ADV: Statistics (Adv), S1 Probability & Probability Distributions (Adv)

Multi-Stage Events (Y11)
Relative Frequency (Y11)
Conditional Probability and Venn Diagrams (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 166.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

HISTORICAL CONTRIBUTION

- *S1 Probability* is a Year 11 topic that contains a mixture of old course and new syllabus content.
- The new *Advanced* course in this topic area has a significant overlap with the *Standard 2* syllabus and also introduces some brand new content.
- *S1 Probability* has been split into four sub-categories for the purposes of this analysis: *1-Multi-Stage Events, 2-Relative Frequency, 3-Conditional Probability and Venn Diagrams and 4-Discrete Probability Distributions.*
- This analysis looks at *Relative Frequency.*

HSC ANALYSIS - What to expect and common pitfalls

- *Relative Frequency* is a sub-topic that has previously only been examined in the *Standard* course but now represents common content with the new *Advanced* syllabus.
- *Relative Frequency* was not examined in the first two *Advanced exams* in 2020-21.
- It has been a substantial contributor to past *Std2* HSC exams, accounting for a very healthy average of 2.7% per year.
- Our database is made up of an extensive history of past HSC questions from the *Std2* course, which can be identified by the "STD2 S2" in their title.
- Although generally well answered, note the sub-50% mean marks in *2018 Adv 26a, 2017 Adv 29c* and in *2016 Adv 23 MC* that deserve attention (note that all mean marks and comments are taken from *Std2* results).

Questions

1. Probability, 2ADV S1 2015 HSC 4 MC

The probability that Mel's soccer team wins this weekend is $\frac{5}{7}$.

The probability that Mel's rugby league team wins this weekend is $\frac{2}{3}$.

What is the probability that neither team wins this weekend?

A. $\frac{2}{21}$

B. $\frac{10}{21}$

C. $\frac{13}{21}$

D. $\frac{19}{21}$

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2. Probability, STD2 S2 2010 HSC 12 MC

A group of 347 people was tested for flu and the results were recorded. The flu test results are not always accurate.

	Test results		
	Test indicated flu	*Test did not indicate flu*	*Total*
People with flu	72	3	75
People without flu	16	256	272
	88	259	347

A person is selected at random from the tested group.

What is the probability that their test result is accurate, to the nearest per cent?

(A) 21%

(B) 22%

(C) 95%

(D) 96%

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3. Probability, 2ADV S1 2019 MET2 11 MC

A and B are events from a sample space such that $P(A) = p$, where $p > 0$, $P(B \mid A) = m$ and $P(B \mid A') = n$.

A and B are independent events when

A. $m = n$

B. $m = 1 - p$

C. $m + n = 1$

D. $m = p$

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4. Probability, 2ADV S1 2019 MET2-N 9 MC

At the start of a particular week, Kim has three red apples and two green apples. She eats one apple everyday. On Monday, Tuesday and Wednesday of that week, she randomly selects an apple to eat. In this three-day period, the probability that Kim does not eat an apple of the same colour on any two consecutive days is

A. $\frac{1}{5}$

B. $\frac{3}{10}$

C. $\frac{2}{5}$

D. $\frac{6}{25}$

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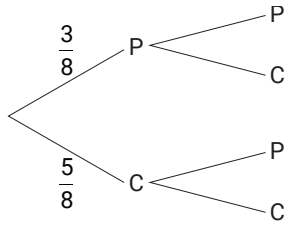
5. Probability, 2ADV S1 2021 HSC 6 MC

There are 8 chocolates in a box. Three have peppermint centres (P) and five have caramel centres (C).

Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.

A partially completed probability tree is shown.



What is the probability that Kim and Sam choose chocolates with different centres?

A. $\frac{15}{64}$

B. $\frac{15}{56}$

C. $\frac{15}{32}$

D. $\frac{15}{28}$

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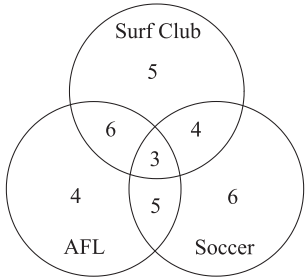
6. Probability, 2ADV S1 SM-Bank 4 MC

In a classroom, students are asked what sports club they are members of and the results are shown in the Venn diagram.



A student who is a member of a soccer club is chosen at random. What is the probability that he/she is also a member of a surf club?

A. $\frac{2}{5}$

B. $\frac{4}{11}$

C. $\frac{2}{9}$

D. $\frac{7}{18}$

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7. Probability, 2ADV S1 2013 HSC 5 MC

A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random without replacement.

What is the probability that at least one of the marbles selected is red?

A. $\frac{1}{6}$

B. $\frac{1}{2}$

C. $\frac{5}{6}$

D. $\frac{29}{30}$

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8. Probability, STD2 S2 2005 HSC 16 MC

On a television game show, viewers voted for their favourite contestant. The results were recorded in the two-way table.

	Male viewers	*Female viewers*
Contestant 1	1372	3915
Contestant 2	2054	3269

One male viewer was selected at random from all of the male viewers.

What is the probability that he voted for Contestant 1?

(A) $\frac{1372}{10\,610}$

(B) $\frac{1372}{5287}$

(C) $\frac{1372}{3426}$

(D) $\frac{1372}{2054}$

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9. Probability, 2ADV S1 2012 MET2 13 MC

A and B are events of a sample space S.

$$P(A \cap B) = \frac{2}{5} \text{ and } P(A \cap B') = \frac{3}{7}$$

$P(B' \mid A)$ is equal to

A. $\frac{6}{35}$

B. $\frac{15}{29}$

C. $\frac{14}{35}$

D. $\frac{29}{35}$

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10. Probability, 2ADV S1 2013 MET2 10 MC

For events A and B, $P(A \cap B) = p$, $P(A' \cap B) = p - \frac{1}{8}$ and $P(A \cap B') = \frac{3p}{5}$.

If A and B are independent, then the value of p is

A. 0

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

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11. Probability, 2ADV S1 2014 HSC 10 MC

Three runners compete in a race. The probabilities that the three runners finish the race in under 10 seconds are $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{2}{5}$ respectively.

What is the probability that at least one of the three runners will finish the race in under 10 seconds?

A. $\frac{1}{60}$

B. $\frac{37}{60}$

C. $\frac{3}{8}$

D. $\frac{5}{8}$

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12. Probability, 2ADV S1 2018 HSC 6 MC

A runner has four different pairs of shoes.

If two shoes are selected at random, what is the probability that they will be a matching pair?

