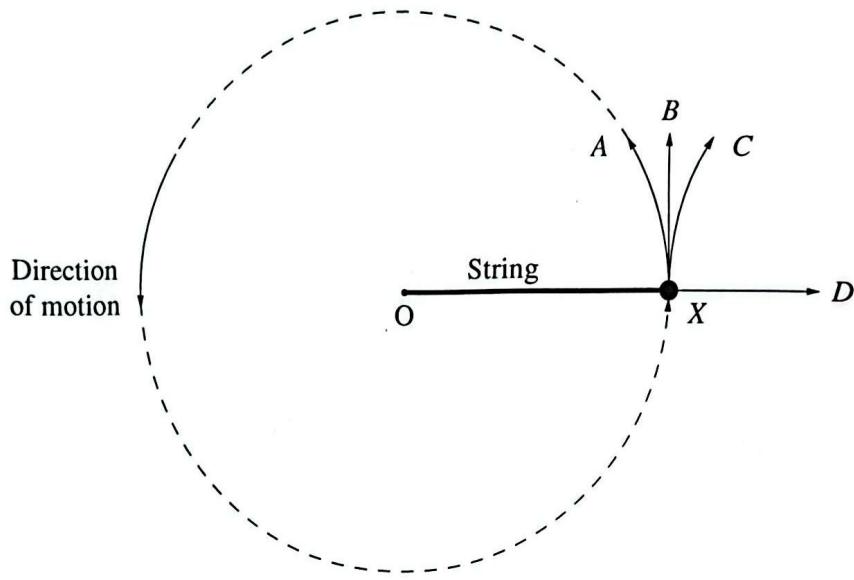


5.2 Circular Motion

Multiple-choice questions: 1 mark each

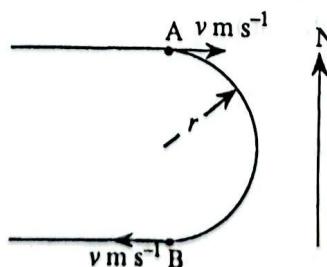
1. A mass attached to a length of string is moving in a circular path around a central point, O, on a flat, horizontal, frictionless table. This is depicted in the diagram below. The string breaks as the mass passes point X.



Which line best depicts the subsequent path of the mass?

2006 HSC Q2

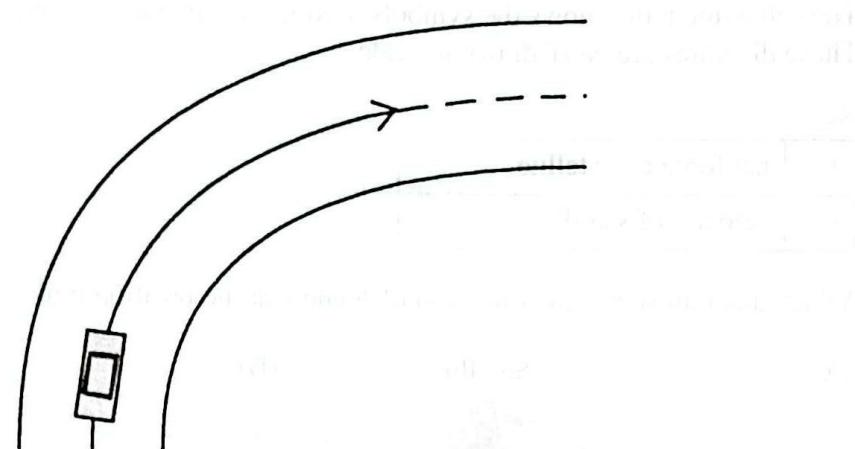
2. A car at A is travelling east at $v \text{ m s}^{-1}$ when it commences a semicircular turn of radius r metres. It emerges from the turn at B after a time t seconds travelling west at $v \text{ m s}^{-1}$, as shown below.



What is the average acceleration of the car between A and B in m s^{-1} ?

1991 HSC Q2 (= HSC Q12 Core 7)

3. A car travelling at constant speed makes a right-hand turn as shown below.



Which of the following statements is correct?

- (A) Both the velocity and acceleration of the car are constant.
- (B) The velocity of the car is constant, but the acceleration is not constant.
- (C) The velocity of the car is not constant, but the acceleration is constant.
- (D) Neither the velocity nor the acceleration of the car is constant.

1992 HSC Q2

4. An object is undergoing circular motion. Which of the following alternatives best describes the object's motion?

	<i>Displacement</i>	<i>Instantaneous speed</i>	<i>Instantaneous velocity</i>
(A)	Constant	Changing	Changing
(B)	Changing	Changing	Constant
(C)	Changing	Constant	Changing
(D)	Constant	Constant	Changing

1997 HSC Q1

5. A satellite is orbiting a planet at a constant speed.

Which of the following statements is correct?

- (A) The satellite is not accelerating.
- (B) The orbit of the satellite has a fixed radius.
- (C) Fuel must be used to supply a constant thrust to the satellite.
- (D) The centripetal force on the satellite is balanced by the gravitational force.

2011 HSC Q16

6. A satellite moves in uniform circular motion around Earth.

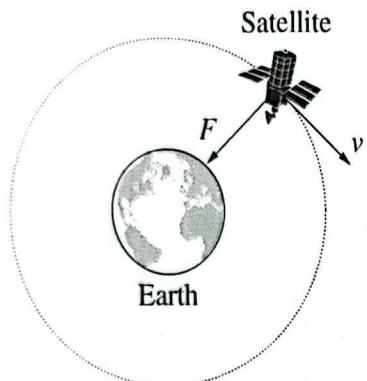
The following table shows the symbols used in the diagrams below.
These diagrams are NOT drawn to scale.

Key

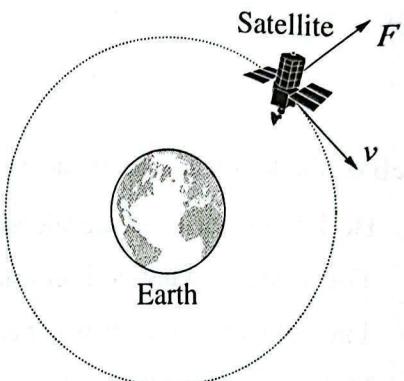
F	net force on satellite
v	velocity of satellite

Which diagram shows the direction of F and v at the position indicated?

