$

7 a $(x + 1)\,e^x$ **b** $(1 - x)\,e^{-x}$
c xe^x **d** $(3x + 4)\,e^{3x-4}$
e $(2x - x^2)\,e^{-x}$ **f** $4xe^{2x}$
g $(x^2 + 2x - 5)\,e^x$ **h** $x^2e^{2x}(3 + 2x)$

8 a $y' = \frac{x - 1}{x^2}e^x$ **b** $y' = (1 - x)\,e^{-x}$
c $y' = \frac{(x - 2)\,e^x}{x^3}$ **d** $y' = (2x - x^2)\,e^{-x}$
e $y' = \frac{x}{(x + 1)^2}e^x$ **f** $y' = -xe^{-x}$
g $y' = (7 - 2x)\,e^{-2x}$ **h** $y' = (x^2 - 2x - 1)\,e^{-x}$

9 a $2e^{2x} + 3e^x$ **b** $4e^{4x} + 2e^{2x}$
c $-2e^{-2x} - 6e^{-x}$ **d** $-6e^{-6x} + 18e^{-3x}$
e $3e^{3x} + 2e^{2x} + e^x$ **f** $12e^{3x} + 2e^{2x} + e^{-x}$

10 a $-5e^x(1 - e^x)^4$ **b** $16e^{4x}(e^{4x} - 9)^3$
c $-\frac{e^x}{(e^x - 1)^2}$ **d** $-\frac{6e^{3x}}{(e^{3x} + 4)^3}$

12 a $f'(x) = 2e^{2x+1}$, $f'(0) = 2e$, $f''(x) = 4e^{2x+1}$, $f''(0) = 4e$

b $f'(x) = -3e^{-3x}$, $f'(1) = -3e^{-3}$, $f''(x) = 9e^{-3x}$, $f''(1) = 9e^{-3}$

c $f'(x) = (1 - x)\,e^{-x}$, $f'(2) = -e^{-2}$, $f''(x) = (x - 2)\,e^{-x}$, $f''(2) = 0$

d $f'(x) = -2xe^{-x^2}$, $f'(0) = 0$, $f''(x) = (4x^2 - 2)\,e^{-x^2}$, $f''(0) = -2$

13 a $y' = ae^{ax}$ **b** $y' = -ke^{-kx}$
c $y' = Ake^{kx}$ **d** $y' = -B\ell e^{-\ell x}$
e $y' = pe^{px+q}$ **f** $y' = pCe^{px+q}$
g $y' = \dfrac{pe^{px} - qe^{-qx}}{r}$ **h** $e^{ax} - e^{-px}$

14 a $3e^x(e^x + 1)^2$

b $4(e^x - e^{-x})\,(e^x + e^{-x})^3$

c $(1 + 2x + x^2)\,e^{1+x} = (1 + x)^2 e^{1+x}$

d $(2x^2 - 1)\,e^{2x-1}$

e $\frac{e^x}{(e^x + 1)^2}$

f $-\frac{2e^x}{(e^x - 1)^2}$

15 a $y' = -e^{-x}$

b $y' = e^x$

c $y' = e^{-x} - 4e^{-2x}$

d $y' = -12e^{-4x} - 3e^{-3x}$

e $y' = e^x - 9e^{3x}$

f $y' = -2e^{-x} - 2e^{-2x}$

17 a $y' = \frac{1}{2}\sqrt{e^x}$ **b** $y' = \frac{1}{3}\sqrt{e^x}$
c $y' = -\frac{1}{2\sqrt{e^x}}$ **d** $y' = -\frac{1}{3\sqrt[3]{e^x}}$
e $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$ **f** $-\frac{1}{2\sqrt{x}}e^{-\sqrt{x}}$

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g $-\frac{1}{x^2}e^{\frac{1}{x}}$ **h** $\frac{1}{x^2}e^{-\frac{1}{x}}$

i $\left(1 + \frac{1}{x^2}\right)e^{x-\frac{1}{x}}$ **j** $e^x e^{e^x} = e^{x+e^x}$

20 a -5 or 2

b $-\frac{1}{2}\left(1 + \sqrt{5}\right)$ or $-\frac{1}{2}\left(1 - \sqrt{5}\right)$

Exercise 6C

1 a $A = \left(\frac{1}{2}, 1\right)$ **b** $y' = 2e^{2x-1}$ **c** $y = 2x$

2 a $R = \left(-\frac{1}{3}, 1\right)$ **b** $y' = 3e^{3x+1}$

c $-\frac{1}{3}$ **d** $3x + 9y - 8 = 0.$

3 a $x - ey + e^2 + 1 = 0$

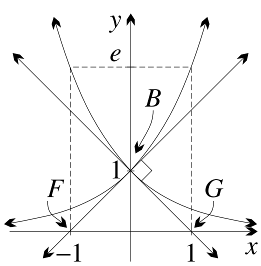
b $x = -e^2 - 1, y = e + e^{-1}$

c $\frac{1}{2}(e^3 + 2e + e^{-1})$

4 a $y = x + 1$ **b** $y = -x + 1$

c $F(-1, 0), G(1, 0)$

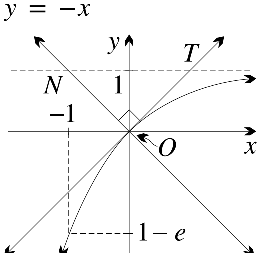
d



e isosceles right triangle, 1 square unit

5 b $y = -x$ **c** $y = 1$

d

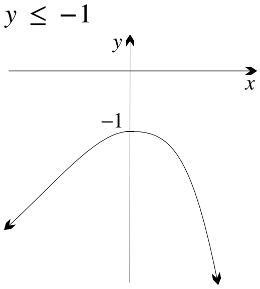


e 1 square unit

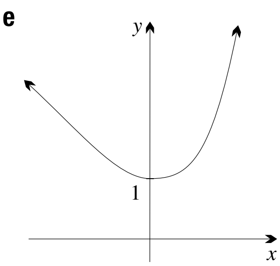
6 a $y' = 1 - e^x, y'' = -e^x$

c maximum turning point at $(0, -1)$

d $y \le -1$



e



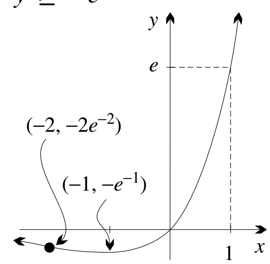
7 b $y = e^t(x - t + 1)$

c The x-intercept of each tangent to $y = e^x$ is 1 unit left of the x-value of the point of contact.

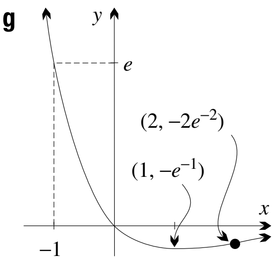
8 a There is a zero at $x = 0$, it is positive for $x > 0$ and negative for $x < 0$. It is neither even nor odd.

e They all tend towards ∞.

f $y \ge -e^{-1}$

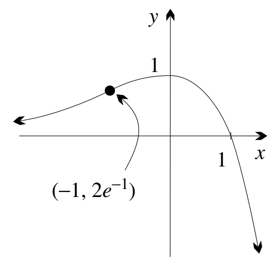


g



9 a

x	0	1	2
y	1	0	$-e^2$
sign	+	0	−



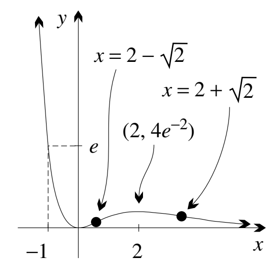
b $y' = -xe^x$,

$y'' = -(x + 1)e^x$

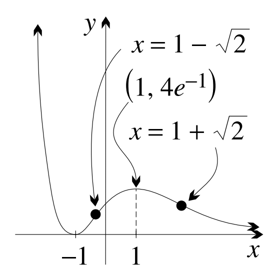
d They all tend to $-\infty$.

e $y \le 1$

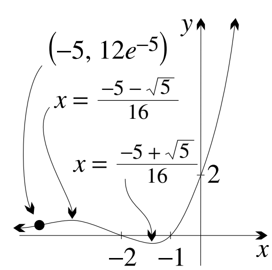
10 d $y \ge 0$



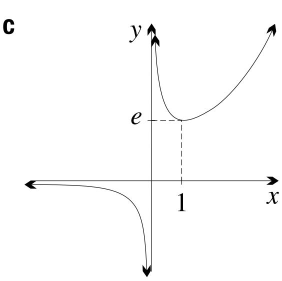
11 d $y \ge 0$



12



13 a $x \ne 0, y < 0$ or $y \ge e$ **c**



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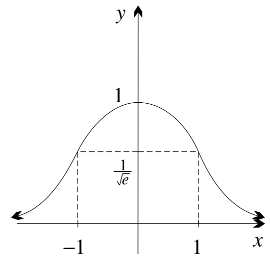
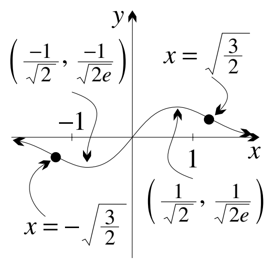
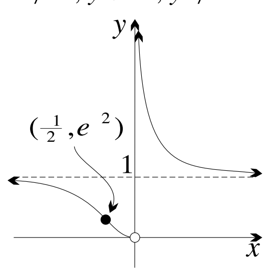
14 a $y' = -xe^{-\frac{1}{2}x^2}$,
$y'' = (x^2 - 1)e^{-\frac{1}{2}x^2}$
d $0 < y \le 1$

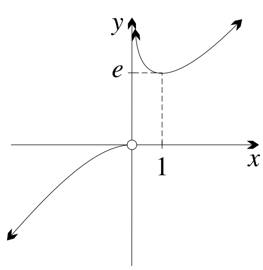


15 d $-\frac{1}{\sqrt{2e}} \le y \le \frac{1}{\sqrt{2e}}$



16 a i $y \to 0$ **ii** $y \to \infty$
b i $y \to -\infty$ **ii** $y \to 0$
17 $x = 1$ or $x = -1$
18 d $x \ne 0, y > 0, y \ne 1$ **e**





Exercise 6D

1 a $\frac{1}{2}e^{2x} + C$ **b** $\frac{1}{3}e^{3x} + C$
c $3e^{3x} + C$ **d** $2e^{2x} + C$
e $5e^{2x} + C$ **f** $4e^{3x} + C$
g $\frac{1}{4}e^{4x+5} + C$ **h** $\frac{1}{4}e^{4x-2} + C$
i $2e^{3x+2} + C$ **j** $e^{4x+3} + C$
k $-\frac{1}{2}e^{7-2x} + C$ **l** $-\frac{1}{6}e^{1-3x} + C$
2 a $e - 1$ **b** $e^2 - e$
c $e - e^{-3}$ **d** $e^2 - 1$
e $\frac{1}{2}(e^4 - 1)$ **f** $4(e^5 - e^{-10})$
g $2(e^{12} - e^{-4})$ **h** $\frac{3}{2}(e^{18} - e^{-6})$
i $\frac{1}{2}(e^3 - e^{-1})$ **j** $\frac{1}{4}(e^{-3} - e^{-11})$
k $\frac{1}{3}(e^{-1} - e^{-4})$ **l** $\frac{e^2}{2}(e^2 - 1)$
m $\frac{e}{3}(e^2 - 1)$ **n** $2e^4(e^3 - 1)$
o $3e^3(e^4 - 1)$ **p** $4e^2(e^3 - 1)$
3 a $-e^{-x} + C$ **b** $-\frac{1}{2}e^{-2x} + C$
c $-\frac{1}{3}e^{-3x} + C$ **d** $e^{-3x} + C$
e $-3e^{-2x} + C$ **f** $4e^{2x} + C$
4 a $f(x) = \frac{1}{2}e^{2x} + C$, for some constant C
b $C = -2\frac{1}{2}$, so $f(x) = \frac{1}{2}e^{2x} - 2\frac{1}{2}$
c $f(1) = \frac{1}{2}e^2 - 2\frac{1}{2}$, $f(2) = \frac{1}{2}e^4 - 2\frac{1}{2}$
5 a $f(x) = x + 2e^x - 1$, $f(1) = 2e$
b $f(x) = 2 + x - 3e^x$, $f(1) = 3 - 3e$
c $f(x) = 1 + 2x - e^{-x}$, $f(1) = 3 - e^{-1}$
d $f(x) = 1 + 4x + e^{-x}$, $f(1) = 5 + e^{-1}$
e $f(x) = \frac{1}{2}e^{2x-1} + \frac{5}{2}$, $f(1) = \frac{1}{2}(e + 5)$
f $f(x) = 1 - \frac{1}{3}e^{1-3x}$, $f(1) = 1 - \frac{1}{3}e^{-2}$
g $f(x) = 2e^{\frac{1}{2}x+1} - 6$, $f(1) = 2e^{\frac{3}{2}} - 6$
h $f(x) = 3e^{\frac{1}{3}x+2} - 1$, $f(1) = 3e^{\frac{7}{3}} - 1$
6 a $\frac{1}{2}e^{2x} + e^x + C$ **b** $\frac{1}{2}e^{2x} - e^x + C$
c $e^{-x} - e^{-2x} + C$ **d** $\frac{1}{2}e^{2x} + 2e^x + x + C$
e $\frac{1}{2}e^{2x} - 2e^x + x + C$ **f** $\frac{1}{2}e^{2x} - 4e^x + 4x + C$
g $\frac{1}{2}(e^{2x} + e^{-2x}) + C$ **h** $\frac{1}{10}(e^{10x} + e^{-10x}) + C$
7 a $\frac{1}{7}e^{7x+q} + C$ **b** $\frac{1}{3}e^{3x-k} + C$
c $\frac{1}{s}e^{sx+1} + C$ **d** $\frac{1}{k}e^{kx-1} + C$
e $e^{px+q} + C$ **f** $e^{mx+k} + C$
g $\frac{A}{s}e^{sx-t} + C$ **h** $\frac{B}{k}e^{kx-\ell} + C$
8 a $-e^{1-x} + C$ **b** $-\frac{1}{3}e^{1-3x} + C$
c $-\frac{1}{2}e^{-2x-5} + C$ **d** $-2e^{1-2x} + C$
e $2e^{5x-2} + C$ **f** $-4e^{5-3x} + C$
9 a $x - e^{-x} + C$ **b** $e^x - e^{-x} + C$
c $\frac{1}{2}e^{-2x} - e^{-x} + C$ **d** $e^{-3x} - \frac{1}{2}e^{-2x} + C$
e $e^{-3x} - e^{-2x} + C$ **f** $e^{-x} - e^{-2x} + C$
10 a $y = e^{x-1}$, $y = e^{-1}$
b $y = e^2 + 1 - e^{2-x}$, $y = e^2 + 1$
c $f(x) = e^x + \frac{x}{e} - 1$, $f(0) = 0$
d $f(x) = e^x - e^{-x} - 2x$
11 a $e^2 - e$
b $\frac{1}{2}(e^2 - e^{-2}) + 4(e - e^{-1}) + 8$
c $e + e^{-1} - 2$
d $\frac{1}{4}(e^4 - e^{-4}) + \frac{1}{2}(e^{-2} - e^2)$
e $e - e^{-1}$
f $e - e^{-1} + \frac{1}{2}(e^{-2} - e^2)$
12 a i $2xe^{x^2+3}$ **ii** $e^{x^2+3} + C$
b i $2(x - 1)e^{x^2-2x+3}$ **ii** $\frac{1}{2}e^{x^2-2x+3} + C$
c i $(6x + 4)e^{3x^2+4x+1}$ **ii** $\frac{1}{2}e^{3x^2+4x+1} + C$
d i $3x^2e^{x^3}$ **ii** $\frac{1}{3}(1 - e^{-1})$
13 a $-\frac{1}{2}e^{-2x} + C$ **b** $-\frac{1}{3}e^{-3x} + C$
c $2e^{\frac{1}{2}x} + C$ **d** $3e^{\frac{1}{3}x} + C$
e $-2e^{-\frac{1}{2}x} + C$ **f** $-3e^{-\frac{1}{3}x} + C$
14 a $y' = xe^x + e^x$, $e^2 + 1$
b $y' = -xe^{-x} + e^{-x}$, $-1 - e^2$
15 a $2e^{\frac{1}{2}x} + \frac{2}{3}e^{-\frac{3}{2}x} + C$ **b** $\frac{3}{2}e^{\frac{2}{3}x} - \frac{3}{4}e^{-\frac{4}{3}x} + C$
16 b 0

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17 a $\frac{1}{2}e^{x^2} + C$ **b** $2e^{x^2-7} + C$
c $\frac{1}{2}e^{3x^2+4x+1} + C$ **d** $\frac{1}{3}e^{x^3-3x^2} + C$
e $-e^{x^{-1}} + C$ **f** $\frac{2}{3}e^{-x\sqrt{x}} + C$

20 b 1.1276
d $e^{0.5} = \alpha + \sqrt{\alpha^2 - 1}, e^{-0.5} = \alpha - \sqrt{\alpha^2 - 1}$

Exercise 6E

1 a i $e - 1 \doteqdot 1.72$
ii $1 - e^{-1} \doteqdot 0.63$
iii $1 - e^{-2} \doteqdot 0.86$
iv $1 - e^{-3} \doteqdot 0.95$
b The total area is exactly 1.

2 a $\frac{1}{2}(e^6 - 1) \doteqdot 201.2$ square units
b $1 - e^{-1} \doteqdot 0.6321$ square units
c $3(1 - e^{-1}) \doteqdot 1.896$ square units

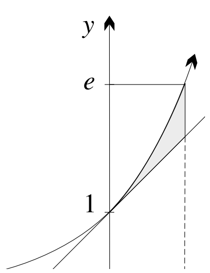
3 a $e(e^2 - 1)\,\text{u}^2$ **b** $\frac{1}{2}(e - e^{-1})\,\text{u}^2$
c $\frac{1}{2}e(e^2 - 1)\,\text{u}^2$ **d** $3e^2(e - 1)\,\text{u}^2$

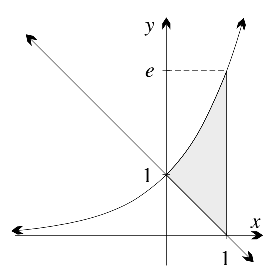
4 a $2(e - e^{-\frac{1}{2}})\,\text{u}^2$ **b** $(1 - e^{-1})\,\text{u}^2$

5 a $2(e^2 - e^{-2}) \doteqdot 14.51\,\text{u}^2$
b $18 + e^3 - e^{-3} \doteqdot 38.04\,\text{u}^2$

6 a $(1 + e^{-2})\,\text{u}^2$ **b** $1\,\text{u}^2$
c $e^{-1}\,\text{u}^2$ **d** $(3 + e^{-2})\,\text{u}^2$
e $1\,\text{u}^2$ **f** $(9 + e^{-2} - e)\,\text{u}^2$

7 a $\int_0^1 (e^x - 1 - x)\,dx = \left(e - 2\frac{1}{2}\right)\text{u}^2$ **b** $\int_0^1 (e^x - 1 + x)\,dx = \left(e - 1\frac{1}{2}\right)\text{u}^2$



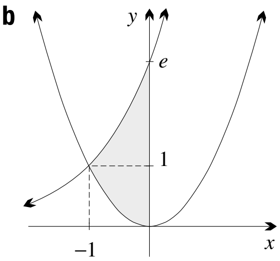


8 a The region is symmetric, so the area is twice the area in the first quadrant.
b $2 - \frac{2}{e}$ square units

9 a The region is symmetric, so the area is twice the area in the first quadrant.
b 2 square units

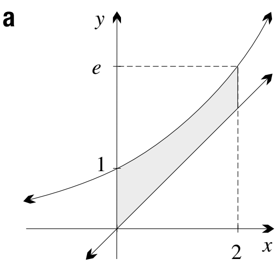
10 b 0
c The region is symmetric, so the area is twice the area in the first quadrant.
d $2(e^3 + e^{-3} - 2)$ square units

11 b



c $\left(e - 1\frac{1}{3}\right)\text{u}^2$

12 a



$(e^2 - 3)$ square units

b intercepts $(0, 7)$ and $(3, 0)$ and area $24 - \frac{7}{\log_e 2}$ square units

13 a $e - 1 \doteqdot 1.7183$
b 1.7539
c The trapezoidal-rule approximation is greater. The curve is concave down, so all the chords are above the curve.

14 a $-\frac{1}{2}e^{-x^2}$
b From $x = 0$ to $x = 2$, area $= \frac{1}{2} - \frac{1}{2}e^{-4}$ square units. The function is odd, so the area (not signed) from $x = -2$ to $x = 2$ is $1 - e^{-4}$ square units.

15 a i $1 - e^N$ **ii** 1
b i $1 - e^{-N}$ **ii** 1
c $\int_0^N 2xe^{-x^2}\,dx = 1 - e^{-N^2}$, thus in the limit as $N \to \infty$ this is just 1.

16 a $2(e - e^{\sqrt{\delta}})$
b It approaches $2(e - 1)$.

17 a $1 - (1 + N)e^{-N}$ **b** 1 **c** 2

Exercise 6F

1 a 2.303 **b** -2.303 **c** 11.72
d -12.02 **e** 3.912 **f** -3.912

2 a ln 20 **b** ln 5 **c** ln 80

3 a 3 **b** -1 **c** -2 **d** $\frac{1}{2}$
e 5 **f** 0.05 **g** 1 **h** e

4 b $1 = e^0$, so $\log_e 1 = \log_e e^0 = 0$.
d $e = e^1$, so $\log_e e = \log_e e^1 = 1$.

5 a $\log_e x = 6$ **b** $x = e^{-2}$ or $x = 1/e^2$
c $e^x = 24$ **d** $x = \log_e \frac{1}{3}$

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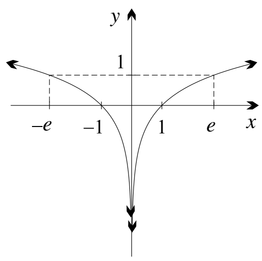
6 a $\dfrac{\log_e 7}{\log_e 2} \doteqdot 2.807$ b $\dfrac{\log_e 25}{\log_e 10} \doteqdot 1.398$

c $\dfrac{\log_e 0.04}{\log_e 3} \doteqdot -2.930$

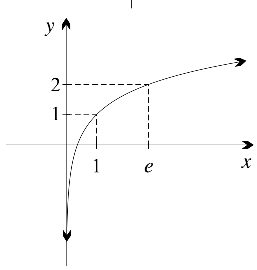
7 a Reflection in $y = x$, which reflects lines with gradient 1 to lines of gradient 1. The tangent to $y = e^x$ at its y-intercept has gradient 1, so its reflection also has gradient 1.

b Reflection in the y-axis, which is also a horizontal dilation with factor -1.

c

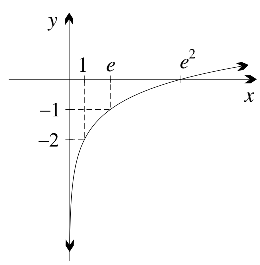


8 a



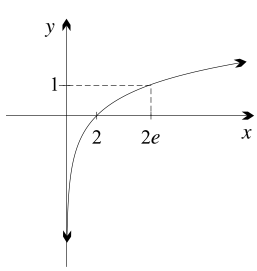
Shift $y = \log_e x$ up 1.

b



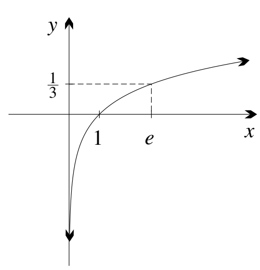
Shift $y = \log_e x$ down 2.

c



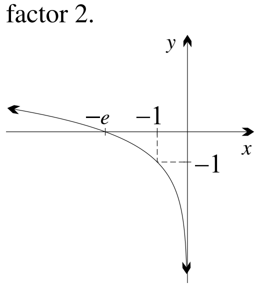
Stretch $y = \log_e x$ horizontally with factor 2.

d



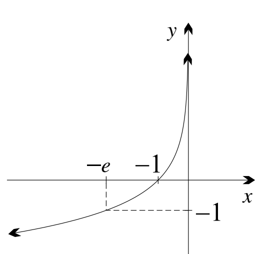
Stretch $y = \log_e x$ vertically with factor $\frac{1}{3}$.

9 a



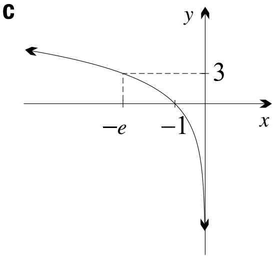
Shift $y = \log_e(-x)$ down 1.

b



Reflect $y = \log_e(-x)$ in the x-axis.

c



Stretch $y = \log_e(-x)$ vertically with factor 3.

10 a e b $-\frac{1}{e}$ c 6

d $\frac{1}{2}$ e $2e$ f 0

g e h 1 i 0

11 It is a horizontal dilation of $y = \log_e(-x)$ with factor $\frac{1}{2}$. Its equation is $y = \log_e(-2x)$.

12 a $x = 1$ or $x = \log_2 7$

b $x = 2$ ($3^x = -1$ has no solutions.)

c i $x = 2$ or $x = 0$

ii $x = 0$ or $x = \log_3 4$

iii $x = \log_3 5$ ($3^x = -4$ has no solutions.)

iv The quadratic has no solutions because $\Delta < 0$

v $x = 2$

vi $x = 1$ or 2

13 a $x = 0$ b $x = \log_e 2$

c $x = 0$ or $x = \log_e 3$ d $x = 0$

14 a $x = 1$ or $x = \log_4 3 \doteqdot 0.792$

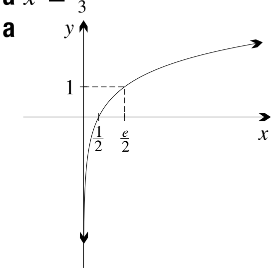
b $x = \log_{10}\frac{1+\sqrt{5}}{2} \doteqdot 0.209$. $\log_{10}\frac{1-\sqrt{5}}{2}$ does not exist because $\frac{1-\sqrt{5}}{2}$ is negative.

c $x = -1$ or $x = \log_{\frac{1}{2}} 2 \doteqdot -0.431$

15 a $x = e$ or $x = e^4$ b $x = 1$ or $x = e^3$

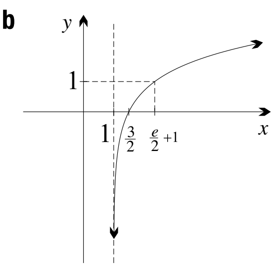
16 a $x = \frac{1}{3}$ b $x = 3$ or 4

17 a



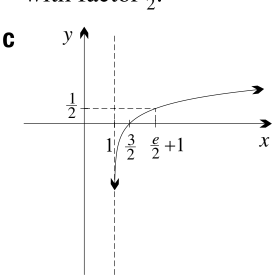
Stretch horizontally with factor $\frac{1}{2}$.

b



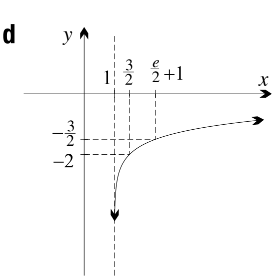
Shift right 1.

c



Stretch vertically with factor $\frac{1}{2}$.

d



Shift down 2.

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18 First, the base must be positive because powers of negative numbers are not well defined when the index is a real number, so a negative number can't be used as a base for logarithms. Secondly, the base cannot be 1 because all powers of 1 are 1, and in any case, $\log_e 1 = 0$ and you can't divide by zero.

19 a Stretch horizontally with factor $\frac{1}{5}$. Alternatively, $y = \log_e x + \log_e 5$, so it is a shift up $\log_e 5$.

b Shift up 2.
Alternatively, $y = \log_e x + \log_e e^2 = \log_e e^2 x$, so it is a dilation horizontally with factor e^{-2}.

20 $2\frac{28}{39}$

Exercise 6G

1 a $y' = \frac{1}{x+2}$ **b** $y' = \frac{1}{x-3}$
c $y' = \frac{3}{3x+4}$ **d** $y' = \frac{2}{2x-1}$
e $y' = \frac{-4}{-4x+1}$ **f** $y' = \frac{-3}{-3x+4}$
g $y' = \frac{-2}{-2x-7} = \frac{2}{2x+7}$ **h** $y' = \frac{6}{2x+4} = \frac{3}{x+2}$
i $y' = \frac{15}{3x-2}$

2 a $y = \log_e 2 + \log_e x$, $y' = \frac{1}{x}$
b $y = \log_e 5 + \log_e x$, $y' = \frac{1}{x}$
c $\frac{1}{x}$ **d** $\frac{1}{x}$ **e** $\frac{4}{x}$ **f** $\frac{3}{x}$ **g** $\frac{4}{x}$ **h** $\frac{3}{x}$

3 a $y' = \frac{1}{x+1}$, $y'(3) = \frac{1}{4}$
b $y' = \frac{2}{2x-1}$, $y'(3) = \frac{2}{5}$
c $y' = \frac{2}{2x-5}$, $y'(3) = 2$
d $y' = \frac{4}{4x+3}$, $y'(3) = \frac{4}{15}$
e $y' = \frac{5}{x+1}$, $y'(3) = \frac{5}{4}$
f $y' = \frac{12}{2x+9}$, $y'(3) = \frac{4}{5}$

4 a $\frac{1}{x}$ **b** $\frac{-1}{x+1}$
c $1 + \frac{4}{x}$ **d** $8x^3 + \frac{3}{x}$
e $\frac{2}{2x-1} + 6x$ **f** $3x^2 - 3 + \frac{5}{5x-7}$

5 a $y = 3\ln x$, $y' = \frac{3}{x}$
b $y = 2\ln x$, $y' = \frac{2}{x}$
c $y = -3\ln x$, $y' = -\frac{3}{x}$
d $y = -2\ln x$, $y' = -\frac{2}{x}$
e $y = \frac{1}{2}\ln x$, $y' = \frac{1}{2x}$
f $y = \frac{1}{2}\ln(x+1)$, $y' = \frac{1}{2(x+1)}$

6 a $\frac{1}{x}$ **b** $\frac{1}{x}$ **c** $\frac{3}{x}$
d $-\frac{6}{x}$ **e** $1 + \frac{1}{x}$ **f** $12x^2 - \frac{1}{x}$

7 a $\frac{2x}{x^2+1}$ **b** $\frac{-2x}{2-x^2}$ **c** $\frac{e^x}{1+e^x}$

8 a $\frac{2x+3}{x^2+3x+2}$ **b** $\frac{6x^2}{1+2x^3}$ **c** $\frac{e^x}{e^x-2}$
d $1 - \frac{2x+1}{x^2+x}$ **e** $2x + \frac{3x^2-1}{x^3-x}$
f $12x^2 - 10x + \frac{4x-3}{2x^2-3x+1}$

9 a $1, 45°$ **b** $\frac{1}{3}, 18°26'$
c $2, 63°26'$ **d** $\frac{1}{4}, 14°2'$

10 a $1 + \log_e x$
b $\frac{2x}{2x+1} + \log_e(2x+1)$
c $\frac{2x+1}{x} + 2\log_e x$
d $x^3(1 + 4\log_e x)$
e $\log_e(x+3) + 1$
f $\frac{2(x-1)}{2x+7} + \log_e(2x+7)$
g $e^x\left(\frac{1}{x} + \log_e x\right)$
h $e^{-x}\left(\frac{1}{x} - \log_e x\right)$

11 a $\frac{1-\log_e x}{x^2}$ **b** $\frac{1-2\log_e x}{x^3}$
c $\frac{\log_e x - 1}{(\log_e x)^2}$ **d** $\frac{x(2\log_e x - 1)}{(\log_e x)^2}$
e $\frac{1 - x\log_e x}{xe^x}$ **f** $\frac{e^x(x\log_e x - 1)}{x(\log_e x)^2}$

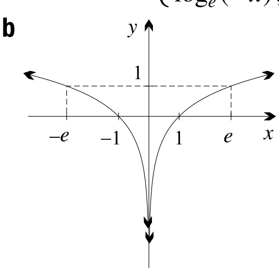
12 a $\frac{3}{x}$ **b** $\frac{1}{3x}$ **c** $-\frac{1}{x}$ **d** $\frac{1}{2x-4}$
e $\frac{1}{1+x} + \frac{1}{1-x}$
f $\frac{1}{x} + \frac{1}{2(x+1)}$
g $y = x\log_e 2$, $y' = \log_e 2$
h $y = x$, $y' = 1$
i $y = x\log_e x$, $y' = 1 + \log_e x$

13 a $f'(x) = \frac{1}{x-1}$, $f'(3) = \frac{1}{2}$, $f''(x) = -\frac{1}{(x-1)^2}$, $f''(3) = -\frac{1}{4}$
b $f'(x) = \frac{2}{2x+1}$, $f'(0) = 2$, $f''(x) = -\frac{4}{(2x+1)^2}$, $f''(0) = -4$
c $f'(x) = \frac{2}{x}$, $f'(2) = 1$, $f''(x) = -\frac{2}{x^2}$, $f''(2) = -\frac{1}{2}$
d $f'(x) = 1 + \log_e x$, $f'(e) = 2$, $f''(x) = \frac{1}{x}$, $f''(e) = \frac{1}{e}$

14 a $\log_e x$, $x = 1$
b $x(1 + 2\log_e x)$, $x = e^{-\frac{1}{2}}$
c $\frac{1-\log_e x}{x^2}$, $x = e$
d $\frac{4(\log_e x)^3}{x}$, $x = 1$
e $\frac{8}{x}(2\log_e x - 3)^3$, $x = e^{\frac{3}{2}}$
f $\frac{-1}{x(\log_e x)^2}$ is never zero.
g $\frac{1}{x\log_e x}$ is never zero.
h $1 + \ln x$, $\left(\frac{1}{e}, -\frac{1}{e}\right)$
i $\frac{x-1}{x^2}$, $(1, 1)$

16 a $\log_e|x| = \begin{cases} \log_e x, & \text{for } x > 0, \\ \log_e(-x), & \text{for } x < 0. \end{cases}$

b



c For $x > 0$, $\log_e|x| = \log_e x$, so $\frac{d}{dx}\log_e x = \frac{1}{x}$.
For $x < 0$, $\log_e|x| = \log_e(-x)$, and using the standard form, $\frac{d}{dx}\log_e(-x) = -\frac{1}{-x} = \frac{1}{x}$.

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d $\log_e 0$ is undefined. In fact, $\log_e x \to -\infty$ as $x = 0$, so $x = 0$ is an asymptote.

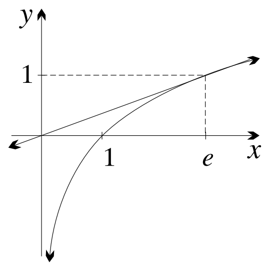
17 d **i** 2 **ii** 2.5937 **iii** 2.7048 **iv** 2.7169 **v** 2.7181

Exercise 6H

1 a $y = \frac{1}{e}x$ **b** $y = x - 1$ **c** $y = ex - 2$

d $y = -x + 1$. When $x = 0$, $y = 1$.

2 a As P moves to the left along the curve, the tangent becomes steeper, so it does not pass through the origin. As P moves right, the angle of the tangent becomes less steep, hence it does not pass through the origin.



b There are no tangents through each point below the curve. There are two tangents through each point above the curve and to the right of the y-axis. There is one tangent through each point on the curve, and through each point on and to the left of the y-axis.

3 a $y = 4x - 4$, $y = -\frac{1}{4}x + \frac{1}{4}$

b $y = x + 2$, $y = -x + 4$

c $y = 2x - 4$, $y = -\frac{1}{2}x - 1\frac{1}{2}$

d $y = -3x + 4$, $y = \frac{1}{3}x + \frac{2}{3}$

4 b $y = 3x - 3, -3$, $y = -\frac{1}{3}x + \frac{1}{3}, \frac{1}{3}$

c $\frac{5}{3}$ square units

5 a $(2, \log_e 2)$, $y = \frac{1}{2}x - 1 + \log_e 2$, $y = -2x + 4 + \log_e 2$

b $\left(\frac{1}{2}, -\log_e 2\right)$, $y = 2x - 1 - \log_e 2$, $y = -\frac{1}{2}x + \frac{1}{4} - \log_e 2$

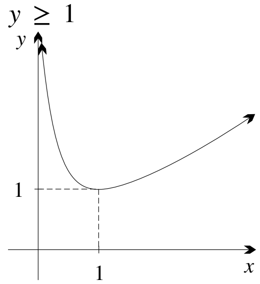
6 a $x > 0$. The domain is not symmetric about the origin, so the function is certainly not even or odd.

b $y' = 1 - \frac{1}{x}$, $y'' = \frac{1}{x^2}$

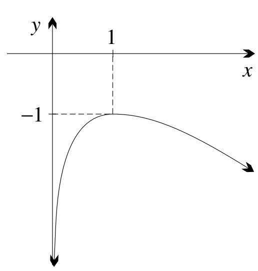
c $y'' > 0$, for all x

d $(1, 1)$

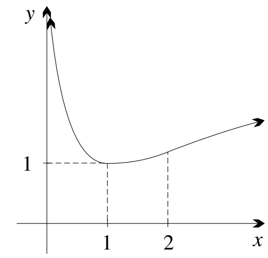
e $y \geq 1$



f



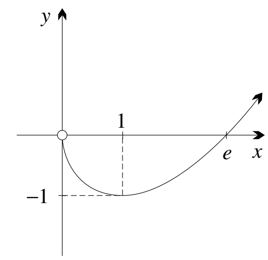
7 a $x > 0$ **d** $y \geq 1$



8 a $x > 0$, $(e, 0)$

b

x	1	e	e^2
y	-1	0	e^2
sign	$-$	0	$+$



c $y'' = \frac{1}{x}$

d $(1, -1)$ is a minimum turning point.

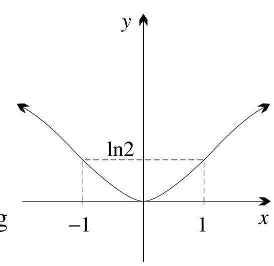
e It is concave up throughout its domain.

f $y \geq -1$

9 a all real x

b Even

c It is zero at $x = 0$, and is positive otherwise because the logs of numbers greater than 1 are positive.



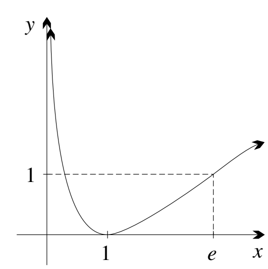
e $(0, 0)$ is a minimum turning point.

f $(1, \log_e 2)$ and $(-1, \log_e 2)$

g $y \geq 0$

10 a $x > 0$

b It is zero at $x = 1$, and is positive otherwise because squares cannot be negative.

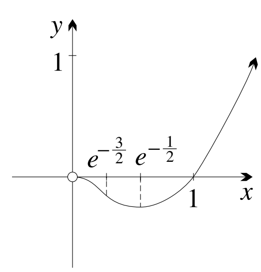


c $y' = \frac{2}{x}\ln x$

d $y \geq 0$

11 a $x > 0$. Minimum at $\left(\frac{1}{\sqrt{e}}, -\frac{1}{2e}\right)$

c $y \to 0$ as $x \to 0^+$, $y' \to 0$ as $x \to 0^+$, hence the graph becomes horizontal approaching the origin.



d $y \geq -\frac{1}{2e}$

12 a $x > 0$. $y \to 0^+$ as $x \to \infty$ so the x-axis is a horizontal asymptote.
$y \to -\infty$ as $x \to 0^+$ so the y-axis is a vertical asymptote.

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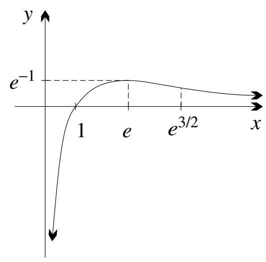
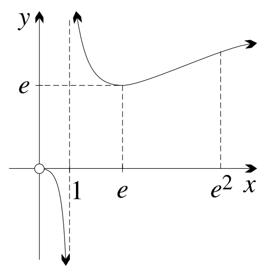
b $y' = \frac{1}{x^2}(1 - \log x)$, $y'' = \frac{1}{x^3}(2\log_e x - 3)$

d $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$

e $y \le e^{-1}$



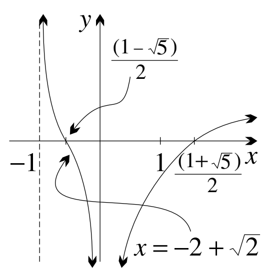
13 $x > 0, x \ne 1, y < 0$ or $y \ge e$. $x = 1$ is a vertical asymptote and the curve becomes horizontal approaching the origin.



14 a $x > -1$ or $x \ne 0$

c $x = -2$ is outside the domain.

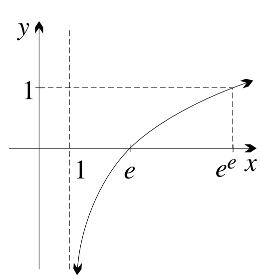
d one at $x = -2 + \sqrt{2}$



15 a $x > 1$

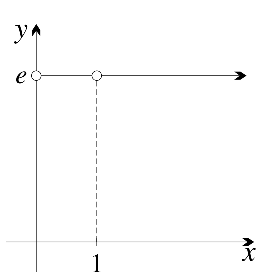
c $y' = \frac{1}{x\ln x}$, which can never be zero, $y'' = -\frac{1 + \ln x}{(x\ln x)^2}$

d The value $x = e^{-1}$ is outside the domain.



16 $\lim\limits_{x\to\infty}\dfrac{\log_e x}{x} = 0$ and $\lim\limits_{x\to 0^+} x\log_e x = 0$

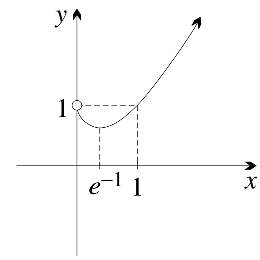
17 $y = e$ for all x in the domain, which is $x > 0$, $x \ne 1$.



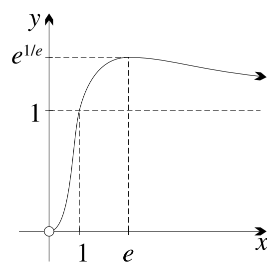
18 a $y' = x^x(1 + \log_e x)$

b Stationary point at $(e^{-1}, e^{-1/e})$, and gradient 1 at $x = 1$.

c domain: $x > 0$ (note that 0^0 is undefined), range: $y \ge e^{-1/e}$



19 b $y' = x^{-2}x^{\frac{1}{x}}(1 - \log_e x)$



Exercise 6I

1 a $2\log_e|x| + C$
b $\frac{1}{3}\log_e|x| + C$
c $\frac{4}{5}\log_e|x| + C$
d $\frac{3}{2}\log_e|x| + C$

2 a $\frac{1}{4}\log_e|4x + 1| + C$
b $\frac{1}{5}\log_e|5x - 3| + C$
c $2\log_e|3x + 2| + C$
d $3\log_e|5x + 1| + C$
e $\log_e|4x + 3| + C$
f $-\log_e|3 - x| + C$
g $-\frac{1}{2}\log_e|7 - 2x| + C$
h $\frac{4}{5}\log_e|5x - 1| + C$
i $-4\log_e|1 - 3x| + C$

3 a $\log_e 5$
b $\log_e 3$
c $\log_e|-2| - \log_e|-8| = -2\log_e 2$
d The integral is meaningless because it runs across an asymptote at $x = 0$.
e $\frac{1}{2}(\log_e 8 - \log_e 2) = \log_e 2$
f $\frac{1}{5}(\log_e|-75| - \log_e|-25|) = -\frac{1}{5}\log_e 3$

4 a $\log_e 2 \doteqdot 0.6931$
b $\log_e 3 - \log_e 5 \doteqdot -0.5108$
c $-\frac{1}{2}\log_e 7 \doteqdot -0.9730$
d $\frac{3}{2}\log_e 3 \doteqdot 1.648$
e $\log_e\frac{5}{2} \doteqdot 0.9163$
f The integral is meaningless because it runs across an asymptote at $x = 5\frac{1}{2}$.

