EXERCISE 6.1 RADIAN MEASURE OF AN ANGLE

2 **(a)** $\frac{3\pi}{4}=\frac{3\times180°}{4}$

$=135°$

(b) $\frac{7\pi}{8}=\frac{7\times180°}{8}$

$=157.5°$

(c) $\frac{6\pi}{5}=\frac{6\times180°}{5}$

$=216°$

(d) $\frac{3\pi}{2}=\frac{3\times180°}{2}$

$=270°$

(e) $\frac{5\pi}{9}=\frac{5\times180°}{9}$

$=100°$

(f) $\frac{11\pi}{12}=\frac{11\times180°}{12}$

$=165°$

(g) $1.8\pi=1.8\times180°$

$=324°$

(h) $\frac{11\pi}{8}=\frac{11\times180°}{8}$

$=247.5°$

4 **(a)** $42°=\frac{\pi\times42°}{180°}$

$=0.7330$

(b) $74°=\frac{\pi\times74°}{180°}$

$=1.2915$

(c) $105°=\frac{\pi\times105°}{180°}$

$=1.8326$

(d) $164°=\frac{\pi\times164°}{180°}$

$=2.8623$

(e) First use your calculator to convert $48°9'$ to decimal degrees.

$48°9'=48.15°$

$48°9'=\frac{\pi\times48°9'}{180°}$

$=0.8404$

(f) $220°=\frac{\pi\times220°}{180°}$

$=3.8397$

(g) First use your calculator to convert $48°9'$ to decimal degrees.

$138°12' = 138.2°$

$$\begin{aligned} 138°12' &= \frac{\pi \times 138.2°}{180°} \\ &= 2.4120 \end{aligned}$$

(h) $72°8' = 72.1333...°$

$$\begin{aligned} 72°8' &= \frac{\pi \times 72.1333...°}{180°} \\ &= 1.2590 \end{aligned}$$

6 **(a)** $$\begin{aligned} 120° &= \frac{120° \times \pi}{180°} \\ &= \frac{2\pi}{3} \end{aligned}$$

(b) $$\begin{aligned} 315° &= \frac{315° \times \pi}{180°} \\ &= \frac{7\pi}{4} \end{aligned}$$

(c) $$\begin{aligned} 210° &= \frac{210° \times \pi}{180°} \\ &= \frac{7\pi}{6} \end{aligned}$$

(d) $$\begin{aligned} 135° &= \frac{135° \times \pi}{180°} \\ &= \frac{3\pi}{4} \end{aligned}$$

(e) $$\begin{aligned} 390° &= \frac{390° \times \pi}{180°} \\ &= \frac{13\pi}{6} \end{aligned}$$

(f) $$\begin{aligned} 405° &= \frac{405° \times \pi}{180°} \\ &= \frac{9\pi}{4} \end{aligned}$$

(g) $$\begin{aligned} 480° &= \frac{480° \times \pi}{180°} \\ &= \frac{8\pi}{3} \end{aligned}$$

(h) $$\begin{aligned} 720° &= \frac{720° \times \pi}{180°} \\ &= 4\pi \end{aligned}$$

(i) $$\begin{aligned} -30° &= \frac{-30° \times \pi}{180°} \\ &= -\frac{\pi}{6} \end{aligned}$$

(j) $$\begin{aligned} -135° &= \frac{-135° \times \pi}{180°} \\ &= -\frac{3\pi}{4} \end{aligned}$$

(k) $$\begin{aligned} 450° &= \frac{450° \times \pi}{180°} \\ &= \frac{5\pi}{2} \end{aligned}$$

(l) $$\begin{aligned} -180° &= \frac{-180° \times \pi}{180°} \\ &= -\pi \end{aligned}$$

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(m) $15^\circ = \dfrac{15^\circ \times \pi}{180^\circ}$

$= \dfrac{\pi}{12}$

(n) $22.5^\circ = \dfrac{22.5 \times \pi}{180^\circ}$

$= \dfrac{\pi}{8}$

(o) $345^\circ = \dfrac{345^\circ \times \pi}{180^\circ}$

$= \dfrac{23\pi}{12}$

(p) $-67.5^\circ = \dfrac{-67.5^\circ \times \pi}{180^\circ}$

$= -\dfrac{3\pi}{8}$

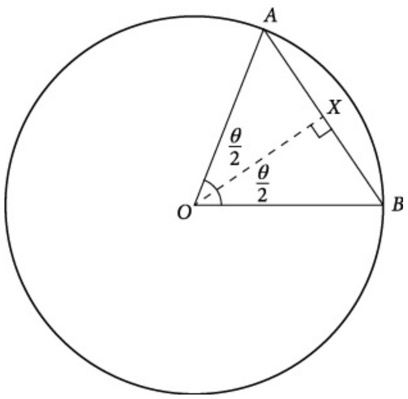
EXERCISE 6.2 ARC LENGTH AND SECTOR AREA OF CIRCLE

2 Convert $70°$ to radians by multiplying by $\frac{\pi}{180}$.

$$\begin{aligned} l &= r\theta \\ &= 15 \times \frac{70 \times \pi}{180} \\ &\approx 18.325... \end{aligned}$$

The arc length, rounded to one decimal place, is $18.3\,\text{cm}$.

4 Let $\angle AOB$ be θ radians.



(a) $A = \frac{1}{2}r^2\theta$

$$\begin{aligned} 240 &= \frac{1}{2} \times 20^2 \times \theta \\ \theta &= \frac{240}{\frac{1}{2} \times 20^2} \\ \theta &= \frac{6}{5} \\ \theta &= 1.2 \end{aligned}$$

$\angle AOB = 1.2$ radians.

(b) $l = r\theta$

$$\begin{aligned} &= 20 \times 1.2 \\ &= 24 \end{aligned}$$

Arc AB is $24\,\text{cm}$.

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(c) $\sin\left(\dfrac{1.2}{2}\right) = \dfrac{AX}{20}$

$AX = 20 \times \sin(0.6)$
$AB = 2AX$
$AB = 2 \times 20 \times \sin(0.6)$
$AB \approx 22.585...$

The chord AB is 22.6 cm, rounded to one decimal place.

6 $\cos\angle AOP = \dfrac{5}{8}$

$\angle AOP = \cos^{-1}\left(\dfrac{5}{8}\right) \approx 0.8956...$

$\angle AOB = 2\angle AOP = 2\cos^{-1}\left(\dfrac{5}{8}\right)$

$\angle AOB \approx 1.7913...$

Reflex $\angle AOB = 2\pi - 2\cos^{-1}\left(\dfrac{5}{8}\right)$

Major arc $AB = r\theta$

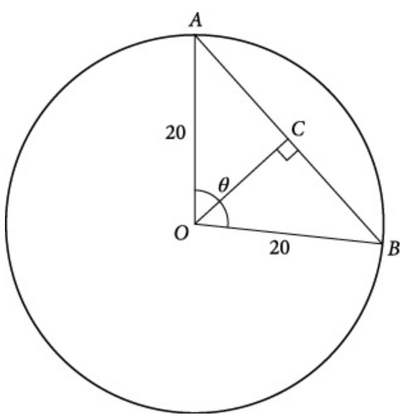
$l = 5 \times \left[2\pi - 2\cos^{-1}\left(\dfrac{5}{8}\right)\right]$

$= 10\pi - 10\cos^{-1}\left(\dfrac{5}{8}\right)$

$\approx 22.459...$

The major arc AB is 22.5 cm, rounded to one decimal place.

8 **(a)**



Let the initial position of the minute hand be A and the final position of the minute hand be B.

