EXTENSION 1

Trigonometry (Ext1), T3 Trig Equations (Y12)

Auxiliary Angles (Ext1)

Identities, Equations and 't' formulae (Ext1)

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Exam Equivalent Time: 79.5 minutes *(based on allocation of 1.5 minutes per mark)*

Trig Equations

Auxiliary Angles
Identities, Equations and 't' Formulae

***SmarterMaths analytics based on the average contribution to Extension 1 exams since 2020.**

HISTORICAL CONTRIBUTION

- *T3 Trig Equations* is easily the biggest sub-topic within Trigonometry and has contributed an impressive average of 8.1% per new syllabus Ext1 exam since its introduction in 2020.
- This topic has been split into two sub-categories for analysis purposes which are: *1-Auxiliary Angles (3.3%) and 2-Identities, Equations and 't' Formulae (4.8%).*
- This analysis look at *Identities, Equations and 't' Formulae.*

HSC ANALYSIS - What to expect and common pitfalls

- *Identities and Equations (4.8%)* was last examined in longer answer questions in 2021 (twice) and 2020. It was allocated a significant 4-marks within a very challenging 7-mark cross topic question in *2020 Ext1 14b*, a question that represents the highest level of difficulty that students can expect.
- A revision focus here is important. The use of compound and double angle identities is not only core knowledge for this topic area, but it will also prove extremely valuable in later applications of calculus.
- NESA Topic Guidance explicitly states that students can be asked to provide "all" solutions to some trig equations. Carefully review *Ext1 T3 EQ-Bank 3* to see a worked solution that does not require knowledge of the general solution formulae.
- The 't' formulae were last examined in 2007 and 2005. It's inclusion in the new syllabus make it a smoky bear in our view - ensure you revise and understand this area!

Questions

1. Trigonometry, EXT1 T3 2015 HSC 11d

Express $5\cos x - 12\sin x$ in the form $A\cos(x+\alpha)$, where $0 \le \alpha \le \dfrac{\pi}{2}$. *(2 marks)*

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2. Trigonometry, EXT1 T3 SM-Bank 10

Given that $\cot(2x) + \dfrac{1}{2}\tan(x) = a\cot(x)$, calculate a. *(3 marks)*

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3. Trigonometry, EXT1 T3 2013 HSC 12a

i. Write $\sqrt{3}\cos x - \sin x$ in the form $2\cos(x+\alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$. *(1 mark)*

ii. Hence, or otherwise, solve $\sqrt{3}\cos x = 1 + \sin x$, where $0 < x < 2\pi$. *(2 marks)*

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4. Trigonometry, EXT1 T3 2007 HSC 2a

By using the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, show that $\dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$. *(2 marks)*

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5. Trigonometry, EXT1 T3 2009 HSC 3c

i. Prove that $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$ provided that $\cos 2\theta \ne -1$. *(2 marks)*

ii. Hence find the exact value of $\tan\dfrac{\pi}{8}$. *(1 mark)*

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6. Trigonometry, EXT1 T3 SM-Bank 13

Find all solutions of $\tan(2\theta) = -\tan\theta$ for $0 \le \theta \le 2\pi$. *(3 marks)*

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7. Trigonometry, EXT1 T3 SM-Bank 12

Solve $\sin(2x) = \sin x$, for $0 \le x \le 2\pi$. *(3 marks)*

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8. Trigonometry, EXT1 T3 2020 HSC 11d

By expressing $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x+a)$, solve $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$, for $0 \le x \le 2\pi$. *(4 marks)*

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9. Trigonometry, EXT1 T3 2021 HSC 11g

By factorising, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$. *(3 marks)*

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10. Trigonometry, EXT1 T3 2023 HSC 11e

Solve $\cos\theta + \sin\theta = 1$ for $0 \le \theta \le 2\pi$. *(3 marks)*

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11. Trigonometry, EXT1 T3 EQ-Bank 4

The current flowing through an electrical circuit can be modelled by the function

$$f(t) = 6\sin 0.05t + 8\cos 0.05t, \quad t \ge 0$$

i. Express the function in the form $f(t) = A\sin(at + b)$, for $0 \le b \le \dfrac{\pi}{2}$. *(2 marks)*

ii. Find the time at which the current first obtains it maximum value. *(1 mark)*

iii. Sketch the graph of $f(t)$. Clearly show its range and label the coordinates of its first maximum value. Do not label x-intercepts. *(1 mark)*

