Answers to exercises

Answers are not provided for certain questions of the type 'show that' or 'prove that'. Please see worked solutions in these cases for a model.

Chapter 1

Exercise 1A

1 a $4x$ **b** $2x$ **c** $-2x$ **d** $-4x$
2 a $5a$ **b** $-a$ **c** $-9a$ **d** $-3a$
3 a 0 **b** $-y$ **c** $-10a$ **d** $-3b$
e $7x$ **f** $-3ab$ **g** $4pq$ **h** $-3abc$
4 a $-6a$ **b** $12a^2$ **c** a^5 **d** a^6
5 a $-2a$ **b** 3 **c** a^6 **d** a
6 a $2t^2$ **b** 0 **c** t^4 **d** 1
7 a $-3x$ **b** $-9x$ **c** $-18x^2$ **d** -2
8 a -4 **b** -12 **c** 18 **d** 2
9 a $x + 3$ **b** $2y - 3$ **c** $2a - 3$
d $8x + 4y$ **e** $-10t - 5$ **f** $4a - 3a^2$
g $-5x^2 - 12x - 3$ **h** $9a - 3b - 5c$
10 a 5 **b** $7m^2$ **c** $-12a$ **d** $-3p^3q^4r$
11 a $2x$ **b** $4x$ **c** $-6a$ **d** $-4b$
12 a $10a$ **b** $-18x$ **c** $-3a^2$
d $6a^3b$ **e** $-8x^5$ **f** $-6p^3q^4$
13 a -2 **b** $3x$ **c** xy
d $-a^4$ **e** $-7ab^3$ **f** $5ab^2c^6$
14 a $6a^5b^6$ **b** $-24a^4b^8$ **c** $9a^6$ **d** $-8a^{12}b^3$
15 a 0 **b** -1 **c** 59 **d** 40
16 a $3a^2$ **b** $5c^4$ **c** a^2bc^6
17 a $2x^5$ **b** $9xy^5$ **c** b^4 **d** $2a^3$
18 a $-x^3 + 3x^2 + 7x - 8$ **b** $-b + 11c$
c $8d - 14c - 2b$ **d** $-18x^{25}y^{22}$
19 a $0 \le x \le 2$
b $x \le -\sqrt{3}$ or $0 \le x \le \sqrt{3}$

Exercise 1B

1 a $3x - 6$ **b** $2x - 6$ **c** $-3x + 6$
d $-2x + 6$ **e** $-3x - 6$ **f** $-2x - 6$
g $-x + 2$ **h** $-2 + x$ **i** $-x - 3$
2 a $3x + 3y$ **b** $-2p + 2q$ **c** $4a + 8b$
d $x^2 - 7x$ **e** $-x^2 + 3x$ **f** $-a^2 - 4a$
g $5a + 15b - 10c$ **h** $-6x + 9y - 15z$
i $2x^2y - 3xy^2$
3 a $x + 2$ **b** $7a - 3$ **c** $2x - 4$
d $4 - 3a$ **e** $2 - x$ **f** $2c$
g $-x - y$ **h** $x + 4$ **i** $5a - 18b$
j $-2s - 10t$ **k** $x^2 + 17xy$ **l** $16a - b$
4 a $x^2 + 5x + 6$ **b** $y^2 + 11y + 28$
c $t^2 + 3t - 18$ **d** $x^2 - 2x - 8$
e $t^2 - 4t + 3$ **f** $2a^2 + 13a + 15$
g $3u^2 - 10u - 8$ **h** $8p^2 - 2p - 15$
i $2b^2 - 13b + 21$ **j** $15a^2 - a - 2$
k $-c^2 + 9c - 18$ **l** $2d^2 + 5d - 12$
6 a $x^2 + 2xy + y^2$ **b** $x^2 - 2xy + y^2$
c $x^2 - y^2$ **d** $a^2 + 6a + 9$
e $b^2 - 8b + 16$ **f** $c^2 + 10c + 25$
g $d^2 - 36$ **h** $49 - e^2$
i $64 + 16f + f^2$ **j** $81 - 18g + g^2$
k $h^2 - 100$ **l** $i^2 + 22i + 121$
m $4a^2 + 4a + 1$ **n** $4b^2 - 12b + 9$
o $9c^2 + 12c + 4$ **p** $4d^2 + 12de + 9e^2$
q $4f^2 - 9g^2$ **r** $9h^2 - 4i^2$
s $25j^2 + 40j + 16$ **t** $16k^2 - 40k\ell + 25\ell^2$
u $16 - 25m^2$ **v** $25 - 30n + 9n^2$
w $49p^2 + 56pq + 16q^2$ **x** $64 - 48r + 9r^2$
7 a $t^2 + 2 + \frac{1}{t^2}$ **b** $t^2 - 2 + \frac{1}{t^2}$ **c** $t^2 - \frac{1}{t^2}$
8 a $10\,404$ **b** $998\,001$ **c** $39\,991$
9 a $a^3 - b^3$ **b** $2x + 3$
c $18 - 6a$ **d** $x^2 + 2x - 1$
e $x^3 - 6x^2 + 12x - 8$ **f** $p^2 + q^2 + r^2$
10 a $x^3 - 6x^2 + 12x - 8$ **b** $x^2 + y^2 + z^2$
c $x^2 - y^2 - z^2 + 2yz$ **d** $a^3 + b^3 + c^3 - 3abc$
11 a $a^2 - b^2 - c^2 + 2bc$ **b** $x^2 - 2x + 3$
c $7x^2 + 16ax + 4a^2$
12 7

Exercise 1C

1 a $2(x + 4)$ **b** $3(2a - 5)$
c $a(x - y)$ **d** $5a(4b - 3c)$
e $x(x + 3)$ **f** $p(p + 2q)$
g $3a(a - 2b)$ **h** $6x(2x + 3)$
i $4c(5d - 8)$ **j** $ab(a + b)$
k $2a^2(3 + a)$ **l** $7x^2y(x - 2y)$
2 a $(p + q)(m + n)$ **b** $(x - y)(a + b)$
c $(x + 3)(a + 2)$ **d** $(a + b)(a + c)$
e $(z - 1)(z^2 + 1)$ **f** $(a + b)(c - d)$

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g $(p - q)(u - v)$ **h** $(x - 3)(x - y)$
i $(p - q)(5 - x)$ **j** $(2a - b)(x - y)$
k $(b + c)(a - 1)$ **l** $(x + 4)(x^2 - 3)$
m $(a - 3)(a^2 - 2)$ **n** $(2t + 5)(t^2 - 5)$
o $(x - 3)(2x^2 - a)$

3 a $(a - 1)(a + 1)$ **b** $(b - 2)(b + 2)$
c $(c - 3)(c + 3)$ **d** $(d - 10)(d + 10)$
e $(5 - y)(5 + y)$ **f** $(1 - n)(1 + n)$
g $(7 - x)(7 + x)$ **h** $(12 - p)(12 + p)$
i $(2c - 3)(2c + 3)$ **j** $(3u - 1)(3u + 1)$
k $(5x - 4)(5x + 4)$ **l** $(1 - 7k)(1 + 7k)$
m $(x - 2y)(x + 2y)$ **n** $(3a - b)(3a + b)$
o $(5m - 6n)(5m + 6n)$ **p** $(9ab - 8)(9ab + 8)$

4 a $(a + 1)(a + 2)$ **b** $(k + 2)(k + 3)$
c $(m + 1)(m + 6)$ **d** $(x + 3)(x + 5)$
e $(y + 4)(y + 5)$ **f** $(t + 2)(t + 10)$
g $(x - 1)(x - 3)$ **h** $(c - 2)(c - 5)$
i $(a - 3)(a - 4)$ **j** $(b - 2)(b - 6)$
k $(t + 2)(t - 1)$ **l** $(u - 2)(u + 1)$
m $(w - 4)(w + 2)$ **n** $(a + 4)(a - 2)$
o $(p - 5)(p + 3)$ **p** $(y + 7)(y - 4)$
q $(c - 3)(c - 9)$ **r** $(u - 6)(u - 7)$
s $(x - 10)(x + 9)$ **t** $(x + 8)(x - 5)$
u $(t - 8)(t + 4)$ **v** $(p + 12)(p - 3)$
w $(u - 20)(u + 4)$ **x** $(t + 25)(t - 2)$

5 a $(3x + 1)(x + 1)$ **b** $(2x + 1)(x + 2)$
c $(3x + 1)(x + 5)$ **d** $(3x + 2)(x + 2)$
e $(2x - 1)(x - 1)$ **f** $(5x - 3)(x - 2)$
g $(5x - 6)(x - 1)$ **h** $(3x - 1)(2x - 3)$
i $(2x - 3)(x + 1)$ **j** $(2x + 5)(x - 1)$
k $(3x + 5)(x - 1)$ **l** $(3x - 1)(x + 5)$
m $(2x + 3)(x - 5)$ **n** $(2x - 5)(x + 3)$
o $(6x - 1)(x + 3)$ **p** $(2x - 3)(3x + 1)$
q $(3x - 2)(2x + 3)$ **r** $(5x + 3)(x + 4)$
s $(5x - 6)(x + 2)$ **t** $(5x - 4)(x - 3)$
u $(5x + 4)(x - 3)$ **v** $(5x - 2)(x + 6)$
w $(3x - 4)(3x + 2)$ **x** $(3x - 5)(x + 6)$

6 a $(a - 5)(a + 5)$ **b** $b(b - 25)$
c $(c - 5)(c - 20)$ **d** $(2d + 5)(d + 10)$
e $(e + 5)(e^2 + 5)$ **f** $(4 - f)(4 + f)$
g $g^2(16 - g)$ **h** $(h + 8)^2$
i $(i - 18)(i + 2)$ **j** $(j + 4)(5j - 4)$
k $(2k + 1)(2k - 9)$ **l** $(k - 8)(2k^2 - 3)$
m $(2a + b)(a - 2)$ **n** $3m^2n^4(2m + 3n)$
o $(7p - 11q)(7p + 11q)$ **p** $(t - 4)(t - 10)$
q $(3t - 10)(t + 4)$ **r** $(5t + 4)(t + 10)$
s $(5t + 8)(t + 5)$ **t** $5t(t^2 + 2t + 3)$
u $(u + 18)(u - 3)$ **v** $(3x - 2y)(x^2 - 5)$
w $(p + q - r)(p + q + r)$ **x** $(2a - 3)^2$

7 a $3(a - 2)(a + 2)$
b $(x - y)(x + y)(x^2 + y^2)$
c $x(x - 1)(x + 1)$ **d** $5(x + 2)(x - 3)$
e $y(5 - y)(5 + y)$
f $(2 - a)(2 + a)(4 + a^2)$
g $2(2x - 3)(x + 5)$ **h** $a(a + 1)(a^2 + 1)$
i $(c + 1)(c - 1)(c + 9)$
j $x(x - 1)(x - 7)$
k $(x - 2)(x + 2)(x^2 + 1)$
l $(x - 1)(x + 1)(a - 2)$

8 a $(2p - q - r)(2p + q + r)$
b $(a - b)(a + b - 1)$
c $a(a - 4b)(a - 6b)$ **d** $x^2(3x - 2)(2x + 1)$
e $(2x - 1)(2x + 1)(x - 3)(x + 3)$
f $2(4 - 5x)(5 + 4x)$
g $(2x - 1)(2x + 1)(x - 3)$
h $(x + a - b)(x + a + b)$
i $(x^2 - x - 1)(x^2 + x + 1)$

9 a $(a + b)(a + b^2)$ **b** $(x - y)(x + y)^3$
c $4ab(a - b)^2$
d $(2x^2 + 3y^2)(2x - 3y)(x + y)$
e $(a - b - c)(a + b + c)(a - b + c)$
$(a + b - c)$
f $(x^2 + y^2)(a^2 + b^2 + c^2)$
g Add and subtract a^2b^2.
$(a^2 - ab + b^2)(a^2 + ab + b^2)$
h Add and subtract $4a^2b^2$.
$(a^2 - 2ab + 2b^2)(a^2 + 2ab + 2b^2)$

Exercise 1D

1 a 1 **b** 2 **c** $\frac{1}{2}$ **d** $\frac{1}{a}$ **e** $\frac{x}{3y}$ **f** $\frac{3}{a}$

2 a 1 **b** $\frac{1}{2}$ **c** $3x$ **d** $\frac{b}{2}$
e $\frac{3}{2x}$ **f** $\frac{1}{2a}$ **g** $\frac{4}{b}$ **h** 6

3 a $\frac{3x}{2}$ **b** $\frac{3y}{4}$ **c** $\frac{2m}{9}$ **d** $\frac{7n}{10}$
e $\frac{3x - 2y}{24}$ **f** $\frac{13a}{6}$ **g** $\frac{b}{15}$ **h** $-\frac{xy}{20}$

4 a $\frac{2}{a}$ **b** $-\frac{1}{x}$ **c** $\frac{3}{2a}$ **d** $\frac{1}{6x}$ **e** $\frac{25}{12a}$ **f** $\frac{1}{2x}$

5 a $\frac{5x + 7}{6}$ **b** $\frac{18x + 11}{20}$ **c** $\frac{x + 1}{4}$
d $\frac{x}{6}$ **e** $\frac{2x + 17}{20}$ **f** $\frac{2x - 3}{6}$

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6 a 2 **b** $\frac{3}{2}$ **c** $\frac{x}{3}$ **d** $\frac{1}{x+y}$ **e** $\frac{3}{2b}$

f $\frac{x}{x-2}$ **g** $\frac{a+3}{a+4}$ **h** $\frac{x+1}{x-1}$ **i** $\frac{x+5}{x+4}$

7 a $\frac{2x+1}{x(x+1)}$ **b** $\frac{1}{x(x+1)}$

c $\frac{2x}{(x+1)(x-1)}$ **d** $\frac{5x-13}{(x-2)(x-3)}$

e $\frac{x-5}{(x+1)(x-1)}$ **f** $\frac{10}{(x+3)(x-2)}$

8 a $\frac{3x}{2(x-1)}$ **b** a **c** $\frac{c+2}{c+4}$

d x **e** $\frac{3x-1}{a+b}$ **f** $\frac{x-7}{3(x+3)}$

9 a -1 **b** $-u-v$ **c** $3-x$

d $\frac{2}{a-b}$ **e** 1 **f** $\frac{-1}{2x+y}$

10 a $\frac{2}{x^2-1}$ **b** $\frac{2x}{(x-2)^2(x+2)}$

c $\frac{3x}{x^2-y^2}$

d $\frac{x+1}{(x-2)(x+3)(x+4)}$

e $\frac{bx}{a(a-b)(a+b)}$

f $\frac{x}{(x-1)(x-2)(x-3)}$

11 a $\frac{1}{3}$ **b** $\frac{7}{13}$ **c** $\frac{3}{11}$ **d** $\frac{1}{5}$

e $\frac{1}{x+2}$ **f** $\frac{t^2-1}{t^2+1}$ **g** $\frac{ab}{a+b}$

h $\frac{x^2+y^2}{x^2-y^2}$ **i** $\frac{x^2}{2x+1}$ **j** $\frac{x-1}{x-3}$

13 a $\frac{a-b+c}{ab}$ **b** $\frac{2x+3}{3x-1}$

c $\frac{4}{x+2y}$ **d** $\frac{2}{(x+1)^2(x-1)}$

14 a 0 **b** 3 **c** $\frac{3n-m}{2}$ **d** $\frac{1}{x}$

Exercise 1E

1 a $x=3$ **b** $p=0$ **c** $a=8$

d $w=-1$ **e** $x=9$ **f** $x=-5$

g $x=-16$ **h** $x=-2$

2 a $n=4$ **b** $b=-1$ **c** $x=4$

d $x=-11$ **e** $a=-\frac{1}{2}$ **f** $y=2$

g $x=\frac{7}{9}$ **h** $x=-\frac{3}{5}$

3 a $a=8$ **b** $y=16$ **c** $x=\frac{1}{3}$

d $a=\frac{2}{5}$ **e** $y=\frac{3}{2}$ **f** $x=-8$

g $a=7$ **h** $x=-\frac{1}{2}$ **i** $a=-5$

j $t=\frac{3}{5}$ **k** $x=-2$ **l** $x=5$

4 a $y=\frac{2}{3}$ **b** $x=15$ **c** $a=-15$

d $x=\frac{9}{2}$ **e** $x=6$ **f** $x=\frac{1}{6}$

g $x=\frac{1}{2}$ **h** $x=20$ **i** $x=-\frac{23}{2}$

j $x=-\frac{7}{3}$

5 a $a=3$ **b** $s=16$ **c** $v=\frac{2}{3}$

d $\ell=21$ **e** $C=35$ **f** $c=-\frac{2}{5}$

6 a $b=\frac{a+d}{c}$ **b** $n=\frac{t-a+d}{d}$

c $r=\frac{p-qt}{t}$ **d** $v=\frac{3}{u-1}$

7 a $x=\frac{19}{6}$ **b** $x=\frac{3}{14}$ **c** $x=-1$ **d** $x=\frac{17}{6}$

8 a $a=-11$ **b** $x=2$ **c** $x=-\frac{7}{3}$ **d** $x=-\frac{5}{2}$

9 a -4 **b** 7 **c** 36 **d** 80 litres

e 15 min **f** 16 **g** 30 km/h **h** 5 hours

10 a $a=-\frac{2b}{3}$ **b** $g=\frac{2fh}{5f-h}$

c $y=\frac{2x}{1-x}$ **d** $b=\frac{4a+5}{a-1}$

11 a $x=\frac{14}{5}$ **b** $a=4$

12 a $x=6$

Exercise 1F

1 a $x=3$ or -3 **b** $y=5$ or -5

c $a=2$ or -2 **d** $c=6$ or -6

e $t=1$ or -1 **f** $x=\frac{3}{2}$ or $-\frac{3}{2}$

g $x=\frac{1}{2}$ or $-\frac{1}{2}$ **h** $a=2\frac{2}{3}$ or $-2\frac{2}{3}$

i $y=\frac{4}{5}$ or $-\frac{4}{5}$

2 a $x=0$ or 5 **b** $y=0$ or -1

c $c=0$ or -2 **d** $k=0$ or 7

e $t=0$ or 1 **f** $a=0$ or 3

g $b=0$ or $\frac{1}{2}$ **h** $u=0$ or $-\frac{1}{3}$

i $x=-\frac{3}{4}$ or 0 **j** $a=0$ or $\frac{5}{2}$

k $y=0$ or $\frac{2}{3}$ **l** $n=0$ or $-\frac{3}{5}$

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3 a $x = -3$ or -1 **b** $x = 1$ or 2
c $x = -4$ or -2 **d** $a = 2$ or 5
e $t = -2$ or 6 **f** $c = 5$
g $n = 1$ or 8 **h** $p = -5$ or 3
i $a = -2$ or 12 **j** $y = -5$ or 1
k $p = -2$ or 3 **l** $a = -11$ or 12
m $c = 3$ or 6 **n** $t = -2$ or 10
o $u = -8$ or 7 **p** $k = -4$ or 6
q $h = -25$ or -2 **r** $a = -22$ or 2

