Neap

Trial Examination 2023

HSC Year 12 Mathematics Advanced

General Instructions	
	• Reading time – 10 minutes
	• Working time – 3 hours
	• Write using black pen
	• Calculators approved by NESA may be used
	• A reference sheet is provided at the back of this paper
	• For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2–6)

• Attempt Questions 1– 10

• Allow about 15 minutes for this section

Section II – 90 marks (pages 7–34)

• Attempt Questions 11– 30

• Allow about 2 hours and 45 minutes for this section

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 HSC Year 12 Mathematics Advanced examination.

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1   Line *l* is shown in the graph.
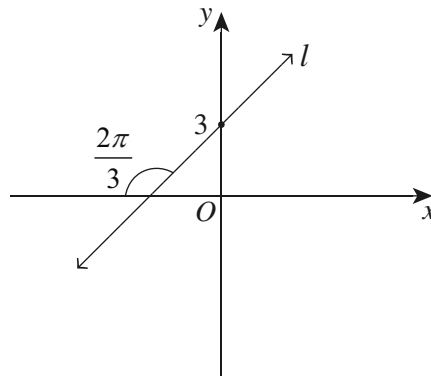
What is the gradient of line *l*?

A. $-\dfrac{1}{\sqrt{3}}$

B. $-\sqrt{3}$

C. $\dfrac{1}{\sqrt{3}}$

D. $\sqrt{3}$

2   What are the solution(s) to $|2x - 1| = 5$?

A. $x = -2$

B. $x = 3$

C. $x = -2$ and $x = 3$

D. $x = -3$ and $x = 2$

3 pH measures the concentration of hydrogen ions, $[H^+]$, in a liquid solution. The formula to calculate the pH of a solution is given by

$$pH = -\log_{10}[H^+].$$

What is the concentration of hydrogen ions in a solution with a pH of 1.5?

A. 1.5^{-10}

B. 1.5^{10}

C. $10^{-1.5}$

D. $10^{1.5}$

4 The graph shows the quadratic function $y = ax^2 + bx + c$.
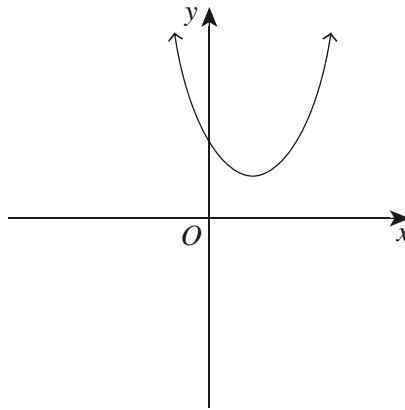
Which of the following statements is correct?

A. $a > 0$, $c > 0$ and $b^2 - 4ac = 0$

B. $a > 0$, $c > 0$ and $b^2 - 4ac > 0$

C. $a > 0$, $c < 0$ and $b^2 - 4ac < 0$

D. $a > 0$, $c > 0$ and $b^2 - 4ac < 0$

5 The duration of each game in a chess tournament was recorded. The data is represented in the cumulative frequency graph.
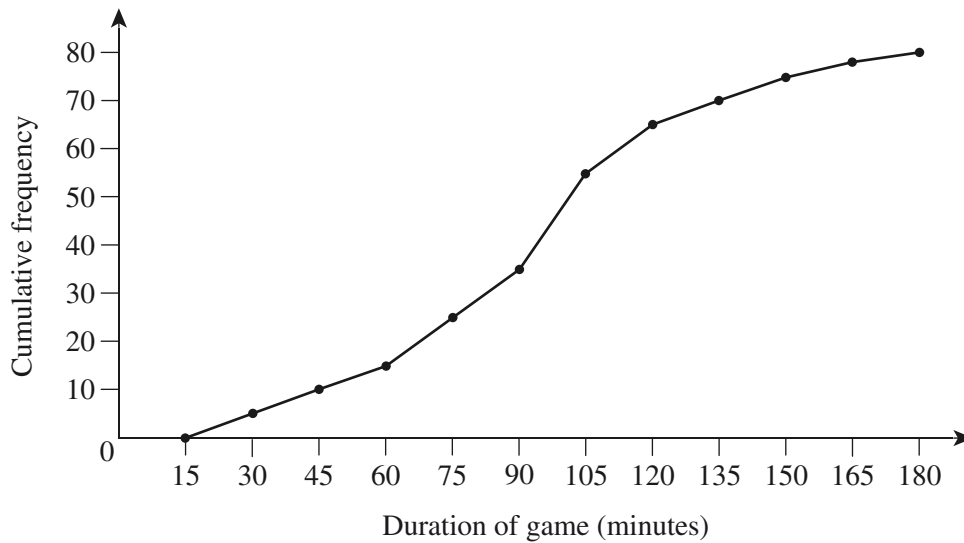
If 25% of the games took less than *n* minutes, what is the value of *n*?