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12. Trigonometry, EXT1 T3 SM-Bank 2

Show that

$\cos 3x = 4\cos^3 x - 3\cos x$. *(3 marks)*

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13. Trigonometry, EXT1 T3 2010 HSC 4b

i. Express $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right)$ in the form $R\cos(\theta + \alpha)$,

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. *(3 marks)*

ii. Hence, or otherwise, solve $2\cos\theta + 2\cos\left(\theta + \dfrac{\pi}{3}\right) = 3$,

for $0 < \theta < 2\pi$. *(2 marks)*

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14. Trigonometry, EXT1 T3 EQ-Bank 1

i. Show that $\sin x + \sin 3x = 2\sin 2x\cos x$. *(2 marks)*

ii. Hence or otherwise, find all values of x that satisfy

$$\sin x + \sin 2x + \sin 3x = 0, \quad x \in [0, 2\pi].$$ *(2 marks)*

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15. Trigonometry, EXT1 T3 EQ-Bank 2

A particular energy wave can be modelled by the function

$$f(t) = \sqrt{5}\sin 0.2t + 2\cos 0.2t, \quad t \in [0, 50]$$

i. Express this function in the form $f(t) = R\sin(nt - \alpha)$, $\alpha \in [0, 2\pi]$. *(2 marks)*

ii. Find the time the wave first attains its maximum value. Give your answer to one decimal place. *(2 marks)*

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16. Trigonometry, EXT1 T3 2005 HSC 4b

By making the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, show that

$\operatorname{cosec}\theta + \cot\theta = \cot\dfrac{\theta}{2}$. *(2 marks)*

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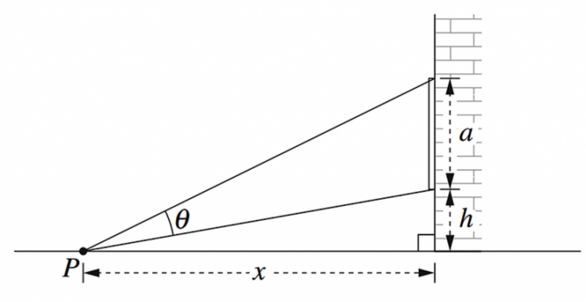
17. Trigonometry, EXT1 T3 SM-Bank 1

A billboard of height a metres is mounted on the side of a building, with its bottom edge h metres above street level. The billboard subtends an angle θ at the point P, x metres from the building.



Use the identity $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$ to show that

$$\theta = \tan^{-1}\left[\frac{ax}{x^2 + h(a+h)}\right].$$ *(2 marks)*

Worked Solutions

1. Trigonometry, EXT1 T3 2015 HSC 11d

$$5\cos x - 12\sin x = A\cos(x+\alpha)$$
$$= A\cos x\cos\alpha - A\sin x\sin\alpha$$

$$\therefore A\cos\alpha = 5,\quad A\sin\alpha = 12$$

$$A^2 = 5^2 + 12^2 = 169$$
$$A = 13$$
$$\Rightarrow 13\cos\alpha = 5$$
$$\cos\alpha = \frac{5}{13}$$
$$\alpha = \cos^{-1}\left(\frac{5}{13}\right) \approx 1.176\ldots \text{ radians}$$

$$\therefore 5\cos x - 12\sin x = 13\cos(x + 1.176\ldots)$$

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2. Trigonometry, EXT1 T3 SM-Bank 10

$$\frac{1}{\tan(2x)} + \frac{\tan(x)}{2} = \frac{a}{\tan(x)},\quad \tan(x) \neq 0$$
$$\frac{1-\tan^2(x)}{2\tan(x)} + \frac{\tan(x)}{2} = \frac{a}{\tan(x)}$$

If $\tan(x) \neq 0$:

$$\frac{1-\tan^2(x)+\tan^2(x)}{2\tan(x)} = \frac{a}{\tan(x)}$$
$$1 = 2a$$
$$\therefore a = \frac{1}{2}$$