4 a $x = -\frac{1}{2}$ or -1 **b** $a = \frac{1}{3}$ or 2
c $y = \frac{1}{4}$ or 1 **d** $x = -5$ or $-\frac{1}{2}$
e $x = -1\frac{1}{2}$ or 1 **f** $n = -1$ or $1\frac{2}{3}$
g $b = -\frac{2}{3}$ or 2 **h** $a = -5$ or $1\frac{1}{2}$
i $y = -2\frac{1}{2}$ or 3 **j** $y = -4$ or $\frac{2}{3}$
k $x = \frac{1}{5}$ or 5 **l** $t = \frac{3}{4}$ or 3
m $t = -\frac{2}{5}$ or 3 **n** $u = -\frac{4}{5}$ or $\frac{1}{2}$
o $x = \frac{1}{5}$ **p** $x = -\frac{2}{3}$ or $-\frac{3}{2}$
q $b = -\frac{3}{2}$ or $-\frac{1}{6}$ **r** $k = -\frac{8}{3}$ or $\frac{1}{2}$

5 a $x = \frac{1 + \sqrt{5}}{2}$ or $\frac{1 - \sqrt{5}}{2}$, $x \fallingdotseq 1.618 \fallingdotseq$ or -0.6180
b $x = \frac{-1 + \sqrt{13}}{2}$ or $\frac{-1 - \sqrt{13}}{2}$, $x \fallingdotseq 1.303$ or -2.303
c $a = 3$ or 4
d $u = -1 + \sqrt{3}$ or $-1 - \sqrt{3}$,
$u \fallingdotseq 0.7321$ or -2.732
e $c = 3 + \sqrt{7}$ or $3 - \sqrt{7}$, $c \fallingdotseq 5.646$ or 0.3542
f $x = -\frac{1}{2}$
g $a = \frac{2 + \sqrt{2}}{2}$ or $\frac{2 - \sqrt{2}}{2}$, $a \fallingdotseq 1.707$ or 0.2929
h $x = -3$ or $\frac{2}{5}$
i $b = \frac{-3 + \sqrt{17}}{4}$ or $\frac{-3 - \sqrt{17}}{4}$, $b \fallingdotseq 0.2808$ or -1.781
j $c = \frac{2 + \sqrt{13}}{3}$ or $\frac{2 - \sqrt{13}}{3}$, $c \fallingdotseq 1.869$ or -0.5352
k $t = \frac{1 + \sqrt{5}}{4}$ or $\frac{1 - \sqrt{5}}{4}$, $t \fallingdotseq 0.8090$ or -0.3090
l no solutions

6 a $x = -1$ or 2 **b** $a = 2$ or 5
c $y = \frac{1}{2}$ or 4 **d** $b = -\frac{2}{5}$ or $\frac{2}{3}$

7 a $x = 1 + \sqrt{2}$ or $1 - \sqrt{2}$
b $x = 2 + \sqrt{3}$ or $2 - \sqrt{3}$
c $a = 1 + \sqrt{5}$ or $1 - \sqrt{5}$
d $m = \frac{2 + \sqrt{14}}{5}$ or $\frac{2 - \sqrt{14}}{5}$

8 a $p = \frac{1}{2}$ or 1 **b** $x = -3$ or 5 **c** $n = 5$
9 a 7 **b** 6 and 9 **c** $x = 15$
10 a $a = 2b$ or $a = 3b$ **b** $a = -2b$ or $a = \frac{b}{3}$
11 a $y = 2x$ or $y = -2x$ **b** $y = \frac{x}{11}$ or $y = -\frac{x}{2}$
12 a $k = -1$ or 3 **b** $u = \frac{4}{3}$ or 4
c $y = 1 + \sqrt{6}$ or $1 - \sqrt{6}$
d $k = \frac{-5 + \sqrt{73}}{4}$ or $\frac{-5 - \sqrt{73}}{4}$
e $a = -\frac{7}{3}$ or 3 **f** $k = -4$ or 15
g $t = 2\sqrt{3}$ or $-\sqrt{3}$
h $m = \frac{1 + \sqrt{2}}{3}$ or $\frac{1 - \sqrt{2}}{3}$
13 a 4 cm **b** 3 cm
c 55 km/h and 60 km/h
14 a $x = 2c$ or $x = \frac{11c}{14}$, where $c \neq 0$.
b $x = a$ or $x = \frac{ab}{a - 2b}$, provided that $a \neq 2b$.

Exercise 1G

1 a $x = 3, y = 3$ **b** $x = 2, y = 4$
c $x = 2, y = 1$ **d** $a = -3, b = -2$
e $p = 3, q = -1$ **f** $u = 1, v = -2$
2 a $x = 3, y = 2$ **b** $x = 1, y = -2$
c $x = 4, y = 1$ **d** $a = -1, b = 3$
e $c = 2, d = 2$ **f** $p = -2, q = -3$
3 a $x = 2, y = 4$ **b** $x = -1, y = 3$
c $x = 2, y = 2$ **d** $x = 9, y = 1$
e $x = 3, y = 4$ **f** $x = 4, y = -1$
g $x = 5, y = 3\frac{3}{5}$ **h** $x = 13, y = 7$
4 a $x = -1, y = 3$ **b** $x = 5, y = 2$
c $x = -4, y = 3$ **d** $x = 2, y = -6$
e $x = 1, y = 2$ **f** $x = 16, y = -24$
g $x = 1, y = 6$ **h** $x = 5, y = -2$
i $x = 5, y = 6$ **j** $x = 7, y = 5$
5 a $x = 1\ \&\ y = 1$, or $x = -2\ \&\ y = 4$
b $x = 2\ \&\ y = 1$, or $x = 4\ \&\ y = 5$
c $x = 0\ \&\ y = 0$, or $x = 1\ \&\ y = 3$
d $x = -2\ \&\ y = -7$, or $x = 3\ \&\ y = -2$
e $x = -3\ \&\ y = -5$, or $x = 5\ \&\ y = 3$
f $x = 1\ \&\ y = 6$, or $x = 2\ \&\ y = 3$

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6 a 53 and 37
b The pen cost 60 c, the pencil cost 15 c.
c Each apple cost 40 c, each orange cost 60 c.
d 44 adults, 22 children
e The man is 36, the son is 12.
f 189 for, 168 against
g 9 \$20 notes, 14 \$10 notes
h 5 km/h, 3 km/h

7 a $x = 12, y = 20$
b $x = 3, y = 2$

8 a $x = 5 \,\&\, y = 10$, or $x = 10 \,\&\, y = 5$
b $x = -8 \,\&\, y = -11$, or $x = 11 \,\&\, y = 8$
c $x = \frac{1}{2} \,\&\, y = 4$, or $x = 10 \,\&\, y = 5$
d $x = 4 \,\&\, y = 5$, or $x = 5 \,\&\, y = 4$
e $x = 1 \,\&\, y = 2$, or $x = \frac{3}{2} \,\&\, y = \frac{7}{4}$
f $x = 2 \,\&\, y = 5$, or $x = \frac{10}{3} \,\&\, y = 3$

9 a $x = 1 \,\&\, y = \frac{5}{4}$
b $x = 2 \,\&\, y = 4$, or $x = -2 \,\&\, y = -4$, or $x = \frac{4}{3} \,\&\, y = 6$, or $x = -\frac{4}{3} \,\&\, y = -6$

10 a $x = 1 \,\&\, y = -2$, or $x = -1 \,\&\, y = 2$, or $x = \frac{7}{3} \,\&\, y = \frac{2}{3}$, or $x = -\frac{7}{3} \,\&\, y = -\frac{2}{3}$

Exercise 1H

1 a 1 **b** 9 **c** 25 **d** 81
e $\frac{9}{4}$ **f** $\frac{1}{4}$ **g** $\frac{25}{4}$ **h** $\frac{81}{4}$

2 a $(x + 2)^2$ **b** $(y + 1)^2$
c $(p + 7)^2$ **d** $(m - 6)^2$
e $(t - 8)^2$ **f** $(x + 10)^2$
g $(u - 20)^2$ **h** $(a - 12)^2$

3 a $x^2 + 6x + 9 = (x + 3)^2$
b $y^2 + 8y + 16 = (y + 4)^2$
c $a^2 - 20a + 100 = (a - 10)^2$
d $b^2 - 100b + 2500 = (b - 50)^2$
e $u^2 + u + \frac{1}{4} = \left(u + \frac{1}{2}\right)^2$
f $t^2 - 7t + \frac{49}{4} = \left(t - \frac{7}{2}\right)^2$
g $m^2 + 50m + 625 = (m + 25)^2$
h $c^2 - 13c + \frac{169}{4} = \left(c - \frac{13}{2}\right)^2$

4 a $x = -1$ or 3 **b** $x = 0$ or 6
c $a = -4$ or -2
d $x = -2 + \sqrt{3}$ or $-2 - \sqrt{3}$
e $x = 5 + \sqrt{5}$ or $5 - \sqrt{5}$
f $y = -5$ or 2 **g** $b = -2$ or 7
h no solution for y
i $a = \frac{-7 + \sqrt{21}}{2}$ or $\frac{-7 - \sqrt{21}}{2}$

5 a $x = \frac{2 + \sqrt{6}}{2}$ or $\frac{2 - \sqrt{6}}{2}$
b $x = \frac{-4 + \sqrt{10}}{2}$ or $\frac{-4 - \sqrt{10}}{2}$
c no solution for x **d** $x = -\frac{3}{2}$ or $\frac{1}{2}$
e $x = \frac{1 + \sqrt{5}}{4}$ or $\frac{1 - \sqrt{5}}{4}$
f $x = \frac{5 + \sqrt{11}}{2}$ or $\frac{5 - \sqrt{11}}{2}$

6 a $a = 3, b = 4$ and $c = 25$
b $A = -5, B = 6$ and $C = 8$

7 a $x^3 + 12x^2 + 48x + 64 = (x + 4)^3$
b $u = x + 4, u^3 - 18u + 12 = 0$

Chapter 1 review exercise

1 a $-6y$ **b** $-10y$ **c** $-16y^2$ **d** -4
2 a $-3a^2$ **b** $-a^2$ **c** $2a^4$ **d** 2
3 a $2t - 1$ **b** $4p + 3q$
c $x - 2y$ **d** $5a^2 - 3a - 18$
4 a $-18k^9$ **b** $-2k^3$ **c** $36k^{12}$ **d** $27k^9$
5 a $14x - 3$ **b** $-4a + 2b$
c $-2a$ **d** $-6x^3 - 10x^2$
e $2n^2 + 11n - 21$ **f** $r^2 + 6r + 9$
g $y^2 - 25$ **h** $6x^2 - 19x + 15$
i $t^2 - 16t + 64$ **j** $4c^2 - 49$
k $16p^2 + 8p + 1$ **l** $9u^2 - 12u + 4$
6 a $18(a + 2)$ **b** $4(5b - 9)$
c $9c(c + 4)$ **d** $(d - 6)(d + 6)$
e $(e + 4)(e + 9)$ **f** $(f - 6)^2$
g $(6 - 5g)(6 + 5g)$ **h** $(h - 12)(h + 3)$
i $(i + 9)(i - 4)$ **j** $(2j + 3)(j + 4)$
k $(3k + 2)(k - 3)$ **l** $(5\ell - 4)(\ell - 2)$
m $(2m - 3)(2m + 5)$ **n** $(n + 1)(m + p)$
o $(p + 9)(p^2 + 4)$ **p** $(q - r)(t - 5)$
q $(u^2 + v)(w - x)$ **r** $(x - y)(x + y + 2)$
7 a $\frac{3x}{4}$ **b** $\frac{x}{4}$ **c** $\frac{x^2}{8}$ **d** 2
e $\frac{13a}{6b}$ **f** $\frac{5a}{6b}$ **g** $\frac{a^2}{b^2}$ **h** $\frac{9}{4}$
i $\frac{x^2 + y^2}{xy}$ **j** $\frac{x^2 - y^2}{xy}$ **k** 1 **l** $\frac{x^2}{y^2}$
8 a $\frac{8x - 13}{15}$ **b** $\frac{8x - 13}{(x + 4)(x - 5)}$
c $\frac{3x + 13}{10}$ **d** $\frac{-3x - 13}{(x + 1)(x - 4)}$

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e $\dfrac{x-3}{4}$ **f** $\dfrac{-2x+6}{x(x+3)}$

9 a $\frac{3}{5}$ **b** $\dfrac{2}{x+y}$ **c** $\dfrac{x+3}{x-4}$

d $\dfrac{x+1}{x^2+1}$ **e** $\dfrac{1}{a+b}$ **f** $\dfrac{x-7}{3x-2}$

10 a $x = 4$ **b** $x = \frac{2}{3}$ **c** $x = 46$ **d** $x = 36$

e $a = 3$ **f** $a = 10$ **g** $a = -17$ **h** $a = -42$

11 a $a = -7$ or 7 **b** $b = -7$ or 0

c $c = -6$ or -1 **d** $d = -7$ or 1

e $e = 2$ or 3 **f** $f = -\frac{3}{2}$ or 2

g $g = \frac{1}{2}$ or 6 **h** $h = -2$ or $\frac{4}{3}$

12 a $x = 2 + \sqrt{3}$ or $2 - \sqrt{3}$

b $y = \frac{-3+\sqrt{21}}{2}$ or $\frac{-3-\sqrt{21}}{2}$

c $y = -3 + \sqrt{5}$ or $-3 - \sqrt{5}$

d $y = \frac{1+\sqrt{7}}{3}$ or $\frac{1-\sqrt{7}}{3}$

e $y = \frac{-5+\sqrt{65}}{4}$ or $\frac{-5-\sqrt{65}}{4}$

f $y = \frac{3+\sqrt{13}}{4}$ or $\frac{3-\sqrt{13}}{4}$

13 a $x = -2 + \sqrt{10}$ or $-2 - \sqrt{10}$

b $x = 3 + \sqrt{6}$ or $3 - \sqrt{6}$

c $x = 1 + \sqrt{13}$ or $1 - \sqrt{13}$

d $x = -5 + 3\sqrt{2}$ or $-5 - 3\sqrt{2}$

14 11

15 $(x^2 - x + 1)(x^2 + x + 1)$

16 $\dfrac{1}{2(x+2)}$

17 $\dfrac{13x}{(x+1)(x+2)(x+3)}$

18 $x = -9$ or $\frac{5}{3}$

19 $x = 2$ & $y = 0$, or $x = -2$ & $y = 0$, or $x = \sqrt{3}$ & $y = -1$, or $x = -\sqrt{3}$ & $y = -1$

Chapter 2

Exercise 2A

1 a $\frac{3}{10}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$ **d** $\frac{1}{20}$

2 a 0.6 **b** 0.27 **c** 0.09 **d** 0.165

3 a 25% **b** 40% **c** 24% **d** 65%

4 a 32% **b** 9% **c** 22.5% **d** 150%

5 a $\frac{1}{3}$ **b** $\frac{4}{5}$ **c** $\frac{2}{3}$ **d** $\frac{3}{4}$ **e** $\frac{2}{5}$

f $\frac{7}{15}$ **g** $\frac{4}{7}$ **h** $\frac{5}{6}$ **i** $\frac{3}{5}$ **j** $\frac{3}{4}$

6 a 0.5 **b** 0.2 **c** 0.6 **d** 0.75

e 0.04 **f** 0.35 **g** 0.125 **h** 0.625

7 a $\frac{2}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{20}$ **d** $\frac{4}{25}$

e $\frac{39}{50}$ **f** $\frac{1}{200}$ **g** $\frac{3}{8}$ **h** $\frac{33}{125}$

8 a $0.\dot{3}$ **b** $0.\dot{6}$ **c** $0.\dot{1}$ **d** $0.\dot{5}$

e $0.\dot{2}\dot{7}$ **f** $0.\dot{0}\dot{9}$ **g** $0.1\dot{6}$ **h** $0.8\dot{3}$

9 a $\frac{3}{4}$ **b** $\frac{7}{10}$ **c** $\frac{5}{6}$ **d** $\frac{4}{15}$

e $\frac{5}{18}$ **f** $\frac{1}{24}$ **g** $\frac{5}{6}$ **h** $\frac{1}{75}$

10 a 5 **b** 8 **c** $\frac{1}{10}$ **d** $\frac{1}{7}$ **e** $\frac{1}{4}$

f 6 **g** $\frac{1}{4}$ **h** $\frac{2}{3}$ **i** 4 **j** $\frac{1}{4}$

11 a $2^3 \times 3$ **b** $2^2 \times 3 \times 5$ **c** $2^3 \times 3^2$

d $2 \times 3^2 \times 7$ **e** $2^3 \times 13$ **f** $3^3 \times 5$

g $3^3 \times 7$ **h** $2 \times 3 \times 7^2$

i $3^2 \times 5 \times 7$ **j** 5×11^2

12 a 60 c **b** 15 kg

c \$7800 **d** 72 min or $1\frac{1}{5}$ h

13 a 0.132 **b** 0.025 **c** 0.3125 **d** 0.3375

e $0.58\dot{3}$ **f** $1.\dot{8}\dot{1}$ **g** $0.1\dot{3}$ **h** $0.2\dot{3}\dot{6}$

14 a $\frac{14}{15}$ **b** $\frac{5}{11}$ **c** $\frac{1}{2000}$

15 a \$800 **b** \$160 **c** \$120

16 a $\frac{1}{11} = 0.\dot{0}\dot{9}$, $\frac{2}{11} = 0.\dot{1}\dot{8}, \ldots, \frac{5}{11} = 0.\dot{4}\dot{5}$, $\frac{6}{11} = 0.\dot{5}\dot{4}, \ldots, \frac{10}{11} = 0.\dot{9}\dot{0}$. The first digit runs from 0 to 9, the second runs from 9 to 0.

b $\frac{1}{7} = 0.\dot{1}4285\dot{7}$, $\frac{2}{7} = 0.\dot{2}8571\dot{4}$, etc. The digits of each cycle are in the same order but start at a different place in the cycle.

