ADVANCED MATHEMATICS

Calculus (Adv), C1 Introduction to Differentiation (Adv)

Rates of Change (Y11)
Standard Differentiation (Y11)
Tangents (Y11)

Logs and Exponentials (Adv), E1 Logs and Exponentials (Adv)

Log/Index Laws and Equations (Y11)
Graphs and Applications (Y11)

Statistics (Adv), S1 Probability & Probability Distributions (Adv)

Discrete Probability Distributions (Y11)
Multi-Stage Events (Y11)

Trigonometry (Adv), T1 Trigonometry and Measure of Angles (Adv)

3D Trigonometry (Y11)
Circular Measure (Y11)
Bearings (Y11)

Trigonometry (Adv), T2 Trig Functions and Identities (Adv)

Trig Identities and Harder Equations (Y11)
Exact Trig Ratios (Y11)

Teacher: Cathyanne Horvat

Exam Equivalent Time: 175.5 minutes *(based on allocation of 1.5 minutes per mark)*

Questions

1. Probability, 2ADV S1 2016 MET2 7 MC

The number of pets, X, owned by each student in a large school is a random variable with the following discrete probability distribution.

x	0	1	2	3
$\Pr(X=x)$	0.5	0.25	0.2	0.05

If two students are selected at random, the probability that they own the same number of pets is

A. 0.3

B. 0.305

C. 0.355

D. 0.405

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2. Probability, 2ADV S1 2019 MET2-N 5 MC

Consider the probability distribution for the discrete random variable X shown in the table below.

x	-1	0	1	2	3
$\Pr(X=x)$	b	b	b	$\dfrac{3}{5}-b$	$\dfrac{3b}{5}$

The value of $E(X)$ is

A. $\dfrac{76}{65}$

B. 1

C. 0

D. $\dfrac{2}{13}$

E. $\dfrac{86}{65}$

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3. Probability, 2ADV S1 2019 HSC 6 MC

A game is played by tossing an ordinary 6-sided die and an ordinary coin at the same time. The game is won if the uppermost face of the die shows an even number or the uppermost face of the coin shows a tail (or both).

What is the probability of winning this game?

A. $\dfrac{1}{4}$

B. $\dfrac{1}{2}$

C. $\dfrac{3}{4}$

D. 1

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4. Probability, 2ADV S1 2009 MET2 10 MC

The discrete random variable X has a probability distribution as shown.

x	0	1	2	3
$\Pr(X=x)$	0.4	0.2	0.3	0.1

The median of X is

A. 0

B. 1

C. 1.1

D. 2

5. Statistics, 2ADV S1 2022 HSC 9 MC

Liam is playing two games. He is equally likely to win each game. The probability that Liam will win at least one of the games is 80%.

Which of the following is closest to the probability that Liam will win both games?

A. 31%

B. 40%

C. 55%

D. 64%

6. Calculus, 2ADV C1 2018 HSC 12d

The displacement of a particle moving along the x-axis is given by

$$x = \frac{t^3}{3} - 2t^2 + 3t,$$

where x is the displacement from the origin in metres and t is the time in seconds, for $t \geq 0$.

i. What is the initial velocity of the particle? ***(1 mark)***

ii. At which times is the particle stationary? ***(2 marks)***

iii. Find the position of the particle when the acceleration is zero. ***(2 marks)***

7. Trigonometry, 2ADV T2 SM-Bank 41

Prove that

$(\sec x + \tan x)(\sec x - \tan x) = 1$. ***(2 marks)***

8. Statistics, 2ADV S1 2023 HSC 12

The table shows the probability distribution of a discrete random variable.

x	0	1	2	3	4
$P(X=x)$	0	0.3	0.5	0.1	0.1

a. Show that the expected value $E(X) = 2$. ***(1 mark)***

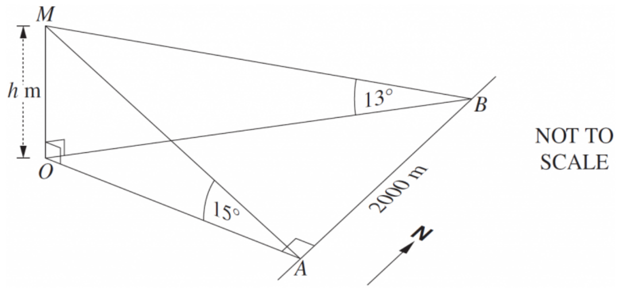
b. Calculate the standard deviation, correct to one decimal place. ***(2 marks)***

9. Trigonometry, 2ADV' T1 2015 HSC 12c

A person walks 2000 metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 15°.

From point B, the angle of elevation to the top of the mountain is 13°.



i. Show that $OA = h \cot 15^\circ$. ***(1 mark)***

ii. Hence, find the value of h. ***(2 marks)***

10. Trigonometry, 2ADV T2 SM-Bank 31

Given $\cot\theta = -\dfrac{24}{7}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, find the exact value of

i. $\sec\theta$ *(2 marks)*

ii. $\sin\theta$ *(1 mark)*

11. Calculus, 2ADV C1 2013 HSC 11b

Evaluate $\lim\limits_{x \to 2} \dfrac{(x-2)(x+2)^2}{x^2-4}$. *(2 marks)*

12. L&E, 2ADV E1 SM-Bank 7

Solve the equation $2\log_3(5) - \log_3(2) + \log_3(x) = 2$ for x. *(2 marks)*

13. L&E, 2ADV E1 SM-Bank 10

Solve the equation $2^{3x-3} = 8^{2-x}$ for x. *(2 marks)*

14. Calculus, 2ADV C1 SM-Bank 11

A particle is moving along the x-axis. Its velocity v at time t is given by

$v = \sqrt{20t - 2t^2}$ metres per second

Find the acceleration of the particle when $t = 4$.

Express your answer as an exact value in its simplest form. ***(3 marks)***

15. Calculus, 2ADV C1 SM-Bank 25

Let $g(x) = \left(2 - x^3\right)^3$.

Evaluate $g\prime(-1)$. ***(2 marks)***

16. L&E, 2ADV E1 2009 HSC 1f

Solve the equation $\ln x = 2$. Give you answer correct to four decimal places. ***(2 marks)***

17. Probability, 2ADV S1 2008 MET1 7

Jane drives to work each morning and passes through three intersections with traffic lights. The number X of traffic lights that are red when Jane is driving to work is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.1	0.2	0.3	0.4

i. What is the mode of X? ***(1 mark)***

ii. Jane drives to work on two consecutive days. What is the probability that the number of traffic lights that are red is the same on both days? ***(2 marks)***

18. Trigonometry, 2ADV T2 SM-Bank 32

Express $5\cot^2 x - 2\text{cosec } x + 2$ in terms of $\text{cosec } x$ and hence solve

$5\cot^2 x - 2\text{cosec } x + 2 = 0$ for $0 < x < 2\pi$. *(3 marks)*

19. Trigonometry, 2ADV T2 SM-Bank 33

Given $\sec\theta = -\dfrac{37}{12}$ for $0 < \theta < \pi$,

find the exact value of $\text{cosec } \theta$. *(2 marks)*

20. Trigonometry, 2ADV T2 SM-Bank 37

Solve the equation $\cos\left(\dfrac{3x}{2}\right) = \dfrac{1}{2}$ for $-\dfrac{\pi}{2} \leq x \leq \dfrac{\pi}{2}$. *(2 marks)*