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3. Trigonometry, EXT1 T3 2013 HSC 12a

i. Write $\sqrt{3}\cos x - \sin x$ in form

$$2\cos(x+\alpha),\quad 0 < \alpha < \frac{\pi}{2}$$
$$2(\cos x\cos\alpha - \sin x\sin\alpha) = \sqrt{3}\cos x - \sin x$$
$$\cos x\cos\alpha - \sin x\sin\alpha = \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$$
$$\Rightarrow \cos\alpha = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \sin\alpha = \frac{1}{2}$$
$$\alpha = \frac{\pi}{6}$$
$$\therefore\ 2\cos\left(x+\frac{\pi}{6}\right) = \sqrt{3}\cos x - \sin x$$

ii. Solve $\sqrt{3}\cos x = 1 + \sin x,\quad 0 < x < 2\pi$

$$\sqrt{3}\cos x - \sin x = 1$$
$$2\cos\left(x+\frac{\pi}{6}\right) = 1\quad \text{(from part (i))}$$
$$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$$
$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Since cos is positive in $1^{\text{st}}/4^{\text{th}}$ quadrants

$$x + \frac{\pi}{6} = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}\qquad (0 < x < 2\pi)$$
$$\therefore\ x = \frac{\pi}{6},\ \frac{3\pi}{2}$$

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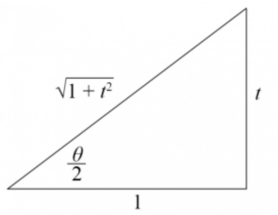
4. Trigonometry, EXT1 T3 2007 HSC 2a



COMMENT: The values of $\sin\theta$ and $\cos\theta$ can be committed to memory or quickly derived using double angle formulae (as shown in the worked solution).

$$\Rightarrow \tan\frac{\theta}{2} = t$$

$$\Rightarrow \sin\theta = 2\cdot\frac{t}{\sqrt{1+t^2}}\cdot\frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2}$$

$$\Rightarrow \cos\theta = \frac{1}{1+t^2} - \frac{t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

Show $\dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$:

$$\begin{aligned}\frac{1-\cos\theta}{\sin\theta} &= \frac{1-\left(\frac{1-t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}} \times \frac{1+t^2}{1+t^2}\\ &= \frac{1+t^2-(1-t^2)}{2t}\\ &= \frac{2t^2}{2t}\\ &= t\\ &= \tan\frac{\theta}{2}\quad \text{...as required}\end{aligned}$$

5. Trigonometry, EXT1 T3 2009 HSC 3c

i. Prove $\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$, $\cos 2\theta \neq -1$

$$\begin{aligned}\text{Using}\ \cos 2\theta &= 2\cos^2\theta - 1\\ &= 1-2\sin^2\theta\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}\\ &= \frac{2\sin^2\theta}{2\cos^2\theta}\\ &= \tan^2\theta\ \text{...as required.}\end{aligned}$$

ii.

$$\begin{aligned}\tan^2\frac{\pi}{8} &= \frac{1-\cos\left(2\times\frac{\pi}{8}\right)}{1+\cos\left(2\times\frac{\pi}{8}\right)}\\ &= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}\times\frac{\sqrt{2}}{\sqrt{2}}\\ &= \frac{\sqrt{2}-1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}\\ &= \frac{\left(\sqrt{2}-1\right)^2}{2-1}\\ &= \left(\sqrt{2}-1\right)^2\end{aligned}$$

$$\therefore\ \tan\frac{\pi}{8} = \sqrt{2}-1 \qquad \left(\tan\left(\frac{\pi}{8}\right) > 0\right)$$

$(\tan\dfrac{\pi}{8} = \sqrt{3-2\sqrt{2}}$ is also a correct answer$)$

6. Trigonometry, EXT1 T3 SM-Bank 13

$$\frac{2\tan\theta}{1-\tan^2\theta} = -\tan\theta$$

Let $\tan\theta = k,$

$$\frac{2k}{1-k^2} = -k$$
$$2k = -k(1-k^2)$$
$$2k + k(1-k^2) = 0$$
$$3k - k^3 = 0$$
$$k(3-k^2) = 0$$

$$k(3-k^2) = 0 \quad \Rightarrow \quad k = 0,\ k = \pm\sqrt{3}$$

If $\tan\theta = 0 \ \Rightarrow\ \theta = 0,\ \pi,\ 2\pi$

If $\tan\theta = \pm\sqrt{3} \Rightarrow\ \theta = \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}$

$\therefore \theta = 0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3},\ 2\pi$

7. Trigonometry, EXT1 T3 SM-Bank 12

$2\sin x\cos x = \sin x$

$\sin x(2\cos x-1) = 0$

$\sin x = 0, \cos x = \frac{1}{2}$

If $\sin x = 0\ \Rightarrow\ x = 0,\ \pi,\ 2\pi$

If $\cos x = \frac{1}{2}\ \Rightarrow\ x = \frac{\pi}{3},\ \frac{5\pi}{3}$

$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

MARKER'S COMMENT: Many students expanded $\sin(2x)$ and then cancelled $\sin x$ on both sides which lost a set of solutions!