17 c $3.0000003 \neq 3$, showing that some fractions are not stored exactly.

18 a If n is divisible by a prime p larger than $\sqrt{n}$, then n is divisible by $\dfrac{n}{p}$. Hence n is divisible by a prime less than or equal to $\dfrac{n}{p}$, which is less than $\sqrt{n}$.

b 2, 3, 5, 7, 11, 13, 17, 19 because $\sqrt{400} = 20$.

c $247 = 13 \times 19$, 241 is prime, $133 = 7 \times 19$, 367 is prime, 379 is prime, $319 = 11 \times 29$

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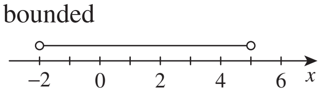
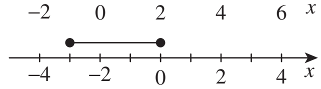
19 a i 1, 2, 4, 5, 7, 8
ii 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24
iii 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31
iv 1, 2, 4, 7, 8, 11, 13, 14, 16, 17, 19, 22, 23, 26, 28, 29, 31, 32, 34, 37, 38, 41, 43, 44

Exercise 2B

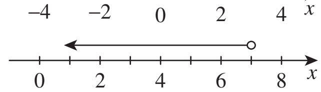
1 a rational, $\frac{-3}{1}$ **b** rational, $\frac{3}{2}$ **c** irrational
d rational, $\frac{2}{1}$ **e** rational, $\frac{3}{1}$ **f** irrational
g rational, $\frac{2}{3}$ **h** rational, $\frac{9}{20}$ **i** rational, $\frac{3}{25}$
j rational, $\frac{333}{1000}$ **k** rational, $\frac{1}{3}$ **l** rational, $\frac{22}{7}$
m irrational **n** rational, $3\frac{7}{50}$ **o** rational, $\frac{0}{1}$
2 a 0.3 **b** 5.7 **c** 12.8 **d** 0.1 **e** 3.0 **f** 10.0
3 a 0.43 **b** 5.4 **c** 5.0
d 0.043 **e** 430 **f** 4300
4 a 3.162 **b** 6.856 **c** 0.563
d 0.771 **e** 3.142 **f** 9.870
5 a 7.62 **b** 5.10 **c** 3840
d 538 000 **e** 0.740 **f** 0.00806
6 a 1 **b** 2 **c** 3 **d** 2 **e** 4
f either 1, 2 or 3
7 a i closed **ii** open **iii** closed
iv neither open nor closed
v open **vi** open **vii** closed
viii neither open nor closed
b i bounded **ii** unbounded **iii** unbounded
iv bounded **v** unbounded **vi** bounded
vii unbounded **viii** bounded
8 a $-2 < x < 5$



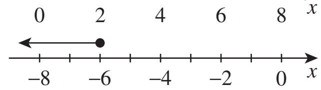
b $-3 \le x \le 0$



c $x < 7$



d $x \le -6$



9 a 45.186 **b** 2.233 **c** 0.054
d 0.931 **e** 0.842 **f** 0.111
10 a 10, rational **b** $\sqrt{41}$, irrational
c 8, rational **d** $\sqrt{5}$, irrational
e $\frac{13}{15}$, rational **f** 45, rational
11 a 0.3981 **b** 0.05263 **c** 1.425
d 5.138 **e** 0.1522 **f** 25 650
g 5.158 **h** 0.7891 **i** 1.388×10^{14}
j 1.134 **k** 0.005892 **l** 1.173

12 a The passage seems to take $\pi \doteqdot 3$.
b 3 significant figures
c Ask the internet.
d 7, with a gap of about 0.3 inches
13 a 9.46×10^{15} m **b** 2.1×10^{22} m
c 4.29×10^{17} seconds **d** 1.3×10^{26} m
14 a 1.836×10^3 **b** 6×10^{26}
16 a Clearly $\dfrac{p+1}{n} > a$.

$$\frac{p+1}{n} = \frac{p}{n} + \frac{1}{n} < a + b - a = b$$

b $n = 63\,293,\ p = 2000$ **c** $\frac{2001}{63\,293}$

Exercise 2C

1 a 4 **b** 6 **c** 9 **d** 11
e 12 **f** 20 **g** 50 **h** 100
2 a $2\sqrt{3}$ **b** $3\sqrt{2}$ **c** $2\sqrt{5}$ **d** $3\sqrt{3}$
e $2\sqrt{7}$ **f** $2\sqrt{10}$ **g** $4\sqrt{2}$ **h** $3\sqrt{11}$
i $3\sqrt{6}$ **j** $10\sqrt{2}$ **k** $2\sqrt{15}$ **l** $5\sqrt{3}$
m $4\sqrt{5}$ **n** $7\sqrt{2}$ **o** $20\sqrt{2}$ **p** $10\sqrt{10}$
3 a $2\sqrt{3}$ **b** $2\sqrt{7}$ **c** $\sqrt{5}$ **d** $-2\sqrt{2}$
e $2\sqrt{3} + 3\sqrt{2}$ **f** $\sqrt{5} - 2\sqrt{7}$ **g** $3\sqrt{6} - 2\sqrt{3}$
h $-3\sqrt{2} - 6\sqrt{5}$ **i** $-4\sqrt{10} + 2\sqrt{5}$
4 a $6\sqrt{2}$ **b** $10\sqrt{3}$ **c** $4\sqrt{6}$ **d** $8\sqrt{11}$
e $9\sqrt{5}$ **f** $12\sqrt{13}$ **g** $20\sqrt{3}$ **h** $8\sqrt{6}$
5 a $\sqrt{20}$ **b** $\sqrt{50}$ **c** $\sqrt{128}$ **d** $\sqrt{108}$
e $\sqrt{125}$ **f** $\sqrt{112}$ **g** $\sqrt{68}$ **h** $\sqrt{490}$
6 a $3\sqrt{2}$ **b** $\sqrt{3}$ **c** $2\sqrt{2}$ **d** $5\sqrt{6}$
e $\sqrt{5}$ **f** $2\sqrt{10}$ **g** $4\sqrt{3}$ **h** $2\sqrt{5}$
i $11\sqrt{2}$ **j** 5 **k** 3 **l** 2
7 a $4\sqrt{6} + 10\sqrt{3}$ **b** $2\sqrt{2} + 6\sqrt{3}$ **c** $4\sqrt{7} - 10\sqrt{35}$
8 a 7 **b** 20 **c** 96 **d** 24
9 b Show that $\sqrt[3]{7} < 2$ and $\sqrt[4]{7} < 2$.
10 a The graph intersects the x-axis at $x = \sqrt{2}$ and $x = -\sqrt{2}$, which are both irrational.

Exercise 2D

1 a 3 **b** $\sqrt{6}$ **c** 7 **d** $\sqrt{30}$
e $6\sqrt{2}$ **f** $10\sqrt{5}$ **g** $6\sqrt{15}$ **h** $30\sqrt{14}$
i 12 **j** 63 **k** 30 **l** 240
2 a $\sqrt{5}$ **b** $\sqrt{7}$ **c** $\sqrt{5}$ **d** 2
e $3\sqrt{2}$ **f** $\sqrt{3}$ **g** $2\sqrt{7}$ **h** $5\sqrt{5}$

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3 a $5 + \sqrt{5}$ **b** $\sqrt{6} - \sqrt{2}$ **c** $2\sqrt{3} - 3$
d $2\sqrt{10} - 4$ **e** $7\sqrt{7} - 14$ **f** $18 - 2\sqrt{30}$

4 a $2\sqrt{3}$ **b** $5\sqrt{2}$ **c** $3\sqrt{5}$
d $4\sqrt{11}$ **e** 24 **f** $12\sqrt{10}$

5 a $2\sqrt{5} - 2$ **b** $3\sqrt{6} + 3\sqrt{2}$ **c** $5\sqrt{3} + 4\sqrt{5}$
d $4\sqrt{3} - 2\sqrt{6}$ **e** $27\sqrt{3} - 9\sqrt{7}$ **f** $21\sqrt{2} - 42$

6 a $\sqrt{6} - \sqrt{3} + \sqrt{2} - 1$
b $\sqrt{35} + 3\sqrt{5} - 2\sqrt{7} - 6$
c $\sqrt{15} + \sqrt{10} + \sqrt{6} + 2$
d $8 - 3\sqrt{6}$ **e** $4 + \sqrt{7}$ **f** $7\sqrt{3} - 4\sqrt{6}$

7 a 4 **b** 2 **c** 1 **d** 7 **e** 15 **f** 29

8 a $4 + 2\sqrt{3}$ **b** $6 - 2\sqrt{5}$ **c** $5 + 2\sqrt{6}$
d $12 - 2\sqrt{35}$ **e** $13 - 4\sqrt{3}$ **f** $29 + 12\sqrt{5}$
g $33 + 4\sqrt{35}$ **h** $30 - 12\sqrt{6}$ **i** $55 + 30\sqrt{2}$

9 a 2 **b** $\frac{3}{5}$ **c** $2\sqrt{3}$ **d** $\frac{5\sqrt{3}}{2}$ **e** 5 **f** 4

10 a 3 **b** 5 **c** 4 **d** 6

11 a $\sqrt{3}$ **b** $\frac{6\sqrt{7}}{13}$

12 a $xy\sqrt{y}$ **b** x^2y^3 **c** $x + 3$
d $(x + 1)\sqrt{x}$ **e** $x(x + 1)y^2$ **f** $x(x + 1)$

13 a If $a = 3$ and $b = 4$, then LHS $= 5$, but RHS $= 7$.
b Squaring both sides gives $2ab = 0$. Thus the statement is true when one of a or b is zero and the other is not negative.

14 a $2\sqrt{3}$ **b** $3\sqrt{11}$ **c** $3 + 2\sqrt{2}$ **d** $7 - 2\sqrt{6}$

15 a $a^2 + 2ab + b^2$
b 2
c $\sqrt{2}$

16 26

Exercise 2E

1 a $\frac{\sqrt{3}}{3}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{3\sqrt{5}}{5}$ **d** $\frac{5\sqrt{2}}{2}$
e $\frac{\sqrt{6}}{3}$ **f** $\frac{\sqrt{35}}{7}$ **g** $\frac{2\sqrt{55}}{5}$ **h** $\frac{3\sqrt{14}}{2}$

2 a $\sqrt{2}$ **b** $\sqrt{5}$ **c** $2\sqrt{3}$ **d** $3\sqrt{7}$
e $\frac{\sqrt{6}}{2}$ **f** $\frac{\sqrt{15}}{3}$ **g** $\frac{4\sqrt{6}}{3}$ **h** $\frac{7\sqrt{10}}{5}$

3 a $\frac{\sqrt{5}}{10}$ **b** $\frac{\sqrt{7}}{21}$ **c** $\frac{3\sqrt{2}}{10}$ **d** $\frac{2\sqrt{3}}{21}$
e $\frac{5\sqrt{2}}{3}$ **f** $\frac{3\sqrt{3}}{4}$ **g** $\frac{\sqrt{30}}{20}$ **h** $\frac{2\sqrt{77}}{35}$

4 a $\frac{\sqrt{3} + 1}{2}$ **b** $\frac{\sqrt{7} - 2}{3}$ **c** $\frac{3 - \sqrt{5}}{4}$
d $\frac{4 + \sqrt{7}}{9}$ **e** $\frac{\sqrt{5} + \sqrt{2}}{3}$ **f** $\frac{\sqrt{10} - \sqrt{6}}{4}$
g $\frac{2\sqrt{3} - 1}{11}$ **h** $\frac{5 + 3\sqrt{2}}{7}$

5 a $\frac{3\sqrt{5} - 3}{4}$ **b** $\frac{8\sqrt{2} + 4\sqrt{3}}{5}$ **c** $\frac{5\sqrt{7} + 7}{18}$
d $\frac{3\sqrt{15} - 9}{2}$ **e** $\frac{28 + 10\sqrt{7}}{3}$ **f** $\sqrt{2} + 1$
g $2 - \sqrt{3}$ **h** $\frac{7 + 2\sqrt{10}}{3}$ **i** $8 - 3\sqrt{7}$
j $\frac{23 + 6\sqrt{10}}{13}$ **k** $4 - \sqrt{15}$ **l** $\frac{93 + 28\sqrt{11}}{5}$

6 a $\sqrt{3} + 1$ **b** $4 - \sqrt{10}$

7 a 3 **b** 1 **c** 7 **d** 2

9 $a = -1, b = 2$

10 a $2\sqrt{2}$ **b** 4 **c** 4 **d** $\dfrac{2}{x - 1}$

11 $\dfrac{\sqrt{x + h} - \sqrt{x}}{h}$

12 a $x^2 + 2 + \dfrac{1}{x^2}$

b ii $x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2 = 28 - 2 = 26$

13 a 8.33 **b** 8.12

14 $\frac{2\sqrt{3} + 3\sqrt{2} - \sqrt{30}}{12}$

Chapter 2 review exercise

1 a rational, $\frac{7}{1}$ **b** rational, $\frac{-9}{4}$ **c** rational, $\frac{3}{1}$
d irrational **e** irrational **f** rational, $\frac{2}{1}$
g rational, $\frac{-4}{25}$ **h** irrational

2 a 4.12, 4.1 **b** 4.67, 4.7 **c** 2.83, 2.8
d 0.77, 0.77 **e** 0.02, 0.019 **f** 542.41, 540

3 a 1.67 **b** 70.1 **c** 1.43
d 0.200 **e** 0.488 **f** 0.496
g 1.27 **h** 1590 **i** 0.978

4 a $2\sqrt{6}$ **b** $3\sqrt{5}$ **c** $5\sqrt{2}$
d $10\sqrt{5}$ **e** $9\sqrt{2}$ **f** $4\sqrt{10}$

5 a $2\sqrt{5}$ **b** 5 **c** 28
d $\sqrt{7} - \sqrt{5}$ **e** $\sqrt{7}$ **f** $3\sqrt{5}$
g 4 **h** $2\sqrt{5}$ **i** $24\sqrt{10}$

6 a $\sqrt{3}$ **b** $7\sqrt{2}$
c $4\sqrt{2}$ **d** $8\sqrt{6} - 6\sqrt{5}$

7 a $3\sqrt{7} - 7$ **b** $2\sqrt{30} + 3\sqrt{10}$
c $3\sqrt{5} - 5\sqrt{15}$ **d** $3\sqrt{2} + 6$

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8 a $\sqrt{5} + 1$ **b** $13 + 7\sqrt{3}$
c $2\sqrt{35} + 4\sqrt{7} - 6\sqrt{5} - 12$
d 1 **e** 13 **f** $11 - 4\sqrt{7}$
g $7 + 2\sqrt{10}$ **h** $34 - 24\sqrt{2}$

9 a $\frac{\sqrt{5}}{5}$ **b** $\frac{3\sqrt{2}}{2}$ **c** $\frac{\sqrt{33}}{11}$
d $\frac{\sqrt{3}}{15}$ **e** $\frac{5\sqrt{7}}{14}$ **f** $\frac{\sqrt{5}}{15}$

10 a $\frac{\sqrt{5} - \sqrt{2}}{3}$ **b** $\frac{3 + \sqrt{7}}{2}$ **c** $\frac{2\sqrt{6} + \sqrt{3}}{21}$
d $\frac{3 - \sqrt{3}}{2}$ **e** $\frac{\sqrt{11} - \sqrt{5}}{2}$ **f** $\frac{6\sqrt{35} + 21}{13}$

11 a $\frac{9 - 2\sqrt{14}}{5}$ **b** $26 + 15\sqrt{3}$

12 $x = 50$ **13** $5\sqrt{5} + 2$

14 $p = 5, q = 2$ **15** $\frac{7}{3}$

16 a $2\sqrt{10}$ **b** 38

18 By rationalising the denominators, the series 'telescopes' to just $\sqrt{16} - \sqrt{1} = 3$.

19 a It is sufficient to show that $\text{LHS}^2 = \text{RHS}^2$, because both sides are positive.
b $\sqrt{3}$

Chapter 3

Exercise 3A

1 a -3 **b** 5 **c** 0 **d** 5

2 a 5 **b** -10 **c** -3 **d** 2

3 a $5, -1, -7$ **b** $0, 4, 0$
c $16, 8, 0$ **d** $4, 1, \frac{1}{4}$

4 a $-4, 4, 12$ **b** $-\frac{1}{3}, 1, \frac{1}{5}$
c $-18, 2, -10$ **d** $1, \sqrt{5}, 3$

5 a $p(x)$: $3, 0, -1, 0, 3$
b $c(x)$: $-15, 0, 3, 0, -3, 0, 15$

6 a 2 **b** -6 **c** 11 **d** 4

7 a $-3, -2, -1, 0, 1, 0, -1$
b $3, 0, 0, 1, 4$

8 a 4 **b** $4\frac{1}{3}$

9 a 0 **b** $2 - 4\sqrt{3}$

10 $C = 50 + 20x$

11 a $y = -\frac{3}{4}x - \frac{5}{4}$ **b** $x = -\frac{4}{3}y - \frac{5}{3}$
c $y = -\dfrac{4}{x}$ **d** $s = \sqrt[3]{V}, s = \sqrt{\dfrac{A}{6}}$

e i $\ell = \dfrac{100}{b}$ **ii** $b = \dfrac{100}{\ell}$

12 a The square root of a negative is undefined.
b The square root of a negative is undefined.
c Division by zero is undefined.
d Division by zero is undefined.

