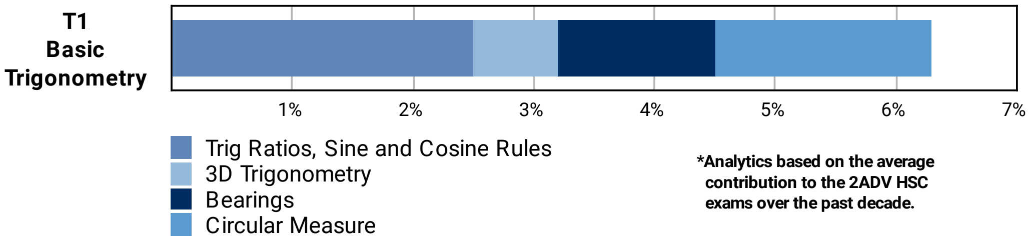
ADV: Trigonometry (Adv), T1 Trigonometry and Measure of Angles (Adv)

Circular Measure (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 82.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

T1 Trigonometry and Measure of Angles



HISTORICAL CONTRIBUTION

- *T1 Trigonometry and Measure of Angles* is a mixture of content that previously belonged to the Standard 2, Mathematics and Ext1 courses. Our analysis has it accounting for an estimated 6.3% of past papers.
- This topic has been split into four sub-topics for analysis purposes: *1-Trig Ratios, Sine and Cosine Rules (2.5%), 2- 3D Trigonometry (0.7%), 3-Bearings (1.3%) and 4-Circular Measure (1.8%).*
- This analysis looks at the sub-topic *Circular Measure*.

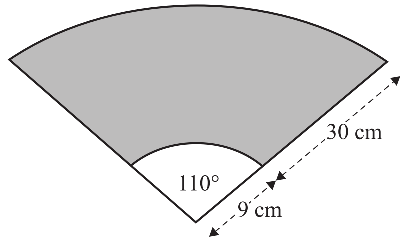
HSC ANALYSIS - What to expect and common pitfalls

- *Circular Measure (1.8%)* has been examined in 7 of the last 9 HSC exams, including both exams of the new Advanced syllabus. Worth noting that the 2020 exam allocated a significant 4-marks to this topic.
- Circular measure questions have been well answered in recent years. We do flag this warning: students are encouraged to carefully revise questions in the period 2005-2010 which represent the highest levels of difficulty in this topic area, producing sub-50% mean marks in 5 of the 6 years.

Questions

1. Trigonometry, 2ADV T1 SM-Bank 1 MC

A windscreen wiper blade can clean a large area of windscreen glass, as shown by the shaded area in the diagram below.



The windscreen wiper blade is 30 cm long and it is attached to a 9 cm long arm.

The arm and blade move back and forth in a circular arc with an angle of 110° at the centre.

The area cleaned by this blade, in square centimetres, is closest to

A. 786

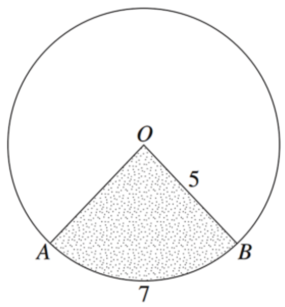
B. 1382

C. 2573

D. 4524

2. Trigonometry, 2ADV T1 2016 HSC 7 MC

The circle centred at O has radius 5. Arc AB has length 7 as shown in the diagram.



What is the area of the shaded sector OAB?

(A) $\dfrac{35}{2}$

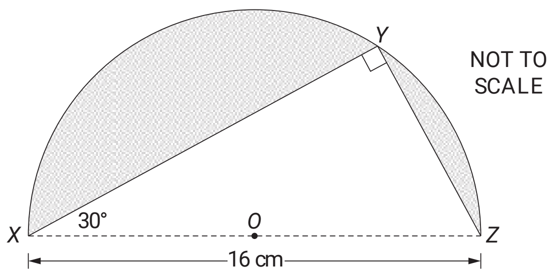
(B) $\dfrac{35}{2}\pi$

(C) $\dfrac{125}{14}$

(D) $\dfrac{125}{14}\pi$

3. Trigonometry, 2ADV T1 2021 HSC 12

A right-angled triangle XYZ is cut out from a semicircle with centre O. The length of the diameter XZ is 16 cm and $\angle YXZ$ = 30°, as shown on the diagram.

