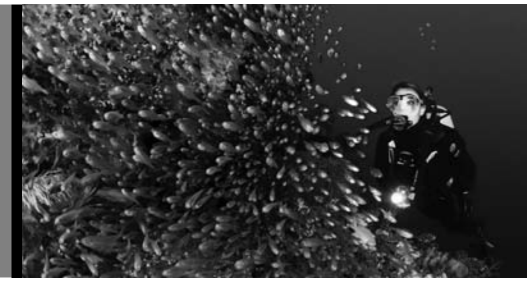
Answers



Answers are not provided for certain questions of the type 'show that' or 'prove that'. Please see worked solutions in these cases for a model.

Chapter 1

Exercise 1A

1 a 850, 1000, 1150, 1300, 1450, 1600, 1750, 1900, 2050, 2200, 2350, 2500, . . .
b 9 months

2 a 20, 25, 30, 35 **b** 36, 46, 56, 66
c 16, 32, 64, 128 **d** 24, 48, 96, 192
e 26, 22, 18, 14 **f** 12, 3, −6, −15
g $3, 1\frac{1}{2}, \frac{3}{4}, \frac{3}{8}$ **h** $3, 1, \frac{1}{3}, \frac{1}{9}$
i 1, −1, 1, −1 **j** 16, 25, 36, 49
k $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$ **l** $-2, 1, -\frac{1}{2}, \frac{1}{4}$

3 a 6, 12, 18, 24 **b** 3, 8, 13, 18
c 2, 4, 8, 16 **d** 5, 25, 125, 625
e 19, 18, 17, 16 **f** 4, 2, 0, −2
g 6, 12, 24, 48 **h** 70, 700, 7000, 70 000
i 1, 8, 27, 64 **j** 2, 6, 12, 20
k −1, 1, −1, 1 **l** −3, 9, −27, 81

4 a 6, 8, 10, 12 **b** 11, 61, 111, 161
c 15, 12, 9, 6 **d** 12, 4, −4, −12
e 5, 10, 20, 40 **f** $\frac{1}{3}, 1, 3, 9$
g $18, 9, 4\frac{1}{2}, 2\frac{1}{4}$ **h** $-100, -20, -4, -\frac{4}{5}$

5 7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, 62
a 5 **b** 11 **c** 4 **d** 8 **e** 52
f 7th term **g** Yes, 17th term.
h No, they all end in 2 or 7.
i 47, the 9th term
j 42, the 8th term

6 $\frac{3}{4}, 1\frac{1}{2}$, 3, 6, 12, 24, 48, 96, 192, 384, 768, 1536
a 6 **b** 10 **c** 3 **d** 10 **e** 384
f 9th term **g** Yes, 8th term.
h No **i** 384, the 10th term
j 48, the 7th term

7 a 13, 14, 15, 16, 17. Add 1.
b 9, 14, 19, 24, 29. Add 5.
c 10, 5, 0, −5, −10. Subtract 5.
d 6, 12, 24, 48, 96. Multiply by 2.
e −7, 7, −7, 7, −7. Multiply by −1.
f $40, 20, 10, 5, 2\frac{1}{2}$. Divide by 2.

8 c $100 = T_{33}$, 200 is not a term, $1000 = T_{333}$.

9 a 16 is not a term, $35 = T_{20}$, $111 = T_{58}$.
b $44 = T_5$, 200 and 306 are not terms.
c 40 is not a term, $72 = T_6$, $200 = T_{10}$.
d $8 = T_3$, 96 is not a term, $128 = T_7$.

10 c 49 **d** $T_{20} = 204$

11 a 52 **b** 73
c $T_{41} = 128$ **d** $T_{21} = 103$

12 a 3, 5, 7, 9 **b** 5, 17, 29, 41
c 6, 3, 0, −3 **d** 12, 2, −8, −18
e 5, 10, 20, 40 **f** 4, 20, 100, 500
g $20, 10, 5, 2\frac{1}{2}$ **h** 1, −1, 1, −1

13 a 1, 0, −1, 0, T_n where n is even.
b 0, −1, 0, 1, T_n where n is odd.
c −1, 1, −1, 1. No terms are zero.
d 0, 0, 0, 0. All terms are zero.

14 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . . The sum of two odd integers is even, and the sum of an even and an odd integer is odd.

Exercise 1B

1 a 14, 18, 22 **b** 18, 23, 28
c 5, −5, −15 **d** −7, −13, −19
e $9, 10\frac{1}{2}, 12$ **f** $6\frac{1}{2}, 6, 5\frac{1}{2}$

2 a 3, 5, 7, 9 **b** 7, 9, 11, 13
c 7, 3, −1, −5 **d** 17, 28, 39, 50
e 30, 19, 8, −3 **f** −9, −5, −1, 3
g $4\frac{1}{2}, 4, 3\frac{1}{2}, 3$ **h** $3\frac{1}{2}, 1\frac{1}{2}, -\frac{1}{2}, -2\frac{1}{2}$
i 0.9, 1.6, 2.3, 3

3 a AP: $a = 3, d = 4$ **b** AP: $a = 11, d = -4$
c AP: $a = 10, d = 7$ **d** not an AP
e AP: $a = 50, d = -15$ **f** AP: $a = 23, d = 11$
g AP: $a = -12, d = 5$ **h** not an AP
i not an AP **j** AP: $a = 8, d = -10$
k AP: $a = -17, d = 17$
l AP: $a = 10, d = -2\frac{1}{2}$

4 a 67 **b** −55 **c** $50\frac{1}{2}$

5 a 29 **b** 51 **c** 29

6 a $a = 6, d = 10$
b 86, 206, 996
c $T_n = 10n - 4$

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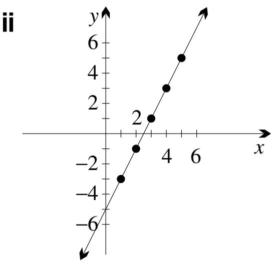
7 a $a = -20, d = 11$ **b** 57, 310, 2169
c $T_n = 11n - 31$
8 a $a = 300, d = -40$ **b** 60, -1700, -39660
c $T_n = 340 - 40n$
9 a $d = 3, T_n = 5 + 3n$
b $d = -6, T_n = 27 - 6n$
c not an AP
d $d = 4, T_n = 4n - 7$
e $d = 1\frac{1}{4}, T_n = \frac{1}{4}(2 + 5n)$
f $d = -17, T_n = 29 - 17n$
g $d = \sqrt{2}, T_n = n\sqrt{2}$
h not an AP
i $d = 3\frac{1}{2}, T_n = \frac{1}{2}(7n - 12) = \frac{7}{2}n - 6$
10 a $T_n = 170 - 5n$ **b** 26 terms
c $T_{35} = -5$
11 a 11 terms **b** 34 terms **c** 16 terms
d 13 terms **e** 9 terms **f** 667 terms
12 a $T_n = 23 - 3n, T_8 = -1$
b $T_n = 55 - 5n, T_{12} = -5$
c $T_n = 74 - 7n, T_{11} = -3$
d $T_n = 85 - 3n, T_{29} = -2$
e $T_n = 353 - 8n, T_{45} = -7$
f $T_n = 25 - \frac{1}{2}n, T_{51} = -\frac{1}{2}$
13 a 11, 15, 19, 23, $a = 11, d = 4$
b $T_{50} + T_{25} = 314, T_{50} - T_{25} = 100$
d $815 = T_{202}$
e $T_{248} = 999, T_{249} = 1003$
f $T_{49} = 203, \ldots, T_{73} = 299$ lie between 200 and 300, making 25 terms.
14 a i $T_n = 8n$
ii $T_{63} = 504, T_{106} = 848$
iii 44 terms
b $T_{91} = 1001, T_{181} = 1991$, 91 terms
c $T_{115} = 805, T_{285} = 1995$, 171 terms
15 a $d = 3$. 7, 10, 13, 16
b $d = -18$. 100, 82, 64, 46, 28
c $d = 8$. $T_{20} = 180$
d $d = -2$. $T_{100} = -166$
16 a \$500, \$800, \$1100, \$1400, . . .
b $a = 500, d = 300$ **c** \$4700
d cost $= 200 + 300n$ **e** 32
17 a 180, 200, 220, . . . **b** $a = 180, d = 20$
c 400 km **d** length $= 160 + 20n$
e 19 months
18 a 9, 6, 3, 0, -3, . . . and $T_n = 12 - 3n$
b i $T_n = 2n - 5, f(x) = 2x - 5$
ii



19 a $d = 4, x = 1$ **b** $d = 6x, x = \frac{1}{3}$
20 a $d = \log_3 2, T_n = n\log_3 2$
b $d = -\log_a 3, T_n = \log_a 2 + (4 - n)\log_a 3$
c $d = x + 4y, T_n = nx + (4n - 7)y$
d $d = -4 + 7\sqrt{5}, T_n = 9 - 4n + (7n - 13)\sqrt{5}$
e $d = -1.88, T_n = 3.24 - 1.88n$
f $d = -\log_a x, T_n = \log_a 3 + (3 - n)\log_a x$
21 a $a = m + b, d = m$
b gradient $= d$, y-intercept $= m - a$

Exercise 1C

1 a 8, 16, 32 **b** 3, 1, $\frac{1}{3}$
c $-56, -112, -224$ **d** $-20, -4, -\frac{4}{5}$
e $-24, 48, -96$ **f** 200, -400, 800
g $-5, 5, -5$ **h** $1, -\frac{1}{10}, \frac{1}{100}$
i 40, 400, 4000
2 a 1, 3, 9, 27 **b** 12, 24, 48, 96
c 5, -10, 20, -40 **d** 18, 6, 2, $\frac{2}{3}$
e 18, -6, 2, $-\frac{2}{3}$ **f** 50, 10, 2, $\frac{2}{5}$
g 6, -3, $1\frac{1}{2}$, $-\frac{3}{4}$ **h** $-13, -26, -52, -104$
i $-7, 7, -7, 7$
3 a GP: $a = 4, r = 2$ **b** GP: $a = 16, r = \frac{1}{2}$
c GP: $a = 7, r = 3$ **d** GP: $a = -4, r = 5$
e not a GP
f GP: $a = -1000, r = \frac{1}{10}$
g GP: $a = -80, r = -\frac{1}{2}$
h GP: $a = 29, r = 1$
i not a GP
j GP: $a = -14, r = -1$
k GP: $a = 6, r = \frac{1}{6}$
l GP: $a = -\frac{1}{3}, r = -3$
4 a 40 **b** $\frac{3}{10}$ **c** -56
d -8 **e** -88 **f** 120
5 a 3^{69} **b** 5×7^{69}
c $8 \times (-3)^{69} = -8 \times 3^{69}$
6 a $a = 7, r = 2$
b $T_6 = 224, T_{50} = 7 \times 2^{49}$
c $T_n = 7 \times 2^{n-1}$

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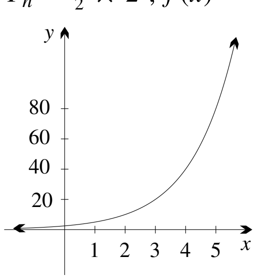
7 a $a = 10, r = -3$
b $T_6 = -2430, T_{25} = 10 \times (-3)^{24} = 10 \times 3^{24}$
c $T_n = 10 \times (-3)^{n-1}$
8 a $a = -80, r = \frac{1}{2}$
b $T_{10} = -\frac{5}{32}, T_{100} = -80 \times \left(\frac{1}{2}\right)^{99}$
c $T_n = -80 \times \left(\frac{1}{2}\right)^{n-1}$
9 a $T_n = 10 \times 2^{n-1}, T_6 = 320$
b $T_n = 180 \times \left(\frac{1}{3}\right)^{n-1}, T_6 = \frac{20}{27}$
c not a GP **d** not a GP
e $T_n = \frac{3}{4} \times 4^{n-1}, T_6 = 768$
f $T_n = -48 \times \left(\frac{1}{2}\right)^{n-1}, T_6 = -1\frac{1}{2}$
10 a $r = -1, T_n = (-1)^{n-1}, T_6 = -1$
b $r = -2, T_n = -2 \times (-2)^{n-1} = (-2)^n, T_6 = 64$
c $r = -3, T_n = -8 \times (-3)^{n-1}, T_6 = 1944$
d $r = -\frac{1}{2}, T_n = 60 \times \left(-\frac{1}{2}\right)^{n-1}, T_6 = -\frac{15}{8}$
e $r = -\frac{1}{2}, T_n = -1024 \times \left(-\frac{1}{2}\right)^{n-1}, T_6 = 32$
f $r = -6, T_n = \frac{1}{16} \times (-6)^{n-1}, T_6 = -486$
11 a $T_n = 2^{n-1}$, 7 terms **b** $T_n = -3^{n-1}$, 5 terms
c $T_n = 8 \times 5^{n-1}$, 7 terms
d $T_n = 7 \times 2^{n-1}$, 6 terms
e $T_n = 2 \times 7^{n-1}$, 5 terms
f $T_n = 5^{n-3}$, 7 terms
12 a $r = 2$. 25, 50, 100, 200, 400
b $r = 2$. 3, 6, 12, 24, 48, 96
c Either $r = 3$, giving 1, 3, 9, 27, 81, or $r = -3$, giving 1, −3, 9, −27, 81.
13 a $r = \frac{1}{9}$ or $-\frac{1}{9}$ **b** $r = 0.1$ or -0.1
c $r = -\frac{3}{2}$ **d** $r = \sqrt{2}$ or $-\sqrt{2}$
14 a 50, 100, 200, 400, 800, 1600, $a = 50, r = 2$
b $6400 = T_8$
c $T_{50} \times T_{25} = 5^4 \times 2^{75}, T_{50} \div T_{25} = 2^{25}$
e The six terms $T_6 = 1600, \ldots, T_{11} = 51\,200$ lie between 1000 and 100000.
15 The successive thicknesses form a GP with 101 terms, and with $a = 0.1$ mm and $r = 2$. Hence thickness $= T_{101} = \frac{2^{100}}{10}$ mm $\doteqdot 1.27 \times 10^{23}$ km $\doteqdot 1.34 \times 10^{10}$ light years, which is close to the present estimate of the distance to the Big Bang.
16 a $\frac{4}{5}$, 4, 20, 100, 500, ... and $T_n = \frac{4}{25} \times 5^n$
b i $T_n = \frac{5}{2} \times 2^n, f(x) = \frac{5}{2} \times 2^x$
ii



17 a $r = \sqrt{2}, T_n = \sqrt{6} \times \left(\sqrt{2}\right)^{n-1} = \sqrt{3} \times \left(\sqrt{2}\right)^{n}$
b $r = ax^2, T_n = a^n x^{2n-1}$
c $r = \frac{y}{x}, T_n = -x^{2-n}y^{n-2}$
18 a $T_n = 2x^n, x = 1$ or -1
b $T_n = x^{6-2n}, x = \frac{1}{3}$ or $-\frac{1}{3}$
c $T_n = 2^{-16} \times 2^{4n-4}x = 2^{4n-20}x, x = 6$
19 a $a = cb, r = b$ **b** $f(x) = \frac{a}{r} \times r^x$

Exercise 1D

1 a 11 **b** 23 **c** −31
d −8 **e** 12 **f** 10
2 a 6 or −6 **b** 12 or −12 **c** 30 or −30
d 14 or −14 **e** 5 **f** −16
3 a 10. 8 or −8 **b** 25. 7 or −7
c $20\frac{1}{2}$. 20 or −20 **d** $-12\frac{1}{2}$. 10 or −10
e −30. 2 **f** 0. 6
g −3. 1 **h** 24. −3
i 40. 45 **j** 84. −16
k $-5\frac{3}{4}$. −36 **l** −21. 7
4 a 7, 14, 21, 28, 35, 42
b 27, 18, 12, 8
c 40, $36\frac{1}{2}$, 33, $29\frac{1}{2}$, 26, $22\frac{1}{2}$, 19, $15\frac{1}{2}$, 12, $8\frac{1}{2}$, 5
d 1, 10, 100, 1000, 10000, 100000, 1000000 or 1, −10, 100, −1000, 10000, −100000, 1000000
e 3, $14\frac{1}{4}$, $25\frac{1}{2}$, $36\frac{3}{4}$, 48
f 3, 6, 12, 24, 48 or 3, −6, 12, −24, 48
5 a $d = 3, a = -9$ **b** $d = 4, a = -1$
c $d = -9, a = 60$ **d** $d = 3\frac{1}{2}, a = -4\frac{1}{2}$
6 a $r = 2, a = 4$ **b** $r = 4, a = \frac{1}{16}$
c $r = 3$ and $a = \frac{1}{9}$, or $r = -3$ and $a = -\frac{1}{9}$
d $r = \sqrt{2}$ and $a = \frac{3}{2}$, or $r = -\sqrt{2}$ and $a = \frac{3}{2}$
7 a $T_8 = 37$ **b** $T_2 = 59$ **c** $T_2 = \frac{3}{8}$
8 a $n = 13$ **b** $n = 8$
c $n = 11$ **d** $n = 8$
9 b $n = 19$ **c** $n = 29$
d $n = 66$ **e** 10 terms
f 37 terms
10 a $T_n = 98 \times \left(\frac{1}{7}\right)^{n-1}$, 10 terms
b $T_n = 25 \times \left(\frac{1}{5}\right)^{n-1} = \left(\frac{1}{5}\right)^{n-3}$, 11 terms
c $T_n = (0.9)^{n-1}$, 132 terms
11 a 78% **b** 152 sheets
12 a $a = 28, d = -1$
b $a = \frac{1}{3}$ and $r = 3$, or $a = \frac{2}{3}$ and $r = -3$
c $T_6 = -2$

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13 a $x = 10$. 9, 17, 25
b $x = -2$. $-2, -6, -10$
c $x = 2$. $-1, 5, 11$
d $x = -4$. $-14, -4, 6$
14 a $x = -\frac{1}{2}$. $-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$
b $x = 1$. 1, 2, 4 or $x = 6$. $-4, 2, -1$
15 a i $x = -48$ **ii** $x = 6$
b i $x = 0.10001$
ii $x = 0.002$ or $x = -0.002$
c i $x = 0.398$ **ii** $x = 20$
d i They can't form an AP. **ii** $x = 9$
e i $x = 2$ **ii** $x = 4$ or $x = 0$
f i $x = \sqrt{5}$ **ii** $x = 2$ or $x = -2$
g i $x = \frac{3}{2}\sqrt{2}$ **ii** $x = 2$ or $x = -2$
h i $x = 40$ **ii** 2^5 or -2^5
i i $x = 0$ **ii** They can't form a GP.
16 a $a = 2$ and $r = 4$
b $a = \log_2 3$ and $d = 1$
17 a $a = 6\frac{1}{4}$ and $b = 2\frac{1}{2}$, or $a = 4$ and $b = -2$
b $a = 1, b = 0$
18 a $T_n = 2^{8-3n}$
19 b $\frac{T_8}{T_1} = \left(\frac{1}{2}\right)^{\frac{7}{12}} \doteq 0.6674 \doteq \frac{2}{3}$
c $\frac{T_5}{T_1} = \left(\frac{1}{2}\right)^{\frac{4}{12}} \doteq 0.7937 \doteq \frac{4}{5}$
d $\frac{T_6}{T_1} = \left(\frac{1}{2}\right)^{\frac{5}{12}} \doteq 0.7491 \doteq \frac{3}{4}$,
$\frac{T_4}{T_1} = \left(\frac{1}{2}\right)^{\frac{3}{12}} \doteq 0.8409 \doteq \frac{5}{6}$
e $\frac{T_3}{T_1} = \left(\frac{1}{2}\right)^{\frac{2}{12}} \doteq 0.8908 \doteq \frac{8}{9}$,
$\frac{T_2}{T_1} = \left(\frac{1}{2}\right)^{\frac{1}{12}} \doteq 0.9439 \doteq \frac{17}{18}$

Exercise 1E

1 a 24 **b** 80 **c** 0 **d** $3\frac{3}{4}$
2 a 450 **b** 24 **c** -54 **d** 15.6
3 a 10, 30, 60, 100, 150 **b** $1, -2, 7, -20, 61$
c 1, 5, 14, 30, 55 **d** $3, 7\frac{1}{2}, 13\frac{1}{2}, 21, 30$
4 a $-2, 3, -3$
b $120, 121, 121\frac{1}{3}$
c 60, 50, 30
d 0.1111, 0.111 11, 0.111 111
5 S_n: 2, 7, 15, 26, 40, 57, 77
S_n: 40, 78, 114, 148, 180, 210, 238
S_n: $2, -2, 4, -4, 6, -6, 8$
S_n: 7, 0, 7, 0, 7, 0, 7
6 T_n: 1, 3, 5, 7, 9, 11, 13
T_n: 2, 4, 8, 16, 32, 64, 128
T_n: $-3, -5, -7, -9, -11, -13, -15$
T_n: $8, -8, 8, -8, 8, -8, 8$
7 T_n: 1, 1, 1, 2, 3, 5, 8, 13
T_n: 3, 1, 3, 4, 7, 11, 18, 29
8 a 42 **b** 75 **c** 15 **d** 174
e 100 **f** 63 **g** 117 **h** -1
i 0 **j** 404 **k** 7 **l** -7
10 a $\sum_{n=1}^{40} n^3$ **b** $\sum_{n=1}^{40} \frac{1}{n}$ **c** $\sum_{n=1}^{20} (n+2)$ **d** $\sum_{n=1}^{12} 2^n$
e $\sum_{n=1}^{10} (-1)^n n$ **f** $\sum_{n=1}^{10} (-1)^{n+1} n$ or $\sum_{n=1}^{10} (-1)^{n-1} n$
11 a $T_1 = 2, T_n = 2^{n-1}$ for $n \geq 2$
b 2, 2, 4, 8, 16, . . .
c The derivative of e^x is the original function e^x. Remove the initial term 2 from the sequence in part **b**, and the successive differences are the original sequence.
12 b $T_1 = 1$ and $T_n = 3n^2 - 3n + 1$ for $n \geq 2$
c $U_1 = 1$ and $U_n = 6n$ for $n \geq 2$
d 1, 7, 19, 37, 61, 91, . . . and 1, 6, 12, 18, 24, 30, . . .
e The derivative of x^3 is the quadratic $3x^2$, and its derivative is the linear function $6x$. Taking successive differences once gives a quadratic, and taking them twice gives a linear function.

Exercise 1F

1 77
2 a $n = 100$, 5050 **b** $n = 50$, 2500
c $n = 50$, 2550 **d** $n = 100$, 15 150
e $n = 50$, 7500 **f** $n = 9000$, 49 504 500
3 a 180 **b** 78 **c** -153 **d** -222
4 a $a = 2, d = 4$, 882
b $a = 3, d = 7$, 1533
c $a = -6, d = 5$, 924
d $a = 10, d = -5$, -840
e $a = -7, d = -3$, -777
f $a = 1\frac{1}{2}, d = 2$, $451\frac{1}{2}$
5 a 222 **b** -630 **c** 78 400
d 0 **e** 65 **f** 30
6 a 101 terms, 10 100 **b** 13 terms, 650
c 11 terms, 275 **d** 100 terms, 15 250
e 11 terms, 319 **f** 10 terms, $61\frac{2}{3}$

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7 a 500 terms, 250 500 **b** 2001 terms, 4 002 000
c 3160 **d** 1440
8 a $S_n = \frac{1}{2}n(5 + 5n)$ **b** $S_n = \frac{1}{2}n(17 + 3n)$
c $S_n = n(1 + 2n)$ **d** $\frac{1}{2}n(5n - 23)$
e $S_n = \frac{1}{4}n(21 - n)$ **f** $\frac{1}{2}n\left(2 + n\sqrt{2} - 3\sqrt{2}\right)$
9 a $\frac{1}{2}n(n + 1)$ **b** n^2
c $\frac{3}{2}n(n + 1)$ **d** $100n^2$
10 a 450 legs. No creatures have the mean number of 5 legs.
b 16 860 years **c** \$352 000
11 a $a = 598, \ell = 200$, 79 800
b $a = 90, \ell = -90$, 0
c $a = -47, \ell = 70$, 460
d $a = 53, \ell = 153$, 2163
12 a $\ell = 22$ **b** $a = -7.1$
c $d = 11$ **d** $a = -3$
13 b i 16 terms **ii** more than 16 terms
c 5 terms or 11 terms
d $n = 18$ or $n = -2$, but n must be a positive integer.
e $n = 4, 5, 6, \ldots, 12$
f Solving $S_n > 256$ gives $(n - 8)^2 < 0$, which has no solutions.
14 a $S_n = n(43 - n)$, 43 terms
b $S_n = \frac{3}{2}n(41 - n)$, 41 terms
c $S_n = 3n(n + 14)$, 3 terms
d $\frac{1}{4}n(n + 9)$, 6 terms
15 a 20 rows, 29 logs on bottom row
b $S_n = 5n^2$, 7 seconds
c 11 trips, deposits are 1 km apart.
16 a $d = -2, a = 11, S_{10} = 20$
b $a = 9, d = -2, T_2 = 7$
c $d = -3, a = 28\frac{1}{2}, T_4 = 19\frac{1}{2}$
17 a 10 terms, $55\log_a 2$
b 11 terms, 0
c 6 terms, $3(4\log_b 3 - \log_b 2)$
d $15(\log_x 2 - \log_x 3)$

Exercise 1G

1 728
2 2801 kits, cats, sacks, wives and man
3 a 1093 **b** 547
4 a $1023, 2^n - 1$ **b** $242, 3^n - 1$
c $-11\,111, -\frac{1}{9}\left(10^n - 1\right)$ **d** $-781, -\frac{1}{4}\left(5^n - 1\right)$
e $-341, \frac{1}{3}\left(1 - (-2)^n\right)$ **f** $122, \frac{1}{2}\left(1 - (-3)^n\right)$
g $-9091, -\frac{1}{11}\left(1 - (-10)^n\right)$
h $-521, -\frac{1}{6} = \left(1 - (-5)^n\right)$
5 a $\frac{1023}{64}, 16\left(1 - \left(\frac{1}{2}\right)^n\right)$ **b** $\frac{364}{27}, \frac{27}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$
c $\frac{605}{9}, \frac{135}{2}\left(1 - \left(\frac{1}{3}\right)^n\right)$ **d** $\frac{211}{24}, \frac{4}{3}\left(\left(\frac{3}{2}\right)^n - 1\right)$
e $\frac{341}{64}, \frac{16}{3}\left(1 - \left(-\frac{1}{2}\right)^n\right)$ **f** $\frac{182}{27}, \frac{27}{4}\left(1 - \left(\frac{1}{3}\right)^n\right)$
g $-\frac{305}{9}, -\frac{135}{4}\left(1 - \left(-\frac{1}{3}\right)^n\right)$ **h** $\frac{55}{24}, \frac{4}{15}\left(1 - \left(-\frac{3}{2}\right)^n\right)$
6 a $5\left((1.2)^n - 1\right)$, 25.96
b $20\left(1 - (0.95)^n\right)$, 8.025
c $100\left((1.01)^n - 1\right)$, 10.46
d $100\left(1 - (0.99)^n\right)$, 9.562
7 a i 2^{63} **ii** $2^{64} - 1$
b 615 km^3
8 a $S_n = \left(\left(\sqrt{2}\right)^n - 1\right)\left(\sqrt{2} + 1\right)$, $S_{10} = 31\left(\sqrt{2} + 1\right)$
b $S_n = \frac{1}{2}\left(1 - \left(-\sqrt{5}\right)^n\right)\left(\sqrt{5} - 1\right)$, $S_{10} = -1562\left(\sqrt{5} - 1\right)$
9 a $a = 6, r = 2$, 762 **b** $a = 9, r = 3$, 3276
c $a = 12, r = \frac{1}{2}, \frac{765}{32}$
10 a $\frac{1}{8} + \frac{3}{4} + \frac{9}{2} + 27 + 162 = 194\frac{3}{8}$ or $\frac{1}{8} - \frac{3}{4} + \frac{9}{2} - 27 + 162 = 138\frac{7}{8}$
b $15\frac{3}{4}$ **c** 1562.496 **d** 7 **e** 640
11 a i 0.01172 tonnes
ii 11.99 tonnes
b 4.9×10^{-3}g
c i $S_n = 10P(1.1^{10} - 1)$
ii \$56.47
12 a 6 terms **b** 8 terms **c** 5 terms **d** 7 terms
13 b $n = 8$ **c** 14 terms
d $S_{14} = 114\,681$
14 a 41 powers of 3 **b** 42 terms

Exercise 1H

1 a $18, 24, 26, 26\frac{2}{3}, 26\frac{8}{9}, 26\frac{26}{27}$
b $S_\infty = 27$
c $S_\infty - S_6 = 27 - 26\frac{26}{27} = \frac{1}{27}$
2 a $24, 12, 18, 15, 16\frac{1}{2}, 15\frac{3}{4}$ **b** $S_\infty = 16$
c $S_\infty - S_6 = 16 - 15\frac{3}{4} = \frac{1}{4}$
3 a $a = 1, S_\infty = 2$ **b** $a = 8, S_\infty = 16$
c $a = -4, S_\infty = -8$
4 a $a = 1, S_\infty = \frac{3}{4}$ **b** $a = 36, S_\infty = 27$
c $a = -60, S_\infty = -45$
5 a $r = \frac{1}{4}, S_\infty = 80$ **b** $r = -\frac{1}{2}, S_\infty = 40$
c $r = -\frac{1}{5}, S_\infty = 50$
6 a $r = -\frac{1}{2}, S_\infty = \frac{2}{3}$ **b** $r = \frac{1}{3}, S_\infty = \frac{3}{2}$
c $r = -\frac{2}{3}, S_\infty = \frac{3}{5}$ **d** $r = \frac{3}{5}, S_\infty = 2\frac{1}{2}$

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e $r = -\frac{3}{2}$, no limiting sum
f $r = \frac{1}{3}, S_\infty = 18$ **g** $r = \frac{1}{10}, S_\infty = 1111\frac{1}{9}$
h $r = -\frac{1}{10}, S_\infty = 909\frac{1}{11}$
i $r = -1$, no limiting sum
j $r = \frac{9}{10}, S_\infty = 1000$ **k** $r = -\frac{1}{5}, S_\infty = -\frac{5}{3}$
l $r = \frac{1}{5}, S_\infty = -\frac{5}{6}$

7 a The successive down-and-up distances form a GP with $a = 12$ and $r = \frac{1}{2}$.
b $S_\infty = 24$ metres

8 a T_n: 10, 10, 10, 10, 10, 10. S_n: 10, 20, 30, 40, 50, 60. $S_n \to \infty$ as $n \to \infty$.
b T_n: 10, −10, 10, −10, 10, −10. S_n: 10, 0, 10, 0, 10, 0. S_n oscillates between 10 and 0 as $n \to \infty$.
c T_n: 10, 20, 40, 80, 160, 320. S_n: 10, 30, 70, 150, 310, 630. $S_n \to \infty$ as $n \to \infty$.
d T_n: 10, −20, 40, −80, 160, −320. S_n: 10, −10, 30, −50, 110, −210. S_n oscillates between larger and larger positive and negative numbers as $n \to \infty$.

9 a $S_\infty - S_4 = 160 - 150 = 10$
b $S_\infty - S_4 = 111\frac{1}{9} - 111\frac{1}{10} = \frac{1}{90}$
c $S_\infty - S_4 = 55\frac{5}{9} - 32\frac{4}{5} = 22\frac{34}{45}$

10 a $a = 2000$ and $r = \frac{1}{5}$
b $S_\infty = 2500$
c $S_\infty - S_4 = 4$

11 a $S_\infty = 10\,000$ **b** $S_\infty - S_{10} \doteqdot 3487$

12 a $S_\infty = \frac{5}{1 - x}, x = \frac{1}{2}$ **b** $S_\infty = \frac{5}{1 - x}, x = -\frac{2}{3}$
c $S_\infty = \frac{5}{1 + x}, x = -\frac{2}{3}$ **d** $S_\infty = \frac{3x}{2}, x = \frac{4}{3}$
e $S_\infty = \frac{3x}{4}, x = \frac{8}{3}$ **f** $S_\infty = 3x, x = \frac{2}{3}$

13 a $-1 < x < 1, \frac{7}{1 - x}$ **b** $-\frac{1}{3} < x < \frac{1}{3}, \frac{2x}{1 - 3x}$
c $0 < x < 2, \frac{1}{2 - x}$ **d** $-2 < x < 0, -\frac{1}{x}$

14 a $r = 1.01$, no limiting sum
b $r = -0.99, S_\infty = \frac{100}{199}$
c $r = (1.01)^{-1}, S_\infty = 101$
d $r = -\frac{1}{6}, S_\infty = \frac{108}{175}$

15 a $r = \frac{1}{4}, S_\infty = \frac{64}{3}\sqrt{5}$
b $r = -\frac{1}{3}, S_\infty = 81\sqrt{7}$ **c** $\frac{7}{6}\left(7 + \sqrt{7}\right)$
d $4\left(2 - \sqrt{2}\right)$ **e** $5\left(5 - 2\sqrt{5}\right)$
f $r = \frac{1}{3}\sqrt{10} > 1$, so there is no limiting sum.
g $\frac{1}{3}\sqrt{3}$ **h** $\frac{1}{2}\left(\sqrt{3} + 1\right)$

16 a $a = \frac{1}{3}, r = \frac{1}{3}, S_\infty = \frac{1}{2}$
b $a = \frac{7}{2}, r = \frac{1}{2}, S_\infty = 7$
c $a = -24, r = -\frac{3}{5}, S_\infty = -15$

17 a $r = \frac{4}{5}$
b $18 + 6 + 2 + \cdots$ or $9 + 6 + 4 + \cdots$
c $r = \frac{5}{6}$
d i $r = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$ ($r = -\frac{1}{2} - \sqrt{5} < -1$, so it is not a possible solution.)
ii $r = \frac{1}{2}$ **iii** $r = \frac{1}{2}\sqrt{2}$ or $-\frac{1}{2}\sqrt{2}$

18 a $-\sqrt{2} < x < \sqrt{2}$ and $x \neq 0, S_\infty = \frac{1}{2 - x^2}$
b $x \neq 0, S_\infty = \frac{1 + x^2}{x^2}$

19 a $w = \frac{1}{1 - v}$ **b** $v = \frac{w}{1 + w}$ **c** v

Exercise 1I

1 a $0.3 + 0.03 + 0.003 + \cdots = \frac{1}{3}$
b $0.1 + 0.01 + 0.001 + \cdots = \frac{1}{9}$
c $0.7 + 0.07 + 0.007 + \cdots = \frac{7}{9}$
d $0.6 + 0.06 + 0.006 + \cdots = \frac{2}{3}$

2 a $0.27 + 0.0027 + 0.000027 + \cdots = \frac{3}{11}$
b $\frac{81}{99} = \frac{9}{11}$ **c** $\frac{1}{11}$ **d** $\frac{4}{33}$ **e** $\frac{26}{33}$
f $\frac{1}{37}$ **g** $\frac{5}{37}$ **h** $\frac{5}{27}$

3 a $12 + (0.4 + 0.04 + \cdots) = 12\frac{4}{9}$
b $7 + 0.81 + 0.0081 + \cdots = 7\frac{9}{11}$
c $8.4 + (0.06 + 0.006 + \cdots) = 8\frac{7}{15}$
d $0.2 + (0.036 + 0.00036 + \cdots) = \frac{13}{55}$

4 a $0.\dot{9} = 0.9 + 0.09 + 0.009 + \cdots = \frac{0.9}{1 - 0.1} = 1$
b $2.7\dot{9} = 2.7 + (0.09 + 0.009 + 0.0009 + \cdots)$
$= 2.7 + \frac{0.09}{1 - 0.1} = 2.7 + 0.1 = 2.8$

5 a $\frac{29}{303}$ **b** $\frac{25}{101}$ **c** $\frac{3}{13}$ **d** $\frac{3}{7}$
e $0.25 + (0.0057 + 0.000057 + \cdots) = \frac{211}{825}$
f $1\frac{14}{135}$ **g** $\frac{1}{3690}$ **h** $7\frac{27}{35}$

6 If $\sqrt{2}$ were a recurring decimal, then we could use the methods of this section to write it as a fraction.

Chapter 1 review exercise

1 14, 5, −4, −13, −22, −31, −40, −49
a 6 **b** 4 **c** −31
d T_8 **e** No **f** $T_{11} = -40$

2 a 52, −62, −542, −5 999 942
b 20 no, $10 = T_8$, $-56 = T_{19}$, −100 no
c $T_{44} = -206$
d $T_{109} = -596$

3 a 4, 7, 7, 7, 7, 7, . . .
b 0, 1, 2, 3, 4, 5, 6, . . .
c $T_1 = 5, T_n = 2n - 1$ for $n > 1$
d $T_1 = 3, T_n = 2^{3n-1}$ for $n > 1$

4 a 82 **b** −15 **c** 1 **d** $\frac{63}{64}$

5 a −5, 5, −5, 5, −5, 5, −5, 5
b −5, 0

6 a AP, $d = 7$ **b** AP, $d = -121$
c neither **d** GP, $r = 3$
e neither **f** GP, $r = -\frac{1}{2}$

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7 a $a = 23, d = 12$
b $T_{20} = 251, T_{600} = 7211$
d $143 = T_{11}$, 173 is not a term.
e $T_{83} = 1007, T_{165} = 1991$
f 83 (Count both T_{83} and T_{165}.)

8 a $a = 20, d = 16$ **b** $T_n = 4 + 16n$
c 12 cases, \$4 change **d** 18

9 a $a = 50, r = 2$
b $T_n = 50 \times 2^{n-1}$ (or 25×2^n)
c $T_8 = 6400, T_{12} = 102\,400$
d $1600 = T_6$, 4800 is not a term.
e 320 000 **f** 18 terms

10 a $a = 486, r = \frac{1}{3}$
b 486, 162, 54, 18, 6, 2 (no fractions)
c 4 **d** $S_6 = 728$ **e** 729

11 a 75 **b** 45 or -45

12 a 11 111 **b** $-16\,400$ **c** 1025

13 a $n = 45, S_{45} = 4995$ **b** $n = 101, S_{101} = 5050$
c $n = 77, S_{77} = 2387$

14 a 189 **b** -1092 **c** $-157\frac{1}{2}$

15 a 300
b $r = -\frac{3}{2} < -1$, so there is no limiting sum.
c $-303\frac{3}{4}$

16 a $-3 < x < -1$ **b** $S_\infty = -\frac{2 + x}{1 + x}$

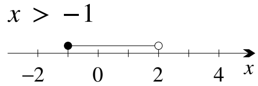
17 a $\frac{13}{33}$ **b** $\frac{52}{111}$ **c** $12\frac{335}{1100} = 12\frac{67}{220}$

18 a $d = 5$, 511 **b** -1450 **c** $r = -2, -24$
d $d = -5$ **e** $n = 2$ or $n = 8$
f $r = -\frac{1}{3}$ **g** 16

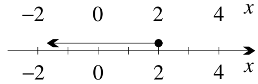
Chapter 2

Exercise 2A

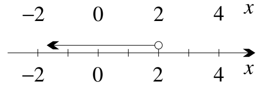
1 a i $-1 \le x \le 2$ **ii** $[-1, 2]$
b i $-1 < x \le 2$ **ii** $(-1, 2]$
c i $x > -1$ **ii** $(-1, \infty)$

2 a i 

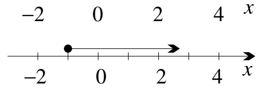
ii $[-1, 2)$

b i 

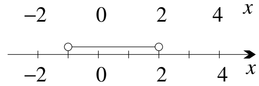
ii $(-\infty, 2]$

c i 

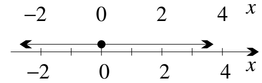
ii $(-\infty, 2)$

3 a i 

ii $x \ge -1$

b i 

ii $-1 < x < 2$

c i 

ii $\mathbb{R}$ (There is no way of writing the interval using inequalities.)

4 a i $2^{15} = 32\,768$ **ii** $5 \times 8 = 40$
iii $2^8 = 256$ **iv** $5 \times 15 = 75$
b i 2^{5x} **ii** 5×2^x
iii 2^{2^x} **iv** $25x$

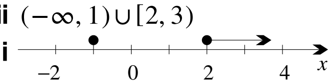
5 a i $x < -1$ or $0 < x < 1$
ii $-1 < x < 0$ or $x > 1$
b i $-5 < x < -2$ or $x > 1$
ii $x < -5$ or $-2 < x < 1$

6 a i $x = -3, -1$ or 2
ii $-3 < x < -1$ or $x > 2$
iii $x < -3$ or $-1 < x < 2$
b i $x = -2, 3$ or 4
ii $-2 < x < 3$ or $x > 4$
iii $x < -2$ or $3 < x < 4$

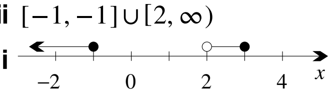
7 a $(-\infty, 1)$, one-to-one
b $(0, 2)$, many-to-one
c $(0, 1)$, one-to-one
d $(4, \infty)$, one-to-one

8 a $x \le 0$ or $1 \le x \le 2$
b $-2 < x < 0$ or $2 < x < 4$
c $0 < x < 3$ or $x > 3$
d $x = 0$ or $x \ge 4$
e $x = -3$ or $x = 3$
f $x = -3$ or $x \ge 0$

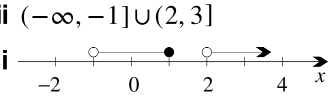
9 a i $-1 < x < 1$ or $2 \le x \le 3$
ii $(-1, 1) \cup [2, 3]$
b i $x < 1$ or $x \ge 2$
ii $(-\infty, 1) \cup [2, \infty)$
c i $x < 1$ or $2 \le x < 3$
ii $(-\infty, 1) \cup [2, 3)$

10 a i 

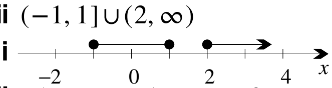
ii $[-1, -1] \cup [2, \infty)$

b i 

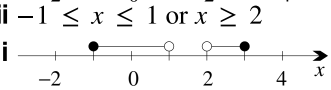
ii $(-\infty, -1] \cup (2, 3]$

c i 

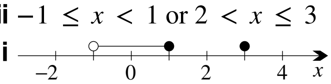
ii $(-1, 1] \cup (2, \infty)$

11 a i 

ii $-1 \le x \le 1$ or $x \ge 2$

b i 

ii $-1 \le x < 1$ or $2 < x \le 3$

c i 

ii $-1 < x \le 1$ or $x = 3$

12 a $(-\infty, 0] \cup [1, 2]$ **b** $(-2, 0) \cup (2, 4)$
c $(0, 3) \cup (3, \infty)$ **d** $[0, 0] \cup (4, \infty]$
e $[-3, -3] \cup [3, 3]$ **f** $[-3, -3] \cup [0, \infty)$

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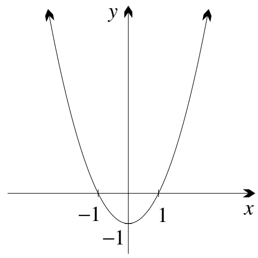
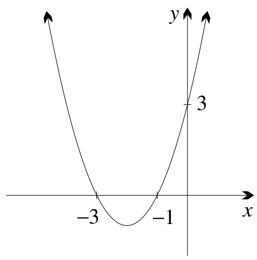
13 y: 3, 0, −1, 0, 3
sign: +, 0, −, 0, +



14 a



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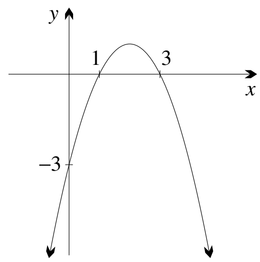

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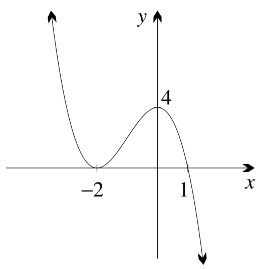
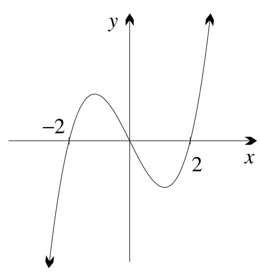


d



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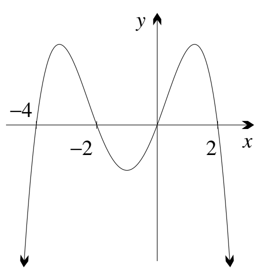

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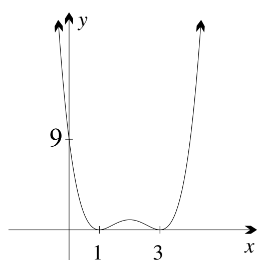
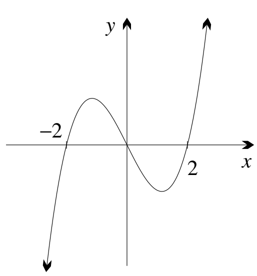
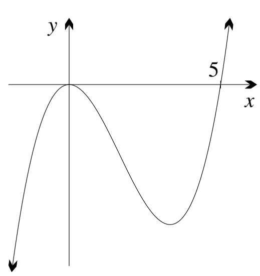


15 a



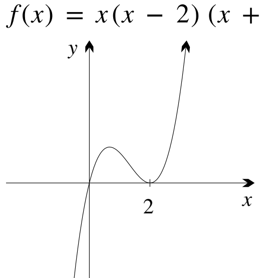
$f(x) = x(x-2)(x+2)$

b



$f(x) = x^2(x-5)$

c

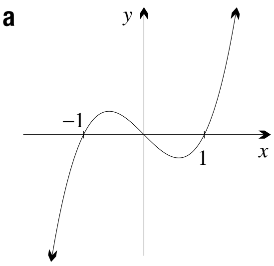


$f(x) = x(x-2)^2$

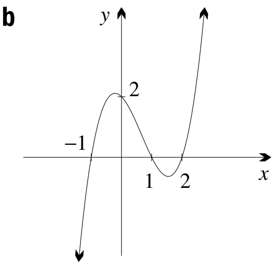
16 a $-2 < x < 0$ or $x > 2$
b $x < 0$ or $0 < x < 5$
c $x \le 0$ or $x = 2$

17 a $y = x(x+1)(x-1),\ x = -1, 0$ or 1
b $y = (x-2)(x-1)(x+1),\ x = -1, 1$ or 2
c $y = (x+2)^2(x-2),\ x = -2$ or 2

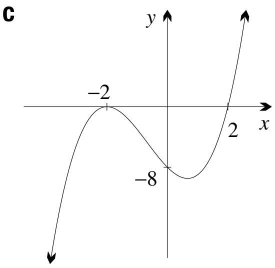
18 a



b



c



19 a zero for $x = 0$, undefined at $x = 3$, positive for $x < 0$ or $x > 3$, negative for $0 < x < 3$
b zero for $x = 4$, undefined at $x = -2$, positive for $x < -2$ or $x > 4$, negative for $-2 < x < 4$
c zero for $x = -3$, undefined at $x = -1$, positive for $x < -3$ or $x > -1$, negative for $-3 < x < -1$
d never zero, undefined at $x = -1$ and at $x = 1$, positive for $x < -1$ or $x > 1$, and negative for $-1 < x < 1$
e zero for $x = -2$ and for $x = 2$, undefined at $x = 0$, positive for $-2 < x < 0$ or $x > 2$, negative for $x < -2$ or $0 < x < 2$
f zero for $x = -2$ and for $x = 2$, undefined at $x = -4$ and at $x = 4$, positive for $x < -4$ or $-2 < x < 2$ or $x > 4$, negative for $-4 < x < -2$ or $2 < x < 4$

20 LHS $= (f \circ g)h(x) = f\big(g(h(x))\big)$,
RHS $= f(g \circ h(x)) = f\big(g(h(x))\big) =$ LHS.

21 a $x < 1$ or $3 < x < 5$
b $-3 \le x \le 1$ or $x \ge 4$
c $x \ne 1$ and $x \ne 3$ (alternatively, $x < 1$ or $1 < x < 3$ or $x > 3$)
d $x < -2$ or $0 < x < 2$ or $x > 4$
e $-3 < x < 0$ or $x > 3$
f $x \le 0$ or $x \ge 5$

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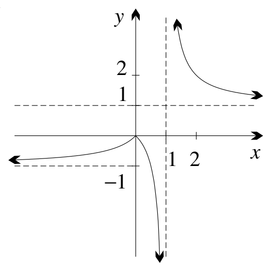
22 a **i** $x \neq 1$

ii $x = 0$

iii $[0, 0] \cup (1, \infty)$

iv

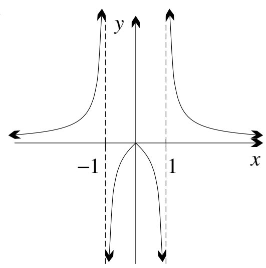


b **i** $x \neq -1$ or 1

ii $x = 0$

iii $(-\infty, -1) \cup [0, 0] \cup (1, \infty)$

iv



23 a It has one endpoint 5, which it contains.

b It contains all its endpoints (there are none).

c It does not contain any of its endpoints (there are none).

Exercise 2B

1 In each case $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

a **i** $x \neq 1$ **ii** $(0, -1)$

iii Dividing top and bottom by x gives $y = \dfrac{\frac{1}{x}}{1 - \frac{1}{x}}$, which has limit zero as $x \to \infty$ and as $x \to -\infty$. Alternatively, there is a constant on the top, so it is clear that $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

iv

x	-1	0	1	2	3
y	$-\frac{1}{2}$	-1	$*$	1	$\frac{1}{2}$

v vertical asymptote: $x = 1$, as $x \to 1^+$, $y > 0$ so $y \to +\infty$, and as $x \to 1^-$, $y < 0$ so $y \to -\infty$

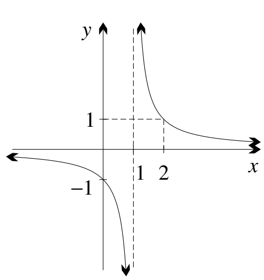
b **i** $x \neq 3$ **ii** $\left(0, \frac{2}{3}\right)$

iv

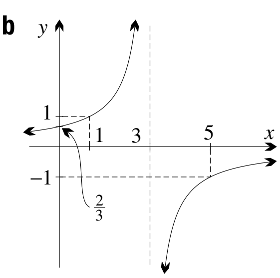
x	0	1	3	5	6
y	$\frac{2}{3}$	1	$*$	-1	$-\frac{2}{3}$

v vertical asymptote: $x = 3$, as $x \to 3^+$, $y < 0$ so $y \to -\infty$, and as $x \to 3^-$, $y > 0$ so $y \to +\infty$

a



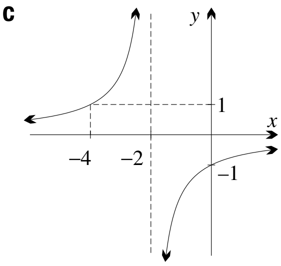
b



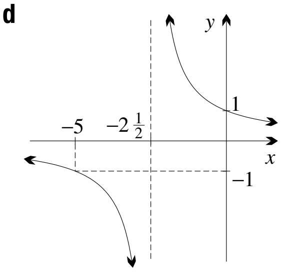
c domain: $x \neq -2$, vertical asymptote: $x = -2$, as $x \to -2^+$, $y < 0$ so $y \to -\infty$, and as $x \to -2^-$, $y > 0$ so $y \to +\infty$

d domain: $x \neq -2\frac{1}{2}$, vertical asymptote: $x = -2\frac{1}{2}$, as $x \to -2\frac{1}{2}^+$, $y > 0$ so $y \to +\infty$, and as $x \to -2\frac{1}{2}^-$, $y < 0$ so $y \to -\infty$

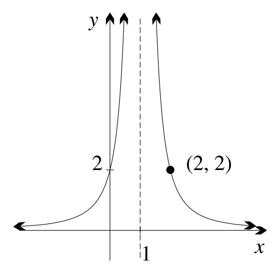
c



d

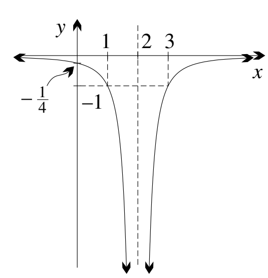


2



domain: $x \neq 1$,
$y \to 0$ as $x \to \infty$ and as $x \to -\infty$
vert'l asymptote $x = 1$, as $x \to 1^+$, $y > 0$ so $y \to \infty$ and as $x \to 1^-$, $y > 0$ so $y \to \infty$

3



domain: $x \neq 2$,
$y \to 0$ as $x \to \infty$ and as $x \to -\infty$
vert'l asymptote $x = 2$, as $x \to 2^+$, $y < 0$ so $y \to -\infty$, as $x \to 2^-$, $y < 0$ so $y \to -\infty$

4 a $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

b $f(x) \to 1$ as $x \to \infty$ and $x \to -\infty$

c $f(x) \to -2$ as $x \to \infty$ and $x \to -\infty$

d $f(x) \to \frac{1}{2}$ as $x \to \infty$ and $x \to -\infty$

e $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

f $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

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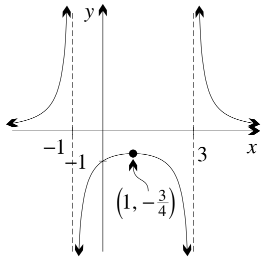
5



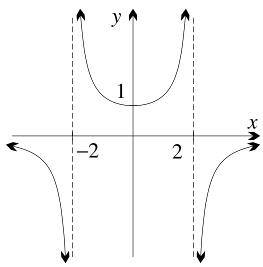
a $x \neq -1, 3$ **b** $(0, -1)$

c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

e as $x \to 3^+$, $y > 0$ so $y \to \infty$, as $x \to 3^-$, $y < 0$ so $y \to -\infty$, as $x \to 1^+$, $y < 0$ so $y \to -\infty$, and as $x \to 1^-$, $y > 0$ so $y \to \infty$

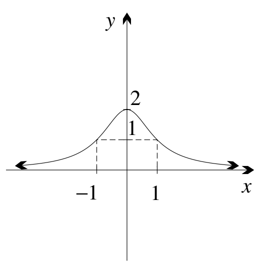
f $y \leq -\frac{3}{4}$, $y > 0$

6 a



b $y < 0$, $y \geq 1$

7



a $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

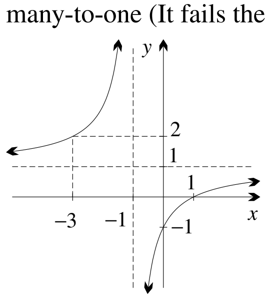
b $x^2 + 1$ is never 0.

c $y' = -4x(x^2 + 1)^{-2}$

e $0 < y \leq 2$

f many-to-one (It fails the horizontal line test.)

