

ADVANCED MATHEMATICS

Functions (Adv), F1 Working with Functions (Adv)

Further Functions and Relations (Y11)

Quadratics and Cubic Functions (Y11)

Linear Functions (Y11)

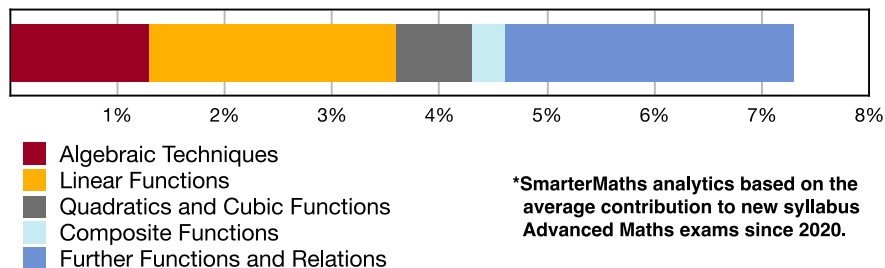
Composite Functions (Y11)



Teacher: Cathyanne Horvat

Exam Equivalent Time: 58.5 minutes (based on allocation of 1.5 minutes per mark)

F1 Working With Functions



HISTORICAL CONTRIBUTION

- *F1 Working with Functions* has contributed a healthy 7.3% per exam since the new syllabus was introduced in 2020.
- We have split the topic into 5 categories for analysis purposes: 1-Algebraic Techniques, 2-Linear Functions, 3-Quadratics and Cubic Functions, 4-Composite Functions and 5-Further Functions and Relations.
- This analysis looks at *Quadratics and Cubic Functions* (0.7%).

HSC ANALYSIS - What to expect and common pitfalls

- *Quadratics and Cubic Functions* have not been a big contributor to Adv exams in the new syllabus era although this area has been examined via multiple choice in the past 2 exams.
- *Quadratic* factorisation has easily been the most common question style in this sub-topic, offering up easy marks in 5 exams within the last decade.
- The range of a quadratic function was tested in 2022 with the answer required in set notation (see 2022 Adv 4 MC).
- Students have also been asked to solve quadratics using the general formula and to find the intersection of quadratic and linear equations.
- A cubic equation was examined in a challenging 2021 question that required students to identify the correct equation of a cubic graph (see 2021 Adv 8 MC).
- The graphic representation of an *odd function* was poorly answered in 2016 and should be reviewed.

Questions

1. Functions, 2ADV F1 2007 HSC 1b

Solve $2x - 5 > -3$ and graph the solution on a number line. (2 marks)

2. Functions, 2ADV F1 2010 HSC 1a

Solve $x^2 = 4x$. (2 marks)

3. Functions, 2ADV F1 SM-Bank 33

- State the domain and range of $y = -\sqrt{12 - x^2}$. (2 marks)
- Sketch the graph. (1 mark)

4. Functions, 2ADV F1 SM-Bank 37

Find all values of x for which $|x - 4| = \frac{x}{2} + 7$. (3 marks)

5. Functions, 2ADV F1 SM-Bank 25

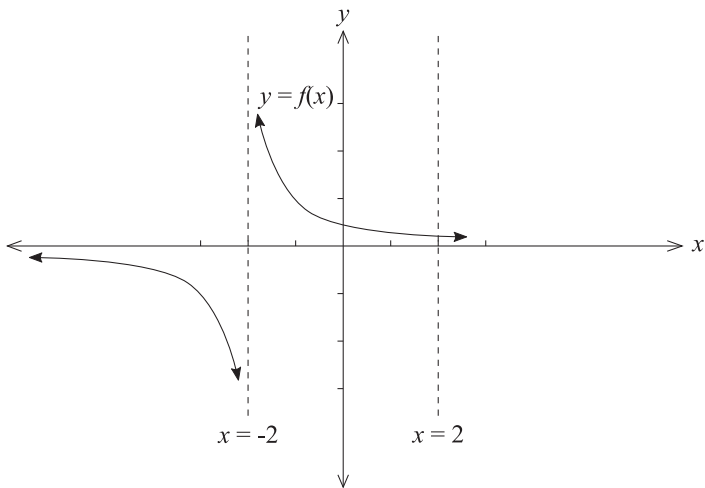
Damon owns a swim school and purchased a new pool pump for \$3250.

