EXERCISE 2.1 REVIEW OF RIGHT-ANGLED TRIANGLES

2 **(a)** $\sin 21°17' = \dfrac{x}{12}$

$$\begin{aligned} x &= 12\sin 21°17' \\ &= 4.3557... \\ &= 4.36 \end{aligned}$$

(b) $\cos 63°14' = \dfrac{x}{10}$

$$\begin{aligned} x &= 10\cos 63°14' \\ &= 4.5035... \\ &= 4.50 \end{aligned}$$

(c) $\tan 22°19' = \dfrac{x}{8}$

$$\begin{aligned} x &= 8\tan 22°19' \\ &= 3.2837... \\ &= 3.28 \end{aligned}$$

(d) $\sin 57°47' = \dfrac{4}{x}$

$$\begin{aligned} x &= \frac{4}{\sin 57°47'} \\ &= 4.7279... \\ &= 4.73 \end{aligned}$$

4 $\tan 56°20' = \dfrac{30}{x}$

$$\begin{aligned} x &= \frac{30}{\tan 56°20'} \\ &= 19.9823... \\ &= 20.0 \text{ m} \end{aligned}$$

$\tan 33°27' = \dfrac{30}{y}$

$$\begin{aligned} y &= \frac{30}{\tan 33°27'} \\ &= 45.4111... \\ &= 45.4 \text{ m} \end{aligned}$$

EXERCISE 2.2 ANGLES OF ANY MAGNITUDE

2 **(a)** $0° < 72° < 90°$

1st quadrant

(b) $90° < 114° < 180°$

2nd quadrant

(c) $90° < 95° < 180°$

2nd quadrant

(d) $180° < 200° < 270°$

3rd quadrant

(e) $270° < 321° < 360°$

4th quadrant

(f) $180° < 183° < 270°$

3rd quadrant

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(g) $0° < 83° < 90°$

1st quadrant

(h) $180° < 216° < 270°$

3rd quadrant

(i) $270° < 300° < 360°$

4th quadrant

(j) $90° < 155° < 180°$

2nd quadrant

4 (a) (i)

$$\sin 125° = 0.8192$$
$$\cos 125° = -0.5736$$
$$\tan 125° = -1.4281$$
$$\operatorname{cosec} 125° = \frac{1}{\sin 125°} = 1.2208$$
$$\sec 125° = \frac{1}{\cos 125°} = -1.7434$$
$$\cot 125° = \frac{1}{\tan 125°} = -0.7002$$

(ii)

$$\sin 152° = 0.4695$$
$$\cos 152° = -0.8829$$
$$\tan 152° = -0.5317$$
$$\operatorname{cosec} 152° = \frac{1}{\sin 152°} = 2.1301$$
$$\sec 152° = \frac{1}{\cos 152°} = -1.1326$$
$$\cot 152° = \frac{1}{\tan 152°} = -1.8807$$

(iii)

$$\sin 117° = 0.8910$$
$$\cos 117° = -0.4540$$
$$\tan 117° = -1.9626$$
$$\operatorname{cosec} 117° = \frac{1}{\sin 117°} = 1.1223$$
$$\sec 117° = \frac{1}{\cos 117°} = -2.2027$$
$$\cot 117° = \frac{1}{\tan 117°} = -0.5095$$

(b) (i)

$$\sin 205° = -0.4226$$
$$\cos 205° = -0.9063$$
$$\tan 205° = 0.4663$$
$$\operatorname{cosec} 205° = \frac{1}{\sin 205°} = -2.3662$$
$$\sec 205° = \frac{1}{\cos 205°} = -1.1034$$
$$\cot 205° = \frac{1}{\tan 205°} = 2.1445$$

(ii)

$$\sin 217° = -0.6018$$
$$\cos 217° = -0.7986$$
$$\tan 217° = 0.7536$$
$$\operatorname{cosec} 217° = \frac{1}{\sin 217°} = -1.6616$$
$$\sec 217° = \frac{1}{\cos 217°} = -1.2521$$
$$\cot 217° = \frac{1}{\tan 217°} = 1.3270$$

(iii)

$$\sin 251° = -0.9455$$
$$\cos 251° = -0.3256$$
$$\tan 251° = 2.9042$$
$$\operatorname{cosec} 251° = \frac{1}{\sin 251°} = -1.0576$$
$$\sec 251° = \frac{1}{\cos 251°} = -3.0716$$
$$\cot 251° = \frac{1}{\tan 251°} = 0.3443$$

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(c) (i)

$$\sin 282° = -0.9781$$
$$\cos 282° = 0.2079$$
$$\tan 282° = -4.7046$$
$$\operatorname{cosec} 282° = \frac{1}{\sin 282°} = -1.0223$$
$$\sec 282° = \frac{1}{\cos 282°} = 4.8097$$
$$\cot 282° = \frac{1}{\tan 282°} = -0.2126$$

(ii)

$$\sin 301° = -0.8572$$
$$\cos 301° = 0.5150$$
$$\tan 301° = -1.6643$$
$$\operatorname{cosec} 301° = \frac{1}{\sin 301°} = -1.1666$$
$$\sec 301° = \frac{1}{\cos 301°} = 1.9416$$
$$\cot 301° = \frac{1}{\tan 301°} = -0.6009$$

(iii)

$$\sin 342° = -0.3090$$
$$\cos 342° = 0.9511$$
$$\tan 342° = -0.3249$$
$$\operatorname{cosec} 342° = \frac{1}{\sin 342°} = -3.2361$$
$$\sec 342° = \frac{1}{\cos 342°} = 1.0515$$
$$\cot 342° = \frac{1}{\tan 342°} = -3.0777$$

(d) (i)

$$\sin(-25°) = -0.4226$$
$$\cos(-25°) = 0.9063$$
$$\tan(-25°) = -0.4663$$
$$\operatorname{cosec}(-25°) = \frac{1}{\sin(-25°)} = -2.3662$$
$$\sec(-25°) = \frac{1}{\cos(-25°)} = 1.1034$$
$$\cot(-25°) = \frac{1}{\tan(-25°)} = -2.1445$$

(ii)

$$\sin(-122°) = -0.8480$$
$$\cos(-122°) = -0.5299$$
$$\tan(-122°) = 1.6003$$
$$\operatorname{cosec}(-122°) = \frac{1}{\sin(-122°)} = -1.1792$$
$$\sec(-122°) = \frac{1}{\cos(-122°)} = -1.8871$$
$$\cot(-122°) = \frac{1}{\tan(-122°)} = 0.6249$$

(iii)

$$\sin(-215°) = 0.5736$$
$$\cos(-215°) = -0.8192$$
$$\tan(-215°) = -0.7002$$
$$\operatorname{cosec}(-215°) = \frac{1}{\sin(-215°)} = 1.7434$$
$$\sec(-215°) = \frac{1}{\cos(-215°)} = -1.2208$$
$$\cot(-215°) = \frac{1}{\tan(-215°)} = -1.4281$$

6 **(a)** $\cot\theta = \dfrac{1}{\tan\theta}$

$= \dfrac{1}{t}$

(b) $\cot\left(90° - \theta\right) = \tan\theta$

$= t$

(c) $\tan\left(180° - \theta\right) = -\tan\theta$

$= -t$

(d) $\tan\left(360° - \theta\right) = -\tan\theta$

$= -t$

(e) $\cot\left(180° - \theta\right) = -\cot\theta$

$= -\dfrac{1}{\tan\theta}$

$= -\dfrac{1}{t}$

(f) $\tan\left(180° + \theta\right) = \tan\theta$

$= t$

8 **(a)** $\tan 305° = -1.4281$

(b) $\sin 212° = -0.5299$

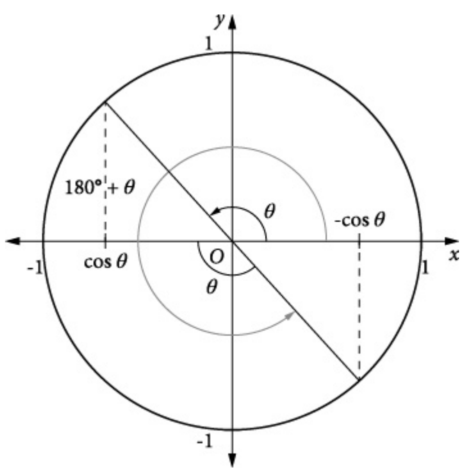
(c) $\cos\left(-140°\right) = -0.7660$

(d) $\sin\left(-160°\right) = -0.3420$

(e) $\cot 42° = \dfrac{1}{\tan 42°} = 1.1106$

(f) $\cos 260° = -0.1736$

10 **(a)**



The two triangles are congruent (AAS) so the horizontal lengths are equal.

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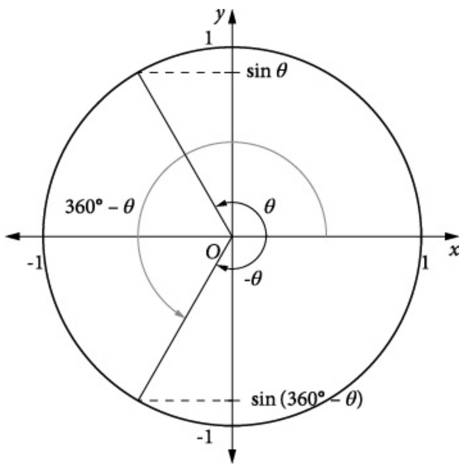
If $90° < \theta < 180°$, then it is an obtuse angle, lies in the second quadrant where $\cos\theta$ is negative . $180° + \theta$ brings it to the fourth quadrant, so $\cos(180° + \theta)$ is positive.

$\cos\theta$ here is negative, so $-\cos\theta$ is positive.

$\therefore \cos(180° + \theta) = -\cos\theta$

(b)



If $90° < \theta < 180°$, then $\sin\theta$ is positive. $\sin(360° - \theta)$ brings it to the third quadrant, so $\sin(360° - \theta)$ is negative.

$\therefore \sin(360° - \theta) = -\sin\theta$

12 (a) $\cos(360° - \theta) = \cos(-\theta) = \cos\theta$

correct

(b) $\sin(180° - \theta) = \sin\theta$

$$= \cos(90° - \theta)$$
$$\neq \cos\theta$$

incorrect

(c) $-\cos(180° + \theta) = -(-\cos\theta)$

$$= \cos\theta$$

correct

(d) $\sin(90° - \theta) = \cos\theta$

correct

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EXERCISE 2.3 TRIGONOMETRIC GRAPHS

2 **(a)** Use the known values for cosine. $\cos 0° = \cos 360° = 1$, $\cos 90° = \cos 270° = 0$,

$\cos 180° = -1$,

$\cos 60° = 0.5$

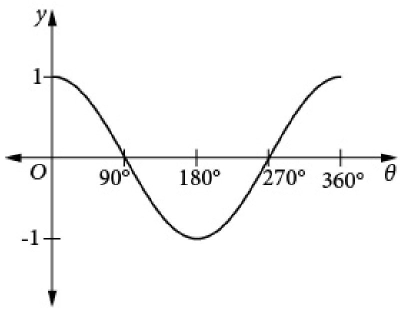
$\cos 120° = \cos(180° - 60°) = -\cos(60°) = -0.5$

$\cos 240° = \cos(180° + 60°) = -\cos(60°) = -0.5$

$\cos 300° = \cos(360° - 60°) = \cos(-60°) = \cos(60°) = 0.5$

θ	0°	60°	90°	120°	180°	240°	270°	300°	360°
$\cos\theta$	1	0.5	0	-0.5	-1	-0.5	0	0.5	1

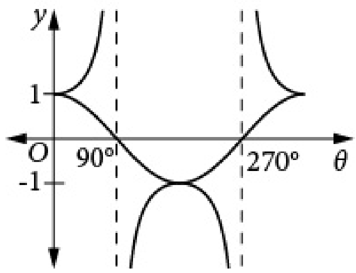
(b)



4 **(a)** $y = \sec x$ will be undefined when $\cos x = 0$, that is when $x = 90^{\circ}, 270^{\circ}$.