8



a $x \neq -1$

b $(0, -1)$

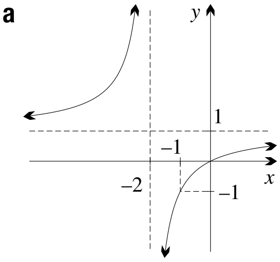
c $y \to 1$ as $x \to \infty$ and as $x \to -\infty$.

d as $x \to 1^+$, $y < 0$ so $y \to -\infty$, as $x \to 1^-$, $y > 0$ so $y \to \infty$

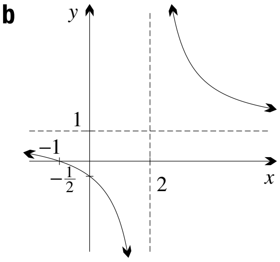
f $y \neq 1$

g one-to-one

9 a



b



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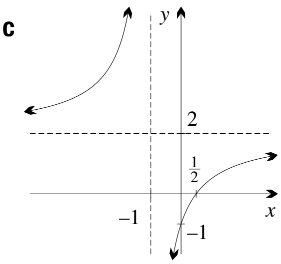
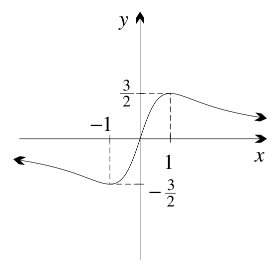
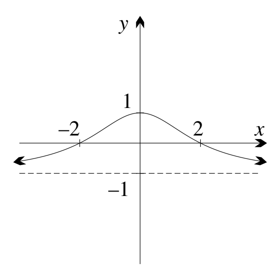


10



11



12 a **i** $y = 2$ **ii** $y = \frac{(x + 2)(x + 3)}{(x - 1)(x - 3)}$

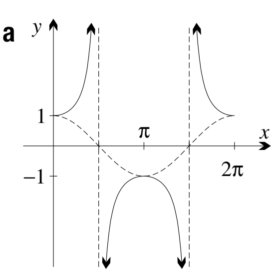
iii $x = 1, x = 3$

b **i** $y = \frac{(x - 1)^2}{(x + 1)(x + 4)}$, $x = -1$, $x = -4$ and $y = 1$

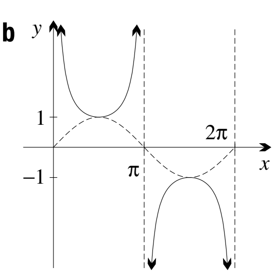
ii $y = \frac{x - 5}{(x - 2)(x + 5)}$, $x = -5$, $x = 2$ and $y = 0$

iii $y = \frac{(1 - 2x)(1 + 2x)}{(1 - 3x)(1 + 3x)}$, $x = \frac{1}{3}$, $x = -\frac{1}{3}$ and $y = \frac{4}{9}$

13 a



b



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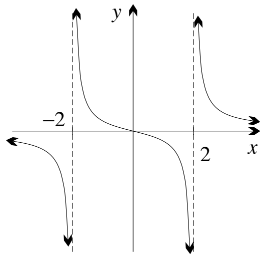
14



a point symmetry in the origin

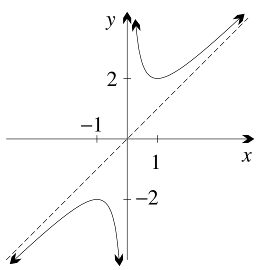
b domain: $x \neq 2$ and $x \neq -2$,
asymptotes: $x = 2$ and $x = -2$

d $y = 0$

f $f'(x) < 0$ for $x \neq 2$ & $x \neq -2$

h all real y

15



a point symmetry in the origin

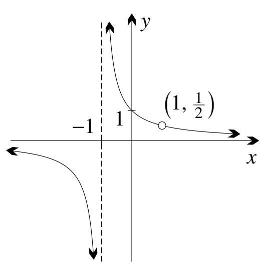
b domain: $x \neq 0$,
asymptote: $x = 0$

e $(-1, -2)$ and $(1, 2)$

g $y \geq 2$ or $y \leq -2$

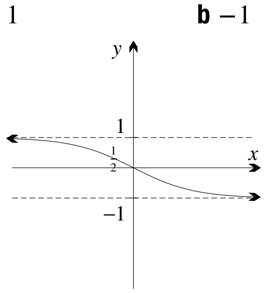
16 a Both numerator and denominator are zero, so y is undefined at $x = 1$. For $x \neq 1$, $y = \frac{1}{x+1}$

b



18 a 1 **b** −1 **c** $(0, 0)$

d



e odd

Exercise 2C

1a $y = \frac{9}{(x-3)(x+3)}$

b $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

c symmetry in the y-axis

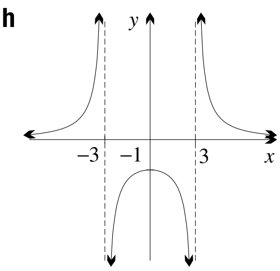
d $(0, -1)$

e $-3 < x < 3$

f $x = -3, x = 3$

g $y = 0$

h



i $y'(0) = 0$

2 a $y = \frac{x}{(2-x)(2+x)}$

b $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

c point symmetry in the origin

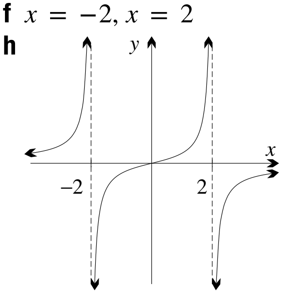
d $(0, 0)$

e $x < -2$ or $0 \leq x < 2$

f $x = -2, x = 2$

g $y = 0$

h



i $y' > 0$ in the domain.

3 a $y = x(x-2)(x+2)$

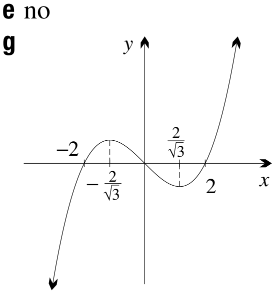
b $(-\infty, \infty)$

c $(-2, 0)$, $(0, 0)$, $(2, 0)$

d point symmetry in the origin

e no

g



4 a $\frac{2(x-2)}{(x-1)(x-4)}$

b $x \neq 1$ and $x \neq 4$

c The domain is not symmetric about $x = 0$.

d $\left(0, -\frac{5}{4}\right)$ and $\left(2\frac{1}{2}, 0\right)$

e $1 < x < 2\frac{1}{2}$ or $x > 4$

f $x = 1$ and $x = 4$

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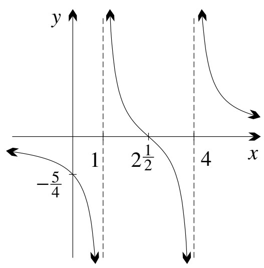
g $y = 0$

h



5 a $y = \frac{3(x-1)}{(x-3)(x+1)}$

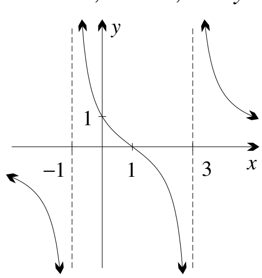
b domain: $x \neq -1$ and $x \neq 3$

intercepts: $(1, 0)$ and $(0, 1)$

c The domain is not symmetric about $x = 0$.

d $x = -1$, $x = 3$, and $y = 0$

e



6 a $y = \frac{(x+1)^2}{(x-1)(x+3)}$

b domain: $x \neq -3$ and $x \neq 1$

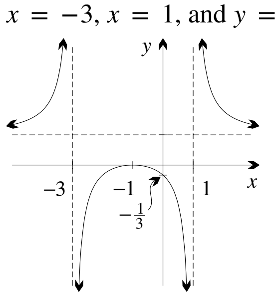
intercepts: $(-1, 0)$ and $\left(0, -\frac{1}{3}\right)$

c The domain is not symmetric about $x = 0$.

d $y < 0$ either side of $x = -1$.

e $x = -3$, $x = 1$, and $y = 1$

f



g $y \leq 0$ or $y > 1$

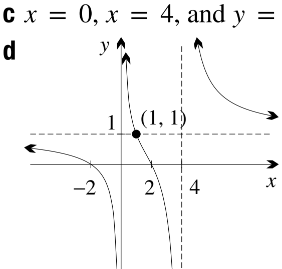
7 a $f(x) = \frac{(x-2)(x+2)}{x(x-4)}$

b domain: $x \neq 0$ and $x \neq 4$

intercepts: $(-2, 0)$ and $(2, 0)$

c $x = 0$, $x = 4$, and $y = 1$

d



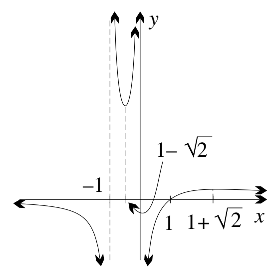
e all real y

8 $y = \frac{x-1}{x(x+1)}$

$y' = \frac{1 + 2x - x^2}{x^2(x+1)^2}$

so $y' = 0$ at $x = 1 + \sqrt{2}$ and $x = 1 - \sqrt{2}$

The graph on the right is not to scale.



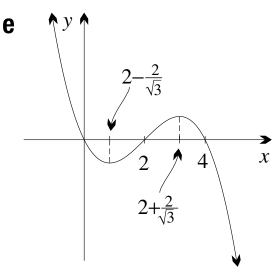
9 $y = x(x-2)(x-4)$

a $-\infty < x < \infty$

b $(0, 0)$, $(2, 0)$, $(4, 0)$

c no

e



10 a all real x

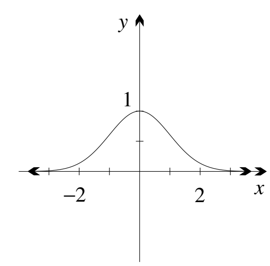
b even

c $(0, 1)$

d $y = 0$

e $(0, 1)$

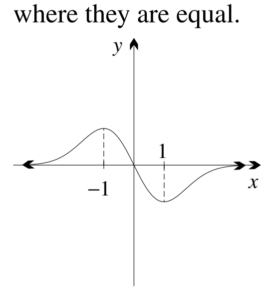
f $0 < y \leq 1$



11 b When $x = 0$, $y' = 0$.

c $e^{-\frac{1}{2}} < 2^{-\frac{1}{2}}$ so $y = 2^{-\frac{1}{2}x^2}$ is higher, except at $x = 0$ where they are equal.

12

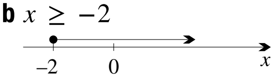


The graph shows $f(x)$ is greatest at $x = -1$ and least at $x = 1$.

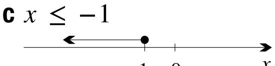
Exercise 2D

1a $x < 5$

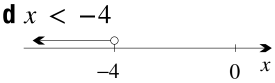
b $x \geq -2$



c $x \leq -1$



d $x < -4$



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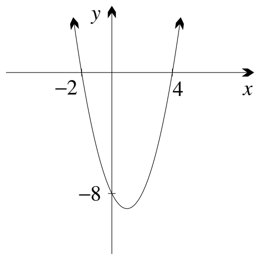
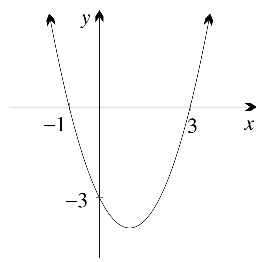
2 a $x > -3,\ (-3, \infty)$
b $x \le 10,\ (-\infty, 10]$
c $x < -2,\ (-\infty, -2)$
d $x \ge -5,\ [-5, \infty)$
e $x \ge -4,\ [-4, \infty)$
f $x < 6,\ (-\infty, 6)$

3 a $0 < x < 4$
b $x \le -1$ or $x \ge 3$
c $x \le 0$ or $x \ge 2$

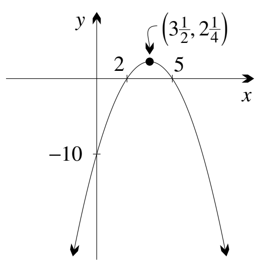
4 a $-2 < x < 4$



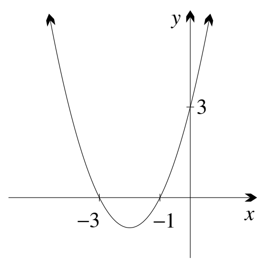
b $x < -1$ or $x > 3$



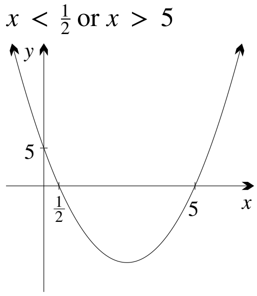
c $2 \le x \le 5$



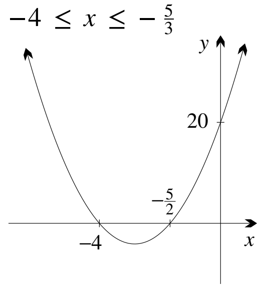
d $x \le -3$ or $x \ge -1$



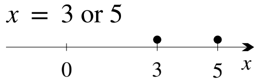
e $x < \frac{1}{2}$ or $x > 5$



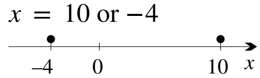
f $-4 \le x \le -\frac{5}{3}$



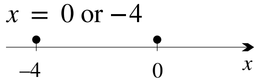
5 a $x = 3$ or 5



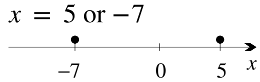
b $x = 10$ or -4



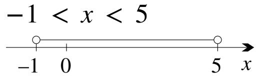
c $x = 0$ or -4



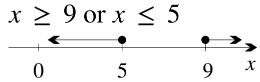
d $x = 5$ or -7



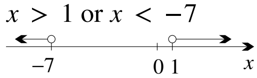
e $-1 < x < 5$



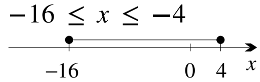
f $x \ge 9$ or $x \le 5$



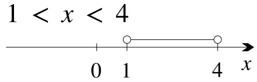
g $x > 1$ or $x < -7$



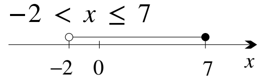
h $-16 \le x \le -4$



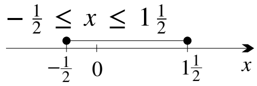
6 a $1 < x < 4$



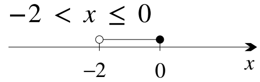
b $-2 < x \le 7$



c $-\frac{1}{2} \le x \le 1\frac{1}{2}$



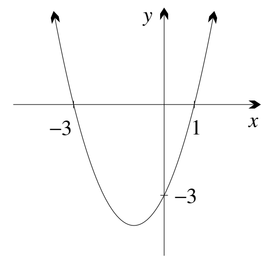
d $-2 < x \le 0$



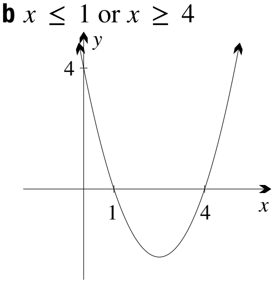
7 a $-4 < x < 2,\ (-4, 2)$
b $-1 \le x \le 2,\ [-1, 2]$
c $\frac{1}{3} < x \le 4,\ \left(-\frac{1}{3}, 4\right]$
d $-6 \le x < 15,\ [-6, 15)$

8 a $x > -10$
b $x \le 4$
c $x \ge -1$
d $x < -4\frac{2}{3}$

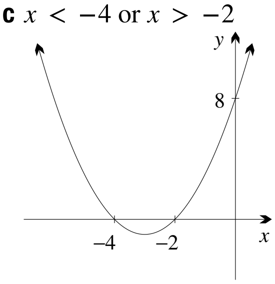
9 a $-3 < x < 1$



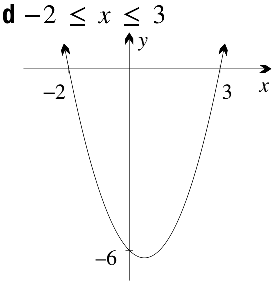
b $x \le 1$ or $x \ge 4$



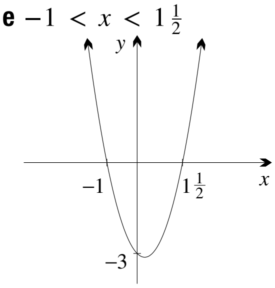
c $x < -4$ or $x > -2$



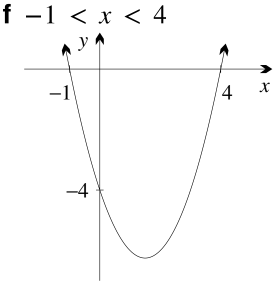
d $-2 \le x \le 3$



e $-1 < x < 1\frac{1}{2}$



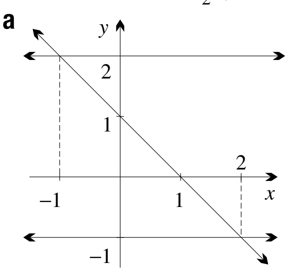
f $-1 < x < 4$



10 a $-1 \le x \le 1$
b $x < 0$ or $x > 3$
c $x \le -12$ or $x \ge 12$
d $x < 0$ or $x > 0$ (or simply $x \ne 0$)
e $x = 3$
f $1 \le x \le 3$

11 $5x - 4 < 7 - \frac{1}{2}x$, with solution $x < 2$

12 a



b $-1 \le x < 2$. The solution of the inequation is where the diagonal line lies between the horizontal lines.

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13 a $x = 5$ or -5 **b** $x = 6$ or -5
c $x = 2$ or $-\frac{8}{7}$ **d** $x = 2$ or $-3\frac{1}{3}$
e $\frac{1}{3} \le x \le 3$ **f** $x > 2$ or $x < \frac{1}{3}$
g $-2 < x < 1$ **h** $x \ge \frac{2}{5}$ or $x \le -2$

14 a $x = 5$ or -5 **b** $x = 8$ or -4
c $x = 2$ or $-3\frac{1}{3}$ **d** $x = \frac{7}{5}$ or $-\frac{11}{5}$
e $-1 < x < 5$ **f** $\frac{1}{3} \le x \le 3$
g $x \ge \frac{2}{5}$ or $x \le -2$ **h** $x > 2$ or $x < \frac{1}{3}$

15 a i $x = 4$ **ii** $2x - 3$ **iii** 5
b i $x = -3$ **ii** 4 **iii** $-2x - 2$
c i $x = -2$ **ii** $x + 9$ **iii** $-3x + 1$
d i $x = 1$ **ii** $4x - 4$ **iii** $-2x + 2$

16 a i $3x - 1 = 0, x = \frac{1}{3}$
ii $-x - 1 = 0, x = -1$
b i $4x - 6 = 4, x = 2\frac{1}{2}$
ii $6 - 2x = 4, x = 1$
c i $\frac{1}{2}x + 1 = 3, x = 4$
ii $-\frac{3}{2}x - 1 = 3, x = -\frac{8}{3}$
d i $3x - 2 = x + 6, x = 4$
ii $-3x + 2 = x + 6, x = -1$

17 a $x \le -2$ or $x \ge 2$ **b** $x < -2$ or $x > 2$

18 a $-2 \le x \le 2$ **b** $-2 < x < 2$
c $x \le -2$ or $x \ge 2$ **d** $x < -2$ or $x > 2$

Exercise 2E

1 a 1 **b** 2 **c** 3 **d** 2 **e** 2 **f** 3

2 a $x = \frac{1}{2}$ **b** $x = -\frac{3\pi}{4}$ or $\frac{\pi}{4}$
c $x \doteqdot -2.1$ or 0.3 or 1.9 **d** $x = 1$ or $x \doteqdot 3.5$
e $x = 1$ or $x \doteqdot -1.9$
f $x = 0$ or $x \doteqdot -1.9$ or 1.9

3 a i $x > 1$ **ii** $x < 1$
b i $x < -3$ or $x > 2$ **ii** $-3 < x < 2$

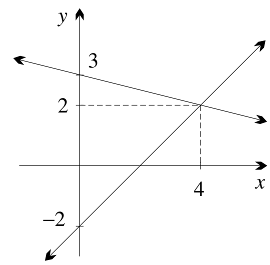
4 a i $x = 0$ or 3 **ii** $0 < x < 3$
iii $x < 0$ or $x > 3$
b i $x = -2$ or 1 **ii** $x < -2$ or $x > 1$
iii $-2 < x < 1$

5 a $x \le -3$ **b** $0 \le x \le 2$ **c** $x = 1$

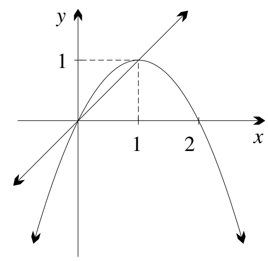
6 a $x < -2$ or $x > 1$ **b** $0 \le x \le 1$
c $-1 < x < 0$ or $x > 1$

7 a $\sqrt{2} \doteqdot 1.4, \sqrt{3} \doteqdot 1.7$
b $x = -1$ or $x = 2$
c $x < -1$ or $x > 2$
d $x = -2$ or $x = 1, -2 \le x \le 1$
e $x \doteqdot 1.62$ or $x \doteqdot -0.62$
f i Draw $y = -x$; $x = 0$ or $x = -1$.
ii Draw $y = x + \frac{1}{2}$; $x \doteqdot 1.37$ or $x \doteqdot -0.37$.
iii Draw $y = \frac{1}{2}x + \frac{1}{2}$; $x = 1$ or $x = -\frac{1}{2}$.

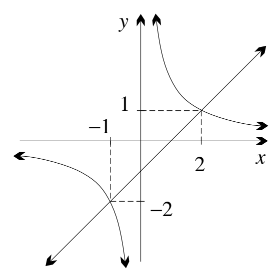
8 a $(4, 2)$, $x - 2 = 3 - \frac{1}{4}x$



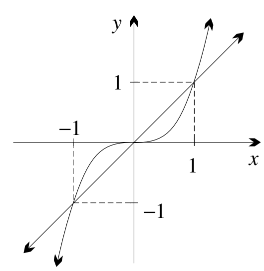
b $(0, 0)$ and $(1, 1)$, $x = 2x - x^2$



c $(-1, -2)$ and $(2, 1)$, $\frac{2}{x} = x - 1$



d $(-1, -1)$, $(0, 0)$ and $(1, 1)$, $x^3 = x$



9 a $x \ge 4$
b $0 < x < 1$
c $x < -1$ or $0 < x < 2$
d $-1 < x < 0$ or $x > 1$

10 a Divide by e^x to get $e^x = e^{1-x}$
b Multiply by $\cos x$ to get $\sin x = \cos x$
c Subtract 1 then divide by x to get $x^2 - 4 = -\frac{1}{x}$

11 a The table below traps the solution between $x = -1.690$ and $x = -1.6905$, so it is $x = -1.690$, correct to three decimal places.

x	-2	-1.7	-1.6	-1.68
2^x	0.25	0.3078	0.3299	0.3121
$x + 2$	0	0.3	0.4	0.32

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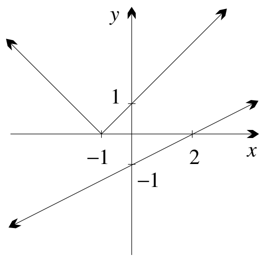
x	-1.69	-1.691	-1.6905
2^x	0.3099	0.3097	0.3098
$x + 2$	0.31	0.309	0.3095

b Part **c**: $x \doteqdot -2.115$. Part **e**: $x \doteqdot -1.872$.

12 a

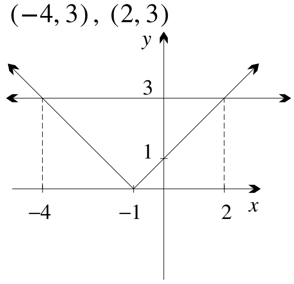


b The graph of $y = |x + 1|$ is always above the graph of $y = \frac{1}{2}x - 1$.

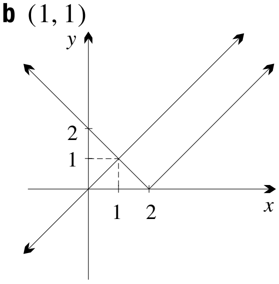
13 a The curve is always above the line.

b The two lines are parallel and thus the first is always below the second.

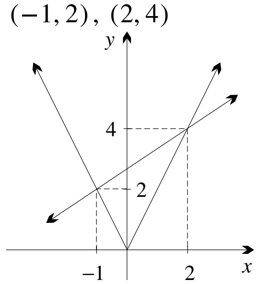
14 a $(-4, 3)$, $(2, 3)$



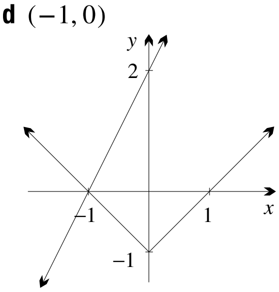
b $(1, 1)$



c $(-1, 2)$, $(2, 4)$



d $(-1, 0)$



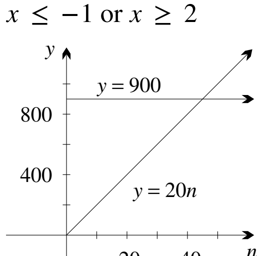
15 a $-4 \le x \le 2$

b $x < 1$

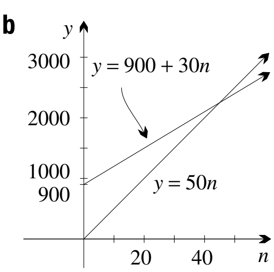
c $x \le -1$ or $x \ge 2$

d $x < -1$

16 a

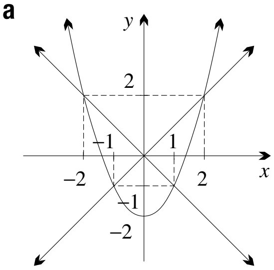


b



In both cases the break-even point is $n = 45$. Total sales are \$2250 at that point.

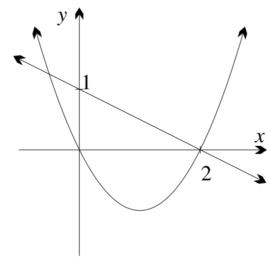
17 a



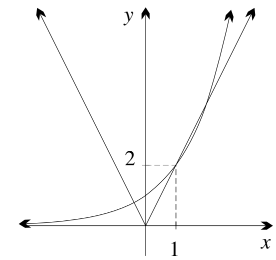
b $x = 2$ or -2

c $x < -2$ or $x > 2$

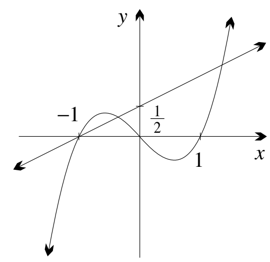
18 a 2 solutions



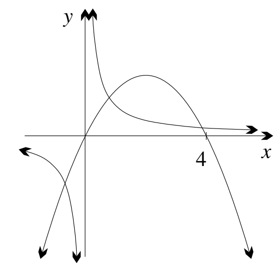
b 3 solutions



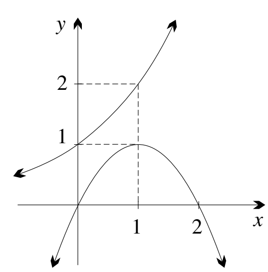
c 3 solutions



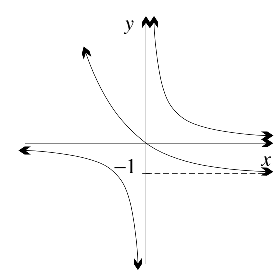
d 3 solutions



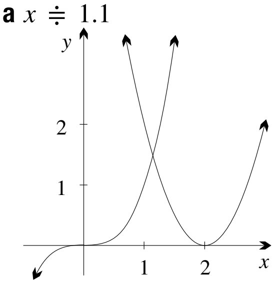
e no solutions



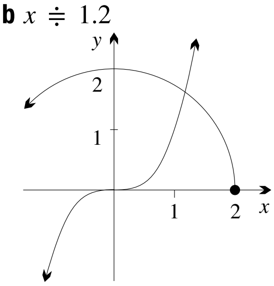
f no solutions



19 a $x \doteqdot 1.1$



b $x \doteqdot 1.2$



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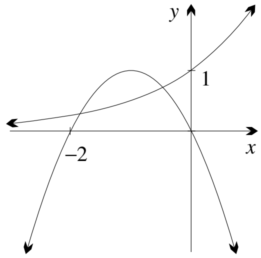
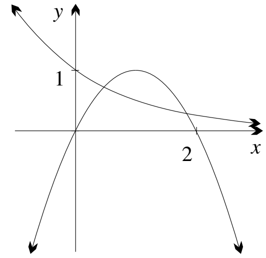
c $x \doteqdot -0.5$ or $x \doteqdot -1.9$

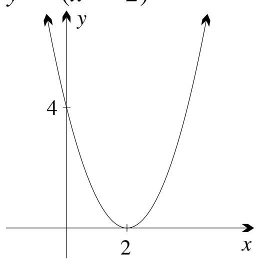


d $x \doteqdot 0.5$ or $x \doteqdot 1.9$.

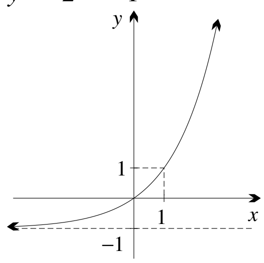


Exercise 2F

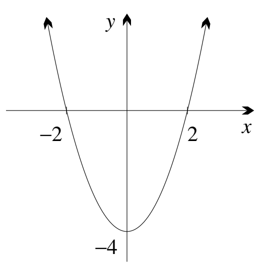
1a $y = (x - 2)^2$



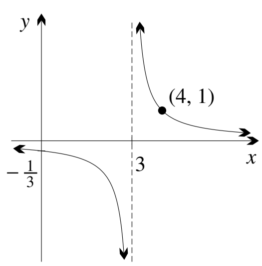
b $y = 2^x - 1$



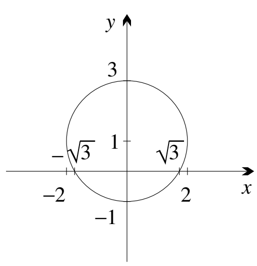
c $y = x^2 - 4$



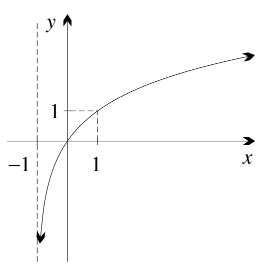
d $y = \frac{1}{x - 3}$



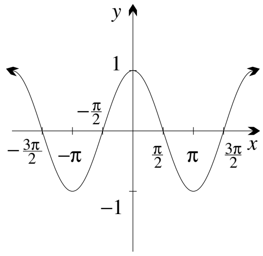
e $x^2 + (y - 1)^2 = 4$



f $y = \log_2 (x + 1)$

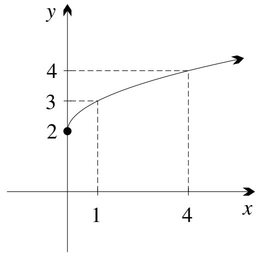


g $y = \sin\left(x + \frac{\pi}{2}\right)$

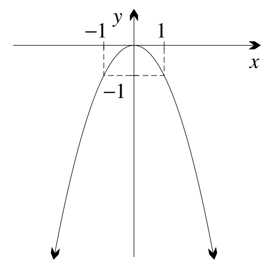


This is also $y = \cos x$.

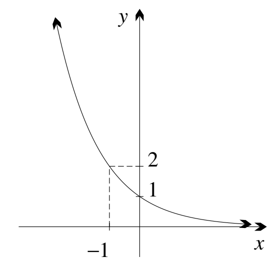
h $y = \sqrt{x} + 2$



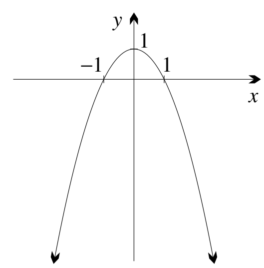
2 a $y = -x^2$



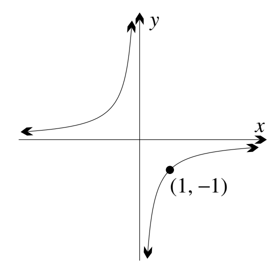
b $y = 2^{-x}$



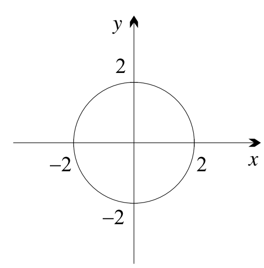
c $y = 1 - x^2$



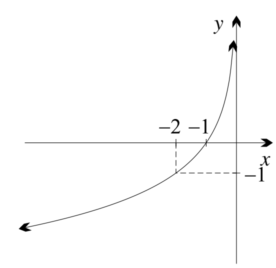
d $y = -\frac{1}{x}$



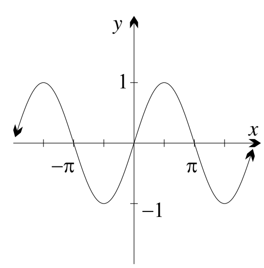
e $x^2 + y^2 = 4$



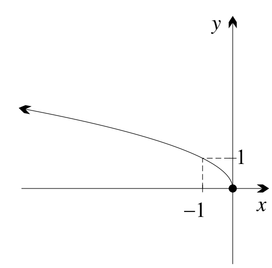
f $y = -\log_2 (-x)$



g $y = \sin x$



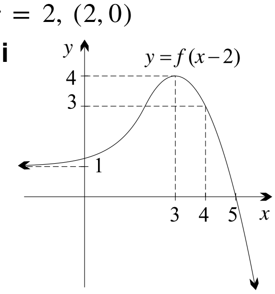
h $y = \sqrt{-x}$



3 In part **e** the circle is symmetric in the y-axis.
In part **g** $y = \sin x$ is an odd function, and so is unchanged by a rotation of 180°.

4 a $r = 2,\ (-1, 0)$ **b** $r = 1,\ (1, 2)$
c $r = 2,\ (2, 0)$ **d** $r = 5,\ (0, 3)$

5 a i



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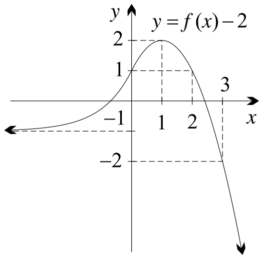
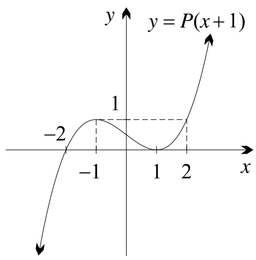
ii



b i



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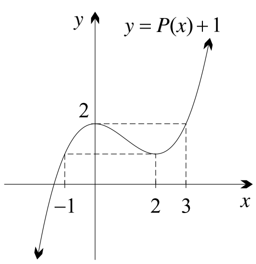
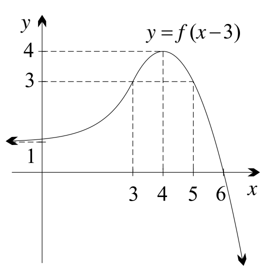


6 a $y = (x + 1)^2 + 2$ **b** $y = \frac{1}{x - 2} + 3$

c $y = \cos\left(x - \frac{\pi}{3}\right) - 2$ **d** $y = e^{x+2} - 1$

7 a i



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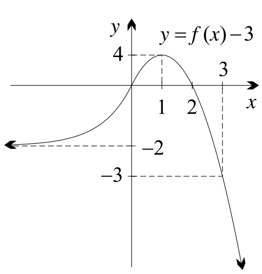
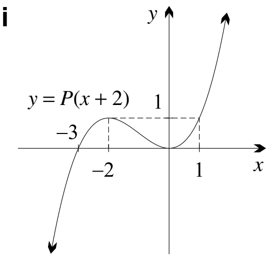


b i



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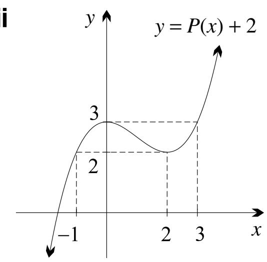
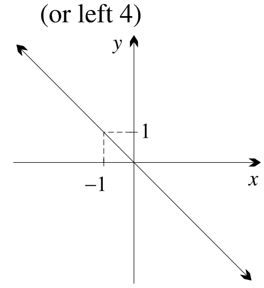


8 a From $y = -x$:

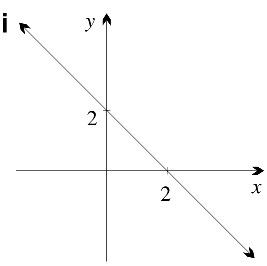
i shift up 2 (or right 2)

ii shift down 2 (or left 2)

iii reflect in x-axis (or y-axis) and shift up 4 (or left 4)



i



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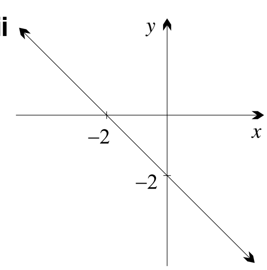
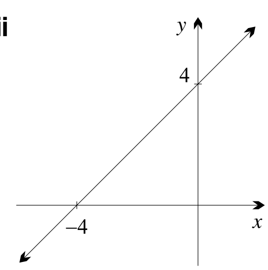


iii

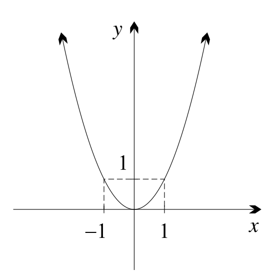


b From $y = x^2$:

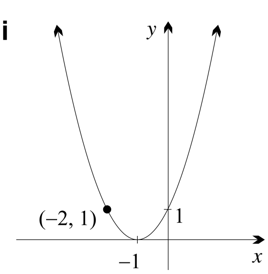
i shift 1 left

ii shift 1 left and reflect in x-axis

iii shift 1 left and shift down 1



i



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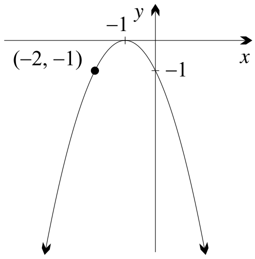
ii



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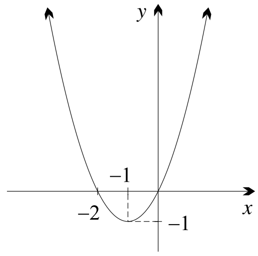
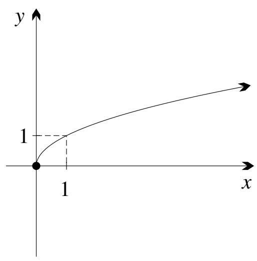


c From $y = \sqrt{x}$:

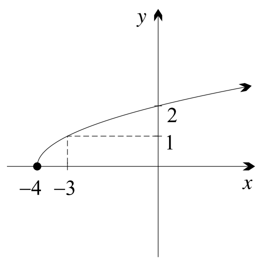
i shift 4 left

ii reflect in x-axis

iii shift 4 left and reflect in x-axis



i



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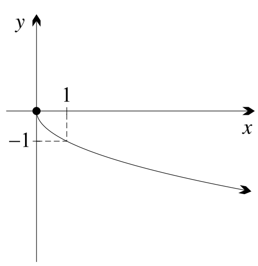

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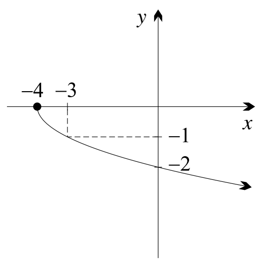
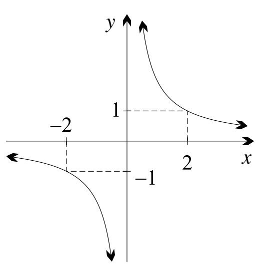


d From $y = \dfrac{2}{x}$:

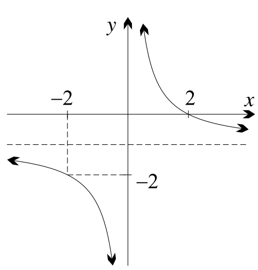
i shift down 1

ii shift down 1, left 2

iii reflect in the x-axis or in the y-axis



i



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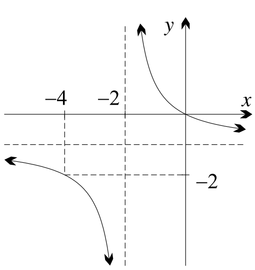
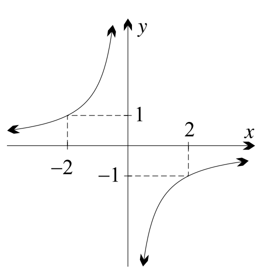


iii

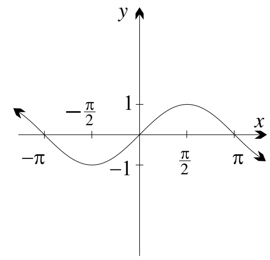


e From $y = \sin x$:

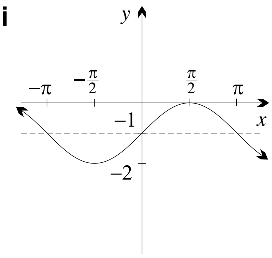
i shift down 1

ii shift down 1, right $\frac{\pi}{4}$

iii reflect in the x-axis or in the y-axis



i



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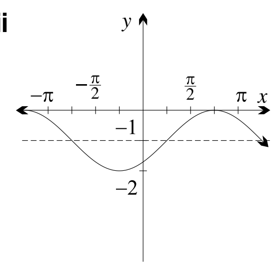

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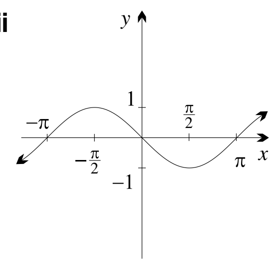
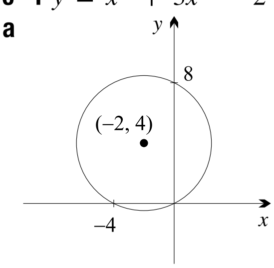


9 a $(1, -2)$ and $(-1, 2)$

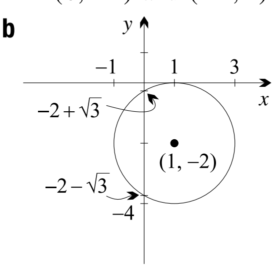
b **i** $y = x^3 - 3x + 1$ **ii** $(1, -1)$ and $(-1, 3)$

c **i** $y = x^3 + 3x^2 - 2$ **ii** $(0, -2)$ and $(-2, 2)$

10 a



b



11 a The parabola $y = x^2$ shifted left 2, down 1.
$y + 1 = (x + 2)^2$

b The hyperbola $xy = 1$ shifted right 2, down 1.
$y + 1 = \frac{1}{x - 2}$

c The exponential $y = 2^x$ reflected in the x-axis, shifted 1 up. $y = 1 - 2^x$

d The curve $y = \cos x$ reflected in the x-axis and shifted 1 up. $y = 1 - \cos x$

12 a The parabola $y = x^2$ reflected in the x-axis, then shifted 3 right and 1 up. $y - 1 = -(x - 3)^2$

b The curve $y = \log_2 x$ reflected in the y-axis, then shifted right 2, down 1. $y + 1 = -\log_2 (x - 2)$

c The half parabola $y = \sqrt{x}$ reflected in the x-axis, then shifted left 4 and 2 up.
$y - 2 = -\sqrt{x + 4}$

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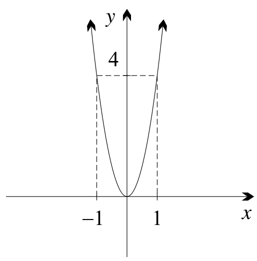
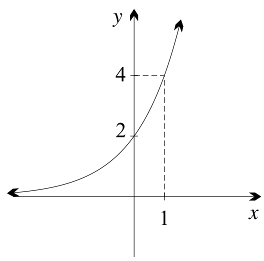
Exercise 2G

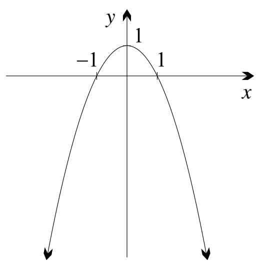
1a $y = 4x^2$



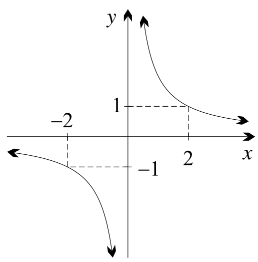
b $y = 2 \times 2^x = 2^{x+1}$



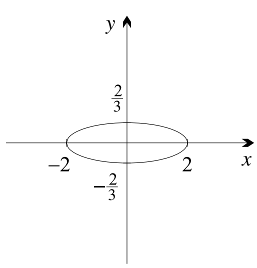
c $y = 1 - x^2$



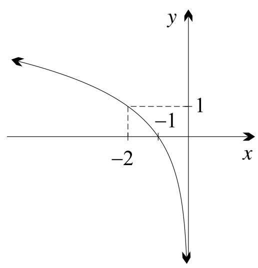
d $y = \frac{2}{x}$



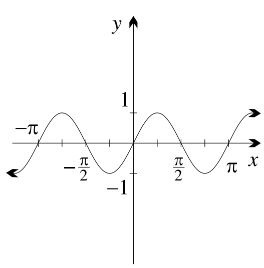
e $x^2 + 9y^2 = 4$



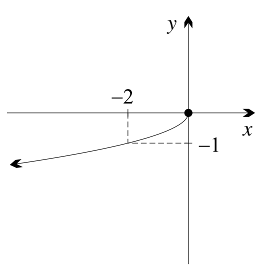
f $y = \log_2(-x)$



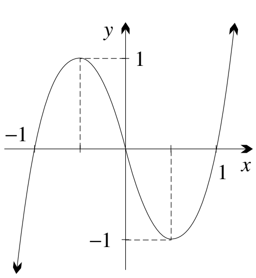
g $y = \sin 2x$



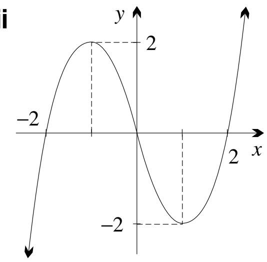
h $y = -2\sqrt{x}$



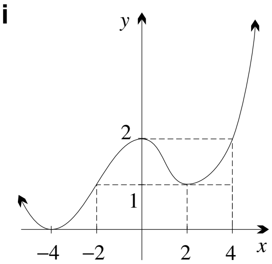
2 a i



ii



b i



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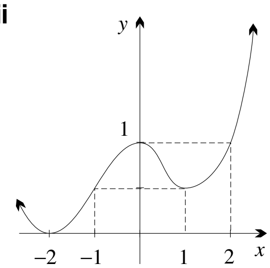
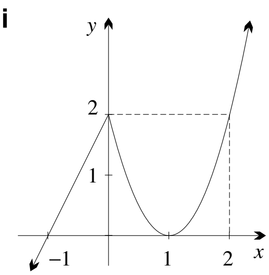


c i



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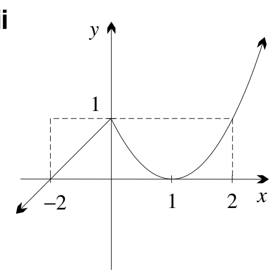
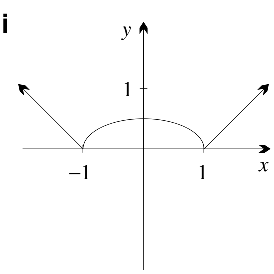


d i



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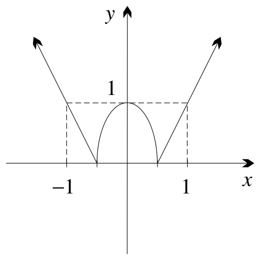
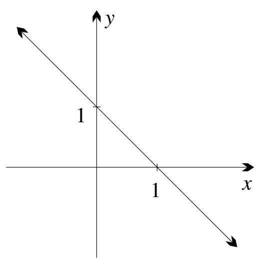
ii

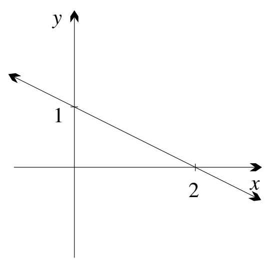


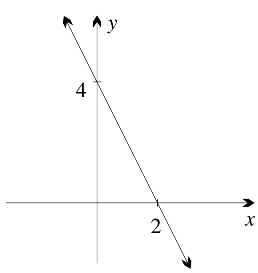
3 a stretch horizontally by factor 2

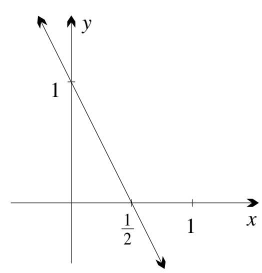
b stretch horizontally by factor 2, vertically by factor 4

c stretch horizontally by factor $\frac{1}{2}$

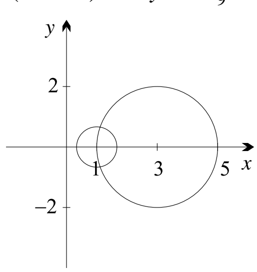


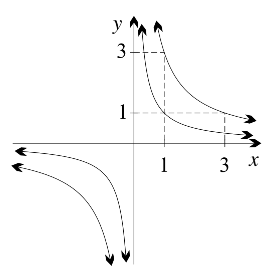




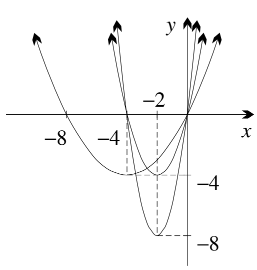


4 a $(x - 1)^2 + y^2 = \frac{4}{9}$ **b** $y = \frac{3}{x}$

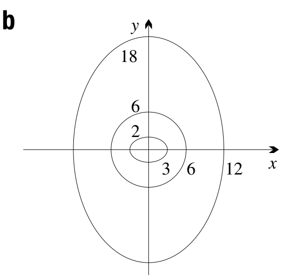




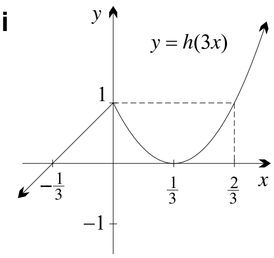
5 a



b



6 a i



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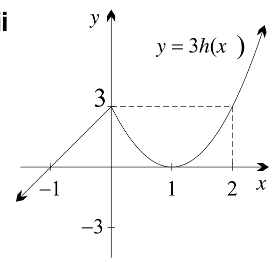
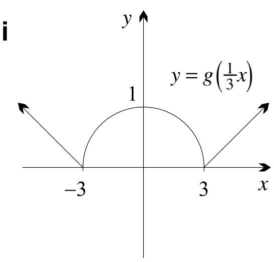
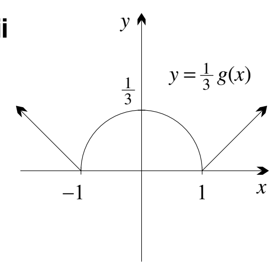


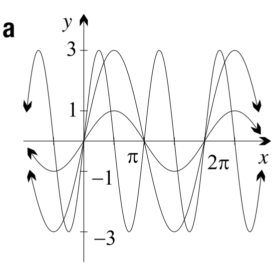
b i



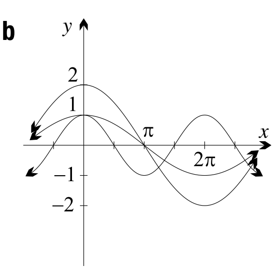
ii



7 a



b



8 a $(1, -2)$ and $(-1, 2)$

b i $y = 2x^3 - 6x$ **ii** $(1, -4)$ and $(-1, 4)$

c i $y = \frac{1}{27}x^3 - x$ **ii** $(3, -2)$ and $(-3, 2)$

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9 a vertical factor 3 **b** horizontal factor $\frac{1}{2}$
c horizontal factor 4 **d** vertical factor 2

10 a $y = \frac{2}{x}$ **b** $y = \frac{2}{x}$
c Both dilations give the same graph.
d yes: by factor $\sqrt{2}$