5 a 1 **b** 2 **c** 3 **d** $\frac{1}{2}$

6 a $x + \log_e|x| + C$
b $\frac{1}{5}x + \frac{3}{5}\log_e|x| + C$
c $\frac{1}{9}\log_e|x| - \frac{8}{9}x + C$
d $3x - 2\log_e|x| + C$
e $x^2 + x - 4\log_e|x| + C$
f $\frac{1}{3}x^3 - \log_e|x| - \frac{2}{x} + C$

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7 a $\log_e|x^2 - 9| + C$
b $\log_e|3x^2 + x| + C$
c $\log_e|x^2 + x - 3| + C$
d $\log_e|2 + 5x - 3x^2| + C$
e $\frac{1}{2}\log_e|x^2 + 6x - 1| + C$
f $\frac{1}{4}\log_e|12x - 3 - 2x^2| + C$
g $\log_e(1 + e^x) + C$
h $-\log_e(1 + e^{-x}) + C$
i $\log_e(e^x + e^{-x}) + C$
The denominators in parts **g–i** are never negative, so the absolute value sign is unnecessary.

8 a $\frac{1}{3}\log_e|3x - k| + C$ **b** $\frac{1}{m}\log_e|mx - 2| + C$
c $\log_e|px + q| + C$ **d** $\frac{4}{s}\log_e|sx - t| + C$

9 a $f(x) = x + 2\ln|x|$, $f(2) = 2 + 2\ln 2$
b $f(x) = x^2 + \frac{1}{3}\ln|x| + 1$, $f(2) = 5 + \frac{1}{3}\ln 2$
c $f(x) = 3x + \frac{5}{2}\ln|2x - 1| - 3$, $f(2) = 3 + \frac{5}{2}\ln 3$
d $f(x) = 2x^3 + 5\ln|3x + 2| - 2$, $f(2) = 14 + 5\ln 8$

10 a $f(x) = x + \ln|x| + \frac{1}{2}x^2$
b $g(x) = x^2 - 3\ln|x| + \frac{4}{x} - 6$

11 a $y = \frac{1}{4}(\log_e|x| + 2)$, $x = e^{-2}$
b $y = 2\log_e|x + 1| + 1$
c $y = \log_e\left|\frac{x^2 + 5x + 4}{10}\right| + 1$, $y(0) = \log_e\frac{4}{10} + 1$
d $y = 2\log_e|x| + x + C$, $y = 2\log_e|x| + x$, $y(2) = \log_e 4 + 2$
e $f(x) = 2 + x - \log_e|x|$, $f(e) = e + 1$

12 a $\log_e|x^3 - 5| + C$
b $\log_e|x^4 + x - 5| + C$
c $\frac{1}{4}\log_e|x^4 - 6x^2| + C$
d $\frac{1}{2}\log_e|5x^4 - 7x^2 + 8| + C$
e $2\log_e 2$
f $\log_e\frac{4(e + 1)}{e + 2}$

13 a i $y' = \log_x$
ii $x\log_e x - x + C$ and $\frac{\sqrt{e}}{2}$
b i $y' = 4x\log_e x$
ii $\frac{1}{2}x^2\log_e x - \frac{1}{4}x^2$ and $2\log 2 - 1 - \frac{e^2}{4}$
c $\frac{2\log_e x}{x}$ and $\dfrac{3}{8}$
d $\ln(\ln x) + C$

14 a $a = e^5$ **b** $a = e^{-4}$
c $a = -e^2$ **d** $a = -e^{-1}$

15 a $\log_e(e^x + 1) + \mathrm{C}$ **b** $\frac{1}{3}(e^3 - e^{-3}) + 2$
c $(x + 1)e^x$

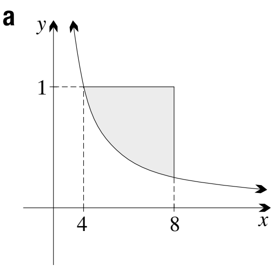
16 The key to all this is that $\log_e|5x| = \log_e 5 + \log_e|x|$, so that $\log_e|x|$ and $\log_e|5x|$ differ only by a constant $\log_e 5$. Thus $C_2 = C_1 - \frac{1}{5}\log_e 5$, and because C_1 and C_2 are arbitrary constants, it does not matter at all. In particular, in a definite integral, adding a constant doesn't change the answer, because it cancels out when we take $F(b) - F(a)$.

17 $y = \begin{cases}\log_e x + 1, & \text{for } x < 0,\\ \log(-x) + 2, & \text{for } x > 0.\end{cases}$

18 d i $\log_e\frac{3}{2} \doteqdot 0.41$
ii $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.$
e $\log_e(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots,$
$\log_e\frac{1}{2} \doteqdot -0.69$
f Using $x = \frac{1}{2}$, $\log_e 3 \doteqdot 1.0986$.

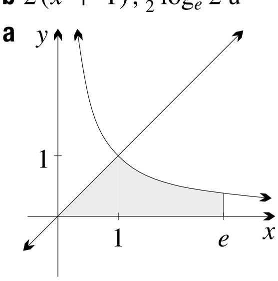
Exercise 6J

1 b $e \doteqdot 2.7$
2 i $\log_e 5 \doteqdot 1.609\,\text{u}^2$
ii 1u^2
iii $2\log_e 2 \doteqdot 1.386\,\text{u}^2$
3 a $(\log_e 3 - \log_e 2)$ square units
b $\log_e 2 - \log_e\frac{1}{2} = 2\log_e 2$ square units
4 a $\frac{1}{3}(\log_e 5 - \log_e 2)\,\text{u}^2$ **b** 9u^2
c $2\log_e 2 + \frac{15}{8}\text{u}^2$ **d** $\log_e 3 + 8\frac{2}{3}\text{u}^2$
5 a $(6 - 3\log_e 3)\,\text{u}^2$ **b** $(4 - \log_e 3)\,\text{u}^2$
6 a $\left(3\frac{3}{4} - 2\log_e 4\right)\text{u}^2$ **b** $\left(3\frac{3}{4} - 2\log_e 4\right)\text{u}^2$
7 a $2\log_e 2\text{u}^2$ **b** $(1 - \log_e 2)\,\text{u}^2$
8 a $(\log_e 4)\,\text{u}^2$ **b** $(6 - 3\log_e 3)\,\text{u}^2$
9 a $\frac{1}{2}\text{u}^2$ **b** $2\log_e\frac{4}{3}\text{u}^2$
10 a



b $(4 - \log_e 2)\,\text{u}^2$

11 a $(\frac{1}{3}, 3)$ and $(1, 1)$ **b** $(\frac{4}{3} - \log_e 3)\,\text{u}^2$
12 a $2x$, $\frac{1}{2}\log_e 5 \doteqdot 0.805\text{u}^2$
b $2(x + 1)$, $\frac{1}{2}\log_e 2\,\text{u}^2$
13 a



b $\frac{3}{2}\text{u}^2$

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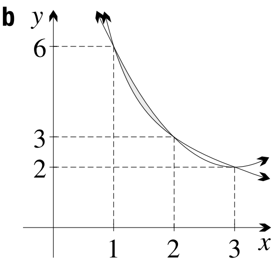
14 a 3.9828 square units

b $5\log_e 5 - 4 \doteqdot 4.0472$ square units

c The estimate is less. The curve is concave down, so the chords are below the curve.

15 b $(2 - \log_e 3)\ \text{u}^2$

16 b

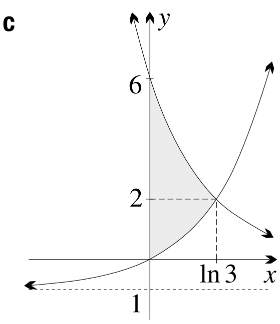


c $(2 - 6\log_e \frac{4}{3})\ \text{u}^2$

17 a The upper rectangle has height 2^{-n}, the lower rectangle has height 2^{-n-1}, both rectangles have width $2^{n+1} - 2^n = 2^n$.

18 b $(\ln 3, 2)$

c



d $(2 + \ln 3)\ \text{u}^2$

19 a $e - 2$ square units **b** e^{-1} square units

c $e - 2 + e^{-1}$ square units

20 $2\log_e(\sqrt{x} + 1) + C$

21 b i $\log_e(1 + \sqrt{2})$ **ii** $\log_e(2 + \sqrt{3})$

23 a i The region is above the x-axis, and contained within the rectangle $ABCO$.

ii

$$0 < \Big[\log_e t\Big]_1^{\sqrt{x}} < \sqrt{x}$$

$$0 < \log_e \sqrt{x} - \log_e 1 < \sqrt{x}$$

$$0 < \tfrac{1}{2}\log_e x < \sqrt{x}$$

$$\boxed{\times \tfrac{2}{x}} \quad 0 < \frac{\log_e x}{x} < \frac{2}{\sqrt{x}}.$$

iii The third term has limit 0 as $x \to \infty$, so the result follows by the sandwiching principle.

Exercise 6K

1 a 1.58 **b** 3.32 **c** 2.02 **d** −4.88

2 a $y' = \frac{1}{x\log_e 2}$ **b** $y' = \frac{1}{x\log_e 10}$ **c** $y' = \frac{3}{x\log_e 5}$

3 a $y' = \frac{1}{x\log_e 3}$ **b** $y' = \frac{1}{x\log_e 7}$ **c** $y' = \frac{5}{x\log_e 6}$

4 a $3^x\log_e 3$ **b** $4^x\log_e 4$ **c** $2^x\log_e 2$

5 a $y' = 10^x\log_e 10$ **b** $y' = 8^x\log_e 8$

c $y' = 3 \times 5^x\log_e 5$

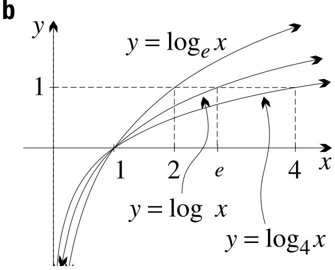
6 a $\frac{2^x}{\log_e 2} + C$ **b** $\frac{6^x}{\log_e 6} + C$

c $\frac{7^x}{\log_e 7} + C$ **d** $\frac{3^x}{\log_e 3} + C$

7 a $\frac{1}{\log_e 2} \doteqdot 1.443$ **b** $\frac{2}{\log_e 3} \doteqdot 1.820$

c $\frac{24}{5\log_e 5} \doteqdot 2.982$ **d** $\frac{15}{\log_e 4} \doteqdot 10.82$

8 b



9 a $\frac{1}{\log_e 2}$ **b** $y = \frac{1}{\log_e 2}(x - 1)$

c i $y = \frac{1}{\log_e 3}(x - 1)$ **ii** $y = \frac{1}{\log_e 5}(x - 1)$

10 a $\frac{6}{\log_e 2} \doteqdot 8.6562$ **b** $2 + \frac{8}{3\log_e 3} \doteqdot 4.4273$

c $\frac{99}{\log_e 10} - 20 \doteqdot 32.9952$

11 $y = \frac{\log_e x}{\log_e 10}$, $y' = \frac{1}{x\log_e 10}$

a $\frac{1}{10\log_e 10}$

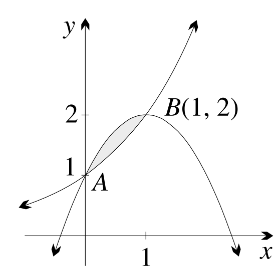
b $x - 10y\log_e 10 + 10(\log_e 10 - 1) = 0$

c $x = \frac{1}{\log_e 10}$

12 a $y = \frac{1}{\log_e 2}\left(\frac{x}{3} - 1 + \log_e 3\right)$, $y = \frac{x}{3} - 1 + \log_e 3$, $y = \frac{1}{\log_e 4}\left(\frac{x}{3} - 1 + \log_e 3\right)$

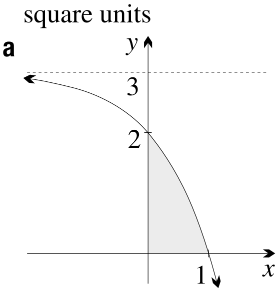
b They all meet the x-axis at $(3 - 3\log_e 3, 0)$

13 b $\left(\frac{5}{3} - \frac{1}{\log_e 2}\right)\ \text{u}^2$



14 intercepts $(0, 7)$ and $(3, 0)$, area $\left(24 - \frac{7}{\log_e 2}\right)$ square units

15 a



b $\left(3 - \frac{2}{\log_e 3}\right)\text{u}^2$

16 b $\displaystyle\int_{-\frac{1}{2}}^{0} x + 1 - 4^x\,dx$ **c** $\frac{3}{8} - \frac{1}{2\log_e 4}$

18 a $x\log_e x - x + C$ **b** $10 - \frac{9}{\log_e 10}$

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19 a **i** $y' = \frac{1}{x \log_e 3}$

ii $y' = \frac{2}{(2x + 3) \log_e 7}$

iii $y' = -\frac{45}{(4 - 9x) \log_e 6}$

b **i** $y' = 10^x \log_e 10$

ii $y' = 4 \times 8^{4x-3} \log_e 8$

iii $y' = -21 \times 5^{2-7x} \log_e 5$

c **i** $\frac{3^{5x}}{5 \log_e 3} + C$ **ii** $\frac{6^{2x+7}}{2 \log_e 6} + C$

iii $-\frac{5 \times 7^{4-9x}}{9 \log_e 7} + C$

20 a $y = e^{kx}$ and $y = \frac{1}{k} \log_e x$

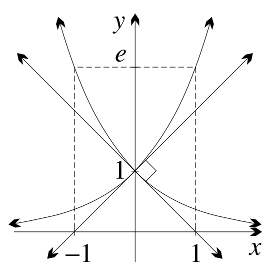
b The functions $y = a^x$ and $y = \log_e x$ are inverse, so they are symmetric in the line $y = x$. The common tangent is therefore the line $y = x$, which has gradient 1. (This argument would be invalid if there were more than one intersection point.)

c $k e^{kx} = 1$ and $\frac{1}{kx} = 1$

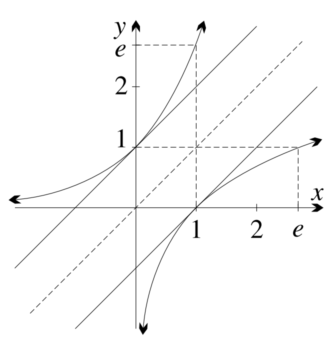
d $k = \frac{1}{e}$, $a = e^{\frac{1}{e}}$

Chapter 6 review exercise

1 a Each graph is reflected onto the other in the line $x = 0$. The tangents have gradients 1 and -1, and are at right angles.



b Each graph is reflected onto the other in the line $y = x$. The tangents both have gradients 1, and are thus parallel.



2 a 54.60 **b** 2.718 **c** 0.2231 **d** 0.6931

e -0.3010 **f** -5.059 **g** 130.6 **h** 0.5925

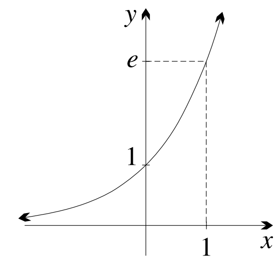
3 a 2.402 **b** 5.672 **c** 5.197 **d** 3.034

4 a e^{5x} **b** e^{6x} **c** e^{-4x} **d** e^{9x}

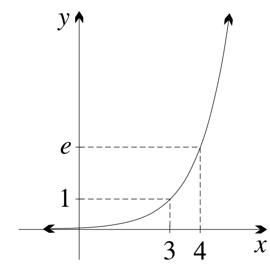
5 a $x = 2$

b $x = \log_e 4 \,(= 2 \log_e 2)$ or $\log_e 7$

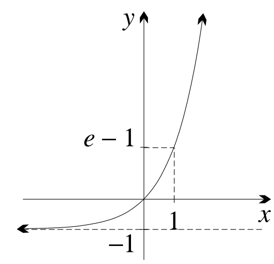
6 a $y > 0$



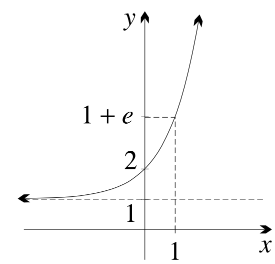
b $y > 0$



c $y > 1$

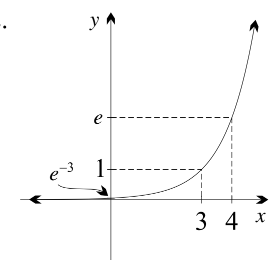


d $y > -1$



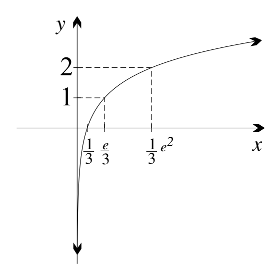
7 a **i** Shift $y = e^x$ right 3 units.

ii $y = e^{-3}e^x$ or $\frac{y}{e^{-3}} = e^x$, so dilate vertically with factor e^{-3}.



b **i** $y = \log_e \frac{x}{1/3}$, so dilate $y = \log_e x$ horizontally with factor $\frac{1}{3}$.

ii $y = \log_e x + \log_e 3$, or $y - \log_e 3 = \log_e x$, so shift $y = \log_e x$ up $\log_e 3$.



8 a e^x **b** $3e^{3x}$ **c** $2e^{2x+3}$ **d** $-e^{-x}$

e $-3e^{-3x}$ **f** $6e^{2x+5}$ **g** $2e^{\frac{1}{2}x}$ **h** $4e^{6x-5}$

9 a $5e^{5x}$ **b** $4e^{4x}$ **c** $-3e^{-3x}$ **d** $-6e^{-6x}$

10 a $3x^2 e^{x^3}$ **b** $(2x - 3)\, e^{x^2 - 3x}$

c $e^{2x} + 2xe^{2x} = e^{2x}(1 + 2x)$

d $6e^{2x}(e^{2x} + 1)^2$

e $\frac{e^{3x}(3x - 1)}{x^2}$

f $2xe^{x^2}(1 + x^2)$

g $5(e^x + e^{-x})(e^x - e^{-x})^4$

h $\frac{4xe^{2x}}{(2x + 1)^2}$

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11 a $y' = 2e^{2x+1}$, $y'' = 4e^{2x+1}$

b $y' = 2xe^{x^2+1}$, $y'' = 2e^{x^2+1}(2x^2 + 1)$

12 $y = e^2x - e^2$, x-intercept 1, y-intercept $-e^2$.

13 a $\frac{1}{3}$

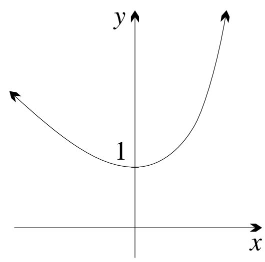
b When $x = 0$, $y'' = 9$, so the curve is concave up there.

14 a $y' = e^x - 1$, $y'' = e^x$

b $(0, 1)$ is a minimum turning point.

c $y'' = e^x$, which is positive for all x.

d Range: $y \geq 1$



15 $\left(\frac{1}{2}, \frac{1}{2e}\right)$ is a maximum turning point.

16 a $\frac{1}{5}e^{5x} + C$ **b** $-2e^{2-5x} + C$

c $5e^{\frac{1}{5}x} + C$ **d** $\frac{3}{5}e^{5x-4} + C$

17 a $e^2 - 1$ **b** $\frac{1}{2}(e^2 - 1)$

c $e - 1$ **d** $\frac{1}{3}(e^2 - 1)$

e $\frac{1}{2}e^2(e - 1)$ **f** $4(e - 1)$

18 a $-\frac{1}{5}e^{-5x} + C$ **b** $\frac{1}{4}e^{4x} + C$

c $-2e^{-3x} + C$ **d** $\frac{1}{6}e^{6x} + C$

e $-\frac{1}{2}e^{-2x} + C$ **f** $e^x - \frac{1}{2}e^{-2x} + C$

g $\frac{1}{3}e^{3x} + e^x + C$ **h** $x - 2e^{-x} - \frac{1}{2}e^{-2x} + C$

19 a $2 - e^{-1}$ **b** $\frac{1}{2}(e^4 + 3)$

c $2(1 - e^{-1})$ **d** $\frac{1}{3}(e - 2)$

e $e - e^{-1}$ **f** $\frac{1}{2}(e^2 + 4e - 3)$

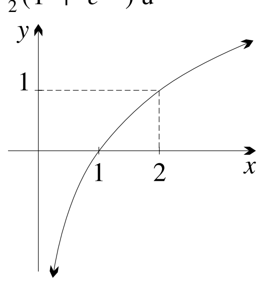
20 $f(x) = e^x + e^{-x} - x + 1$, $f(1) = e + e^{-1}$

21 a $3x^2e^{x^3}$ **b** $\frac{1}{3}(e - 1)$

22 a $3.19\ \text{u}^2$ **b** $0.368\ \text{u}^2$

23 a $\frac{1}{2}(1 + e^{-2})\ \text{u}^2$ **b** $\frac{1}{2}(3 - e)\ \text{u}^2$

24 a



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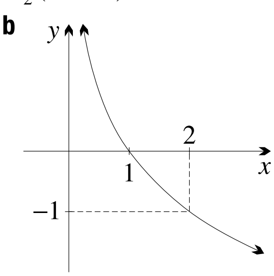
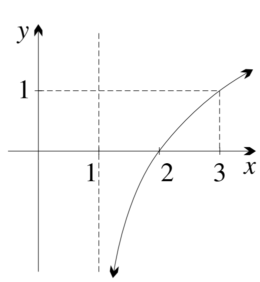
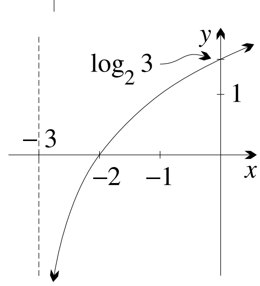


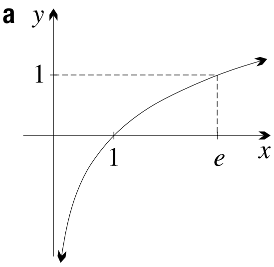
c



d



25 a



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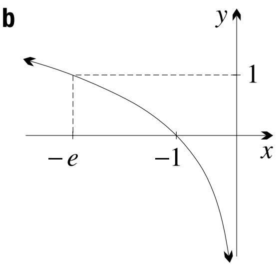

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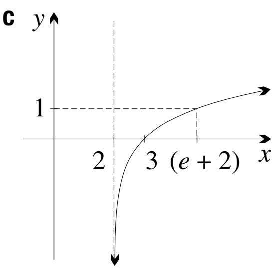
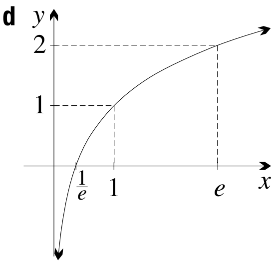


d



26 a e **b** 3 **c** -1 **d** e

27 a $\dfrac{1}{x}$ **b** $\dfrac{1}{x}$ **c** $\dfrac{1}{x + 4}$

d $\dfrac{2}{2x - 5}$ **e** $\dfrac{10}{5x - 1}$ **f** $1 + \dfrac{1}{x}$

g $\dfrac{2x - 5}{x^2 - 5x + 2}$ **h** $\dfrac{15x^4}{1 + 3x^5}$

i $8x - 24x^2 + \dfrac{2x}{x^2 - 2}$

28 a $\dfrac{3}{x}$ **b** $\dfrac{1}{2x}$

c $\dfrac{1}{x} + \dfrac{1}{x + 2}$ **d** $\dfrac{1}{x} - \dfrac{1}{x - 1}$

29 a $1 + \log_e x$ **b** $\dfrac{e^x}{x} + e^x \log_e x$

c $\dfrac{\ln x - 1}{(\ln x)^2}$ **d** $\dfrac{1 - 2\ln x}{x^3}$

30 $y = 3x + 1$

32 a $\log_e |x| + C$ **b** $3\log_e |x| + C$

c $\frac{1}{5}\log_e |x| + C$ **d** $\log_e |x + 7| + C$

e $\frac{1}{2}\log_e |2x - 1| + C$ **f** $-\frac{1}{3}\log_e |2 - 3x| + C$

g $\log_e |2x + 9| + C$ **h** $-2\log_e |1 - 4x| + C$

33 a $\log_e \frac{3}{2}$ **b** $\frac{1}{4}\log_e 13$

c 1 **d** 1

34 a $\log_e(x^2 + 4) + C$

b $\log_e |x^3 - 5x + 7| + C$

c $\frac{1}{2}\log_e |x^2 - 3| + C$

d $\frac{1}{4}\log_e |x^4 - 4x| + C$

35 $\log_e 2\ \text{u}^2$

36 a $12 - 5\log_e 5\ \text{u}^2$

37 a e^x **b** $2^x \log_e 2$

c $3^x \log_e 3$ **d** $5^x \log_e 5$

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38 a $e^x + C$ **b** $\dfrac{2^x}{\log_e 2} + C$

c $\dfrac{3^x}{\log_e 3} + C$ **d** $\dfrac{5^x}{\log_e 5} + C$

39 a $x\log_e x - x$ **b** $xe^x - e^x$

40 a $8\log_e 2$ **b** $\dfrac{1}{8\log_e 2}$

c The curves $y = 2^x$ and $y = \log_2 x$ are reflections of each other in $y = x$. This reflection exchanges A and B, and exchanges their tangents. Because it also exchanges rise and run, the gradients are reciprocals of each other.

41 a $\dfrac{7}{\ln 2}$ and $\dfrac{7}{8\ln 2}$

b When $y = 2^x$ is transformed successively by a vertical dilation with factor 8 and a shift right 3 units, the result is the same graph $y = 2^x$. The region in the second integral is transformed to the region in the first integral by this compound transformation.

Chapter 7

Exercise 7A

1 a The entries under 0.2 are 0.198 669, 0.993 347, 0.202 710, 1.013 550, 0.980 067.

b 1 and 1

3 a $\frac{\pi}{90}$ **b** $\sin 2° = \sin\frac{\pi}{90} \doteqdot \frac{\pi}{90}$ **c** 0.0349

4 a The entries under 5° are 0.087 27, 0.087 16, 0.9987, 0.087 49, 1.003, 0.9962.

b $\sin x < x < \tan x$

c i 1 **ii** 1

d $x \le 0.0774$ (correct to four decimal places), that is, $x \le 4°26'$.

6 a 1 **b** 2 **c** $\frac{1}{2}$ **d** $\frac{3}{2}$ **e** $\frac{5}{3}$ **f** 8

7 87 metres

8 $26'$

13 a $AB^2 = 2r^2(1 - \cos x)$, arc $AB = rx$

b The arc is longer than the chord, so $\cos x$ is larger than the approximation.

15 a $\sin(A - B) = \sin A\cos B - \cos A\sin B$

b 6

Exercise 7B

1 a $\cos x$ **b** $-\sin x$ **c** $\sec^2 x$ **d** $2\cos x$
e $2\cos 2x$ **f** $-3\sin x$ **g** $-3\sin 3x$ **h** $4\sec^2 4x$
i $4\sec^2 x$ **j** $6\cos 3x$ **k** $4\sec^2 2x$ **l** $-8\sin 2x$
m $-2\cos 2x$ **n** $2\sin 2x$ **o** $-2\sec^2 2x$ **p** $\frac{1}{2}\sec^2\frac{1}{2}x$
q $-\frac{1}{2}\sin\frac{1}{2}x$ **r** $\frac{1}{2}\cos\frac{x}{2}$ **s** $\sec^2\frac{1}{5}x$ **t** $-2\sin\frac{x}{3}$
u $4\cos\frac{x}{4}$

2 a $2\pi\cos 2\pi x$ **b** $\frac{\pi}{2}\sec^2\frac{\pi}{2}x$
c $3\cos x - 5\sin 5x$ **d** $4\pi\cos\pi x - 3\pi\sin\pi x$
e $2\cos(2x - 1)$ **f** $3\sec^2(1 + 3x)$
g $2\sin(1 - x)$ **h** $-5\sin(5x + 4)$
i $-21\cos(2 - 3x)$ **j** $-10\sec^2(10 - x)$
k $3\cos\left(\frac{x+1}{2}\right)$ **l** $-6\sin\left(\frac{2x+1}{5}\right)$

3 a $2\cos 2x$, $-4\sin 2x$, $-8\cos 2x$, $16\sin 2x$

b $-10\sin 10x$, $-100\cos 10x$, $1000\sin 10x$, $10\,000\cos 10x$

c $\frac{1}{2}\cos\frac{1}{2}x$, $-\frac{1}{4}\sin\frac{1}{2}x$, $-\frac{1}{8}\cos\frac{1}{2}x$, $\frac{1}{16}\sin\frac{1}{2}x$

d $-\frac{1}{3}\sin\frac{1}{3}x$, $-\frac{1}{9}\cos\frac{1}{3}x$, $\frac{1}{27}\sin\frac{1}{3}x$, $\frac{1}{81}\cos\frac{1}{3}x$

4 $-2\sin 2x$
a 0 **b** -1 **c** $-\sqrt{3}$ **d** -2

5 $\frac{1}{4}\cos\left(\frac{1}{4}x + \frac{\pi}{2}\right)$
a 0 **b** $-\frac{1}{4}$ **c** $\frac{1}{8}\sqrt{2}$ **d** $-\frac{1}{8}\sqrt{2}$

6 a $x\cos x + \sin x$ **b** $2(\tan 2x + 2x\sec^2 2x)$
c $2x(\cos 2x - x\sin 2x)$ **d** $3x^2(\sin 3x + x\cos 3x)$

7 a $\frac{x\cos x - \sin x}{x^2}$ **b** $\frac{-x\sin x - \cos x}{x^2}$
c $\frac{x(2\cos x + x\sin x)}{\cos^2 x}$ **d** $\frac{1 + \sin x - x\cos x}{(1 + \sin x)^2}$

8 a $2x\cos(x^2)$ **b** $-2x\cos(1 - x^2)$
c $-3x^2\sin(x^3 + 1)$ **d** $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$
e $-2\cos x\sin x$ **f** $3\sin^2 x\cos x$
g $2\tan x\sec^2 x$ **h** $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$

9 d $y = \cos x$

11 a $e^{\tan x}\sec^2 x$ **b** $2e^{\sin 2x}\cos 2x$
c $2e^{2x}\cos(e^{2x})$ **d** $-\tan x$
e $\cot x$ **f** $-4\tan 4x$

12 a $\cos^2 x - \sin^2 x$
b $14\sin 7x\cos 7x$
c $-15\cos^4 3x\sin 3x$
d $9\sin 3x(1 - \cos 3x)^2$
e $2(\cos 2x\sin 4x + 2\sin 2x\cos 4x)$
f $15\tan^2(5x - 4)\sec^2(5x - 4)$

13 a $\frac{-\cos x}{(1 + \sin x)^2}$ **b** $\frac{1}{1 + \cos x}$
c $\frac{-1}{1 + \sin x}$ **d** $\frac{-1}{(\cos x + \sin x)^2}$

14 c i The graphs are reflections of each other in the x-axis.
ii The graphs are identical.

16 a $y' = e^x\sin x + e^x\cos x$, $y'' = 2e^x\cos x$
b $y' = -e^{-x}\cos x - e^{-x}\sin x$, $y'' = 2e^{-x}\sin x$

18 a $\log_b P - \log_b Q$

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21 b $\sin\left(\frac{n\pi}{2} + x\right)$

22 b $\frac{1}{2}\Big((m + n)\cos(m + n)x + (m - n)\cos(m - n)x\Big)$

b $\frac{1}{2}\Big(\cos(m + n)x + \cos(m - n)x\Big)$

$= \cos mx \cos nx,$

$-\frac{1}{2}\Big((m + n)\sin(m + n)x + (m - n)\sin(m - n)x\Big)$

Exercise 7C

1 a 1	**b** -1	**c** $\frac{1}{2}$	**d** $-\frac{1}{2}$
e $\frac{1}{\sqrt{2}}$	**f** 1	**g** 2	**h** -2
i $\frac{\sqrt{3}}{4}$	**j** $\frac{1}{4}$	**k** 8	**l** $\sqrt{3}$

3 a $y = -x + \pi$ **b** $2x - y = \frac{\pi}{2} - 1$

c $x + 2y = \frac{\pi}{6} + \sqrt{3}$ **d** $y = -2x + \frac{\pi}{2}$

e $x + y = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ **f** $y = -\pi x + \pi^2$

4 a $\frac{\pi}{2}, \frac{3\pi}{26}$ **b** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c** $\frac{\pi}{6}, \frac{5\pi}{6}$ **d** $\frac{5\pi}{6}, \frac{7\pi}{6}$

6 b 1 and -1

c $x - y = \frac{\pi}{4} - \frac{1}{2}, x + y = \frac{\pi}{4} + \frac{1}{2}$

7 a $y' = \cos x e^{\sin x}$ **b** $\frac{\pi}{2}, \frac{3\pi}{2}$

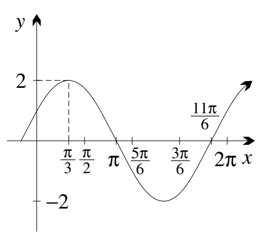
8 a $y' = -\sin x e^{\cos x}$ **b** $0, \pi, 2\pi$

9 a $y' = -\sin x + \sqrt{3}\cos x, y'' = -\cos x - \sqrt{3}\sin x$

b maximum turning point $\left(\frac{\pi}{3}, 2\right)$, minimum turning point $\left(\frac{4\pi}{3}, -2\right)$

c $\left(\frac{5\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$

d

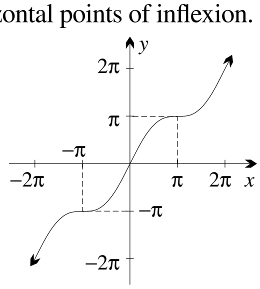


10 a $y' = 1 + \cos x$

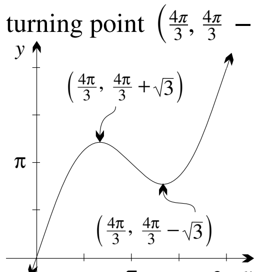
b $(-\pi, -\pi)$ and (π, π) are horizontal points of inflexion.

c $(0, 0)$

d



11 maximum turning point $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$, minimum turning point $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$, inflexion (π, π)

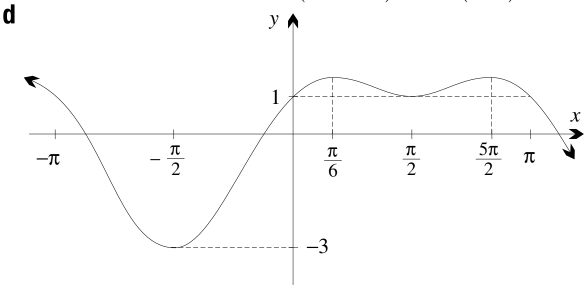


15 b minimum $\sqrt{3}$ when $\theta = \frac{\pi}{6}$, maximum 2 when $\theta = 0$

16 a $y' = 2\cos x - 2\sin 2x, y'' = -2\sin x - 4\cos 2x$

c maximum turning points $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ and $\left(\frac{5\pi}{6}, \frac{3}{2}\right)$, minimum turning points $\left(-\frac{\pi}{2}, -3\right)$ and $\left(\frac{\pi}{2}, 1\right)$

d

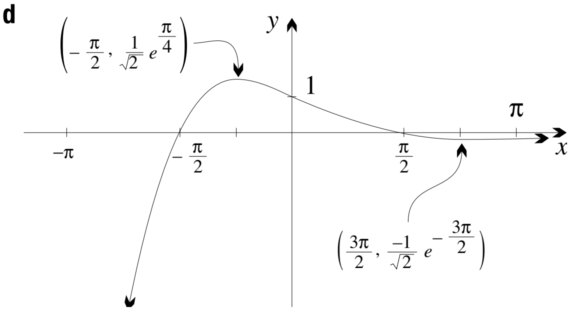


17 a $y' = -e^{-x}(\cos x + \sin x), y'' = 2e^{-x}\sin x$

b minimum turning point $\left(\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}e^{-\frac{3\pi}{4}}\right)$, maximum turning point $\left(-\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}\right)$

c $(-\pi, -e^{\pi}), (0, 1), (\pi, -e^{-\pi})$

d



18 a The angle of inclination is $\pi - \alpha$ and so $m = \tan(\pi - \alpha) = -\tan\alpha$.

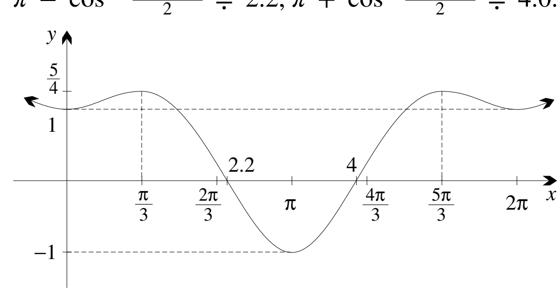
b $P = \left(\frac{1}{\tan\alpha} + 2, 0\right), Q = (0, 2\tan\alpha + 1)$

19 b Over the given domain, the graph of $y = \tan x$ is above the graph of $y = x$.

c $f'(x) = \frac{x\cos x - \sin x}{x^2}$

d From the sketch, we see that $f(x) > \frac{2}{\pi}$ over the given domain.

20 a maximum turning points $\left(\frac{\pi}{3}, \frac{5}{4}\right), \left(\frac{5\pi}{3}, \frac{5}{4}\right)$, minimum turning points $(0, 1), (\pi, -1), (2\pi, 1)$, x-intercepts $\pi - \cos^{-1}\frac{\sqrt{5} - 1}{2} \doteqdot 2.2, \pi + \cos^{-1}\frac{\sqrt{5} - 1}{2} \doteqdot 4.0.$



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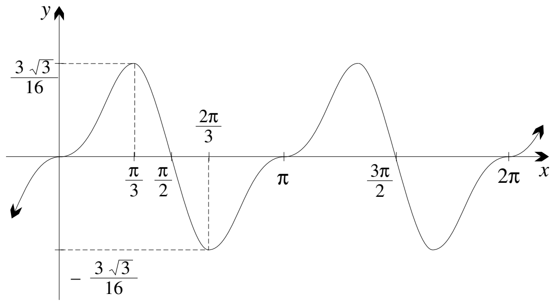
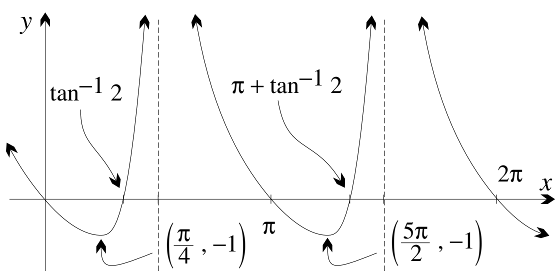
b maximum turning points $\left(\frac{\pi}{3}, \frac{3\sqrt{3}}{16}\right)$, $\left(\frac{4\pi}{3}, \frac{3\sqrt{3}}{16}\right)$,
minimum turning points $\left(\frac{2\pi}{3}, -\frac{3\sqrt{3}}{16}\right)$, $\left(\frac{5\pi}{3}, -\frac{3\sqrt{3}}{16}\right)$
horizontal points of inflexion $(0, 0)$, $(\pi, 0)$, $(2\pi, 0)$



c minimum turning points $\left(\frac{\pi}{4}, -1\right)$, $\left(\frac{5\pi}{4}, -1\right)$, vertical asymptotes $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, x-intercepts $0, \pi, 2\pi$, $\tan^{-1} 2 \doteqdot 1.1$, $\pi + \tan^{-1} 2 \doteqdot 4.25$

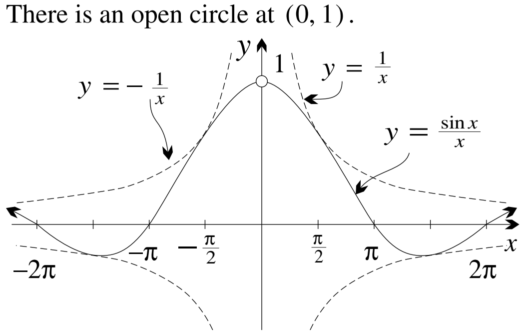


21 a Domain: $x \neq 0$, $f(x)$ is even because it is the ratio of two odd functions, the zeroes are $x = n\pi$ where n is an integer, $\lim_{x\to\infty} f(x) = 0$.

b $f'(x) = \frac{x\cos x - \sin x}{x^2}$, which is zero when $\tan x = x$.

c The graph of $y = x$ crosses the graph of $y = \tan x$ just to the left of $x = \frac{3\pi}{2}$, of $x = \frac{5\pi}{2}$ and of $x = \frac{7\pi}{2}$. Using the calculator, the three turning points of $y = f(x)$ are approximately $(1.43\pi, -0.217)$, $(2.46\pi, 0.128)$ and $(3.47\pi, -0.091)$.

d There is an open circle at $(0, 1)$.



Exercise 7D

1 a $\tan x + C$ **b** $\sin x + C$
c $-\cos x + C$ **d** $\cos x + C$
e $2\sin x + C$ **f** $\frac{1}{2}\sin 2x + C$
g $\frac{1}{2}\sin x + C$ **h** $2\sin\frac{1}{2}x + C$
i $-\frac{1}{2}\cos 2x + C$ **j** $\frac{1}{5}\tan 5x + C$
k $\frac{1}{3}\sin 3x + C$ **l** $3\tan\frac{1}{3}x + C$
m $-2\cos\frac{x}{2} + C$ **n** $-5\sin\frac{1}{5}x + C$
o $2\cos 2x + C$ **p** $-\cos\frac{1}{4}x + C$
q $-36\tan\frac{1}{3}x + C$ **r** $6\sin\frac{x}{3} + C$

2 a 1 **b** $\frac{1}{2}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\sqrt{3}$ **e** 1
f $\frac{3}{4}$ **g** 2 **h** 1 **i** 4

3 a $y = 1 - \cos x$
b $y = \sin x + \cos 2x - 1$
c $y = -\cos x + \sin x - 3$

6 a $\sin(x + 2) + C$ **b** $\frac{1}{2}\sin(2x + 1) + C$
c $-\cos(x + 2) + C$ **d** $-\frac{1}{2}\cos(2x + 1) + C$
e $\frac{1}{3}\sin(3x - 2) + C$ **f** $\frac{1}{5}\cos(7 - 5x) + C$
g $-\tan(4 - x) + C$ **h** $-3\tan\left(\frac{1 - x}{3}\right) + C$
i $3\cos\left(\frac{1 - x}{3}\right) + C$

7 a $2\sin 3x + 8\cos\frac{1}{2}x + C$
b $4\tan 2x - 40\sin\frac{1}{4}x - 36\cos\frac{1}{3}x + C$

8 a $f(x) = \sin\pi x$, $f\left(\frac{1}{3}\right) = \frac{1}{2}\sqrt{3}$
b $f(x) = \frac{1}{2\pi} + \frac{1}{\pi}\sin\pi x$, $f\left(\frac{1}{6}\right) = \frac{1}{\pi}$
c $f(x) = -2\cos 3x + x + \left(1 - \frac{\pi}{2}\right)$

9 a $-\cos(ax + b) + C$ **b** $\pi\sin\pi x + C$
c $\frac{1}{u^2}\tan(v + ux) + C$ **d** $\tan ax + C$

10 a $1 + \tan^2 x = \sec^2 x$, $\tan x - x + C$
b $1 - \sin^2 x = \cos^2 x$, $2\sqrt{3}$

11 a $\log_e f(x) + C$

12 a $\int \tan x = -\ln\cos x + C$

b $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\,dx = \left[\log\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \log 2$

13 a $5\sin^4 x\cos x$, $\frac{1}{5}\sin^5 x + C$
b $3\tan^2 x\sec^2 x$, $\frac{1}{3}\tan^3 x + C$

14 a $\cos x e^{\sin x}$, $e - 1$ **b** $e^{\tan x} + C$, $e - 1$

15 a 1 **b** $\frac{5}{24}$

16 a $\frac{2}{9}$ **b** $\frac{1}{n + 1}$ **c** 0
d $\frac{1}{2^{n+1}(n + 1)}$ **e** $10\frac{1}{8}$ **f** $\frac{1}{n + 1}$

17 b $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, so $\frac{1}{2}\sin^2 x + C$
$= \frac{1}{4} - \frac{1}{4}\cos 2x + C = -\frac{1}{4}\cos 2x + \left(C + \frac{1}{4}\right)$
$= -\frac{1}{4}\cos 2x + D$, where $D = C + \frac{1}{4}$.