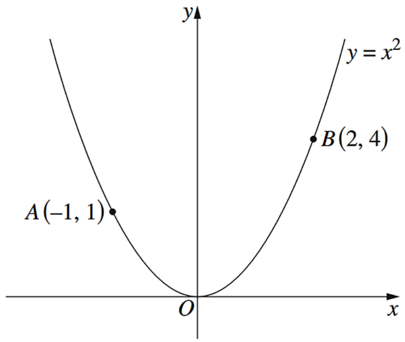
21. Trigonometry, 2ADV T2 SM-Bank 42

Prove that

$$\frac{1 - \sin^2 x \cos^2 x}{\sin^2 x} = \cot^2 x + \sin^2 x. \quad \textit{(2 marks)}$$

22. Calculus, 2ADV C1 2010 HSC 7b

The parabola shown in the diagram is the graph $y = x^2$. The points $A(-1, 1)$ and $B(2, 4)$ are on the parabola.



i. Find the equation of the tangent to the parabola at A. ***(2 marks)***

ii. Let M be the midpoint of AB.
There is a point C on the parabola such that the tangent at C is parallel to AB.
Show that the line MC is vertical. ***(2 marks)***

iii. The tangent at A meets the line MC at T.
Show that the line BT is a tangent to the parabola. ***(2 marks)***

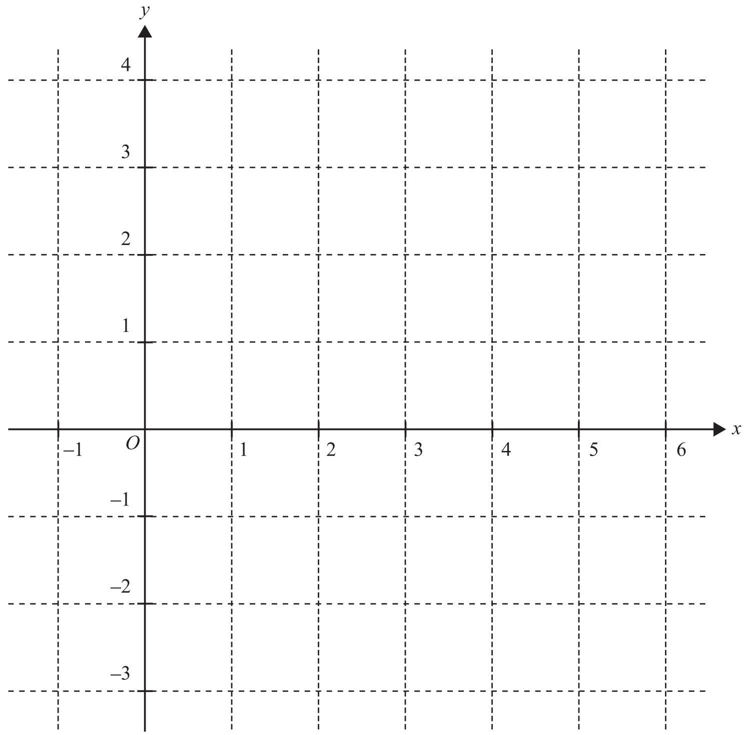
23. L&E, 2ADV E1 2019 MET1 4

Given the function $f(x) = \log_e(x-3) + 2$,

a. State the domain and range of $f(x)$. *(1 mark)*

b. **i.** Find the equation of the tangent to the graph of $f(x)$ at $(4, 2)$. *(2 marks)*

ii. On the axes below, sketch the graph of the function $f(x)$, labelling any asymptote with its equation. Also draw the tangent to the graph of $f(x)$ at $(4, 2)$. *(4 marks)*



24. Calculus, 2ADV C1 2006 HSC 8a

A particle is moving in a straight line. Its displacement, x metres, from the origin, O, at time t seconds, where $t \geq 0$, is given by $x = 1 - \dfrac{7}{t+4}$.

i. Find the initial displacement of the particle. ***(1 mark)***

ii. Find the velocity of the particle as it passes through the origin. ***(3 marks)***

iii. Show that the acceleration of the particle is always negative. ***(1 mark)***

iv. Sketch the graph of the displacement of the particle as a function of time. ***(2 marks)***

25. Probability, 2ADV S1 2019 HSC 15d

The probability that a person chosen at random has red hair is 0.02

a. Two people are chosen at random.
What is the probability that at least ONE has red hair? ***(2 marks)***

b. What is the smallest number of people that can be chosen at random so that the probability that at least ONE has red hair is greater than 0.4? ***(2 marks)***

26. Probability, 2ADV S1 2016 HSC 15b

An eight- sided die is marked with numbers 1, 2, … , 8. A game is played by rolling the die until an 8 appears on the uppermost face. At this point the game ends.

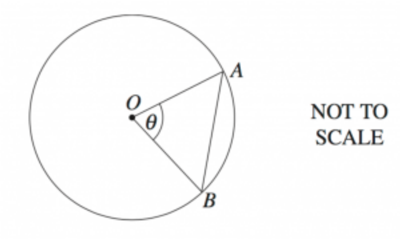
i. Using a tree diagram, or otherwise, explain why the probability of the game ending before the fourth roll is

$$\frac{1}{8}+\frac{7}{8}\times\frac{1}{8}+\left(\frac{7}{8}\right)^2\times\frac{1}{8}.$$ ***(2 marks)***

ii. What is the smallest value of n for which the probability of the game ending before the nth roll is more than $\frac{3}{4}$? ***(3 marks)***

27. Trigonometry, 2ADV T1 2009 HSC 5c

The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



i. There are two possible values of θ for which the area of ΔAOB is $\sqrt{3}$ square centimetres. One value is $\dfrac{\pi}{3}$.

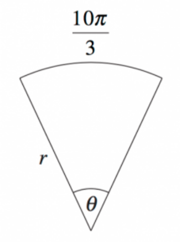
Find the other value. ***(2 marks)***

ii. Suppose that $\theta = \dfrac{\pi}{3}$.

(1) Find the area of sector AOB ***(1 mark)***

(2) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. ***(2 marks)***

28. Trigonometry, 2ADV T1 2008 HSC 7b



The diagram shows a sector with radius r and angle θ where $0 < \theta \leq 2\pi$.

The arc length is $\dfrac{10\pi}{3}$.

i. Show that $r \geq \dfrac{5}{3}$. *(2 marks)*

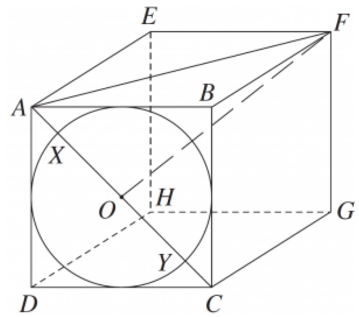
ii. Calculate the area of the sector when $r = 4$. *(2 marks)*

29. Trigonometry, 2ADV T2 2004 HSC 8a

i. Show that $\cos\theta\tan\theta = \sin\theta$. ***(1 mark)***

ii. Hence solve $8\sin\theta\cos\theta\tan\theta = \operatorname{cosec}\,\theta$ for $0 \le \theta \le 2\pi$. ***(2 marks)***

30. Trigonometry, 2ADV' T1 2004 HSC 3d



The length of each edge of the cube $ABCDEFGH$ is 2 metres. A circle is drawn on the face $ABCD$ so that it touches all four edges of the face. The centre of the circle is O and the diagonal AC meets the circle at X and Y.

i. Explain why $\angle FAC = 60^\circ$. ***(1 mark)***

ii. Show that $FO = \sqrt{6}$ metres. ***(1 mark)***

iii. Calculate the size of $\angle XFY$ to the nearest degree. ***(1 mark)***

31. Calculus, 2ADV C1 2019 HSC 14d

The equation of the tangent to the curve $y = x^3 + ax^2 + bx + 4$ at the point where $x = 2$ is $y = x - 4$.

