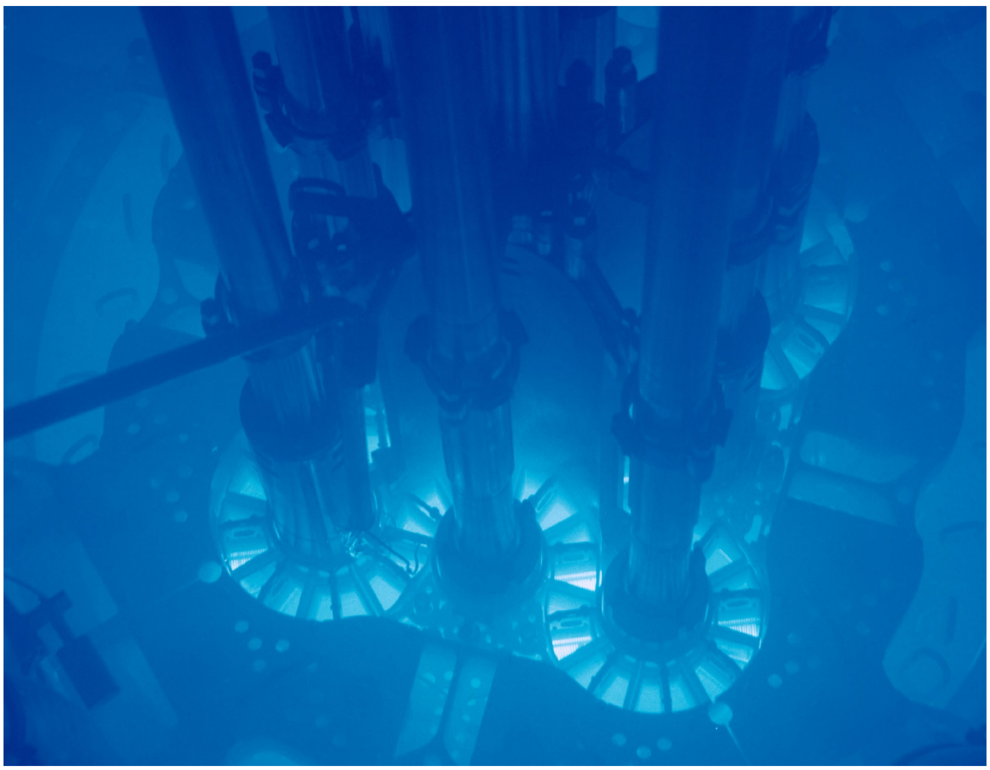
Module 8: From the Universe to the Atom

Part 4: Properties of the Nucleus



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Contents

Cover photo credit: Nuclear reactor core, U.S. Department of Energy

Syllabus content: From the Universe to the Atom

Properties of the Nucleus

Inquiry question: How can the energy of atomic nucleus be harnessed?

Students:

- analyse the spontaneous decay of unstable nuclei, and the properties of the alpha, beta and gamma radiation emitted (ACSPH028, ACSPH030)
- examine the model of half-life in radioactive decay and make quantitative predictions about the activity or amount of a radioactive sample using the following relationships:
  - $N_t = N_0 e^{-\lambda t}$
  - $\lambda = \frac{\ln 2}{t_{1/2}}$
    where N_t = number of particles at time t, N_0 = number of particles present at $t = 0$, = decay constant, $t_{1/2}$ = time for half the radioactive amount to decay (ACSPH029)
- model and explain the process of nuclear fission, including the concepts of controlled and uncontrolled chain reactions, and account for the release of energy in the process (ACSPH033, ACSPH034)
- analyse relationships that represent conservation of mass-energy in spontaneous and artificial nuclear transmutations, including alpha decay, beta decay, nuclear fission and nuclear fusion (ACSPH032)
- account for the release of energy in the process of nuclear fusion (ACSPH035, ACSPH036)
- predict quantitatively the energy released in nuclear decays or transmutations, including nuclear fission and nuclear fusion, by applying: (ACSPH031, ACSPH035, ACSPH036)
  - the law of conservation of energy
  - mass defect
  - binding energy
  - Einstein's mass-energy equivalence relationship $E = mc^2$

Natural Radioactivity

Radioactivity was discovered, almost accidentally, by Henri Becquerel in 1896 when he was examining the emission of a type of radiation emitted by phosphorescent material (a uranium salt) when placed in the sunlight. Becquerel had found that this radiation could pass through paper that was opaque to light and expose photographic film. On one particular day when he prepared some uranium salt with photographic paper to place in the sun, he found that it was an overcast outside and he put the uranium wrapped in photographic paper aside in a dark cupboard. It remained overcast for several days, and when he finally removed it from the cupboard he decided to develop the film anyway - a remarkable decision - he then found that the photographic film had been exposed, even though the uranium salt had not been placed in the sun [1]. The radiation appeared to be coming from the uranium salt itself, in the absence of other sources of energy. As Becquerel said, the effect was "outside the phenomena which one might expect to observe" [1]

Marie and Pierre Curie discovered the existence of radiation (then called Becquerel rays) emanating from other substances such as thorium, polonium and radium, and identified the process as one involving individual atoms. They eventually shared the Nobel prize in 1903 with Becquerel for their discoveries.