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The minute hand moves $\frac{16}{60}$ of one complete revolution.

$$\begin{aligned}\theta &= \frac{16}{60}\times 2\pi\\ &= \frac{8\pi}{15}\end{aligned}$$

$$\begin{aligned}l &= r\theta\\ &= 20\times\frac{8\pi}{15}\\ &\approx 33.51...\end{aligned}$$

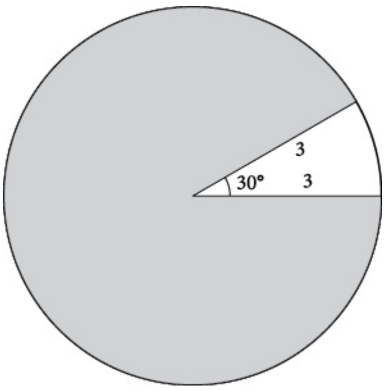
The arc length is 33.5 cm, rounded to one decimal place.

(b) $\sin\left(\frac{1}{2}\times\frac{8\pi}{15}\right)=\frac{AC}{20}$

$$AC = 20\times\sin\left(\frac{8\pi}{30}\right)$$
$$AB = 2AC$$
$$AB = 2\times 20\times\sin\left(\frac{8\pi}{30}\right)$$
$$AB \approx 29.725...$$

The shortest distance is the chord AB with length 29.7 cm, rounded to one decimal place.

10



$$\begin{aligned}A &= \pi r^2-\frac{1}{2}r^2\theta\\ &= \pi\times 3^2-\frac{1}{2}\times 3^2\times\frac{30\times\pi}{180}\\ &= 9\pi-\frac{3\pi}{4}\\ &= \frac{33\pi}{4}\end{aligned}$$

The area remaining is $\frac{33\pi}{4}$ cm^2.

12 (a) Area of large sector $= \frac{1}{2} \times 15^2 \times \frac{45 \times \pi}{180}$

$$= \frac{1}{2} \times 225 \times \frac{\pi}{4}$$

$$= \frac{225\pi}{8}$$

Area of small sector $= \frac{1}{2} \times 10^2 \times \frac{45 \times \pi}{180}$

$$= \frac{1}{2} \times 100 \times \frac{\pi}{4}$$

$$= \frac{100\pi}{8}$$

Shaded area $= \frac{225\pi}{8} - \frac{100\pi}{8} = \frac{125\pi}{8}$

The area of the shaded region is $\frac{125\pi}{8}$ cm^2.

(b) Length of large arc $= 15 \times \frac{45 \times \pi}{180} = \frac{15\pi}{4}$ cm

Length of small arc $= 10 \times \frac{45 \times \pi}{180} = \frac{10\pi}{4}$ cm

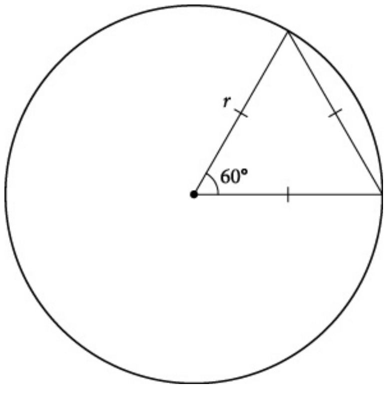
Each straight side is $15 - 10 = 5$ cm

$$P = \frac{15\pi}{4} + \frac{10\pi}{2} + 5 + 5$$

$$= \frac{25\pi}{4} + 10$$

The perimeter of the shaded region is $\frac{25\pi}{4} + 10$ cm.

14



For the chord to equal to the radius, the chord must form an equilateral triangle with the two radius. So each interior angle equals to $60°$.

Convert $60°$ to radians.

$60° = \frac{60 \times \pi}{180} = \frac{\pi}{3}$ radians

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Area of the minor segment:

$$\begin{aligned}A_1 &= \frac{1}{2}r^2(\theta - \sin\theta)\\ &= \frac{1}{2}r^2\left(\frac{\pi}{3} - \sin\frac{\pi}{3}\right)\\ &= \frac{1}{2}r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\\ &= \frac{\pi r^2}{6} - \frac{\sqrt{3}r^2}{4}\\ &= \frac{r^2\left(2\pi - 3\sqrt{3}\right)}{12}\end{aligned}$$

The angle subtended by the major segment is $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$.

Area of the major segment:

$$\begin{aligned}A_2 &= \pi r^2 - \frac{1}{2}r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\\ &= \frac{5\pi r^2}{6} + \frac{\sqrt{3}r^2}{4}\\ &= \frac{r^2\left(10\pi + 3\sqrt{3}\right)}{12}\end{aligned}$$

Ratio of the minor segment to the major segment is:

$$\begin{aligned}&A_1 : A_2\\ &\frac{r^2\left(2\pi - 3\sqrt{3}\right)}{12} : \frac{r^2\left(10\pi + 3\sqrt{3}\right)}{12}\\ &2\pi - 3\sqrt{3} : 10\pi + 3\sqrt{3}\end{aligned}$$

16 The circumference of the circular base of the cone is equal to the arc length of the sector.

$$\begin{aligned}l &= 10 \times \frac{160 \times \pi}{180}\\ &= \frac{80\pi}{9}\\ &\approx 27.9252...\end{aligned}$$

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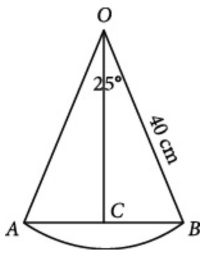
The circumference is 27.9 cm, rounded to one decimal place.

18 (a) Draw a diagram.



$$l = 40\times\frac{25\times\pi}{180}$$
$$= \frac{50\pi}{9}$$
$$\approx 17.4532...$$

The arc length is 17.5 cm, rounded to one decimal place.

(b) $\sin 12.5° = \dfrac{CB}{40}$

$$CB = 40\times\sin 12.5°$$
$$d = 2\times 40\times\sin 12.5°$$
$$d \approx 17.315...$$

The shortest distance between the extreme positions of the bob is 17.3 cm, rounded to one decimal place.

20 The required area is the difference between the area of the triangle and the area of the three equal sectors.

The base, and each side of the triangle, is 10 cm.

The height of the triangle is $10\times\sin 60° = 10\times\dfrac{\sqrt{3}}{2} = 5\sqrt{3}$.

Area of triangle $= \dfrac{1}{2}bh = \dfrac{1}{2}\times 10\times 5\sqrt{3} = 25\sqrt{3}$

Area of the three sectors $= 3\times\dfrac{1}{2}\times 5^2\times\dfrac{60\times\pi}{180} = \dfrac{25\pi}{2}$

Required area $= 25\sqrt{3} - \dfrac{25\pi}{2} \approx\sim 4.0313...$

The required area is 4.0 cm^2, rounded to one decimal place.

EXERCISE 6.3 ANGLES OF ANY MAGNITUDE—RADIANS

2 A

$$\cos(\pi-\theta)=\cos(180°-\theta)=-\cos\theta$$

4 (a) $\sin(\pi-x)=\sin x$

$=0.2$

(b) $\sin(2\pi-x)=-\sin x$

$=-0.2$

(c) $\sin(-x)=-\sin x$

$=-0.2$

(d) $\cos\left(\dfrac{\pi}{2}-x\right)=\sin x$

$=0.2$

(e) $\sin(\pi+x)=-\sin x$

$=-0.2$

(f) $\operatorname{cosec} x=\dfrac{1}{\sin x}$

$=\dfrac{1}{0.2}$

$=5$

6 (a) $\sec x=\dfrac{1}{\cos x}$

$=\dfrac{1}{c}$

(b) $\cos(-x)=\cos x$

$=c$.