$y = \sec x = 1$ when $\cos x = 1$, that is when $x = 0^{\circ}, 180^{\circ}, 360^{\circ}$.

(b)



6 $y = \cot x$ is the graph of $y = \dfrac{1}{\tan x}$.

$y = \cot x$ will be undefined when $\tan x = 0$, that is when $x = 0^{\circ}, 180^{\circ}, 360^{\circ}$.

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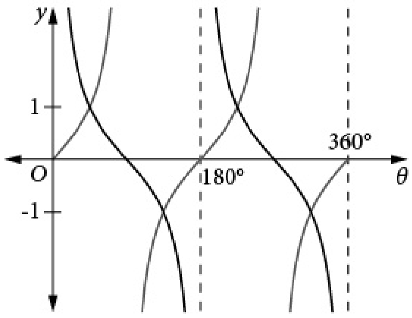
$y = \cot x = 0$ when $\tan x$ is undefined, that is when $x = 90^{\circ}, 270^{\circ}$.

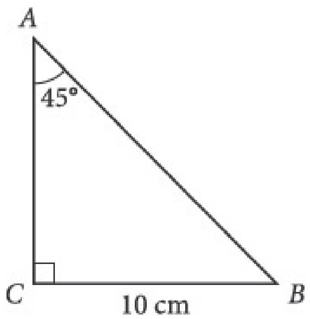
$y = \cot x = \tan x = 1$ when $\tan x$ is undefined, that is when $x = 45^{\circ}, 225^{\circ}$.

$y = \cot x = \tan x = -1$ when $\tan x$ is undefined, that is when $x = 135^{\circ}, 315^{\circ}$.



EXERCISE 2.4 EXACT VALUES OF THE TRIGONOMETRIC RATIOS

2 B



$$\sin 45° = \frac{BC}{AB}$$
$$\frac{1}{\sqrt{2}} = \frac{10}{AB}$$
$$\frac{AB}{10} = \sqrt{2}$$

$AB = 10\sqrt{2}$ cm

$$\tan 45° = \frac{BC}{AC}$$
$$1 = \frac{10}{AC}$$

$AC = 10\,\text{cm}$

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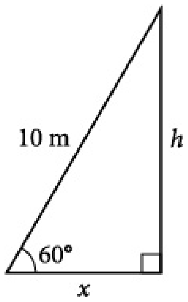
4



(a) Let the height of the wall be h.

$$\sin 60^\circ = \frac{h}{10}$$
$$h = 10 \times \sin 60^\circ$$
$$h = 10 \times \frac{\sqrt{3}}{2}$$
$$h = 5\sqrt{3}\text{ m}$$

(b) Let the distance of the foot of the ladder from the wall be x.

$$\cos 60^\circ = \frac{x}{10}$$
$$x = 10 \times \cos 60^\circ$$
$$x = 10 \times \frac{1}{2}$$
$$x = 5\text{ m}$$

6 **(a)** $\sin 30^\circ = \dfrac{AC}{20}$

$$AC = 20 \times \sin 30^\circ$$
$$AC = 20 \times \frac{1}{2}$$
$$AC = 10\text{ cm}$$

(b) $\cos 30^\circ = \dfrac{DC}{20}$

$$DC = 20 \times \cos 30^\circ$$
$$DC = 20 \times \frac{\sqrt{3}}{2}$$
$$DC = 10\sqrt{3}\text{ cm}$$

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(c) $\cos 45° = \dfrac{AB}{AC}$

$AB = AC \times \cos 45°$

$AB = 10 \times \dfrac{1}{\sqrt{2}}$

$AB = 10 \times \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{10\sqrt{2}}{2}$

$AB = 5\sqrt{2}$ cm

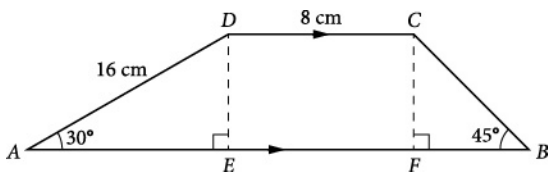
(d) $\tan 45° = \dfrac{BC}{AB}$

$BC = AC \times \tan 45°$

$BC = 5\sqrt{2} \times 1$

$BC = 5\sqrt{2}$ cm

8



Draw two vertical lines, one from *D* and the other from *C* to meet *AB* at *E* and *F*.

$\cos 30° = \dfrac{AE}{16}$

$AE = 16 \times \cos 30°$

$AE = 16 \times \dfrac{\sqrt{3}}{2}$

$AE = 8\sqrt{3}$

$\sin 30° = \dfrac{DE}{16}$

$DE = 16 \times \sin 30°$

$DE = 16 \times \dfrac{1}{2} = 8$

$CF = DE = 8$

$\tan 45° = \dfrac{CF}{FB}$

$FB = \dfrac{CF}{\tan 45°} = \dfrac{8}{1} = 8$

$AB = AE + EF + FB$

$= 8\sqrt{3} + 8 + 8$

$= 16 + 8\sqrt{3}$

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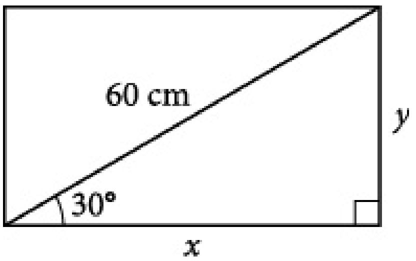
$$\cos 45° = \frac{FB}{BC}$$
$$BC = \frac{FB}{\cos 45°}$$
$$BC = 8 \div \frac{1}{\sqrt{2}}$$
$$BC = 8\sqrt{2}$$

$$\begin{aligned} AB + BC + DC + AD &= 16 + 8\sqrt{3} + 8\sqrt{2} + 8 + 16 \\ &= 40 + 8\sqrt{3} + 8\sqrt{2} \end{aligned}$$

Perimeter is $40 + 8\sqrt{3} + 8\sqrt{2}$ cm.

10



$$\cos 30° = \frac{x}{60}$$
$$x = 60 \times \cos 30°$$
$$x = 60 \times \frac{\sqrt{3}}{2}$$
$$x = 30\sqrt{3}$$

$$\sin 30° = \frac{y}{60}$$
$$y = 60 \times \sin 30°$$
$$y = 60 \times \frac{1}{2}$$
$$y = 30$$

The dimensions of the rectangle are 30 cm by $30\sqrt{3}$ cm.

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EXERCISE 2.5 MORE TRIGONOMETRIC EXACT VALUES

2 **(a)** $\sin 180° = 0$

(b) $\cos 210° = \cos(180° + 30°)$

$$= -\cos 30°$$
$$= -\frac{\sqrt{3}}{2}$$

(c) $\tan 225° = \tan(180° + 45°)$

$$= \tan 45°$$
$$= 1$$

(d) $\cos 240° = \cos(180° + 60°)$

$$= -\cos 60°$$
$$= -\frac{1}{2}$$

(e) $\tan 180° = 0$

4 **(a)** $\sin 360° = 0$

(b) $\cos 390° = \cos(360° + 30°)$

$$= \cos 30°$$
$$= \frac{\sqrt{3}}{2}$$

(c) $\tan 405° = \tan(360° + 45°)$

$$= \tan 45°$$
$$= 1$$

(d) $\cos 450° = \cos(360° + 90°)$

$$= \cos 90°$$
$$= 0$$

(e) $\sin 420° = \sin(360° + 60°)$

$$= \sin 60°$$
$$= \frac{\sqrt{3}}{2}$$

6 **(a)** correct

$$\cos 180° = -1$$

(b) incorrect

$$\sin 45° = \frac{1}{\sqrt{2}}$$

(c) correct

$$\sin 270° = -1$$

(d) correct

$$\begin{aligned} \tan 495° &= \tan(360° + 135°) \\ &= \tan 135° \\ &= \tan(180° - 45°) \\ &= -\tan 45° \\ &= -1 \end{aligned}$$

8 $\tan\theta = 1$ when $\theta = 45°$.

$$\begin{aligned} \tan\theta &= -1 \\ \theta &= 180° - 45°, 360° - 45° \\ \theta &= 135°, 315° \end{aligned}$$

10 $\sin\theta = \cos\theta$

$$\begin{aligned} \frac{\sin\theta}{\cos\theta} &= 1 \\ \tan\theta &= 1 \\ \theta &= 45°, 180° + 45° \\ \theta &= 45°, 225° \end{aligned}$$

12 $2\cos\theta + 1 = 0$

$$2\cos\theta = -1$$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = 180° - 60°, 180° + 60°$$
$$\theta = 120°, 240°$$

14 $\sin\theta + \sqrt{3}\cos\theta = 0$

$$\sin\theta = -\sqrt{3}\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = -\sqrt{3}$$
$$\tan\theta = -\sqrt{3}$$
$$\theta = 180° - 60°, 360° - 60°$$
$$\theta = 120°, 300°$$

16 $0° \le \theta \le 360°$

$$0° \le \frac{\theta}{2} \le 180°$$
$$\cos\frac{\theta}{2} = \frac{1}{2}$$
$$\frac{\theta}{2} = 60°$$
$$\theta = 120°$$

18 $0° \le \theta \le 360°$

$$0° \le \frac{\theta}{2} \le 180°$$
$$\sin\frac{\theta}{2} = -\frac{1}{\sqrt{2}}$$

There is no solution as $\sin\frac{\theta}{2}$ is positive for $0° \le \frac{\theta}{2} \le 180°$.

