



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

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Centre Number

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Student Number

DO NOT REMOVE PAPER FROM EXAM ROOM

2020
**TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION**

Mathematics Advanced

Morning Session
Thursday, 20 August 2020

*with Replacement
Questions*

General Instructions

- Reading time – 10 mins
- Working time – 3 hours
- Write using black pen
- Use Multiple Choice Answer Sheet provided
- NESA-approved calculators may be used
- A reference sheet is provided on a SEPARATE sheet
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks - 100

Section I Pages 2-6

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II Pages 7-26

90 marks

- Attempt Questions 11-37
- Allow about 2 hours and 45 minutes for this section

Disclaimer

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Section I

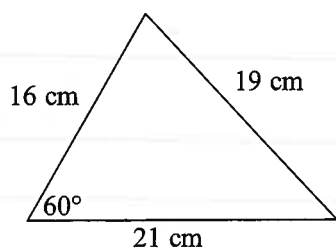
10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1-10.

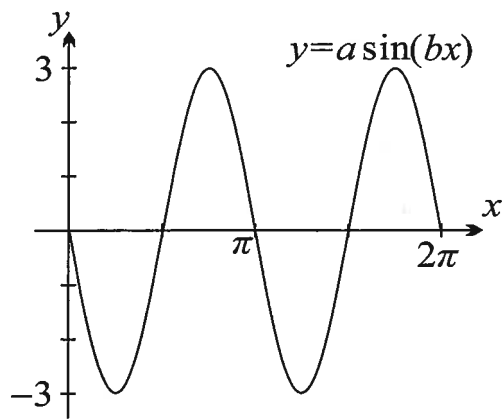
- 1 Which of the following expressions gives the area of the triangle below?



- (A) $\text{Area} = \frac{1}{2} \times 16 \times 19 \times \cos 60^\circ$
- (B) $\text{Area} = \frac{1}{2} \times 16 \times 21 \times \cos 60^\circ$
- (C) $\text{Area} = \frac{1}{2} \times 16 \times 19 \times \sin 60^\circ$
- (D) $\text{Area} = \frac{1}{2} \times 16 \times 21 \times \sin 60^\circ$
- 2 Which of the following is equal to $\frac{1}{3\sqrt{2}-\sqrt{5}}$?

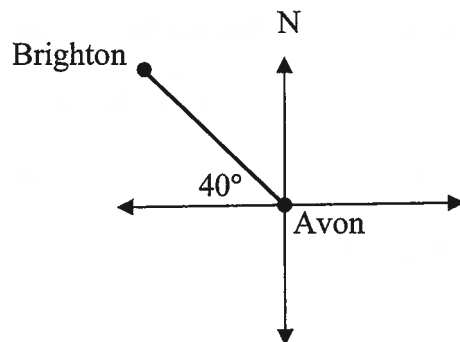
- (A) $3\sqrt{2} - \sqrt{5}$
- (B) $3\sqrt{2} + \sqrt{5}$
- (C) $\frac{3\sqrt{2} - \sqrt{5}}{13}$
- (D) $\frac{3\sqrt{2} + \sqrt{5}}{13}$

- 3 The diagram below is a graph of $y = a \sin(bx)$ in the domain $[0, 2\pi]$.



What is the equation of the curve?

- (A) $y = 3 \sin 2x$
- (B) $y = 2 \sin 3x$
- (C) $y = -3 \sin 2x$
- (D) $y = -2 \sin 3x$
- 4 What is the compass bearing of Avon from Brighton in the diagram shown?