A. $\frac{1}{56}$

B. $\frac{1}{16}$

C. $\frac{1}{7}$

D. $\frac{1}{4}$

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13. Probability, 2ADV S1 2010 MET2 21 MC

Events A and B are mutually exclusive events of a sample space with

$$P(A) = p \text{ and } P(B) = q \text{ where } 0 < p < 1 \text{ and } 0 < q < 1.$$

$P(A' \cap B')$ is equal to

A. $(1-p)(1-q)$

B. $1 - pq$

C. $1 - (p + q)$

D. $1 - (p + q - pq)$

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14. Probability, 2ADV S1 2013 MET2 17 MC

A and B are events of a sample space.

Given that $P(A \mid B) = p$, $P(B) = p^2$ and $P(A) = p^{\frac{1}{3}}$, $P(B \mid A)$ is equal to

A. p^3

B. $p^{\frac{4}{3}}$

C. $p^{\frac{7}{3}}$

D. $p^{\frac{8}{3}}$

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15. Probability, 2ADV S1 SM-Bank 6 MC

A box contains n marbles that are identical in every way except colour, of which k marbles are coloured red and the remainder of the marbles are coloured green. Two marbles are drawn randomly from the box.

If the first marble is **not** replaced into the box before the second marble is drawn, then the probability that the two marbles drawn are the same colour is

A. $\frac{k^2 + (n-k)^2}{n^2}$

B. $\frac{k^2 + (n-k-1)^2}{n^2}$

C. $\frac{2k(n-k-1)}{n(n-1)}$

D. $\frac{k(k-1) + (n-k)(n-k-1)}{n(n-1)}$

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16. Probability, STD2 S2 2016 HSC 23 MC

A group of 485 people was surveyed. The people were asked whether or not they smoke. The results are recorded in the table.

	Smokers	*Non-smokers*	*Total*
Male	88	176	264
Female	68	153	221
	156	329	485

A person is selected at random from the group.

What is the approximate probability that the person selected is a smoker OR is male?

(A) 33%

(B) 18%

(C) 68%

(D) 87%

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17. Probability, 2ADV S1 2004 HSC 1e

A packet contains 12 red, 8 green, 7 yellow and 3 black jellybeans.

One jellybean is selected from the packet at random.

What is the probability that the selected jellybean is red or yellow? ***(2 marks)***

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18. Probability, 2ADV S1 2014 HSC 12c

A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.

i. Draw a tree diagram to show the possible outcomes. Include the probability on each branch. ***(2 marks)***

ii. What is the probability that the two lollies are of different colours? ***(1 mark)***

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19. Probability, 2ADV S1 2012 MET1 2

A car manufacturer is reviewing the performance of its car model X. It is known that at any given six-month service, the probability of model X requiring an oil change is $\frac{17}{20}$, the probability of model X requiring an air filter change is $\frac{3}{20}$ and the probability of model X requiring both is $\frac{1}{20}$.

a. State the probability that at any given six-month service model X will require an air filter change without an oil change. ***(1 mark)***

b. The car manufacturer is developing a new model. The production goals are that the probability of model Y requiring an oil change at any given six-month service will be $\frac{m}{m+n}$, the probability of model Y requiring an air filter change will be $\frac{n}{m+n}$ and the probability of model Y requiring both will be $\frac{1}{m+n}$, where $m, n \in Z^+$.

Determine m in terms of n if the probability of model Y requiring an air filter change without an oil change at any given six-month service is 0.05. ***(2 marks)***

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20. Probability, 2ADV S1 2006 HSC 4c

A chessboard has 32 black squares and 32 white squares. Tanya chooses three different squares at random.

i. What is the probability that Tanya chooses three white squares? ***(2 marks)***

ii. What is the probability that the three squares Tanya chooses are the same colour?. ***(1 mark)***

iii. What is the probability that the three squares Tanya chooses are not the same colour? ***(1 mark)***

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21. Probability, 2ADV S1 2012 HSC 13c

Two buckets each contain red marbles and white marbles. Bucket A contains 3 red and 2 white marbles. Bucket B contains 3 red and 4 white marbles.

Chris randomly chooses one marble from each bucket.

i. What is the probability that both marbles are red? ***(1 mark)***

ii. What is the probability that at least one of the marbles is white? ***(1 mark)***

iii. What is the probability that both marbles are the same colour? ***(2 marks)***

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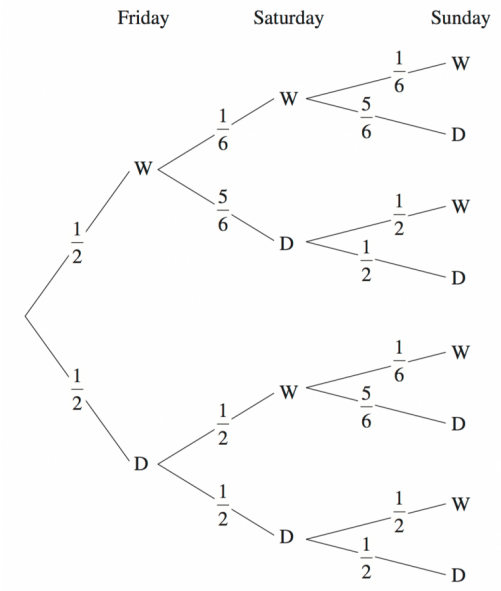
22. Probability, 2ADV S1 2015 HSC 14b

Weather records for a town suggest that:

- if a particular day is wet (W), the probability of the next day being dry is $\frac{5}{6}$
- if a particular day is dry (D), the probability of the next day being dry is $\frac{1}{2}$.

In a specific week Thursday is dry. The tree diagram shows the possible outcomes for the next three days: Friday, Saturday and Sunday.



i. Show that the probability of Saturday being dry is $\frac{2}{3}$. ***(1 mark)***

ii. What is the probability of both Saturday and Sunday being wet? ***(2 marks)***

iii. What is the probability of at least one of Saturday and Sunday being dry? ***(1 mark)***

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23. Probability, STD2 S2 2006 HSC 26c

A new test has been developed for determining whether or not people are carriers of the Gaussian virus.

Two hundred people are tested. A two-way table is being used to record the results.

	Positive	*Negative*
Carrier	74	12
Not a carrier	16	A

i. What is the value of A? ***(1 mark)***

ii. A person selected from the tested group is a carrier of the virus.
What is the probability that the test results would show this? ***(2 marks)***

iii. For how many of the people tested were their test results inaccurate? ***(1 mark)***

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24. Probability, 2ADV S1 SM-Bank 3

In a workplace of 25 employees, each employee speaks either French or German, or both.