20 (a) $\sec A = \dfrac{1}{\cos A} = \dfrac{1}{c}$

(b) $\sec\left(180^\circ - A\right) = \dfrac{1}{\cos\left(180^\circ - A\right)} = \dfrac{1}{-\cos A} = -\dfrac{1}{c}$

(c) $\sec\left(-A\right) = \dfrac{1}{\cos\left(-A\right)} = \dfrac{1}{\cos A} = \dfrac{1}{c}$

(d) $\sec\left(180^\circ + A\right) = \dfrac{1}{\cos\left(180^\circ + A\right)} = \dfrac{1}{-\cos A} = -\dfrac{1}{c}$

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22 D

$$\begin{aligned}\operatorname{cosec}(360°+\theta) &= \frac{1}{\sin(360°+\theta)}\\ &= \frac{1}{\sin\theta}\\ &= \operatorname{cosec}\theta\end{aligned}$$

24 (a) $\operatorname{cosec}\left(90°\right)=\dfrac{1}{\sin 90°}$

$$\begin{aligned}&=\frac{1}{1}\\ &=1\end{aligned}$$

(b) $\sec 150°=\dfrac{1}{\cos 150°}$

$$\begin{aligned}&=\frac{1}{\cos\left(180°-30°\right)}\\ &=-\frac{1}{\cos 30°}\\ &=-\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\\ &=-\frac{2}{\sqrt{3}}\end{aligned}$$

(c) $\cot 240°=\dfrac{1}{\tan 240°}$

$$\begin{aligned}&=\frac{1}{\tan\left(180°+60°\right)}\\ &=\frac{1}{\tan 60°}\\ &=\frac{1}{\sqrt{3}}\end{aligned}$$

(d) $\operatorname{cosec}180°=\dfrac{1}{\sin 180°}$

$$=\frac{1}{0}$$

Undefined

(e) $$\begin{aligned}\sec 300^\circ &= \frac{1}{\cos 300^\circ}\\ &= \frac{1}{\cos\left(360^\circ - 60^\circ\right)}\\ &= \frac{1}{\cos 60^\circ}\\ &= \frac{1}{\left(\frac{1}{2}\right)}\\ &= 2\end{aligned}$$

(f) $$\begin{aligned}\cot 315^\circ &= \frac{1}{\tan 315^\circ}\\ &= \frac{1}{\tan\left(360^\circ - 45^\circ\right)}\\ &= -\frac{1}{\tan 45^\circ}\\ &= -\frac{1}{1}\\ &= -1\end{aligned}$$

(g) $$\begin{aligned}\sec\left(-30^\circ\right) &= \frac{1}{\cos\left(-30^\circ\right)}\\ &= \frac{1}{\cos 30^\circ}\\ &= \frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\\ &= \frac{2}{\sqrt{3}}\end{aligned}$$

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(h) $\operatorname{cosec}510° = \dfrac{1}{\sin(360° + 150°)}$

$$= \frac{1}{\sin 150°}$$
$$= \frac{1}{\sin(180° - 30°)}$$
$$= \frac{1}{\sin 30°}$$
$$= \frac{1}{\left(\frac{1}{2}\right)}$$
$$= 2$$

(i) $\cot(-150°) = \dfrac{1}{\tan(-150°)}$

$$= -\frac{1}{\tan 150°}$$
$$= -\frac{1}{\tan(180° - 30°)}$$
$$= -\frac{1}{-\tan 30°}$$
$$= \frac{1}{\tan 30°}$$
$$= \frac{1}{\left(\frac{1}{\sqrt{3}}\right)}$$
$$= \sqrt{3}$$

(j) $\sec 120° + \operatorname{cosec} 150° = \dfrac{1}{\cos 120°} + \dfrac{1}{\sin 150°}$

$$= \frac{1}{\cos(180° - 60°)} + \frac{1}{\sin(180° - 30°)}$$
$$= -\frac{1}{\cos 60°} + \frac{1}{\sin 30°}$$
$$= -\frac{1}{\left(\frac{1}{2}\right)} + \frac{1}{\left(\frac{1}{2}\right)}$$
$$= 0$$

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26 (a) $\operatorname{cosec}\theta = -\frac{2}{\sqrt{3}}$

$$\frac{1}{\sin\theta} = -\frac{2}{\sqrt{3}}$$
$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$\sin 60° = \frac{\sqrt{3}}{2}$.

$\sin\theta < 0$, so θ is in the third and fourth quadrants.

$$\theta = 180° + 60°, 360° - 60°$$
$$\theta = 240°, 300°$$

(b) $\cot\theta = \sqrt{3}$

$$\frac{1}{\tan\theta} = \sqrt{3}$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$

$\tan 30° = \frac{1}{\sqrt{3}}$.

$\tan\theta > 0$, so θ is in the third and fourth quadrants.

$$\theta = 30°, 180° + 30°$$
$$\theta = 30°, 210°$$

(c) $\sec\theta = -1$

$$\frac{1}{\cos\theta} = -1$$
$$\cos\theta = -1$$

The only solution is $\theta = 180°$.

(d) $\operatorname{cosec}\theta = -\sec\theta$

$$\frac{1}{\sin\theta} = -\frac{1}{\cos\theta}$$
$$-1 = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = -1$$

$\tan 45° = -1$

$\tan\theta < 0$, so θ is in the second and fourth quadrants.

$$\theta = 180° - 45°, 360° - 45°$$
$$\theta = 135°, 315°$$

(e) There is no solution as $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ and cannot equal zero.

(f) $\sec\theta = 2$

$$\frac{1}{\cos\theta} = 2$$
$$\cos\theta = \frac{1}{2}$$

$$\cos 60° = \frac{1}{2}$$

$\cos\theta > 0$, so θ is in the first and fourth quadrants.

$$\theta = 60°, 360° - 60°$$
$$\theta = 60°, 300°$$

(g) $0 \le \theta \le 360°$

$$0 \le 2\theta \le 720°$$

$$\cot 2\theta = \frac{1}{\sqrt{3}}$$
$$\frac{1}{\tan 2\theta} = \frac{1}{\sqrt{3}}$$
$$\tan 2\theta = \sqrt{3}$$

Quadrant 1 is $\tan 60° = \sqrt{3}$

$\tan\theta > 0$, so θ is in the first and third quadrants.

$$2\theta = 60°, 180° + 60°, 360° + 60°, 540 + 60°$$
$$2\theta = 60°, 240°, 420°, 600°$$
$$\theta = 30°, 120°, 210°, 300°$$

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(h) $0 \le \theta \le 360°$

$$0 \le \frac{\theta}{2} \le 180°$$

$$\sec\frac{\theta}{2} = \frac{2}{\sqrt{3}}$$
$$\frac{1}{\cos\frac{\theta}{2}} = \frac{2}{\sqrt{3}}$$
$$\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$$
$$\cos 30° = \frac{\sqrt{3}}{2}$$

$\cos\theta > 0$, and $0 \le \frac{\theta}{2} \le 180°$, so $\frac{\theta}{2}$ must be in the first quadrant.

$$\frac{\theta}{2} = 30°$$
$$\theta = 60°$$

EXERCISE 2.6 DIRECTION AND BEARING

2 **(a)** $\angle ACB = \angle NAC = 45°$

The bearing of A from C is $180° + 45° = 225°$

This is also SW.

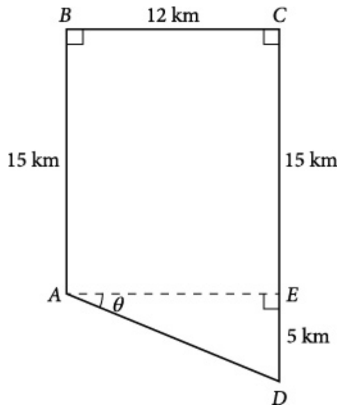
(b) Using alternate angles, $\angle CBA = 180° - 120° = 60°$

The bearing of A from C is $360° - 60° = 300°$

This is also N $60°$ W.

(c) B is due south of C, so $180°$ or S

4



$$AD^2 = 12^2 + 5^2 = 169$$
$$AD = \sqrt{169}$$
$$AD = 13$$

The distance is $13\,\text{km}$.

Let $\angle DAE = \theta$

$$\tan\theta = \frac{5}{12} = 0.4166...$$
$$\theta \approx 22°37'$$

The bearing is $90° + 22°37' = 112°37'$ rounded to the nearest minute.

6 $\angle ALB = 180° - 37° - 53° = 90°$

$$AB^2 = 12^2 + 5^2 = 169$$
$$AB = \sqrt{169}$$
$$AB = 13$$

B is $13\,\text{km}$ from A.

Using alternate angles, $\angle NAL = 180° - 37° = 143°$

$$\tan\angle LAB = \frac{12}{5}$$
$$\angle LAB \approx 67°23'$$

The bearing of B from A is $143° + 67°23' = 210°23'$, rounded to the nearest minute.

8 $\angle CAB = 180° - 145° = 35°$

$\angle ACB = 180° - 35° - 55° = 90°$

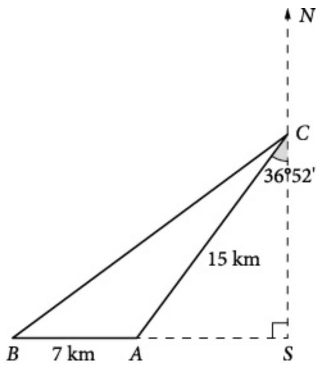
$$\sin 55° = \frac{AC}{20}$$
$$AC = 20 \times \sin 55°$$
$$AC = 16.383...$$

The distance from lighthouse A to the ship is 16.4 km, rounded to 1 decimal place.

$$\cos 55° = \frac{BC}{20}$$
$$BC = 20 \times \cos 55°$$
$$BC = 11.471...$$

The distance from lighthouse B to the ship is 11.5 km, rounded to 1 decimal place.

10 Let C be the starting point at camp.



$$\sin 36°52' = \frac{AS}{15}$$
$$AS = 15 \sin 36°52' = 8.9993...$$
$$BS = BA + AS$$
$$BS = 7 + 8.9993... = 15.9993...$$

Store this value in your calculator's memory.

$$\cos 36°52' = \frac{CS}{15}$$
$$CS = 15 \cos 36°52' = 12.0005...$$

$$BC^2 = BS^2 + CS^2$$
$$BC = \sqrt{(15.9993...)^2 + (12.0005...)^2}$$
$$BC = 19.9997...$$

The distance of his new position from the camp is 20 km, rounded to the nearest km.

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$$\tan\angle BCS = \frac{BS}{CS}$$
$$\tan\angle BCS = \frac{15.9993...}{12.0005...}$$
$$\angle BCS \approx 53°8'$$

The bearing of his new position from the camp is $180° + 53°8' = 233°8'$ to the nearest minute.

EXERCISE 2.7 ANGLES OF ELEVATION AND DEPRESSION

2 Let h be the height of the building.

$$\begin{aligned} h &= 2 + 18\tan 30° \\ &= 2 + 18 \times \frac{1}{\sqrt{3}} \\ &= 12.3923... \end{aligned}$$

The height of the building is 12.4 m, rounded to one decimal place.

4 **(a)** Let h be the height of A from the ground.

$$\tan 36°52' = \frac{h}{PD + CB}$$
$$\tan 36°52' = \frac{h}{60}$$
$$h = 60 \times \tan 36°52'$$
$$h \approx 44.9947...$$

The height of A from the ground is 45.0 m, rounded to one decimal place.

(b) $AP^2 = 60^2 + 45^2 = 5625$

$AP = \sqrt{5625} = 75$

The distance from A to P is 75 m.

(c) $\tan\angle ACB = \dfrac{AB}{BC}$

$$\tan\angle ACB = \frac{12}{5} = 2.4$$
$$\angle ACB \approx 67°23'$$

The angle of elevation of A from C is $67°23'$, rounded to the nearest minute.

6 **(a)** $\tan 20° = \dfrac{50}{AC}$

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$$AC = \frac{50}{\tan 20°} = 137.3738...$$

The horizontal distance between the buildings are 137.4 m, rounded to one decimal place.