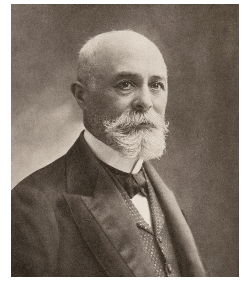

Figure 1: Henri Bequerel. [Public domain] By Paul Nadar - Portrait of Antoine-Henri Becquerel (1852-1908). `https://commons.wikimedia.org/w/index.php?curid=6863939`

[1] Reference: Pais, A. (1986). Inward Bound: Of matter and Forces in the PhysicalWorld. (New York: Oxford University Press). Also see "Uranium: Twisting the Dragon’s tail", 9:45min into https://www.youtube.com/watch?v=CPOijjgYD2I

α decay

Rutherford had identified alpha particles as helium nuclei by 1908 (note that this is before the discovery of the existence of the nucleus). In an alpha decay process a nucleus emits an alpha particle (Helium nucleus) and the atomic number of the emitter A (number of protons in the nucleus) decreases by two and its mass number Z (the total number of protons and neutrons), decreased by four. That is,

$$^{A}_{Z}X \rightarrow^{A-4}_{Z-2} X' +^{4}_{2} He$$

Natural examples of alpha decay include

$$^{238}_{92}U \rightarrow^{234}_{90} Th +^{4}_{2} He$$

and

$$^{232}_{90}Th \rightarrow^{228}_{88} Ra +^{4}_{2} He$$

Alpha decay is modelled in the PhET: `https://phet.colorado.edu/en/simulation/alpha-decay`. Note carefully how sensitive the decay time is to the difference between the energy of the nucleons and the energy required for an particle to escape the nucleus.

β radiation

Walter Kaufmann identified beta particles as electrons in 1902. There are three beta decay processes, the decay of a neutron to produce emission of an electron and an anti-neutrino, the decay of a bound proton to produce the emission of a positron (an anti-electron) and a neutrino and finally 'resonance capture' where one of the inner electrons is captured by the nucleus, producing a neutron and a neutrino.

$$n \rightarrow p^{+} + e^{-} + \bar{\nu}$$
$$p^{+} \rightarrow n + e^{+} + \nu$$
$$p^{+} + e^{-} \rightarrow n + \nu$$

Examples of natural beta decay processes are shown in figure 2.

Table 9.1 Typical β-Decay Processes

Decay	Type	Q (MeV)	$t_{1/2}$
$^{23}Ne \rightarrow ^{23}Na + e^{-} + \bar{\nu}$	β^{-}	4.38	38 s
$^{99}Tc \rightarrow ^{99}Ru + e^{-} + \bar{\nu}$	β^{-}	0.29	2.1×10^{5} y
$^{25}Al \rightarrow ^{25}Mg + e^{+} + \nu$	β^{+}	3.26	7.2 s
$^{124}I \rightarrow ^{124}Te + e^{+} + \nu$	β^{+}	2.14	4.2 d
$^{15}O + e^{-} \rightarrow ^{15}N + \nu$	ε	2.75	1.22 s
$^{41}Ca + e^{-} \rightarrow ^{41}K + \nu$	ε	0.43	1.0×10^{5} y

Figure 2: Examples of beta decay. Table from: Krane, Kenneth. (1988). Introductory Nuclear Physics. (John Wiley Sons).

Examine the process of beta decay in the PhET simulation: `https://phet.colorado.edu/en/simulation/legacy/beta-decay`

γ *radiation*

A third type of radiation emitted during nuclear decay processes is γ radiation, which is a high energy photon. γ rays are most commly emitted as part of an α or β decay process, as a mechanism by which an excited nucleus can relax to a lower energy state. In some nuclear decays, such as that of Molybdenum-99 to Technetium-99m, the nucleus remains in a high energy state, releasing a gamma ray after an extended period of time. These isotope are called "metastable" (and denoted with an 'm', such as Tc_{99m}).

Properties of α*, beta and* γ *radiation*

We will use a cloud chamber to observe $alpha$ radiation from a small piece of Uranium ore, and discuss its properties compared to *beta* and *gamma* radiation.[2]

Complete the table with the properties of α, β and γ radiation.

[2] A YouTube video showing α and β tracks in a cloud chamber. `https://www.youtube.com/watch?v=FS2pKyRKeYs`

Radiation	What it is?	Penetrating power	mass	charge	behaviour in a B field
α					
β					
γ					

Question 1. Radon-222 forms part of a long radioactive decay "chain" from Uranium-238 to Lead-206.

(a) In part of this chain, ${}^{222}_{86}Rn$ undergoes an α decay to produce a new radionuclide. Identify the resultant radionuclide.

(b) After three *more* radioactive decay processes occur, the resulting radionuclide is ${}^{214}_{84}Po$. Identify the nature of these three additional decay processes.

(c) In an earlier part of the decay series, Protactinium-234 (${}^{234}_{91}Pa$) decays to Uranium-234 (${}^{234}_{92}U$). Write an equation for this radioactive decay showing all the decay products.

Half-life

The process of nuclear decay is a statistical process. Each radioactive nucleus has some probability of decaying in a given time interval, however given a collection of nuclei is impossible to tell *which* nucleus will decay and which will remain intact. The probability that a given nucleus will decay in some time interval is independent of the probability of any other nucleus decaying, and independent of how long the nucleus has existed up to that point.

A good analogy is that of continuously rolling a dice. You know that the chance of rolling a 6 is 1/6, but you don't know *when* you will roll a 6 - it could be on your first roll, or it could be on your 20th roll. If you are rolling two die, then the probability of rolling a 6 on one dice is independent of your probability of rolling a 6 on the other. In an analogous way, the probability of any one atom decaying is independent of the probability of any other atom decaying.

Even though we cannot tell when a *particular* nucleus will decay, we can predict the number of nuclei remaining (on average) after a given amount of time has passed.

Extension - Derivation of radioactive decay relationships

The rate at which decays happen is proportional to the number of nuclei present. That is:

$$-\frac{dN}{dt} = \lambda N$$

where N is the number of undecayed nucei and is the decay constant for a particular nuclear decay process.

