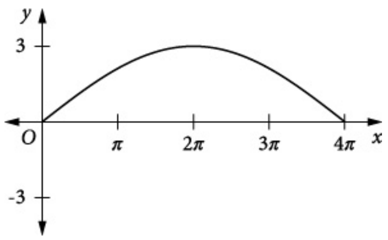
EXERCISE 12.1 TRANSFORMATION OF GRAPHS OF THE TRIGONOMETRIC FUNCTIONS

2 B

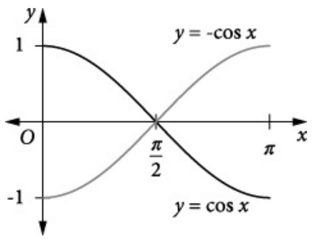
The amplitude is 3 and the period is $\dfrac{2\pi}{\frac{1}{4}} = 8\pi$.

$\therefore 0 \le x \le 4\pi$, will include half of the sine curve.

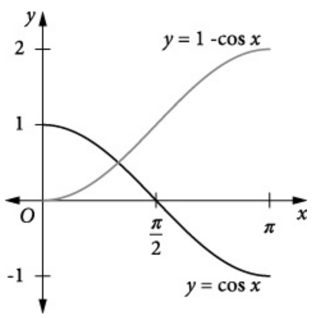


$y = 3\sin\dfrac{x}{4}$

4 (a) This will be half a cycle. The cosine graph starts at $(0,\ 1)$, crosses the x -axis and will end at $(\pi,\ -1)$. $y = -\cos x$ will be a reflection of $y = \cos x$ in the x -axis.



(b) $y = 1 - \cos x$ is $y = -\cos x + 1$ and is the graph $y = -\cos x$ raised by one unit.



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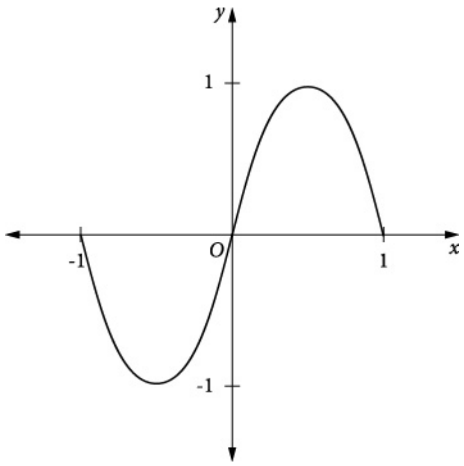
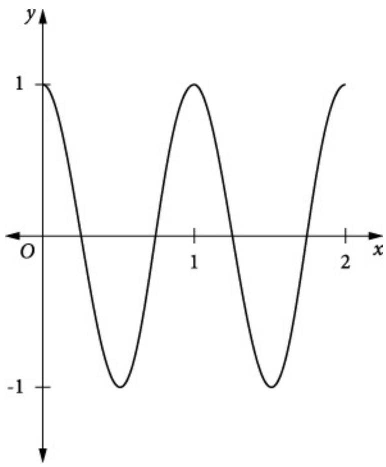
6 (a) The amplitude is 1 and period is $\frac{2\pi}{\pi} = 2$. The graph starts at $(0,\ 0)$.



$y = \sin \pi x$

(b) The amplitude is 1 and period is $\frac{2\pi}{\pi} = 1$. The graph starts at $(0,\ 1)$.



$y = \cos 2\pi x$

(c) There is no amplitude and the period is $\frac{\pi}{\frac{\pi}{2}} = 2$.

There will be a vertical asymptote when $\frac{\pi x}{2} = \frac{\pi}{2} \Rightarrow x = 1$.

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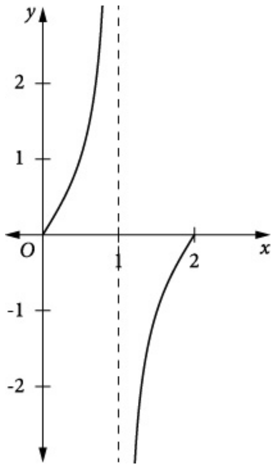
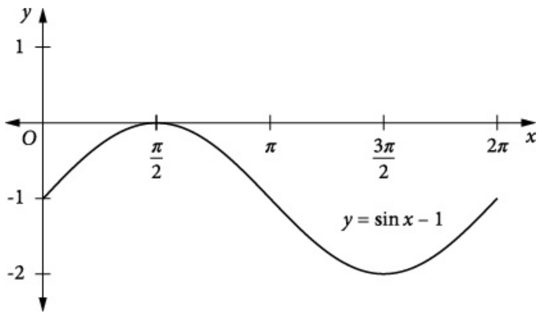
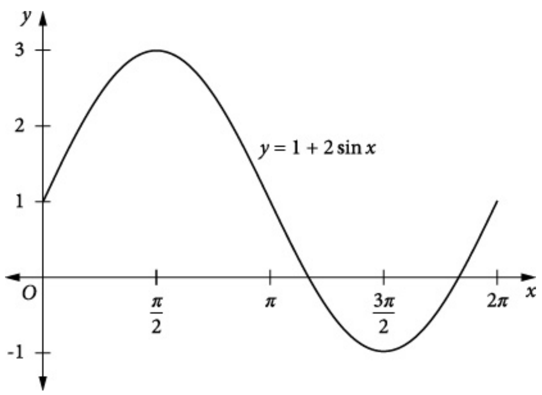


$y = \tan \dfrac{\pi x}{2}$

8 **(a)** This is the graph of $y = \sin x$ moved down one unit.



(b) The graph of $y = 2\sin x$ will be the same as the graph of $y = \sin x$, but with double the amplitude. The graph of $y = 1 + 2\sin x$ will be this graph raised one unit vertically.