8. Trigonometry, EXT1 T3 2020 HSC 11d

$$A\sin(x+\alpha) = \sqrt{3}\sin x + 3\cos x$$
$$A\sin x\cos\alpha + A\cos x\sin\alpha = \sqrt{3}\sin x + 3\cos x$$

Equating co-efficients:

$\Rightarrow A\cos\alpha = \sqrt{3}$

$\Rightarrow A\sin\alpha = 3$

$$A^2 = \left(\sqrt{3}\right)^2 + 3^2 = 12$$

$\therefore A = \sqrt{12}$

$$\frac{A\sin\alpha}{A\cos\alpha} = \frac{3}{\sqrt{3}}$$
$$\tan\alpha = \sqrt{3}$$
$$\alpha = \frac{\pi}{3}$$

$$\sqrt{3}\sin x + 3\cos x = \sqrt{3}$$
$$\sqrt{12}\sin\left(x+\frac{\pi}{3}\right) = \sqrt{3}$$
$$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$$
$$x + \frac{\pi}{3} = \frac{5\pi}{6}, \frac{13\pi}{6}$$
$$\therefore x = \frac{\pi}{2}, \frac{11\pi}{6} \quad (0 \le x \le 2\pi)$$

9. Trigonometry, EXT1 T3 2021 HSC 11g

$$2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$$
$$2\sin^2 x(\sin x+1) - (\sin x+1) = 0$$
$$(2\sin^2 x - 1)(\sin x + 1) = 0$$

$2\sin^2 x = 1 \qquad \sin x = \ = -1$

$$\sin^2 x = \frac{1}{2}$$
$$\sin x = \pm\frac{1}{\sqrt{2}}$$

$$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

10. Trigonometry, EXT1 T3 2023 HSC 11e

Let $t = \tan\dfrac{\theta}{2}$

$$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} = 1 \text{ (see reference sheet)}$$

$$1 - t^2 + 2t = 1 + t^2$$

$$2t^2 - 2t = 0$$

$$2t(t-1) = 0$$

$$t = 0 \text{ or } 1$$

When $\tan\dfrac{\theta}{2} = 0$:

$$\frac{\theta}{2} = 0, \pi \ \Rightarrow\ \theta = 0 \text{ or } 2\pi$$

When $\tan\dfrac{\theta}{2} = 1$:

$$\frac{\theta}{2} = \frac{\pi}{4} \ \Rightarrow\ \theta = \frac{\pi}{2}$$

Test $\theta = \pi$:

$\cos\pi + \sin\pi = -1 + 0 = -1$ (not a solution)

$$\therefore \theta = 0, \frac{\pi}{2}, 2\pi$$

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11. Trigonometry, EXT1 T3 EQ-Bank 4

i. $f(t) = 6\sin 0.05t + 8\cos 0.05t$

$$\begin{aligned} A\sin(at+b) &= A\sin(0.05t+b) \\ &= A\sin(0.05t)\cos(b) + A\cos(0.05t)\sin(b) \end{aligned}$$

$$\Rightarrow A\cos(b) = 6,\ A\sin(b) = 8$$

$$A^2 = 6^2 + 8^2$$

$$A = 10$$

$$\Rightarrow 10\cos(b) = 6$$

$$\cos(b) = \frac{6}{10}$$

$$b = \cos^{-1}0.06 \approx 0.927 \text{ radians}$$

$$\therefore f(t) = 10\sin(0.05t + 0.927)$$

ii. Max occurs at $\sin(0.05t + 0.927) = \sin\dfrac{\pi}{2}$

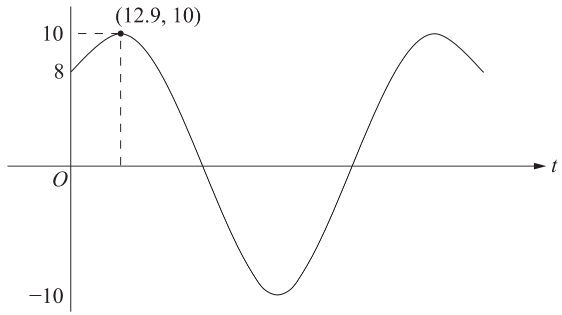
$$0.05t + 0.927 = \frac{\pi}{2}$$

$$0.05t = 0.643\ldots$$

$$\therefore t = 12.87$$

$$= 12.9 \text{ (to 1 d.p.)}$$

iii.