(c) $\cos(\pi-x)=-\cos x$

$=-c$

(d) $\cos(2\pi-x)=\cos x$

$=c$

(e) $\sec(-x)=\dfrac{1}{\cos(-x)}$

$=\dfrac{1}{\cos x}$

$=\dfrac{1}{c}$

(f) $\cos(\pi+x)=-\cos x$

$=-c$

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8 **(a)** $\sin\pi = 0$

(b) $\cos\frac{7\pi}{6} = \cos\left(\pi + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

(c) $\tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4}$

$= 1$

(d) $\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\tan\left(\pi + \frac{\pi}{3}\right)}$

$= \frac{1}{\tan\frac{\pi}{3}}$

$= \frac{1}{\sqrt{3}}$

(e) $\cos\frac{4\pi}{3} = \cos\left(\pi + \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3}$

$= -\frac{1}{2}$

(f) $\sec\left(\frac{5\pi}{4}\right) = \frac{1}{\cos\left(\pi + \frac{\pi}{4}\right)}$

$= \frac{1}{-\cos\left(\frac{\pi}{4}\right)}$

$= -\frac{1}{\frac{1}{\sqrt{2}}}$

$= -\sqrt{2}$

(g) $\tan\pi = 0$

(h) $\sin\frac{7\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right)$

$= -\sin\frac{\pi}{6}$

$= -\frac{1}{2}$

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10 (a) $\sin 2\pi = 0$

(b) $\sin\frac{13\pi}{6} = \sin\left(2\pi + \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

(c) $\tan\frac{9\pi}{4} = \tan\left(2\pi + \frac{\pi}{4}\right)$

$= \tan\frac{\pi}{4}$

$= 1$

(d) $\cot\frac{7\pi}{3} = \frac{1}{\tan\left(2\pi + \frac{\pi}{3}\right)}$

$= \frac{1}{\tan\frac{\pi}{3}}$

$= \frac{1}{\sqrt{3}}$

(e) $\sec\frac{13\pi}{6} = \frac{1}{\cos\left(2\pi + \frac{\pi}{6}\right)}$

$= \frac{1}{\cos\frac{\pi}{6}}$

$= \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

(f) $\cos\frac{5\pi}{2} = \cos\left(2\pi + \frac{\pi}{2}\right)$

$= \cos\frac{\pi}{2}$

$= 0$

(g) $\sin\frac{7\pi}{3} = \sin\left(2\pi + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

(h) $\tan\frac{11\pi}{4} = \tan\left(2\pi + \frac{3\pi}{4}\right)$

$= \tan\frac{3\pi}{4}$

$= \tan\left(\pi - \frac{\pi}{4}\right)$

$= -\tan\frac{\pi}{4}$

$= -1$

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12 (a) $\cos(\pi-\theta)=-\cos\theta$

θ is in the second quadrant, so $\cos\theta$ is negative and $-\cos\theta$ is positive.

$\therefore \cos(\pi-\theta)$ is positive.

(b) $\tan(\pi+\theta)=\tan\theta$

θ is in the second quadrant, so $\tan\theta$ is negative.

$\therefore \tan(\pi+\theta)$ is negative.

(c) $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

θ is in the second quadrant, so $\cos\theta$ is negative.

$\therefore \sin\left(\frac{\pi}{2}-\theta\right)$ is negative.

(d) $\sin(2\pi-\theta)=-\sin\theta$

θ is in the second quadrant, so $\sin\theta$ is positive and $-\sin\theta$ is negative.

$\therefore \sin(2\pi-\theta)$ is negative.

(e) $\cos(\pi+\theta)=-\cos\theta$

θ is in the second quadrant, so $\cos\theta$ is negative and $-\cos\theta$ is positive.

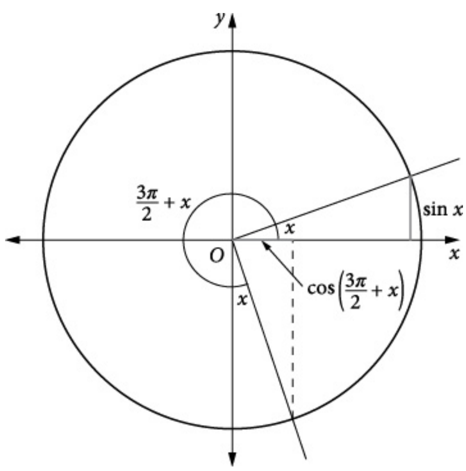
$\therefore \cos(\pi+\theta)$ is positive.

(f) $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta=\frac{1}{\tan\theta}$

θ is in the second quadrant, so $\tan\theta$ is negative.

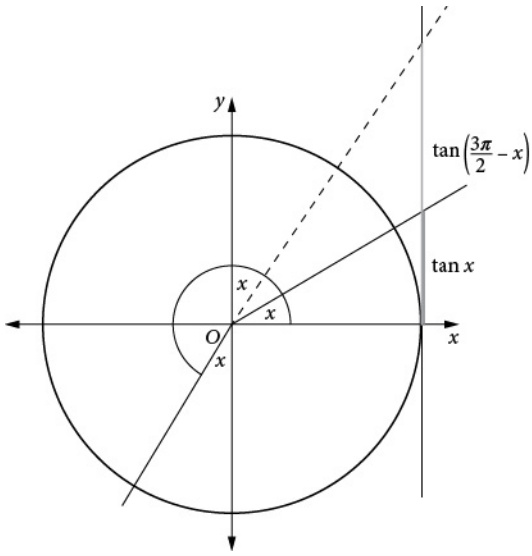
$\therefore \tan\left(\frac{\pi}{2}-\theta\right)$ is negative.

14 (a)



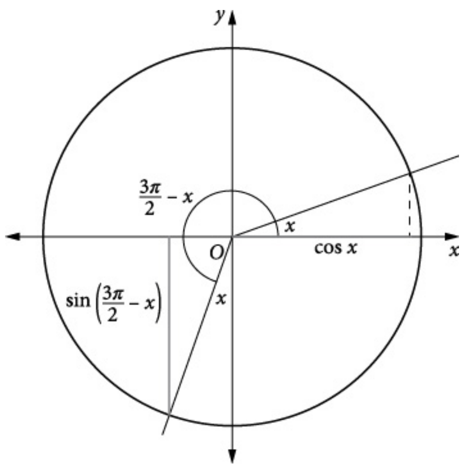
From the diagram, it can be seen that $\cos\left(\frac{3\pi}{2}+x\right)=\sin x$.

(b)



From the diagram, it can be seen that $\tan\left(\dfrac{3\pi}{2}-x\right)=\tan\left(\dfrac{\pi}{2}-x\right)=\cot x.$

(c)

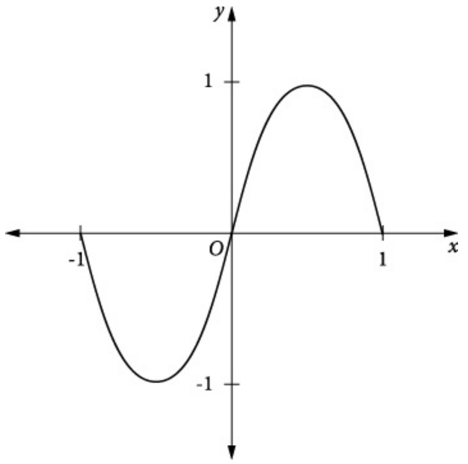


From the diagram, it can be seen that $\sin\left(\dfrac{3\pi}{2}-x\right)=-\cos x.$