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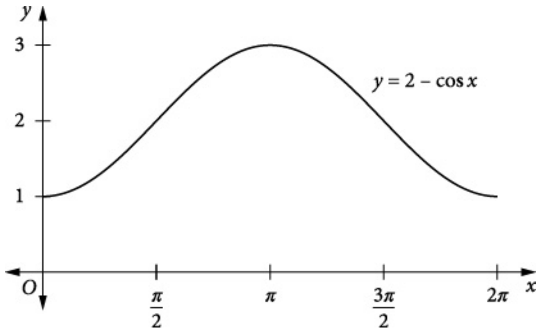
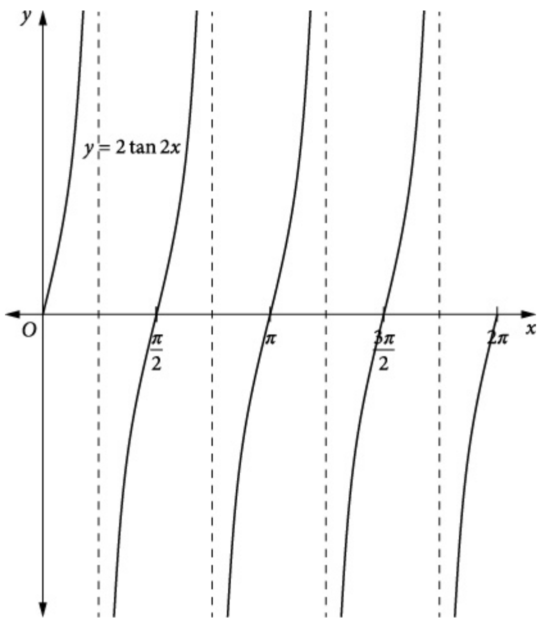
(c) The graph of $y=-\cos x$ is the graph of $y=\cos x$ reflected in the x-axis.

$y=2-\cos x$ is $y=-\cos x+2$ and is the graph $y=-\cos x$ raised by two units.



(d) The period is $\dfrac{\pi}{2}$.

The graph of $y=2\tan 2x$ is twice as high as the graph of $y=\tan 2x$.



(e) The period is $\dfrac{2\pi}{2}=\pi$.

$y=2+\sin 2x$ is $y=\sin 2x+2$ and is the graph of $y=\sin 2x$ raised by two units.

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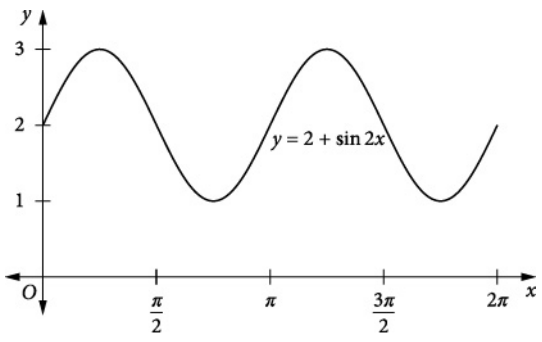
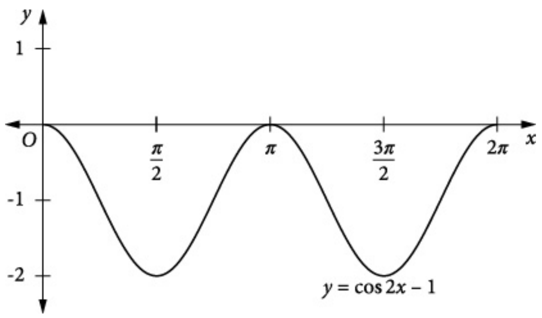


(f) The period is $\frac{2\pi}{2} = \pi$.

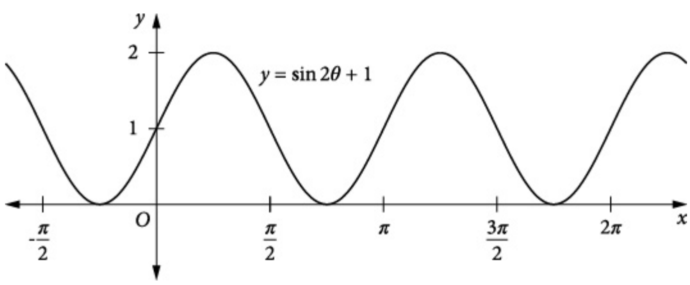
$y = \cos 2x - 1$ is the graph of $y = \cos 2x$ lowered by two units.



10 (a) No domain is given, but graphs should be drawn on both sides of the y-axis and should be for at least one cycle.

The period is $\frac{2\pi}{2} = \pi$.

$y = \sin 2\theta + 1$ is the graph of $y = \sin 2\theta$ raised by one unit.



(b) $y = 3\cos\theta$ has amplitude 3 and will take values from -3 to 3.

$y = 3\cos\theta - 2$ still has amplitude 3 but has been moved down 2 units and will now take values from -5 to 1.

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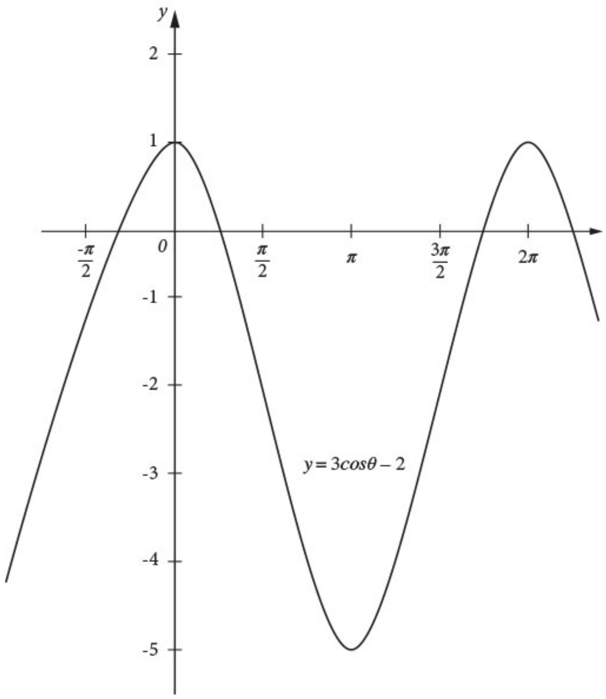
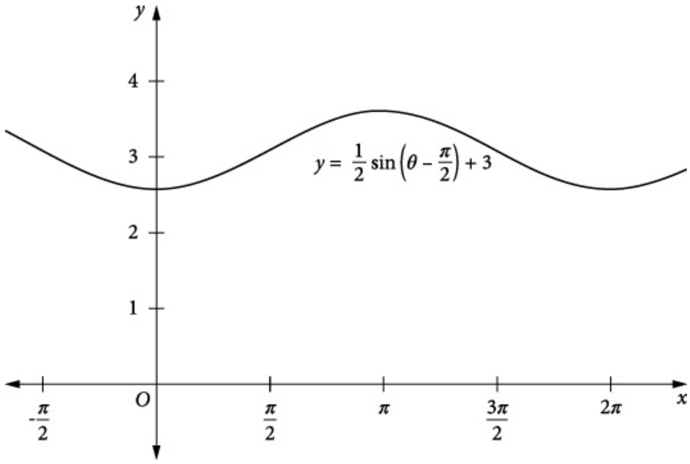


(c) $y = \frac{1}{2}\sin\theta$ will have an amplitude of $\frac{1}{2}$.