If 36% of the employees speak German, and 20% speak both French and German.

i. Calculate the probability one person chosen could speak German if they could speak French. Give your answer to the nearest percent. ***(1 mark)***

ii. Calculate the probability one person chosen could not speak French if they could speak German. Give your answer to the nearest percent. ***(1 mark)***

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25. Probability, 2ADV S1 2009 HSC 5b

On each working day James parks his car in a parking station which has three levels. He parks his car on a randomly chosen level. He always forgets where he has parked, so when he leaves work he chooses a level at random and searches for his car. If his car is not on that level, he chooses a different level and continues in this way until he finds his car.

i. What is the probability that his car is on the first level he searches? ***(1 mark)***

ii. What is the probability that he must search all three levels before he finds his car? ***(1 mark)***

iii. What is the probability that on every one of the five working days in a week, his car is not on the first level he searches? ***(1 mark)***

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26. Probability, STD2 S2 2011 HSC 24b

A die was rolled 72 times. The results for this experiment are shown in the table.

Number obtained	*Frequency*
1	16
2	11
3	A
4	8
5	12
6	15

i. Find the value of A. ***(1 mark)***

ii. What was the relative frequency of obtaining a 4. ***(1 mark)***

iii. If the die was unbiased, which number was obtained the expected number of times? ***(1 mark)***

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27. Probability, 2ADV S1 2020 HSC 14

History and Geography are two of the subjects students may decide to study. For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

a. A student is chosen at random. By a using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography. ***(2 marks)***

b. A students is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History? ***(1 mark)***

c. Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History? ***(2 marks)***

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28. Probability, 2ADV S1 2009 MET1 5

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

i. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1? ***(1 mark)***

ii. What is the probability that the sum of the numbers on the two balls is 5? ***(1 mark)***

iii. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1? ***(2 marks)***

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29. Probability, 2ADV S1 2015 MET1 8

For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

i. Calculate $P(A \cap B)$. ***(1 mark)***

ii. Calculate $P(A' \cap B)$, where A' denotes the complement of A. ***(1 mark)***

iii. If events A and B are independent, calculate $P(A \cup B)$. ***(1 mark)***

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30. Probability, 2ADV S1 2019 HSC 15d

The probability that a person chosen at random has red hair is 0.02

a. Two people are chosen at random.
What is the probability that at least ONE has red hair? ***(2 marks)***

b. What is the smallest number of people that can be chosen at random so that the probability that at least ONE has red hair is greater than 0.4? ***(2 marks)***

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31. Probability, 2ADV S1 2004 HSC 6c

In a game, a turn involves rolling two dice, each with faces marked 0, 1, 2, 3, 4 and 5. The score for each turn is calculated by multiplying the two numbers uppermost on the dice.

i. What is the probability of scoring zero on the first turn? ***(2 marks)***

ii. What is the probability of scoring 16 or more on the first turn? ***(1 mark)***

iii. What is the probability that the sum of the scores in the first two turns is less than 45? ***(2 marks)***

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32. Probability, 2ADV S1 2007 HSC 9b

A pack of 52 cards consists of four suits with 13 cards in each suit.

i. One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table. What is the probability that the second card is from a different suit to the first? ***(1 mark)***

ii. The two cards are replaced and the pack shuffled. Four cards are chosen from the pack and placed side by side on the table. What is the probability that these four cards are all from different suits? ***(2 marks)***

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33. Probability, 2ADV S1 2018 HSC 14e

Two machines, A and B, produce pens. It is known that 10% of the pens produced by machine A are faulty and that 5% of the pens produced by machine B are faulty.

i. One pen is chosen at random from each machine.
What is the probability that at least one of the pens is faulty? ***(1 mark)***

ii. A coin is tossed to select one of the two machines. Two pens are chosen at random from the selected machine.
What is the probability that neither pen is faulty? ***(2 marks)***

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34. Probability, 2ADV S1 2008 HSC 9a

It is estimated that 85% of students in Australia own a mobile phone.

i. Two students are selected at random. What is the probability that neither of them owns a mobile phone? ***(2 marks)***

ii. Based on a recent survey, 20% of the students who own a mobile phone have used their mobile phone during class time. A student is selected at random. What is the probability that the student owns a mobile phone and has used it during class time? ***(1 mark)***

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35. Probability, STD2 S2 2004 HSC 25c

Lie detector tests are not always accurate. A lie detector test was administered to 200 people.

The results were:

• 50 people lied. Of these, the test indicated that 40 had lied;
• 150 people did NOT lie. Of these, the test indicated that 20 had lied.

i. Copy the table into your writing booklet and complete it using the information above

	Test indicated a lie	*Test did not indicate a lie*	*Total*
People who lied			50
People who did NOT lie			150

ii. For how many of the people tested was the lie detector test accurate?

iii. For what percentage of the people tested was the test accurate?

iv. What is the probability that the test indicated a lie for a person who did NOT lie?

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36. Probability, 2ADV S1 2007 MET1 11

There is a daily flight from Paradise Island to Melbourne. The probability of the flight departing on time, given that there is fine weather on the island, is 0.8, and the probability of the flight departing on time, given that the weather on the island is not fine, is 0.6.

In March the probability of a day being fine is 0.4.

Find the probability that on a particular day in March

i. the flight from Paradise Island departs on time ***(2 marks)***

ii. the weather is fine on Paradise Island, given that the flight departs on time. ***(2 marks)***

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37. Probability, 2ADV S1 2011 MET1 8

Two events, A and B, are such that $P = \frac{3}{5}$ and $P = \frac{1}{4}$.

If A' denotes the compliment of A, calculate $P(A' \cap B)$ when

i. $P(A \cup B) = \frac{3}{4}$ ***(2 marks)***

ii. A and B are mutually exclusive. ***(1 mark)***

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38. Probability, 2ADV S1 2014 MET1 9

Sally aims to walk her dog, Mack, most mornings. If the weather is pleasant, the probability that she will walk Mack is $\frac{3}{4}$, and if the weather is unpleasant, the probability that she will walk Mack is $\frac{1}{3}$.

Assume that pleasant weather on any morning is independent of pleasant weather on any other morning.

a. In a particular week, the weather was pleasant on Monday morning and unpleasant on Tuesday morning.