- (A) S50°E
- (B) S40°W
- (C) N40°E
- (D) N50°W

5 What is the value of $(\log a)^2 \div \log(a^2)$?

(A) 1

(B) $\frac{\log a}{2}$

(C) $\log a$

(D) $2\log a$

6 Which of the following is equal to $\frac{4^n - 1}{2^n - 1}$?

(A) $2^n + 1$

(B) $2^n - 1$

(C) 2^2

(D) 2

7 For a normally distributed set of scores, which of the following statements is FALSE?

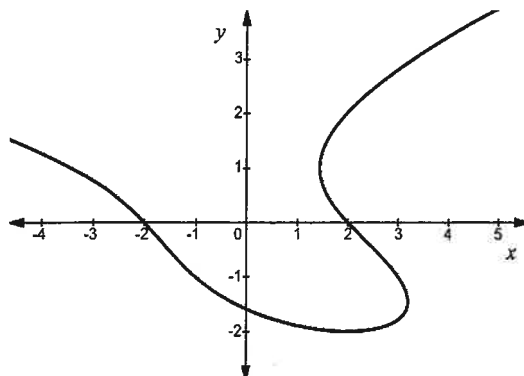
(A) A z-score more than 2 has a higher probability than a z-score more than 1.

(B) The frequency distribution is symmetrical about the mean.

(C) The z-score of a score describes how far that score is from the mean.

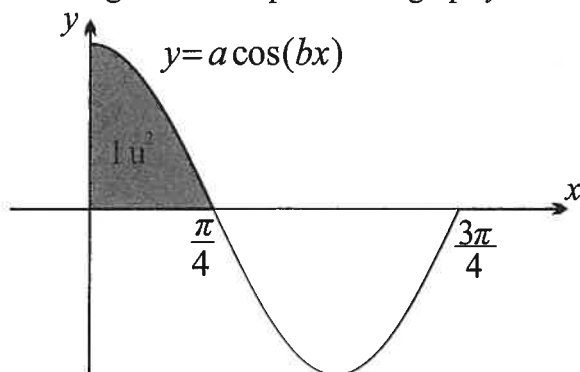
(D) The mean and the median are equal.

- 8 Below is a graph of $x^2 + 2xy - y^3 = 4$.



What type of relation is this?

- (A) one-to-one
 - (B) one-to-many
 - (C) many-to-one
 - (D) many-to-many
- 9 The diagram shows part of the graph $y = a \cos bx$.



The area of the shaded region is equal to 1 unit². What is the value of $\int_0^{\frac{3\pi}{4}} f(x) dx$?

- (A) -2
- (B) -1
- (C) 1
- (D) 3

- 10 The graph of $y = \frac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\frac{1}{2}$. Which of the following gives the equation of the new function?

(A) $\frac{y}{2} = \frac{3}{x-3}$

(B) $2y = \frac{3}{x-4}$

(C) $2y = \frac{3}{x-3}$

(D) $\frac{y}{2} = \frac{3}{x-4}$

End of Section I

The paper continues over the page

Section II

90 marks

Attempt Questions 11-37

Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of your response.
- Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Solve $|2x - 3| \leq 1$.

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Question 12 (2 marks)

Differentiate $\frac{e^x}{x^2 + 1}$.

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Question 13 (2 marks)

Evaluate $\int_{-1}^0 x(3x-4)dx$.

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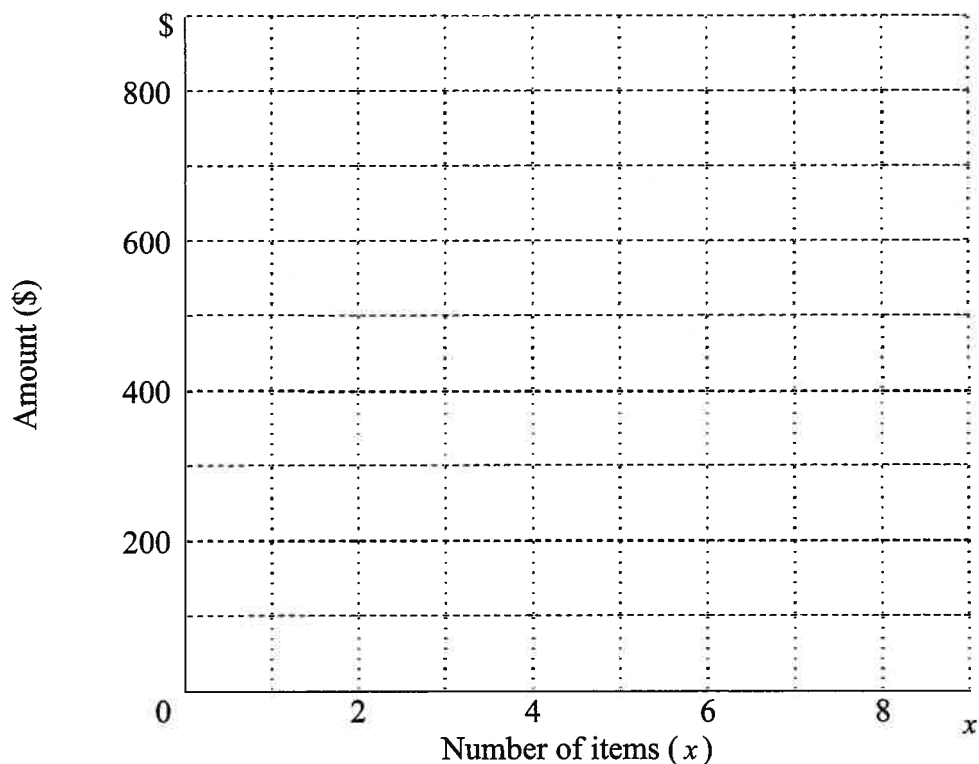
Question 14 (3 marks)