$y = \frac{1}{2}\sin\left(\theta - \frac{\pi}{2}\right)$ is the graph of $y = \frac{1}{2}\sin\theta$ moved $\frac{\pi}{2}$ units right.

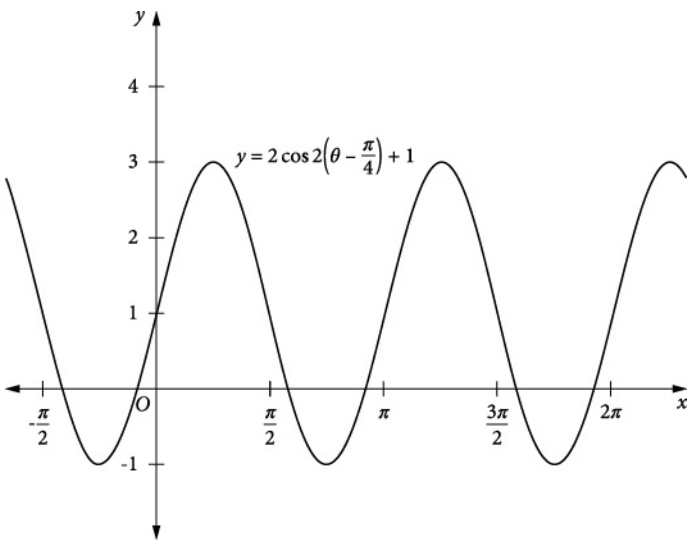
$y = \frac{1}{2}\sin\left(\theta - \frac{\pi}{2}\right) + 3$ is the graph of $y = \frac{1}{2}\sin\left(\theta - \frac{\pi}{2}\right)$ moved 3 units up.



(d) $y = 2\cos 2\theta$ will have an amplitude of 2 and a period of $\frac{2\pi}{2} = \pi$ ..

$y = 2\cos 2\left(\theta - \frac{\pi}{4}\right)$ is the graph of $y = 2\cos 2\theta$ moved $\frac{\pi}{4}$ units right.

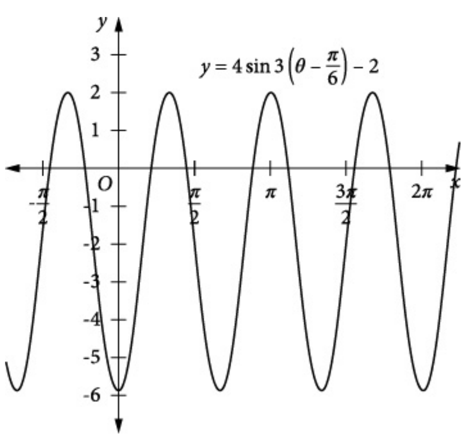
$y = 2\cos 2\left(\theta - \frac{\pi}{4}\right) + 1$ is the graph of $y = 2\cos 2\left(\theta - \frac{\pi}{4}\right)$ moved 1 unit up.



(e) $y = 4\sin 3\theta$ will have an amplitude of 4 and a period of $\frac{2\pi}{3}$.

$y = 4\sin 3\left(\theta - \frac{\pi}{6}\right)$ is the graph of $y = 4\sin 3\theta$ moved $\frac{\pi}{6}$ units right.

$y = 4\sin 3\left(\theta - \frac{\pi}{6}\right) - 2$ is the graph of $y = 4\sin 3\left(\theta - \frac{\pi}{6}\right)$ moved 2 units down.



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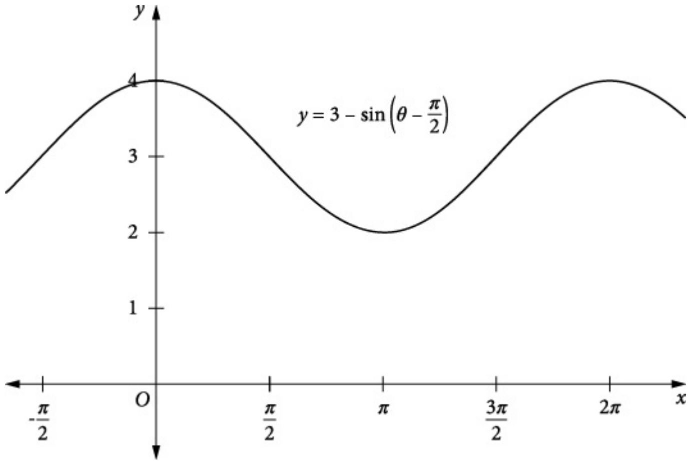
(f) $y=-\sin\theta$ is the graph of $y=\sin\theta$ reflected in the x-axis.

$y=-\sin\left(\theta-\frac{\pi}{2}\right)$ is the graph of $y=-\sin\theta$ moved $\frac{\pi}{2}$ units right.

$y=3-\sin\left(\theta-\frac{\pi}{2}\right)$ is the graph of $y=-\sin\left(\theta-\frac{\pi}{2}\right)$ moved 3 units up.



12 Answers will vary.

EXERCISE 12.2 FURTHER SOLUTION OF TRIGONOMETRIC EQUATIONS

2 (a) $-\pi\le x\le\pi$

$$-\frac{\pi}{2}\le\frac{x}{2}\le\frac{\pi}{2}$$

$$\tan\left(\frac{x}{2}\right)=\frac{1}{\sqrt{3}}\Rightarrow\frac{x}{2}=\frac{\pi}{6}$$

This is the only value for the domain $-\frac{\pi}{2}\le\frac{x}{2}\le\frac{\pi}{2}$.

$$\frac{x}{2}=\frac{\pi}{6}\Rightarrow x=\frac{\pi}{3}$$

(b) $-\pi \le x \le \pi$

$$-\frac{\pi}{3} \le \frac{x}{3} \le \frac{\pi}{3}$$

$$\sin\left(\frac{x}{3}\right) = -\frac{\sqrt{3}}{2} \Rightarrow \frac{x}{3} = -\frac{\pi}{3}$$

This is the only value for the domain $-\frac{\pi}{3} \le \frac{x}{3} \le \frac{\pi}{3}$.

$$\frac{x}{3} = -\frac{\pi}{3} \Rightarrow x = -\pi$$

(c) $-\pi \le x \le \pi$

$$-\frac{\pi}{4} \le \frac{x}{4} \le \frac{\pi}{4}$$

$$\cos\left(\frac{x}{4}\right) = \frac{1}{\sqrt{2}} \Rightarrow \frac{x}{4} = -\frac{\pi}{4}, \frac{\pi}{4}$$