11 a $y = 4x^2$ **b** $y = 4x^2$
c Both dilations give the same graph.
d no

12 a $M(0) = 3$ **b** 53 years
c i The mass has been dilated by factor 2, so $M = 6 \times 2^{-\frac{1}{53}t}$

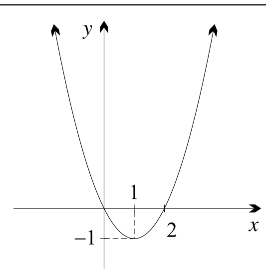
14 a The unit circle $x^2 + y^2 = 1$, horizontally by 3, vertically by 2. $\frac{x^2}{9} + \frac{y^2}{4} = 1$
b The exponential $y = 3^x$, vertically by -2. $y = -2 \times 3^x$
c The curve $y = \tan x$, horizontally by 3, vertically by 2. $y = 2\tan\frac{x}{3}$

15 a i stretch vertically by factor 2, $\frac{y}{2} = 2^x$, or translate left by 1, $y = 2^{(x+1)}$
ii stretch along both axes by k, $\frac{y}{k} = \frac{1}{\frac{x}{k}}$, or stretch horizontally by k^2, $y = \frac{1}{\frac{x}{k^2}}$
iii reciprocal, $y = \frac{1}{3^x}$, or reflect in the y-axis, $y = 3^{-x}$

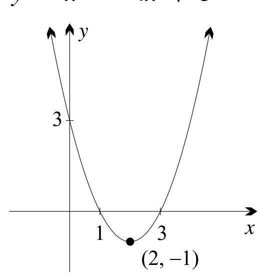
16 vertically by factor a^2

Exercise 2H

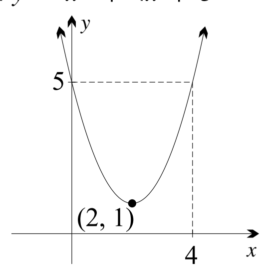
1



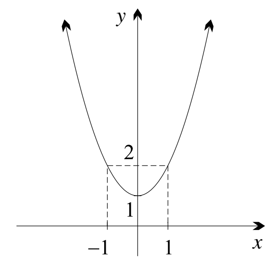
a i $y = x^2 - 4x + 3$



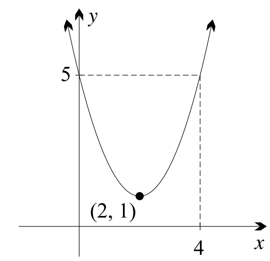
ii $y = x^2 + 4x + 5$



b i $y = x^2 - 2x + 2$

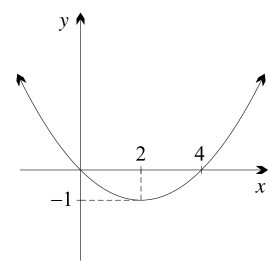


ii $y = x^2 + 4x + 5$

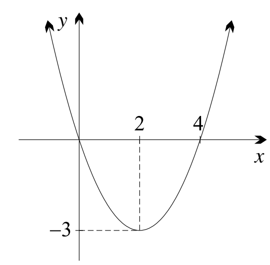


c yes

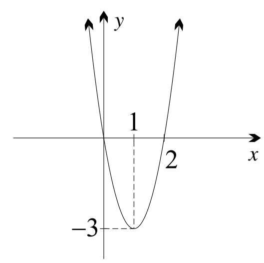
2 a i $y = \frac{1}{4}x^2 - x$



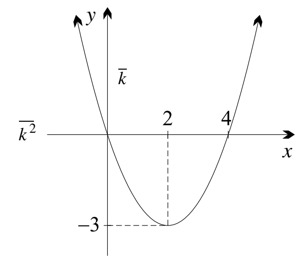
ii $y = \frac{3}{4}x^2 - 3x$



b i $y = 3x^2 - 6x$

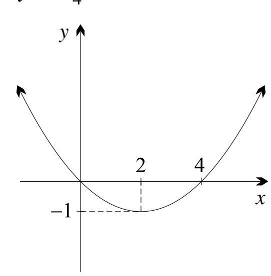


ii $y = \frac{3}{4}x^2 - 3x$

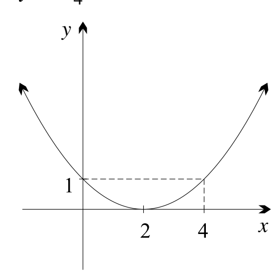


c yes

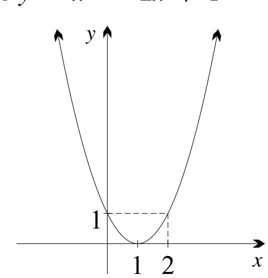
3 a i $y = \frac{1}{4}x^2 - x$



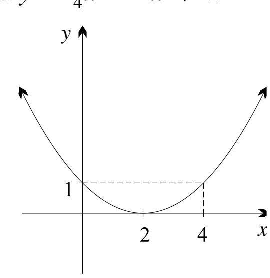
ii $y = \frac{1}{4}x^2 - x + 1$



b i $y = x^2 - 2x + 1$



ii $y = \frac{1}{4}x^2 - x + 1$



c yes

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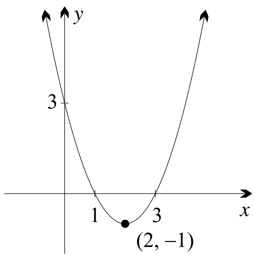
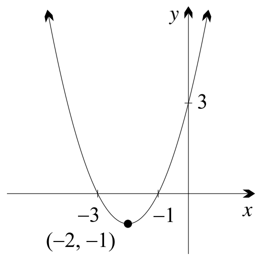
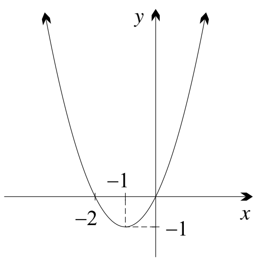
4 a **i** $y = x^2 - 4x + 3$



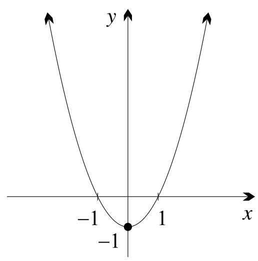
ii $y = x^2 + 4x + 3$



b **i** $y = x^2 + 2x$



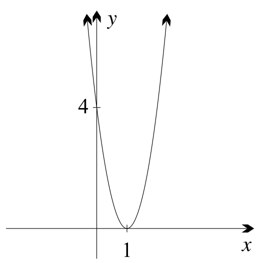
ii $y = x^2 - 1$



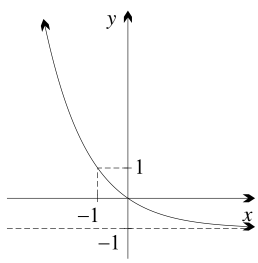
c no: order matters

5 a no **b** no **c** yes **d** yes **e** no **f** yes

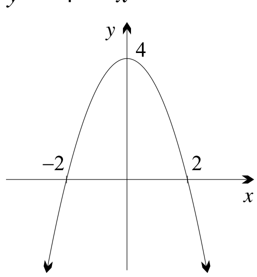
6 a $y = 4(x - 1)^2$



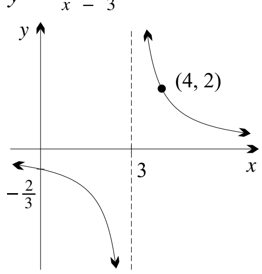
b $y = 2^{-x} - 1$



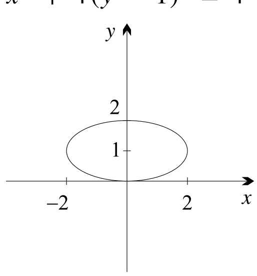
c $y = 4 - x^2$



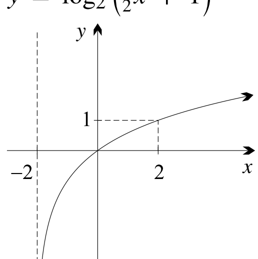
d $y = \frac{2}{x - 3}$



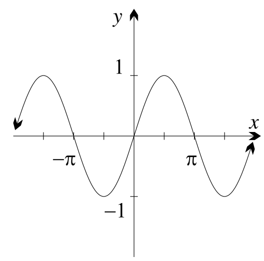
e $x^2 + 4(y - 1)^2 = 4$



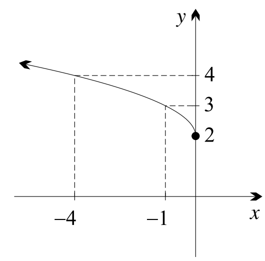
f $y = \log_2\left(\frac{1}{2}x + 1\right)$



g $y = -\sin(x + \pi)$

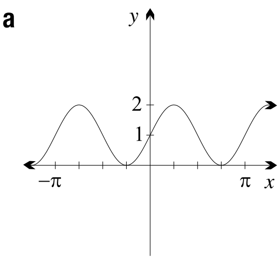


h $y = -\sqrt{x} + 2$

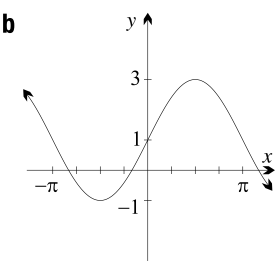


This is also $y = \sin x$.

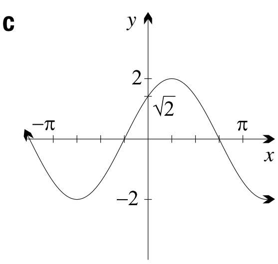
7 a



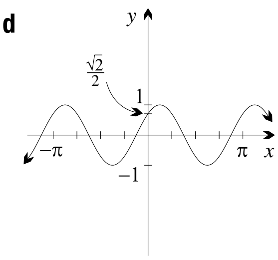
b



c



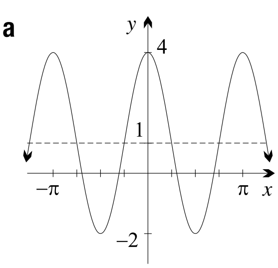
d



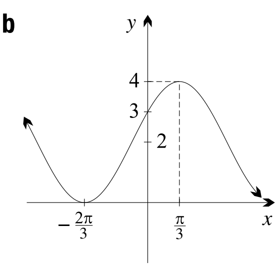
8 a $y = \frac{1}{4}(x + 2)^2 - 4$ **b** $y = \frac{1}{4}(x + 1)^2 - 4$

c $y = 2 - 2^x$ **d** $y = \frac{2}{x - 2} + 1$

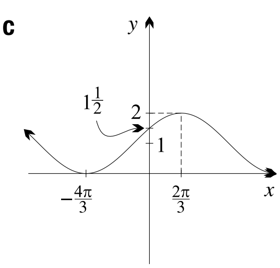
9 a



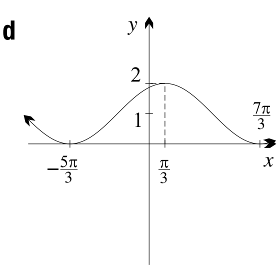
b



c



d



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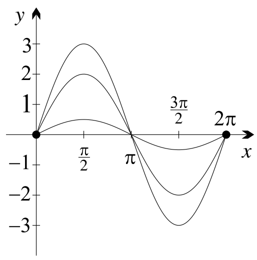
Exercise 2I

1a

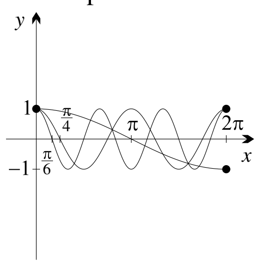


i $\frac{1}{2}$ **ii** 2 **iii** 3

b The graph $y = \sin x$ is stretched vertically by a factor of k.

c The amplitude increases. The bigger the amplitude, the steeper the wave.

2 a

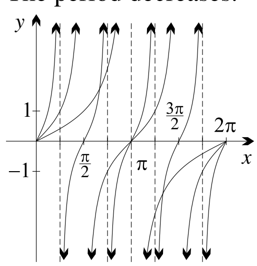


i 4π **ii** π **iii** $\frac{2\pi}{3}$

b The graph $y = \cos x$ is stretched horizontally by a factor of $\frac{1}{n}$.

c The period decreases.

3 a

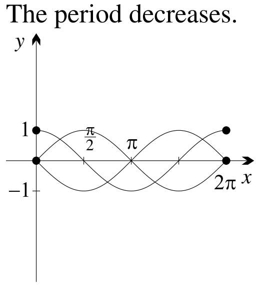


i π **ii** 2π **iii** $\frac{\pi}{2}$

b The graph $y = \tan x$ is stretched horizontally by a factor of $\frac{1}{a}$.

c The period decreases.

4 a

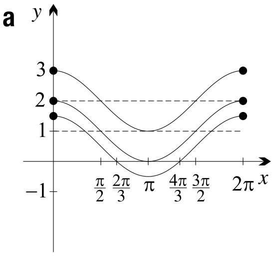


i $\frac{\pi}{2}$ **ii** π **iii** 2π or 0

b The graph $y = \sin x$ is shifted α units to the left.

c The graph stays the same, because $y = \sin x$ has period 2π.

5 a



i Range: $0 \leq y \leq 2$ or $[0, 2]$, mean value: 1

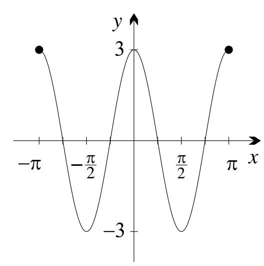
ii Range: $[1, 3]$, mean value: 2

iii Range: $\left[-\frac{1}{2}, \frac{3}{2}\right]$, mean value: $\frac{1}{2}$

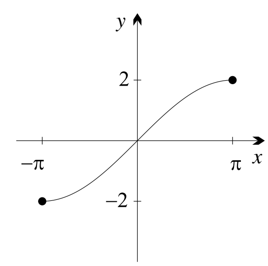
b The graph $y = \cos x$ is shifted c units up, and the mean value is c.

c It moves up.

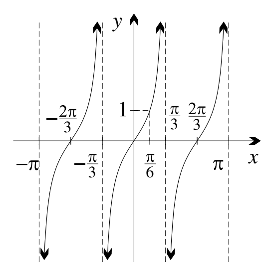
6 a period $= \pi$, amplitude $= 3$



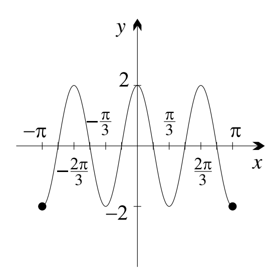
b period $= 4\pi$, amplitude $= 2$



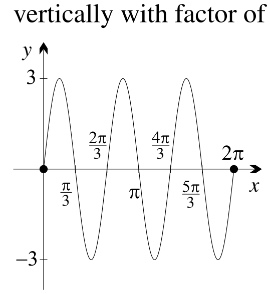
c period $= \frac{2\pi}{3}$, no amplitude



d period $= \frac{2\pi}{3}$, amplitude $= 2$



7 a Stretch horizontally by a factor of $\frac{1}{3}$, then stretch vertically with factor of 3.



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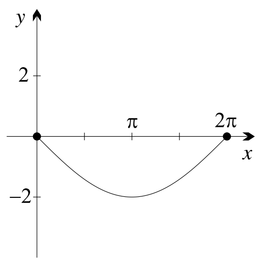
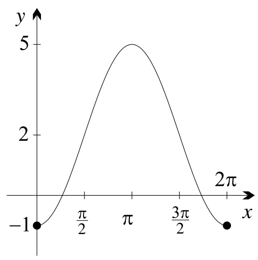
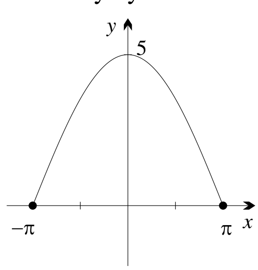
b Stretch horizontally with factor 2, then stretch vertically with factor 2, then reflect in the x-axis.



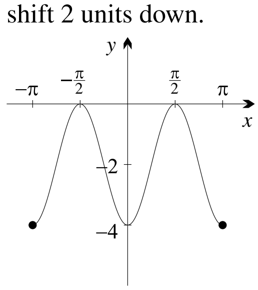
c Shift $\frac{\pi}{2}$ units right, then stretch vertically by a factor of 3, then shift 2 units up.



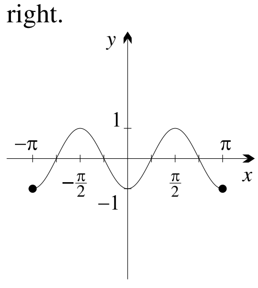
8 a Stretch horizontally by a factor 2, then stretch vertically by a factor of 5.



b Stretch horizontally by a factor of $\frac{1}{2}$, then stretch vertically by a factor 2 then reflect in the x-axis, then shift 2 units down.



c Stretch horizontally by a factor of $\frac{1}{2}$, then shift $\frac{\pi}{2}$ units right.



9 a Stretch horizontally by a factor of $\frac{1}{3}$, then shift $\frac{\pi}{6}$ units left.

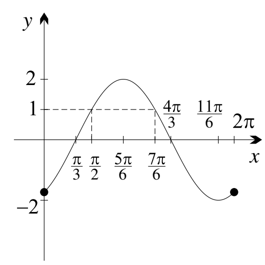
b Stretch horizontally by a factor of $\frac{1}{4}$, then shift $\frac{\pi}{4}$ units right, then stretch vertically by a factor of $\frac{1}{4}$, then shift 4 units down.

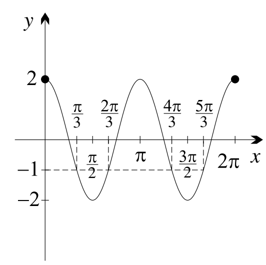
c Stretch horizontally by a factor of 2, then shift $\frac{\pi}{2}$ units left, then stretch vertically by a factor of 6, then reflect in the x-axis.

10 a Part **a**: period $= \frac{2\pi}{3}$, phase $= 0 + \frac{\pi}{2} = \frac{\pi}{2}$.
Part **b**: period $= \frac{2\pi}{4} = \frac{\pi}{2}$, phase $= -\pi$ (but this is twice the period, so we can also say that phase $= 0$).
Part **c**: period $= 4\pi$, phase $= \frac{\pi}{4}$.

b **i** period $= \pi$, phase $= 2\left(0 - \frac{\pi}{3}\right) = -\frac{2\pi}{3}$
ii period $= 6\pi$, phase $= \frac{\pi}{3}$
iii period $= \frac{\pi}{3}$, phase $= \frac{3\pi}{8}$

11 a $x = \frac{\pi}{2}$ or $\frac{7\pi}{6}$ **b** $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$

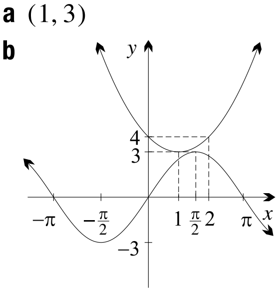




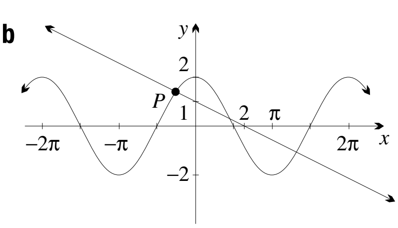
12 a $x \doteqdot 1.675$ **b** $x \doteqdot 0.232$ or 1.803

13 a $(1, 3)$

b



14 a, b



c 3
d P is in the second quadrant

15 a 4 **c** the origin **d** $m > \frac{1}{4}$ *or* $m = 0$

16 a 10 metres **b** 6 metres
c 2 pm **d** 9:20 am

17 a **ii** 1 **iii** $0 < k < 1$
b **ii** 1.3 **iii** $\angle AOB = 2\theta \doteqdot 2.6$ radians
c **ii** $\ell > 300$

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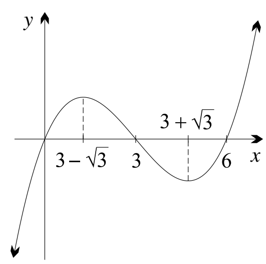
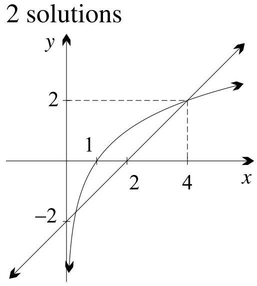
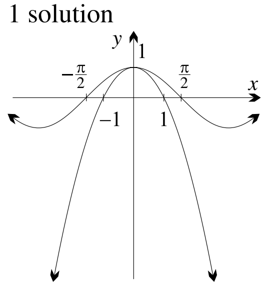
Chapter 2 review exercise

1a **i** $-1 < x < 2$ **ii** $(-1, 2)$
b **i** $-1 \le x < 2$ **ii** $[-1, 2)$
c **i** $x \le 2$ **ii** $(-\infty, 2]$
2 a **i** 0 **ii** 4 **iii** 8 **iv** 0
b **i** $x^2 + 2x$ **ii** x^2
iii $x^4 - 2x^2$ **iv** $x + 2$
3 a $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$
b $f(x) \to \frac{1}{2}$ as $x \to \infty$ and $x \to -\infty$
c $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$
4 $y = x(x - 3)(x - 6)$
a $-\infty < x < \infty$ **b** $(0, 0), (3, 0), (6, 0)$
c no asymptotes
e $y' = 3(x^2 - 6x + 6)$
so $y' = 0$ at $x = 3 - \sqrt{3}$ or $x = 3 + \sqrt{3}$

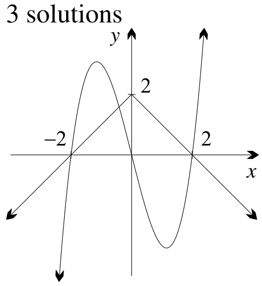


5 a $-4 \le x < 2$, $[-4, 2)$ **b** $-\frac{3}{2} < x < 0$, $\left(-\frac{3}{2}, 0\right)$
c $-3 \le x < \frac{1}{2}$, $\left[-3, \frac{1}{2}\right)$ **d** $-4 \le x \le 10$, $[-6, 10]$
6 a 2 solutions **b** 1 solution

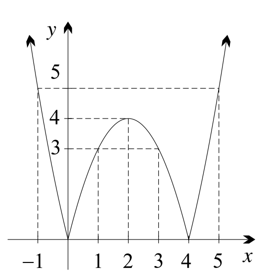




c 3 solutions



7 a **i**



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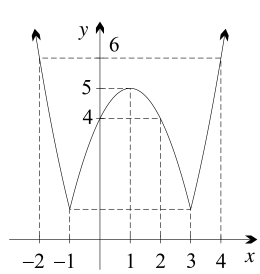
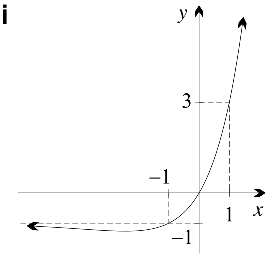


b **i**



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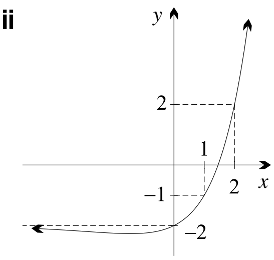
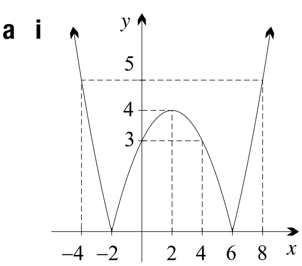


8 a **i**



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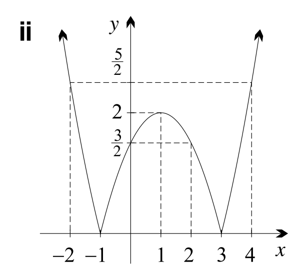
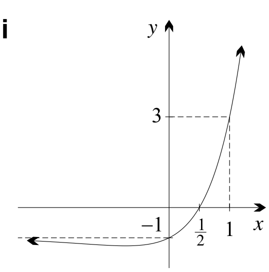


b **i**



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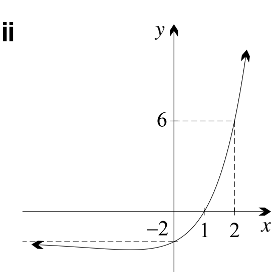

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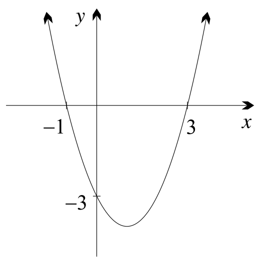
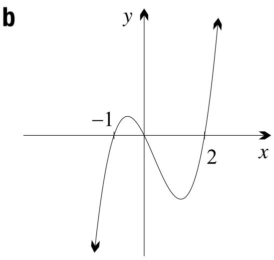
9 a



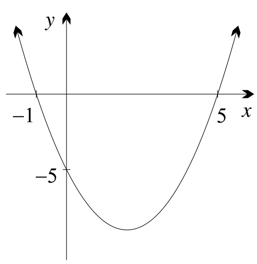
$-1 \le x \le 3$

b



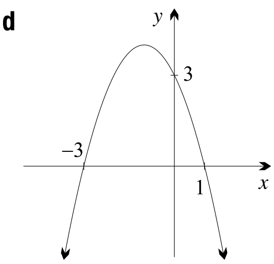
$x \le -1$ or $0 \le x \le 2$

c



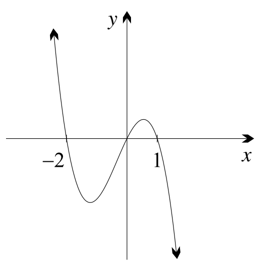
$-1 \le x \le 5$

d



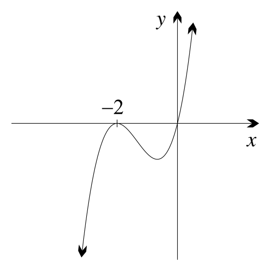
$x \le -3$ or $x \ge 1$

e



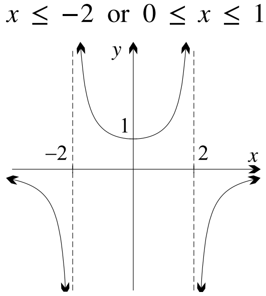
$x \le -2$ or $0 \le x \le 1$

f



$x \le 0$

10



a $x \neq -2, 2$

b $(0, 1)$

c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$

e As $x \to 2^+$, $y < 0$ so $y \to -\infty$, as $x \to 2^-$, $y > 0$ so $y \to \infty$, as $x \to -2^+$, $y > 0$ so $y \to \infty$, and as $x \to -2^-$, $y < 0$ so $y \to -\infty$

f $(-\infty, 0) \cup [1, \infty)$

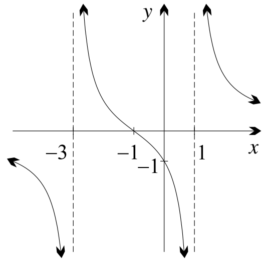
11 a $y = \frac{3(x+1)}{(x+3)(x-1)}$

b domain: $x \neq 1$ and $x \neq -3$
intercepts: $(-1, 0)$ and $(0, -1)$

c The domain is not symmetric about $x = 0$.

d $x = -3$, $x = 1$, and $y = 0$

e



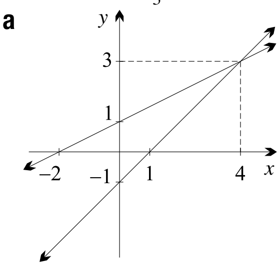
12 a $x = -3\frac{1}{2}$ or $3\frac{1}{2}$

b $x = \frac{1}{3}$ or 1

c $-3 \le x \le \frac{1}{3}$

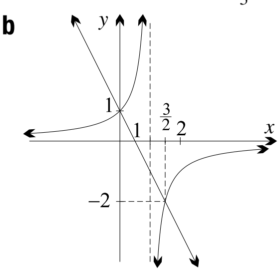
d $x < -2$ or $x > -\frac{1}{3}$

13 a



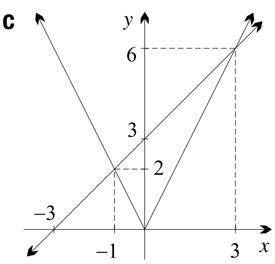
$x \ge 4$

b



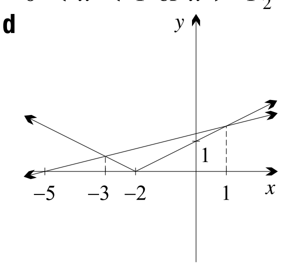
$0 < x < 1$ or $x > 1\frac{1}{2}$

c



$-1 \le x \le 3$

d



$x < -3$ or $x > 1$

14 a $y = (x-2)^2 + 1$

b $y = \frac{1}{x+2} - 3$

c $y = \sin\left(x + \frac{\pi}{6}\right) - 1$

d $y = e^{x-2} + 1$

15 a horizontal factor 2

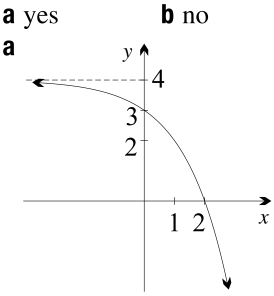
b vertical factor $\frac{1}{2}$

c vertical factor $\frac{1}{3}$

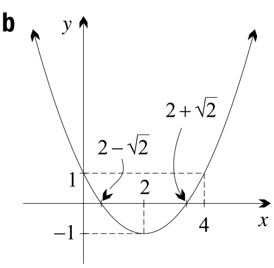
d horizontal factor 2

16 a yes **b** no **c** no **d** yes

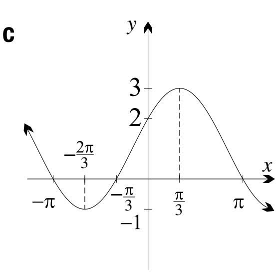
17 a



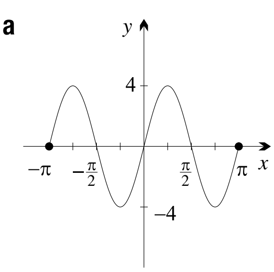
b



c

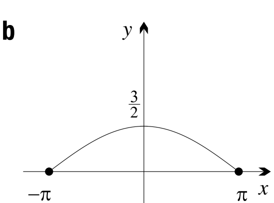


18 a



amplitude is 4, period is π

b



amplitude is $\frac{3}{2}$, period is 4π

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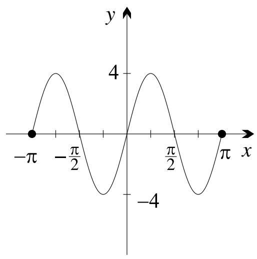
19 a Reflect in the x-axis, then shift up 1 unit.

b



20 a Reflect in the y-axis, then stretch vertically with factor 3, then shift down 2 units. Actually, the first transformation, reflect in the y-axis, is unnecessary because $y = \cos x$ is even.

b Stretch horizontally with factor $\frac{1}{4}$, and vertically with factor 4. There is no need to shift left $\frac{\pi}{2}$ units because the period is $\frac{\pi}{2}$.

c Stretch horizontally with factor $\frac{1}{2}$, then shift right $\frac{\pi}{6}$ units.

21 a 0

b $4\left(0 + \frac{\pi}{2}\right) = 2\pi$, or more simply 0

c $0 - \frac{\pi}{3} = -\frac{\pi}{3}$

Chapter 3

Exercise 3A

1 a A, G and I **b** C and E **c** B, D, F and H

2 a increasing **b** stationary **c** decreasing **d** decreasing **e** increasing **f** stationary

3 a $2x - 6$

b i decreasing **ii** decreasing **iii** stationary **iv** increasing **v** decreasing

4 a $3x^2 - 12x + 9$

b i increasing **ii** stationary **iii** decreasing **iv** stationary **v** increasing

5 a $x = 1$ **b** $x = 2$ **c** $x = -3$ **d** $x = 4$ **e** $x = 0$ or $x = 2$ **f** $x = 2$ or $x = -2$

6 a The derivative is always negative.

b The derivative is always positive.

c $f'(x) = 3x^2$, which is positive except at $x = 0$.

d $f'(x) = 2x$, which is positive if $x > 0$ and negative if $x < 0$. At $x = 0$ the function is stationary.

7 a increasing **b** decreasing **c** stationary

8 a stationary **b** increasing **c** decreasing

9 a increasing **b** decreasing **c** increasing

10 a increasing **b** decreasing **c** increasing

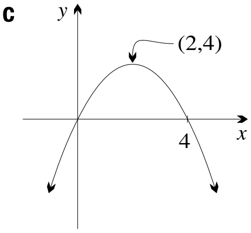
11 a $4 - 2x$

b i $x < 2$ **ii** $x > 2$ **iii** $x = 2$

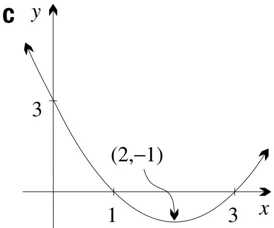
12 a $2x - 4$

b i $x > 2$ **ii** $x < 2$ **iii** $x = 2$

11 c



12 c



14 a $3x^2 + 4x + 1$ **b** $-\frac{1}{3}$ and -1 **d** $-1 < x < -\frac{1}{3}$

15 a $x > 2$ **b** $x < -3$ **c** $x > 1$ or $x < -1$ **d** $x < 0$ or $x > 2$

16 a III **b** I **c** IV **d** II

17 a $\dfrac{1}{x^2}$

b The function is not continuous at $x = 0$.

18 a $-\dfrac{6}{(x - 3)^2}$

b $f'(x)$ is negative for $x \neq 3$.

19 a $\dfrac{x^2(x^2 + 3)}{(x^2 + 1)^2}$

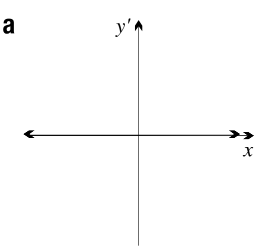
b $f'(x)$ is positive for $x \neq 0$.

20 a $x^2 + 2x + 5$

b $f'(x) > 0$ for all values of x.

c $f(-3) = -8$, $f(0) = 7$, $f(x)$ is increasing for all x. Hence the curve crosses the x-axis exactly once between $x = -3$ and $x = 0$ and nowhere else.

21 a



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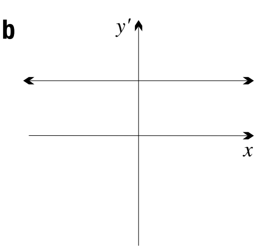

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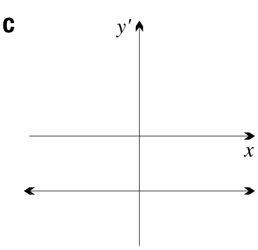
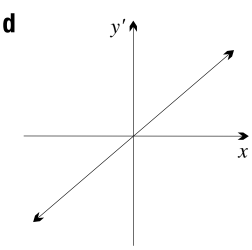


d



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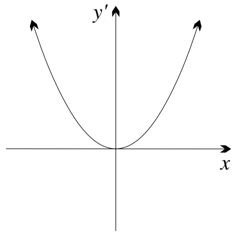

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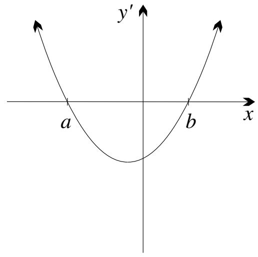

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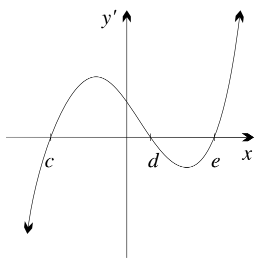

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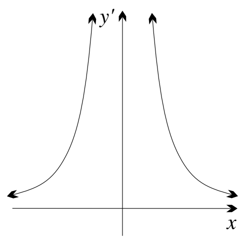
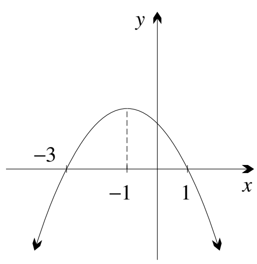


22 a $f'(x) = -3x^2 + 4x - 5$

b For $f'(x)$, $\triangle = -44 < 0$, so $f'(x)$ has no zeroes. Also $f'(0) = -5 < 0$, so $f'(x)$ is negative for all values of x.

c 1

23 a



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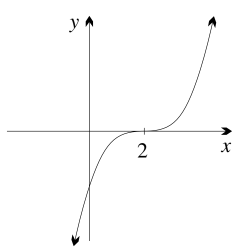

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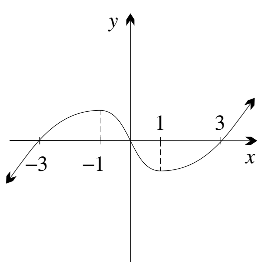
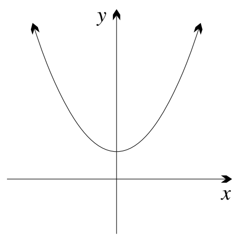


d



Exercise 3B

1a $x = 3$ **b** $x = -2$

c $x = 1$ or -1

2 a $(2, 3)$

b $(4, 0)$

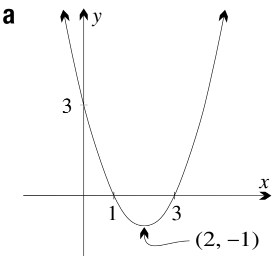
c $(1, -2)$

d $(1, 0)$

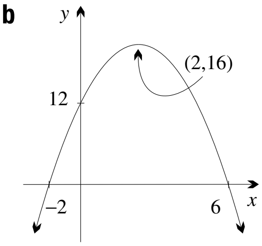
e $(0, 0)$ and $(2, -4)$

f $(1, -2)$

3 a



b



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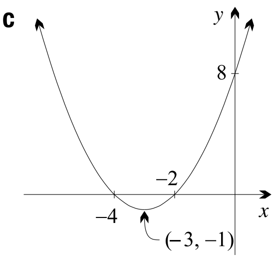
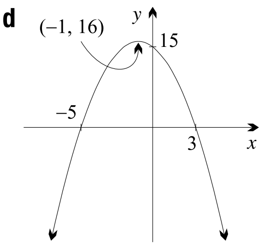


d



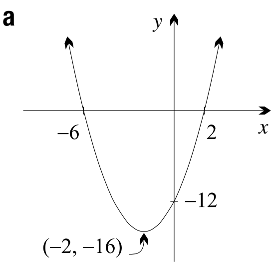
4 a minimum

b maximum

c minimum

d horizontal (or stationary) point of inflection

5 a



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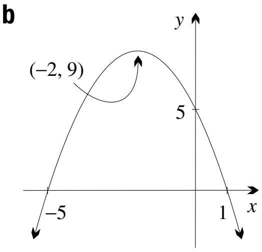

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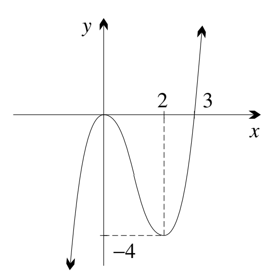
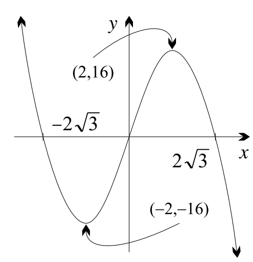
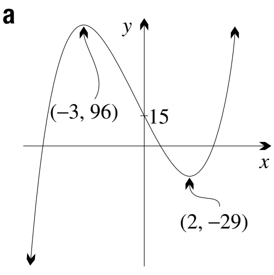


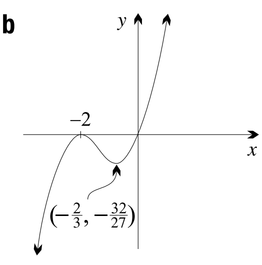
7



8 a



b



Answers 3A–3B

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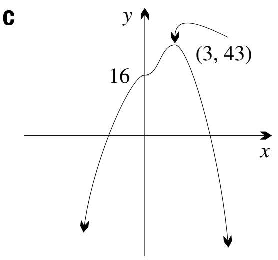
c



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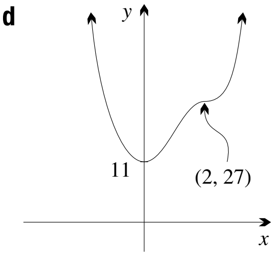

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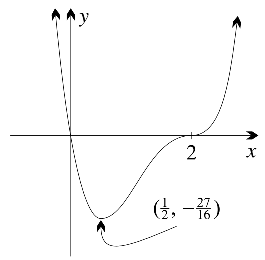

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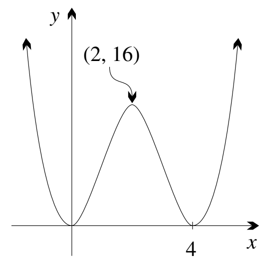

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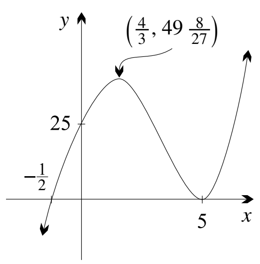


12 a $a = -8$ **b** $a = 2$

13 a $a = 2$ and $c = 3$

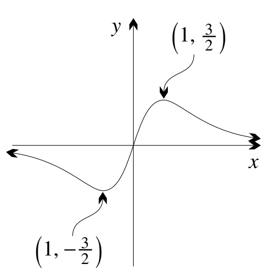
b $b = -3$ and $c = -24$

14 b $a = b = -1, c = 6$

15 a The curve passes through the origin.

c $a = -1$

16 c



d i no roots **ii** 1 root

iii 2 roots **iv** 1 root

17 b $9a - 3b + 3c = -27$

c $a = 2, b = 3, c = -12$

d $d = 7$

Exercise 3C

1a $3x^2, 6x, 6$ **b** $10x^9, 90x^8, 720x^7$

c $7x^6, 42x^5, 210x^4$ **d** $2x, 2, 0$

e $8x^3, 24x^2, 48x$ **f** $15x^4, 60x^3, 180x^2$

g $-3, 0, 0$ **h** $2x - 3, 2, 0$

i $12x^2 - 2x, 24x - 2, 24$

j $20x^4 + 6x^2, 80x^3 + 12x, 240x^2 + 12$

2 a $2x + 3, 2$

b $3x^2 - 8x, 6x - 8$

c $2x - 1, 2$

d $6x - 13, 6$

e $30x^4 - 36x^3, 120x^3 - 108x^2$

f $32x^7 + 40x^4, 224x^6 + 160x^3$

3 a $0.3x^{-0.7}, -0.21x^{-1.7}, 0.357x^{-2.7}$

b $-\frac{1}{x^2}, \frac{2}{x^3}, -\frac{6}{x^4}$ **c** $-\frac{2}{x^3}, \frac{6}{x^4}, -\frac{24}{x^5}$

d $-\frac{15}{x^4}, \frac{60}{x^5}, -\frac{300}{x^6}$ **e** $2x - \frac{1}{x^2}, 2 + \frac{2}{x^3}, -\frac{6}{x^4}$

4 a $-\frac{3}{x^4}, \frac{12}{x^5}$ **b** $-\frac{4}{x^5}, \frac{20}{x^6}$

c $-\frac{6}{x^3}, \frac{18}{x^4}$ **d** $-\frac{6}{x^4}, \frac{24}{x^5}$

5 a $2(x + 1), 2$

b $9(3x - 5)^2, 54(3x - 5)$

c $8(4x - 1), 32$

d $-11(8 - x)^{10}, 110(8 - x)^9$

6 a $\frac{-1}{(x + 2)^2}, \frac{2}{(x + 2)^3}$ **b** $\frac{2}{(3 - x)^3}, \frac{6}{(3 - x)^4}$

c $\frac{-15}{(5x + 4)^4}, \frac{300}{(5x + 4)^5}$ **d** $\frac{12}{(4 - 3x)^3}, \frac{108}{(4 - 3x)^4}$

7 a $\frac{1}{2\sqrt{x}}, \frac{-1}{4x\sqrt{x}}$ **b** $\frac{1}{3}x^{-\frac{2}{3}}, -\frac{2}{9}x^{-\frac{5}{3}}$

c $\frac{3}{2}\sqrt{x}, \frac{3}{4\sqrt{x}}$ **d** $-\frac{1}{2}x^{-\frac{3}{2}}, \frac{3}{4}x^{-\frac{5}{2}}$

e $\frac{1}{2\sqrt{x + 2}}, \frac{-1}{4(x + 2)^{\frac{3}{2}}}$ **f** $\frac{-2}{\sqrt{1 - 4x}}, \frac{-4}{(1 - 4x)^{\frac{3}{2}}}$

8 a $f'(x) = 3x^2 + 6x + 5, f''(x) = 6x + 6$

b i 5 **ii** 14 **iii** 6 **iv** 12

9 a i 15 **ii** 12 **iii** 6 **iv** 0

b i -8 **ii** 48 **iii** -192 **iv** 384

10 a $\frac{1}{(x + 1)^2}, \frac{-2}{(x + 1)^3}$

b $\frac{7}{(2x + 5)^2}, \frac{-28}{(2x + 5)^3}$

11 $(x - 1)^3(5x - 1), 4(x - 1)^2(5x - 2)$

12 a $1, -1$ **b** $-\frac{1}{3}$

13 a $nx^{n-1}, n(n - 1)x^{n-2}, n(n - 1)(n - 2)x^{n-3}$

b $n(n - 1)(n - 2)\ldots1, 0$

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Exercise 3D

1

Point	A	B	C	D	E	F	G	H	I
y'	0	+	0	–	0	–	0	+	0
y''	+	0	–	0	0	0	+	0	0

2 a concave down **b** concave up
  c concave up **d** concave down

3 a minimum **b** maximum
  c minimum **d** minimum

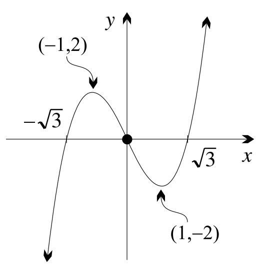
4 a $y'' = 2$, so $y'' > 0$ for all values of x.
  b $y'' = -6$, so $y'' < 0$ for all values of x.

5 a $y'' = 6x - 6$
  b i $x > 1$ **ii** $x < 1$

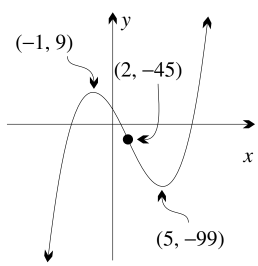
6 a $y'' = 6x - 2$
  b i $x > \frac{1}{3}$ **ii** $x < \frac{1}{3}$

7 $x = 0$ and $x = 2$, but not $x = -3$.

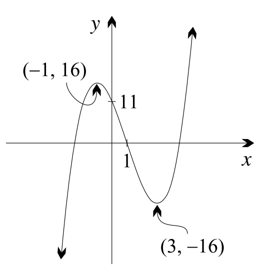
8 e



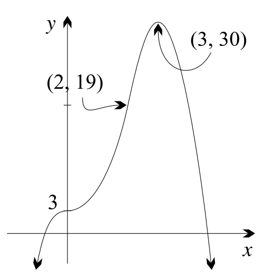
9 d



10 d



11 d



12 a $x > 2$ or $x < -1$ **b** $-1 < x < 2$
  c $x > \frac{1}{2}$ **d** $x < \frac{1}{2}$

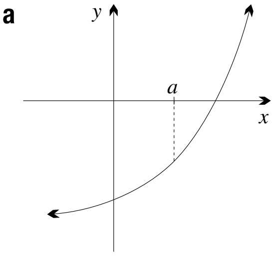
13 a $y' = 3x^2 + 6x - 72$, $y'' = 6x + 6$
  d $75x + y - 13 = 0$

14 a $f'(x) = 3x^2$, $f''(x) = 6x$, $g'(x) = 4x^3$, $g''(x) = 12x^2$
  b $f''(x) = g''(x) = 0$, no
  c $f(x)$ has a horizontal (or stationary) point of inflection, $g(x)$ has a minimum turning point.

15 a $y'' = 6x - 2a$, $a = 6$
  b $y'' = 6x + 4a$, $a > 1\frac{1}{2}$
  c $y'' = 12x^2 + 6ax + 2b$, $a = -5$, $b = 6$
  d $a > -\frac{2}{3}$

16 a Increasing. **b** Concave down.

17 a



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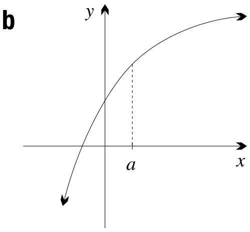

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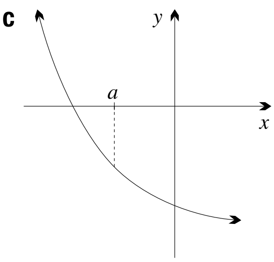
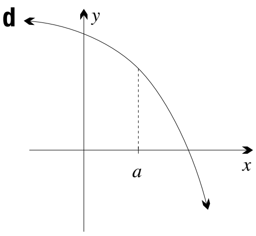


d



18 a $y' = (x - 3)^2 + 2 \geq 2$ for all real x.
  b There is a point of inflection at $x = 3$.
  c One, because the function is continuous and increasing for all real x.

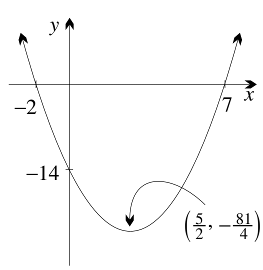
19 $a = 2$, $b = -3$, $c = 0$ and $d = 5$

Exercise 3E

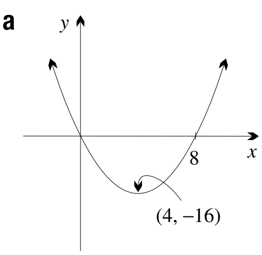
1a $(6, 0)$ **b** $(4, 32)$ **c** $(2, 16)$

2 a $x = -1$ or $x = 2$ **b** $x = 0$
  c $-1 < x < 2$ **d** $x < 0$

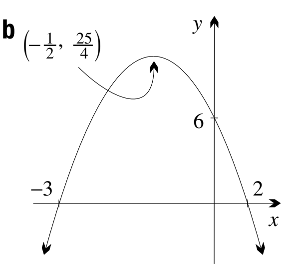
3



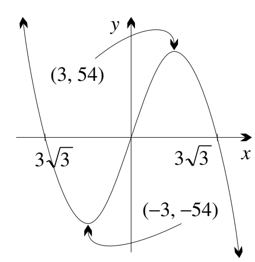
4 a



b



5



a Show that $f(-x) = -f(x)$. Point symmetry in the origin.
e When $x = 0$, $y' = 27$.

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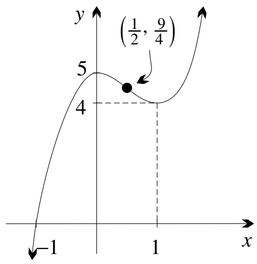
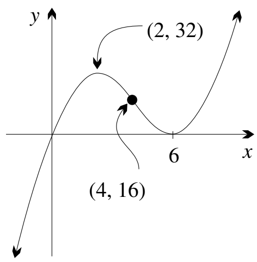
6 When $x = \frac{1}{2}$, $y' = 1\frac{1}{2}$.



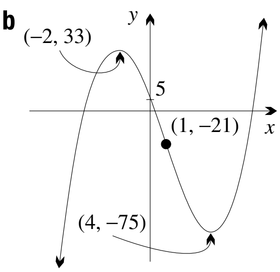
c $f''(0 = -6$, $f''\left(\frac{1}{2}\right) = 0$ and $f''(1) = 6$, so the concavity changes sign around $x = \frac{1}{2}$.

b Also $f'\left(\frac{1}{2}\right) = -1\frac{1}{2}$.