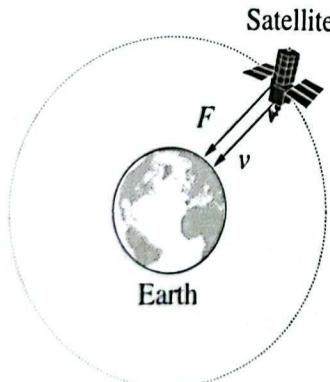
(A)



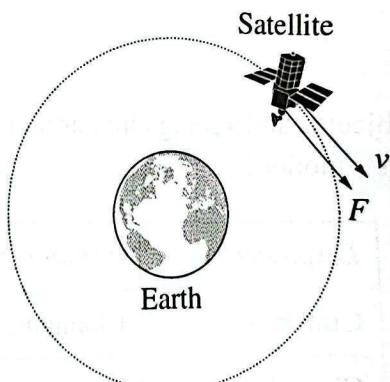
(B)



(C)



(D)



2003 HSC Q

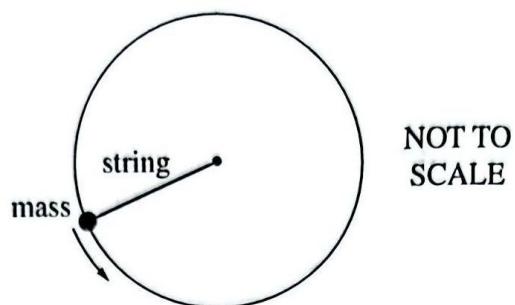
7. The Moon has a mass of 7.2×10^{22} kg. It moves round the Earth at a distance of 3.8×10^8 m. Its speed round the Earth is 1.0×10^3 m s⁻¹.

The gravitational force of the Earth on the Moon is

- (A) 7.0×10^{23} N
 (B) 1.9×10^{20} N
 (C) 2.7×10^{-3} N
 (D) 4.8×10^{-6} N

1996 HSC Q

8. A 200 g mass is swung in a horizontal circle as shown. It completes 5 revolutions in 3 seconds. The circle has a 2 m diameter.

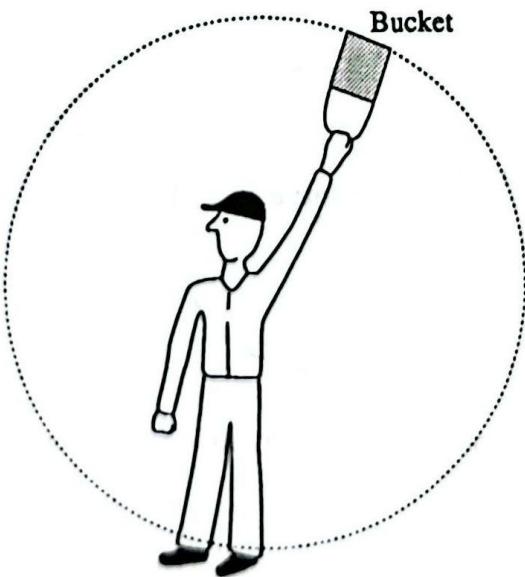


Which of the following forces is closest to that required to keep the mass moving in this circle?

- (A) 0.50 N
- (B) 2.5 N
- (C) 10 N
- (D) 20 N

2010 HSC Q5

9. A boy swings a plastic bucket of water in a vertical circular path over his head, as shown.

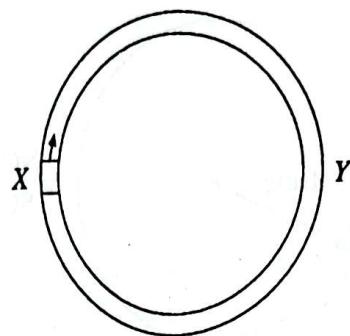


When the bucket of water is near the top of the circle, the water does not fall out because:

- (A) gravity does not act on the water inside the bucket when it is being rotated.
- (B) the water has no weight at the top of the circle.
- (C) a force pulls the water outwards towards the bottom of the bucket.
- (D) the bucket and the water are accelerating at the same rate.

1996 HSC Q3

10. A car moves round a horizontal circular track with a constant speed. Points X and Y are on opposite sides of the track.

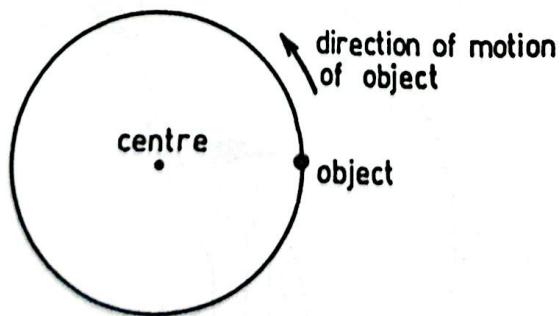


One of the statements below is *incorrect*. Choose the statement that is NOT correct.

- (A) At point X on the track, the acceleration of the car is zero.
- (B) Over one complete revolution of the circuit (from X to Y and back again to X) the average velocity of the car is zero.
- (C) Over one complete revolution of the circuit (from X to Y and back again to X) the displacement of the car is zero.
- (D) In moving from point X to point Y, the change of speed of the car is zero.

1996 HSC Q5

11. An object is travelling around in a circular track at a constant speed of 15 m s^{-1} , as shown below.



The vectors that best represent the velocity and acceleration of the object at the position shown are:

- (A) A vertical vector labeled 'v' pointing upwards and to the left, and a horizontal vector labeled 'a' pointing to the right.
- (B) A vertical vector labeled 'v' pointing upwards and to the left, and a horizontal vector labeled 'a' pointing to the left.
- (C) A vertical vector labeled 'v' pointing upwards and to the left, and a horizontal vector labeled 'a = 0' pointing downwards.
- (D) A vertical vector labeled 'v' pointing upwards and to the left, and a horizontal vector labeled 'a' pointing upwards and to the right.

1990 HSC Q5

12. Figure 1 below shows a railway carriage rounding a curve in a level track. The wheels of the carriage are shaped so that they sit on the rails, as shown in Figure 2.