Find the probability that Sally walked Mack on at least one of these two mornings. ***(2 marks)***

b. In the month of April, the probability of pleasant weather in the morning was $\frac{5}{8}$.

i. Find the probability that on a particular morning in April, Sally walked Mack. ***(2 marks)***

ii. Using your answer from **part b.i.**, or otherwise, find the probability that on a particular morning in April, the weather was pleasant, given that Sally walked Mack that morning. ***(2 marks)***

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39. Probability, 2ADV S1 2017 MET1 8

For events A and B from a sample space, $P(A|B) = \frac{1}{5}$ and $P(B|A) = \frac{1}{4}$. Let $P(A \cap B) = p$.

i. Find $P(A)$ in terms of p. ***(1 mark)***

ii. Find $P(A' \cap B')$ in terms of p. ***(2 marks)***

iii. Given that $P(A \cup B) \leq \frac{1}{5}$, state the largest possible interval for p. ***(2 marks)***

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40. Probability, 2ADV S1 2010 HSC 8b

Two identical biased coins are tossed together, and the outcome is recorded.

After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? ***(2 marks)***

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41. Probability, 2ADV S1 2018 HSC 16b

A game involves rolling two six-sided dice, followed by rolling a third six-sided die. To win the game, the number rolled on the third die must lie between the two numbers rolled previously. For example, if the first two dice show 1 and 4, the game can only be won by rolling a 2 or 3 with the third die.

i. What is the probability that a player has no chance of winning before rolling the third die? ***(2 marks)***

ii. What is the probability that a player wins the game? ***(2 marks)***

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42. Probability, 2ADV S1 2007 MET1 6

Two events, A and B, from a given event space, are such that $P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{3}$.

i. Calculate $P(A' \cap B)$ when $P(A \cap B) = \frac{1}{8}$. ***(1 mark)***

ii. Calculate $P(A' \cap B)$ when A and B are mutually exclusive events. ***(1 mark)***

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43. Probability, 2ADV S1 2009 HSC 9a

Each week Van and Marie take part in a raffle at their respective workplaces.

The probability that Van wins a prize in his raffle is $\frac{1}{9}$. The probability that Marie wins a prize in her raffle is $\frac{1}{16}$.

What is the probability that, during the next three weeks, at least one of them wins a prize? ***(2 marks)***

Worked Solutions

1. Probability, 2ADV S1 2015 HSC 4 MC

$$P(\text{win at soccer}) = \frac{5}{7}$$

$$\therefore P(\text{not win at soccer}) = 1 - \frac{5}{7} = \frac{2}{7}$$

$$P(\text{win at league}) = \frac{2}{3}$$

$$\therefore P(\text{not win at league}) = \frac{1}{3}$$

$$\therefore P(\text{not win at both}) = \frac{2}{7} \times \frac{1}{3}$$

$$= \frac{2}{21}$$

$$\Rightarrow A$$

---

2. Probability, STD2 S2 2010 HSC 12 MC

$$P\,(\text{Test accurate}) = \frac{\text{Accurate readings}}{\text{Total tested}}$$

$$= \frac{72 + 256}{347}$$

$$= 94.5244...\,\%$$

$$\Rightarrow C$$

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3. Probability, 2ADV S1 2019 MET2 11 MC

Since A and B are independent,

$P(B \mid A) = P(B \mid A')$

$\therefore m = n$

$\Rightarrow A$

Worked Solutions

4. Probability, 2ADV S1 2019 MET2-N 9 MC

$$
\begin{aligned}
&P(\text{alternate colours}) \\
&\quad = P(RGR) + P(GRG) \\
&\quad = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{2}{3} + \frac{2}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} \\
&\quad = \frac{12}{60} + \frac{6}{60} \\
&\quad = \frac{3}{10}
\end{aligned}
$$

$\Rightarrow B$

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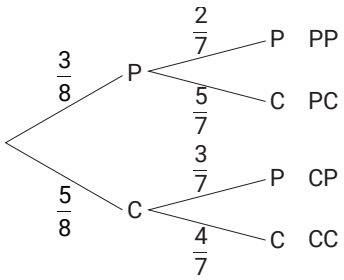
5. Probability, 2ADV S1 2021 HSC 6 MC



$$
\begin{aligned}
P(\text{different centres}) &= P(\text{PC}) + P(\text{CP}) \\
&= \frac{3}{8} \cdot \frac{5}{7} + \frac{5}{8} \cdot \frac{3}{7} \\
&= \frac{15}{56} + \frac{15}{56} \\
&= \frac{15}{28}
\end{aligned}
$$

$\Rightarrow D$

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6. Probability, 2ADV S1 SM-Bank 4 MC

$$
\begin{aligned}
P(\text{Surf} \mid \text{Soccer}) &= \frac{n(\text{Surf} \cap \text{Soccer})}{n(\text{Soccer})} \\
&= \frac{3+4}{3+4+5+6} \\
&= \frac{7}{18}
\end{aligned}
$$

$\Rightarrow D$

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7. Probability, 2ADV S1 2013 HSC 5 MC

$$
\begin{aligned}
\text{P(at least 1 red)} &= 1 - P(\text{none red}) \\
&= 1 - P(B_1) \times P(B_2) \times P(B_3) \\
&= 1 - \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} \\
&= 1 - \frac{120}{720} \\
&= 1 - \frac{1}{6} \\
&= \frac{5}{6}
\end{aligned}
$$

$\Rightarrow C$

---

8. Probability, STD2 S2 2005 HSC 16 MC

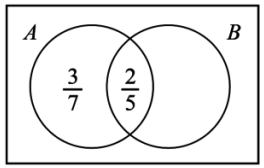
$$
\begin{aligned}
\text{Total male viewers} &= 1372 + 2054 \\
&= 3426
\end{aligned}
$$

$$
\begin{aligned}
&P\ (\text{Male viewer chosen voted for C1}) \\
&\quad = \frac{\text{Males who voted for C1}}{\text{Total male viewers}} \\
&\quad = \frac{1372}{3426}
\end{aligned}
$$

$\Rightarrow C$

---

9. Probability, 2ADV S1 2012 MET2 13 MC



$$
\begin{aligned}
P(B' \mid A) &= \frac{P(B' \cap A)}{P(A)} \\
&= \frac{P(B' \cap A)}{P(B' \cap A) + P(A \cap B)} \\
&= \frac{\frac{3}{7}}{\frac{3}{7} + \frac{2}{5}} \\
&= \frac{15}{29}
\end{aligned}
$$

$\Rightarrow \ B$

10. Probability, 2ADV S1 2013 MET2 10 MC

$$
\begin{aligned}
P(A) &= P(A \cap B) + P(A \cap B') \\
&= p + \frac{3p}{5} \\
&= \frac{8p}{5}
\end{aligned}
$$

$$
\begin{aligned}
P(B) &= P(B \cap A) + P(B \cap A') \\
&= p + p - \frac{1}{8} \\
&= 2p - \frac{1}{8}
\end{aligned}
$$