A. 10

B. 67.5

C. 70

D. 112.5

6 Which of the following is the derivative of $y = \ln\sqrt{\dfrac{x+1}{x-1}}$?

A. $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{x+1}{x-1}\right)$

B. $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{1}{x+1} - \dfrac{1}{x-1}\right)$

C. $\dfrac{dy}{dx} = 2\left(\dfrac{1}{x+1} - \dfrac{1}{x-1}\right)$

D. $\dfrac{dy}{dx} = \dfrac{1}{2}(\ln(x+1) - \ln(x-1))$

7 The function $y = f(x)$ is transformed to $y = g(x)$, as shown in the diagram.
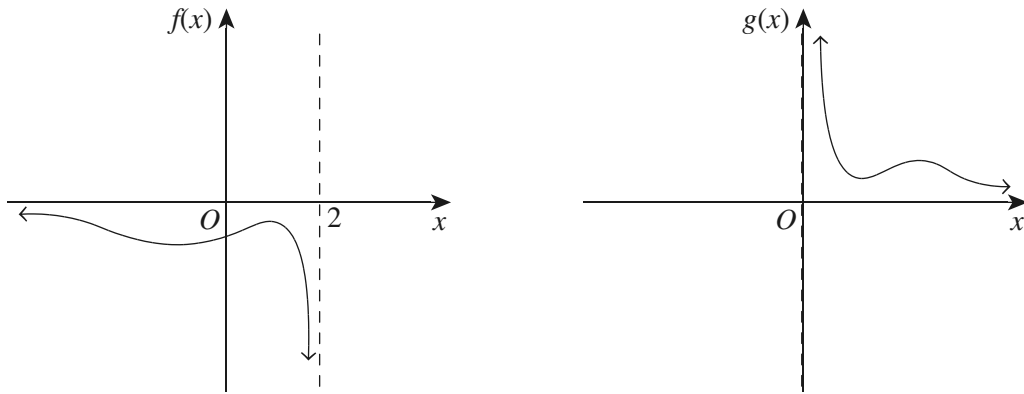
Which of the following equations best represents the transformed function?

A. $g(x) = -f(2 - x)$

B. $g(x) = -f(x - 2)$

C. $g(x) = f(2 - x)$

D. $g(x) = f(x - 2)$

8 Consider the simultaneous equations.

$$ax + y - 4 = 0$$

$$x + 2y - a = 0$$

If the equations have no real solutions, what is the value of a?

A. $a = \dfrac{1}{2}$

B. $a = 1$

C. $a = 4$

D. $a = 8$

9 Consider the information about events A and B.

- $P(B) = 0.6$
- $P(A \,|\, B) = 0.4$
- $P(A \,|\, \bar{B}) = 0.8$

What is the value of $P(B \,|\, A)$?