13 a $2a - 4, -2a - 4, 2a - 2$
b $2 - a, 2 + a, 1 - a$
c $a^2, a^2, a^2 + 2a + 1$
d $\dfrac{1}{a - 1}, \dfrac{1}{-a - 1} = -\dfrac{1}{a + 1}, \dfrac{1}{a}$

14 a $5t, 5t - 8$ **b** $\sqrt{t} - 2, \sqrt{t - 2}$
c $t^2 + 2t - 2, t^2 - 2t$ **d** $-t^2, -t^2 + 4t - 2$

15 a $7 + h$ **b** $p + q + 5$ **c** $2x + h + 5$

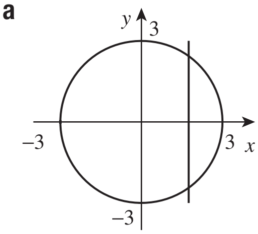
17 a 64 **b** 28 **c** $(x + 3)^2$ **d** $x^2 + 3$

18 It approaches 2.72.

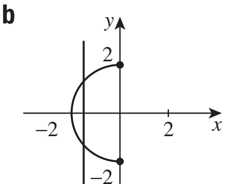
Exercise 3B

1 Notice that the y-axis is such a line in every case. Shown below are some other vertical lines that intersect at least twice.

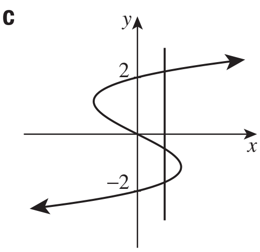
a



b



c



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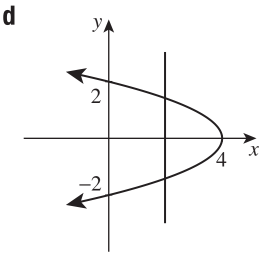
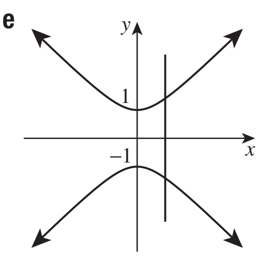
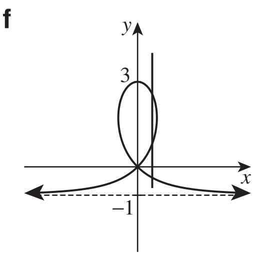


e



f



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2 a, c, f, h

3 a domain: all real x, range: $y \geq -1$

b domain: $-2 \leq x \leq 2$, range: $-2 \leq y \leq \sqrt{3}$

c domain: all real x, range: all real y

d domain: $-1 \leq x$, range: all real y

e domain: $-2 \leq x \leq 2$, range: $-3 \leq y \leq 3$

f domain: all real x, range: all real y

g domain: $0 \leq x \leq 2$, range: $-2 \leq y \leq 2$

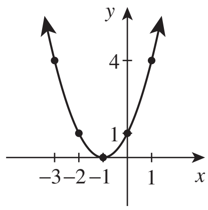
h domain: all real x, range: $y < 1$

4 a $x \neq 0$ **b** $x \neq 3$ **c** $x \neq -2$

5 a $x \geq 0$ **b** $x \geq 2$ **c** $x \geq -5$

6 a i 4, 1, 0, 1, 4

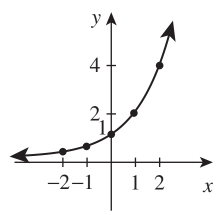
ii



iii domain: all real x, range: $y \geq 0$

b i $\frac{1}{4}, \frac{1}{2}$, 1, 2, , 4

ii



iii domain: all real x, range: $y > 0$

7 a $(0, 3)$ and $(0, -3)$ **b** $(0, 1)$ and $(0, -1)$

c $(2, 1)$ and $(2, 5)$ **d** $(2, 2)$ and $(2, -2)$

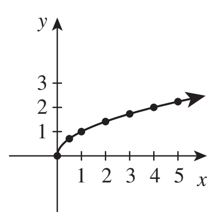
8 a all real x **b** $x \neq \frac{1}{2}$ **c** $x \geq -4$

d $x \leq 2$ **e** $x < 1$ **f** $x > 1\frac{1}{2}$

9 a $x \geq 0$

b 0, 0.7, 1, 1.4, 1.7, 2, 2.2

c It is the top half of a concave right parabola.

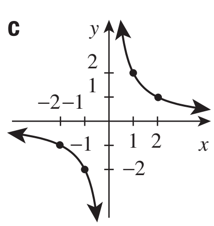


10 a $x \neq 0$

b $-\frac{1}{2}$, -1, -2, -4, *, 4, 2, 1, $\frac{1}{2}$

Division by zero is undefined.

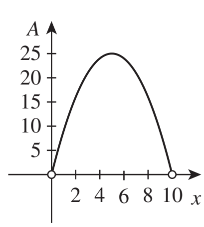
c



11 a $A = x(10 - x)$

b Both $10 - x > 0$ and $x > 0$. Thus $10 > x$ and $x > 0$. Hence the domain is $0 < x < 10$.

c



12 a $y = 2x + 3$

b $y = \dfrac{4}{x}$

c $y = \dfrac{3}{x - 2}$ **d** $y = -2 + \sqrt{9 - x^2}$

e $y = \sqrt[3]{x - 1}$ **f** $y = \dfrac{2x - 3}{x + 1}$

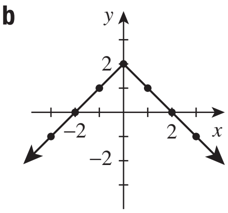
13 a $x > -2$ **b** $x \neq 2$ and $x \neq -2$

c $x \neq -1$ and $x \neq 0$ **d** $x \neq 2$ and $x \neq 3$

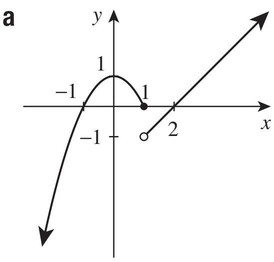
e $x \leq -2$ or $x \geq 2$ **f** $-1 < x < 1$

14 a -1, 0, 1, 2, 1, 0, -1

b



15 a



b There is a break. The proper way to say this is that $f(x)$ is not continuous at $x = 1$.

16 a i domain: all real x, range: $y \leq 4$

ii domain: $-2 \leq x \leq 2$, range: $0 \leq y \leq 2$

iii domain: $-2 < x < 2$, range: $y \geq \frac{1}{2}$

b i domain: $-3 < x < 1$, range $y > \frac{1}{2}$

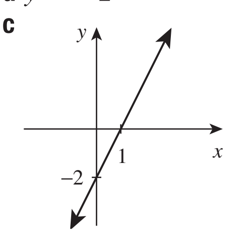
ii domain: all real x, range: $0 < y \leq \frac{1}{\sqrt{2}}$

17 a $-1 < x < 1$

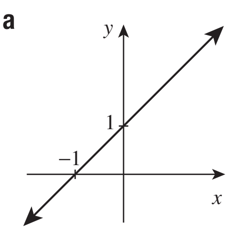
Exercise 3C

1 a $y = -2$ **b** $x = 1$

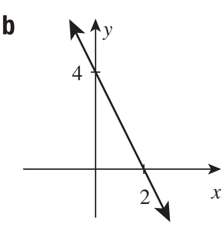
c



2 a



b



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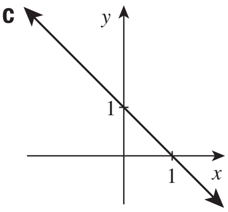
c


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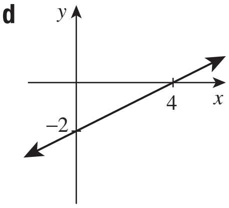

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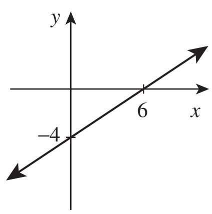

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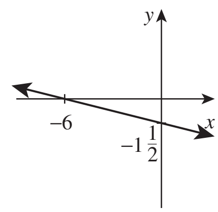
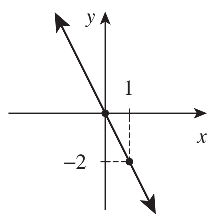
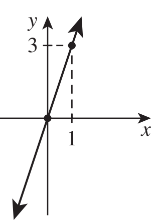


3 a When $x = 0$, $y = 0$.

b $(1, -2)$



4 a


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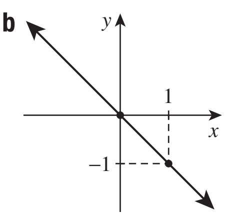

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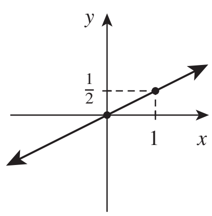
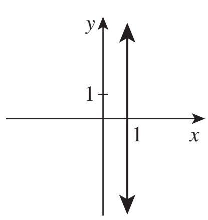


5 a


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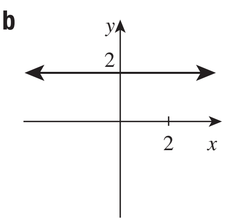

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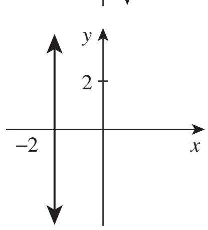

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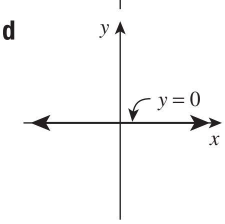

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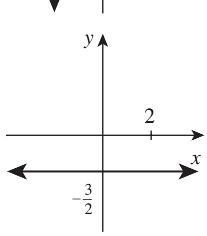
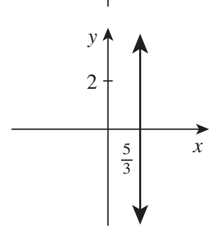


f


6 a a, c, f

b $(1, 0)$ and $(1, 1)$ are on $x = 1$.

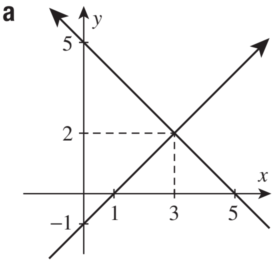
c $(-2, 0)$ and $(-2, 1)$ are on $x = -2$.

d $\left(\frac{5}{3}, 0\right)$ and $\left(\frac{5}{3}, 1\right)$ are on $3x = 5$.

7 c $y = 1 - x$ **d** $y = \frac{1}{2}x - 2$

e $y = \frac{2}{3}x - 4$ **f** $y = -\frac{1}{4}x - \frac{3}{2}$

8 a yes **b** no **c** yes **d** yes **e** yes **f** no

9 a


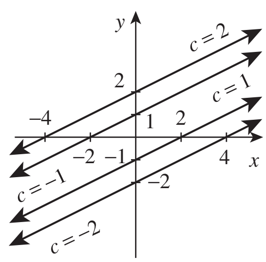
b $(3, 2)$

10 a $(-1, 3)$ **b** $(1, -2)$ **c** $(-2, -1)$

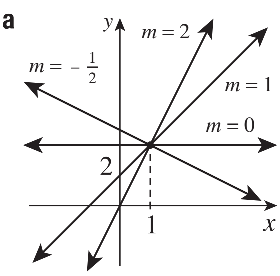
11 a $C(n) = 10 + 50n$

b i $D(n) = 8 + 2n$

ii $T = C + D$ so $T(n) = 18 + 52n$

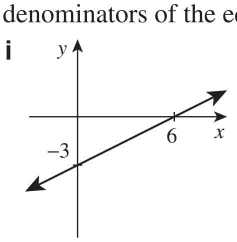
12 a


b They are parallel. The value of c gives the y-intercept.

13 a


b $(1, 2)$

14 a $(a, 0)$ and $(0, b)$. The intercepts appear in the denominators of the equation.

b i


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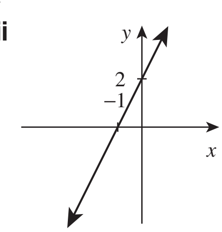

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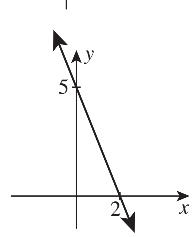

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15 a $\left(3, 1\frac{1}{2}\right)$

c $5x - 2y - 12 = 0$

16 a $Ax_1 + By_1 + C = 0, Ax_2 + By_2 + C = 0,$
$Ax + By + C = 0$

b See worked solutions

c $(2 - y)(3 - x) = (4 + y)(x - 1)$

Exercise 3D

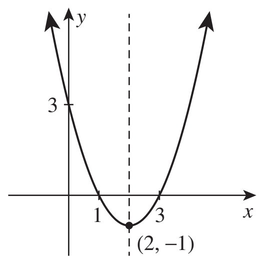
1 a i $y = 3$

ii $x = 1, 3$

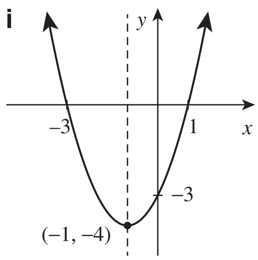
iii $x = 2$

iv $(2, -1)$

v



b i



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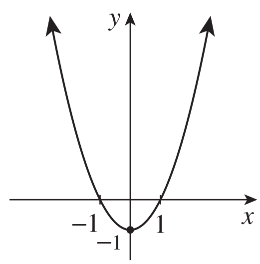
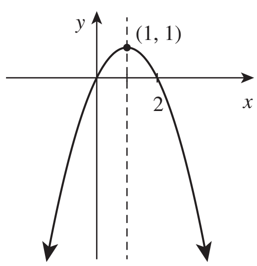
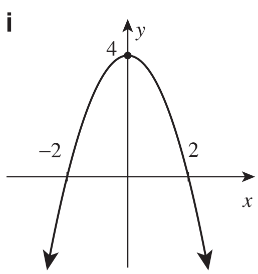


2 a



b i



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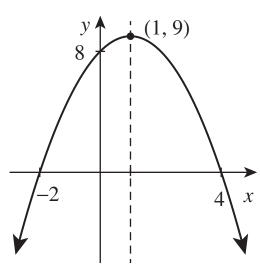
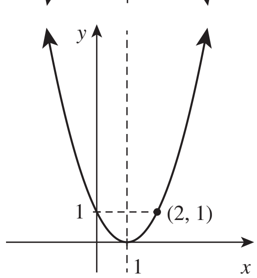
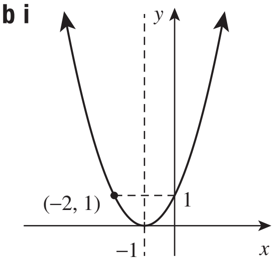


3 a



b i



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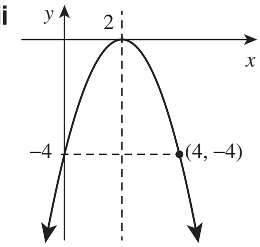
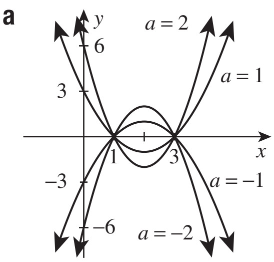


4 a



b $(1, 0)$ and $(3, 0)$

5 a $y = (x - 4)(x - 6)$

b $y = x(x - 3)$

c $y = (x + 3)(x - 5)$

d $y = (x + 6)(x + 1)$

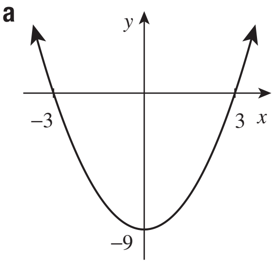
6 a $y = x(x - 3)$

b $y = (x + 2)(x - 1)$

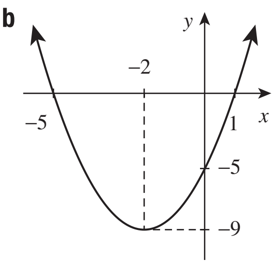
c $y = -(x + 1)(x - 3)$

d $y = -(x + 2)(x + 5)$

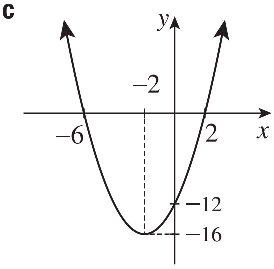
7 a



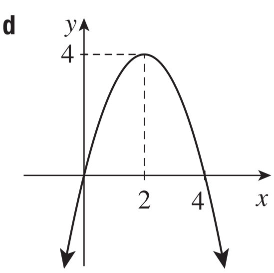
b



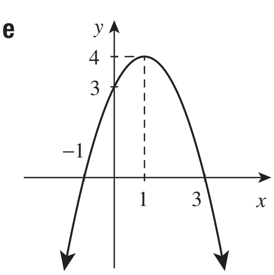
c



d



e



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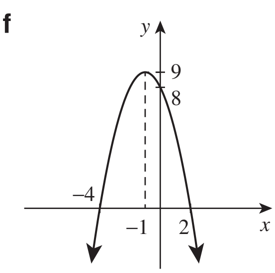