He writes down the value of the pool pump by 8% of the original price each year.

- Construct a function to represent the value of the pool pump after t years. (1 mark)
- Draw the graph of the function and state its domain and range. (2 marks)

6. Functions, 2ADV F1 SM-Bank 36

Consider the function $f(x) = \frac{1}{x+2}$.



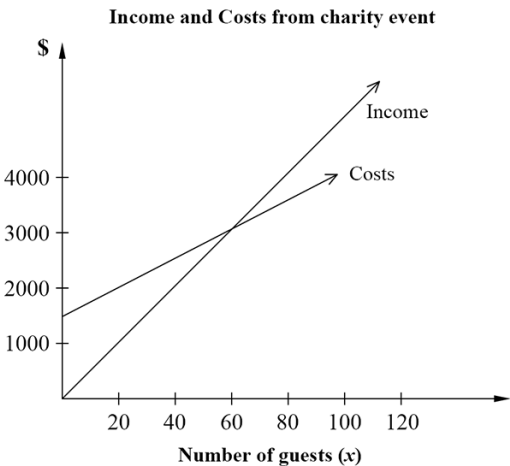
- i. Sketch the graph $y = f(-x)$. (2 marks)
- ii. On the same graph, sketch $y = -f(x)$. (2 marks)

7. Functions, 2ADV F1 2019 HSC 13e

- i. Sketch the graph of $y = |x - 1|$ for $-4 \leq x \leq 4$. (1 mark)
- ii. Using the sketch from part i, or otherwise, solve $|x - 1| = 2x + 4$. (2 marks)

8. Algebra, STD2 A4 SM-Bank 27

- Fiona and John are planning to hold a fund-raising event for cancer research. They can hire a function room for \$650 and a band for \$850. Drinks will cost them \$25 per person.
- i. Write a formula for the cost (\$C) of holding the charity event for x people. (1 mark)
 - ii. The graph below shows the planned income and costs if they charge \$50 per ticket. Estimate the number of guests they need to break even. (1 mark)



- iii. How much profit will Fiona and John make if 80 people attend their event? (1 mark)

9. Functions, 2ADV F1 EQ-Bank 11

Given the function $f(x) = \sqrt{3-x}$ and $g(x) = x^2 - 2$, sketch $y = g(f(x))$ over its natural domain. (2 marks)

10. Functions, 2ADV F1 EQ-Bank 12

Two archers play a game where each can aim for a large target or a small target.

If an arrow hits the large target it scores L points, and if it hits the small target, it scores S points.

The results of a game are shown in the table below.

Archer	Number of large target hits	Number of small target hits	Total Score
1	5	8	71
2	12	5	71

By forming a pair of simultaneous equations, or otherwise, find the value of L and S . (3 marks)

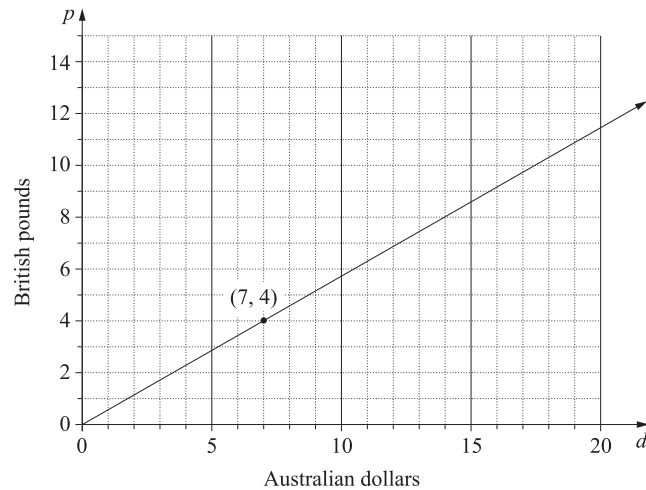
11. Functions, 2ADV F1 EQ-Bank 8

Jacques is a marine biologist and finds that the mass of a crab is directly proportional to the cube of the diameter of its shell.

