Name: ____________________ Due: Solutions

Year 12 Advanced Term 2 – Assignment 2

Section 1 – Multiple Choice

1. What is $\int \frac{3x}{x^2-4}dx$?

$3\int \frac{x}{x^2-4}dx = \frac{3}{2}\int \frac{2x}{x^2-4}dx$

$= \frac{3}{2}\ln|x^2-4| + c$

$\Rightarrow$ B

A. $\frac{2}{3}\ln|x^2-4| + c$

B. $\frac{3}{2}\ln|x^2-4| + c$

C. $2\ln|x^2-4| + c$

D. $\frac{1}{3}\ln|x^3-4x| + c$

2. Which interval gives the range of the function $y = -(1 + 3\sin 2x)$?

$-3 \le 3\sin 2x \le 3$

$-3+1 \le 1+3\sin 2x \le 3+1$

$-2 \le 1+3\sin 2x \le 4$

$-2^{x-1} \le -(1+3\sin 2x) \le 4^{x-1}$

$-4 \le -(1+3\sin 2x) \le 2 \Rightarrow$ D

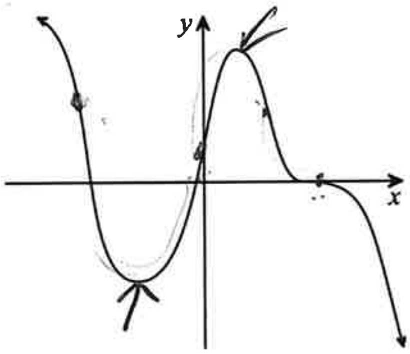
A. $-4 \le y \le -2$

B. $-5 \le y \le -1$

C. $-1 \le y \le 5$

D. $-4 \le y \le 2$

3. The graph of $y = f'(x)$ is shown.



How many inflection points does $y = f(x)$ have?

It has 2 tp $\Rightarrow \therefore$ 2 POI

A. 0

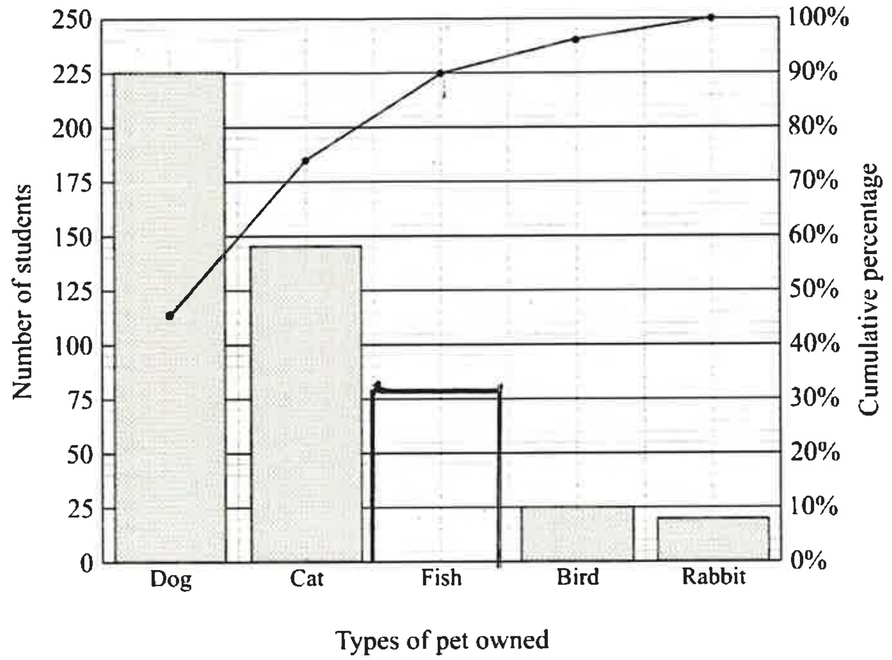
B. 1

C. 2

D. 3

Section 2 – Show full working!

1. A group of students was surveyed and data relating to the types of pets they owned was collected. The Pareto chart shows the data collected. The column representing the number of students owning pet fish has been removed.



(a) How many students own a pet dog or cat?

$225 + 145 = 370$

(b) Complete the Pareto chart, showing the number of students who own a pet fish.