This is the only value for the domain $-\frac{\pi}{4} \le \frac{x}{4} \le \frac{\pi}{4}$.

$$\frac{x}{4} = -\frac{\pi}{4}, \frac{\pi}{4} \Rightarrow x = -\pi, \pi$$

4 **(a)** $-\pi \le x \le \pi$

$$-2\pi \le 2x \le 2\pi$$

$$\sqrt{2}\cos(2x) = 1$$

$$\cos(2x) = \frac{1}{\sqrt{2}}$$

$$2x = \frac{\pi}{4}, -\frac{\pi}{4}, 2\pi - \frac{\pi}{4}, -\left(2\pi - \frac{\pi}{4}\right)$$

$$2x = \frac{\pi}{4}, -\frac{\pi}{4}, \frac{7\pi}{4}, -\frac{7\pi}{4}$$

$$x = \frac{\pi}{8}, -\frac{\pi}{8}, \frac{7\pi}{8}, -\frac{7\pi}{8}$$

$$x = -\frac{7\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{7\pi}{8}$$

(b) $-\pi \le x \le \pi$

$$-2\pi \le 2x \le 2\pi$$

$$-\frac{5\pi}{2} \le 2x - \frac{\pi}{2} \le \frac{3\pi}{2}$$

$$\cos\left(2x - \frac{\pi}{2}\right) = 1$$

$$2x - \frac{\pi}{2} = 0,\ \ -2\pi$$

$$2x = \frac{\pi}{2}\ ,-\frac{3\pi}{2}$$

$$x = \frac{\pi}{4}\ ,-\frac{3\pi}{4}$$

(c) $-\pi \le x \le \pi$

$$-2\pi \le 2x \le 2\pi$$

$$-2\pi + \frac{\pi}{6} \le 2x + \frac{\pi}{6} \le 2\pi + \frac{\pi}{6}$$

$$-\frac{11\pi}{6} \le 2x + \frac{\pi}{6} \le \frac{13\pi}{6}$$

$$\sin\left(2x + \frac{\pi}{6}\right) = -1$$

$$2x + \frac{\pi}{6} = \frac{3\pi}{2},\ \ -\frac{\pi}{2}$$

$$2x = \frac{3\pi}{2} - \frac{\pi}{6},\ \ -\frac{\pi}{2} - \frac{\pi}{6}$$

$$2x = \frac{8\pi}{6}\ ,-\frac{4\pi}{6}$$

$$2x = \frac{4\pi}{3}\ ,-\frac{2\pi}{3}$$

$$x = \frac{2\pi}{3}\ ,-\frac{\pi}{3}$$

(d) $-\pi \le x \le \pi$

$$-3\pi \le 3x \le 3\pi$$

$$-3\pi - \frac{\pi}{4} \le 3x - \frac{\pi}{4} \le 3\pi - \frac{\pi}{4}$$

$$-\frac{13\pi}{4} \le 3x - \frac{\pi}{4} \le \frac{11\pi}{4}$$

$$3\tan\left(3x-\frac{\pi}{4}\right)=\sqrt{3}$$

$$\tan\left(3x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{3}=\frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$3x-\frac{\pi}{4}=-3\pi+\frac{\pi}{6},\quad -2\pi+\frac{\pi}{6},\quad -\pi+\frac{\pi}{6},\frac{\pi}{6},\ \pi+\frac{\pi}{6},2\pi+\frac{\pi}{6}$$

$$3x-\frac{\pi}{4}=-\frac{17\pi}{6},\ -\frac{11\pi}{6},-\frac{5\pi}{6},\ \frac{\pi}{6},\ \frac{7\pi}{6},\ \frac{13\pi}{6},$$

$$3x=-\frac{17\pi}{6}+\frac{\pi}{4},\ -\frac{11\pi}{6}+\frac{\pi}{4},-\frac{5\pi}{6}+\frac{\pi}{4},\ \frac{\pi}{6}+\frac{\pi}{4},\ \frac{7\pi}{6}+\frac{\pi}{4},\ \frac{13\pi}{6}+\frac{\pi}{4}$$

$$3x=-\frac{31\pi}{12},\ -\frac{19\pi}{12},-\frac{7\pi}{12},\ \frac{5\pi}{12},\ \frac{17\pi}{12},\ \frac{29\pi}{12}$$

$$x=-\frac{31\pi}{36},\ -\frac{19\pi}{36},-\frac{7\pi}{36},\ \frac{5\pi}{36},\ \frac{17\pi}{36},\ \frac{29\pi}{36}$$

(e) $-\pi\le x\le\pi$

$$-\frac{\pi}{2}\le\frac{x}{2}\le\frac{\pi}{2}$$

$$-\frac{\pi}{2}+\frac{\pi}{18}\le\frac{x}{2}+\frac{\pi}{18}\le\frac{\pi}{2}+\frac{\pi}{18}$$

$$-\frac{8\pi}{18}\le\frac{x}{2}+\frac{\pi}{18}\le\frac{10\pi}{18}$$

$$-\frac{4\pi}{9}\le\frac{x}{2}+\frac{\pi}{18}\le\frac{5\pi}{9}$$

$$\sin\left(\frac{x}{2}+\frac{\pi}{18}\right)=\frac{1}{2}$$

$$\frac{x}{2}+\frac{\pi}{18}=\frac{\pi}{6}$$

$$\frac{x}{2}=\frac{\pi}{6}-\frac{\pi}{18}=\frac{2\pi}{18}=\frac{\pi}{9}$$

$$x=\frac{2\pi}{9}$$

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6 (a) Divide both sides by $\cos\left(2x+\dfrac{\pi}{6}\right)$.