7 a



b



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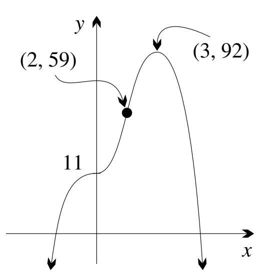

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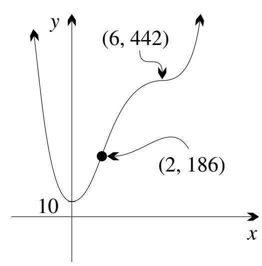

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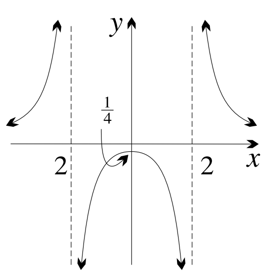


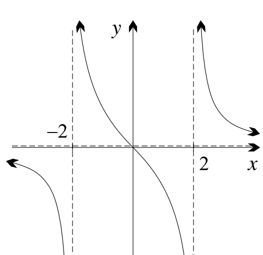
c line symmetry in the y-axis

d domain: $x \neq 2$ and $x \neq -2$, asymptotes: $x = 2$ and $x = -2$

e $y = 0$

g $y > 0$ or $y \leq -\frac{1}{4}$

11



c gradient $= -\frac{1}{4}$

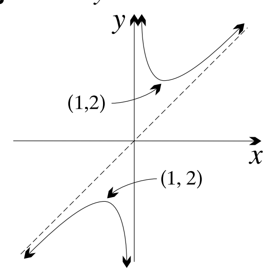
d domain: $x \neq 2$ and $x \neq -2$, asymptotes: $x = 2$ and $x = -2$

e $y = 0$

f point symmetry in the origin

g all real y

12



c point symmetry in the origin

d domain: $x \neq 0$, asymptote: $x = 0$

g $y \geq 2$ or $y \leq -2$

Exercise 3F

1a A local maximum, B local minimum

b C global maximum, D local minimum, E local maximum, F global minimum

c G global maximum, H horizontal point of inflection

d I horizontal point of inflection, J global minimum

2 a 0, 4 **b** 2, 5 **c** 0, 4 **d** 0, 5

e $0, 2\sqrt{2}$ **f** $-1, -\frac{1}{4}$ **g** $-1, 2$

3 a $-1, 8$ **b** $-49, 5$ **c** 0, 4 **d** 0, 9

4 a local maximum $y = 4$ at $x = 1$, global minimum -5, global maximum 20

b local minimum $y = 4$ at $x = 1$, global minimum -5, local maximum 11, global maximum 139

c global minimum 4, global maximum 11

Exercise 3G

1b $3\frac{1}{8}$ metres

2 b 3 **c** 18

3 b 4 **c** 32

4 c 5 **d** 25 cm^2

5 c 10 **d** 200 m^2

6 After 2 hours and 40 minutes.

7 d 24 cm

8 b $x = 30$ m and $y = 20$ m

9 c $h = 2$, $w = \frac{3}{2}$

10 a $\frac{x}{4}, \frac{10 - x}{4}$ **c** 5 **d** $\frac{25}{8}$ cm^2

11 a $R = x(47 - \frac{1}{3}x)$

b $-\frac{8}{15}x^2 + 32x - 10$

c 30

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12 c $\frac{10}{3\pi}$ **d** $\frac{1000}{27\pi}$ m^3

13 c $20\sqrt{10}\,\pi$ cm^3

14 c 4 cm by 4 cm by 2 cm

15 c $\frac{10}{3}$

16 a $S = 16x + 4y$

c 27 m by 9 m by 18 m

17 b Width $16\sqrt{3}$ cm and depth $16\sqrt{6}$ cm.

18 b 15 cm by 5 cm by 3.75 cm

19 d $r = 8$

20 c 48 cm^2

21 d $2\left(\sqrt{10} + 1\right)$ cm by $4\left(\sqrt{10} + 1\right)$ cm

22 b 80 km/h **c** \$400

23 b When $x = 0$, $T2.75$ hours, and when $x = 20$, $T \doteqdot 2.61$ hours.

d 2.45 hours

Exercise 3H

1 a $\frac{1}{7}x^7 + C$ **b** $\frac{1}{4}x^4 + C$ **c** $\frac{1}{11}x^{11} + C$

d $\frac{3}{2}x^2 + C$ **e** $5x + C$ **f** $\frac{1}{2}x^{10} + C$

g $3x^7 + C$ **h** C

2 a $\frac{1}{3}x^3 + \frac{1}{5}x^5 + C$ **b** $x^4 - x^5 + C$

c $\frac{2}{3}x^3 + \frac{5}{8}x^8 + C$ **d** $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + C$

e $3x - 2x^2 + 2x^8 + C$ **f** $x^3 - x^4 - x^5 + C$

3 a $\frac{1}{3}x^3 - \frac{3}{2}x^2 + C$ **b** $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + C$

c $x^3 + \frac{11}{2}x^2 - 4x + C$ **d** $\frac{5}{6}x^6 - x^4 + C$

e $x^8 + \frac{1}{2}x^4 + C$

f $\frac{1}{2}x^2 + \frac{1}{4}x^4 - 3x - x^3 + C$

4 a i $y = x^2 + 3x + 3$

ii $y = x^2 + 3x + 4$

b i $y = 3x^3 + 4x + 1$

ii $y = 3x^3 + 4x - 2$

c i $y = x^3 - 2x^2 + 7x$

ii $y = x^3 - 2x^2 + 7x - 7$

5 a $-\dfrac{1}{x} + C$ **b** $-\dfrac{1}{2x^2} + C$

c $\dfrac{1}{x^2} + C$ **d** $\dfrac{1}{x^3} + C$

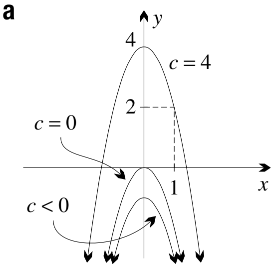
e $-\dfrac{1}{x} + \dfrac{1}{2x^2} + C$

6 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $2\sqrt{x} + C$ **c** $\frac{3}{4}x^{\frac{4}{3}} + C$

d $4\sqrt{x} + C$ **e** $\frac{5}{8}x^{\frac{8}{5}} + C$

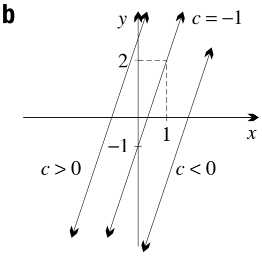
7 a $y = \frac{2}{3}x^{\frac{3}{2}} + 1$ **b** $y = \frac{2}{3}x^{\frac{3}{2}} - 16$

8 a



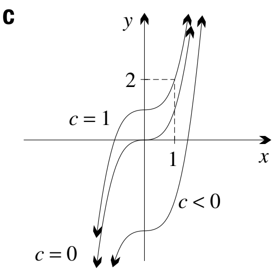
$y = -2x^2 + c$,
$y = 4 - 2x^2$

b



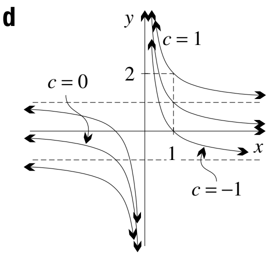
$y = 3x + c$,
$y = 3x - 1$

c



$y = x^3 + c$,
$y = x^3 + 1$

d



$y = \dfrac{1}{x} + c$,
$y = \dfrac{1}{x} + 1$

9 a $\frac{1}{4}(x + 1)^4 + C$ **b** $\frac{1}{6}(x - 2)^6 + C$

c $\frac{1}{3}(x + 5)^3 + C$ **d** $\frac{1}{10}(2x + 3)^5 + C$

e $\frac{1}{21}(3x - 4)^7 + C$ **f** $\frac{1}{20}(5x - 1)^4 + C$

g $-\frac{1}{4}(1 - x)^4 + C$ **h** $-\frac{1}{28}(1 - 7x)^4 + C$

i $\dfrac{-1}{3(x - 2)^3} + C$ **j** $\dfrac{1}{9(1 - x)^9} + C$

10 a $\frac{2}{3}(x + 1)^{\frac{3}{2}} + C$ **b** $\frac{2}{3}(x - 5)^{\frac{3}{2}} + C$

c $-\frac{2}{3}(1 - x)^{\frac{3}{2}} + C$ **d** $\frac{1}{3}(2x - 7)^{\frac{3}{2}} + C$

e $\frac{2}{9}(3x - 4)^{\frac{3}{2}} + C$

11 a $y = \frac{1}{5}(x - 1)^5$

b $\frac{1}{8}(2x + 1)^4 - \frac{9}{8}$

c $y = \frac{1}{3}(2x + 1)^{\frac{3}{2}}$

12 a $y = \frac{3}{5}x^5 - \frac{1}{4}x^4 + x$

b $y = -\frac{1}{4}x^4 + x^3 + 2x - 2$

c $y = -\frac{1}{20}(2 - 5x)^4 + \frac{21}{20}$

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13 30

14 The rule gives the primitive of x^{-1} as $\dfrac{x^0}{0}$, which is undefined. This problem will be addressed in Chapter 5.

15 $y = x^3 + 2x^2 - 5x + 6$

16 $y = -x^3 + 4x^2 + 3$

17 $y = \begin{cases} \dfrac{1}{x} + 4\frac{1}{2}, & \text{for } x > 0, \\ \dfrac{1}{x} + 1, & \text{for } x < 0. \end{cases}$

Chapter 3 review exercise

1a C and H **b** A and F
c B, D, E and G **d** A, B, G and H
e D **f** C, E and F

2 a $f'(x) = 3x^2 - 2x - 1$
b **i** decreasing **ii** stationary
iii increasing **iv** increasing

3 a $2x - 4$
b **i** $x > 2$ **ii** $x < 2$ **iii** $x = 2$

4 a $y' = 3x^2$, increasing
b $y' = 2x - 1$, increasing
c $y' = 5(x - 1)^4$, stationary
d $y' = -\dfrac{4}{(x - 3)^2}$, decreasing

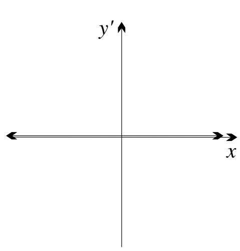
5 a $7x^6, 42x^5$
b $3x^2 - 8x, 6x - 8$
c $5(x - 2)^4, 20(x - 2)^3$
d $-\dfrac{1}{x^2}, \dfrac{2}{x^3}$

6 a concave up
b concave down

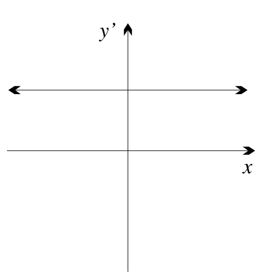
7 a $12x - 6$
b **i** $x > \frac{1}{2}$ **ii** $x < \frac{1}{2}$

8 a $x < 1$ or $x > 3$ **b** $1 < x < 3$
c $x > 2$ **d** $x < 2$

9 a



b



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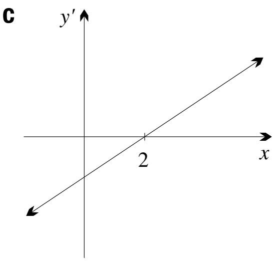

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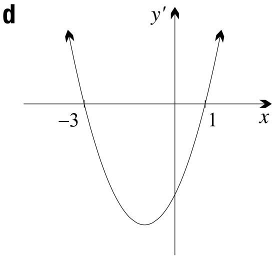
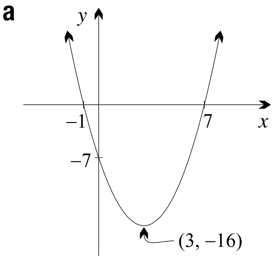


10 a $P(-1, 3)$, $Q\left(\frac{1}{3}, \frac{49}{27}\right)$ **b** $x > -\frac{1}{3}$
c $\frac{49}{27} < k < 3$

11 a



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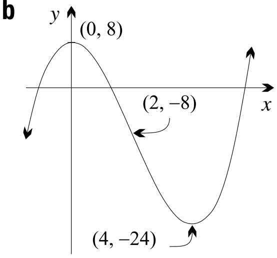
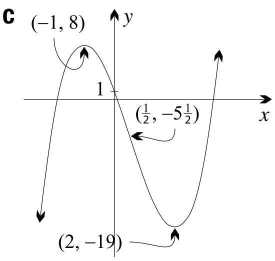
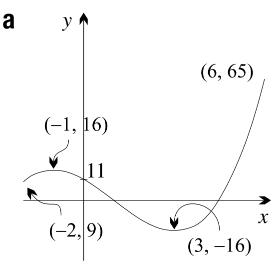


c



12 a



b 65 and -16

13 a $a = -2$ **b** $a = 3$ and $b = 6$

14 b -16

15 a 175 **b** 0 **c** 256

16 b $\frac{1600}{27}$ cm^3

17 b $r = 8$ m

18 a $\frac{1}{8}x^8 + C$ **b** $x^2 + C$
c $4x + C$ **d** $2x^5 + C$
e $4x^2 + x^3 - x^4 + C$

19 a $x^3 - 3x^2 + C$
b $\frac{1}{3}x^3 - 2x^2 - 5x + C$
c $\frac{4}{3}x^3 - 6x^2 + 9x + C$

20 a $\frac{1}{6}(x + 1)^6 + C$
b $\frac{1}{8}(x - 4)^8 + C$
c $\frac{1}{8}(2x - 1)^4 + C$

21 a $-\frac{1}{x} + C$ **b** $\frac{2}{3}x^{\frac{2}{3}} + C$

22 $f(x) = x^3 - 2x^2 + x + 3$

23 25

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Chapter 4

Exercise 4A

1 a $\frac{1}{2}u^2$
b The area under the curve is less than the area of the triangle.
2 a $\frac{1}{16}u^2$ **b** $\frac{5}{16}u^2$
c The area under the curve is less than the combined area of the triangle and trapezium.
3 b The gaps between the upper line segments and the curve are getting smaller.
4 a 6 **b** 12 **c** 8 **d** 9
e 2 **f** $\frac{25}{2}$ **g** 6 **h** 20
5 a 8 **b** 25 **c** 9 **d** 24
e 36 **f** 24 **g** 9 **h** 8
6 a $\frac{7}{32}u^2$ **b** $\frac{15}{32}u^2$
c The sum of the areas of the lower rectangles is less than the exact area under the curve which is less than the sum of the areas of the upper rectangles. Note that $\int_0^1 x^2\,dx = \frac{1}{3}$.
7 d As the number of rectangles increases, the interval within which the exact area lies becomes smaller. Note that $\int_0^1 2^x\,dx = \dfrac{1}{\ln 2} \doteqdot 1.44$.
8 d As the number of rectangles increases, the interval within which the exact area lies becomes smaller. Note that $\int_2^4 \ln x\,dx = 6\ln 2 - 2 \doteqdot 2.16$.
9 a 15 **b** 15 **c** 25 **d** 40
e $\frac{25}{2}$ **f** 12 **g** 16 **h** 24
i 8 **j** 18 **k** 4 **l** 16
m 4 **n** 16 **o** $\frac{25}{2}$ **p** $\frac{25}{2}$
10 a 8π **b** $\frac{25}{4}\pi$
11 a You should count approximately 133 squares. $\frac{133}{400} \doteqdot 0.33$. We shall see later that $\int_0^1 x^2\,dx = \frac{1}{3}$.
b The exact values are:
i $\frac{1}{24}$ **ii** $\frac{7}{24}$
12 b 0.79 **c** 3.16
13 e The interval is getting smaller.
f Yes, they appear to be getting closer and closer to the exact value.

Exercise 4B

1 a 1 **b** 15 **c** 16 **d** 84 **e** 19
f 243 **g** 62 **h** 2 **i** 1
2 a **i** 4 **ii** 25
iii 1 (Note that $\int_4^5 dx$ means $\int_4^5 1\,dx$.)
b Each function is a horizontal line, so each integral is a rectangle.
3 a 30 **b** 6 **c** 33 **d** 18 **e** 132
f 2 **g** 23 **h** 44 **i** 60
4 a 2 **b** 2 **c** 9
d 30 **e** 96 **f** 10
5 a $13\frac{1}{2}$ **b** $4\frac{2}{3}$ **c** $29\frac{1}{4}$
d 2 **e** $20\frac{5}{6}$ **f** 98
6 a 24 **b** 18 **c** $2\frac{2}{3}$
d 21 **e** $\frac{1}{4}$ **f** $\frac{8}{15}$
7 a 42 **b** 14 **c** 62
d $8\frac{1}{3}$ **e** $6\frac{2}{3}$ **f** 6
8 a $\frac{1}{24}$ **b** $\frac{20}{27}$ **c** $\frac{7}{8}$
9 a **i** $\frac{1}{10}$ **ii** $\frac{5}{36}$ **iii** 15
b **i** $\frac{1}{2}$ **ii** $\frac{15}{32}$ **iii** 7
10 a **ii** 8 **b** **ii** 6
11 a $k = 1$ **b** $k = 4$ **c** $k = 8$
d $k = 3$ **e** $k = 3$ **f** $k = 2$
12 a $1 + \frac{\pi}{2}$ **b** $2\frac{1}{2}$
13 a $\frac{3}{2}$ **b** $\frac{5}{8}$ **c** $42\frac{1}{3}$
14 a $13\frac{1}{3}$ **b** $8\frac{59}{120}$ **c** $\frac{1}{24}$
15 a x^2 is never negative.
b The function has an asymptote $x = 0$, which lies in the given interval. Hence the integral is meaningless and the use of the fundamental theorem is invalid.
c Part **ii** is meaningless because it crosses the asymptote at $x = 3$.
16 a **i** x^2 **ii** $x^3 + 3x$ **iii** $\frac{1}{x}$ **iv** $(x^3 - 3)^4$

Exercise 4C

1 The values are 6 and −6, which differ by a factor of −1.
2 a LHS = RHS = 2
b LHS = RHS = $6\frac{3}{4}$
c LHS = RHS = 0
3 a The interval has width zero.
b $y = x$ is an odd function.
4 a The area is below the x-axis.
b The area is above the x-axis.
c The areas above and below the x-axis are equal.
d The area below the x-axis is greater than the area above.
5 a The area is above the x-axis.
b The area is below the x-axis.

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c $y = 1 - x^2$ is an even function, so is symmetrical about the y-axis.

d The area under the parabola from 0 to $\frac{1}{2}$ is greater than the area from $\frac{1}{2}$ to 1.

6 a -7 **b** -5

7 a $1\frac{1}{3}$

b The integral from -1 to 1 is negative because the curve is below the x-axis and the integral runs forwards. But when the limits are reversed and the integral runs backwards, the value is the opposite and so is positive.

8 The area under the line $y = 2x$ from $x = 0$ to $x = 1$ is greater than the area under $y = x$.

9 The area below the x-axis is greater than the area above.

10 a i 6

ii -6. The integrals are opposites because the limits have been reversed.

b i 5

ii 5. The factor 20 can be taken out of the integral.

c i 45 **ii** 30

iii 15. An integral of a sum is the sum of the integrals.

d i 48 **ii** 3

iii 45. The interval $0 \le x \le 2$ can be dissected into the intervals $0 \le x \le 1$ and $1 \le x \le 2$.

e i 0

ii 0. An integral over an interval of zero width is zero.

11 a 0. The interval has zero width.

b 0. The interval has zero width.

c 0. The integrand is odd.

d 0. The integrand is odd.

e 0. The integrand is odd.

f 0. The integrand is odd.

12 a The curves meet at $(0, 0)$ and at $(1, 1)$.

b In the interval $0 \le x \le 1$, the curve $y = x^3$ is below the curve $y = x^2$.

c $\frac{1}{4}$ and $\frac{1}{3}$

13 a $\frac{\pi}{2} - \frac{1}{2}$ **b** $1 - \frac{\pi}{2}$

14 a The function is odd, so the integral is zero.

b The function is even, so its graph is symmetrical about the y-axis.

15 a false **b** true **c** false **d** false

Exercise 4D

1 a $A(x) = \frac{3}{2}x^2$

b $A'(x) = 3x$

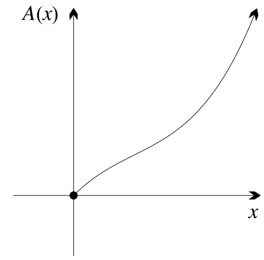
2 a $y = 3, A(x) = 3x, A'(x) = 3$

b $y = 2t, A(x) = x^2, A'(x) = 2x$

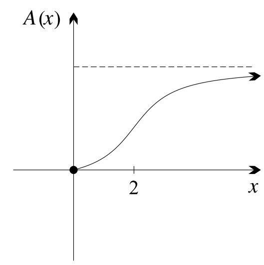
c $y = 2 + t, A(x) = 2x + \frac{1}{2}x^2, A'(x) = 2 + x$

d $y = 5 - t, A(x) = 5x - \frac{1}{2}x^2, A'(x) = 5 - x$

3 a $A(x)$ is increasing at a decreasing rate in the interval $0 \le x < 2$, and increasing at an increasing rate for $x > 2$. It has an inflection at $x = 2$.



b $A(x)$ is increasing at an increasing rate in the interval $0 \le x \le 2$, and increasing at a decreasing rate for $x \ge 2$. It has a point of inflection at $x = 2$.



4 a $A'(x) = \dfrac{1}{x}$ **b** $A'(x) = \dfrac{1}{1 + t^3}$

c $A'(x) = e^{-\frac{1}{2}x^2}$

5 a $A'(x) = 3x^2 - 12, A(x) = x^3 - 12x + 11$

b $A'(x) = x^3 + 4x, A(x) = \frac{1}{4}x^4 + 2x^2 - 12$

c $A'(x) = \frac{1}{x^2}, A(x) = \frac{1}{2} - \frac{1}{x}$

6 a The curve $y = A(x)$ looks like $y = e^x - 1$. The curve is increasing at an increasing rate.

b The curve is zero at $x = 1$, and is increasing at an increasing rate.

c The curve is zero at $x = 1$, and is increasing at a decreasing rate.

7 a The values are 0, 1, 0, -1, 0. The curve looks like $y = \sin x$, and this suggests that $\sin x$ has derivative $\cos x$.

b The values are 0, 1, 2, 1, 0. The graph looks like $y = 1 - \cos x$, which suggest that the derivative of $\cos x$ is $-\sin x$.

8 a $A(x)$ is increasing when $f(t)$ is positive, that is, for $t < c$, and is decreasing for $t > c$.

b $A(x)$ has a maximum turning point at $x = c$, and no minimum turning points.

c $A(x)$ has inflections when $f'(t)$ changes sign, that is, at $x = b$ and $x = d$.

d Because of the point symmetry of $f(t)$, there are two zeroes of $A(x)$ are $x = a$ and $x = e$.

e $A(x)$ is positive for $a < x < e$ and negative for $x < a$ and for $x > e$.

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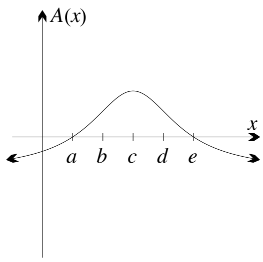
f



9 a The function is continuous at every real number, so it is a continuous function.

b The domain is $x \neq 2$, and y is continuous at every value in its domain, so it is a continuous function.

c Zero now lies in the domain, and y is not continuous at $x = 0$, so it is not a continuous function.

d The domain is $x \geq 0$, and y is continuous at every value in its domain, so it is a continuous function.

e The domain is $x > 0$, and y is continuous at every value in its domain, so it is a continuous function.

f The domain is $x \geq 0$, and y is not continuous at $x = 0$, so it is not a continuous function.

Exercise 4E

1 a $4x + C$ **b** $x + C$ **c** C
d $-2x + C$ **e** $\frac{x^2}{2} + C$ **f** $\frac{x^3}{3} + C$
g $\frac{x^4}{4} + C$ **h** $\frac{x^8}{8} + C$

2 a $x^2 + C$ **b** $2x^2 + C$ **c** $x^3 + C$
d $x^4 + C$ **e** $x^{10} + C$ **f** $\frac{x^4}{2} + C$
g $\frac{2x^6}{3} + C$ **h** $\frac{x^9}{3} + C$

3 a $\frac{x^2}{2} + \frac{x^3}{3} + C$ **b** $\frac{x^5}{5} - \frac{x^4}{4} + C$
c $\frac{x^8}{8} + \frac{x^{11}}{11} + C$ **d** $x^2 + x^5 + C$
e $x^9 - 11x + C$ **f** $\frac{x^{14}}{2} + \frac{x^9}{3} + C$
g $4x - \frac{3x^2}{2} + C$ **h** $x - \frac{x^3}{3} + \frac{x^5}{5} + C$
i $x^3 - 2x^4 + \frac{7x^5}{5} + C$

4 a $-x^{-1} + C$ **b** $-\frac{1}{2}x^{-2} + C$ **c** $-\frac{1}{7}x^{-7} + C$
d $-x^{-3} + C$ **e** $-x^{-9} + C$ **f** $-2x^{-5} + C$

5 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $\frac{3}{4}x^{\frac{4}{3}} + C$ **c** $\frac{4}{5}x^{\frac{5}{4}} + C$
d $\frac{3}{5}x^{\frac{5}{3}} + C$ **e** $2x^{\frac{1}{2}} + C$ **f** $\frac{8}{3}x^{\frac{3}{2}} + C$

6 a $\frac{1}{3}x^3 + x^2 + C$ **b** $2x^2 - \frac{1}{4}x^4 + C$
c $\frac{5}{3}x^3 - \frac{3}{4}x^4 + C$ **d** $\frac{1}{5}x^5 - \frac{5}{4}x^4 + C$
e $\frac{1}{3}x^3 - 3x^2 + 9x + C$ **f** $\frac{4}{3}x^3 + 2x^2 + x + C$
g $x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + C$ **h** $4x - 3x^3 + C$
i $\frac{1}{3}x^3 - \frac{1}{2}x^4 - 3x + 3x^2 + C$

7 a $\frac{1}{2}x^2 + 2x + C$ **b** $\frac{1}{2}x^2 + \frac{1}{3}x^3 + C$
c $\frac{1}{6}x^3 - \frac{1}{16}x^4 + C$

8 a $-\dfrac{1}{x} + C$ **b** $-\dfrac{1}{2x^2} + C$ **c** $-\dfrac{1}{4x^4} + C$
d $-\dfrac{1}{9x^9} + C$ **e** $-\dfrac{1}{x^3} + C$ **f** $-\dfrac{1}{x^5} + C$
g $-\dfrac{1}{x^7} + C$ **h** $-\dfrac{1}{3x} + C$ **i** $-\dfrac{1}{28x^4} + C$
j $\dfrac{1}{10x^2} + C$ **k** $\dfrac{1}{4x^4} - \dfrac{1}{x} + C$
l $-\dfrac{1}{2x^2} - \dfrac{1}{3x^3} + C$

9 a $\frac{2}{3}x^{\frac{3}{2}} + C$ **b** $\frac{3}{4}x^{\frac{4}{3}} + C$
c $2\sqrt{x} + C$ **d** $\frac{3}{5}x^{\frac{5}{3}} + C$

10 a 18 **b** 12 **c** 4 **d** $\frac{3}{5}$

11 a $\frac{1}{6}(x + 1)^6 + C$ **b** $\frac{1}{4}(x + 2)^4 + C$
c $-\frac{1}{5}(4 - x)^5 + C$ **d** $-\frac{1}{3}(3 - x)^3 + C$
e $\frac{1}{15}(3x + 1)^5 + C$ **f** $\frac{1}{32}(4x - 3)^8 + C$
g $-\frac{1}{14}(5 - 2x)^7 + C$ **h** $-\frac{1}{40}(1 - 5x)^8 + C$
i $\frac{1}{24}(2x + 9)^{12} + C$ **j** $\frac{3}{22}(2x - 1)^{11} + C$
k $\frac{4}{35}(5x - 4)^7 + C$ **l** $-\frac{7}{8}(3 - 2x)^4 + C$

12 a $\frac{3}{5}\left(\frac{1}{3}x - 7\right)^5 + C$ **b** $\frac{4}{7}\left(\frac{1}{4}x - 7\right)^7 + C$
c $-\frac{5}{4}\left(1 - \frac{1}{5}x\right)^4 + C$

13 a $-\dfrac{1}{2(x + 1)^2} + C$ **b** $-\dfrac{1}{3(x - 5)^3} + C$
c $-\dfrac{1}{3(3x - 4)} + C$ **d** $\dfrac{1}{4(2 - x)^4} + C$
e $-\dfrac{3}{5(x - 7)^5} + C$ **f** $-\dfrac{1}{2(4x + 1)^4} + C$
g $\dfrac{2}{15(3 - 5x)^3} + C$ **h** $\dfrac{1}{5 - 20x} + C$
i $-\dfrac{7}{96(3x + 2)^4} + C$

14 a $\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + C$ **b** $\frac{1}{2}x^2 - 4x + C$
c $2x^2 - \frac{8}{3}x^{\frac{3}{2}} + x + C$

15 a i $\frac{2}{3}$ **ii** 2 **iii** 12
b i $5\frac{1}{3}$ **ii** $96\frac{4}{5}$ **iii** 4

16 a 2 **b** $-\frac{13}{6}$ **c** $12\frac{1}{6}$

17 $\displaystyle\int x^{-1}dx = \frac{x^0}{0} + C$ is meaningless. Chapter 5 deals with the resolution of this problem.

18 a $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + C$ **b** $-\frac{1}{6}(7 - 4x)^{\frac{3}{2}} + C$
c $\frac{3}{16}(4x - 1)^{\frac{4}{3}} + C$ **d** $\frac{2}{3}\sqrt{3x + 5} + C$

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19 a $\frac{242}{5}$ **b** 0 **c** $121\frac{1}{3}$
d 1 **e** $\frac{13}{6}$ **f** 2
g 0 **h** $\frac{112}{9}$ **i** $8\frac{2}{5}$

Exercise 4F

1 a 4u^2 **b** 26u^2 **c** 81u^2 **d** 12u^2
e 9u^2 **f** $6\frac{2}{3}\text{u}^2$ **g** $\frac{128}{3}\text{u}^2$ **h** 6u^2
i $\frac{1}{4}\text{u}^2$ **j** $57\frac{1}{6}\text{u}^2$ **k** 36u^2 **l** 60u^2

2 a 25u^2 **b** 8u^2

3 a 4u^2 **b** 108u^2 **c** $\frac{9}{2}\text{u}^2$
d $34\frac{2}{3}\text{u}^2$ **e** 18u^2 **f** 2u^2

4 a $\frac{4}{3}\text{u}^2$ **b** $\frac{27}{2}\text{u}^2$ **c** $\frac{81}{4}\text{u}^2$ **d** $46\frac{2}{5}\text{u}^2$

5 a $\frac{9}{2}\text{u}^2$ **b** $\frac{4}{3}\text{u}^2$ **c** $\frac{45}{4}\text{u}^2$ **d** 9u^2

6 b $4\frac{1}{2}\text{u}^2$ **c** 2u^2 **d** $6\frac{1}{2}\text{u}^2$
e $2\frac{1}{2}$. This is the area above the x-axis minus the area below it.

7 b $10\frac{2}{3}\text{u}^2$ **c** $2\frac{1}{3}\text{u}^2$ **d** 13u^2
e $-8\frac{1}{3}$. This is the area above the x-axis minus the area below it.

8 b $2\frac{2}{3}\text{u}^2$ **c** $\frac{5}{12}\text{u}^2$ **d** $3\frac{1}{12}\text{u}^2$
e $-2\frac{1}{4}$. This is the area above the x-axis minus the area below it.

9 a $11\frac{2}{3}\text{u}^2$ **b** $128\frac{1}{2}\text{u}^2$ **c** 4u^2
d $8\frac{1}{2}\text{u}^2$ **e** $32\frac{3}{4}\text{u}^2$ **f** $11\frac{1}{3}\text{u}^2$

10 a 13u^2 **b** $2\frac{1}{2}\text{u}^2$ **c** $9\frac{1}{3}\text{u}^2$ **d** $7\frac{1}{3}\text{u}^2$

11 a i 64u^2 **ii** 128u^2 **iii** $64\frac{4}{5}\text{u}^2$
b i 50u^2 **ii** 18u^2 **iii** $\frac{32}{3}\text{u}^2$

12 16u^2

13 a $(2, 0)$, $\left(0, 4\sqrt{2}\right)$, $\left(0, -4\sqrt{2}\right)$
b $\frac{16\sqrt{2}}{3}\text{u}^2$
ii $x = 2 - \frac{y^2}{16}$

14 a $y = \frac{1}{3}x^3 - 2x^2 + 3x$
b The curve passes through the origin, $(1, 1\frac{1}{3})$ is a maximum turning point and $(3, 0)$ is a minimum turning point.
c $\frac{4}{3}\text{u}^2$

Exercise 4G

1 a $\frac{1}{6}\text{u}^2$ **b** $\frac{1}{4}\text{u}^2$ **c** $\frac{3}{10}\text{u}^2$ **d** $\frac{1}{12}\text{u}^2$
e $\frac{2}{35}\text{u}^2$ **f** $20\frac{5}{6}\text{u}^2$ **g** 36u^2 **h** $20\frac{5}{6}\text{u}^2$

2 a $\frac{4}{3}\text{u}^2$ **b** $\frac{1}{6}\text{u}^2$ **c** $\frac{4}{3}\text{u}^2$ **d** $4\frac{1}{2}\text{u}^2$

3 a $5\frac{1}{3}\text{u}^2$ **b** $\frac{9}{4}\text{u}^2$

4 a $16\frac{2}{3}\text{u}^2$ **b** $9\frac{1}{3}\text{u}^2$

5 c $4\frac{1}{2}\text{u}^2$

6 c $\frac{4}{3}\text{u}^2$

7 c 36u^2

8 a $4\frac{1}{2}\text{u}^2$ **b** $20\frac{5}{6}\text{u}^2$ **c** $2\frac{2}{3}\text{u}^2$

9 c 36u^2

10 c $\frac{4}{3}\text{u}^2$

11 a $4\frac{1}{2}\text{u}^2$ **b** $20\frac{5}{6}\text{u}^2$ **c** $21\frac{1}{3}\text{u}^2$

12 c $\frac{1}{3}\text{u}^2$

13 b $y = x - 2$ **c** $5\frac{1}{3}\text{u}^2$

14 c 108u^2

15 b The points are $(-4, -67)$, $(1, -2)$, and $(2, 5)$.
c $73\frac{5}{6}\text{u}^2$

Exercise 4H

1 a 40 **b** 22 **c** -26

2 a 164

3 a 30

4 a The curve is concave up, so the chord is above the curve, and the area under the chord will be greater than the area under the curve.
b The curve is concave down, so the chord is underneath the curve, and the area under the chord will be less than the area under the curve.

5 b 10
c $10\frac{2}{3}$, the curve is concave down.
d $6\frac{1}{4}\%$

6 b $10\frac{1}{10}$
c y'' is positive in the interval $1 \le x \le 5$, so the curve is concave up.

7 b 12.660
c $12\frac{2}{3}$. y'' is negative in the interval $4 \le x \le 9$, so the curve is concave down.

8 a 0.73 **b** 4.5 **c** 3.4 **d** 37

9 a 1.12 **b** 0.705 **c** 22.9 **d** 0.167

10 9.2 metres

11 550 m^2

12 5900

13 a 0.7489
b $\pi \doteqdot 3.0$, the approximation is less than the integral, because the curve is concave down.

15 a 2
b Let $A = (-1, 0)$, $P = (0, 1)$ and $B = (1, 2)$. Then the curve has point symmetry in its y-intercept $P(0, 1)$, and the trapezoidal rule gives the area under the chord APB. The result now follows by symmetry.

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Exercise 4I

1 a $8(2x + 3)^3$
b i $(2x + 3)^4 + C$ **ii** $2(2x + 3)^4 + C$

2 a $9(3x - 5)^2$
b i $(3x - 5)^3 + C$ **ii** $3(3x - 5)^3 + C$

3 a $20(1 + 4x)^4$
b i $(1 + 4x)^5 + C$ **ii** $\frac{1}{2}(1 + 4x)^5 + C$

4 a $-8(1 - 2x)^3$
b i $(1 - 2x)^4 + C$ **ii** $\frac{1}{4}(1 - 2x)^4 + C$

5 a $-4(4x + 3)^{-2}$
b i $(4x + 3)^{-1} + C$
ii $-\frac{1}{4}(4x + 3)^{-1} + C$

6 a $(2x - 5)^{-\frac{1}{2}}$
b i $(2x - 5)^{\frac{1}{2}} + C$ **ii** $\frac{1}{3}(2x - 5)^{\frac{1}{2}} + C$

7 a $8x(x^2 + 3)^3$
b i $(x^2 + 3)^4 + C$ **ii** $5(x^2 + 3)^4 + C$

8 a $15x^2(x^3 - 1)^4$
b i $(x^3 - 1)^5 + C$ **ii** $\frac{1}{5}(x^3 - 1)^5 + C$

9 a $\dfrac{2x}{\sqrt{2x^2 + 3}}$
b i $\sqrt{2x^2 + 3} + C$ **ii** $\frac{1}{2}\sqrt{2x^2 + 3} + C$

10 a $\dfrac{3\left(\sqrt{x} + 1\right)^2}{2\sqrt{x}}$
b i $\left(\sqrt{x} + 1\right)^3 + C$ **ii** $\frac{2}{3}\left(\sqrt{x} + 1\right)^3 + C$

11 a $12(x^2 + 2x)(x^3 + 3x^2 + 5)^3$
b i $(x^3 + 3x^2 + 5)^4 + C$
ii $\frac{1}{12}(x^3 + 3x^2 + 5)^4 + C$

12 a $-7(2x + 1)(5 - x^2 - x)^6$
b i $(5 - x^2 - x)^7 + C$
ii $-\frac{1}{7}(5 - x^2 - x)^7 + C$

13 a $\frac{1}{4}(5x + 4)^4 + C$ **b** $\frac{1}{6}(1 - 3x)^6 + C$
c $\frac{1}{8}(x^2 - 5)^8 + C$ **d** $\frac{1}{5}(x^3 + 7)^5 + C$
e $\frac{-1}{3x^2 + 2} + C$ **f** $2\sqrt{9 - 2x^3} + C$

14 a $\frac{1}{3}(5x^2 + 3)^3 + C$ **b** $\frac{1}{4}(x^2 + 1)^4 + C$
c $\frac{1}{6}(1 + 4x^3)^6 + C$ **d** $\frac{1}{30}(1 + 3x^2)^5 + C$
e $-\frac{1}{32}(1 - x^4)^8 + C$ **f** $\frac{2}{3}(x^3 - 1)^{\frac{3}{2}} + C$
g $\frac{1}{15}(5x^2 + 1)^{\frac{3}{2}} + C$ **h** $2\sqrt{x^2 + 3} + C$
i $\frac{1}{4}\sqrt{4x^2 + 8x + 1} + C$ **j** $-\dfrac{1}{4(x^2 + 5)^2} + C$

15 a $\frac{32}{15}$ **b** $\frac{7}{144}$ **c** $\frac{1}{12}$ **d** 936

Chapter 4 review exercise

1 a 1 **b** $\frac{3}{2}$ **c** 609 **d** $\frac{2}{5}$ **e** -12
f $8\frac{2}{3}$ **g** 8 **h** -10 **i** $21\frac{1}{3}$

2 a $4\frac{2}{3}$ **b** $-1\frac{2}{3}$ **c** $\frac{1}{3}$

3 a $-1\frac{1}{2}$ **b** 15 **c** $-6\frac{1}{6}$

4 a ii $k = 6$
b ii $k = 3$

5 a 0. The integral has zero width.
b 0. The integrand is odd.
c 0. The integrand is odd.

6 a 8 **b** $\frac{3}{2}$

7 a i $4x - \frac{1}{2}x^2 + 10$ **ii** $\frac{1}{2} - x^{-1}$
b i $4 - x$ **ii** x^{-2}
c i $x^5 - 5x^3 + 1$ **ii** $\dfrac{x^2 + 4}{x^2 - 1}$

8 a $\frac{x^2}{2} + 2x + C$
b $\frac{x^4}{4} + x^3 - \frac{5x^2}{2} + x + C$
c $\frac{x^3}{3} - \frac{x^2}{2} + C$
d $-\frac{x^3}{2} + \frac{5x^2}{2} - 6x + C$
e $-x^{-1} + C$
f $-\frac{1}{6x^6} + C$
g $\dfrac{2x^{\frac{2}{3}}}{3} + C$
h $\frac{1}{5}(x + 1)^5 + C$
i $\frac{1}{12}(2x - 3)^6 + C$

9 a $9\frac{1}{3}$ u^2 **b** 4 u^2 **c** $\frac{4}{3}$ u^2 **d** 1 u^2
e $\frac{1}{6}$ u^2 **f** $\frac{4}{15}$ u^2 **g** $\frac{1}{6}$ u^2 **h** $4\frac{1}{2}$ u^2

10 b $\frac{4}{3}$ u^2

11 a 9 **b** 0.56

12 a $18(3x + 4)^5$
b i $(3x + 4)^6 + C$
ii $\frac{1}{2}(3x + 4)^6 + C$

13 a $6x(x^2 - 1)^2$
b i $(x^2 - 1)^3 + C$
ii $\frac{1}{6}(x^2 - 1)^3 + C$

14 a $\frac{1}{5}(x^3 + 1)^5 + C$
b $-\frac{1}{2(x^2 - 5)^2} + C$

16 a There is an asymptote at $x = 1$.
b $\sqrt{9 - 3x}$ is undefined for $x > 3$.
c $\log_e(x - 2)$ is undefined for $x \le 2$ (or, there is an asymptote at $x = 2$).
d There is an asymptote at $x = 1$.

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Chapter 5

Exercise 5A

1 a 2^{10} **b** e^7 **c** 2^4
d e^3 **e** 2^{12} **f** e^{30}

2 a e^{7x} **b** e^{2x} **c** e^{10x}
d e^{-5x} **e** e^{-3x} **f** e^{-12x}

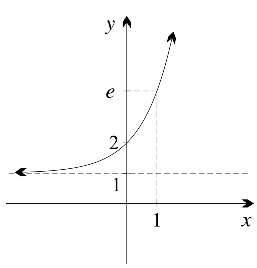
3 a 7.389 **b** 0.04979 **c** $e^1 \doteqdot 2.718$
d $e^{-1} \doteqdot 0.3679$ **e** $e^{\frac{1}{2}} \doteqdot 1.649$ **f** $e^{-\frac{1}{2}} \doteqdot 0.6065$

4 a $y' = e^x$ and $y'' = e^x$
b 'The curve $y = e^x$ is always concave up, and is always increasing at an increasing rate.'

5 a gradient $= e$, $y = ex$.
b $y = x + 1$
c $y = \frac{1}{e}(x + 2)$

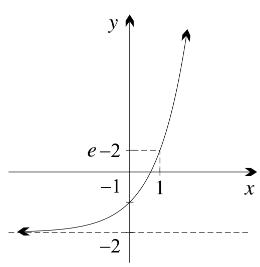
6 a $P = (1, e - 1)$
b $\dfrac{dy}{dx} = e^x$. When $x = 1$, $\dfrac{dy}{dx} = e$.
c tangent: $ex - y - 1 = 0$,
normal: $x + ey - e^2 + e - 1 = 0$

7 a



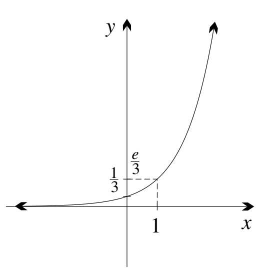
Shift e^x up 1

b



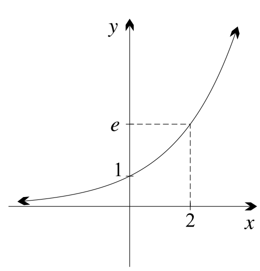
Shift e^x down 2

c



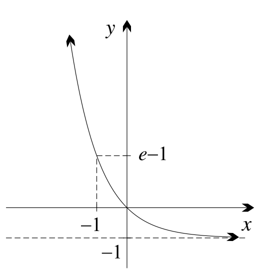
Stretch e^x vertically with factor $\frac{1}{3}$

d



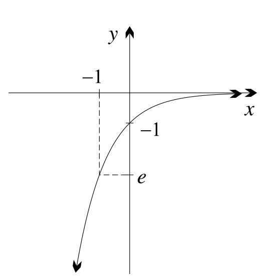
Stretch e^x horizontally with factor 2

8 a



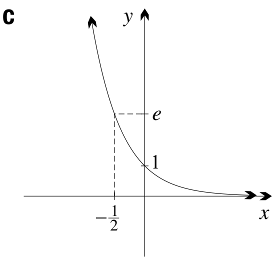
Shift e^{-x} down 1

b



Reflect e^{-x} in x-axis

c



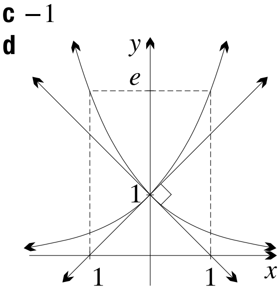
Stretch e^{-x} horizontally with factor $-\frac{1}{2}$

9 It is a vertical dilation of $y = e^x$ with factor $-\frac{1}{3}$. Its equation is $y = -\frac{1}{3}e^x$.

10 a $e^{2x} - 1$ **b** $e^{6x} + 3e^{4x} + 3e^{2x} + 9$
c $1 - 2e^{3x}$ **d** $e^{-4x} + 2 + e^{4x}$

11 a $e^{2x} + e^x$ **b** $e^{-2x} - e^{-x}$
c $e^{20x} + 5e^{30x}$ **d** $2e^{-4x} + 3e^{-5x}$

12 a 1
b Reflection in y-axis
c -1
d



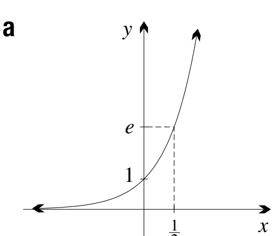
e Horizontal dilation with factor -1

13 a e^x, e^x, e^x, e^x
b $e^x + 3x^2, e^x + 6x, e^x + 6, e^x$
c $4e^x, 4e^x, 4e^x, 4e^x$
d $5e^x + 10x, 5e^x + 10, 5e^x, 5e^x$. In part **c**, the gradient equals the height.

14 a 1, 45° **b** e, 69°48′
c e^{-2}, 7°42′ **d** e^5, 89°37′

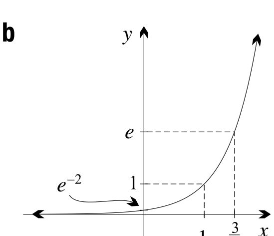
15 a $e - 1$
b $\dfrac{dy}{dx} = e^x$. When $x = 1$, $\dfrac{dy}{dx} = e$.
c $y = ex - 1$

16 a



Stretch horizontally with factor $\frac{1}{2}$.

b



Shift right 1.

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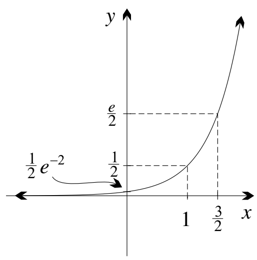
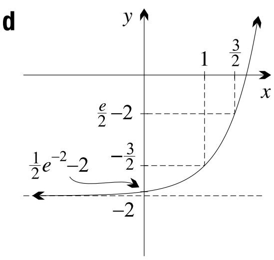
c



Stretch vertically with factor $\frac{1}{2}$.

d



Shift down 2.

17 a Shift left 2. Alternatively, $y = e^2e^x$, so it is a vertical dilation with factor e^2.

b Stretch vertically with factor 2. Alternatively, $y = e^{\log_e 2}e^x = e^{x+\log_e 2}$, so it is a shift left $\log_e 2$.

Exercise 5B

1 a $7e^{7x}$ **b** $12e^{3x}$ **c** $-5e^{5x}$ **d** $2e^{\frac{1}{3}x}$ **e** $7e^{-7x}$ **f** e^{-2x}

2 a $y' = e^{x-3}$ **b** $y' = 3e^{3x+4}$ **c** $y' = 2e^{2x-1}$ **d** $y' = 4e^{4x-3}$ **e** $y' = -3e^{-3x+4}$ **f** $y' = -2e^{-2x-7}$

3 a $e^x - e^{-x}$ **b** $2e^{2x} + 3e^{-3x}$ **c** $\frac{e^x + e^{-x}}{2}$ **d** $\frac{e^x - e^{-x}}{3}$ **e** $e^{2x} + e^{3x}$ **f** $e^{4x} + e^{5x}$

4 a $y' = 3e^{3x}$ **b** $y' = 2e^{2x}$ **c** $y' = 2e^{2x}$ **d** $y' = 6e^{6x}$ **e** $y' = 3e^{3x}$ **f** $y' = -e^{-x}$ **g** $y' = -3e^{-3x}$ **h** $y' = -5e^{-5x}$

5 a i $-e^{-x}, e^{-x}, -e^{-x}, e^{-x}$

ii Successive derivatives alternate in sign. More precisely, $f^{(n)}(x) = \begin{cases} e^{-x}, & \text{if } n \text{ is even,} \\ -e^{-x}, & \text{if } n \text{ is odd.} \end{cases}$

b i $2e^{2x}, 4e^{2x}, 8e^{2x}, 16e^{2x}$

ii Each derivative is twice the previous one. More precisely, $f^{(n)}(x) = 2^n e^{2x}$.

6 a $2e^{2x} + e^x$ **b** $e^{-x} - 4e^{-2x}$ **c** $2e^{2x} + 2e^x$ **d** $2e^{2x} + 6e^x$ **e** $2e^{2x} - 2e^x$ **f** $2e^{2x} - 4e^x$ **g** $2(e^{2x} + e^{-2x})$ **h** $10(e^{10x} + e^{-10x})$

7 a $2e^{2x+1}$ **b** $2xe^{x^2}$ **c** $-xe^{-\frac{1}{2}x}$ **d** $2xe^{x^2+1}$ **e** $-2xe^{1-x^2}$ **f** $2(x+1)e^{x^2+2x}$ **g** $(1-2x)e^{6+x-x^2}$ **h** $(3x-1)e^{3x^2-2x+1}$

8 a $(x+1)e^x$ **b** $(1-x)e^{-x}$ **c** xe^x **d** $(3x+4)e^{3x-4}$ **e** $(2x-x^2)e^{-x}$ **f** $4xe^{2x}$ **g** $(x^2+2x-5)e^x$ **h** $x^2e^{2x}(3+2x)$

9 a $y' = \frac{x-1}{x^2}e^x$ **b** $y' = (1-x)e^{-x}$ **c** $y' = \frac{(x-2)e^x}{x^3}$ **d** $y' = (2x-x^2)e^{-x}$ **e** $y' = \frac{x}{(x+1)^2}e^x$ **f** $y' = -xe^{-x}$ **g** $y' = (7-2x)e^{-2x}$ **h** $y' = (x^2-2x-1)e^{-x}$

10 a $2e^{2x} + 3e^x$ **b** $4e^{4x} + 2e^{2x}$ **c** $-2e^{-2x} - 6e^{-x}$ **d** $-6e^{-6x} + 18e^{-3x}$ **e** $3e^{3x} + 2e^{2x} + e^x$ **f** $12e^{3x} + 2e^{2x} + e^{-x}$

11 a $-5e^x(1-e^x)^4$ **b** $16e^{4x}(e^{4x}-9)^3$ **c** $-\frac{e^x}{(e^x-1)^2}$ **d** $-\frac{6e^{3x}}{(e^{3x}+4)^3}$

13 a $f'(x) = 2e^{2x+1}$, $f'(0) = 2e$, $f''(x) = 4e^{2x+1}$, $f''(0) = 4e$

b $f'(x) = -3e^{-3x}$, $f'(1) = -3e^{-3}$, $f''(x) = 9e^{-3x}$, $f''(1) = 9e^{-3}$

c $f'(x) = (1-x)e^{-x}$, $f'(2) = -e^{-2}$, $f''(x) = (x-2)e^{-x}$, $f''(2) = 0$

d $f'(x) = -2xe^{-x^2}$, $f'(0) = 0$, $f''(x) = (4x^2-2)e^{-x^2}$, $f''(0) = -2$

14 a $y' = ae^{ax}$ **b** $y' = -ke^{-kx}$ **c** $y' = Ake^{kx}$ **d** $y' = -B\ell e^{-\ell x}$ **e** $y' = pe^{px+q}$ **f** $y' = pCe^{px+q}$ **g** $y' = \dfrac{pe^{px} - qe^{-qx}}{r}$ **h** $e^{ax} - e^{-px}$

15 a $3e^x(e^x+1)^2$

b $4(e^x - e^{-x})(e^x + e^{-x})^3$

c $(1 + 2x + x^2)e^{1+x} = (1+x)^2e^{1+x}$

d $(2x^2 - 1)e^{2x-1}$

e $\frac{e^x}{(e^x+1)^2}$

f $-\frac{2e^x}{(e^x-1)^2}$

16 a $y' = -e^{-x}$

b $y' = e^x$

c $y' = e^{-x} - 4e^{-2x}$

d $y' = -12e^{-4x} - 3e^{-3x}$

e $y' = e^x - 9e^{3x}$

f $y' = -2e^{-x} - 2e^{-2x}$

17 a $y' = \frac{1}{2}\sqrt{e^x}$ **b** $y' = \frac{1}{3}\sqrt{e^x}$ **c** $y' = -\frac{1}{2\sqrt{e^x}}$ **d** $y' = -\frac{1}{3\sqrt[3]{e^x}}$ **e** $\frac{1}{2\sqrt{x}}e^{\sqrt{x}}$ **f** $-\frac{1}{2\sqrt{x}}e^{-\sqrt{x}}$ **g** $-\frac{1}{x^2}e^{\frac{1}{x}}$ **h** $\frac{1}{x^2}e^{-\frac{1}{x}}$ **i** $\left(1 + \frac{1}{x^2}\right)e^{x-\frac{1}{x}}$ **j** $e^xe^{e^x} = e^{x+e^x}$

20 a -5 or 2

b $-\frac{1}{2}\left(1+\sqrt{5}\right)$ or $-\frac{1}{2}\left(1-\sqrt{5}\right)$

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Exercise 5C

1 a $A = \left(\frac{1}{2}, 1\right)$ **b** $y' = 2e^{2x-1}$ **c** $y = 2x$

2 a $R = \left(-\frac{1}{3}, 1\right)$ **b** $y' = 3e^{3x+1}$

c $-\frac{1}{3}$ **d** $3x + 9y - 8 = 0.$

3 a $-e$ **b** $\frac{1}{e}$

c $x - ey + e^2 + 1 = 0$

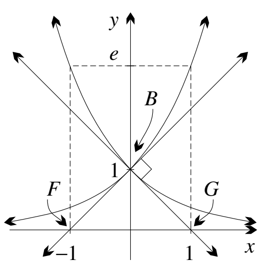
d $x = -e^2 - 1, y = e + e^{-1}$

e $\frac{1}{2}(e^3 + 2e + e^{-1})$

4 a 1 **b** $y = x + 1$

c -1 **d** $y = -x + 1$

e



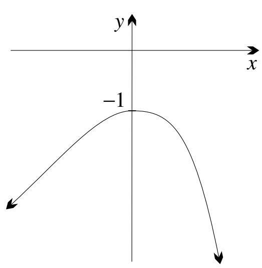
f isosceles right triangle, 1 square unit

5 a $1 - e$ **b** $y = (1 - e)x$

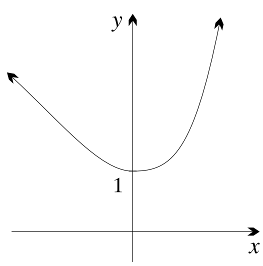
6 a $y' = 1 - e^x, y'' = -e^x$

c maximum turning point at $(0, -1)$

d $y \le -1$



e



7 a $y' = -xe^x$

b $y = e^{-1}(x + 3)$

c -3

8 a The x-intercept is -1 and the y-intercept is e.

b Area $= \frac{1}{2}e$

9 a $y' = 3e^{3x-6}, y' = 9e^{3x-6}$

b $3e^{3x-6}$ and $9e^{3x-6}$ are always positive.

c $(2, 1)$ **d** $3e^{-6}, -\frac{1}{3}e^6$

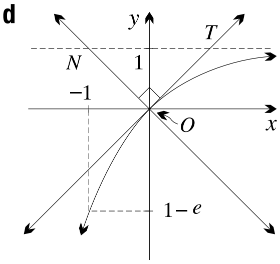
10 a $y' = -2xe^{-x^2}$

b $ex - 2y + (2e^{-1} - e) = 0$

c $1 - 2e^{-2}$

11 b $y = -x$ **c** $y = 1$

d



e 1 square unit

12 b $y = e^t(x - t + 1)$

c The x-intercept of each tangent to $y = e^x$ is 1 unit left of the x-value of the point of contact.