This equation can be rearranged and integrated as follows:

$$\frac{dN}{N} = -\lambda dt$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt$$

Evaluating the integral we obtain:

$$\ln \frac{N}{N_0} = -\lambda t$$

Taking the natural logarithm of both sides:

$$\frac{N}{N_0} = e^{-\lambda t}$$

Radioactive decay relationships

The number of atoms of a radioactive substance remaining, N_t after time t can be shown (see previous section) to be:

$$N_t = N_0 e^{-\lambda t} \tag{1}$$

where λ is the known as the decay constant.

One way to characterise how quickly a radioactive substance decays to to calculate its **half-life**. This is the time taken for exactly half of the original number of atoms to decay.

To find the half-life, we rearrange equation 1 to make t the subject, and set $N_t/N_0 = 1/2$, to obtain:

$$t_{1/2} = -\frac{1}{\lambda} \ln \frac{1}{2} = \frac{\ln 2}{\lambda}$$

Rearranging this equation, we can obtain an expression for the decay constant in terms of the half-life of the radioactive substance:

$$\lambda = \frac{ln2}{t_{1/2}} \tag{2}$$

Modelling radioactive decay using MMs

We will simulate the process of radioactive decay using MMs.

Begin by making a table for your results. There should be a column to record the time period and another column to record the number of remaining MMs for your group.

Everyone will receive a small packet of MMs, a paper napkin and a clean plastic cup. Pour your MMs onto your paper towel and count them. Combine the count of everyone in your group and enter this as the initial number N_0 in the "time period 0" row of your table.

Tip your MMs into your cup and shake for 1 second, then tip onto your napkin. Any MMs landing face-up (i.e. with the "MMs" writing showing) have decayed during the time you were shaking.

Remove any decayed atoms (by any means you like... ;), record the total remaining number of atoms for your whole group and repeat this process until there are no remaining un-decayed atoms.

Time period	N	$\ln(N/N_0)$
0		
1		
2		
3		
4		
5		
6		

Figure 3: Amount of Tc_{99m} remaining over time.

1. Draw a graph of N_t versus time period.
2. Graph $ln(N/N_0)$ and comment on the shape of the graph.
3. What are the limitations of this model in simulating real radioactive decay?

Question 2. Technetium-99m is a gamma emitter that is used as a radioactive tracer. It is injected into patients for procedures which use gamma ray imaging such as Single Photon Emission Computed Tomography (SPECT), used to detect cancer.

In a procedure, a patient is injected with $N_0 = 1 \times 10^{-10}$g of Tc_{99m} before a procedure. The amount of Tc_{99m} remaining in their body over time is shown in the table on the right.

(a) By completing the table appropriately, graph the data in a form that will produce a straight line, and thereby determine the decay constant of Tc_{99m}.

(b) Calculate the half-life of Tc_{99m} and comment on why this contributes to its usefulness as a radioactive tracer for imaging procedures.

$t(hours)$	$N(\times 10^{10}\text{g})$	
0	1	
2.5	0.75	
5	0.56	
7.5	0.42	
10	0.31	
12.5	0.23	
15	0.18	

Figure 4: Amount of Tc_{99m} remaining over time.

Nuclear stability

Most elements exist as a number of **isotopes**, that is, atoms with the same atomic number but different numbers of neutrons. Some isotopes are stable, while others undergo radioactive decay.

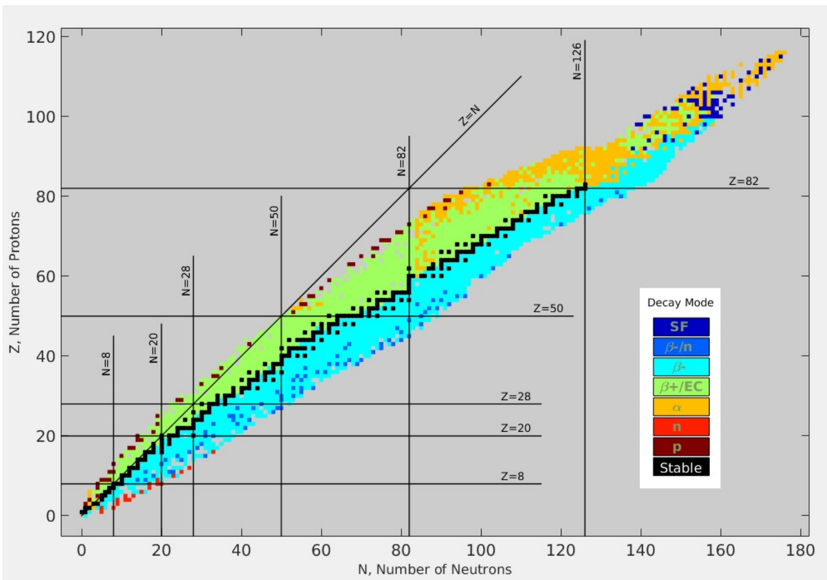

Figure 5: Modes of radioactive decay for isotopes of the elements. An online interactive version is available: `https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`. Image credit: By Bdushaw - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=61302798`

Figure 5 shows the mechanisms by which unstable isotopes decay. The key can be read as:

- SF - spontaneous fission
- β^-/n: decay in which a neutron decays into a proton, expelling both an electron as well a neutron
- β^-: decay in which a neutron decays into a proton, releasing an electron
- β^+/EC: decay in which an inner electron is "captured" by the nucleus (electron capture), converting a proton into a neutron.
- α: decay releasing a helium nucleus
- n: direct emission of a neutron
- p: direct emission of a proton
- Stable: nucleus does not undergo radioactive decay

Note the presence of a "Valley of stability" (the nuclei coloured black), which corresponds to the line $Z = N$ (the number of neutrons equals the number of protons) for small atomic numbers, then lies in the region where $N > Z$ for large atomic numbers.