Find the values of a and b. *(3 marks)*

32. Calculus, 2ADV C3 2010 HSC 8d

Let $f(x) = x^3 - 3x^2 + kx + 8$, where k is a constant.

Find the values of k for which $f(x)$ is an increasing function. *(2 marks)*

33. Trigonometry, 2ADV T2 SM-Bank 40

Let $(\tan\theta - 1)\left(\sin\theta - \sqrt{3}\cos\theta\right)\left(\sin\theta + \sqrt{3}\cos\theta\right) = 0$.

i. State all possible values of $\tan\theta$. *(1 mark)*

ii. Hence, find all possible solutions for $(\tan\theta - 1)\left(\sin^2\theta - 3\cos^2\theta\right) = 0$, where $0 \leq \theta \leq \pi$. *(2 marks)*

34. Algebra, STD2 A4 2012 HSC 30c

In 2010, the city of Thagoras modelled the predicted population of the city using the equation

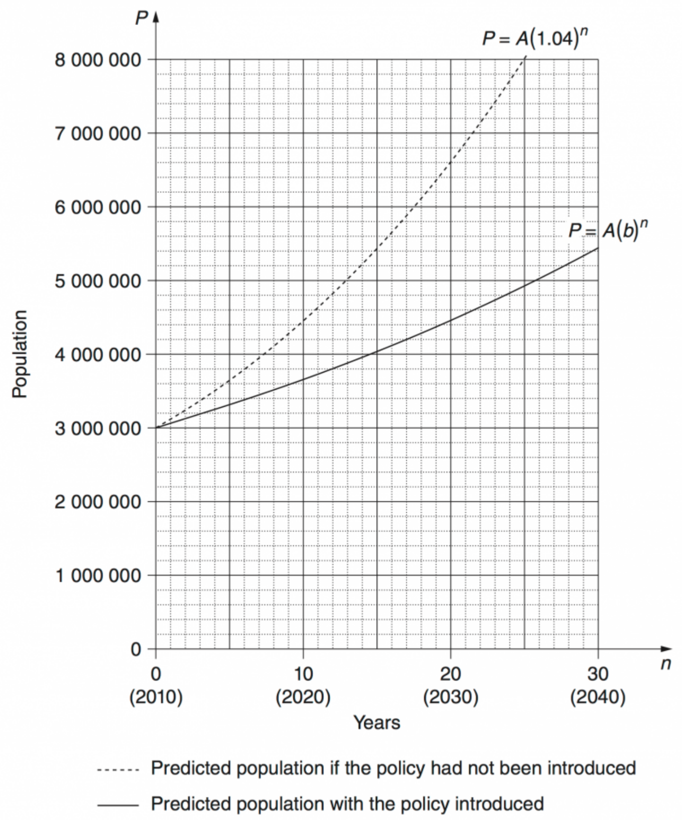
$$P = A(1.04)^n.$$

That year, the city introduced a policy to slow its population growth. The new predicted population was modelled using the equation

$$P = A(b)^n.$$

In both equations, P is the predicted population and n is the number of years after 2010.

The graph shows the two predicted populations.



i. Use the graph to find the predicted population of Thagoras in 2030 if the population policy had NOT been introduced. *(1 mark)*

ii. In each of the two equations given, the value of A is 3 000 000.
What does A represent? *(1 mark)*

iii. The guess-and-check method is to be used to find the value of b, in $P = A(b)^n$.

(1) Explain, with or without calculations, why 1.05 is not a suitable first estimate for b. *(1 mark)*

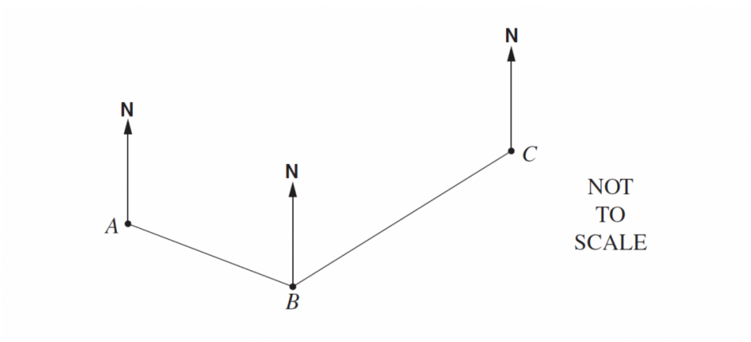
(2) With $n = 20$ and $P = 4\ 460\ 000$, use the guess-and-check method and the equation $P = A(b)^n$ to estimate the value of b to two decimal places. Show at least TWO estimate values for b, including calculations and conclusions. *(2 marks)*

iv. The city of Thagoras was aiming to have a population under 7 000 000 in 2050. Does the model indicate that the city will achieve this aim?

Justify your answer with suitable calculations. *(2 marks)*

35. Trigonometry, 2ADV* T1 2011 HSC 24c

A ship sails 6 km from A to B on a bearing of 121°. It then sails 9 km to C. The size of angle ABC is 114°.



Copy the diagram into your writing booklet and show all the information on it.

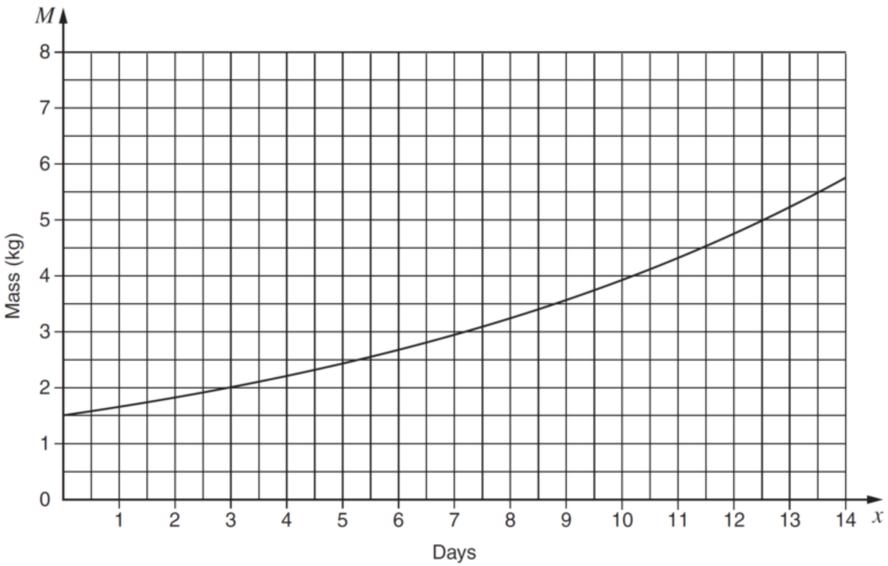
i. What is the bearing of C from B? *(1 mark)*

ii. Find the distance AC. Give your answer correct to the nearest kilometre. *(2 marks)*

iii. What is the bearing of A from C? Give your answer correct to the nearest degree. *(3 marks)*

36. Algebra, STD2 A4 2016 HSC 29b

The mass M kg of a baby pig at age x days is given by $M = A(1.1)^x$ where A is a constant. The graph of this equation is shown.



i. What is the value of A? *(1 mark)*

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ii. What is the daily growth rate of the pig's mass? Write your answer as a percentage. *(1 mark)*

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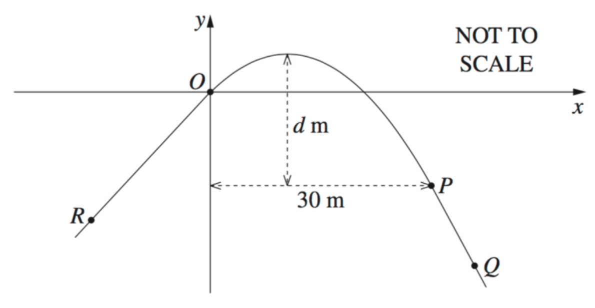
....