$$\frac{\sin\left(2x+\frac{\pi}{6}\right)}{\cos\left(2x+\frac{\pi}{6}\right)}=\sqrt{3}\Rightarrow\tan\left(2x+\frac{\pi}{6}\right)=\sqrt{3}$$

$$0\le x\le 2\pi$$
$$0\le 2x\le 4\pi$$
$$\frac{\pi}{6}\le 2x+\frac{\pi}{6}\le 4\pi+\frac{\pi}{6}$$
$$\frac{\pi}{6}\le 2x+\frac{\pi}{6}\le\frac{25\pi}{6}$$

$$\tan\left(2x+\frac{\pi}{6}\right)=\sqrt{3}$$

$$2x+\frac{\pi}{6}=\frac{\pi}{3},\ \pi+\frac{\pi}{3},\ 2\pi+\frac{\pi}{3},\ 3\pi+\frac{\pi}{3}$$
$$2x+\frac{\pi}{6}=\frac{\pi}{3},\ \frac{4\pi}{3},\ \frac{7\pi}{3},\ \frac{10\pi}{3}$$
$$2x=\frac{\pi}{3}-\frac{\pi}{6},\ \frac{4\pi}{3}-\frac{\pi}{6},\ \frac{7\pi}{3}-\frac{\pi}{6},\ \frac{10\pi}{3}-\frac{\pi}{6}$$
$$2x=\frac{\pi}{6},\ \frac{7\pi}{6},\ \frac{13\pi}{6},\ \frac{19\pi}{6}$$
$$x=\frac{\pi}{12},\ \frac{7\pi}{12},\ \frac{13\pi}{12},\ \frac{19\pi}{12}$$

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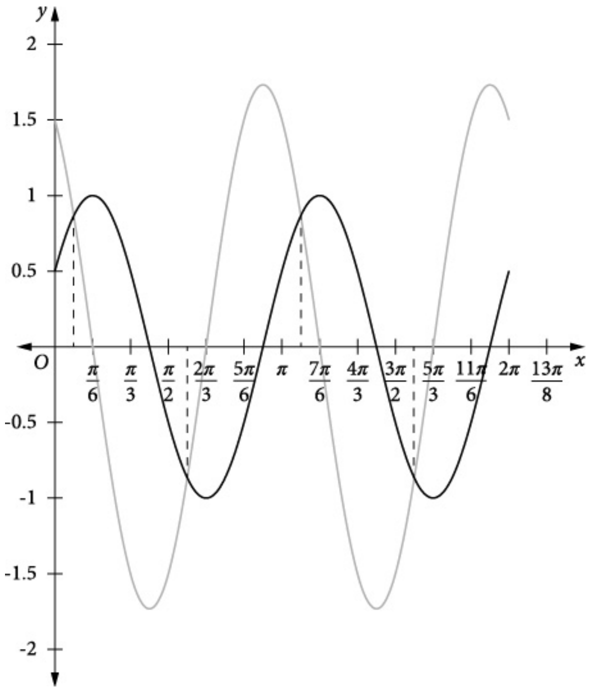
(b)

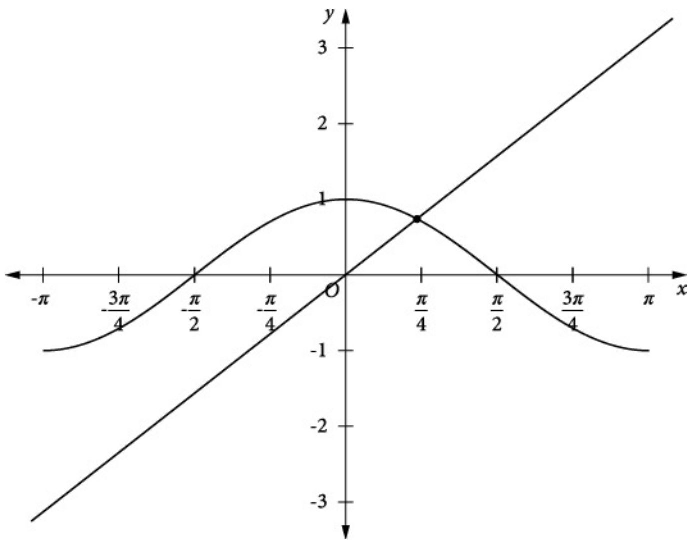


$$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$$

EXERCISE 12.3 GRAPHICAL SOLUTION OF EQUATIONS

2 **(a)** Draw the graph of $y = x$ on the same graph as $y = \cos x$.

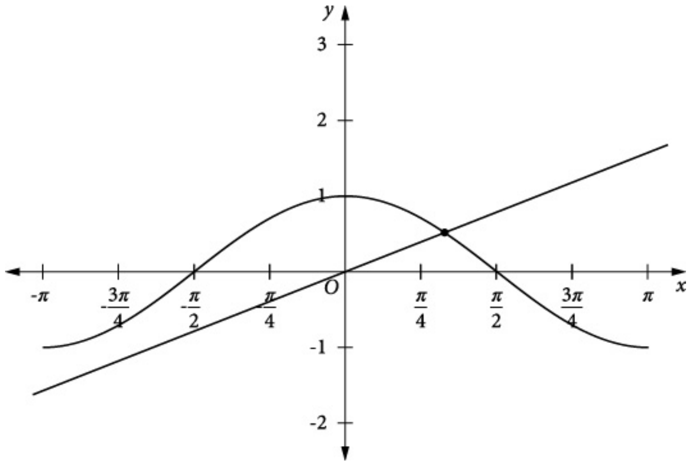
Find the x-coordinate(s) of the intersection(s).



$$\cos x = x$$
$$x \approx 0.74$$

(b) Draw the graph of $y = \frac{x}{2}$ on the same graph as $y = \cos x$.

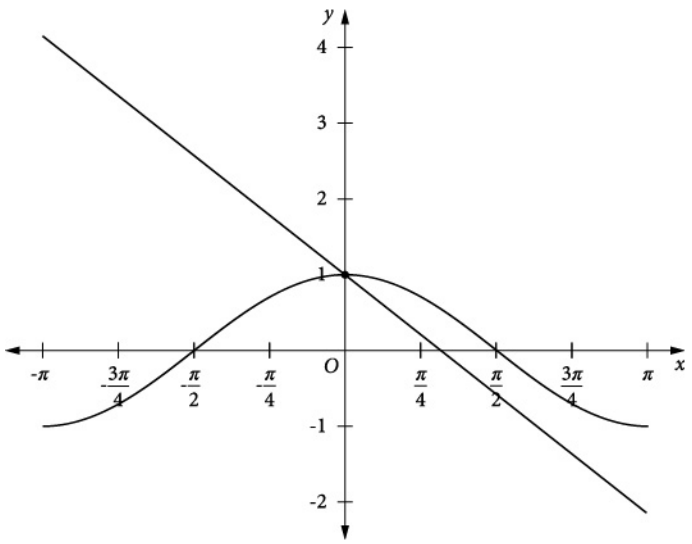
Find the x-coordinate(s) of the intersection(s).



$$\cos x = \frac{x}{2}$$
$$x \approx 1.03$$

(c) Draw the graph of $y = 1 - x$ on the same graph as $y = \cos x$.