12. Trigonometry, EXT1 T3 SM-Bank 2

$$\begin{aligned}
\text{LHS} &= \cos(2x+x) \\
&= \cos 2x \cos x - \sin 2x \sin x \\
&= \left(\cos^2 x - \sin^2 x\right)\cos x - 2\sin x \cos x \sin x \\
&= \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x \\
&= \cos^3 x - \left(1 - \cos^2 x\right)\cos x - 2\left(1 - \cos^2 x\right)\cos x \\
&= \cos^3 x - \cos x + \cos^3 x - 2\cos x + 2\cos^3 x \\
&= 4\cos^3 x - 3\cos x
\end{aligned}$$

COMMENT: High achieving students know the 3 variants of $\cos 2x$ back to front. Here, $\cos 2x = \cos^2 x - \sin^2 x$ breaks the back of this problem.

13. Trigonometry, EXT1 T3 2010 HSC 4b

i. $2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right)$

$$= 2\cos\theta + 2\left(\cos\theta\cos\left(\frac{\pi}{3}\right) - \sin\theta\sin\left(\frac{\pi}{3}\right)\right)$$

$$= 2\cos\theta + 2\cos\theta \times \frac{1}{2} - 2\sin\theta \times \frac{\sqrt{3}}{2}$$

$$= 2\cos\theta + \cos\theta - \sqrt{3}\sin\theta$$

$$= 3\cos\theta - \sqrt{3}\sin\theta$$

$$R\cos(\theta + \alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$$

$$R\cos\alpha = 3 \qquad R\sin\alpha = \sqrt{3}$$

$$\cos\alpha = \frac{3}{R} \qquad \sin\alpha = \frac{\sqrt{3}}{R}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\therefore \alpha = \frac{\pi}{6} \quad \left(0 < \alpha < \frac{\pi}{2}\right)$$

$$R^2 = 3^2 + \left(\sqrt{3}\right)^2$$

$$= 9 + 3$$

$$R = \sqrt{12} = 2\sqrt{3}$$

$$\therefore\ 2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right) = 2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right)$$

ii. $2\cos\theta + 2\cos\left(\theta + \frac{\pi}{3}\right) = 3$

$$2\sqrt{3}\cos\left(\theta + \frac{\pi}{6}\right) = 3$$

$$\cos\left(\theta + \frac{\pi}{6}\right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

Since cos is positive in 1st and 4th quadrants,

$$\theta + \frac{\pi}{6} = \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$$

$$\therefore \theta$$

♦ *Mean mark part (ii) 49%*
MARKER'S COMMENT: Many students did not check their answers against the stated domain for θ.

$$= \frac{5\pi}{3} \qquad (0 < \theta < 2\pi)$$

14. Trigonometry, EXT1 T3 EQ-Bank 1

i. $\sin x + \sin 3x = \sin x + \sin 2x\cos x + \cos 2x \sin x$

$$= \sin x + 2\sin x\cos^2 x + \left(\cos^2 x - \sin^2 x\right)\sin x$$

$$= \sin x + 2\sin x\cos^2 x + \cos^2 x\sin x - \sin^3 x$$

$$= \sin x + 3\sin x\left(1 - \sin^2 x\right) - \sin^3 x$$

$$= \sin x + 3\sin x - 3\sin^3 x - \sin^3 x$$

$$= 4\sin x\left(1 - \sin^2 x\right)$$

$$= 4\sin x\cos^2 x$$

$$= 2\sin 2x\cos x$$

$$= \text{RHS}$$

ii. $\sin x + \sin 2x + \sin 3x = 0$

$$\sin 2x + 2\sin 2x\cos x = 0$$

$$\sin 2x(1 + 2\cos x) = 0$$

If $\sin 2x = 0$:

$$2x = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$\therefore x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