18 $\sin 2x + 2x\cos 2x$, $\frac{\pi - 2}{8}$

19 b $\frac{4}{3}$

20 b i $\frac{6}{5}$ **ii** $-\frac{6}{7}$

21 a $A = 5, B = 3$

Exercise 7E

1 a 1 square unit **b** $\frac{1}{2}$ square unit
2 a 1 square unit **b** $\sqrt{3}$ square units

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3 a $1 - \frac{1}{\sqrt{2}}$ square units **b** $1 - \frac{\sqrt{3}}{2}$ square units

4 a $\frac{1}{2}\sqrt{3}$ u^2 **b** $\frac{1}{2}\sqrt{3}$ u^2

5 a $\frac{1}{2}$ u^2 **b** $\frac{1}{2}$ u^2

c $1 - \frac{\sqrt{3}}{2} = \frac{1}{2}\left(2 - \sqrt{3}\right)$ u^2

d $\frac{1}{3}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{1}{6}\left(2 - \sqrt{2}\right)$ u^2

e $\frac{2}{3}\sqrt{3}$ u^2

f 4 u^2

6 a $\left(\sqrt{2} - 1\right)$ u^2 **b** $\frac{1}{4}$ u^2

c $\left(\frac{\pi^2}{8} - 1\right)$ u^2 **d** $(\pi - 2)$ u^2

7 a $\left(2 - \sqrt{2}\right)$ u^2 **b** $1\frac{1}{2}$ u^2

8 a 2 u^2 **b** 1 u^2

9 a 2 u^2 **b** $\sqrt{2}$ u^2 **c** 2 u^2

d $\frac{1}{2}$ u^2 **e** 4 u^2 **f** 1 u^2

10 b $\frac{4}{\pi}$ u^2

11 3.8 m^2

12 4 u^2

14 b $\frac{1}{2}\left(3 + \sqrt{3}\right)$ u^2

15 c $\frac{3}{4}\sqrt{3}$ u^2

16 b $2\sqrt{2}$ u^2

17 b They are all 4 u^2.

18 b The curve is below $y = 1$ just as much as it is above $y = 1$, so the area is equal to the area of a rectangle n units long and 1 unit high.

19 12

20 a Since $x^2 > 0$ for $0 < x < \frac{\pi}{2}$, $x^2 \sin x < x^2 \times x < x^2 \tan x$.

21 b $\cos x$ and $(1 + \sin x)^2$ are both positive in the given domain, so y' is negative there.

22 a 0, since the integrand is odd.

b 0, since the integrand is odd.

c 2, since the integrand is even.

d $6\sqrt{3}$, since the integrand is even.

e 6π. The first term is even, the other two are odd.

f $-\frac{2}{3} + \frac{\pi^2}{4}$. The first term is odd, the other two are even.

Chapter 7 review exercise

1 a $5\cos x$ **b** $5\cos 5x$

c $-25\sin 5x$ **d** $5\sec^2(5x - 4)$

e $\sin 5x + 5x\cos 5x$ **f** $\frac{-5x\sin 5x\ -\ \cos 5x}{x^2}$

g $5\sin^4 x\cos x$ **h** $5x^4\, 3\sec^2(x^5)$

i $-5\sin 5x e^{\cos 5x}$ **j** $\frac{5\cos 5x}{\sin 5x} = 5\cot 5x$

2 $-\sqrt{3}$

3 a $y = 4x + \sqrt{3} - \frac{4\pi}{3}$ **b** $y = -\frac{\pi}{2}x + \frac{\pi^2}{4}$

4 a $\frac{\pi}{2}$ **b** $\frac{3\pi}{4}, \frac{7\pi}{4}$

5 a $4\sin x + C$ **b** $-\frac{1}{4}\cos 4x + C$

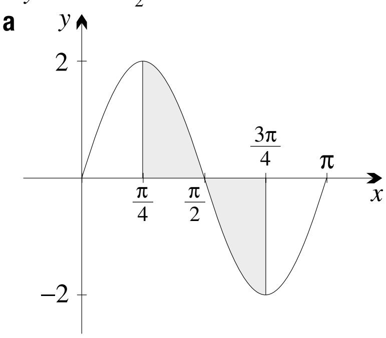
c $4\tan\frac{1}{4}x + C$

6 a $\sqrt{3} - 1$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

7 0.089

8 $y = 2\sin\frac{1}{2}x - 1$

9 a



b 2 u^2

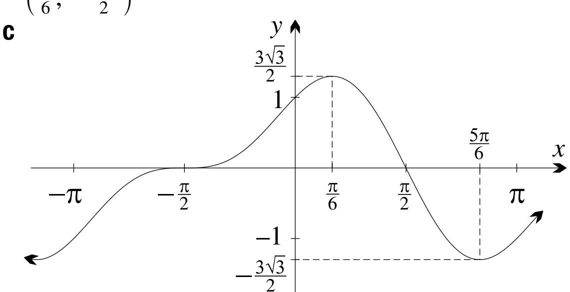
10 a $\frac{1}{2}$u^2 **b** $\frac{3\sqrt{3}}{4}$u^2

11 a $\tan x = \frac{\sin x}{\cos x}$ **b** $\frac{1}{2}\ln 2$u^2

12 a $\left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), (0, 2)$

b Horizontal point of inflexion $\left(-\frac{\pi}{2}, 0\right)$, maximum turning point $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$, minimum turning point $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

c

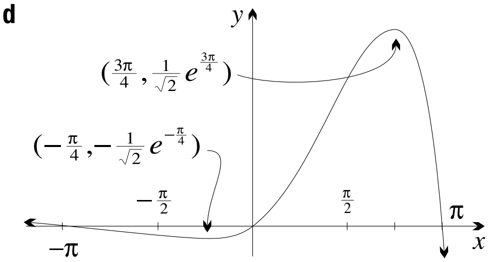


13 a $y' = e^x(\cos x + \sin x)$, $y'' = 2e^x\cos x$

b Minimum turning point $\left(-\frac{\pi}{4}, -\frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}}\right)$, maximum turning point $\left(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}}\right)$.

c $\left(-\frac{\pi}{2}, -e^{-\frac{\pi}{2}}\right), \left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right)$

d



14 b $\frac{9\sqrt{2}}{10}$ cm^2/ min **c** $\theta = \pi$

16 a $\frac{1}{2}\sin e^{2x} + C$ **b** $\frac{1}{2}\cos e^{-2x} + C$

c $\frac{1}{3}\log_e(3\tan x + 1) + C$

d $-\frac{3}{5}\log_e(4 + 5\cos x) + C$

e $\tan x - \sin x + C$

f $\frac{2}{3}$

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17 b **i** $\frac{1}{2}\tan^2 x + \log_e(\cos x) + C$
ii $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \log_e(\cos x) + C$

18 b $\left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right)\text{u}^2$

19 b **i** $1\frac{1}{5}$ **ii** 0

c **i** $\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C$
ii $\frac{\sin(m+n)x}{2(m+n)} + x + C$

Chapter 8

Exercise 8A

1 a 60 km, 090°T **b** 7 km, 146°T
c 37 km, 073°T

2 b 24.8 km, 050°T

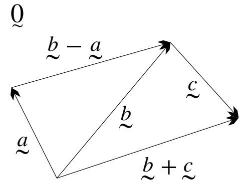
3 The opposite sides WX and ZY are parallel and equal, so $WXYZ$ is a parallelogram.

4 The opposite sides BA and CD are parallel and equal, and $\angle BAD = 90°$. So $ABCD$ is a parallelogram with an interior angle of 90°, so it is a rectangle.

5 a The 4 sides are equal, so $PQRS$ is a rhombus.
b The opposite sides of a rhombus are parallel, so $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have opposite directions.

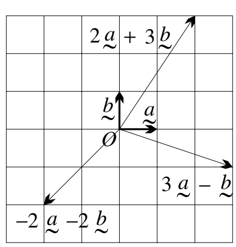
7 $\underset{\sim}{0}$

8



9 a $\overrightarrow{AD}$ **b** $\overrightarrow{BA}$ **c** $\overrightarrow{BD}$ **d** $\overrightarrow{AB}$

10



11 a $\underset{\sim}{f}$ **b** $\underset{\sim}{d}$ **c** $\underset{\sim}{h}$ **d** $\underset{\sim}{b}$
e $\underset{\sim}{c}$ **f** $\underset{\sim}{a}$ **g** $\underset{\sim}{e}$ **h** $\underset{\sim}{g}$

12 a $\underset{\sim}{u} - \underset{\sim}{v}$ **b** $\underset{\sim}{u} - \frac{1}{2}\underset{\sim}{v}$

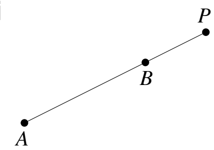
13 a $\overrightarrow{AM}$ and $\overrightarrow{MB}$ have the same length and direction. Similarly $\overrightarrow{PN}$ and $\overrightarrow{NQ}$ have the same length and direction.
b $\underset{\sim}{a} = \underset{\sim}{p} + \underset{\sim}{u} - \underset{\sim}{v}, \underset{\sim}{b} = \underset{\sim}{p} - \underset{\sim}{u} + \underset{\sim}{v}$

14 a $\overrightarrow{BC} = \underset{\sim}{q} - \underset{\sim}{p}, \overrightarrow{BP} = \frac{1}{3}(\underset{\sim}{q} - \underset{\sim}{p})$

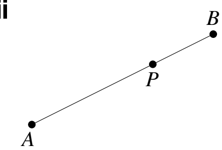
15 a $\overrightarrow{MB} = \frac{1}{2}(\underset{\sim}{u} - \underset{\sim}{v} - \underset{\sim}{w})$
b $\overrightarrow{MA} = \frac{1}{2}(\underset{\sim}{u} + \underset{\sim}{v} - \underset{\sim}{w})$

16 a $\overrightarrow{WX} = \underset{\sim}{x} - \underset{\sim}{w}, \overrightarrow{WP} = \frac{1}{2}(\underset{\sim}{x} - \underset{\sim}{w})$
b $\overrightarrow{RP} = \frac{1}{2}(\underset{\sim}{w} + \underset{\sim}{x})$ **c** $\overrightarrow{RQ} = \frac{1}{2}(\underset{\sim}{y} + \underset{\sim}{z})$

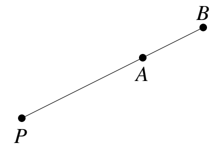
17 a **i**



ii



iii



b **i** $k = \frac{3}{5}$
ii $k = 3$
iii $k = -2$

18 The triangles are similar by the SAS similarity test — the angles between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, and between $\lambda\underset{\sim}{a}$ and $\lambda\underset{\sim}{b}$ are equal, and the matching sides are in ratio $1:\lambda$. It now follows that the head of the vector $\lambda\underset{\sim}{b}$ is the head of the vector $\lambda(\underset{\sim}{a} + \underset{\sim}{b})$.

19 a Two zero vectors each have zero length and no direction, and so are equal.
b Rome for administration (in the distant past), Greenwich UK for longitude, Jerusalem and Mecca for religious ceremonies, the North and South Poles for maps. The obelisk in Macquarie Place, Sydney, remains the origin for road distances in NSW. It is inscribed on the front,
'This Obelisk was erected in Macquarie Place A.D. 1818, to Record that all the public roads leading to the interior of the colony are measured from it. L. Macquarie Esq Governor'

20 c The three medians of a triangle are concurrent, and their point of intersection trisects each median. (A *median* of a triangle is the line joining a vertex to the midpoint of the opposite side.)

21 b $\overrightarrow{PQ} = \frac{1}{4}(3\underset{\sim}{c} + \underset{\sim}{d} - 3\underset{\sim}{a} - \underset{\sim}{b})$

Exercise 8B

1 a 10 **b** $16\underset{\sim}{i} + 12\underset{\sim}{j}$ **c** 20
d $-40\underset{\sim}{i} - 30\underset{\sim}{j}$ **e** 50

2 a $3\underset{\sim}{i} - \underset{\sim}{j}$ **b** $\sqrt{10}$ **c** $\underset{\sim}{i} + 7\underset{\sim}{j}$
d $5\sqrt{2}$ **e** $-8\underset{\sim}{i} - \underset{\sim}{j}$ **f** $\sqrt{65}$

3 a $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$ **b** $5\sqrt{2}$ **c** $\begin{bmatrix} 12 \\ 20 \end{bmatrix}$ **d** $4\sqrt{34}$

4 a $\underset{\sim}{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \underset{\sim}{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $\underset{\sim}{u} + \underset{\sim}{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$
b $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}, \underset{\sim}{b} = 4\underset{\sim}{i} + \underset{\sim}{j}, \underset{\sim}{a} - \underset{\sim}{b} = -\underset{\sim}{i} + \underset{\sim}{j}$

5 a $\frac{1}{\sqrt{5}}\underset{\sim}{i} + \frac{2}{\sqrt{5}}\underset{\sim}{j}$ **b** $-\frac{4}{5}\underset{\sim}{i} + \frac{3}{5}\underset{\sim}{j}$ **c** $-\frac{3}{\sqrt{34}}\underset{\sim}{i} + \frac{5}{\sqrt{34}}\underset{\sim}{j}$

7 a Yes. The intervals PQ and QR both have gradient -1.
b No. The intervals PQ and QR have different gradients.

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8 a $4\underset{\sim}{i} - 7\underset{\sim}{j}$ **b** $6\underset{\sim}{i} + 3\underset{\sim}{j}$ **c** $5\underset{\sim}{i} - 2\underset{\sim}{j}$

9 a $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ **b** $\begin{bmatrix} 11 \\ -7 \end{bmatrix}$ **c** $\begin{bmatrix} -7 \\ 6 \end{bmatrix}$ **d** $\begin{bmatrix} 7 \\ -6 \end{bmatrix}$

10 a $6\underset{\sim}{i} + 4\underset{\sim}{j}$ **b** $2\sqrt{13}$ **c** $\frac{3}{\sqrt{13}}\underset{\sim}{i} + \frac{2}{\sqrt{13}}\underset{\sim}{j}$

11 a $2\sqrt{2}, \dfrac{\pi}{4}$ **b** $2, -\dfrac{\pi}{3}$

c $6, \dfrac{5\pi}{6}$ **d** $2\sqrt{3}, -\dfrac{3\pi}{4}$

12 a $2\sqrt{2}\underset{\sim}{i} - 2\sqrt{2}\underset{\sim}{j}$ **b** $-\sqrt{6}\underset{\sim}{i} + 3\sqrt{2}\underset{\sim}{j}$

c $-\sqrt{3}\underset{\sim}{i} - \underset{\sim}{j}$

d $\left(\sqrt{3} - 1\right)\underset{\sim}{i} + \left(\sqrt{3} + 1\right)\underset{\sim}{j}$

13 $\lambda_1 = 6, \lambda_2 = -4$

14 a $\overrightarrow{AB} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ and $\overrightarrow{CB} = \begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix}$

b $|\overrightarrow{AB}| = |\overrightarrow{CB}| = 2$

c Equilateral, because each side has length 2.

15 The opposite sides AD and BC are parallel and equal.

16 c It is rhombus, because it is a parallelogram with adjacent sides equal.

17 b It is a parallelogram, because its diagonals bisect each other.

18 $a = -8$ and $b = 11$

19 $\underset{\sim}{a} + \underset{\sim}{b} - \underset{\sim}{c}, \underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{a}, \underset{\sim}{c} + \underset{\sim}{a} - \underset{\sim}{b}$

Exercise 8C

1 a 10 **b** 22 **c** 0 **d** $2x^2 - 2x - 3$

2 a 15 **b** $6\sqrt{2}$

3 a $\cos\theta = -\frac{1}{2}, \theta = 120°$

b $\cos\theta = 0.8, \theta \doteqdot 37°$

4 a 0 **b** 0 **c** 8 **d** -15

5 a 20 **b** 34 **c** -14

6 a no **b** yes **c** yes

7 a $\overrightarrow{AB} = 3\underset{\sim}{i} + 9\underset{\sim}{j}, \overrightarrow{AC} = -4\underset{\sim}{i} + 8\underset{\sim}{j}$

b 60

8 a $\overrightarrow{PQ} = 2\sqrt{3}\underset{\sim}{i} + 6\underset{\sim}{j}, \overrightarrow{PR} = 4\sqrt{3}\underset{\sim}{i} + 4\underset{\sim}{j}$

b 48

9 a $\frac{3}{5}$ **b** $\frac{1}{\sqrt{5}}$ **c** $\frac{10}{\sqrt{221}}$

10 $\lambda = -\frac{2}{3}$ or 2

11 a i 6 **ii** -60 **iii** 0 **iv** 42 **v** 10 **vi** 32

b i -6 **ii** 60 **iii** 0 **iv** 30 **v** -2 **vi** 32

c i 0 **ii** 0 **iii** 0 **iv** 36 **v** 4 **vi** 32

12 c It is a rectangle, because it is a parallelogram with a right-angle.

13 c It is a rhombus, because its diagonals bisect each other at 90°.

14 a $\overrightarrow{AP} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \overrightarrow{AQ} = \begin{bmatrix} 13 \\ -3 \end{bmatrix}$

b 63°

15 58°8′

17 b $\dfrac{87}{\sqrt{7738}}$ **c** 43.5 u^2

18 b $\overrightarrow{PB} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \begin{bmatrix} -8 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \end{bmatrix}$

20 a $\overrightarrow{PR} = \underset{\sim}{a} - 4\underset{\sim}{b}$

21 a i $(\underset{\sim}{c} - \underset{\sim}{a}) \cdot (\underset{\sim}{d} - \underset{\sim}{b}) = 0$

ii $|\underset{\sim}{c} - \underset{\sim}{a}| = |\underset{\sim}{d} - \underset{\sim}{b}|$ (other answers are possible)

b $m = 4$ and $n = 1$ or $m = 6$ and $n = 15$

22 a A dot product is negative when the angle is obtuse.

c i They are both 3. When calculating the RHS, be careful to take the exterior angles as the angles between the vectors

ii They are both 4. **iii** They are both 8.

Exercise 8D

1 a $\overrightarrow{AB} = \underset{\sim}{a} + \underset{\sim}{b}$ **b** $\overrightarrow{PQ} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

c $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AB}$ so $PQ \parallel AB$ and $PQ = \frac{1}{2}AB$.

2 a $\overrightarrow{AC} = \underset{\sim}{a} + \underset{\sim}{b} = \underset{\sim}{d} + \underset{\sim}{c}$ **b** $\overrightarrow{PQ} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

c $\overrightarrow{PQ} = \frac{1}{2}(\underset{\sim}{d} + \underset{\sim}{c})$

3 a $\overrightarrow{AC} = \underset{\sim}{a} + \underset{\sim}{b}, \overrightarrow{BD} = \underset{\sim}{b} - \underset{\sim}{c}$

b $\underset{\sim}{a} \cdot \underset{\sim}{a} = x^2, \underset{\sim}{a} \cdot \underset{\sim}{b} = 0, \underset{\sim}{a} \cdot \underset{\sim}{c} = -x^2, \underset{\sim}{a} \cdot \underset{\sim}{a} = 0.$

c $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = x^2 + y^2,$
$(\underset{\sim}{b} + \underset{\sim}{c}) \cdot (\underset{\sim}{b} + \underset{\sim}{c}) = y^2 + x^2$

d The diagonals of a rectangle have equal length.

4 a $\overrightarrow{AC} = \underset{\sim}{a} + \underset{\sim}{b}, \overrightarrow{BD} = \underset{\sim}{b} - \underset{\sim}{a}$

b $a \cdot a = \ell^2, a \cdot b = 0, a \cdot c = -\ell^2, a \cdot d = 0$

c 0

d The diagonals of a square meet at right angles.

5 a The sides of a rhombus are equal.

c $\overrightarrow{OC} = \underset{\sim}{a} + \underset{\sim}{b}$ and $\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$

e They are perpendicular.

6 a The opposite sides of a parallelogram are parallel and equal.

b $\overrightarrow{OB} = \underset{\sim}{c} + \underset{\sim}{a}$ **c** $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

d The diagonals OB and AC are equal.

f It is a rectangle.

7 a $\overrightarrow{OB} = -\underset{\sim}{a}$

b $\overrightarrow{AP} = \underset{\sim}{p} - \underset{\sim}{a}$ and $\overrightarrow{BP} = \underset{\sim}{p} + \underset{\sim}{a}$

d An angle in a semi-circle is a right-angle.

8 a P lies on the altitude from A to BC.

b P lies on the altitude from B to CA.

d From **iii**, $\overrightarrow{CP} \cdot \overrightarrow{BA} = 0$, so $\overrightarrow{CP}$ is perpendicular to $\overrightarrow{BA}$, so P lies on the altitude from C to BA.

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Exercise 8E

1 a $\underset{\sim}{i}$ **b** $2\underset{\sim}{j}$ **c** $-3\underset{\sim}{i}$
2 a 2 **b** 4 **c** $6\sqrt{2}$
3 a $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ **b** $3\underset{\sim}{j}$ **c** $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$
4 a $3\sqrt{3}$ **b** $6\sqrt{3}$
6 a $\begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$ **b** $\frac{3}{5}\underset{\sim}{i} - \frac{1}{5}\underset{\sim}{j}$ **c** $\begin{bmatrix} -\frac{21}{5} \\ \frac{28}{5} \end{bmatrix}$
7 a $\frac{6}{5}\underset{\sim}{i} + \frac{2}{5}\underset{\sim}{j}$ **b** $\frac{27}{10}\underset{\sim}{i} + \frac{9}{10}\underset{\sim}{j}$
8 a $\frac{6}{\sqrt{13}}$ **b** $\frac{14}{\sqrt{5}}$
9 $-\frac{36}{13}\underset{\sim}{i} + \frac{24}{13}\underset{\sim}{j}$
10 $7\sqrt{5}$
11 $\lambda = \frac{40}{3}$ or -10
13 a $-\frac{1}{3}$
b $-3\underset{\sim}{i} + \underset{\sim}{j}$ is one such vector.
d $23\underset{\sim}{i} - \underset{\sim}{j}$ **e** $7\sqrt{10}$ **f** $2\sqrt{10}$

Exercise 8F

1 $10\sqrt{3}$ m/s, 10 m/s
2 $25\underset{\sim}{i} - 9\underset{\sim}{j}$
3 $\sqrt{10}$ N
4 34 N at 28° to the 30 N force
5 a $1000\cos 15° \doteqdot 966$ N
b $1000\sin 15° \doteqdot 259$ N
6 28.3 N
7 a 12 N **b** $12\sqrt{3}$ N
8 a $\overrightarrow{OP} = (20\cos 25°)\underset{\sim}{i} + (20\sin 25°)\underset{\sim}{j}$
$\overrightarrow{OQ} = (16\cos 50°)\underset{\sim}{i} + (16\sin 50°)\underset{\sim}{j}$
b 35 N, 36° above the horizontal
9 $F = 49$
10 37°
11 5.4 N, 5.2° north of east
12 5 m/s^2 at an angle of $\tan^{-1}\frac{7}{24}$ above the horizontal
13 a $19\underset{\sim}{i} + 88\underset{\sim}{j}$ **b** $-84\underset{\sim}{i} + 288\underset{\sim}{j}$
14 a 49 N **b** 60 N **c** 6°
15 a $\left(3 - 2\sqrt{2}\right)\underset{\sim}{i} + \left(5 - 2\sqrt{2}\right)\underset{\sim}{j}$
b 2.2 m/s, 4.5°T
16 c 20 kg
17 a $T = 44.1$ **b** $m = 9$
18 $2\sqrt{3}$ and 4
19 a $3a = T - 3g\sin\theta$ **b** $2a = 2g\sin 2\theta - T$

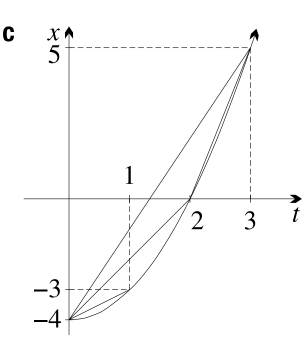
Chapter 8 review exercise

1 211.5 km, 088°T
2 a $\overrightarrow{AD}$ **b** $\overrightarrow{AC}$ **c** $\overrightarrow{DC}$ **d** 0 **e** $\overrightarrow{CD}$ **f** $\overrightarrow{DB}$
3 a $\underset{\sim}{b} - \underset{\sim}{a}$ **b** $\frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ **c** $\frac{1}{6}(3\underset{\sim}{b} - \underset{\sim}{a})$
4 a $\overrightarrow{PA} = \frac{3}{4}\underset{\sim}{a}$ **b** $\overrightarrow{AQ} = \frac{3}{7}(\underset{\sim}{b} - \underset{\sim}{a})$
5 a $6\underset{\sim}{i} + 8\underset{\sim}{j}$ **b** 10 **c** $\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$
6 a $\sqrt{5}\,a$ **b** $\begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$ **c** $\begin{bmatrix} 4a \\ 2a \end{bmatrix}$ **d** $5a^2$
7 a yes **b** no, unless $x = 0$
8 137°44′
9 c a rectangle
10 a $\begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$ **b** $\frac{33}{10}\underset{\sim}{i} + \frac{11}{10}\underset{\sim}{j}$
11 $\frac{84}{\sqrt{153}}$
12 36°
13 a $\overrightarrow{MA} = \frac{1}{2}\underset{\sim}{a}$ **b** $\overrightarrow{AN} = \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$
d $\overrightarrow{MN} = \overrightarrow{PB} = \frac{1}{2}\underset{\sim}{b}$, so a pair of opposite sides are parallel and equal.
14 a $\underset{\sim}{p} = \underset{\sim}{b} - \underset{\sim}{a}, m = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$
15 a $|a|^2 = |c|^2$
b $|\overrightarrow{AB}|^2 = |\overrightarrow{CB}|^2$
16 a $\overrightarrow{AC} = \underset{\sim}{a} + \underset{\sim}{b}, \overrightarrow{BD} = \underset{\sim}{b} - \underset{\sim}{a}$
b $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = x^2 + y^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b}$ and $(b - a) \cdot (b - a) = x^2 + y^2 - 2a \cdot b$
c If the parallelogram is a rectangle, then $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$, so the diagonals are equal. Conversely, if the diagonals are equal, then $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$, so the parallelogram is a rectangle.
17 a $\overrightarrow{OP} = (37\cos 50°)\underset{\sim}{i} + (37\sin 50°)\underset{\sim}{j}$,
$\overrightarrow{OQ} = (23\cos 25°)\underset{\sim}{i} - (23\sin 25°)\underset{\sim}{j}$
b 48.4 N, 22.7° above the horizontal
18 7.08 km/h, 133°T
19 14 N
20 a 4.9 m/s^2 **b** $T = 14.7$

Chapter 9

Exercise 9A

1a $x = -4, -3, 0, 5$
b **i** 1 m/s
ii 2 m/s
iii 3 m/s
iv 5 m/s
c



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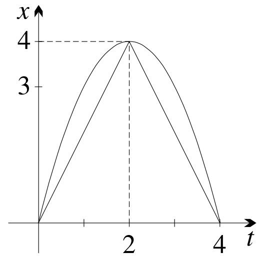
2 a $x = 0, 3, 4, 3, 0$

c The total distance travelled is 8 metres.
The average speed is $2\,\text{m/s}$.

d **i** $2\,\text{m/s}$ **ii** $-2\,\text{m/s}$ **iii** $0\,\text{m/s}$

e



3 a $x = 0, 120, 72, 0$ **b** 240 metres

c 20 m/s

d **i** 30 m/s **ii** -15 m/s **iii** 0 m/s

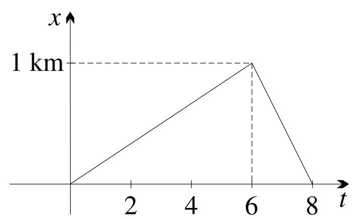
4 a **i** 6 minutes

ii 2 minutes

c 15 km/hr

d 20 km/hr

b



5 a $x = 0, 3, 1, 4, 2, 5, 3, 6$

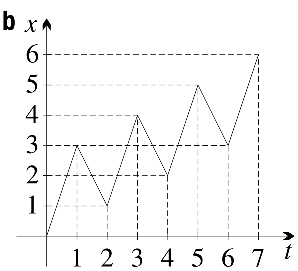
c 7 hours

d 18 metres, $2\frac{4}{7}$ m/hr

e $\frac{6}{7}$ m/hr

f Those between 1 and 2 metres high or between 4 and 5 metres high

b



6 a $t = 0, 1, 4, 9, 16$

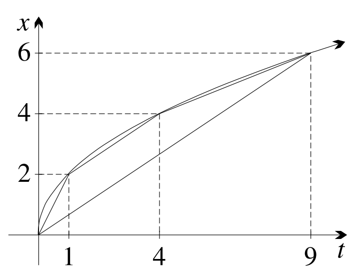
b **i** 2 cm/s

ii $\frac{2}{3}$ cm/s

iii $\frac{2}{5}$ cm/s

iv $\frac{2}{3}$ cm/s

c They are parallel.



7 a **i** $-1\,\text{m/s}$

ii 4 m/s

iii -2 m/s

b 40 metres, $1\frac{1}{3}$ m/s

c 0 metres, 0 m/s

d $2\frac{2}{19}$ m/s

8 a **i** once

ii three times

iii twice

b **i** when $t = 4$ and when $t = 14$

ii when $0 \le t < 4$ and when $4 < t < 14$

c It rises 2 metres, at $t = 8$.

d It sinks 1 metre, at $t = 17$.

e As $t \to \infty$, $x \to 0$, meaning that eventually it ends up at the surface.

f **i** $-1\,\text{m/s}$ **ii** $\frac{1}{2}$ m/s **iii** $-\frac{1}{3}$ m/s

g **i** 4 metres **ii** 6 metres

iii 9 metres **iv** 10 metres

h **i** 1 m/s **ii** $\frac{3}{4}$ m/s **iii** $\frac{9}{17}$ m/s

9 b $x = 3$ and $x = -3$ **c** $t = 4, t = 20$

d $t = 8, t = 16$ **e** $8 < t < 16$

f 12 cm, $\frac{3}{4}$ cm/s

10 a amplitude: 4 metres, period: 12 seconds

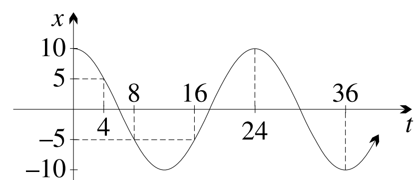
b 10 times

c $t = 3, 15, 27, 39, 51$

d It travels 16 metres with average speed $1\frac{1}{3}$ m/s.

e $x = 0$, $x = 2$ and $x = 4$, 2 m/s and 1 m/s

11 amplitude: 10 metres, period: 24 seconds



c It is at $x = 0$ when $t = 6$, 18 and 30.

d When $t = 0$, $x = 10$. The maximum distance is 20 metres, when $t = 12$ and 36.

e 60 metres, $1\frac{2}{3}$ m/s

f 10, 5, -5, -10, -5, 5, 10

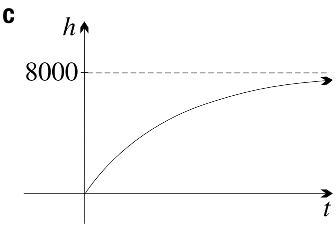
g $-1\dfrac{1}{4}$ m/s, $-2\dfrac{1}{2}$ m/s, $-1\dfrac{1}{4}$ m/s

h $x = -5$ when $t = 8$ or $t = 16$, $x < -5$ when $8 < t < 16$.

12 a When $t = 0$, $h = 0$. As $t \to \infty$, $h \to 8000$.

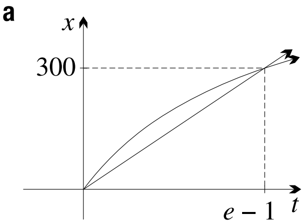
b 0, 3610, 5590, 6678

c



d 361 m/ min , 198 m/ min , 109 m/ min

13 a



c The maximum distance is
$300\log(e - 1) - \dfrac{300(e - 2)}{e - 1} \doteqdot 37$ metres when $t = e - 2 \doteqdot 43''$.

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Exercise 9B

1 a $v = -2t$

b $a = -2$

c $x = 11$ metres, $v = -6$ m/s, $a = -2$ m/s^2

d distance from origin: 11 metres, speed: 6 m/s

2 a $v = 2t - 10$

b displacement: -21 cm, distance from origin: 21 cm, velocity: $v = -4$ cm/s, speed: $|v| = 4$ cm/s

c When $v = 0$, $t = 5$ and $x = -25$.

3 a $v = 3t^2 - 12t$, $a = 6t - 12$

b When $t = 0$, $x = 0$ cm, $|v| = 0$ cm/s and $a = -12$ cm/s^2.

c left $(x = -27 \text{ cm})$

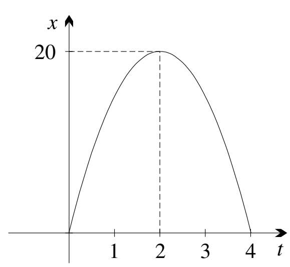
d left $(v = -9 \text{ cm/s})$

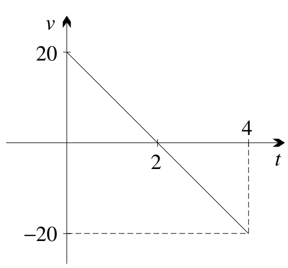
e right $(a = 6 \text{ cm/s}^2)$

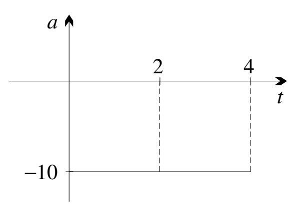
f When $t = 4$, $v = 0$ cm/s and $x = -32$ cm.

g When $t = 6$, $x = 0$, $v = 36$ cm/s and $|v| = 36$ cm/s.

4 a $x = 5t(4 - t)$
$v = 20 - 10t$
$a = -10$







b 20 m/s

c It returns at $t = 4$; both speeds are 20 m/s.

d 20 metres after 2 seconds

e -10 m/s^2. Although the ball is stationary, its velocity is changing, meaning that its acceleration is non-zero.

5 $\dot{x} = -4e^{-4t}$, $\ddot{x} = 16e^{-4t}$

a e^{-4t} is positive, for all t, so $\dot{x}$ is always negative and $\ddot{x}$ is always positive.

b **i** $x = 1$ **ii** $x = 0$

c **i** $\dot{x} = -4$, $\ddot{x} = 16$ **ii** $\dot{x} = 0$, $\ddot{x} = 0$

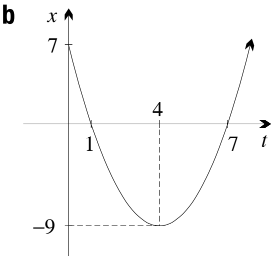
6 $v = 2\pi \cos \pi t$, $a = -2\pi^2 \sin \pi t$

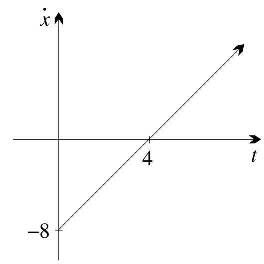
a When $t = 1$, $x = 0$, $v = -2\pi$ and $a = 0$.

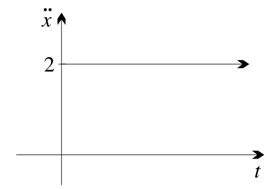
b **i** right $(v = \pi)$ **ii** left $\left(a = -\pi^2\sqrt{3}\right)$

7 a $x = (t - 7)(t - 1)$
$\dot{x} = 2(t - 4)$
$\ddot{x} = 2$

b







c **i** $t = 1$ and $t = 7$ **ii** $t = 4$

d **i** 7 metres when $t = 0$

ii 9 metres when $t = 4$

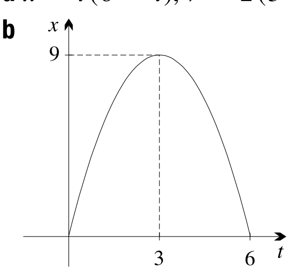
iii 27 metres when $t = 10$

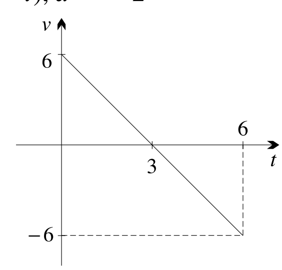
e -1 m/s, $t = 3\frac{1}{2}$, $x = -8\frac{3}{4}$

f 25 metres, $3\frac{4}{7}$ m/s

8 a $x = t(6 - t)$, $v = 2(3 - t)$, $a = -2$

b





c **i** When $t = 2$, it is moving upwards and accelerating downwards.

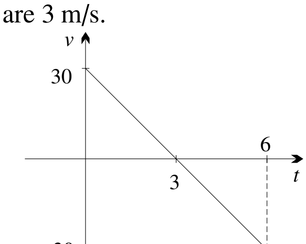
ii When $t = 4$, it is moving downwards and accelerating downwards.

d $v = 0$ when $t = 3$. It is stationary for zero time, 9 metres up the plane, and is accelerating downwards at 2 m/s^2.

e 4 m/s. When $v = 4$, $t = 1$ and $x = 5$.

f All three average speeds are 3 m/s.

9 a 45 metres, 3 seconds, 15 m/s



b 30 m/s, 20, 10, 0, -10, -20, -30

c 0 seconds

d The acceleration was always negative.

e The velocity was decreasing at a constant rate of 10 m/s every second.

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10 a 8 metres, when $t = 3$

b i when $t = 3$ and $t = 9$ (because the gradient is zero)

ii when $0 \le t < 3$ and when $t > 9$ (because the gradient is positive)

iii when $3 < t < 9$ (because the gradient is negative)

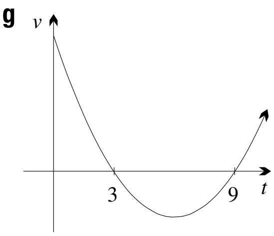
c $x = 0$ again when $t = 9$. Then $v = 0$ (because the gradient is zero) and it is accelerating to the right (because the concavity is upwards).

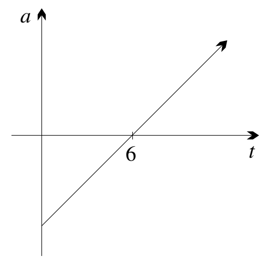
d at $t = 6$ (at the point of inflection the second derivative is zero), $x = 4$, moving to the left

e $0 \le t < 6$

f i $t \doteqdot 4, 12$ **ii** $t \doteqdot 10$

g





11 a $x = 4\cos\frac{\pi}{4}t$, $v = -\pi\sin\frac{\pi}{4}t$, $a = -\frac{1}{4}\pi^2\cos\frac{\pi}{4}t$

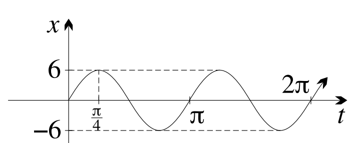
b maximum displacement: $x = 4$ when $t = 0$ or $t = 8$, maximum velocity: π m/s when $t = 6$, maximum acceleration: $\frac{1}{4}\pi^2$ m/s^2 when $t = 4$

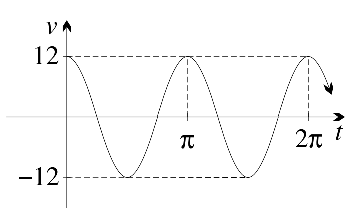
c 40 metres, 2 m/s

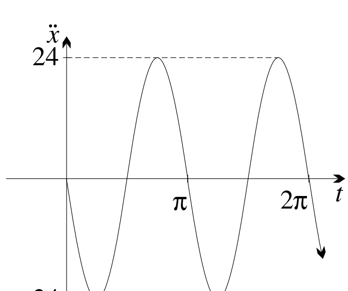
d $1\frac{1}{3} < t < 6\frac{2}{3}$

e i $t = 0$, $t = 4$ and $t = 8$ **ii** $4 < t < 8$

12 a $x = 6\sin 2t$
$v = 12\cos 2t$
$\ddot{x} = -24\sin 2t$







b $\ddot{x} = -4x$

c i $x = 0$ when $t = 0, \frac{\pi}{2}$ or π.

ii $v = 0$ when $t = \frac{\pi}{4}$ or $\frac{3\pi}{4}$.

iii same as **i**

d i $x < 0$ when $\frac{\pi}{2} < t < \pi$.

ii $v < 0$ when $\frac{\pi}{4} < t < \frac{3\pi}{4}$.

iii $\ddot{x} < 0$ when $0 < t < \frac{\pi}{2}$.

e i $t = \frac{\pi}{12}$

ii $t = \frac{\pi}{6}$

13 a i $0 \le t < 8$

ii $0 \le t < 4$ and $t > 12$

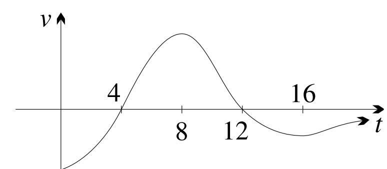
iii roughly $8 < t < 16$

b roughly $t = 8$

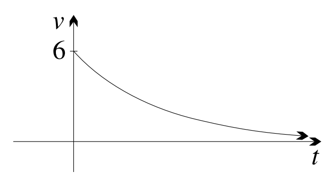
c i $t \doteqdot 5, 11, 13$ **ii** $t \doteqdot 13, 20$

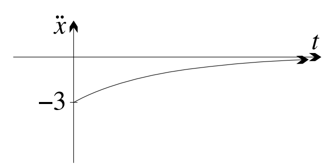
d twice **e** 17 units

f



14 a $\dot{x} = 6e^{-0.5t}$, $\ddot{x} = -3e^{-0.5t}$





b i downwards (downwards is positive here.)

ii upwards

c The velocity and acceleration tend to zero and the position tends to 12 metres below ground level.

d $x = 6$ when $e^{-0.5t} = \frac{1}{2}$, that is, $t = 2\log_e 2$ minutes. The speed then is 3 m/min (half the initial speed of 6 m/min) and the acceleration is $-1\frac{1}{2}$ m/min^2 (half the initial acceleration of -3 m/min^2).

e 19 minutes. When $t = 18$, $x \doteqdot 11.9985$ metres. When $t = 19$, $x \doteqdot 11.9991$ metres.

15 a $0 \le x \le 2r$

b i $\dfrac{dx}{d\theta} = \dfrac{2r\sin\theta}{\sqrt{5 - 4\cos\theta}}$. M is travelling upwards when $0 < \theta < \pi$.

ii M is travelling downwards when $\pi < \theta < 2\pi$.

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c The speed is maximum when $\theta = \frac{\pi}{3}$ (when $\dfrac{dx}{d\theta} = r$) and when $\theta = \frac{5\pi}{3}$ (when $\dfrac{dx}{d\theta} = -r$).

d When $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$, $\angle APC$ is a right angle, so AP is a tangent to the circle. At these places, P is moving directly towards A or directly away from A, and so the distance AP is changing at the maximum rate. Again because AP is a tangent, $\dfrac{dx}{d\theta}$ at these points must equal the rate of change of arc length with respect to θ, which is r or $-r$ when $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ respectively.

16 $\sin\alpha = \dfrac{2}{g} \doteqdot 0.204\,08$, $\alpha \doteqdot 11^\circ 47'$

Exercise 9C

1 a $x = t^3 - 3t^2 + 4$

b When $t = 2$, $x = 0$ metres and $v = 0$ m/s.

c $a = 6t - 6$

d When $t = 1$, $a = 0$ m/s^2 and $x = 2$ metres.

2 a $v = 10t$, $x = 5t^2$

b 4 seconds, 40 m/s

c After 2 seconds, it has fallen 20 metres and its speed is 20 m/s.

d It is halfway down after $2\sqrt{2}$ seconds, and its speed then is $20\sqrt{2}$ m/s.