Find the x-coordinate(s) of the intersection(s).



$$\cos x = 1 - x$$
$$x = 0$$

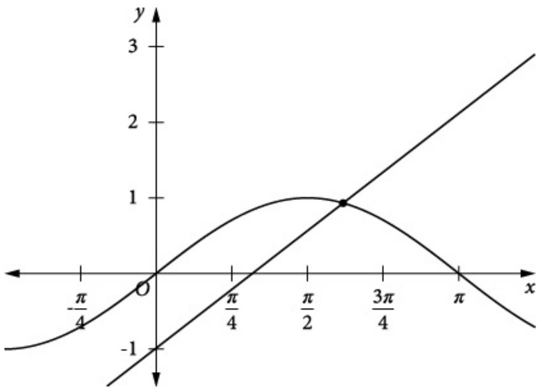
4 B

The graphs drawn should be $y = \sin x$ and $y = x - 1$.

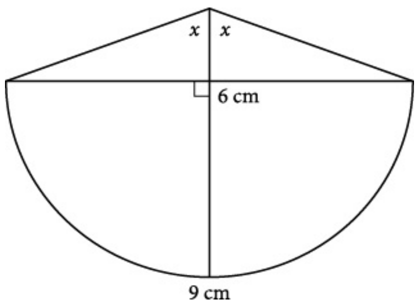
A and **C** both incorrectly have the graph of $y = \cos x$.

The straight line in **D** (and **C**) has a negative gradient and cannot be $y = x - 1$.

Only **B** has the correct graphs.



6 (a) First draw a diagram.



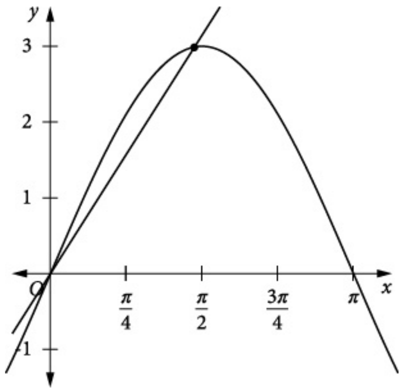
$$l = r\theta$$
$$9 = r \times 2x$$
$$9 = 2rx$$
$$r = \frac{9}{2x}$$

$$\sin x = \frac{3}{r}$$
$$r = \frac{3}{\sin x}$$

$$\frac{9}{2x} = \frac{3}{\sin x}$$
$$9\sin x = 6x$$
$$3\sin x = 2x$$

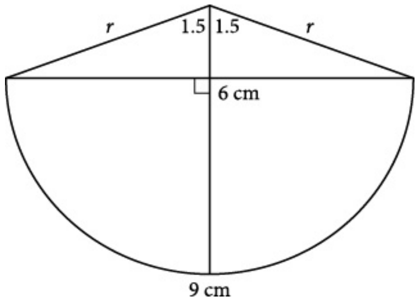
(b) Draw the graphs of $y = 3\sin x$ and $y = 2x$ on the same axes as.

Find the x-coordinate(s) of the intersection(s).



Since x is positive, there is only one solution at $x \approx 1.5$.

(c)



$$\begin{aligned}
\text{Area} &= \frac{1}{2}r^2(\theta - \sin\theta) \\
&= \frac{1}{2}\times\left(\frac{9}{2x}\right)^2 \times (2x - \sin 2x) \\
&= \frac{1}{2}\times\left(\frac{9}{2\times1.5}\right)^2 \times \left[2\times1.5 - \sin(2\times1.5)\right] \\
&\approx 12.86...
\end{aligned}$$

Area is $12.9\ \text{cm}^2$, rounded to one decimal place.

8 **(a)** $\frac{1}{2}r^2(\theta - \sin\theta) = \frac{1}{4}A$

$$\frac{1}{2}r^2\left[\left(\frac{\pi}{2}+x\right)-\sin\left(\frac{\pi}{2}+x\right)\right]=\frac{1}{4}\pi r^2$$

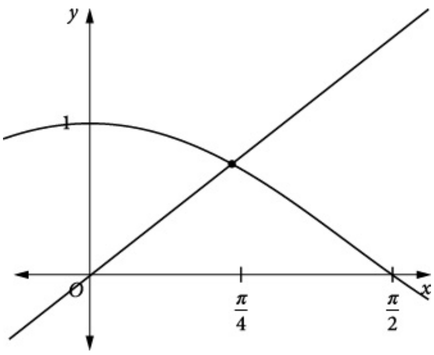
$$\frac{\pi}{2}+x-\sin\left(\frac{\pi}{2}+x\right)=\frac{\pi}{2}$$

$$x-\cos x=0$$

$$x=\cos x$$

(b) Draw the graph of $y = x$ on the same axes as $y = \cos x$.

Find the x-coordinate(s) of the intersection(s).



$x = \cos x$

$x = 0.74$

EXERCISE 12.4 APPLICATIONS INVOLVING TRIGONOMETRIC FUNCTIONS AND GRAPHS

2 **(a)** $T = 15\sin\left(\frac{\pi t}{12}\right)+20$

This is a positive sin graph, so the starting point $(t=0)$ is at the median value of 20, when

$15\sin\left(\frac{\pi t}{12}\right)=0$.

The amplitude is 15.

The median is 20.

The period is

$$\frac{2\pi}{\pi/12}=2\pi\times\frac{12}{\pi}=24$$

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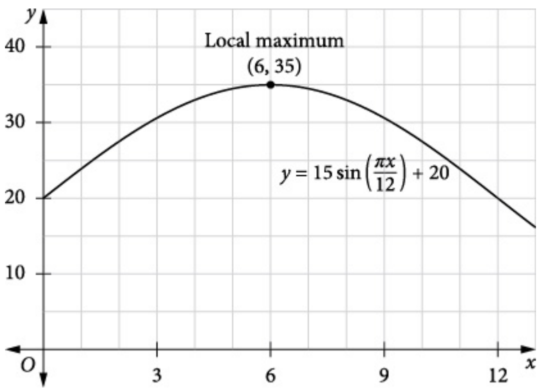
which corresponds to the number of hours in a day.

The maximum temperature will be $15°\text{C} + 20°\text{C} = 35°\text{C}$ and will occur after a quarter of the period, i.e. 6 hours.

T- intercept $(0, 20)$

The domain is limited to 12 hours, so only the first half of the cycle should be shown on the graph.