Mass defect

When nuclear decay occurs, large amounts of energy (relative to the energy released during chemical reactions) are released from the nucleus. The apparent lack of any external energy source to account for the energy released by radiation initially caused great consternation among scientists [3]. The answer was eventually found in Einstein's relationship between the rest mass of an object and its energy content

$$E = m_0 c^2$$

[3] Pais, A. (1986). Inward Bound: Of matter and Forces in the PhysicalWorld. (New York: Oxford University Press).

The difference in mass of the products of a nuclear reaction and the reactants is called the mass defect and can be used to calculate the energy released (or absorbed) during a reaction.

Binding Energy

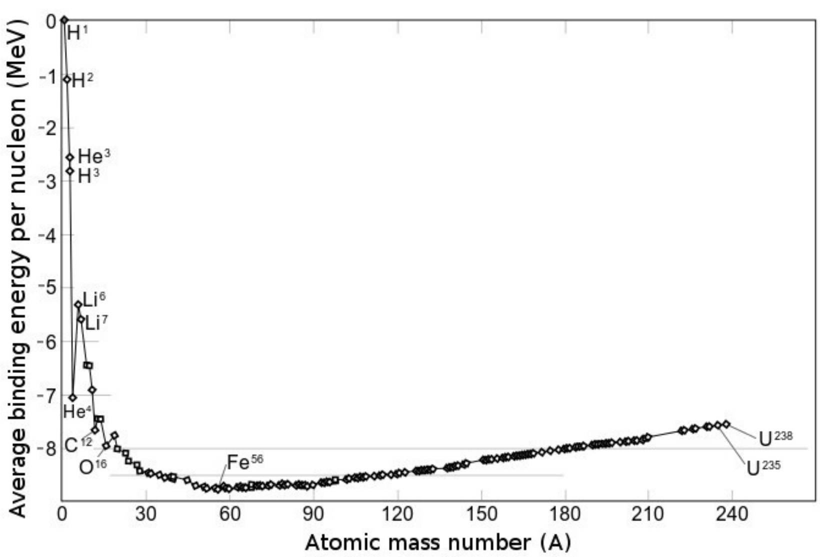

Figure 6: A graph of the binding energy per nucleon as a function of atomic number. Values are negative as energy is required to separate nucleons (This graph also commonly appears with *magnitude* of binding energy per nucleon on the vertical axis). The greater the *magnitude* of the binding energy per nucleon, the more stable the nucleus. Image credit: By Bdushaw - Own work, CC BY-SA 4.0, `https://commons.wikimedia.org/w/index.php?curid=50341087`

The **binding energy** is the energy equivalent of the mass defect between a nucleus and its constituent protons and neutrons.

$$\Delta E_{be} = [\Sigma m - M]c^2$$

where M is the mass of the nucleus, Σm is the sum of the masses of all the individual protons and neutrons that comprise the nucleus, and

$$\Sigma m - M$$

is the *mass defect*.

A meaningful measure of the stability of a nucleus can be obtained by calculating the **binding energy per nucleon**, which is the ratio of the total binding energy to the number of nucleons in the nucleus:

$$\Delta E_{ben} = \frac{\Delta E_{be}}{A}$$

The magnitude of the **binding energy per nucleon** is the **average energy per nucleon required to completely separate the nucleus into its constituent nucleons**.

The greater the magnitude of ΔE_{ben}, the more energy per nucleon that is required to break apart the nucleus. In other words, the higher the magnitude of ΔE_{ben}, the more *stable* the nucleus.

Figure 6 shows a graph of the binding energy per nucleon as a function of atomic number. Notice that iron (Fe) nuclei have the most negative value of ΔE_{ben} (i.e. the largest magnitude of ΔE_{ben}). This means that more energy is release *per nucleon* if a nucleus of iron is formed, than is released *per nucleon* if a nucleus of Uranium or a nucleus of Carbon is formed.

The significance of this observation is that energy would be released overall if a nucleus with an atomic number much larger than Iron were to be broken apart and reformed into two smaller nuclei with atomic numbers closer to iron (a process known as *fission*). Similarly, energy would be released overall if two small nuclei with atomic numbers much less than iron were to be broken apart and reformed into a single nucleus with an atomic number closer to iron (a process known as *fusion*).

Question 3.

(a) What is the binding energy of Uranium 235?

(b) What is the binding energy *per nucleon* of Uranium 235?

Mass of $^{235}_{92}U = 235.0439299$u

Mass of $p = 1.007825$u

Mass of $n = 1.008665$u

Nuclear fission

Extremely large nuclei can undergo spontaneous fission by a process visualised in figure 5. Other large elements can be caused to undergo fission by bombarding them with neutrons. *Slow* neutrons are much more effective at producing fission than fast neutrons. In nuclear reactors, neutrons are slowed using a *moderator*, usually water.