37. Calculus, 2ADV C1 2009 HSC 6c

The diagram illustrates the design for part of a roller-coaster track. The section RO is a straight line with slope 1.2, and the section PQ is a straight line with slope – 1.8. The section OP is a parabola $y = ax^2 + bx$. The horizontal distance from the y-axis to P is 30 m.



In order that the ride is smooth, the straight line sections must be tangent to the parabola at O and at P.

i. Find the values of a and b so that the ride is smooth. *(3 marks)*

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ii. Find the distance d, from the vertex of the parabola to the horizontal line through P, as shown on the diagram. *(2 marks)*

....

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Worked Solutions

1. Probability, 2ADV S1 2016 MET2 7 MC

$$\begin{aligned}P(0,0)+P(1,1)+P(2,2)+P(3,3)\\ &= 0.5^2+0.25^2+0.2^2+0.05^2\\ &= 0.355\end{aligned}$$

$\Rightarrow\ C$

2. Probability, 2ADV S1 2019 MET2-N 5 MC

$$1 = b+b+b+\frac{3}{5}-b+\frac{3b}{5}$$

$$\frac{2}{5} = \frac{13b}{5}$$

$$b = \frac{2}{13}$$

$$\begin{aligned}E(X) &= -\frac{2}{13}+0+\frac{2}{13}+2\left(\frac{3}{5}-\frac{2}{13}\right)+3\left(\frac{6}{65}\right)\\ &= \frac{76}{65}\end{aligned}$$

$\Rightarrow\ A$

3. Probability, 2ADV S1 2019 HSC 6 MC

Game lost only if an odd and a head show.

$$\begin{aligned}\therefore P(W) &= 1-P(\text{odd})\cdot P(\text{head})\\ &= 1-\frac{3}{6}\cdot\frac{1}{2}\\ &= \frac{3}{4}\end{aligned}$$

$\Rightarrow\ C$

4. Probability, 2ADV S1 2009 MET2 10 MC

$$P(X\le 0) = 0.4$$

$$P(X\le 1) = 0.6$$

$\therefore$ Median $= 1$

$\Rightarrow\ B$

5. Statistics, 2ADV S1 2022 HSC 9 MC

$$P(\text{at least 1 W}) = 1 - P(\text{LL}) = 0.8$$

$$P(\text{LL}) = 0.2$$

$$P(\text{L}) = \sqrt{0.2} = 0.447$$

... Mean marks/comments here

♦♦♦ Mean mark 27%.

$$P(\text{W}) = 1 - 0.447 = 0.553$$

$$P(\text{WW}) = (0.553)^2 = 0.31$$

$\Rightarrow A$

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6. Calculus, 2ADV C1 2018 HSC 12d

i. $x = \dfrac{t^3}{3} - 2t^2 + 3t$

$$v = \frac{dx}{dt} = t^2 - 4t + 3$$

Find v when $t = 0$:

$$v = 0 - 0 + 3 = 3\ \text{ms}^{-1}$$

ii. Particle is stationary when $v = 0$

$$t^2 - 4t + 3 = 0$$

$$(t-3)(t-1) = 0$$

$t = 1$ or 3 seconds

iii. $a = \dfrac{dv}{dt} = 2t - 4$

Find t when $a = 0$

$$2t - 4 = 0$$

$$t = 2$$

$$x(2) = \frac{2^3}{3} - 2(2^2) + 3(2) = \frac{8}{3} - 8 + 6 = \frac{2}{3}$$

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7. Trigonometry, 2ADV T2 SM-Bank 41

$$\begin{aligned}\text{LHS} &= (\sec x + \tan x)(\sec x - \tan x)\\ &= \sec^2 x - \tan^2 x\\ &= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1 - \sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x}\\ &= 1\\ &= \text{RHS}\end{aligned}$$

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8. Statistics, 2ADV S1 2023 HSC 12

a.
$$\begin{aligned}E(X) &= 0 + 1 \times 0.3 + 2 \times 0.5 + 3 \times 0.1 + 4 \times 0.1\\ &= 0.3 + 1 + 0.3 + 0.4\\ &= 2\end{aligned}$$

b.
$$\begin{aligned}\text{Var}(X) &= E(X^2) - [E(X)]^2\\ &= \left(0 + 1^2 \times 0.3 + 2^2 \times 0.5 + 3^2 \times 0.1 + 4^2 \times 0.1\right) - 2^2\\ &= (0.3 + 2 + 0.9 + 1.6) - 4\\ &= 0.8\end{aligned}$$

$$\begin{aligned}\therefore \sigma &= \sqrt{0.8}\\ &= 0.8944\ldots\\ &= 0.9\ \ (\text{to 1 d.p.})\end{aligned}$$

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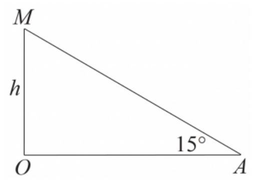
9. Trigonometry, 2ADV' T1 2015 HSC 12c

i. Show $OA = h \cot 15°$



In ΔMOA,

$$\tan 15° = \frac{h}{OA}$$

$$OA = \frac{h}{\tan 15°}$$

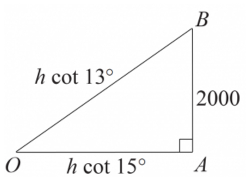
$$= h \cot 15° \quad \text{...as required}$$

ii. In ΔMOB

$$\tan 13° = \frac{h}{OB}$$

$$OB = \frac{h}{\tan 13°}$$

$$= h \cot 13°$$



In ΔAOB:

$$OA^2 + AB^2 = OB^2$$

$$OB^2 - OA^2 = AB^2$$

$$(h \cot 13°)^2 - (h \cot 15°)^2 = 2000^2$$

$$h^2\left[(\cot^2 13° - \cot^2 15°)\right] = 2000^2$$

$$h^2 = \frac{2000^2}{\cot^2 13° - \cot^2 15°}$$

$$\therefore h = \sqrt{\frac{2000^2}{\cot^2 13° - \cot^2 15°}}$$

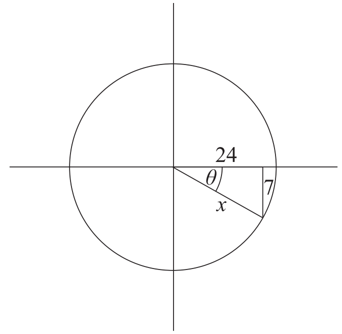
$$= 909.704\ldots$$

$$= 910 \text{ m (nearest metre)}$$

10. Trigonometry, 2ADV T2 SM-Bank 31

i. $\cot\theta = -\dfrac{24}{7} \quad \Rightarrow \quad \tan\theta = -\dfrac{7}{24}$

Graphically, given $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$:



$$x = \sqrt{24^2 + 7^2} = 25$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{1}{\frac{24}{25}}$$

$$= \frac{25}{24}$$

ii. $\sin\theta = -\dfrac{7}{25}$

11. Calculus, 2ADV C1 2013 HSC 11b

$$\lim_{x \to 2} \frac{(x-2)(x+2)^2}{x^2-4}$$

$$= \lim_{x \to 2} \frac{(x-2)(x+2)^2}{(x-2)(x+2)}$$

$$= \lim_{x \to 2} (x+2)$$

$$= 4$$

COMMENT: This question has been simplified as students no longer need to factorise the difference between 2 cubes ($x^3 - 2^3$).