Michael has a small manufacturing business.

The cost of manufacturing is given by the equation $C = 50x + 200$ and the income earned is given by the equation $I = 100x$, where x is the number of items that the business has manufactured.

- (a) Graph each of the two equations on the grid below.

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- (b) How many items need to be manufactured for the business to break even?

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Question 15 (2 marks)

Find the gradient of the tangent to the curve $y = e^x \sin x$ as it passes through the origin.

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Question 16 (2 marks)

Two ordinary 6-sided dice, one red and one blue, are rolled and the numbers on the uppermost face are recorded. The blue die shows a number less than five. What is the probability that the sum of the two numbers is greater than seven?

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Question 17 (2 marks)

A set of scores is normally distributed. A score of 12 has z-score of -2 while a score of 21 has a z-score of $+1$.

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Calculate the mean of the scores.

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Question 18 (4 marks)

A cup of coffee is cooling according to the following exponential formula

$C = 21 + (74 \times 3^{-0.2t})$, where C is the temperature in degrees Celsius and t is the time in minutes since the coffee was poured.

- (a) Calculate the initial temperature of the coffee.

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- (b) Calculate the temperature of the coffee after 10 minutes, correct to the nearest degree.

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- (c) After how many minutes, to the nearest minute, will the coffee first reach 50°C ?

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Question 19 (2 marks)

Show that $\frac{\sec \theta - \sec \theta \cos^4 \theta}{1 + \cos^2 \theta} = \sin \theta \tan \theta$.

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Question 20 (3 marks)

The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n(3n+7)}{2}.$$

- (a) Find the first three terms of the sequence.

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- (b) Find an expression for the n -th term of the sequence.

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Question 21 (3 marks)

Solve $\sin\left(x + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$.

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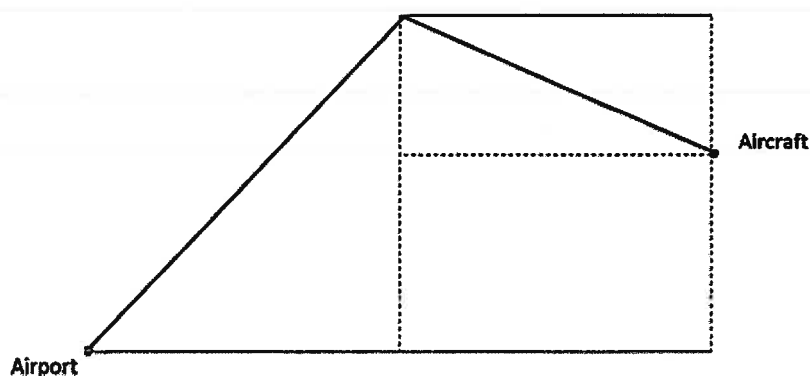
Question 22 (5 marks)

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An aircraft took off from an airport on an angle of elevation of 30° until it reached a maximum height of 12 000 metres.

The aircraft then descended on an angle of depression of 20° to a new height of 9000 metres.

Find the straight-line distance of the aircraft to the airport when it first reaches the new height of 9000 metres, correct to the nearest metre.