EXERCISE 6.4 GRAPHS OF TRIGONOMETRIC FUNCTIONS USING RADIANS

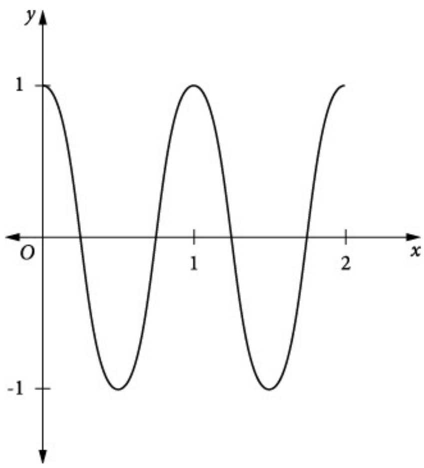
2 (a) $y = \sin \pi x$

The amplitude is 1 and the period is $\dfrac{2\pi}{\pi} = 2$



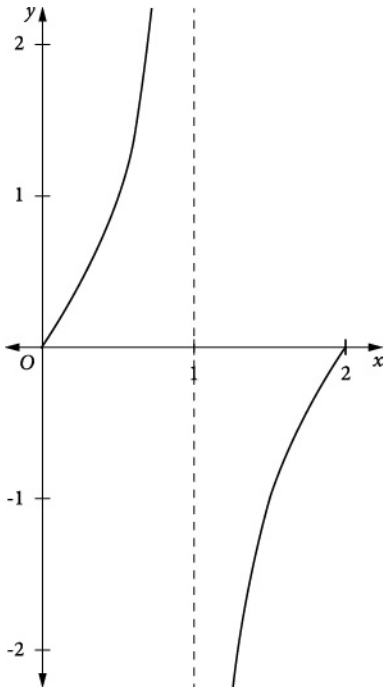
(b) $y = \cos 2\pi x$

The amplitude is 1 and the period is $\dfrac{2\pi}{2\pi} = 1$



(c) $y = \tan\frac{\pi x}{2}$

There is no amplitude and the period is $\dfrac{\pi}{\frac{\pi}{2}} = 2$



EXERCISE 6.5 TRIGONOMETRIC IDENTITIES AND PROOFS

2 Since θ is acute, θ must be in the first quadrant, so all the trigonometric ratios will be positive.

$$\sin\theta = \frac{5}{13}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{25}{169} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\cos\theta = \frac{12}{13}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$$

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$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{13} \div \frac{12}{13} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$$

(a) correct

(b) incorrect

(c) correct

(d) incorrect

4 A

Since $\cos u$ is positive and u is not in the first quadrant, then u must be in the fourth quadrant. $\sin\theta$ and $\tan\theta$ will be negative.

$$\cos u = \frac{2}{3}$$

$$\begin{aligned}\sin^2 u + \cos^2 u &= 1\\ \sin^2 u + \frac{4}{9} &= 1\\ \sin^2 u &= 1 - \frac{4}{9} = \frac{5}{9}\\ \sin u &= -\frac{\sqrt{5}}{3}\end{aligned}$$

$$\tan u = \frac{\sin u}{\cos u} = -\frac{\sqrt{5}}{3} \div \frac{2}{3} = -\frac{\sqrt{5}}{3} \times \frac{3}{2} = -\frac{\sqrt{5}}{2}$$

$$\cot u = \frac{1}{\tan u} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Note: Rationalising the denominator will make the next lot of working easier.

$$\begin{aligned}\frac{\cos u - 2\cot u}{\tan u - 3\sin u} &= \frac{\frac{2}{3} - 2 \times -\frac{2\sqrt{5}}{5}}{-\frac{\sqrt{5}}{2} - 3 \times -\frac{\sqrt{5}}{3}}\\ &= \frac{\frac{2}{3} + \frac{4\sqrt{5}}{5}}{-\frac{\sqrt{5}}{2} + \sqrt{5}}\end{aligned}$$

Multiply numerator and denominator by the LCM, 30.

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$$\begin{aligned}\frac{\cos u - 2\cot u}{\tan u - 3\sin u} &= \frac{20+24\sqrt{5}}{-15\sqrt{5}+30\sqrt{5}}\\ &= \frac{20+24\sqrt{5}}{15\sqrt{5}}\\ &= \frac{20+24\sqrt{5}}{15\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}\\ &= \frac{20\sqrt{5}+120}{75}\\ &= \frac{4\sqrt{5}+24}{15}\\ &= \frac{4\left(\sqrt{5}+6\right)}{15}\end{aligned}$$

6 (a) For $x = a\sec\theta$

$$\begin{aligned}\frac{x^2}{\sqrt{x^2-a^2}} &= \frac{a^2\sec^2\theta}{\sqrt{a^2\sec^2\theta - a^2}}\\ &= \frac{a^2\sec^2\theta}{\sqrt{a^2\left(\sec^2\theta - 1\right)}}\\ &= \frac{a^2\sec^2\theta}{\sqrt{a^2\tan^2\theta}}\\ &= \frac{a^2\sec^2\theta}{a\tan\theta}\\ &= \frac{\frac{a}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}}\\ &= \frac{a}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta}\\ &= \frac{a}{\sin\theta\cos\theta}\\ &= a\operatorname{cosec}\theta\sec\theta\end{aligned}$$

(b) For $x = a\cos\theta$

$$\begin{aligned}\sqrt{a^2-x^2} &= \sqrt{a^2 - a^2\cos^2\theta}\\ &= \sqrt{a^2\left(1-\cos^2\theta\right)}\\ &= \sqrt{a^2\sin^2\theta}\\ &= a\sin\theta\end{aligned}$$

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8 $\sin\theta = p$

$\sin^2\theta = p^2$

$$
\begin{aligned}
\cos^2\theta + \sin^2\theta &= 1 \\
\cos^2\theta &= 1 - \sin^2\theta \\
\cos^2\theta &= 1 - p^2 \\
\cos\theta &= \pm\sqrt{1-p^2}
\end{aligned}
$$

Since $180° < \theta < 270°$, it is in the third quadrant and $\cos\theta$ is negative.

$\therefore \cos\theta = -\sqrt{1-p^2}$

10 $\sin\theta = x$

$$
\begin{aligned}
\sin^2\theta &= x^2 \\
\cos^2\theta &= 1 - \sin^2\theta \\
&= 1 - x^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{1-\cos^2\theta}{\sec^2\theta} &= \frac{\sin^2\theta}{\sec^2\theta} \\
&= \sin^2\theta\cos^2\theta \\
&= x^2\left(1-x^2\right)
\end{aligned}
$$

12 $x = a\sin\theta$

$$
\begin{aligned}
\frac{x}{a} &= \sin\theta \\
\frac{x^2}{a^2} &= \sin^2\theta \\
y &= b\cos\theta \\
\frac{y}{b} &= \cos\theta \\
\frac{y^2}{b^2} &= \cos^2\theta
\end{aligned}
$$

$$
\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1
\end{aligned}
$$

14 (a)
$$
\begin{aligned}
\left(1+\tan^2 u\right)\left(1-\sin^2 u\right) &= \sec^2 u \times \cos^2 u \\
&= \frac{1}{\cos^2 u} \times \cos^2 u \\
&= 1
\end{aligned}
$$

(b)
$$
\begin{aligned}
\frac{1}{1-\sin V} + \frac{1}{1+\sin V} &= \frac{1+\sin V + 1 - \sin V}{1-\sin^2 V} \\
&= \frac{2}{\cos^2 V} \\
&= 2\sec^2 V
\end{aligned}
$$

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(c) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta+(1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$

$$=\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$
$$=\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$
$$=2\operatorname{cosec}\theta$$

(d) $\dfrac{1-\sin\theta}{1+\cos\theta}\times\dfrac{1+\sin\theta}{1-\cos\theta}=\dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$$=\frac{\cos^2\theta}{\sin^2\theta}$$
$$=\cot^2\theta$$