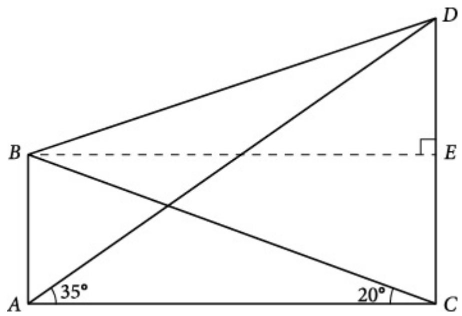
(b) $\tan 35° = \frac{CD}{AC}$

Use the unrounded value of AC.

$$CD = 137.3738... \times \tan 35°$$
$$CD \approx 96.1902...$$

The height of CD is 96.2 m, rounded to one decimal place.

(c)



$$\tan \angle EBD = \frac{DE}{BE}$$
$$\tan \angle EBD = \frac{CD - AB}{AC}$$
$$\tan \angle EBD = \frac{96.1902... - 50}{137.3738...}$$
$$\angle EBD \approx 18°35'$$

The angle of elevation of D as seen from B is $18°35'$ to the nearest minute.

8 **(a)** Let the horizontal distance from the point of observation to the wall be of x.

$$\tan 21° = \frac{5}{x}$$
$$x = \frac{5}{\tan 21°} = 13.0254...$$

The horizontal distance from the point of observation to the wall is 13.0 m, rounded to one decimal place. Keep the exact value in your calculator.

(b) Let the height of the wall be h.

$$h = 5 + 13.0254... \times \tan 32° = 13.1392...$$

The height of the wall is 13 m, rounded to the nearest metre.

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10 (a) Let the height of O above street level be x.

$$\tan 50° = \frac{x}{20}$$
$$x = 20 \times \tan 50° = 23.835...$$

Keep the exact value in your calculator.

The height of O above street level is $45 - 23.835... = 21.16$ m, rounded to two decimal places.

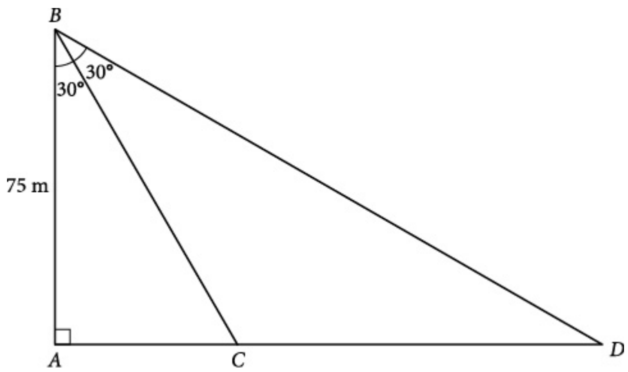
(b) Let the angle of elevation of O from the foot of the first building be θ.

The length of the side opposite θ is $45 - 23.835... = 21.165...$ m.

$$\tan\theta = \frac{21.165...}{20}$$
$$\theta \approx 46°37'$$

The angle of elevation of O from the foot of the first building is $46°37'$ rounded to the nearest minute.

12 (a)



Let C and D be the positions of the two buoys.

$$\tan 30° = \frac{AC}{75}$$
$$AC = 75 \times \tan 30°$$
$$= 75 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= 75 \times \frac{\sqrt{3}}{3}$$
$$= 25\sqrt{3}$$

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$$\tan 60° = \frac{AD}{75}$$
$$AD = 75 \times \tan 60°$$
$$AD = 75 \times \sqrt{3}$$
$$AD = 75\sqrt{3}$$

The distance of each buoy from the light house is $25\sqrt{3}$ m and $75\sqrt{3}$ m respectively.

(b) $75\sqrt{3} - 25\sqrt{3} = 50\sqrt{3}$

The distance between the two buoys is $50\sqrt{3}$ m.

EXERCISE 2.8 THE SINE RULE

2 $A = 180° - 45° - 120°$
$= 15°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 15°} = \frac{10}{\sin 45°}$$
$$a = 10\sqrt{2} \times \sin 15° = 3.6602...$$

$a = 3.7$, rounded to one decimal place.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 120°} = \frac{10}{\sin 45°}$$
$$\frac{c}{\frac{\sqrt{3}}{2}} = \frac{10}{\frac{1}{\sqrt{2}}}$$
$$= 10\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{6} = 12.2474...$$

$c = 5\sqrt{6}$ or 12.2, rounded to one decimal place.

4 $\dfrac{b}{\sin 45°} = \dfrac{5}{\sin 60°}$

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$$\frac{b}{\frac{1}{\sqrt{2}}}=\frac{5}{\frac{\sqrt{3}}{2}}$$

$$\begin{aligned}b&=5\times\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{2}}\\&=\frac{10}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}\\&=\frac{10\sqrt{6}}{6}\\&=\frac{5\sqrt{6}}{3}\end{aligned}$$

6 $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$$\frac{3}{\sin A}=\frac{5}{\sin 2A}$$
$$5\sin A=3\sin 2A$$

8 $\angle C=180°-42°-28°$

$$\begin{aligned}&=110°\\a&=BC=6\\AB&=c\end{aligned}$$

$$\frac{AB}{\sin C}=\frac{BC}{\sin A}$$
$$\frac{AB}{\sin 110°}=\frac{6}{\sin 42°}$$

Since $110°=180°-70°$, $\sin 110°=\sin 70°$

$$\frac{AB}{\sin 70°}=\frac{6}{\sin 42°}$$

(a) incorrect

(b) correct

(c) incorrect

(d) correct

10 $\dfrac{\sin B}{13}=\dfrac{\sin 20°}{10}$

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$$\sin B = \frac{13 \times \sin 20°}{10}$$

$\angle ABS$ is obtuse,

$\therefore B = 180° - 26°23'57.93'$ or $180° - 26.3994...°$

$B \approx 153°36'2.07"$ or $153.6005...°$

Do not round this value.

$S = 180° - 20° - 153°36'2.07'' = 6°23'57.03''$, or

$S = 180° - 20° - 153.6005...° = 6.3994...°$ or

$$\begin{aligned} \frac{s}{\sin S} &= \frac{a}{\sin A} \\ \frac{s}{\sin 6.3994...°} &= \frac{10}{\sin 20°} \\ s &= \frac{10 \times \sin 6.3994...°}{\sin 20°} \\ AB &= 3.2588... \end{aligned}$$

The closest point to A that the goat can reach is B. This distance is $AB = 3.26$ m, rounded to two decimal places.

12 $\dfrac{\sin \angle RPQ}{QR} = \dfrac{\sin \angle PRQ}{PQ}$

$$\begin{aligned} \frac{\sin \angle RPQ}{22} &= \frac{\sin 15°}{20} \\ \sin \angle RPQ &= \frac{22 \times \sin 15°}{20} \\ \angle RPQ &\approx 16°32'27.51" \end{aligned}$$

If $\angle RPQ$ is acute, then $\angle RPQ = 16°32'$ to the nearest minute. If $\angle RPQ$ is obtuse, then $\angle RPQ = 163°28'$ to the nearest minute.

When $\angle RPQ = 16°32'$, then

$$\begin{aligned} \angle PQR &= 180° - 16°32' - 15° \\ &= 148°28' \end{aligned}$$

$$\begin{aligned} \frac{PR}{\sin 148°28'} &= \frac{20}{\sin 15°} \\ PR &= \frac{20 \times \sin 148°28'}{\sin 15°} \\ PR &\approx 40.413... \end{aligned}$$

$PR = 40.41$ cm round to two decimal places.

When $\angle RPQ = 163°28'$, then

$$\angle PQR = 180° - 163°28' - 15° \\ = 1°32'$$

$$\frac{PR}{\sin 1°32'} = \frac{20}{\sin 15°}$$
$$PR = \frac{20 \times \sin 1°32'}{\sin 15°}$$
$$PR \approx 2.067...$$

$PR = 2.07$ cm round to two decimal places.

14 (a) $\dfrac{BC}{AC} = \dfrac{2}{1}$

$BC = 2AC$

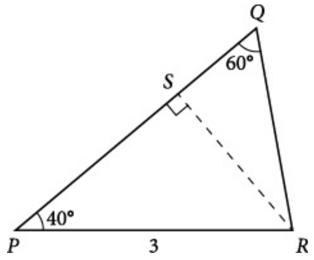
Note that $\angle BAC = A,\ BC = a,\ AC = b.$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ABC}{AC}$$
$$\frac{\sin \angle BAC}{2AC} = \frac{\frac{1}{4}}{AC}$$
$$\sin \angle BAC = \frac{\frac{1}{4}}{AC} \times \frac{2AC}{1} = \frac{1}{2}$$

(b) $\sin \angle BAC = \dfrac{1}{2}$

$$\angle BAC = 30° \text{ or } 180° - 30°$$
$$\angle BAC = 30° \text{ or } 150°$$

16



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(a) $\angle PRQ = 180° - 40° - 60°$

$= 80°$

$$\frac{r}{\sin 80°} = \frac{3}{\sin 60°}$$
$$r = \frac{3 \times \sin 80°}{\sin 60°} = 3.4114...$$

$$\frac{p}{\sin 40°} = \frac{3}{\sin 60°}$$
$$p = \frac{3 \times \sin 40°}{\sin 60°} = 2.2266...$$

Perimeter $\approx 3.411 + 2.227 + 3 = 8.6$ cm (1 d.p.)

(b) $\sin 40° = \dfrac{RS}{3}$

$RS = 3 \times \sin 40° = 1.928...$

The length of the perpendicular line from R to PQ is 1.9 cm, rounded to one decimal place.

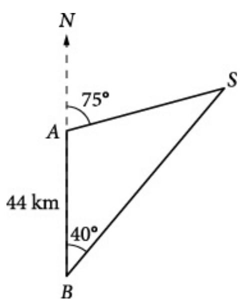
18 $\angle XZY = 180° - 68° - 82°$

$= 30°$

$$\frac{XZ}{\sin 82°} = \frac{5}{\sin 30°}$$
$$XZ = \frac{5 \times \sin 82°}{\sin 30°} = 9.902...$$

The length of XZ is 9.90 m, rounded to two decimal places.

20



$A = 180° - 75° = 105°$
$S = 180° - 105° - 40° = 35°$

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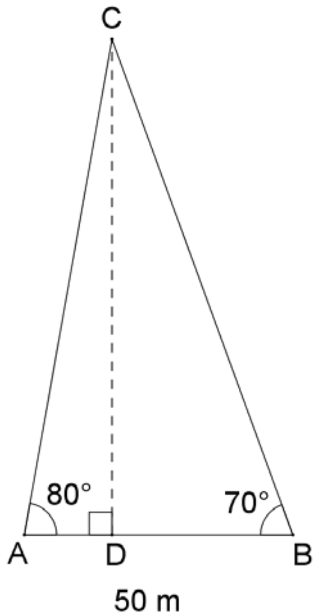
$$\frac{a}{\sin 105°} = \frac{44}{\sin 35°}$$

$$a = \frac{44 \times \sin 105°}{\sin 35°} = 74.097...$$

The distance of the ship from B is 74 km, to the nearest km.

22



$$C = 180° - 80° - 70° = 30°$$

$$\frac{b}{\sin 70°} = \frac{50}{\sin 30°}$$

$$AC = \frac{50 \times \sin 70°}{\sin 30°}$$

$$\frac{CD}{AC} = \sin 80°$$

$$CD = AC \times \sin 80°$$

$$CD = \frac{50 \times \sin 70°}{\sin 30°} \times \sin 80° = 92.54...$$

The width of the river is 93 m, to the nearest metre.