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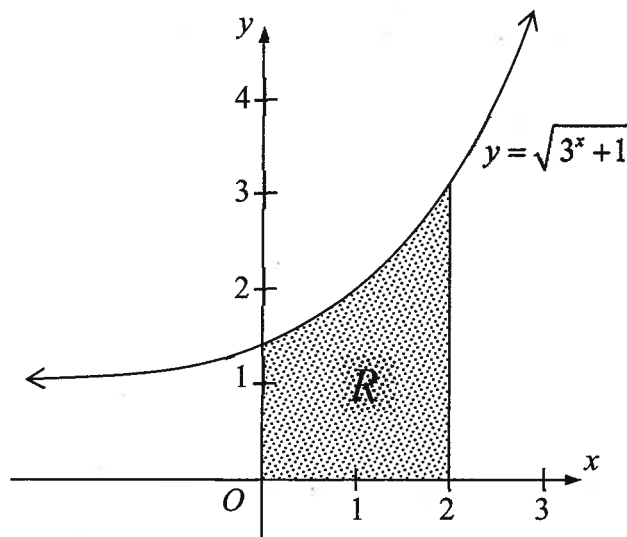
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Question 23 (3 marks)

The diagram below shows the region R which is bounded by $y = \sqrt{3^x + 1}$, the x -axis and the lines $x = 0$ and $x = 2$.



- (a) Use the Trapezoidal Rule with five function values to find an approximation for the area R . Give your answer to two decimal places.

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- (b) With reference to the curve, give a reason whether your approximation is an over-estimation or underestimation.

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Question 24 (5 marks)

Seven students, who are still on their Learner's Permit, recorded the number of driving hours they have logged so far and the mark they scored in their Visual Art assessment task.

The results are recorded in the table below:

Name	Ally	Bree	Cain	Dan	Elle	Frank	Guy
Hours of driving (x)	63	68	72	76	82	84	91
Visual Art mark (y)	73	71	71	85	92	80	81

- (a) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. **1**

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- (b) Calculate the value, to two decimal places, of the gradient and the y -intercept. Use these values to write the equation of the least-squares line of best fit. **2**

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- (c) Another student, who has logged 55 hours of driving and who scored 94 in Visual Art, is added to the data. **1**

Describe the impact this addition to the data will have on Pearson's correlation coefficient.

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- (d) A student made the statement, "Doing well in Visual Art depends on how much driving you have done." **1**

Based on the data, justify whether you agree or disagree with the statement.

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Question 25 (4 marks)

The displacement of a particle moving along the x -axis is given by $x = t + e^{-2t}$, where x is the displacement from the origin in metres and t is the time in seconds, for $t \geq 0$.

- (a) Find an expression, in terms of t , for the velocity of the particle.

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- (b) Find the time when the particle is closest to the origin.

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Question 26 (2 marks)

Let $f(x) = 5x - 2$ and $g(x) = \frac{1}{\sqrt{x}}$. State the domain of $g(f(x))$.

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Question 27 (7 marks)

Consider the curve $y = (x+1)^2(x-5)$.

- (a) Find the stationary points and determine their nature.

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- (b) Given that the point $(1, -16)$ lies on the curve, show that it is a point of inflection.

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Question 27 continues over the page

Question 27 (continued)

- (c) Sketch the curve $y = (x+1)^2(x-5)$, showing the stationary points and the point of inflection.

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Question 28 (3 marks)

Consider the circle given by the equation $x^2 + 8x + y^2 - 4y - 29 = 0$.

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Give the equation of the circle in the form $(x-h)^2 + (y-k)^2 = r^2$, if it is translated up by three units and right by five units.

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Question 29 (6 marks)

The time t minutes that a customer spends waiting in a queue has a probability density function:

$$f(x) = \begin{cases} \frac{x(k-x)}{36} & \text{for } 0 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the value of k is 6.

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- (b) Write down the value of $E(X)$.

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- (c) Calculate $\text{Var}(X)$.