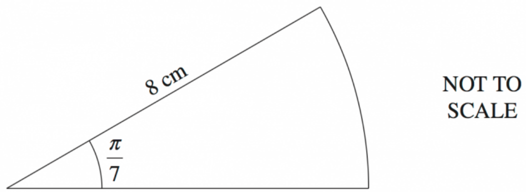


a. Find the length of XY in centimetres, correct to two decimal places. ***(2 marks)***

b. Hence, find the area of the shaded region in square centimetres, correct to one decimal place. ***(3 marks)***

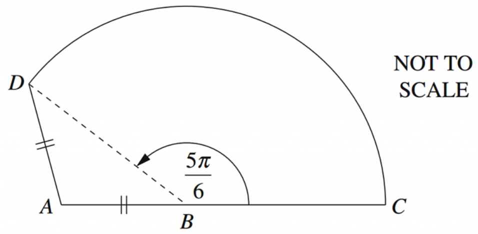
4. Trigonometry, 2ADV T1 2014 HSC 11g

The angle of a sector in a circle of radius 8 cm is $\dfrac{\pi}{7}$ radians, as shown in the diagram.



Find the exact value of the perimeter of the sector. ***(2 marks)***

5. Trigonometry, 2ADV T1 2006 HSC 4a



In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \dfrac{5\pi}{6}$

The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

Copy or trace the diagram into your writing booklet.

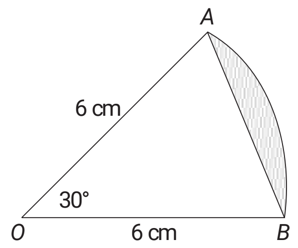
i. Show that $\angle DAB = \dfrac{2\pi}{3}$. ***(1 mark)***

ii. Find the length of BD. ***(2 marks)***

iii. Find the area of the garden $ABCD$. ***(2 marks)***

6. Trigonometry, 2ADV T1 2017 HSC 11e

In the diagram, OAB is a sector of the circle with centre O and radius 6 cm, where $\angle AOB = 30°$.



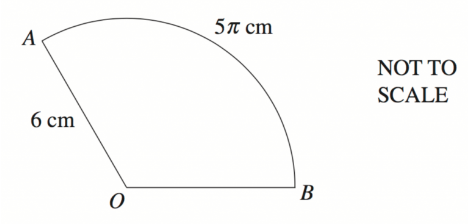
NOT TO SCALE

i. Find the exact value of the area of the triangle OAB. ***(1 mark)***

ii. Find the exact value of the area of the shaded segment. ***(1 mark)***

---

7. Trigonometry, 2ADV T1 2004 HSC 4a



NOT TO SCALE

AOB is a sector of a circle, centre O and radius 6 cm.

The length of the arc AB is 5π cm.

Calculate the exact area of the sector AOB. ***(2 marks)***

---

8. Trigonometry, 2ADV T1 2012 HSC 11f

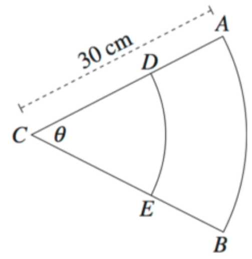
The area of the sector of a circle with a radius of 6 cm is 50 cm².

Find the length of the arc of the sector. ***(2 marks)***

---

9. Trigonometry, 2ADV T1 2013 HSC 13c

The region ABC is a sector of a circle with radius 30 cm, centred at C. The angle of the sector is θ. The arc DE lies on a circle also centred at C, as shown in the diagram.



The arc DE divides the sector ABC into two regions of equal area.