A. $\dfrac{1}{5}$

B. $\dfrac{2}{5}$

C. $\dfrac{3}{7}$

D. $\dfrac{4}{5}$

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.
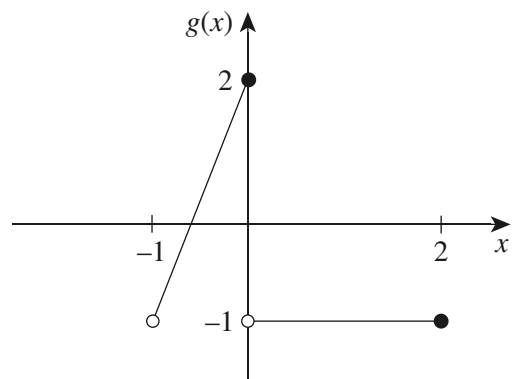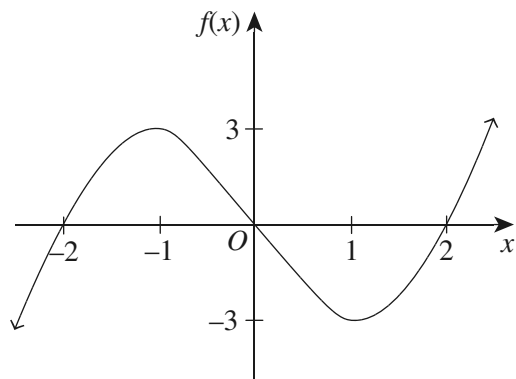
What is the domain and range for $y = f(g(x))$?

	Domain	Range
A.	$(-1, 2]$	$[-3, 3]$
B.	$(-1, 2]$	$(-1, 2]$
C.	$(-\infty, \infty)$	$[-3, 3)$
D.	$(-\infty, \infty)$	$(-1, 2]$

HSC Year 12 Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–30
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve $9^{2x-3} = 27^x$. **2**

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Question 12 (3 marks)

The probability distribution table for the discrete random variable X is shown. **3**

x	0	1	2	3	4	5
$P(X = x)$	0.35	a	b	0.15	0.05	0.01

If $E(X) = 1.5$, find the values of a and b.

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Question 13 (6 marks)

The diagram shows the first four figures in a pattern.

F_1 F_2 F_3 F_4
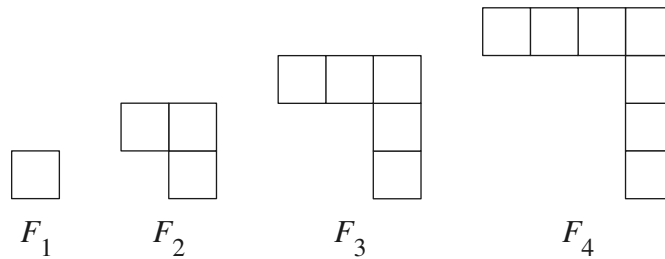
(a) Determine the number of squares required to construct F_{15}. **2**

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(b) Is it possible for a figure in the sequence to have 175 squares? **2**

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(c) Determine the number of squares required to make the first 50 figures. **2**

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Question 14 (2 marks)

Differentiate $y = (3x^2 + 1)^3$ in terms of x. Give your answer in the simplest form. **2**

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Question 15 (2 marks)

Find $\int \dfrac{5x^3 - 2x}{x^5} dx$. **2**

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Question 16 (2 marks)

Evaluate $\displaystyle\int_1^4 2\sqrt{x} + \frac{3}{x}\,dx.$

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Question 17 (8 marks)

The data set shows Mathematics test results of a sample of 20 students.

| 35% | 55% | 65% | 65% | 66% | 68% | 69% | 70% | 71% | 72% |
| 72% | 74% | 75% | 77% | 79% | 80% | 81% | 82% | 88% | 92% |

(a) Calculate the interquartile range. **2**

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(b) Show that the result of 35% is an outlier. **1**