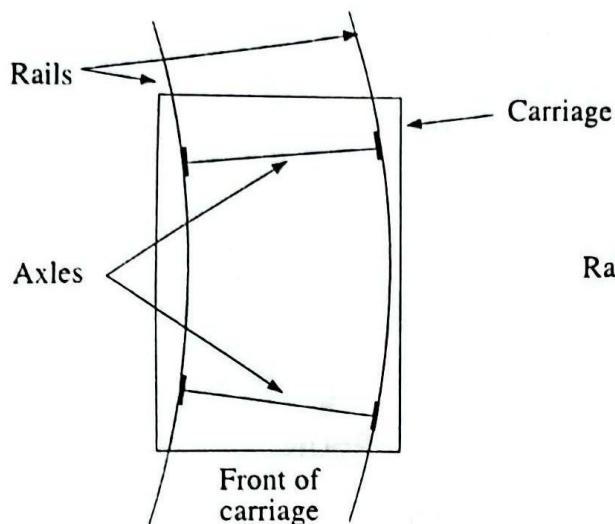


FIG. 1. VIEW FROM ABOVE

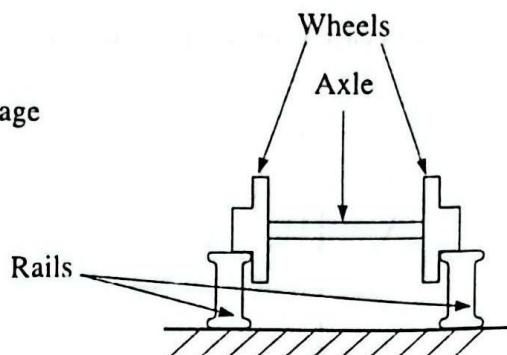
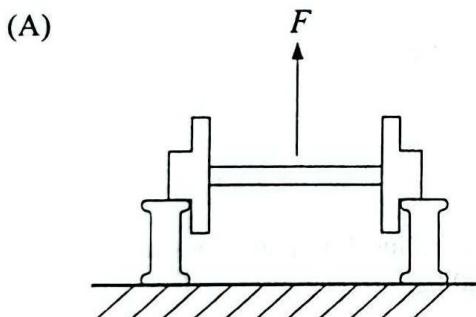
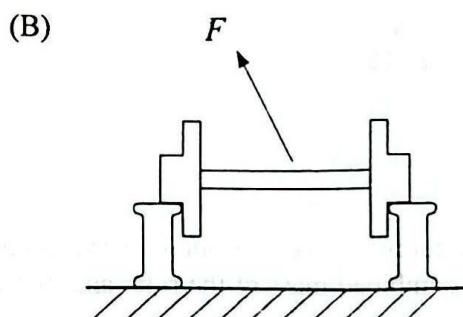


FIG. 2. VIEW FROM THE FRONT

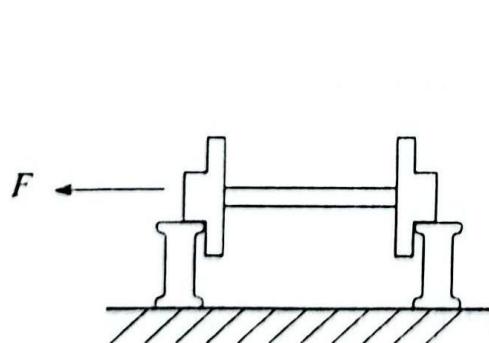
Which alternative below shows the direction of the force F exerted *on the wheels by the track* as the carriage rounds the curve?



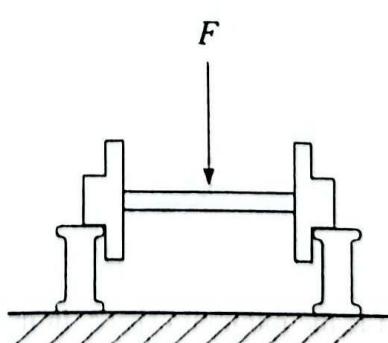
VIEW FROM THE FRONT



VIEW FROM THE FRONT



VIEW FROM THE FRONT



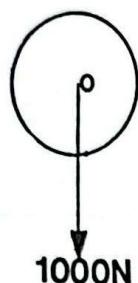
VIEW FROM THE FRONT

1997 HSC Q6

13. Each of the diagrams below shows a uniform disc, of diameter 6 m, which can rotate about its fixed centre, O . Different forces, whose magnitudes and lines of action are shown, are applied to the disc.

In which case is the torque about O the greatest?

(A)



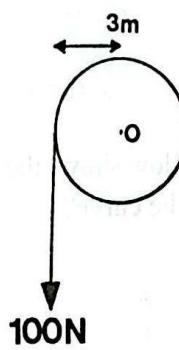
(B)



(C)

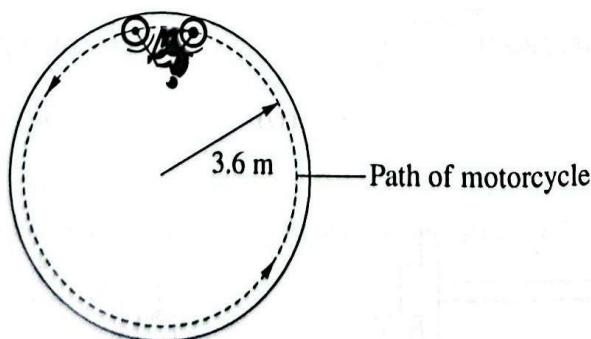


(D)



1982 Elective 3(a)

14. A motorcycle travels around a vertical circular path of radius 3.6 m at a constant speed. The combined mass of the rider and motorcycle is 200 kg.

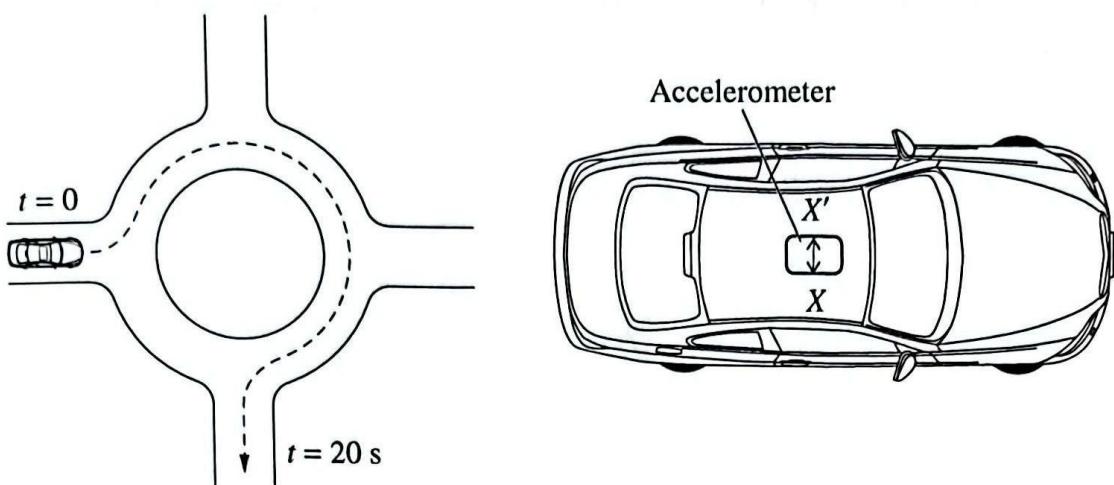


What is the minimum speed, in m s^{-1} , at which the motorcycle must travel to maintain the circular path?

