EXERCISE 9.1 INTRODUCTION TO PROBABILITY

2 **(a)** Answers will vary.

(b) No, because all numbers in the same area start with the same number, or only one or two numbers.

(c) The distribution would not be uniform as some letters would start names more than others.

4 C

Many babies are born in Sydney every hour, so this has the probability closest to 1, although housing prices increase nearly all the time, but do have periods where they drop. In the middle of a boom period, you could argue for this one.

6 The number of hearts drawn and relative frequency will vary.

The expected number of hearts is recorded in the table. As one quarter of the cards are hearts, you would expect that the number of hearts to be about one quarter of the number of withdrawals.

Number of withdrawals	20	40	60	80	100
Number of hearts	$20\times\frac{1}{4}=5$	$40\times\frac{1}{4}=10$	$60\times\frac{1}{4}=15$	$80\times\frac{1}{4}=20$	$100\times\frac{1}{4}=25$

As you increase the number of withdrawals, you would expect the relative frequency to get closer to $\frac{1}{4}$, or 0.25. 0.7 is a much higher relative frequency, so it is extremely unlikely that anyone's results would be even near this number.

8 **(a)** Any of the 12 eggs are equally likely to be selected, and 3 are brown.

$$P(\text{brown})=\frac{3}{12}=\frac{1}{4}$$

(b) Any of the 12 eggs are equally likely to be selected, and 9 are white.

$$P(\text{white})=\frac{9}{12}=\frac{3}{4}$$

10 **(a)** Divisible by 3: $3,\ 6,\ 9, 12,\ 15,\ 18$

Any of the 20 cards are equally likely to be selected, and 6 are divisible by 3.

$$P(\text{divisible by } 3)=\frac{6}{20}=\frac{3}{10}$$

(b) Divisible by 5: $5,\ 10,\ 15,\ 20$

Any of the 20 cards are equally likely to be selected, and 4 are divisible by 5.

$$P(\text{divisible by } 5) = \frac{4}{20} = \frac{1}{5}$$

(c) Divisible by 3 or 5 or both: $3,\ 5,\ 6,\ 9, 10,\ 12,\ 15,\ 18,\ 20$

Any of the 20 cards are equally likely to be selected, and 9 are divisible by 3 or 5 or both.

$$P(\text{divisible by 3 or 5 or both}) = \frac{9}{20}$$

(d) Divisible by 3 and 5: 15

$$P(\text{divisible by 3 and 5}) = \frac{1}{20}$$

12 (a) Half the cards are even, and the other half are odd.

$$P(\text{even}) = \frac{1}{2}$$

(b) $P(\text{odd}) = \frac{1}{2}$

(c) All the cards are either even or odd.

$$P(\text{odd or even}) = 1$$

(d) Greater than 7: $8,\ 9,\ 10,\ 11,\ 12$

$$P(\text{number} > 7) = \frac{5}{10} = \frac{1}{2}$$

(e) Divisible by 3: $3,\ 6,\ 9,\ 12$

$$P(\text{divisible by } 3) = \frac{4}{10} = \frac{2}{5}$$

(f) Even and divisible by 3: $6,\ 12$

$$P(\text{even and divisible by } 3) = \frac{2}{10} = \frac{1}{5}$$

(g) Divisible by 5: $5,\ 10$

$$P(\text{divisible by } 5) = \frac{2}{10} = \frac{1}{5}$$

(h) Odd or divisible by 5, but not both: $3,\ 7,\ 9,\ 10,\ 11$

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$$P(\text{Odd or divisible by 5, but not both}) = \frac{5}{10} = \frac{1}{2}$$

14 (a) There are 13 hearts in a standard pack of 52 playing cards.

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

(b) There are 4 kings in a standard pack of 52 playing cards.

$$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$$

(c) There are 13 hearts and 4 kings, but one king is also a heart, so there are 13 + 3 = 16 favourable outcomes.

$$P(\text{heart or king}) = \frac{16}{52} = \frac{4}{13}$$

(d) There is only one card that is a heart and a king, the king of hearts.

$$P(\text{king and heart}) = \frac{1}{52}$$

(e) There is only one queen of diamonds in a standard pack of 52 playing cards.

$$P(\text{queen of diamonds}) = \frac{1}{52}$$

16 (a) There are $5+3+2=10$ marbles altogether.

$$P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$$

(b) $P(\text{red}) = \dfrac{3}{10}$

(c) $P(\text{green}) = \dfrac{2}{10} = \dfrac{1}{5}$

$$P(\text{not green}) = 1 - P(\text{green}) = 1 - \frac{1}{5} = \frac{4}{5}$$

(d) There are $5+2=7$ marbles that are blue or green.

$$P(\text{blue or green}) = \frac{7}{10}$$

(e) $P(\text{not red, nor blue}) = P(\text{green}) = \dfrac{2}{10} = \dfrac{1}{5}$

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18 **(a)** $P(\text{red})=\dfrac{6}{10}=\dfrac{3}{5}$

(b) $P(\text{not yellow})=1-P(\text{yellow})=1-\dfrac{3}{10}=\dfrac{7}{10}$

(c) There are $1+3=4$ marbles that are blue or green.

$P(\text{white or yellow})=\dfrac{1}{10}+\dfrac{3}{10}=\dfrac{4}{10}=\dfrac{2}{5}$

(d) $P(\text{neither red nor white})=P(\text{yellow})=\dfrac{3}{10}$

(e) $P(\text{not red})=1-P(\text{red})=1-\dfrac{3}{5}=\dfrac{2}{5}$

20 **(a)** $P(\text{some children})=6\%+38\%+42\%+10\%=96\%$

(b) $P(\text{no children})=1-P(\text{some children})=1-96\%=4\%$

(c) $P(\text{at least 2 children})=38\%+42\%+10\%=90\%$

(d) $P(\text{not more than 2 children})=4\%+6\%+38\%=48\%$

22 **(a)** The area more than 5 m from the fence is a rectangle of area 40 m by 10 m, or 400 m^2.

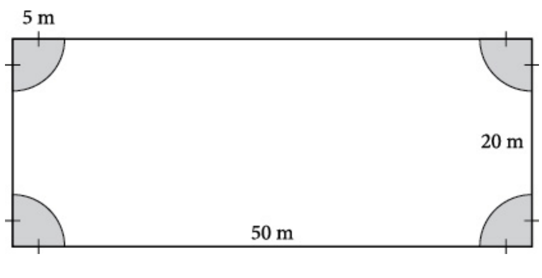
The area of the paddock is a rectangle of area 50 m by 20 m, or 1000 m^2.