$370 = 74\%$

$\frac{370}{74} = 1\%$

$\frac{370}{74} \times 16 = 16\%$ (Change in %)

$80 =$

2. Find $\int \frac{x}{\sqrt{9-x^2}}\,dx$.

$= \int x\left(9-x^2\right)^{-\frac{1}{2}} dx$

Major player $\Rightarrow$ $\left(9-x^2\right)^{\frac{1}{2}}$

$\frac{d}{dx}\left(9-x^2\right)^{\frac{1}{2}} = \frac{1}{2}\left(9-x^2\right)^{-\frac{1}{2}} \times -2x$

$= -x\left(9-x^2\right)^{-\frac{1}{2}}$

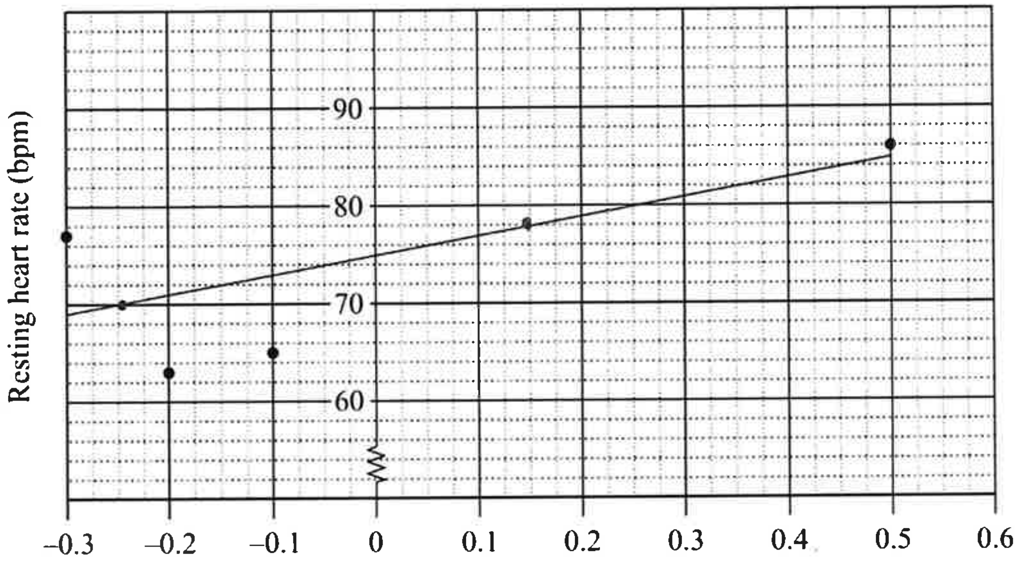
$\therefore \int x\left(9-x^2\right)^{-\frac{1}{2}} dx = -\left(9-x^2\right)^{\frac{1}{2}} + C$

3. A healthy human body temperature is 37.0° C.

Eight randomly selected people were examined by medical staff. The difference in their body temperature from 37.0° C (in degrees) and resting heart rate (in beats per minute) were recorded.

Temperature difference from 37°C (x)	–0.2	–0.3	–0.3	–0.2	–0.1	0	0.2	0.5
Heart rate (y)	63	77	70	74	65	78	79	86

(a) Complete the scatterplot by adding the last four points from the table.



(b) The least-squares regression line has been plotted in the graph in part (a). Find the equation of this line.

$m = \dfrac{78 - 70}{0.15 - (-0.25)} = \dfrac{8}{0.4} = 20$

$y = 20x + 75$

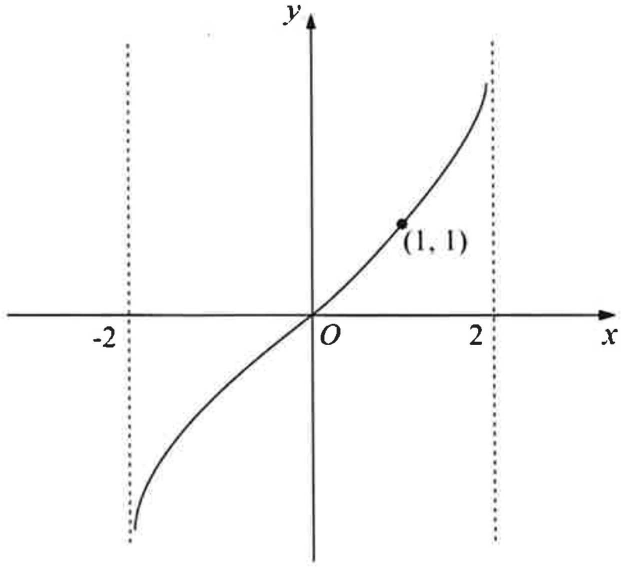
(c) By using the equation of the regression line, predict the resting heart rate of a person with a body temperature of 37.3° C. Give your answer correct to the nearest whole number.