3 a $a = -10$, $v = -10t - 25$, $x = -5t^2 - 25t + 120$

b 3 seconds, 55 m/s **c** 40 m/s

4 a i $\dot{x} = 3t^2$, $x = t^3$

ii $\dot{x} = -\frac{1}{3}e^{-3t} + \frac{1}{3}$, $x = \frac{1}{9}e^{-3t} + \frac{1}{3}t - \frac{1}{9}$

iii $\dot{x} = \frac{1}{\pi}\sin \pi t$, $x = -\frac{1}{\pi^2}\cos \pi t + \frac{1}{\pi^2}$

iv $\dot{x} = -12(t+1)^{-1} + 12$, $x = -12\log_e(t+1) + 12t$

b i $a = 0$, $x = -4t - 2$

ii $a = \frac{1}{2}e^{\frac{1}{2}t}$, $x = 2e^{\frac{1}{2}t} - 4$

iii $a = 16\cos 2t$, $x = -4\cos 2t + 2$

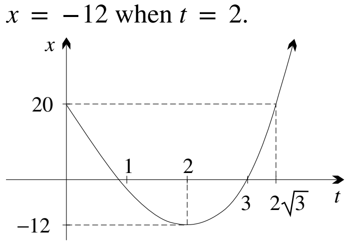
iv $a = \frac{1}{2}t^{-\frac{1}{2}}$, $x = \frac{2}{3}t^{\frac{3}{2}} - 2$

5 a $\dot{x} = 6t^2 - 24$, $x = 2t^3 - 24t + 20$

b $t = 2\sqrt{3}$, speed: 48 m/s

c $x = -12$ when $t = 2$.

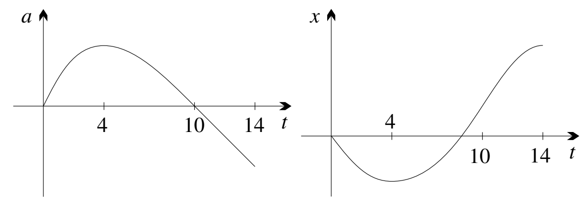
d



6 a $4 < t < 14$ **b** $0 < t < 10$ **c** $t = 14$

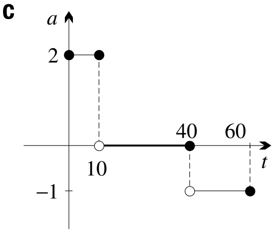
d $t = 4$ **e** $t \doteqdot 8$

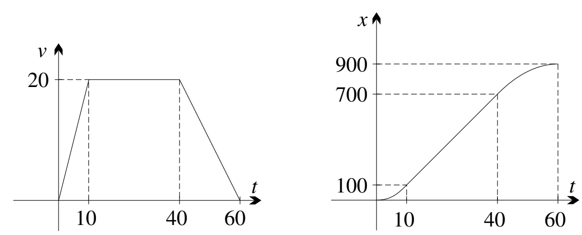
f



7 a 20 m/s

c





8 a $a = -4$, $x = 16t - 2t^2 + C$

b $x = C$ after 8 seconds, when the speed is 16 cm/s.

c $\dot{x} = 0$ when $t = 4$. Maximum distance right is 32 cm when $t = 4$, maximum distance left is 40 cm when $t = 10$. The acceleration is -4 cm/s^2 at all times.

d 104 cm, 10.4 cm/s

9 a $x = t^2(t-6)^2$, after 6 seconds, 0 cm/s

b 162 cm, 27 cm/s

c $\ddot{x} = 12(t^2 - 6t + 6)$, $24\sqrt{3}$ cm/s after $3 - \sqrt{3}$ and $3 + \sqrt{3}$ seconds.

d The graphs of x, v and $\ddot{x}$ are all unchanged by reflection in $t = 3$, but the mouse would be running backwards!

10 a $\ddot{x} = 6t$, $v = 3t^2 - 9$

b $x = t^3 - 9t + C_1$, 3 seconds

11 $e - 1$ seconds, $v = 1/e$, $\ddot{x} = -1/e^2$.
The velocity and acceleration approach zero, but the particle moves to infinity.

12 a $\dot{x} = -5 + 20e^{-2t}$, $x = -5t + 10 - 10e^{-2t}$, $t = \log_e 2$ seconds

b It rises $7\frac{1}{2} - 5\log_e 2$ metres, when the acceleration is 10 m/s^2 downwards.

c The velocity approaches a limit of 5 m/s downwards, called the *terminal velocity.*

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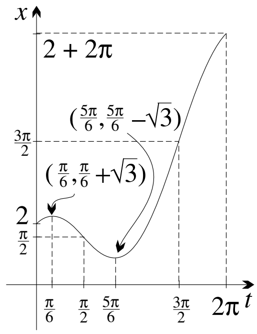
13 a $v = 1 - 2\sin t$, $x = t + 2\cos t$

b $\frac{\pi}{2} < t < \frac{3\pi}{2}$

c $t = \frac{\pi}{6}$ when $x = \frac{\pi}{6} + \sqrt{3}$, and $\frac{5\pi}{6}$ when $x = \frac{5\pi}{6} - \sqrt{3}$, $\frac{\pi}{6} < t < \frac{5\pi}{6}$.

d 3 m/s when $t = \frac{3\pi}{2}$ and $x = \frac{3\pi}{2}$, -1 m/s when $t = \frac{\pi}{2}$ and $x = \frac{\pi}{2}$.



e 2π metres, 1 m/s

f $4\sqrt{3} + \frac{2\pi}{3}$ metres, $\frac{1}{3} + \frac{2}{\pi}\sqrt{3}$ m/s

14 a Thomas, by 15 m/s

b $x_T = 20\log\ (t + 1)$, $x_H = 5t$

c during the 10th second, $3\frac{2}{11}$ m/s

d after 3 seconds, by 13 metres

15 a For $V \geq 30$ m/s, they collide after $180/V$ seconds, $\frac{180}{V^2}(V^2 - 900)$ metres above the valley floor.

b $V = 30\sqrt{2}$ m/s $3\sqrt{2}$ seconds

16 a $v = 5(e^{-2t} - 1)$, $x = \frac{5}{2}(1 - e^{-2t}) - 5t$

b The speed gradually increases with limit 5 m/s (the terminal velocity).

18 a $x_1 = 2 + 6t + t^2$, $x_2 = 1 + 4t - t^2$, $D = 1 + 2t + 2t^2$

b D is never zero, the minimum distance is 1 metre at $t = 0$ (t cannot be negative).

c $v_M = 5$ m/s, $12\frac{1}{2}$ metres

Exercise 9D

1 a 80 tonnes **b** When $t = 0$, $V = 0$.

c 360 tonnes **d** 20 tonnes/minute

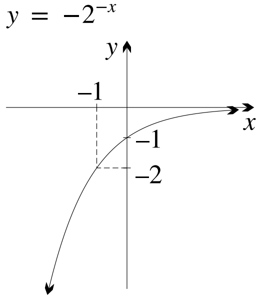
2 a 80 000 litres **b** 5 000 litres

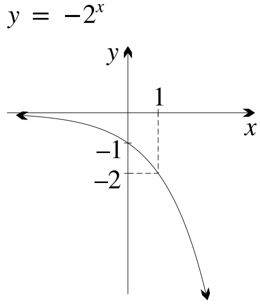
c 0 min $\leq$ t $\leq$ 20 min **d** 6000 litres/min

e The tank is emptying, so F is decreasing.

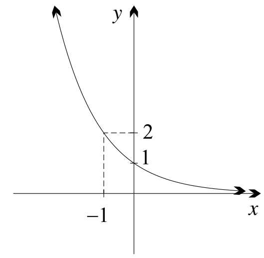
3 a 1500 **b** 300 **c** 15 minutes

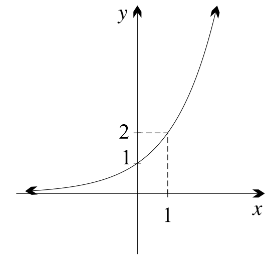
4 a $y = -2^{-x}$ **b** $y = -2^{x}$



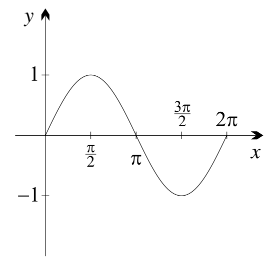


c $y = 2^{-x}$ **d** $y = 2^{x}$





5



a i $0 \leq x \leq \frac{\pi}{2}$

ii $\frac{\pi}{2} \leq x \leq \pi$

iii $\pi \leq x \leq \frac{3\pi}{2}$

iv $\frac{3\pi}{2} \leq x \leq 2\pi$

b i $\pi \leq x \leq 2\pi$

ii $0 \leq x \leq \pi$

6 a $h = 60e^{-\frac{t}{3}} - 30$

b 30 m/s upwards

c $h = 27.62$ m at $3\ln 2 \doteqdot 2.08$ seconds

d $h \doteqdot 10.23$ m and speed is 15 m/s downwards

e 30 m/s downwards

7 a i 12 kg/min

ii $10\frac{2}{3}$ kg/min

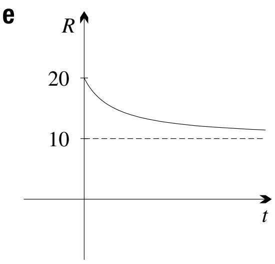
b 10 kg/min

c $\dot{R} = \frac{-20}{(1 + 2t)^2}$,

$\ddot{R} = \frac{80}{(1 + 2t)^3}$

d R is decreasing at a decreasing rate

e



8 a $(0, 0)$ and $(9, 81e^{-9}) \doteqdot (9, 0.0)$

b $\dot{M} = 9(1 - t)e^{-t}$, $(1, 9e^{-1}) \doteqdot (1, 3.3)$

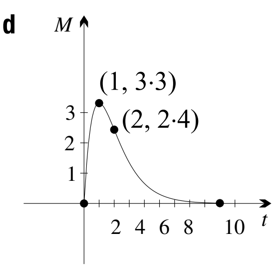
c $\ddot{M} = 9(t - 2)e^{-t}$, $(2, 18e^{-2}) \doteqdot (2, 2.4)$

e $t = 1$

f $t = 0$

g $t = 2$

d

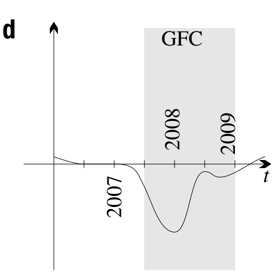


9 a The graph is steepest in January 2008.

b It levels out in 2009?

c The LIBOR reduced at a decreasing rate. It may have been mistaken as indicating the crisis was ending.

d



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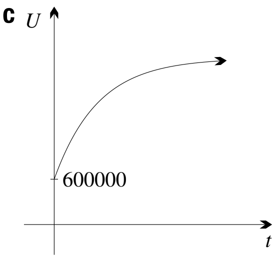
10 a Unemployment was increasing.

b The rate of increase was decreasing.

c



11 a $A = 9 \times 10^5$ **b** $N(1) = 380087$

c When t is large, N is close to 4.5×10^5.

d $\dot{N} = \frac{9 \times 10^5 e^{-t}}{(2 + e^{-t})^2}$

12 a $I = \dfrac{300t\left(2 - \frac{1}{5}t\right)}{200 + 3t^2 - \frac{1}{5}t^3}\%$ **b** $I(4) \doteqdot 6.12\%$

c $t = 0$ or 10. The latter is rejected because the model is only valid for 8 years.

13 b exponentials are always positive.

c $\phi(0) = \frac{1}{\sqrt{2\pi}}, \lim\limits_{x \to \infty} \phi(x) = 0$

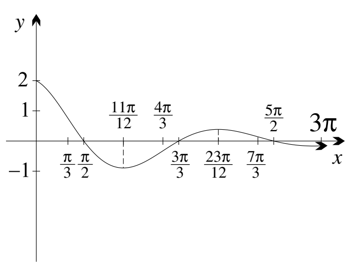
d $\phi'(x) < 0$ for $x > 0$ (decreasing)

e at $x = 1$ and $x = -1$, where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$.

h The curve approaches the horizontal asymptote more slowly for larger x.

14 a $y = 2$ and $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

f



Exercise 9E

1 b $1\ \text{m}^2/\text{s}$ **c** 7 metres **d** $9\ \text{m}^2$

2 a $A = \frac{1}{2}\ell^2$

c i $5\ \text{m}^2/\text{s}$ **ii** $3\ \text{m}^2/\text{s}$

d 34 metres

3 a $15.1\ \text{m}^3/\text{s}$ **b** $30.2\ \text{m}^2/\text{s}$

4 b $\frac{2}{9\pi}$ cm/s **c** $\frac{10}{\sqrt{\pi}}$ cm, $\frac{4000}{3\sqrt{\pi}}\ \text{cm}^3$

5 a $V = \frac{2}{3}\pi r^3$ **b** $\frac{1}{10\pi}$ cm/s

6 b 5 degrees per second

7 2 degrees per minute

8 a $V = 100h^2x\ \text{cm}^3$ **b** $\dfrac{dh}{dt} = 0.1$ cm/day

10 a $-2\sqrt{1 - x^2}$

b -2 m/s — as the point crosses the y-axis it is travelling horizontally at a speed of 2 m/s.

11 a $\dfrac{2CV^2}{L^2}\ \text{m/s}^2$

c As L decreases, the speed passing the truck increases, so the driver should wait as long as possible before beginning to accelerate. A similar result is obtained if the distance between car and truck is increased. Optimally, the driver should allow both L to decrease and C to increase.

d 950 metres

12 b This is just two applications of the chain rule.

d 6

13 c $x = h = 50(\sqrt{3} + 1)$ metres

d 200 km/h

Exercise 9F

1 b $t = 4$ **c** 57 **d** $t = 2$

2 a 25 minutes **c** 3145 litres

3 a $P = 6.8 - 2\log_e(t + 1)$

b approximately 29 days

4 a $-2\ \text{m}^3/\text{s}$

b 20 s

c $V = 520 - 2t + \frac{1}{20}t^2$

d $20\ \text{m}^3$

e 2 minutes and 20 seconds

5 a no **c** $t \doteqdot 1.28$ **d** $x = \frac{5}{2}$

6 a 0 **b** 250 m/s

c $x = 1450 - 250(5e^{-0.2t} + t)$

7 a $I = 18\,000 - 5t + \frac{48}{\pi}\sin\frac{\pi}{12}t$

b $\dfrac{dI}{dt}$ has a maximum of -1, so it is always negative.

c There will be 3600 tonnes left.

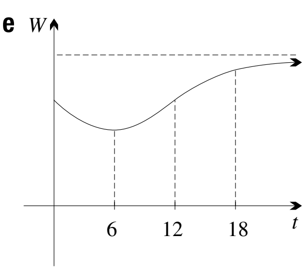
8 a It was decreasing for the first 6 months and increasing thereafter.

b after 6 months

c after 12 months

d It appears to have stabilised, increasing towards a limiting value.

e



9 a $\theta = \tan^{-1} t + \frac{\pi}{4}$

b $t = \tan\ (\theta - \frac{\pi}{4})$

c As $t \to \infty$, $\tan^{-1} t \to \frac{\pi}{2}$, and so $\theta \to \frac{3\pi}{4}$.

10 a $1200\ \text{m}^3$ per month at the beginning of July

b $W = 0.7t - \frac{3}{\pi}\sin\frac{\pi}{6}t$

11 b $k = \frac{5}{24}$

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12 a $V = \frac{1}{3}\pi r^3$ **b** $\frac{dr}{dt} = \frac{1}{2\pi r^2}$

c $t = \frac{2\pi}{3}(r^3 - 1000)$ **d** 25 minutes 25 seconds

13 a $V = \frac{\pi}{3}(128 - 48h + h^3)$

b **i** $A = \pi(16 - h^2)$ **ii** 1 hour 20 minutes

Chapter 9 review exercise

1 a $x = 24, x = 36$, 6 cm/s

b $x = 16, x = 36$, 10 cm/s

c $x = -8, x = -8$, 0 cm/s

d $x = 9, x = 81$, 36 cm/s

2 a $v = 40 - 2t, a = -2$, 175 m, 30 m/s, -2 m/s^2

b $v = 3t^2 - 25, a = 6t$, 0 m, 50 m/s, 30 m/s^2

c $v = 8(t - 3), a = 8$, 16 m, 16 m/s, 8 m/s^2

d $v = -4t^3, a = -12t^2$, -575 m, -500 m/s, -300 m/s^2

e $v = 4\pi \cos \pi t, a = -4\pi^2 \sin \pi t$, 0 m, -4π m/s, 0 m/s^2

f $v = 21\, e^{3t-15}, a = 63\, e^{3t-15}$, 7 m, 21 m/s, 63 m/s^2

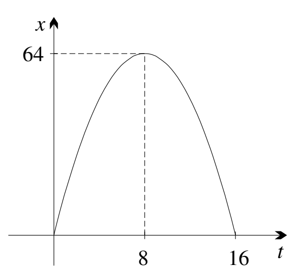
3 a $v = 16 - 2t, a = -2$

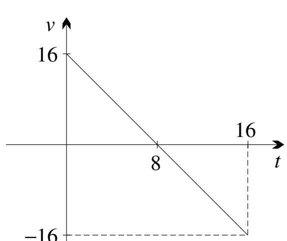
b 60 m, -4 m/s, 4 m/s, -2 m/s^2

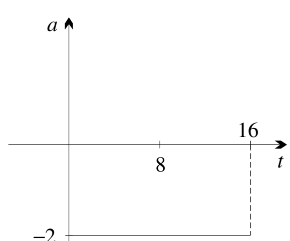
c $t = 16$s, $v = -16$ m/s

d $t = 8$ s, $x = 64$ m

e







4 a $a = 0, x = 7t + 4$

b $a = -18t, x = 4t - 3t^3 + 4$

c $a = 2(t - 1), x = \frac{1}{3}(t - 1)^3 + 4\frac{1}{3}$

d $a = 0, x = 4$

e $a = -24 \sin 2t, x = 4 + 6 \sin 2t$

f $a = -36e^{-3t}, x = 8 - 4e^{-3t}$

5 a $v = 3t^2 + 2t, x = t^3 + t^2 + 2$

b $v = -8t, x = -4t^2 + 2$

c $v = 12t^3 - 4t, x = 3t^4 - 2t^2 + 2$

d $v = 0, x = 2$

e $v = 5 \sin t, x = 7 - 5 \cos t$

f $v = 7e^t - 7, x = 7e^t - 7t - 5$

6 a $\dot{x} = 3t^2 - 12, x = t^3 - 12t$

b When $t = 2, \dot{x} = 0$.

c 16 cm

d $2\sqrt{3}$ seconds, 24 cm/s, $12\sqrt{3}$ cm/s^2

e As $t \to \infty, x \to \infty$ and $v \to \infty$.

7 a The acceleration is 10 m/s^2 downwards.

b $v = -10t + 40, x = -5t^2 + 40t + 45$

c 4 seconds, 125 metres

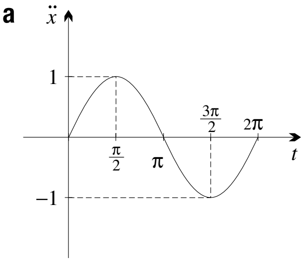
d When $t = 9, x = 0$.

e 50 m/s

f 80 metres, 105 metres

g 25 m/s

8 a



b $t = \pi$ and $t = 2\pi$

c $\dot{x} = -\cos t$

d $t = \frac{\pi}{2}$

e **i** $x = 5 - \sin t$

ii $x = 4$

9 a $v = 20$ m/s

b $20\, e^{-t}$ is always positive.

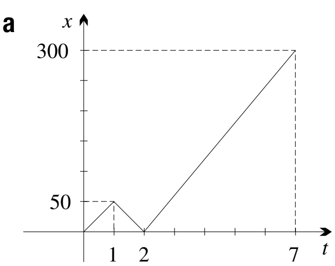
c $a = -20\, e^{-t}$

d -20 m/s^2

e $x = 20 - 20\, e^{-t}$

f As $t \to \infty, a \to 0, v \to 0$ and $x \to 20$.

10 a



b 400 km

c $57\frac{1}{7}$ km/hr

11 a $x = 20$ m, $v = 0$

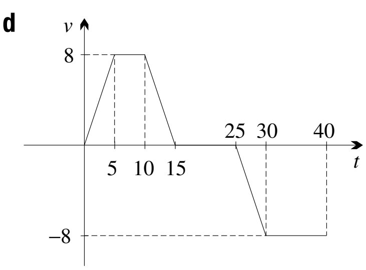
b **i** 8 m/s **ii** 0 **iii** -8 m/s

c **i** north (The graph is concave up.)

ii south (The graph is concave down.)

iii south (The graph is concave down.)

d



12 a at $t = 5$

b at $t = 12, 0 < t < 12, t > 12$

c $0 < t < 5, t > 5$

d at $t = 12$, when the velocity was zero

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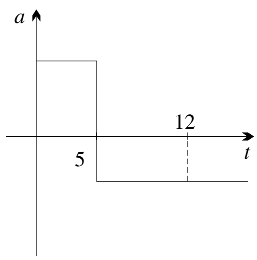
e


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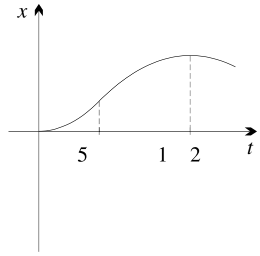
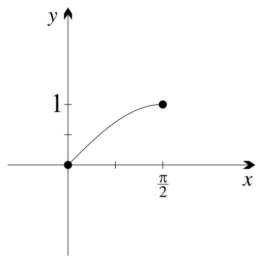


13 a i


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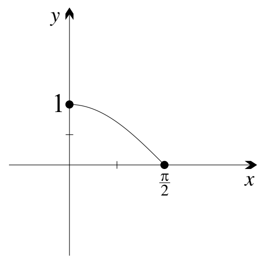

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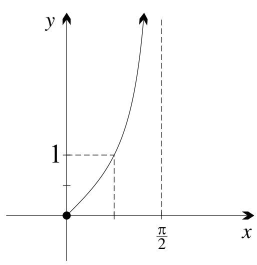

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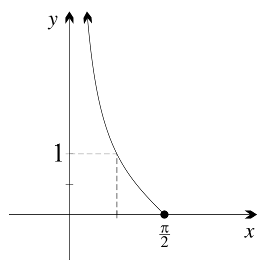
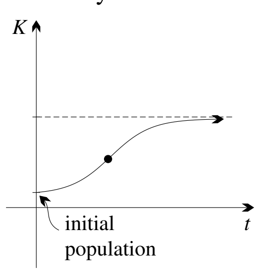
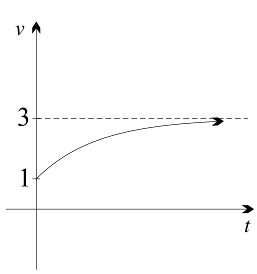


b **i** $y = \sin x$ **ii** $y = \cos x$
iii $y = \cot x$ **iv** $y = \tan x$

14 a Initially K increases at an increasing rate so the grpah is concave up. Then K increases at a decreasing rate so is concave down. The change in concavity coincides with the inflection point.

b


15 a 7500 L
b $\dot{V} = -12(50 - 2t)$
c $\dot{V}$ is negative in the given domain.
d $\dot{V}$ is negative and $\ddot{V} = 24$ is positive, so the outflow decreases.

16 a


b $\dot{x}$ increases so it accelerates.
$\ddot{x} = \frac{2}{5}e^{-\frac{1}{5}t}$ which is always positive.
c $\lim_{t\to\infty} \dot{x} = 3$ m/s
d $x = 3t + 10\left(e^{-\frac{1}{5}t} - 1\right)$

17 a $V = \frac{1}{5}t^2 - 20t + 500$
c $t = 50 - 25\sqrt{2} \doteqdot 15$ seconds. Discard the other answer $t = 50 + 25\sqrt{2}$ because after 50 seconds the bottle is empty.

18 a $-\frac{35}{24}$ cm/s
b $-\frac{1}{96}$ radians per second

Chapter 10

Exercise 10A

1 b $x = 30t,\ y = -5t^2 + 30t$
c $t = 6$ seconds. **d** 180 m
e $t = 3$ seconds **f** 45 m
2 b $x = 20t\sqrt{3},\ y = -5t^2 + 20t$
c **i** $t = 4$ seconds. **ii** $80\sqrt{3}$ m
iii 20 m
3 b $x = 10t,\ y = -5t^2 + 10t\sqrt{3}$
c **i** 15.9 m **ii** 12.4 m/s
4 b $x = 36t,\ y = -5t^2 + 48t$
c 146.5 m
5 a $\underset{\sim}{v} = 8\underset{\sim}{i} + (6 - 10t)\underset{\sim}{j}$
b $\underset{\sim}{r} = (8t)\,\underset{\sim}{i} + (6t - 5t^2)\underset{\sim}{j}$
c **i** 10 m/s **ii** $16\underset{\sim}{i} - 8\underset{\sim}{j}$
iii $4.8\underset{\sim}{i} + 1.8\underset{\sim}{j}$
6 a $\dot{x} = 40,\ \dot{y} = 25$
7 a $\underset{\sim}{v} = 4\underset{\sim}{i} + 4\underset{\sim}{j}$ **b** $2\underset{\sim}{i} + 0.75\underset{\sim}{j}$
9 b $x = 6t\sqrt{3},\ y = -5t^2 - 6t$
c 1 second **d** 10.4 m
10 a 1.19 seconds **b** 5.03 m
11 26 m/s
13 They collide 1 second after P_2 is projected.
14 c $\theta = 45°$ or $\theta \doteqdot 81°52'$
15 b **i** $\sqrt{5}$ s, $\sqrt{85}$ s
ii $20\sqrt{5} \doteqdot 44.7$ m/s in both cases, at angles of 0° and 76.0° to the horizontal.
18 c $\theta = 60°$

Exercise 10B

1 a 13.8 m **b** 18 m or 68.4 m
c $\frac{dy}{dx} = -\frac{5}{162}x + \frac{4}{3}$

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2 b **i** 192 m **ii** 20 m **iii** 22.6°

iv 6° below the horizontal

3 a $\sqrt{10}$

d **i** 9.5° above the horizontal

ii 9.5° below the horizontal

4 a $y = -\frac{1}{5}x^2$ **b** 10 m

c 76° below the horizontal

5 a $\dot{x} = 12\sqrt{3}, \dot{y} = -10t + 12,$

$x = 12t\sqrt{3}, y = -5t^2 + 12t$

d $D \doteqdot 58.7$

6 b **i** 13.5 m/s

ii 71° below the horizontal

7 b $\theta \doteqdot 28.2°$ or $61.8°$

8 b 60°

9 b 62°22′ or 37°5′

11 a $x = \frac{1}{2}Vt\sqrt{2}, y = -\frac{1}{2}gt^2 + \frac{1}{2}Vt\sqrt{2}$

12 c **ii** $R = 18$ metres

Chapter 10 review exercise

1 b $x = 60t\cos 40°, y = -5t^2 + 60t\sin 40°$

c **i** $t \doteqdot 7.7$ seconds.

ii 354.5 m **iii** 74.4 m

2 a 33.75 m **b** 96.58 m **c** 41.23 m/s

3 b $x = 15t, y = -5t^2 + 20t$

c $15\sqrt{2}$ m

4 a $\dot{x} = 5\sqrt{2}, x = 5t\sqrt{2}, \dot{y} = -10t + 5\sqrt{2},$

$y = -5t^2 + 5t\sqrt{2}$

b range: 10 metres, maximum height: 2.5 metres when $x = 5$

c **i** 1.6 metres

ii $\tan^{-1}\frac{3}{5}$ below the horizontal

d **i** $x = 3$

ii $\tan^{-1}\frac{2}{5}$ above the horizontal

5 a $\underset{\sim}{v} = 24\underset{\sim}{i} + (18 - 10t)\underset{\sim}{j}$

b $\underset{\sim}{r} = (24t)\underset{\sim}{i} + (18t - 5t^2)\underset{\sim}{j}$

c **i** 30 m/s

ii $96\underset{\sim}{i} - 8\underset{\sim}{j}$ **iii** $43.2\underset{\sim}{i} + 16.2\underset{\sim}{j}$

6 a $\underset{\sim}{v} = 12\sqrt{5}\underset{\sim}{i} + 24\underset{\sim}{j}$ **b** $36\sqrt{5}\underset{\sim}{i} + 27\underset{\sim}{j}$

c It is falling, because the height was 28 m after 2 seconds (or show that $\dot{y} = -6 < 0$).

7 b 11.25 m

8 a Initially, $\dot{x} = \sqrt{5}$ and $\dot{y} = 2\sqrt{5}$.

b $\dot{x} = \sqrt{5}, x = t\sqrt{5}, \dot{y} = -10t + 2\sqrt{5},$

$y = -5t^2 + 2t\sqrt{5}$

d 1 metre **e** 2 metres

f $\dot{x} = \sqrt{5}, \dot{y} = -2\sqrt{5}, v = 5$ m/s, $\theta = -\tan^{-1} 2$

g $y = 2x - x^2$

9 46 m/s

11 b $y = -\frac{4}{45}x^2 + \sqrt{3}x$

c **i** 15 m **ii** $\sqrt{3}$ seconds

Chapter 11

Exercise 11A

1 b $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ or $\frac{3\pi}{2}$

2 b $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ or 2π

3 b $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

4 a $\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$ **b** $\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

c $\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}$ or $\frac{17\pi}{9}$

d $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ or 2π

5 a $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ or 2π **b** $x = \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}$ or $\frac{5\pi}{3}$

c $x = \frac{\pi}{6}, \frac{\pi}{2}$ or $\frac{5\pi}{6}$ **d** $x = \frac{2\pi}{3}, \pi$ or $\frac{4\pi}{3}$

e $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ or 2π

f $x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$ or 2π

6 a $\theta = 90°, 194°29', 270°$ or $345°31'$

b $\theta = 60°, 120°, 240°$ or $300°$

c $\theta = 60°$ or $300°$

d $\theta = 22°30', 67°30', 112°30', 157°30', 202°30', 247°30', 292°30'$ or $337°30'$

e $\theta = 41°49', 138°11', 210°$ or $330°$

f $\theta = 54°44', 125°16', 234°44'$ or $305°16'$

g $\theta = 106°16'$

h $\theta = 0°, 60°, 300°$ or $360°$

i $\theta = 30°, 90°, 150°, 210°, 270°$ or $330°$

j $\theta = 45°, 63°26', 225°$ or $243°26'$

7 b $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

8 b $x = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}$ or $\frac{9\pi}{5}$

9 b $\theta = 0, \frac{\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}$ or π

10 b $x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ or 2π

11 b $\theta = \frac{7\pi}{24}$ or $\frac{19\pi}{24}$

12 a $\alpha = 60°$ or $300°$

b $\alpha = 63°26', 135°, 243°26'$ or $315°$

c $\alpha = 15°, 75°, 105°, 165°, 195°, 255°, 285°$ or $345°$

d $\alpha = 180°$ or $240°$

e $\alpha = 30°, 60°, 210°$ or $240°$

f $\alpha = 60°$ or $300°$

13 b $\frac{\pi}{5}, \frac{\pi}{3}, \frac{3\pi}{5}$ or π

14 b $\theta = 160°55'$ or $289°5'$

15 c $x \doteqdot -2.571, -1.368$ or 3.939

16 e $x = \tan\frac{\pi}{10}, -\tan\frac{\pi}{10}, \tan\frac{3\pi}{10}$ or $-\tan\frac{3\pi}{10}$

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Exercise 11B

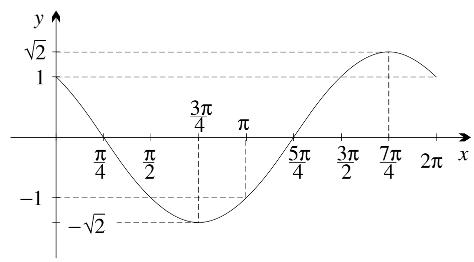
1 a $R = 2, \alpha = \frac{\pi}{3}$ **b** $R = 3\sqrt{2}, \alpha = \frac{\pi}{4}$

2 a $R = 13, \alpha \doteqdot 22°37'$ **b** $R = 2\sqrt{5}, \alpha \doteqdot 63°26'$

3 b $A = \sqrt{2}$ **c** $\alpha = \frac{\pi}{4}$

d Maximum is $\sqrt{2}$, when $x = \frac{7\pi}{4}$. Minimum is $-\sqrt{2}$, when $x = \frac{3\pi}{4}$.

e $x = \frac{\pi}{2}$ or π **f** amplitude: $\sqrt{2}$, period: 2π

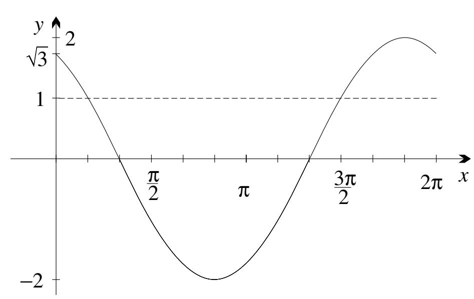


5 b $B = 2$ **c** $\theta = \frac{\pi}{6}$

d Maximum is 2, when $x = -\frac{\pi}{6}$. Minimum is -2, when $x = \frac{5\pi}{6}$.

e $x = \frac{\pi}{6}, \frac{3\pi}{2}$

f



6 c $x \doteqdot 126°52'$

7 b $x = 90°$ or $x \doteqdot 323°8'$

8 b $x = 270°$ or $x \doteqdot 306°52'$

9 a $3\sin\left(x + \tan^{-1}\frac{2}{\sqrt{5}}\right)$ **b** $x = 180°$ or $x \doteqdot 276°23'$

10 a $x \doteqdot 77°39'$ or $344°17'$ **b** $x \doteqdot 103°29'$ or $156°8'$

c $x \doteqdot 30°41'$ or $297°26'$ **d** $x \doteqdot 112°37'$ or $323°8'$

11 a $A = 2, \alpha = \frac{5\pi}{6}$ **b** $A = 5\sqrt{2}, \alpha = \frac{5\pi}{4}$

12 a $A = \sqrt{41}, \alpha \doteqdot 321°20'$

b $A = 5\sqrt{5}, \alpha \doteqdot 259°42'$

13 a i $2\cos\left(x + \frac{11\pi}{6}\right)$ **ii** $x = \frac{\pi}{2}$ or $\frac{11\pi}{6}$

b i $\sqrt{2}\sin\left(x + \frac{3\pi}{4}\right)$ **ii** $x = 0$ or $\frac{3\pi}{2}$

c i $2\sin\left(x + \frac{5\pi}{3}\right)$ **ii** $x = \frac{\pi}{6}$ or $\frac{3\pi}{2}$

d i $\sqrt{2}\cos\left(x - \frac{5\pi}{4}\right)$ **ii** $x = \pi$ or $\frac{3\pi}{2}$

14 a i $\sqrt{5}\sin\left(x + 116°34'\right)$

ii $x = 270°$ or $x \doteqdot 36°52'$

b i $5\cos\left(x - 3.7851\right)$ **ii** $x \doteqdot 2.63$ or 4.94

15 a $x \doteqdot 313°36'$ **b** $x \doteqdot 79°6'$ or $218°59'$

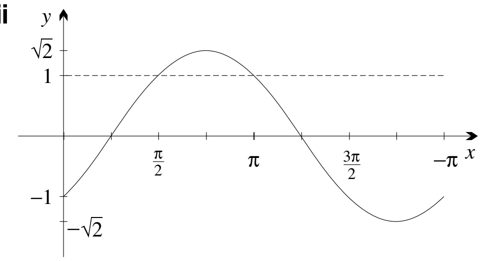
16 b $\theta = 0, \frac{3\pi}{4}, \frac{3\pi}{2}$ or $\frac{7\pi}{4}$

17 a $x = \frac{7\pi}{12}, \frac{11\pi}{12}$ **b** $x = \frac{\pi}{3}, \frac{4\pi}{3}$

c $x = 0, \frac{\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \pi, \frac{9\pi}{8}, \frac{3\pi}{2}, \frac{13\pi}{8}, 2\pi$

18 b $x = -\frac{5\pi}{6}, -\frac{\pi}{12}, \frac{\pi}{6}$ or $\frac{11\pi}{12}$

19 a ii



iii $\frac{\pi}{2} < x < \pi$

b i $\frac{\pi}{2} \le x \le \frac{11\pi}{6}$

ii $0 \le x < \frac{\pi}{6}$ or $\frac{3\pi}{2} < x \le 2\pi$

iii $\frac{2\pi}{3} < x < \pi$ or $\frac{5\pi}{3} < x < 2\pi$

iv $0 \le x \le \frac{\pi}{12}$ or $\frac{17\pi}{12} \le x \le 2\pi$

20 b $\sin x + \sqrt{3}\cos x = 2\sin\left(x - \frac{5\pi}{3}\right)$ or $2\cos\left(x - \frac{\pi}{6}\right)$ or $2\cos\left(x + \frac{11\pi}{6}\right)$

c $\cos x - \sin x = \sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)$ or $\sqrt{2}\sin\left(x + \frac{3\pi}{4}\right)$ or $\sqrt{2}\sin\left(x - \frac{5\pi}{4}\right)$

Exercise 11C

1 c $x = 0, \frac{3\pi}{2}, 2\pi$

2 b $x = 0, \frac{2\pi}{3}, 2\pi$

3 b $x = 90°$ or $x \doteqdot 298°4'$

4 b $x = 180°$ or $x \doteqdot 67°23'$

6 a $x = 90°$ or $x \doteqdot 12°41'$

b $x \doteqdot 36°52'$ or $241°56'$

c $x \doteqdot 49°48'$ or $197°35'$

d $x = 180°$ or $x \doteqdot 280°23'$

8 $x = 45°, 225°$ or $x \doteqdot 18.4°, 198.4°$

9 b $x \doteqdot 36°52'$

Chapter 11 review exercise

1 a $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ or 2π **b** $x = \frac{\pi}{3}, \pi$ or $\frac{5\pi}{3}$

c $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ **d** $x = \frac{\pi}{3}$ or $\frac{4\pi}{3}$

2 a $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ **b** $x = \frac{3\pi}{4}$

3 a $2\cos\left(x - \frac{\pi}{6}\right)$ **b** $x = \frac{5\pi}{6}$ or $\frac{3\pi}{2}$

4 a $3\sin\left(x + \tan^{-1}\frac{\sqrt{5}}{2}\right)$ **b** $x \doteqdot 41.8°$

5 a $\sqrt{13}\cos\left(x + \tan^{-1}\frac{2}{3}\right)$ **b** $x \doteqdot 40°12'$ or $252°25'$

6 $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ or 2π

8 $x \doteqdot 1.20$ or 2.87

9 b $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$ or $\frac{11\pi}{6}$

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Chapter 12

Exercise 12A

2 a $\frac{-1}{\sqrt{1-x^2}}$ **b** $\frac{1}{1+x^2}$
c $\frac{2}{\sqrt{1-4x^2}}$ **d** $\frac{3}{1+9x^2}$
e $\frac{-5}{\sqrt{1-25x^2}}$ **f** $\frac{-1}{\sqrt{1-x^2}}$
g $\frac{2x}{\sqrt{1-x^4}}$ **h** $\frac{3x^2}{1+x^6}$
i $\frac{1}{x^2+4x+5}$ **j** $\frac{1}{\sqrt{2x-x^2}}$
k $\sin^{-1}x+\frac{x}{\sqrt{1-x^2}}$ **l** $2x\tan^{-1}x+1$
m $\frac{1}{\sqrt{25-x^2}}$ **n** $\frac{4}{16+x^2}$
o $\frac{-1}{2\sqrt{x-x^2}}$ **p** $\frac{1}{2(1+x)\sqrt{x}}$
q $\frac{-1}{1+x^2}$

3 a 2 **b** 2 **c** 1 **d** -1

4 a Tangent is $y=-6x+\pi$, normal is $y=\frac{1}{6}x+\pi$.
b Tangent is $y=\frac{1}{\sqrt{2}}x+\frac{\pi}{4}-1$, normal is $y=-\sqrt{2}x+\frac{\pi}{4}+2$.

5 b $\frac{\pi}{2}$

6 a π **b** $\frac{\pi}{2}$

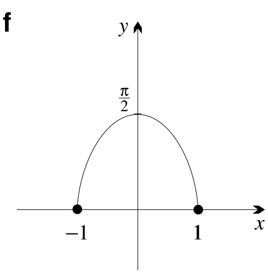
7 b concave up

9 a $\cos^{-1}x$ **b** $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$
c $\frac{2}{\sqrt{7+12x-4x^2}}$ **d** $\frac{1}{x^2-2x+2}$
e $\frac{e^x}{\sqrt{1-e^{2x}}}$ **f** $\frac{1}{2\sqrt{1-x^2}\sin^{-1}x}$
g $\frac{1}{2x\sqrt{\log_e x(1-\log_e x)}}$ **h** $\frac{\sin^{-1}\sqrt{1-x}}{2\sqrt{x}}-\frac{1}{2\sqrt{1-x}}$
i $\frac{1}{1+x^2}$

11 a $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
b $x=\frac{1}{2}\sin y$
c $\frac{dx}{dy}=\frac{1}{2}\cos y\ge 0$ for $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
d $\frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$

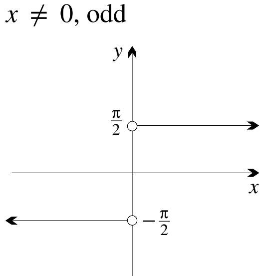
12 a $\frac{1}{\sqrt{4-x^2}}$ **b** $\frac{-1}{\sqrt{2x-x^2}}$ **c** $\frac{1}{2(1+x)\sqrt{x}}$

13 a $-1\le x\le 1$, even
b The y-axis, because the function is even.
c $\frac{-2x}{\sqrt{1-x^4}}$
e The tangents at $x=1$ and $x=-1$ are vertical.
f

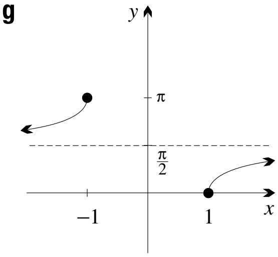


15 c $\frac{1}{45}$ rad/s

16 a $x\ne 0$, odd
c



17 a $x\ge 1$ or $x\le -1$
c They are undefined.
d When $x>1$,
$f'(x)=\frac{1}{x\sqrt{x^2-1}}$,
and when $x<-1$,
$f'(x)=\frac{-1}{x\sqrt{x^2-1}}$.
e $f'(x)>0$ for $x>1$ and for $x<-1$.
f i $\frac{\pi}{2}$ **ii** $\frac{\pi}{2}$
g



18 a $\frac{3e^{3x}}{1+e^{6x}}$ **b** $\frac{3x^2}{\sqrt{1-x^6}}$
c $-\frac{1}{x\sqrt{1-(\log_e x)^2}}$

19 a $-1\le x\le 1$
c $g(x)=\frac{\pi}{2}$ for $0\le x\le 1$.

20 $\tan^{-1}\frac{x+2}{1-2x}$ is $\tan^{-1}x+\tan^{-1}2$ for $x<\frac{1}{2}$, and is $\tan^{-1}x+\tan^{-1}2-\pi$ for $x>\frac{1}{2}$.

21 a domain: all real x, range: $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$, odd
c No, because $\frac{0}{0}$ is undefined.
d $f'(x)=1$ when $\cos x>0$, and $f'(x)=-1$ when $\cos x<0$.