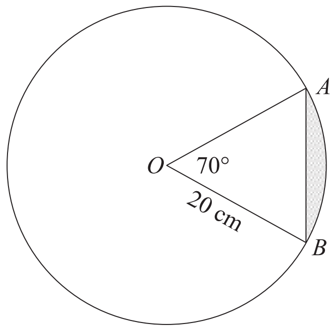
Find the exact length of the interval CD. ***(2 marks)***

---

10. Trigonometry, 2ADV T1 2019 HSC 13b

The diagram shows a circle with centre O and radius 20 cm.

The points A and B lie on the circle such that $\angle AOB = 70°$.



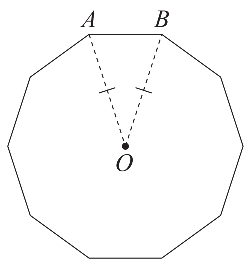
NOT TO SCALE

Find the perimeter of the shaded segment, giving your answer correct to one decimal place. ***(3 marks)***

---

11. Trigonometry, 2ADV T1 2020 HSC 22

The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal). The decagon has centre O.

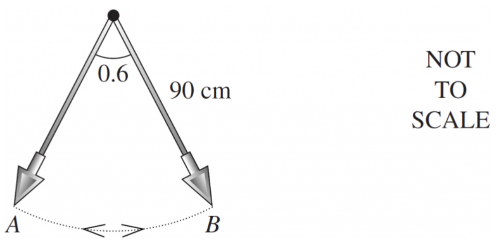


The perimeter of the shape is 80 cm.

By considering triangle OAB, calculate the area of the ten-sided shape. Give your answer in square centimetres correct to one decimal place. ***(4 marks)***

---

12. Trigonometry, 2ADV T1 2005 HSC 4a



A pendulum is 90 cm long and swings through an angle of 0.6 radians. The extreme positions of the pendulum are indicated by the points A and B in the diagram.

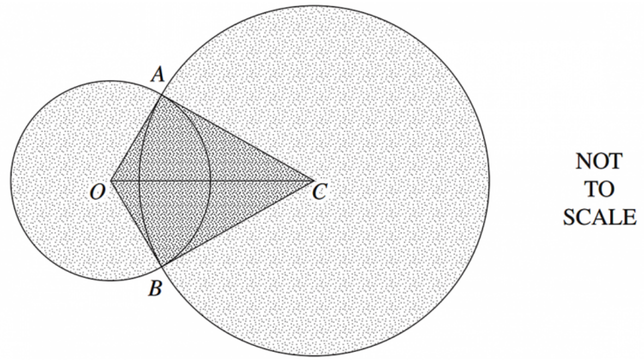
i. Find the length of the arc AB. ***(1 mark)***

ii. Find the straight-line distance between the extreme positions of the pendulum. ***(2 marks)***

iii. Find the area of the sector swept out by the pendulum. ***(1 mark)***

---

13. Trigonometry, 2ADV T1 2007 HSC 4c



An advertising logo is formed from two circles, which intersect as shown in the diagram.

The circles intersect at A and B and have centres at O and C.

The radius of the circle centred at O is 1 metre and the radius of the circle centred at C is $\sqrt{3}$ metres. The length of OC is 2 metres.

i. Use Pythagoras' theorem to show that $\angle OAC = \dfrac{\pi}{2}$. ***(1 mark)***

ii. Find $\angle ACO$ and $\angle AOC$. ***(2 marks)***

iii. Find the area of the quadrilateral $AOBC$. ***(1 mark)***

iv. Find the area of the major sector ACB. ***(1 mark)***

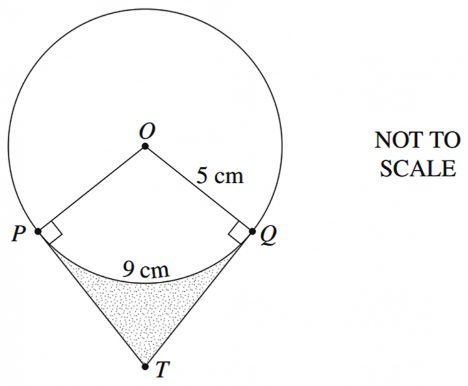
v. Find the total area of the logo (the sum of all the shaded areas). ***(2 marks)***

---

14. Trigonometry, 2ADV T1 2010 HSC 6b

The diagram shows a circle with centre O and radius 5 cm.

