ADVANCED MATHEMATICS

Trigonometry (Adv), T1 Trigonometry and Measure of Angles (Adv)

Trig Ratios, Sine and Cosine Rules (Y11)

3D Trigonometry (Y11)

Bearings (Y11)

Trigonometry (Adv), T2 Trig Functions and Identities (Adv)

Exact Trig Ratios (Y11)

Trig Identities and Harder Equations (Y11)

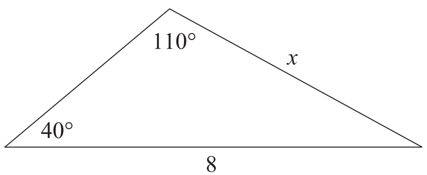
Teacher: Cathyanne Horvat

Exam Equivalent Time: 54 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Trigonometry, 2ADV T1 2019 HSC 11a

Using the sine rule, find the value of x correct to one decimal place. ***(2 marks)***



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2. Trigonometry, 2ADV T2 2009 HSC 1e

Find the exact value of θ such that $2\cos\theta = 1$, where $0 \le \theta \le \frac{\pi}{2}$. ***(2 marks)***

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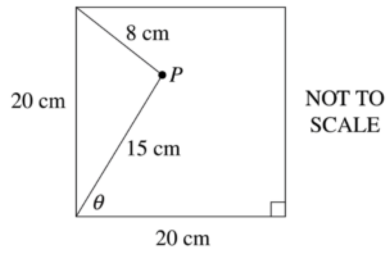
3. Trigonometry, 2ADV T1 2016 HSC 12c

Square tiles of side length 20 cm are being used to tile a bathroom.

The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.

To locate the point P the tiler needs to know the size of the angle θ shown in the diagram.



Find the size of the angle θ to the nearest degree. ***(3 marks)***

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4. Trigonometry, 2ADV T2 SM-Bank 41

Prove that

$$(\sec x + \tan x)(\sec x - \tan x) = 1. \quad \textit{(2 marks)}$$

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5. Trigonometry, 2ADV T2 SM-Bank 43

Find the exact value of

$$\cot\left(-\frac{5\pi}{6}\right). \quad \textit{(2 marks)}$$

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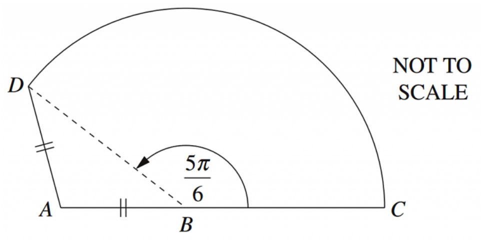
6. Trigonometry, 2ADV T1 2006 HSC 4a



In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \dfrac{5\pi}{6}$

The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

Copy or trace the diagram into your writing booklet.

i. Show that $\angle DAB = \dfrac{2\pi}{3}$. *(1 mark)*

ii. Find the length of BD. *(2 marks)*

iii. Find the area of the garden $ABCD$. *(2 marks)*

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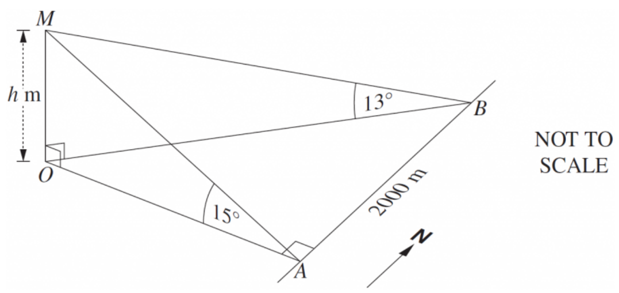
7. Trigonometry, 2ADV' T1 2015 HSC 12c

A person walks 2000 metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 15°.