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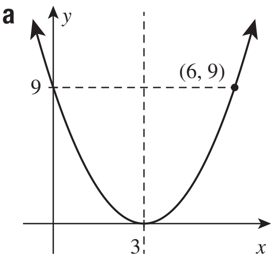
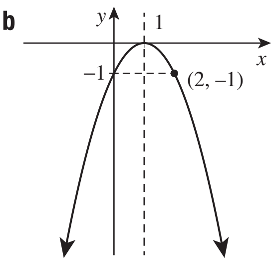


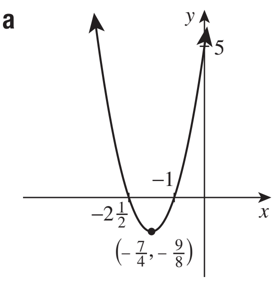
$y = (x - 3)^2$

b



$y = -(x - 1)^2$

9 a



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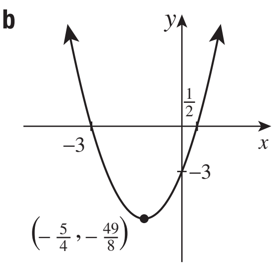

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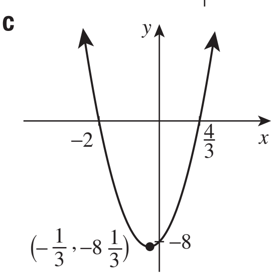

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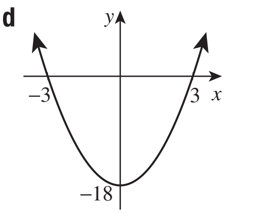

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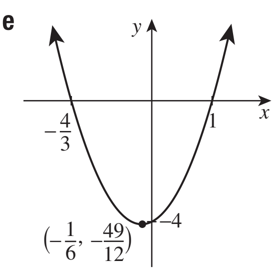

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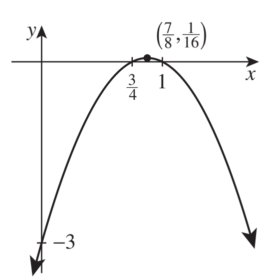


10 a $y = (x + 1)(x - 2)$
b $y = -(x + 3)(x - 2)$
c $y = 3(x + 2)(x - 4)$
d $y = -\frac{1}{2}(x - 2)(x + 2)$

11 a $y = 2(x - 1)(x - 3)$
b $y = -2(x + 2)(x - 1)$
c $y = -3(x + 1)(x - 5)$
d $y = \frac{1}{4}(x + 2)(x + 4)$

12 a $y = 3(x - 2)(x - 8)$
b $y = -(x - 2)(x - 8)$
c $y = \frac{4}{3}(x - 2)(x - 8)$
d $y = -\frac{20}{7}(x - 2)(x - 8)$

13 a $y = x(x + 3)$
b $y = \frac{3}{2}x^2$
c $y = -4x(x - 2)$
d $y = -2x(x + 6)$

14 a $a = \dfrac{c}{\alpha\beta}$
b $a = -\dfrac{b}{\alpha + \beta}$
c $a = \dfrac{2}{(1 - \alpha)(1 - \beta)}$

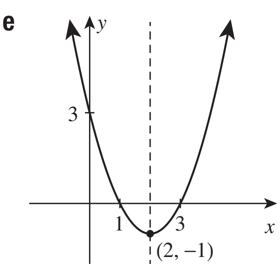
15 a $f(x) = (x - 4)(x + 2)$, so the axis is $x = 1$.
b i Both $f(1 + h) = h^2 - 9$ and $f(1 - h) = h^2 - 9$.
ii The parabola is symmetric in the line $x = 1$.

16 a $(-1 + p), (-1 - p), x = -1$
b $(p - 1), (p + 1), x = p$
c $(2 + p), -p, x = 1$

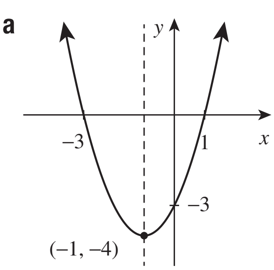
17 a The value of the function is the same h units right $\left(\frac{1}{2}(\alpha + \beta) + h\right)$ or left $\left(\frac{1}{2}(\alpha + \beta) - h\right)$ of the axis.
b The result follows from part **a** by putting $h = \left(\frac{1}{2}(\alpha + \beta) + x\right)$

Exercise 3E

1 a $a = 1$, concave up
b $y = 3$
c $x = 1, 3$
d $x = 2, V(2, -1)$
e



2 a



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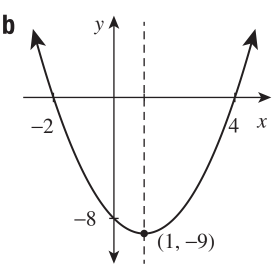

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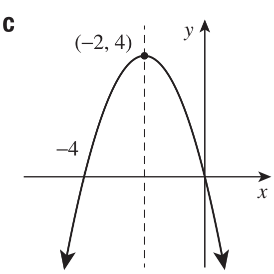
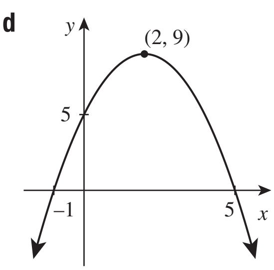


d



3 a $f(x) = (x - 2)^2 + 1$
b $f(x) = (x + 3)^2 + 2$
c $f(x) = (x - 1)^2 + 7$
d $f(x) = (x - 5)^2 - 24$
e $f(x) = (x + 1)^2 - 6$
f $f(x) = (x + 2)^2 - 5$

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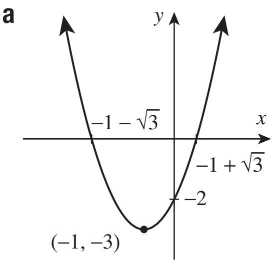
4 a



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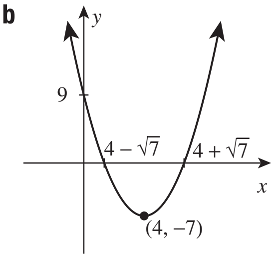

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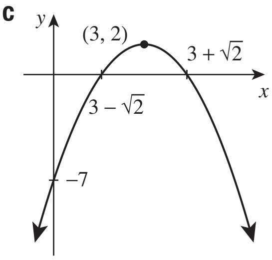

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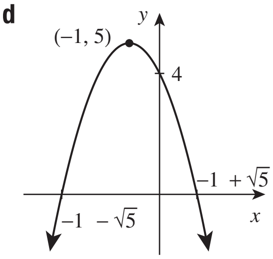


5 a $x = 1, 3$ **b** $x = -3, 1$ **c** $x = -1, 2$

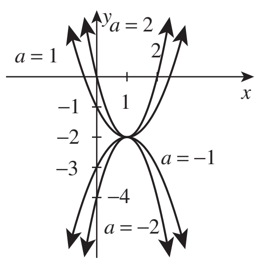
6 a $y = (x-1)^2 + 2$ **b** $y = (x+2)^2 - 3$

c $y = -(x-3)^2 + 4$ **d** $y = -(x-2)^2 - 1$

7 a $y = (x-2)^2 + 5$ **b** $y = x^2 - 3$

c $y = (x+1)^2 + 7$ **d** $y = (x-3)^2 - 11$

8 a

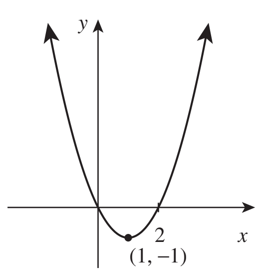


b $(1, -2)$

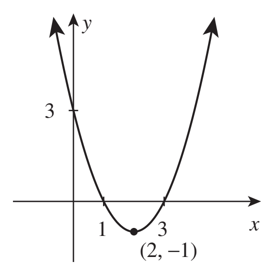
c $a > 0$

d The vertex is below the x-axis. Thus the parabola will only intersect the x-axis if it is concave up.

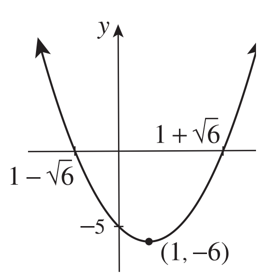
9 a $y = (x-1)^2 - 1$



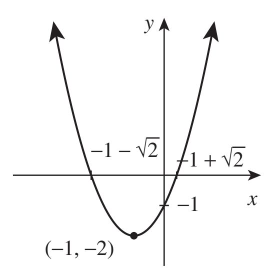
b $y = (x-2)^2 - 1$



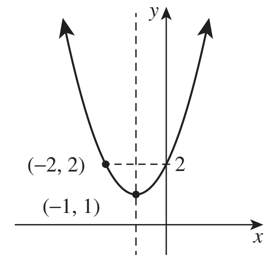
c $y = (x-1)^2 - 6$



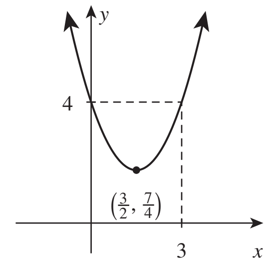
d $y = (x+1)^2 - 2$



e $y = (x+1)^2 + 1$



f $y = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$



10 Put $h = -4$ and $k = 2$ into the formula $y = a(x-h)^2 + k$.

a $y = (x+4)^2 + 2$ **b** $y = 3(x+4)^2 + 2$

c $y = \frac{7}{8}(x+4)^2 + 2$ **d** $y = -\frac{1}{8}(x+4)^2 + 2$

11 a $V = (3, -5)$, concave up, two x-intercepts.

b $V = (-1, 3)$, concave down, two x-intercepts.

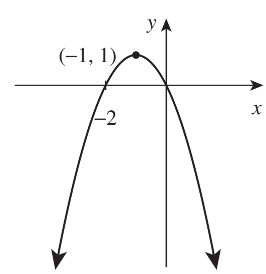
c $V = (-2, -1)$, concave down, no x-intercepts.

d $V = (4, 3)$, concave up, no x-intercepts.

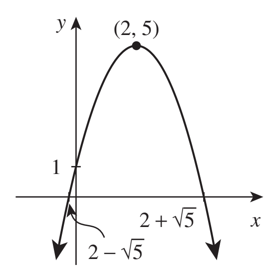
e $V = (-1, 0)$, concave up, one x-intercept.

f $V = (3, 0)$, concave down, one x-intercept.

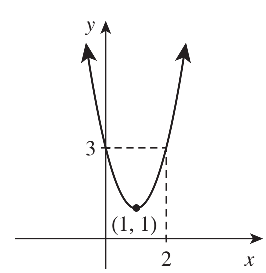
12 a $y = -(x+1)^2 + 1$



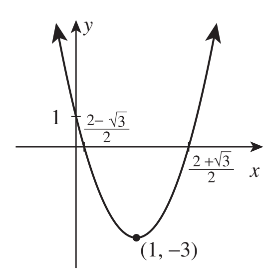
b $y = -(x-2)^2 + 5$



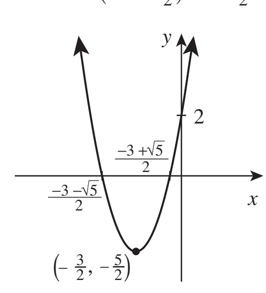
c $y = 2(x-1)^2 + 1$



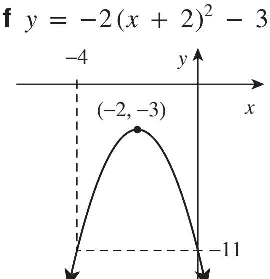
d $y = 4(x-1)^2 - 3$



e $y = 2\left(x + \frac{3}{2}\right)^2 - \frac{5}{2}$



f $y = -2(x+2)^2 - 3$



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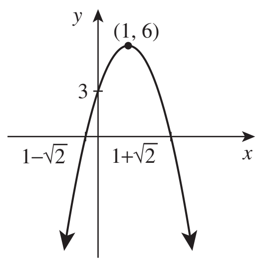
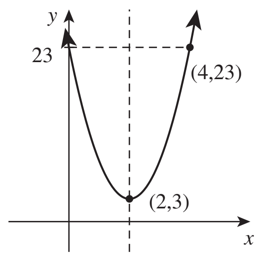
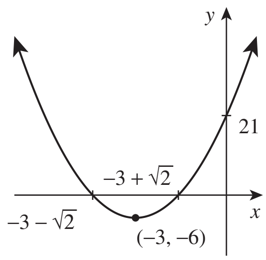
g $y = -3(x - 1)^2 + 6$



h $y = 5(x - 2)^2 + 3$



i $y = 3(x + 3)^2 - 6$



13 a $f(x) = \left(x + 1 + \sqrt{2}\right)\left(x + 1 - \sqrt{2}\right)$

b $f(x) = \left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right)$

c $f(x) = -\left(x + 1 + \sqrt{5}\right)\left(x + 1 - \sqrt{5}\right)$

14 $y = (x + 2)^2 + k$

a $y = (x + 2)^2 - 4$ **b** $y = (x + 2)^2 - 48$

c $y = (x + 2)^2 - 9$ **d** $y = (x + 2)^2 - 10$

e $y = (x + 2)^2 - 2$ **f** $y = (x + 2)^2 + 7$

15 a $y = 2(x - 1)^2 + 1$ **b** $y = -(x - 3)^2 + 2$

c $y = \frac{1}{2}(x + 2)^2 - 4$ **d** $y = -3(x + 1)^2 + 4$

16 a $-d - \sqrt{e}, -d + \sqrt{e}$ **b** $2\sqrt{e}$

c $e = 1$. They have vertex on the line $y = -1$.

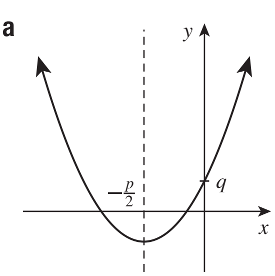
17 $h_1 = h_2$, but $k_1 \neq k_2$. The two curves have the same axis of symmetry, but different vertices.

18 b The vertex is $\left(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a}\right)$ and the axis of symmetry is $x = -\dfrac{b}{2a}$.

c $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

d $y = a\left(x + \dfrac{b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x + \dfrac{b - \sqrt{b^2 - 4ac}}{2a}\right)$

19 a



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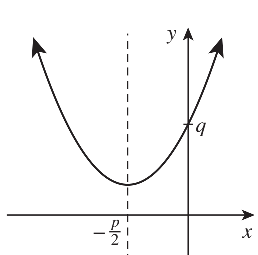

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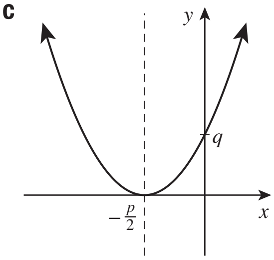


20 a The value of the function is the same t units right or left of the axis.

b The result follows from part **a** by putting $t = h - x$.