The length of the arc PQ is 9 cm. Lines drawn perpendicular to OP and OQ at P and Q respectively meet at T.



i. Find $\angle POQ$ in radians. ***(1 mark)***

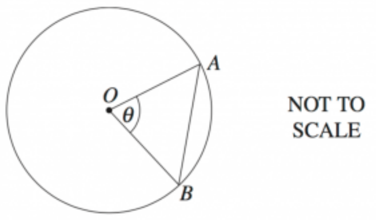
ii. Prove that ΔOPT is congruent to ΔOQT. ***(2 marks)***

iii. Find the length of PT. ***(1 mark)***

iv. Find the area of the shaded region. ***(2 marks)***

15. Trigonometry, 2ADV T1 2009 HSC 5c

The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



i. There are two possible values of θ for which the area of ΔAOB is $\sqrt{3}$ square centimetres. One value is $\dfrac{\pi}{3}$.

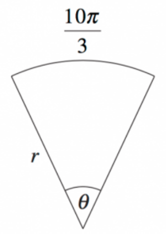
Find the other value. ***(2 marks)***

ii. Suppose that $\theta = \dfrac{\pi}{3}$.

(1) Find the area of sector AOB ***(1 mark)***

(2) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. ***(2 marks)***

16. Trigonometry, 2ADV T1 2008 HSC 7b



The diagram shows a sector with radius r and angle θ where $0 < \theta \leq 2\pi$.

The arc length is $\dfrac{10\pi}{3}$.

i. Show that $r \geq \dfrac{5}{3}$. ***(2 marks)***

ii. Calculate the area of the sector when $r = 4$. ***(2 marks)***

Copyright © 2004-21 The State of New South Wales (Board of Studies, Teaching and Educational Standards NSW)

Worked Solutions

1. Trigonometry, 2ADV T1 SM-Bank 1 MC

$$\begin{aligned}\text{Area cleaned} &= \text{large sector} - \text{small sector}\\ &= \frac{110}{360}\times\pi\times 39^2 \ - \frac{110}{360}\times\pi\times 9^2\\ &= 1382.3\ldots\ \text{cm}^2\end{aligned}$$

$\Rightarrow B$

---

2. Trigonometry, 2ADV T1 2016 HSC 7 MC

$$\begin{aligned}\text{Arc} &= \frac{\theta}{2\pi}\times 2\pi r = r\theta\\ 7 &= 5\theta\\ \therefore \theta &= \frac{7}{5}\end{aligned}$$

Area of shaded sector

$$\begin{aligned}&= \frac{1}{2}\theta r^2\\ &= \frac{1}{2}\times\frac{7}{5}\times 5^2\\ &= \frac{35}{2}\ \text{u}^2\end{aligned}$$

$\Rightarrow A$

Worked Solutions

3. Trigonometry, 2ADV T1 2021 HSC 12

a. $$\begin{aligned}\cos 30^\circ &= \frac{XY}{16}\\ XY &= 16\cos 30^\circ\\ &= 13.8564\\ &= 13.86\ \text{cm (2 d.p.)}\end{aligned}$$

b. $$\begin{aligned}\text{Area of semi-circle} &= \frac{1}{2}\times\pi r^2\\ &= \frac{1}{2}\pi\times 8^2\\ &= 100.531\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of }\Delta XYZ &= \frac{1}{2}ab\sin C\\ &= \frac{1}{2}\times 16\times 13.856\times\sin 30^\circ\\ &= 55.42\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Shaded Area} &= 100.531 - 55.42\\ &= 45.111\\ &= 45.1\ \text{cm}^2\ \text{(1 d.p.)}\end{aligned}$$