From point B, the angle of elevation to the top of the mountain is 13°.



i. Show that $OA = h \cot 15^\circ$. *(1 mark)*

ii. Hence, find the value of h. *(2 marks)*

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8. Trigonometry, 2ADV T2 SM-Bank 31

Given $\cot\theta = -\dfrac{24}{7}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, find the exact value of

i. $\sec\theta$ *(2 marks)*

ii. $\sin\theta$ *(1 mark)*

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9. Trigonometry, 2ADV T2 2011 HSC 2b

Find the exact values of x such that $2\sin x = -\sqrt{3}$, where $0 \le x \le 2\pi$. *(2 marks)*

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10. Trigonometry, 2ADV T2 SM-Bank 2

Find all solutions of the equation $2\cos\theta = \sqrt{3}\cot\theta$, for $0 \le \theta \le 2\pi$ *(3 marks)*

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11. Trigonometry, 2ADV T2 SM-Bank 32

Express $5\cot^2 x - 2\operatorname{cosec} x + 2$ in terms of $\operatorname{cosec} x$ and hence solve

$5\cot^2 x - 2\operatorname{cosec} x + 2 = 0$ for $0 < x < 2\pi$. *(3 marks)*

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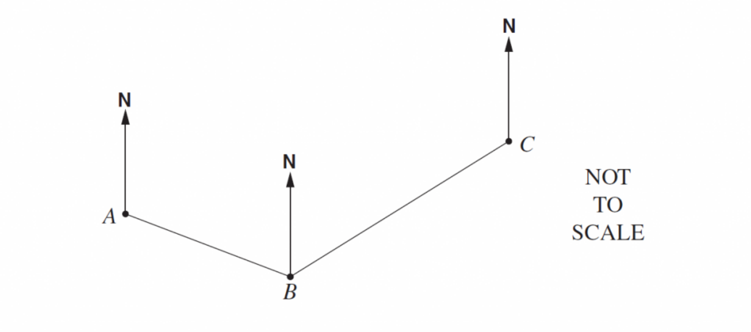
12. Trigonometry, 2ADV* T1 2011 HSC 24c

A ship sails 6 km from A to B on a bearing of 121°. It then sails 9 km to C. The size of angle ABC is 114°.



Copy the diagram into your writing booklet and show all the information on it.

i. What is the bearing of C from B? *(1 mark)*

ii. Find the distance AC. Give your answer correct to the nearest kilometre. *(2 marks)*

iii. What is the bearing of A from C? Give your answer correct to the nearest degree. *(3 marks)*

Worked Solutions

1. Trigonometry, 2ADV T1 2019 HSC 11a

$$\begin{aligned}\frac{x}{\sin 40^\circ} &= \frac{8}{\sin 110^\circ}\\ x &= \frac{8\times\sin 40^\circ}{\sin 110^\circ}\\ &= 5.47\\ &= 5.5\ (1\text{ d.p.})\end{aligned}$$

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2. Trigonometry, 2ADV T2 2009 HSC 1e

$$\begin{aligned}2\cos\theta &= 1\\ \cos\theta &= \frac{1}{2}\\ \therefore\ \theta &= \frac{\pi}{3},\quad 0\le\theta\le\frac{\pi}{2}\end{aligned}$$

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3. Trigonometry, 2ADV T1 2016 HSC 12c

$\alpha+\theta = 90$

Using the cosine rule,

$$\begin{aligned}\cos\alpha &= \frac{20^2+15^2-8^2}{2\times 20\times 15}\\ &= 0.935\\ \alpha &= 20.7\ldots^\circ\end{aligned}$$

$$\begin{aligned}\therefore\theta &= 90-20.7\ldots\\ &= 69.22\ldots\\ &= 69^\circ\ (\text{nearest degree})\end{aligned}$$

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4. Trigonometry, 2ADV T2 SM-Bank 41

$$\begin{aligned}\text{LHS} &= (\sec x+\tan x)(\sec x-\tan x)\\ &= \sec^2 x-\tan^2 x\\ &= \frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1-\sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x}\\ &= 1\\ &= \text{RHS}\end{aligned}$$

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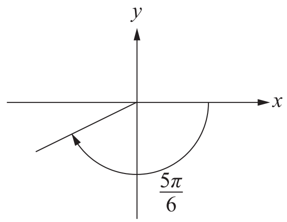
5. Trigonometry, 2ADV T2 SM-Bank 43



$$\begin{aligned}\cot\left(-\frac{5\pi}{6}\right) &= \frac{1}{\tan\left(-\frac{5\pi}{6}\right)}\\ &= \frac{1}{\tan\left(\frac{\pi}{6}\right)}\\ &= \frac{1}{\frac{1}{\sqrt{3}}}\\ &= \sqrt{3}\end{aligned}$$

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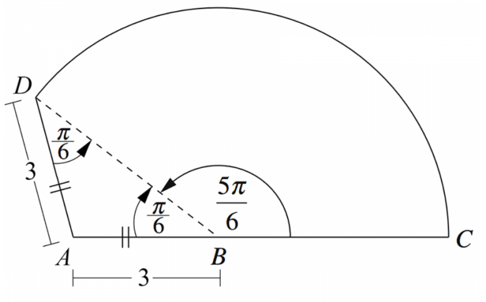
6. Trigonometry, 2ADV T1 2006 HSC 4a

i.



