EXERCISE 4.1 FUNCTIONS AND RELATIONS

2 C

$f(t) = t^2 - 9$

There is no restriction on the domain, so the domain is the set of all real numbers.

Since $t^2 \ge 0$, $t^2 - 9 \ge -9$

The range is $f(t) \ge -9$

4 (a)
$$\begin{aligned} f(1) &= 3\times 1 - 6 \\ &= -3 \end{aligned}$$

$$\begin{aligned} f(-2) &= 3\times(-2) - 6 \\ &= -12 \end{aligned}$$

$$\begin{aligned} f(a) &= 3\times a - 6 \\ &= 3a - 6 \end{aligned}$$

(b) $f(a) = a$

$$\begin{aligned} 3a - 6 &= a \\ 2a &= 6 \\ a &= 3 \end{aligned}$$

(c) $f(x) > x$

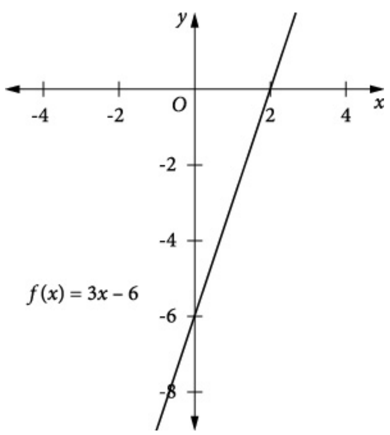
$$\begin{aligned} 3x - 6 &> x \\ 2x &> 6 \\ x &> 3 \end{aligned}$$

(d) The graph will have a gradient of 3 and the *y*-intercept will be -6.

Find any other point, e.g. the *x*-intercept.

$$\begin{aligned} f(x) &= 0 \\ 3x - 6 &= 0 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

Draw the straight line with *y*-intercept -6 and *x*-intercept 2.



6 **(a)** $g(1) = \sqrt{1}$

$= 1$

correct

(b) $g(9) = \sqrt{9}$

$= 3$

incorrect

(c) $g(x^2) = \sqrt{x^2}$

$= |x|$

Correct (since $x \geq 0$)

(d) $g(x+2) = \sqrt{x+2}$

correct

8 **(a)** $f(x) = \sqrt{x-2}$

$x - 2 \geq 0$
$x \geq 2$

Domain: $x \geq 2$

The square root sign signifies the positive square root.

Range: $y \geq 0$

(b) $f(x) = \sqrt{3-x}$

$3 - x \geq 0$
$-x \geq -3$
$x \leq 3$

Domain: $x \leq 3$

The square root sign signifies the positive square root.

Range: $y \geq 0$

(c) $f(x) = \sqrt{x^2 - 9}$

$x^2 - 9 \geq 0$
$x^2 \geq 9$

Domain: All values of x where $x \geq 3$ or $x \leq -3$.

The square root sign signifies the positive square root.

Range: $y \geq 0$

(d) $g(x) = \dfrac{1}{x}$

$g(x)$ is defined for all values of x except 0.

Domain: All real x, $x \neq 0$

$\dfrac{1}{x}$ can take all values except 0.

Range: All real $g(x)$, $g(x) \neq 0$

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(e) $h(t) = t^3$

Domain: All real t

$h(t)$ can equal any real number.

Range: All real numbers

(f) $g(k) = 5 - k^2$

$$k^2 \ge 0$$
$$5 - k^2 \le 5$$

Domain: All real numbers

Range: $g(k) \le 5$

10 (a) For $x = 0$, the second function applies.

$$f(0) = x = 0$$

(b) For $x = 2$, the second function applies.

$$f(2) = x = 2$$

(c) For $x = -2$, the first function applies.

$$\begin{aligned} f(-2) &= \frac{1}{-2} \\ &= -\frac{1}{2} \end{aligned}$$

(d) $a^2 \ge 0$, so the second function applies.

$$f(a^2) = a^2$$

EXERCISE 4.2 SKETCHING BASIC FUNCTIONS

2 (a) $m = 3.$ The gradient is $3,$ and is therefore always positive; the function is increasing.

(b) $3x + 2y - 6 = 0$

$$\begin{aligned} 2y &= -3x + 6 \\ y &= -\frac{3}{2}x + 3 \\ m &= -\frac{3}{2} \end{aligned}$$

The gradient is always negative; the function is always decreasing.

(c) $m = -2.$ The gradient is $-2,$ and is therefore always negative; the function is always decreasing.

(d) $m = 1.$ The gradient is $1,$ and is therefore always positive; the function is always increasing.

(e) $4x - y - 8 = 0$

$$\begin{aligned} 4x - 8 &= y \\ m &= 4 \end{aligned}$$

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The gradient is positive; the function is always increasing.

(f) $m=-1$. The gradient is -1, and is therefore always negative; the function is always decreasing.

(g) This is a horizontal straight line, so the gradient is zero; the function is neither increasing nor decreasing.

(h) This is a vertical straight line, so the gradient is undefined; the function is neither increasing nor decreasing.

(i) $x+2y+5=0$

$$\begin{aligned}2y&=-x-5\\ y&=-\frac{x}{2}-5\\ m&=-\frac{1}{2}\end{aligned}$$

The gradient is always negative; the function is always decreasing.

4 **(a)** $f(x)=x$

$$\begin{aligned}f(-x)&=-x\\ &=-f(x)\end{aligned}$$

$\therefore f(x)$ is odd

(b) $f(x)=-x^2$

$$\begin{aligned}f(-x)&=-(-x)^2\\ &=-x^2\\ &=f(x)\end{aligned}$$

$\therefore f(x)$ is even

(c) $f(x)=x^3$

$$\begin{aligned}f(-x)&=-x^3\\ &=-f(x)\end{aligned}$$

$\therefore f(x)$ is odd

(d) $f(x)=-\frac{1}{x}$

$$\begin{aligned}f(-x)&=-\frac{1}{-x}\\ &=\frac{1}{x}\\ &=-f(x)\end{aligned}$$

$\therefore f(x)$ is odd

(e) $f(x)=x^4$

$$\begin{aligned}f(-x)&=(-x)^4\\ &=x^4\\ &=f(x)\end{aligned}$$

$\therefore f(x)$ is even

(f) $f(x)=\frac{1}{x^2}$

$$\begin{aligned}f(-x)&=\frac{1}{(-x)^2}\\ &=\frac{1}{x^2}\\ &=f(x)\end{aligned}$$

$\therefore f(x)$ is even

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6 For the equation $y = mx + c$, the gradient is m and the y-intercept is c.

Therefore, for the equation $y = 2x - 3$, the gradient is 2 and the y-intercept is -3.

Because the gradient is (always) positive, y is an increasing function.

Find the x-intercept.

$$\begin{aligned} y &= 0 \\ 2x - 3 &= 0 \\ x &= \frac{3}{2} \end{aligned}$$

The x-intercept is $\dfrac{3}{2}$.

(a) correct

(b) incorrect

(c) correct

(d) incorrect

EXERCISE 4.3 SQUARE ROOTS AND ABSOLUTE VALUE

2
$$\begin{aligned} \sqrt{(-2.5)^2} &= \sqrt{6.25} \\ &= 2.5 \end{aligned}$$

4 Since $\sqrt{3} - 2 < 0$

$$\begin{aligned} \left|\sqrt{3} - 2\right| &= -\left(\sqrt{3} - 2\right) \\ &= 2 - \sqrt{3} \end{aligned}$$

6 If $x \geq 0$, $y \geq 0$

$$|x| + |y| = x + y$$

If $x \geq 0$, $y < 0$

$$|x| + |y| = x - y$$

If $x < 0$, $y \geq 0$

$$\begin{aligned} |x| + |y| &= -x + y \\ &= y - x \end{aligned}$$

If $x < 0$, $y < 0$

$$\begin{aligned} |x| + |y| &= -x - y \\ &= -(x + y) \end{aligned}$$

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8 If $x \geq 5$

$$\begin{aligned}|x-5|+|x+5| &= x-5+x+5\\ &= 2x\end{aligned}$$

If $-5 \leq x < 5$

$$\begin{aligned}|x-5|+|x+5| &= -(x-5)+(x+5)\\ &= -x+5+x+5\\ &= 10\end{aligned}$$

If $x < -5$

$$\begin{aligned}|x-5|+|x+5| &= -(x-5)-(x+5)\\ &= -x+5-x-5\\ &= -2x\end{aligned}$$

10 $2x+3 \geq 0$ when $x \geq -\frac{3}{2}$.

$2x+3 \leq 0$ when $x \leq -\frac{3}{2}$.