Since A and B are independent events,

$$
\begin{aligned}
P(A \cap B) &= P(A) \times P(B) \\
p &= \frac{8p}{5}\left(2p - \frac{1}{8}\right) \\
5p &= 16p^2 - p \\
16p^2 - 6p &= 0 \\
2p(8p - 3) &= 0 \\
\therefore p &= \frac{3}{8}, \quad p \neq 0
\end{aligned}
$$

$\Rightarrow \ C$

11. Probability, 2ADV S1 2014 HSC 10 MC

♦♦ Mean mark 26%

$$\text{P}(R_1 < 10 \text{ secs}) = \frac{1}{4} \Rightarrow \text{P}(\overline{R}_1) = \frac{3}{4}$$

$$\text{P}(R_2 < 10 \text{ secs}) = \frac{1}{6} \Rightarrow \text{P}(\overline{R}_2) = \frac{5}{6}$$

$$\text{P}(R_3 < 10 \text{ secs}) = \frac{2}{5} \Rightarrow \text{P}(\overline{R}_3) = \frac{3}{5}$$

$$\begin{aligned}\therefore \text{P}(&\text{at least } 1 < 10 \text{ secs}) \\ &= 1 - \text{P}(\text{all} \geq 10 \text{ secs}) \\ &= 1 - \frac{3}{4} \times \frac{5}{6} \times \frac{3}{5} \\ &= 1 - \frac{45}{120} \\ &= \frac{5}{8}\end{aligned}$$

$\Rightarrow D$

12. Probability, 2ADV S1 2018 HSC 6 MC

♦♦ Mean mark 31%.

Strategy One:

Choose 1 shoe then find the probability the next choice is matching.

$$P = 1 \times \frac{1}{7}$$

$$= \frac{1}{7}$$

$$P = \frac{\text{Number of desired outcomes}}{\text{Number of possibilities}}$$

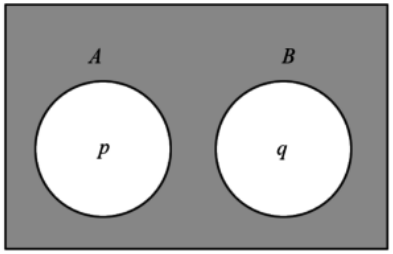
$$= \frac{4}{{}^8C_2}$$

$$= \frac{4}{28}$$

$$= \frac{1}{7}$$

$\Rightarrow C$

13. Probability, 2ADV S1 2010 MET2 21 MC



♦ Mean mark 43%.

$$\begin{aligned}P(A' \cap B') &= 1 - p - q \\ &= 1 - (p + q)\end{aligned}$$

$\Rightarrow C$

14. Probability, 2ADV S1 2013 MET2 17 MC

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$p = \frac{P(A \cap B)}{p^2}$$

$$\therefore P(A \cap B) = p^3$$

♦ Mean mark 49%.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

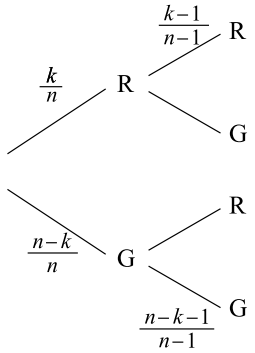
$$= \frac{p^3}{p^{\frac{1}{3}}}$$

$$\therefore P(B \mid A) = p^{\frac{8}{3}}$$

$\Rightarrow D$

15. Probability, 2ADV S1 SM-Bank 6 MC

n marbles $\Rightarrow$ k red, $(n-k)$ green



$$\therefore P(\text{same colour}) = \frac{k}{n}\cdot\frac{(k-1)}{(n-1)} + \frac{(n-k)}{n}\cdot\frac{(n-k-1)}{(n-1)}$$
$$= \frac{k(k-1)+(n-k)(n-k-1)}{n(n-1)}$$

$\Rightarrow$ D

16. Probability, STD2 S2 2016 HSC 23 MC

$P(\text{Smoker or a male})$

$$= \frac{\text{Total males + female smokers}}{\text{Total surveyed}}$$
$$= \frac{264+68}{485}$$
$$= 0.684\ldots$$

$\Rightarrow C$

♦♦ Mean mark 34%.

17. Probability, 2ADV S1 2004 HSC 1e

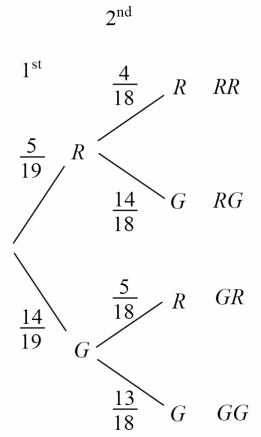
12 R, 8 G, 7 Y, 3 B

Total jellybeans $= 30$

$$P(\text{R or Y}) = \frac{(\#\text{ Red + Yellow})}{\text{Total jellybeans}}$$
$$= \frac{12+7}{30}$$
$$= \frac{19}{30}$$

18. Probability, 2ADV S1 2014 HSC 12c

i.

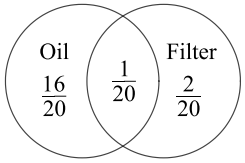


ii. $P(\text{different colours})$

$$= P(RG) + P(GR)$$
$$= \frac{5}{19}\times\frac{14}{18} + \frac{14}{19}\times\frac{5}{18}$$
$$= \frac{70}{342} + \frac{70}{342}$$
$$= \frac{140}{342}$$
$$= \frac{70}{171}$$

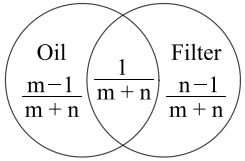
19. Probability, 2ADV S1 2012 MET1 2

a.



$$\begin{aligned}\Pr(F \cap O') &= \Pr(F) - \Pr(F \cap O) \\ &= \frac{3}{20} - \frac{1}{20} \\ &= \frac{1}{10}\end{aligned}$$

b.



$$\begin{aligned}\Pr(F \cap O') &= \frac{n}{m+n} - \frac{1}{m+n} \\ \frac{1}{20} &= \frac{n-1}{m+n} \\ m+n &= 20n - 20 \\ m &= 19n - 20\end{aligned}$$