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Question 30 (3 marks)

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Chickens are sized according to their weight. For example, a size 16 chicken weighs 1.6 kg. At a particular farm, the chickens they sell as size 15 have weights which are normally distributed with a mean of 1.56 kg and a standard deviation of 0.025 kg. Chickens that weigh less than 1.5 kg should be classified size 14 and chickens that weigh more than 1.6 kg should be classified size 16.

The following table gives the probability of scores with a z-score less than the given value.

z	first decimal place									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.500	0.540	0.579	0.618	0.655	0.692	0.726	0.758	0.788	0.816
1.	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
2.	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
3.	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000

For example, the probability of a score with a z-score of less than 2.6 is 0.995, as highlighted above.

What is the probability that a randomly selected size 15 chicken has been incorrectly classified? Give your answer correct to the nearest 3 decimal places.

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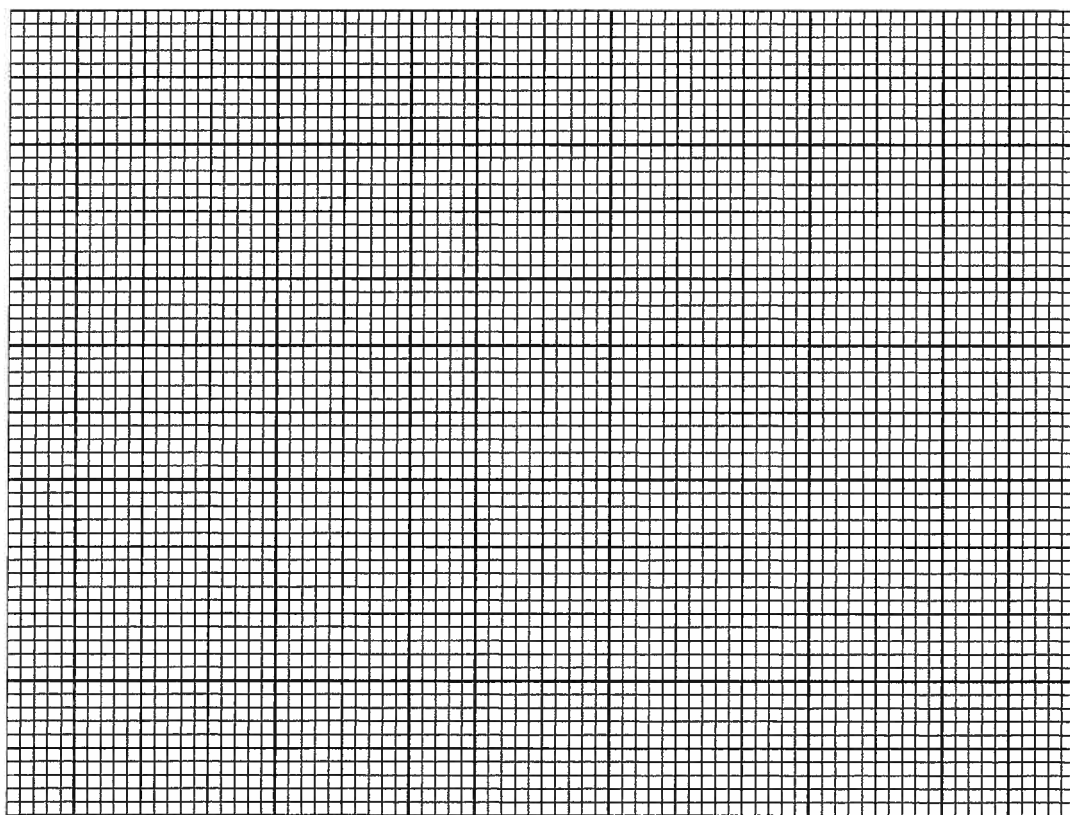
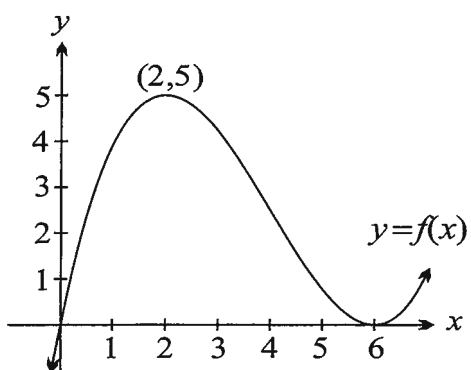
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Question 31 (2 marks)

The sketch shows $y = f(x)$. Draw a sketch of $f(2-x)$.