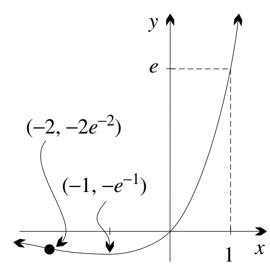
13 b $\lim\limits_{x \to -\infty} xe^x = 0$

d $\lim\limits_{x \to \infty} xe^{-x} = 0$

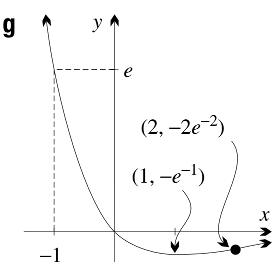
14 a There is a zero at $x = 0$, it is positive for $x > 0$ and negative for $x < 0$. It is neither even nor odd.

e They all tend towards ∞.

f $y \ge -e^{-1}$



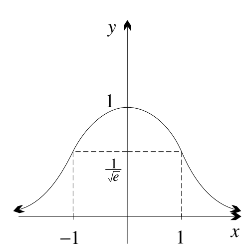
g



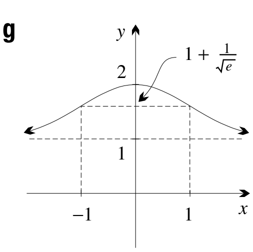
15 a It is always positive.

d $(1, e^{\frac{1}{2}})$ and $(-1, e^{\frac{1}{2}})$

f $0 < y \le 1$



g



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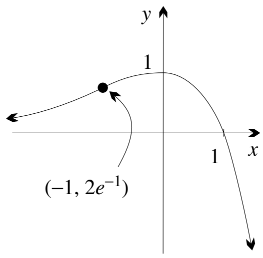
16 a

x	0	1	2
y	1	0	$-e^2$
sign	+	0	−

d They all tend to ∞.

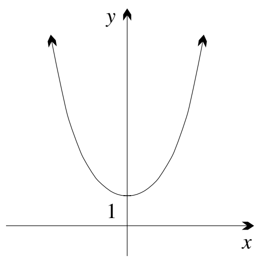
e $y \le 1$



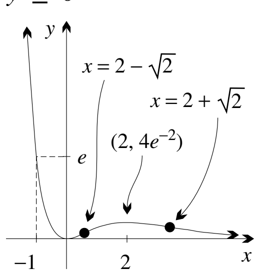
17 b $y' = \frac{e^x - e^{-x}}{2}$

d As $x \to \infty$, $y \to \infty$.

e



18 d $y \ge 0$



Exercise 5D

1 a $\frac{1}{2}e^{2x} + C$ **b** $\frac{1}{3}e^{3x} + C$
c $3e^{\frac{1}{3}x} + C$ **d** $2e^{\frac{1}{2}x} + C$
e $5e^{2x} + C$ **f** $4e^{3x} + C$

2 a $\frac{1}{4}e^{4x+5} + C$ **b** $\frac{1}{4}e^{4x-2} + C$
c $2e^{3x+2} + C$ **d** $e^{4x+3} + C$
e $-\frac{1}{2}e^{7-2x} + C$ **f** $-\frac{1}{6}e^{1-3x} + C$

3 a $e - 1$ **b** $e^2 - e$
c $e - e^{-3}$ **d** $e^2 - 1$
e $\frac{1}{2}(e^4 - 1)$ **f** $\frac{1}{3}(e^3 - e^{-3})$
g $4(e^5 - e^{-10})$ **h** $2(e^{12} - e^{-4})$
i $\frac{3}{2}(e^{18} - e^{-6})$

4 a $e - e^{-1}$ **b** $\frac{1}{2}(e^3 - e^{-1})$
c $\frac{1}{4}(e^{-3} - e^{-11})$ **d** $\frac{1}{3}(e^{-1} - e^{-4})$
e $\frac{e^2}{2}(e^2 - 1)$ **f** $\frac{e}{3}(e^2 - 1)$
g $2e^4(e^3 - 1)$ **h** $3e^3(e^4 - 1)$
i $4e^2(e^3 - 1)$

5 a $-e^{-x} + C$ **b** $-\frac{1}{2}e^{-2x} + C$
c $-\frac{1}{3}e^{-3x} + C$ **d** $e^{-3x} + C$
e $-3e^{-2x} + C$ **f** $4e^{2x} + C$

6 a $f(x) = \frac{1}{2}e^{2x} + C$, for some constant C
b $C = -2\frac{1}{2}$, so $f(x) = \frac{1}{2}e^{2x} - 2\frac{1}{2}$
c $f(1) = \frac{1}{2}e^2 - 2\frac{1}{2}$, $f(2) = \frac{1}{2}e^4 - 2\frac{1}{2}$

7 a $f(x) = x + 2e^x - 1$, $f(1) = 2e$
b $f(x) = 2 + x - 3e^x$, $f(1) = 3 - 3e$
c $f(x) = 1 + 2x - e^{-x}$, $f(1) = 3 - e^{-1}$
d $f(x) = 1 + 4x + e^{-x}$, $f(1) = 5 + e^{-1}$
e $f(x) = \frac{1}{2}e^{2x-1} + \frac{5}{2}$, $f(1) = \frac{1}{2}(e + 5)$
f $f(x) = 1 - \frac{1}{3}e^{1-3x}$, $f(1) = 1 - \frac{1}{3}e^{-2}$
g $f(x) = 2e^{\frac{1}{2}x+1} - 6$, $f(1) = 2e^{\frac{3}{2}} - 6$
h $f(x) = 3e^{\frac{1}{3}x+2} - 1$, $f(1) = 3e^{\frac{7}{3}} - 1$

8 a $\frac{1}{2}e^{2x} + e^x + C$
b $\frac{1}{2}e^{2x} - e^x + C$
c $e^{-x} - e^{-2x} + C$
d $\frac{1}{2}e^{2x} + 2e^x + x + C$
e $\frac{1}{2}e^{2x} + 6e^x + 9x + C$
f $\frac{1}{2}e^{2x} - 2e^x + x + C$
g $\frac{1}{2}e^{2x} - 4e^x + 4x + C$
h $\frac{1}{2}(e^{2x} + e^{-2x}) + C$
i $\frac{1}{10}(e^{10x} + e^{-10x}) + C$

9 a $\frac{1}{2}e^{2x+b} + C$ **b** $\frac{1}{7}e^{7x+q} + C$
c $\frac{1}{3}e^{3x-k} + C$ **d** $\frac{1}{6}e^{6x-\lambda} + C$
e $\frac{1}{a}e^{ax+3} + C$ **f** $\frac{1}{s}e^{sx+1} + C$
g $\frac{1}{m}e^{mx-2} + C$ **h** $\frac{1}{k}e^{kx-1} + C$
i $e^{px+q} + C$ **j** $e^{mx+k} + C$
k $\frac{A}{s}e^{sx-t} + C$ **l** $\frac{B}{k}e^{kx-\ell} + C$

10 a $-e^{1-x} + C$ **b** $-\frac{1}{3}e^{1-3x} + C$
c $-\frac{1}{2}e^{-2x-5} + C$ **d** $-2e^{1-2x} + C$
e $2e^{5x-2} + C$ **f** $-4e^{5-3x} + C$

11 a $x - e^{-x} + C$ **b** $e^x - e^{-x} + C$
c $\frac{1}{2}e^{-2x} - e^{-x} + C$ **d** $e^{-3x} - \frac{1}{2}e^{-2x} + C$
e $e^{-3x} - e^{-2x} + C$ **f** $e^{-x} - e^{-2x} + C$

12 a $y = e^{x-1}$, $y = e^{-1}$
b $y = e^2 + 1 - e^{2-x}$, $y = e^2 + 1$
c $f(x) = e^x + \frac{x}{e} - 1$, $f(0) = 0$
d $f(x) = e^x - e^{-x} - 2x$

13 a $e^2 - e$
b $\frac{1}{2}(e^2 - e^{-2}) + 4(e - e^{-1}) + 8$
c $e + e^{-1} - 2$
d $\frac{1}{4}(e^4 - e^{-4}) + \frac{1}{2}(e^{-2} - e^2)$

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e $e - e^{-1}$

f $e - e^{-1} + \frac{1}{2}(e^{-2} - e^{2})$

14 a **i** $2xe^{x^2+3}$ **ii** $e^{x^2+3} + C$

b **i** $2(x - 1)\,e^{x^2-2x+3}$ **ii** $\frac{1}{2}e^{x^2-2x+3} + C$

c **i** $(6x + 4)\,e^{3x^2+4x+1}$ **ii** $\frac{1}{2}e^{3x^2+4x+1} + C$

d **i** $3x^2e^{x^3}$ **ii** $\frac{1}{3}(1 - e^{-1})$

15 a $-\frac{1}{2}e^{-2x} + C$ **b** $-\frac{1}{3}e^{-3x} + C$

c $2e^{\frac{1}{2}x} + C$ **d** $3e^{\frac{1}{3}x} + C$

e $-2e^{-\frac{1}{2}x} + C$ **f** $-3e^{-\frac{1}{3}x} + C$

16 a **i** $y' = xe^{x}$ **ii** $e^{2} + 1$

b **i** $y' = -xe^{-x}$ **ii** $-1 - e^{2}$

17 a $2e^{\frac{1}{2}x} + \frac{2}{3}e^{-\frac{3}{2}x} + C$ **b** $\frac{3}{2}e^{\frac{2}{3}x} - \frac{3}{4}e^{-\frac{4}{3}x} + C$

18 b 0

Exercise 5E

1 a **i** $e - 1 \doteqdot 1.72$ **ii** $1 - e^{-1} \doteqdot 0.63$

iii $1 - e^{-2} \doteqdot 0.86$ **iv** $1 - e^{-3} \doteqdot 0.95$

b The total area is exactly 1.

2 a $(1 - e^{-1})$ square units

b $e(e^{2} - 1)$ square units

c $(e - e^{-1})$ square units

d $e - e^{-2}$ square units

3 a **i** $\frac{1}{2}(e^{6} - 1) \doteqdot 201.2$ square units

ii $\frac{1}{2}(1 - e^{-6}) \doteqdot 0.4988$ square units

b **i** $1 - e^{-1} \doteqdot 0.6321$ square units

ii $e - 1 \doteqdot 1.718$ square units

c **i** $3(e - 1) \doteqdot 5.155$ square units

ii $3(1 - e^{-1}) \doteqdot 1.896$ square units

4 a $e(e^{2} - 1)\,\text{u}^2$ **b** $e(e^{2} - 1)\,\text{u}^2$

c $\frac{1}{2}(e - e^{-1})\,\text{u}^2$ **d** $\frac{1}{3}(e - e^{-2})\,\text{u}^2$

e $(e^{2} - 1)\,\text{u}^2$ **f** $\frac{1}{2}e(e^{2} - 1)\,\text{u}^2$

g $3e^{2}(e - 1)\,\text{u}^2$ **h** $2(1 - e^{-2})\,\text{u}^2$

5 a $(e^{2} - 1)\,\text{u}^2$ **b** $2(e - e^{-\frac{1}{2}})\text{u}^2$

c $(1 - e^{-1})\,\text{u}^2$ **d** $2(e^{\frac{1}{2}} - e^{-1})\text{u}^2$

6 a $(3 - e^{-2}) \doteqdot 2.865\,\text{u}^2$ **b** $e^{-1} \doteqdot 0.3679\,\text{u}^2$

c $2(e^{2} - e^{-2}) \doteqdot 14.51\,\text{u}^2$

d $18 + e^{3} - e^{-3} \doteqdot 38.04\,\text{u}^2$

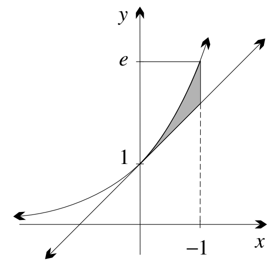
7 a $(1 + e^{-2})\,\text{u}^2$ **b** $1\,\text{u}^2$

c $e^{-1}\,\text{u}^2$ **d** $(3 + e^{-2})\,\text{u}^2$

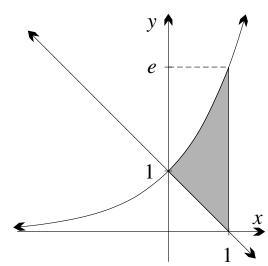
e $1\,\text{u}^2$

f $(9 + e^{-2} - e)\,\text{u}^2$

8 a $\int_0^1 (e^{x} - 1 - x)\,dx = \left(e - 2\frac{1}{2}\right)\text{u}^2$



b $\int_0^1 (e^{x} - 1 + x)\,dx = \left(e - 1\frac{1}{2}\right)\text{u}^2$



9 a The region is symmetric, so the area is twice the area in the first quadrant.

b $2 - \frac{2}{e}$ square units

10 a The region is symmetric, so the area is twice the area in the first quadrant.

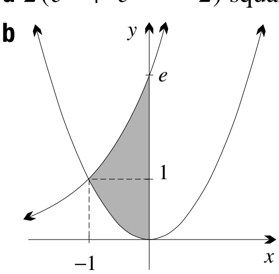
b 2 square units

11 b 0

c The region is symmetric, so the area is twice the area in the first quadrant.

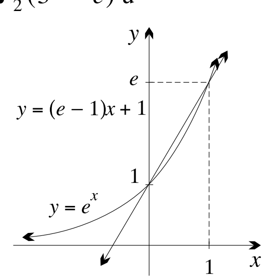
d $2(e^{3} + e^{-3} - 2)$ square units

12 b



c $\left(e - 1\frac{1}{3}\right)\text{u}^2$

13 b $\frac{1}{2}(3 - e)\,\text{u}^2$



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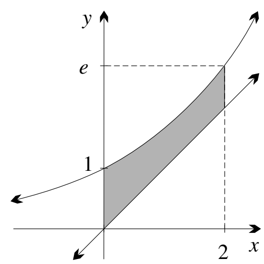
14 $(e^2 - 3)$ square units



15 a $e - 1 \doteqdot 1.7183$ **b** 1.7539

c The trapezoidal-rule approximation is greater. The curve is concave down, so all the chords are above the curve.

16 a 0.8863 square units **b** 3.5726 square units

17 a i $1 - e^N$ **ii** 1

b i $1 - e^{-N}$ **ii** 1

18 a $-\frac{1}{2}e^{-x^2}$

b From $x = 0$ to $x = 2$, area $= \frac{1}{2} - \frac{1}{2}e^{-4}$ square units. The function is odd, so the area (not signed) from $x = -2$ to $x = 2$ is $1 - e^{-4}$ square units.

Exercise 5F

1 a 2.303 **b** -2.303 **c** 11.72

d -12.02 **e** 3.912 **f** -3.912

2 a 3 **b** -1 **c** -2 **d** $\frac{1}{2}$

e 5 **f** 0.05 **g** 1 **h** e

3 b $1 = e^0$, so $\log_e 1 = \log_e e^0 = 0$.

d $e = e^1$, so $\log_e e = \log_e e^1 = 1$.

4 a $\log_e x = 6$ **b** $x = e^{-2}$ or $x = 1/e^2$

c $e^x = 24$ **d** $x = \log_e \frac{1}{3}$

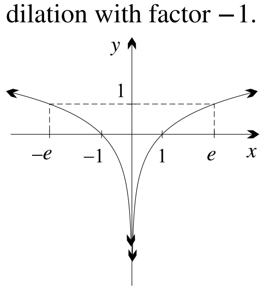
5 a $\dfrac{\log_e 7}{\log_e 2} \doteqdot 2.807$ **b** $\dfrac{\log_e 25}{\log_e 10} \doteqdot 1.398$

c $\dfrac{\log_e 0.04}{\log_e 3} \doteqdot -2.930$

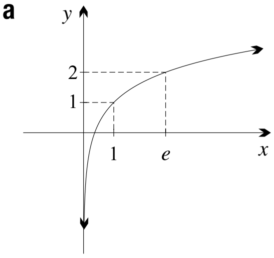
6 a Reflection in $y = x$, which reflects lines with gradient 1 to lines of gradient 1. The tangent to $y = e^x$ at its y-intercept has gradient 1, so its reflection also has gradient 1.

b Reflection in the y-axis, which is also a horizontal dilation with factor -1.

c

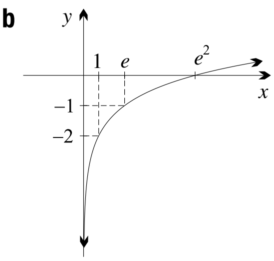


7 a



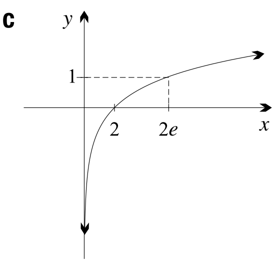
Shift $y = \log_e x$ up 1.

b



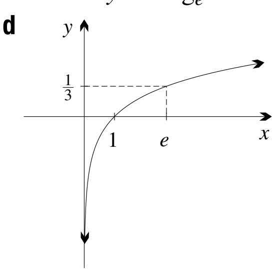
Shift $y = \log_e x$ down 2.

c



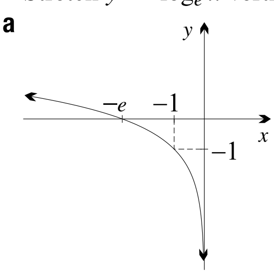
Stretch $y = \log_e x$ horizontally with factor 2.

d



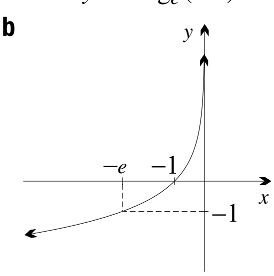
Stretch $y = \log_e x$ vertically with factor $\frac{1}{3}$.

8 a



Shift $y = \log_e(-x)$ down 1.

b



Reflect $y = \log_e(-x)$ in the x-axis.

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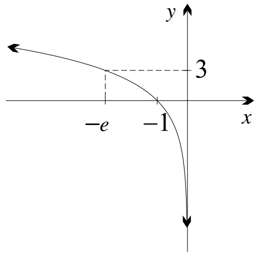
c



Stretch $y = \log_e(-x)$ vertically with factor 3.

9 It is a horizontal dilation of $y = \log_e(-x)$ with factor $\frac{1}{2}$. Its equation is $y = \log_e(-2x)$.

10 a $x = 1$ or $x = \log_2 7$

b $x = 2$ ($3^x = -1$ has no solutions.)

c **i** $x = 2$ or $x = 0$

ii $x = 0$ or $x = \log_3 4$

iii $x = \log_3 5$ ($3^x = -4$ has no solutions.)

iv The quadratic has no solutions because $\triangle < 0$

v $x = 2$

vi $x = 1$ or 2

11 a $x = 0$

b $x = \log_e 2$

c $x = 0$ or $x = \log_e 3$

d $x = 0$

12 a $x = e$ or $x = e^4$ **b** $x = 1$ or $x = e^3$

13 a **i** e **ii** 1 **iii** 0

b **i** ln 20 **ii** ln 5

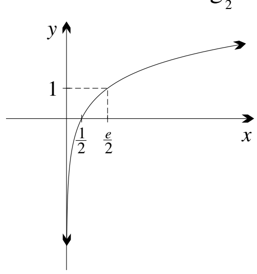
iii ln 8 or 3 ln 2

14 a $x = 1$ or $x = \log_4 3 \doteqdot 0.792$

b $x = \log_{10} \frac{1 + \sqrt{5}}{2} \doteqdot 0.209$. $\log_{10} \frac{1 - \sqrt{5}}{2}$ does not exist because $\frac{1 - \sqrt{5}}{2}$ is negative.

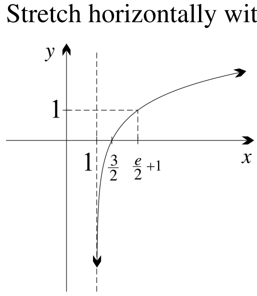
c $x = -1$ or $x = \log_{\frac{1}{2}} 2 \doteqdot -0.431$

15 a



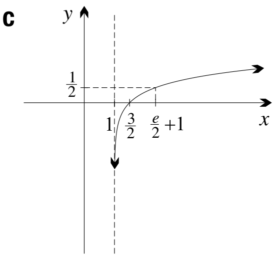
Stretch horizontally with factor $\frac{1}{2}$.

b



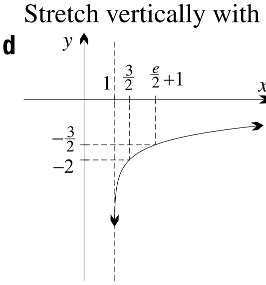
Shift right 1.

c



Stretch vertically with factor $\frac{1}{2}$.

d



Shift down 2.

16 a Stretch horizontally with factor $\frac{1}{5}$. Alternatively, $y = \log_e x + \log_e 5$, so it is a shift up $\log_e 5$.

b Shift up 2. Alternatively, $y = \log_e x + \log_e e^2 = \log_e e^2 x$, so it is a dilation horizontally with factor e^{-2}.

Exercise 5G

1 a $y' = \frac{1}{x + 2}$ **b** $y' = \frac{1}{x - 3}$

c $y' = \frac{3}{3x + 4}$ **d** $y' = \frac{2}{2x - 1}$

e $y' = \frac{-4}{-4x + 1}$ **f** $y' = \frac{-3}{-3x + 4}$

g $y' = \frac{-2}{-2x - 7} = \frac{2}{2x + 7}$

h $y' = \frac{6}{2x + 4} = \frac{3}{x + 2}$

i $y' = \frac{15}{3x - 2}$

2 a $y = \log_e 2 + \log_e x$, $y' = \frac{1}{x}$

b $y = \log_e 5 + \log_e x$, $y' = \frac{1}{x}$

c $\frac{1}{x}$ **d** $\frac{1}{x}$ **e** $\frac{4}{x}$ **f** $\frac{3}{x}$ **g** $\frac{4}{x}$ **h** $\frac{3}{x}$

3 a $y' = \frac{1}{x + 1}$, $y'(3) = \frac{1}{4}$

b $y' = \frac{2}{2x - 1}$, $y'(3) = \frac{2}{5}$

c $y' = \frac{2}{2x - 5}$, $y'(3) = 2$

d $y' = \frac{4}{4x + 3}$, $y'(3) = \frac{4}{15}$

e $y' = \frac{5}{x + 1}$, $y'(3) = \frac{5}{4}$

f $y' = \frac{12}{2x + 9}$, $y'(3) = \frac{4}{5}$

4 a $\frac{1}{x}$ **b** $\frac{-1}{x + 1}$

c $1 + \frac{4}{x}$ **d** $8x^3 + \frac{3}{x}$

e $\frac{2}{2x - 1} + 6x$ **f** $3x^2 - 3 + \frac{5}{5x - 7}$

5 a $y = 3 \ln x$, $y' = \frac{3}{x}$

b $y = 2 \ln x$, $y' = \frac{2}{x}$

c $y = -3 \ln x$, $y' = -\frac{3}{x}$

d $y = -2 \ln x$, $y' = -\frac{2}{x}$

e $y = \frac{1}{2} \ln x$, $y' = \frac{1}{2x}$

f $y = \frac{1}{2} \ln(x + 1)$, $y' = \frac{1}{2(x + 1)}$

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6 a $\frac{1}{x}$ **b** $\frac{1}{x}$ **c** $\frac{3}{x}$
d $-\frac{6}{x}$ **e** $1+\frac{1}{x}$ **f** $12x^2-\frac{1}{x}$

7 a $\frac{2x}{x^2+1}$ **b** $\frac{-2x}{2-x^2}$ **c** $\frac{e^x}{1+e^x}$

8 a $\frac{2x+3}{x^2+3x+2}$ **b** $\frac{6x^2}{1+2x^3}$ **c** $\frac{e^x}{e^x-2}$
d $1-\frac{2x+1}{x^2+x}$ **e** $2x+\frac{3x^2-1}{x^3-x}$
f $12x^2-10x+\frac{4x-3}{2x^2-3x+1}$

9 a $1, 45°$ **b** $\frac{1}{3}, 18°26'$
c $2, 63°26'$ **d** $\frac{1}{4}, 14°2'$

10 a $1+\log_e x$ **b** $\frac{2x}{2x+1}+\log_e(2x+1)$
c $\frac{2x+1}{x}+2\log_e x$ **d** $x^3(1+4\log_e x)$
e $\log_e(x+3)+1$ **f** $\frac{2(x-1)}{2x+7}+\log_e(2x+7)$
g $e^x\left(\frac{1}{x}+\log_e x\right)$ **h** $e^{-x}\left(\frac{1}{x}-\log_e x\right)$

11 a $\frac{1-\log_e x}{x^2}$ **b** $\frac{1-2\log_e x}{x^3}$ **c** $\frac{\log_e x-1}{(\log_e x)^2}$
d $\frac{x(2\log_e x-1)}{(\log_e x)^2}$ **e** $\frac{1-x\log_e x}{xe^x}$ **f** $\frac{e^x(x\log_e x-1)}{x(\log_e x)^2}$

12 a $\frac{3}{x}$ **b** $\frac{4}{x}$ **c** $\frac{1}{3x}$ **d** $\frac{1}{4x}$
e $-\frac{1}{x}$ **f** $-\frac{1}{x}$ **g** $\frac{1}{2x-4}$ **h** $\frac{5}{10x+4}$

13 a $f'(x)=\frac{1}{x-1}, f'(3)=\frac{1}{2}, f''(x)=-\frac{1}{(x-1)^2}$,
$f''(3)=-\frac{1}{4}$
b $f'(x)=\frac{2}{2x+1}, f'(0)=2, f''(x)=-\frac{4}{(2x+1)^2}$,
$f''(0)=-4$
c $f'(x)=\frac{2}{x}, f'(2)=1, f''(x)=-\frac{2}{x^2}, f''(2)=-\frac{1}{2}$
d $f'(x)=1+\log x, f'(e)=2, f''(x)=\frac{1}{x}$,
$f''(e)=\frac{1}{e}$

14 a $\log_e x, x=1$
b $x(1+2\log_e x), x=e^{-\frac{1}{2}}$
c $\frac{1-\log_e x}{x^2}, x=e$
d $\frac{2\log_e x}{x}, x=1$
e $\frac{4(\log_e x)^3}{x}, x=1$
f $\frac{-1}{x(1+\log_e x)^2}$ is never zero.
g $\frac{8}{x}(2\log_e x-3)^3, x=e^{\frac{3}{2}}$
h $\frac{-1}{x(\log_e x)^2}$ is never zero.
i $\frac{1}{x\log_e x}$ is never zero.

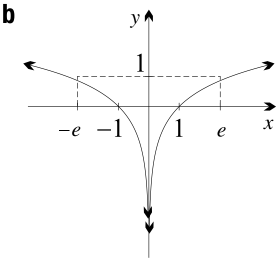
15 a $\left(\frac{1}{e}, -\frac{1}{e}\right)$ **b** $(1, 1)$

16 a $y'=\dfrac{\ln x-1}{(\ln x)^2}$

17 a $\frac{1}{x+2}+\frac{1}{x+1}$ **b** $\frac{1}{x+5}+\frac{3}{3x-4}$ **c** $\frac{1}{1+x}+\frac{1}{1-x}$
d $\frac{3}{3x-1}-\frac{1}{x+2}$ **e** $\frac{2}{x-4}-\frac{3}{3x+1}$ **f** $\frac{1}{x}+\frac{1}{2(x+1)}$

18 a $y=x\log_e 2, y'=\log_e 2$
b $y=x, y'=1$
c $y=x\log_e x, y'=1+\log_e x$

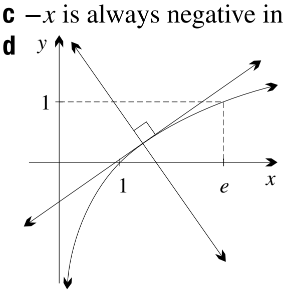
19 a $\log_e|x|=\begin{cases}\log_e x, & \text{for } x>0,\\ \log_e(-x), & \text{for } x<0.\end{cases}$
b



c For $x>0$, $\log_e|x|=\log_e x$, so $\dfrac{d}{dx}\log_e x=\dfrac{1}{x}$.
For $x<0$, $\log_e|x|=\log_e(-x)$, and using the standard form, $\dfrac{d}{dx}\log_e(-x)=-\dfrac{1}{-x}=\dfrac{1}{x}$.
d $\log_e 0$ is undefined. In fact, $\log_e x\to-\infty$ as $x=0$, so $x=0$ is an asymptote.

Exercise 5H

1 a $y'=\frac{1}{x}$ **b** $\frac{1}{e}$ **c** $y=\frac{1}{e}x$
2 a 1 **b** $y=x-1$
3 a e **b** $y=ex-2$
4 a 1
c $y=-x+1$. When $x=0$, $y=1$.
5 a $y=4x-4, y=-\frac{1}{4}x+\frac{1}{4}$
b $y=x+2, y=-x+4$
c $y=2x-4, y=-\frac{1}{2}x-1\frac{1}{2}$
d $y=-3x+4, y=\frac{1}{3}x+\frac{2}{3}$
6 b $3, -\frac{1}{3}$
c $y=3x-3, -3, y=-\frac{1}{3}x+\frac{1}{3}, \frac{1}{3}$
d $\frac{5}{3}$ square units
7 a $\frac{1}{2}$ **b** $y=\frac{1}{2}x$
8 a $y=-\ln 2\times(x-2)$ **b** $2\ln 2$
9 a $x>0, y=\frac{1}{x}$
b $\frac{1}{x}$ is always positive in the domain.
c $-x$ is always negative in the domain.
d



e $y''=-\frac{1}{x^2}$. It is always concave down.
10 a $(2, \log_e 2), y=\frac{1}{2}x-1+\log_e 2$,
$y=-2x+4+\log_e 2$
b $\left(\frac{1}{2}, -\log_e 2\right), y=2x-1-\log_e 2$,
$y=-\frac{1}{2}x+\frac{1}{4}-\log_e 2$

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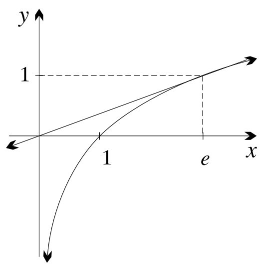
11 a As P moves left along the curve, the tangent becomes steeper, so it does not pass through the origin.
As P moves right, the angle of the tangent becomes less steep, hence it does not pass through the origin.



b There are no tangents through each point below the curve. There are two tangents through each point above the curve and to the right of the y-axis. There is one tangent through each point on the curve, and through each point on and to the left of the y-axis.

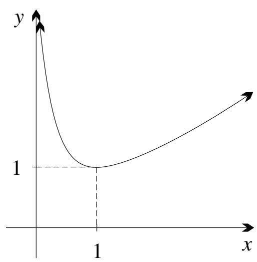
12 a $x > 0$. The domain is not symmetric about the origin, so the function is certainly not even or odd.

b $y' = 1 - \frac{1}{x}$, $y'' = \frac{1}{x^2}$

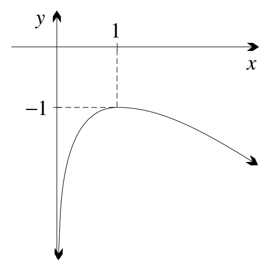
c $y'' > 0$, for all x

d $(1, 1)$

e $y \ge 1$

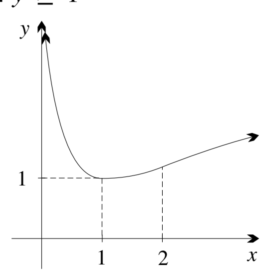


f



13 a $x > 0$

d $y \ge 1$



14 a $\lim\limits_{x \to \infty} \dfrac{\log_e x}{x} = 0$

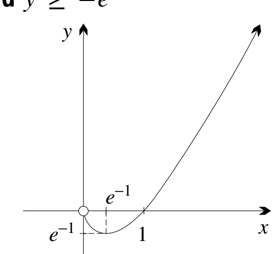
b $\lim\limits_{x \to 0^+} x \log_e x = 0$

15 a $x > 0$, $(1, 0)$

b $y'' = \frac{1}{x}$

c $(e^{-1}, -e^{-1})$ is a minimum turning point.

d $y \ge -e^{-1}$



16 a $x > 0$, $(e, 0)$

b

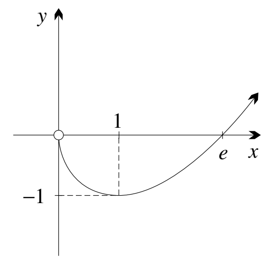
x	1	e	e^2
y	-1	0	e^2
sign	$-$	0	$+$

c $y'' = \dfrac{1}{x}$

d $(1, -1)$ is a minimum turning point.

e It is concave up throughout its domain.

f $y \ge -1$



17 a all real x

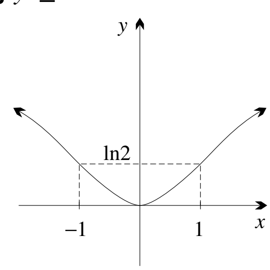
b Even

c It is zero at $x = 0$, and is positive otherwise because the logs of numbers greater than 1 are positive.

e $(0, 0)$ is a minimum turning point.

f $(1, \log_e 2)$ and $(-1, \log_e 2)$

g $y \ge 0$

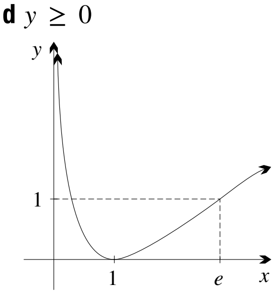


18 a $x > 0$

b It is zero at $x = 1$, and is positive otherwise because squares cannot be negative.

c $y' = \frac{2}{x} \ln x$

d $y \ge 0$



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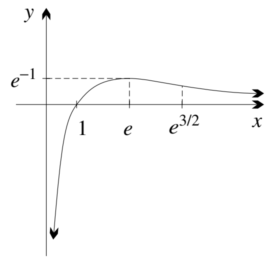
19 a $x > 0$

c (e, e^{-1}) is a maximum turning point.

d $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$

e $y \le e^{-1}$



Exercise 5I

1 a $2\log_e|x| + C$ **b** $5\log_e|x| + C$

c $\frac{1}{2}\log_e|x| + C$ **d** $\frac{1}{3}\log_e|x| + C$

e $\frac{4}{5}\log_e|x| + C$ **f** $\frac{3}{2}\log_e|x| + C$

2 a $\frac{1}{4}\log_e|4x + 1| + C$ **b** $\frac{1}{5}\log_e|5x - 3| + C$

c $2\log_e|3x + 2| + C$ **d** $3\log_e|5x + 1| + C$

e $\log_e|4x + 3| + C$ **f** $-\log_e|3 - x| + C$

g $-\frac{1}{2}\log_e|7 - 2x| + C$ **h** $\frac{4}{5}\log_e|5x - 1| + C$

i $-4\log_e|1 - 3x| + C$

3 a $\log_e 5$

b $\log_e 3$

c $\log_e|-2| - \log_e|-8| = -2\log_e 2$

d The integral is meaningless because it runs across an asymptote at $x = 0$.

e $\frac{1}{2}(\log_e 8 - \log_e 2) = \log_e 2$

f $\frac{1}{5}(\log_e|-75| - \log_e|-25|) = -\frac{1}{5}\log_e 3$

4 a $\log_e 2 \doteqdot 0.6931$

b $\log_e 3 - \log_e 5 \doteqdot -0.5108$

c $3\log_e 2 \doteqdot 2.079$ **d** $\frac{2}{3}\log_e 2 \doteqdot 0.4621$

e $-\frac{1}{2}\log_e 7 \doteqdot -0.9730$ **f** $\frac{3}{2}\log_e 3 \doteqdot 1.648$

g $\log_e\frac{5}{2} \doteqdot 0.9163$ **h** $\frac{3}{2}\log_e 5 \doteqdot 2.414$

i The integral is meaningless because it runs across an asymptote at $x = 5\frac{1}{2}$.

5 a 1 **b** 2 **c** 3 **d** $\frac{1}{2}$

6 a $x + \log_e|x| + C$ **b** $\frac{1}{5}x + \frac{3}{5}\log_e|x| + C$

c $\frac{2}{3}\log_e|x| - \frac{1}{3}x + C$ **d** $\frac{1}{9}\log_e|x| - \frac{8}{9}x + C$

e $3x - 2\log_e|x| + C$

f $x^2 + x - 4\log_e|x| + C$

g $\frac{3}{2}x^2 + 4\log_e|x| + \frac{1}{x} + C$

h $\frac{1}{3}x^3 - \log_e|x| - \frac{2}{x} + C$

7 a $\log_e|x^2 - 9| + C$

b $\log_e|3x^2 + x| + C$

c $\log_e|x^2 + x - 3| + C$

d $\log_e|2 + 5x - 3x^2| + C$

e $\frac{1}{2}\log_e|x^2 + 6x - 1| + C$

f $\frac{1}{4}\log_e|12x - 3 - 2x^2| + C$

g $\log_e(1 + e^x) + C$

h $-\log_e(1 + e^{-x}) + C$

i $\log_e(e^x + e^{-x}) + C$

The denominators in parts **g**–**i** are never negative, so the absolute value sign is unnecessary.

8 a $f(x) = x + 2\ln|x|, f(2) = 2 + 2\ln 2$

b $f(x) = x^2 + \frac{1}{3}\ln|x| + 1, f(2) = 5 + \frac{1}{3}\ln 2$

c $f(x) = 3x + \frac{5}{2}\ln|2x - 1| - 3$, $f(2) = 3 + \frac{5}{2}\ln 3$

d $f(x) = 2x^3 + 5\ln|3x + 2| - 2$, $f(2) = 14 + 5\ln 8$

9 a $y = \frac{1}{4}(\log_e|x| + 2), x = e^{-2}$

b $y = 2\log_e|x + 1| + 1$

c $y = \log_e\left|\frac{x^2 + 5x + 4}{10}\right| + 1, y(0) = \log_e\frac{4}{10} + 1$

d $y = 2\log_e|x| + x + C, y = 2\log_e|x| + x$, $y(2) = \log_e 4 + 2$

e $f(x) = 2 + x - \log_e|x|, f(e) = e + 1$

10 a $\frac{1}{2}\log_e|2x + b| + C$ **b** $\frac{1}{3}\log_e|3x - k| + C$

c $\frac{1}{a}\log_e|ax + 3| + C$ **d** $\frac{1}{m}\log_e|mx - 2| + C$

e $\log_e|px + q| + C$ **f** $\frac{A}{S}\log_e|sx - t| + C$

11 a $\log_e|x^3 - 5| + C$

b $\log_e|x^4 + x - 5| + C$

c $\frac{1}{4}\log_e|x^4 - 6x^2| + C$

d $\frac{1}{2}\log_e|5x^4 - 7x^2 + 8| + C$

e $2\log_e 2$

f $\log_e\frac{4(e + 1)}{e + 2}$

12 a $f(x) = x + \ln|x| + \frac{1}{2}x^2$

b $g(x) = x^2 - 3\ln|x| + \frac{4}{x} - 6$

13 $\frac{1}{3}(e^3 - e^{-3}) + 2$

14 a $y' = \log_x$

b i $x\log_e x - x + C$ **ii** $\frac{\sqrt{e}}{2}$

15 b $\frac{1}{2}x^2\log_e x - \frac{1}{4}x^2$ **c** $2\log 2 - 1 - \frac{e^2}{4}$

16 a $\frac{2\log_e x}{x}$ **b** $\dfrac{3}{8}$

17 $\ln(\ln x) + C$

18 The key to all this is that $\log_e|5x| = \log_e 5 + \log_e|x|$, so that $\log_e|x|$ and $\log_e|5x|$ differ only by a constant $\log_e 5$. Thus $C_2 = C_1 - \frac{1}{5}\log_e 5$, and because C_1 and C_2 are arbitrary constants, it does not matter at all. In particular, in a definite integral, adding a constant doesn't change the answer, because it cancels out when we take $F(b) - F(a)$.

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19 a i $a = e^5$ ii $a = e^{-4}$

b i $a = -e^2$ ii $a = -e^{-1}$

20 a $\log_e\left(1 + \sqrt{2}\right)$ **b** $\log_e\left(2 + \sqrt{3}\right)$

Exercise 5J

1 b $e \doteqdot 2.7$

2 a i 1u^2 ii $\log_e 5 \doteqdot 1.609\,\text{u}^2$

b i 1u^2 ii $2\log_e 2 \doteqdot 1.386\,\text{u}^2$

iii 14u^2 iv $14\log_e 5 \doteqdot 22.53\,\text{u}^2$

3 a $\log_e 2$ square units

b $(\log_e 3 - \log_e 2)$ square units

c $-\log_e \frac{1}{3} = \log_e 3$ square units

d $\log_e 2 - \log_e \frac{1}{2} = 2\log_e 2$ square units

4 a i $\frac{1}{2}(\log_e 11 - \log_e 5) \doteqdot 0.3942\,\text{u}^2$

ii $\frac{1}{2}\log_e 3 \doteqdot 0.5493\,\text{u}^2$

b i $\frac{1}{3}(\log_e 5 - \log_e 2) \doteqdot 0.3054\,\text{u}^2$

ii $\frac{1}{3}\log_e 10 \doteqdot 0.7675\,\text{u}^2$

c i $\frac{1}{2}\log_e 3 \doteqdot 0.5493\,\text{u}^2$

ii $\log_e 3 \doteqdot 1.099\,\text{u}^2$

d i 9u^2

ii $3(\log_e 11 - \log_e 2) \doteqdot 5.114\,\text{u}^2$

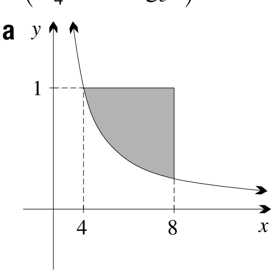
5 a $\log_e 2 + 1\,\text{u}^2$ **b** $2\log_e 2 + \frac{15}{8}\text{u}^2$

c $\log_e 3 + 8\frac{2}{3}\text{u}^2$

6 a $(6 - 3\log_e 3)\,\text{u}^2$ **b** $(4 - \log_e 3)\,\text{u}^2$

7 a $\left(3\frac{3}{4} - 2\log_e 4\right)\text{u}^2$ **b** $\left(3\frac{3}{4} - 2\log_e 4\right)\text{u}^2$

8 a



b $4(1 - \log_e 2)\,\text{u}^2$

9 a $2\log_e 2\text{u}^2$ **b** $(1 - \log_e 2)\,\text{u}^2$

10 a $(\log_e 4)\,\text{u}^2$ **b** $(6 - 3\log_e 3)\,\text{u}^2$

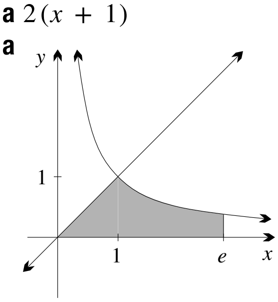
11 a $\frac{1}{2}\text{u}^2$ **b** $2\log_e \frac{4}{3}\text{u}^2$

12 a $\left(\frac{1}{3}, 3\right)$ and $(1, 1)$ **b** $\left(\frac{4}{3} - \log_e 3\right)\text{u}^2$

13 a $2x$ **b** $\frac{1}{2}\log_e 5 \doteqdot 0.805\,\text{u}^2$

14 a $2(x + 1)$ **b** $\frac{1}{2}\log_e 2\text{u}^2$

15 a



b $\frac{3}{2}\text{u}^2$

16 a $\ln 2 \doteqdot 0.693$ **b** $\frac{17}{24} \doteqdot 0.708$

17 a $\ln 3 \doteqdot 1.0986$ square units

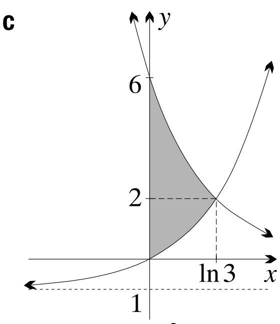
b 1.1667 square units

18 a 3.9828 square units

19 b $(e - 1)\,\text{u}^2$ **c** $1\,\text{u}^2$

20 b $(\ln 3, 2)$

c



d $(2 + \ln 3)\,\text{u}^2$

21 a $e - 2$ square units **b** e^{-1} square units

c $e - 2 + e^{-1}$ square units

Exercise 5K

1 a 1.58 **b** 3.32 **c** 2.02 **d** −4.88

2 a $y' = \frac{1}{x\log_e 2}$ **b** $y' = \frac{1}{x\log_e 10}$ **c** $y' = \frac{3}{x\log_e 5}$

3 a $y' = \frac{1}{x\log_e 3}$ **b** $y' = \frac{1}{x\log_e 7}$ **c** $y' = \frac{5}{x\log_e 6}$

4 a $3^x\log_e 3$ **b** $4^x\log_e 4$ **c** $2^x\log_e 2$

5 a $y' = 10^x\log_e 10$ **b** $y' = 8^x\log_e 8$

c $y' = 3 \times 5^x\log_e 5$

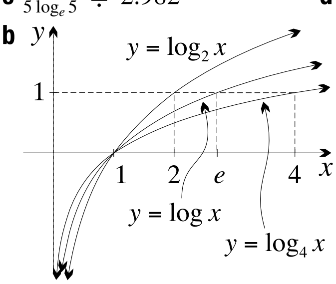
6 a $\frac{2^x}{\log_e 2} + C$ **b** $\frac{6^x}{\log_e 6} + C$

c $\frac{7^x}{\log_e 7} + C$ **d** $\frac{3^x}{\log_e 3} + C$

7 a $\frac{1}{\log_e 2} \doteqdot 1.443$ **b** $\frac{2}{\log_e 3} \doteqdot 1.820$

c $\frac{24}{5\log_e 5} \doteqdot 2.982$ **d** $\frac{15}{\log_e 4} \doteqdot 10.82$

8 b



9 a $\frac{1}{\log_e 2}$ **b** $y = \frac{1}{\log_e 2}(x - 1)$

c i $y = \frac{1}{\log_e 3}(x - 1)$ ii $y = \frac{1}{\log_e 5}(x - 1)$

10 a $\frac{6}{\log_e 2} \doteqdot 8.6562$ **b** $2 + \frac{8}{3\log_e 3} \doteqdot 4.4273$

c $\frac{99}{\log_e 10} - 20 \doteqdot 32.9952$

11 $y = \frac{\log_e x}{\log_e 10}$, $y' = \frac{1}{x\log_e 10}$

a $\frac{1}{10\log_e 10}$

b $x - 10y\log_e 10 + 10(\log_e 10 - 1) = 0$

c $x = \frac{1}{\log_e 10}$

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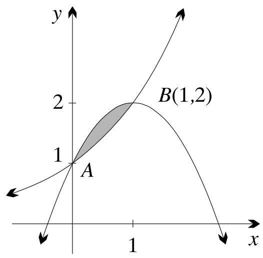
12 a $y = \frac{1}{\log_e 2}\left(\frac{x}{3} - 1 + \log_e 3\right)$, $y = \frac{x}{3} - 1 + \log_e 3$, $y = \frac{1}{\log_e 4}\left(\frac{x}{3} - 1 + \log_e 3\right)$

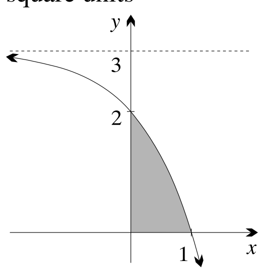
b They all meet the x-axis at $(3 - 3\log_e 3, 0)$

13 b $\left(\frac{5}{3} - \frac{1}{\log_e 2}\right)$ u^2



14 intercepts $(0, 7)$ and $(3, 0)$, area $\left(24 - \frac{7}{\log_e 2}\right)$ square units

15 a



b $\left(3 - \frac{2}{\log_e 3}\right)$ u^2

16 a $\int_{-\frac{1}{2}}^{0} x + 1 - 4^x dx$ **b** $\frac{3}{8} - \frac{1}{2\log_e 4}$

18 a $x\log_e x - x + C$ **b** $10 - \frac{9}{\log_e 10}$

19 a i $y' = \frac{1}{x\log_e 3}$ **ii** $y' = \frac{2}{(2x + 3)\log_e 7}$

iii $y' = -\frac{45}{(4 - 9x)\log_e 6}$

b i $y' = 10^x \log_e 10$

ii $y' = 4 \times 8^{4x-3} \log_e 8$

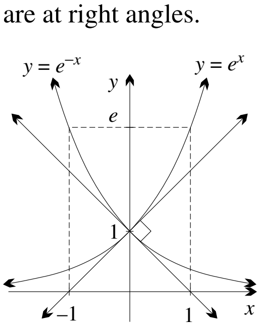
iii $y' = -21 \times 5^{2-7x} \log_e 5$

c i $\frac{3^{5x}}{5\log_e 3} + C$ **ii** $\frac{6^{2x+7}}{2\log_e 6} + C$

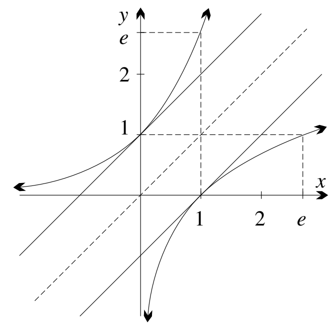
iii $-\frac{5 \times 7^{4-9x}}{9\log_e 7} + C$

Chapter 5 review exercise

1 a Each graph is reflected onto the other in the line $x = 0$. The tangents have gradients 1 and -1, and are at right angles.



b Each graph is reflected onto the other in the line $y = x$. The tangents both have gradients 1, and are thus parallel.

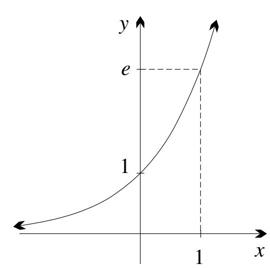


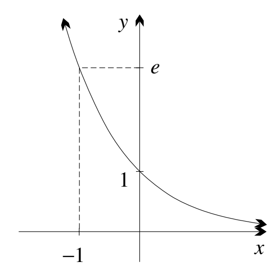
2 a 54.60	**b** 2.718	**c** 0.2231	**d** 0.6931
e -0.3010	**f** -5.059	**g** 130.6	**h** 0.5925
3 a 2.402	**b** 5.672	**c** 5.197	**d** 3.034
4 a e^{5x}	**b** e^{6x}	**c** e^{-4x}	**d** e^{9x}

5 a $x = 2$

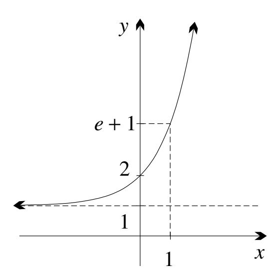
b $x = \log_e 4 (= 2\log_e 2)$ or $\log_e 7$

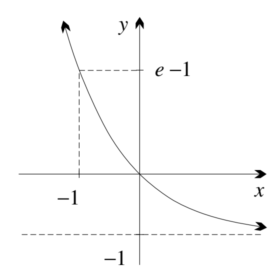
6 a $y > 0$ **b** $y > 0$





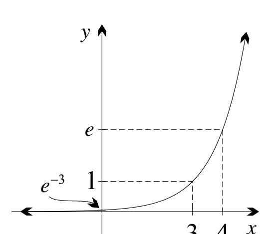
c $y > 1$ **d** $y > -1$





7 a i Shift $y = e^x$ right 3 units.

ii $y = e^{-3}e^x$ or $\frac{y}{e^{-3}} = e^x$, so dilate vertically with factor e^{-3}.



b i $y = \log_e \frac{x}{1/3}$, so dilate $y = \log_e x$ horizontally with factor $\frac{1}{3}$.