---

4. Trigonometry, 2ADV T1 2014 HSC 11g

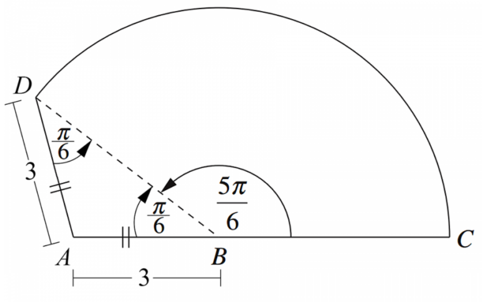
$$\begin{aligned}\text{Arc length} &= \frac{\theta}{2\pi}\times 2\pi r\\ &= \frac{\pi}{7}\times 8\\ &= \frac{8\pi}{7}\ \text{cm}\end{aligned}$$

$$\begin{aligned}\text{Sector perimeter} &= \text{arc} + 2\times\text{radius}\\ &= \frac{8\pi}{7} + 16\ \text{cm}\end{aligned}$$

---

5. Trigonometry, 2ADV T1 2006 HSC 4a

i.



Show $\angle DAB = \dfrac{2\pi}{3}$

$$\angle DBA = \pi - \frac{5\pi}{6} \quad (\pi \text{ radians in straight angle } ABC)$$

$$= \frac{\pi}{6} \text{ radians}$$

$$\therefore \angle BDA = \frac{\pi}{6} \text{ radians} \quad (\text{base angles of isosceles } \Delta ADB)$$

$$\therefore \angle DAB = \pi - \left(\frac{\pi}{6} + \frac{\pi}{6}\right) \quad (\text{angle sum of } \Delta ADB)$$

$$= \frac{2\pi}{3} \text{ radians} \ldots \text{as required}$$

ii. Using the cosine rule:

$$BD^2 = AD^2 + AB^2 - 2 \times AD \times AB \times \cos\frac{2\pi}{3}$$

$$= 9 + 9 - (2 \times 3 \times 3 \times \ -0.5)$$

$$= 27$$

$$\therefore BD = \sqrt{27}$$

$$= 3\sqrt{3} \text{ m}$$

iii. Area of $\Delta ADB = \dfrac{1}{2}ab\sin C$

$$= \frac{1}{2} \times 3 \times 3 \times \sin\frac{2\pi}{3}$$

$$= \frac{9}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{9\sqrt{3}}{4} \text{ m}^2$$

Area of sector BCD

$$= \frac{\frac{5\pi}{6}}{2\pi} \times \pi r^2$$

$$= \frac{5\pi}{12} \times \left(3\sqrt{3}\right)^2$$

$$= \frac{45\pi}{4} \text{ m}^2$$

$\therefore$ Area of garden $ABCD$

$$= \frac{9\sqrt{3}}{4} + \frac{45\pi}{4}$$

$$= \frac{9\sqrt{3} + 45\pi}{4} \text{ m}^2$$

---

6. Trigonometry, 2ADV T1 2017 HSC 11e

i. Area $\Delta OAB = \dfrac{1}{2}ab\sin C$

$$= \frac{1}{2} \times 6^2 \times \sin 30^\circ$$

$$= 9 \text{ cm}^2$$

ii. Area segment $=$ Area sector $-$ Area ΔOAB

$$= \frac{30}{360} \times \pi \times 6^2 - 9$$

$$= 3\pi - 9 \text{ cm}^2$$

---

7. Trigonometry, 2ADV T1 2004 HSC 4a

$$\text{Arc length} = \frac{\theta}{2\pi} \times 2\pi r = r\theta$$
$$5\pi = 6\theta$$
$$\therefore\ \theta = \frac{5\pi}{6} \text{ radians}$$