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Question 32 (6 marks)

Mimi borrows \$650 000 to buy a house at 6 % per annum with interest compounded monthly. The loan is to be repaid with equal monthly repayments of \$4200. Let A_n be the amount owing after n months.

- (a) Show that $A_n = 840\,000 - 190\,000 \times 1.005^n$.

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- (b) After 15 years, the amount owing is \$373 722 to the nearest dollar. At this time Mimi borrows a further \$200 000 to build an extension. She adds this onto her previous mortgage. If the monthly repayments remain the same, what is the minimum remaining number of months it will take her to pay off the balance of the loan?

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Question 33 (6 marks)

- (a) Expand and simplify the expression $(x+4)(x-1)(x-3)$.

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- (b) Consider the curves defined by the equations $y = \frac{12}{x}$ and $y = 13 - x^2$.

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Using part (a), or otherwise, find the points of intersection between these two curves and find the exact area enclosed between the two curves in the first quadrant.

[illegible]

Question 34 (2 marks)

Solve $\log_3 x + \log_9 x = 12$.

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Question 35 (2 marks)

Sophie rolls two dice and observes that the sum of the uppermost numbers is six.
What is the probability that the number four has appeared at least once?

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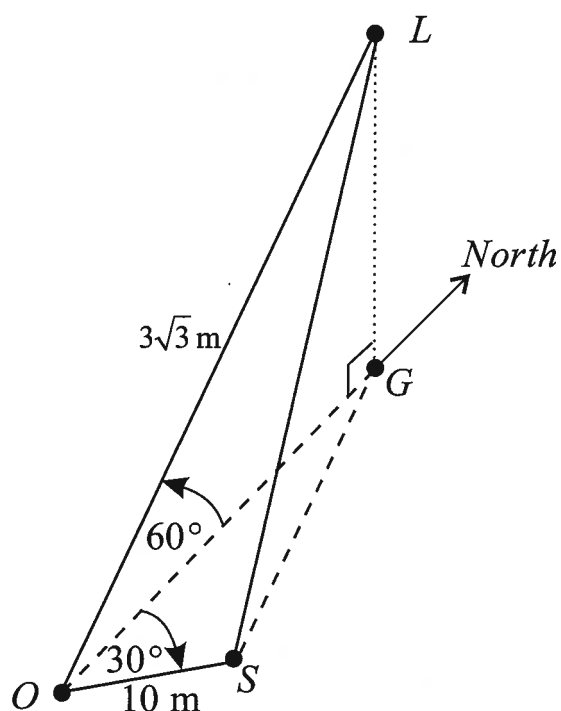
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Question 36 (3 marks)

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During the festival of lanterns Oscar releases a lantern from point O . It floats up into the sky due north in a straight line with an angle of elevation of 60° . Sarina is located at S , 10 metres from O at a bearing of $N30^\circ E$ from O .

The lantern has travelled $3\sqrt{3}$ metres from O when Sarina first observes it at L . The point G is on the ground directly below L .



Let SL be y . Find the distance y . Give your answer correct to the nearest metre.

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EXAMINERS

Svetlana Onisczenko (Convenor)
David Houghton
Christopher Dobell-Brown
Sue Wymer
Michael Zaouk

Sydney Grammar School, Darlinghurst
Sydney Grammar School, Darlinghurst
St Aloysius' College, Milsons Point
SCEGGS, Darlinghurst
St Ursula's College, Kingsgrove

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Mathematics Advanced Exam Paper Replacement for Question 29 Page 19

Question 29 (6 marks)

Consider the random variable X whose probability density function is

$$f(x) = \begin{cases} \frac{x(k-x)}{36} & \text{for } 0 \leq x \leq k \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Show that the value of k is 6. 2

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- (b) What is the mode and why? 2

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- (c) Find the cumulative distribution function of X . 2

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Mathematics Advanced Exam Paper Replacement for Question 11 Page 7

Question 11 (2 marks)

Solve $|2x - 3| = 1$.

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