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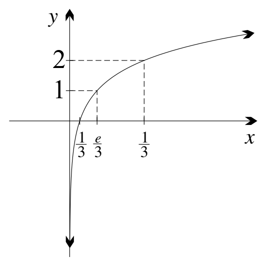
ii $y = \log_e x + \log_e 3$, or $y - \log_e 3 = \log_e x$, so shift $y = \log_e x$ up $\log_e 3$.



8 a e^x **b** $3e^{3x}$ **c** $2e^{2x+3}$ **d** $-e^{-x}$
e $-3e^{-3x}$ **f** $6e^{2x+5}$ **g** $2e^{\frac{1}{2}x}$ **h** $4e^{6x-5}$

9 a $5e^{5x}$ **b** $4e^{4x}$ **c** $-3e^{-3x}$ **d** $-6e^{-6x}$

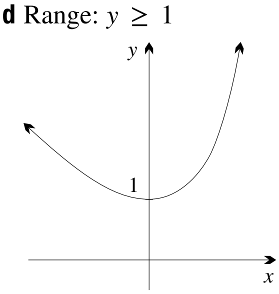
10 a $3x^2e^{x^3}$ **b** $(2x - 3)e^{x^2-3x}$
c $e^{2x} + 2xe^{2x} = e^{2x}(1 + 2x)$
d $6e^{2x}(e^{2x} + 1)^2$
e $\frac{e^{3x}(3x - 1)}{x^2}$
f $2xe^{x^2}(1 + x^2)$
g $5(e^x + e^{-x})(e^x - e^{-x})^4$
h $\frac{4xe^{2x}}{(2x + 1)^2}$

11 a $y' = 2e^{2x+1}$, $y'' = 4e^{2x+1}$
b $y' = 2xe^{x^2+1}$, $y'' = 2e^{x^2+1}(2x^2 + 1)$

12 $y = e^2x - e^2$, x-intercept 1, y-intercept $-e^2$.

13 a $\frac{1}{3}$
b When $x = 0$, $y'' = 9$, so the curve is concave up there.

14 a $y' = e^x - 1$, $y'' = e^x$
b $(0, 1)$ is a minimum turning point.
c $y'' = e^x$, which is positive for all x.
d Range: $y \geq 1$



15 $\left(\frac{1}{2}, \frac{1}{2e}\right)$ is a maximum turning point.

16 a $\frac{1}{5}e^{5x} + C$ **b** $-2e^{2-5x} + C$
c $5e^{\frac{1}{5}x} + C$ **d** $\frac{3}{5}e^{5x-4} + C$

17 a $e^2 - 1$ **b** $\frac{1}{2}(e^2 - 1)$
c $e - 1$ **d** $\frac{1}{3}(e^2 - 1)$
e $\frac{1}{2}e^2(e - 1)$ **f** $4(e - 1)$

18 a $-\frac{1}{5}e^{-5x} + C$ **b** $\frac{1}{4}e^{4x} + C$
c $-2e^{-3x} + C$ **d** $\frac{1}{6}e^{6x} + C$
e $-\frac{1}{2}e^{-2x} + C$ **f** $e^x - \frac{1}{2}e^{-2x} + C$
g $\frac{1}{3}e^{3x} + e^x + C$ **h** $x - 2e^{-x} - \frac{1}{2}e^{-2x} + C$

19 a $2 - e^{-1}$ **b** $\frac{1}{2}(e^4 + 3)$
c $2(1 - e^{-1})$ **d** $\frac{1}{3}(e - 2)$
e $e - e^{-1}$ **f** $\frac{1}{2}(e^2 + 4e - 3)$

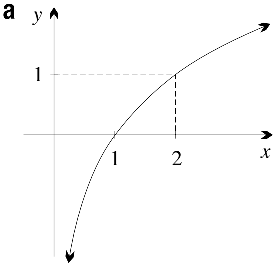
20 $f(x) = e^x + e^{-x} - x + 1$, $f(1) = e + e^{-1}$

21 a $3x^2e^{x^3}$ **b** $\frac{1}{3}(e - 1)$

22 a $3.19\ \text{u}^2$ **b** $0.368\ \text{u}^2$

23 a $\frac{1}{2}(1 + e^{-2})\ \text{u}^2$ **b** $\frac{1}{2}(3 - e)\ \text{u}^2$

24 a



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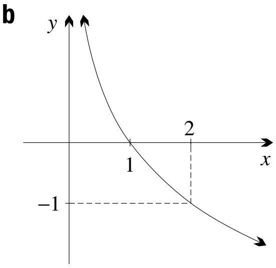

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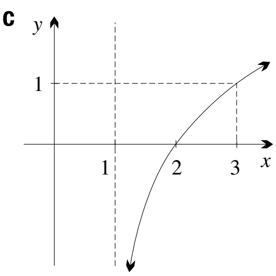
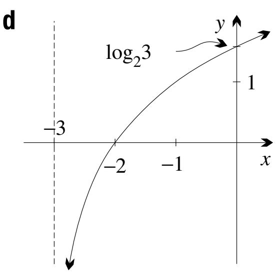
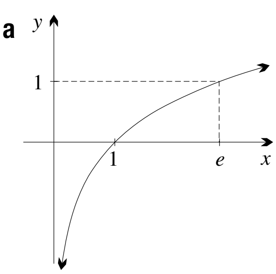


d



25 a



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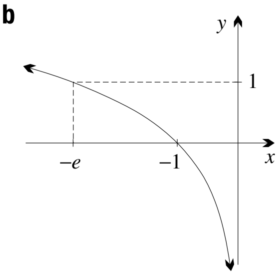

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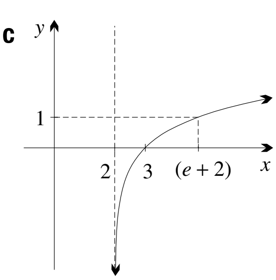

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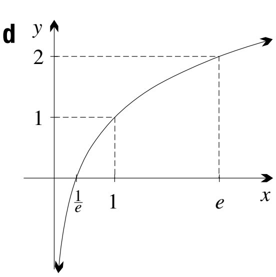

26 a e **b** 3 **c** -1 **d** e

27 a $\dfrac{1}{x}$ **b** $\dfrac{1}{x}$ **c** $\dfrac{1}{x + 4}$
d $\dfrac{2}{2x - 5}$ **e** $\dfrac{10}{5x - 1}$ **f** $1 + \dfrac{1}{x}$
g $\dfrac{2x - 5}{x^2 - 5x + 2}$ **h** $\dfrac{15x^4}{1 + 3x^5}$
i $8x - 24x^2 + \dfrac{2x}{x^2 - 2}$

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28 a $\dfrac{3}{x}$ **b** $\dfrac{1}{2x}$

c $\dfrac{1}{x} + \dfrac{1}{x+2}$ **d** $\dfrac{1}{x} - \dfrac{1}{x-1}$

29 a $1 + \log x$ **b** $\dfrac{e^x}{x} + e^x \log x$

c $\dfrac{\ln x - 1}{(\ln x)^2}$ **d** $\dfrac{1 - 2\ln x}{x^3}$

30 $y = 3x + 1$

32 a $\log_e|x| + C$ **b** $3\log_e|x| + C$

c $\frac{1}{5}\log_e|x| + C$ **d** $\log_e|x + 7| + C$

e $\frac{1}{2}\log_e|2x - 1| + C$ **f** $-\frac{1}{3}\log_e|2 - 3x| + C$

g $\log_e|2x + 9| + C$ **h** $-2\log_e|1 - 4x| + C$

33 a $\log_e\frac{3}{2}$ **b** $\frac{1}{4}\log_e 13$ **c** 1 **d** 1

34 a $\log_e(x^2 + 4) + C$

b $\log_e|x^3 - 5x + 7| + C$

c $\frac{1}{2}\log_e|x^2 - 3| + C$

d $\frac{1}{4}\log_e|x^4 - 4x| + C$

35 $\log_e 2\,\text{u}^2$

36 a $12 - 5\log_e 5\,\text{u}^2$

37 a e^x **b** $2^x\log_e 2$ **c** $3^x\log_e 3$ **d** $5^x\log_e 5$

38 a $e^x + C$ **b** $\dfrac{2^x}{\log_e 2} + C$

c $\dfrac{3^x}{\log_e 3} + C$ **d** $\dfrac{5^x}{\log_e 5} + C$

39 a $x\log_e x - x$ **b** $xe^x - e^x$

40 a $8\log_e 2$ **b** $\dfrac{1}{8\log_e 2}$

c The curves $y = 2^x$ and $y = \log_2 x$ are reflections of each other in $y = x$. This reflection exchanges A and B, and exchanges their tangents. Because it also exchanges rise and run, the gradients are reciprocals of each other.

41 a $\dfrac{7}{\ln 2}$ and $\dfrac{7}{8\ln 2}$

b When $y = 2^x$ is transformed successively by a vertical dilation with factor 8 and a shift right 3 units, the result is the same graph $y = 2^x$. The region in the second integral is transformed to the region in the first integral by this compound transformation.

Chapter 6

Exercise 6A

1 a The entries under 0.2 are 0.198 669, 0.993 347, 0.202710, 1.013550, 0.980067.

b 1 and 1

3 a $\frac{\pi}{90}$ **b** $\sin 2° = \sin\frac{\pi}{90} \doteqdot \frac{\pi}{90}$ **c** 0.0349

4 a The entries under 5° are 0.087 27, 0.087 16, 0.9987, 0.08749, 1.003, 0.9962.

b $\sin x < x < \tan x$

c i 1 **ii** 1

d $x \le 0.0774$ (correct to four decimal places), that is, $x \le 4°26'$.

6 87 metres

7 $26'$

12 a $AB^2 = 2r^2(1 - \cos x)$, arc $AB = rx$

b The arc is longer than the chord, so $\cos x$ is larger than the approximation.

Exercise 6B

1 a $\cos x$ **b** $-\sin x$ **c** $\sec^2 x$ **d** $2\cos x$

e $2\cos 2x$ **f** $-3\sin x$ **g** $-3\sin 3x$ **h** $4\sec^2 4x$

i $4\sec^2 x$ **j** $6\cos 3x$ **k** $4\sec^2 2x$ **l** $-8\sin 2x$

m $-2\cos 2x$ **n** $2\sin 2x$ **o** $-2\sec^2 2x$ **p** $\frac{1}{2}\sec^2\frac{1}{2}x$

q $-\frac{1}{2}\sin\frac{1}{2}x$ **r** $\frac{1}{2}\cos\frac{x}{2}$ **s** $\sec^2\frac{1}{5}x$ **t** $-2\sin\frac{x}{3}$

u $4\cos\frac{x}{4}$

2 a $2\pi\cos 2\pi x$ **b** $\frac{\pi}{2}\sec^2\frac{\pi}{2}x$

c $3\cos x - 5\sin 5x$ **d** $4\pi\cos\pi x - 3\pi\sin\pi x$

e $2\cos(2x - 1)$ **f** $3\sec^2(1 + 3x)$

g $2\sin(1 - x)$ **h** $-5\sin(5x + 4)$

i $-21\cos(2 - 3x)$ **j** $-10\sec^2(10 - x)$

k $3\cos\left(\frac{x+1}{2}\right)$ **l** $-6\sin\left(\frac{2x+1}{5}\right)$

3 a $2\cos 2x, -4\sin 2x, -8\cos 2x, 16\sin 2x$, amplitudes: 2, 4, 8, 16

b $-10\sin 10x, -100\cos 10x, 1000\sin 10x$, $10000\cos 10x$

c $\frac{1}{2}\cos\frac{1}{2}x, -\frac{1}{4}\sin\frac{1}{2}x, -\frac{1}{8}\cos\frac{1}{2}x, \frac{1}{16}\sin\frac{1}{2}x$

d $-\frac{1}{3}\sin\frac{1}{3}x, -\frac{1}{9}\cos\frac{1}{3}x, \frac{1}{27}\sin\frac{1}{3}x, \frac{1}{81}\cos\frac{1}{3}x$, amplitudes: $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}$

4 $-2\sin 2x$

a 0 **b** -1 **c** $-\sqrt{3}$ **d** -2

5 $\frac{1}{4}\cos\left(\frac{1}{4}x + \frac{\pi}{2}\right)$

a 0 **b** $-\frac{1}{4}$ **c** $\frac{1}{8}\sqrt{2}$ **d** $-\frac{1}{8}\sqrt{2}$

6 a $x\cos x + \sin x$ **b** $2(\tan 2x + 2x\sec^2 2x)$

c $2x(\cos 2x - x\sin 2x)$ **d** $3x^2(\sin 3x + x\cos 3x)$

7 a $\frac{x\cos x - \sin x}{x^2}$ **b** $\frac{-x\sin x - \cos x}{x^2}$

c $\frac{x(2\cos x + x\sin x)}{\cos^2 x}$ **d** $\frac{1 + \sin x - x\cos x}{(1 + \sin x)^2}$

8 a $2x\cos(x^2)$ **b** $-2x\cos(1 - x^2)$

c $-3x^2\sin(x^3 + 1)$ **d** $-\frac{1}{x^2}\cos(\frac{1}{x})$

e $-2\cos x\sin x$ **f** $3\sin^2 x\cos x$

g $2\tan x\sec^2 x$ **h** $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$

9 d $y = \cos x$

11 a $e^{\tan x}\sec^2 x$ **b** $2e^{\sin 2x}\cos 2x$ **c** $2e^{2x}\cos(e^{2x})$

d $-\tan x$ **e** $\cot x$ **f** $-4\tan 4x$

12 a $\cos^2 x - \sin^2 x$ **b** $14\sin 7x\cos 7x$

c $-15\cos^4 3x\sin 3x$ **d** $9\sin 3x(1 - \cos 3x)^2$

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e $2(\cos 2x \sin 4x + 2\sin 2x \cos 4x)$

f $15\tan^2(5x - 4)\sec^2(5x - 4)$

13 a $\frac{-\cos x}{(1 + \sin x)^2}$ **b** $\frac{1}{1 + \cos x}$

c $\frac{-1}{1 + \sin x}$ **d** $\frac{-1}{(\cos x + \sin x)^2}$

14 c i The graphs are reflections of each other in the x-axis.

ii The graphs are identical.

d i $y = e^x$ **ii** $y = e^x, y = e^{-x}$

iii $y = e^x$ **iv** $y = e^x, y = e^{-x}, y = \sin x$

16 a $y' = e^x \sin x + e^x \cos x, y'' = 2e^x \cos x$

b $y' = -e^{-x}\cos x - e^{-x}\sin x, y'' = 2e^{-x}\sin x$

18 a $\log_b P - \log_b Q$

Exercise 6C

1 a 1 **b** -1 **c** $\frac{1}{2}$ **d** $-\frac{1}{2}$

e $\frac{1}{\sqrt{2}}$ **f** 1 **g** 2 **h** -2

i $\frac{\sqrt{3}}{4}$ **j** $\frac{1}{4}$ **k** 8 **l** $\sqrt{3}$

3 a $y = -x + \pi$ **b** $2x - y = \frac{\pi}{2} - 1$

c $x + 2y = \frac{\pi}{6} + \sqrt{3}$ **d** $y = -2x + \frac{\pi}{2}$

e $x + y = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ **f** $y = -\pi x + \pi^2$

4 a $\frac{\pi}{2}, \frac{3\pi}{2}$ **b** $\frac{\pi}{3}, \frac{5\pi}{3}$ **c** $\frac{\pi}{6}, \frac{5\pi}{6}$ **d** $\frac{5\pi}{6}, \frac{7\pi}{6}$

6 b 1 and -1

c $x - y = \frac{\pi}{4} - \frac{1}{2}, x + y = \frac{\pi}{4} + \frac{1}{2}$

7 a $y' = \cos x e^{\sin x}$ **b** $\frac{\pi}{2}, \frac{3\pi}{2}$

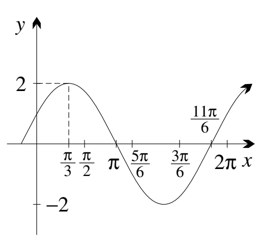
8 a $y' = -\sin x e^{\cos x}$ **b** $0, \pi, 2\pi$

9 a $y' = -\sin x + \sqrt{3}\cos x, y'' = -\cos x - \sqrt{3}\sin x$

b maximum turning point $\left(\frac{\pi}{3}, 2\right)$, minimum turning point $\left(\frac{4\pi}{3}, -2\right)$

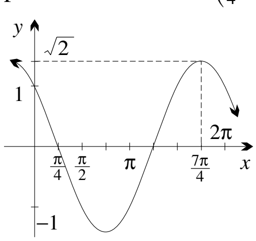
c $\left(\frac{5\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$

d



10 a $y' = -\sin x - \cos x, y'' = -\cos x + \sin x$, minimum turning point $\left(\frac{3\pi}{4}, -\sqrt{2}\right)$, maximum turning point $\left(\frac{7\pi}{4}, \sqrt{2}\right)$, points of inflection $\left(\frac{\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right)$

b

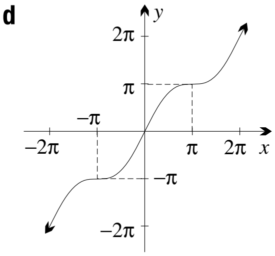


11 a $y' = 1 + \cos x$

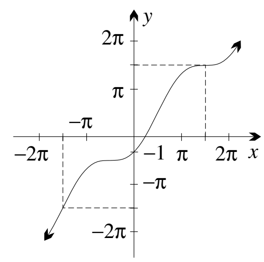
b $(-\pi, -\pi)$ and (π, π) are horizontal points of inflection.

c $(0, 0)$

d

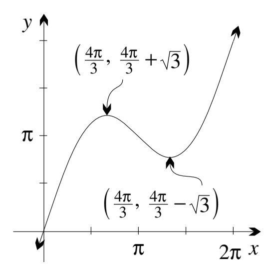


12 $y' = 1 + \sin x, y'' = \cos x$, horizontal points of inflection $\left(-\frac{\pi}{2}, -\frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$, points of inflection $\left(-\frac{3\pi}{2}, -\frac{3\pi}{2}\right), \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$



13 c $\sin^2\theta = \frac{1}{3}, \theta \doteqdot 19.47°$ **d** $2\pi\sqrt{3}\text{ m}^3$

14 maximum turning point $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$, minimum turning point $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$, inflection (π, π)



16 b minimum $\sqrt{3}$ when $\theta = \frac{\pi}{6}$, maximum 2 when $\theta = 0$

Exercise 6D

1 a $\tan x + C$ **b** $\sin x + C$

c $-\cos x + C$ **d** $\cos x + C$

e $2\sin x + C$ **f** $\frac{1}{2}\sin 2x + C$

g $\frac{1}{2}\sin x + C$ **h** $2\sin\frac{1}{2}x + C$

i $-\frac{1}{2}\cos 2x + C$ **j** $\frac{1}{5}\tan 5x + C$

k $\frac{1}{3}\sin 3x + C$ **l** $3\tan\frac{1}{3}x + C$

m $-2\cos\frac{x}{2} + C$ **n** $-5\sin\frac{1}{5}x + C$

o $2\cos 2x + C$ **p** $-\cos\frac{1}{4}x + C$

q $-36\tan\frac{1}{3}x + C$ **r** $6\sin\frac{x}{3} + C$

2 a 1 **b** $\frac{1}{2}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\sqrt{3}$ **e** 1

f $\frac{3}{4}$ **g** 2 **h** 1 **i** 4

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3 a **i** $y = 1 - \cos x$ **ii** $y = 3 - \cos x$
iii Shift up 2.
b $y = \sin x + \cos 2x - 1$
c $y = -\cos x + \sin x - 3$

6 a $\sin(x + 2) + C$ **b** $\frac{1}{2}\sin(2x + 1) + C$
c $-\cos(x + 2) + C$ **d** $-\frac{1}{2}\cos(2x + 1) + C$
e $\frac{1}{3}\sin(3x - 2) + C$ **f** $\frac{1}{5}\cos(7 - 5x) + C$
g $-\tan(4 - x) + C$ **h** $-3\tan\left(\frac{1 - x}{3}\right) + C$
i $3\cos\left(\frac{1 - x}{3}\right) + C$

7 a $2\sin 3x + 8\cos\frac{1}{2}x + C$
b $4\tan 2x - 40\sin\frac{1}{4}x - 36\cos\frac{1}{3}x + C$

8 a $f(x) = \sin \pi x$, $f\left(\frac{1}{3}\right) = \frac{1}{2}\sqrt{3}$
b $f(x) = \frac{1}{2\pi} + \frac{1}{\pi}\sin \pi x$, $f\left(\frac{1}{6}\right) = \frac{1}{\pi}$
c $f(x) = -2\cos 3x + x + (1 - \frac{\pi}{2})$

9 a $-\cos(ax + b) + C$ **b** $\pi \sin \pi x + C$
c $\frac{1}{u^2}\tan(v + ux) + C$ **d** $\tan ax + C$

10 a $1 + \tan^2 x = \sec^2 x$, $\tan x - x + C$
b $1 - \sin^2 x = \cos^2 x$, $2\sqrt{3}$

11 a $\log_e |f(x)| + C$

12 a $\int \tan x = -\ln \cos x + C$
b $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\,dx = \left[\log|\sin x|\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \log 2$

13 a **i** $2x\cos x^2$ **ii** $\sin x^2 + C$
b **i** $-3x^2\sin x^3$ **ii** $-\frac{1}{3}\cos x^3 + C$
c **i** $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$ **ii** $2\tan\sqrt{x} + C$

14 a $5\sin^4 x\cos x$, $\frac{1}{5}\sin^5 x + C$
b $3\tan^2 x\sec^2 x$, $\frac{1}{3}\tan^3 x + C$

15 a $\cos x e^{\sin x}$, $e - 1$ **b** $e^{\tan x} + C$, $e - 1$

16 a 1 **b** $\frac{5}{24}$

17 $\sin 2x + 2x\cos 2x$, $\frac{\pi - 2}{8}$

18 All three integrals are meaningless because:
a $\sec x$ has an asymptote at $x = \frac{\pi}{2}$,
b $\tan x$ has an asymptote at $x = \frac{\pi}{2}$,
c $\cot x$ has an asymptote at $x = 0$.

Exercise 6E

1 a 1 square unit **b** $\frac{1}{2}$ square unit
2 a 1 square unit **b** $\sqrt{3}$ square units
3 a $1 - \frac{1}{\sqrt{2}}$ square units **b** $1 - \frac{\sqrt{3}}{2}$ square units
4 a $\frac{1}{2}\sqrt{3}$ u^2 **b** $\frac{1}{2}\sqrt{3}$ u^2
5 a $\frac{1}{2}$ u^2 **b** $\frac{1}{2}$ u^2
c $1 - \frac{\sqrt{3}}{2} = \frac{1}{2}\left(2 - \sqrt{3}\right)$ u^2
d $\frac{1}{3}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{1}{6}\left(2 - \sqrt{2}\right)$ u^2
e $\frac{2}{3}\sqrt{3}$ u^2
f 4 u^2

6 a $\left(\sqrt{2} - 1\right)$ u^2 **b** $\frac{1}{4}$ u^2
c $\left(\frac{\pi^2}{8} - 1\right)$ u^2 **d** $(\pi - 2)$ u^2

7 a $\left(2 - \sqrt{2}\right)$ u^2 **b** $1\frac{1}{2}$ u^2
8 a 2 u^2 **b** 1 u^2
9 a 2 u^2 **b** $\sqrt{2}$ u^2 **c** 2 u^2
d $\frac{1}{2}$ u^2 **e** 4 u^2 **f** 1 u^2
10 b $\frac{4}{\pi}$ u^2
11 3.8 m^2
12 4 u^2
14 b $\frac{1}{2}\left(3 + \sqrt{3}\right)$ u^2
15 b $\frac{3}{4}\sqrt{3}$ u^2
16 They are all 4 u^2.
17 b The curve is below $y = 1$ just as much as it is above $y = 1$, so the area is equal to the area of a rectangle n units long and 1 unit high.

Chapter 6 review exercise

1 a $5\cos x$ **b** $5\cos 5x$
c $-25\sin 5x$ **d** $5\sec^2(5x - 4)$
e $\sin 5x + 5x\cos 5x$ **f** $\frac{-5x\sin 5x - \cos 5x}{x^2}$
g $5\sin^4 x\cos x$ **h** $5x^4\sec^2(x^5)$
i $-5\sin 5x e^{\cos 5x}$ **j** $\frac{5\cos 5x}{\sin 5x} = 5\cot 5x$

2 $-\sqrt{3}$

3 a $y = 4x + \sqrt{3} - \frac{4\pi}{3}$ **b** $y = -\frac{\pi}{2}x + \frac{\pi^2}{4}$

4 a $\frac{\pi}{2}$ **b** $\frac{3\pi}{4}$, $\frac{7\pi}{4}$

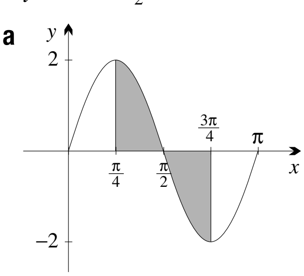
5 a $4\sin x + C$ **b** $-\frac{1}{4}\cos 4x + C$ **c** $4\tan\frac{1}{4}x + C$

6 a $\sqrt{3} - 1$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

7 0.089

8 $y = 2\sin\frac{1}{2}x - 1$

9 a



b 2 u^2

10 a $\frac{1}{2}$ u^2 **b** $\frac{3\sqrt{3}}{4}$ u^2

11 a $\tan x = \frac{\sin x}{\cos x}$ **b** $\frac{1}{2}\ln 2$ u^2

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Chapter 7

Exercise 7A

1 a $x = 2$ **b** $x = 18$ **c** 4 m/s

2 a $x = 0, 20$; 10 cm/s **b** $x = 4, 0$; -2 cm/s

c $x = 3, 3$; 0 cm/s **d** $x = 1, 4$; $1\frac{1}{2}$ cm/s

3 a $x = 0, -3, 0, 15, 48$

b i -3 cm/s **ii** 3 cm/s

iii 15 cm/s **iv** 33 cm/s

4 a $x = -4, -3, 0, 5$

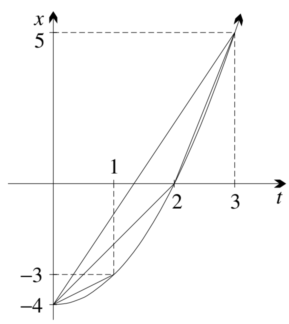
b i 1 m/s

ii 2 m/s

iii 3 m/s

iv 5 m/s

c



5 a $x = 0, 120, 72, 0$ **b** 240 metres

c 20 m/s

d i 30 m/s **ii** -15 m/s **iii** 0 m/s

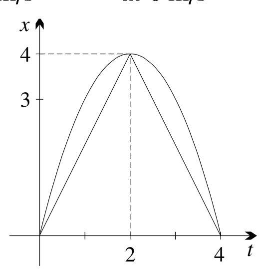
6 a $x = 0, 3, 4, 3, 0$

c The total distance travelled is 8 metres.
The average speed is 2 m/s.

d i 2 m/s

ii -2 m/s

iii 0 m/s



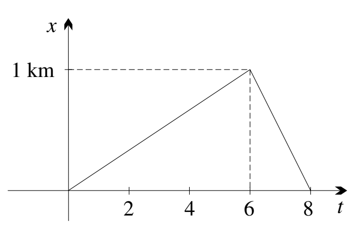
7 a i 6 minutes

ii 2 minutes

c 15 km/hr

d 20 km/hr

b



8 a $x = 0, 3, 1, 4, 2, 5, 3, 6$

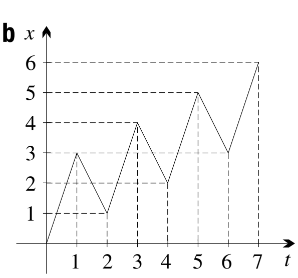
c 7 hours

d 18 metres, $2\frac{4}{7}$ m/hr

e $\frac{6}{7}$ m/hr

f Those between 1 and 2 metres high or between 4 and 5 metres high

b



9 a $t = 0, 1, 4, 9, 16$

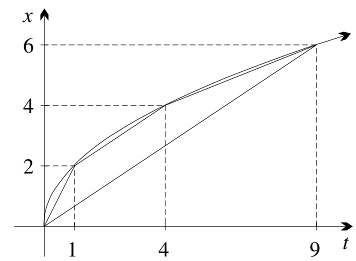
b i 2 cm/s

ii $\frac{2}{3}$ cm/s

iii $\frac{2}{5}$ cm/s

iv $\frac{2}{3}$ cm/s

c They are parallel.



10 a i -1 m/s

ii 4 m/s

iii -2 m/s

b 40 metres, $1\frac{1}{3}$ m/s

c 0 metres, 0 m/s

d $2\frac{2}{19}$ m/s

11 a i once

ii three times

iii twice

b i when $t = 4$ and when $t = 14$

ii when $0 \le t < 4$ and when $4 < t < 14$

c It rises 2 metres, at $t = 8$.

d It sinks 1 metre, at $t = 17$.

e As $t \to \infty$, $x \to 0$, meaning that eventually it ends up at the surface.

f i -1 m/s **ii** $\frac{1}{2}$ m/s **iii** $-\frac{1}{3}$ m/s

g i 4 metres **ii** 6 metres

iii 9 metres **iv** 10 metres

h i 1 m/s **ii** $\frac{3}{4}$ m/s **iii** $\frac{9}{17}$ m/s

12 b $x = 3$ and $x = -3$ **c** $t = 4, t = 20$

d $t = 8, t = 16$ **e** $8 < t < 16$

f 12 cm, $\frac{3}{4}$ cm/s

13 a amplitude: 4 metres, period: 12 seconds

b 10 times

c $t = 3, 15, 27, 39, 51$

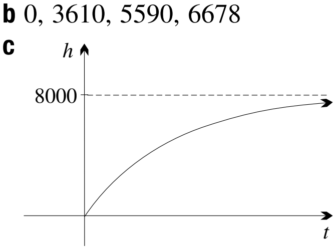
d It travels 16 metres with average speed $1\frac{1}{3}$ m/s.

e $x = 0$, $x = 2$ and $x = 4$, 2 m/s and 1 m/s

14 a When $t = 0$, $h = 0$. As $t \to \infty$, $h \to 8000$.

b 0, 3610, 5590, 6678

c



d 361 m/ min , 198 m/ min , 109 m/ min

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Exercise 7B

1 a $v = -2t$

b $a = -2$

c $x = 11$ metres, $v = -6$ m/s, $a = -2$ m/s^2

d distance from origin: 11 metres, speed: 6 m/s

2 a $v = 10t - 10$, $a = 10$. When $t = 1$, $x = -5$ metres, $v = 0$ m/s, $a = 10$ m/s^2.

b $v = 3 - 6t^2$, $a = -12t$. When $t = 1$, $x = 1$ metre, $v = -3$ m/s, $a = -12$ m/s^2.

c $v = 4t^3 - 2t$, $a = 12t^2 - 2$. When $t = 1$, $x = 4$ metres, $v = 2$ m/s, $a = 10$ m/s^2.

3 a $v = 2t - 10$

b displacement: -21 cm, distance from origin: 21 cm, velocity: $v = -4$ cm/s, speed: $|v| = 4$ cm/s

c When $v = 0$, $t = 5$ and $x = -25$.

4 a $v = 3t^2 - 12t$, $a = 6t - 12$

b When $t = 0$, $x = 0$ cm, $|v| = 0$ cm/s and $a = -12$ cm/s^2.

c left $(x = -27 \text{ cm})$

d left $(v = -9 \text{ cm/s})$

e right $(a = 6 \text{ cm/s}^2)$

f When $t = 4$, $v = 0$ cm/s and $x = -32$ cm.

g When $t = 6$, $x = 0$, $v = 36$ cm/s and $|v| = 36$ cm/s.

5 a $v = \cos t$, $a = -\sin t$, 1 cm, 0 cm/s, -1 cm/s^2

b $v = -\sin t$, $a = -\cos t$, 0 cm, -1 cm/s, 0 cm/s^2

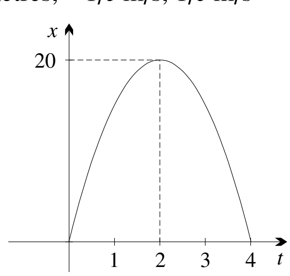
6 a $v = e^t$, $a = e^t$, e metres, e m/s, e m/s^2

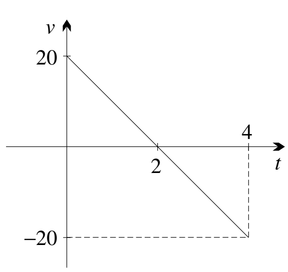
b $v = -e^{-t}$, $a = e^{-t}$, $1/e$ metres, $-1/e$ m/s, $1/e$ m/s^2

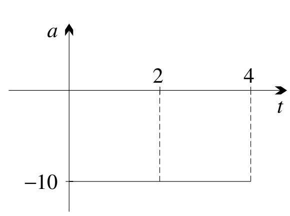
7 a $x = 5t(4 - t)$

$v = 20 - 10t$

$a = -10$







b 20 m/s

c It returns at $t = 4$; both speeds are 20 m/s.

d 20 metres after 2 seconds

e -10 m/s^2. Although the ball is stationary, its velocity is changing, meaning that its acceleration is non-zero.

8 $\dot{x} = -4e^{-4t}$, $\ddot{x} = 16e^{-4t}$

a e^{-4t} is positive, for all t, so $\dot{x}$ is always negative and $\ddot{x}$ is always positive.

b i $x = 1$ **ii** $x = 0$

c i $\dot{x} = -4$, $\ddot{x} = 16$ **ii** $\dot{x} = 0$, $\ddot{x} = 0$

9 $v = 2\pi \cos \pi t$, $a = -2\pi^2 \sin \pi t$

a When $t = 1$, $x = 0$, $v = -2\pi$ and $a = 0$.

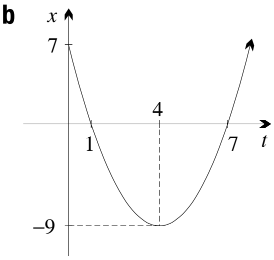
b i right $(v = \pi)$ **ii** left $\left(a = -\pi^2\sqrt{3}\right)$

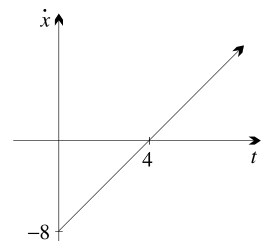
10 a $x = (t - 7)(t - 1)$

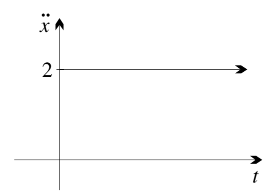
$\dot{x} = 2(t - 4)$

$\ddot{x} = 2$

b







c i $t = 1$ and $t = 7$ **ii** $t = 4$

d i 7 metres when $t = 0$

ii 9 metres when $t = 4$

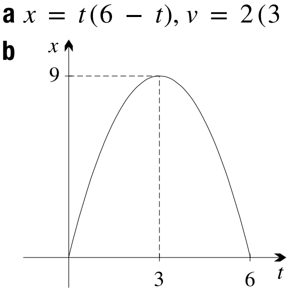
iii 27 metres when $t = 10$

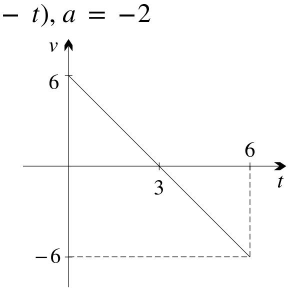
e -1 m/s, $t = 3\frac{1}{2}$, $x = -8\frac{3}{4}$

f 25 metres, $3\frac{4}{7}$ m/s

11 a $x = t(6 - t)$, $v = 2(3 - t)$, $a = -2$

b





c i When $t = 2$, it is moving upwards and accelerating downwards.

ii When $t = 4$, it is moving downwards and accelerating downwards.

d $v = 0$ when $t = 3$. It is stationary for zero time, 9 metres up the plane, and is accelerating downwards at 2 m/s^2.

e 4 m/s. When $v = 4$, $t = 1$ and $x = 5$.

f All three average speeds are 3 m/s.

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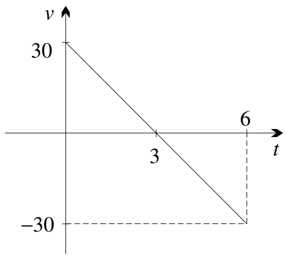
12 a 45 metres, 3 seconds, 15 m/s

b 30 m/s, 20, 10, 0, $-10, -20, -30$



c 0 seconds

d The acceleration was always negative.

e The velocity was decreasing at a constant rate of 10 m/s every second.

13 a 8 metres, when $t = 3$

b i when $t = 3$ and $t = 9$ (because the gradient is zero)

ii when $0 \le t < 3$ and when $t > 9$ (because the gradient is positive)

iii when $3 < t < 9$ (because the gradient is negative)

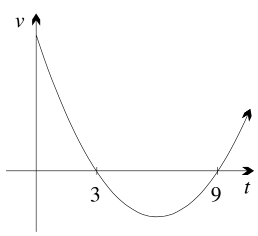
c $x = 0$ again when $t = 9$. Then $v = 0$ (because the gradient is zero) and it is accelerating to the right (because the concavity is upwards).

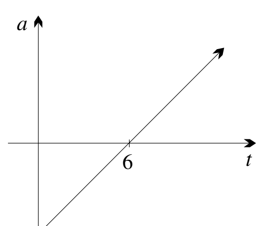
d at $t = 6$ (at the point of inflection the second derivative is zero), $x = 4$, moving to the left

e $0 \le t < 6$

f i $t \doteqdot 4, 12$ **ii** $t \doteqdot 10$

g





14 a $x = 4\cos\frac{\pi}{4}t,\ v = -\pi\sin\frac{\pi}{4}t,\ a = -\frac{1}{4}\pi^2\cos\frac{\pi}{4}t$

b maximum displacement: $x = 4$ when $t = 0$ or $t = 8$, maximum velocity: π m/s when $t = 6$, maximum acceleration: $\frac{1}{4}\pi^2$ m/s^2 when $t = 4$

c 40 metres, 2 m/s

d $1\frac{1}{3} < t < 6\frac{2}{3}$

e i $t = 0, t = 4$ and $t = 8$ **ii** $4 < t < 8$

15 a i $0 \le t < 8$ **ii** $0 \le t < 4$ and $t > 12$

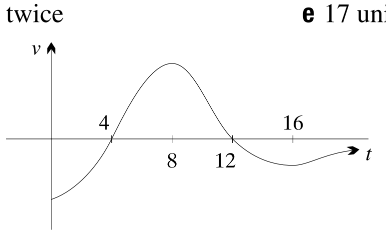
iii roughly $8 < t < 16$

b roughly $t = 8$

c i $t \doteqdot 5, 11, 13$ **ii** $t \doteqdot 13, 20$

d twice **e** 17 units

f



16 b i downwards (Downwards is positive here.)

ii upwards

c The velocity and acceleration tend to zero and the position tends to 12 metres below ground level.

d $t = 2\log_e 2$ minutes. The speed then is 3 m/min (half the initial speed of 6 m/min) and the acceleration is $-1\frac{1}{2}$ m/min^2 (half the initial acceleration of -3 m/min^2).

e When $t = 18$, $x \doteqdot 11.9985$ metres.
When $t = 19$, $x \doteqdot 11.9991$ metres.

Exercise 7C

1 a $x = t^3 - 3t^2 + 4$

b When $t = 2$, $x = 0$ metres and $v = 0$ m/s.

c $a = 6t - 6$

d When $t = 1$, $a = 0$ m/s^2 and $x = 2$ metres.

2 a $v = -3t^2$

b When $t = 5$, $v = -75$ cm/s and $|v| = 75$ cm/s.

c $x = -t^3 + 8$

d When $t = 2$, $x = 0$ cm and $a = -12$ cm/s^2.

3 a $v = 8t - 16$

b $x = 4t^2 - 16t + 16$

c When $t = 0$, $x = 16$, $v = -16$ and $|v| = 16$.

4 a $v = 6t - 30$ **b** 5 seconds **c** $a = 0$

5 a $v = 2t - 20$

b $x = t^2 - 20t$

c When $v = 0$, $t = 10$ and $x = -100$.

d When $x = 0$, $t = 0$ or 20. When $t = 20$, $v = 20$ m/s.

6 a $v = 10t$, $x = 5t^2$

b When $t = 4$, $x = 80$ metres, which is at the bottom, and $v = 40$ m/s.

c After 2 seconds, it has fallen 20 metres and its speed is 20 m/s.

d When $t = 2\sqrt{2}$, $x = 40$ metres, which is halfway down, and $v = 20\sqrt{2}$ m/s.

7 a $a = -10, v = -10t - 25, x = -5t^2 - 25t + 120$

b 3 seconds **c** 55 m/s **d** 40 m/s

8 a $\dot{x} = -4t,\ x = -2t^2$

b $\dot{x} = 3t^2,\ x = t^3$

c $\dot{x} = 2e^{\frac{1}{2}t} - 2,\ x = 4e^{\frac{1}{2}t} - 2t - 4$

d $\dot{x} = -\frac{1}{3}e^{-3t} + \frac{1}{3},\ x = \frac{1}{9}e^{-3t} + \frac{1}{3}t - \frac{1}{9}$

e $\dot{x} = -4\cos 2t + 4,\ x = -2\sin 2t + 4t$

f $\dot{x} = \frac{1}{\pi}\sin \pi t,\ x = -\frac{1}{\pi^2}\cos \pi t + \frac{1}{\pi^2}$

g $\dot{x} = \frac{2}{3}t^{\frac{3}{2}},\ x = \frac{4}{15}t^{\frac{5}{2}}$

h $\dot{x} = -12(t + 1)^{-1} + 12,$
$x = -12\log_e(t + 1) + 12t$

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9 a $a = 0, x = -4t - 2$

b $a = 6, x = 3t^2 - 2$

c $a = \frac{1}{2}e^{\frac{1}{2}t}, x = 2e^{\frac{1}{2}t} - 4$

d $a = -3e^{-3t}, x = -\frac{1}{3}e^{-3t} - 1\frac{2}{3}$

e $a = 16\cos 2t, x = -4\cos 2t + 2$

f $a = -\pi\sin\pi t, x = \frac{1}{\pi}\sin\pi t - 2$

g $a = \frac{1}{2}t^{-\frac{1}{2}}, x = \frac{2}{3}t^{\frac{3}{2}} - 2$

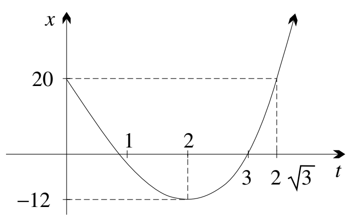
h $a = -24(t + 1)^{-3}, x = -12(t + 1)^{-1} + 10$

10 a $\dot{x} = 6t^2 - 24, x = 2t^3 - 24t + 20$

b $t = 2\sqrt{3}$, speed: 48 m/s

c $x = -12$ when $t = 2$.

d



11 a $k = 6$ and $C = -9$, hence $a = 6t$ and $v = 3t^2 - 9$.

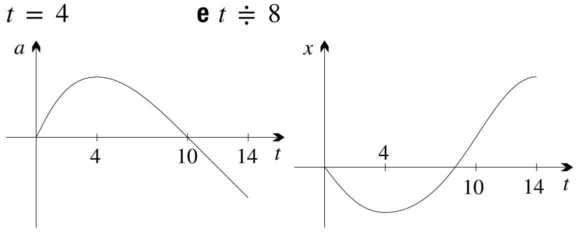
b i $x = t^3 - 9t + 2$

ii at $t = 3$ seconds (Put $x = 2$ and solve for t.)

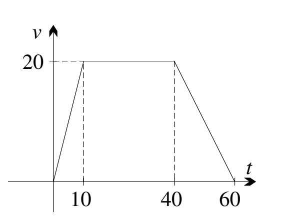
12 a $4 < t < 14$ **b** $0 < t < 10$ **c** $t = 14$

d $t = 4$ **e** $t \doteqdot 8$

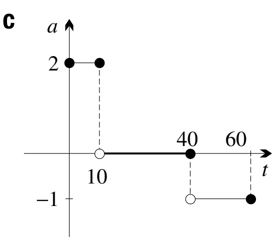
f

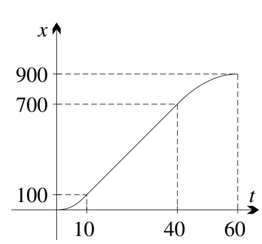


13 a 20 m/s



c





14 a $\ddot{x} = -4, x = 16t - 2t^2 + C$

b $x = C$ after 8 seconds, when the speed is 16 cm/s.

c $\dot{x} = 0$ when $t = 4$. Maximum distance right is 32 cm when $t = 4$, maximum distance left is 40 cm when $t = 10$. The acceleration is -4 cm/s^2 at all times.

d 104 cm, 10.4 cm/s

15 a $x = \log_e(t + 1) - 1, a = -\dfrac{1}{(t + 1)^2}$

b $e - 1$ seconds, $v = 1/e, a = -1/e^2$

c The velocity and acceleration approach zero, but the particle moves to infinity.

16 a $\dot{x} = -5 + 20e^{-2t}, x = -5t + 10 - 10e^{-2t}$, $t = \log_e 2$ seconds

b It rises $7\frac{1}{2} - 5\log_e 2$ metres, when the acceleration is 10 m/s^2 downwards.

c The velocity approaches a limit of 5 m/s downwards, called the *terminal velocity.*

17 a $v = 1 - 2\sin t, x = t + 2\cos t$

b $\frac{\pi}{2} < t < \frac{3\pi}{2}$

c $t = \frac{\pi}{6}$ when $x = \frac{\pi}{6} + \sqrt{3}$, and $\frac{5\pi}{6}$ when $x = \frac{5\pi}{6} - \sqrt{3}$.

d 3 m/s when $t = \frac{3\pi}{2}$, and -1 m/s when $t = \frac{\pi}{2}$

Exercise 7D

1 a 80 tonnes **b** When $t = 0, V = 0$.

c 360 tonnes **d** 20 tonnes/minute

2 a 80 000 litres **b** 5 000 litres

c 20 minutes, $0 \le t \le 20$ **d** 6000 L/min

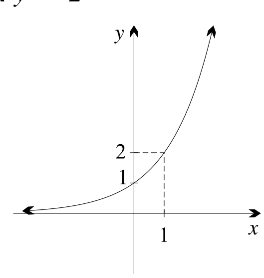
e The tank is emptying, so F is decreasing.

f average rate $= \frac{80000}{20} = 4000$ L/min

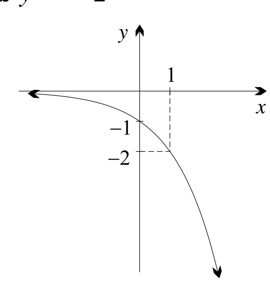
3 a 1500 **b** 300 **c** 15 minutes

d average rate $= \frac{60000 - 1500}{15} = 300$ L/min (This is, of course, just the flow rate, which is constant.)

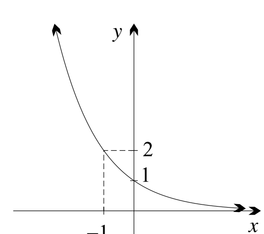
4 a $y = 2^x$



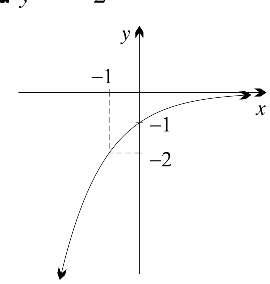
b $y = -2^x$



c $y = 2^{-x}$



d $y = -2^{-x}$



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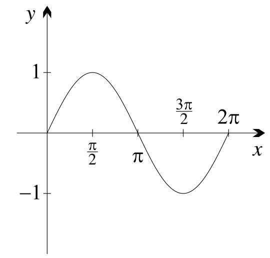
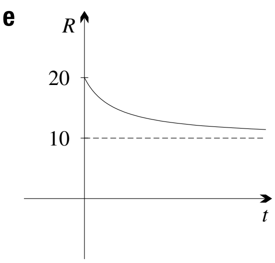


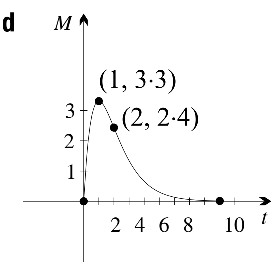
a **i** $0 \le x \le \frac{\pi}{2}$
ii $\frac{\pi}{2} \le x \le \pi$
iii $\pi \le x \le \frac{3\pi}{2}$
iv $\frac{3\pi}{2} \le x \le 2\pi$
b **i** $\pi \le x \le 2\pi$
ii $0 \le x \le \pi$

6 a $\dot{h} = 60e^{-\frac{t}{3}} - 30$
b 30 m/s upwards
c $h = 27.62$ m at $3 \ln 2 \doteqdot 2.079$ seconds
d $h \doteqdot 10.23$ m and speed is 15 m/s downwards
e 30 m/s downwards

7 a **i** 12 kg/min
ii $10\frac{2}{3}$ kg/min
b 10 kg/min
c $\dot{R} = \frac{-20}{(1 + 2t)^2}$,
$\ddot{R} = \frac{80}{(1 + 2t)^3}$
d R is decreasing at a decreasing rate
e

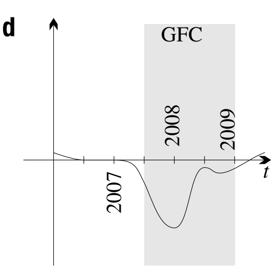


8 a $(0, 0)$ and $(9, 81e^{-9}) \doteqdot (9, 0.0)$
b $\dot{M} = 9(1 - t)e^{-t}$, $(1, 9e^{-1}) \doteqdot (1, 3.3)$
c $\ddot{M} = 9(t - 2)e^{-t}$, $(2, 18e^{-2}) \doteqdot (2, 2.4)$
d



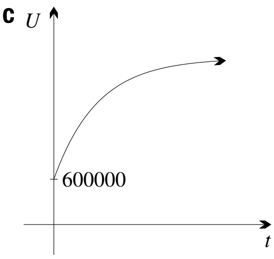
e $t = 1$
f $t = 0$
g $t = 2$

9 a The graph is steepest in January 2008.
b It levels out in 2009?
c The LIBOR reduced at a decreasing rate. It may have been mistaken as indicating the crisis was ending.
d



10 It took time for the scheme to start working with maximum pollution in the river reached in 2016. Then the level of pollution decreased at an increasing rate. But after 2017 the rate at which the pollution decreased gradually slowed down and was almost zero in 2020. A new scheme would have been required to remove the remaining pollution.

11 a Unemployment was increasing.
b The rate of increase was decreasing.
c

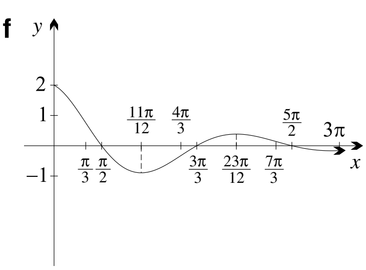


12 a $A = 9 \times 10^5$ **b** $N(1) = 380087$
c When t is large, N is close to 4.5×10^5.
d $\dot{N} = \frac{9 \times 10^5 e^{-t}}{(2 + e^{-t})^2}$

13 a $I = \dfrac{300t\left(2 - \frac{1}{5}t\right)}{200 + 3t^2 - \frac{1}{5}t^3}\%$ **b** $I(4) \doteqdot 6.12\%$
c $t = 0$ or 10. The latter is rejected because the model is only valid for 8 years.

14 b Exponentials are always positive.
c $\phi(0) = \frac{1}{\sqrt{2\pi}}, \lim_{x \to \infty} \phi(x) = 0$
d $\phi'(x) < 0$ for $x > 0$ (decreasing)
e At $x = 1$ and $x = -1$, where $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$.
g decreasing at increasing rate for $0 \le x \le 1$, decreasing at decreasing rate for $x \ge 1$.
h The curve approaches the horizontal asymptote more slowly for larger x.

15 a $y = 2$ and $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
f



Exercise 7E

1 a $y = 3t - 1$ **b** $y = 2 + t - t^2$
c $y = \sin t + 1$ **d** $y = e^t - 1$
2 b 15 min
3 a 25 minutes **c** 3145 litres
4 a **i** 3 cm^3/min **ii** 13 cm^3/min
b $E = \frac{1}{2}t^2 + 3t$
c **i** 80 cm^3 **i** 180 cm^3
5 b $t = 4$ **c** 57 **d** $t = 2$
6 a $P = 6.8 - 2 \log_e(t + 1)$
b approximately 29 days
7 b $k = \frac{5}{24}$

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8 a no **b** $t \doteqdot 1.28$ **c** $x = \frac{5}{2}$

9 a 0 **b** 250 m/s

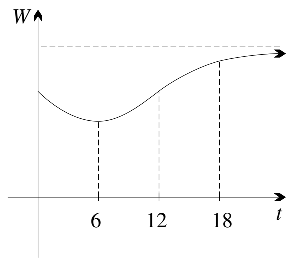
c $x = 1450 - 250(5e^{-0.2t} + t)$

10 a It was decreasing for the first 6 months and increasing thereafter.

b after 6 months

c after 12 months

d It appears to have stabilised, increasing towards a limiting value.

e


11 a $-2\ m^3/s$ **b** 20 s

c $V = 520 - 2t + \frac{1}{20}t^2$ **d** $20\ m^3$

e 2 minutes and 20 seconds

12 a $V = \frac{1}{5}t^2 - 20t + 500$

b $t = 50 - 25\sqrt{2} \doteqdot 15$ seconds. Discard the other answer $t = 50 + 25\sqrt{2}$ because after 50 seconds the bottle is empty.