$P(\text{more than 5 m from fence})=\dfrac{400}{1000}=\dfrac{2}{5}$

(b) $P(\text{less than 5 m from fence})=1-P(\text{more than 5 m from fence})=1-\dfrac{2}{5}=\dfrac{3}{5}$

Note: We consider the event ‘exactly five metres from the fence’ to have a probability of zero.

(c)



$$P(<5\text{ m from the corners})=\frac{\pi\times 5^2}{1000}\approx 0.0785$$

EXERCISE 9.2 VENN DIAGRAMS

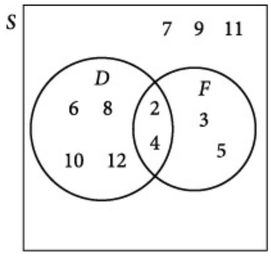
2 The sum of two dice ranges from 2 to 12.

The even numbers are $2,\ 4,\ 6,\ 8,\ 10,\ 12$.

The numbers less than 6 are $2,\ 3,\ 4,\ 5,\ 6$.

The leftover numbers are $7,\ 9,\ 11$.

Place the numbers according to whether they are in sets D, F, both or neither.



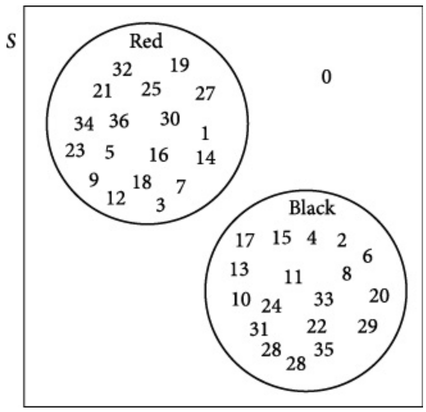
(a) $D \cup F$ contains all numbers either in D or F or both sets.

$D \cup F = \{2,\ 3,\ 4,\ 5,\ 6,\ 8,\ 10,\ 12\}$

(b) $D \cap F$ contains all numbers in both D and F.

$D \cap F = \{2,\ 4\}$

4



(a) $B \cup R$ contains all numbers either in B or R (no numbers in both sets).

$B \cup R = \{1,\ 2,\ 3,\ ...36\}$

(b) As there are no numbers in both sets, $B \cap R = \{\ \}$

(c) $n(B \cup R) = 36$

(d) $n(B \cap R) = 0$

(e) $n(S) = 37$

6 (a) There are $25 - 5 = 20$ students who study either French or German or both.

If we add the numbers studying French and the numbers studying German, we get $18 + 12 = 30$.

This means we have counted $30 - 20 = 10$ students twice, because they study both French and German.

Of the 18 students who study French, 10 also study German, leaving 8 students who study French only.

(b) Of the 12 students who study German, 10 also study French, leaving 2 students who study German only.

2 students only study German.

(c) $8 + 2 = 10$

10 students only study one language.

8 (a) There are no overlapping even and odd numbers.

$A \cap B = \{\ \}$

(b) All the numbers are either odd or even.

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

(c) To be in both these sets, the numbers must be odd and greater than 4.

$B \cap C = \{5\}$

(d) To be in both these sets, the numbers must be even and greater than 4.

$A \cap C = \{6\}$

$n(A \cap C) = 1$

10 (a) The set S contains all numbers from 1 to 14. Incorrect

(b) Check which numbers are in either or both circles. This list is correct.

(c) Check which numbers are in both circles. This list is correct.

(d) The set S contains all numbers from 1 to 14. Correct

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EXERCISE 9.3 FINITE SAMPLE SPACES

2 **(a)** Multiple of 3: $3,\ 6,\ 9,\ 12,\ 15$

$$P(A)=\frac{5}{17}$$

(b) Multiple of 8: $8,\ 16$

$$P(B)=\frac{2}{17}$$

(c) Multiple of 5: $5,\ 10,\ 15$

$$P(C)=\frac{3}{17}$$

(d) $P\left(\overline{A}\right)=1-P(A)=1-\frac{5}{17}=\frac{12}{17}$

(e) $P\left(\overline{B}\right)=1-P(B)=1-\frac{2}{17}=\frac{15}{17}$

(f) A and B are mutually exclusive events.

$$P(A \text{ or } B)=P(A)+P(B)=\frac{5}{17}+\frac{2}{17}=\frac{7}{17}$$

(g) A and C are not mutually exclusive events.

They have one number in common, 15, so $P(A \text{ and } C)=\dfrac{1}{17}$.

$$\begin{aligned}P(A \text{ or } C)&=P(A)+P(C)-P(A \text{ and } C)\\&=\frac{5}{17}+\frac{12}{17}-\frac{1}{17}\\&=\frac{7}{17}\end{aligned}$$

(h) $\overline{A}$ and B are not mutually exclusive events.

The numbers which are not a multiple of 3, but a multiple of 8, are $8,\ 16$.

$$\therefore P\left(\overline{A} \text{ and } B\right)=\frac{2}{17}$$

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$$\begin{aligned}P\left(\overline{A}\text{ or }B\right)&=P\left(\overline{A}\right)+P\left(B\right)-P\left(\overline{A}\text{ and }B\right)\\&=\frac{12}{17}+\frac{2}{17}-\frac{2}{17}\\&=\frac{12}{17}\end{aligned}$$

(i) A and $\overline{C}$ are not mutually exclusive events.

The numbers which are a multiple of 3, but not a multiple of 5, are $3,\ 6,\ 9,\ 12$.

$$\therefore P\left(A\text{ and }\overline{C}\right)=\frac{4}{17}$$

$$\begin{aligned}P\left(A\text{ or }\overline{C}\right)&=P\left(A\right)+P\left(\overline{C}\right)-P\left(A\text{ and }\overline{C}\right)\\&=\frac{5}{17}+\frac{14}{17}-\frac{4}{17}\\&=\frac{15}{17}\end{aligned}$$

(j) $P\left(\overline{B}\right)=1-P\left(B\right)=1-\frac{2}{17}=\frac{15}{17}$ (see part **(d)**)

$$P\left(\overline{C}\right)=1-P\left(C\right)=1-\frac{3}{17}=\frac{14}{17}$$

$\overline{B}$ and $\overline{C}$ means 'not a multiple of 8 and not a multiple of 5'.

$\overline{B}$ and $\overline{C}$: $1,\ 2,\ 3,\ 4,\ 6,\ 7,\ 9,\ 11,\ 12,\ 13,\ 14,\ 17$

$$\therefore P\left(\overline{B}\text{ and }\overline{C}\right)=\frac{12}{17}$$

$$\begin{aligned}P\left(\overline{B}\text{ or }\overline{C}\right)&=P\left(\overline{B}\right)+P\left(\overline{C}\right)-P\left(\overline{B}\text{ and }\overline{C}\right)\\&=\frac{15}{17}+\frac{14}{17}-\frac{12}{17}\\&=1\end{aligned}$$

(k) A and B are mutually exclusive, B and C are also mutually exclusive.