If a crab with a shell diameter of 15 cm weighs 680 grams, what will be the diameter of a crab that weighs 1.1 kilograms? Give your answer to 1 decimal place. **(2 marks)**

12. Algebra, STD2 A2 2019 HSC 34

The relationship between British pounds (***p***) and Australian dollars (***d***) on a particular day is shown in the graph.



- Write the direct variation equation relating British pounds to Australian dollars in the form **$p = md$** . Leave **m** as a fraction. **(1 mark)**
- The relationship between Japanese yen (***y***) and Australian dollars (***d***) on the same day is given by the equation **$y = 76d$** . Convert 93 100 Japanese yen to British pounds. **(2 marks)**

13. Functions, 2ADV F1 SM-Bank 3

Let **$f(x) = \sqrt{x+1}$** for **$x \geq 0$**

- State the range of **$f(x)$** . **(1 mark)**
- Let **$g(x) = x^2 + 4x + 3$** , where **$x \leq c$** and **$c \leq 0$** .

Find the largest possible value of **c** such that the range of **$g(x)$** is a subset of the domain of **$f(x)$** . **(2 marks)**

14. Functions, 2ADV F1 2020 HSC 24

The circle of **$x^2 - 6x + y^2 + 4y - 3 = 0$** is reflected in the **x** -axis.

Sketch the reflected circle, showing the coordinates of the centre and the radius. **(3 marks)**

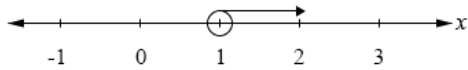
Worked Solutions

1. Functions, 2ADV F1 2007 HSC 1b

$$2x - 5 > -3$$

$$2x > 2$$

$$x > 1$$



2. Functions, 2ADV F1 2010 HSC 1a

$$x^2 = 4x$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$\therefore x = 0 \text{ or } 4$$

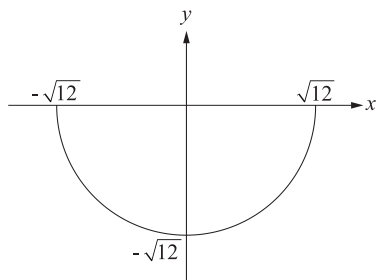
3. Functions, 2ADV F1 SM-Bank 33

i. $y = -\sqrt{12 - x^2}$

Domain: $-\sqrt{12} \leq x \leq \sqrt{12}$

Range: $-\sqrt{12} \leq y \leq 0$

ii.



4. Functions, 2ADV F1 SM-Bank 37

$$x - 4 = \frac{x}{2} + 7 \quad \text{or} \quad -(x - 4) = \frac{x}{2} + 7$$

$$2x - 8 = x + 14$$

$$-2x + 8 = x + 14$$

$$x = 22$$

$$3x = -6$$

$$x = -2$$

$$\therefore x = 22 \text{ or } -2$$

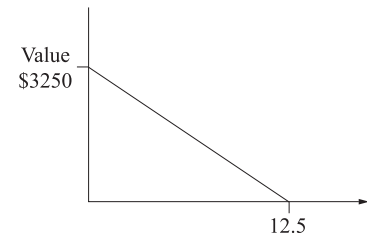
5. Functions, 2ADV F1 SM-Bank 25

i. Depreciation each year = $8\% \times 3250$

$$= \$260$$

$$\therefore \text{Value} = 3250 - 260t$$

ii.