Area of sector AOB

$$= \frac{\theta}{2\pi} \times \pi r^2$$
$$= \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 6^2 \times \frac{5\pi}{6}$$
$$= 15\pi \text{ cm}^2$$

8. Trigonometry, 2ADV T1 2012 HSC 11f

Area of sector, radius 6 cm $= 50$ cm²

$$\frac{\theta}{2\pi} \times \pi r^2 = 50$$
$$\frac{1}{2} r^2 \theta = 50$$
$$\frac{1}{2} \times 6^2 \times \theta = 50$$
$$\theta = \frac{50}{18} = \frac{25}{9} \text{ radians}$$

TIP: Many students find it easier to think of the area of a sector by calculating $\frac{\theta}{2\pi}$ multiplied by the area of a circle rather than remembering a formula.

$$\therefore \text{ Length of Arc} = \frac{\theta}{2\pi} \times 2\pi r$$
$$= \theta \times r$$
$$= \frac{25}{9} \times 6$$
$$= \frac{50}{3} \text{ cm}$$

9. Trigonometry, 2ADV T1 2013 HSC 13c

$$\text{Area of sector } ABC = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 30^2 \times \theta$$
$$= 450\theta$$

Let $CD = x$

$$\text{Area of sector } CDE = \frac{1}{2} x^2 \theta$$

Since DE divides sector ABC in half,

$$\text{Area sector } CDE = \frac{1}{2} \times \text{Area sector } ABC$$
$$\frac{1}{2} x^2 \theta = \frac{1}{2} \times 450\theta$$
$$x^2 = 450$$
$$x = \sqrt{450} \qquad (x > 0)$$
$$= 15\sqrt{2} \text{ cm}$$

$\therefore$ The exact length of interval CD is $15\sqrt{2}$ cm.

MARKER'S COMMENT: Simply finding the area of sector ABC achieved half marks in this challenging question! ***Show your working.***

10. Trigonometry, 2ADV T1 2019 HSC 13b

$$\text{Arc } AB = \frac{70}{360} \times 2\pi \times 20$$
$$= 24.43\ldots$$

Using cosine rule,

$$AB^2 = 20^2 \times 20^2 - 2 \cdot 20 \cdot 20 \times \cos 70$$
$$= 526.383\ldots$$
$$AB = 22.94\ldots$$

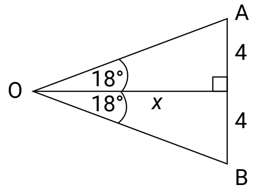
$$\therefore \text{ Perimeter} = 24.43 + 22.94\ldots$$
$$= 47.37$$
$$= 47.4 \text{ cm (1 d.p.)}$$

11. Trigonometry, 2ADV T1 2020 HSC 22

$$\angle AOB = \frac{360}{10} = 36^\circ$$

$$AB = \frac{80}{10} = 8 \text{ cm}$$

ΔAOB is made up of 2 identical right-angled triangles



$$\tan 18^\circ = \frac{4}{x}$$

$$x = \frac{4}{\tan 18^\circ}$$

$$\therefore \text{ Area of decagon} = 20 \times \frac{1}{2} \times \frac{4}{\tan 18^\circ} \times 4$$

$$= 492.429\ldots$$

$$= 492.4 \text{ cm}^2 \text{ (to 1 d.p.)}$$

---

12. Trigonometry, 2ADV T1 2005 HSC 4a

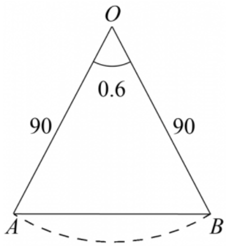
i. $\text{Arc } AB = \dfrac{\theta}{2\pi} \times 2\pi r$

$$= r\theta$$

$$= 90 \times 0.6$$

$$= 54 \text{ cm}$$

ii.