13 a $I = 18000 - 5t + \frac{48}{\pi}\sin\frac{\pi}{12}t$

b $\frac{dI}{dt}$ has a maximum of -1, so it is always negative.

c There will be 3600 tonnes left.

14 a $1200\ m^3$ per month at the beginning of July

b $W = 0.7t - \frac{3}{\pi}\sin\frac{\pi}{6}t$

Exercise 7F

1 a 4034 **b** 2.3 **c** 113.4 **d** 603

2 a 0.2695 **b** 2.77 **c** -12.5 **d** -2.7

3 a 20 **b** 66

c 24th **d** 5 rabbits per month

4 a 100 kg **b** 67 kg **c** 45 kg

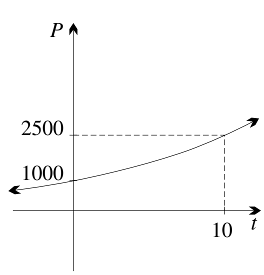
d 75 **e** 0.8 kg/year **f** 2 kg/year

5 a $k = \frac{1}{10}\log_e \frac{5}{2}$
$\doteqdot 0.092$

b 8230

c during 2020

d $\frac{dP}{dt} = kP$
$\doteqdot 905$



6 b 1350 **c** 135 per hour **d** 23 hours

7 c 6.30 grams, 1.46 grams per minute

d 6 minutes 58 seconds

e $20\,g$, $20e^{-k} \doteqdot 15.87\,g$, $20e^{-2k} \doteqdot 12.60\,g$,

$20e^{-3k} = 10\,g$, $r = e^{-k} = 2^{-\frac{1}{3}} \doteqdot 0.7937$

8 b $-\frac{1}{5}\log_e\frac{7}{10}$ **b** 10290

c At $t \doteqdot 8.8$, that is, some time in the fourth year from now.

9 b $h_0 = 100$ **c** $k = -\frac{1}{5}\log_e\frac{2}{5} \doteqdot 0.18$

d 6.4°C

10 b $k = \dfrac{\log_e 2}{5750} \doteqdot 1.21 \times 10^{-4}$

c $t = \frac{1}{k}\log_e\frac{100}{15} \doteqdot 16000$ years, correct to the nearest 1000 years.

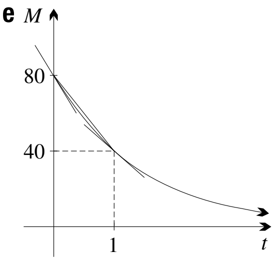
11 b 30

c i 26 **ii** $\frac{1}{5}\log_e\frac{15}{13}$ (or $-\frac{1}{5}\log_e\frac{13}{15}$)

12 a 80 g, 40 g, 20 g, 10 g

b 40 g, 20 g, 10 g. During each hour, the average mass loss is 50%.

c $M_0 = 80$,
$k = \log 2 \doteqdot 0.693$

e


d 55.45 g/hr, 27.73 g/hr, 13.86 g/hr, 6.93 g/hr

13 a 72% **b** 37% **c** 7%

14 a $k = \dfrac{\log_e 2}{1690} \doteqdot 4.10 \times 10^{-4}$

b $t = \dfrac{\log_e 5}{k} \doteqdot 3924$ years

15 b $\mu_1 = 1.21 \times 10^{-4}$

c $\mu_2 = 1.16 \times 10^{-4}$

d The values of μ differ so the data are inconsistent.

e i 625.5 millibars

ii 1143.1 millibars

iii 19205 metres

16 b $L = \frac{1}{2}$

17 a 34 minutes **b** 2.5%

18 b $C_0 = 20000$, $k = \frac{1}{5}\log_e\frac{9}{8} \doteqdot 0.024$

c 64946 ppm

d i 330 metres from the cylinder

ii If it had been rounded down, then the concentration would be above the safe level.

19 a $p = \frac{1}{10}\log_e\frac{5}{4} \doteqdot 0.022$, $q = \frac{1}{10}\log_e\frac{6}{5} \doteqdot 0.018$

b $t = \frac{\log_e 2}{p + q} \doteqdot 17.10$ years, that is during 2017.

Chapter 7 review exercise

1 a $x = 24$, $x = 36$, 6 cm/s

b $x = 16$, $x = 36$, 10 cm/s

c $x = -8$, $x = -8$, 0 cm/s

d $x = 9$, $x = 81$, 36 cm/s

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2 a $v = 40 - 2t, a = -2$, 175 m, 30 m/s, -2 m/s^2

b $v = 3t^2 - 25, a = 6t$, 0 m, 50 m/s, 30 m/s^2

c $v = 8(t - 3), a = 8$, 16 m, 16 m/s, 8 m/s^2

d $v = -4t^3, a = -12t^2$, -575 m, -500 m/s, -300 m/s

e $v = 4\pi \cos \pi t$ $a = -4\pi^2 \sin \pi t$, 0 m, -4π m/s, 0 m/s^2

f $v = 21\, e^{3t-15}, a = 63\, e^{3t-15}$, 7 m, 21 m/s, 63 m/s^2

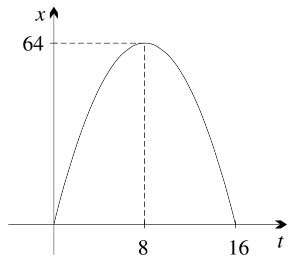
3 a $v = 16 - 2t, a = -2$

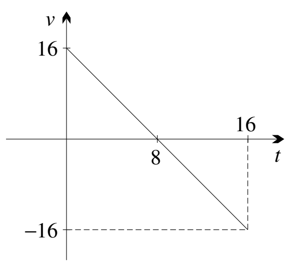
b 60 m, -4 m/s, 4 m/s, -2 m/s^2

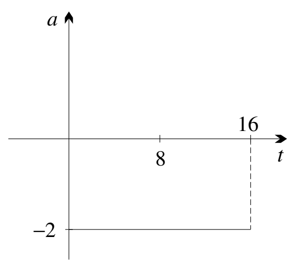
c $t = 16$ s, $v = -16$ m/s

d $t = 8$ s, $x = 64$ m

e







4 a $a = 0, x = 7t + 4$

b $a = -18t, x = 4t - 3t^3 + 4$

c $a = 2(t - 1), x = \frac{1}{3}(t - 1)^3 + 4\frac{1}{3}$

d $a = 0, x = 4$

e $a = -24 \sin 2t, x = 4 + 6 \sin 2t$

f $a = -36e^{-3t}, x = 8 - 4e^{-3t}$

5 a $v = 3t^2 + 2t, x = t^3 + t^2 + 2$

b $v = -8t, x = -4t^2 + 2$

c $v = 12t^3 - 4t, x = 3t^4 - 2t^2 + 2$

d $v = 0, x = 2$

e $v = 5 \sin t, x = 7 - 5 \cos t$

f $v = 7e^t - 7, x = 7e^t - 7t - 5$

6 a $\dot{x} = 3t^2 - 12, x = t^3 - 12t$

b When $t = 2, \dot{x} = 0$.

c 16 cm

d $2\sqrt{3}$ seconds, 24 cm/s, $12\sqrt{3}$ cm/s^2

e As $t \to \infty, x \to \infty$ and $v \to \infty$.

7 a The acceleration is 10 m/s^2 downwards.

b $v = -10t + 40, x = -5t^2 + 40t + 45$

c 4 seconds, 125 metres

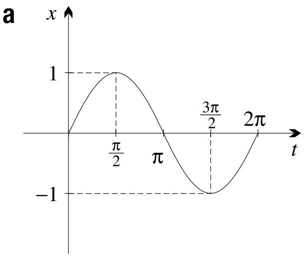
d When $t = 9, x = 0$.

e 50 m/s

f 80 metres, 105 metres

g 25 m/s

8 a



b $t = \pi$ and $t = 2\pi$

c $\dot{x} = -\cos t$

d $t = \frac{\pi}{2}$

e **i** $x = 5 - \sin t$

ii $x = 4$

9 a $v = 20$ m/s

b $20\, e^{-t}$ is always positive.

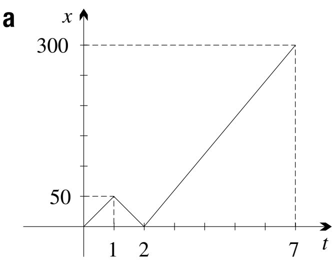
c $a = -20\, e^{-t}$

d -20 m/s^2

e $x = 20 - 20\, e^{-t}$

f As $t \to \infty, a \to 0, v \to 0$ and $x \to 20$.

10 a



b 400 km

c $57\frac{1}{7}$ km/hr

11 a $x = 20$ m, $v = 0$

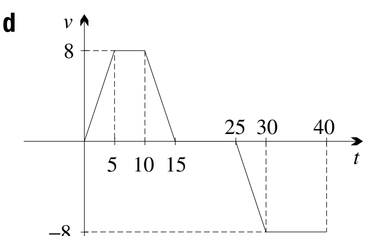
b **i** 8 m/s **ii** 0 **iii** -8 m/s

c **i** north (The graph is concave up.)

ii south (The graph is concave down.)

iii south (The graph is concave down.)

d



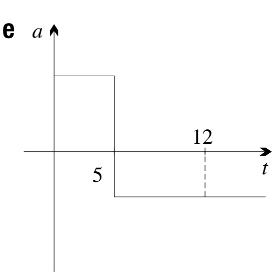
12 a at $t = 5$

b at $t = 12, 0 < t < 12, t > 12$

c $0 < t < 5, t > 5$

d at $t = 12$, when the velocity was zero

e



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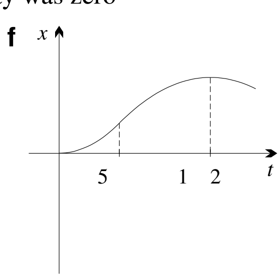

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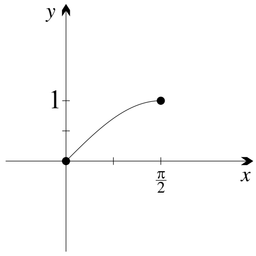
13 a i



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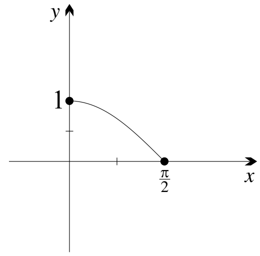

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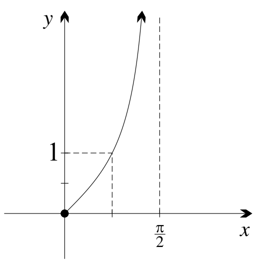

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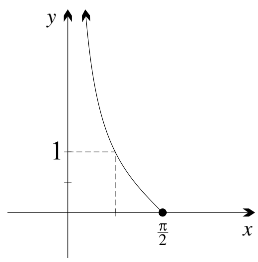
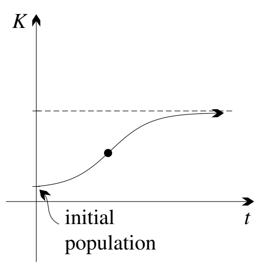


b i $y = \sin x$ **ii** $y = \cos x$
iii $y = \cot x$ **iv** $y = \tan x$

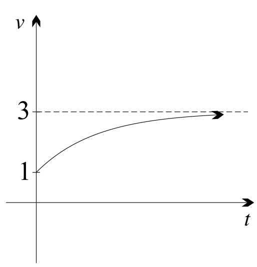
14 a Initially K increases at an increasing rate so the grpah is concave up. Then K increases at a decreasing rate so is concave down. The change in concavity coincides with the inflection point.

b



15 a 7500 L
b $\dot{V} = -12(50 - 2t)$
c $\dot{V}$ is negative in the given domain.
d $\dot{V}$ is negative and $\ddot{V} = 24$ is positive, so the outflow decreases.

16 a



b $\dot{x}$ increases so it accelerates.
c $\ddot{x} = \frac{2}{5}e^{-\frac{1}{5}t}$ which is always positive.
d $\lim\limits_{t \to \infty} \dot{x} = 3\,\text{m/s}$

$x = 3t + 10\left(e^{-\frac{1}{5}t} - 1\right)$

17 a 13 000 **b** 0.088 **c** 31 400
18 a 56 min 47 s **b** 48 grams

Chapter 8

Exercise 8A

1 a $T_3 - T_2 = T_2 - T_1 = 7$ **b** $a = 8, d = 7$
c 358 **d** 1490
2 a $a = 2, d = 2$ **b** 250 500
3 a $\frac{T_3}{T_2} = \frac{T_2}{T_1} = 2$ **b** $a = 5, r = 2$
c 320 **d** 635
4 a $\frac{T_3}{T_2} = \frac{T_2}{T_1} = \frac{1}{2}$ **b** $a = 96, r = \frac{1}{2}$
c $\frac{3}{4}$ **d** $191\frac{1}{4}$
e $-1 < r < 1, S_\infty = 192$
5 a i $a = 52, d = 6$
ii 14 **iii** 1274
b i 125 **ii** $-\frac{25}{49}$
c i $d = -3$
ii $T_{35} = -2$
iii $S_n = \frac{1}{2}n(203 - 3n)$
6 a i 1.01
ii $T_{20} = 100 \times 1.01^{19} \doteqdot 120.81$
iii 2201.90
b i $\frac{3}{2}$
ii 26 375
iii $|r| = \frac{3}{2} > 1$
c i $\frac{1}{3}$
ii $|r| = \frac{1}{3} < 1, S_\infty = 27$
7 a \$96 000, \$780 000 **b** the 7th year
8 a $r = 1.05$ **b** \$124 106, \$1 006 232
9 a i All the terms are the same.
ii The terms are decreasing.
b If $r = 0$, then $T_2 \div T_1 = 0$, so $T_2 = 0$. Hence $T_3 \div T_2 = T_3 \div 0$ is undefined.
c i The terms alternate in sign.
ii All the terms are the same.
iii The terms are $a, -a, a, -a, \ldots$.
iv The terms are decreasing in absolute value.
10 a \$50 000, \$55 000, \$60 000, $d = \$5000$
b \$40 000, \$46 000, \$52 900, $r = 1.15$
c For Lawrence $T_5 = \$70000$ and $T_6 = \$75000$. For Julian $T_5 \doteqdot \$69960.25$ and $T_6 \doteqdot \$80454.29$. The difference in T_6 is about \$5454.
11 a i $T_n = 47000 + 3000n$
ii the 18th year
b \$71 166

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12 a 12 metres, 22 metres, 32 metres
b $10n + 2$
c i 6 **ii** 222 metres
13 a 18 times **b** 1089 **c** Monday
14 a 85000 **b** 40000
15 a $D = 6400$ **b** $D = 7600$ **c** the 15th year
d $S_{13} = \$1092000$, $S_{14} = 1204000$
16 a 40m, 20m, 10m, $a = 40$, $r = \frac{1}{2}$
b 80m
c The GP has ratio $r = 2$ and hence does not converge. Thus Stewart would never stop running.
17 a $\left(\frac{1}{2}\right)^{\frac{1}{4}}$
b $S_\infty = \dfrac{F}{1 - \left(\frac{1}{2}\right)^{\frac{1}{4}}} \doteqdot 6.29F$
18 a i $r = \cos^2 x$ **ii** $x = 0, \pi, 2\pi$
iv When $\cos x = 0$, the series is not a GP because the ratio cannot be zero. But the series is then $1 + 0 + 0 + \cdots$, which trivially converges to 1. When $\cos x = 0$, then $\sin x = 1$ or -1, so $\operatorname{cosec}^2 x = 1$, which means that the given formula for S_∞ is still correct.
b i $r = \sin^2 x$ **ii** $x = \frac{\pi}{2}, \frac{3\pi}{2}$
iv When $\sin x = 0$, the series is not a GP because the ratio cannot be zero. But the series is then $1 + 0 + 0 + \cdots$, which trivially converges to 1. When $\sin x = 0$, then $\cos x = 1$ or -1, so $\sec^2 x = 1$, which means that the given formula for S_∞ is still correct.
19 b at $x = 16$
c i at $x = 18$, halfway between the original positions
ii 36 metres, the original distance between the bulldozers

Exercise 8B

1 a 5 **b** 14 **c** 23 **d** 3
e 9 **f** 15 **g** 4 **h** 8
i 14 **j** 2 **k** 5 **l** 11
2 a $\dfrac{T_3}{T_2} = \dfrac{T_2}{T_1} = 1.1$
b $a = 10$, $r = 1.1$
c $T_{15} = 10 \times 1.1^{14} \doteqdot 37.97$
d 19
3 a $r = 1.05$
b \$62053, \$503116
c the 13th year
4 the 19th year
7 a SC 50: 50%, SC 75: 25%, SC 90: 10%
c 4 **d** at least 7
8 a $T_n = 3 \times \left(\dfrac{2}{3}\right)^{n-1}$ **b** 4.5 metres
c ii 16
9 a the 10th year **b** the 7th year
10 a Increasing by 100% means doubling, increasing by 200% means trebling, increasing by 300% means multiplying by 4, and so on.
b Solve $(1.25)^n > 4$. The smallest integer solution is $n = 7$.
11 a $S_n = \dfrac{3\left(1 - \left(\frac{2}{3}\right)^n\right)}{1 - \frac{2}{3}} = 9\left(1 - \left(\frac{2}{3}\right)^n\right)$
b The common ratio is less than 1. $S = 9$
c $n = 17$

Exercise 8C

1 a i \$900 **ii** \$5900
b i \$120 **ii** \$420
c i \$3750 **ii** \$13750
d i \$5166 **ii** \$17166
2 a i \$5955.08 **ii** \$955.08
b i \$443.24 **ii** \$143.24
c i \$14356.29 **ii** \$4356.29
d i \$18223.06 **ii** \$6223.06
3 a i \$4152.92 **ii** \$847.08
b i \$199.03 **ii** \$100.97
c i \$6771.87 **ii** \$3228.13
d i \$7695.22 **ii** \$4304.78
4 a \$507.89 **b** \$1485.95
c \$1005.07 **d** \$10754.61
5 a \$6050 **b** \$25600 **c** 11 **d** 5.5%
6 a $A_n = 10000(1 + 0.065 \times n)$
b $A_{15} = \$19750$, $A_{16} = \$20400$
7 a \$101608.52 **b** \$127391.48
8 a Howard — his is \$21350 and hers is \$21320.
b Juno — hers is now \$21360.67 so is better by \$10.67.
9 a \$1120 **b** \$1123.60 **c** \$1125.51 **d** \$1126.83
10 a \$8000 **b** \$12000 **c** \$20000
11 \$19990
12 a \$7678.41 **b** \$1678.41
c 9.32% per annum
13 a \$12209.97
b 4.4% per annum
c Solve $10000 \times \left(\frac{1.04}{12}\right)^n > 15000$. The smallest integer solution is $n = 122$ months.
14 \$1110000
15 a 24 **b** 14 **c** 7 **d** 9
16 $A_n = 6000 \times 1.12^n$ **a** 7 years
b 10 years **c** 13 years **d** 21 years

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17 8 years and 6 months

18 a $C = C_0 \times 1.01^t$

i $1.01^{12} - 1 \doteqdot 12.68\%$

ii $\log_{1.01} 2 \doteqdot 69.66$ months

b $k = \log_e 1.01$

i $e^{12k} - 1 \doteqdot 12.68\%$

ii $\frac{1}{k}\log_e 2 \doteqdot 69.66$ months

19 7.0%

20 a \$5250 **b** \$20 250

c 6.19% per annum

21 a \$40 988 **b** \$42 000

22 b 3 years

Exercise 8D

1 a **i** \$732.05 **ii** \$665.50 **iii** \$605

iv \$550 **v** \$2552.55

b **i** \$550, \$605, \$665.50, \$732.05

ii $a = 550, r = 1.1, n = 4$

iii \$2552.55

2 a **i** \$1531.54 **ii** \$1458.61

iii \$1389.15, \$1323, \$1260 **vi** \$6962.30

b **i** \$1260, \$1323, \$1389.15, \$1458.61, \$1531.54

ii $a = 1260, r = 1.05, n = 5$

iii \$6962.30

3 a **i** $\$1500 \times 1.07^{15}$

ii $\$1500 \times 1.07^{14}$

iii $\$1500 \times 1.07$

iv $A_{15} = (1500 \times 1.07) + (1500 \times 1.07^2) + \cdots + (1500 \times 1.07^{15})$

b \$40 332

4 a **i** $\$250 \times 1.005^{24}$

ii $\$250 \times 1.005^{23}$

iii $\$250 \times 1.005$

iv $A_{24} = (250 \times 1.005) + (250 \times 1.005^2) + \cdots + (250 \times 1.005^{24})$

b \$6390

5 a **i** $\$3000 \times 1.065^{25}$

ii $\$3000 \times 1.065^{24}$

iii $\$3000 \times 1.065$

iv $A_{25} = (3000 \times 1.065) + (3000 \times 1.065^2) + \cdots + (3000 \times 1.065^{25})$

c \$188 146 and \$75 000

6 b \$669 174.36 **c** \$429 174.36 **e** \$17 932.55

7 c iii 18

8 a \$200 000 **b** \$67 275 **c** \$630 025

d **i** $A_n = 100000 \times 1.1 \times ((1.1)^n - 1)$

iii 25

e $\frac{100000}{630025} \times 10000 \doteqdot \$15872$

9 a \$360 **b** \$970.27

10 a \$31 680 **b** \$394 772 **c** \$1 398 905

11 a \$134 338 **b** \$309 281

12 \$3086

13 a \$286 593

b **i** \$107 355 **ii** \$152 165

14 a \$27 943.29 **b** the 19th year

15 a 18

16 The function FV calculates the value just after the last premium has been paid, not at the end of that year.

17 c $A_2 = 1.01M + 1.01^2M$,
$A_3 = 1.01M + 1.01^2M + 1.01^3M$,
$A_n = 1.01M + 1.01^2M + \cdots + 1.01^nM$

e \$4350.76 **f** \$363.70

18 b $A_2 = 1.002 \times 100 + 1.002^2 \times 100$,
$A_3 = 1.002 \times 100 + 1.002^2 \times 100 + 1.002^3 \times 100$,
$A_n = 1.002 \times 100 + 1.002^2 \times 100 + \cdots + 1.002^n \times 100$

Exercise 8E

1 b **i** \$210.36 **ii** \$191.24 **iii** \$173.86

iv \$158.05 **v** \$733.51

c **i** \$158.05, \$173.86, \$191.24, \$210.36

ii $a = 158.05, r = 1.1, n = 4$

iii \$733.51

2 b **i** \$1572.21 **ii** \$1497.34

iii \$1426.04, \$1358.13, \$1293.46

iv \$7147.18

c **i** \$1293.46, \$1358.13, \$1426.04, \$1497.34, \$1572.21

ii $a = 1293.46, r = 1.05, n = 5$

iii \$7147.18

3 a **ii** $\$1646.92 \times 1.07^{14}$ **iii** $\$1646.92 \times 1.07^{13}$

iv $\$1646.92 \times 1.07$ **v** \$1646.92

vi $A_{15} = 15000 \times (1.07)^{15} - (1646.92 + 646.92 \times 1.07 + \cdots + 1646.92 \times (1.07)^{13} + 1646.92 \times (1.07)^{14})$

c \$0

4 a **i** $100000 \times 1.005^{240}$

ii $M \times 1.005^{239}$

iii $M \times 1.005^{238}$ and M

iv $A_{240} = 100000 \times 1.005^{240} - (M + 1.005M + 1.005^2M + \cdots + 1.005^{239}M)$

c The loan is repaid. **d** \$716.43

e \$171 943.20

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5 a **i** 10000×1.015^{60}
ii $M \times 1.015^{59}$
iii $M \times 1.015^{58}$ and M
iv $A_{60} = 10000 \times 1.015^{n} - (M + 1.015M + \cdots + 1.015^2 M + 1.015^{59} M)$
c \$254
6 a $A_{180} = 165000 \times 1.0075^{180} - (1700 + 1700 \times 1.0075 + 1700 \times 1.0075^2 + \cdots + 1700 \times 1.0075^{179})$
c −\$10012.67
7 a $A_n = 250000 \times 1.006^n - (2000 + 2000 \times 1.006 + 2000 \times 1.006^2 + \cdots + 2000 \times 1.006^{n-1})$
c \$162498, which is more than half.
d −\$16881 **f** 8 months
8 c It will take 57 months, but the final payment will only be \$5490.41.
9 a The loan is repaid in 25 years.
c \$1226.64 **d** \$367993
e \$187993 and 4.2% pa
10 b \$345
11 a \$4202 **b** $A_{10} = \$6.66$
c Each instalment is approximately 48 cents short because of rounding.
12 b \$216511
13 a \$2915.90 **b** \$84.10
14 a \$160131.55
b \$1633.21 < \$1650, so the couple can afford the loan.
15 b zero balance after 20 years
c \$2054.25
16 \$44131.77
17 b 57
18 c $A_2 = 1.005^2 P - M - 1.005M$,
$A_3 = 1.005^3 P - M - 1.005M - 1.005^2 M$,
$A_n = 1.005^n P - M - 1.005M - \cdots - 1.005^{n-1} M$
e \$1074.65 **f** \$34489.78
19 b $A_2 = 1.008^2 P - M - 1.008M$,
$A_3 = 1.008^3 P - M - 1.008M - 1.008^2 M$,
$A_n = 1.008^n P - M - 1.008M - \cdots - 1.008^{n-1} M$
d \$136262
e $n = \log_{1.008} \dfrac{125M}{125M - P}$, 202 months

Chapter 8 review exercise

1 a $a = 31, d = 13$ **b** 16 **c** 2056
2 a $\frac{1}{2}$ **b** $|r| = \dfrac{1}{2} < 1$ **c** $S_\infty = 48$
3 a $n = 11$ **b** $n = 99$ **c** $n = 228$ **d** $n = 14$
4 27000 litres
5 'Increasing by 2000%' means that the profit is 21 times larger. The smallest integer solution of $(1.14)^n > 21$ is $n = 24$.
6 a $r = 1.04$ **b** \$49816, \$420214
7 a $T_n = 43000 + 4000n$ **b** 2017
8 2029
9 a \$15593.19 **b** \$3593.19 **c** 5.99%
10 a $\$25000 \times (0.88)^4 \doteqdot 14992$
b \$2502 per year
c $\$25000 \div (0,88)^4 \doteqdot \$41688$
d \$4172 per year
11 b \$224617.94
c \$104617.94
d The value is \$277419.10, with contributions of \$136000.00.
13 a $A_{180} = 159000 \times 1.005625^{180} - (1415 + 1415 \times 1.005625 + 1415 \times 1.005625^2 + \cdots + 1415 \times 1.005625^{179})$
c −\$2479.44
d \$1407.01
14 a $A_n = 1700000 \times 1.00375^n - (18000 + 18000 \times 1.00375 + 18000 \times 1.00375^2 + \cdots + 18000 \times 1.00375^{n-1})$
c \$919433, which is more than half.
d −\$57677.61
f 3 months

Chapter 9

Exercise 9A

1 a categorical
b numeric and continuous. But 'height correct to the nearest mm' is numeric and discrete.
c numeric and continuous. But 'age in years' is numeric and discrete.
d categorical by party or political code. This would need to be defined carefully — if a person can be affiliated to two parties, it would not be a function.
e categorical
f categorical
g numeric and discrete
h Shoe sizes are often arranged into categories.
i These are frequently integers from 1–100, that is, numeric and discrete. If results are reported by a grade, for example, A, B, C, . . . , this might be considered categorical.

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2 a median 14, mode 14, range 8

b median 10, every score is trivially a mode, range 12

c median 8, mode 3, range 12

d median 6.5, mode 4 & 6, range 6

e median 4, mode 4, range 7

f median 5.5, mode 2 & 3 & 9, range 8

3 a

score x	1	2	3	4	5	6	7	8
frequency f	4	3	4	2	1	1	1	6
cumulative	4	7	11	13	14	15	16	22

b 3.5

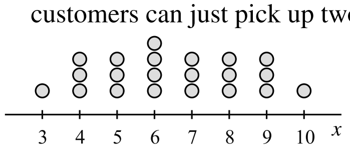
c 8

d i This is a median, but it might be more useful to use the mode in this case. It may be easier to develop a square box for four cupcakes rather than three.

ii See the previous comments. It is also common for sales to package a larger box to encourage customers to overbuy.

iii This is the mode, but if a box of four is marketed, customers can just pick up two boxes of four.

4 a



b

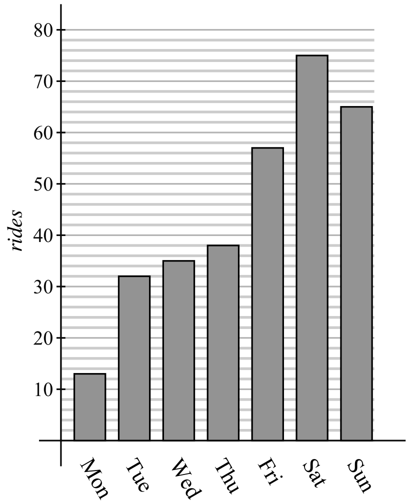
score x	3	4	5	6	7	8	9	10
frequency f	1	3	3	4	3	3	3	1
cumulative	1	4	7	11	14	18	20	21

c 6 hoops

d 6.5 hoops

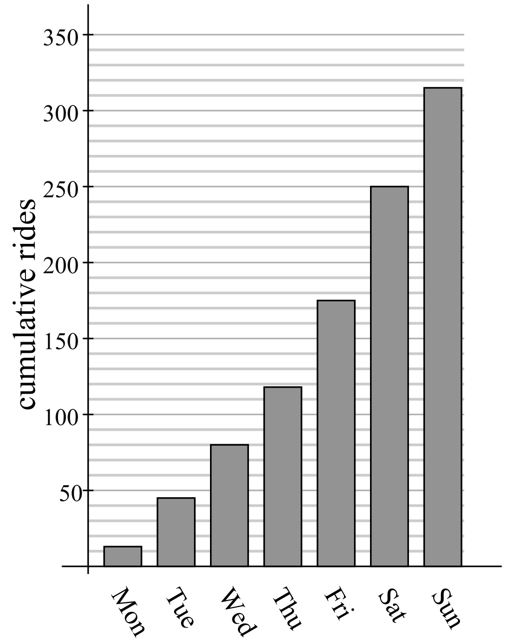
e Not really. If the scores are ordered by time, his scores improve over the sessions. This information is lost in the table and plot.

5 a



b

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
frequency	13	32	35	38	57	75	65
cumulative	13	45	80	118	175	250	315



6 a Blond hair and blue eyes. Different results might be expected in a different part of the world.

b Red hair and green eyes

c 45%

d 17%

e $25 \div 54 \doteqdot 46\%$

f $90 \div 247 \doteqdot 36\%$

g $671 \div 753 \doteqdot 89\%$

h These two results would suggest so. Geneticists link this to various pigment genes that affect both characteristics.

i The proportion of the various eye and hair colours will vary in different genetic populations and ethnic groups. Studies such as this may be done with a relatively non-diverse population to prevent the clouding effects of differing genetics.

7 a 80

b

salad	pie	soup	panini	burger
32.5%	12.5%	8.75%	20%	26.25%

c

salad	pie	soup	panini	burger
$130	$60	$70	$96	$168

d $524

e It returns more money than the more popular pie option. It is probably also important for the café to include a vegetarian option on the menu to cater for such customers or for groups with such customers.

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8 a In 2002 the price was $400 thousand, and in 2017 it was $1 million.

b Prices increased by 150%.

c $40 thousand per year

d They will increase another $13 \times \$40000 = \$520000$ to around $1.5 million.

e From 2014 to 2015, median house prices increased $120 thousand.

f From 2010 to 2011, median house prices decreased $40 thousand. How much did prices change?

9 a 35%, 140 dogs **b** 11%

c 75% **d** 15%

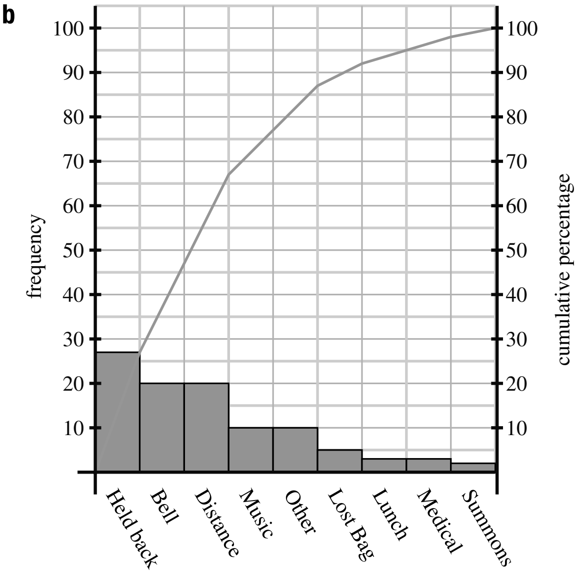
e This is quite a large category, and it may be that more investigation should be done to see if there were any other popular types of pets lumped into this category.

f Some pets may require more care and attention. For example, dogs may require frequent exercise and attention. This may give an opportunity for 'value adding' if owners are willing to pay for it. They should also consider what other pet boarding facilities are in the area, because it may be better to pick up a niche market, not covered by other pet boarding houses. Some pets may also be able to use the same types of accommodation, for example, rabbits and guinea pigs.

10 a

Reason	frequency	cumulative
Held back	27	27
Bell	20	47
Distance	20	67
Music	10	77
Other	10	87
Lost bag	5	92
Lunch	3	95
Medical	3	98
Summons	2	100

b



c The categories are arranged in descending order, so the function will be increasing (if every frequency is greater than zero), but by less at nearly every stage, causing it to curve downwards.

d 67%

e Remind teachers to release students promptly, increase the volume of the bell or the number of locations where the bell sounds, timetable students in rooms closer together where possible.

11 a 6% **b** 64% **c** 5%

d Care is needed when the graph is read in a hurry. Compare this with the Pareto chart later in this exercise where both axes are the same scale.

12 a The vertical origin is not at a 0% unemployment rate. This exaggerates the scale of the graph, which only shows a variation of 0.25%. This is still potentially significant, but it is only shown over a four-month period, so it is impossible to examine long-term trends. There are natural cycles — for example, there may be a rise when school pupils enter the employment market, and a drop when Christmas provides short-term retail employment. January may be a low point in economic indicators, before businesses return from holidays and begin to hire staff.

b There has been a significant increase over this five-year period, but more questions need to be asked by someone viewing the graph. What does the vertical scale represent — is it spending per citizen or spending per household? If it is per household, have the household structures changed over the period, such as more larger households? Is this a

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small community, in which case the data won't be very robust to changes in population? Is the data collected from sales at local shops, and does it include tourists and people passing through — has there been an increase in tourism, and was the data collected at the same time of year (more takeaways may be sold at the height of the tourist season)? What is included in the category of 'takeaway food' — if this is a health study, takeaway salads may be considered healthier than takeaway burgers (which the graphic is trying to suggest). Finally, note that the eye interprets the increase by the size of the graphic, but in fact it is the height that holds information, suggesting a greater increase than was actually the case.

c **i** People who do not have access to the internet, or do not feel as comfortable accessing and filling in an online survey, will not be represented. This may be more prevalent amongst older demographics.

ii The group should look at other hospitals, unless they particularly want to investigate the change in costs at their local hospital. Hospital costs could be influenced by government policy increasing the staffing numbers at the hospital, by purchase of new expensive diagnostic equipment, by opening and closing particular hospital wards (possibly relocating them to other hospitals), by quality control improvements, by industrial action of staff, and so on. The group likely will want to investigate the cause of any changes to overall expenses and may want to produce graphs of particular expenses, such as doctors' fees. They need to be clear what questions they actually want to ask — for example, are they concerned that medical treatment is getting more expensive for certain sections of the community who cannot afford it?

13 a 58%

b Around 3 billion

c About 0.92 billion

d 5.2%

e It may be of some use if choosing a major world language is a consideration, but there are often other considerations in deciding what language to learn. For example, you may have relatives who speak Malay, or a girl-friend who is French, or you may want to learn Japanese because of Japan's importance to Australia's economy. Others learn languages for academic reasons, such as Latin because of its historical and linguistic importance, or Russian to study Russian literature. When deciding a language on the number of speakers, it is probably more useful to consider the total number of speakers, not merely those who speak it as a first language — close to a billion people speak English, but only a third of them do so as a first language.

14 a **i** 15°C **ii** 30°C

b **i** 17°C **ii** 23°

c Around 6−7°C in December–January

d September and May

e November–February

f June–August

g Colour-blind readers may find the colours difficult to distinguish. Using dashes and colour also provides two visuals cues for the bulk of readers, making the graph easier to read.

15 a 30

b 73% and 27%

c Bill on Essay writing, Claire on Interpretation, Ellie on all sections.

d 80%

e 40%

f Aaron and Dion. Notice that Claire has not reached 50% in the Interpretation section.

16 a Provided that similar levels of postgraduates survive to the 55–64 age bracket.

b 61.1% **c** 21.6%

Exercise 9B

1 a $\bar{x} = 7$, Var $= 3.6$, $s \doteqdot 1.90$

x	f	xf	$(x - \bar{x})^2$	$(x - \bar{x})^2 f$
3	1	3	16	16
5	1	5	4	4
6	1	6	1	1
7	3	21	0	0
8	2	16	1	2
9	1	9	4	4
10	1	10	9	9
Total	10	70		36

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b $\bar{x} = 7$, Var $= 3.6$, $s \doteqdot 1.90$

x	f	xf	x^2f
3	1	3	9
5	1	5	25
6	1	6	36
7	3	21	147
8	2	16	128
9	1	9	81
10	1	10	100
Total	10	70	526

2 a $\bar{x} = 18$, $s \doteqdot 3.67$ **b** $\bar{x} = 7$, $s \doteqdot 3.06$

c $\bar{x} = 55$, $s \doteqdot 7.58$ **d** $\bar{x} = 11$, $s \doteqdot 1.88$

3 a $\bar{x} \doteqdot 7.17$, $s \doteqdot 3.18$ **b** $\bar{x} = 5.7$, $s \doteqdot 1.73$

c $\bar{x} = 3.03$, $s \doteqdot 0.94$ **d** $\bar{x} \doteqdot 42.88$, $s \doteqdot 10.53$

4 a 34 **b** $\mu \doteqdot 3.26$, $\sigma \doteqdot 1.75$

c

class	0–2	3–5	6–8
centre	1	4	7
freq	12	18	4

d $\mu \doteqdot 3.29$, $\sigma \doteqdot 1.93$

e Information is lost when data are grouped, causing the summary statistics to change.

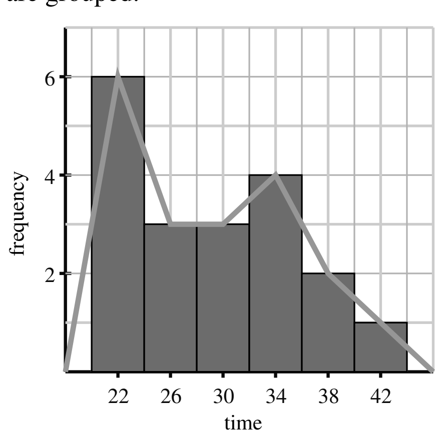
5 a 29.5

b

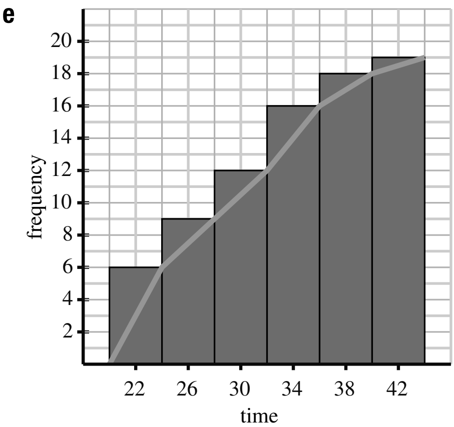
class	20–24	24–28	28–32	32–36	36–40	40–44
centre	22	26	30	34	38	40
freq	6	3	3	4	2	1
c.f.	6	9	12	16	18	20

c 30. No, because information is lost when the data are grouped.

d



e



6 a

x	152	154	155	157	158	159	162	163
f	1	2	1	1	2	3	2	3

x	164	165	166	168	170
f	2	2	3	1	1

b 162.5

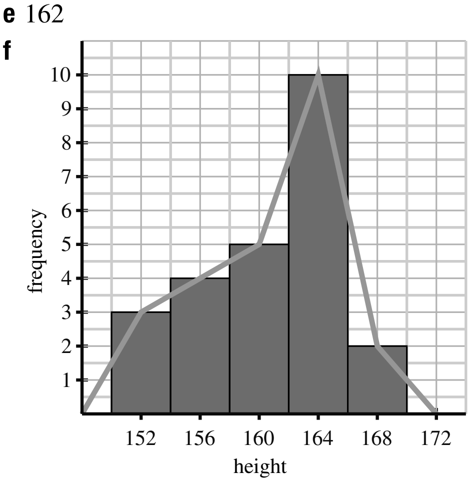
c Trends are less clear when the data are not grouped, because it is less visually clear that the data are falling in certain zones on the domain.

d

group	150–154	154–158	158–162	162–166	166–170
centre	152	156	160	164	168
freq	3	4	5	10	2

e 162

f



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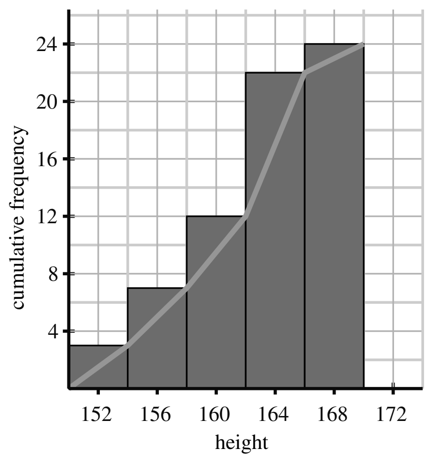
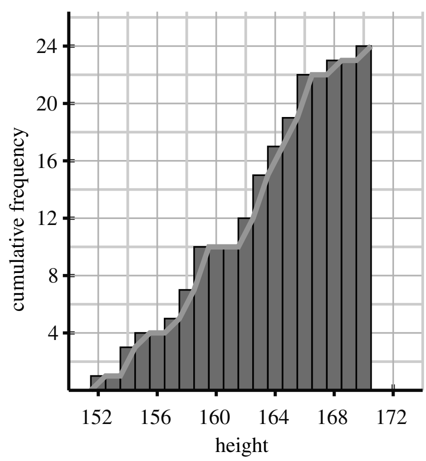


h



i The cumulative frequency polygon and ogive are much less sensitive to the grouping process than the frequency histogram and ogive. The graphs in parts **g** and **h** look very similar in shape.

7 a **i** 14.3 (1 decimal place)
ii 13.7 (1 decimal place)
iii 13.6 (1 decimal place)
b 0.005%

Exercise 9C

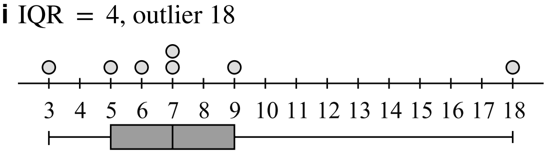
1 a mean 6.9, median 8, mode 8, range 10
b mean 21.4, median 22.5, mode 12, range 18
c mean 5.2, median 5.5, mode 7, range 5
d mean 62.3, median 61, trimodal: 54, 61, 73, range 19

2 a $Q_1 = 7, Q_2 = 13, Q_3 = 17$, IQR $= 10$
b $Q_1 = 12.5, Q_2 = 18.5, Q_3 = 25.5$, IQR $= 13$
c $Q_1 = 7.5, Q_2 = 11, Q_3 = 18$, IQR $= 10.5$
d $Q_1 = 5, Q_2 = 8.5, Q_3 = 13$, IQR $= 8$
e $Q_1 = 4, Q_2 = 7, Q_3 = 13$, IQR $= 9$
f $Q_1 = 10, Q_2 = 15, Q_3 = 21$, IQR $= 11$
g $Q_1 = 5, Q_2 = 9, Q_3 = 13.5$, IQR $= 8.5$
h $Q_1 = 12, Q_2 = 14, Q_3 = 18$, IQR $= 6$

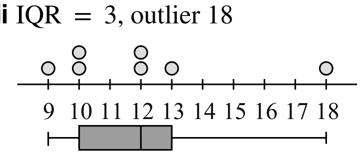
3 a $Q_1 = 4, Q_2 = 12, Q_3 = 16$, IQR $= 12$
b $Q_1 = 1, Q_2 = 6.5, Q_3 = 11$, IQR $= 10$
c $Q_1 = 7, Q_2 = 9, Q_3 = 12$, IQR $= 5$
d $Q_1 = 2.5, Q_2 = 5, Q_3 = 7$, IQR $= 4.5$
e $Q_1 = 7, Q_2 = 7, Q_3 = 10$, IQR $= 3$
f $Q_1 = 4, Q_2 = 5, Q_3 = 9$, IQR $= 5$
g $Q_1 = 2.5, Q_2 = 4, Q_3 = 9.5$, IQR $= 7$
h $Q_1 = 4.5, Q_2 = 9, Q_3 = 12$, IQR $= 7.5$

4 a Answers may differ here, but 40 and 92 are likely.
b 40, 54, 59, 69, 92
c IQR $= 15$, $Q_1 - 1.5 \times$ IQR $= 31.5$ and $Q_3 + 1.5 \times$ IQR $= 91.5$. Thus 92 is the only outlier by the IQR criterion.
d Some may identify 40 as an outlier by eye — this shows the advantage of plotting values, where it becomes evident that this score is well separated from other scores. A student receiving 40 in this cohort should be noted as someone needing extra attention and assistance.
e **i** 54, 60, 70.5, IQR $= 16.5$
ii 53.5, 58, 68.5, IQR $= 14.5$
iii 54, 59, 68.5, IQR $= 14.5$
f In this case, with a reasonably sized dataset, the middle of the data is fairly stable and removing an extreme value has only a small effect on the quartiles and IQR. With a large dataset and tightly clustered values in the middle two quarters of the data, the difference would be even smaller.
g **i** 60.8, 11.1 **ii** 61.6, 10.5
iii 59.5, 9.4 **iv** 60.3. 8.7
h 2.4 is 22% of 11.1. Any deviation from the mean is exaggerated by the standard deviation because the deviation from the mean is squared when calculating the variance.

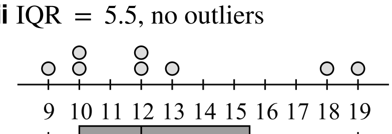
5 a **i** IQR $= 4$, outlier 18



ii IQR $= 3$, outlier 18



iii IQR $= 5.5$, no outliers



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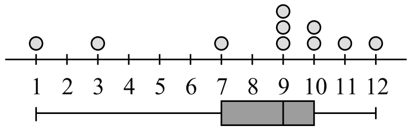
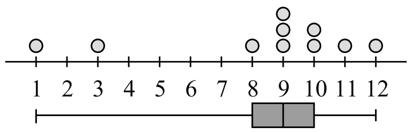
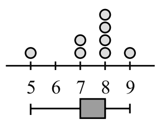
iv IQR $= 3$, outlier 1



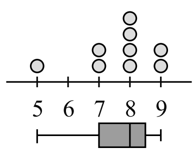
v IQR $= 2$, outliers 1, 3



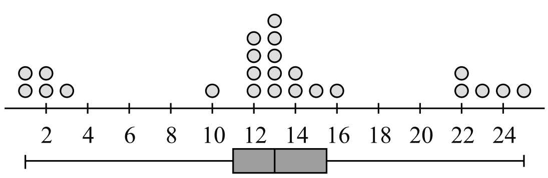
vi IQR $= 1$, outlier 5



vii IQR $= 1.5$, no outliers



viii IQR $= 4.5$, outliers 1, 1, 2, 2, 3, 23, 24, 25



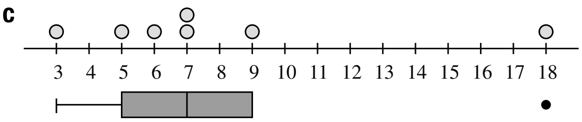
b It must be noted that some of the pathologies in these examples come about because of the small datasets. Statistics is always more accurate and reliable with a large dataset.

Generally the definition picks up the values that appear extreme on the dot plots. Notably (in these small datasets), it picks up single extreme values — if more values are a long way from the mean, they may not be marked as outliers.

Datasets with a small IQR may need a closer inspection — in part **vi** and **vii**, the value at 5 is not so extreme and the datasets are not so different, yet in one case it is marked as an outlier, but in the other it is not. The final dataset has a very tight subset of data between the Q_1 and Q_3, giving a small interquartile range. This definition of outliers gives 8 values in 24 (one third of the data) as outliers. Furthermore, 23–25 are outliers, but 22 is not. The issue here is the unusual shape of the distribution.

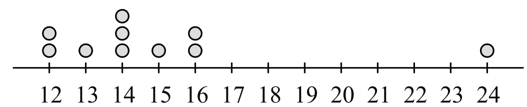
Rules such as this IQR criterion for outliers should be an invitation to inspect the values that have been flagged more closely, rather than following a rule blindly.

c



6 a $\bar{x} = 15$, $s = 3.29$

b The value 24 appears to be an outlier



c IQR $= 3$ and $Q_3 = 16$.
Because $24 > 16 + 1.5 \times 3$, this definition also labels 24 an outlier.

d $\bar{x} = 14$, $s = 1.41$

e This does not have much effect on the mean, but it has a big percentage effect on the standard deviation — removing the outlier more than halves the standard deviation. The operation of squaring $(x - \bar{x})$ means that values well separated from the mean have an exaggerated effect on the size of the variance.

f No effect at all!

g If there are significant outliers, or at least values spread far from the mean, this can have a big influence on the IQR. The IQR is a good measure if you are more interested in the spread of the central 50% of the data.

7 a Emily got less than 62

b Around 50% (and no more than 50%)

c The mathematics results were more spread out, and the centre of the data (by median) was 5 marks higher. The interquartile range of both distributions, however, was the same. Clearly the mathematics cohort has some students who perform much more strongly, and others who perform much weaker, than the majority of their peers.

d Xavier was placed in the upper half of the English cohort, but in the lower half of the mathematics cohort. The English result was thus more impressive.

e i 45

ii The bottom 25% of English scores show a spread of 6 marks (51–57). The bottom 25% of mathematics scores show a spread of 8 marks (53−61). The spread of the lower half is now much more comparable.

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8 a The results are not paired. Just becausey Genjo received the lowest score in the writing task does not mean that he received the lowest score in the speaking task. Thus we cannot answer the question, although we might make conjectures, given that Genjo is obviously struggling significantly with English.

b **i** mean 66.1, median 68, range 56

ii IQR $= 73 - 60 = 13$, 91 and 35 are outliers.

c **i** mean 64.4, median 65.5, range 56

ii $IQR = 71 - 57.5 = 13.5$, 37 and 93 are outliers.

d It is difficult to say. Students have found the second task more challenging, evidenced by the lower mean and median. This could be due to the construction of the task, or simply because it is a type of task that some students find more difficult.

Exercise 9D

1 a **i** height **ii** weight

b **i** radius

ii area. It is natural to think that the area of the circle is determined by the radius chosen when it is drawn, but mathematically we could write $r = \sqrt{\frac{A}{\pi}}$, reversing the natural relationship.

c **i** weight

ii price. Note that the price may change when meat is bought in bulk, so there is a deeper relationship between these two quantities than simply price $=$ weight $\times$ cost per kg.

d **i** world rank **ii** placing

e **i** temperature

ii power consumption. Power consumption increases with the use of air conditioners (higher temperatures) or heaters (colder weather).

f It is natural to take x as the independent variable and y as the dependent variable. Note in this case the relationship cannot naturally be reversed, because there are multiple x-values resulting from the same y-value.

2 a strong positive **b** virtually none.

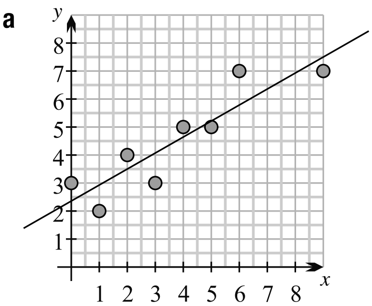
c strong negative **d** strong negative

e moderate positive **f** weak positive

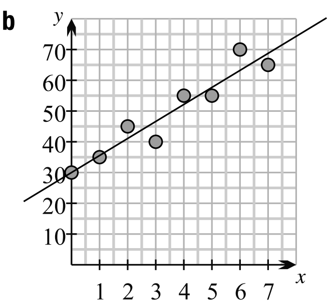
g strong negative **h** strong positive

i moderate negative

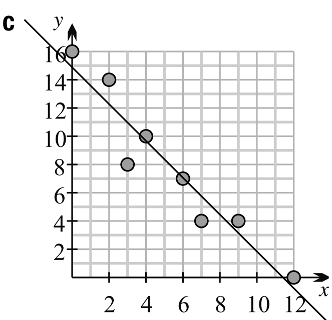
3 a



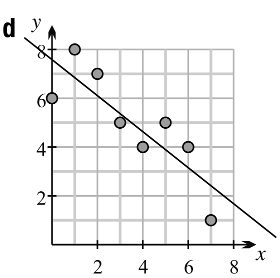
b



c

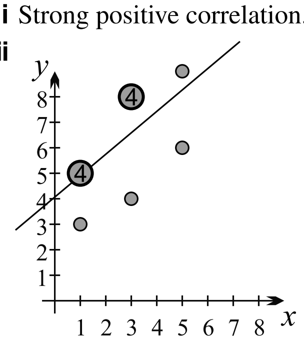


d



4 a **i** Strong positive correlation.

ii



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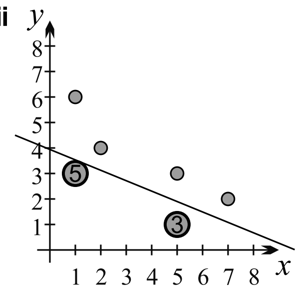
b **i** Strong negative correlation.

ii



5 a A quadratic relationship (a parabola).

b A square root. **c** A hyperbola.

d A circle. **e** An exponential.

f No obvious relationship.