Find t when value = 0

$$3250 - 260t = 0$$

$$t = \frac{3250}{260}$$

$$= 12.5 \text{ years}$$

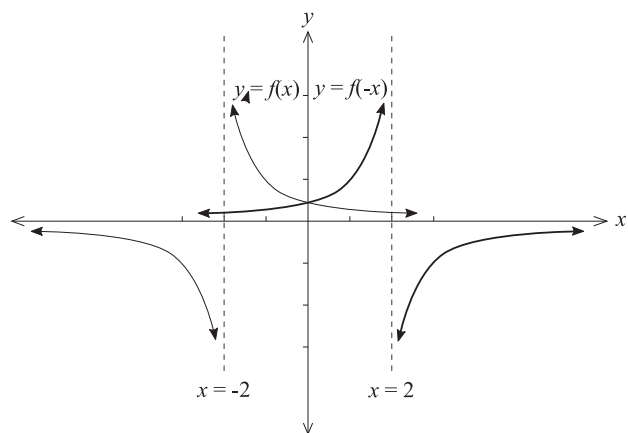
Domain $\{t: 0 \leq t \leq 12.5\}$

Range $\{y: 0 \leq y \leq 3250\}$

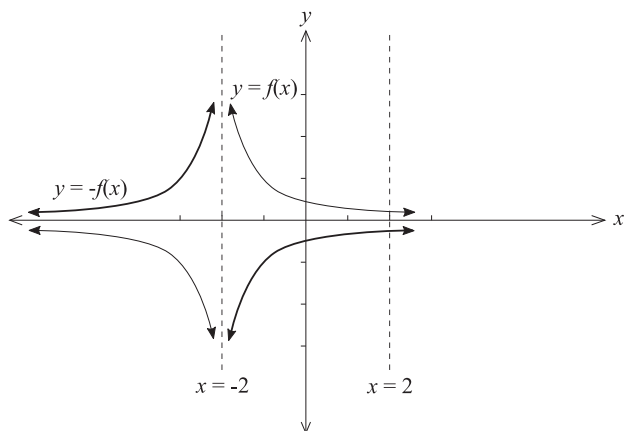
6. Functions, 2ADV F1 SM-Bank 36

i. Sketch $y = \frac{1}{x+2}$

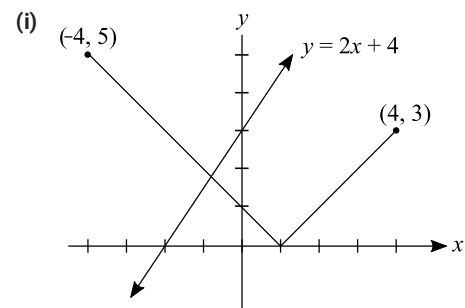
$y = f(-x) \Rightarrow$ reflect $y = \frac{1}{x+2}$ in the y -axis.



ii. $y = -f(x) \Rightarrow$ reflect $y = \frac{1}{x+2}$ in the x -axis.



7. Functions, 2ADV F1 2019 HSC 13e



ii. By inspection, intersection when $x = -1$

Test:

$$|-1 - 1| = -2 + 4$$

$$2 = 2$$

\therefore Intersection at $(-1, 2)$

8. Algebra, STD2 A4 SM-Bank 27

i. **Fixed Costs** = $650 + 850$
 $= \$1500$

Variable Costs = $\$25x$
 $\therefore \$C = 1500 + 25x$

ii. **From the graph**

Costs = Income when $x = 60$
 (i.e. where graphs intersect)

iii. **When** $x = 80$:

Income = 80×50
 $= \$4000$

$\$C = 1500 + 25 \times 80$
 $= \$3500$

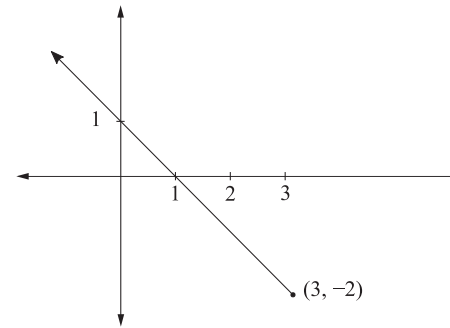
\therefore **Profit** = $4000 - 3500$
 $= \$500$

9. Functions, 2ADV F1 EQ-Bank 11

$$g(x) = x^2 - 2, \quad f(x) = \sqrt{3-x}$$

$$\begin{aligned} g(f(x)) &= (\sqrt{3-x})^2 - 2 \\ &= 3 - x - 2 \\ &= 1 - x \end{aligned}$$

Since $f(x) = \sqrt{3-x}$,
 \Rightarrow **Domain:** $x \leq 3$



10. Functions, 2ADV F1 EQ-Bank 12

$$5L + 8S = 71 \quad \dots (1)$$

$$12L + 5S = 71 \quad \dots (2)$$

Multiply $(1) \times 5$

$$25L + 40S = 355 \quad \dots (3)$$

Multiply $(2) \times 8$

$$96L + 40S = 568 \quad \dots (4)$$

Subtract $(4) - (3)$

$$71L = 213$$

$$\therefore L = 3$$

Substitute $L = 3$ **into** (1)

$$8S = 56$$

$$\therefore S = 7$$

11. Functions, 2ADV F1 EQ-Bank 8

$$M \propto d^3$$

$$M = kd^3$$

When $M = 680$, $d = 15$

$$680 = k \times 15^3$$

$$k = 0.201481\dots$$

Find d when $M = 1100$:

$$1100 = 0.20148\dots \times d^3$$

$$d = \sqrt[3]{\frac{1100}{0.20148\dots}}$$

$$= 17.608\dots$$

$$= 17.6 \text{ cm (to 1 d.p.)}$$

12. Algebra, STD2 A2 2019 HSC 34

a. $m = \frac{\text{rise}}{\text{run}} = \frac{4}{7}$

$$p = \frac{4}{7}d$$

♦ Mean mark 42%.

b. Yen to Australian dollars:

$$y = 76d$$

$$93\,100 = 76d$$

$$d = \frac{93\,100}{76}$$

$$= 1225$$

Aust dollars to pounds:

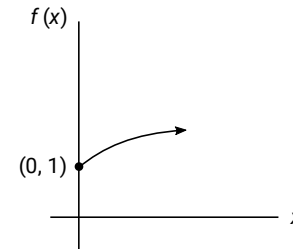
$$p = \frac{4}{7} \times 1225$$

$$= 700 \text{ pounds}$$

$$\therefore 93\,100 \text{ Yen} = 700 \text{ pounds}$$

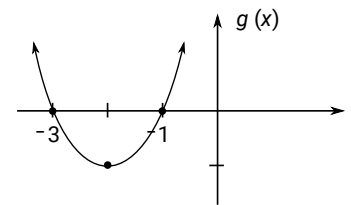
13. Functions, 2ADV F1 SM-Bank 3

i. Sketch of $f(x)$:



\therefore Range: $y \geq 1$

ii. Sketch $g(x) = (x+1)(x+3)$



Domain of $f(x)$: $x \geq 0$

Find domain of $g(x)$ such that range $g(x)$: $y \geq 0$

Graphically, this occurs when $g(x)$ has domain:

$$x \leq -3 \text{ and } x \geq -1$$

$$\therefore c = -3$$

14. Functions, 2ADV F1 2020 HSC 24

$$x^2 - 6x + y^2 + 4y - 3 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 - 16 = 0$$

$$(x-3)^2 + (y+2)^2 = 16$$

\Rightarrow Original circle has centre $(3, -2)$, radius = 4

Reflect in x -axis:

Centre $(3, -2) \rightarrow (3, 2)$

◆ Mean mark 48%.

