

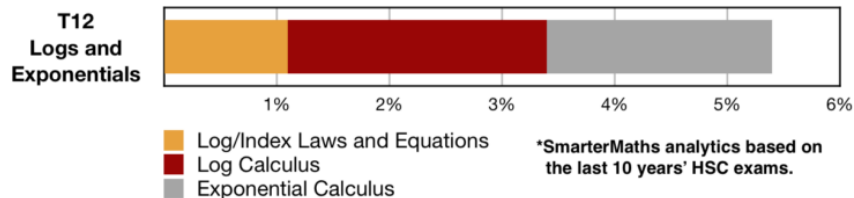
ADVANCED MATHEMATICS
Logs and Exponentials (Adv), E1 Logs and Exponentials (Adv)
Log Calculus (Y12)
Exponential Calculus (Y12)

Teacher: Cathyanne Horvat

Exam Equivalent Time: 30 minutes *(based on allocation of 1.5 minutes per mark)*



T12 Logs and Exponentials



HISTORICAL CONTRIBUTION

- T12 Logs and Exponentials* has contributed an average of 5.4% to the HSC exam per year, although this *does not* include its regular contribution to cross-topic problems involving other sub-topics such as Area Under Curves and Tangents.
- This topic has been split into three sub-categories for this analysis which are: *1-Index Laws and Other Equations (1.1%), 2-Log Calculus (2.3%), and 3-Exponential Calculus (2.0%)*.

PAST HSC ANALYSIS - What to expect and common pitfalls

- Topic 12 L&E* has historically been a great topic area for students to score highly in.
- Index Laws and Other Equations (1.1%)* represent a small area that examiners have been consistently testing within multiple choice since this mode of question was included in 2012 (note it was omitted in 2018 and poorly answered in 2017).
- Log Calculus (2.3%)* is the biggest contributor to Topic 12. The integration of a fraction that results in a logarithm is a 2-3 mark question type that has appeared in 5 of the last 9 years, but has been notably absent in the last 3 years. Revision focus recommended here.
- Exponential Calculus (2.0%)* has been examined every year since 2011 (with a 4 mark allocation in 2018). Easy marks available for well prepared students here.

Questions

1. Calculus, 2ADV C2 2008 HSC 2aii

Differentiate with respect to x :

$$x^2 \log_e x \quad (2 \text{ marks})$$

2. Calculus, 2ADV C2 SM-Bank 7

Let $f(x) = \frac{e^x}{(x^2 - 3)}$.

Find $f'(x)$. *(2 marks)*

3. Calculus, 2ADV C2 SM-Bank 8

Let $y = (x + 5)\log_e(x)$.

Find $\frac{dy}{dx}$ when $x = 5$. (2 marks)

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4. Calculus, 2ADV C2 2019 MET1 1a

Let $y = \frac{2e^{2x} - 1}{e^x}$.

Find $\frac{dy}{dx}$. (2 marks)

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5. Calculus, 2ADV C2 2012 HSC 11d

Differentiate $(3 + e^{2x})^5$. (2 marks)

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6. Calculus, 2ADV C2 2015 HSC 11f

Differentiate $y = (x + 4)\ln x$. (2 marks)

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7. Calculus, 2ADV C2 2017 HSC 11d

Differentiate $x^3 \ln x$. (2 marks)

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8. Calculus, 2ADV C2 EQ-Bank 2

Differentiate with respect to x :

$e^{\tan(2x)}$ (2 marks)

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9. Calculus, 2ADV C2 EQ-Bank 1

Differentiate $\log_2 x^2$ with respect to x . (2 marks)

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10. Calculus, 2ADV C2 EQ-Bank 3

Differentiate $3x 6^x$. (2 marks)

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Worked Solutions

1. Calculus, 2ADV C2 2008 HSC 2aii

$$\begin{aligned}y &= x^2 \log_e x \\ \frac{dy}{dx} &= x^2 \cdot \frac{1}{x} + 2x \cdot \log_e x \\ &= x + 2x \log_e x\end{aligned}$$