Exercise 3F

1 a i concave up

ii $y = -1$

iii $x = 1$

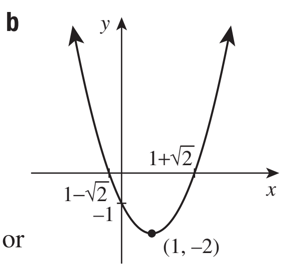
iv $= (1, -2)$

v $\Delta = 8$

vi $\Delta > 0$

vii $x = 1 - \sqrt{2} \doteqdot -0.41$, or $1 + \sqrt{2} \doteqdot 2.41$.

b



2 a $-1 - \sqrt{3}$ or $-1 + \sqrt{3}$, -2.73 or 0.73

b $2 - \sqrt{3}$ or $2 + \sqrt{3}$, 0.27 or 3.73

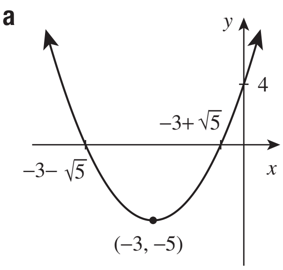
c $\frac{1}{2}\left(3 - \sqrt{17}\right)$ or $\frac{1}{2}\left(3 + \sqrt{17}\right)$, -0.56 or 3.56

d $-1 - \sqrt{5}$ or $-1 + \sqrt{5}$, -3.24 or 1.24

e $\frac{1}{3}\left(1 - \sqrt{7}\right)$ or $\frac{1}{3}\left(1 + \sqrt{7}\right)$, -0.55 or 1.22

f $\frac{1}{2}\left(-2 - \sqrt{6}\right)$ or $\frac{1}{2}\left(-2 + \sqrt{6}\right)$, -2.22 or 0.22

3 a



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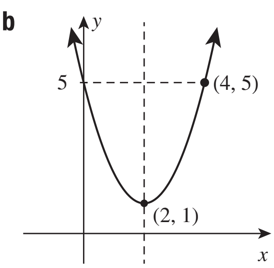

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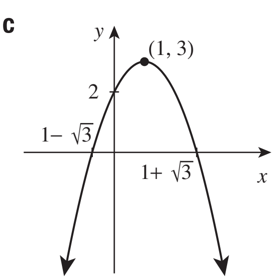

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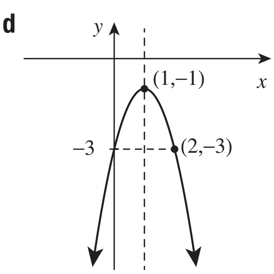

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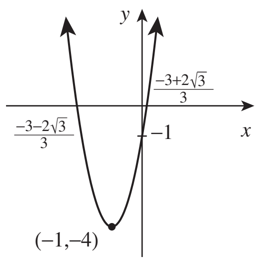

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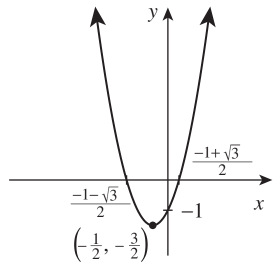
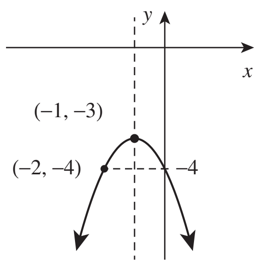


g



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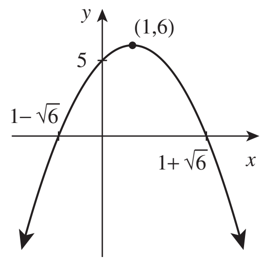

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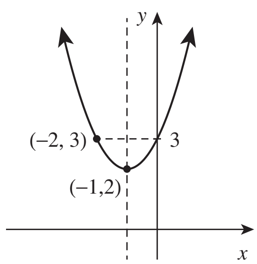


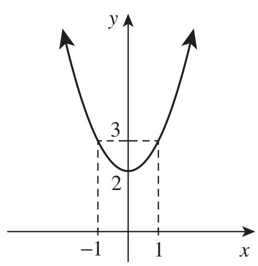
4 a $x = -1, 4$ **b** $x = 2, 3$ **c** $x = -2, 6$

5 a i $\Delta = -8 < 0$

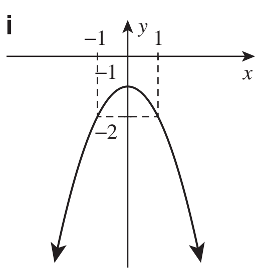
ii Both equal $(0, 2)$.

iii $(1, 3)$

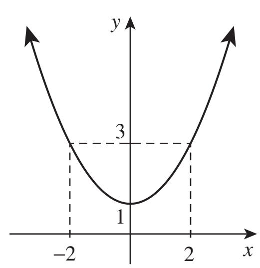
iv $(-1, 3)$



b i



ii



6 a $3 - 2\sqrt{2}, 3 + 2\sqrt{2}$

b $1 - \sqrt{5}, \ + \sqrt{5}$

c $\frac{5 - \sqrt{10}}{3}, \frac{5 + \sqrt{10}}{3}$

7 a $\Delta = 17$, two zeroes

b $\Delta = 0$, one zero

c $\Delta = -7$, no zeroes

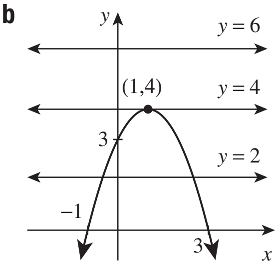
8 a i $x = 1 - \sqrt{2}$ and $x = 1 + \sqrt{2}$

ii $x = 1$

iii There are none.

c $k < 4$

b



9 a $f(x) = (x - 3 + \sqrt{5})(x - 3 - \sqrt{5})$

b $f(x) = (x + 1 + \sqrt{2})(x + 1 - \sqrt{2})$

c $f(x) = \left(x - \frac{3 - \sqrt{5}}{2}\right)\left(x - \frac{3 + \sqrt{5}}{2}\right)$

d $f(x) = 3\left(x + \frac{3 + \sqrt{3}}{3}\right)\left(x + \frac{3 - \sqrt{3}}{3}\right)$

e $f(x) = -\left(x - \frac{3 - \sqrt{13}}{2}\right)\left(x - \frac{3 + \sqrt{13}}{2}\right)$

f $f(x) = -2(x + 1)\left(x - \frac{1}{2}\right)$

10 b i axis: $x = -2$, vertex: $(-2, -3)$

ii axis: $x = 3$, vertex: $(3, 1)$

iii axis: $x = -2$, vertex: $(-2, 13)$

11 $\frac{1}{2}p\left(-1 + \sqrt{5}\right)$

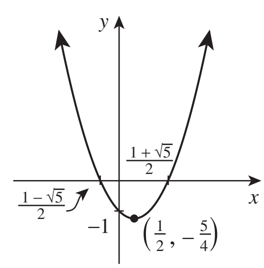
12 a $x = h + \sqrt{-k}$ or $h - \sqrt{-k}$

13 a $x = -\frac{b}{2}$, vertex $\left(-\frac{b}{2}, \frac{1}{4}(4c - b^2)\right)$

b Difference between zeroes is $\sqrt{b^2 - 4c}$.

c $b^2 - 4c = 1$

14

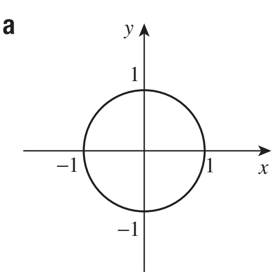


Exercise 3G

1 a $(0, 0)$, 4 units **b** $(0, 0)$, 7 units

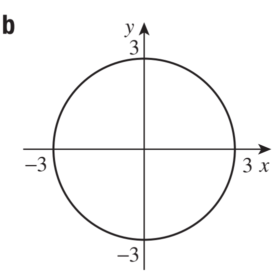
c $(0, 0)$, $\frac{1}{3}$ units **d** $(0, 0)$, 1.2 units

2 a



$-1 \le x \le 1$
$-1 \le y \le 1$

b



$-3 \le x \le 3$
$-3 \le y \le 3$

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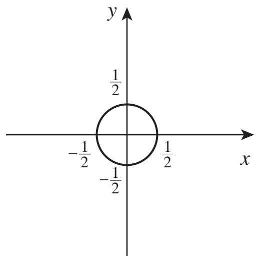
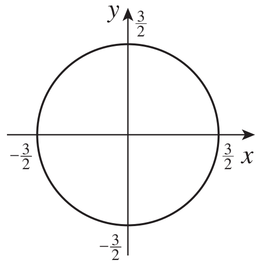
c



$-\frac{1}{2} \le x \le \frac{1}{2}$

$-\frac{1}{2} \le y \le \frac{1}{2}$

d

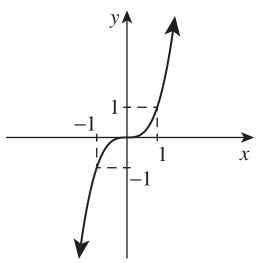


$-\frac{3}{2} \le x \le \frac{3}{2}$

$-\frac{3}{2} \le y \le \frac{3}{2}$

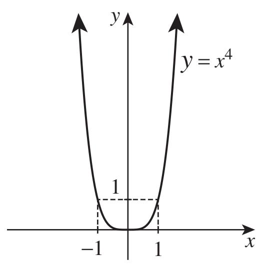
3 a $-3.375, -1, -0.125, 0, 0.125, 1, 3.375$

b

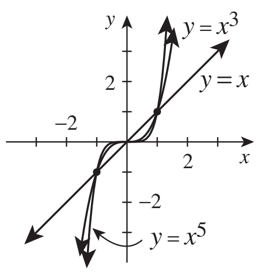


4 a $5.0625, 1, 0.0625, 0, 0.0625, 1, 5.0625$

b



5 a



b $(-1, -1)$, $(0, 0)$ and $(1, 1)$

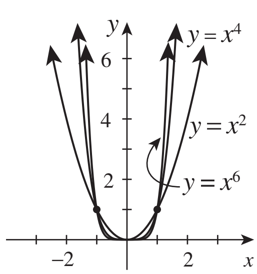
c i $y = x^5$ **ii** $y = x$

d i $y = x^5$ **ii** $y = x$

e In each case, the result is the same curve.

f Every index is odd.

6 a



b $(-1, 1)$, $(0, 0)$ and $(1, 1)$

c i $y = x^6$ **ii** $y = x^2$

d i $y = x^6$ **ii** $y = x^2$

e In each case, the result is the same curve.

f Every index is even.

7 a degree 1, coefficient 2

b degree 3, coefficient 0

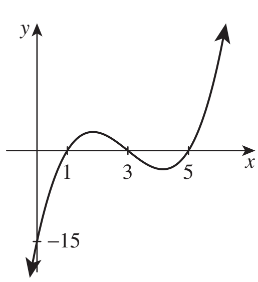
c not a polynomial

d not a polynomial

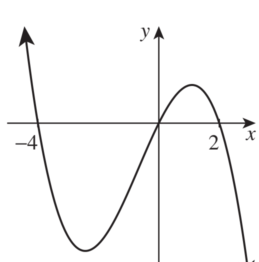
e degree 3, coefficient -1

f not a polynomial

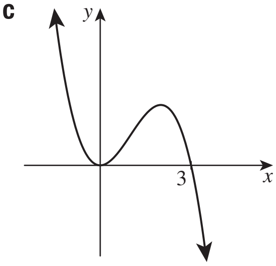
8 a



b



c



9 a $x^2 + y^2 = 4$

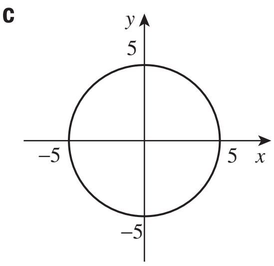
b $x^2 + y^2 = 5$

c $x^2 + y^2 = 25$

d $x^2 + y^2 = 10$

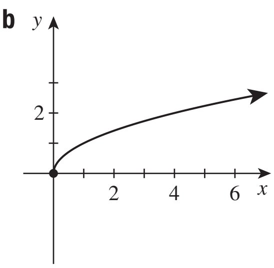
10 a 5 or -5, 4.9 or -4.9, 4.6 or -4.6, 4 or -4, 3 or -3, 0.

c

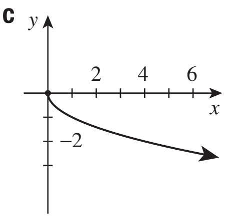


11 a 0, 0.5, 1, 1.5, 2, 2.5

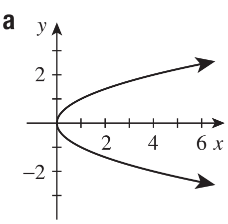
b



c



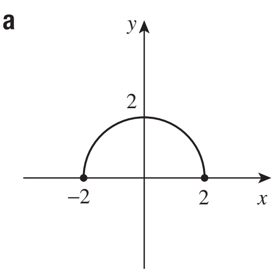
12 a



b It is a concave-right parabola.

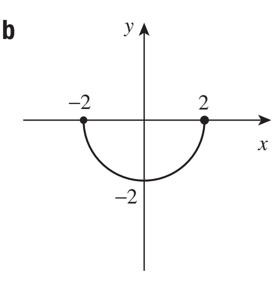
c In both cases, squaring gives $x = y^2$. This is the result of swapping x and y in $y = x^2$.

13 a



domain: $-2 \le x \le 2$,
range: $0 \le y \le 2$

b



domain: $-2 \le x \le 2$,
range: $-2 \le y \le 0$

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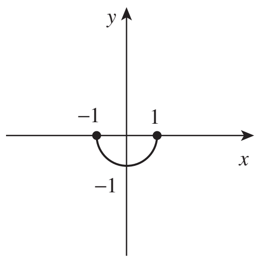
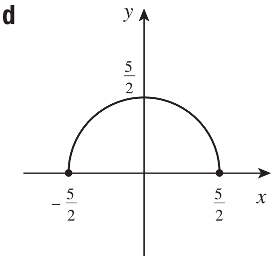
c



domain: $-1 \le x \le 1$,

range: $-1 \le y \le 0$

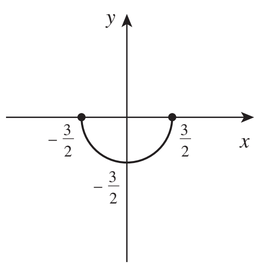
d



domain: $-\frac{5}{2} \le x \le \frac{5}{2}$,

range: $0 \le y \le \frac{5}{2}$

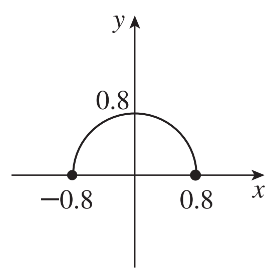
e



domain: $-\frac{3}{2} \le x \le \frac{3}{2}$,

range: $-\frac{3}{2} \le y \le 0$

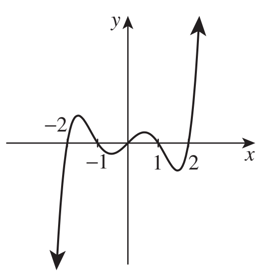
f



domain: $-0.8 \le x \le 0.8$,

range: $0 \le y \le 0.8$

14 a



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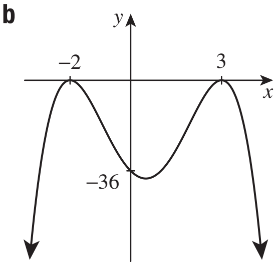

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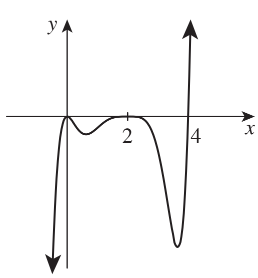
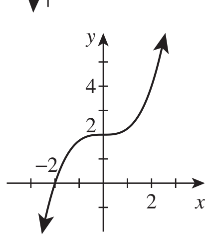


15 a i



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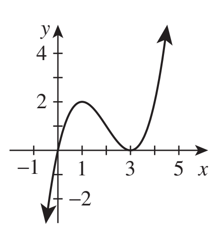
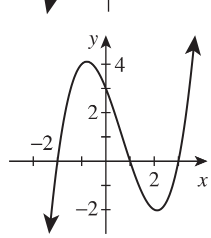


iii



d The product of the zeroes is $-\dfrac{d}{a}$.

16 a $y = -3(x + 1)(x - 1)(x - 4)$

b $y = -(x + 1)^2(x - 1)^3(x - 3)^2$

17 a $r = \sqrt{5}, (2, 1), (1, 2), (-1, 2), (-2, 1), (-2, -1), (-1, -2), (1, -2), (2, -1)$

b $r = \sqrt{2}, (1, -1), (-1, -1)$

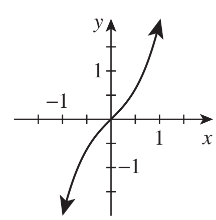
c $r = \sqrt{10}, (3, 1), (1, 3), (1, -3), (3, -1)$

d $r = \sqrt{17}, (4, 1), (1, 4), (-1, 4), (-4, 1), (-4, -1), (-1, -4), (1, -4), (4, -1)$

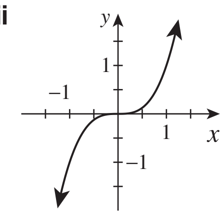
18 a $\left(0, 2\sqrt{\lambda^2 - \alpha^2}\right)$ **b** $r = \lambda$

c Lie the ladder on the ground and the midpoint is λ from the wall.

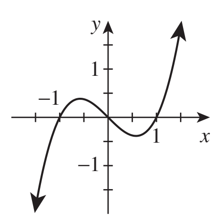
19 a i



ii



iii



b i 1st and 3rd

ii In each case, the result is the same curve.

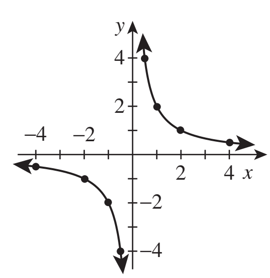
iii Every index is odd.

c The slope:
$x^3 + x$ is upwards,
x^3 is horizontal,
$x^3 - x$ is downwards.

Exercise 3H

1 a i $-\frac{1}{2}, -1, -2,$
$-4, 4, 2, 1, \frac{1}{2}$

ii



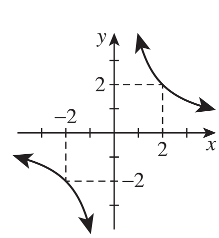
iii 1st and 3rd

iv the x-axis ($y = 0$) and the y-axis ($x = 0$)

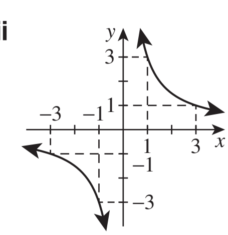
v domain: $x \neq 0$, range: $y \neq 0$

b In each case, the domain is $x \neq 0$, the range is $y \neq 0$. The asymptotes are $y = 0$ and $x = 0$. The branches are in quadrants 1 and 3.

i



ii



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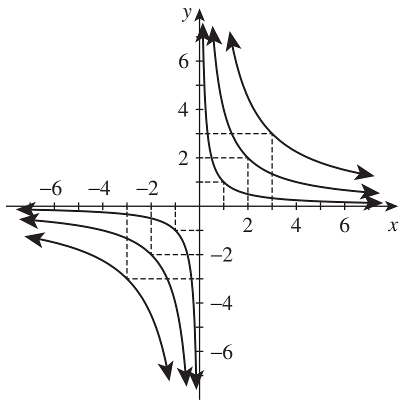
2



a 1st and 3rd

b the x-axis ($y = 0$) and the y-axis ($x = 0$)

c $x \neq 0, y \neq 0$

d $(1, 1)$ and $(-1, -1)$ on $y = \dfrac{1}{x}$

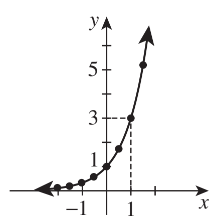
$(2, 2)$ and $(-2, -2)$ on $y = \dfrac{4}{x}$

$(3, 3)$ and $(-3, -3)$ on $y = \dfrac{9}{x}$

The values are the square roots of the numerator.