(A and C are not mutually exclusive as they contain the common element 15.)

4 There are $8+7+5=20$ marbles in the container. The events, 'red marble', 'red marble' and 'red marble' are all mutually exclusive.

(a) incorrect

$P(\text{red or black}) = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$

(b) incorrect

$P(\text{not white}) = 1 - P(\text{white}) = 1 - \frac{7}{20} = \frac{13}{20}$

(c) correct

$P(\text{neither black nor white}) = P(\text{red}) = \frac{8}{20} = \frac{2}{5}$

(d) correct

$P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{8}{20} = \frac{12}{20} = \frac{3}{5}$

6 **(a)** A number divisible by 2 and divisible by 3 is divisible by 6.

A and B: 6, 12, 18, 24, 30, 36, 42, 48

$P(A \text{ and } B) = \frac{8}{50} = \frac{4}{25}$

(b) A number divisible by 2 and divisible by 5 is divisible by 10.

A and C: 10, 20, 30, 40, 50

$P(A \text{ and } C) = \frac{5}{50} = \frac{1}{10}$

(c) A number divisible by 3 and divisible by 5 is divisible by 15.

B and C: 15, 30, 45

$P(B \text{ and } C) = \frac{3}{50}$

(d) A number divisible by 2, divisible by 3 and divisible by 5 is divisible by 30.

$P(A \text{ and } B \text{ and } C) = \frac{1}{50}$

(e) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= \frac{25}{50} + \frac{16}{50} - \frac{8}{50}$$
$$= \frac{33}{50}$$

(f) $P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$

$$= \frac{25}{50} + \frac{10}{50} - \frac{1}{10}$$
$$= \frac{3}{5}$$

(g) $P(B \text{ or } C) = P(B) + P(C) - P(B \text{ and } C)$

$$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50}$$
$$= \frac{23}{50}$$

(h) $P(A \text{ or } B \text{ or } C) = P([A \text{ or } B] \text{ or } C)$

$= P(A \text{ or } B) + P(C) - P([A \text{ or } B] \text{ and } C)$

$[A \text{ or } B]$ and C means it is divisible by 5 and also divisible by either 2 or 3.

This means all numbers divisible by 10, together with all numbers divisible by 15.

$[A \text{ or } B]$ and C: 10, 15, 20, 30, 40, 45, 50

Use the results from part (e) and that there are 10 multiples of 5, so $P(C) = \frac{10}{50}$,

$$\begin{aligned} P(A \text{ or } B \text{ or } C) &= P([A \text{ or } B] \text{ or } C) \\ &= P(A \text{ or } B) + P(C) - P([A \text{ or } B] \text{ and } C) \\ &= \frac{33}{50} + \frac{10}{50} - \frac{7}{50} \\ &= \frac{36}{50} \\ &= \frac{18}{25} \end{aligned}$$

This question could also be solved using a Venn diagram with three overlapping sets.

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8 Drawing a table is useful.

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(a) There are 26 totals greater than 5. Do not include totals of 5.

$$P(A)=\frac{26}{36}=\frac{13}{18}$$

(b) There are 21 totals less than 8. Do not include totals of 8.

$$P(B)=\frac{21}{36}=\frac{7}{12}$$

(c) 'Greater than 5' and 'less than 8' can only be 6 or 7. There are five totals of 6 and six totals of 7. $P(A \text{ and } B)=\frac{11}{36}$

(d) All numbers must be either be greater than 5 or less than 8 or both.

$\therefore P(A \text{ or } B)=1$

(e) $P(A)+P(B)-P(A \text{ and } B)=\frac{13}{18}+\frac{7}{12}-\frac{11}{36}$

$$=1$$
$$=P(A \text{ or } B)$$

$$\therefore P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$$

A and B are not mutually exclusive since two dice can sum up to either 6 or 7, which belongs to both sets.

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10 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$\frac{15}{16} = \frac{3}{8} + P(B) - \frac{1}{8}$$
$$P(B) = \frac{15}{16} - \frac{3}{8} + \frac{1}{8}$$
$$= \frac{11}{16}$$

12 (a) Since 20 students study both subjects, then $50 - 20 = 30$ study History only, and $30 - 20 = 10$ study English only.

$$P(\text{only History or only English or Both}) = \frac{30}{100} + \frac{10}{100} + \frac{20}{100} = \frac{3}{5}$$

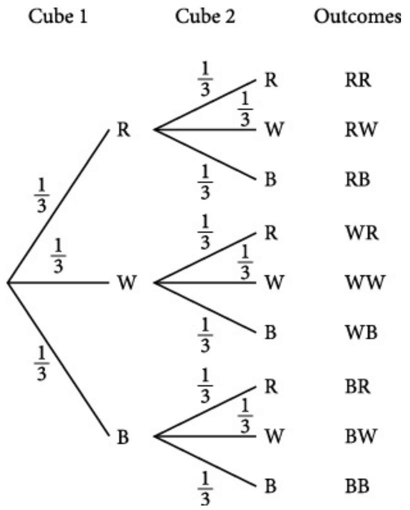
(b) $P(\text{History but not English}) = P(\text{History only}) = \frac{30}{100} = \frac{3}{10}$

(c) A total of 30 students study English and of these 20 also study History (study both).

$$P(\text{History, given that the student studies English}) = \frac{20}{30} = \frac{2}{3}$$

EXERCISE 9.4 SUCCESSIVE OUTCOMES

2



(a) There are 9 possible outcomes (3×3). 3 of these are the same colour (RR, WW, BB).