12. L&E, 2ADV E1 SM-Bank 7

$$\begin{aligned}\log_3(5)^2-\log_3(2)+\log_3(x)&=2\\ \log_3(25x)-\log_3(2)&=2\\ \log_3\left(\frac{25x}{2}\right)&=2\\ \frac{25x}{2}&=3^2\\ \therefore x&=\frac{18}{25}\end{aligned}$$

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13. L&E, 2ADV E1 SM-Bank 10

$$\begin{aligned}2^{3x-3}&=2^{3(2-x)}\\ 3x-3&=6-3x\\ 6x&=9\\ \therefore x&=\frac{3}{2}\end{aligned}$$

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14. Calculus, 2ADV C1 SM-Bank 11

$$v=\sqrt{20t-2t^2}$$

$$\begin{aligned}\alpha&=\frac{dv}{dt}\\ &=\frac{1}{2}\cdot\left(20t-2t^2\right)^{-\frac{1}{2}}\cdot(20-4t)\end{aligned}$$

When $t=4$,

$$\begin{aligned}\alpha&=\frac{1}{2}\left(20\cdot 4-2\cdot 4^2\right)^{-\frac{1}{2}}(20-16)\\ &=\frac{2}{\sqrt{48}}\\ &=\frac{2}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}\\ &=\frac{\sqrt{3}}{6}\ \text{ms}^{-2}\end{aligned}$$

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15. Calculus, 2ADV C1 SM-Bank 25

Using Chain Rule:

$$\begin{aligned}g\prime(x)&=3\left(2-x^3\right)^2\left(-3x^2\right)\\ &=-9x^2\left(2-x^3\right)^2\\ \therefore g\prime(1)&=-9(-1)^2\left[2-(-1)^3\right]^2\\ &=-81\end{aligned}$$

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16. L&E, 2ADV E1 2009 HSC 1f

$$\begin{aligned}\ln x&=2\\ \log_e x&=2\\ x&=e^2\\ &=7.38905\ldots\\ &=7.3891\quad\text{(to 4 d.p.)}\end{aligned}$$

MARKER'S COMMENT: Students are reminded to write answers to more decimal places than required before rounding up.

---

17. Probability, 2ADV S1 2008 MET1 7

i. 3

ii. $P(0,0)+P(1,1)+P(2,2)+P(3,3)$

$$\begin{aligned}&=0.1^2+0.2^2+0.3^2+0.4^2\\ &=0.3\end{aligned}$$

18. Trigonometry, 2ADV T2 SM-Bank 32

$$\begin{aligned}\cot^2 x &= \frac{\cos^2 x}{\sin^2 x}\\ &= \frac{1-\sin^2 x}{\sin^2 x}\\ &= \text{cosec}^2 x - 1\end{aligned}$$

$$\begin{aligned}5\cot^2 x - 2\text{cosec } x + 2 &= 0\\ 5\left(\text{cosec}^2 x - 1\right) - 2\text{cosec } x + 2 &= 0\\ 5\text{cosec}^2 x - 2\text{cosec } x - 3 &= 0\\ (5\text{cosec } x + 3)(\text{cosec } x - 1) &= 0\end{aligned}$$

$$\text{cosec } x = -\frac{3}{5} \qquad \text{cosec } x = 1$$

$$\sin x = -\frac{5}{3} \qquad \sin x = 1$$

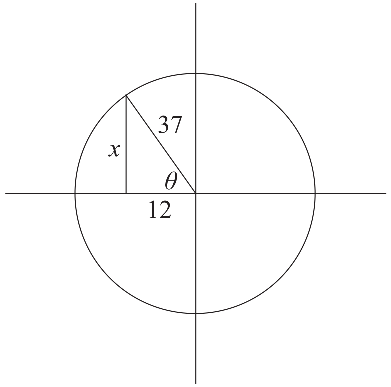
$$(\text{no solution}) \qquad x = \frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{2}$$

19. Trigonometry, 2ADV T2 SM-Bank 33

$$\sec\theta = -\frac{37}{12} \Rightarrow \cos\theta = -\frac{12}{37}$$

Graphically:



$$x = \sqrt{37^2 - 12^2} = 35$$

$$\begin{aligned}\therefore\ \text{cosec }\theta &= \frac{1}{\sin\theta}\\ &= \frac{1}{\frac{35}{37}}\\ &= \frac{37}{35}\end{aligned}$$

20. Trigonometry, 2ADV T2 SM-Bank 37

$$\cos\left(\frac{3x}{2}\right) = \frac{1}{2}$$

$$\Rightarrow \text{ Base angle } = \frac{\pi}{3}$$

$$\frac{3x}{2} = \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$$

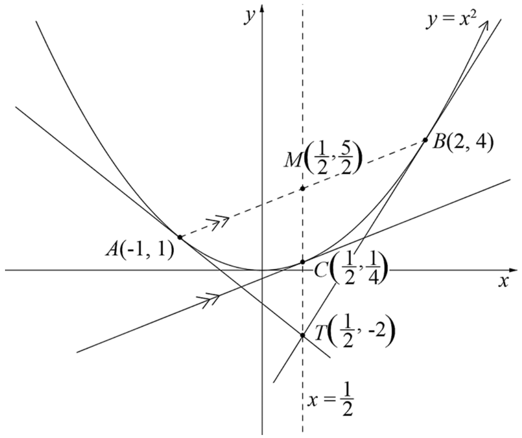
$$\begin{aligned}\therefore x &= \frac{-2\pi}{9}, \frac{2\pi}{9}, \frac{10\pi}{9}\\ &= \frac{-2\pi}{9}, \frac{2\pi}{9} \quad \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)\end{aligned}$$

21. Trigonometry, 2ADV T2 SM-Bank 42

$$\begin{aligned}
\text{RHS} &= \frac{\cos^2 x}{\sin^2 x} + \sin^2 x \\
&= \frac{\cos^2 x + \sin^4 x}{\sin^2 x} \\
&= \frac{\cos^2 x + \sin^2 x\left(1 - \cos^2 x\right)}{\sin^2 x} \\
&= \frac{\cos^2 x + \sin^2 x - \sin^2 x \cos^2 x}{\sin^2 x} \\
&= \frac{1 - \sin^2 x \cos^2 x}{\sin^2 x} \\
&= \text{LHS}
\end{aligned}$$

22. Calculus, 2ADV C1 2010 HSC 7b

i.