When temp = 37.3, the resting heart rate is 81

(d) Explain why the least-squares regression line would not be reliable to predict the resting heart rate of a person with a body temperature of 37.6° C?

The data only goes up to 37.5°C, so 37.6 is an extrapolation

4. The graph of $y = a \tan bx$ is shown below for $-2 < x < 2$.



Find the values of a and b.

$y = \tan x$ has $-\frac{\pi}{2} < x < \frac{\pi}{2}$

So $y = \tan bx$ has $-\frac{\pi}{2} < bx < \frac{\pi}{2}$

But $\frac{\pi}{2} \times \frac{1}{b} = 2$

$\frac{\pi}{4} = b$

also $(1,1)$ lies on

$y = a \tan \frac{\pi}{4} x$

$\therefore 1 = a \tan \frac{\pi}{4} \Rightarrow 1 = a \times 1$

$a = 1$

5. The point $A(8, 23)$ lies on the graph of $y = g(x)$.

It is known that the graph of $y = g(x)$ is obtained by transforming the graph of $y = f(x)$ such that $g(x) = -2f(4x) + 1$.

Find the coordinates of the point on the graph of $y = f(x)$ which is mapped to the point A.

$(8, 23)$ lies on $y = g(x)$. You need to go backwards

$(8, 23) \Rightarrow (8, 23 - 1) = (8, 22)$

$(8, 22) \Rightarrow (8, 22 \times -\frac{1}{2}) = (8, -11)$

$(8, -11) \Rightarrow (8 \times 4, -11) = (32, -11)$

$\boxed{\text{AH}}$: Let required point be (x, y)

$\therefore \frac{x}{4} = 8 \Rightarrow x = 32$

$-2y + 1 = 23 \Rightarrow y = -11$ $\therefore (32, -11)$

6. The share price ($P) of a renewable energy company is increasing such that

$$\frac{dP}{dt} = \frac{3}{t+1},$$

where t is the time in months.

If the initial price of the share price is $2.00, find the price after 6 months. Write your answer to the nearest cent.

When $t=0$, $P=2$, Need P when $t=6$

$$\frac{dP}{dt} = \frac{3}{t+1}$$

$$P = 3\ln(t+1) + c$$

when $t=0$, $P=2 \Rightarrow$ $2 = \cancel{3\ln 1} + c$

$$2 = c$$

$$\therefore P = 3\ln(t+1) + 2$$

when $t=6$, $P = 3\ln 7 + 2$

$$\doteq \$7.84$$

7. Let $f(x) = x^3 + x^2 + 3x + 3$.

(a) Show that the graph of $y = f(x)$ has no stationary points.

$f'(x) = 3x^2 + 2x + 3$

Need to show $f'(x) \neq 0$ for any x.

this means $\Delta < 0$

$\Rightarrow \Delta = 2^2 - 4(3)(3) = 4 - 36 = -32$

Since $\Delta = -32 < 0$, No sol'n for $f(x)$

(b) Find any points of inflexion.

$f''(x) = 6x + 2$

$f''(x) = 0$ for P.O.I. :