If $\cos x = -\frac{1}{2}$:

$$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

15. Trigonometry, EXT1 T3 EQ-Bank 2

i. $f(t) = \sqrt{5}\sin 0.2t + 2\cos 0.2t$

$$\begin{aligned} R\sin(nt - \alpha) &= R\sin(0.2t - \alpha) \\ &= R\sin 0.2t\cos\alpha - R\cos 0.2t\sin\alpha \end{aligned}$$

$\Rightarrow R\cos\alpha = \sqrt{5}, \quad R\sin\alpha = -2$

$R^2 = \left(\sqrt{5}\right)^2 + (-2)^2 = 9$

$R = 3$

$\cos\alpha = \dfrac{\sqrt{5}}{3}, \sin\alpha = -\dfrac{2}{3}$

$\Rightarrow \alpha$ is in 4th quadrant

$\text{Base angle} = \cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right) = 0.7297$

$$\begin{aligned} \therefore \alpha &= 2\pi - 0.7297 \\ &= 5.553\ldots \end{aligned}$$

$\therefore f(t) = 3\sin(0.2t - 5.553)$

ii. Max value occurs when $\sin(0.2t - 5.553) = 1$

$$\begin{aligned} 0.2t - 5.553 &= \frac{\pi}{2} \\ 0.2t &= 7.124\ldots \\ t &= 35.62\ldots \end{aligned}$$

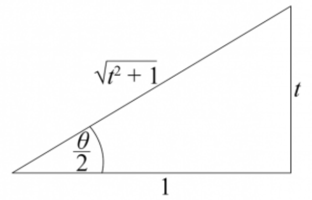
Test if $t > 0$ for:

$$\begin{aligned} 0.2t - 5.553 &= -\frac{3\pi}{2} \\ 0.2t &= 0.8406 \\ t &= 4.20\ldots \end{aligned}$$

$\therefore$ Time of 1st maximum: $t = 4.2$

16. Trigonometry, EXT1 T3 2005 HSC 4b

Show $\text{cosec}\ \theta + \cot\ \theta = \cot\ \dfrac{\theta}{2}$



$\tan\ \dfrac{\theta}{2} = t$

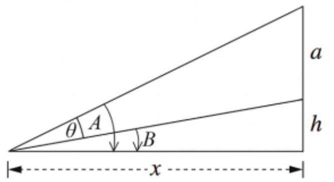
$\Rightarrow \sin\theta = 2\sin\ \dfrac{\theta}{2}\ \cos\ \dfrac{\theta}{2} = \dfrac{2t}{t^2+1}$

$\Rightarrow \cos\theta = \cos^2\ \dfrac{\theta}{2} - \sin^2\ \dfrac{\theta}{2} = \dfrac{1-t^2}{t^2+1}$

$\Rightarrow \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{2t}{1-t^2}$

$$\begin{aligned} \text{LHS} &= \frac{1}{\sin\theta} + \frac{1}{\tan\theta} \\ &= \frac{t^2+1}{2t} + \frac{1-t^2}{2t} \\ &= \frac{t^2+1+1-t^2}{2t} \\ &= \frac{1}{t} \\ &= \frac{1}{\tan\ \frac{\theta}{2}} \\ &= \cot\ \frac{\theta}{2} \\ &= \text{RHS} \ \ldots \text{ as required.} \end{aligned}$$

17. Trigonometry, EXT1 T3 SM-Bank 1



MARKER'S COMMENT: Answers that included a diagram and clearly labelled angles were generally successful.

Show $\theta = \tan^{-1}\left[\dfrac{ax}{x^2 + h(a+h)}\right]$

$$\tan A = \frac{a+h}{x}$$

$$\tan B = \frac{h}{x}$$

$$\begin{aligned}\tan(A-B) &= \frac{\frac{a+h}{x} - \frac{h}{x}}{1 + \left(\frac{a+h}{x}\right)\left(\frac{h}{x}\right)} \times \frac{x^2}{x^2} \\ &= \frac{x(a+h) - xh}{x^2 + h(a+h)} \\ &= \frac{ax}{x^2 + h(a+h)}\end{aligned}$$

Since $\theta = A - B$

$$\theta = \tan^{-1}\left[\frac{ax}{x^2 + h(a+h)}\right] \quad \ldots \text{as required.}$$