Exercise 12B

2 a $\cos^{-1}x+C$ **b** $\sin^{-1}\frac{x}{2}+C$
c $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$ **d** $\sin^{-1}\frac{3x}{2}+C$
e $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+C$ **f** $\cos^{-1}\frac{x}{\sqrt{5}}+C$

3 a $\frac{\pi}{2}$ **b** $\frac{\pi}{8}$ **c** $\frac{\pi}{4}$ **d** $\frac{\pi}{12}$ **e** $\frac{\pi}{6}$ **f** $\frac{5\pi}{12}$

4 a $y=\sin^{-1}x+\pi$ **b** $y=\tan^{-1}\frac{x}{4}+\frac{\pi}{4}$

5 a $\frac{\pi}{3}$ **b** $\frac{\pi}{4}$

6 a $\frac{1}{2}\sin^{-1}2x+C$
b $\frac{1}{4}\tan^{-1}4x+C$
c $\frac{1}{\sqrt{2}}\cos^{-1}\sqrt{2}x+C$
d $\frac{1}{3}\sin^{-1}\frac{3x}{2}+C$
e $\frac{1}{15}\tan^{-1}\frac{3x}{5}+C$
f $\frac{1}{2}\cos^{-1}\frac{2x}{\sqrt{3}}+C$

7 a $\frac{\pi}{18}$ **b** $\frac{\pi}{12}$ **c** $\frac{2\pi}{9}\sqrt{3}$
d $\frac{5\pi}{24}$ **e** $\frac{\pi}{12}\sqrt{3}$ **f** $\frac{\pi}{120}\sqrt{10}$

8 c $\left(\frac{\pi}{12}+\frac{1}{2}\sqrt{3}-1\right)$ unit^2

9 b $\left(1-\frac{1}{2}\sqrt{3}\right)$ unit^2

10 b $\frac{\pi}{2}$

11 a $\frac{6x^2}{4+x^6}$ **b** $\frac{1}{6}\tan^{-1}\frac{x^3}{2}+C$

12 a $\tan^{-1}x+\frac{x}{1+x^2}$ **b** $\frac{\pi}{4}-\frac{1}{2}\ln 2$

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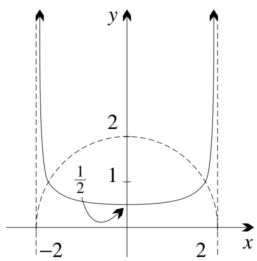
13 a 0 **b** 0 **c** $\frac{3\pi}{4}$
d 0 **e** 0 **f** 18π

14 a i 0
b i $f(0) = 0$ and $f'(x) < 0$ for $x > 0$.
ii $\frac{\pi - 2}{8}$

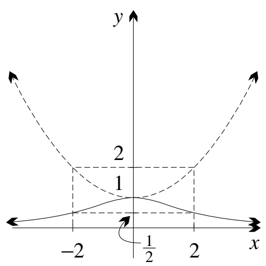
15



c domain: $-2 \le x \le 2$, range: $y \ge \frac{1}{2}$, even

d $\frac{\pi}{3}$ unit^2 **e** π unit^2

16



a The y-axis, because it is an even function.
b domain: all real x, range: $0 < y \le 1$
d 0
e π unit^2
f $4\tan^{-1}\frac{a}{2}$ unit^2
g 2π unit^2

18 a $\frac{5323}{6800}$

20 a $2\tan^{-1}\sqrt{x} + C$ **b** $\tan^{-1}e - \frac{\pi}{4}$

21 b 0.153 unit^2
c The integrand is well-defined in the interval $[0, 7]$, and lies between $\frac{1}{4}$ and $\frac{1}{9}$, so the area lies between $\frac{7}{4}$ and $\frac{7}{9}$, which is much larger than the answer of 0.153 that was calculated in part **b**. The primitive, however, is undefined at two values within the interval, at $x = \frac{\pi}{2}$ and at $x = \frac{3\pi}{2}$, which renders the argument completely invalid.

22 g $\pi \doteqdot 3.092$, error $\doteqdot 0.050$

23 a $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 + \cdots + \tan^{-1}n$
b $x\tan^{-1}x - \frac{1}{2}\ln(1 + x^2)$

Exercise 12C

2 a $\frac{2 + \sqrt{3}}{4}$ **b** $\frac{2 + \sqrt{3}}{4}$ **c** $-\frac{1}{4}$ **d** $\frac{2 - \sqrt{2}}{4}$

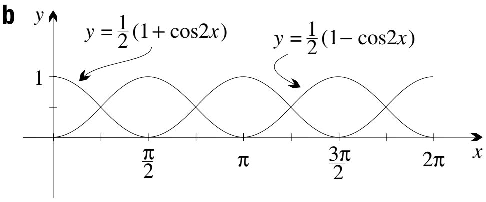
3 a $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ **b** $\frac{1}{2}x - \frac{1}{8}\sin 4x + C$
c $\frac{1}{2}x - \sin\frac{1}{2}x + C$ **d** $\frac{1}{2}x - \frac{1}{12}\sin 6x + C$

4 a $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$ **b** $\frac{1}{2}x + \frac{1}{24}\sin 12x + C$
c $\frac{1}{2}x + \frac{1}{2}\sin x + C$ **d** $\frac{1}{2}x + \frac{1}{40}\sin 20x + C$

5 a $\frac{\pi}{2}$ **b** $\frac{1}{8}(\pi + 2)$
c $\frac{1}{12}(\pi - 3)$ **d** $\frac{1}{32}(\pi + 2\sqrt{2})$
e $\frac{1}{24}(4\pi + 9)$ **f** $\frac{1}{24}(2\pi - 3\sqrt{3})$

6 a $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$
b $\frac{1}{4}\cos 2x - \frac{1}{8}\cos 4x + C$
c $\frac{2\sqrt{2}}{3}$ **d** $\frac{\sqrt{3}}{28}$

7 b



8 a $\frac{1}{4}\sin^4 x + C$ **b** $\frac{1}{7}\sin^7 x + C$
c $-\frac{1}{6}\cos^6 x + C$ **d** $-\frac{1}{9}\cos^9 x + C$
e $-\cos e^x$ **f** $\frac{1}{5}\sin 5e^x + C$
g $-\log_e|\cos x| + C$ **h** $\frac{1}{7}\log_e|\sin 7x| + C$

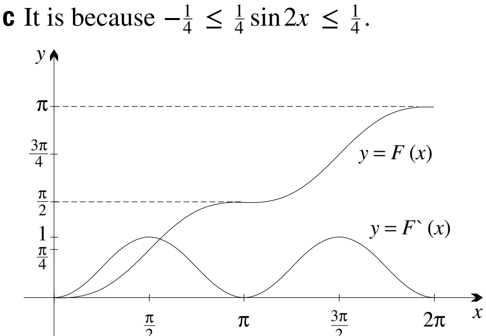
9 a $\cos x \sin x = \frac{1}{2}\sin 2x$, so $-\frac{1}{2} \le y \le \frac{1}{2}$.
b i $-\frac{1}{4}\cos 2x + C$
ii $-\frac{1}{2}\frac{1}{2}\sin^2 x + D$ using $u = \sin x$, and $-\frac{1}{2}\cos^2 x + E$ using $u = \cos x$
c The three constants C, D and E are different. The answers in part **ii** can be reconciled with each other using the Pythagorean identity, and they can be reconciled with part **i** using the $\cos 2x$ formulae.

10 b $\cos^4 x = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$
c i $\frac{3\pi}{8}$ **ii** $\frac{1}{32}(3\pi + 8)$
d $\frac{5}{24}$

11 a $\frac{1}{2}\tan 2x - x + C$ **b** $-2\cot\frac{1}{2}x - x + C$
c $\sqrt{3} - 1 - \frac{\pi}{12}$ **d** $\frac{1}{4}\sqrt{3} - \frac{\pi}{12}$

12 a $\frac{1}{2}\tan^2 x + C$ **b** $x - \sin x + C$
c $\tan x + \sin x + C$

13 b i $x = 0, \pi$ or 2π
ii $0 < x < \pi$ or $\pi < x < 2\pi$
iii no values of x
c It is because $-\frac{1}{4} \le \frac{1}{4}\sin 2x \le \frac{1}{4}$.



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Cambridge Maths Stage 6

d $(\frac{\pi}{2}, \frac{\pi}{4})$ and $(\frac{3\pi}{2}, \frac{3\pi}{4})$ are points of inflection, while $(0, 0)$, $(\pi, \frac{\pi}{2})$ and $(2\pi, \pi)$ are stationary (or horizontal) points of inflection.

f **i** $k = 3\pi$

ii $k = n\pi$, where n is an integer.

14 $\frac{1}{2}$

Exercise 12D

1 c $\frac{1}{4}(1 + x^2)^4 + C$

2 a $\frac{1}{4}(2x + 3)^4 + C$ **b** $\frac{1}{5}(1 + x^3)^5 + C$

c $\frac{-1}{1 + x^2} + C$ **d** $2\sqrt{3x - 5} + C$

e $\frac{1}{4}\sin^4 x + C$ **f** $\log_e(1 + x^4) + C$

3 c $-\sqrt{1 - x^2} + C$

4 a $\frac{1}{24}(x^4 + 1)^6 + C$ **b** $\frac{2}{9}(x^3 - 1)^{\frac{3}{2}} + C$

c $\frac{1}{3}e^{x^3} + C$ **d** $\frac{-1}{(1 + \sqrt{x})^2} + C$

e $\frac{1}{6}\tan^3 2x + C$ **f** $-e^{\frac{1}{x}} + C$

5 a $\frac{65}{12}$ **b** $\sqrt{2} - 1$ **c** $\frac{1}{3}$

d $\frac{1}{24}$ **e** 2 **f** $\frac{1}{2}(e^2 - 1)$

g $\frac{1}{10}$ **h** $\frac{\pi^4}{64}$ **i** 3

j $\frac{1}{2}\ln 3$

6 $\frac{\pi}{12}$ units^2

7 a $\log_e \frac{3}{2}$ **b** $\frac{\pi}{4}$ **c** $\frac{2}{3}$ **d** $\frac{4}{3}$

8 a $\sqrt{1 + e^{2x}} + C$ **b** $\ln(\ln x) + C$

c $-\ln(\ln \cos x) + C$ **d** $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

9 a $y = \frac{1}{2}\tan^{-1} e^{2x}$ **b** $y = \sin^{-1}\frac{x}{2} + \frac{x}{2} + \frac{1}{2}$

10 b **i** $\frac{2}{\ln 2}$ **ii** $\frac{1}{5}(4\sqrt{2} - 1)$

11 a $\ln 2$ **b** $\dfrac{e}{e + 1}$

12 $\tan^{-1}\sqrt{x - 1} + C$

13 a $2\sin^{-1}\sqrt{x} + C_1$ **b** $\sin^{-1}(2x - 1) + C_2$

Exercise 12E

1 c $\frac{1}{7}(x - 1)^7 + \frac{1}{6}(x - 1)^6 + C$

2 a $\frac{2}{3}(x - 1)^{\frac{3}{2}} + 2(x - 1)^{\frac{1}{2}} + C$

b $\ln|x - 1| - \frac{1}{x - 1} + C$

3 c $\frac{2}{5}(x + 1)^{\frac{5}{2}} - \frac{2}{3}(x + 1)^{\frac{3}{2}} + C$

4 a $\frac{2}{7}(x + 1)^{\frac{7}{2}} - \frac{4}{5}(x + 1)^{\frac{5}{2}} + \frac{2}{3}(x + 1)^{\frac{3}{2}} + C$

b $\frac{4}{3}(x + 1)^{\frac{3}{2}} + 2(x + 1)^{\frac{1}{2}} + C$

5 a $(x + 2) - 4\ln(x + 2) + C$

b $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + 2(2x - 1)^{\frac{1}{2}} + C$

c $\frac{3}{40}(4x - 5)^{\frac{5}{2}} + \frac{5}{8}(4x - 5)^{\frac{3}{2}} + C$

d $2(1 + \sqrt{x}) - 2\ln(1 + \sqrt{x}) + C$

6 a $\frac{49}{20}$ **b** $2\ln 2 - \frac{1}{2}$

c $\frac{8}{9}$ **d** $\frac{1}{9}$

e $\frac{128}{15}$ **f** $\frac{4}{3}$

g $4 - 6\ln\frac{5}{3}$ **h** $\frac{2517}{40}$

7 a $\sin^{-1}\frac{x + 2}{3} + C$

b **i** $\frac{1}{\sqrt{3}}\tan^{-1}\frac{x + 1}{\sqrt{3}} + C$ **ii** $\sin^{-1}\frac{x + 1}{\sqrt{5}} + C$

iii $\frac{\pi}{6}$ **iv** $\frac{\pi}{16}$

8 b **i** $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$ **ii** $\cos^{-1}\frac{x}{\sqrt{3}} + C$

iii $\frac{1}{2}\sin^{-1} 2x + C$ **iv** $\frac{1}{4}\tan^{-1} 4x + C$

v $\frac{\pi}{6}$ **vi** $\frac{\pi}{24}$

9 b **i** $\frac{x}{4\sqrt{4 + x^2}} + C$ **ii** $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$

iii π **iv** $-\frac{\sqrt{25 - x^2}}{25x} + C$

v $-\frac{\sqrt{9 + x^2}}{9x} + C$ **vi** $\frac{\sqrt{3}}{8}$

10 $y = \sqrt{x^2 - 9} - 3\tan^{-1}\frac{\sqrt{x^2 - 9}}{3}$

11 $\frac{1}{3}\left(6\sqrt{3} - 7\sqrt{2}\right)$ units^2

13 b $\frac{8}{3}$

Exercise 12F

1 b $81\pi\text{ u}^3$

2 b $36\pi\text{ u}^3$

3 a $16\pi\text{ u}^3$ **b** $9\pi\text{ u}^3$ **c** $\frac{32\pi}{5}\text{ u}^3$ **d** $6\pi\text{ u}^3$

e $\frac{16\pi}{3}\text{ u}^3$ **f** $\frac{\pi}{7}\text{ u}^3$ **g** $9\pi\text{ u}^3$ **h** $16\pi\text{ u}^3$

4 a $3\pi\text{ u}^3$ **b** $\frac{256\pi}{3}\text{ u}^3$ **c** $\frac{3093\pi}{5}\text{ u}^3$ **d** $\frac{\pi}{2}\text{ u}^3$

e $\frac{256\pi}{3}\text{ u}^3$ **f** $\frac{243\pi}{5}\text{ u}^3$ **g** $\frac{16\pi}{15}\text{ u}^3$ **h** $\frac{16\pi}{3}\text{ u}^3$

5 $\frac{\pi}{2}(e^2 - 1)$ cubic units

6 $\pi\ln 2\text{ u}^3$

7 $\pi\ln 6\text{ u}^3$

8 a $\tan^2 x = \sec^2 x - 1$ **b** $\pi\left(\sqrt{3} - \frac{\pi}{3}\right)\text{ u}^3$

9 a $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ **b** $\frac{\pi^2}{4}\text{ u}^3$

10 a $\frac{296\pi}{3}\text{ u}^3$ **b** $\frac{119\pi}{6}\text{ u}^3$ **c** $\frac{625\pi}{6}\text{ u}^3$ **d** $\frac{16\pi}{105}\text{ u}^3$

11 a $\frac{\pi}{3}\text{ u}^3$ **b** $\frac{28\pi}{15}\text{ u}^3$ **c** $\frac{81\pi}{10}\text{ u}^3$ **d** $\frac{\pi}{2}\text{ u}^3$

14 71.62 mL

15 a $256\pi\text{ u}^3$ **b** $128\pi\text{ u}^3$ **c** $128\pi\text{ u}^3$

16 a $\frac{32\pi}{5}\text{ u}^3$, $8\pi\text{ u}^3$ **b** $\frac{50\pi}{3}\text{ u}^3$, $\frac{5\pi}{3}\text{ u}^3$

c $8\pi\text{ u}^3$, $\frac{128\pi}{5}\text{ u}^3$ **d** $\frac{24\pi}{5}\text{ u}^3$, $\frac{\pi}{2}\text{ u}^3$

17 b **i** $\frac{2\pi}{35}\text{ u}^3$ **ii** $\frac{\pi}{10}\text{ u}^3$

18 b $\frac{64\pi}{3}\text{ u}^3$

19 $\pi(8\ln 2 - 5)\text{ u}^3$

20 a $y = 3x$

c **i** $\frac{15\pi}{7}\text{ u}^3$ **ii** $\frac{2\pi}{5}\text{ u}^3$

21 $\frac{\pi}{2}(8\ln 2 - 3)\text{ u}^3$

23 a $\ln(\sec\theta + \tan\theta) + C$

24 a $(0, 0)$ and $(1, 1)$

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c It is the cone formed by rotating the line $y = x$ from $x = 0$ to $x = 1$ about the $x -$ axis.

d $\frac{\pi}{3}\,\text{u}^3$

25 c $8\pi\,\text{u}^3$

Chapter 12 review exercise

1 a $\frac{3}{\sqrt{1 - 9x^2}}$ **b** $\frac{3}{9 + x^2}$

c $\frac{1}{\sqrt{2x - x^2}}$ **d** $2x\tan^{-1}x + \frac{x^2}{1 + x^2}$

e $\frac{2}{x^2 + 4x + 8}$ **f** $\frac{-1}{\sqrt{x^4 - x^2}}$

3 a They each have derivative $\frac{-1}{\sqrt{1 - x^2}}$.

b The functions differ by at most a constant. This constant is zero, so $\cos^{-1}x = \sin^{-1}\sqrt{1 - x^2}$ for $0 \le x \le 1$.

4 a $\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C$ **b** $\sin^{-1}\frac{x}{\sqrt{3}} + C$

c $\frac{1}{6}\tan^{-1}\frac{2x}{3} + C$ **d** $\frac{1}{3}\cos^{-1}\frac{3x}{4} + C$

5 a $\frac{\pi}{36}$ **b** $\frac{\pi}{3}$

6 a $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$ **b** $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$

c $\frac{1}{2}x + \frac{1}{8}\sin 4x + C$ **d** $\frac{1}{2}x - \frac{1}{16}\sin 8x + C$

7 a $\frac{\pi}{6}$ **b** $\frac{1}{12}(\pi + 3)$

9 a $\frac{1}{6}(5x - 1)^6 + C$ **b** $\frac{1}{3}(x^2 + 2)^3 + C$

c $\frac{-1}{x^4 + 1} + C$ **d** $\frac{1}{2}\sqrt{4x + 3} + C$

e $\frac{1}{3}\sin^3 x + C$ **f** $\frac{1}{4}\tan^4 x + C$

10 a $\frac{1}{15}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$

d $\frac{1}{3}$ **e** $e^2 - e$ **f** $\frac{1}{2}\ln 2$

11 a $x - 1 + \ln(x - 1) + C$

b $\frac{2}{3}(x + 2)^{\frac{3}{2}} - 6(x + 2)^{\frac{1}{2}} + C$

c $\frac{1}{10}(2x + 1)^{\frac{5}{2}} - \frac{1}{6}(2x + 1)^{\frac{3}{2}} + C$

d $2(4 + \sqrt{x}) - 8\ln(4 + \sqrt{x}) + C$

12 a $\frac{11}{30}$ **b** $4 - 3\ln 2$ **c** 36 **d** $\frac{13}{15}$

13 a $x \le 9, y \ge 0$ **b** $18\,\text{u}^2$

d i $\frac{81\pi}{2}\,\text{u}^3$ **ii** $\frac{648\pi}{5}\,\text{u}^3$

14 $4\pi\ln 2\,\text{u}^3$

15 $\frac{2688\pi}{5}\,\text{u}^3$

16 a $\cos^2 2x = \frac{1}{2} + \frac{1}{2}\cos 4x$

17 $8.49\,\text{u}^3$

18 b $\pi(1 - \ln 2)\,\text{u}^3$

19 a Minimum turning point at $(1, 2)$, maximum turning point at $(-1, -2)$.

c $\frac{9\pi}{4}\,\text{u}^3$

20 b $72 - \frac{9\pi}{2}\,\text{u}^3$ **c** $\frac{5004\pi}{5}\,\text{u}^3$

21 a $\pi\int_1^3 2^{2x+2}\,dx$ **b** $180\pi\,\text{u}^3$

c $\frac{120\pi}{\log_e 2} \doteqdot 173\pi\,\text{u}^3$ The curve is concave up.

Chapter 13

Exercise 13A

1 a first **b** first

c second **d** first

e second **f** first

g first **h** second **i** second

2 a linear **b** non-linear **d** linear

f non-linear **g** linear

3 a 1 **b** 1 **c** 2 **d** 1 **e** 2

f 1 **g** 1 **h** 2 **i** 2

5 a $y = x^2 - 3x + C$

b $y = -6e^{-2x} + 4x + C$

c $y = \tan x + C$

d $y = 3\sin 2x - 3\cos 3x + C$

e $y = \frac{2}{15}(1 - 5x)^{\frac{3}{2}} + C$

f $y = 2\sin x^2 + C$

8 a $y = x^2 + Ax + B$

b $y = -\frac{1}{4}\cos 2x + Ax + B$

c $y = 4e^{\frac{1}{2}x} + Ax + B$

d $y = -\log|\cos x| + Ax + B$

11 a $y = x - 1$

b $y = x^2 - 3x + 2$

c $y = x^3 + 3x^2 - 9x + 7$

d $y = 2 - \cos x$

e $y = 3(e^{2x} - 1)$

f $y = 2x\sqrt{x} - 2x - 1$

12 a ii $y = \log\left|\frac{x}{1 - x}\right| + C$

iii $y = \log\left|\frac{x}{1 - x}\right|$

b ii $y = \log\left|\frac{2 + x}{2 - x}\right| + C$

iii $y = \log\left|\frac{2 + x}{2 - x}\right| + 1$

13 a i $2x + 2yy' = 0$

iii y' is undefined at $(3, 0)$ and $(-3, 0)$ where the tangent to the circle is vertical.

b There may be other answers that are equivalent to these.

i $y' = \frac{1}{2y}$ **ii** $\frac{dy}{dx} = -\frac{y}{x}$

iii $\frac{dy}{dx} = -\frac{9x}{16y}$ **iv** $\frac{dy}{dx} = \frac{x}{4y}$

v $\frac{dy}{dx} = \frac{y}{2y - x}$ **vi** $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$

14 b When $y = 12$, $y'' = -y = -12$.

c $f''(x)$ is the opposite of $f(x)$, so it is a reflection in the x-axis.

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15 $y = 1 + x - \log(\cos x)$

17 a $\lambda = 1$ or 3 **b** $\lambda = -1$

c No real solution.

18 a $y' - y\tan x = 0,\ y(0) = 1$

b $(y')^2 = y^2(y^2 - 1),\ y(0) = 1$

19 a $y(0) = 1$

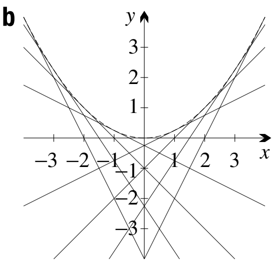
b $y'(0) = -2 \times 0 \times 1 = 0$

c i $y'' = -2y - 2xy'$ by the product rule, then substitute the expression for y'.

ii $y''(0) = -2$, so it is concave down.

d $y''' = (12x - 8x^3)y$ so $y'''(0) = 0$

20 b



They seem to form the outline of a curve (shown dotted.)

c $(2p - h, p^2 - ph)$

e $y = \frac{1}{4}x^2$; this is called a *singular* solution.

Each line in **b** is tangent to the parabola. Thus every point in the parabola is also a point in a general solution. Hence the parabola itself is also a solution.

21 $y = \dfrac{x^n}{n!} + \dfrac{A_1x^{n-1}}{(n-1)!} + \cdots + \dfrac{A_{n-1}x^1}{1!} + A_n.$

There are n arbitrary constants.

Exercise 13B

1 a -1 **b** 1 **c** 3 **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** -3

2 a

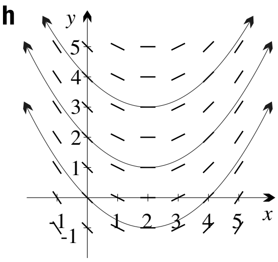
y \ x	-1	0	1	2	3	4	5
3	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
1	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
0	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
-1	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
-3	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$

d Every entry in that column is the same.

e Every vertical line $x = k$ is an isocline.

f concave up **g** a parabola

h



3 a $y' = x - y$

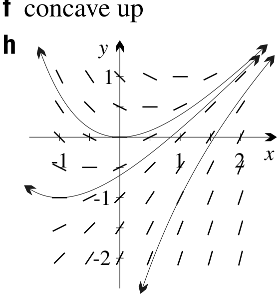
b

y \ x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
1	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$\frac{1}{2}$	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
0	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
-1	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$-\frac{3}{2}$	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$
-2	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4

e The lines $y = x + k$ are isoclines.

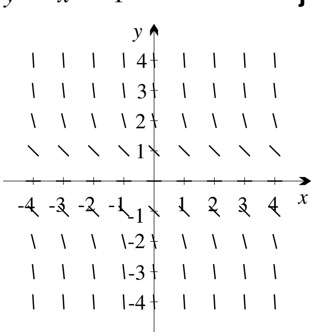
f concave up

h



i $y = x - 1$ **j** It is a solution.

4 a



i x-axis

ii any horizontal line

iii decrease to zero then increase

iv it is an isocline

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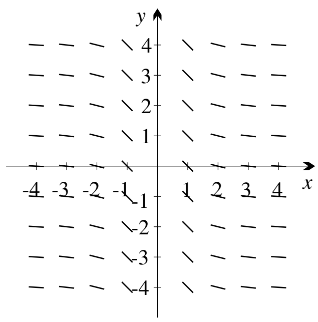
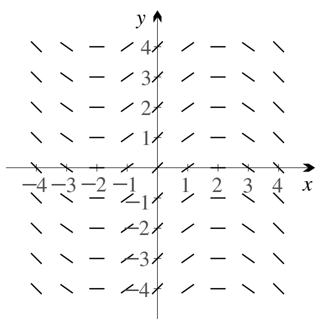


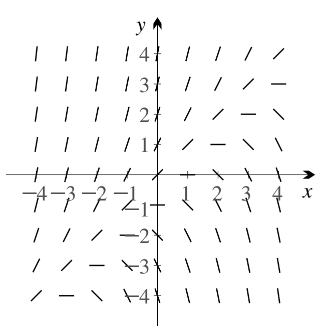
i none
ii any vertical line
iii it is an isocline
iv decrease to vertical then increase

c



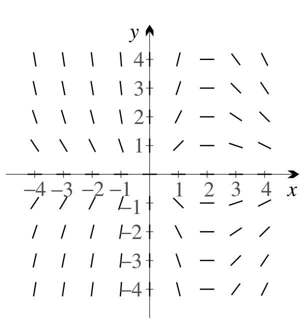
i $x = -2, 2$
ii any vertical line
iii it is an isocline
iv increase from -1 to 1 then back

d



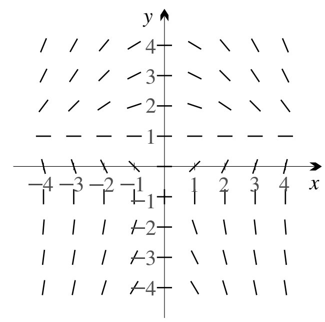
i $y = x - 1$
ii $y = x + C$
iii increase
iv decrease

e



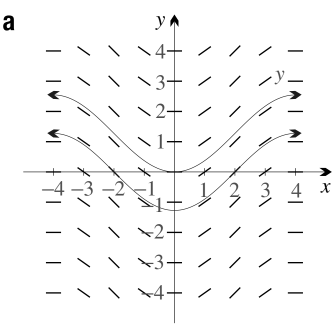
i $x = 2$
ii $y = 0$
iii increase
iv decreasing, but undefined at y-axis

f



i $y = 1$ or $x = 0$
ii undefined at $y = -1$
iii decreasing, but undefined at $y = -1$
iv decrease

5 a



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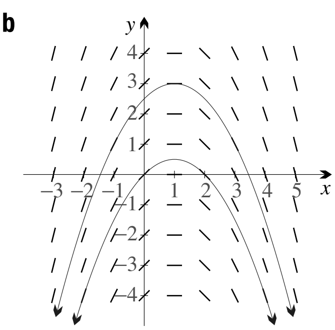

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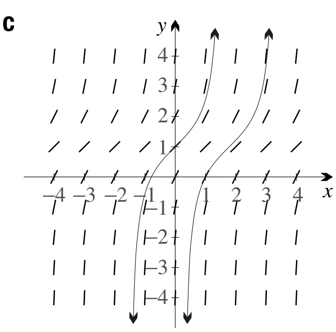

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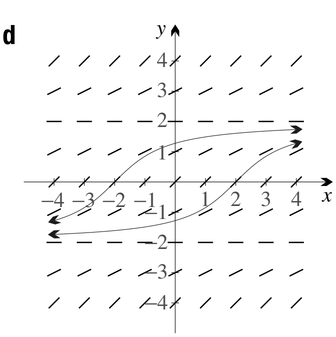

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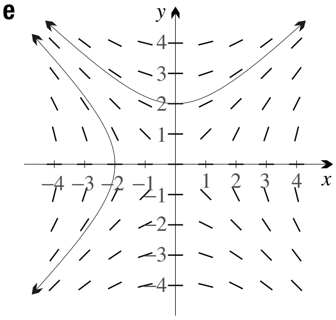
e



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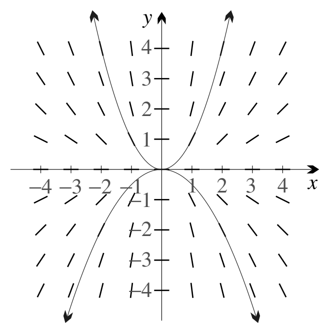


6 a vertical isoclines so $y' = f(x)$

b vertical isoclines so $y' = f(x)$

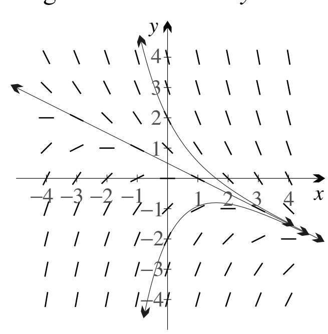
c horizontal isoclines so $y' = g(y)$

d horizontal isoclines so $y' = g(y)$

e diagonal isoclines so y' is a combination.

f diagonal isoclines so y' is a combination.

7



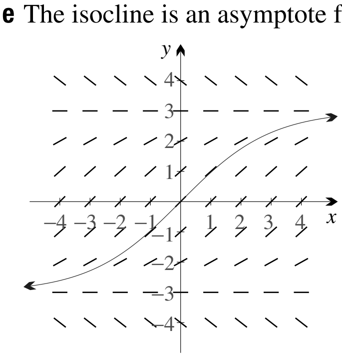
b **i** decrease

ii closer

c $y' = -\frac{1}{2}$ everywhere on that line.

e The isocline is an asymptote for each one.

8



b $y = -2, 2$ **c** yes

d **i** converge **ii** diverge **iii** yes

iv They are asymptotes for the solution curves.

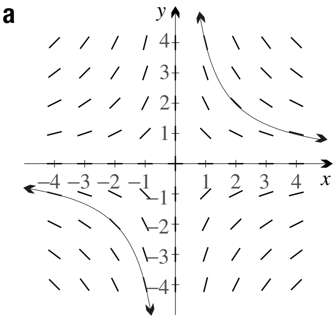
9 D. $y' = -1 - y$

10 C. $y' = \frac{1}{3}(3 - x^2)$

11 B. $y' = 1 - \dfrac{x}{y}$

12 a B. $y' = x - \frac{1}{2}y$ **b** D. $y' = \dfrac{-2xy}{1 + x^2}$

13 a



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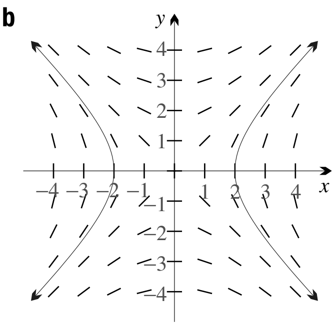
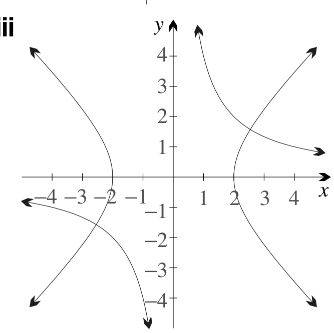
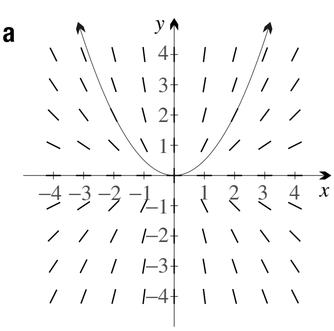


c iii



d The product of the derivatives is -1.

14 a



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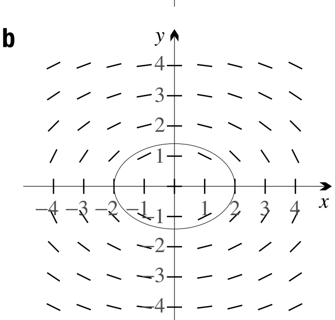

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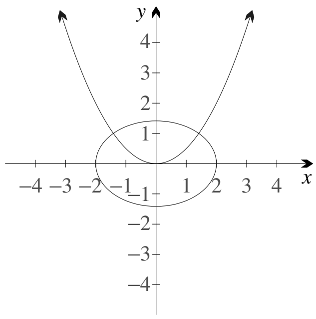
c iii



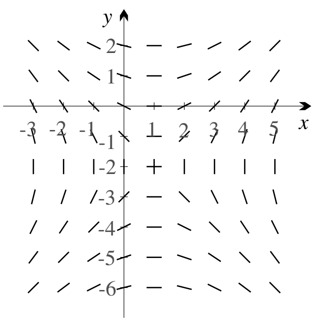
d The product of the derivatives is -1.

15 a $x^2 + y^2 = 4$

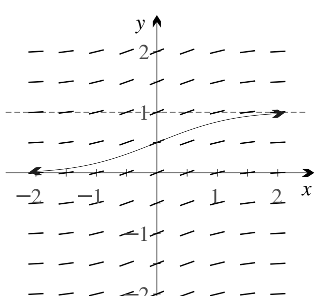
b $(x - 3)^2 + (y - 1)^2 = 4$

d i $\dfrac{dy}{dx} = \dfrac{(x - 1)}{(y + 2)}$

ii

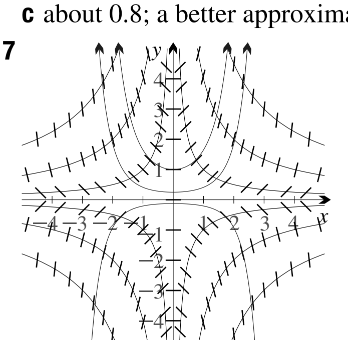


16



c about 0.8; a better approximation is 0.8413

17



a $x = 0$ and $y = 0$

b the rectangular hyperbola $xy = C$

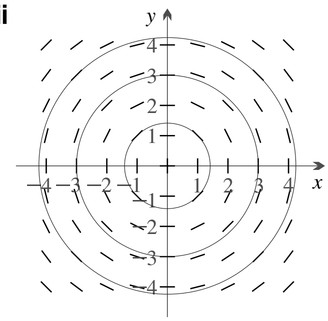
j i The exact gradient of the integral curve is known as an isocline is crossed.

ii It takes a lot of time to sketch the isoclines.

18 a i They are horizontal.

ii They are vertical.

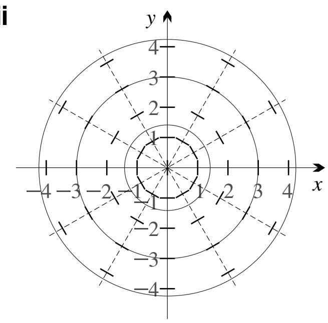
iii



iv circle

b i $y = -\frac{1}{C}x$

ii



The straight lines at 30° and 60° to the x-axis.

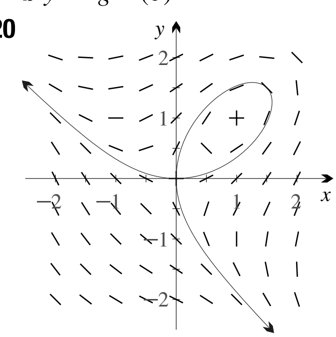
iii The product of the gradients is $C \times \frac{-1}{C} = -1$.

v Yes: notice that the innermost line elements almost join up to give the outline of a circle.

19 a $x = f^{-1}(c)$

b $y = g^{-1}(c)$

20



c $y = x^2$

d $x = y^2$

f The curve crosses itself, horizontally and vertically.

g $y = -x - 1$

h $y = x$

i The equation is symmetric in x and y.

ii Swapping x and y yields the reciprocal, $\dfrac{dx}{dy}$.

Exercise 13C

1 a $\displaystyle\int (y + 1)\,dy = \int (x - 1)\,dx$

b $(y + 1)^2 = (x - 1)^2 + D$

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2 a $y = \log\left|\frac{1}{2}x^2 + C\right|$

b $y = \tan\,(x^4 + C)$

3 a Substitute $y = 0$, where $x \neq 0$. Then LHS and RHS are both zero.

b $y = \dfrac{1}{C + \log|x|}$

4 a $\displaystyle\int y\,dy = \int -x\,dx$ **b** $y^2 = -x^2 + D$

c $y^2 + x^2 = 4$

5 a $y^2 = x^2 + 1$

b $y = \tan\left(\frac{1}{2}(x + 1)^2\right)$

c $y = \dfrac{1}{x^2 + 1}$

d $y = \log|1 - \tan x|$ (A more precise answer would exclude the points where $y = 0$.)

6 a $y = -2$

b $y = Cx^2 - 2$, where $C \neq 0$

c Allow $C = 0$.

7 a $y = 0$

b $y = Ce^{-\frac{1}{2}x^2}$, where $C \neq 0$

c Allow $C = 0$.

8 a $y = \frac{C}{x} + 2$ **b** $y^2 = C(1 + x^2)$

c $y = \frac{C}{x^2}$ **d** $y = Ce^{-\cos x}$

e $y = Ce^{-3/x}$ **f** $y = Cxe^{-x}$

9 a $\cos\,y = 0$, that is, $y = -\dfrac{3\pi}{2}, y = -\dfrac{\pi}{2}, y = \dfrac{\pi}{2}$ and $y = \dfrac{3\pi}{2}$

b $y = \tan^{-1}(x^3 + C)$

10 a $y = 0$ **b** $y = C(x - 1)^2$

c $y = (x - 1)^2$

11 a $y = 1$ **b** $y = 1 + C\sec\,x$

c $y = 1 + \sqrt{2}\sec\,x$

12 a $y = \frac{1}{2}x$ **b** $y = 2\sqrt{x}$

c $y = \dfrac{4}{1 + x^2}$ **d** $y = \frac{2}{x}$

e $y = e^{\sin x - 1}$ **f** $y = e^{1 - \frac{2}{x}x}$

13 b $y = C\log x$

14 b $y = \dfrac{4e^x}{(2 + x)^2}$

15 a $1 + \cos 2x$

b $y^2 = \sin 2x + 2x + 2$

16 a $y' = u + xu'$

b i $u' = \dfrac{2 + u}{x}$ **ii** $u = Cx - 2$

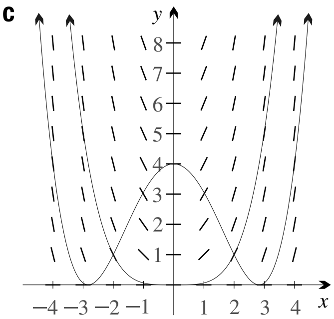
iii $y = Cx^2 - 2x$

17 a $\tan^{-1}y + \tan^{-1}x = C$

b Take the tangent of both sides and use the compound angle formula.

c $y = \dfrac{1 - x}{1 + x}$

18 c



d Substitute y_2 to get LHS $= \frac{1}{4}x(x^2 - 8)$ and RHS $= \frac{1}{4}x|x^2 - 8|$. These two expressions differ whenever $0 < |x| < 2\sqrt{2}$, as they have opposite sign. The correct solution is obtained by separation of variables.

19 a This is just a re-arrangement of the fundamental theorem of calculus.

b i $y = e^{-\frac{1}{2}x^2}$

ii $y_1(x) = 1 - \frac{1}{2}x^2$

$y_2(x) = 1 - \frac{1}{2}x^2 + \frac{1}{8}x^4$

$y_3(x) = 1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6$

$y_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6 + \frac{1}{384}x^8$

iii $e \doteqdot 2.7183$

Exercise 13D

1 a $y = 0$ **b** $\log\,|y| = -x + C$

c $y = Ae^{-x}$ **d** put $A = 0$

e $y = 2e^{-x}$

2 a $y = 0$ **b** $\dfrac{dx}{dy} = \dfrac{1}{3y}$

c $x = \frac{1}{3}\log|y| + C$ **d** $y = Ae^{3x}$ where A is a real number.

e put $A = 0$ **f** $y = -e^{3x}$

3 a $y = -3e^x$ **b** $y = e^{-2x}$

c $y = -2e^{-3x}$ **d** $y = -e^{2x}$

4 a $y = 2$

b $\log|y - 2| = -x + C$

c $y = 2 + Ae^{-x}$

d put $A = 0$

e $y = 2 + e^{-x}$

5 a $y = 1 + 2e^{-x}$ **b** $y = 1 - e^x$

c $y = -1 + 2e^{\frac{1}{2}x}$ **d** $y = 3 + e^{-2x}$

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6 a $y = \dfrac{3}{1 - 6x}$ **b** $y = \dfrac{1}{1 + x}$

c $y = \tan x$ **d** $y = -\log(1 - x)$

e $y = \log(x - 2)$ **f** $y = \left(\dfrac{3 + x}{3}\right)^3$

7 a $y = Ae^{kx}$ **b** $y = 20e^{kx}$ **c** $k = -\log 2$

d $y = 20 \times 2^{-x}, y(3) = 2\frac{1}{2}$

8 a $y = Ae^{kx}$ **b** $y = 8e^{kx}$ **c** $k = \log \frac{3}{2}$

d $y = 8 \times \left(\frac{3}{2}\right)^x, y(4) = 40\frac{1}{2}$

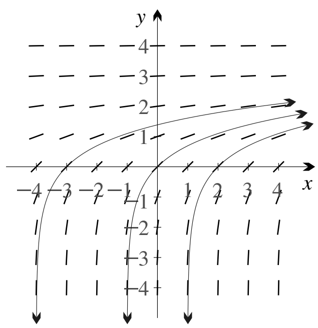
9 b $C = 0$

c ii $y = D \sin \frac{n\pi}{10} x$

10 a $y = \log(x + C)$

b Log curves with different x-intercepts.

c



iii Shift left or right.

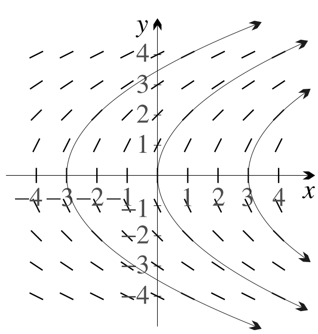
iv The isoclines are horizontal lines.

d $y = \log(x + e)$

11 a $y^2 = 4x + C$

b Concave right parabolas with vertex on the x-axis.

c



iii Shift left or right.

iv The isoclines are horizontal lines.

d $y^2 = 4x + 1$

12 a $\left(0, \frac{1}{2}\right)$ **iii** $\left(0, \frac{1}{2}\right)$

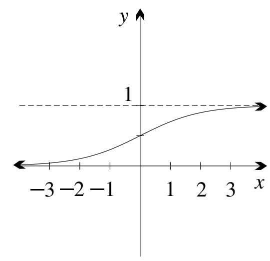
b $e^{-x} > 0$ for all x.

c 1 and 0

d $L' = \dfrac{e^{-x}}{(1 + e^{-x})^2}$ so $L' \neq 0$

e ii $L'' = \dfrac{(e^{\frac{x}{2}} - e^{-\frac{x}{2}})}{(e^{\frac{x}{2}} + e^{-\frac{x}{2}})^3}$

f



13 b i $y = 0$ and $y = 1$. **ii** $y = \dfrac{1}{1 + Be^{-x}}$

c $\log B$ to the right

d i yes: $y = 0$ **ii** yes: $y = 1$

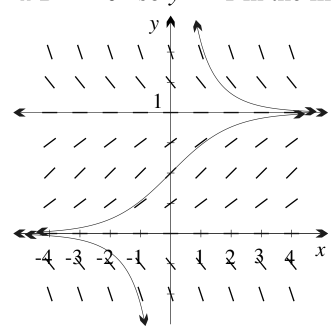
14 a $y = 0$ and $y = 1$

b $y = \dfrac{1}{1 + Be^{-rx}}$ **c** $B = \frac{1}{y_0} - 1$

e i $B \to \infty$ so $y = 0$ in the limit.

ii $B \to 0^+$ so $y = 1$ in the limit.

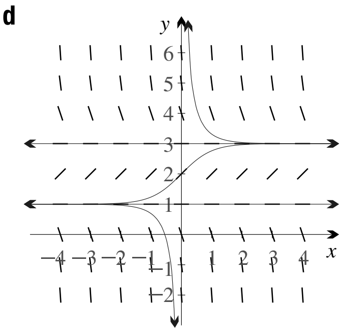
15



16 b i $y = 1$ and $y = 3$ **ii** $y = \dfrac{3 + Be^{-2x}}{1 + Be^{-2x}}$

c i $y = 3$ **ii** $y = 1$

d



e i $y'' = -2(1 - y)(2 - y)(3 - y)$

ii $(0, 2)$

17 b $v = 1 + Be^{-rx}$ **c** $y = \dfrac{1}{1 + Be^{-rx}}$

18 a $v' = 2(1 - v)$ **b** $v(0) = 0$

c $v = 1 - e^{-2x}$ **d** $y' = 1 - e^{-2x}$

e $y = \frac{1}{2} + x + \frac{1}{2}e^{-2x}$

19 a From the DE $f'(x) = g\big(f(x)\big)$ so by shifting $f'(x - C) = g\big(f(x - C)\big)$.

b Shift right by C. **c** horizontal lines

d If a graph is shifted right, its gradient at a given height is unchanged by the shift.