(e) $\sin\theta\cos(90°-\theta)+\cos\theta\sin(90°-\theta)=\sin\theta\times\sin\theta+\cos\theta\times\cos\theta$

$$=\sin^2\theta+\cos^2\theta$$
$$=1$$

(f) $1-\dfrac{\sin A\cos A}{\tan A}=1-\sin A\cos A\times\dfrac{\cos A}{\sin A}$

$$=1-\cos^2 A$$
$$=\sin^2 A$$

(g) $2\cos^2 30°-1=2\times\left(\dfrac{\sqrt{3}}{2}\right)^2-1$

$$=2\times\frac{3}{4}-1$$
$$=\frac{1}{2}$$

(h) $1-\sin\theta\cos(90°-\theta)=1-\sin\theta\times\sin\theta$

$$=1-\sin^2\theta$$
$$=\cos^2\theta$$

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16 $LHS = (\cot t + \text{cosec} t)^2$

$$
\begin{aligned}
&= \left(\frac{\cos t}{\sin t} + \frac{1}{\sin t}\right)^2 \\
&= \left(\frac{\cos t + 1}{\sin t}\right)^2 \\
&= \frac{(1+\cos t)^2}{\sin^2 t} \\
&= \frac{(1+\cos t)^2}{1-\cos^2 t} \\
&= \frac{(1+\cos t)^2}{(1-\cos t)(1+\cos t)} \\
&= \frac{1+\cos t}{1-\cos t} \\
&= RHS
\end{aligned}
$$

$$\therefore (\cot t + \cos \text{ec} t)^2 = \frac{1+\cos t}{1-\cos t}$$

18 $LHS = \sec\theta + \tan\theta$

$$
\begin{aligned}
&= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \\
&= \frac{1+\sin\theta}{\cos\theta} \\
&= RHS
\end{aligned}
$$

$$\therefore \sec\theta + \tan\theta = \frac{1+\sin\theta}{\cos\theta}$$

20 $LHS = \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)$

$$\begin{aligned}
&= \sin\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)+\cos\theta\left(1+\frac{\cos\theta}{\sin\theta}\right)\\
&= \sin\theta+\frac{\sin^2\theta}{\cos\theta}+\cos\theta+\frac{\cos^2\theta}{\sin\theta}\\
&= \frac{\sin^2\theta\cos\theta+\sin^3\theta+\sin\theta\cos^2\theta+\cos^3\theta}{\sin\theta\cos\theta}\\
&= \frac{\sin^2\theta(\cos\theta+\sin\theta)+\cos^2\theta(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}\\
&= \frac{(\sin^2\theta+\cos^2\theta)(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}\\
&= \frac{1\times(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}\\
&= \frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\\
&= RHS
\end{aligned}$$

$$\therefore \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)=\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}$$

22 $LHS = \dfrac{\cot\theta\cos\theta}{\cot\theta+\cos\theta}$

$$\begin{aligned}
&= \frac{\frac{\cos\theta}{\sin\theta}\times\cos\theta}{\frac{\cos\theta}{\sin\theta}+\cos\theta}\times\frac{\sin\theta}{\sin\theta}\\
&= \frac{\cos^2\theta}{\cos\theta+\sin\theta\cos\theta}\\
&= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}\\
&= \frac{\cos\theta}{1+\sin\theta}\\
&= RHS
\end{aligned}$$

$$\therefore \frac{\cot\theta\cos\theta}{\cot\theta+\cos\theta}=\frac{\cos\theta}{1+\sin\theta}$$

24 $LHS = \dfrac{(\cos t \cot t - \sin t \tan t)\sin t \cos t}{\cos t - \sin t}$

$$= \frac{\left(\cos t \times \dfrac{\cos t}{\sin t} - \sin t \times \dfrac{\sin t}{\cos t}\right)\sin t \cos t}{\cos t - \sin t}$$

$$= \frac{\cos^3 t - \sin^3 t}{\cos t - \sin t}$$

$$= \frac{(\cos t - \sin t)(\cos^2 t + \cos t \sin t + \sin^2 t)}{\cos t - \sin t}$$

$$= \sin^2 t + \cos^2 t + \cos t \sin t$$

$$= 1 + \sin t \cos t$$

$$= RHS$$

$$\therefore \frac{(\cos t \cot t - \sin t \tan t)\sin t \cos t}{\cos t - \sin t} = 1 + \sin t \cos t$$

EXERCISE 6.6 SOLVING TRIGONOMETRIC EQUATIONS

2 D

$$\sin x + \cos x = 0$$

$$\sin x = -\cos x$$

$$\frac{\sin x}{\cos x} = -1$$

$$\tan x = -1$$

$$x = 135°$$

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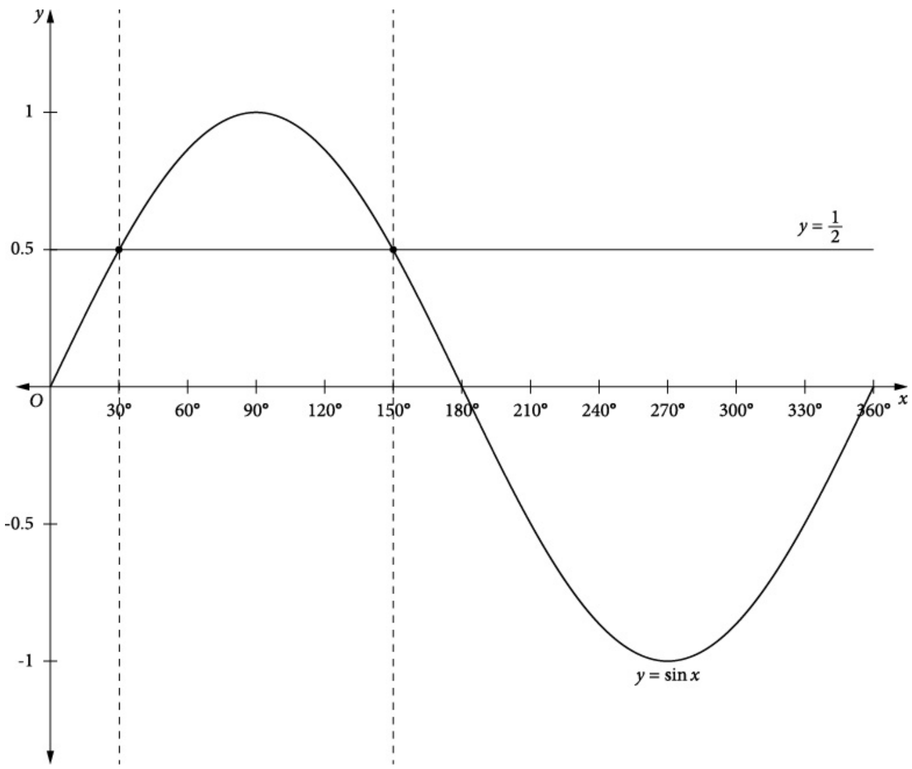
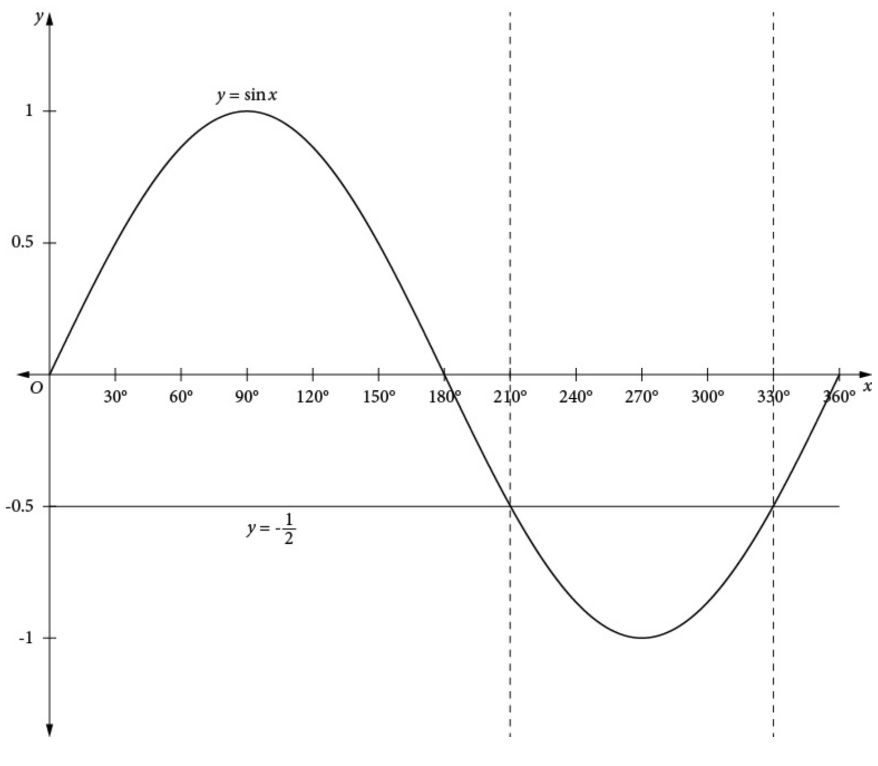
4 (a)