$$P(\text{same colour}) = \frac{3}{9} = \frac{1}{3}$$

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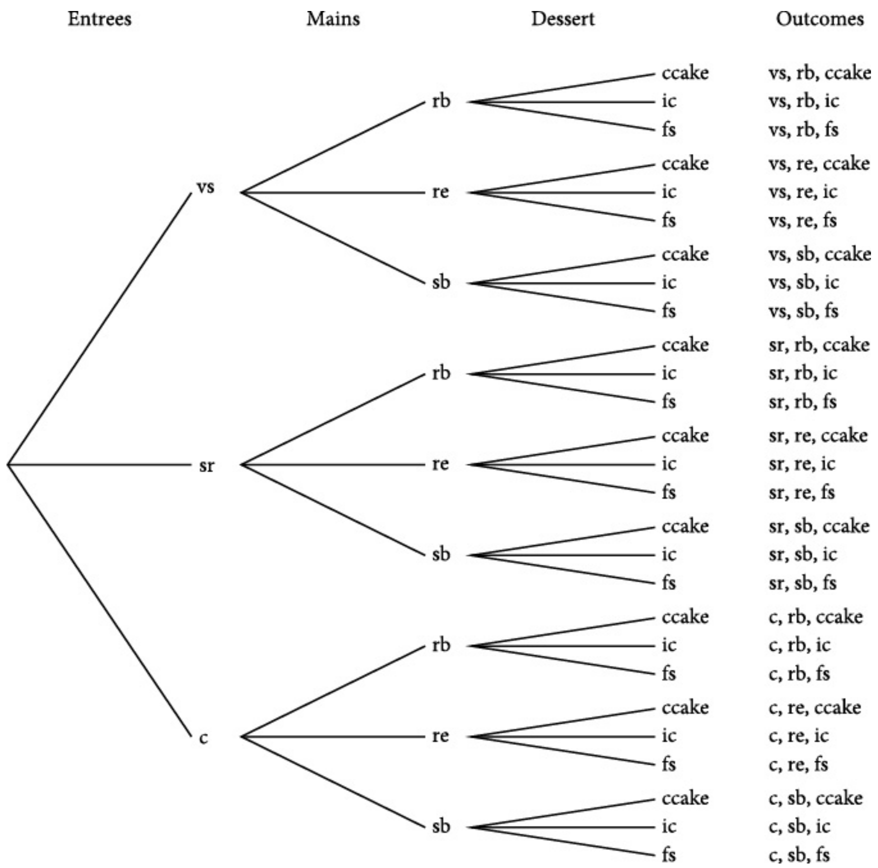
(b) This can occur in two ways: red and then white, or white and then red.

$$P(\text{one red, one white}) = \frac{2}{9}$$

4



(a) $3 \times 3 \times 3 = 27$

There are 27 different meal choices.

(b) There are two ways this can occur: One entrée, one main, and two choices for dessert.

$$p = \frac{2}{27}$$

(c) Two possibilities: One choice for entrées, two choices for main and one choice for dessert.

$$p = \frac{2}{27}$$

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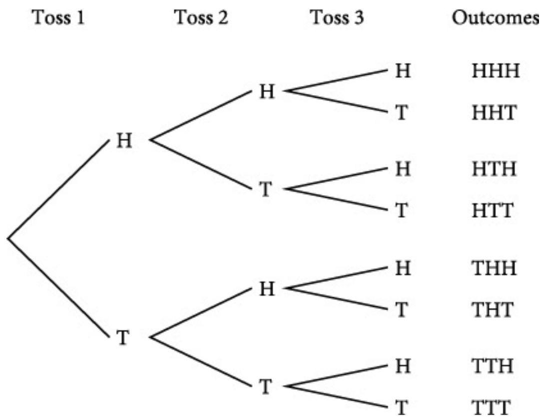
6



(a) There are 8 equally likely possibilities $(2\times2\times2)$. Three tails can only occur one way.

$$P(\text{TTT})=\frac{1}{8}$$

(b) There are 8 equally likely possibilities. One head occurs in three of these (HTT or THT or TTH).

$$P(\text{one head})=\frac{3}{8}$$

(c) There are 8 equally likely possibilities. Two heads occurs in three of these (HHT or HTH or THH).

$$P(\text{two heads})=\frac{3}{8}$$

(d) Three heads can only be HHH.

$$P(\text{HHH})=\frac{1}{8}$$

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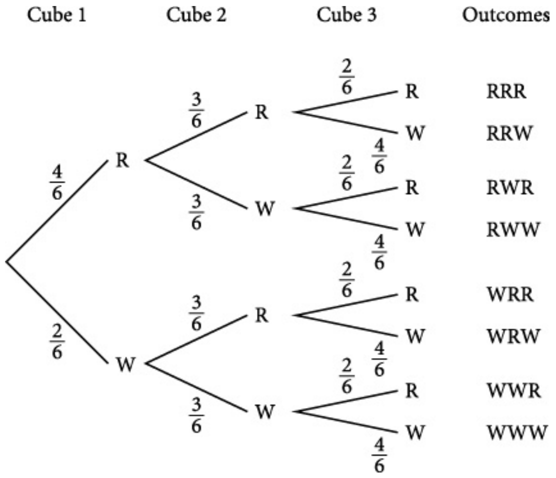
8 (a)



(b) $P(\text{RRR}) = \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} = \frac{1}{9}$

(c) $P(\text{WRR}) + P(\text{RWR}) + P(\text{RRW}) = \frac{2}{6} \times \frac{3}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{3}{6} \times \frac{4}{6}$

$$= \frac{1}{18} + \frac{1}{9} + \frac{2}{9}$$
$$= \frac{7}{18}$$

(d) $P(\text{WRR}) + P(\text{RWR}) + P(\text{RRW}) + P(\text{RRR}) = \frac{7}{18} + \frac{1}{9} = \frac{1}{2}$

Some clever students may deduce that, and because of the ‘symmetry’ (the question would be equivalent if you swapped the word ‘red’ and ‘white’ around), then the probability of ‘more white’ is the same as the probability of ‘more red’ and because you cannot have an equal number of red and white faces, then

$$P(\text{more red than white}) = P(\text{more red than white}) = \frac{1}{2}$$

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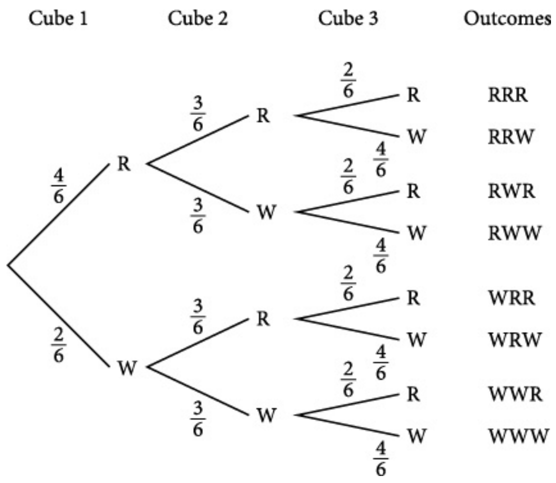
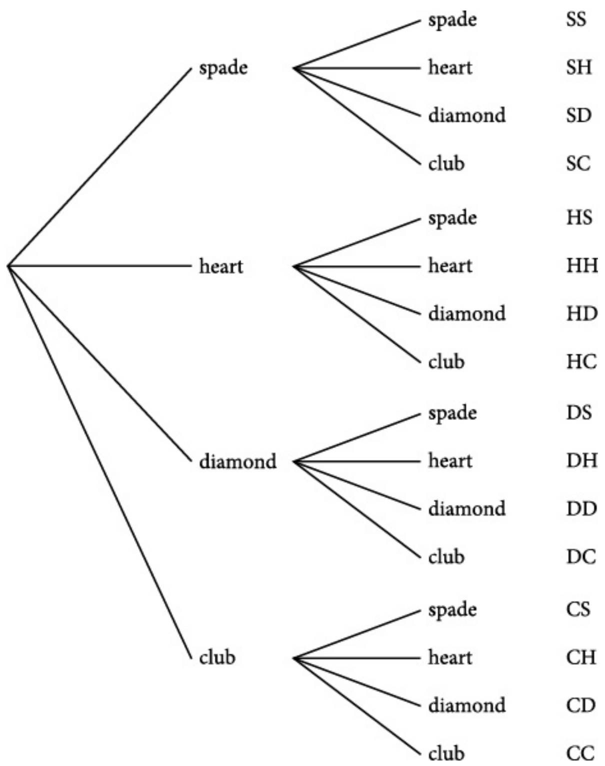
10