3 a i 0.1, 0.2, 0.3, 0.6, 1, 1.7, 3, 5.2, 9

ii



iii $(0, 1)$

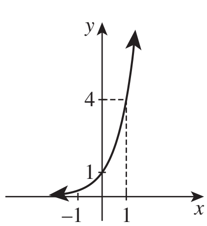
iv 3, the base

v the x-axis ($y = 0$)

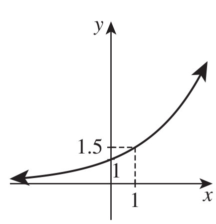
vi domain: all real x, range: $y > 0$

b In each case, the domain is all real x, the range is $y > 0$. The asymptote is $y = 0$. The y-intercept is $(0, 1)$. At $x = 1$, $y =$ the base.

i



ii



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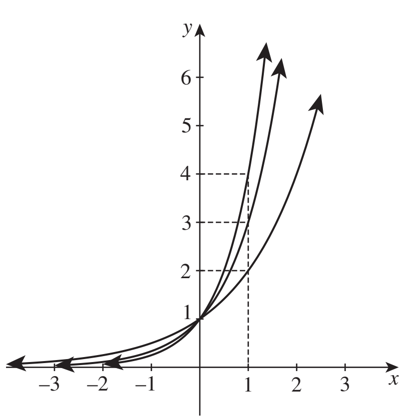


a $(0, 1)$

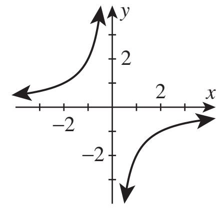
b the x-axis ($y = 0$)

c all real x, $y > 0$

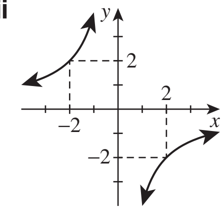
e $y = 4^x$, it has the greater base.

f $y = 4^x$ again, it has the greater base.

5 a i



ii



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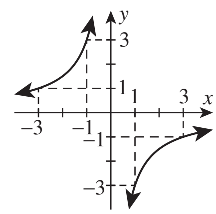
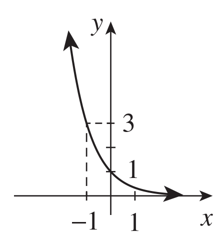


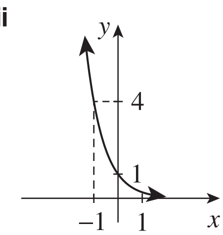
b i quadrants 2 and 4

ii The minus sign has caused the quadrants to change.

6 a i



ii



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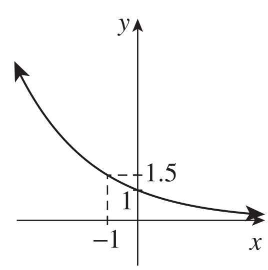


b i No: it is $(0, 1)$.

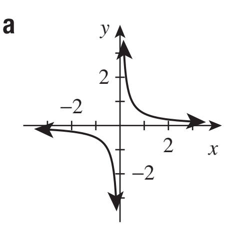
ii No: it is the x-axis.

iii $x = -1$

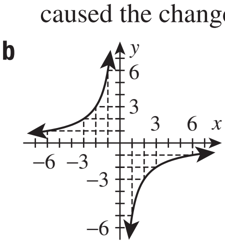
iv In Questions 4 and 5, the y-values grow. In these questions they decay away.

v The minus sign has caused the changes.

7 a



b



8 a i $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

ii There are no points with integer coordinates.

b i $\left(-\sqrt{6}, \sqrt{6}\right)$ and $\left(\sqrt{6}, -\sqrt{6}\right)$

ii $(-6, 1)$, $(-3, 2)$, $(-2, 3)$, $(-1, 6)$, $(1, -6)$, $(2, -3)$, $(3, -2)$, $(6, -1)$

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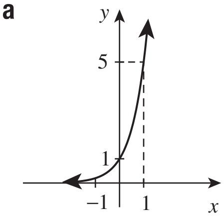
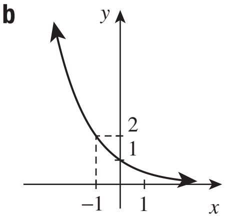
9 a



b

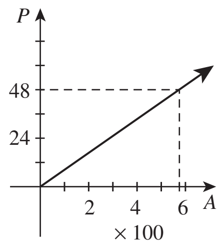


10 a $P = kA$

b $k = \frac{1}{12}$

c $55\frac{2}{3}$ L

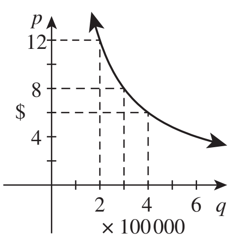
d 1 bucket, 4 tins



11 a $T = 2400000$

b 300 000

c Sales will halve.



12 a $y \to 0$ as $x \to -\infty$.

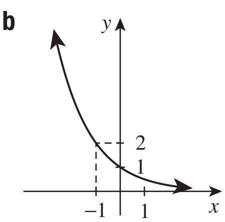
b $y \to 0$ as $x \to \infty$.

c $y \to 0$ as $x \to \infty$ and as $x \to -\infty$,
$y \to \infty$ as $x \to 0$, $y \to -\infty$ as $x \to 0^-$.

13 a $\left(\frac{1}{2}\right)^x = (2^{-1})^x$

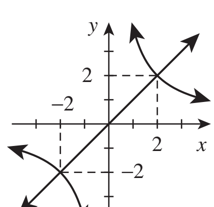
so $\left(\frac{1}{2}\right)^x = 2^{-x}$

b



14 a (c, c) and $(-c, -c)$

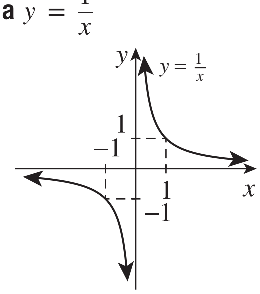
b



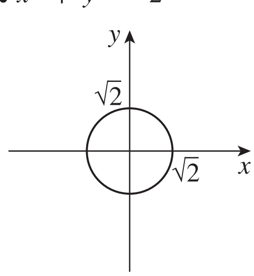
15 4 m × 12 m or 6 m × 8 m

16 No, because the only points that satisfy the equation lie on the x and y axes. The equation represents the two coordinate axes.

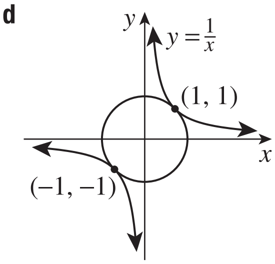
17 a $y = \dfrac{1}{x}$



b $x^2 + y^2 = 2$

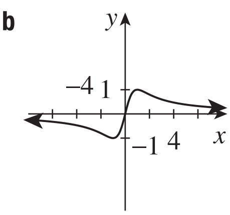


d



18 a $-\frac{16}{65}, -\frac{8}{17}, -\frac{4}{5},$
$-1, -\frac{4}{5}, 0, \frac{4}{5}, 1, \frac{4}{5}, \frac{8}{17}, \frac{16}{65}$

b



c x-axis ($y = 0$)

d (0, 0)

19 a $P\left(\dfrac{2}{b}, \dfrac{b}{2}\right)$

c 2 units^2

20 a $a = \frac{1}{4}, b = \frac{3}{4}, c = 1$

b $\sqrt{2} \doteqdot \frac{23}{16}, \frac{1}{\sqrt{2}} \doteqdot \frac{11}{16}$

Exercise 3I

1 a Vertical line test: Yes. It is a function.

b Horizontal line test: No. Many-to-one

c 10:00 pm on Saturday to 10:00 pm on Sunday

d 3 ft and 4 ft

e i 10:00 pm, 6:00 am, 10:30 am and 3:30 pm

ii 11:00 pm, 4:45 am and 1:00 pm

iii Never

f 0, 1, 2, 3 and 4

2 a It passes the vertical line test, so it is a function. Also, it fails the horizontal line test, so it is many-to-one.

b 1°C

c It was never 20°C. It was 8°C at 1:00 am, 8:00 am and 10:30 pm on the first day, and at about 3:30 pm on the second day.

d 0, 2, 3, 4, 5 (Whether 1 is omitted depends on how accurately you are supposed to read the graph.)

3 a and **e i** Vertical line test: No. Horizontal line test: Yes. One-to-many

ii Vertical line test: No. Horizontal line test: No. Many-to-many

iii Vertical line test: Yes. Horizontal line test: No. Many-to-one

iv Vertical line test: No. Horizontal line test: No. Many-to-many

v Vertical line test: Yes. Horizontal line test: Yes. One-to-one

vi Vertical line test: No. Horizontal line test: Yes. One-to-many

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b parts **iii**, **v** **c** parts **i**, **v**, **vi** **d** part **v**

4 a one-to-many **b** many-to-many
c one-to-many **d** many-to-one
e one-to-one **f** many-to-many

5 a i When $y = 0, x = 2$ or -2
ii When $y = 3, x = 0$ or 6
iii When $y = 0, x = 1$ or 0 or -1
iv When $y = 2, x = 1$ or -1
b iv They are all one-to-many, because x and y are reversed.

6 a i $x = \frac{1}{3}y + \frac{1}{3}$ **ii** $x = -\frac{1}{2}y + \frac{5}{2}$
iii $x = \frac{1}{2}\sqrt[3]{y}$ **iv** $x = \dfrac{5}{y}$
b iv They are all one-to-one also, because x and y are reversed.

7 a When $x = 3, y = 4$ or -6. When $y = -1$, $x = 8$ or -2
b When $x = 0, y = 3$ or -3. When $y = 0$, $x = 2$ or -2
c When $x = 2, y = \sqrt{3}$ or $-\sqrt{3}$. When $y = 0$, $x = 1$ or -1

8 a It passes neither test, and is thus many-to-many.
b Vertical line test: Yes. Horizontal line test: No. It is many-to-one, and therefore a function.

9 a It is a function, but it may be one-to-one or many-to-one.
b If there are two or more students with the same preferred name, it is many-to-one. Otherwise it is one-to-one.

10 a $\ldots, -270°, 90°, 450°, \ldots$
b one-to-many **c** many-to-one

11 a Probably many-to-many, but just possibly one-to-one.
b The condition to be one-to-one is that every flat has no more than one occupant, and in this case, every inhabitant is mapped to himself or herself, that is, $f(x) = x$, for every inhabitant x. Otherwise the relation is many-to-many.
c The relation is then the *empty relation*, which is discussed later in Section 4E. This empty relation is a one-to-one function, because it trivially passes the vertical and horizontal line tests.

12 a many-to-one **b** one-to-many **c** one-to-one
d one-to-one (trivially because the graph has only one point)
e many-to-many
f one-to-many (factor as $x = (y-2)(y-3)$
g many-to-one (factor as $y = x(x-3)(x-4)$
h one-to-one **i** one-to-one
j one-to-one **k** many-to-many
l one-to-one

13 a $f(a) = f(b)$ because $g(x)$ is one-to-one. Hence $a = b$ because $f(x)$ is one-to-one.
b The composition of two one-to-one functions is one-to-one.

14 a One-to-one. Every even integer n is $f\left(\frac{1}{2}n\right)$, and is not the image of any other number.
Odd integers are not the image of anything.
Every other real number is only the image of itself.
b Many-to-one, $f(3) = 1\frac{1}{2} = f\left(1\frac{1}{2}\right)$.
c One-to-one. Every rational number x is cubed, and x^3 is again a rational number, and the cubes of two distinct numbers are never equal.
d Many-to-one, $f\left(\sqrt[3]{2}\right) = 2 = f(2)$.

Chapter 3 review exercise

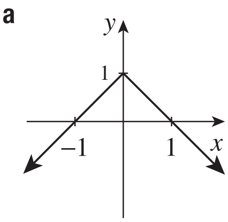
1 a not a function **b** function
c function **d** not a function

2 a $-2 \le x \le 0, -2 \le y \le 2$
b all real x, all real y
c $x \ne 0, y \ne 0$ **d** $x = 2$, all real y

3 a $21, -4$ **b** $5, -15$

4 a $x \ne 2$ **b** $x \ge 1$
c $x \ge -\dfrac{2}{3}$ **d** $x < 2$

5 a $2a + 2, 2a + 1$
b $a^2 - 3a - 8, a^2 - 5a - 3$

6 a


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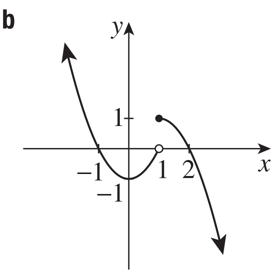
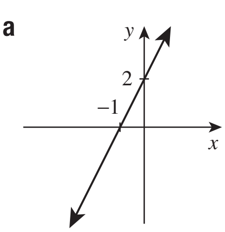


7 a


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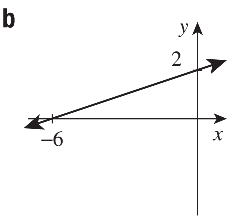

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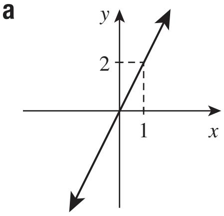
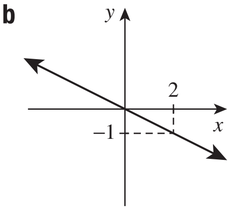
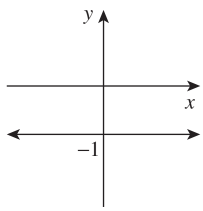
8 a



b



9 a i



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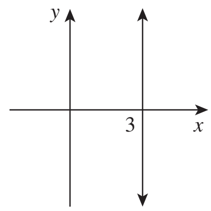
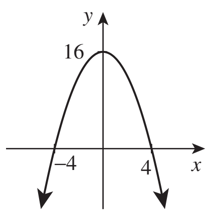


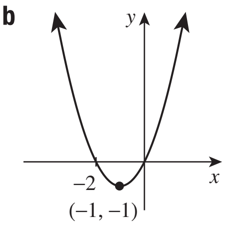
b $(3, -1)$

10 a



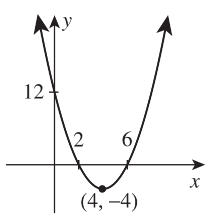
domain: all real x,
range: $y \le 16$

b



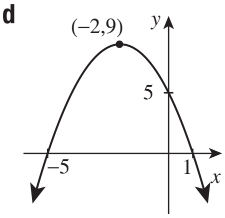
domain: all real x,
range: $y \ge -1$

c



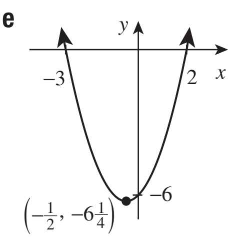
domain: all real x,
range: $y \ge -4$

d



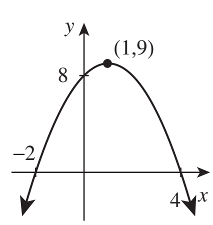
domain: all real x,
range: $y \le 9$

e



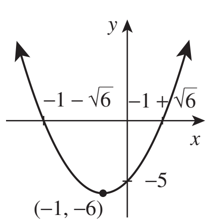
domain: all real x,
range: $y \ge -6\frac{1}{4}$

f



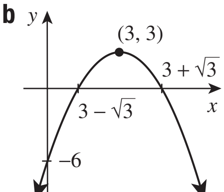
domain: all real x,
range: $y \le 9$

11 a



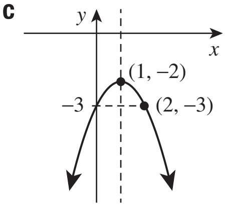
$y = (x + 1)^2 - 6$

b



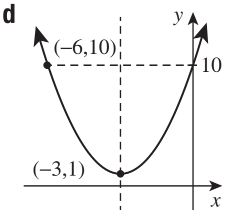
$y = -(x - 3)^2 + 3$

c



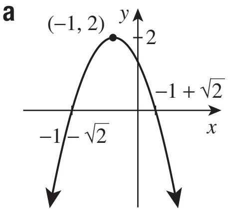
$y = -(x - 1)^2 - 2$

d



$y = (x + 3)^2 + 1$

12 a



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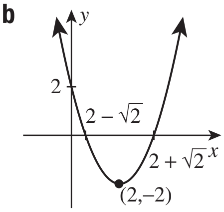

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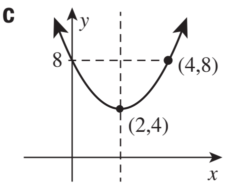

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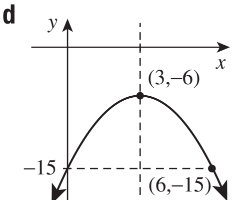
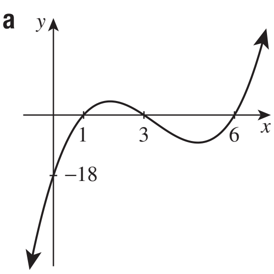


13 a



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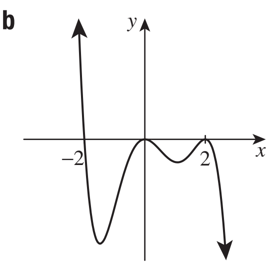
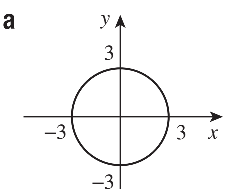