If $x \geq -\frac{3}{2}$, $\sqrt{(2x+3)^2} = 2x+3$

If $x < -\frac{3}{2}$, $\sqrt{(2x+3)^2} = -(2x+3) = -2x-3$

12 (a) $x-2=3$

$x=5$

OR

$$\begin{aligned}-(x-2) &= 3\\ -x+2 &= 3\\ -x &= 1\\ x &= -1\end{aligned}$$

(b) $x+3=7$

$x=4$

OR

$$\begin{aligned}-(x+3) &= 7\\ -x-3 &= 7\\ -x &= 10\\ x &= -10\end{aligned}$$

(c) $4-x=5$

$$\begin{aligned}-x &= 1\\ x &= -1\end{aligned}$$

OR

$$\begin{aligned}-(4-x) &= 5\\ -4+x &= 5\\ x &= 9\end{aligned}$$

(d) $x+7=2$

$x=-5$

OR

$$\begin{aligned}-(x+7) &= 2\\ -x-7 &= 2\\ -x &= 9\\ x &= -9\end{aligned}$$

(e) $|x-6|=0$

$$\begin{aligned}x-6 &= 0\\ x &= 6\end{aligned}$$

(f) $x-5=1$

$x=6$

OR

$$\begin{aligned}-(x-5) &= 1\\ -x+5 &= 1\\ -x &= -4\\ x &= 4\end{aligned}$$

(g) $|x+1|=0$

$$x+1=0$$
$$x=-1$$

(h) $10+x=3$

$$x=-7$$

OR

$$-(10+x)=3$$
$$-10-x=3$$
$$-x=13$$
$$x=-13$$

(i) $2x+1=2$

$$2x=1$$
$$x=\frac{1}{2}$$

OR

$$-(2x+1)=2$$
$$-2x-1=2$$
$$-2x=3$$
$$x=-\frac{3}{2}$$

(j) $2x-5=3$

$$2x=8$$
$$x=4$$

OR

$$-(2x-5)=3$$
$$-2x+5=3$$
$$-2x=-2$$
$$x=1$$

(k) $5x+1=4$

$$5x=3$$
$$x=\frac{3}{5}$$

OR

$$-(5x+1)=4$$
$$-5x-1=4$$
$$-5x=5$$
$$x=-1$$

(l) $3x-4=5$

$$3x=9$$
$$x=3$$

OR

$$-(3x-4)=5$$
$$-3x+4=5$$
$$-3x=1$$
$$x=-\frac{1}{3}$$

(m) $|3x+1|=0$

$$3x+1=0$$
$$3x=-1$$
$$x=-\frac{1}{3}$$

(n) $6x+1=7$

$$6x=6$$
$$x=1$$

OR

$$-(6x+1)=7$$
$$-6x-1=7$$
$$-6x=8$$
$$x=-\frac{4}{3}$$

(o) $|4x-1|=0$

$$4x-1=0$$
$$4x=1$$
$$x=\frac{1}{4}$$

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(p) $2x-9=13$

$2x=22$

$x=11$

OR

$-(2x-9)=13$

$-2x+9=13$

$-2x=4$

$x=-2$

14 (a) $x-1<3$

$x<4$

OR

$-(x-1)<3$

$-x+1<3$

$-x<2$

$x>-2$

$-2<x$

$\therefore -2<x<4$

(b) $y+2>4$

$y>2$

OR

$-(y+2)>4$

$-y-2>4$

$-y>6$

$y<-6$

$\therefore y<-6$ or $y>2$

(c) $t-6\le 2$

$t\le 8$

OR

$-(t-6)\ge 2$

$-t+6\le 2$

$-t\le -4$

$t\ge 4$

$4\le t$

$\therefore 4\le t\le 8$

(d) $x+4\ge 2$

$x\ge -2$

OR

$-(x+4)\ge 2$

$-x-4\ge 2$

$-x\ge 6$

$x\le -6$

$\therefore x\le -6$ or $x\ge -2$

(e) The absolute value will always be greater or equal to zero for all values, hence all real m.

(f) $3-x\le 5$

$3-x\le 5$

$-x\le 2$

$x\ge -2$

$-2\le x$

OR

$-(3-x)\le 5$

$-3+x\le 5$

$x\le 8$

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$\therefore -2 \le x \le 8$

(g) The absolute value will always be greater or equal to zero so it can only be less than or equal to zero if the value inside equals zero.

$y+1=0 \Rightarrow y=-1$

The only solution to $|y+1| \le 0$ is $y=-1$

(h) $7+x<3$

$x<-4$

OR

$-(7+x)<3$

$-7-x<3$

$-x<10$

$x>-10$

$\therefore -10<x<-4$

(i) $2x+1>3$

$2x>2$

$x>1$

OR

$-(2x+1)>3$

$-2x-1>3$

$-2x>4$

$x<-2$

$\therefore x<-2$ or $x>1$

(j) $3z-5<1$

$3z<6$

$z<2$

OR

$-(3z-5)<1$

$-3z+5<1$

$-3z<-4$

$z>\frac{4}{3}$

$\therefore \frac{4}{3}<z<2$

(k) $4x+3 \ge 5$

$4x \ge 2$

$x \ge \frac{1}{2}$

OR

$-(4x+3) \ge 5$

$-4x-3 \ge 5$

$-4x \ge 8$

$x \le -2$

$\therefore x \le -2$ or $x \ge \frac{1}{2}$

(l) $3t-2<5$

$3t<7$

$t<\frac{7}{3}$

OR

$-(3t-2)<5$

$-3t+2<5$

$-3t<3$

$t>-1$

$\therefore -1<t<\frac{7}{3}$

(m) The absolute value function cannot be negative, so there is no solution

(n) $5x+4>9$

$5x>5$

$x>1$

(o) $|1-2x|>0$ for all values of x except for when $1-2x=0$.

(p) $2x-7 \ge 11$

$2x \ge 18$

$x \ge 9$

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OR

$$-(5x+4)>9$$
$$-5x-4>9$$
$$-5x>13$$
$$x<-\frac{13}{5}$$

$$\therefore x<-\frac{13}{5} \text{ or } x>1$$

$$1-2x=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$$

$$\therefore \text{All real } x,\ x\neq\frac{1}{2}$$

OR

$$-(2x-7)\geq 11$$
$$-2x+7\geq 11$$
$$-2x\geq 4$$
$$x\geq -2$$

$$\therefore x\leq -2 \text{ or } x\geq 9$$

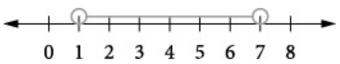
16 $y-4<3$

$$y<7$$

OR

$$-(y-4)<3$$
$$-y+4<3$$
$$-y<-1$$
$$y>1$$

$$\therefore 1<y<7$$



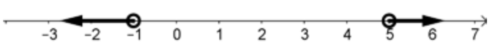
18 $y-2>3$

$$y>5$$

OR

$$-(y-2)>3$$
$$-y+2>3$$
$$-y>1$$
$$y<-1$$

$$\therefore y<-1 \text{ or } y>5$$



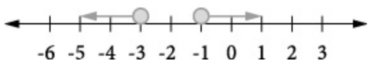
20 $x+2 \geq 1$

$x \geq -1$

OR

$$\begin{aligned} -(x+2) &\geq 1 \\ -x-2 &\geq 1 \\ -x &\geq 3 \\ x &\leq -3 \end{aligned}$$

$\therefore x \leq -3$ or $x \geq -1$



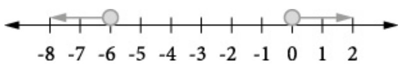
22 $3+x \geq 3$

$x \geq 0$

OR

$$\begin{aligned} -(3+x) &\geq 3 \\ -3-x &\geq 3 \\ -x &\geq 6 \\ x &\leq -6 \end{aligned}$$

$\therefore x \leq -6$ or $x \geq 0$



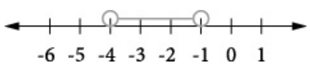
24 $2x+5 < 3$

$$\begin{aligned} 2x &< -2 \\ x &< -1 \end{aligned}$$

OR

$$\begin{aligned} -(2x+5) &< 3 \\ -2x-5 &< 3 \\ -2x &< 8 \\ x &> -4 \end{aligned}$$

$\therefore -4 < x < -1$



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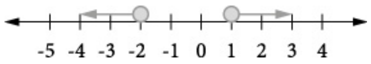
26 $2+4x \geq 6$

$$\begin{aligned} 4x &\geq 4 \\ x &\geq 1 \end{aligned}$$

OR

$$\begin{aligned} -(2+4x) &\geq 6 \\ -2-4x &\geq 6 \\ -4x &\geq 8 \\ x &\leq -2 \end{aligned}$$

$\therefore x \leq -2$ or $x \geq 1$



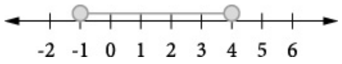
28 $2x-3 \leq 5$

$$\begin{aligned} 2x &\leq 8 \\ x &\leq 4 \end{aligned}$$

OR

$$\begin{aligned} -(2x-3) &\leq 5 \\ -2x+3 &\leq 5 \\ -2x &\leq 2 \\ x &\geq -1 \end{aligned}$$