- (A) 0.42
- (B) 1.9
- (C) 5.9
- (D) 35

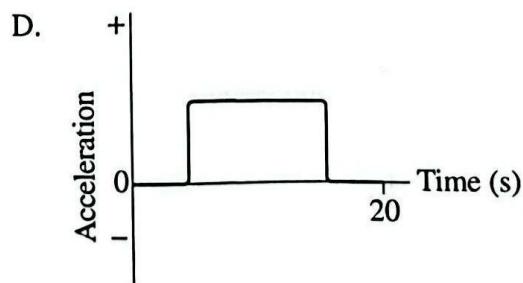
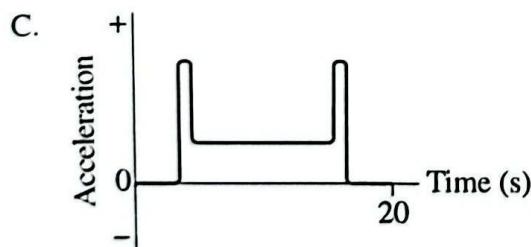
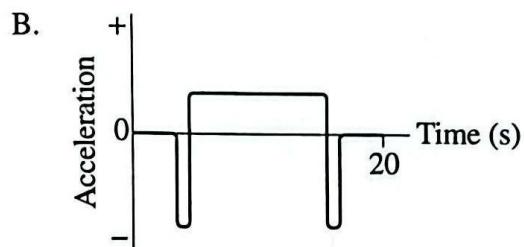
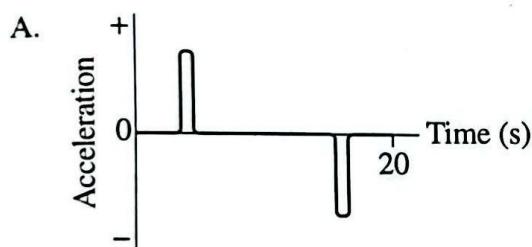
2016 HSC Q18

15. A car travelling at a constant speed follows the path shown.



An accelerometer that measures acceleration along the $X-X'$ direction is fixed in the car.

Which graph shows the measurements recorded by the accelerometer over the 20-second interval?



2017 HSC Q15

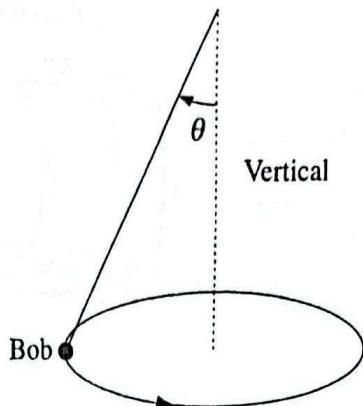
16. A satellite is moving in a circular orbit of radius $7.0 \times 10^6\text{ m}$ around Earth.

If the speed of the satellite is $8.1 \times 10^3\text{ m s}^{-1}$, what is its centripetal acceleration?

- (A) 9.4 m s^{-2}
- (B) 9.8 m s^{-2}
- (C) $5.6 \times 10^{25}\text{ m s}^{-2}$
- (D) $3.9 \times 10^{32}\text{ m s}^{-2}$

2009 HSC Q2

17. The diagram below shows a conical pendulum consisting of a bob and string. The bob is rotating with constant speed in a horizontal circle. The string makes an angle θ with the vertical.



Which diagram best shows the directions of the applied forces acting on the pendulum bob?

(A)



(B)



(C)



(D)



1998 HSC Q6

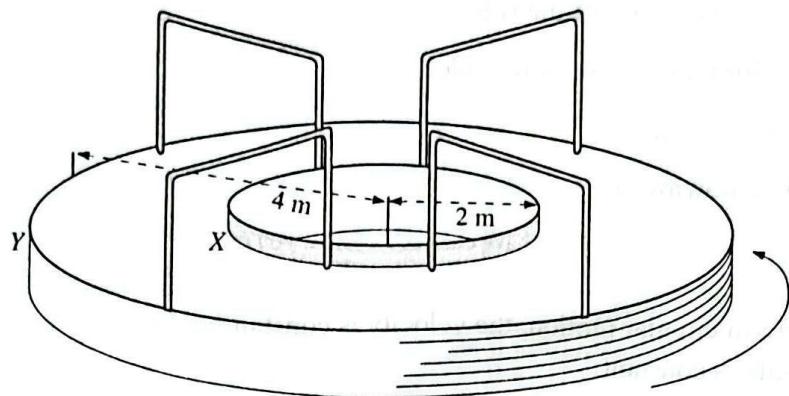
18. An object moves so that the distance it travels continues to increase, but the magnitude of its displacement from a fixed point remains constant.

Which type of motion listed below is the object undergoing?

- (A) Straight line motion
- (B) Projectile motion
- (C) Circular motion
- (D) Free fall due to gravity

1984 HSC Q1 (adapted)

19. A merry-go-round may be represented as shown below. X is a point on the inner edge and Y is a point on the outer edge.



When the merry-go-round is spinning, which pair of the following ratios is correct?

	$\frac{\text{Angular velocity of } X}{\text{Angular velocity of } Y}$	$\frac{\text{Linear velocity of } X}{\text{Linear velocity of } Y}$
(A)	1 : 1	1 : 1
(B)	1 : 1	1 : 2
(C)	1 : 2	1 : 2
(D)	1 : 2	1 : 1

1999 HSC Q2

20. Which of the following statements is incorrect about the work being done on an object undergoing circular motion in a horizontal plane?

- (A) No work is being done as the total energy is constant due to the potential energy and kinetic energy being constant.
- (B) Force and displacement are perpendicular at each instant, so there is no displacement in the direction of the force and so $W = Fs = 0$.
- (C) No work is being done as the total energy is constant due to the displacement being in the direction of the velocity.
- (D) Displacement is in the direction of the force and so $W = Fs = \text{a constant}$, causing work to be done on the object.

21. When a fun park ride such as the Ferris Wheel at Luna Park Sydney is rotating at maximum speed, in what direction does the centripetal force act on the riders?
- (A) Towards the centre of the ride.
(B) Away from the centre of the ride.
(C) Straight upwards.
(D) Straight downwards.
22. During uniform circular motion, the velocity is constantly changing, but the magnitude of the velocity is constant.
- Which of the following best describes an object that is undergoing circular motion in a horizontal plane?
- (A) Changing speed, changing kinetic energy, changing potential energy.
(B) Constant speed, constant kinetic energy, constant potential energy.
(C) Changing speed, changing kinetic energy, constant potential energy.
(D) Constant speed, constant kinetic energy, changing potential energy.

Short-answer questions

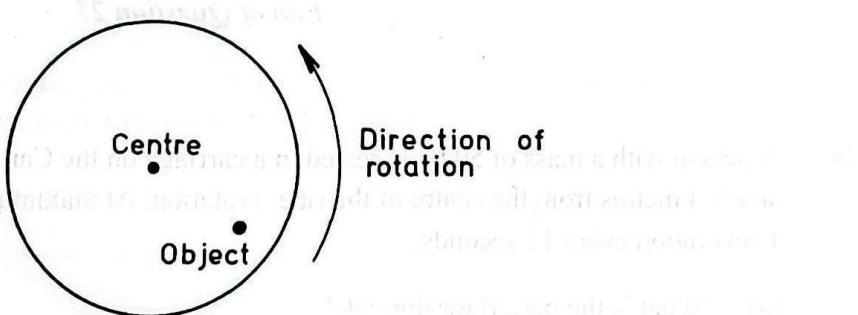
23. A spaceship is in a circular orbit just above the surface of the Moon, where the gravitational acceleration is 1.5 m s^{-2} . The Moon's radius is $1.5 \times 10^6 \text{ m}$.