$$\sin x = \frac{1}{2}$$
$$x = 30°,\ 150°$$

(b) Draw the line $y = -\frac{1}{2}$.



$$\sin x = -\frac{1}{2}$$
$$x = 210°,\ 330°$$

6 A

$$\operatorname{cosec} x = 3$$

$$\sin x = \frac{1}{3}$$
$$x = 0.3398...,\ \pi - 0.3398...$$
$$x \approx 0.340,\ 2.802$$

8 (a) $\tan x = 2\sin x$

$$\frac{\sin x}{\cos x} = 2\sin x$$
$$\sin x = 2\sin x\cos x$$
$$\sin x - 2\sin x\cos x = 0$$
$$\sin x(1 - 2\cos x) = 0$$

$$\sin x = 0 \quad \text{or } 1 - 2\cos x = 0$$

$$\sin x = 0 \Rightarrow x = 0,\ \pi,\ 2\pi$$

$$1 - 2\cos x = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3},\ 2\pi - \frac{5\pi}{3}$$
$$x = \frac{\pi}{3},\ \frac{5\pi}{3}$$

$$x = 0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3},\ 2\pi$$

(b) $5\cos^2 x + 2\sin x - 2 = 0$

$$5\left(1 - \sin^2 x\right) + 2\sin x - 2 = 0$$
$$5 - 5\sin^2 x + 2\sin x - 2 = 0$$
$$5\sin^2 x - 2\sin x - 3 = 0$$
$$(5\sin x + 3)(\sin x - 1) = 0$$

$$5\sin x + 3 = 0$$
$$\sin x = -\frac{3}{5}$$

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The first quadrant solution of $\sin x = \dfrac{3}{5}$ is $x = 0.6435...$.

$$x = \pi + 0.6435..., 2\pi - 0.6435...,$$
$$x \approx 3.786, 5.640$$

or

$$\sin x - 1 = 0$$
$$\sin x = 1$$
$$x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2},\ x \approx 3.786,\ 5.640$$

(c) $3\sec^2 x - 5\tan x = 5$

$$3\left(1 + \tan^2 x\right) - 5\tan x = 5$$
$$3 + 3\tan^2 x - 5\tan x = 5$$
$$3\tan^2 x - 5\tan x - 2 = 0$$
$$\left(3\tan x + 1\right)\left(\tan x - 2\right) = 0$$

$$3\tan x + 1$$
$$\tan x = -\frac{1}{3}$$

The first quadrant solution of $\tan x = \dfrac{1}{3}$ is $x = 0.3217...$.

$$x = \pi - 0.3217...,\ 2\pi - 0.3217...$$
$$x \approx 2.820,\ 5.961$$

Or

$$\tan x - 2 = 0$$
$$\tan x = 2$$
$$x = 1.1071...,\ \pi + 1.1071...$$
$$x \approx 1.107,\ 4.249$$

$$x \approx 1.107,\ 2.820,\ 4.249,\ 5.961$$

(d) $6\tan x = 5\operatorname{cosec} x$

$$\frac{6\sin x}{\cos x} = \frac{5}{\sin x}$$
$$6\sin^2 x = 5\cos x$$
$$6\left(1-\cos^2 x\right) - 5\cos x = 0$$
$$6 - 6\cos^2 x - 5\cos x = 0$$
$$6\cos^2 x + 5\cos x - 6 = 0$$
$$\left(2\cos x + 3\right)\left(3\cos x - 2\right) = 0$$

$$2\cos x + 3 = 0$$
$$\cos x = -\frac{3}{2}$$

No solutions.

or

$$3\cos x - 2 = 0$$
$$\cos x = \frac{2}{3}$$

$$x \approx 0.841,\ 2\pi - 0.841$$
$$x \approx 0.841,\ 5.442$$

(e) $7\sin x\cos x + \cos^2 x = 1$

$$7\sin x\cos x = 1 - \cos^2 x$$
$$7\sin x\cos x = \sin^2 x$$
$$\sin^2 x - 7\sin x\cos x = 0$$
$$\sin x\left(7\cos x - \sin x\right) = 0$$
$$\sin x = 0$$
$$x = 0,\ \pi,\ 2\pi$$
$$7\cos x - \sin x = 0$$
$$\sin x = 7\cos x$$
$$\frac{\sin x}{\cos x} = 7$$
$$\tan x = 7$$
$$x = 1.4288...,\ \pi + 1.4288...$$
$$x \approx 1.429,\ 4.570$$

$$x = 0,\ 1.429,\ \pi,\ 4.570,\ 2\pi$$

(f) $\sin^2 x + 2\sin x\cos x = 3\cos^2 x$

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$$\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$$
$$(\sin x - \cos x)(\sin x + 3\cos x) = 0$$
$$\sin x = \cos x,\ \ \sin x = -3\cos x$$
$$\tan x = 1,\ \ \tan x = -3$$

$$\tan x = 1$$
$$x = \frac{\pi}{4},\ \pi + \frac{\pi}{4}$$
$$x = \frac{\pi}{4},\ \frac{3\pi}{4}$$

Or, $\tan x = -3$

The first quadrant solution of $\tan x = -3$ is $x = 1.2490...$.

$$x = \pi - 1.2490...,\ 2\pi - 1.2490...,$$
$$x \approx 1.893,\ 5.034$$

$$x = \frac{\pi}{4},\ 1.893,\ \frac{5\pi}{4},\ 5.034$$

10 (a) $0 \le x \le 2\pi$

$$0 \le \frac{x}{2} \le \pi$$

$$\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$$
$$\frac{x}{2} = \frac{\pi}{3},\ \frac{2\pi}{3}$$
$$x = \frac{2\pi}{3},\ \frac{4\pi}{3}$$

(b) $0 \le x \le 2\pi$

$$0 \le \frac{x}{2} \le \pi$$

$$\tan\frac{x}{2} = -\frac{1}{\sqrt{3}}$$
$$\frac{x}{2} = \frac{5\pi}{6}$$
$$x = \frac{5\pi}{3}$$

(c) $0 \le x \le 2\pi$

$$0 \le \frac{x}{3} \le \frac{2\pi}{3}$$

$$\cos\frac{x}{3} = 1$$
$$\frac{x}{3} = 0$$
$$x = 0$$

12 (a) $0 \le x \le \pi$

$$0 \le \frac{x}{3} \le \frac{\pi}{3}$$

Calculate $\frac{\pi}{3}$. $\frac{\pi}{3} = 1.047...$

$$\sin\frac{x}{3} = 0.4$$
$$\frac{x}{3} \approx 0.41151...$$
$$x = 1.23$$

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(b) $0 \le x \le \pi$

$0 \le 3x \le 3\pi$

The first quadrant solution of $\cos 3x = 0.7$ is $3x = 0.7953\ldots$.