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The teacher wanted to determine whether there was a relationship between the students' school attendance and their Mathematics test results. She drew a scatterplot to show the relationship.
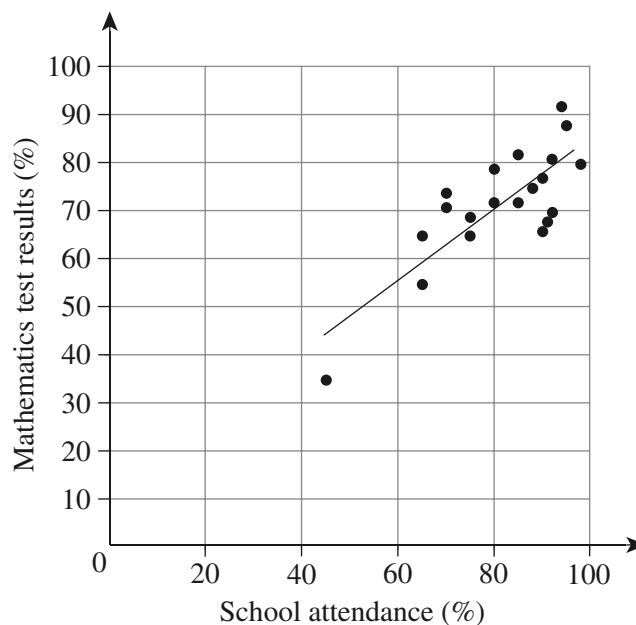
The least-squares regression line has the equation $y = 0.7341x + 12.151$.

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Question 17 (continued)

(c) The teacher initially calculated the correlation coefficient for this dataset to be $r = -2.6577$. **2**
She realised she had made a mistake in her calculation.

Without calculating the correlation coefficient, give TWO reasons why $r = -2.6577$ is incorrect.

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(d) The teacher recalculated the correlation coefficient and found that $r = 0.8031$. **2**

Describe the strength and direction of the correlation between the students' school attendance and Mathematics test results.

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(e) Robin, who was absent on the day of the Mathematics test, has an attendance of 88%. **1**

Use the equation of the least-squares regression line to estimate that Robin would have achieved a result of approximately 77%.

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End of Question 17

Question 18 (3 marks)

The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = -2$. **3**
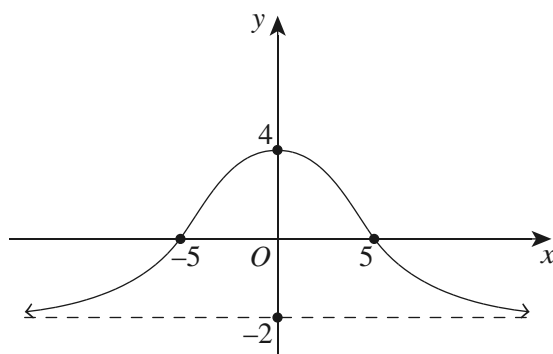
On the axes below, sketch the graph of $y = f'(x)$, clearly showing the behaviour at the intercepts and any asymptotes.
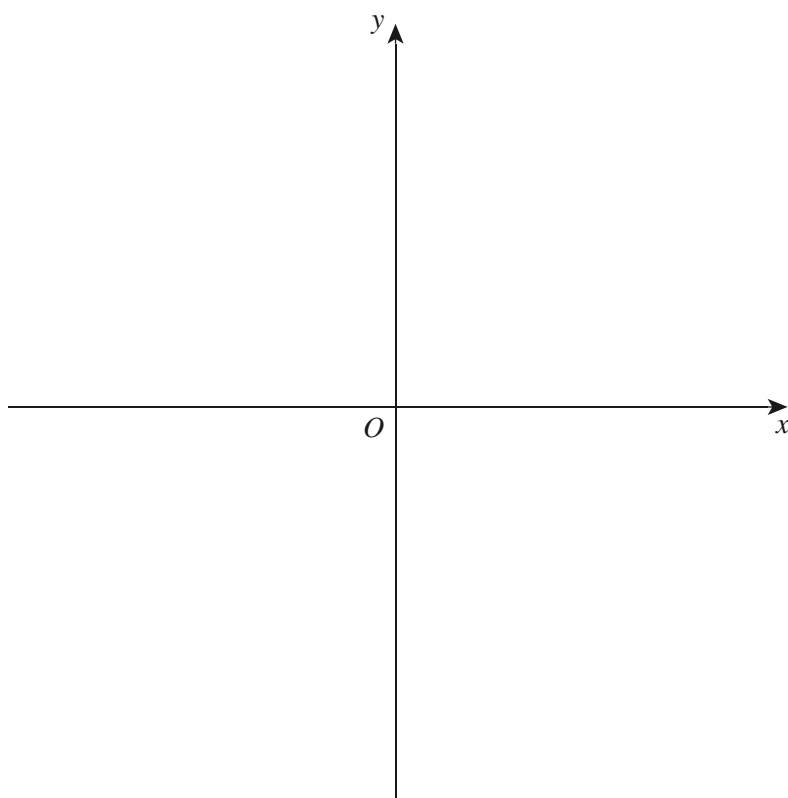
Question 19 (3 marks)

The function $y = \sin(x)$ undergoes a series of graphical transformations and becomes $y = 5\sin\left(2x + \dfrac{\pi}{3}\right)$. **3**

Outline the transformations that were applied to $y = \sin(x)$ in the correct order.