$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

At $A(-1,1) \Rightarrow \dfrac{dy}{dx} = -2$

Tangent has $m = -2$, through $(-1,1)$:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -2(x+1)$$

$$y - 1 = -2x - 2$$

$$2x + y + 1 = 0$$

$\therefore$ Tangent at A is $2x + y + 1 = 0$

ii. $A(-1,1) \quad B(2,4)$

$$M = \left(\frac{-1+2}{2}, \frac{1+4}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{5}{2}\right)$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-1}{2+1} = 1$$

When $\dfrac{dy}{dx} = 1$

♦ *Mean mark 37%.*
IMPORTANT: The critical understanding required for this question is that the gradient of AB needs to be equated to the gradient function (i.e. $\frac{dy}{dx}$).

$$2x = 1$$

$$x = \frac{1}{2}$$

$$\therefore\ C\left(\frac{1}{2}, \frac{1}{4}\right)$$

$\Rightarrow M$ and C both have x-value $= \dfrac{1}{2}$

$\therefore\ MC$ is vertical ... as required

iii. T is point on tangent when $x = \dfrac{1}{2}$

♦♦ *Mean mark 29%.*

Tangent $2x + y + 1 = 0$

At $x = \dfrac{1}{2}$

$$2 \times \left(\frac{1}{2}\right) + y + 1 = 0$$

$$\Rightarrow y = -2$$

$$\therefore\ T\left(\frac{1}{2}, -2\right)$$

Given $B(2,4)$

$$m_{BT} = \frac{4+2}{2 - \frac{1}{2}}$$

$$= 4$$

At $B(2,4)$, find gradient of tangent:

$$\frac{dy}{dx} = 2x = 2 \times 2 = 4$$

$\therefore m_{\text{tangent}} = 4 = m_{BT}$

$\therefore BT$ is a tangent

23. L&E, 2ADV E1 2019 MET1 4

a. Domain: $x > 3$

Range: $y \in R$

b.i. $g(x) = \log_e(x-3) + 2$

$g\prime(x) = \dfrac{1}{x-3}$

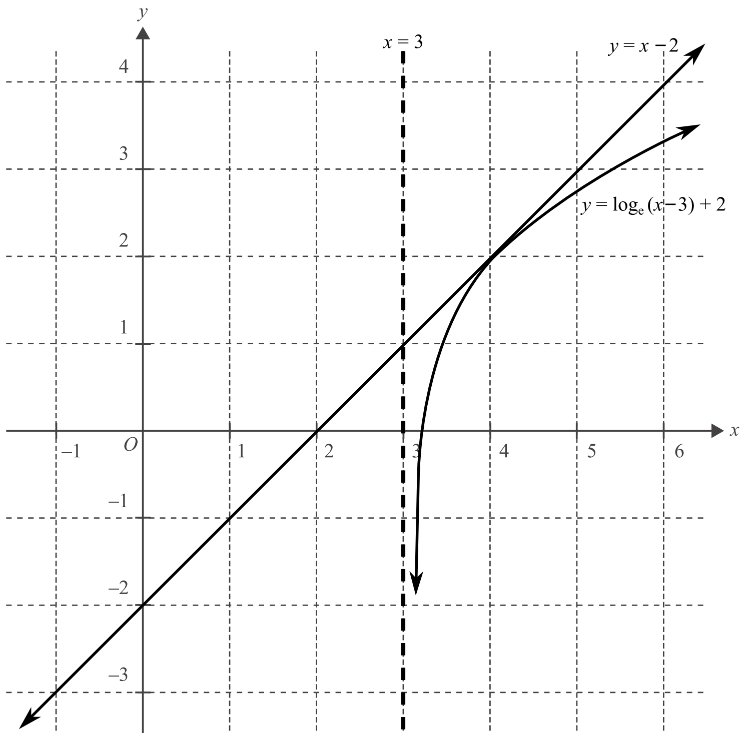
$g\prime(4) = 1$

Equation of tangent, $m = 1$ through $(4, 2)$:

$y-2 = 1(x-4)$

$y = x-2$

b.ii.



24. Calculus, 2ADV C1 2006 HSC 8a

i. $x = 1-\dfrac{7}{t+4}$

When $t = 0$,

$$x = 1-\frac{7}{4}$$

$$= -\frac{3}{4} \text{ m}$$

$\therefore$ Initial displacement is $\dfrac{3}{4}$ metres to the left of the origin.

ii. $x = 1-\dfrac{7}{t+4} = 1-7(t+4)^{-1}$

$$\dot{x} = (-1)\ -7(t+4)^{-2} \times \frac{d}{dt}(t+4)$$

$$= 7(t+4)^{-2} \times 1$$

$$= \frac{7}{(t+4)^2}$$

Find t when $x = 0$

$$0 = 1-\frac{7}{t+4}$$

$$\frac{7}{t+4} = 1$$

$$7 = (t+4)$$

$$t = 3$$

When $t = 3$

$$\dot{x} = \frac{7}{(3+4)^2}$$

$$= \frac{1}{7} \text{ ms}^{-1}$$

$\therefore$ The velocity of the particle as it passes through the origin is $\dfrac{1}{7}$ ms^{-1}.

iii. $\dot{x} = 7(t+4)^{-2}$

$$\ddot{x} = \frac{d\dot{x}}{dt} = -14\ (t+4)^{-3}$$

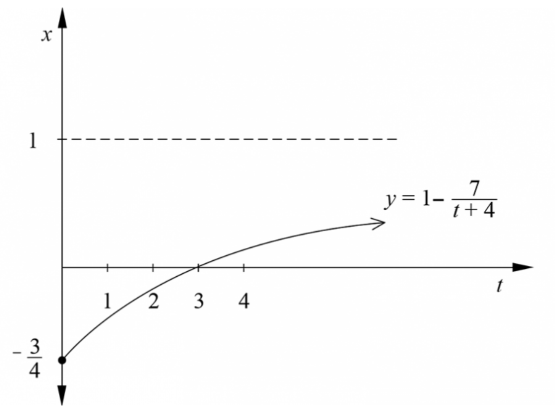
Given $t \geq 0$

$\Rightarrow \ (t+4)^{-3} \geq 0$

$\Rightarrow -14(t+4)^{-3} \leq 0$

$\therefore \ddot{x}$ is always negative.

(iv)



25. Probability, 2ADV S1 2019 HSC 15d

a. $P(R) = 0.02$

$P\left(\overline{R}\right) = 0.98$

P (At least 1 has red hair)

$$= 1 - P\left(\overline{R}, \overline{R}\right)$$
$$= 1 - 0.98 \times 0.98$$
$$= 0.0396$$

b. Find n such that

$$1 - 0.98^n > 0.4$$
$$0.98^n < 0.6$$
$$\ln 0.98^n < \ln 0.6$$
$$n \ln 0.98 < \ln 0.6$$
$$n > \frac{\ln 0.6}{\ln 0.98}, \quad (\ln 0.98 < 0)$$
$$> 25.28\ldots$$

$\therefore$ 26 people must be chosen.