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20 a ii $v\dfrac{dv}{dy} = -y$

b $v^2 + y^2 = C$

c It represents a circle for $C = r^2 > 0$, a single point, the origin, when $C = 0$ and has no solutions when $C < 0$.

d $v = r\cos x,\ y = r\sin x$

e $y = r\sin(x - D)$

21 a $\cos^{-1} y = x + C$

b $C = 0$ so $\cos^{-1} y = x$

c LHS $= -\sin x$, RHS $= -\sqrt{1 - \cos^2 x} = -\sin x$
These are unequal when $\sin x < 0$.

d $y = \cos x$ with domain $0 \le x \le \pi$.

Exercise 13E

1 a i $a = -2,\ b = 1$ so that $y = x - 2x^2$

ii $a = -1,\ b = -1$ so that
$y = -e^{-x}(\cos x + \sin x)$

iii $a = 1,\ b = -1$ so that $y = x - 1 + 3e^{-x}$

b i $a = \frac{1}{2},\ b = -2,\ c = -1$ so that
$y = \frac{1}{2}x^2 - 2x - 1 + 4e^{-2x}$

ii $a = \frac{1}{4},\ b = \frac{5}{4},\ c = -\frac{1}{8}$ so that
$y = \frac{1}{8}(2x^2 + 10x - 1) - \sin 2x$

c $\lambda = -3$ or -2 so that $y = 5e^{-3x}$ or $y = 5e^{-2x}$

2 a $R = Ae^{kt}$

b $A = 100$

c $k = -\frac{1}{4}\log 5$

d $\frac{4}{5}$ gram

3 a $H = 25 - Ae^{kt}$

b $H = 25 - 20e^{kt}$

c $k = -\frac{1}{10}\log 2$

d 43 min

4 a $\dfrac{dV}{dt} = k\pi r^2$, for some constant k.

b $\frac{dr}{dt} = \frac{1}{3}k$

c $r = \frac{1}{3}kt + 4$

d $k = -\frac{1}{4}$, so $r = 4 - \frac{1}{12}t$

e $V = \pi(4 - \frac{1}{12}t)^3$, for $0 \le t \le 48$

5 a $h(0) = 400$; the height decreases so $\dfrac{dh}{dt} < 0$.

b $h(t) = \frac{1}{4}(kt + 40)^2$

c $k = -1$

d 40 mins

e No: $h(t)$ increases for $t > 40$, which is impossible.

6 a $\dfrac{y}{x}$

b $\dfrac{dy}{dx} = \dfrac{y}{x}$

c $y = Cx$

d $x = 0$ because then y' undefined.

7 a $\dfrac{y}{x}$

b $\dfrac{dy}{dx} = -\dfrac{x}{y}$

c $x^2 + y^2 = r^2$

8 a y

b $\dfrac{dy}{dx} = y$

c $y = Ae^x$

d $y = e^x$

9 a $P = P_0e^{kh}$

b $k = -\dfrac{\log 2}{4000}$

c $160\sqrt{2} \doteqdot 226$ kPa

10 a $A = (2x, 0),\ B = (0, 2y)$

b $\dfrac{dy}{dx} = -\dfrac{y}{x}$

c the hyperbola $xy = C$

11 a $C = S + Ae^{-kt}$

b $C = S + (C_0 - S)e^{-kt}$

12 b $N = \dfrac{1000}{1 + Be^{-1000kt}}$

c $N = \dfrac{1\,000}{1 + 24e^{-1000kt}}$

d 7.357×10^{-4}

e 623

13 b $N = \dfrac{P}{1 + Be^{-kPt}}$

c $N = \dfrac{23.2 \times 187.5}{23.2 + 164.3e^{-187.5kt}}$

d $k = 1.702 \times 10^{-4}$ correct to four significant figures.

e $N(80) = 120.9$ million

f The mathematical model needs to be revised. Clearly the carrying capacity is much larger that the figure estimated in 1850.

14 a $N = \dfrac{P}{1 + Be^{-kPt}}$

c $k = \dfrac{1}{t_1P}\log\left(\dfrac{N_1(P - N_0)}{N_0(P - N_1)}\right)$

d $(P - N_1)^2N_2N_0 = (P - N_2)(P - N_0)N_1^2$

15 a It represents the harvest.

b $y(0) = 3$

d $y = \dfrac{12(1 - 3e^{-\frac{1}{3}t})}{1 - 9e^{-\frac{1}{3}t}}$

e $t = 3\log 3 \doteqdot 3.3$ years

f i $y(0) = 5$

ii $\lim\limits_{t\to\infty} y = 12$, that is, the fish population approaches a stable figure of 1200. This suggests the harvest should be stopped for one year to save the species.

16 a i $I = \displaystyle\int \frac{du}{u}$

ii $I = \log(u) + C = \log(\log x) + C$

b $y = e^{Ae^{kt}}$

17 $\dfrac{dM}{dt} = -$outflow $-$decay, that is
$\dfrac{dM}{dt} = -\dfrac{5M}{100} - \dfrac{10M}{100} = -\dfrac{3M}{20}$ with $M(0) = 200$,
hence $M = 200e^{-\frac{3}{20}t}$

18 a $h(t) = 20(1 + 4e^{kt})$ where $k = \frac{1}{10}\log\frac{3}{4}$

b i $\dfrac{dH}{dt} = k(H - 20 + t),\ H(0) = 80$

ii $\dfrac{dy}{dt} = ky + 1,\ y(0) = 60$

iii $y = -\frac{1}{k} + (60 + \frac{1}{k})e^{kt}$
$H(t) = 20 - \frac{1}{k} - t + (60 + \frac{1}{k})e^{kt}$

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19 a $\dfrac{dy}{dt} = ry(1 - y),\ y(0) = y_0$ where $y_0 = \dfrac{N_0}{P}$

b $\dfrac{dy}{dx} = y(1 - y),\ y(0) = y_0$

c $v' = 1 - v$

d $v = 1 + Ae^{-x}$ so $y = \dfrac{1}{1 + Ae^{-x}}$

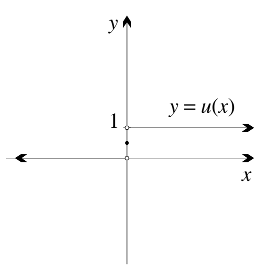
e $y = \dfrac{N_0}{N_0 + (P - N_0)e^{-x}}$

f $N = \dfrac{N_0 P}{N_0 + (P - N_0)e^{-kPt}}$

20 a The curves all have an asymptote $y = 0$ on the left and an asymptote $y = 1$ on the right, and the curves become steeper at $\left(0, \frac{1}{2}\right)$ as the value of r increases.

b **i** 1 **ii** $\frac{1}{2}$ **iii** 0

c



Chapter 13 review exercise

1 a 1st-order, linear, one arbitrary constant

b 2nd-order, linear, two arbitrary constants

c 3rd-order, non-linear, three arbitrary constants

2 a $y' = \frac{y}{x}(1 - x^2)$

b

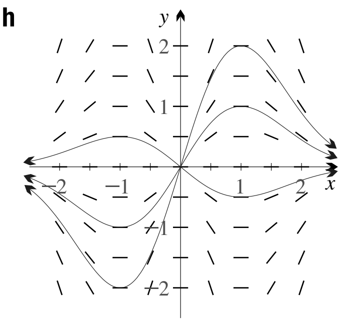
$y \backslash x$	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
2	3	$\frac{4}{3}$	0	-3	*	3	0	$-\frac{4}{3}$	-3
$\frac{3}{2}$	$\frac{9}{4}$	$\frac{5}{4}$	0	$-\frac{5}{4}$	*	$\frac{5}{4}$	0	$-\frac{5}{4}$	$-\frac{9}{4}$
1	$\frac{3}{2}$	$\frac{5}{6}$	0	$-\frac{3}{2}$	*	$\frac{3}{2}$	0	$-\frac{5}{6}$	$-\frac{3}{2}$
$\frac{1}{2}$	$\frac{3}{4}$	1	0	$-\frac{5}{12}$	*	$\frac{5}{12}$	0	$-\frac{5}{12}$	$-\frac{3}{4}$
0	0	0	0	0	*	0	0	0	0
$-\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{5}{12}$	0	$\frac{5}{12}$	*	$-\frac{5}{12}$	0	$\frac{5}{12}$	$\frac{3}{4}$
-1	$-\frac{3}{2}$	$-\frac{5}{6}$	0	$\frac{3}{2}$	*	$-\frac{3}{2}$	0	$\frac{5}{6}$	$\frac{3}{2}$
$-\frac{3}{2}$	$-\frac{9}{4}$	$-\frac{5}{4}$	0	$\frac{5}{4}$	*	$-\frac{5}{4}$	0	$\frac{5}{4}$	$\frac{9}{4}$
-2	-3	$-\frac{4}{3}$	0	3	*	-3	0	$\frac{4}{3}$	3

e The lines $y = 0$, $x = 1$ and $x = -1$ are isoclines.

f $y = 0$

g Odd: y' is unchanged when x is replaced with $-x$ and y is replaced with $-y$.

h



4 a $y = 0$

b $\dfrac{dx}{dy} = \dfrac{-2}{y}$

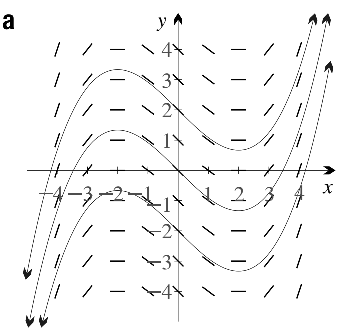
c $x = -2\log|y| + C$

d $y = Ae^{-\frac{1}{2}x}$ where A is a real number.

e put $A = 0$

f $y = 3e^{-\frac{1}{2}x}$

5 a

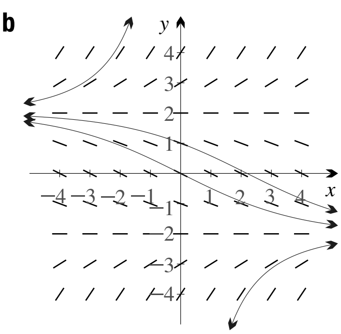


i $x = 2$ or $x = -2$

ii it is an isocline

iii decrease to -1 then increase

b



i $y = 2$ or $y = -2$

ii decrease to $-\frac{1}{2}$ then increase

iii it is an isocline

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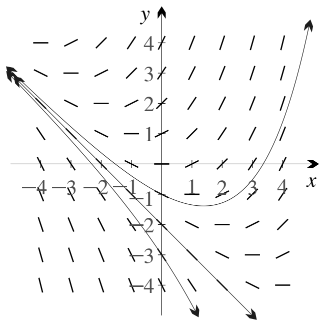
c



i $y = -x$ **ii** decrease **iii** increase

7 a $y = \dfrac{C}{1 + x^2}$

b $(x - 1)^2 + (y + 2)^2 = C$

c $y = \dfrac{Ce^x}{x}$

8 a $y = 1 + e^{-\frac{1}{2}x}$ **b** $y = 5 - 3e^{-\frac{1}{2}x}$

9 b $y = C\sqrt{\dfrac{1 + x}{1 - x}}$

10 a $x \neq 0$; there are no intercepts.

b $\lim\limits_{x \to \infty} L(x) = 1$ and $\lim\limits_{x \to -\infty} L(x) = 0$

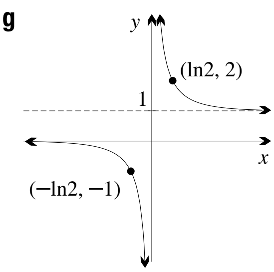
c $L(x) \to \infty$ as $x \to 0^+$ and $L(x) \to -\infty$ as $x \to 0^-$

d $(\log 2, 2)$, $(-\log 2, -1)$

e ii $L'' = \dfrac{(e^{\frac{x}{2}} + e^{\frac{x}{2}})}{(e^{\frac{x}{2}} - e^{\frac{x}{2}})^3}$

f concave up for $x > 0$, concave down for $x < 0$

g



11 C. $y' = \frac{1}{4}(x^2 + y^2)$

12 B. $y' = 1 + \dfrac{y}{x}$

13 a $y = \dfrac{2}{x}$

b $y = \log(x^2 + x - 1)$

c $y = \dfrac{-1}{1 + 2\sqrt{x}}$

14 b i $y = 0$ and $y = 1$.

ii $y = \dfrac{1}{1 + Be^{-x}}$ **iii** $y = \dfrac{1}{1 + 3e^{-x}}$

15 b i $y = 2$ and $y = 3$.

ii $y = \dfrac{3 - 4e^{-\frac{1}{5}x}}{1 - 2e^{-\frac{1}{5}x}}$ **iii** $x = 5\log\dfrac{4}{3} \doteqdot 1.44$

16 b $N = \dfrac{5}{1 + Be^{-5kt}}$ **c** $N = \dfrac{5}{1 + 4e^{-5kt}}$

d 4.418×10^{-2} **e** 233 000

17 a $y'' = 1 - 2y - 2xy''$

b $y' = 0$ and $y'' = -1$

c $y = \dfrac{1}{2}(1 + e^{-x^2})$

18 a The solutions are $y = F(x) + C$ where $F'(x) = f(x)$. Each is a vertical shift of the other.

b The solutions are $y = G^{-1}(x + C)$ where $G'(y) = \dfrac{1}{g(y)}$.

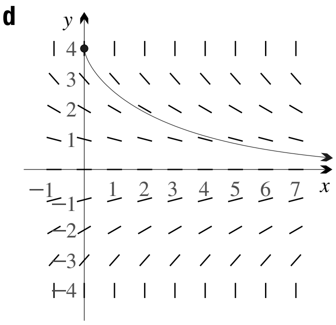
Each is a horizontal shift of the other.

19 a $y = 1$ and $y = -1$ **d** $B = A + \dfrac{\pi}{2}$

20 b $-4 \le y \le 4$ with $y(0) = 4$

c $y = 0$ does not satisfy $y(0) = 4$.

d



Chapter 14

Exercise 14A

1 a 300 500 **b** 125

c i $d = -3$

ii $T_{35} = -2$

iii $S_n = \frac{1}{2}n(203 - 3n)$

2 a i $\frac{3}{2}$ **ii** 26 375

iii $|r| = \frac{3}{2} > 1$

b i $\frac{1}{3}$

ii $|r| = \frac{1}{3} < 1$, $S_\infty = 27$

3 a \$96 000, \$780 000 **b** the 7th year

4 a $r = 1.05$ **b** \$124 106, \$1 006 232

5 a i All the terms are the same.

ii The terms are decreasing.

b If $r = 0$, then $T_2 \div T_1 = 0$, so $T_2 = 0$. Hence $T_3 \div T_2 = T_3 \div 0$ is undefined.

c i The terms alternate in sign.

ii All the terms are the same.

iii The terms are $a, -a, a, -a, \ldots$.

iv The terms are decreasing in absolute value.

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6 a \$50 000, \$55 000, \$60 000, $d = \$5000$

b \$40 000, \$46 000, \$52 900, $r = 1.15$

c For Lawrence $T_5 = \$70000$ and $T_6 = \$75000$. For Julian $T_5 \doteqdot \$69960.25$ and $T_6 \doteqdot \$80454.29$. The difference in T_6 is about \$5454.

7 a i $T_n = 47000 + 3000n$

ii the 18th year

b \$71 166

8 a 12 metres, 22 metres, 32 metres

b $10n + 2$

c i 6 **ii** 222 metres

9 a 18 times **b** 1089 **c** Monday

10 a 85 000 **b** 40 000

11 a $D = 6400$ **b** $D = 7600$ **c** the 15th year

d $S_{13} = \$1092000$, $S_{14} = 1204000$

12 $r = \left(\frac{1}{2}\right)^{\frac{1}{4}}$, $S_\infty = \dfrac{F}{1 - \left(\frac{1}{2}\right)^{\frac{1}{4}}} \doteqdot 6.29F$

13 a i $-\frac{\pi}{4} < x < \frac{\pi}{4}$

ii $S_\infty = \cos^2 x$

iii When $x = 0$, the series is not a GP because the ratio $-\tan^2 x$ cannot be zero. But the series is then $1 + 0 + 0 + \cdots$, which trivially converges to 1. Because $\cos^2 x = 1$, the given formula for S_∞ is still correct.

b i $r = \cos^2 x$ **ii** $x = 0, \pi, 2\pi$

iv When $\cos x = 0$, the series is not a GP because the ratio cannot be zero. But the series is then $1 + 0 + 0 + \cdots$, which trivially converges to 1. When $\cos x = 0$, then $\sin x = 1$ or -1, so $\operatorname{cosec}^2 x = 1$, which means that the given formula for S_∞ is still correct.

c i $r = \sin^2 x$ **ii** $x = \frac{\pi}{2}, \frac{3\pi}{2}$

iv When $\sin x = 0$, the series is not a GP because the ratio cannot be zero. But the series is then $1 + 0 + 0 + \cdots$, which trivially converges to 1. When $\sin x = 0$, then $\cos x = 1$ or -1, so $\sec^2 x = 1$, which means that the given formula for S_∞ is still correct.

14 b at $x = 16$

c i at $x = 18$, halfway between the original positions

ii 36 metres, the original distance between the bulldozers

15 a 125 metres **b** 118.75 metres

d $a = 118.75$, $d = -6.25$, $\ell = 6.25$ and $n = 19$

e $2 \times S_{19} + 125 = 20 \times 125$, which is $2\frac{1}{2}$ km.

Exercise 14B

1 a 5 **b** 14 **c** 3 **d** 15

e 4 **f** 8 **g** 14 **h** 11

2 a 13 **b** 10 **c** 8 **d** 8

3 a $\frac{T_3}{T_2} = \frac{T_2}{T_1} = 1.1$

b $a = 10$, $r = 1.1$

c $T_{15} = 10 \times 1.1^{14} \doteqdot 37.97$

d 19

4 a $r = 1.05$

b \$62 053, \$503 116

c the 13th year

5 the 19th year

6 a SC50: 50%, SC75: 25%, SC90: 10%

c 4 **d** at least 7

7 a $T_n = 3 \times \left(\frac{2}{3}\right)^{n-1}$

b 4.5 metres

c ii 16

8 a 2000 **b** 900 **c** 10 years

9 a the 10th year **b** the 7th year

10 a Increasing by 100% means doubling, increasing by 200% means trebling, increasing by 300% means multiplying by 4, and so on.

b Solve $(1.25)^n > 4$. The smallest integer solution is $n = 7$.

11 a $S_n = \dfrac{3\left(1 - \left(\frac{2}{3}\right)^n\right)}{1 - \frac{2}{3}} = 9\left(1 - \left(\frac{2}{3}\right)^n\right)$

b The common ratio is less than 1. $S = 9$

c $n = 17$

12 a $\frac{1}{2}\cos\theta\sin\theta$ **b** $\sin^2\theta$

13 a $\frac{1}{\sqrt{n}}$

b No — the spiral keeps turning without bound.

Exercise 14C

1 a i \$900 **ii** \$5900

b i \$5166 **ii** \$17 166

2 a i \$5955.08 **ii** \$955.08

b i \$18 223.06 **ii** \$6223.06

3 a i \$4152.92 **ii** \$847.08

b i \$7695.22 **ii** \$4304.78

4 a \$507.89 **b** \$10 754.61

5 a $A_n = 10000(1 + 0.065 \times n)$

b $A_{15} = \$19750$, $A_{16} = \$20400$

6 a \$101 608.52 **b** \$127 391.48

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7 a Howard — his is \$21 350 and hers is \$21 320.
b Juno — hers is now \$21 360.67 so is better by \$10.67.
8 a \$6050 **b** \$25 600 **c** 11 **d** 5.5%
9 a \$8000 **b** \$12 000 **c** \$20 000
10 \$19 990
11 a \$7678.41 **b** \$1678.41
c 9.32% per annum
12 a \$12 209.97 **b** 4.4% per annum
c Solve $10000 \times \left(\frac{1.04}{12}\right)^n > 15000$. The smallest integer solution is $n = 122$ months.
13 \$1 110 000
14 7.0%
15 a 21 years **b** 8 years and 6 months
c 14 years
16 3 years
17 Sid
18 a \$5250 **b** \$20 250
c 6.19% per annum
19 a \$40 988 **b** \$42 000
20 a i \$1120 **ii** \$1125.51
iii \$1126.83 **iv** \$1127.47
b amount $= 1000 \times e^{0.12} =$ \$1127.50
c annual compounding :\$3105.85,
continuous compounding, \$3320.12
21 a $A_n = P + PRn$
c P is the principal, PRn is the simple interest and $\sum_{k=2}^{n} {}^nC_k R^k$ is the result of compound interest over and above simple interest.

Exercise 14D

1 a i \$732.05 **ii** \$665.50 **iii** \$605
iv \$550 **v** \$2552.55
b i \$550, \$605, \$665.50, \$732.05
ii $a = 550, r = 1.1, n = 4$
iii \$2552.55
2 a i \$1531.54 **ii** \$1458.61
iii \$1389.15, \$1323, \$1260 **iv** \$6962.30
b i \$1260, \$1323, \$1389.15, \$1458.61, \$1531.54
ii $a = 1260, r = 1.05, n = 5$
iii \$6962.30
3 a i $\$1500 \times 1.07^{15}$
ii $\$1500 \times 1.07^{14}$
iii $\$1500 \times 1.07$
iv $A_{15} = (1500 \times 1.07) + (1500 \times 1.07^2) + \cdots + (1500 \times 1.07^{15})$
b \$40 332
4 a i $\$250 \times 1.005^{24}$
ii $\$250 \times 1.005^{23}$
iii $\$250 \times 1.005$
iv $A_{24} = (250 \times 1.005) + (250 \times 1.005^2) + \cdots + (250 \times 1.005^{24})$
b \$6390
5 a i $\$3000 \times 1.065^{25}$
ii $\$3000 \times 1.065^{24}$
iii $\$3000 \times 1.065$
iv $A_{25} = (3000 \times 1.065) + (3000 \times 1.065^2) + \cdots + (3000 \times 1.065^{25})$
c \$188 146 and \$75 000
6 b \$669 174.36 **c** \$429 174.36 **e** \$17 932.55
7 c iii 18
8 a \$200 000 **b** \$67 275 **c** \$630 025
d i $A_n = 100000 \times 1.1 \times ((1.1)^n - 1)$
iii 25
e $\frac{1000000}{630025} \times 10000 \doteqdot \$15872$
9 a \$360 **b** \$970.27
10 a \$31 680 **b** \$394 772 **c** \$1 398 905
11 a \$134 338 **b** \$309 281
12 \$3086
13 a \$286 593
b i \$107 355 **ii** \$152 165
14 a \$27 943.29 **b** the 19th year
15 a 18
16 The function FV calculates the value just after the last premium has been paid, not at the end of that year.
17 c $A_2 = 1.01M + 1.01^2M$,
$A_3 = 1.01M + 1.01^2M + 1.01^3M$,
$A_n = 1.01M + 1.01^2M + \ldots + 1.01^nM$
e \$4350.76 **f** \$363.70
18 b $A_2 = 1.002 \times 100 + 1.002^2 \times 100$,
$A_3 = 1.002 \times 100 + 1.002^2 \times 100 + 1.002^3 \times 100$,
$A_n = 1.002 \times 100 + 1.002^2 \times 100 + \cdots + 1.002^n \times 100$
d about 549 weeks

Exercise 14E

1 b i \$210.36 **ii** \$191.24 **iii** \$173.86
iv \$158.05 **v** \$733.51
c i \$158.05, \$173.86, \$191.24, \$210.36
ii $a = 158.05, r = 1.1, n = 4$
iii \$733.51

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2 b **i** \$1572.21 **ii** \$1497.34

iii \$1426.04, \$1358.13, \$1293.46

iv \$7147.18

c **i** \$1293.46, \$1358.13, \$1426.04, \$1497.34, \$1572.21

ii $a = 1293.46, r = 1.05, n = 5$

iii \$7147.18

3 a **ii** $\$1646.92 \times 1.07^{14}$ **iii** $\$1646.92 \times 1.07^{13}$

iv $\$1646.92 \times 1.07$ **v** \$1646.92

vi $A_{15} = 15000 \times (1.07)^{15} - \big(1646.92 + 1646.92 \times 1.07 + \cdots + 1646.92 \times (1.07)^{13} + 1646.92 \times (1.07)^{14}\big)$

c \$0

4 a **i** $100000 \times 1.005^{240}$

ii $M \times 1.005^{239}$

iii $M \times 1.005^{238}$ and M

iv $A_{240} = 100000 \times 1.005^{240} - (M + 1.005M + 1.005^2M + \ldots + 1.005^{239}M)$

c The loan is repaid. **d** \$716.43

e \$171 943.20

5 a **i** 10000×1.015^{60}

ii $M \times 1.015^{59}$

iii $M \times 1.015^{58}$ and M

iv $A_{60} = 10000 \times 1.015^n - (M + 1.015M + 1.015^2M + \cdots + 1.015^{59}\mathrm{M})$

c \$254

6 a $A_{180} = 165000 \times 1.0075^{180} - (1700 + 1700 \times 1.0075 + 1700 \times 1.0075^2 + \cdots + 1700 \times 1.0075^{179})$

c $-\$10012.67$

7 a $A_n = 250000 \times 1.006^n - (2000 + 2000 \times 1.006 + 2000 \times 1.006^2 + \cdots + 2000 \times 1.006^{n-1})$

c \$162 498, which is more than half.

d $-\$16881$

f 8 months

8 c It will take 57 months, but the final payment will only be \$5490.41.

9 a The loan is repaid in 25 years.

c \$1226.64 **d** \$367 993

e \$187 993 and 4.2% pa

10 b \$345

11 a \$4202 **b** $A_{10} = \$6.66$

c Each instalment is approximately 48 cents short because of rounding.

12 b \$216 511

13 a \$2915.90 **b** \$84.10

14 a \$160 131.55

b \$1633.21 < \$1650, so the couple can afford the loan.

15 b zero balance after 20 years

c \$2054.25

16 \$44 131.77

17 b 57

18 c $A_2 = 1.005^2P - M - 1.005M$,
$A_3 = 1.005^3P - M - 1.005M - 1.005^2M$,
$A_n = 1.005^nP - M - 1.005M - \cdots - 1.005^{n-1}M$

e \$1074.65 **f** \$34 489.78

19 b $A_2 = 1.008^2P - M - 1.008M$,
$A_3 = 1.008^3P - M - 1.008M - 1.008^2M$,
$A_n = 1.008^nP - M - 1.008M - \cdots - 1.008^{n-1}M$

d \$136 262

e $n = \log_{1.008}\dfrac{125M}{125M - P}$, 202 months

Chapter 14 review exercise

1 a $a = 31, d = 13$ **b** 16 **c** 2056

2 a $\frac{1}{2}$ **b** $|r| = \frac{1}{2} < 1$ **c** $S_\infty = 48$

3 a $n = 11$ **b** $n = 99$ **c** $n = 228$ **d** $n = 14$

4 27 000 litres

5 ‘Increasing by 2000%’ means that the profit is 21 times larger. The smallest integer solution of $(1.14)^n > 21$ is $n = 24$.

6 a $r = 1.04$ **b** \$49 816, \$420 214

7 a $T_n = 43000 + 4000n$ **b** 2017

8 2029

9 a \$15 593.19 **b** \$3593.19 **c** 5.99%

10 a $\$25000 \times (0.88)^4 \doteqdot 14992$

b \$2502 per year

c $\$25000 \div (0,88)^4 \doteqdot \$41688$

d \$4172 per year

11 b \$224 617.94 **c** \$104 617.94

d The value is \$277 419.10, with contributions of \$136 000.00.

13 a $A_{180} = 159000 \times 1.005625^{180} - (1415 + 1415 \times 1.005625 + 1415 \times 1.005625^2 + \cdots + 1415 \times 1.005625^{179})$

c $-\$2479.44$ **d** \$1407.01

14 a $A_n = 1700000 \times 1.00375^n - (18000 + 18000 \times 1.00375 + 18000 \times 1.00375^2 + \cdots + 18000 \times 1.00375^{n-1})$

c \$919 433, which is more than half.

d $-\$57677.61$ **f** 3 months

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Chapter 15

Exercise 15A

1 a categorical
b numeric and continuous. But 'height correct to the nearest mm' is numeric and discrete.
c numeric and continuous. But 'age in years' is numeric and discrete.
d categorical by party or political code. This would need to be defined carefully — if a person can be affiliated to two parties, it would not be a function.
e categorical
f categorical
g numeric and discrete
h Shoe sizes are often arranged into categories.
i These are frequently integers from 1–100, that is, numeric and discrete. If results are reported by a grade, for example, A, B, C, . . . , this might be considered categorical.

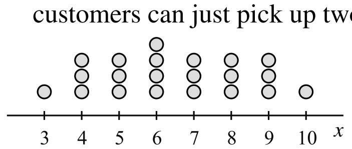
2 a median 14, mode 14, range 8
b median 10, every score is trivially a mode, range 12
c median 8, mode 3, range 12
d median 6.5, mode 4 & 6, range 6
e median 4, mode 4, range 7
f median 5.5, mode 2 & 3 & 9, range 8

3 a

score x	1	2	3	4	5	6	7	8
frequency f	4	3	4	2	1	1	1	6
cumulative	4	7	11	13	14	15	16	22

b 3.5
c 8
d i This is a median, but it might be more useful to use the mode in this case. It may be easier to develop a square box for four cupcakes rather than three.
ii See the previous comments. It is also common for sales to package a larger box to encourage customers to overbuy.
iii This is the mode, but if a box of four is marketed, customers can just pick up two boxes of four.

4 a

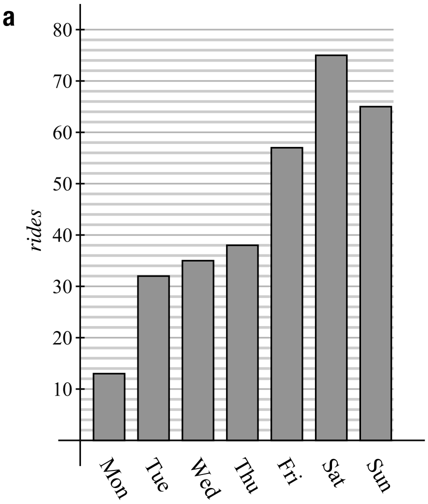


b

score x	3	4	5	6	7	8	9	10
frequency f	1	3	3	4	3	3	3	1
cumulative	1	4	7	11	14	18	20	21

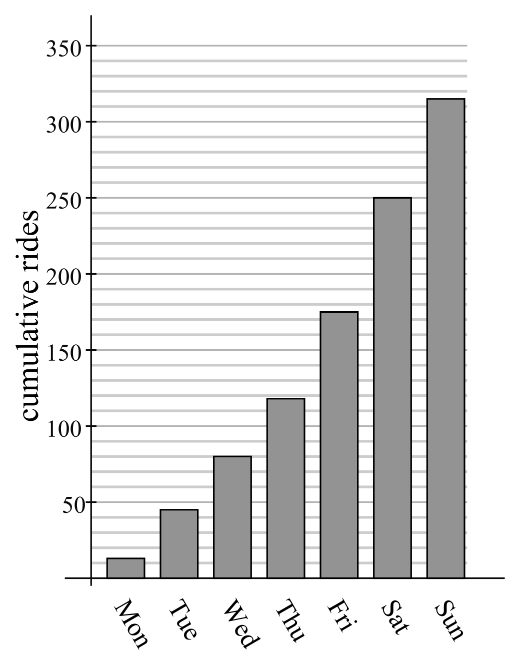
c 6 hoops
d 6.5 hoops
e Not really. If the scores are ordered by time, his scores improve over the sessions. This information is lost in the table and plot.

5 a



b

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
frequency	13	32	35	38	57	75	65
cumulative	13	45	80	118	175	250	315



6 a Blond hair and blue eyes. Different results might be expected in a different part of the world.
b Red hair and green eyes
c 45%
d 17%
e $25 \div 54 \doteqdot 46\%$
f $90 \div 247 \doteqdot 36\%$
g $671 \div 753 \doteqdot 89\%$

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h These two results would suggest so. Geneticists link this to various pigment genes that affect both characteristics.

i The proportion of the various eye and hair colours will vary in different genetic populations and ethnic groups. Studies such as this may be done with a relatively non-diverse population to prevent the clouding effects of differing genetics.

7 a 80

b

salad	pie	soup	panini	burger
32.5%	12.5%	8.75%	20%	26.25%

c

salad	pie	soup	panini	burger
\$130	\$60	\$70	\$96	\$168

d \$524

e It returns more money than the more popular pie option. It is probably also important for the café to include a vegetarian option on the menu to cater for such customers or for groups with such customers.

8 a In 2002 the price was \$400 thousand, and in 2017 it was \$1 million.

b Prices increased by 150% .

c \$40 thousand per year

d They will increase another $13 \times \$40\,000 = \$520\,000$ to around \$1.5 million.

e From 2014 to 2015, median house prices increased \$120 thousand.

f From 2010 to 2011, median house prices decreased \$40 thousand. How much did prices change?

9 a 35%, 140 dogs **b** 11%

c 75% **d** 15%

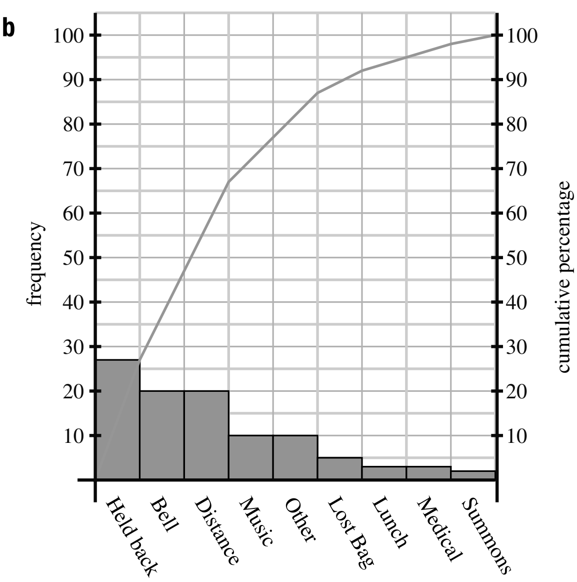
e This is quite a large category, and it may be that more investigation should be done to see if there were any other popular types of pets lumped into this category.

f Some pets may require more care and attention. For example, dogs may require frequent exercise and attention. This may give an opportunity for 'value adding' if owners are willing to pay for it. They should also consider what other pet boarding facilities are in the area, because it may be better to pick up a niche market, not covered by other pet boarding houses. Some pets may also be able to use the same types of accommodation, for example, rabbits and guinea pigs.

10 a

Reason	frequency	cumulative
Held back	27	27
Bell	20	47
Distance	20	67
Music	10	77
Other	10	87
Lost bag	5	92
Lunch	3	95
Medical	3	98
Summons	2	100

b



c The categories are arranged in descending order, so the function will be increasing (if every frequency is greater than zero), but by less at nearly every stage, causing it to curve downwards.

d 67%

e Remind teachers to release students promptly, increase the volume of the bell or the number of locations where the bell sounds, timetable students in rooms closer together where possible.

11 a 6% **b** 64% **c** 5%

d Care is needed when the graph is read in a hurry. Compare this with the Pareto chart later in this exercise where both axes are the same scale.

12 a The vertical origin is not at a 0% unemployment rate. This exaggerates the scale of the graph, which only shows a variation of 0.25%. This is still potentially significant, but it is only shown over a four-month period, so it is impossible to examine long-term trends. There are natural cycles — for

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example, there may be a rise when school pupils enter the employment market, and a drop when Christmas provides short-term retail employment. January may be a low point in economic indicators, before businesses return from holidays and begin to hire staff.

b There has been a significant increase over this five-year period, but more questions need to be asked by someone viewing the graph. What does the vertical scale represent — is it spending per citizen or spending per household? If it is per household, have the household structures changed over the period, such as more larger households? Is this a small community, in which case the data won't be very robust to changes in population? Is the data collected from sales at local shops, and does it include tourists and people passing through — has there been an increase in tourism, and was the data collected at the same time of year (more takeaways may be sold at the height of the tourist season)? What is included in the category of 'takeaway food' — if this is a health study, takeaway salads may be considered healthier than takeaway burgers (which the graphic is trying to suggest). Finally, note that the eye interprets the increase by the size of the graphic, but in fact it is the height that holds information, suggesting a greater increase than was actually the case.

c **i** People who do not have access to the internet, or do not feel as comfortable accessing and filling in an online survey, will not be represented. This may be more prevalent amongst older demographics.

ii The group should look at other hospitals, unless they particularly want to investigate the change in costs at their local hospital. Hospital costs could be influenced by government policy increasing the staffing numbers at the hospital, by purchase of new expensive diagnostic equipment, by opening and closing particular hospital wards (possibly relocating them to other hospitals), by quality control improvements, by industrial action of staff, and so on. The group likely will want to investigate the cause of any changes to overall expenses and may want to produce graphs of particular expenses, such as doctors' fees. They need to be clear what questions they actually want to ask — for example, are they concerned that medical treatment is getting more expensive for certain sections of the community who cannot afford it?

13 a 58%

b Around 3 billion

c About 0.92 billion

d 5.2%

e It may be of some use if choosing a major world language is a consideration, but there are often other considerations in deciding what language to learn. For example, you may have relatives who speak Malay, or a girl-friend who is French, or you may want to learn Japanese because of Japan's importance to Australia's economy. Others learn languages for academic reasons, such as Latin because of its historical and linguistic importance, or Russian to study Russian literature. When deciding a languageon the number of speakers, it is probably more useful to consider the total number of speakers, not merely those who speak it as a first language — close to a billion people speak English, but only a third of them do so as a first language.

14 a **i** 15°C **ii** 30°C

b **i** 17°C **ii** 23°

c Around 6−7°C in December–January

d September and May

e November–February

f June–August

g Colour-blind readers may find the colours difficult to distinguish. Using dashes and colour also provides two visuals cues for the bulk of readers, making the graph easier to read.

15 a 30

b 73% and 27%

c Bill on Essay writing, Claire on Interpretation, Ellie on all sections.

d 80%

e 40%

f Aaron and Dion. Notice that Claire has not reached 50% in the Interpretation section.

16 a Provided that similar levels of postgraduates survive to the 55-64 age bracket.

b 61.1% **c** 21.6%

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Exercise 15B

1 a $\bar{x} = 7$, Var $= 3.6$, $s \doteqdot 1.90$

x	f	xf	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
3	1	3	16	16
5	1	5	4	4
6	1	6	1	1
7	3	21	0	0
8	2	16	1	2
9	1	9	4	4
10	1	10	9	9
Total	10	70		36

b $\bar{x} = 7$, Var $= 3.6$, $s \doteqdot 1.90$

x	f	xf	$x^2 f$
3	1	3	9
5	1	5	25
6	1	6	36
7	3	21	147
8	2	16	128
9	1	9	81
10	1	10	100
Total	10	70	526

2 a $\bar{x} = 18$, $s \doteqdot 3.67$ **b** $\bar{x} = 7$, $s \doteqdot 3.06$
c $\bar{x} = 55$, $s \doteqdot 7.58$ **d** $\bar{x} = 11$, $s \doteqdot 1.88$

3 a $\bar{x} \doteqdot 7.17$, $s \doteqdot 3.18$ **b** $\bar{x} = 5.7$, $s \doteqdot 1.73$
c $\bar{x} = 3.03$, $s \doteqdot 0.94$ **d** $\bar{x} \doteqdot 42.88$, $s \doteqdot 10.53$

4 a 34 **b** $\mu \doteqdot 3.26$, $\sigma \doteqdot 1.75$

c

class	0–2	3–5	6–8
centre	1	4	7
freq	12	18	4

d $\mu \doteqdot 3.29$, $\sigma \doteqdot 1.93$

e Information is lost when data are grouped, causing the summary statistics to change.

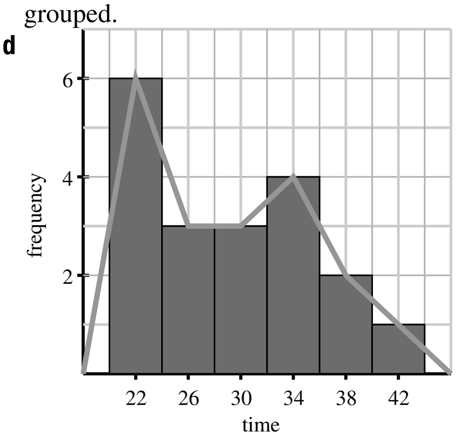
5 a 29.5

b

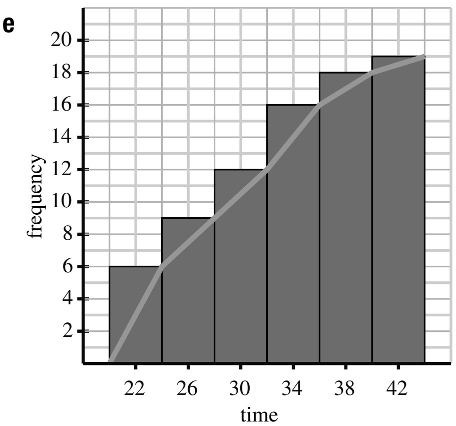
class	20–24	24–28	28–32	32–36	36–40	40–44
centre	22	26	30	34	38	40
freq	6	3	3	4	2	1
c.f.	6	9	12	16	18	20

c 30. No, because information is lost when the data are grouped.

d



e



6 a

x	152	154	155	157	158	159	162	163
f	1	2	1	1	2	3	2	3

x	164	165	166	168	170
f	2	2	3	1	1

b 162.5

c Trends are less clear when the data are not grouped, because it is less visually clear that the data are falling in certain zones on the domain.

d

group	150–154	154–158	158–162	162–166	166–170
centre	152	156	160	164	168
freq	3	4	5	10	2

e 162

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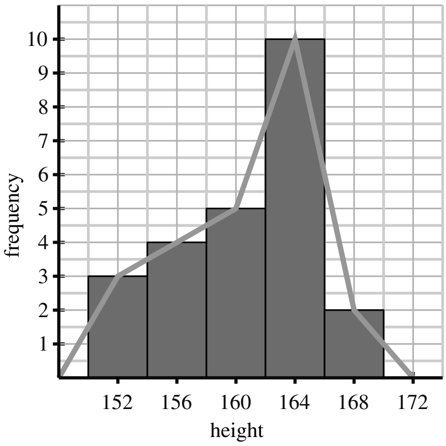
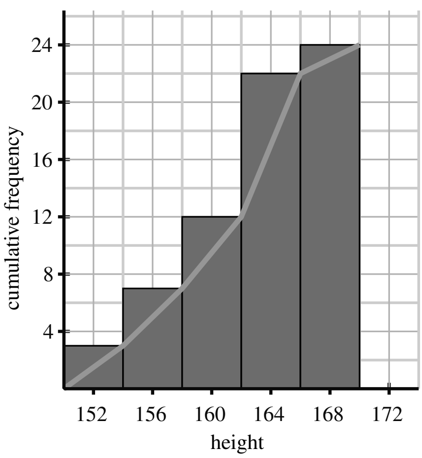
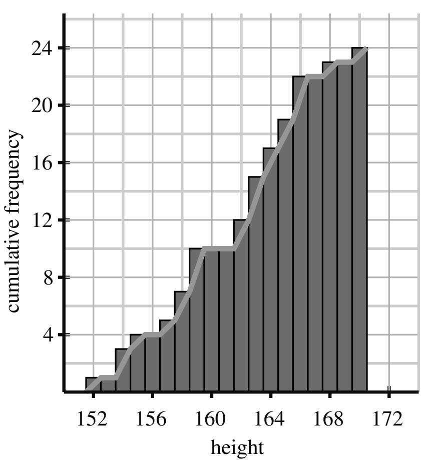
f





h



i The cumulative frequency polygon and ogive are much less sensitive to the grouping process than the frequency histogram and ogive. The graphs in parts (g) and (h) look very similar in shape.