Neutron induced fission

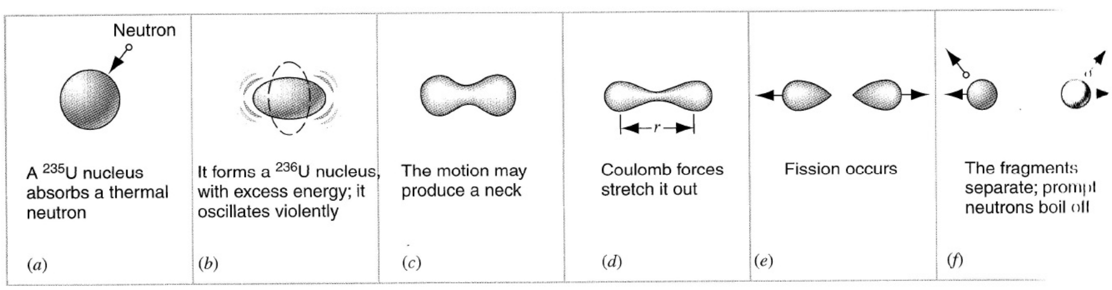

Figure 7: The 'liquid drop' model of how the impact of a neutron into a nucleus can cause instability and then

In neutron-induced fission, a neutron is absorbed by a nucleus, which becomes unstable due to the excess energy. If this instability causes the nucleus to become sufficiently distended that a 'neck' is produced, then electrostatic repulsion can cause the two halves to fly apart, sometimes emitting 'secondary' neutrons in the process, as shown in figure.

A typical neutron-induced fission reaction is:

$$^{235}_{92}U +^{1}_{0} n \rightarrow^{93}_{37} Rb +^{141}_{55} Cs + 2^{1}_{0}n$$

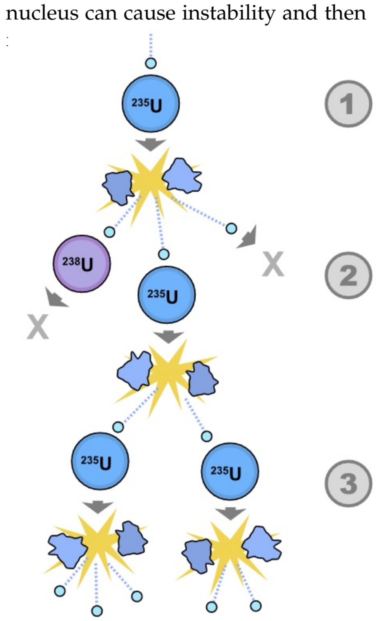

Figure 8: An example of a nuclear chain reaction. Figure credit: Wikimedia commons

Nuclear chain reactions

Many neutron-induced fission reactions produce secondary neutrons, and these can go on to induce more fission reactions in other atoms, producing a **chain reaction**. If less than one of these secondary neutrons goes on to produce another fission event, then the chain-reaction is sub-critical, and fission reactions will naturally stop. If exactly one (on average) of the secondary neutrons produces another fission event, then the chain-reaction is critical, and is at the point of being self-sustaining. If more than one (on average) neutrons from a fission event goes on to produce more fission events then the reaction is super-critical, and the rate of fission will increase. We can define the *neutron reproduction factor* k, where $k < 1$ for sub-critical, $k = 1$ for critical and $k > 1$ for super-critical reactions.

Uncontrolled (super-critical) chain reactions

Uncontrolled chain reactions are used in nuclear explosives. In this case the excess neutrons from each reaction go on to produce reactions themselves, resulting in an uncontrolled chain reaction which releases an exponentially increasing amount of energy.

To make a nuclear explosive, it is necessary to collect sub-critical pieces of fissionable material into a super-critical assembly quickly. Two basic designs were originally employed during the Manhattan project, one was the "gun-type" which was used with $^{2}35U$ where a subcritical hollow "bullet" was fired into a subcritical target to create a supercritical combination. A second design used conventional explosives to produce an implosion, where a spherical shock wave compresses the fissionable material into a super-critical state.

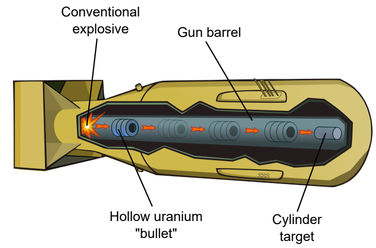

Figure 9: An example of "gun-type" nuclear explosive design, similar to that used in the "Little Boy" uranium bomb.
Figure credit: Vector version by Dake with English labels by Papa Lima Whiskey, lines modified by Mfield, CC BY-SA 3.0, via Wikimedia Commons.

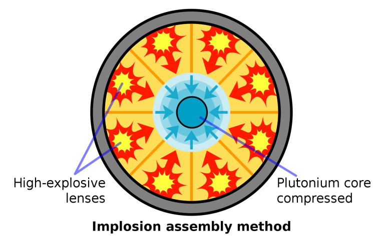

Figure 10: An example of an "implosion" nuclear explosive design, similar to that used in the "Fat man" Plutonium bomb. Figure credit: Wikimedia commons, Public domain.

Question 4.

(a) Calculate the mass defect for the fission reaction

$$^{235}_{92}U +^{1}_{0} n \rightarrow^{93}_{37} Rb +^{141}_{55} Cs + 2^{1}_{0}n$$

(b) What is the energy released when this reaction occurs?
Mass of $^{141}_{55}Cs = 140.920046$u
Mass of $^{93}_{37}Rb = 92.922042$u
Mass of $^{235}_{92}U = 235.0439299$u
Mass of $p = 1.007825$u
Mass of $n = 1.008665$u

(c) Calculate the binding energy *per nucleon* of $^{93}_{37}Rb$ and compare it to the binding energy per nucleon of Uranium-235 (calculated in the previous question).

(d) Calculate the binding energy of $^{93}_{37}Rb$ and compare it to the binding energy of Uranium-235

Controlled chain reactions

The overall aim of a controlled nuclear reaction, such as that in a nuclear power plant, is to maintain the reaction at a critical state. There are three main types of nuclear fission reactors.