$$6x + 2 = 0$$

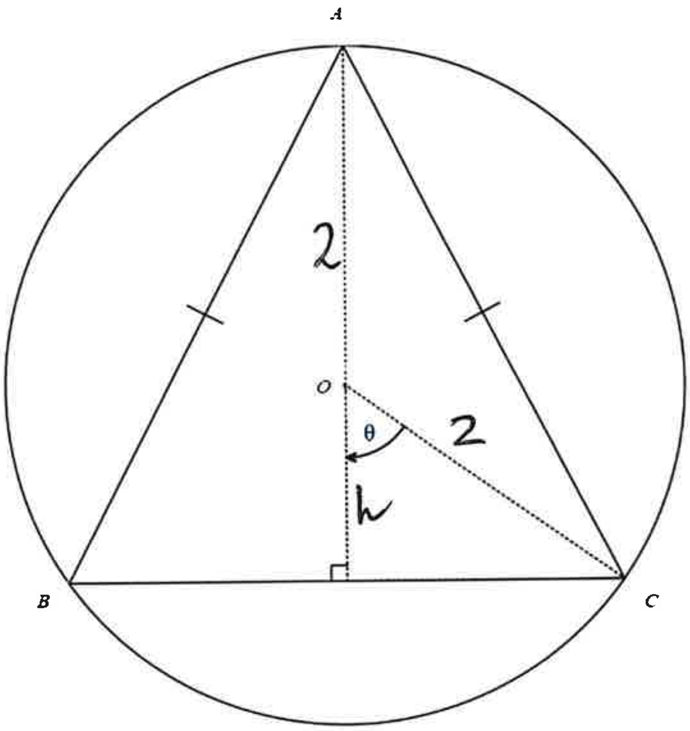
$$x = -\frac{1}{3}$$

Show change in concavity :

x	-1	$-\frac{1}{3}$	0
$f''(x)$	-4	0	2

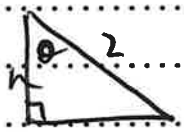
Since theres a change in concavity, $x = -\frac{1}{3}$ is a POI

8. An isosceles triangle ΔABC is inscribed inside a circle of fixed radius 2 and centre O. Let θ be defined as in the diagram below.



(a) Show that the area of the triangle ΔABC is given by $A = 4\sin\theta(1+\cos\theta)$.

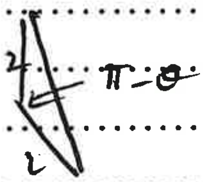
$\cos\theta = \frac{h}{2}$

In 

$h = 2\cos\theta$

$\therefore A = \frac{1}{\cancel{2}} \times h \times \cancel{2} \times \sin\theta = h\sin\theta$

$A = 2\cos\theta\sin\theta$

In 

$A = \frac{1}{\cancel{2}} \times \cancel{2} \times 2 \times \sin(\pi - \theta)$

$= 2\sin(\pi - \theta)$

$A = 2\sin\theta$

However, we need to double these:

$\therefore A_{\Delta ABC} = 2\left[2\cos\theta\sin\theta + 2\sin\theta\right]$

$\therefore A = 4\sin\theta\left[\cos\theta + 1\right]$

(b) Find the value of θ that maximises the area of ΔABC.

$A = 4\sin\theta\left[\cos\theta + 1\right]$

Need $A' = 0$ for S.P.:

$A' = 4\sin\theta(-\sin\theta) + (\cos\theta + 1)(4\cos\theta)$

$= -4\sin^2\theta + 4\cos^2\theta + 4\cos\theta$

$= -4(1 - \cos^2\theta) + 4\cos^2\theta + 4\cos\theta$

$= -4 + 4\cos^2\theta + 4\cos^2\theta + 4\cos\theta$

$A' = 8\cos^2\theta + 4\cos\theta - 4$

Need $A' = 0$: $8\cos^2\theta + 4\cos\theta - 4 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

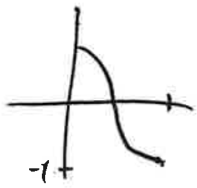
$x - 2$

$+ 1$

$2, -1$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

$\theta = \frac{\pi}{3}$ θ has no solution here



Test nature of $\theta = \frac{\pi}{3}$:

$A'' = -16\cos\theta\sin\theta - 4\sin\theta$

when $\theta = \frac{\pi}{3}$, $A'' = -16\cos\frac{\pi}{3}\sin\frac{\pi}{3} - 4\sin\frac{\pi}{3}$

$= -\cancel{16}^{\,8} \times \frac{1}{\cancel{2}} \times \frac{\sqrt{3}}{\cancel{2}} - \cancel{4}^{\,2} \times \frac{\sqrt{3}}{\cancel{2}}$

$= -4\sqrt{3} - 2\sqrt{3}$

$= -6\sqrt{3}$

$A'' < 0$ $\therefore$ concave down

$\therefore$ local max at $\theta = \frac{\pi}{3}$

$\therefore$ Max. area is when $\theta = \frac{\pi}{3}$