(b) Since t is the number of hours after 9.00, then at 9.00 $t = 0$.

$$T = 15\sin\left(\frac{\pi \times 0}{12}\right) + 20$$
$$T = 15\sin(0) + 20$$
$$T = 20$$

The temperature in the room is $20°\text{C}$ at 9.00 am.

(c) Midday is 3 hours after 9.00 am.

$$T = 15\sin\left(\frac{\pi \times 3}{12}\right) + 20$$
$$T = 15\sin\left(\frac{\pi}{4}\right) + 20$$
$$T = \frac{15\sqrt{2}}{2} + 20$$
$$T \approx 30.6$$

The temperature in the room is $30.6\ ^\circ\text{C}$ at midday.

(d) $15\sin\left(\frac{\pi t}{12}\right)+20=30$

$$15\sin\left(\frac{\pi t}{12}\right)=10$$
$$\sin\left(\frac{\pi t}{12}\right)=\frac{10}{15}=\frac{2}{3}$$
$$\frac{\pi t}{12}=0.7297...,\ \pi-0.7297...$$
$$t=0.7297...\times\frac{12}{\pi},\ 2.4118...\times\frac{12}{\pi}$$
$$t=2.7873...,\ 9.2126...$$

2.78735 hours is 2 h 0.78735×60 min.

This rounds to 2 h 47 min.

$9.2126...$ hours is 9 h $0.2126...\times 60$ min.

This rounds to 2 h 13 min.

Alternatively, since the period is 24 hours, another solution will be

12 h – 2 h 47 min = 9 h 13 min.

These times are after 9.00 am, so the solutions are 11.47 am and 6.13 pm.

(e) Time at least $30°\text{C}=9.2126...-2.78735...=6.425...$ (Use more accurate values.)

$$\frac{6.425...}{12}\times 100\%\approx 53.5\%$$

The temperature was at least $30\,^{\circ}\text{C}$ for 53.3% of the time.

4 **(a)** The median value is half way between the minimum and maximum value

Since the voltage oscillates between -240 and 240, the median value is 0.

The amplitude is the magnitude of the wave's variation from the median so it is 240.

Hence, $k=240$ and $c=0$.

(b) A frequency of 50 cycles per second means the graph repeats 50 times each second, so each repeat, or the period, is $\frac{1}{50}=0.02$ seconds,

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$$\frac{2\pi}{a} = 0.02$$
$$\frac{2\pi}{0.02} = a$$
$$a = 100\pi$$

(c) Substitute the values of the constants.

$k = 240$, $a = 100\pi$, $c = 0$.

$V = k\sin at + c$
$V = 240\sin(100\pi t)$

The ± is used as no information is given to determine where the function starts in the cycle.

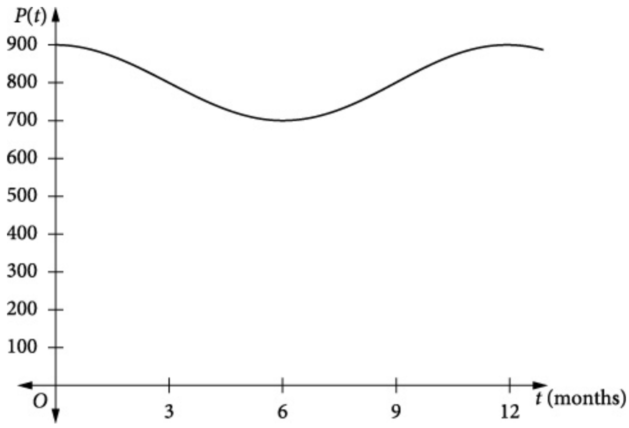
6 **(a)** The maximum value is 900, when $t = 0.12$.

The minimum value is 700, when $t = 6$.

Going from maximum to minimum is half a cycle, so a period, or full cycle, will be twice this value, a year or 12 months.

Plot the points and join them in a smooth curve.

The median will be the average of the maximum and minimum, .



(b) $P(t) = k\cos(at) + c$

The median is in the middle of the range,

$$\frac{900 + 700}{2} = 800$$

So, $c = 800$.

The amplitude is half the variation, so $k = \frac{200}{2} = 100$.

The period is 12 months, so

$$12 = \frac{2\pi}{a}$$
$$12a = 2\pi$$
$$a = \frac{\pi}{6}$$

So, $P(t) = k\cos(at) + c$ becomes $\boldsymbol{P(t) = 100\cos\left(\frac{\pi t}{6}\right) + 800}$.

(c) Some approximations of dates will be used to make this question reasonable.

For 31st of March, use April 1st, $t = 3$.

$$P(3) = 100\cos\left(\frac{3\pi}{6}\right) + 800 = 800$$

For the 20 June use $t = 5\frac{2}{3} = \frac{17}{3}$:

$$P\left(\frac{17}{3}\right) = 100\cos\left(\frac{17\pi}{3\times 6}\right) + 800 \approx 702$$

For the 23 October use $t = 9\frac{23}{31} = 9.7419...$

$$P(9.7419...) = 100\cos\left(\frac{9.7419...\times\pi}{6}\right) + 800 \approx 838$$

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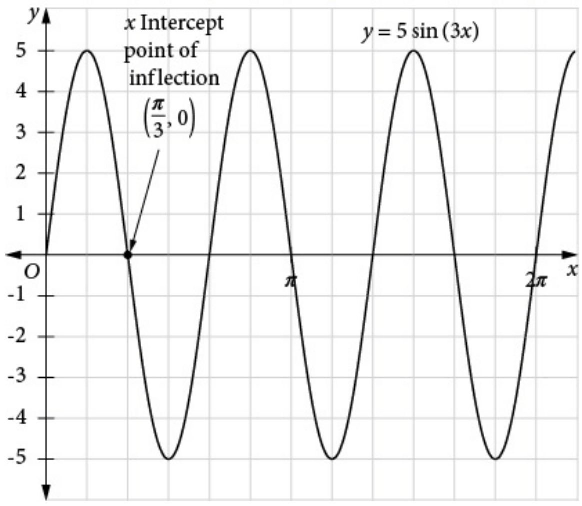
CHAPTER REVIEW 12