Using the cosine rule

Distance AB in straight line

$$AB^2 = 90^2 \times 90^2 - 2 \times 90 \times 90 \times \cos 0.6$$

$$= 2829.563\ldots$$

$$\therefore AB = 53.193\ldots$$

$$= 53.2 \text{ cm (to 1 d.p.)}$$

iii. Area of sector

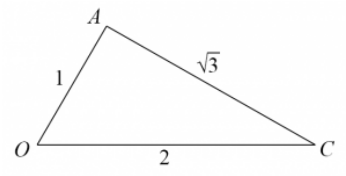
$$= \frac{0.6}{2\pi} \times \pi r^2$$

$$= 0.3 \times 90^2$$

$$= 2430 \text{ cm}^2$$

13. Trigonometry, 2ADV T1 2007 HSC 4c

i.



In ΔAOC

$$\begin{aligned} AO^2 + AC^2 &= 1^2 + \sqrt{3}^2 \\ &= 1 + 3 \\ &= 4 \\ &= OC^2 \end{aligned}$$

$$\therefore \Delta AOC \text{ is right-angled and } \angle OAC = \frac{\pi}{2}$$

ii. $\sin \angle ACO = \dfrac{1}{2}$

$$\therefore \angle ACO = \frac{\pi}{6}$$

$$\sin \angle AOC = \frac{\sqrt{3}}{2}$$

$$\therefore \angle AOC = \frac{\pi}{3}$$

iii. Area $AOBC$

$$\begin{aligned} &= 2 \times \text{Area } \Delta AOC \\ &= 2 \times \frac{1}{2} \times b \times h \\ &= 2 \times \frac{1}{2} \times 1 \times \sqrt{3} \\ &= \sqrt{3} \text{ m}^2 \end{aligned}$$

iv. $\angle ACB = \dfrac{\pi}{6} + \dfrac{\pi}{6} = \dfrac{\pi}{3}$

$$\begin{aligned} \therefore \angle ACB \text{ (reflex)} &= 2\pi - \frac{\pi}{3} \\ &= \frac{5\pi}{3} \end{aligned}$$

Area of major sector ACB

$$= \frac{\theta}{2\pi} \times \pi r^2$$
$$= \frac{\frac{5\pi}{3}}{2\pi} \times \pi\left(\sqrt{3}\right)^2$$
$$= \frac{5\pi}{6} \times 3$$
$$= \frac{5\pi}{2}\ \text{m}^2$$

v. $\angle AOB = \dfrac{\pi}{3} + \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

$$\therefore \angle AOB\ (\text{reflex}) = 2\pi - \frac{2\pi}{3}$$
$$= \frac{4\pi}{3}$$

Area of major sector AOB

$$= \frac{\frac{4\pi}{3}}{2\pi} \times \pi \times 1^2$$
$$= \frac{2\pi}{3}\ \text{m}^2$$

$\therefore$ Total area of the logo

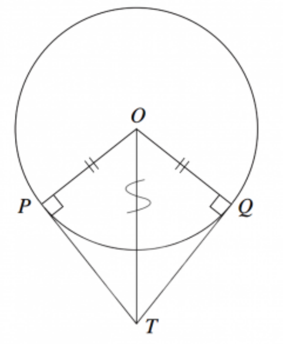
$$= \frac{5\pi}{2} + \frac{2\pi}{3} + \text{Area } AOBC$$
$$= \frac{15\pi + 4\pi}{6} + \sqrt{3}$$
$$= \left(\frac{19\pi + 6\sqrt{3}}{6}\right)\ \text{m}^2$$

14. Trigonometry, 2ADV T1 2010 HSC 6b

i. Length of Arc $= r\theta$

$$9 = 5 \times \angle POQ$$
$$\therefore\ \angle POQ = \frac{9}{5}\ \text{radians}$$

ii.