1. In power reactors the heat energy produced by the fission reaction can be used to heat steam to drive a steam turbine generator to produce electricity.
2. In research reactors the aim is to produce neutrons for research purposes, or other useful radioactive materials such as those used in medicine or industry. ANSTO at Lucus Heights in Sydney is such a reactor.
3. Converters (or breeder reactors) are designed to convert non-fissionable material into fissionable material suitable for use in power reactors.

There are several aspects to achieving an operating nuclear reactor.

Minimise neutron leakage (need for large volume-to-surface area ratio).

Even though the neutron-induced fission of U-235 produces, on average, 2.47 secondary neutrons, there are several mechanisms by which these neutrons can be lost so that they don't go on to produce another fission reaction. One of these is due to the finite surface area of the reactor core - some neutrons simply escape through the surface. This is reduced to an acceptable level by using a sufficiently large volume to surface area ratio.

Neutron energy issues (need for a moderator)

The neutrons emerging from a fission reaction are fast, however as discovered by Fermi, the most effective neutrons for producing nuclear fission are slow. It is therefore necessary to introduce a moderator material, which can slow the neutrons down. The most commonly used material is water, as it can also function as a fluid to transfer heat from the reactor core to the water used to run the steam turbine.

Control rods

Finally, it is necessary to fine-tune the reaction rate to ensure that the reactor is operating at a critical (not super-critical!) level. This is achieved by the use of control rods made of a neutron absorbing material such as cadmium. These are inserted into the reactor core, and can be raised or lowered as required to adjust the rate of reaction.

Heat exchanger

In order for the nuclear reactor to perform its function of producing hot steam to drive a steam turbine to produce electricity, it is necessary for there to be a fluid that carries thermal energy from the core out to where it can be used to produce steam. Usually this is water, which can also perform the job of a moderating material.

Shielding

To maintain the safety of workers, the core must be shielded. Commonly lead, concrete or a deep pool of water may be used (see ANSTO website).

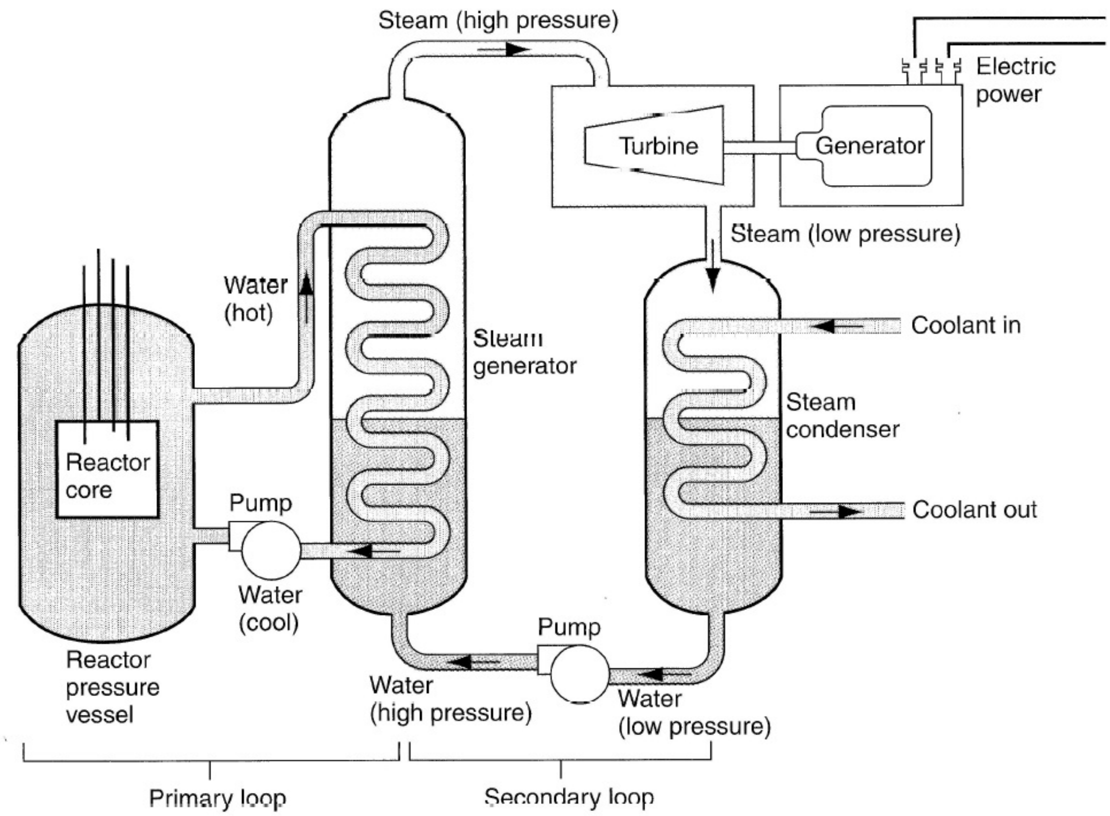

Figure 11: Schematic of a nuclear reactor. From Halliday, D., Resnick, R. Krane, K.S., (1960). Physics: Volume 2 (5th Ed. New York: John Wiley Sons, Inc.)

Nuclear fusion

The graph of binding energy per nucleon shown in figure 6 shows that energy can be released if two light nuclei fuse together to create a larger nuclei, as long as that nuclei is no larger than iron.

Fusion of light elements into heaver elements is the process by which the sun (and all other stars) release energy. We will examine this process in detail in the section "Origins of the Elements".

Scientists are attempting to produce sustained fusion reactions as a potential future energy source. The largest project of this kind is the ITER thermonuclear fusion reactor. Located in France, this project aims to produce sustained fusion using a reaction between two isotopes of hydrogen, deuterium and tritium.