$\cos 3x = -0.7$

$3x = \pi - 0.7953\ldots,\ \pi + 0.7953\ldots,\ 3\pi - 0.7953\ldots$

$3x = 2.3461\ldots,\ 3.9369\ldots,\ 8.6293\ldots$

$x \approx 0.782,\ 1.312,\ 2.876$

(c) $0 \le x \le \pi$

$0 \le \frac{x}{4} \le \frac{\pi}{4}$

$\tan \frac{x}{4} = 1.5 > 1$

$\therefore \frac{x}{4} > \frac{\pi}{4}$

No solution in the given domain.

14 (a) $5 \cos^2 x + 8 \sin x - 8 = 0$

$5(1 - \sin^2 x) + 8 \sin x - 8 = 0$

$5 - 5 \sin^2 x + 8 \sin x - 8 = 0$

$5 \sin^2 x - 8 \sin x - 3 = 0$

$(5 \sin x + 3)(\sin x - 1) = 0$

$\sin x = -\frac{3}{5}$ or $\sin x = 1$

$x = \sin^{-1}\left(-\frac{3}{5}\right)$ or $x = \sin^{-1}(1)$

$x = 0.644, 2.498$ or $x = \frac{\pi}{2}$

$x = \frac{\pi}{2}, 0.644, 2.498$

(b) $6 \tan x = 5 \cot x$

$6 \tan x = \frac{5}{\tan x}$

$6\tan^2 x = 5$

$\tan^2 x = \dfrac{5}{6}$

$\tan x = \pm\sqrt{\dfrac{5}{6}}$

$\tan x = \sqrt{\dfrac{5}{6}}$ or $\tan x = -\sqrt{\dfrac{5}{6}}$

$x = \tan^{-1}\sqrt{\dfrac{5}{6}}$ or $x = \tan^{-1}\left(-\sqrt{\dfrac{5}{6}}\right)$

$x = 0.740, 3.881$ or $x = 2.402, 5.543$

$x = 0.740, 2.402, 3.881, 5.543$

16 (a) $\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ or $\tan x = -1$

$x = 0, \pi, 2\pi$ or $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

$x = 0, \dfrac{3\pi}{4}, \pi, \dfrac{7\pi}{4}, 2\pi$

(b) $\sin^2 x - \sin x\cos x = 0$

$\sin x(\sin x - \cos x) = 0$

$\sin x = 0$ or $\sin x = \cos x$

$\sin x = 0$ or $\tan x = 1$

$x = 0, \pi, 2\pi$ or $x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

$x = 0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}, 2\pi$

(c) $2\sin^2\theta + \cos\theta = 1$

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$$2\left(1-\cos^2\theta\right)+\cos\theta=1$$
$$2-2\cos^2\theta+\cos\theta=1$$
$$2\cos^2\theta-\cos\theta-1=0$$
$$\left(2\cos\theta+1\right)\left(\cos\theta-1\right)=0$$
$$\cos\theta=-\frac{1}{2}\text{ or }\cos\theta=1$$
$$\theta=\pi-\frac{\pi}{3},\ \pi+\frac{\pi}{3}\text{ or }\theta=0,\ 2\pi$$
$$\theta=\frac{2\pi}{3},\ \frac{4\pi}{3}\text{ or }\theta=0,\ 2\pi$$
$$\theta=0,\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ 2\pi$$

(d) $3\cos^2\theta-2\sin\theta=2$

$$3\left(1-\sin^2\theta\right)-2\sin\theta=2$$
$$3-3\sin^2\theta-2\sin\theta=2$$
$$3\sin^2\theta+2\sin\theta-1=0$$
$$\left(3\sin\theta-1\right)\left(\sin\theta+1\right)=0$$
$$\sin\theta=\frac{1}{3}\text{ or }\sin\theta=-1$$
$$\theta=0.3398...,\ \pi-0.3398...\text{ or }\theta=\frac{3\pi}{2}$$
$$\theta\approx0.340,\ 2.802,\ \theta=\frac{3\pi}{2}$$

(e) $\left(2\cos x+1\right)\left(\sin x-1\right)=0$

$$\cos x=-\frac{1}{2}\text{ or }\sin x=1$$
$$x=\pi-\frac{\pi}{3},\ \pi+\frac{4\pi}{3}\text{ or }x=\frac{\pi}{2}$$
$$x=\frac{2\pi}{3},\ \frac{4\pi}{3}\text{ or }x=\frac{\pi}{2}$$
$$x=\frac{\pi}{2},\ \frac{2\pi}{3},\ \frac{4\pi}{3}$$

(f) $5\sec^2 x+2\tan x=8$

$$5\left(1+\tan^2 x\right)+2\tan x=8$$
$$5+5\tan^2 x+2\tan x=8$$
$$5\tan^2 x+2\tan x-3=0$$
$$(5\tan x-3)(\tan x+1)=0$$
$$\tan x=\frac{3}{5} \text{ or } \tan x=-1$$
$$x=0.5404..., \ \pi-0.5404... \text{ or } x=\pi-\frac{\pi}{4}, \ 2\pi-\frac{\pi}{4}$$
$$x\approx 0.540, \ 3.682, \ \text{or } x=\frac{3\pi}{4}, \ \frac{7\pi}{4}$$

18 (a) $3\tan^3\theta-3\tan^2\theta-\tan\theta+1=0$

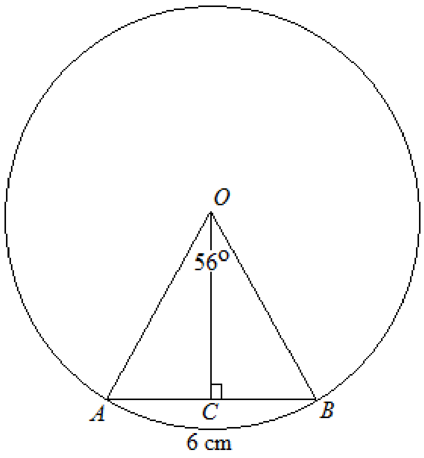
$$3\tan^2\theta(\tan\theta-1)-1(\tan\theta-1)=0$$
$$\left(3\tan^2\theta-1\right)(\tan\theta-1)=0$$
$$\tan^2\theta=\frac{1}{3} \text{ or } \tan\theta=1$$
$$\tan\theta=\pm\sqrt{\frac{1}{3}} \text{ or } \tan\theta=1$$
$$\tan\theta=\frac{1}{\sqrt{3}} \text{ or } \tan\theta=-\frac{1}{\sqrt{3}} \text{ or } \tan\theta=1$$
$$\theta=\frac{\pi}{6}, \ \frac{7\pi}{6} \text{ or } \theta=\frac{5\pi}{6}, \ \frac{11\pi}{6} \text{ or } \theta=\frac{\pi}{4}, \ \frac{5\pi}{4}$$
$$\theta=\frac{\pi}{6}, \ \frac{\pi}{4}, \ \frac{5\pi}{6}, \ \frac{7\pi}{6}, \ \frac{5\pi}{4}, \ \frac{11\pi}{6}$$

(b) $\cos^3\theta-2\cos^2\theta+\cos\theta=0$

$$\cos\theta\left(\cos^2\theta-2\cos\theta+1\right)=0$$
$$\cos\theta(\cos\theta-1)^2=0$$
$$\cos\theta=0 \text{ or } \cos\theta=1$$
$$\theta=\frac{\pi}{2},\frac{3\pi}{2} \text{ or } \theta=0,2\pi$$
$$\theta=0,\frac{\pi}{2},\frac{3\pi}{2},2\pi$$

CHAPTER REVIEW 6

2 (a)



$$6 = r \times \frac{56 \times \pi}{180}$$
$$6 = r \times \frac{14\pi}{45}$$
$$r = 6 \times \frac{45}{14\pi}$$
$$r = \frac{135}{7\pi}$$
$$r \approx 6.138...$$

The radius is 6.1 cm, rounded to one decimal place.