Show $\angle DAB = \dfrac{2\pi}{3}$

$\angle DBA = \pi - \dfrac{5\pi}{6}$ (π radians in straight angle ABC)

$= \dfrac{\pi}{6}$ radians

$\therefore \angle BDA = \dfrac{\pi}{6}$ radians (base angles of isosceles ΔADB)

$\therefore \angle DAB = \pi - \left(\dfrac{\pi}{6} + \dfrac{\pi}{6}\right)$ (angle sum of ΔADB)

$= \dfrac{2\pi}{3}$ radians . . . as required

ii. Using the cosine rule:

$$BD^2 = AD^2 + AB^2 - 2 \times AD \times AB \times \cos\frac{2\pi}{3}$$

$$= 9 + 9 - (2 \times 3 \times 3 \times -0.5)$$

$$= 27$$

$$\therefore BD = \sqrt{27}$$

$$= 3\sqrt{3} \text{ m}$$

iii. Area of $\Delta ADB = \dfrac{1}{2}ab\sin C$

$$= \frac{1}{2} \times 3 \times 3 \times \sin\frac{2\pi}{3}$$

$$= \frac{9}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{9\sqrt{3}}{4} \text{ m}^2$$

Area of sector BCD

$$= \frac{\frac{5\pi}{6}}{2\pi} \times \pi r^2$$

$$= \frac{5\pi}{12} \times \left(3\sqrt{3}\right)^2$$

$$= \frac{45\pi}{4} \text{ m}^2$$

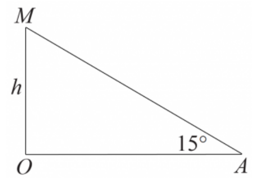
$\therefore$ Area of garden $ABCD$

$$= \frac{9\sqrt{3}}{4} + \frac{45\pi}{4}$$

$$= \frac{9\sqrt{3} + 45\pi}{4} \text{ m}^2$$

7. Trigonometry, 2ADV' T1 2015 HSC 12c

i. Show $OA = h\cot 15^\circ$



In ΔMOA,

$$\tan 15^\circ = \frac{h}{OA}$$

$$OA = \frac{h}{\tan 15^\circ}$$

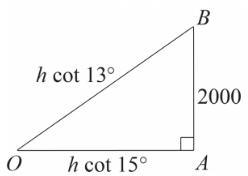
$$= h\cot 15^\circ \quad \text{...as required}$$

ii. In ΔMOB

$$\tan 13^\circ = \frac{h}{OB}$$

$$OB = \frac{h}{\tan 13^\circ}$$

$$= h\cot 13^\circ$$



In ΔAOB:

$$OA^2 + AB^2 = OB^2$$

$$OB^2 - OA^2 = AB^2$$

$$(h\cot 13^\circ)^2 - (h\cot 15^\circ)^2 = 2000^2$$

$$h^2\left[\left(\cot^2 13^\circ - \cot^2 15^\circ\right)\right] = 2000^2$$

$$h^2 = \frac{2000^2}{\cot^2 13^\circ - \cot^2 15^\circ}$$

$$\therefore h = \sqrt{\frac{2000^2}{\cot^2 13^\circ - \cot^2 15^\circ}}$$

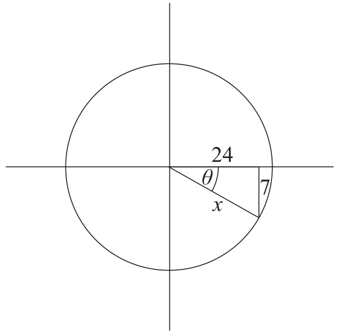
$$= 909.704\ldots$$

$$= 910 \text{ m (nearest metre)}$$

8. Trigonometry, 2ADV T2 SM-Bank 31

i. $\cot\theta = -\dfrac{24}{7} \quad \Rightarrow \quad \tan\theta = -\dfrac{7}{24}$

Graphically, given $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$:



$$x = \sqrt{24^2 + 7^2} = 25$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{\frac{24}{25}}$$

$$= \frac{25}{24}$$

ii. $\sin\theta = -\dfrac{7}{25}$

9. Trigonometry, 2ADV T2 2011 HSC 2b

$$2\sin x = -\sqrt{3} \quad \text{where} \quad 0 \le x \le 2\pi$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Since $\sin x$ is negative in $3^{\text{rd}}/4^{\text{th}}$ quadrants

$$x = \pi + \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}$$

$$= \frac{4\pi}{3},\ \frac{5\pi}{3} \quad \text{radians}$$

MARKER'S COMMENT: Better responses found the reference angle and then identified the correct quadrants, as shown clearly in the worked solution.