There are 16 equally likely possibilities (4×4).

(a) $P(\text{SS})=\dfrac{1}{16}$

(b) 16 of the outcomes have no hearts.

$$P(\text{no hearts})=\frac{9}{16}$$

Students may realise that the probability that each card is not a heart is $1-\dfrac{1}{4}=\dfrac{3}{4}$, so

$$P(\text{no hearts}) = \frac{3}{4} \times \frac{3}{4}$$
$$= \frac{9}{16}$$

(c) $P(\text{SS}) + P(\text{CC}) + P(\text{HH}) + P(\text{DD}) = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4}$

EXERCISE 9.5 INDEPENDENT EVENTS

2 **(a)** correct

There are $25 + 15 = 40$ students in the class.

$$P(\text{boy}) = \frac{25}{40}$$
$$= \frac{5}{8}$$

(b) correct

15 students wear glasses.

$$P(\text{wears glasses}) = \frac{15}{40} = \frac{3}{8}$$

(c) correct

$$P(\text{boy and wears glasses}) = \frac{8}{40} = \frac{1}{5}$$

(d) incorrect

There are $15 - 7 = 8$ girls who wear glasses.

$$P(\text{girl and does not wear glasses}) = \frac{8}{40} = \frac{1}{5}$$

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4 (a)

$$P(1,1)+P(2,2)+P(3,3)+P(4,4)+P(5,5)+P(6,6)=\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}$$
$$=\frac{1}{36}\times 6$$
$$=\frac{1}{6}$$

(b) There are 3 sums of 10, 2 sums of 11 and 1 sum of 12 out of a total of 36 sums.

$$P(\text{sum bigger than }9)=\frac{3+2+1}{36}=\frac{1}{6}$$

(c) $P(3,4)=\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

(d) $P(\text{even,odd})=\frac{3}{6}\times\frac{3}{6}=\frac{1}{4}$

(e) The smallest number on each die is 1, so the smallest possible sum is $1+1=2$.

It is impossible to have a sum less than 2, so $P(\text{sum}<2)=0$.

6 (a) There are two even numbers out of the set containing five numbers.

$$P(A)=\frac{2}{5}$$

(b) $P(B)=\frac{3}{6}=\frac{1}{2}$

(c) There are 2 sums of 10 and 1 sum of 11 out of a total of 30 equally likely sums.

$$P(C)=\frac{2+1}{30}=\frac{1}{10}$$

(d) This can happen in two ways (5, 5 and 6, 4) out of a total of 30 equally likely sums.

$$P(D)=\frac{2}{30}=\frac{1}{15}$$

8 We will assume that the two events are independent, so that the colour of hair does not affect the likelihood of riding a bicycle to school.

(a) $p=40\%\times 25\%=\frac{40}{100}\times\frac{25}{100}=\frac{4}{10}\times\frac{1}{4}=\frac{1}{10}$

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(b) $p = (1-40\%)\times(1-25\%) = 60\%\times 75\% = \dfrac{9}{20}$

(c) $P = 40\%\times(1-25\%) = 40\%\times 75\% = \dfrac{3}{10}$

(d) $P = (1-40\%)\times 25\% = 60\%\times 25\% = \dfrac{3}{20}$

10 (a) $P(D,D) = \dfrac{1}{4}\times\dfrac{1}{4} = \dfrac{1}{16}$

(b) $P(\text{not } D, \text{not } D) = \left(1-\dfrac{1}{4}\right)\times\left(1-\dfrac{1}{4}\right) = \dfrac{3}{4}\times\dfrac{3}{4} = \dfrac{9}{16}$

(c) $P(D, \text{not } D) + P(\text{not } D, D) = \dfrac{1}{4}\times\dfrac{3}{4} + \dfrac{3}{4}\times\dfrac{1}{4} = \dfrac{3}{8}$

(d) $P(D, \text{not } D) = \dfrac{1}{4}\times\dfrac{3}{4} = \dfrac{3}{16}$

(e) $P(\text{not } D, D) = \dfrac{3}{4}\times\dfrac{1}{4} = \dfrac{3}{16}$

(f) $P(\text{at least 1 } D) = P(D, \text{not } D) + P(\text{not } D, D) + P(D,D)$

$$= \frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}$$
$$= \frac{7}{16}$$

12 (a) $P(\text{WWW}) = \dfrac{1}{2}\times\dfrac{1}{3}\times\dfrac{1}{4} = \dfrac{1}{24}$

(b) $P(\text{LLL}) = \left(1-\dfrac{1}{2}\right)\times\left(1-\dfrac{1}{3}\right)\times\left(1-\dfrac{1}{4}\right) = \dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{3}{4} = \dfrac{1}{4}$

(c) $P(\text{WLL}) = \dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{3}{4} = \dfrac{1}{4}$

(d) $P(\text{LLW}) = \dfrac{1}{2}\times\dfrac{2}{3}\times\dfrac{1}{4} = \dfrac{1}{12}$

(e) $P(\text{WWL}) = \dfrac{1}{2}\times\dfrac{1}{3}\times\dfrac{3}{4}$

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$=\frac{1}{8}$

14 (a) $P(\text{HHHH})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$

(b) $P(\text{TTTT})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$

(c) $P(\text{HTHT})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$

(d) $P(\text{HHHT})=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$

(e) $P(\text{at least one H})=1-P(\text{TTTT})$

$=1-\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$

$=\frac{15}{16}$

or

$$\begin{aligned}P(\text{H in any one of the four tosses})&=P(\text{HTTT})+P(\text{THTT})+P(\text{TTHT})+P(\text{TTTH})\\&=\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}\\&=\frac{1}{4}\end{aligned}$$