(b) A diagram may be helpful.

$$\sin 28° = \frac{BC}{r}$$
$$BC = r\sin 28°$$
$$AB = 2r\sin 28°$$
$$= 2 \times \frac{135}{7\pi} \times \sin 28°$$
$$= 3.3260...$$

The length of AB is 5.73 cm, rounded to two decimal places.

4 $6\cos^2 x - 5\cos x + 1 = 0$

$$(2\cos x - 1)(3\cos x - 1) = 0$$

$$2\cos x - 1 = 0 \text{ or } 3\cos x - 1 = 0$$

$$\cos x = \frac{1}{2} \text{ or } \cos x = \frac{1}{3}$$

$$x = -\frac{\pi}{3}, \frac{\pi}{3} \text{ or } x = 1.2309..., \ -1.2309...$$

$$x = -\frac{\pi}{3}, \frac{\pi}{3} \text{ or } x \approx 1.231, \ -1.231$$

6 **(a)** $\sin\theta = -0.5$

$\sin\theta < 0$ so θ is in the 3rd and 4th quadrants:

$$\theta = \pi + \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$$
$$\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}$$

(b) $\cos\theta = 0$

$$\theta = \frac{\pi}{2}, \ \frac{3\pi}{2}$$

(c) $\tan\theta = -1$

$\tan\theta < 0$ so θ is in the 2nd and 4th quadrants:

$$= \pi - \frac{\pi}{4}, \ 2\pi - \frac{\pi}{4}$$
$$= \frac{3\pi}{4}, \ \frac{7\pi}{4}$$

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(d) $\sec\theta = \frac{2}{\sqrt{3}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\cos\theta > 0,$ so θ is in the 1st and 4th quadrants:

$\theta = \frac{\pi}{6},\ 2\pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{6},\ \frac{11\pi}{6}$

(e) $\cot\theta = \sqrt{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta > 0$ so θ is in the 1st and 3rd quadrants:

$\theta = \frac{\pi}{6},\ \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6},\ \frac{7\pi}{6}$

(f) $\operatorname{cosec}\theta = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\sin\theta > 0$ so θ is in the 1st and 2nd quadrants:

$\theta = \frac{\pi}{4},\ \pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4},\ \frac{3\pi}{4}$

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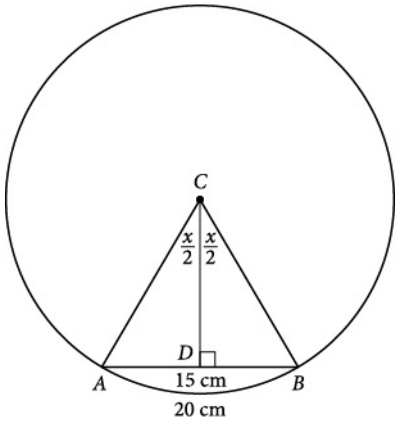
8 (a) Draw a diagram.

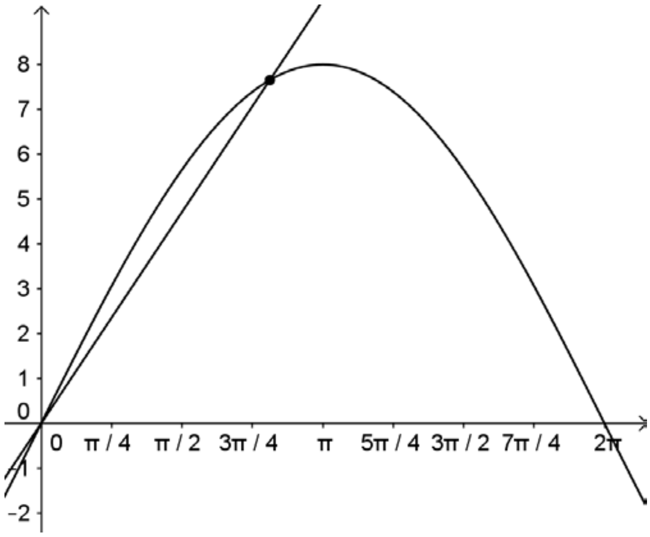


$$l = r\theta$$
$$20 = rx$$
$$r = \frac{20}{x}$$

$$\sin\frac{x}{2} = \frac{\frac{15}{2}}{r}$$
$$r = \frac{7.5}{\sin\frac{x}{2}}$$

$$\frac{20}{x} = \frac{7.5}{\sin\frac{x}{2}}$$
$$20\sin\frac{x}{2} = 7.5x$$
$$40\sin\frac{x}{2} = 15x$$
$$8\sin\frac{x}{2} = 3x$$

(b) On the same axes, draw the graphs of $y = 8\sin\dfrac{x}{2}$ and $y = 3x$.



$x = 0,\ 2.6,\ -2.6$

If the answers are to relate to the circle in part **(a)**, the only meaningful solution is $x = 2.6$, although there is a trivial solution $x = 0$.

10 (a) $65° = 65 \times \dfrac{\pi}{180} \approx 1.1345$

(b) $281° = 281 \times \dfrac{\pi}{180} \approx 4.9044$

(c) $-100° = -100 \times \dfrac{\pi}{180} \approx -1.7453$

(d) $-326° = -326 \times \dfrac{\pi}{180} \approx -5.6898$

12 (a) $2.6 = 2.6 \times \dfrac{180°}{\pi} = 180°58'$

(b) $-1.4 = -1.4 \times \dfrac{180°}{\pi} = -80°13'$

(c) $0.341 = 0.341 \times \dfrac{180°}{\pi} = 19°32'$

(d) $-3 = -3 \times \dfrac{180°}{\pi} = -171°53'$

14 (a) On the unit circle, π has the position $(-1, 0)$.

$\therefore \cos\pi = -1$

(b) $\tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right)$

$= \tan\frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}$

(c) $\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

(d) $\cos\frac{5\pi}{3} = \cos\left(2\pi - \frac{\pi}{3}\right)$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

16 (a) $\left(\frac{\pi}{2} - x\right)$ is complementary to x.

The ratios sine and cosine are a complementary pair.

Therefore, $\sin\left(\frac{\pi}{2} - x\right) = \cos x$.

(b) $\cos\left(\frac{3\pi}{2} - x\right) = \cos\left(\pi + \frac{\pi}{2} - x\right)$

$= -\cos\left(\frac{\pi}{2} - x\right)$

$= -\sin x$

(c) $\tan\left(\frac{\pi}{2} + x\right) = \tan\left(\pi - \frac{\pi}{2} + x\right)$

$= \tan\left(\pi - \left(\frac{\pi}{2} - x\right)\right)$

$= -\tan\left(\frac{\pi}{2} - x\right)$

$\left(\frac{\pi}{2} - x\right)$ is complementary to x.

The ratios tangent and cotangent are a complementary pair.

Hence, $\tan\left(\frac{\pi}{2} + x\right) = -\tan\left(\frac{\pi}{2} - x\right) = -\cot x$