20. Probability, 2ADV S1 2006 HSC 4c

i. $\mathrm{P}(WWW) = \frac{32}{64} \times \frac{31}{63} \times \frac{30}{62}$

$$= \frac{5}{42}$$

ii. $\mathrm{P}(\text{same colour})$

$$\begin{aligned}&= P(WWW) + P(BBB) \\ &= \frac{5}{42} + \frac{32}{64} \times \frac{31}{63} \times \frac{30}{62} \\ &= \frac{5}{42} + \frac{5}{42} \\ &= \frac{5}{21}\end{aligned}$$

iii. $\mathrm{P}(\text{not all the same colour})$

$$\begin{aligned}&= 1 - \mathrm{P}(\text{same colour}) \\ &= 1 - \frac{5}{21} \\ &= \frac{16}{21}\end{aligned}$$

21. Probability, 2ADV S1 2012 HSC 13c

i. $P\{(\perp hred)\} = P(R_1) \times P(R_2)$

$$= \frac{3}{5} \times \frac{3}{7}$$

$$= \frac{9}{35}$$

ii. $P(\text{at least one white}) = 1 - P(\text{none white})$

$$= 1 - P(R_1) \times P(R_2)$$

$$= 1 - \frac{9}{35}$$

$$= \frac{26}{35}$$

STRATEGY: When the term "at least" appears in a probability question, it is likely that $\mathbf{1 - P(\text{complement})}$ will solve the question more efficiently and with less chance of error, as shown in part (ii).

iii. $P(\text{same colour}) = P(R_1R_2) + P(W_1W_2)$

$$= \frac{9}{35} + \left(\frac{2}{5} \times \frac{4}{7}\right)$$

$$= \frac{9}{35} + \frac{8}{35}$$

$$= \frac{17}{35}$$

---

22. Probability, 2ADV S1 2015 HSC 14b

i. Show $P(\text{Sat dry}) = \frac{2}{3}$

$P(\text{Sat dry})$

$$= P(W, D) + P(D, D)$$

$$= \left(\frac{1}{2} \times \frac{5}{6}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{5}{12} + \frac{1}{4}$$

$$= \frac{2}{3} \quad \ldots \text{ as required}$$

ii. $P(\text{Sat and Sun wet})$

$$= P(WWW) + P(DWW)$$

$$= \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}\right)$$

$$= \frac{1}{72} + \frac{1}{24}$$

$$= \frac{1}{18}$$

iii. $P(\text{At least Sat or Sun dry})$

$$= 1 - P(\text{Sat and Sun both wet})$$

$$= 1 - \frac{1}{18}$$

$$= \frac{17}{18}$$

23. Probability, STD2 S2 2006 HSC 26c

i. $A = 200 - (74 + 12 + 16)$

$= 98$

ii. $P = \dfrac{\#\text{ Positive carriers}}{\text{Total carriers}}$

$= \dfrac{74}{86}$

$= \dfrac{37}{43}$

iii. $\#$ People with inaccurate results

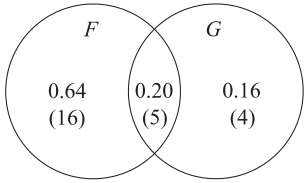
$= 12 + 16$

$= 28$

---

24. Probability, 2ADV S1 SM-Bank 3

i. **Expressing in a Venn diagram:**



$$P(G \mid F) = \frac{P(G \cap F)}{P(F)}$$

$$= \frac{0.20}{0.84}$$

$$= 0.238\ldots$$

$$= 24\%$$

ii. $P(\text{not } F \mid G) = \dfrac{P(\text{not } F \cap G)}{P(G)}$

$= \dfrac{0.16}{0.36}$

$= 0.444\ldots$

$= 44\%$

---

25. Probability, 2ADV S1 2009 HSC 5b

i. $P(\text{1st chosen}) = \dfrac{1}{3}$

ii. $P(\text{search 3 levels})$

$$\begin{aligned} &= P(\text{not 1st}) \times P(\text{not 2nd}) \\ &= \frac{2}{3} \times \frac{1}{2} \\ &= \frac{1}{3} \end{aligned}$$

♦♦ *Mean marks of 31% and 39% for part (ii) and (iii) respectively.*

iii. $P(\text{not 1st for 5 days})$

$$\begin{aligned} &= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \\ &= \frac{32}{243} \end{aligned}$$

26. Probability, STD2 S2 2011 HSC 24b

i. Since die rolled 72 times

$$\begin{aligned} \therefore A &= 72 - (16 + 11 + 8 + 12 + 15) \\ &= 72 - 62 \\ &= 10 \end{aligned}$$

ii. $\text{Relative frequency of } 4 = \dfrac{8}{72}$

$$= \frac{1}{9}$$

♦ *Mean mark 38%*
IMPORTANT: Many students confused 'relative frequency' with 'frequency' and incorrectly answered 8.

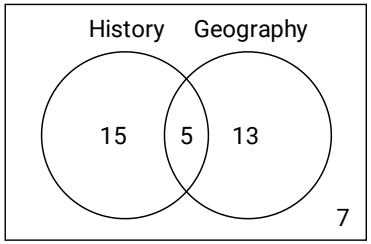
iii. Expected frequency of any number

$$\begin{aligned} &= \frac{1}{6} \times 72 \\ &= 12 \end{aligned}$$

$\therefore$ 5 was obtained the expected number of times.

27. Probability, 2ADV S1 2020 HSC 14

a.



$$\begin{aligned} P(\text{H and G}) &= \frac{5}{40} \\ &= \frac{1}{8} \end{aligned}$$

b. $P(\overline{\text{H}} \mid \text{G}) = \dfrac{P(\overline{\text{H}} \cap \text{G})}{P(\text{G})}$

$$= \frac{13}{18}$$

♦ *Mean mark part (b) 49%.*

c. $P(\text{H}, \overline{\text{H}}) = \dfrac{20}{40} \times \dfrac{20}{39}$

$$= \frac{10}{39}$$

28. Probability, 2ADV S1 2009 MET1 5

i. $P(4,1) = \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

ii. $P(\text{Sum} = 5)$

$= P(1,4) + P(2,3) + P(3,2) + P(4,1)$

$= 4 \times \left(\frac{1}{4} \times \frac{1}{3}\right)$

$= \frac{1}{3}$

iii. **Conditional Probability**

$P(2^{\text{nd}} = 1 \mid \text{Sum} = 5)$

$= \frac{P(4,1)}{P(\text{Sum} = 5)}$

$= \frac{\frac{1}{12}}{\frac{1}{3}}$

$= \frac{1}{4}$

♦ *Mean mark 46%.*

---

29. Probability, 2ADV S1 2015 MET1 8

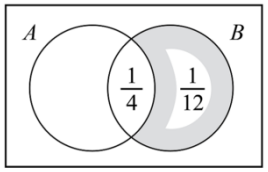
i. **Using Conditional Probability:**

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\frac{3}{4} = \frac{P(A \cap B)}{\frac{1}{3}}$

$\therefore P(A \cap B) = \frac{1}{4}$

ii.