16 You can assume the events are independent.

(a) $P(\text{FFF})=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$

(b) $P(\text{NNN})=\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$

(c) $P(\text{FNF})=\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}=\frac{4}{27}$

18 (a) $P(\text{AA})=\frac{2}{3}\times\frac{3}{5}=\frac{2}{5}$

(b) $P(\text{AN})+P(\text{NA})=\frac{2}{3}\times\left(1-\frac{2}{5}\right)+\left(1-\frac{1}{3}\right)\times\frac{3}{5}$

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$$= \frac{2}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{5}$$
$$= \frac{4}{15} + \frac{3}{15}$$
$$= \frac{7}{15}$$

(c) $P(\text{at least one alive}) = 1 - P(\text{NN})$

$$= 1 - \frac{1}{3} \times \frac{2}{5}$$
$$= \frac{13}{15}$$

20 (a) Every day is an independent event.

$$P(\text{miss bus}) = \frac{1}{8 \times 5} = \frac{1}{40}$$

$$P(\text{not miss bus}) = 1 - P(\text{miss}) = 1 - \frac{1}{40} = \frac{39}{40}$$

(b) $P(\text{not miss, not miss}) = \frac{39}{40} \times \frac{39}{40}$

$$= \left(\frac{39}{40}\right)^2$$
$$= \frac{1521}{1600}$$

(c) $P(\text{not miss for 5 days}) = \frac{39}{40} \times \frac{39}{40} \times \frac{39}{40} \times \frac{39}{40} \times \frac{39}{40} = \left(\frac{39}{40}\right)^5 \approx 0.88$

(d) $P(\text{miss at least 1}) = 1 - P(\text{not miss for 5 days}) = 1 - \left(\frac{39}{40}\right)^5 \approx 0.12$

22 (a) $P(\text{double}) = \frac{1}{13} \times \frac{1}{16} = \frac{1}{208}$

(b) $P(\text{quinella first race}) = \frac{1}{13} \times \frac{1}{12} + \frac{1}{13} \times \frac{1}{12} = \frac{1}{78}$

(c) $P(\text{quinella second race}) = \frac{1}{16} \times \frac{1}{15} + \frac{1}{16} \times \frac{1}{15} = \frac{1}{120}$

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(d) $P(\text{quinella first race and quinella second race}) = \frac{1}{78} \times \frac{1}{120} = \frac{1}{9360}$

EXERCISE 9.6 DEPENDENT EVENTS

2 B

The probabilities that the first ball is green and that the first ball is white are $\frac{6}{10}$ and $\frac{4}{10}$.

If the first ball is green, there will be 9 balls left, of which 5 are green and 4 are white.

$\therefore P(\text{G}|\text{G}) = \frac{5}{9}$

If the first ball is white, there will be 9 balls left, of which 6 are green and 3 are white.

$\therefore P(\text{W}|\text{W}) = \frac{3}{9}$

$$\begin{aligned} P(\text{same colour}) &= P(\text{GG}) + P(\text{WW}) \\ &= P(\text{G}) \times P(\text{G}|\text{G}) + P(\text{W}) \times P(\text{W}|\text{W}) \\ &= \frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{3}{9} \\ &= \frac{30+12}{90} \\ &= \frac{42}{90} \\ &= \frac{7}{15} \end{aligned}$$

4 If the first egg is a double yolk, there will be two double yolks in the remaining 11 eggs. If the next is also a double yolk, there will be only one double yolk in the remaining 10 eggs.

$$\begin{aligned} P(\text{3 double yolks}) &= P(\text{DY}) \times P(\text{DY}|\text{DY}) \times P(\text{DY}|\text{DY,DY}) \\ &= \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} \\ &= \frac{1}{220} \end{aligned}$$

6 **(a)** correct

After taking out a diamond, there will be 12 diamonds left in a pack containing now 51 cards.

$$\begin{aligned}P(DD) &= P(D)\times P(D\mid D)\\ &= \frac{1}{4}\times\frac{12}{51}\\ &= \frac{1}{17}\end{aligned}$$

(b) incorrect

$$\begin{aligned}P(\text{same suit}) &= P(DD)+P(HH)+P(SS)+P(CC)\\ &= \frac{13}{52}\times\frac{12}{51}+\frac{13}{52}\times\frac{12}{51}+\frac{13}{52}\times\frac{12}{51}+\frac{13}{52}\times\frac{12}{51}\\ &= \frac{1}{17}\times 4\\ &= \frac{4}{17}\end{aligned}$$

(c) correct

After taking out a spade, there will be 13 clubs left in a pack containing now 51 cards.

After taking out a club, there will be 13 spades left in a pack containing now 51 cards.

$$P(\text{SC})+P(\text{CS}) = \frac{1}{4}\times\frac{13}{51}+\frac{1}{4}\times\frac{13}{51} = \frac{26}{204} = \frac{13}{102}$$

(d) correct

$$P(\text{different suits}) = 1-(\text{same suit}) = 1-\frac{4}{17} = \frac{13}{17}$$

8 **(a)** There are initially $7+5=12$ people. If a teacher is chosen, there will be 6 teachers and 5 students out of the 11 remaining. If a student is chosen, there will be 7 teachers and 3 students left.

$$P(\text{both teachers}) = \frac{7}{12}\times\frac{6}{11} = \frac{7}{22}$$

(b) $P(\text{both pupils}) = \dfrac{5}{12}\times\dfrac{4}{11} = \dfrac{5}{33}$

(c) $P(\text{teacher, pupil})+P(\text{pupil, teacher}) = \dfrac{7}{12}\times\dfrac{5}{11}+\dfrac{5}{12}\times\dfrac{7}{11}$

$$= \frac{35}{66}$$

(d) The only possible outcomes are ‘one teacher and one pupil’, ‘both teachers’ and ‘both pupils’. Therefore ‘one teacher and one pupil’ is the complementary event to ‘both teachers’ and ‘both pupils’.

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$$\begin{aligned} P(\text{teacher, pupil}) + P(\text{pupil, teacher}) &= 1 - P(\text{both teachers}) - P(\text{both pupils}) \\ &= 1 - \frac{7}{22} - \frac{5}{33} \\ &= \frac{35}{66} \end{aligned}$$

10 $P(\text{WW}) = \frac{1}{10} \times \frac{1}{9}$

$= \frac{1}{90}$

12 (a) $P(\text{gorilla A, gorilla A}) = \frac{10}{15} \times \frac{9}{14}$

$= \frac{3}{7}$

(b) $P(\text{chimpanzee B, chimpanzee B}) = \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$

(c) $P(\text{gorilla A, chimpanzee B}) + P(\text{chimpanzee A, gorilla B}) = \frac{10}{15} \times \frac{6}{10} + \frac{5}{15} \times \frac{4}{10}$

$= \frac{8}{15}$

14 $P(\text{even, even, even}) = \frac{20}{40} \times \frac{19}{39} \times \frac{18}{38}$

$= \frac{3}{26}$

16 $P(\text{1st, 2nd, 3rd}) = \frac{1}{9} \times \frac{1}{8} \times \frac{1}{7}$

$= \frac{1}{504}$

18 (a) If there are a large number of coins, we can ignore the small change caused by withdrawing a coin.