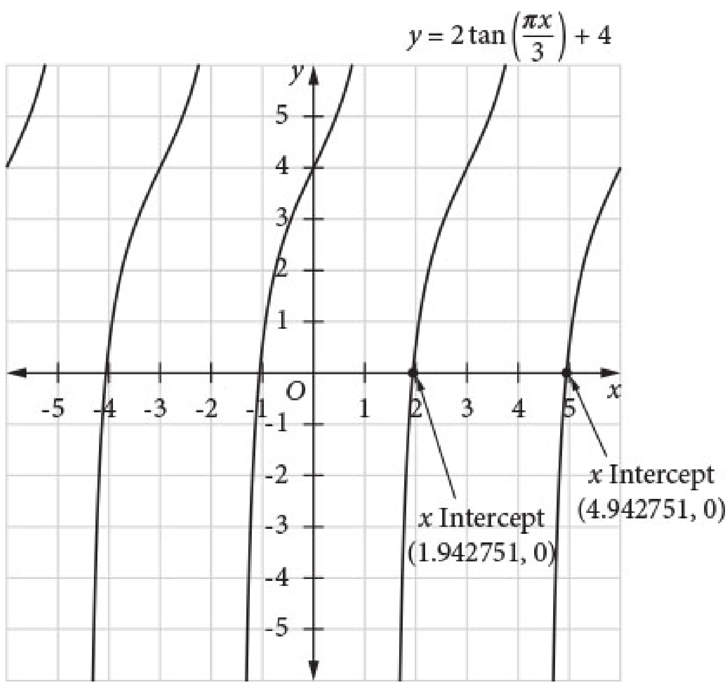
2 (a) The period is $\frac{2\pi}{3}$ and the amplitude is 5.



(b) There is no amplitude, but the graph will be stretched vertically by a factor of 2.

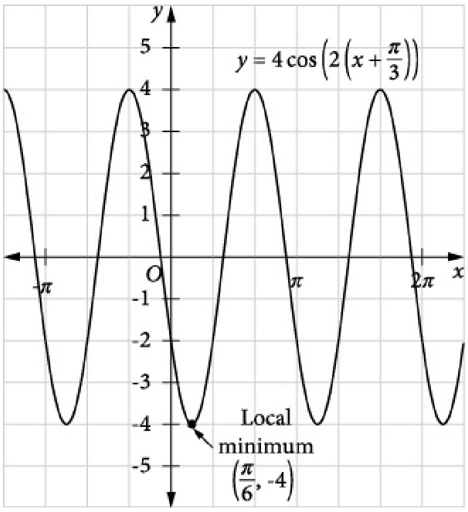
The period is $\frac{\pi}{\pi/3} = 3$.

The graph has been moved up 4 units.

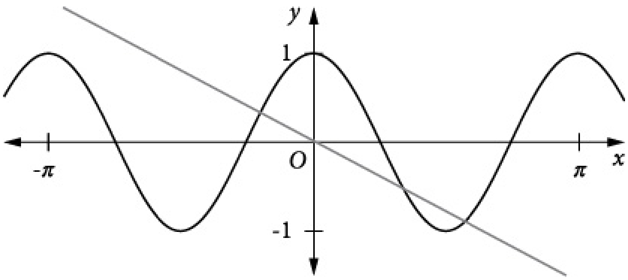


(c) The amplitude is 4 and the period is $\dfrac{2\pi}{2} = \pi$

The graph has been shifted $\dfrac{\pi}{3}$ units to the left.



4



$\cos 2x = -\dfrac{x}{2}$ when $x = -0.63,\ 1.07,\ 1.80$

6 **(a)** c is the median, the average of the maximum and the minimum.

The minimum is -4 so $c = \dfrac{12 + (-4)}{2} = \dfrac{8}{2} = 4$.

(b) The amplitude of the function is the difference between the maximum (or the minimum) and the median, and is $12 - 4 = 8$.

Note that the graph is a negative sine, so $k = -8$.

(c) The graph completes a full cycle in 24 hours so the period of the graph is 24.

(d) $a = \dfrac{2\pi}{24} = \dfrac{\pi}{12}$

(e) Reading from the graph, the minimum temperature is $-4\,°\text{C}$.

(f) Substitute $k = -8,\ a = \dfrac{\pi}{12},\ c = 4$ in the equation.

$$f(x) = 8\sin\left(\frac{\pi}{12}x\right) + 4$$

Solve for $f(x) = 0$.

$$\begin{aligned}
-8\sin\frac{\pi x}{12} + 4 &= 0\\
\sin\frac{\pi x}{12} &= \frac{1}{2}\\
\frac{\pi}{12}x &= \frac{\pi}{6}, \frac{5\pi}{6}\\
x &= 2, 10
\end{aligned}$$

The pond has a temperature of $0°\text{C}$ at 2 am and 10 am.

8 **(a)** Find the value of $h(0)$:

$$\begin{aligned}
h(0) &= -8\cos(0) + 9\\
&= 1
\end{aligned}$$

Oscar is 1 m off the ground at the beginning of the ride.

(b) The amplitude of the function is 8, so the difference from the lowest point of the ride and the highest point of the ride is 16 metres. Given Oscar starts the ride 1 m above the ground, the greatest distance Oscar is from the ground during the course of the ride is 17 m.

(c) The period of the function equates to one full rotation of the ride.

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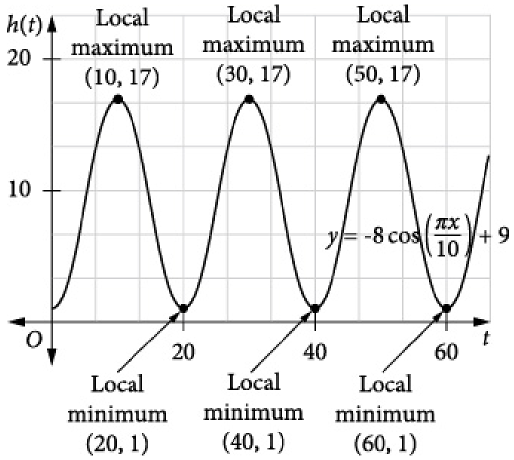
Period $= 2\pi \div \frac{\pi}{10} = 2\pi \times \frac{10}{\pi} = 20\,\text{s}$

(d) The domain of the function is $[0,180]$ and the ride takes 20 seconds for a full rotation.

$180 \div 20 = 9$

So, 9 rotations are completed before the ride comes to a stop.

(e) Use the information about period and amplitude, etc.



(f) Solve the equation $h(t) = 9$:

$$9 = -8\cos\left(\frac{\pi t}{10}\right) + 9$$

$$0 = \cos\left(\frac{\pi t}{10}\right)$$

$$\frac{\pi t}{10} = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$t = 5, 15$$

For each rotation, the ride is at least 9 metres off the ground for 10 seconds $(15-5)$.

Therefore, during the 3 minutes Oscar can see the ocean for 90 seconds (10 sec $\times$ 9 rotations) during the three-minute ride.

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