$P(A' \cap B) = P(B) - P(A \cap B)$

$= \frac{1}{3} - \frac{1}{4}$

$= \frac{1}{12}$

iii. **If A, B independent**

$P(A \cap B) = P(A) \times P(B)$

$\frac{1}{4} = P(A) \times \frac{1}{3}$

$\therefore P(A) = \frac{3}{4}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

$\therefore P(A \cup B) = \frac{5}{6}$

♦♦ *Mean mark 28%.*
MARKER'S COMMENT: A lack of understanding of independent events was clearly evident.

30. Probability, 2ADV S1 2019 HSC 15d

a. $P(R) = 0.02$

$P(\overline{R}) = 0.98$

$$\begin{aligned} &P\text{ (At least 1 has red hair)} \\ &\quad = 1 - P(\overline{R}, \overline{R}) \\ &\quad = 1 - 0.98 \times 0.98 \\ &\quad = 0.0396 \end{aligned}$$

b. Find n such that

♦♦ *Mean mark 24%.*

$$\begin{aligned} 1 - 0.98^n &> 0.4 \\ 0.98^n &< 0.6 \\ \ln 0.98^n &< \ln 0.6 \\ n \ln 0.98 &< \ln 0.6 \\ n &> \frac{\ln 0.6}{\ln 0.98}, \quad (\ln 0.98 < 0) \\ &> 25.28\ldots \end{aligned}$$

$\therefore$ 26 people must be chosen.

---

31. Probability, 2ADV S1 2004 HSC 6c

i.

Die 2 \ Die 1	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

MARKER'S COMMENT: Students who drew up the table for the sample space were "overwhelmingly" more successful in all parts of this question.

$$\therefore P(0) = \frac{11}{36}$$

ii. $P(\geq 16) = \dfrac{4}{36} = \dfrac{1}{9}$

iii. $P(\text{Sum} < 45) = 1 - P(\text{Sum} \geq 45)$

$$\begin{aligned} P(\text{Sum} \geq 45) &= P(20, 25) + P(25, 20) + P(25, 25) \\ &= \left(\frac{2}{36} \times \frac{1}{36}\right) + \left(\frac{2}{36} \times \frac{1}{36}\right) + \left(\frac{1}{36} \times \frac{1}{36}\right) \\ &= \frac{2}{1296} + \frac{2}{1296} + \frac{1}{1296} \\ &= \frac{5}{1296} \end{aligned}$$

$$\begin{aligned} \therefore P(\text{Sum} < 45) &= 1 - \frac{5}{1296} \\ &= \frac{1291}{1296} \end{aligned}$$

32. Probability, 2ADV S1 2007 HSC 9b

i. After 1st card is drawn

$$\begin{aligned}
&\#\text{ Cards left from another suit} = 39 \\
&\#\text{ Cards left in pack} = 51 \\
&\therefore P(\text{2nd card from the same suit}) \\
&\quad = \frac{39}{51} \\
&\quad = \frac{13}{17}
\end{aligned}$$

ii. $P(\text{all 4 cards from different suits})$

$$\begin{aligned}
&= \frac{52}{52} \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49} \\
&= \frac{2179}{20\,825} \\
&= 0.1046\ldots \\
&= 0.105 \quad (\text{to 3 d.p.})
\end{aligned}$$

---

33. Probability, 2ADV S1 2018 HSC 14e

i. $\text{P(at least 1 faulty)} = 1 - \text{P(both faulty)}$

$$\begin{aligned}
&= 1 - 0.9 \times 0.95 \\
&= 1 - 0.855 \\
&= 0.145
\end{aligned}$$

ii. $\text{P(2 non-faulty pens)}$

♦ *Mean mark 48%.*

$$\begin{aligned}
&= (\text{choose A, NF, NF}) + P(\text{choose B, NF, NF}) \\
&= \frac{1}{2} \times 0.9 \times 0.9 + \frac{1}{2} \times 0.95 \times 0.95 \\
&= 0.405 + 0.45125 \\
&= 0.85625
\end{aligned}$$

---

34. Probability, 2ADV S1 2008 HSC 9a

i. $\text{P}(M) = 0.85$

$$\begin{aligned}
&\text{P}(M^c) = 1 - 0.85 = 0.15 \\
&\therefore \text{P}(M^c, M^c) = \frac{15}{100} \cdot \frac{15}{100} \\
&\qquad = \frac{225}{10\,000} \\
&\qquad = \frac{9}{400}
\end{aligned}$$

COMMENT: M^c is syllabus notation for the complement of event M.

ii. $\text{P(owns mobile and used it)}$

$$\begin{aligned}
&= \text{P}(M) \times \text{P(used it)} \\
&= \frac{17}{20} \times \frac{20}{100} \\
&= \frac{17}{100}
\end{aligned}$$

---

35. Probability, STD2 S2 2004 HSC 25c

i.

	Test indicated a lie	*Test did not indicate a lie*	*Total*
People who lied	40	10	50
People who did NOT lie	20	130	150

ii. $\#$ **Accurate readings**

$$\begin{aligned} &= 40 + 130 \\ &= 170 \end{aligned}$$

iii. Percentage of people with accurate readings

$$\begin{aligned} &= \frac{\#\text{ Accurate readings}}{\text{Total readings}} \times 100 \\ &= \frac{170}{200} \\ &= 85\% \end{aligned}$$

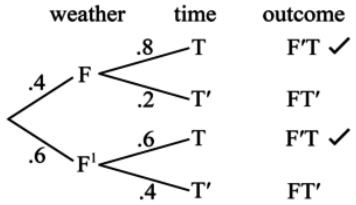
iv. P(lie detected when NOT a lie)

$$\begin{aligned} &= \frac{20}{150} \\ &= \frac{2}{15} \end{aligned}$$

---

36. Probability, 2ADV S1 2007 MET1 11

i.



$$\begin{aligned} &P(\text{FT}) + P(\text{F'T}) \\ &\quad = 0.4 \times 0.8 + 0.6 \times 0.6 \\ &\quad = 0.32 + 0.36 \\ &\quad = 0.68 \end{aligned}$$

ii. Conditional probability:

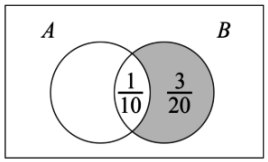
$$\begin{aligned} P(F \mid T) &= \frac{P(F \cap T)}{P(T)} \\ &= \frac{0.32}{0.68} \end{aligned}$$

$$\therefore P(F \mid T) = \frac{8}{17}$$

♦♦ *Mean mark 29%.*
MARKER'S COMMENT: Students continue to struggle with conditional probability. Attention required here.