24 Before starting this question, students should note that sides AB and AC are opposite both angles D and that both angles D add to $180°$, so the value of their sines will be equal.

Let $\angle BAD = \theta$, then $\angle CAD = \theta$

Let $\angle BDA = \phi$, then $\angle CDA = 180° - \theta$

In ΔABD, $\dfrac{BD}{\sin\theta}=\dfrac{AB}{\sin\phi}$

$$BD=\frac{AB\sin\theta}{\sin\phi}$$

In ΔADC, $\dfrac{DC}{\sin\theta}=\dfrac{AC}{\sin\left(180°-\phi\right)}=\dfrac{AC}{\sin\phi}$

$$DC=\frac{AC\sin\theta}{\sin\phi}$$

$$\frac{BD}{DC}=\frac{\dfrac{AB\sin\theta}{\sin\phi}}{\dfrac{AC\sin\theta}{\sin\phi}}$$

$$\frac{BD}{DC}=\frac{AB\sin\theta}{\sin\phi}\times\frac{\sin\phi}{AC\sin\theta}$$

$$\frac{BD}{DC}=\frac{AB}{AC}$$

26 $b>a\Rightarrow B>A$

$\therefore A$ cannot be obtuse.

Therefore a unique triangle can be certain if $\angle B$ is given.

28 $\dfrac{\sin B}{8.5}=\dfrac{\sin 40°}{6.5}$

$\sin B=\dfrac{8.5\times\sin 40°}{6.5}$

$B\approx 57°12'$ or $B\approx 180°-57°12'=122°48'$

Check the second solution is possible.

When $B=122°48'$, $C=180°-122°48'-40°>0$, so both solutions are possible.

$B=57°12'$ or $122°48'$

30 $\dfrac{\sin B}{6.7}=\dfrac{\sin 53°40'}{5.4}$

$\sin B=\dfrac{6.7\times\sin 53°40'}{5.4}$

$B\approx 88°14'$ or $180°-88°14'=91°46'$

If $B=88°14'$, $C=180°-53°40'-88°14'=38°6'$

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$$\frac{c}{\sin 38°6'} = \frac{5.4}{\sin 53°40'}$$

$$c = \frac{5.4 \times \sin 38°6'}{\sin 53°40'} = 4.136... \approx 4.1$$

If $B = 91°46'$, $C = 180° - 53°40' - 91°46' = 34°34'$

$$\frac{c}{\sin 33°34'} = \frac{5.4}{\sin 53°40'}$$

$$c = \frac{5.4 \times \sin 34°34'}{\sin 53°40'} = 3.803... \approx 3.8$$

Two possible triangles can be formed.

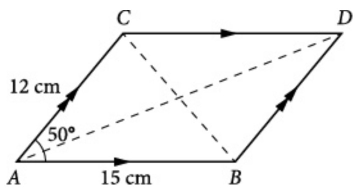
Either $B = 88°14'$, $C = 38°6'$, $c = 4.1$ cm or $B = 91°46'$, $C = 34°34'$, $c = 3.8$ cm.

EXERCISE 2.9 THE COSINE RULE

2 The smallest angle of a triangle is opposite the shortest side.

$$\cos\theta = \frac{11^2 + 13^2 - 9^2}{2 \times 11 \times 13} = \frac{19}{26}$$

4



$$CB^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \times \cos 50°$$

$$CB = \sqrt{369 - 360\cos 50°} = 11.73...$$

$$\angle ACD = 180° - 50°$$
$$= 130°$$

$$AD^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \times \cos 130°$$

$$AD = \sqrt{369 - 360\cos 130°} = 24.50...$$

The lengths of the two diagonals are 11.7 m and 24.5 m, rounded to one decimal place.

6 $x^2 = 3.2^2 + 4.8^2 - 2 \times 3.2 \times 4.8 \times \cos 65°$

$$x^2 = 3.2^2 + 4.8^2 - 30.72\cos 65°$$

$$x = \sqrt{3.2^2 + 4.8^2 - 30.72\cos 65°} = 4.505... \approx 4.5$$

(a) incorrect

(b) incorrect

(c) correct

(d) correct

8 $\cos\theta = \dfrac{5^2 + 6^2 - 9^2}{2\times5\times6}$

$$\cos\theta = -\frac{1}{3}$$
$$\theta \approx 109°28'$$

The largest angle is $109°28'$ round to the nearest minute.

10 (a) The angle required will be opposite the 14 cm diagonal.

$$\cos\theta = \frac{8^2 + 10^2 - 14^2}{2\times8\times10}$$
$$\cos\theta = -\frac{1}{5}$$
$$\theta \approx 101°32'$$

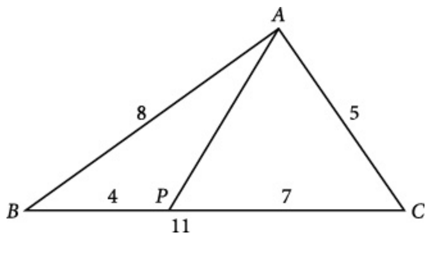
The size of the angles of the parallelogram are $101°32'$ and $180° - 101°32' = 78°28'$, rounded to the nearest minute.

(b) $x^2 = 8^2 + 10^2 - 2\times8\times10\times\cos 78°28'$

$$x = \sqrt{164 - 160\cos 78°28'} \approx 11.489...$$

The length of the other diagonal is 11.5 cm, rounded to one decimal place.

12



$$\cos\angle ABC = \frac{8^2 + 11^2 - 5^2}{2\times8\times11}$$
$$\cos\angle ABC = \frac{10}{11}$$

$$AP^2 = 8^2 + 4^2 - 2\times8\times4\times\cos\left(\angle ABC\right)$$
$$= 64 + 16 - 64\times\frac{10}{11} = \frac{240}{11}$$

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$$AP = \sqrt{\frac{240}{11}} \neq 5$$

$$\cos \angle ACB = \frac{5^2 + 11^2 - 8^2}{2 \times 5 \times 11}$$

$$\cos \angle ACB = \frac{41}{55}$$

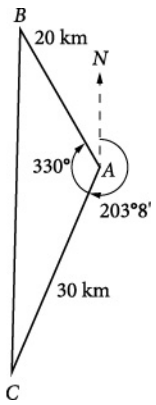
(a) incorrect

(b) correct

(c) correct

(d) correct

14



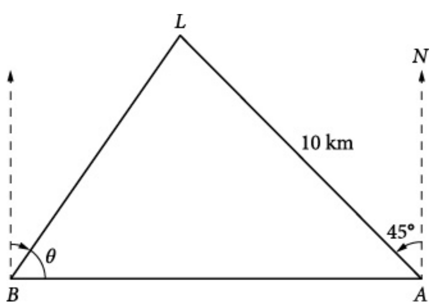
$$A = 330° - 203°8' = 126°52'$$

$$BC^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \times \cos 126°52'$$

$$BC = \sqrt{1300 - 1200 \cos 126°52'} \approx 44.94...$$

The distance from B to C is 44.9 km, rounded to one decimal place.

16



(a) Let A be the initial position of the ship, and B be the position of the ship after 45 minutes.

$$AB = 16 \times \frac{3}{4} = 12$$

Keep this value in your calculator.

After 45 minutes, the ship is 8.6 km from the lighthouse, rounded to one decimal place.

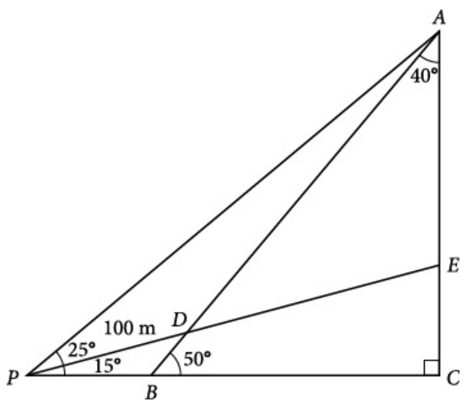
(b) $\cos\theta = \dfrac{BL^2 + 12^2 - 10^2}{2 \times BL \times 12}$

$= \dfrac{8.619...^2 + 12^2 - 10^2}{2 \times 8.619... \times 12}$ $\quad BL^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \times \cos 45°$

$\theta \approx 55°7'$ $\quad BL = 8.619...$

The bearing of the lighthouse from the ship is $90° - 55°7' = 34°53'$, rounded to the nearest minute.

18



$$\begin{aligned}\angle PAC &= 180° - \angle APC - \angle PCA \\ &= 180° - 40° - 90° \\ &= 50°\end{aligned}$$

$$\begin{aligned}\angle BAC &= 180° - \angle ABC - \angle BCA \\ &= 180° - 50° - 90° \\ &= 40°\end{aligned}$$

$$\begin{aligned}\angle PAB &= \angle PAC - \angle BAC \\ &= 50° - 40° \\ &= 10°\end{aligned}$$

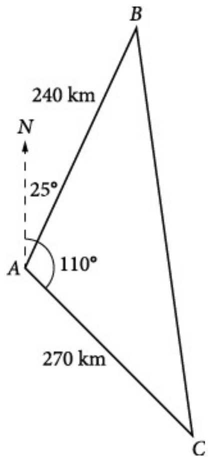
$$\begin{aligned}\angle PDA &= 180° - \angle DPA - \angle PAD \\ &= 180° - 25° - 10° \\ &= 145°\end{aligned}$$

$$\frac{AP}{\sin 145°} = \frac{DP}{\sin 10°}$$
$$AP = \frac{100 \times \sin 145°}{\sin 10°}$$

$$\sin 40° = \frac{AC}{AP}$$
$$AC = \frac{100 \times \sin 145° \times \sin 40°}{\sin 10°}$$
$$AC = 212.318...$$

The vertical height of A above P is 212.3 m, rounded to one decimal place.

20



$$\angle BAC = 135° - 25° = 110°$$

$$AB = 80 \times 3 = 240$$
$$AC = 90 \times 3 = 270$$

$$BC^2 = 240^2 + 270^2 - 2 \times 240 \times 270 \times \cos 110°$$
$$BC = \sqrt{130\ 500 - 129\ 600 \cos 110°}$$
$$= 418.12...$$

They are 418 m apart after 3 hours, rounded to the nearest metre.

EXERCISE 2.10 AREA OF A TRIANGLE

2 Area $= \frac{1}{2} \times 4 \times 5 \times \sin 53°8' = 8.00...$

The area is 8 cm^2, rounded to the nearest square centimetre.

4 B

$$\cos C = \frac{6^2 + 7^2 - 11^2}{2 \times 6 \times 7}$$

$$\cos C = -\frac{3}{7}$$

$$C \approx 115°22'36.96''$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2} \times 6 \times 7 \times \sin 115°22'36.96'' \\ &\approx 18.97...\end{aligned}$$

6 (a) $\cos \angle APB = \dfrac{10^2 + 8^2 - 14^2}{2 \times 10 \times 8}$

$$\cos \angle APB = -\frac{1}{5}$$

$$\angle APB \approx 101.5369...°$$

$$\cos(\angle APC) = \frac{12^2 + 8^2 - 18^2}{2 \times 12 \times 8}$$

$$\cos \angle APC = -\frac{29}{48}$$

$$\angle APC \approx 127.1688...°$$

$$\begin{aligned}\angle BPC &= 360° - 101.5369...° - 127.1688...° \\ &= 131.2941...°\end{aligned}$$

$$BC^2 = 12^2 + 10^2 - 2 \times 12 \times 10 \times \cos 131.2941...°$$

$$BC = \sqrt{402.3819...}$$

$$BC \approx 20.0594...$$

Keep this value in your calculator.