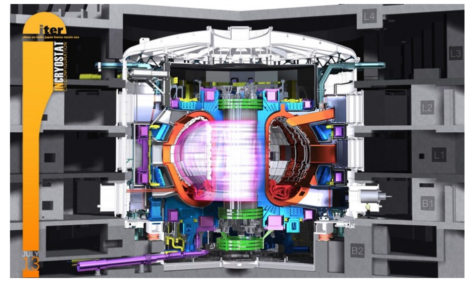

Figure 12: Schematic of the ITER tokamak. Image credit: ITER

$$^2_1H +^3_1 H \rightarrow^4_2 He +^1_0 n + 17.59\text{MeV}$$

by confining a plasma of these isotopes at a temperature of 150 million °C in a "magnetic bottle" called a Tokamak.

Question 5.

Every cubic metre of seawater contains 33 grams of deuterium. Tritium exists only in trace amounts naturally, but can be produced within a fusion tokamak by allowing the escaping neutrons from the fusion reaction to bombard lithium, which then recombines into an atom of tritium and an atom of helium). The mass of one atom of deuterium is 2.01410177811u.

(a) How much energy could be released from a cubic metre of seawater if all the deuterium were to undergo fusion with tritium?

(b) Australians use 9,324 kWh of electricity per capita (i.e. per person) every year, which equates to 229.40 billion kWh for all of Australia. Neglecting energy losses within the fusion reactor and in conversion of the heat to electricity, how many grams of deuterium would need to undergo fusion to provide this amount of energy?

Answers

Answer 1. Radon-222 forms part of a long radioactive decay "chain" from Uranium-238 to Lead-206.

(a) In part of this chain, $^{222}_{86}Rn$ undergoes an α decay to produce a new radionuclide. Identify the resultant radionuclide.

$^{215}_{84}Po$

(b) After three *more* radioactive decay processes occur, the resulting radionuclide is $^{214}_{84}Po$. Identify the nature of these three additional decay processes.

One α and two β^{-1} decays have occured.

(c) In an earlier part of the decay series, Protactinium-234 ($^{234}_{91}Pa$) decays to Uranium-234 ($^{234}_{92}U$). Write an equation for this radioactive decay showing all the decay products.

$$^{234}_{91}Pa \rightarrow^{234}_{92} U + e^{-} + \bar{\nu}$$

Answer 2. See figure 13

Answer 3.
(a) What is the binding energy of Uranium 235?
Mass defect is

$$92 \times 1.007825\text{u} + (235 - 92) \times 1.008665\text{u} - 235.0439299\text{u} = 236.959 - 235.0439299\text{u} = 1.915\text{u}$$

which is equivalent to

$$E_{be} = 1.915 \times 931.5MeV = 1.78GeV$$

(b) What is the binding energy *per nucleon* of Uranium 235?
The binding energy per nucleon is:

$$E_{ben} = 1.78GeV/235 = 7.59MeV$$

Mass of $^{235}_{92}U = 235.0439299\text{u}$

Mass of $p = 1.007825\text{u}$
Mass of $n = 1.008665\text{u}$

Answer 4.
(a) Calculate the binding energy of $^{93}_{37}Rb$ and compare it to the binding energy of Uranium-235

Question 2. Technetium-99m is a gamma emitter that is used as a radioactive tracer. It is injected into patients for procedures which use gamma ray imaging such as Single Photon Emission Computed Tomography (SPECT), used to detect cancer.

In a procedure, a patient is injected with $N_0 = 1 \times 10^{-10}$g of Tc_{99m} before a procedure. The amount of Tc_{99m} remaining in their body over time is shown in the table on the right.

(a) By completing the table appropriately, graph the data in a form that will produce a straight line, and thereby determine the decay constant of Tc_{99m}.

(b) Calculate the half-life of Tc_{99m} and comment on why this contributes to its usefulness as a radioactive tracer for imaging procedures.

$t(hours)$	$N(\times 10^{10}\text{g})$	$\ln\left(\frac{N}{N_0}\right)$
0	1	0
2.5	0.75	-0.29
5	0.56	-0.58
7.5	0.42	-0.87
10	0.31	-1.17
12.5	0.23	-1.47
15	0.18	-1.71

Figure 4: Amount of Tc_{99m} remaining over time.