$\therefore -1 \leq x \leq 4$



30 $x^2 - 1 \leq 4$

$$\begin{aligned} x^2 &\leq 5 \\ -\sqrt{5} \leq x &\leq \sqrt{5} \end{aligned}$$

OR

$$\begin{aligned} -(x^2-1) &\leq 4 \\ -x^2+1 &\leq 4 \\ -x^2 &\leq 3 \\ x^2 &\leq -3 \end{aligned}$$

No real solutions

$\therefore -\sqrt{5} \leq x \leq \sqrt{5}$

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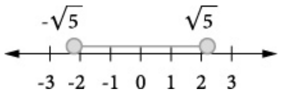


32 Solve $|2x+5|<3$.

$$2x+5<3$$
$$2x<-2$$
$$x<-1$$

OR

$$-(2x+5)<3$$
$$-2x-5<3$$
$$-2x<8$$
$$x>-4$$

$$-4<x<-1$$

Solve $|2+4x|\geq 6$.

$$2+4x\geq 6$$
$$4x\geq 4$$
$$x\geq 1$$

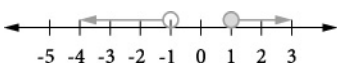
OR

$$-(2+4x)\geq 6$$
$$-2-4x\geq 6$$
$$-4x\geq 8$$
$$x\leq -2$$

$x\leq -2$ or $x\geq 1$

The full solution is $x\leq -2$ or $x\geq 1$ or $-4<x<-1$.

This is equivalent to $x<-1$ or $x\geq 1$.



34 $\dfrac{\sqrt{x^2}}{|x|}=\dfrac{|x|}{|x|}$

$$=1$$
$$x\neq 0$$

36 $\sqrt{\dfrac{(x-4)^2}{x^2}}=\dfrac{\sqrt{(x-4)^2}}{\sqrt{x^2}}=\dfrac{|x-4|}{|x|}$

The function is undefined when $x=0$.

If $x\geq 4$, $\dfrac{|x-4|}{|x|}=\dfrac{x-4}{x}=1-\dfrac{4}{x}$

If $0<x<4$, $\dfrac{|x-4|}{|x|}=\dfrac{-(x-4)}{x}=\dfrac{4-x}{x}=\dfrac{4}{x}-1$

If $x<0$, $\dfrac{|x-4|}{|x|}=\dfrac{-(x-4)}{-x}=\dfrac{x-4}{x}=1-\dfrac{4}{x}$

38 $\sqrt{x^2-10x+25}=\sqrt{(x-5)^2}$

$=|x-5|$

If $x<5$, $|x-5|=-(x-5)=5-x$

If $x\geq 5$, $|x-5|=x-5$

40 (a) (i) $|xy|=|5\times 2|$

$=10$

$|x|\times|y|=|5|\times|2|$

$=10$

The statement is true.

(ii) $|x|+|y|=|5|+|2|$

$=7$

$|x+y|=|5+2|$

$=7$

$\leq|x|+|y|$

The statement is true. (Something that is equal to 7 is also less than or equal to 7).

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(b) (i) $|xy| = |3\times -2|$

$= |-6|$
$= 6$

$|x|\times|y| = |3|\times|-2|$
$= 3\times 2$
$= 6$

The statement is true.

(ii) $|x|+|y| = |3|+|-2|$

$= 3+2$
$= 5$

$|x+y| = |3-2| = 1 \le |x|+|y|$

The statement is true.

(c) (i) $|xy| = |-6\times 8|$

$= |-48|$
$= 48$

$|x|\times|y| = |-6|\times|8|$
$= 6\times 8$
$= 48$

The statement is true.

(ii) $|x|+|y| = |-6|+|8|$

$= 6+8$
$= 14$

$|x+y| = |-6+8| = 2 \le |x|+|y|$

The statement is true.

(d) (i) $|xy| = |-4\times -3|$

$= 12$

$|x|\times|y| = |-4|\times|-3|$
$= 4\times 3$
$= 12$

The statement is true.

(ii) $|x|+|y| = |-4|+|-3|$

$= 4+3$
$= 7$

$|x+y| = |-4-3|$
$= |-7|$
$= 7$
$\le |x|+|y|$

The statement is true.

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EXERCISE 4.4 ABSOLUTE VALUE FUNCTIONS

2 **C**

$$y = |3x - 2|$$

$$3x - 2 = 0$$
$$x = \frac{2}{3}$$

The *x*-intercept (and the vertex, or 'point') is at $x = \frac{2}{3}$.

When $3x - 2 > 0$, $x > \frac{2}{3}$, $y = 3x - 2$

When $3x - 2 < 0$,, $x < \frac{2}{3}$, $y = -3x + 2$

Find the *y*-intercept. When $x = 0$,

$$\begin{aligned} y &= |3 \times 0 - 2| \\ &= |-2| \\ &= 2 \end{aligned}$$

The graph is always positive.

The only option is C.

4 **(a)** Domain: all real x

$$\begin{aligned} f(x) &= x \\ f(-x) &= -x \\ &= -f(x) \end{aligned}$$

$\therefore f(x)$ is odd

(b) Domain: all real x

$$\begin{aligned} f(x) &= x + 1 \\ f(-x) &= -x + 1 \\ f(-x) &\neq f(x) \\ f(-x) &\neq -f(x) \end{aligned}$$

$\therefore f(x)$ is neither odd nor even.

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(c) Domain: all real x

$$\begin{aligned} f(x) &= |x| \\ f(-x) &= |-x| \\ &= |x| \\ &= f(x) \end{aligned}$$

$\therefore f(x)$ is even

(d) Domain: all real x

$$\begin{aligned} f(x) &= x^3 + x \\ f(-x) &= (-x)^3 + (-x) \\ &= -x^3 - x \\ &= -(x^3 + x) \\ &= -f(x) \end{aligned}$$

$\therefore f(x)$ is odd

(e) Domain: all real x

$$\begin{aligned} f(x) &= 4 - x^2 \\ f(-x) &= 4 - (-x)^2 \\ &= 4 - x^2 \\ &= f(x) \end{aligned}$$

$\therefore f(x)$ is even

(f) Domain: all real x

$$\begin{aligned} f(x) &= (x-2)^2 \\ f(-x) &= (-x-2)^2 \\ &= \left[(-1)(x+2)\right]^2 \\ &= (x+2)^2 \\ f(-x) &\neq f(x) \\ f(-x) &\neq -f(x) \end{aligned}$$

$\therefore f(x)$ is neither odd nor even

(g) $4 - x^2 \geq 0$

$$-x^2 \geq -4$$
$$x^2 \leq 4$$
$$-2 \leq x \leq 2$$

Domain: $-2 \leq x \leq 2$

$$\begin{aligned} f(x) &= \sqrt{4 - x^2} \\ f(-x) &= \sqrt{4 - (-x)^2} \\ &= \sqrt{4 - x^2} \\ &= f(x) \end{aligned}$$

$\therefore f(x)$ is even

(h) $x^2 - 1 \neq 0$

$$x \neq \pm 1$$

Domain: $x \neq \pm 1$

$$\begin{aligned} f(x) &= \frac{x}{x^2 - 1} \\ f(-x) &= \frac{-x}{(-x)^2 - 1} \\ &= -\frac{x}{x^2 - 1} \\ &= -f(x) \end{aligned}$$

$\therefore f(x)$ is odd

(i) Domain: all real x

$$\begin{aligned} f(x) &= x^2 + x \\ f(-x) &= (-x)^2 + (-x) \\ &= x^2 - x \\ f(-x) &\neq -f(x) \\ f(-x) &\neq f(x) \end{aligned}$$

$\therefore f(x)$ is neither odd nor even

6 $f(x) = |2x - 5|$

x-intercept:

$$2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

There will be a vertex or sharp point at $x = \frac{5}{2}$

y-intercept:

Where $x = 0$

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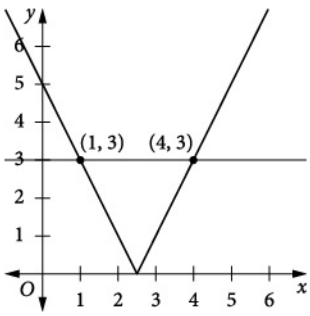
$$\begin{aligned} y &= |2\times 0-5| \\ &= |-5| \\ &= 5 \end{aligned}$$

When $2x-5>0,\ x>\dfrac{5}{2},\ f(x)=2x-5$.

When $2x-5<0,\ x<\dfrac{5}{2},\ f(x)=-(2x-5)=-2x+5$.



These points can be checked.

When $x=1,\ f(x)=|2\times 1-5|=|-1|=1$.

When $x=3,\ f(x)=|2\times 3-5|=|1|=1$.

8 $|x|+|y|=2$

When $x>0,\ y>0$

$$\begin{aligned} x+y&=2 \\ x+y-2&=0 \end{aligned}$$

(a) is correct.