6 a **ii** 6L and 10L

b $V = 2t$

c The V-intercept is zero. In no minutes, zero water will flow through the pipe.

d This is the flow rate of the water, 2 L/s.

e Negative time makes little sense here, because he cannot measure the volume of water that flowed for say -3 minutes.

f Experimental error could certainly be a factor, but it may simply be that the flow rate of water is not constant. It may vary due to factors in, for example, the pumping system.

g 60 L. The extrapolation seems reasonable provided that the half-hour chosen is at about the same time of day that he performed his experiment.

h 22.5 minutes

i Yasuf's experiments were all carried out in a period of several hours during the day. It may be that the flow rate changes at certain times of the day, for example, at peak demands water pressure may be lower and the flow rate may decrease. The flow rate may also be different at night — for example, the water pump may only operate during the day. More information and experimentation is required.

7 a 1000

b **i** $P = 0.9t + 5$

ii It looks fairly good.

iii Predicted $P = 13.1$, actual $P = 15.4$, so the error was 230 people.

c **i** The new model predicts $P = 16.4$, so it is certainly much better.

ii Population is growing very strongly in Hammonsville. Investigators should be looking into the cause of the growth, which may change over the next few years. For example, it may be due to a short-term mining boom. Eventually there may be other constraining factors, such as available land for housing.

d Extrapolation can be dangerous. Provided, however, that the independent variable is constrained to a small enough interval, linear predictions may well have validity. This is the idea behind calculus, where curves are approximated locally by a tangent.

8 a 99 in assessment 1, 98 in assessment 2. They were obtained by the same student, but another student also got 99 in assessment 1.

b 27 in assessment 1, 33 in assessment 2. They were the same student.

c Students getting below about 77 marks in assessment 1 do better in assessment 2, students above 77 marks in assessment 1 get a lower mark in assessment 2, according to the line of best fit. Perhaps the second assessment started easier, but was harder at the end.

d **i** 50 **ii** 65 **iv** 26

v A negative score! Clearly the model breaks down for small scores.

e $y = 0.74x + 20$

f A more accurate method would incorporate data from more than one assessment task in estimating their missing score. This is a question better tackled using standard deviation and the techniques of the next chapter.

9 a The maximum vertical difference between a plotted point and the line of best fit is about $0.8\,\text{s}^2$.

b It could be experimental error. For example, the string could have been twisted or released poorly, the experiment could have been incorrectly timed, or there could have been a recording error.

c They may have measured 10 periods and then divided by 10 before recording the length of one period. Errors could then arise if the motion was *damped*, that is, if the pendulum slowed down significantly over a short time period.

d By this model, $T^2 = \dfrac{2\pi^2}{g}L \doteqdot 4.03L$. These results are in pretty good agreement with the theory.

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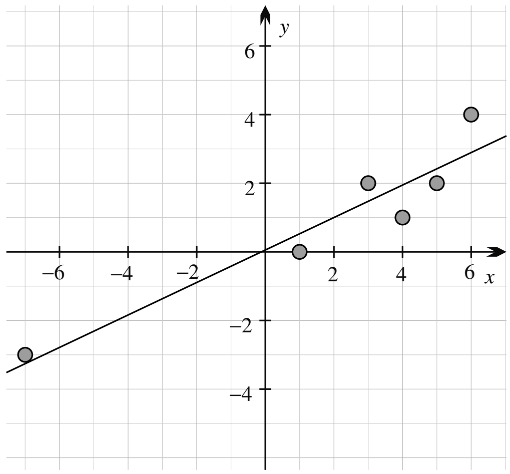
Exercise 9E

1 a There appears to be a fairly strong correlation, though note the small dataset.

b



c

							Sum
x	−7	1	3	4	5	6	12
y	−3	0	2	1	2	4	6
$x - \bar{x}$	−9	−1	1	2	3	4	0
$y - \bar{y}$	−4	−1	1	0	1	3	0
$(x - \bar{x})^2$	81	1	1	4	9	16	112
$(y - \bar{y})^2$	16	1	1	0	1	9	28
$(x - \bar{x})(y - \bar{y})$	36	1	1	0	3	12	53

d $(\bar{x}, \bar{y}) = (2, 1)$

e See above

f $53 \div \sqrt{112 \times 28} = 0.95$

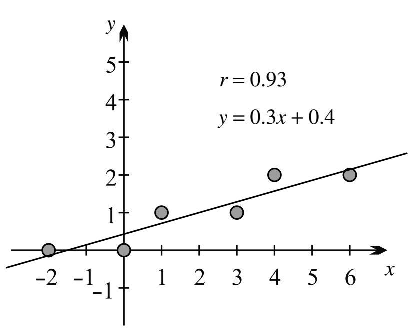
g It is a good fit.

h $53 \div 112 = 0.47$

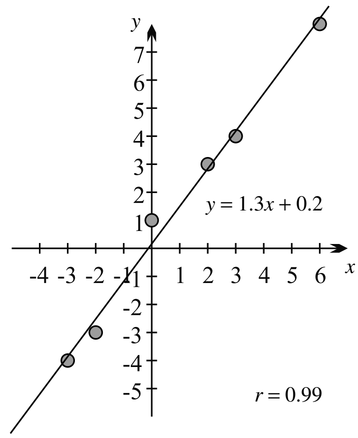
i $b = 1 - 0.47 \times 2 = 0.06$

j $y = \frac{1}{2}x + 0.$

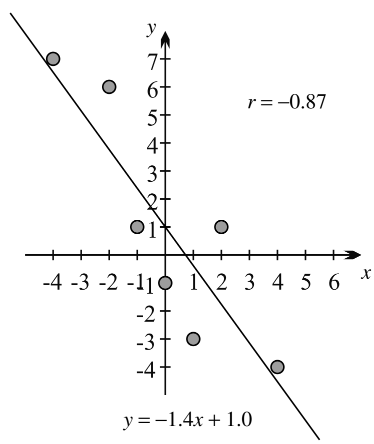
2 a



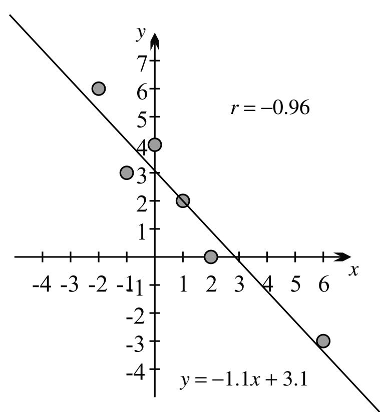
b



c



d



Exercise 9F

1 a $r = 0.96$, $y = 0.96x + 0.47$

b $r = 0.79$, $y = 0.45x + 2.6$

c $r = -0.86$, $y = -1.05x + 8.75$

d $r = -0.53$, $y = -0.41x + 4.70$

e $r = 0.96$, $y = 1.38x + 0.75$

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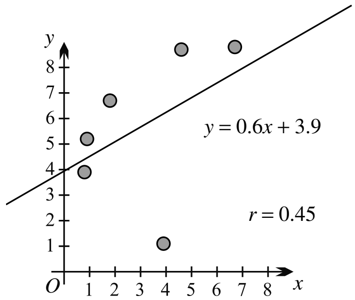
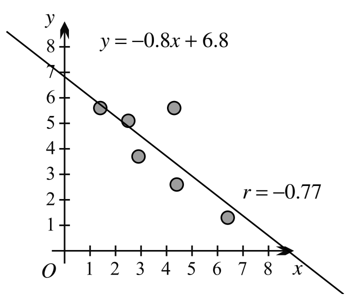
2 a $r = 0.45$, $y = 0.58x + 3.94$



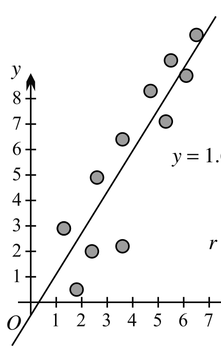
If the outlier at $(3.9, 1.1)$ is removed, then $r = 0.91$, $y = 0.75x + 4.43$.

b $r = -0.77$, $y = -0.78x + 6.83$



If the outlier at $(4.3, 5.6)$ is removed, then $r = -0.97$, $y = -0.89x + 6.79$.

c $r = 0.89$, $y = 1.62x - 0.51$



If the outlier at $(3.6, 2.2)$ is removed, then $r = 0.93$, $y = 1.61x - 0.19$.

3 Because the dataset was larger, the effect of the single outlier was mitigated by the other data points.

4 a Dataset 1:

i $y = 1x + 1.4$, $r = 0.86$

ii $y = 0.8x + 1.9$, $r = 0.79$

Dataset 2:

i $y = 0.7x + 3.0$, $r = 0.76$

ii $y = 0.7x + 2.5$, $r = 0.82$

b In all cases the correlation is strong. In part **a**, the repeated point has strengthened the correlation, but in the second example it has weakened it. Note that a strong correlation doesn't indicate that the data are correct. In part **a**, for example, leaving out 4 of the 9 points still gave a strong correlation, but a very different equation of line of best fit.

c The effect is less in the larger dataset, as expected. The gradient is unchanged (correct to one decimal place) and the y-intercept only differs by 20%, rather than by 26%. In a larger (more realistically sized) dataset, the effect would likely be less again. The effect of the repeated point will also depend on its place on the graph (central versus on the extremes of the data) and how close it is to the line of best fit.

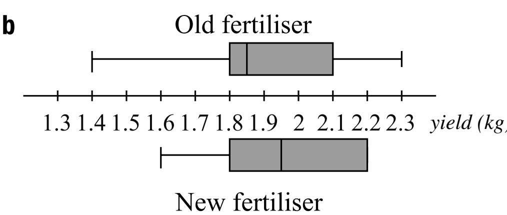
Chapter 9 review exercise

1 a mean 5, median 4.5, mode 4, range 8

b mean 15, median 15, mode 15 and 16 (it is bimodal), range 7

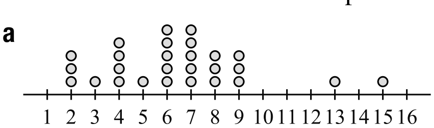
2 a Old fertiliser: 1.8, 1.85, 2.1,
New fertiliser: 1.8, 1.95, 2.2

b



c The fertiliser does appear to increase his yield — the median yield has increased by 100 g. Probably more more data are required because the lower quarter $(0–25\%)$ shows an increase, but the maximum has reduced. These claims, however, are each being made on the basis of one data point.

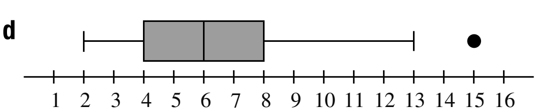
3 a



b By eye, 13 and 15 look like outliers.

c $\text{IQR} = 4$. By the IQR criterion, 15 is an outlier, but 13 is not.

d



4 a mean $\doteqdot 11.82\,\text{s}$, standard deviation $\doteqdot 0.537\,\text{s}$

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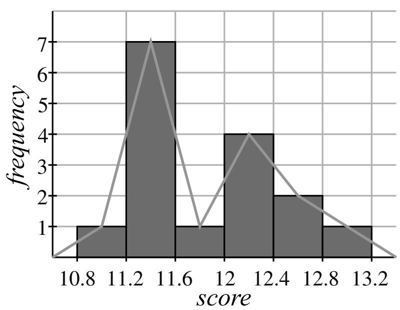
b

group	10.8–11.2	11.2–11.6	11.6–12.0
centre	11.0	11.4	11.8
freq	1	7	1
group	12.0–12.4	12.4–12.8	12.8–13.2
centre	12.2	12.6	13.0
freq	4	2	1

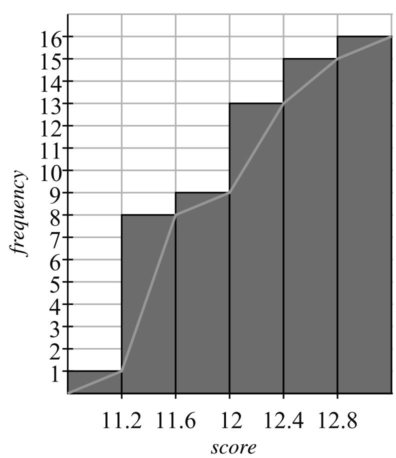
c mean $\doteqdot$ 11.85 s, standard deviation $\doteqdot$ 0.563 s. Agreement is reasonable, but as expected, the answers are not exactly the same.

d



e 0.5 seconds is a big difference in the time of a 100 metre sprint — the scale would be too coarse.

f



g The line at 50% of the data (frequency 8) meets the polygon where the sprint time is 11.6 seconds. You can confirm that this agrees with the result for splitting the grouped ordered data into two equal sets.

5 a

	first	second	Total
order entrée	45	42	87
no entreé	38	28	66
Total	83	70	153

b 153

c $87 \div 153 \doteqdot 57\%$

d 54%

e $P(\text{order entrée} \mid \text{attend first}) = 45 \div 83 \doteqdot 54\%$.
$P(\text{order entrée} \mid \text{attend second}) = 42 \div 70 = 60\%$.
No, it is not correct.

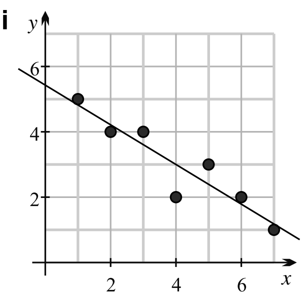
f $P(\text{attended first} \mid \text{ordered an entrée})$
$= 45 \div 87 \doteqdot 52\%$.

g $90 \times 60\% = 54$

h Those attending the first session may prefer a quick meal before heading out to the theatre or some other event. There may also be more family groups operating on a tighter budget.

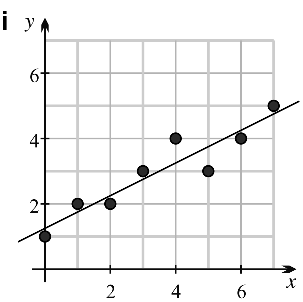
i If they can estimate the demand on certain dishes, then they may be able to prepare parts of the dish in advance, for example, preparing the garnishes or chopping the ingredients.

6 a i



ii $r = -0.93$, $y = -0.61x + 5.43$

b i



ii $r = 0.94$, $y = 0.5x + 1.25$

7 a 120 000

b 94 000, 62 000, 80 000, 80 000

c 316 000 and 79 000

d The arrivals may vary over the year because of seasonal or other effects. Government policy may consider an annual immigration quota, allowing a higher rate in one quarter to be balanced by a low rate in a subsequent quarter. As in 2000, examining the average for each quarter balances out such effects.

e 84 000

f It would be important to know the emigration rate of those leaving the country. The Net Overseas Migration (NOM) may be the better measure for

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many purposes. Other information of interest might include country of origin, destination within Australia, and whether they're intending to stay permanently or for a limited period.

g **i** 71 600

ii Rounding error has affected these calculations — a discrepancy in the second decimal place of the gradient is multiplied by 2000, resulting in an answer that is out by as much as $0.05 \times 2000 = 100$ thousand.

iii 84.300, which is in agreement with part **d**.

iv 660 000

v $660 \div 316 \times 100\% \doteqdot 209\%$, which is a 108% increase.

Chapter 10

Exercise 10A

1 **A** and **C**

2 a

x	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

b **i** $\frac{3}{8}$ **ii** $\frac{1}{16}$ **iii** 0 **iv** 1

c **i** $\frac{3}{16}$ **ii** $\frac{1}{2}$ **iii** $\frac{15}{16}$ **iv** $\frac{3}{8}$

3 a

score x	1	2	3	4	5	Total
f_r	0.1	0.2	0.45	0.15	0.1	1
xf_r	0.1	0.4	1.35	0.6	0.5	2.95
x^2f_r	0.1	0.8	4.05	2.4	2.5	9.85

b The sum of probabilities is 1.

c $\bar{x} = 2.95$

d The sample mean $\bar{x}$ is a measure of the centre of the dataset.

e $s^2 \doteqdot 1.15$ **f** $\sigma \doteqdot 1.07$

g The sample standard deviation s is a measure of the spread of the dataset.

h They are estimates of the mean $\mu = \mathrm{E}(X)$ and the standard deviation σ of the probability distribution.

i The sample mean $\bar{x}$ is 2.95 , so after 100 throws, 295 is a reasonable estimate of the sum.

4 a $\bar{x} = 5.26$, $x \doteqdot 1.07$

b The centre of the data is about 2.3 units greater, but the spread is about the same, according to the standard deviation.

5 a 3.5 and 4

b

score x	1	2	3	4	5	6	Total
frequency f	2	4	4	8	2	0	10
$P(X = x)$	0.1	0.2	0.2	0.4	0.1	0	1
$x \times P(x)$	0.1	0.4	0.6	1.6	0.5	0	3.2
$x^2 \times P(x)$	0.1	0.8	1.8	6.4	2.5	0	11.6

c $E(X) = 3.2$

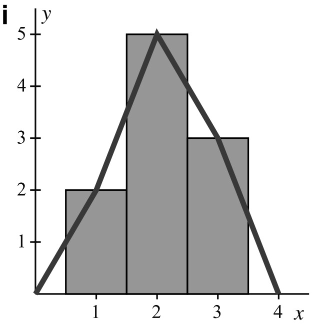
d $\mathrm{Var}(X) = 11.6 - (3.2)^2 = 1.36$

e 1.17

f It is usual to expect that for a quiz (covering recent work and including short easy questions) the marks will be high. These marks don't look impressive.

g $\mathrm{E}(X) = 16$, $\mathrm{Var}(X) = 34$, standard deviation 5.83

6 a **i**



ii 10

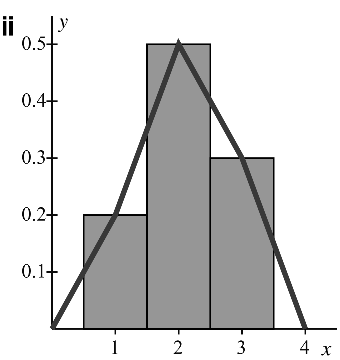
iii 10

iv Both areas are the same and equal to the total frequency, that is the number of scores.

b **i**

score x	1	2	3
frequency f	2	5	3
relative frequency f_r	0.2	0.5	0.3

ii



iii 1

iv 1

v Both areas are the same and equal to the total 1, that is the sum of the relative frequencies. (This will only happen when the rectangles have width 1.)

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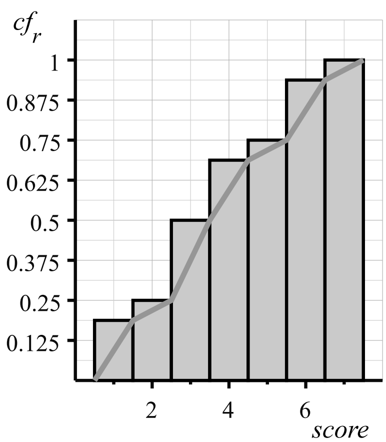
vi The relative frequencies are estimates of the probabilities. Note that both add to 1, both are non-negative, and both measure the chance that a random value will lie within the given rectangle of the histogram. A relative frequency is the *experimental* probability of an outcome, and is an *estimate* of the theoretical probability.

7 a

x	1	2	3	4	5	6	7	Total
f	3	1	4	3	1	3	1	16
f_r	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	1
cf	3	4	8	11	12	15	16	—
cf_r	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{8}{16}$	$\frac{11}{16}$	$\frac{12}{16}$	$\frac{15}{16}$	1	—

b

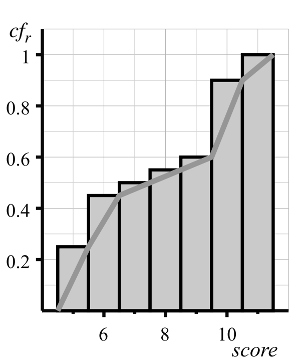


c $Q_1 = 2.5, Q_2 = 3.5, Q_3 = 5.5$

8 a

x	5	6	7	8	9	10	11	Total
f	5	4	1	1	1	6	2	16
f_r	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$	$\frac{1}{20}$.	$\frac{1}{20}$	$\frac{6}{20}$	$\frac{2}{20}$	1
cf	5	9	10	11	12	15	20	—
cf_r	$\frac{5}{20}$	$\frac{9}{20}$	$\frac{10}{20}$	$\frac{11}{20}$	$\frac{12}{20}$	$\frac{18}{20}$	1	—

b



c $Q_1 = 5.5, Q_2 = 7.5, Q_3 = 10$

9 a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** 0.1

d 12.5% of the households have 3 or more cars, so the town planners will not recommend additional on-street parking.

e

x	0	1	2	3	4
$P(X = x)$	0.25	0.5	0.125	0.10	0.025

f The area of the histogram is exactly the sum of the probabilities, because the width of each bar is 1 in this graph.

g The triangles cut off above the polygon fit into the spaces below the polygon.

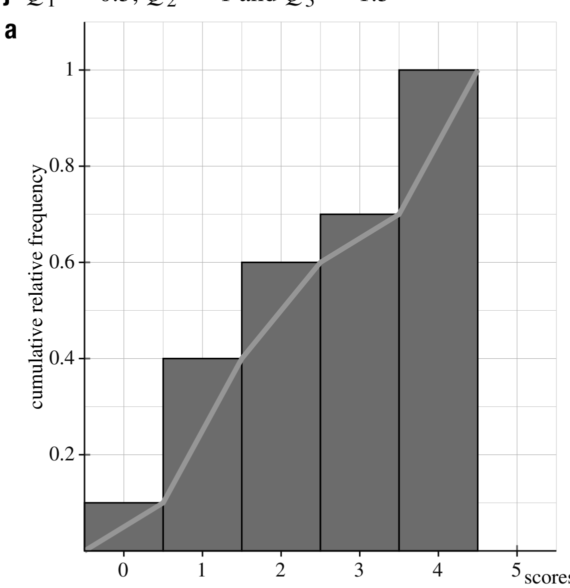
h This is an average, and is best understood by saying that for a large sample of *n* houses, we would expect them to have about 1.15*n* cars between them — see the next part.

i 115 cars. We are assuming that streets in the suburb are uniform with respect to car ownership. Actually, streets closer to train stations may manage with fewer cars because people catch the train to work, more affluent streets may own more cars, people may adjust car ownership to allow for availability of off-street or on-street parking.

x	0	1	2	3	4
$P(X \le x)$	0.25	0.75	0.875	0.975	1

j $Q_1 = 0.5, Q_2 = 1$ and $Q_3 = 1.5$

10 a



b 3.5 **c** 1, 2

d $Q_3 \doteqdot 3.7$ **e** 3.67. They agree.

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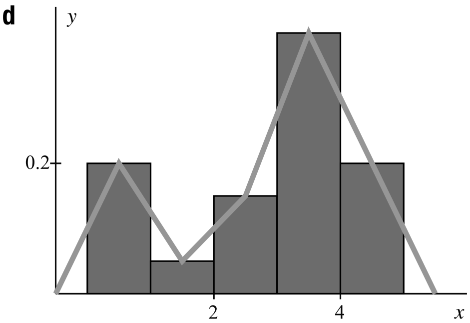
11 a median 3.5, mode 3.5

b

spent	0–1	1–2	2–3	3–4	4–5	Total
cc x	0.50	1.50	2.50	3.50	4.50	—
f	20	5	15	40	20	100
f_r	0.20	0.05	0.15	0.40	0.20	1

c mean 2.85, variance 1.9275, standard deviation 1.39

d



e **i** 0.2 **ii** 0.05 **iii** 0.15
iv 0.4 **v** 0.2

f The area of the relative frequency polygon, or the area under the frequency polygon bounded by the x-axis (they are the same). This only happens because the rectangles have width 1.

g **i** Equally likely
ii They are twice as likely to have spent between \$3–\$4.

h $\mathrm{E}(Y) = 4.85$, same variance

12 a The histogram covers 40 grid rectangles.

b **i** $0.3 \times 0.5 = 0.15$
ii 0.1 **iii** 0.3

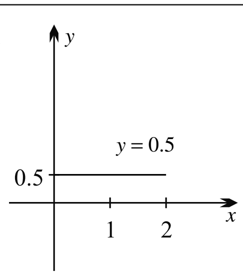
c **i** It is twice as likely to be 20°C.
ii In the class $19.25 - 19.75$

d **i** 0.1 **ii** 0.3

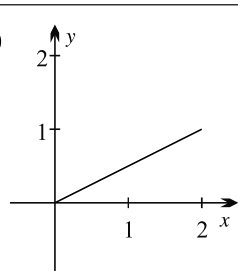
e First, the histogram only records the maximum daily temperature. Secondly, it recorded 20 consectuve days, but there will be natural variation over the year, and even within a season.

Exercise 10B

1 a



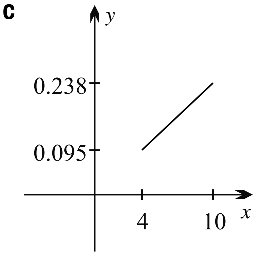
b



i $\int_0^2 f(x)\,dx = \text{area rectangle} = 1$

ii $\int_0^2 f(x)\,dx = \text{area triangle} = 1$

c



i $\int_4^{10} f(x)\,dx = \text{area trapezium}$

$= \frac{1}{2} \times 6\left(\frac{4}{42} + \frac{10}{42}\right) = 1$

2 a Yes, mode is $x = 1$

b No, the integral is 3

c No. The integral is 1, but $f(x) < 0$ if $x > 2$.

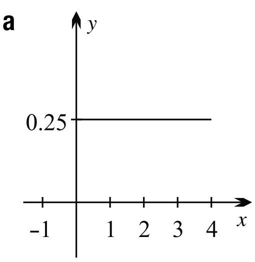
d Yes, provided that $n \geq 0$. Then mode is $x = 1$.

e Yes, mode is $x = \frac{\pi}{2}$

f Yes, mode is $x = 2$

3 b $f(x) = \frac{3}{4}(x - 3)(x - 1) < 0$, for $1 < x < 3$

4 a



b Clear from the graph

c **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$
iv 0 **v** $\frac{3}{4}$ **vi** $\frac{3}{4}$

d LHS $= \frac{1}{4}$, RHS $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

5 a $F(x) = \frac{1}{64}x^2$

b $F(x) = \frac{1}{16}(x^3 + 8)$

c $F(x) = \frac{x}{2}(3 - x^2)$

d $F(x) = \frac{1}{e}(e^x + x - 1)$

6 a $Q_2 = 4\sqrt{2}$, $Q_1 = 4$, $Q_3 = 4\sqrt{3}$

b $Q_2 = 0$, $Q_1 = -\sqrt[3]{4}$, $Q_3 = \sqrt[3]{4}$

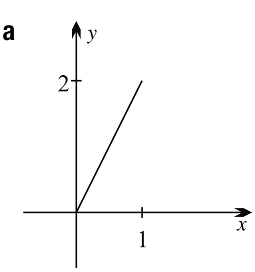
7 a $\frac{20}{170} \doteqdot 12\%$

b $\frac{4\pi}{170} \doteqdot 7\%$

c $\frac{20+4\pi}{170} \doteqdot 19\%$

d $1 - \frac{20+4\pi}{170} \doteqdot 81\%$

8 a



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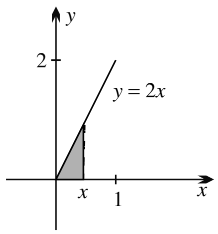
c **i** Area $= \frac{1}{2}x \times 2x$



d $Q_1 = \frac{1}{2}, Q_2 = \frac{1}{\sqrt{2}}, Q_3 = \frac{\sqrt{3}}{2}$

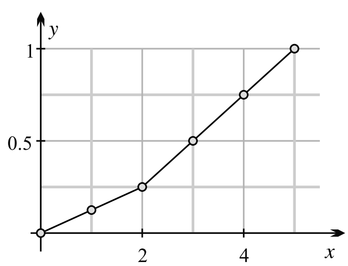
9 a $\frac{5}{243}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10}$ **d** $\frac{1}{2}$

10 a Clearly $f(x) \geq 0$ for all x, and the area under the graph is $2 \times 0.125 + 3 \times 0.25 = 1$.

b

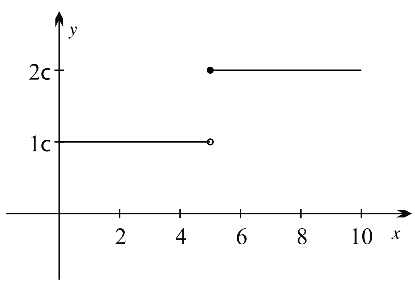
x	0	1	2	3	4	5
$P(X \leq x)$	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

c



d $F(x) = \begin{cases} \frac{1}{8}x, & \text{for } 0 \leq x < 2, \\ \frac{1}{4}x - \frac{1}{4}, & \text{for } 2 \leq x \leq 5. \end{cases}$

11 a



b Area $= 15c$, so $c = \frac{1}{15}$

c $F(x) = \begin{cases} cx, & \text{for } 0 \leq x < 5, \\ 2cx - 5c, & \text{for } 5 \leq x \leq 10. \end{cases}$

d $P(1 < X < 7) = F(7) - F(1) = 8c = \frac{8}{15}$

12 a The mode is $x = 2$ (where the vertex is).

c Symmetric about $x = 2$, $P(X = 2) = 0$, and total area is 1.

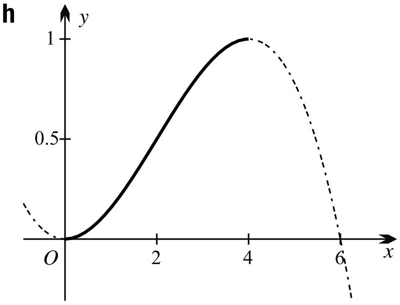
d $\frac{5}{32}$ and $\frac{27}{32}$ are complementary probabilities.

e $\frac{11}{256}$. The symmetry of the graph means that the areas are the same.

f $\frac{1}{32}x^2(6 - x)$

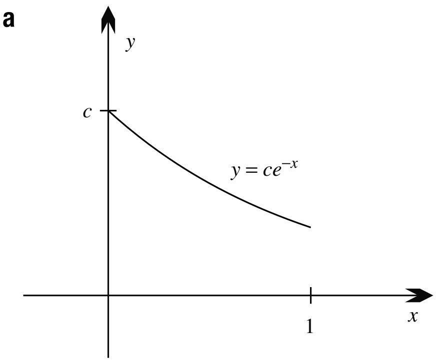
g **i** $\frac{81}{256}$ **ii** $\frac{41}{256}$ **iii** $\frac{29}{256}$ **iv** $\frac{47}{256}$

h



i $Q_1 = 1.3$, $Q_3 = 2.7$.

13 a



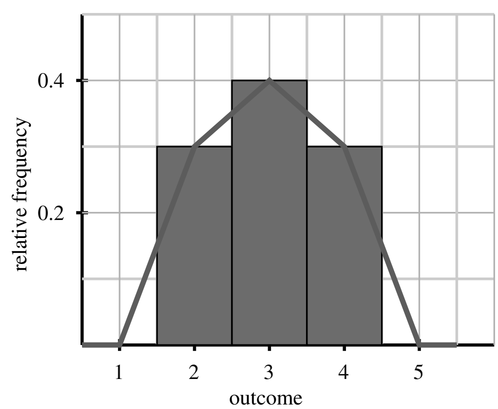
b $c = \frac{e}{e - 1}$

c $F(x) = \frac{e}{e - 1}(1 - e^{-x})$

d $Q_1 = \ln \frac{4e}{3e + 1}, Q_2 = \ln \frac{2e}{e + 1}, Q_3 = \ln \frac{4e}{e + 3}$

14 a See part **c**.

b Both areas are 1.



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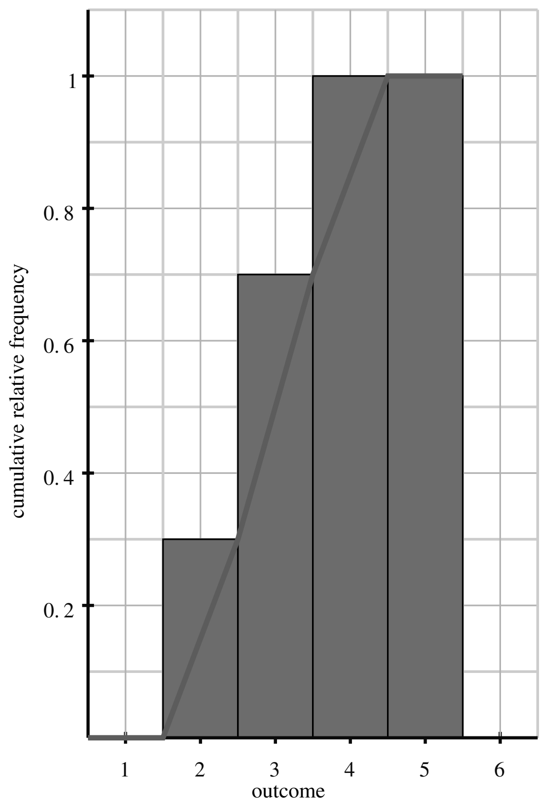
c

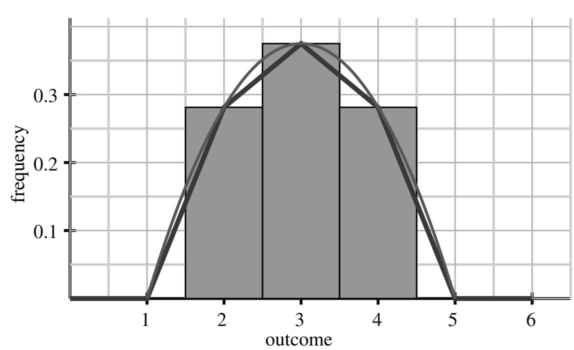


d 2.3, 3, 3.7

e i $\int_1^5 f(x)\,dx = 1$, and $f(x) \geq 0$ for $1 \leq x \leq 5$

ii

x	1	2	3	4	5
$f(x)$	0	0.28	0.375	0.28	0



iii $\frac{1}{32}(-x^3 + 9x^2 - 15x + 7)$

iv $P(X \leq 2.3) \doteqdot 0.25$, $P(X \leq 3) = 0.5$, $P(X \leq 3.7) \doteqdot 0.75$

v 2.3, 3 and 3.7 still seem good approximations.

15 b $F(x) = 1 - \frac{1}{x}$

c Total probability is 1.

d $\frac{4}{3}$, 2, 4

16 b $F(x) = 1 - e^{-x}$

c $Q_1 = \ln\frac{4}{3}$, $Q_2 = \ln 2$, $Q_3 = \ln 4$

Exercise 10C

1 a $f(x) \geq 0$ and by area formula or integration, $\int_0^{10} f(x)\,dx = 1$.

b $E(X) = 5$

c Yes — in the centre of this distribution interval $[0, 10]$

d $\mathrm{Var} = \frac{25}{3}$, $\sigma = \frac{5}{3}\sqrt{3}$

e $E(X^2) = \frac{100}{3}$ and $\mathrm{Var} = \frac{25}{3}$

3 a The function is never negative, and the integral over $[-1, 1]$ is 1.

b $E(X) = 0$

c $\mathrm{Var}(X) = \frac{3}{5}$, $\sigma = \frac{\sqrt{15}}{5}$

d $\frac{3\sqrt{15}}{25} \doteqdot 0.46$

4 a $E(X) = \frac{2}{3}$, $\mathrm{Var}(X) = \frac{1}{18}$, $\sigma = \frac{\sqrt{2}}{6}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) = \frac{4\sqrt{2}}{9}$

b $E(X) = 0$, $\mathrm{Var}(X) = \frac{1}{2}$, $\sigma = \frac{1}{\sqrt{2}}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) = \frac{1}{2}$

c $E(X) = 3$, $\mathrm{Var}(X) = \frac{3}{5}$, $\sigma = \frac{\sqrt{15}}{5}$, $P(\mu - \sigma \leq X \leq \mu + \sigma) \doteqdot 0.668$

5 a Yes.

b $E(X) = \frac{c}{2}$, as expected for a measure of the centre of this uniform distribution.

c $\frac{c^2}{12}$

d The answer agrees for this special case with $c = 10$.

e $E(X) = \frac{c}{2} + h = \frac{c + 2h}{2}$, variance unchanged.

f Put $h + c = k$ in the previous result: $E(X) = \frac{k + h}{2}$, $\mathrm{Var}(X) = \frac{(k - h)^2}{12}$

6 b $E(X) = \frac{23}{8}$, $E(X^2) = \frac{121}{12}$, $\mathrm{Var}(X) = \frac{349}{192} \doteqdot 1.82$

7 LHS $= \int_a^b x^2 f(x)\,dx - \int_a^b 2\mu x f(x)\,dx + \int_a^b \mu^2 f(x)\,dx$

By the definition of a PDF,

Term 3 $= \mu^2 \int_a^b f(x)\,dx = \mu^2$.

By the formula for the mean,

Term 2 $= -2\mu \int_a^b x f(x)\,dx = -2\mu^2$.

8 b $E(X) = 2.1$ **c** Agrees.

d Not only do both satisfy the condition that the area under the curve is 1, but they give the same result for the expected value.

9 b $E(X) = \frac{3}{2}$, $E(X^2) = 3$, $\mathrm{Var}(X) = \frac{3}{4}$

c i $1 - \frac{1}{4^3}$ **ii** $\frac{1}{8}$ **iii** $\frac{117}{1000}$

d $F(x) = 1 - \frac{1}{x^3}$

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10 a $\frac{d}{dx}xe^{-x} = e^{-x} - xe^{-x}$,

so $\int xe^{-x}\,dx = -e^{-x} - xe^{-x}$.

b $\mathrm{E}(X) = 1$

c The derivative is $(2xe^{-x} - x^2e^{-x})$
$+ (2e^{-x} - 2xe^{-x}) - 2e^{-x} = -x^2e^{-x}$,

so $\int x^2e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

d $\mathrm{E}(X^2) = 2$ and $\mathrm{Var}(X) = 1$.

Exercise 10D

1 a 0.5 **b** 0.8413 **c** 0.9972 **d** 0.9332
e 0.6554 **f** 0.9893 **g** 0.8849 **h** 1.0000

2 The total area under the curve is 1, so the areas of regions to the right and left of $z = a$ add to 1. This identity is true for any probability distribution.
a 0.5 **b** 0.1587 **c** 0.0228 **d** 0.0082
e 0.0968 **f** 0.2420 **g** 0.0548 **h** 0.0000

3 a From the even symmetry of the graph,
$P(Z < -a) = P(Z > a) = 1 - P(Z \le a)$.
(The result also holds for $a \le 0$, but this is not useful to us.) This result is certainly not true for all probability distributions.

b **i** 0.1151 **ii** 0.0107 **iii** 0.4207
iv 0.0007 **v** 0.0000 **vi** 0.2420
vii 0.0548 **viii** 0.0808 **ix** 0.5000

4 b **i** 0.4032 **ii** 0.4918 **iii** 0.2580
iv 0.4918 **v** 0.3643 **vi** 0.2580
vii 0.4452 **viii** 0.4032 **ix** 0.5000

c **i** 0.8064 **ii** 0.9836 **iii** 0.5762
iv 0.9962 **v** 0.3108 **vi** 0.8664

5 h **a** and **e**, **b** and **g**, **c** and **h**, **d** and **f**

6 h **a** and **c**, **b** and **g**, **d** and **f**, **e** and **h**

7 a This is evident from a graph by subtraction of areas.

b **i** 0.0483 **ii** 0.4100 **iii** 0.2297
iv 0.0923 **v** 0.4207 **vi** 0.1552

c **i** 0.9193 **ii** 0.7008 **iii** 0.9013

8 a 0.5 **b** 0 **c** 0.0359
d 0.8849 **e** 0.1151 **f** 0.3849
g 0.0359 **h** 0.8849 **i** 0.0792
j 0.8490

9 a 0.9032 **b** 0 **c** 0.3446
d 0.9554 **e** 0.9032 **f** 0.4332
g 0.2119 **h** 0.4207 **i** 0.0689
j 0.8893

10 a 0.9208 **b** 0.0792 **c** 0.6341 **d** 0.0364

12 a 50% **b** 84%

c 97.5% (Note the inaccuracy here. From the tables it should be 97.72.)
d 16% **e** 49.85% **f** 34%
g 47.5% **h** 2.35% **i** 68%
j 83.85% **k** 81.5% **l** 97.5%

13 a $b = 1$ **b** $b = 2$ **c** $b = -1$
d $b = 1$ **e** $b = 1$ **f** $b = 4$

14 a 0.6 **b** 2.3 **c** 1.2
d -0.8 **e** 1.1 **f** 2.6

15 a **i** $P(-1 < Z < 1) \doteqdot 68\%$
ii $P(Z < 2) \doteqdot 97.5\%$
iii $P(Z\langle -3 \text{ or } Z\rangle 3) = 0.3\%$

b Around 0.7 centimetres.

16 Mathematically, $P(Z = a) = \int_a^a f(x)\,dx$, which is an area of zero width. Practically, this represents the probability of getting a value exactly $Z = a$ for a continuous distribution, for example a height of exactly 1.7142435345345 … metres. In a continuous distribution, all such probabilities are zero.

17 a **i** all real values **ii** Even
iii $x = 0$ **iv** 1
v $z = -1$ and $z = 1$ **vi** $\left(0, \frac{1}{\sqrt{2\pi}}\right)$
vii There are no z-intercepts.

b **i** 0 **ii** 0 **iii** 0 **iv** 1

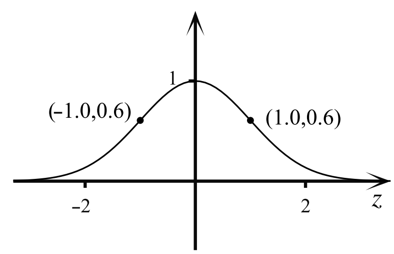
c $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$

18 c Stationary point $(0, 1)$. It is a maximum.

d Inflections at $(1, e^{-0.5})$ and $(-1, e^{-0.5})$

e $f(x) \to 0$ as $x \to \infty$ and as $x \to -\infty$

f



g See the graph at the top of this exercise.

19 a **i** $P(0 \le Z \le 1) = 0.3401$
ii $P(-1 \le Z \le Z) = 0.6802$
iii The graph is concave up on $[0, 1]$ — the concavity changes at the point of inflection at $z = 1$. Thus the polygonal path of the trapezoidal rule will lie below the exact curve.
iv This is good agreement with the empirical rule (68) and the table (0.6826).

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b **i** $P(-2 < Z < 2) = 2 \times 0.4750 = 0.95$

ii $P(-3 < Z < 3) = 2 \times 0.4981 = 0.9962$

20 a $E(Z) = \int_{-\infty}^{\infty} z \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\, dz$, which is the integral of an odd function on a symmetric domain, so $E(Z) = 0$.

b $\frac{d}{dz}\left(ze^{-\frac{1}{2}z^2}\right) = 1 \times e^{-\frac{1}{2}z^2} + z \times -ze^{-\frac{1}{2}z^2}$

$$ze^{-\frac{1}{2}z^2} = \int e^{-\frac{1}{2}z^2}\, dz - \int z \times ze^{-\frac{1}{2}z^2}\, dz$$

$$\left[ze^{-\frac{1}{2}z^2}\right]_{-\infty}^{\infty} = \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\, dz - \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\, dz$$

The LHS is 0, so

$$\int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\, dz = \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\, dz$$

and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2}\, dz = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2}\, dz$

$= 1$

Thus we have shown that $E(Z^2) = 1$.

c Using the previous part,

$$\begin{aligned}\text{Var}(Z) &= E(Z^2) - E(Z)^2\\ &= 1 - 0\\ &= 1\end{aligned}$$

Exercise 10E

1 a $z = 1$, 1 SD above **b** $z = -2$, 2 SD below

c $z = 1$, 1 SD above **d** $z = -2$, 2 SD below

e $z = 5$, 5 SD above **f** $z = -3$, 3 SD below

2 a **i** $+2.5$ **ii** -3 **iii** $+5.5$ **iv** $+\frac{1}{4}$

b **i** iii **ii** i, iii, iv

iii ii **iv** iv

v i, ii, iii

3 a $P(Z \le 0.5)$ **b** $P(Z > 0.25)$

c $P(Z \le -1)$ **d** $P(Z \ge -1.5)$

e $P(-2 \le Z \le -0.5)$ **f** $P(-1.75 \le Z \le 0.25)$

4 a $P(Z \ge 0) = 0.5$

b $P(-1 \le Z \le 1) = 0.68$

c $P(Z \le 2) = 0.975$

d $P(Z \ge -2) = 0.975$

e $P(-3 \le Z \le 1) = 0.8385$

f $P(-2 \le Z \le -1) = 0.1475$

5 a $P(-1 \le Z \le 3) = 0.8385$

b $P(Z \ge 1) = 0.16$

c $P(Z \ge 2) = 0.025$

6 a $P(-2.5 \le Z \le 2.5) = 0.9876$

b $P(Z \ge 1.6) = 0.0548$

c $P(Z \le -0.8) = 0.2119$

d $P(Z \ge -1.3) = 0.9032$

e $P(Z < 1.6) = 0.9452$

f $P(-2.5 < Z \le -1.5) = 0.0606$

7 a The score is above the mean.

b The score is below the mean.

c The score is equal to the mean.

8 a 69, 80

b 69, 80, 95, 50, 90, 52, 45

c 43, 45, 50, 52

d 95, 98

e It doesn't look very normal ('bell shaped').

Here is the stem-and-leaf plot:

4	3 5
5	0 2
6	9
7	
8	0
9	0 5 8

9 a **i** z-score for English (2.5) and maths (2). English is more impressive.

ii z-score for English (-0.8) and maths (-0.6). Maths is more impressive.

iii z-score for English (1.5) and maths (1). English is more impressive.

b 95 is 2.2 standard deviations above the mean.

$P(Z > 2.2) \doteqdot 1 - 0.9861 \doteqdot 1.4\%$

c The mathematics mean of 62 is 0.3 English standard deviations below the English mean 65%.

$P(Z > -0.3) = P(Z < 0.3) = 0.6179 \doteqdot 0.62$

10 a About 408 scores will lie within one SD from the mean, that is, in $[40, 60]$. About 570 scores will lie within two SDs from the mean, that is, in $[30, 70]$. About 598 scores will lie within three SDs from the mean, that is, in $[20, 80]$.

b **i** 415 **ii** 260 **iii** 462

c 4

11 a **i** $-1, -1.5, -2$

ii 1.5 standard deviations below the mean

iii 45

iv Some assessments may be harder than others — simply averaging his other results takes no account of this.

v Jack may perform better in certain types of assessments, for example, in Biology lab experiments, or he may perform better at certain times of the year. For example, his results may improve towards the end of the year.

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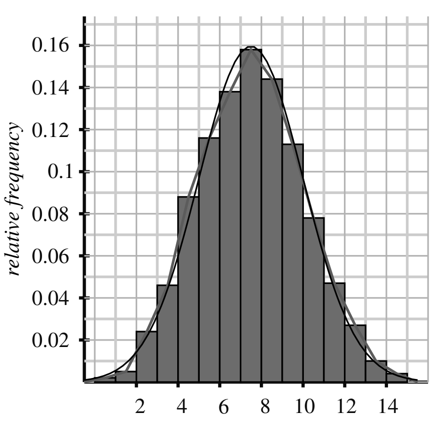
b z-scores 0.4, 0.625, 1, average 0.675.
Jill's estimate is 71.1

Exercise 10F

1 a 97.5% **b** 84%
2 a 3 **b** 50 **c** 1630
3 a 2.5%
b $2400 \times 0.15 \div 100 = 3.6$ screws (perhaps round to 4)
4 5%
5 a 0.26% **b** 65 000
6 about 31%
7 b 186 cm
c i Interpolate between 1.6 and 1.7
ii 197 cm
8 The boxes need to be marked with a mean weight of 496.7 grams — this would probably be rounded down to 496, which is actually two standard deviations below the mean, so 97.5 of boxes weigh above this value.
9 a 69%
b i 33% **ii** 33%
10 a 12% **b** about 0.07%
c i 4.5% **ii** 0.6%
11 2.5 standard deviations is 15 g, so 1 standard deviation is 6 g. The mean weight is 112 g.

Exercise 10G

1 a $\mu = 7.5, \sigma = 2.5$
c



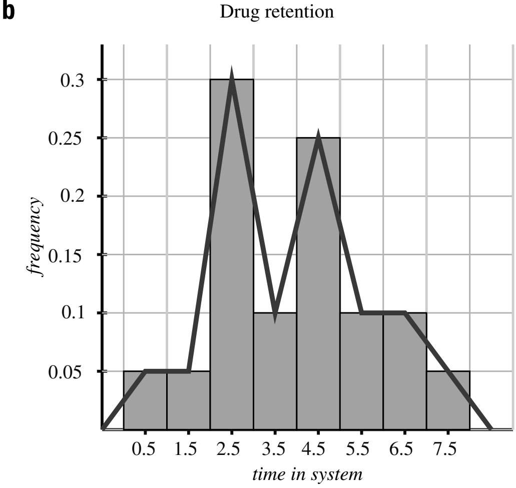
d Either perform the experiment more than 1000 times, or average more than three random numbers at each stage.
4 c The mean should be about 5 and the standard deviation about 1.6.
8 d $\left(0, \frac{1}{\sqrt{2\pi}}\right)$

Chapter 10 review exercise

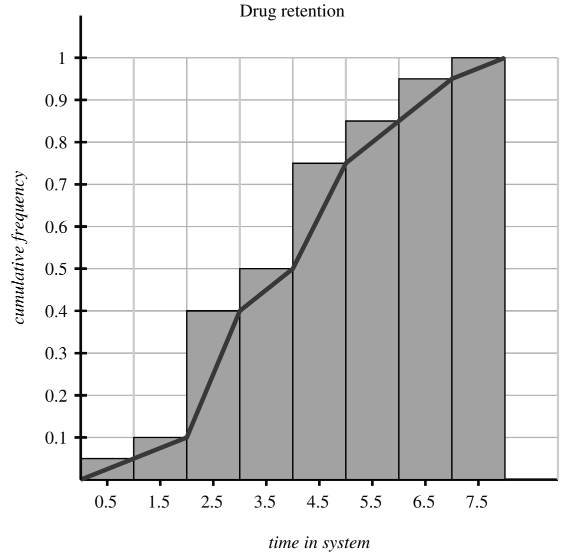
1 a

x	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8
cc	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
f	1	1	6	2	5	2	2	1
cf	1	2	8	10	15	17	19	20
f_r	0.05	0.05	0.3	0.1	0.25	0.1	0.1	0.05
cf_r	0.05	0.1	0.4	0.5	0.75	0.85	0.95	1

b



c



d i 4.0
ii $Q_1 = 2.5, Q_3 = 5.0$
iii 6.5
iv 6.0
e This was only a preliminary experiment, and a larger dataset may resolve the unusual outcomes. It may be worth investigating any common links between patients falling in the two intervals associated with the two modes — perhaps different sexes react differently to the drug, perhaps it was administered

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differently, or perhaps the two groups are behaving differently after medication, for example, changing their levels of follow-up exercise or their food intake.

2 a False, it joins to the right end.

b False. The area under the relative frequency polygon is 1 if the rectangles each have width 1.

c True. **d** True. **e** True.

f The empirical rule says 99.7% and only applies to a normal distribution, so false in general.

3 b A uniform probability distribution with a uniform probability density function.

c $\mathrm{E}(X) = 0$

d $\mathrm{E}(X^2) = \mathrm{Var}(X) = \frac{100}{3}, \sigma = \frac{10\sqrt{3}}{3}$

4 b $F(x) = \frac{1}{16}x(12 - x^2), 0 \le x \le 2$

c **i** 0.34, 0.7, 1.1 **ii** 0.85

iii 0.8 **iv** 0.78

v $P(X \le 0.4) - P(x \le 0.2) = 0.3 - 0.15 = 0.15$

5 a 0.5 **b** 0.9032 **c** 0.9282

d 0.3085 **e** 0.4207 **f** 0.4247

6 a 97.5% **b** 84% **c** 81.5%

d 2.35%

7 a 0.6915 **b** 0.1151 **c** 0.5125

d 0.1760

8 a Let X be the lifetime in years of a machine.
$P(X > 8) = P(Z > 1.3) = 9.7\%$

b $P(X < 5) = P(Z < -1.07) = 14.2\%$. This is probably an unacceptable risk for the manufacturer, and they should increase the mean life, or decrease the standard deviation, or adjust the length of their advertised warranty.

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