10. Trigonometry, 2ADV T2 SM-Bank 2

$$2\cos\theta = \sqrt{3}\cot\theta$$
$$2\cos\theta - \sqrt{3}\cot\theta = 0$$
$$2\cos\theta - \sqrt{3}\frac{\cos\theta}{\sin\theta} = 0$$
$$\left(2 - \frac{\sqrt{3}}{\sin\theta}\right)\cos\theta = 0$$

If $\cos\theta = 0$,

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

If $2 - \dfrac{\sqrt{3}}{\sin\theta} = 0 \quad \Rightarrow \quad \sin\theta = \dfrac{\sqrt{3}}{2}$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

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11. Trigonometry, 2ADV T2 SM-Bank 32

$$\begin{aligned}\cot^2 x &= \frac{\cos^2 x}{\sin^2 x}\\ &= \frac{1 - \sin^2 x}{\sin^2 x}\\ &= \text{cosec}^2 x - 1\end{aligned}$$

$$\begin{aligned}5\cot^2 x - 2\text{cosec } x + 2 &= 0\\ 5\left(\text{cosec}^2 x - 1\right) - 2\text{cosec } x + 2 &= 0\\ 5\text{cosec}^2 x - 2\text{cosec } x - 3 &= 0\\ (5\text{cosec } x + 3)(\text{cosec } x - 1) &= 0\end{aligned}$$

$\text{cosec } x = -\dfrac{3}{5} \qquad \text{cosec } x = 1$

$\sin x = -\dfrac{5}{3} \qquad \sin x = 1$

(no solution) $\qquad x = \dfrac{\pi}{2}$

$$\therefore x = \frac{\pi}{2}$$

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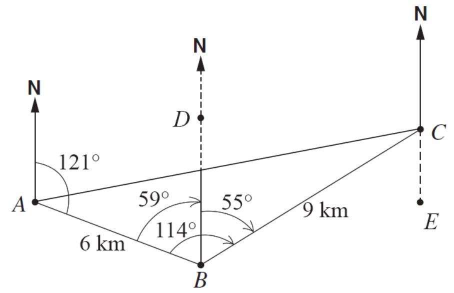
12. Trigonometry, 2ADV* T1 2011 HSC 24c

i.



> **STRATEGY:** This deserves repeating again: *Draw North-South parallel lines through major points to make the angle calculations easier!*

Let point D be due North of point B

$$\begin{aligned}\angle ABD &= 180 - 121 \text{ (cointerior with } \angle A)\\ &= 59^\circ\\ \angle DBC &= 114 - 59\\ &= 55^\circ\end{aligned}$$

$\therefore$ Bearing of C from B is 055°

ii. Using cosine rule:

$$\begin{aligned}AC^2 &= AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC\\ &= 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 114^\circ\\ &= 160.9275\ldots\end{aligned}$$

$$\begin{aligned}\therefore AC &= 12.685\ldots \quad (\text{Noting } AC > 0)\\ &= 13 \text{ km (nearest km)}\end{aligned}$$

iii. Need to find $\angle ACB$ (see diagram)

$$\begin{aligned}\cos\angle ACB &= \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}\\ &= \frac{(12.685\ldots)^2 + 9^2 - 6^2}{2 \times (12.685..) \times 9}\\ &= 0.9018\ldots\\ \angle ACB &= 25.6^\circ \text{ (to 1 d.p.)}\end{aligned}$$

> **MARKER'S COMMENT:** The best responses showed clear working on the diagram.

From diagram,

$\angle BCE = 55^\circ$ (alternate angle, $DB \parallel CE$)

$\therefore$ Bearing of A from C

$$\begin{aligned}&= 180 + 55 + 25.6\\ &= 260.6\end{aligned}$$

$$= \mathbf{261^\circ} \text{ (nearest degree)}$$