When $x>0,\ y>0$

$$\begin{aligned} x-y&=2 \\ x-y-2&=0 \end{aligned}$$

(c) is correct.

When $x<0,\ y>0$

$$\begin{aligned} -x+y&=2 \\ x-y+2&=0 \end{aligned}$$

(d) is correct.

When $x<0,\ y<0$

$$\begin{aligned} -x-y&=2 \\ x+y+2&=0 \end{aligned}$$

(b) is correct.

(a) correct **(b)** correct **(c)** correct **(d)** correct

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EXERCISE 4.5 CIRCLES

2 B

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-4))^2+(y-4)^2=6^2$$
$$(x+4)^2+(y-4)^2=36$$

4 (a) $x^2+y^2-6x+4y-3=0$

$$x^2-6x+9+y^2+4y+4=3+9+4$$
$$(x-3)^2+(y+2)^2=16$$
$$(x-3)^2+(y+2)^2=4^2$$

Centre $(3,-2)$, radius 4

(b) $x^2+y^2+4x+2y-4=0$

$$x^2+4x+4+y^2+2y+1=4+4+1$$
$$(x+2)^2+(y+1)^2=9$$
$$(x+2)^2+(y+1)^2=3^2$$

Centre $(-2,-1)$, radius 3

(c) $(x-3)^2+y^2=3$

$$(x-3)^2+y^2=\left(\sqrt{3}\right)^2$$

Centre $(3,0)$, radius $\sqrt{3}$

(d) $(x+a)^2+(y-b)^2=8$

$$(x+a)^2+(y-b)^2=\left(\sqrt{8}\right)^2$$
$$(x+a)^2+(y-b)^2=\left(2\sqrt{2}\right)^2$$

Centre $(-a,b)$, radius $2\sqrt{2}$

(e) $x^2+y^2-5x+3y-1=0$

$$x^2-5x+\frac{25}{4}+y^2+3y+\frac{9}{4}=1+\frac{25}{4}+\frac{9}{4}$$
$$\left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{19}{2}$$
$$\left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\left(\sqrt{\frac{19}{2}}\right)^2$$
$$\left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\left(\frac{\sqrt{38}}{2}\right)^2$$

Centre $\left(\frac{5}{2},-\frac{3}{2}\right)$, radius $\frac{\sqrt{38}}{2}$

(f) $x^2+y^2+4x+2y-5=0$

$$x^2+4x+4+y^2+2y+1=5+4+1$$
$$(x+2)^2+(y+1)^2=10$$
$$(x+2)^2+(y+1)^2=\left(\sqrt{10}\right)^2$$

Centre $(-2,-1)$, radius $\sqrt{10}$

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(g) $2x^2+2y^2-8x+5y+3=0$

$$x^2+y^2-4x+\frac{5}{2}y+\frac{3}{2}=0$$
$$x^2-4x+4+y^2+\frac{5}{2}y+\frac{25}{16}=-\frac{3}{2}+4+\frac{25}{16}$$
$$(x-2)^2+\left(y+\frac{5}{4}\right)^2=\frac{65}{16}$$
$$(x-2)^2+\left(y+\frac{5}{4}\right)^2=\left(\frac{\sqrt{65}}{4}\right)^2$$

Centre $\left(2,-\frac{5}{4}\right)$, radius $\frac{\sqrt{65}}{4}$

(h) $3x^2+3y^2+9x-4y-24=0$

$$x^2+y^2+3x-\frac{4}{3}y-8=0$$
$$x^2+3x+\frac{9}{4}+y^2-\frac{4}{3}y+\frac{4}{9}=8+\frac{9}{4}+\frac{4}{9}$$
$$\left(x+\frac{3}{2}\right)^2+\left(y-\frac{2}{3}\right)^2=\frac{385}{36}$$
$$\left(x+\frac{3}{2}\right)^2+\left(y-\frac{2}{3}\right)^2=\left(\frac{\sqrt{385}}{6}\right)^2$$

Centre $\left(-\frac{3}{2},\frac{2}{3}\right)$, radius $\frac{\sqrt{385}}{6}$

6 $x^2+y^2-6x+2y+10=0$

$$x^2-6x+9+y^2+2y+1=-10+9+1$$
$$(x-3)^2+(y+1)^2=0$$

Centre $(3,-1)$, radius 0.

(a) incorrect

(b) incorrect

(c) incorrect

(d) correct

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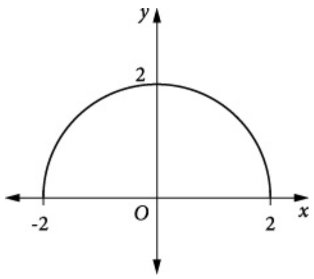
8 (a) The circle's centre is $(0,\ 0)$ and the radius is $\sqrt{4}=2$.

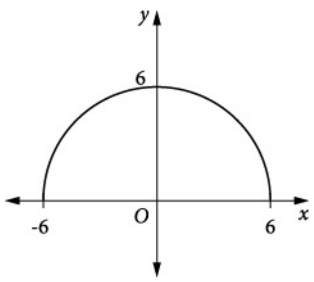
$$\begin{aligned} x^2+y^2&=4 \\ y^2&=4-x^2 \\ y&=\sqrt{4-x^2} \end{aligned}$$



Range: $0 \le y \le 2$

(b) The circle's centre is $(0,\ 0)$ and the radius is $\sqrt{36}=6$.

$$\begin{aligned} x^2+y^2&=36 \\ y^2&=36-x^2 \\ y&=\sqrt{36-x^2} \end{aligned}$$



Range: $0 \le y \le 6$

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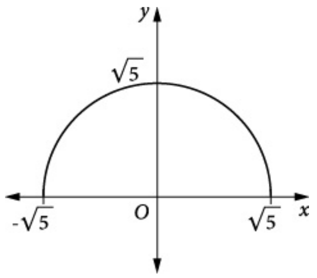
(c) The circle's centre is $(0,\ 0)$ and the radius is $\sqrt{5} \approx 2.23$.

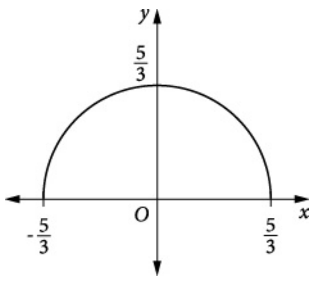
$$
\begin{aligned}
x^2 + y^2 &= 5 \\
y^2 &= 5 - x^2 \\
y &= \sqrt{5 - x^2}
\end{aligned}
$$



Range: $0 \le y \le \sqrt{5}$

(d) The circle's centre is $(0,\ 0)$ and the radius is $\sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$.

$$
\begin{aligned}
x^2 + y^2 &= \frac{25}{9} \\
y^2 &= \frac{25}{9} - x^2 \\
y &= \sqrt{\frac{25}{9} - x^2}
\end{aligned}
$$



Range: $0 \le y \le \dfrac{5}{3}$

10 (a) $x^2 + y^2 + 2x - 6y + 1 = 0$

$x^2 + 2x + 1 + y^2 - 6y + 9 = -1 + 1 + 9$

$(x+1)^2 + (y-3)^2 = 9$

Centre $(-1,\ 3)$, radius $\sqrt{9} = 3$

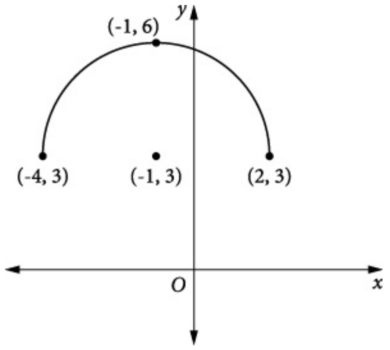
(b) $(x+1)^2 + (y-3)^2 = 9$

$(y-3)^2 = 9 - (x+1)^2$

$y - 3 = \sqrt{9-(x+1)^2}$

$y = 3 + \sqrt{9-(x+1)^2}$

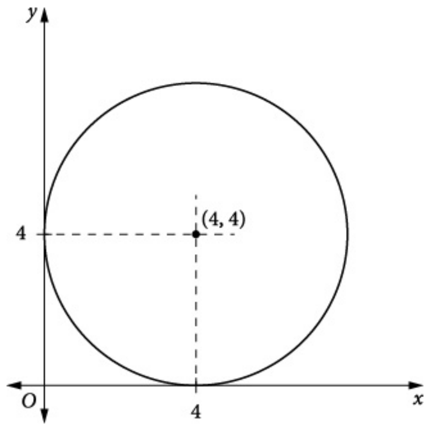
(c) The graph will be the top half of a circle of centre $(-1,\ 3)$, radius $\sqrt{9} = 3$.



(d) From the graph, the domain will be $[-4,\ 2]$ or $-4 \le x \le\ 2$, and the range will be $[3,\ 6]$ or $3 \le y \le\ 6$.