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Question 20 (4 marks)

Find the sum of the geometric sequence $6 - 12 + 24 - \ldots + 1536$. **4**

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Question 21 (3 marks)

The table shows the future value interest factors for an annuity of $1. The contributions are made at the beginning of each year.

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Period	Interest rate per period				
	1%	2%	3%	4%	5%
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019

Duncan deposits $5000 into a savings account at the beginning of each year for eight years. During the first six years, the interest rate is 3% per annum, compounding annually. During the seventh and eighth years, the interest rate is 2.5% per annum, compounding annually.

What is the total amount in Duncan's account at the end of eight years?

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Question 22 (3 marks)

The diagram shows a 75 m vertical tower, represented by line *DC*. Points *A* and *B* are in the **3** same horizontal plane as the base of the tower, point *C*, and point *A* is west of point *B*. The angle of elevation from point *A* to point *D* is 28°, and the angle of elevation from point *B* to point *D* is 35°.
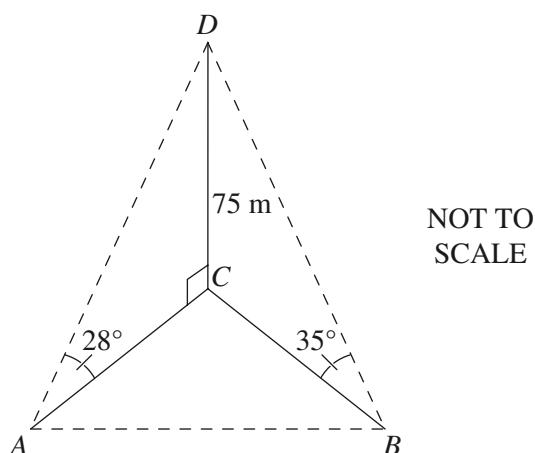
The bearing of point *C* from point *A* is 050°T, and the bearing of point *C* from point *B* is 300°T.

Find the distance between points *A* and *B*, correct to the nearest metre.

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Question 23 (6 marks)

The continuous random variable, t, represents the time it takes, in years, to construct a high-rise building. The probability density function for t is given by

$$f(t) = \begin{cases} kt(5-t) & 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Show that $k = \dfrac{6}{125}$. **2**

(b) Find the mode of the distribution. **2**

(c) What percentage of high-rise buildings are constructed within a year? **2**

Question 24 (7 marks)

(a)   Show that $\dfrac{d}{dx}(x\ln x - x) = \ln x$.   **2**

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(b)   Show that $y = \dfrac{1}{e}x$ is the equation of the tangent to the curve $y = \ln x$ at the point $(e, 1)$.   **2**

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Question 24 continues on page 20

Question 24 (continued)

(c)   The graph shows the function $y = \ln x$ and the tangent $y = \dfrac{1}{e}x$.   **3**
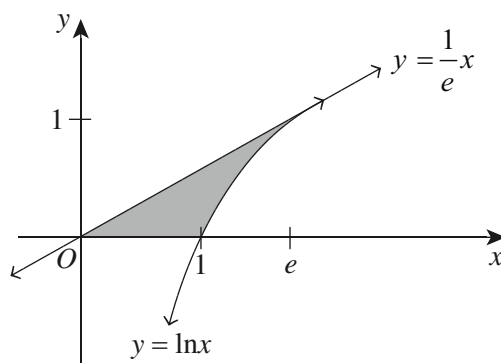
Find the shaded area.

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End of Question 24

Question 25 (7 marks)

A speedometer is a device that measures the speed of vehicles. The data recorded by a speedometer on a motorway that has a speed limit of 110 km/h is normally distributed with a mean of 110 km/h and a standard deviation of 5 km/h.