2. Calculus, 2ADV C2 SM-Bank 7

$$\begin{aligned}\text{Let } u &= e^x \Rightarrow u' = e^x \\ v &= (x^2 - 3) \Rightarrow v' = 2x \\ f'(x) &= \frac{e^x (x^2 - 3) - 2xe^x}{(x^2 - 3)^2} \\ &= \frac{e^x (x^2 - 2x - 3)}{(x^2 - 3)^2}\end{aligned}$$

3. Calculus, 2ADV C2 SM-Bank 8

$$\begin{aligned}\frac{dy}{dx} &= 1 \times \log_e x + (x + 5) \cdot \frac{1}{x} \\ &= \log_e x + \frac{x + 5}{x} \\ \therefore \frac{dy}{dx} \Big|_{x=5} &= \log_e 5 + 2\end{aligned}$$

4. Calculus, 2ADV C2 2019 MET1 1a

Method 1

$$\begin{aligned}y &= 2e^x - e^{-x} \\ \frac{dy}{dx} &= 2e^x + e^{-x}\end{aligned}$$

Method 2

$$\begin{aligned}\frac{dy}{dx} &= \frac{4e^{2x} \cdot e^x - (2e^{2x} - 1)e^x}{(e^x)^2} \\ &= \frac{4e^{3x} - 2e^{3x} + e^x}{e^{2x}} \\ &= \frac{2e^{2x} + 1}{e^x}\end{aligned}$$

5. Calculus, 2ADV C2 2012 HSC 11d

$$\begin{aligned}y &= (3 + e^{2x})^5 \\ \frac{dy}{dx} &= 5(3 + e^{2x})^4 \times \frac{d}{dx}(3 + e^{2x}) \\ &= 5(3 + e^{2x})^4 \times 2e^{2x} \\ &= 10e^{2x}(3 + e^{2x})^4\end{aligned}$$

6. Calculus, 2ADV C2 2015 HSC 11f

$$\begin{aligned}y &= (x + 4) \ln x \\ \text{Using the product rule} \\ \frac{dy}{dx} &= \frac{d}{dx}(x + 4) \cdot \ln x + (x + 4) \frac{d}{dx} \ln x \\ &= \ln x + (x + 4) \frac{1}{x} \\ &= \ln x + \frac{4}{x} + 1\end{aligned}$$

7. Calculus, 2ADV C2 2017 HSC 11d

$$\begin{aligned}y &= x^3 \ln x \\ \text{Using the product rule:} \\ \frac{dy}{dx} &= 3x^2 \cdot \ln x + x^3 \cdot \frac{1}{x} \\ &= x^2(3 \ln x + 1)\end{aligned}$$

8. Calculus, 2ADV C2 EQ-Bank 2

$$\begin{aligned}y &= e^{\tan(2x)} \\ \frac{dy}{dx} &= \frac{d}{dx} \tan(2x) \times e^{\tan(2x)} \\ &= 2 \sec^2(2x) \cdot e^{\tan(2x)}\end{aligned}$$

9. Calculus, 2ADV C2 EQ-Bank 1

$$y = \log_2 x^2$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\ln x^2}{\ln 2} \right)$$

$$= \frac{1}{\ln 2} \cdot \frac{d}{dx} (\ln x^2)$$

$$= \frac{1}{\ln 2} \cdot \frac{2x}{x^2}$$

$$= \frac{2}{x \ln 2}$$

TIP: The new Advanced reference sheet can be used here!

10. Calculus, 2ADV C2 EQ-Bank 3

$$y = 3x \cdot 6^x$$

$$\frac{dy}{dx} = 3 \cdot 6^x + \ln 6 \cdot 6^x \cdot 3x$$

$$= 3 \cdot 6^x (1 + x \ln 6)$$

COMMENT: See HSC exam reference sheet when differentiating 6^x .