7 a **i** 14.3 (1 decimal place)
ii 13.7 (1 decimal place)
iii 13.6 (1 decimal place)
b 0.005%

Exercise 15C

1 a mean 6.9, median 8, mode 8, range 10
b mean 21.4, median 22.5, mode 12, range 18
c mean 5.2, median 5.5, mode 7, range 5
d mean 62.3, median 61, trimodal: 54, 61, 73, range 19

2 a $Q_1 = 7, Q_2 = 13, Q_3 = 17$, IQR $= 10$
b $Q_1 = 12.5, Q_2 = 18.5, Q_3 = 25.5$, IQR $= 13$
c $Q_1 = 7.5, Q_2 = 11, Q_3 = 18$, IQR $= 10.5$
d $Q_1 = 5, Q_2 = 8.5, Q_3 = 13$, IQR $= 8$
e $Q_1 = 4, Q_2 = 7, Q_3 = 13$, IQR $= 9$
f $Q_1 = 10, Q_2 = 15, Q_3 = 21$, IQR $= 11$
g $Q_1 = 5, Q_2 = 9, Q_3 = 13.5$, IQR $= 8.5$
h $Q_1 = 12, Q_2 = 14, Q_3 = 18$, IQR $= 6$

3 a $Q_1 = 4, Q_2 = 12, Q_3 = 16$, IQR $= 12$
b $Q_1 = 1, Q_2 = 6.5, Q_3 = 11$, IQR $= 10$
c $Q_1 = 7, Q_2 = 9, Q_3 = 12$, IQR $= 5$
d $Q_1 = 2.5, Q_2 = 5, Q_3 = 7$, IQR $= 4.5$
e $Q_1 = 7, Q_2 = 7, Q_3 = 10$, IQR $= 3$
f $Q_1 = 4, Q_2 = 5, Q_3 = 9$, IQR $= 5$
g $Q_1 = 2.5, Q_2 = 4, Q_3 = 9.5$, IQR $= 7$
h $Q_1 = 4.5, Q_2 = 9, Q_3 = 12$, IQR $= 7.5$

4 a Answers may differ here, but 40 and 92 are likely.
b 40, 54, 59, 69, 92
c IQR $= 15$, $Q_1 - 1.5 \times$ IQR $= 31.5$ and $Q_3 + 1.5 \times$ IQR $= 91.5$. Thus 92 is the only outlier by the IQR criterion.
d Some may identify 40 as an outlier by eye — this shows the advantage of plotting values, where it becomes evident that this score is well separated from other scores. A student receiving 40 in this cohort should be noted as someone needing extra attention and assistance.
e **i** 54, 60, 70.5, IQR $= 16.5$
ii 53.5, 58, 68.5, IQR $= 14.5$
iii 54, 59, 68.5, IQR $= 14.5$
f In this case, with a reasonably sized dataset, the middle of the data is fairly stable and removing an extreme value has only a small effect on the quartiles and IQR. With a large dataset and tightly clustered values in the middle two quarters of the data, the difference would be even smaller.
g **i** 60.8, 11.1 **ii** 61.6, 10.5
iii 59.5, 9.4 **iv** 60.3. 8.7
h 2.4 is 22% of 11.1. Any deviation from the mean is exaggerated by the standard deviation because the deviation from the mean is squared when calculating the variance.

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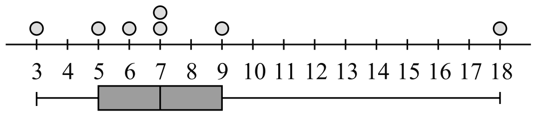
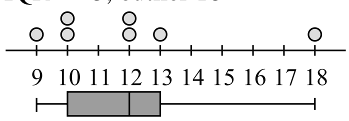
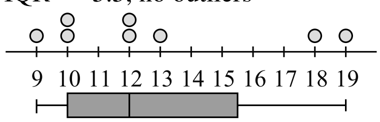
5 a i IQR $= 4$, outlier 18



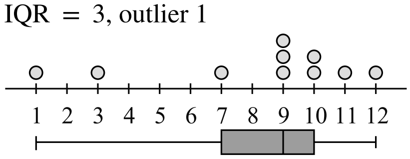
ii IQR $= 3$, outlier 18



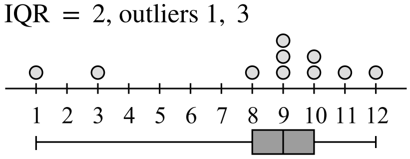
iii IQR $= 5.5$, no outliers



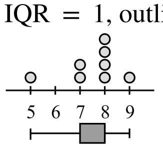
iv IQR $= 3$, outlier 1



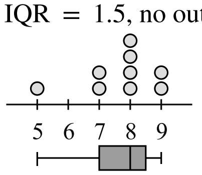
v IQR $= 2$, outliers 1, 3



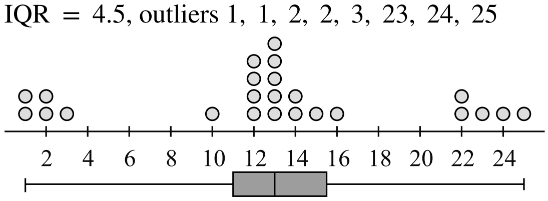
vi IQR $= 1$, outlier 5



vii IQR $= 1.5$, no outliers



viii IQR $= 4.5$, outliers 1, 1, 2, 2, 3, 23, 24, 25



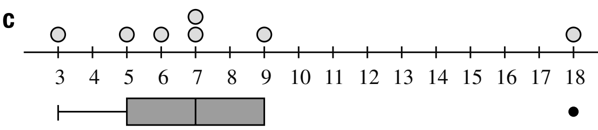
b It must be noted that some of the pathologies in these examples come about because of the small datasets. Statistics is always more accurate and reliable with a large dataset.

Generally the definition picks up the values that appear extreme on the dot plots. Notably (in these small datasets), it picks up single extreme values — if more values are a long way from the mean, they may not be marked as outliers.

Datasets with a small IQR may need a closer inspection — in part (vi) and (vii), the value at 5 is not so extreme and the datasets are not so different, yet in one case it is marked as an outlier, but in the other it is not. The final dataset has a very tight subset of data between the Q_1 and Q_3, giving a small interquartile range. This definition of outliers gives 8 values in 24 (one third of the data) as outliers. Furthermore, 23–25 are outliers, but 22 is not. The issue here is the unusual shape of the distribution.

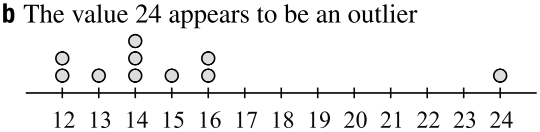
Rules such as this IQR criterion for outliers should be an invitation to inspect the values that have been flagged more closely, rather than following a rule blindly.

c



6 a $\bar{x} = 15$, $s = 3.29$

b The value 24 appears to be an outlier



c IQR $= 3$ and $Q_3 = 16$.
Because $24 > 16 + 1.5 \times 3$, this definition also labels 24 an outlier.

d $\bar{x} = 14$, $s = 1.41$

e This does not have much effect on the mean, but it has a big percentage effect on the standard deviation — removing the outlier more than halves the standard deviation. The operation of squaring $(x - \bar{x})$ means that values well separated from the mean have an exaggerated effect on the size of the variance.

f No effect at all!

g If there are significant outliers, or at least values spread far from the mean, this can have a big influence on the IQR. The IQR is a good measure if you are more interested in the spread of the central 50% of the data.

7 a Emily got less than 62

b Around 50% (and no more than 50%)

c The mathematics results were more spread out, and the centre of the data (by median) was 5 marks higher. The interquartile range of both distributions, however, was the same. Clearly the mathematics cohort has some students who perform much more

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strongly, and others who perform much weaker, than the majority of their peers.

d Xavier was placed in the upper half of the English cohort, but in the lower half of the mathematics cohort. The English result was thus more impressive.

e i 45

ii The bottom 25% of English scores show a spread of 6 marks (51–57). The bottom 25% of mathematics scores show a spread of 8 marks (53–61). The spread of the lower half is now much more comparable.

8 a The results are not paired. Just becausey Genjo received the lowest score in the writing task does not mean that he received the lowest score in the speaking task. Thus we cannot answer the question, although we might make conjectures, given that Genjo is obviously struggling significantly with English.

b i mean 66.1, median 68, range 56

ii IQR $= 73 - 60 = 13$, 91 and 35 are outliers.

c i mean 64.4, median 65.5 , range 56

ii $IQR = 71 - 57.5 = 13.5$, 37 and 93 are outliers.

d It is difficult to say. Students have found the second task more challenging, evidenced by the lower mean and median. This could be due to the construction of the task, or simply because it is a type of task that some students find more difficult.

Exercise 15D

1 a i height **ii** weight

b i radius **ii** area. It is natural to think that the area of the circle is determined by the radius chosen when it is drawn, but mathematically we could write $r = \sqrt{\dfrac{A}{\pi}}$, reversing the natural relationship.

c i weight **ii** price. Note that the price may change when meat is bought in bulk, so there is a deeper relationship between these two quantities than simply price $=$ weight $\times$ cost per kg.

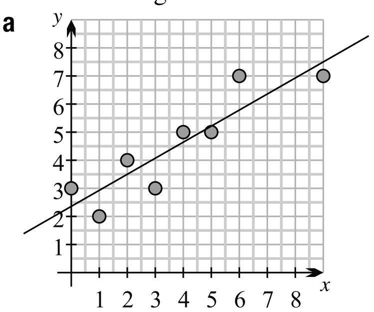
d i world rank **ii** placing

e i temperature **ii** power consumption. Power consumption increases with the use of air conditioners (higher temperatures) or heaters (colder weather).

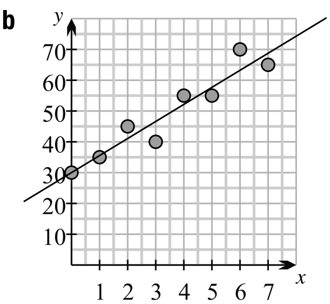
f It is natural to take x as the independent variable and y as the dependent variable. Note in this case the relationship cannot naturally be reversed, because there are multiple x-values resulting from the same y-value.

2 a strong positive **b** virtually none.
c strong negative **d** strong negative
e moderate positive **f** weak positive
g strong negative **h** strong positive
i moderate negative

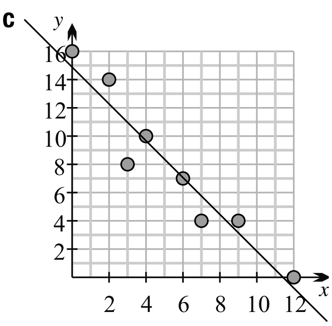
3 a



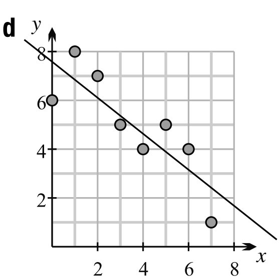
b



c



d



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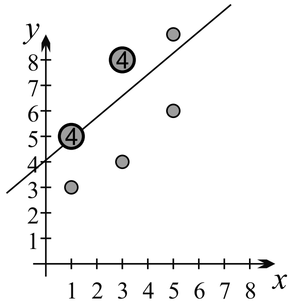
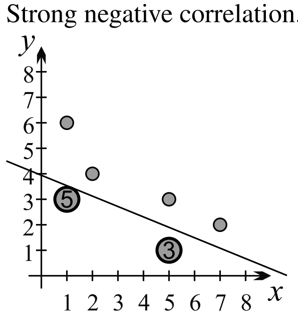
4 a Strong positive correlation.



b Strong negative correlation.



5 a A quadratic relationship (a parabola).

b A square root. **c** A hyperbola.

d A circle. **e** An exponential.

f No obvious relationship.

6 a ii 6 L and 10 L

b $V = 2t$

c The V-intercept is zero. In no minutes, zero water will flow through the pipe.

d This is the flow rate of the water, 2 L/s.

e Negative time makes little sense here, because he cannot measure the volume of water that flowed for say -3 minutes.

f Experimental error could certainly be a factor, but it may simply be that the flow rate of water is not constant. It may vary due to factors in, for example, the pumping system.

g 60 L. The extrapolation seems reasonable provided that the half-hour chosen is at about the same time of day that he performed his experiment.

h 22.5 minutes

i Yasuf's experiments were all carried out in a period of several hours during the day. It may be that the flow rate changes at certain times of the day, for example, at peak demands water pressure may be lower and the flow rate may decrease. The flow rate may also be different at night — for example, the water pump may only operate during the day. More information and experimentation is required.

7 a 1000

b i $P = 0.9t + 5$

ii It looks fairly good.

iii Predicted $P = 13.1$, actual $P = 15.4$, so the error was 230 people.

c i The new model predicts $P = 16.4$, so it is certainly much better.

ii Population is growing very strongly in Hammonsville. Investigators should be looking into the cause of the growth, which may change over the next few years. For example, it may be due to a short-term mining boom. Eventually there may be other constraining factors, such as available land for housing.

d Extrapolation can be dangerous. Provided, however, that the independent variable is constrained to a small enough interval, linear predictions may well have validity. This is the idea behind calculus, where curves are approximated locally by a tangent.

8 a 99 in assessment 1, 98 in assessment 2. They were obtained by the same student, but another student also got 99 in assessment 1.

b 27 in assessment 1, 33 in assessment 2. They were the same student.

c Students getting below about 77 marks in assessment 1 do better in assessment 2, students above 77 marks in assessment 1 get a lower mark in assessment 2, according to the line of best fit. Perhaps the second assessment started easier, but was harder at the end.

d i 50 **ii** 65 **iv** 26

v A negative score! Clearly the model breaks down for small scores.

e $y = 0.74x + 20$

f A more accurate method would incorporate data from more than one assessment task in estimating their missing score. This is a question better tackled using standard deviation and the techniques of the next chapter.

9 a The maximum vertical difference between a plotted point and the line of best fit is about 0.8 s^2.

b It could be experimental error. For example, the string could have been twisted or released poorly, the experiment could have been incorrectly timed, or there could have been a recording error.

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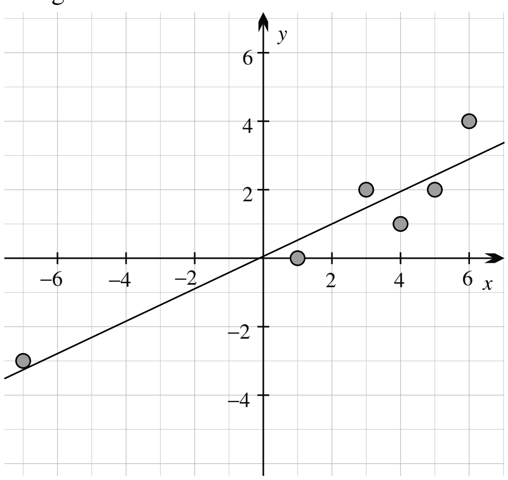
c They may have measured 10 periods and then divided by 10 before recording the length of one period. Errors could then arise if the motion was *damped*, that is, if the pendulum slowed down significantly over a short time period.

d By this model, $T^2 = \dfrac{2\pi^2}{g}L \doteqdot 4.03L$. These results are in pretty good agreement with the theory.

Exercise 15E

1 a There appears to be a fairly strong correlation, though note the small dataset.

b



c

							Sum
x	-7	1	3	4	5	6	12
y	-3	0	2	1	2	4	6
$x - \bar{x}$	-9	-1	1	2	3	4	0
$y - \bar{y}$	-4	-1	1	0	1	3	0
$(x - \bar{x})^2$	81	1	1	4	9	16	112
$(y - \bar{y})^2$	16	1	1	0	1	9	28
$(x - \bar{x})(y - \bar{y})$	36	1	1	0	3	12	53

d $(\bar{x}, \bar{y}) = (2, 1)$

e See above

f $53 \div \sqrt{112 \times 28} = 0.95$

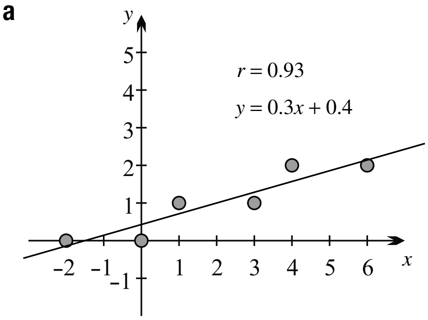
g It is a good fit.

h $53 \div 112 = 0.47$

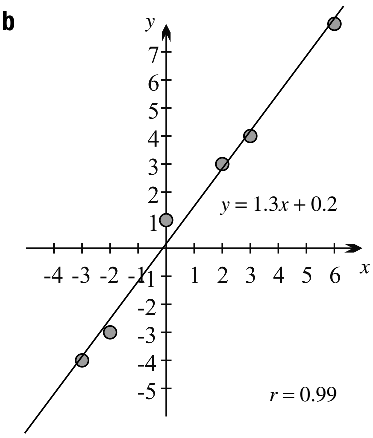
i $b = 1 - 0.47 \times 2 = 0.06$

j $y = \frac{1}{2}x + 0.$

2 a



b



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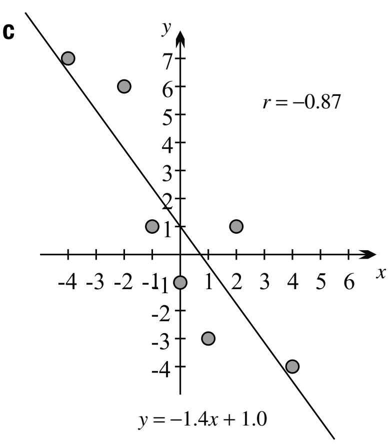
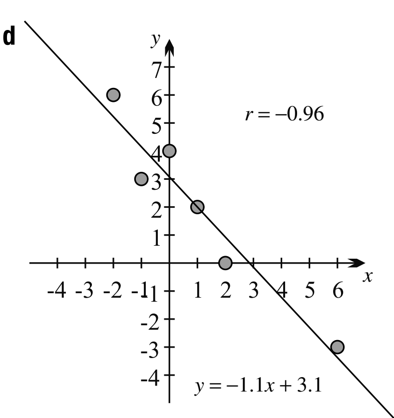


d



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Exercise 15F

1 a $r = 0.96$, $y = 0.96x + 0.47$

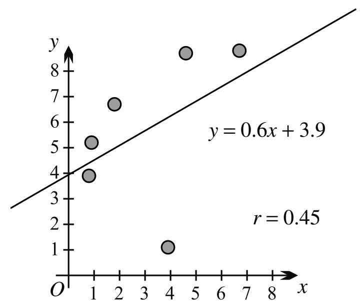
b $r = 0.79$, $y = 0.45x + 2.6$

c $r = -0.86$, $y = -1.05x + 8.75$

d $r = -0.53$, $y = -0.41x + 4.70$

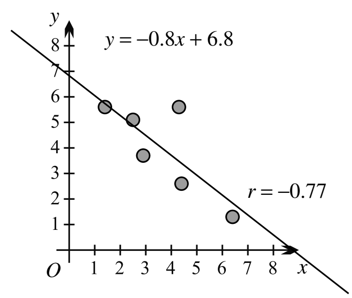
e $r = 0.96$, $y = 1.38x + 0.75$

2 a $r = 0.45$, $y = 0.58x + 3.94$



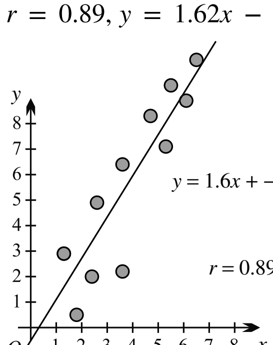
If the outlier at $(3.9,\ 1.1)$ is removed, then $r = 0.91$, $y = 0.75x + 4.43$.

b $r = -0.77$, $y = -0.78x + 6.83$



If the outlier at $(4.3, 5.6)$ is removed, then $r = -0.97$, $y = -0.89x + 6.79$.

c $r = 0.89$, $y = 1.62x - 0.51$



If the outlier at $(3.6,\ 2.2)$ is removed, then $r = 0.93$, $y = 1.61x - 0.19$.

3 Because the dataset was larger, the effect of the single outlier was mitigated by the other data points.

4 a Dataset 1:

i $y = 1x + 1.4$, $r = 0.86$

ii $y = 0.8x + 1.9$, $r = 0.79$

Dataset 2:

i $y = 0.7x + 3.0$, $r = 0.76$

ii $y = 0.7x + 2.5$, $r = 0.82$

b In all cases the correlation is strong. In part **a**, the repeated point has strengthened the correlation, but in the second example it has weakened it. Note that a strong correlation doesn't indicate that the data are correct. In part **a**, for example, leaving out 4 of the 9 points still gave a strong correlation, but a very different equation of line of best fit.

c The effect is less in the larger dataset, as expected. The gradient is unchanged (correct to one decimal place) and the y-intercept only differs by 20%, rather than by 26%. In a larger (more realistically sized) dataset, the effect would likely be less again. The effect of the repeated point will also depend on its place on the graph (central versus on the extremes of the data) and how close it is to the line of best fit.

Chapter 15 review exercise

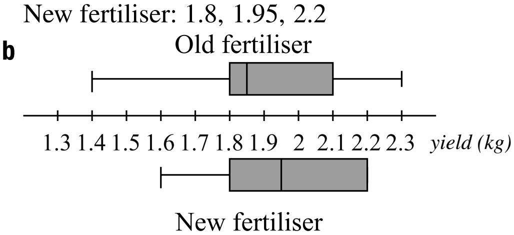
1 a mean 5, median 4.5, mode 4, range 8

b mean 15, median 15, mode 15 and 16 (it is bimodal), range 7

2 a Old fertiliser: 1.8, 1.85, 2.1,

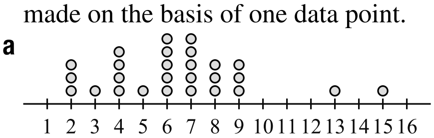
New fertiliser: 1.8, 1.95, 2.2

b



c The fertiliser does appear to increase his yield — the median yield has increased by 100 g. Probably more more data are required because the lower quarter $(0–25\%)$ shows an increase, but the maximum has reduced. These claims, however, are each being made on the basis of one data point.

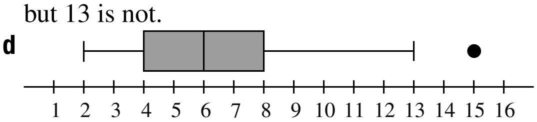
3 a



b By eye, 13 and 15 look like outliers.

c $\text{IQR} = 4$. By the IQR criterion, 15 is an outlier, but 13 is not.

d



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4 a mean $\doteqdot$ 11.82 *s*, standard deviation $\doteqdot$ 0.537 s

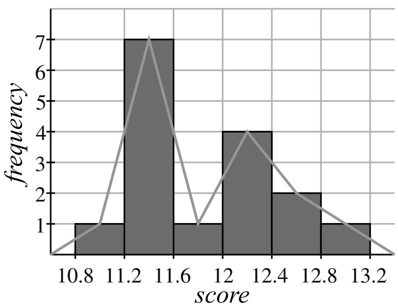
b

group	10.8–11.2	11.2–11.6	11.6–12.0
centre	11.0	11.4	11.8
freq	1	7	1

group	12.0–12.4	12.4–12.8	12.8–13.2
centre	12.2	12.6	13.0
freq	4	2	1

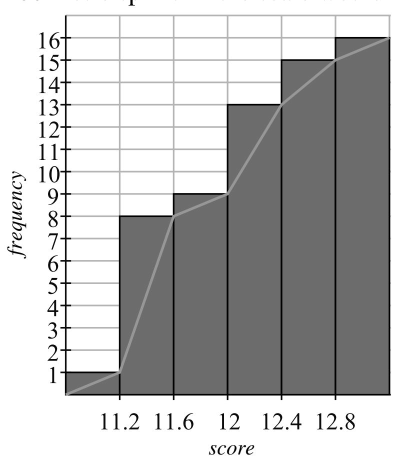
c mean $\doteqdot$ 11.85 s, standard deviation $\doteqdot$ 0.563 s. Agreement is reasonable, but as expected, the answers are not exactly the same.

d



e 0.5 seconds is a big difference in the time of a 100 metre sprint — the scale would be too coarse.

f



g The line at 50% of the data (frequency 8) meets the polygon where the sprint time is 11.6 seconds. You can confirm that this agrees with the result for splitting the grouped ordered data into two equal sets.

5 a

	first	second	Total
order entrée	45	42	87
no entreé	38	28	66
Total	83	70	153

b 153

c $87 \div 153 \doteqdot 57\%$

d 54%

e P(order entrée | attend first) $= 45 \div 83 \doteqdot 54\%$. P(order entrée | attend second) $= 42 \div 70 = 60\%$. No, it is not correct.

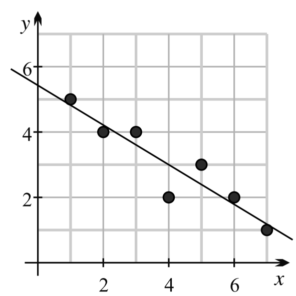
f P(attended first | ordered an entrée) $= 45 \div 87 \doteqdot 52\%$.

g $90 \times 60\% = 54$

h Those attending the first session may prefer a quick meal before heading out to the theatre or some other event. There may also be more family groups operating on a tighter budget.

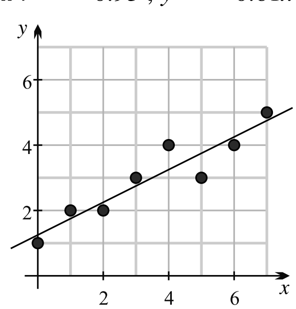
i If they can estimate the demand on certain dishes, then they may be able to prepare parts of the dish in advance, for example, preparing the garnishes or chopping the ingredients.

6 a i



ii $r = -0.93$, $y = -0.61x + 5.43$

b i



ii $r = 0.94$, $y = 0.5x + 1.25$

7 a 120 000

b 94 000, 62 000, 80 000, 80 000

c 316 000 and 79 000

d The arrivals may vary over the year because of seasonal or other effects. Government policy may consider an annual immigration quota, allowing a higher rate in one quarter to be balanced by a low rate in a subsequent quarter. As in 2000, examining the average for each quarter balances out such effects.

e 84 000

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f It would be important to know the emigration rate of those leaving the country. The Net Overseas Migration (NOM) may be the better measure for many purposes. Other information of interest might include country of origin, destination within Australia, and whether they're intending to stay permanently or for a limited period.

g i 71 600

ii Rounding error has affected these calculations — a discrepancy in the second decimal place of the gradient is multiplied by 2000, resulting in an answer that is out by as much as $0.05 \times 2000 = 100$ thousand.

iii 84.300, which is in agreement with part **d**.

iv 660 000

v $660 \div 316 \times 100\% \doteqdot 209\%$, which is a 108% increase..

Chapter 16

Exercise 16A

1 **a** and **c**

2 a

x	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

b i $\frac{3}{8}$ **ii** $\frac{1}{16}$ **iii** 0 **iv** 1

c i $\frac{3}{16}$ **ii** $\frac{1}{2}$ **iii** $\frac{15}{16}$ **iv** $\frac{3}{8}$

3 a

score x	1	2	3	4	5	Total
f_r	0.1	0.2	0.45	0.15	0.1	1
xf_r	0.1	0.4	1.35	0.6	0.5	2.95
x^2f_r	0.1	0.8	4.05	2.4	2.5	9.85

b The sum of probabilities is 1.

c $\bar{x} = 2.95$

d The sample mean $\bar{x}$ is a measure of the centre of the dataset.

e $s^2 \doteqdot 1.15$ **f** $\sigma \doteqdot 1.07$

g The sample standard deviation s is a measure of the spread of the dataset.

h They are estimates of the mean $\mu = \mathrm{E}(X)$ and the standard deviation σ of the probability distribution.

i The sample mean $\bar{x}$ is 2.95, so after 100 throws, 295 is a reasonable estimate of the sum.

4 a $\bar{x} = 5.26$, $x \doteqdot 1.07$

b The centre of the data is about 2.3 units greater, but the spread is about the same, according to the standard deviation.

5 a 3.5 and 4

b

score x	1	2	3	4	5	6	Total
frequency f	2	4	4	8	2	0	10
$P(X = x)$	0.1	0.2	0.2	0.4	0.1	0	1
$x \times P(x)$	0.1	0.4	0.6	1.6	0.5	0	3.2
$x^2 \times P(x)$	0.1	0.8	1.8	6.4	2.5	0	11.6

c $E(X) = 3.2$

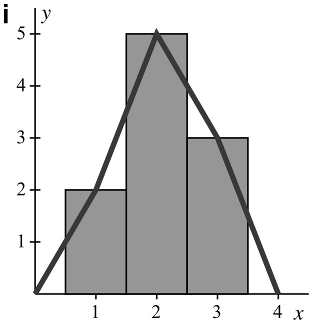
d $\mathrm{Var}(X) = 11.6 - (3.2)^2 = 1.36$

e 1.17

f It is usual to expect that for a quiz (covering recent work and including short easy questions) the marks will be high. These marks don't look impressive.

g $\mathrm{E}(X) = 16$, $\mathrm{Var}(X) = 34$, standard deviation 5.83

6 a i



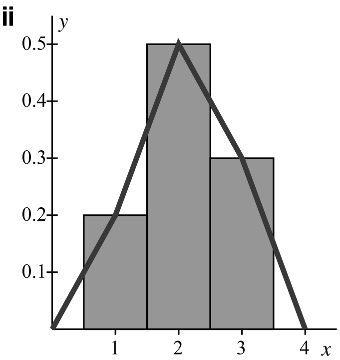
ii 10 **iii** 10

iv Both areas are the same and equal to the total frequency, that is the number of scores.

b i

score x	1	2	3
frequency f	2	5	3
relative frequency f_r	0.2	0.5	0.3

ii



iii 1 **iv** 1

v Both areas are the same and equal to the total 1, that is the sum of the relative frequencies. (This will only happen when the rectangles have width 1.)

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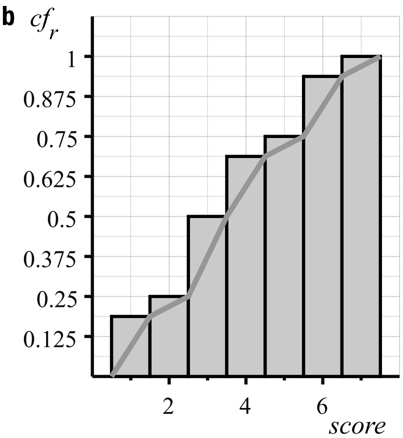
vi The relative frequencies are estimates of the probabilities. Note that both add to 1, both are non-negative, and both measure the chance that a random value will lie within the given rectangle of the histogram. A relative frequency is the *experimental* probability of an outcome, and is an *estimate* of the theoretical probability.

7 a

x	1	2	3	4	5	6	7	Total
f	3	1	4	3	1	3	1	16
f_r	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	1
cf	3	4	8	11	12	15	16	—
cf_r	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{8}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{15}{16}$	1	—

b

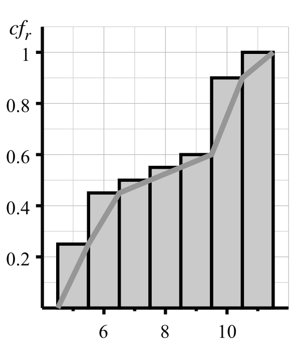


c $Q_1 = 2.5, Q_2 = 3.5, Q_3 = 5.5$

8 a

x	5	6	7	8	9	10	11	Total
f	5	4	1	1	1	6	2	16
f_r	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{6}{20}$	$\frac{2}{20}$	1
cf	5	9	10	11	12	15	20	—
cf_r	$\frac{5}{20}$	$\frac{9}{20}$	$\frac{10}{20}$	$\frac{11}{20}$	$\frac{12}{20}$	$\frac{18}{20}$	1	—

b



c $Q_1 = 5.5, Q_2 = 7.5, Q_3 = 10$

9 a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** 0.1

d 12.5% of the households have 3 or more cars, so the town planners won't recommend additional on-street parking.

e

x	0	1	2	3	4
$P(X = x)$	0.25	0.5	0.125	0.10	0.025

f The area of the histogram is exactly the sum of the probabilities, because the width of each bar is 1 in this graph.

g The triangles cut off above the polygon fit into the spaces below the polygon.

h This is an average, and is best understood by saying that for a large sample of n houses, we would expect them to have about $1.15n$ cars between them — see the next part.

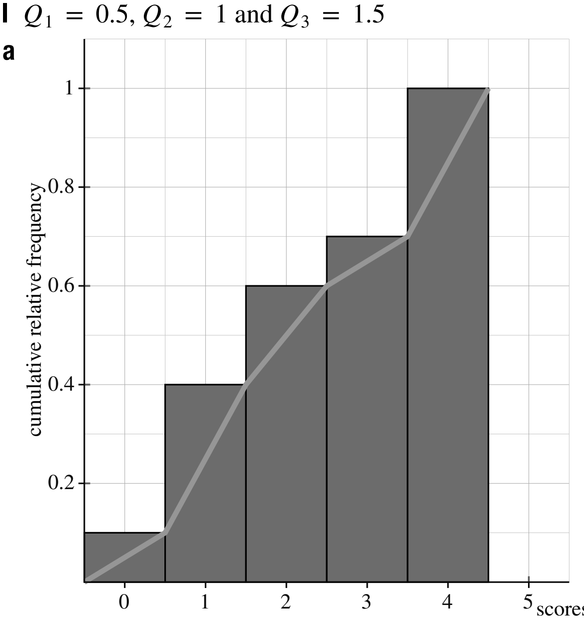
i 115 cars. We are assuming that streets in the suburb are uniform with respect to car ownership. Actually, streets closer to train stations may manage with fewer cars because people catch the train to work, more affluent streets may own more cars, people may adjust car ownership to allow for availability of off-street or on-street parking.

j

x	0	1	2	3	4
$P(X \le x)$	0.25	0.75	0.875	0.975	1

l $Q_1 = 0.5, Q_2 = 1$ and $Q_3 = 1.5$

10 a



b 3.5 **c** 1, 2

d $Q_3 \doteqdot 3.7$ **e** 3.67. They agree.

11 a median 3.5, mode 3.5

b

spent	0–1	1–2	2–3	3–4	4–5	Total
cc x	0.50	1.50	2.50	3.50	4.50	—
f	20	5	15	40	20	100
f_r	0.20	0.05	0.15	0.40	0.20	1

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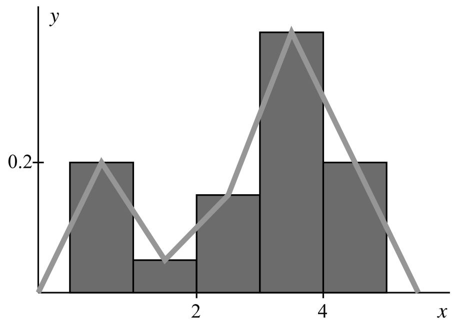
c mean 2.85, variance 1.9275, standard deviation 1.39

d



e **i** 0.2 **ii** 0.05 **iii** 0.15
iv 0.4 **v** 0.2

f The area of the relative frequency polygon, or the area under the frequency polygon bounded by the x-axis (they are the same). This only happens because the rectangles have width 1.

g **i** Equally likely
ii They are twice as likely to have spent between \$3–\$4.

h $E(Y) = 4.85$, same variance

12 a The histogram covers 40 grid rectangles.

b **i** $0.3 \times 0.5 = 0.15$
ii 0.1 **iii** 0.3

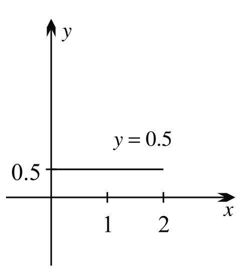
c **i** It is twice as likely to be 20°C.
ii In the class 19.25–19.75

d **i** 0.1 **ii** 0.3

e First, the histogram only records the maximum daily temperature. Secondly, it recorded 20 consectuve days, but there will be natural variation over the year, and even within a season.

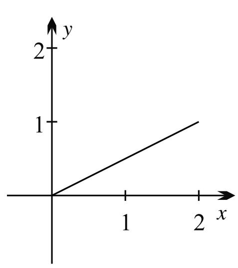
Exercise 16B

1 a



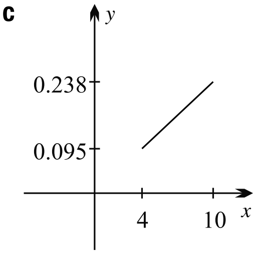
ii $\int_0^2 f(x)\,dx$
$=$ area rectangle $= 1$

b



ii $\int_0^2 f(x)\,dx$
$=$ area triangle $= 1$

c



ii $\int_4^{10} f(x)\,dx =$ area trapezium
$= \frac{1}{2} \times 6\left(\frac{4}{42} + \frac{10}{42}\right) = 1$

2 a Yes, mode is $x = 1$

b No, the integral is 3.

c No. The integral is 1, but $f(x) < 0$ if $x > 2$.

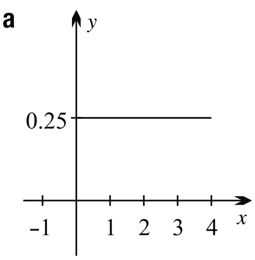
d Yes, provided that $n \geq 0$. Then mode is $x = 1$.

e Yes, mode is $x = \frac{\pi}{2}$

f Yes, mode is $x = 2$

3 b $f(x) = \frac{3}{4}(x - 3)(x - 1) < 0$, for $1 < x < 3$

4 a



b Clear from the graph

c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$
iv 0 **v** $\frac{3}{4}$ **vi** $\frac{3}{4}$

d LHS $= \frac{1}{4}$, RHS $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

5 a $F(x) = \frac{1}{64}x^2$

b $F(x) = \frac{1}{16}(x^3 + 8)$

c $F(x) = \frac{x}{2}(3 - x^2)$

d $F(x) = \frac{1}{e}(e^x + x - 1)$

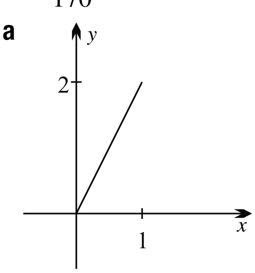
6 a $Q_2 = 4\sqrt{2}$, $Q_1 = 4$, $Q_3 = 4\sqrt{3}$

b $Q_2 = 0$, $Q_1 = -\sqrt[3]{4}$, $Q_3 = \sqrt[3]{4}$

7 a $\dfrac{20}{170} \doteqdot 12\%$ **b** $\dfrac{4\pi}{170} \doteqdot 7\%$

c $\dfrac{20 + 4\pi}{170} \doteqdot 19\%$ **d** $1 - \dfrac{20 + 4\pi}{170} \doteqdot 81\%$

8 a



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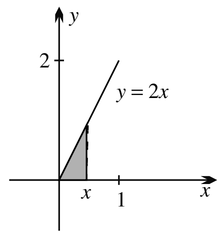
c **i** Area $= \frac{1}{2}x \times 2x$



d $Q_1 = \frac{1}{2}, Q_2 = \frac{1}{\sqrt{2}}, Q_3 = \frac{\sqrt{3}}{2}$

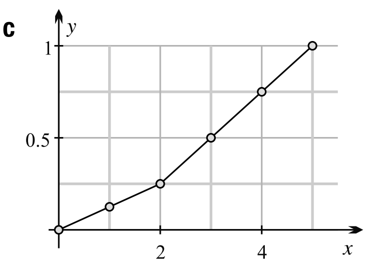
9 a $\frac{5}{243}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10}$ **d** $\frac{1}{2}$

10 a Clearly $f(x) \geq 0$ for all x, and the area under the graph is $2 \times 0.125 + 3 \times 0.25 = 1$.

b

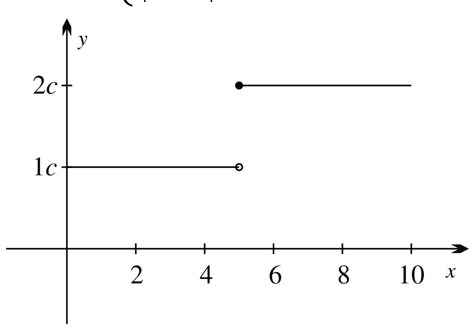
x	0	1	2	3	4	5
$P(X \leq x)$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

c



d $F(x) = \begin{cases} \frac{1}{8}x, & \text{for } 0 \leq x < 2, \\ \frac{1}{4}x - \frac{1}{4}, & \text{for } 2 \leq x \leq 5, \end{cases}$

11 a



b Area $= 15c$, so $c = \frac{1}{15}$

c $F(x) = \begin{cases} cx, & \text{for } 0 \leq x < 5, \\ 2cx - 5c, & \text{for } 2 \leq x \leq 5, \end{cases}$

d $P(1 < X < 7) = F(7) - F(1) = 8c = \frac{8}{15}$

12 a The mode is $x = 2$ (where the vertex is).

c Symmetric about $x = 2$, $P(X = 2) = 0$, and total area is 1.

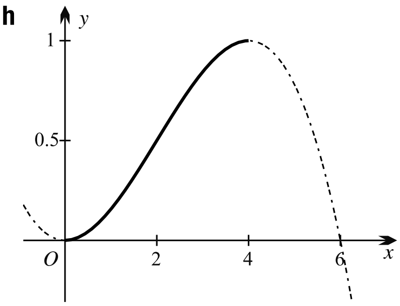
d $\frac{5}{32}$ and $\frac{27}{32}$ are complementary probabilities.

e $\frac{11}{256}$. The symmetry of the graph means that the areas are the same.

f $\frac{1}{32}x^2(6 - x)$

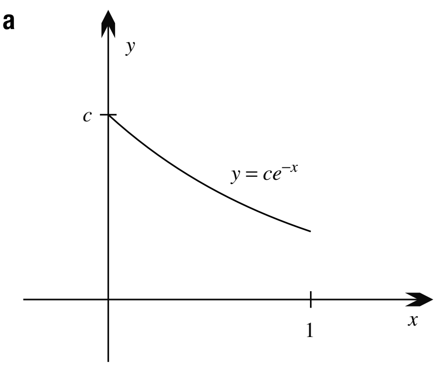
g **i** $\frac{81}{256}$ **ii** $\frac{41}{256}$ **iii** $\frac{29}{256}$ **iv** $\frac{47}{256}$

h



j $Q_1 = 1.3, Q_3 = 2.7$.

13 a



b $c = \frac{e}{e - 1}$

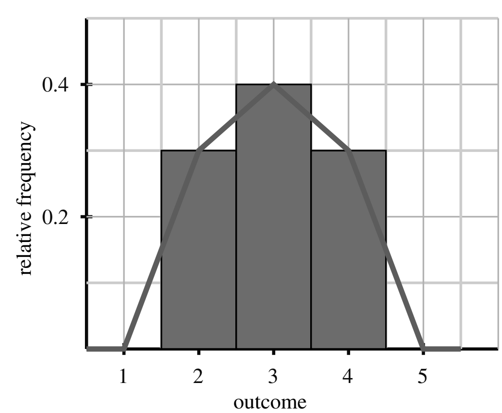
c $F(x) = \dfrac{e}{e - 1}(1 - e^{-x})$

d $Q_1 = \ln \dfrac{4e}{3e + 1}, Q_2 = \ln \dfrac{2e}{e + 1},$

$Q_3 = \ln \dfrac{4e}{e + 3}$

14 a See part **c**.

b Both areas are 1.