(a)   If drivers travelling at a speed of more than 120 km/h are considered to be driving at a dangerous   **1**
      speed, what percentage of drivers travel at a dangerous speed?

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(b)   The probability density function for a normal distribution is   **2**

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.$$

Explain why the probability of drivers travelling at a speed between 102.5 km/h and 120 km/h can be found by evaluating

$$\int_{-1.5}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx.$$

Support your answer with relevant calculations.

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Question 25 (continued)

(c) Using the integral from part (b), use three applications of the trapezoidal rule to find the probability that a driver travels at a speed between 102.5 km/h and 120 km/h. **2**

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(d) Hence, find the approximate probability of a driver travelling faster than 102.5 km/h. **2**

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End of Question 25

Question 26 (4 marks)

Find the value of k such that $\displaystyle\int_0^k e^x \left(e^x - 2\right) dx = \frac{3}{2}$.   **4**

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Question 27 (5 marks)

A particle travels in a line such that its displacement is given by the function

$$x(t) = \begin{cases} 5t & 0 \le t \le 1 \\ 6\sqrt{t} - \dfrac{1}{t} & t > 1 \end{cases},$$

where x is the particle's displacement from the origin, in metres, and t is the time, in seconds.

(a)   Show that the particle begins its journey at the origin.   **1**

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(b)   Show that the particle is never at rest.   **3**

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(c)   Find the distance travelled in the first four seconds of the particle's journey.   **1**

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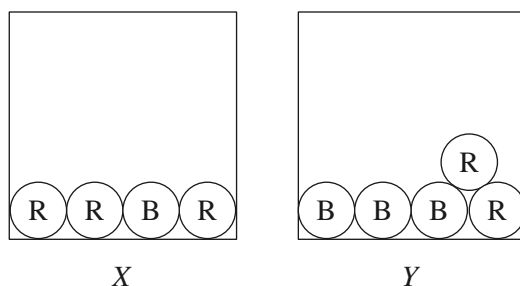..

Question 28 (6 marks)

Edie and Catriona are playing a game. They take turns picking a coloured ball from boxes X and Y. The first player to choose a blue ball wins the game. If the ball is red, the ball is placed back into the box and the game continues.

X Y

In each round of the game, players have to choose a ball from a particular box.

- In the first round, both players must choose a ball from box X.

- If the game continues to the second round, both players must choose a ball from box Y.

- If the game continues to the third round, both players must choose another ball from box X.

Edie takes the first turn in each round.

(a) Find the probability that Edie wins in the first round. **1**

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(b) Show that the probability that Edie wins in the first OR second round is $\dfrac{1}{4}+\left(\left(\dfrac{3}{4}\right)^{2}\times\dfrac{3}{5}\right)$. **2**

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Question 28 continues on page 26

Question 28 (continued)

(c)   The probability of Edie eventually winning the game can be expressed as the series   **3**

$$\frac{1}{4}+\left(\left(\frac{3}{4}\right)^{2}\times\frac{3}{5}\right)+\left(\left(\frac{3}{4}\right)^{2}\times\left(\frac{2}{5}\right)^{2}\times\frac{1}{4}\right)+\left(\left(\frac{3}{4}\right)^{4}\times\left(\frac{2}{5}\right)^{2}\times\frac{3}{5}\right)+\dots$$

(Do NOT prove this result.)

Show that Edie has a greater chance of winning the game than Catriona.

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End of Question 28

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Question 29 (4 marks)

Three circles with radius *r* are drawn such that their centres lie on the circumference of the other two circles, as shown in the diagram.

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Points *A*, *B* and *C* are the centres of the three circles.

Prove that the area of the shaded region is $\frac{1}{2}r^2\left(\pi - \sqrt{3}\right)$.

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Question 30 (10 marks)

In the circulatory system, the heart pumps blood to the various organs in the human body. It is ideal that the heart uses the minimum amount of energy required to pump blood. The energy required is reduced when the resistance of the blood is lowered. The resistance of the blood can be determined by the formula

$$R = k\,\frac{L}{r^4},$$

where L is the length of the blood vessel, r is the radius of the blood vessel and k is a constant such that $k \geq 0$.