12 The circle touches the x-axis at $(4,0)$ so the *x*-axis is a tangent, and the centre must lie on the line $x = 4$.

The circle touches the y-axis at $(0,4)$ so the y-axis is a tangent, and the centre must lie on the line $y = 4$.



From the diagram, the centre is $(4,\ 4)$ and the radius is 4.

$(x-4)^2+(y-4)^2=16$

14 Substitute $(0,0)$ into $x^2+y^2-5x+3y+2$

$0^2+0^2-5\times0+3\times0+2=2>0$

$\therefore(0,0)$ is outside of the circle.

OR

$$\begin{aligned}
&x^2+y^2-5x+3y+2=0\\
&x^2-5x+\left(\frac{5}{2}\right)^2+y^2+3y+\left(\frac{3}{2}\right)^2=-2+\frac{25}{4}+\frac{9}{4}\\
&\left(x-\frac{5}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{26}{4}
\end{aligned}$$

The circle has centre $\left(\frac{5}{2},\ \frac{3}{2}\right)$ and radius $\frac{\sqrt{26}}{2}$.

Calculate the distance of the origin from the centre of the circle.

$$d^2=\left(0-\frac{5}{2}\right)^2+\left(0+\frac{3}{2}\right)^2=\frac{25}{4}+\frac{9}{4}=\frac{34}{4}$$

The distance of the origin from the centre is $\frac{\sqrt{34}}{2}$, which is greater than the length of the radius, so the origin must be outside the circle.

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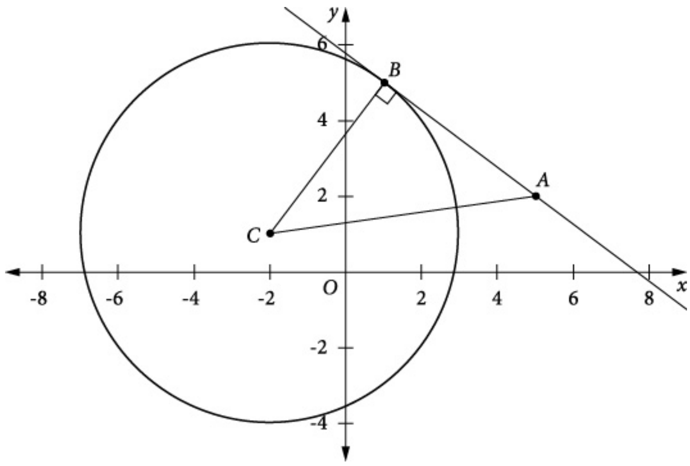
16 (a) $x^2 + y^2 + 4x - 2y - 20 = 0$

$$\begin{aligned} x^2 + 4x + 4 + y^2 - 2y + 1 &= 20 + 4 + 1 \\ (x+2)^2 + (y-1)^2 &= 25 \end{aligned}$$

The centre is $C(-2,1)$, the radius is 5.



Let the point of contact of the tangent and the circle be $B(x, y)$

$BC = 5$ since it is the radius of the circle

$$\begin{aligned} AC &= \sqrt{(5+2)^2 + (2-1)^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

The tangent from $A(5, 2)$ is perpendicular to a radius that is drawn to the point of contact.

Using Pythagoras' theorem

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ 50 &= AB^2 + 25 \\ AB^2 &= 50 - 25 \\ AB^2 &= 25 \\ AB &= 5 \end{aligned}$$

The length of the tangent to the circle from the point $(5, 2)$ is 5 units.

(b) Let $x = 0$

$$\begin{aligned}(0+2)^2 + (y-1)^2 &= 25\\ 4 + (y-1)^2 &= 25\\ (y-1)^2 &= 21\\ y-1 &= \pm\sqrt{21}\\ y &= 1 \pm \sqrt{21}\end{aligned}$$

The distance between these two points is $1+\sqrt{21}-(1-\sqrt{21}) = 2\sqrt{21}$

The length of the intercept on the y-intercept is $2\sqrt{21}$ units.

18 (a) The midpoint M of AB is $\left(\frac{-1+5}{2}, \frac{3+7}{2}\right) = (2, 5)$

(b) The distance between the midpoint M and A or B is the radius of the circle

$$\begin{aligned}d_{AM} &= \sqrt{(2-(-1))^2 + (5-3)^2}\\ &= \sqrt{13}\\ r &= \sqrt{13}\end{aligned}$$

The midpoint M is the circle centre

The equation of the circle is:

$$\begin{aligned}(x-2)^2 + (y-5)^2 &= 13\\ x^2 - 4x + 4 + y^2 - 10y + 25 - 13 &= 0\\ x^2 + y^2 - 4x - 10y + 16 &= 0\end{aligned}$$

(c) On the y-axis, $x = 0$.

$$\begin{aligned}(0-2)^2 + (y-5)^2 &= 13\\ 4 + (y-5)^2 &= 13\\ (y-5)^2 &= 9\\ y-5 &= \pm 3\\ y &= 5-3, y = 5+3\\ y &= 2, y = 8\end{aligned}$$

The two intersection points are $(0, 2)$ and $(0, 8)$

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20 (a) (i) The general form is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the centre of the circle and r is the radius.

$h=2,\ k=1$ and $r=6-2=4$

$$(x-2)^2+(y-1)^2=4^2$$
$$(x-2)^2+(y-1)^2=16$$

(ii) To find the general form this equation needs to be expanded and simplified.

$$\begin{aligned}(x-2)^2+(y-1)^2&=16\\ x^2-4x+4+y^2-2y+1&=16\\ x^2+y^2-4x-2y+5&=16\\ x^2+y^2-4x-2y-11&=0\end{aligned}$$

(b) (i) The general form is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the centre of the circle and r is the radius.

$h=-2,\ k=-1$ and $r=6-2=4$

Use Pythagoras' theorem to find r^2.

$$\begin{aligned}(-2-(-5))^2+(-1-3)^2&=r^2\\ 3^2+(-4)^2&=r^2\\ 9+16&=r^2\\ r^2&=25\end{aligned}$$

The equation is

$$\begin{aligned}(x-(-2))^2+(y-(-1))^2&=25\\ (x+2)^2+(y+1)^2&=25\end{aligned}$$

(ii) To find the general form this equation needs to be expanded and simplified.

$$\begin{aligned}(x+2)^2+(y+1)^2&=25\\ x^2+4x+4+y^2+2y+1&=25\\ x^2+y^2+4x+2y+5&=25\\ x^2+y^2+4x+2y-20&=0\end{aligned}$$

(c) (i) The general form is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the centre of the circle and r is the radius.

$h=-3,\ k=2$

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Use Pythagoras' theorem to find r^2:

$$\begin{aligned}(-3-0)^2+(2-1)^2&=r^2\\(-3)^2+1^2&=r^2\\9+1&=r^2\\r^2&=10\end{aligned}$$

The equation is:

$$\begin{aligned}(x-(-3))^2+(y-2)^2&=10\\(x+3)^2+(y-2)^2&=10\end{aligned}$$

(ii) To find the general form this equation needs to be expanded and simplified.

$$\begin{aligned}(x+3)^2+(y-2)^2&=10\\x^2+6x+9+y^2-4y+4&=10\\x^2+y^2+6x-4y+13&=10\\x^2+y^2+6x-4+3&=0\end{aligned}$$

(d) (i) The general form is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the centre of the circle and r is the radius.

$h=-2,\ k=-1$

Use Pythagoras' theorem to find r^2:

$$\begin{aligned}(1-3)^2+(-2-(-4))^2&=r^2\\(-2)^2+(2)^2&=r^2\\4+4&=r^2\\r^2&=8\end{aligned}$$

The equation is

$$\begin{aligned}(x-3)^2+(y-(-4))^2&=8\\(x-3)^2+(y+4)^2&=8\end{aligned}$$

(ii) To find the general form this equation needs to be expanded and simplified.

$$\begin{aligned}(x-3)^2+(y+4)^2&=8\\x^2-6x+9+y^2+8y+16&=8\\x^2+y^2-6x+8y+25&=8\\x^2+y^2-6x+8y+17&=0\end{aligned}$$

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EXERCISE 4.6 CUBIC POLYNOMIALS

2 **(a)** $(x+2)^3=0$

$$x+2=0$$
$$x=-2$$

(b) $(x+2)^3=1$

$$x+2=1$$
$$x=-1$$

(c) $(x+2)^3=8$

$$x+2=2$$
$$x=0$$

(d) $(x+2)^3=2$

$$x+2=\sqrt[3]{2}$$
$$x=\sqrt[3]{2}-2$$

(e) $(x-1)^3=4$

$$x-1=\sqrt[3]{4}$$
$$x=\sqrt[3]{4}+1$$

(f) $3(x-4)^3=5$

$$(x-4)^3=\frac{5}{3}$$
$$x-4=\sqrt[3]{\frac{5}{3}}$$
$$x=\sqrt[3]{\frac{5}{3}}+4$$
$$=\frac{\sqrt[3]{5}}{\sqrt[3]{3}}\times\left(\frac{\sqrt[3]{3}}{\sqrt[3]{3}}\right)^2+4$$
$$=\frac{\sqrt[3]{45}}{3}+4$$