Calculate the orbital speed of the spaceship.

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1982 HSC Q14 – 3 marks

24. An object is placed near the edge of a disc that is rotating at a constant speed in the horizontal plane, as shown below.

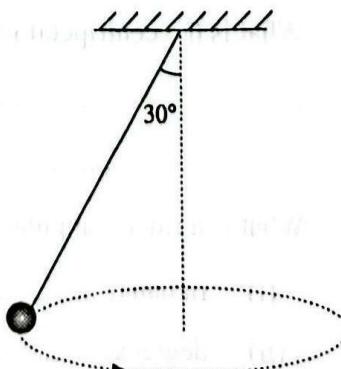


- (a) On the diagram above, draw labelled vectors that correctly show the directions of the object's linear velocity and acceleration.
(b) The disc rotates at 1.5 revolutions per second. What is its angular velocity in rad s^{-1} ?

.....
.....

1986 HSC Q13 – 1 + 2 + 3 marks

25. A ball of mass m hangs from a string and moves with a constant speed in a horizontal circle as shown. The string makes an angle of 30° with the vertical.



Question 25 continues

Question 25 (continued)

- (a) Draw a vector diagram of the forces that act on the ball.

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- (b) What is the acceleration of the ball?

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- (c) Write an equation for the tension in the string under these conditions?

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.....

End of Question 25

1995 HSC Q18 – 2 + 2 + 2 = 6 marks

26. A person with a mass of 50 kg is seated in a carriage on the Carousel ride at Luna Park Sydney and is 4 metres from the centre of the ride's rotation. At maximum speed, the Carousel makes 1 revolution every 12 seconds.

- (a) What is the period for this ride?

.....
.....

- (b) What is the rider's orbital (linear) velocity?

.....
.....

- (c) What is the rider's angular velocity?

.....
.....

- (d) What is the centripetal force acting on the rider?

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.....

- (e) What is a rider's angular displacement after 3 seconds in:

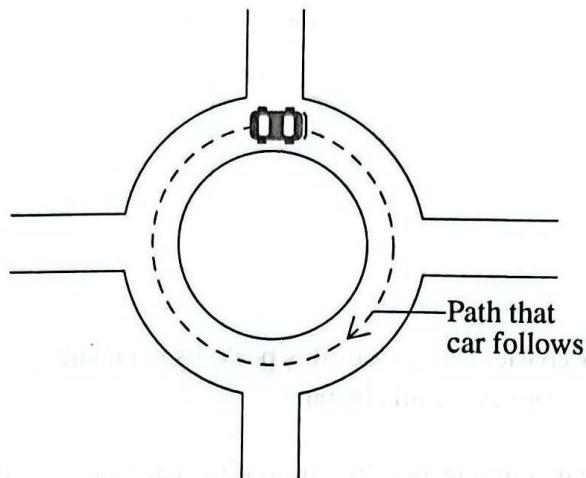
(i) radians?

(ii) degrees?

1 + 1 + 1 + 1 + 2 = 6 marks

27. A car with a mass of 800 kg travels at a constant speed of 7.5 m s^{-1} on a roundabout so that it follows a circular path with a radius of 16 m.

4



A person observing this situation makes the following statement.

'There is no net force acting on the car because the speed is constant and the friction between the tyres and the road balances the centripetal force acting on the car.'

Assess this statement. Support your answer with an analysis of the horizontal forces acting on the car, using the numerical data provided above.

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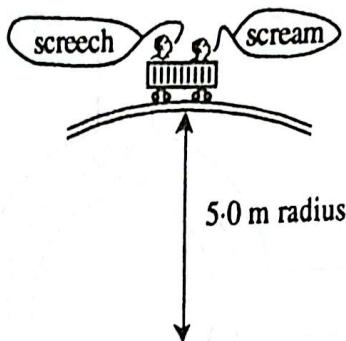
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2004 HSC Q18 – 4 marks

28. The track of a roller coaster forms a circular arc of radius 5.0 m at one of its peaks, as shown below.



As one of the roller coaster cars passes this peak, its occupants just 'float' off their seats, making them feel weightless at this instant.

- (a) What are the magnitude and direction of the car's acceleration as it passes the peak?

.....

- (b) Calculate the speed of the car along the track at this point.

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1992 HSC Q14 - 2 + 2 = 4 marks

29. (a) The Carousel ride at Luna Park Sydney has an inner row of horses and an outer row of horses. This ride has uniform circular motion.

Assume this ride has reached maximum speed and ignore the up and down movement of the horses to answer the following questions:

- (i) Does the orbital (linear) velocity of a horse change or remain the same? Explain your answer.

.....

- (ii) Does the magnitude of a horse's acceleration change or remain the same at different points around one revolution of the Carousel? Explain your answer.

.....

.....

- (b) Compare the angular velocity of an inner horse to and the angular velocity of an outer horse.

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2 + 2 + 2 = 6 marks

30. A truck laden with heavy goods has a mass of 3500 kg. It is travelling horizontally at 75 km h^{-1} around a bend in the road. The radius of curvature of the bend is 300 metres.
- (a) At what angle should the bend in the road be banked so that the truck can turn the corner (assuming no friction force is acting)?

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.....

- (b) What is the net force acting on the truck?

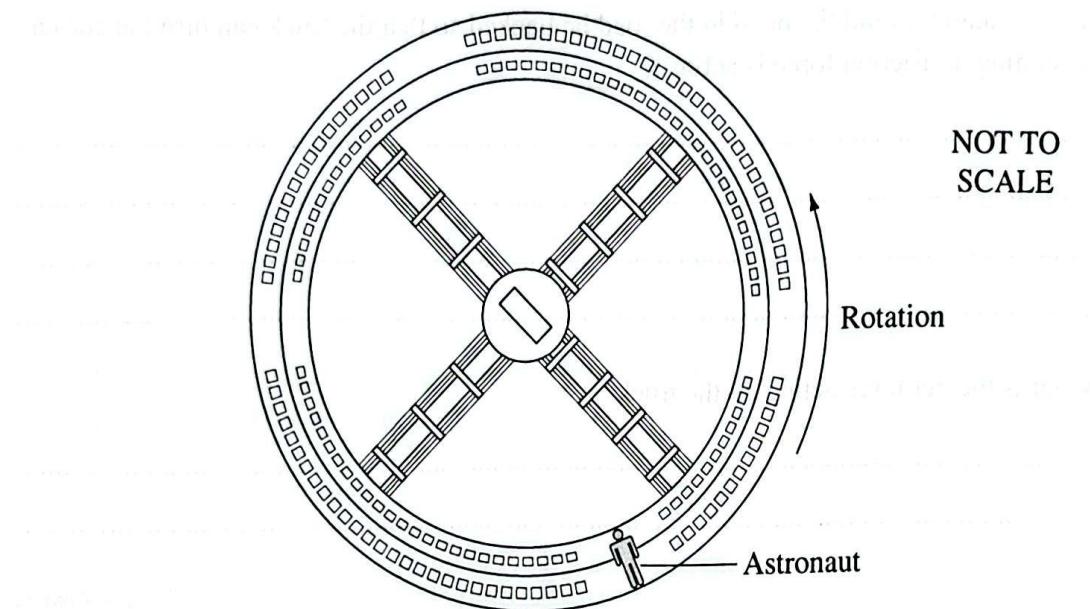
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$2 + 2 = 4 \text{ marks}$