The diagram shows a section of a blood vessel in the circulatory system.

NOT TO SCALE
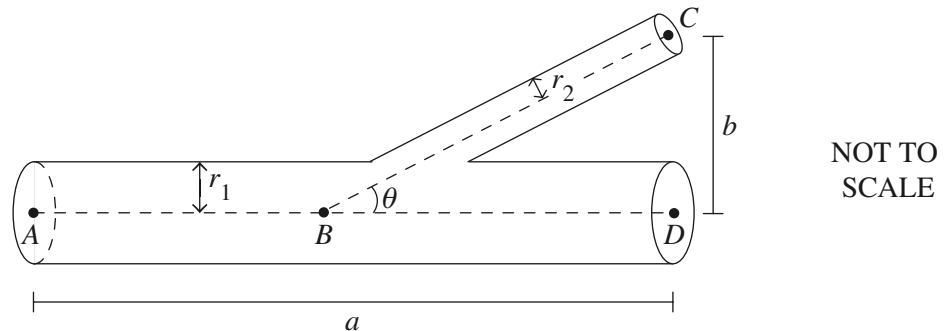
A, B, C and D are various points in the blood vessel. It is given that $r_1 > r_2$.

(a)   Show that the total resistance of blood that travels along path ABC is **2**

$$R = k\left(\frac{a - b\cot\theta}{\left(r_1\right)^4} + \frac{b\operatorname{cosec}\theta}{\left(r_2\right)^4}\right).$$

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Question 30 continues on page 29

Question 30 (continued)

(b) Show that $\dfrac{d}{dx}(\cot\theta) = -\operatorname{cosec}^2\theta.$ **2**

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Question 30 continues on page 30

Question 30 (continued)

(c)   Assume that the following is known about the resistance of the blood in the blood vessel.   **4**

- $\dfrac{d}{dx}(\operatorname{cosec}\theta) = -\cot\theta\operatorname{cosec}\theta$

- $\dfrac{d^2 R}{d\theta^2} > 0$ when $\cos\theta = \left(\dfrac{r_2}{r_1}\right)^4$

(Do NOT prove these results.)

Show that the resistance of the blood is minimised when $\cos\theta = \left(\dfrac{r_2}{r_1}\right)^4$.

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Question 30 continues on page 31

Question 30 (continued)

(d) Find the branching angle, θ, such that the length of r_2 is $\dfrac{3}{4}$ of the length of r_1 when the resistance of the blood is minimised.

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Section II extra writing space

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MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi r h$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, r \neq 1$$

$$S = \frac{a}{1-r}, |r| < 1$$

Logarithmic and exponential functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$
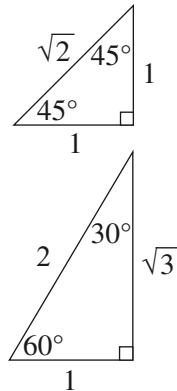
Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score less than $Q_1 - 1.5 \times IQR$ or more than $Q_3 - 1.5 \times IQR$

Normal distribution
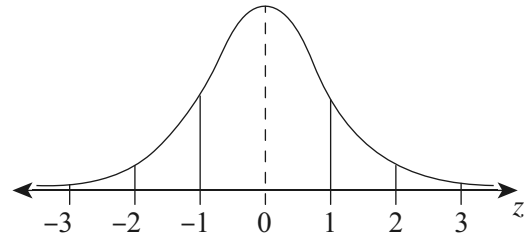
- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r\, p^r (1 - p)^{n - r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n - x}, \quad x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx}\ \ f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - [f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + [f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n\, dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$
$$\text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\, dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\, dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\, dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\, dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}\, dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}\, dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}\, dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\, dx = uv - \int v\frac{du}{dx}\, dx$$

$$\int_a^b f(x)\, dx$$
$$\approx \frac{b-a}{2n}\left\{ f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1}) \right] \right\}$$
where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^r + \ldots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$
$$\text{where } \underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$$
$$\text{and } \underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$