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$$P(5,5,10)+P(5,10,5)+P(10,5,5)=\frac{2}{5}\times\frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{3}{5}\times\frac{2}{5}+\frac{3}{5}\times\frac{2}{5}\times\frac{2}{5}$$
$$=\frac{36}{125}$$

(b) $$P(5,5,10)+P(5,10,5)+P(10,5,5)+P(5,5,5)=\frac{36}{125}+\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}$$
$$=\frac{44}{125}$$

(c) 'Not more than two of them are five-cent coins' is the complementary event to 'all of them are five-cent coins'.

$$P(\text{not more than two } 5)=1-P(5,5,5)$$
$$=1-\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}$$
$$=\frac{117}{125}$$

20 $P(\text{Ace C, not, not})+P(\text{not, Ace C, not})+P(\text{not, not, Ace C})$

$$=\frac{1}{52}\times\frac{51}{51}\times\frac{50}{50}+\frac{51}{52}\times\frac{1}{51}\times\frac{50}{50}+\frac{51}{52}\times\frac{50}{51}\times\frac{1}{50}$$
$$=\frac{3}{52}$$

22 (a) $P(\text{even, even})=\frac{5}{10}\times\frac{4}{9}=\frac{2}{9}$

(b) $P(\text{even, odd})+P(\text{odd, even})=\frac{5}{10}\times\frac{5}{9}+\frac{5}{10}\times\frac{5}{9}=\frac{50}{90}=\frac{5}{9}$

(c) The combinations of two numbers that give a sum of 12:

$2+10,\ 10+2,\ 3+9,\ 9+3,\ 4+8,\ 8+4,\ 5+7,\ 7+5$

There are no doubles of any number since it is without replacement.

The probability of any of these combinations is $\frac{1}{10}\times\frac{1}{9}=\frac{1}{90}$

$$P(\text{sum }=12)=8\times\frac{1}{90}=\frac{4}{45}$$

24 A ratio of $9:1$ means that out of every 10 shots, 9 hit the bullseye. Let success be S and failure F.

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$P(\mathrm{S})=\frac{9}{10}$ and $P(\mathrm{F})=1-\frac{9}{10}=\frac{1}{10}$

(a) $P(\mathrm{SSS})=\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}=\frac{729}{1000}=0.729$

(b) $P(\text{at least 2 S})=P(\mathrm{SSF})+P(\mathrm{SFS})+P(\mathrm{FSS})+P(\mathrm{SSS})$

$$\begin{aligned}&=\frac{9}{10}\times\frac{9}{10}\times\frac{1}{10}+\frac{9}{10}\times\frac{1}{10}\times\frac{9}{10}+\frac{1}{10}\times\frac{9}{10}\times\frac{9}{10}+\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\\&=\frac{972}{1000}\\&=\frac{243}{250}\\&=0.972\end{aligned}$$

(c) $P(\text{not more than 1 S})=1-P(\text{at least 2 }S)$

$$\begin{aligned}&=1-\frac{243}{250}\\&=\frac{7}{250}\\&=0.028\end{aligned}$$

26 (a) Because there is a large number of cubes, we can consider the events independent.

$P(\mathrm{W})=60\%=\frac{60}{100}=\frac{3}{5}$ and $P(\mathrm{B})=40\%=\frac{40}{100}=\frac{2}{5}$.

$$\begin{aligned}P(\mathrm{WW})+P(\mathrm{BB})&=\frac{3}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{2}{5}\\&=\frac{13}{25}\end{aligned}$$

(b) $P(\text{different colours})=1-\left[P(WW)+P(BB)\right]$

$$\begin{aligned}&=1-\frac{13}{25}\\&=\frac{12}{25}\end{aligned}$$

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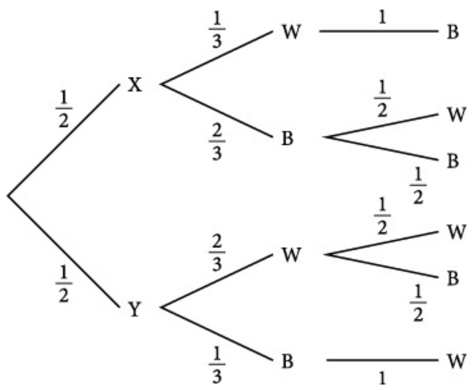
28

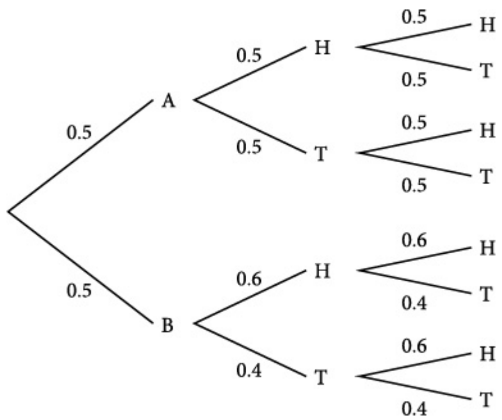


(a) $P(\mathrm{BB})+P(\mathrm{WW})=\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}+\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}=\frac{1}{3}$

(b) $P(\text{different colours})=1-\left[P(BB)+P(WW)\right]=1-\frac{1}{3}=\frac{2}{3}$

30 The probability of choosing one of the two coins is $\frac{1}{2}$.

Draw a tree diagram.



$P(\mathrm{AHH})+P(\mathrm{BHH})=0.5\times0.5\times0.5+0.5\times0.6\times0.6=0.305$

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CHAPTER REVIEW 9

2 **(a)** These events are independent.

$$P(BB)+P(WW)+P(GG)+P(RR)=\frac{1}{4}\times\frac{1}{6}+\frac{1}{4}\times\frac{1}{6}+\frac{1}{4}\times\frac{1}{6}+\frac{1}{4}\times\frac{1}{6}=\frac{1}{6}$$

(b) $P(\text{different colours})=1-\left[P(BB)+P(WW)+P(GG)+P(RR)\right]$

$$=1-\frac{1}{6}$$
$$=\frac{5}{6}$$

(c) $P(\text{at least 1 } B)=1-P(\text{no } B)$

$$=1-\frac{3}{4}\times\frac{5}{6}$$
$$=\frac{3}{8}$$

(d) $P(\text{neither black nor white})=\frac{2}{4}\times\frac{4}{6}=\frac{1}{3}$

4 On average, for every 5 buttons drawn out, 3 will be red and 2 will be white.

$P(\text{R})=\frac{3}{5}$ and $P(\text{W})=\frac{2}{5}$.