37. Probability, 2ADV S1 2011 MET1 8

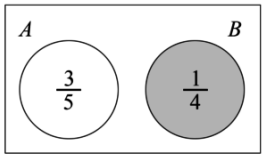
i. Sketch Venn Diagram



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{3}{4} = \frac{3}{5} + \frac{1}{4} - P(A \cap B)$$
$$P(A \cap B) = \frac{1}{10}$$

$$\therefore\ P(A' \cap B) = \frac{1}{4} - \frac{1}{10} = \frac{3}{20}$$

ii.



$P(A \cap B) = 0$ **(mutually exclusive),**

$$\therefore\ P(A' \cap B) = P(B) = \frac{1}{4}$$

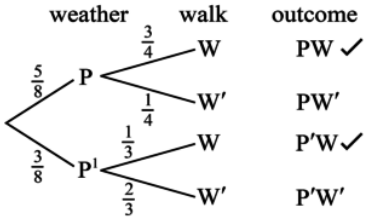
---

38. Probability, 2ADV S1 2014 MET1 9

a. $P(\text{at least 1 walk}) = 1 - P(\text{no walk})$

$$= 1 - \frac{1}{4} \times \frac{2}{3}$$
$$= \frac{5}{6}$$

b.i. Construct tree diagram:



$$P(PW) + P(P'W) = \frac{5}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{3}$$
$$= \frac{19}{32}$$

b.ii. $P(P \mid W) = \dfrac{P(P \cap W)}{P(W)}$

$$= \frac{\frac{5}{8} \times \frac{3}{4}}{\frac{19}{32}}$$
$$= \frac{15}{32} \times \frac{32}{19}$$
$$= \frac{15}{19}$$

♦ *Part (b)(ii) mean mark 38%.*

---

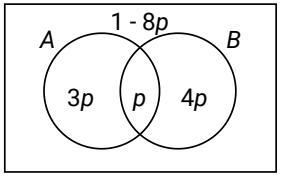
39. Probability, 2ADV S1 2017 MET1 8

i. $P(A) = \dfrac{P(A \cap B)}{P(B|A)}$

$= \dfrac{p}{\frac{1}{4}}$

$= 4p$

ii. **Consider the Venn diagram:**



$P(A' \cap B') = 1 - 8p$

♦ *Mean mark 40%.*
MARKER'S COMMENT: The most successful answers used a Venn diagram or table.

iii. Given $P(A \cup B) = 8p$

$\Rightarrow 0 < 8p \leq \dfrac{1}{5}$

$\therefore 0 < p \leq \dfrac{1}{40}$

♦ *Mean mark 37%.*

40. Probability, 2ADV S1 2010 HSC 8b

$P(H_1H_2) = P(H_1) \times P(H_2)$

$= 0.36$

Since coins are identical:

$P(H) = \sqrt{0.36}$

$= 0.6$

$P(T) = 1 - P(H)$

$= 0.4$

$\therefore P(T_1T_2) = 0.4 \times 0.4$

$= 0.16$

♦♦ *Mean mark 28%.*
NOTE: The most common error was $P(T_1T_2) = 1 - 0.36 = 0.64$. Ensure you understand why this does **not** apply.

41. Probability, 2ADV S1 2018 HSC 16b

i. **Construct a sample space of the number of possible winning rolls:**

	1	2	3	4	5	6
1	–	–	1	2	3	4
2	–	–	–	1	2	3
3	1	–	–	–	1	2
4	2	1	–	–	–	1
5	3	2	1	–	–	–
6	4	3	2	1	–	–

♦ *Mean mark 40%.*
COMMENT: Constructing the full sample space is a critical step here..

$\text{P(no chance)} = \dfrac{\text{number of pairs with no gap}}{\text{total possibilities}}$

$= \dfrac{16}{36}$

$= \dfrac{4}{9}$

ii. **The sample space in the table shows:**

$\rightarrow$ 8 combinations leave a gap for a single winning number,

$\rightarrow$ 6 combinations leave a gap for two winning numbers,

$\vdots$

$\therefore \text{P(winning)} = \dfrac{1}{36}\left[8 \times \dfrac{1}{6} + 6 \times \dfrac{2}{6} + 4 \times \dfrac{3}{6} + 2 \times \dfrac{4}{6}\right]$

$= \dfrac{1}{36}\left(\dfrac{8}{6} + \dfrac{12}{6} + \dfrac{12}{6} + \dfrac{8}{6}\right)$

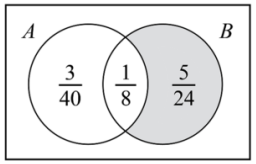
$= \dfrac{1}{36}\left(\dfrac{40}{6}\right)$

$= \dfrac{5}{27}$

♦♦♦ *Mean mark 7%.*

42. Probability, 2ADV S1 2007 MET1 6

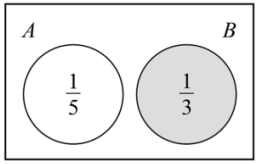
i. Sketch Venn diagram:



♦♦ *Mean mark 31%.*
MARKER'S COMMENTS: Students who drew a Venn diagram or Karnaugh map were the most successful.

$$\therefore P(A' \cap B) = P(B) - P(A \cap B)$$
$$= \frac{1}{3} - \frac{1}{8}$$
$$= \frac{5}{24}$$

ii.



♦♦ *Mean mark 23%.*

Mutually exclusive means $P(A \cap B) = 0$,

$$\therefore P(A' \cap B) = \frac{1}{3}$$

43. Probability, 2ADV S1 2009 HSC 9a

$$P(\text{Van loses}) = 1 - \frac{1}{9} = \frac{8}{9}$$
$$P(\text{Marie loses}) = 1 - \frac{1}{16} = \frac{15}{16}$$
$$P(\text{both lose}) = \frac{8}{9} \times \frac{15}{16} = \frac{5}{6}$$

$\text{P(At least 1 wins)}$
$$= 1 - P(\text{both lose for 3 weeks})$$
$$= 1 - \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$$
$$= 1 - \frac{125}{216}$$
$$= \frac{91}{216}$$