The length of BC is 20 cm to the nearest cm.

(b) $\cos(\angle CAB) = \dfrac{18^2 + 14^2 - 20.0594...^2}{2 \times 18 \times 14} = 0.2333...$

$$\angle CAB = 76.5044...°$$

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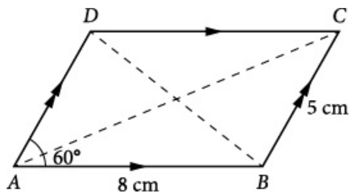
$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin A\\ &= \frac{1}{2}\times 18\times 14\times \sin 76.5044...^\circ\\ &= 122.5209...\end{aligned}$$

The area is $122.5\ \text{cm}^2$, rounded to one decimal place.

8



(a) $BD^2 = 8^2 + 5^2 - 2\times 8\times 5\times \cos 60^\circ$

$$BD = \sqrt{49}$$
$$BD = 7$$

$$\begin{aligned}\angle ADC &= 180^\circ - 60^\circ\\ &= 120^\circ\end{aligned}$$

$$AC^2 = 8^2 + 5^2 - 2\times 8\times 5\times \cos 120^\circ$$
$$AC = \sqrt{129}$$
$$AC = 11.35...$$

$AC = 11.4$ cm, rounded to one decimal place, and $BD = 7$ cm.

(b) Area of $\Delta ABD = \Delta DBC$

$$\begin{aligned}\text{Area} &= 2\times\frac{1}{2}\times 8\times 5\times \sin 60^\circ\\ &= 40\times\frac{\sqrt{3}}{2}\\ &= 20\sqrt{3}\end{aligned}$$

The area of $ABCD$ is $20\sqrt{3}$ cm^2, or 34.6 cm^2, rounded to one decimal place.

10 Find the angle opposite the 80 m side. Any angle may be chosen.

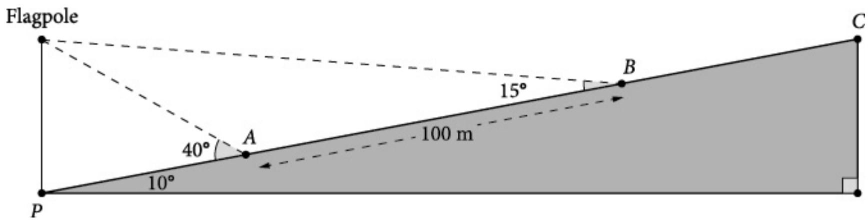
$$\cos\theta = \frac{100^2 + 90^2 - 80^2}{2\times100\times90}$$
$$\cos\theta = \frac{13}{20}$$
$$\theta = 49.458...^\circ$$

$$\text{Area} = \frac{1}{2}\times100\times90\times\sin 49.458...^\circ$$
$$= 3419.7039...$$

The area of this triangular field is $3420\ \text{m}^2$, rounded to the nearest square metre.

EXERCISE 2.11 APPLIED TRIGONOMETRY

2 Draw a diagram.



$\angle FAP = 10^\circ + 30^\circ = 40^\circ$

$\angle FAB = 180^\circ - 30^\circ - 10^\circ = 140^\circ$

$\angle FBA = 10^\circ + 5^\circ = 15^\circ$

$\therefore \angle AFB = 180^\circ - 140^\circ - 15^\circ = 25^\circ$

Us the sine rule in ΔABF :

$$\frac{AF}{\sin 15^\circ} = \frac{100}{\sin 25^\circ}$$
$$AF = \frac{100}{\sin 25^\circ}\times\sin 15^\circ \approx 61.2418$$

$\angle FPA = 90^\circ - 10^\circ = 80^\circ$

Use the sine rule in ΔAFP .

$$\frac{h}{\sin 40^\circ} = \frac{61.2418}{\sin 80^\circ}$$
$$h = \frac{61.2418}{\sin 80^\circ}\times\sin 40^\circ \approx 39.9727$$

The height of the flagpole is 39.97 m (2 d.p.).

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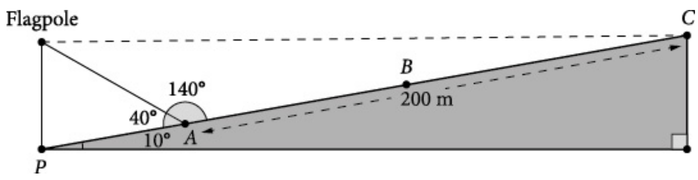


Use the cosine rule in ΔAFC to find FC.

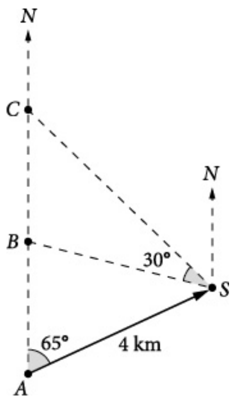
$$FC^2 = 61.2418^2 + 100^2 - 2(61.2418)(100)\cos 140°$$
$$FC \approx 250.032$$

Use the sine rule in ΔAFC to find the magnitude of $\angle ACF$:

$$\frac{\sin(\angle ACF)}{61.2418} = \frac{\sin 140°}{250.032}$$
$$\sin(\angle ACF) = \frac{\sin 140°}{250.032} \times 61.2418$$
$$\angle ACF = 9°4'$$

Since $\angle ACF < 10°,$ the top of the of the flagpole is at an angle of depression from C of $10° - 9°4' = 0°56'.$

4 Draw a diagram.



$$\angle CSN = 360° - 315° = 45°$$

Using co-interior angles, $\angle ASN = 180° - 65° = 115°$.

$$\angle ASC = 115° - 45° = 70°$$

Using alternate angles, $\angle ACS = \angle CSN = 45°$.

Using the sine rule in ΔASC,

$$\frac{AC}{\sin 70°} = \frac{4}{\sin 45°}$$
$$AC = \frac{4\sin 70°}{\sin 45°}$$

$$\angle BSN = 360° - 285° = 75°$$

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$\angle ASB = 115° - 75° = 40°$

Using alternate angles, $\angle ABS = \angle BSN = 75°$.

Using the sine rule in ΔASC,

$$\frac{AB}{\sin 40°} = \frac{4}{\sin 75°}$$
$$AB = \frac{4\sin 40°}{\sin 75°}$$

$$\begin{aligned} BC &= AC - AB \\ &= \frac{4\sin 70°}{\sin 45°} - \frac{4\sin 40°}{\sin 75°} \\ &= 4\left(\frac{\sin 70°}{\sin 45°} - \frac{\sin 40°}{\sin 75°}\right) \end{aligned}$$

One correct option is **B**.

There are other ways of finding an answer.

Using the sine rule in ΔASB,

$$\frac{SB}{\sin 65°} = \frac{4}{\sin 75°}$$
$$SB = \frac{4\sin 65°}{\sin 75°}$$

Using the sine rule in ΔCBS,

$$\frac{CB}{\sin 30°} = \frac{SB}{\sin 45°} = \frac{4\sin 65°}{\sin 75° \sin 45°}$$

$$\begin{aligned} CB &= \frac{4\sin 65° \sin 30°}{\sin 75° \sin 45°} \\ &= \frac{4\sin 65° \times 0.5}{\sin 75° \sin 45°} \\ &= \frac{2\sin 65°}{\sin 75° \sin 45°} \end{aligned}$$

The other correct option is **C**.

Alternatively, use the sine rule in ΔASC.

$$\frac{SC}{\sin 65°} = \frac{4}{\sin 45°}$$
$$SC = \frac{4\sin 65°}{\sin 45°}$$

Using the sine rule in ΔCBS,

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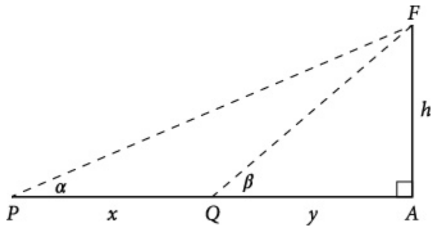
$$\frac{CB}{\sin 30°} = \frac{SC}{\sin 45°} = \frac{4\sin 65°}{\sin 45° \sin 75°}$$

$$CB = \frac{4\sin 65° \sin 30°}{\sin 75° \sin 45°} = \frac{2\sin 65°}{\sin 75° \sin 45°}$$

6 Draw a diagram.



$$\tan\alpha° = \frac{h}{x+y}$$

$$\tan\beta° = \frac{h}{y} \Rightarrow y = h\tan\beta°$$

Substitute $y = h\tan\beta°$ into $\tan\alpha° = \dfrac{h}{x+y}$

$$\tan\alpha° = \frac{h}{x+\left(\dfrac{h}{\tan\beta°}\right)}$$

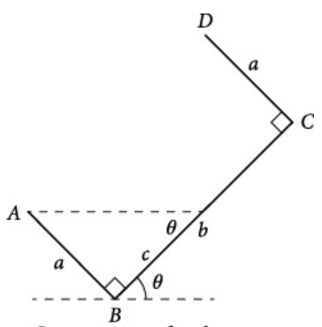
$$x + \frac{h}{\tan\beta°} = \frac{h}{\tan\alpha°}$$

$$x + h\cot\beta° = h\cot\alpha°$$

$$x = h\cot\alpha° - h\cot\beta°$$

$$x = h\left(\cot\alpha° - \cot\beta°\right)$$

8 Draw a diagram.



Cross-section of tank

$$\frac{a}{c} = \tan\theta$$

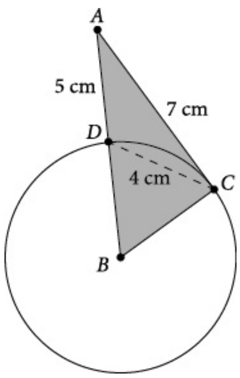
$$c = \frac{a}{\tan\theta} = a\cot\theta$$

Triangular area $= \frac{1}{2} \times a \times a\cot\theta = \frac{1}{2}a^2\cot\theta$

Depth (when level) $= \frac{\text{area}}{\text{base}} = \frac{a^2\cot(\theta)}{2b}$

10 A

Draw a diagram.