4 **(a)** $(x-1)(x+1)(x+2)=0$

$$(x-1)=0,\ (x+1)=0,\ (x+2)=0$$
$$x=1,\ x=-1,\ x=-2$$

(b) $(x-1)(x+1)(x+2)=-2$

Expand and simplify:

$$(x^2-1)(x+2)=-2$$
$$x^3+2x^2-x-2=-2$$
$$x^3+2x^2-x=0$$
$$x(x^2+2x-1)=0$$

$$x=0 \text{ or } x=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm2\sqrt{2}}{2}$$

$$x=0 \text{ or } x=-1\pm\sqrt{2}$$

The solutions are $x=0,-1+\sqrt{2},-1-\sqrt{2}$

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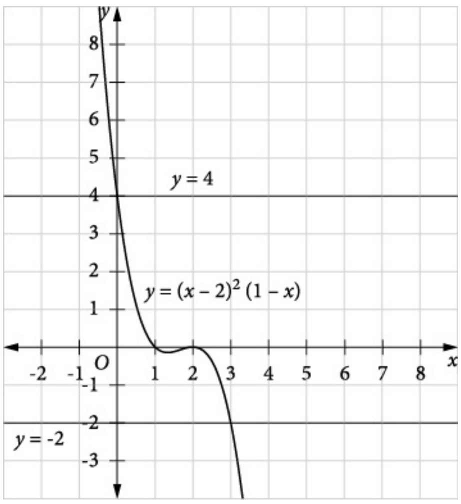
6 $y = (x-2)^2(1-x)$

There is a double root at $x = 2$ and a single root at $x = 1$

The y-intercept is $(0-2)^2(1-0) = 4$.

As $x \to \infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to \infty$.



(a) Reading from the x-axis,

$(x-2)^2(1-x) = 0$ when $x = 1, x = 2$

(b) Reading from the line $y = 4$,

$(x-2)^2(1-x) = 4$ when $x = 0$

(c) Reading from the line $y = -2$,

$(x-2)^2(1-x) = -2$ when $x = 3$

(d) To have three distinct roots, horizontal lines can be drawn approximately between $y = -0.15$ and $y = 0$.

So $-0.15 < c < 0$

(e) The x^3 term comes from $x \times x \times (-x) = -x^3$.

The coefficient of x^3 is -1

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8 There is a double root at $-a$, so $(x+a)^2$ must be a factor.

There is a single root at b, so $(x-b)$ or $(b-x)$ must be a factor.

The y-intercept is positive, so the factor containing b must be $(b-x)$.

B $y=(x+a)^2(b-x)$

10 Using the null factor law, the cubic function that cuts the x-axis at $x=-1,2,3$ can be written

$y=a(x+1)(x-2)(x-3)$.

The y-intercept, $(0,6)$, occurs when $x=0$.

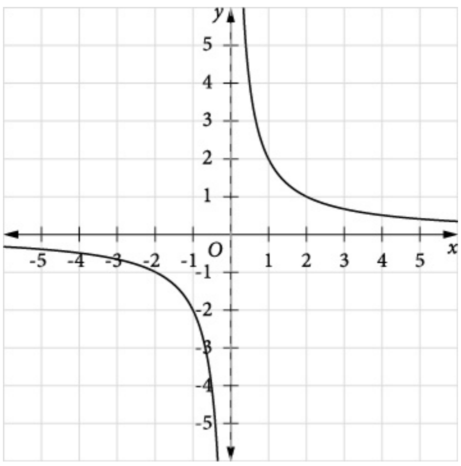
$$6=a(1)(-2)(-3)$$
$$6=6a$$
$$a=1$$

$y=(x+1)(x-2)(x-3)$.

EXERCISE 4.7 THE EQUATION $y = k / x$ AND INVERSE VARIATION

2 This is a rectangular hyperbola, and the axes will be the asymptotes.

Since k is positive, it will be in the first and third quadrants.



Vertical asymptote: $x=0$

Horizontal asymptote: $y=0$

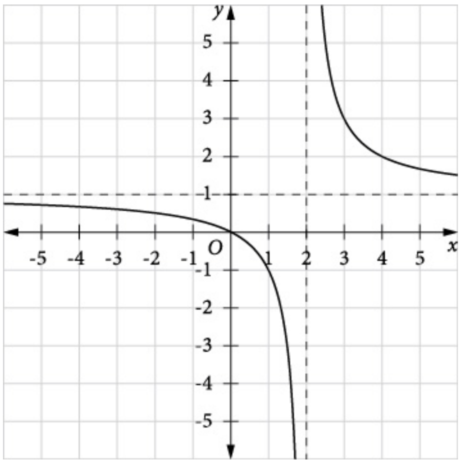
4 $y = \dfrac{x}{x-2}$

$= \dfrac{x-2+2}{x-2}$

$= \dfrac{x-2}{x-2} + \dfrac{2}{x-2}$

$= 1 + \dfrac{2}{x-2}$

This is the graph of $y = \dfrac{2}{x}$ translated 2 units to the right and 1 unit up.



Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

The domain is real x, $x \neq 2$.

The range is real y, $y \neq 1$

6 **(a)** $PV = k$

$15.28 \times 2 = k$

$k = 30.56$

(b) $P = \dfrac{k}{V}$

$= \dfrac{30.56}{4}$

$= 7.64$

7.64 atmospheres

(c) $V = \dfrac{k}{P}$

$= \dfrac{30.56}{90}$

$= 0.3395...$

0.34 litres

EXERCISE 4.8 WORKING WITH FUNCTIONS

2 D

$$
\begin{aligned}
f(x) &= x^2 + 7 \\
g(x) &= 5 - 2x \\
f(x)g(x) &= (x^2 + 7)(5 - 2x) \\
&= 5x^2 - 2x^3 + 35 - 14x \\
&= -2x^3 + 5x^2 - 14x + 35
\end{aligned}
$$

4 $f(x) = x + 4$

$g(x) = x^2 - 6$

(a) $f(x) + g(x) = x + 4 + x^2 - 6$

$= x^2 + x - 2$

Domain: Real x

Range: Use $x = -\dfrac{b}{2a}$ to find the axis of symmetry.

$-\dfrac{b}{2a} = -\dfrac{1}{2}$

When $x = -\dfrac{1}{2}$:

$$
\begin{aligned}
x^2 + x - 2 &= \left(-\frac{1}{2}\right)^2 - \frac{1}{2} - 2 \\
&= \frac{1}{4} - \frac{1}{2} - 2 \\
&= -2\frac{1}{4}
\end{aligned}
$$

This represents the minimum value of the quadratic, so the range is $y \geq -2\dfrac{1}{4}$.

(b) $f(x) - g(x) = x + 4 - (x^2 - 6)$

$= x + 4 - x^2 + 6$
$= -x^2 + x + 10$

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Domain: Real x

Range: Use $x = -\frac{b}{2a}$ to find the axis of symmetry.

$$-\frac{b}{2a} = -\frac{1}{-2}$$
$$= \frac{1}{2}$$

When $x = \frac{1}{2}$:

$$-x^2 + x + 10 = -\left(\frac{1}{2}\right)^2 - \frac{1}{2} + 10$$
$$= -\frac{1}{4} - \frac{1}{2} + 10$$
$$= 9\frac{3}{4}$$

This represents the maximum value of the quadratic, so the range is $y \le 9\frac{3}{4}$.

6 $f(x) = x$

$g(x) = x + 4$

(a) $f(x)g(x) = x(x+4)$

$$= x^2 + 4x$$

Domain: Real x

Range: x-intercepts at 0 and -4 so the axis of symmetry is $x = -2$.

When $x = -2$:

$$x^2 + 4x = (-2)^2 + 4(-2)$$
$$= 4 - 8$$
$$= -4$$

This represents the minimum value of the quadratic, so the range is $[-4, \infty)$.

(b) $\frac{g(x)}{f(x)}=\frac{x+4}{x}$

$$=\frac{x}{x}+\frac{4}{x}$$
$$=1+\frac{4}{x}$$

Domain: $\frac{4}{x}$ is undefined when $x=0$, so the domain is real x, $x\neq 0$.

Range: The graph of $y=\frac{4}{x}$ has a horizontal asymptote at $y=0$.

This graph is translated 1 unit up, so the asymptote is $y=1$. Thus, the range is real y, $y\neq 1$.

(c) $\frac{f(x)}{g(x)}=\frac{x}{x+4}$

$$=\frac{x+4-4}{x+4}$$
$$=\frac{x+4}{x+4}-\frac{4}{x+4}$$
$$=1-\frac{4}{x+4}$$

Domain: $\frac{4}{x+4}$ is undefined when $x=-4$, so the domain is real x, $x\neq -4$.