31. The Wild Mouse at Luna Park Sydney is a rollercoaster ride. A roller coaster car travels through the bottom of a dip of radius 8.0 metres at a speed of 12 m s^{-1} .
- (a) At the bottom of the dip, what is the net force acting on a passenger with a mass of 64 kg?
-
- (b) At the bottom of the dip, what is the normal reaction force on the passenger by the seat?
-
.....
- (c) Compare the size of the reaction force acting on the passenger to the weight force when at the bottom of the dip.
-
.....

$2 + 2 + 2 = 6 \text{ marks}$

- 32.** The diagram shows a futuristic space station designed to stimulate gravity in a weightless environment.



- (a) Explain how rotating the space station simulates gravity for the astronaut.

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- (b) Calculate the rotational speed that a space station with a diameter of 550 m would need for astronauts to experience 1 g of acceleration.

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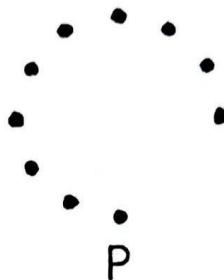
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2013 HSC Q30 – 2 + 2 = 4 marks

33. The diagram below represents a stroboscopic photograph of a body moving anticlockwise around a circular path.



The strobe frequency was set at 1.0 Hz.

- (a) What indicates that the motion is uniform?

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.....

- (b) Calculate the angular velocity of the object, in rad s^{-1} .

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- (c) At the point marked P, the stroboscope frequency instantaneously changes to 2.0 Hz. On the diagram above, show by crosses (x) the next 3 recorded positions of the object.

- (d) What is now the angular velocity of the object?

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.....

1985 HSC Q20 – 2 + 2 + 2 + 2 = 8 marks

34. A plane is flying in a circular path of radius 2000 m with a constant speed of 40 m s^{-1} .

- (a) How long will it take the plane to fly one complete circle? [Use $\pi = 3.14$]

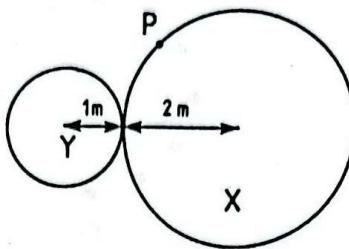
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- (b) What is the *magnitude* and *direction* of the acceleration of the plane?

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1983 HSC Q13 – 2 + 2 = 4 marks

35. Two wheels are arranged as shown in the diagram below. The wheels are touching and rotate without slipping, so that the points in contact have the same linear velocity.



The wheel X has a radius of 2 m, the wheel Y has a radius of 1 m.

The wheel Y is rotating at 10 revolutions per second.

If P is a point on the rim of wheel X, calculate:

- (a) the linear speed of point P.

.....
.....

- (b) the magnitude and direction of the acceleration of point P.

.....
.....

- (c) the magnitude of the angular velocity of point P.

.....
.....

1984 HSC Q20 – 2 + 2 + 2 = 6 marks

36. On a ride at Luna Park Sydney, the riders move in a circle of radius 6.9 m at a speed of 10 m s^{-1} . This ride has uniform circular motion.

- (a) At what angular speed do the riders move?

.....

- (b) Once the ride is operating at maximum speed, what is the angular displacement of a rider over a 2 minute period, in radians?

.....
.....

- (c) Calculate the period for this ride and how many revolutions the riders will complete in 2 minutes.

.....
.....

2 + 2 + 2 = 6 marks

5.2 Circular Motion

Multiple choice: 1 mark each

1. B 2. B 3. D 4. C 5. C 6. A 7. B 8. D
9. D 10. A 11. A 12. B 13. C 14. C 15. B 16. A
17. D 18. C 19. B 20. D 21. A 22. B

Explanations:

1. **B** The mass is undergoing circular motion in a horizontal plane. (The reaction force of the table upwards balances the gravitational force downwards.) It initially has a force acting on it inwards towards the centre. When the string breaks, the centripetal force is no longer acting on the mass, so it continues its motion from point *X* in a straight line towards *B* (Newton's First Law). So (B) is the answer.
2. **B** Between points A and B, the car has varying velocity, but constant speed $v \text{ m s}^{-1}$ and is undergoing uniform circular motion. So, during this period, the acceleration has a constant magnitude that is always directed towards the centre of the semicircle. Acceleration can be given as: $a = \frac{\Delta v}{t} = \frac{v - (-v)}{t} = \frac{2v}{t}$ west as in (B). The acceleration is also given by: $a = \frac{v^2}{r}$ towards the west, not towards the east. So (A) is incorrect.
3. **D** As the car travels around the bend, its velocity is continuously changing in direction, so the velocity is changing. Its acceleration is always perpendicular to its velocity towards the centre of the curve, so it is also changing. Only (D) has both changing and so is the answer.
4. **C** 'Uniform' circular motion means that speed is constant, so (A) and (B) are incorrect. Since the object is constantly changing its position relative to the centre of the circle, the direction (but not the magnitude) of both its displacement and its velocity are continually changing as in (C), but not in (D).
5. **C** A satellite orbiting a planet at constant speed ($v = \frac{2\pi r}{T}$) will travel at a fixed radius from the planet, so (B) is the answer. The satellite will be accelerating towards the centre of the planet, so (A) is incorrect. The gravitational force of the planet supplies the necessary energy, so (C) is incorrect. The centripetal force is supplied by gravity rather than balanced by it, so (D) is incorrect.

6. A To maintain any circular motion, an INWARDS force is always needed. For uniform circular motion, the force is continually towards the centre, called centripetal force. The direction of the velocity at any instant is the direction of the motion at that instant – that is, a tangent to the circle. Only answer (A) shows both the force and velocity correctly.

7. B $F = \frac{mv^2}{r} = \frac{7.2 \times 10^{22} \times (1 \times 10^3)^2}{3.8 \times 10^8} = 1.89 \times 10^{20} \approx 1.9 \times 10^{20}$... as in (B).