Use the cosine rule in ΔADC to find the magnitude of $\angle ADC$.

$$\cos(\angle ADC) = \frac{5^2 + 4^2 - 7^2}{2 \times 5 \times 4} = -\frac{1}{5}$$

$$\angle ADC = 101.5369...^\circ$$

$$\angle BDC = 180^\circ - 101.5369...^\circ = 78.4630...^\circ$$

$BD = BC$ (radius of the circle), therefore ΔBCD is an isosceles triangle with

$$\angle BDC = \angle BCD = 78.4630...^\circ$$

$$\angle CBD = 180^\circ - 2 \times 78.4630...^\circ = 23.0739...^\circ$$

Use the sine rule in ΔBCD to find BC.

$$\frac{BC}{\sin 78.4630...^\circ} = \frac{4}{\sin 23.0739...^\circ}$$

$$BC = \frac{4}{\sin 23.0739...^\circ} \times \sin 78.4630...^\circ = 9.999...$$

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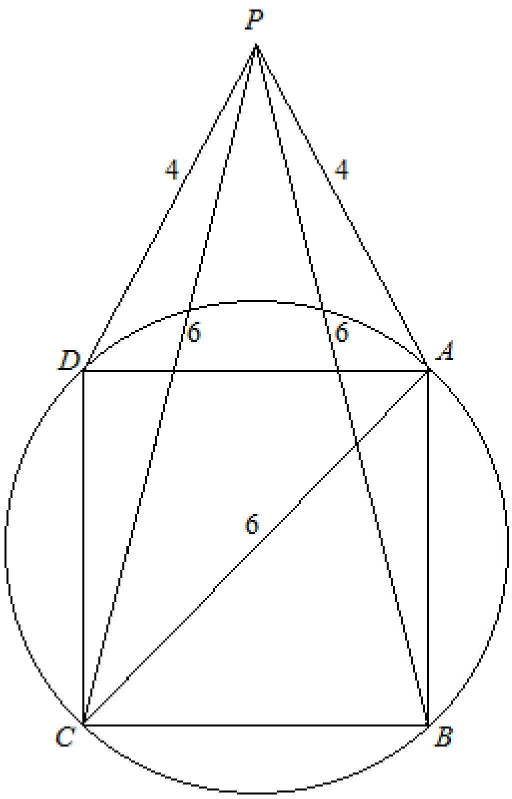
12 Draw a diagram. Note: This is a three-dimensional problem. P is *above* the circle.

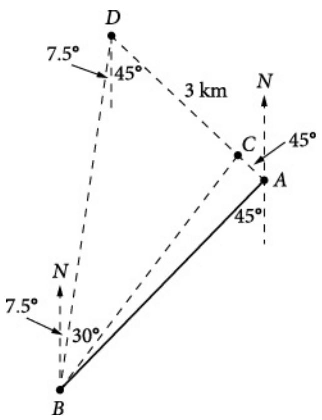


AC is a diameter so $AC = 2 \times 3 = 6$ cm.

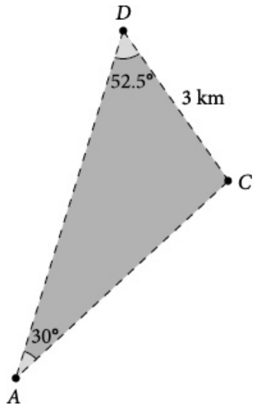
Use the cosine rule in ΔAPC.

$$\cos(\angle APC) = \frac{6^2 + 4^2 - 6^2}{2 \times 6 \times 4} = \frac{16}{48} = \frac{1}{3}$$
$$\angle APC \approx 70°32'$$

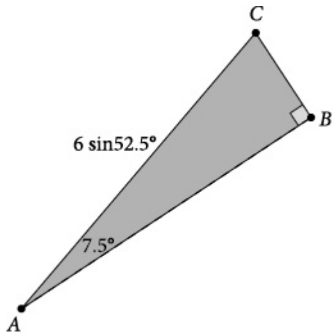
14 Draw a diagram.



Note that $\angle DBA = 37.5° - 7.5° = 30°$, $\angle BDC = 45° + 7.5° = 52.5°$, $\angle BAC = 90°$.



$$\frac{AC}{\sin 52.5°} = \frac{3}{\sin 30°} = \frac{3}{0.5} = 6$$
$$AC = 6\sin 52.5°$$

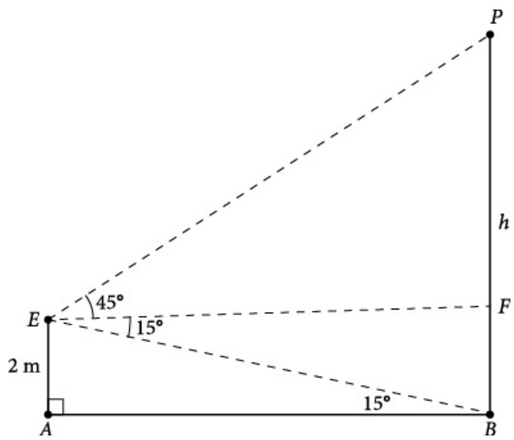


$$\cos 7.5° = \frac{AB}{6\sin 52.5°}$$
$$\therefore AB = 6\sin 52.5° \cos 7.5° \approx 4.7194$$

$$\text{Average speed} = 4.7194 \div \left(\frac{4}{60}\right) = 70.791 \text{ km/h}$$

The average speed is approximately 71 km/h.

16 Draw a diagram.



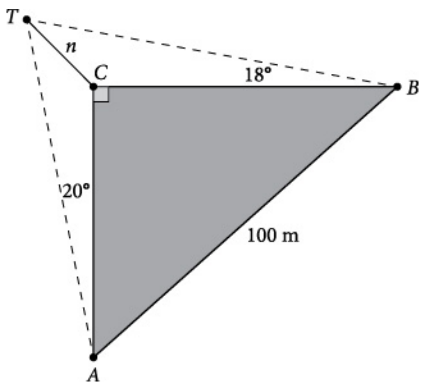
$$\frac{AB}{2} = \tan 15°$$
$$AB = 2\tan 15°$$

Note that $EF = AB = 2\tan 15°$ and $\angle PEF = 45° - 15° = 30°$.

$$\frac{PF}{EF} = \tan 30°$$
$$\frac{PF}{2\tan 75°} = \tan 30°$$
$$PF = 2\tan 30° \tan 75°$$

$$\begin{aligned} h &= BF + EF \\ &= 2 + 2\tan 30° \tan 75° \\ &= 2\left(1 + \tan 30° \tan 75°\right) \end{aligned}$$

18 Draw a diagram.



$$\frac{CB}{h} = \tan\left(90° - 18°\right) = \tan 72°$$
$$CB = h\tan 72°$$

$$\frac{AC}{h} = \tan\left(90° - 20°\right) = \tan 70°$$
$$AC = h\tan 70°$$

Use Pythagoras' theorem in ΔABC..

$$\left(h\tan 72°\right)^2 + \left(h\tan 70°\right)^2 = 100^2$$
$$h^2\left(\tan^2 72° + \tan^2 70°\right) = 100^2$$
$$h = \sqrt{\frac{100^2}{\tan^2 72° + \tan^2 70°}}$$
$$\therefore h = \frac{100}{\sqrt{\tan^2 72° + \tan^2 70°}}$$

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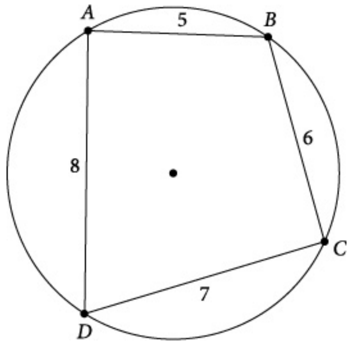
20 Draw a diagram.



Use the cosine rule in ΔADC:

$$AC^2 = 8^2 + 7^2 - 2\times 8\times 7\times \cos(\angle ADC)$$
$$AC^2 = 113 - 112\cos(\angle ADC)$$

Use the cosine rule in ΔABC:

$$AC^2 = 5^2 + 6^2 - 2\times 5\times 6\times \cos(180° - \angle ADC)$$

$$\cos(180° - \theta) = -\cos(\theta)$$

So $AC^2 = 41 + 60\cos(\angle ADC)$

Solve the equations simultaneously for $\cos(\angle ADC)$

$$113 - 112\cos(\angle ADC) = 61 + 60\cos(\angle ADC)$$
$$52 = 172\cos(\angle ADC)$$
$$\cos(\angle ADC) = \frac{52}{172} = \frac{13}{43}$$

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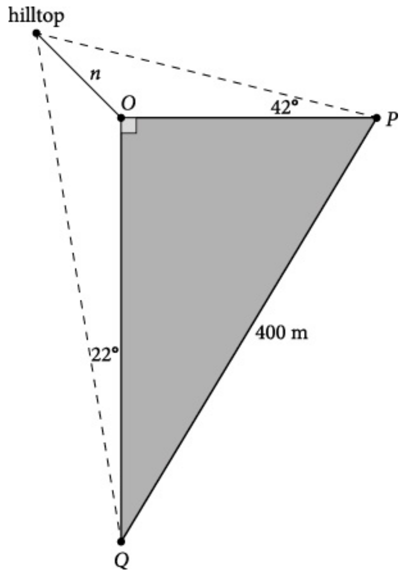
22 Draw a diagram.



$$\frac{h}{OP} = \tan 42°$$

$$OP = \frac{h}{\tan 42°}$$

$$\frac{h}{OQ} = \tan 22°$$

$$OQ = \frac{h}{\tan 22°}$$

Use Pythagoras' theorem in ΔPOQ.

$$OP^2 + OQ^2 = 400^2$$

$$\left(\frac{h}{\tan 42°}\right)^2 + \left(\frac{h}{\tan 22°}\right)^2 = 400^2$$

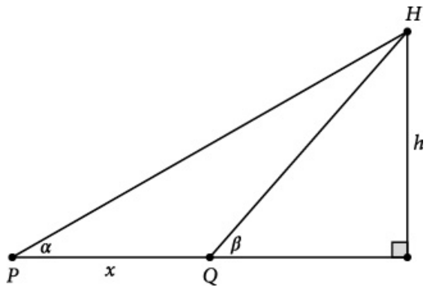
$$h^2 \cot^2 42° + h^2 \cot^2 22° = 400^2$$

$$h^2 (\cot^2 42° + \cot^2 22°) = 400^2$$

$$h = \sqrt{\frac{400^2}{\cot^2 42° + \cot^2 22°}} \approx 147.447$$

The height of the hill is 147.4 metres (1 d.p.).

24 Draw a diagram.



Since β is an exterior angle of ΔPQH, $\angle PHQ = \beta - \alpha$.

Use the sine rule in ΔPQH.

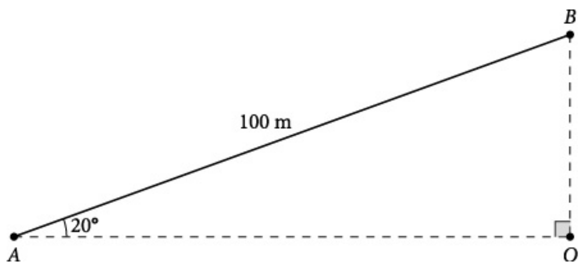
$$\frac{PQ}{\sin(\beta-\alpha)^\circ} = \frac{QH}{\sin\alpha}$$

$$QH = \frac{PQ\sin\alpha}{\sin(\beta-\alpha)^\circ} = \frac{x\sin\alpha}{\sin(\beta-\alpha)^\circ}$$

In ΔPQH, $\dfrac{h}{QH} = \sin\beta^\circ$

$$\begin{aligned} h &= QH\sin\beta^\circ \\ &= \frac{x\sin\alpha\sin\beta}{\sin(\beta-\alpha)^\circ} \end{aligned}$$

26 Draw a diagram.



(a) $OB = 100\sin 20^\circ \approx 34.2020 \approx 34.2$ m.