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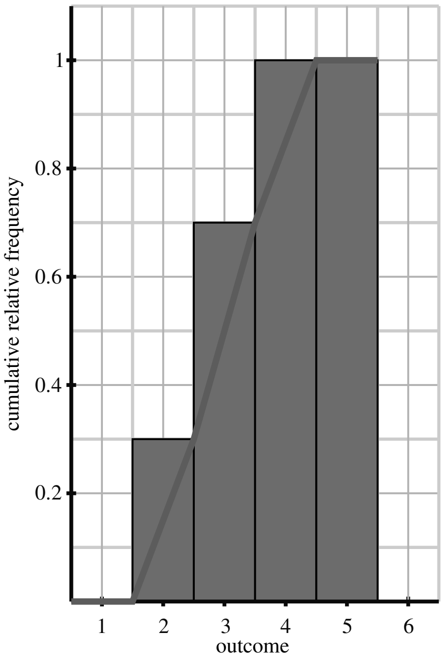
c

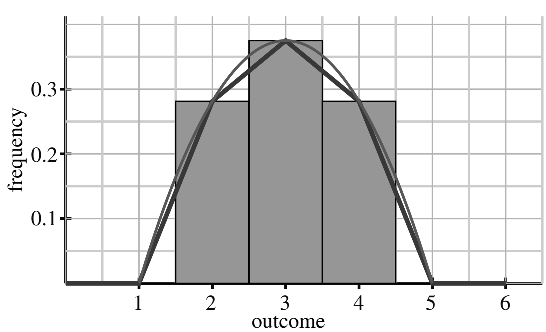


d 2.3, 3, 3.7

e i $\int_1^5 f(x)\,dx = 1$, and $f(x) \geq 0$ for $1 \leq x \leq 5$

ii

x	1	2	3	4	5
$f(x)$	0	0.28	0.375	0.28	0



iii $\frac{1}{32}(-x^3 + 9x^2 - 15x + 7)$

iv $P(X \leq 2.3) \doteqdot 0.25$, $P(X \leq 3) = 0.5$, $P(X \leq 3.7) \doteqdot 0.75$

v 2.3, 3 and 3.7 still seem good approximations.

15 b $F(x) = 1 - \dfrac{1}{x}$

c Total probability is 1.

d $\frac{4}{3}$, 2, 4

16 b $F(x) = 1 - e^{-x}$

c $Q_1 = \ln\frac{4}{3}$, $Q_2 = \ln 2$, $Q_3 = \ln 4$

17 b i This returns the square of the distance from a random point in the square to the centre of the square and circle.

ii 1 **iii** 0

c The code measures the relative frequency of points lying in the circle, that is, the probability that the point will lie in the circle. The value in cell C1 should approach π.

Exercise 16C

1 a $f(x) \geq 0$ and by area formula or integration, $\int_0^{10} f(x)\,dx$.

b $E(X) = 5$

c Yes — in the centre of this distribution interval [0, 10]

d $\text{Var} = \frac{25}{3}$, $\sigma = \frac{5}{3}\sqrt{3}$

e $E(X^2) = \frac{100}{3}$ and $\text{Var} = \frac{25}{3}$

3 a The function is never negative, and the integral over $[-1, 1]$ is 1.

b $E(X) = 0$ **c** $\text{Var}(X) = \frac{3}{5}$, $\sigma = \frac{\sqrt{15}}{5}$

d $\dfrac{3\sqrt{15}}{25} \doteqdot 0.46$

4 a $E(X) = \frac{2}{3}$, $\text{Var}(X) = \frac{1}{18}$, $\sigma = \frac{\sqrt{2}}{6}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) = \frac{4\sqrt{2}}{9}$

b $E(X) = 0$, $\text{Var}(X) = \frac{1}{2}$, $\sigma = \frac{1}{\sqrt{2}}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) = \frac{1}{2}$

c $E(X) = 3$, $\text{Var}(X) = \frac{3}{5}$, $\sigma = \frac{\sqrt{15}}{5}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) \doteqdot 0.668$

5 a Yes.

b $E(X) = \frac{c}{2}$, as expected for a measure of the centre of this uniform distribution.

c $\frac{c^2}{12}$

d The answer agrees for this special case with $c = 10$.

e $E(X) = \frac{c}{2} + h = \frac{c + 2h}{2}$, variance unchanged.

f Put $h + c = k$ in the previous result: $E(X) = \frac{k + h}{2}$, $\text{Var}(X) = \frac{(k - h)^2}{12}$

6 b $E(X) = \frac{23}{8}$, $E(X^2) = \frac{121}{12}$, $\text{Var}(X) = \frac{349}{192} \doteqdot 1.82$

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7 LHS $= \int_a^b x^2 f(x)\,dx - \int_a^b 2\mu x f(x)\,dx + \int_a^b \mu^2 f(x)\,dx.$

By the definition of a PDF,

Term 3 $= \mu^2 \int_a^b f(x)\,dx = \mu^2.$

By the formula for the mean,

Term 2 $= -2\mu \int_a^b x f(x)\,dx = -2\mu^2.$

8 b $\mathrm{E}(X) = 2.1$ **c** Agrees.

d Not only do both satisfy the condition that the area under the curve is 1, but they give the same result for the expected value.

9 b $\mathrm{E}(X) = \frac{3}{2}$, $\mathrm{E}(X^2) = 3$, $\mathrm{Var}(X) = \frac{3}{4}$

c i $1 - \frac{1}{4^3}$ **ii** $\frac{1}{8}$ **iii** $\frac{117}{1000}$

d $F(x) = 1 - \frac{1}{x^3}$

10 a $\frac{d}{dx} xe^{-x} = e^{-x} - xe^{-x}$,

so $\int xe^{-x}\,dx = -e^{-x} - xe^{-x}$.

b $\mathrm{E}(X) = 1$

c The derivative is

$(2xe^{-x} - x^2e^{-x}) + (2e^{-x} - 2xe^{-x}) - 2e^{-x}$
$= -x^2e^{-x}$,

so $\int x^2 e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

d $\mathrm{E}(X^2) = 2$ and $\mathrm{Var}(X) = 1$.

12 b Clearly $f(x) \ge 0$, and part (a) proves that the integral is 1.

c $\mathrm{E}(X) = 0$

d $\mathrm{E}(X^2)$ is unbounded (infinite).

Exercise 16D

1 a 0.5 **b** 0.8413 **c** 0.9972 **d** 0.9332
e 0.6554 **f** 0.9893 **g** 0.8849 **h** 1.0000

2 The total area under the curve is 1, so the areas of regions to the right and left of $z = a$ add to 1. This identity is true for any probability distribution.

a 0.5 **b** 0.1587 **c** 0.0228 **d** 0.0082
e 0.0968 **f** 0.2420 **g** 0.0548 **h** 0.0000

3 a From the even symmetry of the graph, $P(Z < -a) = P(Z > a) = 1 - P(Z \le a)$. (The result also holds for $a \le 0$, but this is not useful to us.) This result is certainly not true for all probability distributions.

b i 0.1151 **ii** 0.0107 **iii** 0.4207
iv 0.0007 **v** 0.0000 **vi** 0.2420
vii 0.0548 **viii** 0.0808 **ix** 0.5000

4 b i 0.4032 **ii** 0.4918 **iii** 0.2580
iv 0.4918 **v** 0.3643 **vi** 0.2580
vii 0.4452 **viii** 0.4032 **ix** 0.5000

c i 0.8064 **ii** 0.9836 **iii** 0.5762
iv 0.9962 **v** 0.3108 **vi** 0.8664

5 **a** & **e**, **b** & **g**, **c** & **h**, **d** & **f**

6 **a** & **c**, **b** & **g**, **d** & **f**, **e** & **h**

7 a This is evident from a graph by subtraction of areas.

b i 0.0483 **ii** 0.4100 **iii** 0.2297
iv 0.0923 **v** 0.4207 **vi** 0.1552

c i 0.9193 **ii** 0.7008 **iii** 0.9013

8 a 0.5 **b** 0 **c** 0.0359
d 0.8849 **e** 0.1151 **f** 0.3849
g 0.0359 **h** 0.8849 **i** 0.0792
j 0.8490

9 a 0.9032 **b** 0 **c** 0.3446
d 0.9554 **e** 0.9032 **f** 0.4332
g 0.2119 **h** 0.4207 **i** 0.0689
j 0.8893

10 a 0.9208 **b** 0.0792 **c** 0.6341 **d** 0.0364

12 a 50% **b** 84%

c 97.5% (Note the inaccuracy here. From the tables it should be 97.72.)

d 16% **e** 49.85% **f** 34%
g 47.5% **h** 2.35% **i** 68%
j 83.85% **k** 81.5% **l** 97.5%

13 a $b = 1$ **b** $b = 2$ **c** $b = -1$
d $b = 1$ **e** $b = 1$ **f** $b = 4$

14 a 0.6 **b** 2.3 **c** 1.2
d -0.8 **e** 1.1 **f** 2.6

15 a i $P(-1 < Z < 1) \doteqdot 68\%$

ii $P(Z < 2) \doteqdot 97.5\%$

iii $P(Z < -3 \text{ or } Z > 3) = 0.3\%$

b Around 0.7 centimetres.

16 Mathematically, $P(Z = a) = \int_a^a f(x)\,dx$, which is an area of zero width. Practically, this represents the probability of getting a value exactly $Z = a$ for a continuous distribution, for example a height of exactly 1.7142435345345 … metres. In a continuous distribution, all such probabilities are zero.

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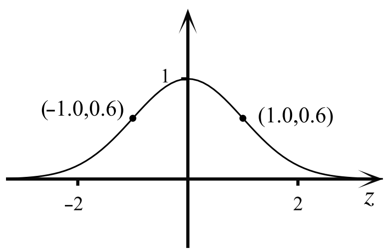
17 a **i** all real values **ii** Even
iii $x = 0$ **iv** 1
v $z = -1$ and $z = 1$ **vi** $\left(0, \dfrac{1}{\sqrt{2\pi}}\right)$
vii There are no z-intercepts.
b **i** 0 **ii** 0 **iii** 0 **iv** 1
c $\phi(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

18 c Stationary point $(0, 1)$. It is a maximum.
d Inflections at $(1, e^{-0.5})$ and $(-1, e^{-0.5})$
e $f(x) \to 0$ as $x \to \infty$ and as $x \to -\infty$
f



g See the graph at the top of this exercise.

19 a **i** $P(0 \le Z \le 1) = 0.3401$
ii $P(-1 \le Z \le Z) = 0.6802$
iii The graph is concave up on $[0, 1]$ — the concavity changes at the point of inflection at $z = 1$. Thus the polygonal path of the trapezoidal rule will lie below the exact curve.
iv This is good agreement with the empirical rule (68%) and the table (0.6826).
b **i** $P(-2 < Z < 2) = 2 \times 0.4750 = 0.95$
ii $P(-3 < Z < 3) = 2 \times 0.4981 = 0.9962$

20 a $E(Z) = \int_{-\infty}^{\infty} z \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz$, which is the integral of an odd function on a symmetric domain, so $E(Z) = 0$.

b $\dfrac{d}{dz}\left(ze^{-\frac{1}{2}z^2}\right) = 1 \times e^{-\frac{1}{2}z^2} + z \times -ze^{-\frac{1}{2}z^2}$

$$ze^{-\frac{1}{2}z^2} = \int e^{-\frac{1}{2}z^2}\,dz - \int z \times ze^{-\frac{1}{2}z^2}\,dz$$

$$\left[ze^{-\frac{1}{2}z^2}\right]_{-\infty}^{\infty} = \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\,dz - \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\,dz$$

The LHS is 0, so

$$\int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\,dz = \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\,dz$$

and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\,dz = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\,dz$
$= 1$

Thus we have shown that $E(Z^2) = 1$.

c Using the previous part,

$$\begin{aligned}\mathrm{Var}(Z) &= E(Z^2) - E(Z)^2 \\ &= 1 - 0 \\ &= 1\end{aligned}$$

Exercise 16E

1 a $z = 1$, 1 SD above **b** $z = -2$, 2 SD below
c $z = 1$, 1 SD above **d** $z = -2$, 2 SD below
e $z = 5$, 5 SD above **f** $z = -3$, 3 SD below

2 a **i** $+2.5$ **ii** -3 **iii** $+5.5$ **iv** $+\frac{1}{4}$
b **i** iii **ii** i, iii, iv
iii ii **iv** iv
v i, ii, iii

3 a $P(Z \le 0.5)$ **b** $P(Z > 0.25)$
c $P(Z \le -1)$ **d** $P(Z \ge -1.5)$
e $P(-2 \le Z \le -0.5)$
f $P(-1.75 \le Z \le 0.25)$

4 a $P(Z \ge 0) = 0.5$
b $P(-1 \le Z \le 1) = 0.68$
c $P(Z \le 2) = 0.975$
d $P(Z \ge -2) = 0.975$
e $P(-3 \le Z \le 1) = 0.8385$
f $P(-2 \le Z \le -1) = 0.1475$

5 a $P(-1 \le Z \le 3) = 0.8385$
b $P(Z \ge 1) = 0.16$
c $P(Z \ge 2) = 0.025$

6 a $P(-2.5 \le Z \le 2.5) = 0.9876$
b $P(Z \ge 1.6) = 0.0548$
c $P(Z \le -0.8) = 0.2119$
d $P(Z \ge -1.3) = 0.9032$
e $P(Z < 1.6) = 0.9452$
f $P(-2.5 < Z \le -1.5) = 0.0606$

7 a The score is above the mean.
b The score is below the mean.
c The score is equal to the mean.

8 a 69, 80
b 69, 80, 95, 50, 90, 52, 45
c 43, 45, 50, 52
d 95, 98
e It doesn't look very normal ('bell shaped').
Here is the stem-and-leaf plot:

4	3 5
5	0 2
6	9
7	
8	0
9	0 5 8

9 a **i** z-score for English (2.5) and maths (2). English is more impressive.
ii z-score for English (-0.8) and maths (-0.6). Maths is more impressive.

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iii z-score for English (1.5) and maths (1). English is more impressive.

b 95% is 2.2 standard deviations above the mean.
$P(Z > 2.2) \doteqdot 1 - 0.9861 \doteqdot 1.4\%$

c The mathematics mean of 62% is 0.3 English standard deviations below the English mean 65%.
$P(Z > -0.3) = P(Z < 0.3) = 0.6179 \doteqdot 0.62$

10 a About 408 scores will lie within one SD from the mean, that is, in [40, 60]. About 570 scores will lie within two SDs from the mean, that is, in [30, 70]. About 598 scores will lie within three SDs from the mean, that is, in [20, 80].

b **i** 415 **ii** 260 **iii** 462 **iv** 4

11 a **i** -1, -1.5, -2

ii 1.5 standard deviations below the mean

iii 45

iv Some assessments may be harder than others — simply averaging his other results takes no account of this.

v Jack may perform better in certain types of assessments, for example, in Biology lab experiments, or he may perform better at certain times of the year. For example, his results may improve towards the end of the year.

b z-scores 0.4, 0.625, 1, average 0.675.
Jill's estimate is 71.1

Exercise 16F

1 a 97.5% **b** 84%

2 a 3 **b** 50 **c** 1630

3 a 2.5%

b $2400 \times 0.15 \div 100 = 3.6$ screws (perhaps round to 4)

4 5%

5 a 0.26% **b** 65 000

6 about 31%

7 b 186 cm

c **i** Interpolate between 1.6 and 1.7

ii 197 cm

8 The boxes need to be marked with a mean weight of 496.7 grams — this would probably be rounded down to 496, which is actually two standard deviations below the mean, so 97.5% of boxes weigh above this value.

9 a 69%

b **i** 33% **ii** 33%

10 a 12% **b** about 0.07%

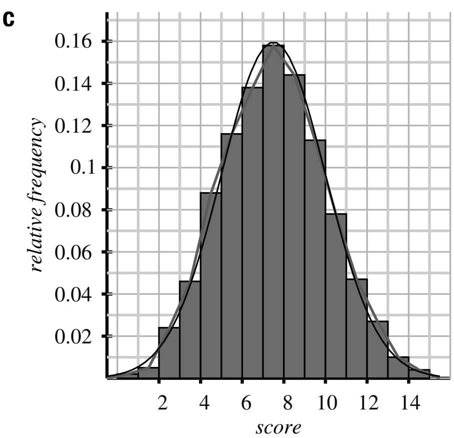
c **i** 4.5% **ii** 0.6%

11 2.5 standard deviations is 15 g, so 1 standard deviation is 6 g. The mean weight is 112 g.

Exercise 16G

1 a $\mu = 7.5, \sigma = 2.5$

c



d Either perform the experiment more than 1000 times, or average more than three random numbers at each stage.

4 c The mean should be about 5 and the standard deviation about 1.6.

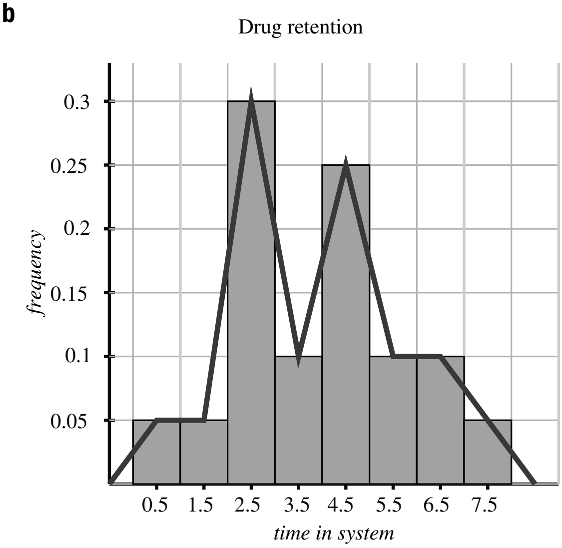
8 d $\left(0, \frac{1}{\sqrt{2\pi}}\right)$

Chapter 16 review exercise

1 a

x	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8
cc	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
f	1	1	6	2	5	2	2	1
cf	1	2	8	10	15	17	19	20
f_r	0.05	0.05	0.3	0.1	0.25	0.1	0.1	0.05
cf_r	0.05	0.1	0.4	0.5	0.75	0.85	0.95	1

b



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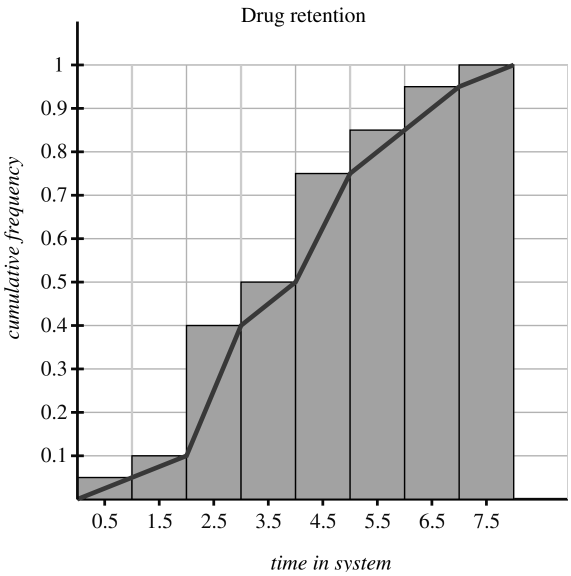
c



d **i** 4.0 **ii** $Q_1 = 2.5, Q_3 = 5.0$
iii 6.5 **iv** 6.0

e This was only a preliminary experiment, and a larger dataset may resolve the unusual outcomes. It may be worth investigating any common links between patients falling in the two intervals associated with the two modes — perhaps different sexes react differently to the drug, perhaps it was administered differently, or perhaps the two groups are behaving differently after medication, for example, changing their levels of follow-up exercise or their food intake.

2 a False, it joins to the right end.
b False. The area under the relative frequency polygon is 1 if the rectangles each have width 1.
c True. **d** True. **e** True
f The empirical rule says 99.7%, and only applies to a normal distribution, so false in general.

3 b A uniform probability distribution with a uniform probability density function.
c $E(X) = 0$
d $E(X^2) = \text{Var}(X) = \frac{100}{3}, \sigma = \frac{10\sqrt{3}}{3}$

4 b $F(x) = \frac{1}{16}x(12 - x^2), 0 \le x \le 2$
c **i** 0.34, 0.7, 1.1 **ii** 0.85
iii 0.8 **iv** 0.78
v $P(X \le 0.4) - P(x \le 0.2) = 0.3 - 0.15 = 0.15$

5 a 0.5 **b** 0.9032 **c** 0.9282
d 0.3085 **e** 0.4207 **f** 0.4247

6 a 97.5% **b** 84% **c** 81.5% **d** 2.35%

7 a 0.6915 **b** 0.1151 **c** 0.5125 **d** 0.1760

8 a Let X be the lifetime in years of a machine.
b $P(X > 8) = P(Z > 1.3) = 9.7\%$
$P(X < 5) = P(Z < -1.07) = 14.2\%$. This is probably an unacceptable risk for the manufacturer, and they should increase the mean life, or decrease the standard deviation, or adjust the length of their advertised warranty.

Chapter 17

Exercise 17A

1 a $\frac{1}{32}$ **b** $\frac{5}{16}$ **c** $\frac{5}{32}$ **d** $\frac{31}{32}$

2 ${}^6C_2 \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right)^2$

3 a Because of the large size of the roll and the large number of males on the roll, the probability is fairly constant across the selection.
b $(0.65)^{12}$

4 a ${}^{12}C_3 \left(\frac{5}{6}\right)^9 \left(\frac{1}{6}\right)^3$ **b** ${}^{12}C_8 \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^8$
c ${}^{12}C_{10} \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^{10} + {}^{12}C_{11} \left(\frac{5}{6}\right)^1 \left(\frac{1}{6}\right)^{11} + \left(\frac{1}{6}\right)^{12}$

5 a 0.2009 **b** 0.7368 **c** 0.2632

6 a $\left(\frac{9}{10}\right)^{20} + {}^{20}C_1 \left(\frac{9}{10}\right)^{19} \left(\frac{1}{10}\right) + {}^{20}C_2 \left(\frac{9}{10}\right)^{18} \left(\frac{1}{10}\right)^2$
b $1 - \left(\frac{9}{10}\right)^{20}$

7 a $0.91^{10} + {}^{10}C_1 \times 0.91^9 \times 0.09$
$+ {}^{10}C_2 \times 0.91^8 \times 0.09^2$
b $1 - 0.91^{10} - {}^{10}C_1 \times .91^9 \times 0.09$

8 b HHTT, HTHT, THHT, HTTH, THTH, TTHH.
The probability that she wins is $\frac{6}{2^4} = \frac{3}{8}$.
c See the answer to part **b**. There are ${}^4C_2 = 6$ arrangements.
d The number of ways of choosing the two coins that are to be heads up is ${}^4C_2$.

9 0.593

10 ${}^{31}C_3 \times 0.95^{28} \times 0.05^3$

11 a **i** 0.10764 **ii** 0.11372
b 0.78549

12 a 17 **b** 7

13 a **i** 0.487 **ii** 0.031
b First, we have assumed that boys and girls are equally likely. Secondly, we have assumed that in any one family, the events 'having a boy' and 'having a girl' are independent. Thirdly, we have assumed that every child is a boy or a girl.

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14 a The argument is invalid. Normally, mathematics books are grouped together, so that once the shelf is chosen, one would expect all or none of the books to be mathematics books, thus the five stages are not independent events. The result would be true if the books were each chosen at random from the library.

b The argument is invalid. People in a particular neighbourhood tend to vote more similarly than the population at large, so the four events are not independent. This method also oversamples small streets, which may introduce an additional bias.

15 a 0.03456 **b** 0.68256

16 a 0.409600 **b** 0.001126 **c** 0.000869

17 a 0.0060 **b** 0.0303

18 a $\dfrac{3}{250}$

b i $\left(\frac{3}{250}\right)^{10}$

ii ${}^{10}C_5 \left(\frac{3}{250}\right)^5 \left(\frac{247}{250}\right)^5$

iii $\left(\frac{247}{250}\right)^{10} + 10\left(\frac{247}{250}\right)^9 \left(\frac{3}{250}\right)$

19 a $\frac{7}{16}$

b i ${}^{8}C_3 \left(\frac{9}{16}\right)^5 \left(\frac{7}{16}\right)^3$

ii $1 - \left(\frac{9}{16}\right)^8 - 8\left(\frac{9}{16}\right)^7 \left(\frac{7}{16}\right)^1 - {}^{8}C_2 \left(\frac{9}{16}\right)^6 \left(\frac{7}{16}\right)^2$

20 a 34 **b** 22

21 a 0.2048 **b** 0.26272

c i $\dfrac{n(n-1)(n-2)(n-3)}{20 \times 19 \times 18 \times 17}$

22 a $a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2 + 6abc$

b i 0.102 96 **ii** 0.13133

iii 0.89704

Exercise 17B

1 a The events are not independent — if it rains one day, it is more likely to rain the next day because rainy days tend to come in groups.

b Yes, and X is the number of throws that were less than 5.

c The possible number of trials is not finite, and the stages are not independent because if she wins, then the game stops.

d Let X be the number of heads turning up in 20 throws. Then X is a random variable with a binomial distribution.

e Strictly, the pens are not replaced, so the probability changes as each pen is removed and tested. If the population of pens is large, then p is almost constant with each selection, and it could be modelled with a binomial distribution.

f No. There are not two outcomes at each stage. It could be modified to 'arrives on time' or 'takes less than 20 minutes', but the events may still not be independent.

g Yes. Note that while the experiment is different at each stage, the probabilities at each stage are independent and have the same probability 0.01 of success.

2 a

heads x	0	1	2	3	4
ways	1	6	15	20	15
p	0.016	0.094	0.234	0.313	0.23
xp	0	0.094	0.469	0.938	0.938
x^2p	0	0.094	0.938	2.813	3.75

heads x	5	6	Total
ways	6	1	64
p	0.094	0.016	1
xp	0.469	0.0940	3
x^2p	2.344	0.563	10.5

b 3 heads

c $\mu = \sum xp = 3$,

varience $= \sum x^2p - \mu^2 = 10.5 - 9 = 1.5$

d Results agree.

e The distribution is symmetric, thus the centre of the distribution is exactly at the midpoint.

3 a

x	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{32}$	$\frac{4}{32}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$	$\frac{1}{32}$

mode $=$ 2 or 3, mean $= \frac{5}{2}$, SD $= \frac{\sqrt{5}}{2}$

b

x	0	1	2	3	4	5
$P(X = x)$	$\frac{32}{243}$	$\frac{80}{243}$	$\frac{80}{243}$	$\frac{40}{243}$	$\frac{10}{243}$	$\frac{1}{243}$

mode $=$ 1 or 2, mean $= \frac{5}{3}$, SD $= \frac{\sqrt{10}}{3}$

c

x	0	1	2	3	4	5
$P(X = x)$	0.269	0.404	0.242	0.073	0.011	0.001

mode $= 1$, mean $\doteqdot 1.15$, SD $\doteqdot 0.94$

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4 a The expected value is 9.6 wins and the standard deviation is 2.4.

b Larry's result is 1.5 standard deviations below the mean

5 a 26%; $\mathrm{E}(X) = 1, \sigma = 0.91$

b i 62%; $\mathrm{E}(X) = 2, \sigma = 1.29$

ii 93%; $\mathrm{E}(X) = 4, \sigma = 1.83$

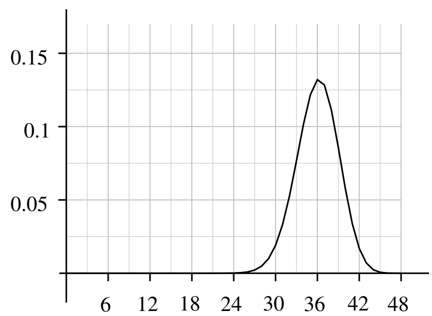
6 a right skewed

b mean 12, standard deviation 3.

c The mode is 12, which has a probability of about 0.13.

d Shade the region bounded by [6, 18] on the horizontal axis.

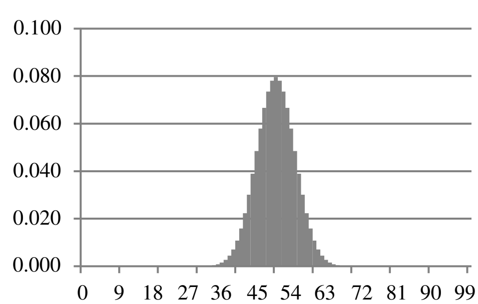
e (This is the graph for $p = 0.25$ reflected horizontally in $x = 24$.)



7 e 0.028 **f** 0.864 **g** 50 heads

h $i = 3$, and the interval is [47, 53]

i



It looks bell shaped, like a normal distribution.

8 a $\mathrm{E}(X) = 12$. This is the number a student might expect to get right by guessing alone.

b 3 **c** 4 **d** 20, 4

e This time she is five standard deviations above the mean, which is even more unusual than her previous result.

f 75% is 36/48, which is 8 standard deviations above the mean. 60% is 60/100, which is 10 standard deviations above the mean. Thus it should be harder to get 60% in the second test than 75% in the first, just by chance. Note, however, that both results are almost impossible to achieve just by guessing.

9 Team A: $\mu = 35, \sigma = 3.2$, and 45 is 3.1σ above the mean.
Team B: $\mu = 63, \sigma = 4.3$, and 74 is 2.6σ above the mean.
Thus Team A's changes to the drug's delivery has shown stronger evidence for an improvement. Further trials should be carried out to check the validity of this result (see Section 17D on sampling).

10 a mean $= 15$, standard deviation $\doteqdot 3.57$

b That is, the probability of 14, 15 or 16 people voting WTP. This is approximately 32.5%.

11 a $\sigma^2 = np(1 - p) = -np^2 + np$

b It is a parabola, symmetric in its axis of symmetry $p = \frac{1}{2}$.

d This is the vertex, occurring halfway between the roots $p = 0$ and $p = 1$, that is, at $p = \frac{1}{2}$.

e It is clear from the quadratic graph in (a) and $\sigma = \sqrt{\sigma^2}$ has the same behaviour.

12 a 4 **b** 6 **c** 8

13 $2:\sqrt{3}$

14 d If an experiment testing a certain result is repeated enough times, it is to be expected that the hypothesis will be upheld eventually. If 99 times it fails and once it succeeds, then only publishing the success gives a skewed picture of the truth.

15 b The probability of being chosen is $p = \frac{1}{20}$, thus the mean waiting time is $\mu = \dfrac{1}{p} = 20$ time periods, or $5 \times 20 = 100$ minutes.

Exercise 17C

1 a $B(20,\ 0.3)$ **b** 10.82%

d 10.75% **e** 0.65%.

The result is very accurate!

2 a Answers for this question may vary slightly depending on the accuracy used to determine normal probability values (table or other technology).
$B(50,\ 0.5)$, $P(18 \le X \le 20) = 8.49\%$
$np = 25$, $nq = 25$, normal approximation using $N(25,\ 12.5)$ is 8.46%, with percentage error 0.1%

b $B(20,\ 0.4)$, $P(8 \le X \le 9) = 33.94\%$,
$np = 8$, $nq = 12$, normal approximation using $N(8,\ 4.8)$ is 34.3%, with percentage error 1%

c $B(30,\ 0.3)$, $P(5 \le X \le 7) = 25.12\%$,
$np = 9$, $nq = 21$, normal approximation using $N(9,\ 6.3)$ is 23.8%, with percentage error 5%

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d $B(40,\ 0.2)$, $P(9 \le X \le 12) = 36.36\%$, $np = 8$, $nq = 32$, normal approximation using $N(8,\ 6.4)$ is 38.3% , with percentage error 5%

e $B(22,\ 0.6)$, $P(13 \le X \le 15) = 46.59\%$, $np = 13.2$, $nq = 8.8$, normal approximation using $N(13.2,\ 5.28)$ is 46.0%, with percentage error 1%

f $B(80, 0.1)$, $P(10 \le X \le 13) = 24.98\%$, $np = 8$, $nq = 72$, normal approximation using $N(8,\ 7.2)$ is 26.8% , with percentage error 7%

g $B(500,\ 0.25)$, $P(100 \le X \le 103) = 0.84\%$, $np = 125$, $nq = 375$, normal approximation using $N(125,\ 93.75)$ is 0.9%, with percentage error 6%

h $B(200,\ 0.9)$, $P(170 \le X \le 172) = 3.39\%$, $np = 180$, $nq = 20$, normal approximation using $N(180,\ 18)$ is 3.2%, with percentage error 7%

3 a There are two possible outcomes, pink or blue.

b There are n stages, each independent, and with the same probability of success.

c If the counter is not returned, the stages of the experiment will not be independent. With the large number of counters, however, the probability will not change much, and we could approximate the experiment as binomial.

d $p = 0.6$, $\mu = np = 12$, $\sigma^2 = np(1 - p) = 4.8$, so $\sigma \doteqdot 2.19$.

e $\binom{20}{14}(0.6)^{14}(0.4)^6 \doteqdot 0.124$

f ii $P(13.5 < X < 14.5) \doteqdot P(0.68 < Z < 1.14) \doteqdot 0.12$. Correct to two significant figures, both are 12%.

4 a $p = \frac{1320}{3000} = 0.44$

b $\mu = 6.6$, $\sigma \doteqdot 1.92$

c 14.04%

d $np = 6.6$, $nq = 8.4$. Yes.

e 14%

f Less than 1%

5 a $n > 10$ **b** $n > 20$ **c** $n > 40$

d $n > 500$ **e** $n > 20$ **f** $n > 40$

g $n > 50$ **h** $n > 11$

6 a Very large numbers are involved. A normal calculator cannot handle numbers such as ${}^{854}C_{76}$. Other calculating devices may not be accurate dealing with the large numbers involved. There are also a large number of cases to consider — from 60 to 76 successes.

b It is hard to get a representative sample of the whole world, because different ethnic groups will have different tendencies to colour blindness.

c $\mu \doteqdot 68.32$, $\sigma \doteqdot 7.93$,
$P(60 < X < 76)$
$= P(-1.05 < Z < 0.97) \doteqdot 68.7\%$

d $P(0.97 < Z < 1.03) = 1.5\%$

7 a $\left(\frac{3}{4}\right)^{15} \doteqdot 1\%$ **b** 99% **c** 77%

8 Interpreting this as $P(11 \le X \le 20)$, a normal approximation gives 25%.

9 $P(X > 20) = P(21 \le X \le 30) \doteqdot 17.6\%$. The underlying Bernoulli distribution is not applied with replacement, because the same person will not be in the park twice at the same gathering. If the population of Nashville is large, it should be reasonable to neglect this fact. It is also assumed that the visitors to the park are a random cross-section of Nashville. Groups with similar musical tastes may arrive together.

10 b There are still 100 trials, but the basic Bernoulli trial has changed. It could be that an extremely biased coin is tossed, or a card labelled 1 is selected (with replacement) from a pack of cards labelled 1–10.

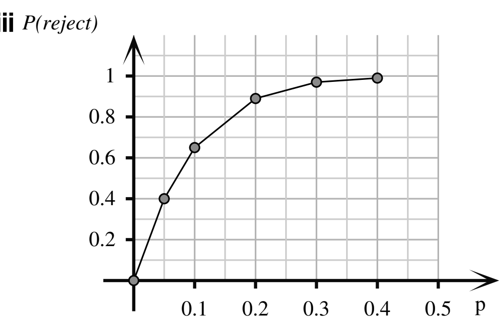
c The graphs are all bell-shaped curves. Smaller probabilities give a curve centred to the left (skewed to the right), and larger probabilities give a curve centred to the right (skewed to the left). Probabilities further from 0.5 give a narrower curve (distribution).

12 a i 0.4

ii

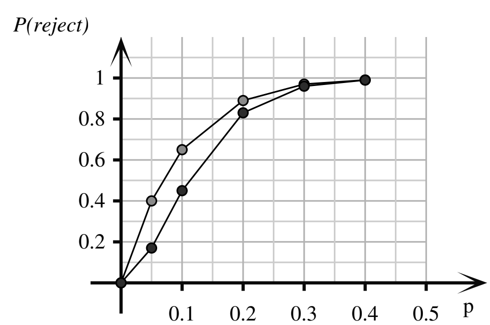
p	0	0.05	0.1	0.2	0.3	0.4
P(reject)	0	0.4	0.65	0.89	0.97	0.99

iii



b

p	0	0.05	0.1	0.2	0.3	0.4
P(reject)	0	0.17	0.45	0.83	0.96	0.99



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c The second method is more forgiving if there are a few punnets that need to be rejected. Both methods are strongly likely to reject the batch if p is high, indeed the curves approach one another closely by the time p reaches 20%.

Exercise 17D

1 a $\frac{2}{5}$ **b** $\frac{4}{10}$ **c** $\frac{3}{4}$

2 a

X	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

b

$\hat{h}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
$P(\hat{p})$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

c 0.5

d It is the probability p in each Bernoulli trial, that is, it is the probability of a coin landing heads.

3 a

$\hat{p}$	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
f_r	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{2}{20}$	$\frac{6}{20}$	$\frac{7}{20}$

b 0.72

c It is an estimate of the probability that a shopper chosen at random lives in the suburb.

4 a 8, 9 or 10 heads **b** 5.47%

5 a 6.4%

b 7.3% This result is surprisingly accurate because 9% of 50 is 4.5, so we are effectively applying a continuity correction.

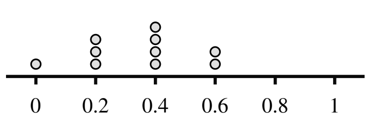
6 a $12 \div 32 \doteqdot 0.375$

b 0.2

c

$\hat{p}$	0	0.2	0.4	0.6	0.8	1.0
Frequency	1	3	4	2	0	0

d



7 13%

8 a Let X record the number of hearts selected. Then $P(15.5 \le X \le 24.5) \doteqdot 75\%$

b Let $\mu = 0.25$, and $\sigma = \sqrt{0.25 \times 0.75 \div 80} \doteqdot 0.0484$. Thus $P(0.20 \le \hat{p} \le 0.30) = P(-1.03 \le Z \le 1.03)$ $= 2 \times P(Z \le 1.03) - 1 = 0.8485 \times 2$ $- 1 \doteqdot 70\%$. (The exact answer is 76% so part **a** is more accurate).

9 a 97% **b** 94%

10 a i 0.6% **ii** 3.7%

b i The mean and standard deviation for $\hat{p}$ are $\mu = 0.05$, and $\sigma = \sqrt{0.05 \times 0.95 \div 1000} \doteqdot 0.0069$. Thus $P(X < 0.03) = P(Z < -2.90)$ $= 1 - P(Z < 2.90) = 1 - 0.998 \doteqdot 0.2\%$

ii This result is significantly different from the previous claim that 5% of patients will have a reaction. They should check whether the sample was random — perhaps it consisted of patients more resistant to the side effects of the medication. Theyshould also check whether there have been any changes to the medication to reduce patient reactions. It is also possibly just chance that this result occurred, but the likelihood of this is small.

11 a $\mu = 0.6, \sigma^2 = 0.012$, so $\sigma = 0.1095$.

c $P(\hat{p} \le 0.4) \doteqdot P(Z \le -1.83)$
$\doteqdot 3.4\%$

d No, there is an error of 40%. The sample is too small and we are not using any continuity correction.

e $P(\hat{p} \le 0.75) \doteqdot P(Z < 1.37)$
$\doteqdot 91.5\%$

f 3.7%. The curve is flatter at the top end and varies less with $\hat{p}$. Percentage difference is also exaggerated by small values, such as at the left end of the curve.

12 b

n	1000	500	100	50	25
exact	0.9026	0.8262	0.6914	0.6641	0.6550
appr	0.896	0.8143	0.655	0.610	0.579
error	0.7	1.6	5.21	8.1	11.6

(answers may vary if you have used technology in place of the provided normal tables.)

c The accuracy improves as n increases, and is quite good for large samples.

13 For the sample proportion $\hat{p}$, $\mu = 0.5$, and $\sigma = \sqrt{0.5 \times 0.5 \div 50} \doteqdot 0.071$. Thus $P(X > 0.6) = P(Z > 1.41) = 1 - P(Z < 1.41)$ $= 1 - 0.9207 \doteqdot 8\%$. It appears that people strongly prefer the branded version, even though it is identical. There may be an expectation that the branded version is superior, or they may prefer the packaging.

14 In the population of those with the disease, let $p = 0.3$ be the probability of a positive response to the drug. Let X be the binomial random variable

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measuring how many respond positively to the drug in the trial of $n = 100$ and let $\hat{p}$ be the sample proportion $\hat{p} = \frac{X}{n}$. The mean and standard deviation of $\hat{p}$ are $\mu = p = 0.3$ and $\sigma = \sqrt{p(1-p)/n} = 0.046$. Then $P(\hat{p} \geq 40\%) = P(Z \geq 2.17) = 1.5\%$. It is unlikely to be by chance.

15 a **i** 132 **ii** 234 **iii** 525 **iv** 2100 **v** $\frac{2100}{k^2}$

b **i** 100 **ii** 178 **iii** 400 **iv** 1600 **v** $\frac{1600}{k^2}$

c **i** 157 **ii** 278 **iii** 625 **iv** 2500 **v** $\frac{2500}{k^2}$

16 a **i** 0.205 **ii** $40 \times 0.205 \doteqdot 8$

17 a $p = \frac{5}{18}$

b $P(X = 0 \leq X \leq 4) = 0.3096$

c 0.2149 **d** 0.2968

e 0.2149. The sample proportion distribution is just the binomial stretched vertically by a factor n and compressed horizontally by a factor $\frac{1}{n}$, thus the corresponding areas will be the same.

f 0.2968. The factor $\frac{1}{40}$ corresponds to half an interval on the histogram and thus applies the same continuity correction as part **d**.

18 b **i** $\sigma \doteqdot 0.047$

ii Margin of error $= 0.09 = 9\%$, and the 95% confidence interval for the probability p is $[58\%, 76\%]$.

iii $n > 8493$

Chapter 17 review exercise

1 a ${}^4C_3 \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)$ **b** ${}^4C_2 \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2$

2 $\left(\frac{5}{6}\right)^{15} + {}^{15}C_1 \left(\frac{5}{6}\right)^{14} \left(\frac{1}{6}\right) + {}^{15}C_2 \left(\frac{5}{6}\right)^{13} \left(\frac{1}{6}\right)^2$

3 0.000786

4 We need $1 - \left(\frac{3}{4}\right)^n > 0/95$.
Thus $n > \log(0.05)/\log(0.75) > 10$.
The experiment needs to be repeated 11 times.

5 a $\frac{1}{8}$

b

x	0	1	2	3
$P(X = x)$	0.4488	0.3847	0.1374	0.0262

x	4	5	6
$P(X = x)$	0.0028	0.0002	0

c 26 times **d** 29 times

6 a Yes. $p = 0.5, q = 0.5$

b Yes, if success is interpreted as the player winning. Probability of success $p = \frac{1}{12}$ and of failure $q = \frac{11}{12}$.

c Yes, with experimental probability $p = 0.004$, $q = 0.996$

d No, because a Bernoulli trial must have only two possible outcomes.

7 a 4, 3.2, 1.79 **b** 35, 17.5, 4.18

c 4.8, 0.96, 0.98 **d** 48, 28.8, 5.37

e 30, 27, 5.2 **f** 1.25, 0.9375, 0.97

8 a 3, 2.85, 1.7

b $P(X = 2, 3 \text{ or } 4) = 62.8\%$

c $P(0 \leq X \leq 4) \doteqdot 82\%$. The probability of rejection is 18%

d mean 1.2, standard deviation 1.08, $P(X = 0, 1 \text{ or } 2) = 88\%$. The probability of rejection is 12%

9 a 25.62% **b** 40, $\sqrt{20}$

c $np = nq = 40 > 5$, so our criterion suggests that a normal approximation is appropriate.

d This is called a continuity correction and occurs because we are approximating a discrete distribution by a continuous curve.

e 25.6% **f** less than 0.1%

g 1.7%

10 a $\frac{1}{6}$

b If $X \sim B\left(80, \frac{1}{6}\right)$, then $P(X \geq 15) = 36\%$

11 a 0.35

b Not neccessarily.

c A binomial distribution with $n = 100$ and p equal to the (unknown) proportion of the population intending to vote for the WTP.

12 a

$\hat{p}$	0	0.2	0.4	0.6	0.8	1
$P(\hat{p})$	$\frac{32}{3125}$	$\frac{48}{625}$	$\frac{144}{625}$	$\frac{216}{625}$	$\frac{162}{625}$	$\frac{243}{3125}$

b **i** $\frac{272}{3125}$ **ii** $\frac{992}{3125}$

c mean $\mu_{\hat{p}} = \frac{3}{5}$, variance $\frac{6}{125}$, $\sigma_{\hat{p}} = \frac{1}{25}\sqrt{30}$.

13 a $\mu = p = \frac{1}{6}$; $\sigma = \frac{1}{60}$

b The z-score is $\left(\frac{70}{500} - \frac{1}{6}\right) \div \frac{1}{6} = 1.6$ standard deviations below the mean.

14 $P(X > 0.53) \doteqdot 31\%$

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