8. D $F = \frac{mv^2}{r} = \frac{0.200 \times \left(\frac{2\pi r \times 5}{3}\right)^2}{1} = \frac{0.200 \times \left(\frac{10\pi}{3}\right)^2}{1} = 0.200 \times \frac{100}{9} \times \pi^2 = 21.9 \text{ N}$

So 20 N is the best alternative, as in (D).

9. D The centripetal acceleration of both the bucket and the water are the same and greater than g , as in (D). Gravity acts constantly causing the weight force, so (A) and (B) are incorrect. Centrifugal force is an imaginary force due to inertia of the water, so (C) is incorrect.

10. A To move in a circle at constant speed, the car is continually accelerated towards the centre of the circular path, so (A) is an incorrect statement. In each half of the revolution, the velocity is equal in magnitude, but opposite in direction to the opposite point, so the average is zero, as in (B). The car returns to where it started, so displacement is zero, as in (C). The car is moving with constant speed so there is no change in its speed, as in (D). So (A) is the only incorrect statement.

11. A The velocity of the object is in a direction tangential to the circular track. So at the point shown, it is in a direction vertically up the page as shown in all four alternatives. The acceleration is towards the centre of the circular track, so is to the left of the page. Only (A) has this and so is the answer.

12. B The rails of the railway track must be providing:
- an upwards reaction force equal in magnitude to the weight of the carriage, and
 - a horizontal centripetal force to turn the carriage around the curve.
- The sum of these two components is given in (B).

13. C Torque, $\tau = r \perp F$

In A, $\tau = 0 \perp 1000 = 0 \text{ Nm}$ (force is acting through the centre), in B, $\tau = 1 \perp 250 = 250 \text{ Nm}$, in C, $\tau = 2 \perp 100 = 400 \text{ Nm}$, while in D, $\tau = 3 \perp 100 = 300 \text{ Nm}$. Hence (C) is the greatest and so is the answer.

14. C $F = mg = \frac{mv^2}{r} \quad \therefore v^2 = gr = 9.8 \times 3.6 = 35.28$

So $v = \sqrt{35.28} = 5.9 \text{ m s}^{-1}$... as in (C).

15. **B** Just before the car enters the roundabout and again after it leaves the roundabout, the car is travelling in a straight line with constant speed, so its acceleration is zero. As the car enters the roundabout, it changes direction briefly and accelerates towards X' . Travelling around the roundabout, a constant centripetal force causes acceleration towards the centre of the roundabout. This is positive towards X . As it leaves the roundabout, the car changes direction briefly and accelerates towards X' . The acceleration towards X' is in the opposite direction to X and so appears on the graph as negative acceleration. Only (B) shows this whole sequence correctly.

16. **A**
$$a = \frac{v^2}{r} = \frac{(8.1 \times 10^3)^2}{7.0 \times 10^6} = 9.37 \approx 9.4 \text{ m s}^{-1}$$
 ... as in (A).

17. **D** There are only two applied forces acting on the pendulum bob ... the weight force acting vertically downwards, and the tension in the string acting upwards at angle θ to the vertical, as shown in (D). The resultant of these two forces is the centripetal force causing the circular motion. (A) incorrectly shows the resultant force and not the tension. (B) incorrectly shows the resultant force as well as the applied forces. (C) incorrectly shows an additional force outwards.
18. **C** In most forms of motion, both displacement and distance travelled will increase together. However, in circular motion, while the distance travelled increases, the displacement from the centre of motion remains constant. So (C) is the answer.

19. **B** The merry-go-round rotates as a solid object, so all points on it have the same angular velocity, ω . \therefore the ratio, $\omega_X : \omega_Y = 1:1$... as in (A) or (B). Linear velocity, $v = \omega r$. Since $r_X = 2 \text{ m}$ and $r_Y = 4 \text{ m}$, then $v_X:v_Y = 1:2$... as in (B).

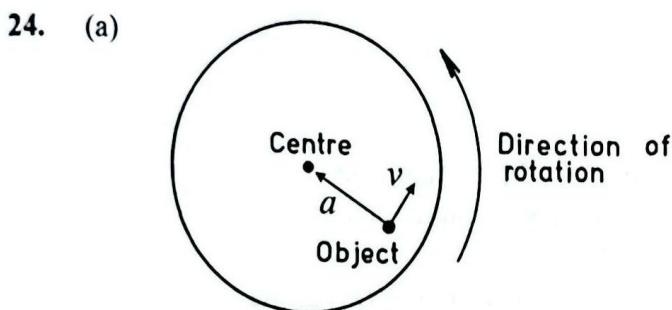
20. **D** (A), (B) and (C) all correctly describe how the work being done on an object undergoing circular motion in a horizontal plane is zero. (D) is incorrect as it claims work is done on the object and gives the wrong direction of the displacement.

21. **A** $F_c = \frac{mv^2}{r}$ and is perpendicular to the v towards the centre of the ride, as in (A).

22. **B** Speed, kinetic energy and potential energy are all scalar quantities. So they remain constant when an object is undergoing uniform circular motion, as in (B).

Short-answer questions

23. $a_c = \frac{v^2}{r}$
- $$\therefore 1.5 \text{ m s}^{-2} = \frac{v^2}{1.5 \times 10^6}$$
- $$v = \sqrt{1.5 \times 1.5 \times 10^6} = 1.5 \times 10^3 \text{ m s}^{-1}$$
- \therefore orbital speed is $1.5 \times 10^3 \text{ m s}^{-1}$
- [Note: The only force acting on the spaceship is gravity.
So, a_c is gravitational acceleration.]



(b) $\omega = 1.5 \text{ rev s}^{-1} = 1.5 \times 2\pi$ [Remember: 2π radians = 360°]
 $= 3\pi \text{ rad s}^{-1}$
 \therefore angular velocity, $\omega = 9.4 \text{ rad s}^{-1}$

25. (a)
-
- [Note: Only two forces act: weight downwards, mg , and the tension (F_T) in the string.
The resultant of these two forces, F_c produces the centripetal acceleration giving uniform circular motion.]