Range: The graph of $y=\frac{4}{x+4}$ has a horizontal asymptote at $y=0$.

This graph is translated 1 unit up, so the asymptote is $y=1$. Thus, the range is real y, $y\neq 1$.

8 $f(x)=x$

$g(x)=x^2+4$

(a) $f(x)g(x)=x(x^2+4)$

$$=x^3+4x$$

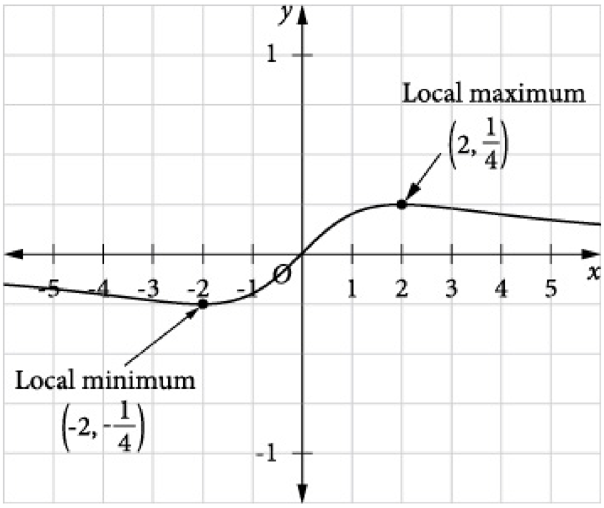
Domain: Real x

Range: Real numbers

(b) $\frac{f(x)}{g(x)}=\frac{x}{x^2+4}$

Domain: The denominator is always non-zero, so the domain is real x.

Range: This function cannot be rearranged into a format that lends itself to analysis. Hence, draw the graph. Either draw a careful graph and estimate the maximum and minimum values, or use graphing software.



The range is $[-0.25, 0.25]$

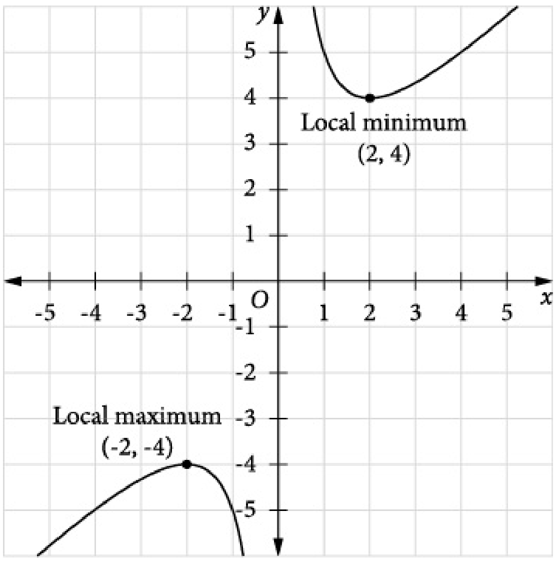
(c) $\dfrac{g(x)}{f(x)} = \dfrac{x^2+4}{x}$

$$= \frac{x^2}{x} + \frac{4}{x}$$

$$= x + \frac{4}{x}$$

Domain: Real x, $x \neq 0$

Range: This function cannot be rearranged into a format that lends itself to analysis. Hence, draw the graph. Either draw a careful graph and estimate the maximum and minimum values, or use graphing software.



The range is $(-\infty,-4]$ and $[4,\infty)$.

10 $f(x)=x+4$

$g(x)=x^2-16$

(a) $f(x)g(x)=(x+4)(x^2-16)$

$$=x^3-16x+4x^2-64$$
$$=x^3+4x^2-16x-64$$

Domain: Real x

Range: The graph is a cubic, so the range is the set of real numbers

(b) $\dfrac{g(x)}{f(x)}=\dfrac{x^2-16}{x+4}$

$$=\frac{(x-4)(x+4)}{x+4}$$
$$=x-4, x\neq -4$$

Domain: Real x, $x\neq -4$

Range: Since $x\neq -4$, $\dfrac{g(x)}{f(x)}\neq -4-4$.

So, the range is the set of all real numbers except -8.

(c) $\dfrac{f(x)}{g(x)}=\dfrac{x+4}{x^2-16}$

$$=\frac{x+4}{(x+4)(x-4)}$$

This function is defined for all values of x except

If $x \neq \pm 4$, $\dfrac{x+4}{(x+4)(x-4)}=\dfrac{1}{x-4}$.

Domain: Real x, $x \neq \pm 4$

Range: When $x=-4$, $\dfrac{f(x)}{g(x)}=\dfrac{1}{-4-4}=-\dfrac{1}{8}$

$y=\dfrac{1}{x-4}$ will have a horizontal asymptote $y=0$.

So the range is the set of all real numbers except for 0 and $-\dfrac{1}{8}$.

12 (a) $f(x)g(x)=(x+3)(x^2-5)$

$$=x^3-5x+3x^2-15$$
$$=x^3+3x^2-5x-15$$

This is a cubic function.

Domain: The set of real numbers

Range: The set of real numbers

(b) $g(x)h(x)=(x^2-5)\sqrt{x-4}$

$\sqrt{x-4}$ is defined for $x \geq 4$

Domain: $[4,\infty)$

Range: In the restricted domain x^2-5 has a minimum value of 11 when $x=4$.

The minimum value of $\sqrt{x-4}$ is 0 and occurs when $x=4$.

So, at $x=4$, the minimum value of $g(x)h(x)$ is 0 when $x=4$.

Hence, the range is $[0,\infty)$.

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(c) $\dfrac{f(x)}{g(x)} = \dfrac{x+3}{x^2-5}$

Domain: $x^2 - 5 = 0$ when $x = \pm\sqrt{5}$. Hence, the domain is Real x, $x \neq \pm\sqrt{5}$.

Range:

For $-\sqrt{5} < x < \sqrt{5}, f(x) > 0,\ g(x) < 0$ so $\dfrac{f(x)}{g(x)} < 0$

For $x > \sqrt{5}, f(x) > 0,\ g(x) > 0$ so $\dfrac{f(x)}{g(x)} > 0$

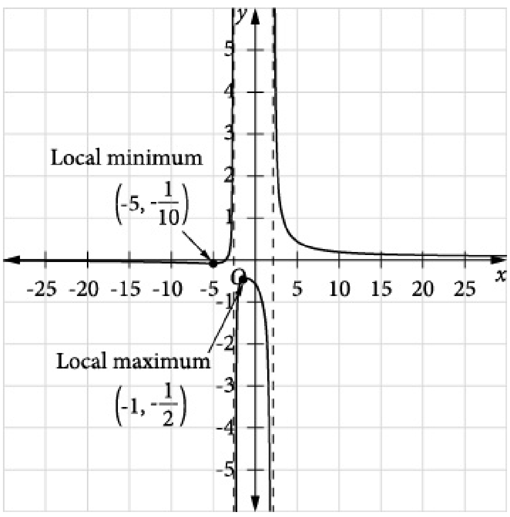
For $-3 < x < -\sqrt{5}, f(x) > 0,\ g(x) > 0$ so $\dfrac{f(x)}{g(x)} > 0$

For $x = -3,\ f(x) = 0$ so $\dfrac{f(x)}{g(x)} = 0$

For $x < -3,\ f(x) < 0,\ g(x) > 0$ so $\dfrac{f(x)}{g(x)} < 0,$ but as $x \to -\infty,$ $\dfrac{f(x)}{g(x)} \to 0$ from below. For these values it would seem safe to say $\dfrac{f(x)}{g(x)} \geq 0$.

For $x = 0,$ $\dfrac{f(x)}{g(x)} = -0.6$ and it seems safe to say that $\dfrac{f(x)}{g(x)} \leq -0.6$.

To determine what happens to the range between -0.6 and 0 requires a graph.



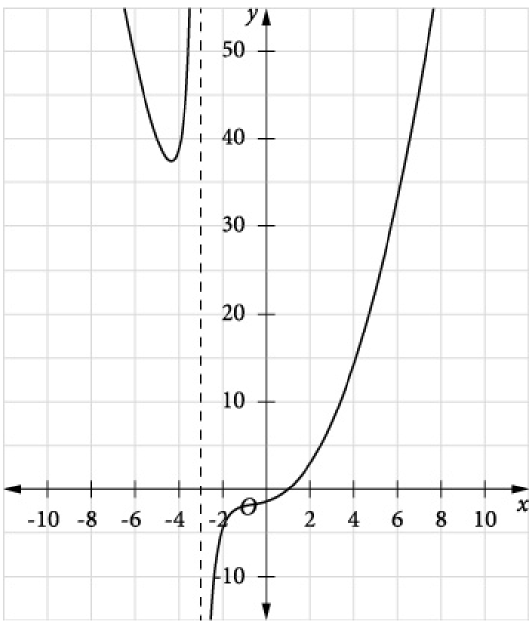
So, the range is $(-\infty, -0.5]$ and $[-0.1, \infty)$

(d) $\dfrac{k(x)}{f(x)} = \dfrac{x^3 + 2x^2 - 6}{x + 3}$

Domain: Real x, $x \neq -3$

Range: It looks as if the answer is the set of real numbers since when $x < -3$, $\dfrac{k(x)}{f(x)} > 0$ and when $x > -3$, $\dfrac{k(x)}{f(x)}$ can be positive or negative, with $k(x) = 0$ for some values of x.