Prove $\Delta OPT \cong \Delta OQT$

OT is common

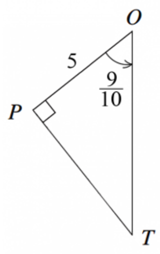
$\angle OPT = \angle OQT = 90^\circ$ (given)

$OP = OQ$ (radii)

$\therefore\ \Delta OPT \cong \Delta OQT$ (RHS)

MARKER'S COMMENT: Know the difference between the congruency proof of RHS and SAS. Incorrect identification will lose a mark.

iii.



$$\angle POT = \frac{1}{2} \times \angle POQ \quad (\text{from part (ii)})$$
$$= \frac{1}{2} \times \frac{9}{5}$$
$$= \frac{9}{10}\ \text{radians}$$

$$\tan\angle POT = \frac{PT}{OP}$$

♦♦ *Mean mark below 30%.*
MARKER'S COMMENT: Many students struggled to work in

$$\tan\left(\frac{9}{10}\right) = \frac{PT}{5}$$

$$PT = 5 \times \tan\left(\frac{9}{10}\right)$$

$$= 6.3007...$$

$$= 6.3 \text{ cm} \quad (\text{to 1 d.p.})$$

radians. Make sure you understand this concept.

iv. Shaded Area $=$ Area $OQTP$ $-$ Area sector OQP

♦ *Mean mark 35%.*

$$\text{Area } OQTP = 2 \times \text{Area } \Delta OPT$$

$$= 2 \times \frac{1}{2} \times OP \times PT$$

$$= 5 \times 6.3007$$

$$\approx 31.503...$$

$$\approx 31.5 \text{ cm}^2$$

$$\text{Area sector } OQP = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times 25 \times \frac{9}{5}$$

$$= 22.5 \text{ cm}^2$$

$$\therefore \text{ Shaded Area} = 31.503 - 22.5$$

$$= 9.003...$$

$$= 9.0 \text{ cm}^2 \quad (\text{to 1 d.p.})$$

---

15. Trigonometry, 2ADV T1 2009 HSC 5c

i. $\text{Area } \Delta AOB = \frac{1}{2}ab\sin\theta$

$$= \frac{1}{2} \times 2 \times 2 \times \sin\theta$$

$$= 2\sin\theta$$

$$2\sin\theta = \sqrt{3} \quad (\text{given})$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = \frac{\pi}{3},\ \pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3},\ \frac{2\pi}{3}$$

$$\therefore \text{ The other value of } \theta \text{ is } \frac{2\pi}{3} \text{ radians}$$

ii. (1) $\text{Area of sector } AOB = \pi r^2 \times \frac{\theta}{2\pi}$

$$= \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \text{ cm}^2$$

ii. (2) Using the cosine rule:

$$AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \times \cos\theta$$

$$= 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos\left(\frac{\pi}{3}\right)$$

$$= 4 + 4 - 4$$

$$= 4$$

$$\therefore AB = 2$$

$$\text{Arc } AB = 2\pi r \times \frac{\theta}{2\pi}$$

$$= r\theta$$

$$= \frac{2\pi}{3} \text{ cm}$$

$$\therefore \text{ Perimeter} = \left(2 + \frac{2\pi}{3}\right) \text{ cm}$$

16. Trigonometry, 2ADV T1 2008 HSC 7b

i. Show $r \geq \dfrac{5}{3}$

Arc length $= r\theta$ where $0 < \theta \leq 2\pi$

$$r\theta = \frac{10\pi}{3}$$

$$\therefore \theta = \frac{10\pi}{3r}$$

Using $0 \leq \theta \leq 2\pi$

$$0 \leq \frac{10\pi}{3r} \leq 2\pi$$

$$\frac{10\pi}{3} \leq 2\pi r$$

$$\frac{5}{3} \leq r$$

$$\therefore r \geq \frac{5}{3} \text{ ... as required.}$$

ii. $\text{Area} = \dfrac{1}{2}r^2\theta$

$$= \frac{1}{2} \times 4^2 \times \frac{10\pi}{3 \times 4}$$

$$= \frac{20\pi}{3} \text{ u}^2$$

Copyright © 2016-2022 M2 Mathematics Pty Ltd (SmarterMaths.com.au)