(b) $\tan 30^\circ = \frac{ma_c}{mg} = \frac{a_c}{g}$ [Note: Other methods could be used.]
 $a_c = g \tan 30^\circ = 9.8 \tan 30^\circ = 5.658$
 \therefore acceleration of ball, $a_c = 5.7 \text{ m s}^{-2}$ (towards centre of circle)

(c) $\cos 30^\circ = \frac{mg}{F_T}$
 \therefore tension in string, $F_T = \frac{9.8 \text{ m}}{\cos 30^\circ} = (11.3 \text{ m}) \text{ Newtons}$

26. (a) 1 revolution takes 12 s, so $T = 12$ s
- (b) Linear velocity, $v = \frac{2\pi r}{T} = \frac{2\pi 4}{12} = 2.1 \text{ m s}^{-1}$
- (c) Angular velocity, $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = 0.52 \text{ rad s}^{-1}$
- (d) $F_c = \frac{mv^2}{r} = 54.8 \text{ N}$
- (e) (i) $3 \text{ s} = 0.25 \times T$ Angular displacement $= \frac{2\pi}{4} = 0.5\pi = 1.6 \text{ radians}$
(ii) $1.6 \text{ radians} = 90^\circ$
27. The statement is incorrect. The car is travelling in uniform circular motion as it travels around the roundabout. The car is continually changing its direction so according to Newton's First Law a net horizontal force must be acting on the car.
For circular motion, this net horizontal force is a centripetal force (F_c) acting towards the centre of the roundabout, which provides the car's centripetal acceleration, a_c . This is given by the equation:
- $$F_c = ma_c = \frac{mv^2}{r} = \frac{800 \times 7.5^2}{16} = 2812.5 \text{ N} \approx 2813 \text{ N}$$
- The frictional force between the tyres and the road provides this horizontal centripetal force. So it is not a balanced force, as stated in the quote.
28. (a) Acceleration $= 9.8 \text{ m s}^{-2}$ towards centre of circle
- (b) $a_c = \frac{v^2}{r} \quad 9.8 \text{ m s}^{-2} = \frac{v^2}{5}$
 $\therefore v = 7 \text{ m s}^{-1}$
29. (a) (i) It changes – the Carousel has uniform circular motion, so although the magnitude of v is constant, \bar{v} keeps changing in direction.
(ii) a_c is constant (as $a_c = \frac{v^2}{r}$) – but its direction changes as a_c is always perpendicular to \bar{v} towards the centre of the ride.
- (b) Every point on the Carousel moves through the same angle at the same rate and so every point on the ride has the same angular velocity (ω). Hence ω is the same for both the inner and outer horses.

30. (a) $mg \tan \theta = \frac{mv^2}{r}$ $m = 3500 \text{ kg}$ $v = 75 \text{ km h}^{-1} = 20.8 \text{ m s}^{-1}$

$$3500 \times 9.8 \times \tan \theta = 3500 \times \frac{20.8^2}{300}$$

$$\tan \theta = \frac{3500}{3500 \times 9.8} \times \frac{20.8^2}{300} = 0.1472$$

\therefore angle of banking, $\theta = 8.4^\circ$

(b) $F_{net} = mg \tan \theta$
 $= 3500 \times 9.8 \times \tan 8.4^\circ = 3500 \times 9.8 \times 0.1472$
 $\therefore F_{net} = 5048 \text{ N}$

31. (a) $F_{net} = F_c = \frac{mv^2}{r} = \frac{64 \times 12^2}{8.0} = 1152 \text{ N}$

(b) $F_R = F_{net} + mg = 1152 + (64 \times 9.8)$
 $\therefore F_R \approx 1779 \text{ N}$

(c) $F_R = 1779 \text{ N}$, whereas $F_W = mg = 64 \times 9.8 \approx 627 \text{ N}$.
 $\therefore F_R$ is approximately $2.8 \times$ greater than F_W .

32. (a) When a space station rotates, a centripetal force acts towards the centre of rotation. The reaction force to this centripetal force acts outwards to the outer wall of the space station. This simulates gravity for the astronaut, as they feel the force towards the centre acting through their feet.

(b) $F = \frac{mv^2}{r}$

Substituting, $1 \times 9.8 = \frac{1 \times v^2}{225}$

$$\therefore v = \sqrt{9.8 \times 225} = 51.9 \text{ m s}^{-1}$$

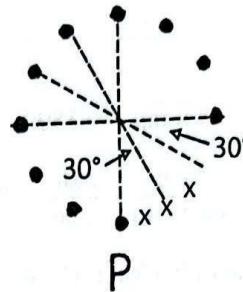
33. (a) Each strobe image is an equal distance apart and the time of strobe flashes is constant. So angular velocity (ω) is constant. Hence the motion is uniform.

(b) $f = 1.0 \text{ Hz}$ $\therefore T = \frac{1}{f} = 1 \text{ flash/second}$

From diagram, $\theta = 30^\circ = \frac{2\pi}{12} = \frac{\pi}{6}$

$$\therefore \omega = \frac{\theta}{t} = \frac{\pi}{6} = 0.52 \text{ rad s}^{-1}$$

- (c) $f = 2.0 \text{ Hz} \quad \therefore T = \frac{1}{f} = \frac{1}{2} \text{ second}$
 So from P, the strobe shows 2x more images than before.



- (d) Changing the stroboscope frequency does not affect the angular velocity.
 So ω remains constant at 0.52 rad s^{-1} .

34. (a) Circumference of circle, $c = 2\pi r \quad v = \frac{2\pi r}{t}$

$$\therefore t = \frac{2\pi r}{v} = \frac{2\pi \times 2000}{40} = 314 \text{ s}$$

(b) $a_c = \frac{v^2}{r} = \frac{40^2}{2000}$

$\therefore a_c = 0.8 \text{ m s}^{-2}$ towards centre of circle

35. (a) Linear speed of P = linear speed of Y

$$v_Y = \frac{2\pi}{T} r = \frac{2\pi}{0.1} \times 1 = 20\pi \approx 63 \text{ m s}^{-1}$$

\therefore Linear speed of P is $20\pi \text{ m s}^{-1} \approx 63 \text{ m s}^{-1}$

(b) At P, $a_c = \frac{v^2}{r} = \frac{(20\pi)^2}{2} = 200\pi^2 \approx 1974 \text{ m s}^{-2}$ towards the centre of Wheel X

(c) $v = \frac{2\pi r}{T} = \omega r \quad \text{So } \omega = \frac{v}{r}$

$$\therefore \text{at P: angular velocity, } \omega = \frac{v}{r} = \frac{20\pi}{2} = 10\pi \text{ rad s}^{-1} \approx 31.4 \text{ rad s}^{-1}$$

An alternative method:

Wheel Y is rotating at 10 rev s^{-1} , so $\omega_Y = 10 \times 2\pi = 20\pi \text{ rad s}^{-1}$

\therefore point P on wheel X has $\omega_X = 10\pi \text{ rad s}^{-1} \approx 31.4 \text{ rad s}^{-1}$

[as $X_{radius} = 2 \times Y_{radius} \dots$ so X has 2x circumference of Y]

36. (a) Angular speed, $\omega = \frac{v}{r} = \frac{10}{6.9} = 1.45 \text{ rad s}^{-1}$

(b) $\omega = \frac{\Delta\theta}{t} \quad \Delta\theta = \omega t = 1.45 \times 120 = 174 \text{ rad}$

\therefore angular displacement is 174 rad

(c) $v = \frac{2\pi r}{T} \quad \text{So, } 10 = \frac{2\pi r}{T}$

$$\therefore \text{Period, } T = \frac{2\pi r}{10} = 4.35 \text{ s}$$