♦♦ *Mean mark 24%.*

26. Probability, 2ADV S1 2016 HSC 15b

i. $P(\text{game ends before 4th roll})$

$$= P(8) + P(\text{not } 8, 8) + P(\text{not } 8, \text{not } 8, 8)$$
$$= \frac{1}{8} + \frac{7}{8} \cdot \frac{1}{8} + \frac{7}{8} \cdot \frac{7}{8} \cdot \frac{1}{8}$$
$$= \frac{1}{8} + \frac{7}{8} \cdot \frac{1}{8} + \left(\frac{7}{8}\right)^2 \cdot \frac{1}{8} \quad \ldots \text{ as required}$$

ii. $\frac{1}{8} + \frac{7}{8} \cdot \frac{1}{8} + \left(\frac{7}{8}\right)^2 \cdot \frac{1}{8} + \ldots$

$\Rightarrow$ GP where $a = \frac{1}{8}, \quad r = \frac{7}{8}$

Find n such that $S_{n-1} > \frac{3}{4}$,

$$S_{n-1} = \frac{a\left(1 - r^{n-1}\right)}{1 - r}$$
$$\frac{3}{4} < \frac{1}{8} \times \frac{\left(1 - \left(\frac{7}{8}\right)^{n-1}\right)}{1 - \frac{7}{8}}$$
$$\frac{3}{4} < 1 - \left(\frac{7}{8}\right)^{n-1}$$
$$\left(\frac{7}{8}\right)^{n-1} < \frac{1}{4}$$
$$(n-1) \cdot \ln \frac{7}{8} < \ln \frac{1}{4}$$
$$n - 1 > \frac{\ln \frac{1}{4}}{\ln \frac{7}{8}}$$
$$> 11.38\ldots$$

$\therefore n = 12$

♦♦ *Mean mark 29%.*
ALGEBRA: Note that dividing by $\ln \frac{7}{8}$ reverses the < sign as it is dividing by a negative number.

27. Trigonometry, 2ADV T1 2009 HSC 5c

i. Area $\Delta AOB = \dfrac{1}{2}ab\sin\theta$

$= \dfrac{1}{2} \times 2 \times 2 \times \sin\theta$

$= 2\sin\theta$

$2\sin\theta = \sqrt{3}$ (given)

$\sin\theta = \dfrac{\sqrt{3}}{2}$

$\therefore \theta = \dfrac{\pi}{3}, \ \pi - \dfrac{\pi}{3}$

$= \dfrac{\pi}{3}, \ \dfrac{2\pi}{3}$

$\therefore$ The other value of θ is $\dfrac{2\pi}{3}$ radians

ii. (1) Area of sector $AOB = \pi r^2 \times \dfrac{\theta}{2\pi}$

$= \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times 2^2 \times \dfrac{\pi}{3}$

$= \dfrac{2\pi}{3}$ cm²

ii. (2) Using the cosine rule:

$AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \times \cos\theta$

$= 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos\left(\dfrac{\pi}{3}\right)$

$= 4 + 4 - 4$

$= 4$

$\therefore \ AB = 2$

Arc $AB = 2\pi r \times \dfrac{\theta}{2\pi}$

$= r\theta$

$= \dfrac{2\pi}{3}$ cm

$\therefore$ Perimeter $= \left(2 + \dfrac{2\pi}{3}\right)$ cm

28. Trigonometry, 2ADV T1 2008 HSC 7b

i. Show $r \geq \dfrac{5}{3}$

Arc length $= r\theta$ where $0 < \theta \leq 2\pi$

$r\theta = \dfrac{10\pi}{3}$

$\therefore \theta = \dfrac{10\pi}{3r}$

Using $0 \leq \theta \leq 2\pi$

$0 \leq \dfrac{10\pi}{3r} \leq 2\pi$

$\dfrac{10\pi}{3} \leq 2\pi r$

$\dfrac{5}{3} \leq r$

$\therefore \ r \geq \dfrac{5}{3}$ ... as required.

ii. Area $= \dfrac{1}{2}r^2\theta$

$= \dfrac{1}{2} \times 4^2 \times \dfrac{10\pi}{3 \times 4}$

$= \dfrac{20\pi}{3}$ u²

29. Trigonometry, 2ADV T2 2004 HSC 8a

i. Prove $\cos\theta\tan\theta = \sin\theta$

$$\begin{aligned}\text{LHS} &= \cos\theta\tan\theta \\ &= \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) \\ &= \sin\theta \\ &= \text{ RHS}\end{aligned}$$

ii. $8\sin\theta\cos\theta\tan\theta = \text{cosec } \theta$

$$\begin{aligned}\therefore 8\sin\theta(\sin\theta) &= \text{cosec } \theta, \quad \text{(part (i))} \\ 8\sin^2\theta &= \frac{1}{\sin\theta} \\ 8\sin^3\theta &= 1 \\ \sin^3\theta &= \frac{1}{8} \\ \sin\theta &= \frac{1}{2} \\ \therefore \theta &= \frac{\pi}{6}, \frac{5\pi}{6}. \quad (\text{for } 0 \le \theta \le 2\pi)\end{aligned}$$

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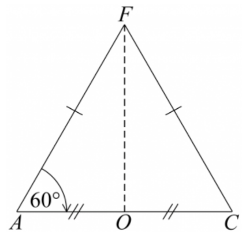
30. Trigonometry, 2ADV' T1 2004 HSC 3d

i.



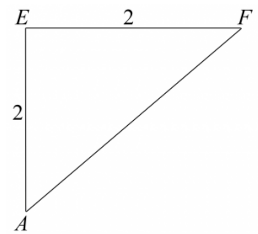
Since FA, AC and FC are all diagonals of sides of a cube,

$FA = AC = FC$

$\therefore \Delta FAC$ is equilateral

$\therefore \angle FAC = 60°$

ii.



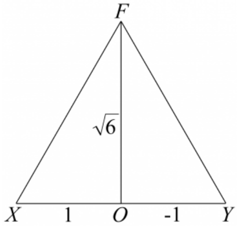
In ΔAEF

$$\begin{aligned}AF^2 &= EF^2 + EA^2 \\ &= 2^2 + 2^2 \\ &= 8 \\ AF &= \sqrt{8} \\ &= 2\sqrt{2}\end{aligned}$$

In ΔAFO

$$\begin{aligned}\sin 60° &= \frac{FO}{AF} \\ \frac{\sqrt{3}}{2} &= \frac{FO}{2\sqrt{2}} \\ FO &= \frac{\sqrt{3}}{2} \times 2\sqrt{2} \\ &= \sqrt{6} \text{ metres} \ldots \text{as required.}\end{aligned}$$

iii.



XY is the diameter of a circle AND the width of the cube.

$\therefore XY = 2$

$\therefore OX = OY = 1$

$$\tan \angle OFX = \frac{1}{\sqrt{6}}$$

$$\angle OFX = 22.207\ldots^\circ$$

$$\begin{aligned}\therefore \angle XFY &= 2 \times 22.407\ldots \\ &= 44.415\ldots \\ &= 44^\circ \text{ (nearest degree)}\end{aligned}$$

---

31. Calculus, 2ADV C1 2019 HSC 14d

$$y = x^3 + ax^2 + bx + 4$$

$$\frac{dy}{dx} = 3x^2 + 2ax + b$$

When $x = 2$, $\dfrac{dy}{dx} = 1$

$$12 + 4a + b = 1$$

$$4a + b = -11 \ldots (1)$$

♦ *Mean mark 46%.*

The point $(2, -2)$ lies on y:

$$8 + 4a + 2b + 4 = -2$$

$$4a + 2b = -14 \ldots (2)$$

Subtract $(2)-(1)$

$$b = -3$$

Substitute into (1)

$$\begin{aligned}4a - 3 &= -11 \\ 4a &= -8 \\ a &= -2\end{aligned}$$

---

32. Calculus, 2ADV C3 2010 HSC 8d

$$f(x) = x^3 - 3x^2 + kx + 8$$

$$f'(x) = 3x^2 - 6x + k$$

$f(x)$ is increasing when $f'(x) > 0$

$\Rightarrow 3x^2 - 6x + k > 0$

♦♦ *Mean mark 28%.*
MARKER'S COMMENT: The arithmetic required to solve $36 - 12k < 0$ proved the undoing of *too many* students in this question. *TAKE CARE!*

$f'(x)$ is always positive

$\Rightarrow f'(x)$ is a positive definite.

i.e. when $a > 0$ and $\Delta < 0$

$$a = 3 > 0$$

$$\Delta = b^2 - 4ac$$

$$\begin{aligned}\therefore (-6)^2 - (4 \times 3 \times k) &< 0 \\ 36 - 12k &< 0 \\ 12k &> 36 \\ k &> 3\end{aligned}$$

$\therefore$ $f(x)$ is increasing when $k > 3$.