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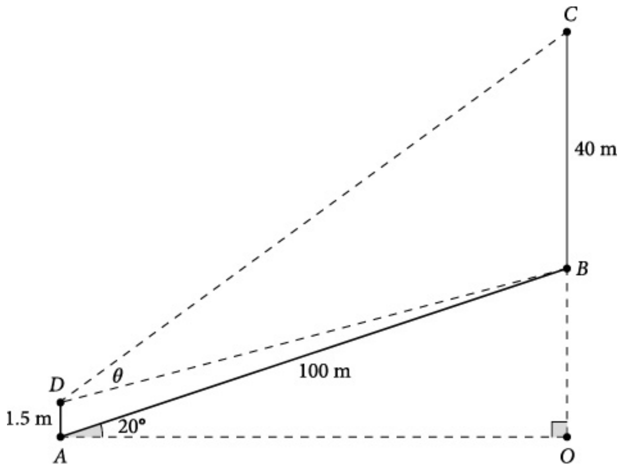
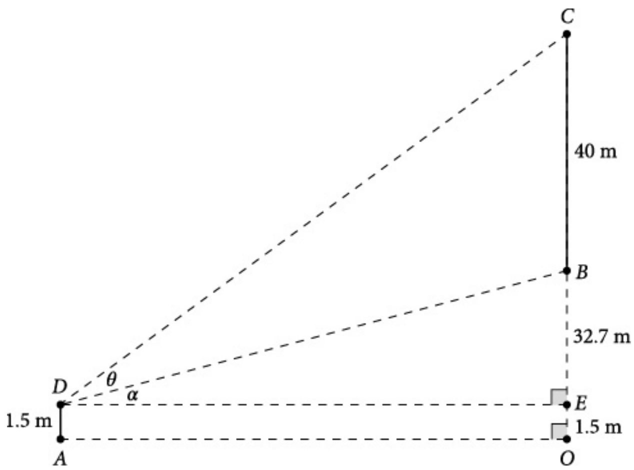
(b) Draw a diagram.



Add a horizontal dotted line from eye level (D) to a new point E 1.5 m above the base line AO.



$BE = 34.2020... - 1.5 \approx 32.7020...$
$CE = 32.7020... + 40 \approx 72.7020...$

Using the first diagram,

$DE = AO = 100\cos 20° \approx 93.9693$

$$\begin{aligned}
\tan(\theta+\alpha) &= \frac{CE}{DE} \approx \frac{72.7020}{93.9693} \\
\theta+\alpha &\approx 37.7276° \\
\tan\alpha &= \frac{BE}{DE} \approx \frac{32.7020}{93.9693} \\
\alpha &\approx 19.1872° \\
\theta &\approx 37.7276° - 19.1872° \\
&= 18.5404° \\
&= 18°32'
\end{aligned}$$

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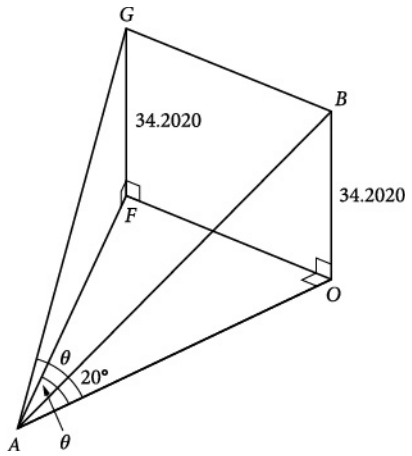
(c) Draw a diagram.



The slope is $\frac{GF}{AF} = \frac{1}{10}$

$AF = 10 \times GF \approx 342.020$

Use Pythagoras' rule in ΔAFG.

$$\begin{aligned} AG^2 &= AF^2 + FG^2 \\ &\approx 342.020^2 + 34.2020^2 \\ &= 118\,135.040... \\ AG &\approx 343.708 \end{aligned}$$

Use Pythagoras' rule in ΔAOF.

$$\begin{aligned} OF^2 &= AF^2 - AO^2 \\ &\approx 342.020^2 - 93.9693^2 \\ &= 108\,147.451... \\ OF &\approx 328.858 \\ BG = OF &\approx 328.858 \end{aligned}$$

Use the cosine rule in ΔABG.

$$\begin{aligned} \cos(\angle BAG) &= \frac{AB^2 + AG^2 - BG^2}{2 \times AB \times AG} \\ &= \frac{100^2 + 343.708^2 - 328.858^2}{2 \times 100 \times 343.708} \\ &= 0.2907... \\ \angle BAG &\approx 73°6' \end{aligned}$$

The road will make an angle of $73°6'$ with the line of steepest slope, correct to the nearest minute.

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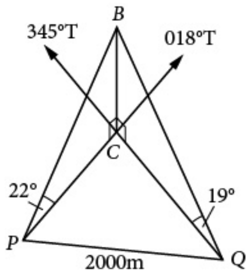
28



$\angle PCQ = (360 - 345)° + 18° = 33°$

Use the cosine rule in ΔPCQ:

$2000^2 = PC^2 + QC^2 - 2 \times PC \times QC \cos 33°$

Find expressions for PC and QC in terms of BC.

$$\tan 22° = \frac{BC}{PC} \Rightarrow PC = \frac{BC}{\tan 22°}$$
$$\tan 19° = \frac{BC}{QC} \Rightarrow QC = \frac{BC}{\tan 19°}$$

Substitute the expressions for PC and QC into the above equation and solve for BC.

$$2000^2 = PC^2 + QC^2 - 2 \times PC \times QC \cos 33°$$
$$2000^2 = \left(\frac{BC}{\tan 22°}\right)^2 + \left(\frac{BC}{\tan 19°}\right)^2 - 2\left(\frac{BC}{\tan 22°}\right)\left(\frac{BC}{\tan 19°}\right)\cos 33°$$
$$2000^2 \approx 6.12605BC^2 + 8.43444BC^2 - 12.057BC^2$$
$$2000^2 \approx 2.50349BC^2$$
$$BC \approx \frac{2000}{\sqrt{2.50349}} \approx 1264.03$$

The height of the balloon is 1264 m (to the nearest metre).

CHAPTER REVIEW 2

2 (a) $\tan 315° = \tan(360° - 45°)$

$= -\tan 45°$

$= -1$

(b) $\sin 225° = \sin(180° - 45°)$

$= -\sin 45°$

$= -\dfrac{1}{\sqrt{2}}$

(c) $\cos 180° = -1$

$= -\cos 30°$

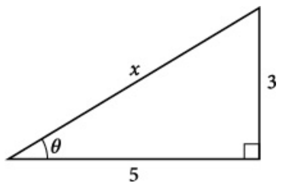
$= -\dfrac{\sqrt{3}}{2}$

(d) $\tan 360° = 0$

(e) $\sin 60° = \dfrac{\sqrt{3}}{2}$

(f) $\cos 210° = \cos(180° + 30°)$

4



(a) $x^2 = 3^2 + 5^2$

$x = \sqrt{34}$

Since $180° < \theta < 270°$ ¸ $\sin\theta$ is negative.

$\sin\theta = -\dfrac{3}{\sqrt{34}}$

(b) Since $180° < \theta < 270°$ ¸ $\cos\theta$ is negative.

$\cos\theta = -\dfrac{5}{\sqrt{34}}$

6 (a) $\tan(90° - \theta) = \cot\theta$

$= \dfrac{1}{t}$

(b) $\tan(180° + \theta) = \tan\theta$

$= t$

(c) $\cot(180° - \theta) = -\cot\theta$

$= -\dfrac{1}{t}$

(d) $\tan(360° - \theta) = -\tan\theta$

$= -t$

(e) $\tan(-\theta) = -\tan\theta$

$= -t$

(f) $\tan(90° + \theta) = -\tan(180° - (90° - \theta))$

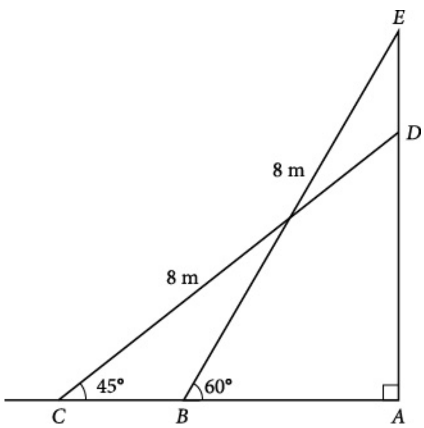
$$= -\tan(90° - \theta)$$
$$= -\cot\theta$$
$$= -\frac{1}{t}$$

8 The largest angle is opposite the longest side in a triangle (8).

$$\cos\theta = \frac{5^2 + 6^2 - 8^2}{2 \times 5 \times 6}$$
$$\cos\theta = -\frac{1}{20}$$
$$\theta \approx 92°52'$$

The largest angle is $92°52'$, rounded to the nearest minute. Since it is greater than $90°$, this triangle is obtuse-angled.

10 Draw a diagram.

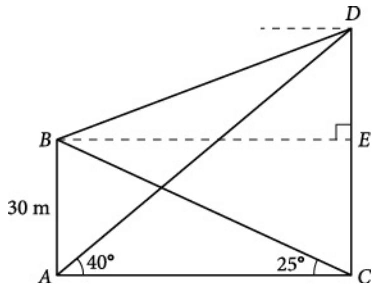


$$\sin 60° = \frac{AE}{8}$$
$$AE = 8\sin 60° = 4\sqrt{3}$$

$$\sin 45° = \frac{AD}{8}$$
$$AD = 8\sin 45° = 4\sqrt{2}$$

Slip distance $= ED = AE - AD = 4\sqrt{3} - 4\sqrt{2} = 4\left(\sqrt{3} - \sqrt{2}\right)$

12



(a) $\tan 25° = \dfrac{30}{AC}$

$$AC = \frac{30}{\tan 25°}$$
$$AC = 64.3352...$$

The horizontal distance between the buildings is 64.3 m, rounded to one decimal place.

(b) $\tan 40° = \dfrac{CD}{AC}$

$$CD = 64.3352... \times \tan 40°$$
$$CD = 53.95...$$

The height of CD is 54 m, rounded to the nearest metre.

(c) The angle of depression of B from D is the same as $\angle DBE$.

$$\begin{aligned} DE &= CD - EC \\ &= 53.95... - 30 \\ &= 23.95... \end{aligned}$$

$$\tan \angle DBE = \frac{DE}{BE}$$
$$\tan \angle DBE \approx \frac{23.95...}{64.335}$$
$$\angle DBE \approx 20°25'$$

The angle of depression of B from D is $20°25'$, rounded to the nearest minute.

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14 (a) $r^2 = (100-50t)^2 + (80t)^2 - 4(100-50t)\times 80t \times \cos 60°$

$$= (100-50t)^2 + (80t)^2 - 4(100-50t)\times 80t \times \frac{1}{2}$$

$$= (100-50t)^2 + (80t)^2 - 2(100-50t)\times 80t$$

$$= \left((100-50t)-80t\right)^2$$

$$r^2 = \left(100-130t\right)^2$$

$$\therefore r = 100-130t$$

(b) $r = 100 - 130 \times \dfrac{30}{43}$

$$= \frac{160000}{1849} = 86.53$$

$$\therefore r = 87$$