Because the jar contains a large number of buttons, we may assume the events are independent.

(a) $P(\text{RRW})+P(\text{RWR})+P(\text{WRR})=\frac{3}{5}\times\frac{3}{5}\times\frac{2}{5}+\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{3}{5}\times\frac{3}{5}$

$$=\frac{54}{125}$$

(b) $P(\text{not more than one W})=P(\text{RRR})+P(\text{RRW})+P(\text{RWR})+P(\text{WRR})$

$$=\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}+\frac{3}{5}\times\frac{3}{5}\times\frac{2}{5}+\frac{3}{5}\times\frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{3}{5}\times\frac{3}{5}$$
$$=\frac{81}{125}$$

6 The number of students studying at least one language is $25-5=20$ students.

If we add the French students and German students together we get $18+12=30$, but there are only 20 students studying a language in total. This means we have counted $35-25=10$ students twice. 10 students study both French and German.

(a) The number of students who study French only is $18-10=8$.

$$P(\text{French only})=\frac{8}{25}$$

(b) The number of students who study German only is $12-10=2$

$$P(\text{German only})=\frac{2}{25}$$

(c) Number of students study French or German or both is $8+2+10=20$.

Alternatively, it is $25-5=20$ students.

$$P(\text{French or German or both})=\frac{20}{25}=\frac{4}{5}$$

(d) $P(\text{both French and German})=\dfrac{10}{25}=\dfrac{2}{5}$

8 (a) $P(B\text{ and even})=\dfrac{1}{5}\times\dfrac{2}{5}=\dfrac{2}{25}$

(b) $P(C\text{ or }D\text{ and odd})=\dfrac{2}{5}\times\dfrac{3}{5}=\dfrac{6}{25}$

(c) $P(E\text{ and even})+P(C\text{ and number}>3)=\dfrac{1}{5}\times\dfrac{2}{5}+\dfrac{1}{5}\times\dfrac{2}{5}=\dfrac{4}{25}$

(d) There is only one number that is greater than 2 that hasn't already been counted in the probability of $P(\text{consonant and odd})$.

$$P(\text{consonant and odd})+P(\text{consonant and number}>2)=\frac{3}{5}\times\frac{3}{5}+\frac{3}{5}\times\frac{1}{5}=\frac{12}{25}$$

10 (a) $P(\text{WWW})=\dfrac{2}{3}\times\dfrac{2}{3}\times\dfrac{2}{3}=\dfrac{8}{27}$

(b) For any one match, $P(\text{L})=1-\dfrac{2}{3}=\dfrac{1}{3}$.

$$P(\text{LLL})=\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}=\frac{1}{27}$$

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(c) $P(\text{WLW}) = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$

(d) $P(\text{LWW}) = \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$

12 (a) $P(\text{A, defective}) = 25\% \times 5\% = \frac{1}{4} \times \frac{1}{20} = \frac{1}{80} = 0.0125 \text{ or } 1.25\%$

(b) $P(B, \text{not defective}) = 35\% \times 96\% = \frac{7}{20} \times \frac{24}{25} = \frac{42}{125} = 0.336 \text{ or } 33.6\%$

14 (a) $P(\text{defective}) = \frac{3}{10}$

(b) If the first is defective, there will be 2 defectives out of 9 lights left.

$P(\text{defective, defective}) = \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$

(c) $P(\text{not defective, not defective}) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$

(d) $P(\text{exactly 1 defective}) = P(\text{defective, not defective}) + P(\text{not defective, defective})$

$$\begin{aligned}
&= \frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9} \\
&= \frac{21 + 21}{90} \\
&= \frac{7}{15}
\end{aligned}$$

16 (a) $P(\text{RRR}) + P(\text{WWW}) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} + \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{5}$

(b) $P(\text{RRR}) + P(\text{WWW}) = \frac{6}{10} \times \frac{6}{10} \times \frac{6}{10} + \frac{4}{10} \times \frac{4}{10} \times \frac{4}{10} = \frac{7}{25}$

18 (a) 'No more than two rejects' is the complement to all three being rejects.

Note that $20\% = \frac{1}{5}$.

$$\begin{aligned} P(\text{no more than 2 rejects}) &= 1 - P(3\text{ rejects}) \\ &= 1 - 20\% \times 20\% \times 20\% \\ &= 1 - \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \\ &= \frac{124}{125} \end{aligned}$$

(b) $P(\text{at least 2 rejects}) = P(2\text{ rejects}) + P(3\text{ rejects})$

$$\begin{aligned} &= \frac{1}{5}\times\frac{1}{5}\times\frac{4}{5}+\frac{1}{5}\times\frac{4}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\times\frac{1}{5}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5} \\ &= \frac{13}{125} \end{aligned}$$

20 (a) Because there is a large sample of people, we can consider the events independent.

A ratio of $7:3$ means that out of every 10 people, 7 will be in favour and 3 will be against.. Let in favour be F and against A.

$P(\text{F}) = \frac{7}{10}$ and $P(\text{A}) = 1 - \frac{7}{10} = \frac{3}{10}$

$$P(\text{FFF}) = \frac{7}{10}\times\frac{7}{10}\times\frac{7}{10} = \frac{343}{1000} = 0.343$$

(b) $P(\text{FFA}) + P(\text{FAF}) + P(\text{AFF}) + P(\text{FFF})$

$$\begin{aligned} &= \frac{7}{10}\times\frac{7}{10}\times\frac{3}{10}+\frac{7}{10}\times\frac{3}{10}\times\frac{7}{10}+\frac{3}{10}\times\frac{7}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{7}{10}\times\frac{7}{10} \\ &= \frac{784}{1000} \\ &= \frac{98}{125} \\ &= 0.784 \end{aligned}$$

(c) $P(\text{not more than two against}) = 1 - P(\text{AAA}) = 1 - \frac{3}{10}\times\frac{3}{10}\times\frac{3}{10} = \frac{973}{1000} = 0.973$

22 (a) These events are not independent.

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$$\begin{aligned} P(\mathrm{RR}) &= \frac{5}{10} \times \frac{4}{9} \\ &= \frac{2}{9} \end{aligned}$$

(b) $P(\text{at least one green}) = 1 - P(\mathrm{RR}) = 1 - \frac{2}{9} = \frac{7}{9}$

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