---

33. Trigonometry, 2ADV T2 SM-Bank 40

i. $(\tan\theta - 1)\left(\sin\theta - \sqrt{3}\cos\theta\right)\left(\sin\theta + \sqrt{3}\cos\theta\right) = 0$

$\Rightarrow \tan\theta = 1$

$$\Rightarrow \sin\theta - \sqrt{3}\cos\theta = 0$$
$$\sin\theta = \sqrt{3}\cos\theta$$
$$\tan\theta = \sqrt{3}$$

♦ *Mean mark 42%.*

$$\Rightarrow \sin\theta + \sqrt{3}\cos\theta = 0$$
$$\sin\theta = -\sqrt{3}\cos\theta$$
$$\tan\theta = -\sqrt{3}$$

$\therefore \tan\theta = 1 \text{ or } \tan\theta = \pm\sqrt{3}$

ii. $(\tan\theta - 1)\left(\sin^2\theta - 3\cos^2\theta\right) = 0$

Using part a:

$(\tan\theta - 1)\left(\sin\theta - \sqrt{3}\cos\theta\right)\left(\sin\theta + \sqrt{3}\cos\theta\right) = 0$

♦ *Mean mark 42%.*

$\Rightarrow \tan\theta = 1 \quad \text{or} \quad \tan\theta = \pm\sqrt{3}$

$\theta = \dfrac{\pi}{4} \qquad \theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

$\therefore \theta = \dfrac{\pi}{4}, \dfrac{\pi}{3} \text{ or } \dfrac{2\pi}{3} \quad (0 \leq \theta \leq \pi)$

---

34. Algebra, STD2 A4 2012 HSC 30c

i. 2030 occurs at $n = 20$ on the x-axis.

Expected population (no policy) $= 6\ 600\ 000$

COMMENT: Common ADV/STD2 content in new syllabus.

ii. A represents the population when $n = 0$ which is the population in 2010.

iii. (1) $P = A(1.05)^n$ would be steeper and lie above $P = A(1.04)^n$ since $1.05 > 1.04$

♦ *Mean mark part (ii) 48%*

iii. (2) Let $b = 1.03$

$$P = 3\ 000\ 000 \times 1.03^{20}$$
$$= 5\ 418\ 000$$

Let $b = 1.02$

$$P = 3\ 000\ 000 \times 1.02^{20}$$
$$= 4\ 457\ 800$$

$\therefore b = 1.02$

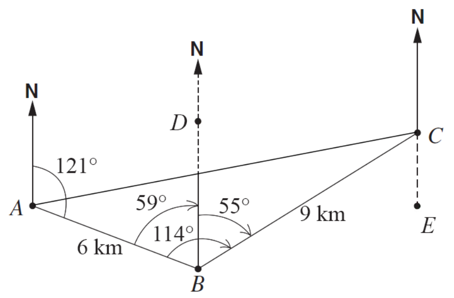
iv. In 2050, $n = 40$

$$P = 3\ 000\ 000 \times 1.02^{40}$$
$$= 6\ 624\ 119 \quad \text{(nearest whole)}$$

Since the population is below 7 million, the model will achieve the aim.

35. Trigonometry, 2ADV* T1 2011 HSC 24c

i.



Let point D be due North of point B

$$\begin{aligned}\angle ABD &= 180 - 121 \text{ (cointerior with } \angle A)\\ &= 59^\circ\\ \angle DBC &= 114 - 59\\ &= 55^\circ\end{aligned}$$

$\therefore$ Bearing of C from B is 055°

STRATEGY: This deserves repeating again: *Draw North-South parallel lines through major points to make the angle calculations easier!*

ii. Using cosine rule:

$$\begin{aligned}AC^2 &= AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC\\ &= 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 114^\circ\\ &= 160.9275\ldots\\ \therefore AC &= 12.685\ldots \quad (\text{Noting } AC > 0)\\ &= 13 \text{ km (nearest km)}\end{aligned}$$

iii. Need to find $\angle ACB$ (see diagram)

$$\begin{aligned}\cos\angle ACB &= \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC}\\ &= \frac{(12.685\ldots)^2 + 9^2 - 6^2}{2 \times (12.685..) \times 9}\\ &= 0.9018\ldots\\ \angle ACB &= 25.6^\circ \text{ (to 1 d.p.)}\end{aligned}$$

MARKER'S COMMENT: The best responses showed clear working on the diagram.

From diagram,
$\angle BCE = 55^\circ$ (alternate angle, $DB \,||\, CE$)
$\therefore$ Bearing of A from C

$$\begin{aligned}&= 180 + 55 + 25.6\\ &= 260.6\\ &= 261^\circ \text{ (nearest degree)}\end{aligned}$$

36. Algebra, STD2 A4 2016 HSC 29b

i. When $x = 0$,

$$\begin{aligned}1.5 &= A(1.1)^0\\ \therefore A &= 1.5 \text{ kg}\end{aligned}$$

♦ *Mean mark 48%.*
COMMENT: Common ADV/STD2 content in new syllabus.

ii. Daily growth rate

$$\begin{aligned}&= 0.1\\ &= 10\%\end{aligned}$$

♦♦♦ *Mean mark part (ii) 6%.*
MARKER'S COMMENT:
Interpretation of the exponential was very poorly understood.

37. Calculus, 2ADV C1 2009 HSC 6c

i.

$$y = ax^2 + bx$$

$$\frac{dy}{dx} = 2ax + b$$

At $x = 0$,

$$\frac{dy}{dx} = b$$

We need m at $O = 1.2$

$\therefore\ b = 1.2$

At $P,\ x = 30$

$$\frac{dy}{dx} = 2 \times a \times 30 + 1.2$$

$$= 60a + 1.2$$

We need m at $P = -1.8$

$$60a + 1.2 = -1.8$$

$$60a = -3$$

$$a = -\frac{3}{60}$$

$$= -0.05$$

$\therefore$ For a smooth ride, $a = -0.05$ and $b = 1.2$

♦♦♦ *Exact data unavailable.*
MARKER'S COMMENT: Successful students equated the curve gradient to the straight section as a requirement for a smooth ride.

ii.

$$y = -0.05x^2 + 1.2x$$

$$\frac{dy}{dx} = -0.1x + 1.2$$

Find x when $\frac{dy}{dx} = 0$

$$-0.1x + 1.2 = 0$$

$$x = \frac{1.2}{0.1}$$

$$= 12$$

$\therefore$ MAX when $x = 12$

When $x = 12$

$$y = -0.05 \times 12^2 + 1.2 \times 12$$

$$= -7.2 + 14.4$$

$$= 7.2$$

When $x = 30$

$$y = -0.05 \times 30^2 + 1.2 \times 30$$

$$= -45 + 36$$

$$= -9$$

$$\therefore\ d = 7.2 + |-9|$$

$$= 16.2\ \text{m}$$