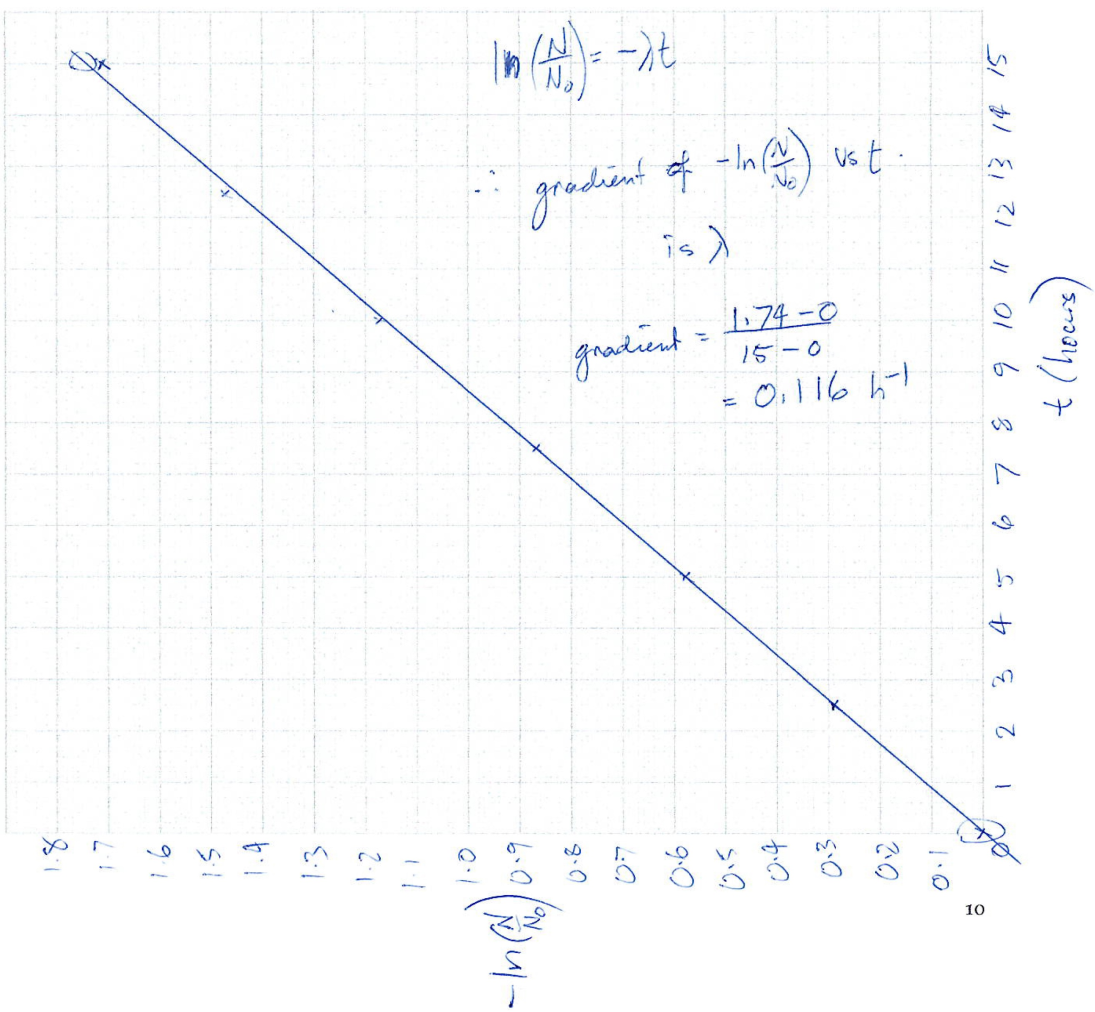

Figure 13: Answer for Q2.

Mass defect is

$$37 \times 1.007825\text{u} + (93-37) \times 1.008665\text{u} - 92.922042\text{u} = 93.775 - 92.922042 = 0.852723\text{u}$$

which is equivalent to

$$E_{be} = 0.852723 \times 931.5 MeV = 794 MeV$$

The total binding energy of Rb-93 is less than the total binding energy of Uranium-235.

(b) Calculate the binding energy *per nucleon* of $^{93}_{37}Rb$ and compare it to the binding energy per nucleon of Uranium-235

The binding energy per nucleon of Rubidium-93 is

$$E_{ben} = 794 MeV/93 = 8.54 MeV$$

which is *higher* than the binding energy per nucleon for Uranium-235. This means it takes more energy *per nucleon* to pull apart Rubidium than it does to pull apart Uranium-235.

(c) Calculate the mass defect for the fission reaction

$$^{235}_{92}U + ^1_0 n \rightarrow ^{93}_{37} Rb + ^{141}_{55} Cs + 2^1_0 n$$

The mass defect is

$$235.0439299 - 92.922042 - 140.920046 - 1.008665\text{u} = 0.193\text{u}$$

(d) What is the energy released when this reaction occurs?

The energy released is equal to

$$0.193 \times 931.5 = 180 MeV$$

Mass of $^{141}_{55}Cs = 140.920046$u

Mass of $^{93}_{37}Rb = 92.922042$u
Mass of $^{235}_{92}U = 235.0439299$u
Mass of $p = 1.007825$u
Mass of $n = 1.008665$u

Answer 5.

Every cubic metre of seawater contains 33 grams of deuterium. Tritium exists only in trace amounts naturally, but can be produced within a fusion tokamak by allowing the escaping neutrons from the fusion reaction to bombard lithium, which then recombines into an atom of tritium and an atom of helium). The mass of one atom of deuterium is 2.01410177811u

(a) How much energy could be released from a cubic metre of seawater if all the deuterium were to undergo fusion with tritium?

33 grams of deuterium contains $0.033/(2.0141 \times 1.661 \times 10^{-27}) = 9.86 \times 10^{24}$ atoms of deuterium. This means that the total energy available from one cubic meter of seawater is $9.86 \times 10^{24} \times 17.59 \times 10^{6} \times 1.6 \times 10^{-19} = 2.78 \times 10^{13} J = 28TJ$

(b) Australians use 9,324 kWh of electricity per capita (i.e. per person) every year, which equates to 229.40 billion kWh for all of Australia. Neglecting energy losses within the fusion reactor and in conversion of the heat to electricity, how many grams of deuterium would need to undergo fusion to provide this amount of energy?

We need to convert the energy usage in kWh to Joules:

$$229.4 \times 10^{9} kWh = 229.4 \times 10^{9} \times 3.6 \times 10^{6} J = 8.26 \times 10^{17} J = 825840TJ$$

So the amount of seawater we would need to supply Australia's entire energy requirements would be $30000m^3$. An olympiad swimming pool holds $2500m^3$, so this is equivalent to 12 olympic sized swimming pools of water.

Even if the efficiency of a fusion reactor is quite low - it would be mean a revolution for the world's energy needs...