Draw a graph to check.



This confirms the range is the set of real numbers.

(e) $\dfrac{h(x)}{f(x)} = \dfrac{\sqrt{x-4}}{x+3}$

Domain: $\sqrt{x-4}$ defined for $x \geq 4$ and the denominator is not defined for $x = -3$. So, the domain is $[4, \infty)$.

Range: The numerator is always positive or zero in the restricted domain. However, it is difficult to further analyse the function without drawing the graph.

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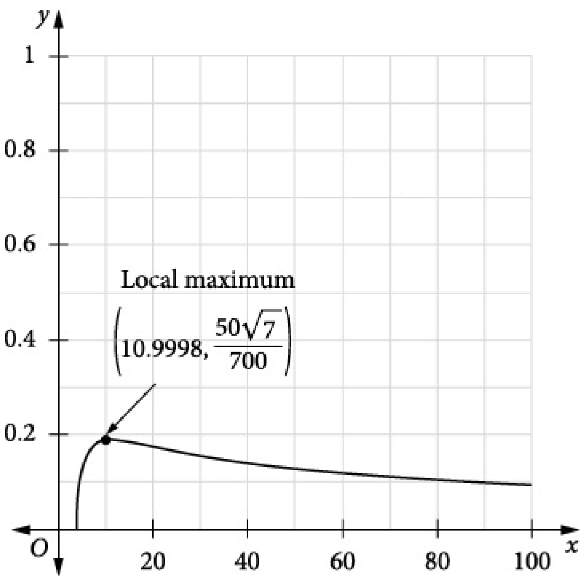
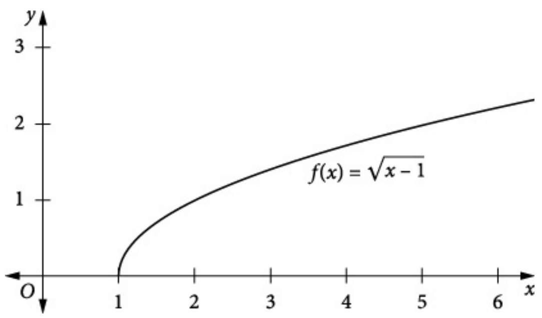


$\frac{50\sqrt{7}}{700} \approx 0.189$ so the range is $[0, 0.189]$

CHAPTER REVIEW 4

2 (a) $f(x) = \sqrt{x-1}$

$x \geq 1$. The minimum value of x is 1.



(b) $f(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

There will be vertical asymptotes at $x = 2$ and $x = -2$.

As $x \to \infty$, $f(x) \to 0$ from above.

As $x \to -\infty$, $f(x) \to 0$ from above.

When As $-4 \leq x \leq 4$, $f(x) < 0$. from above.

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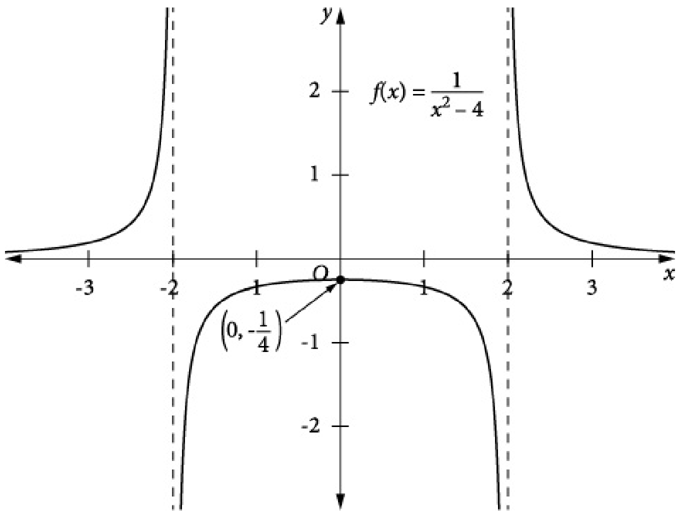
The minimum value of $x^2 - 4$ is -4, so the maximum value of $f\left(x\right)$ for $-4 \le x \le 4$ is $-\dfrac{1}{4}$.



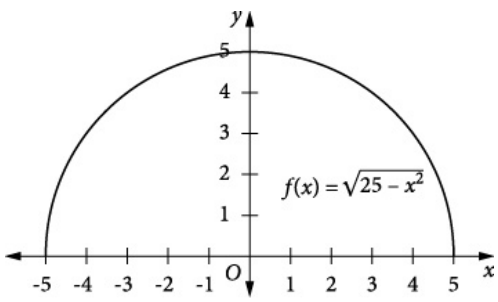
(c) $f\left(x\right) = \sqrt{25 - x^2}$

Note that $f\left(x\right) \ge 0$.

It may be easier to let $f\left(x\right) = y$.

$$y = \sqrt{25 - x^2}$$
$$y^2 = 25 - x^2$$
$$x^2 + y^2 = 25$$

This is the top half of a circle with centre the origin and radius 5 units.



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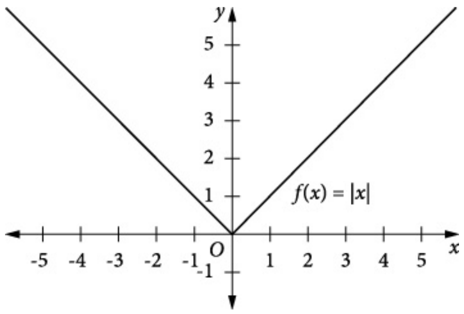
(d) $f(x)=|x|$

When $x \geq 0$, $f(x)=x$.

When $x<0$, $f(x)=-x$.



4 $y=1-|x|$

When $x \geq 0$, $f(x)=1-x$.

When $x<0$, $f(x)=1-(-x)=x+1$.

Domain: all real x

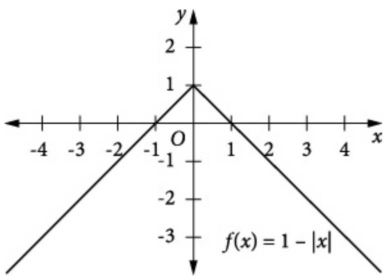
Since $|x| \geq 0$, $y \leq 1$

Range: $y \leq 1$

There is a sharp turning point at $(0,\ 1)$

x-intercept when $y=0$

$$\begin{aligned}1-|x|&=0\\|x|&=1\\x&=\pm 1\end{aligned}$$



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6 **(a)** $f(x)=\sqrt{x+3}+\sqrt{2-x}$

For $\sqrt{x+3}$ is only defined if $x \geq -3$.

For $\sqrt{2-x}$ is only defined if $x \leq 2$.

Domain: $-3 \leq x \leq 2$

(b) $g(x)=\dfrac{x}{|x-1|}$

$g(x)$ is only undefined when $x-1=0 \Rightarrow x=1$

Domain: all real x, $x \neq 1$

8 Where $x \geq 0$, $f(x)=x+1$.

Where $x<0$, $f(x)=-x+1=1-x$.

$y=|x|+1$

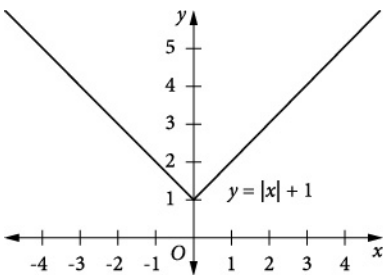
Domain: all real x

Since $|x| \geq 0$, $y \geq 1$

Range: $y \geq 1$

There is a sharp turning point at $(0,\ 1)$

There will be no x-intercept since $y \geq 1$.



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10 If the line $y = x - 4$ is a tangent of the circle, $x^2 + y^2 = 8$, there will only be one point of intersection. To find the point(s) of intersection, substitute $y = x - 4$ into $x^2 + y^2 = 8$.

$$\begin{aligned} x^2 + (x-4)^2 &= 8 \\ x^2 + x^2 - 8x + 16 &= 8 \\ 2x^2 - 8x + 8 &= 0 \\ x^2 - 4x - 4 &= 0 \\ (x-2)^2 &= 0 \\ x &= 2 \end{aligned}$$

There is only one point of intersection, so the line $y = x - 4$ must be a tangent to the circle $x^2 + y^2 = 8$. At $x = 2$, $y = 2 - 4 = -2$, the point of intersection is $(2, -2)$.

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