14 a



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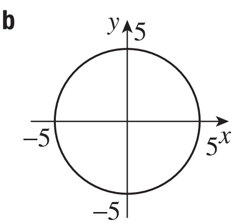
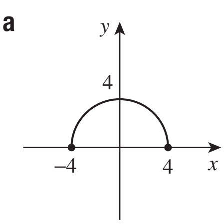
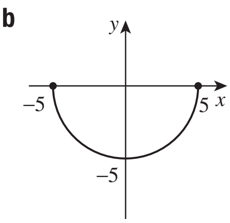


15 a



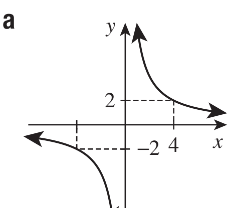
domain: $-4 \le x \le 4$,
range: $0 \le y \le 4$

b



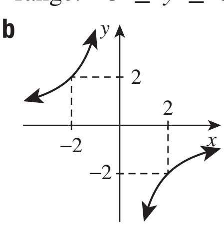
domain: $-5 \le x \le 5$,
range: $-5 \le y \le 0$

16 a



domain: $x \ne 0$,
range: $y \ne 0$

b



domain: $x \ne 0$,
range: $y \ne 0$

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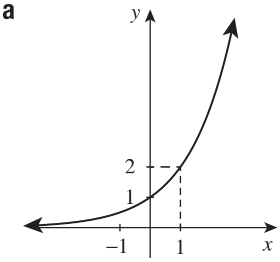
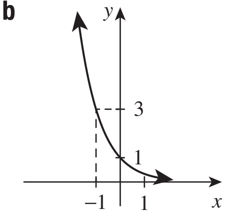
17 a



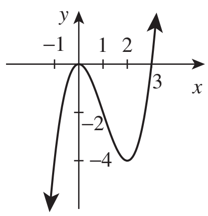
domain: all real x,
range: $y > 0$

b



domain: all real x,
range: $y > 0$

18 a



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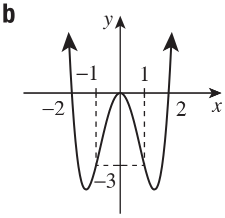

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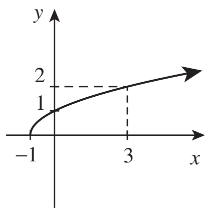
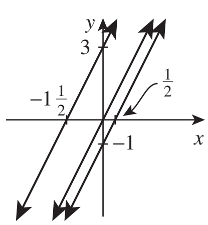


19 a



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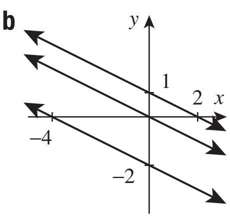

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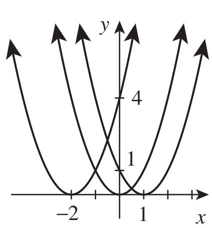

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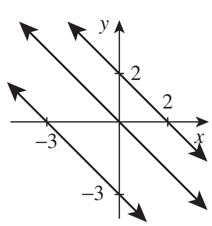

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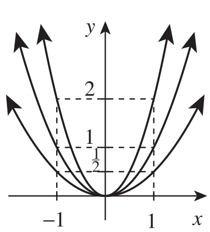

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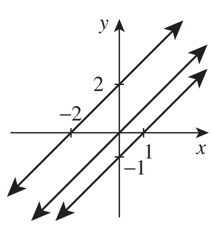

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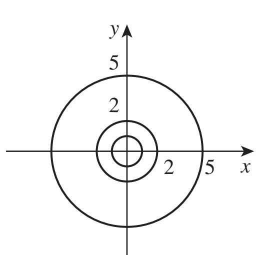

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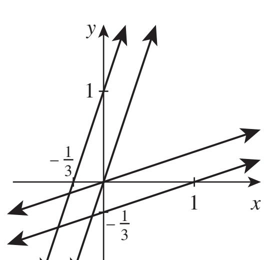

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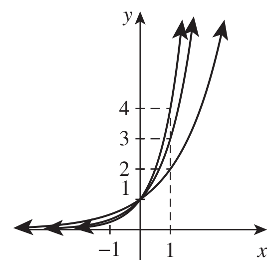

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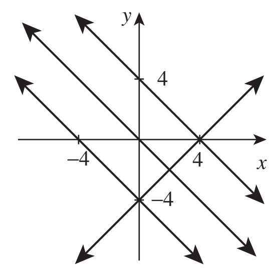

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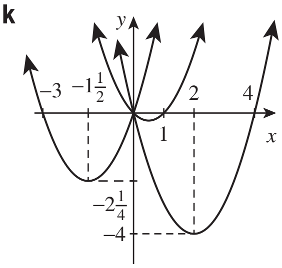
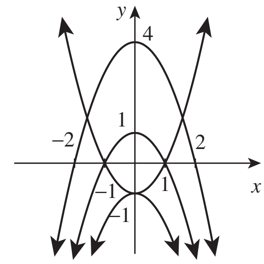
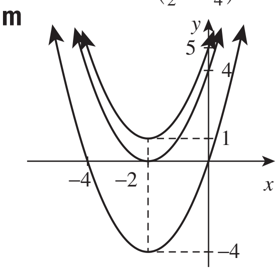


Top vertex: $\left(\frac{1}{2}, -\frac{1}{4}\right)$

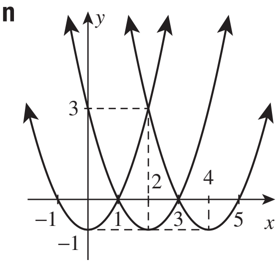
l



m



n



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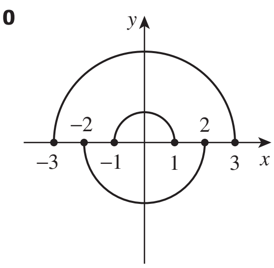

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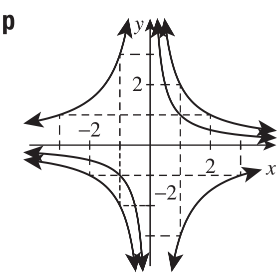

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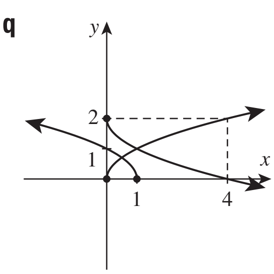

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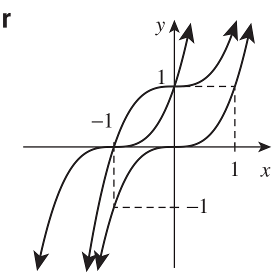

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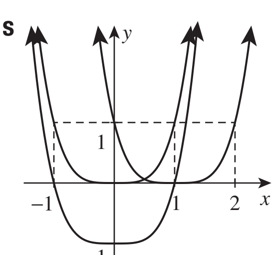

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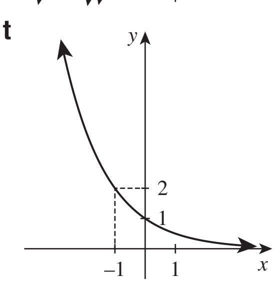

All three are the same.

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20 a one-to-one **b** many-to-many

c one-to-many **d** many-to-one

21 a It is probably a many-to-one function, but it is possibly a one-to-one function

b If every person was born in a different country, the function is one-to-one. Otherwise it is many-to-one.

Chapter 4

Exercise 4A

1 a x^2: 4, 1, 0, 1, 4, 9

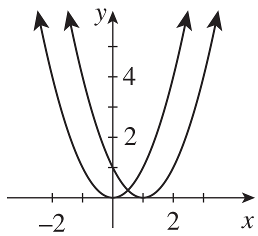
$(x-1)^2$: 9, 4, 1, 0, 1, 4

b $y = x^2$, $V = (0, 0)$.

$y = (x-1)^2$, $V = (1, 0)$.

c Here x is replaced by $(x-1)$, so it is a shift right by 1 unit.

b



2 a $\frac{1}{4}x^3$: $-6\frac{3}{4}$, -2, $-\frac{1}{4}$, 0, $\frac{1}{4}$, 2, $6\frac{3}{4}$

$\left(\frac{1}{4}x^3 + 2\right)$: $-4\frac{3}{4}$, 0, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, 4, $8\frac{3}{4}$

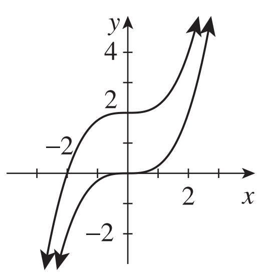
b $(0, 0)$ and $(0, 2)$

c The second equation is also

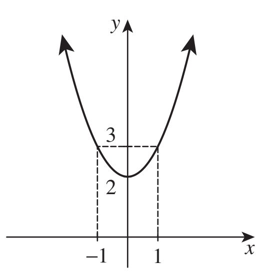
$y - 2 = \frac{1}{4}x^3$.

Here y is replaced by $(y-2)$, so it is a shift up by 2 units.

b

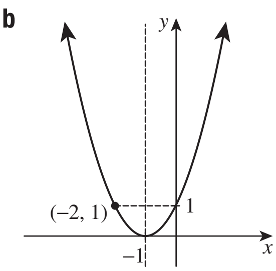


3 a



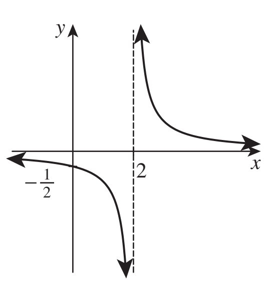
up 2 units

b



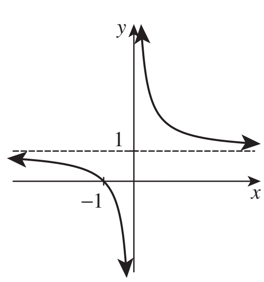
left 1 unit

c



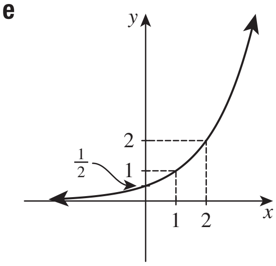
right 2 units

d



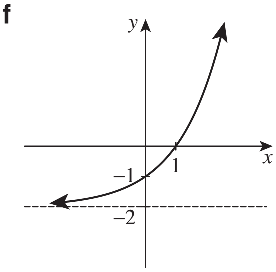
up 1 unit

e



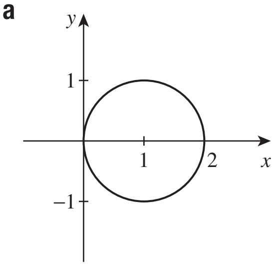
right 1 unit

f

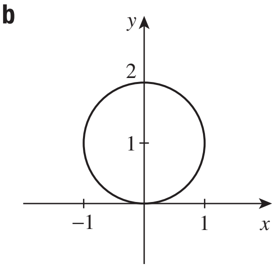


down 2 units

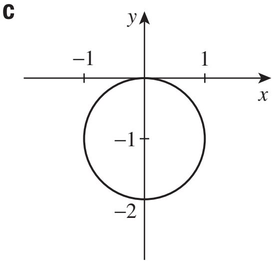
4 a



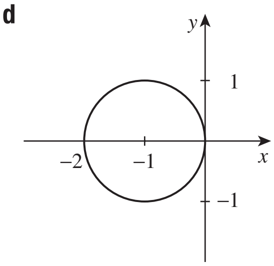
b



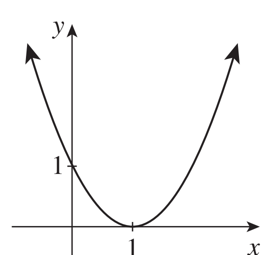
c



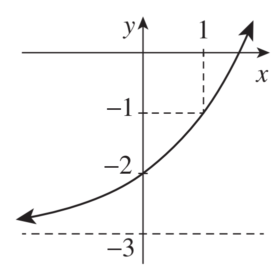
d



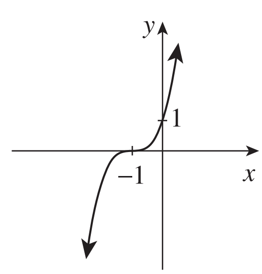
5 a $y = (x-1)^2$



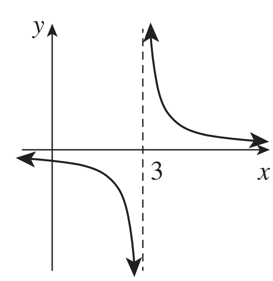
b $y = 2^x - 3$



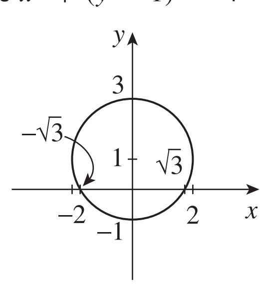
c $y = (x+1)^3$



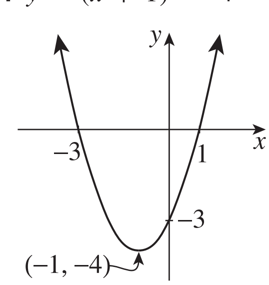
d $y = \dfrac{1}{x-3}$



e $x^2 + (y-1)^2 = 4$



f $y = (x+1)^2 - 4$



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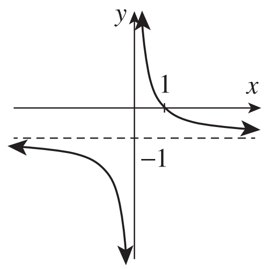
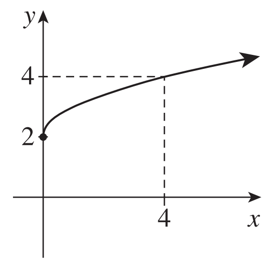
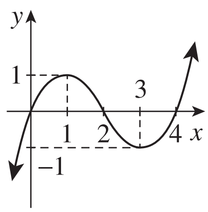
g $x(y + 1) = 1$



h $y = \sqrt{x} + 2$



6 a i



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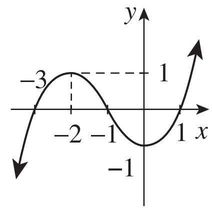
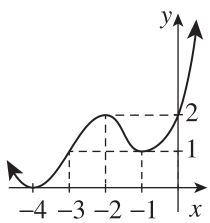


b i



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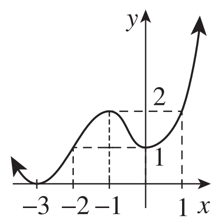
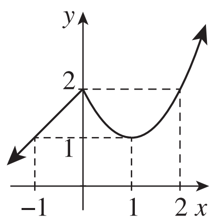


c i



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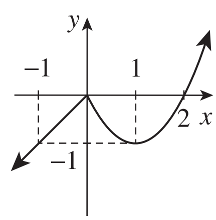
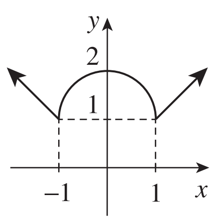


d i



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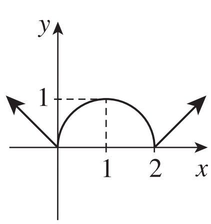


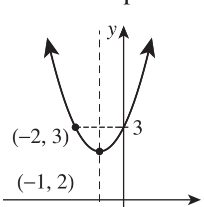
7 a $r = 2,\ (-1, 0)$ **b** $r = 1,\ (1, 2)$

c $r = 3,\ (1, 2)$ **d** $r = 5,\ (-3, 4)$

e $r = 3,\ (5, -4)$ **f** $r = 6,\ (-7, 1)$

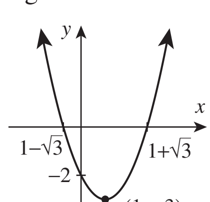
8 a $y = (x + 1)^2 + 2$

This is $y = x^2$ shifted left 1 and up 2.



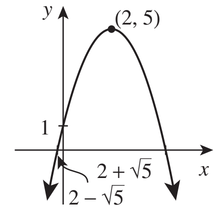
b $y = (x - 1)^2 - 3$

This is $y = x^2$ shifted right 1 and down 3.



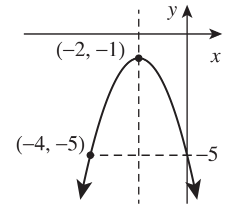
c $y = -(x - 2)^2 + 5$

This is $y = -x^2$ shifted right 2 and up 5.



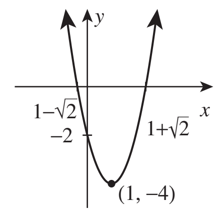
d $y = -(x + 2)^2 - 1$

This is $y = -x^2$ shifted left 2 and down 1.



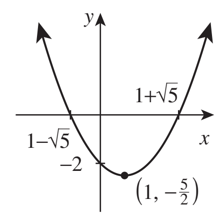
e $y = 2(x - 1)^2 - 4$

This is $y = 2x^2$ shifted right 1 and down 4.



f $y = \frac{1}{2}(x - 1)^2 - \frac{5}{2}$

This is $y = \frac{1}{2}x^2$ shifted right 1 and down $\frac{5}{2}$.



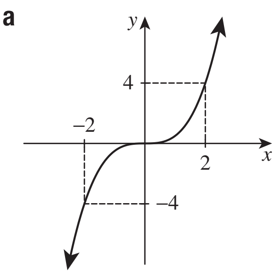
9 a the circle $x^2 + y^2 = 1$ translated right 2, up 3, $(x - 2)^2 + (y - 3)^2 = 1$

b the circle $x^2 + y^2 = 4$ translated left 2, down 1, $(x + 2)^2 + (y + 1)^2 = 4$

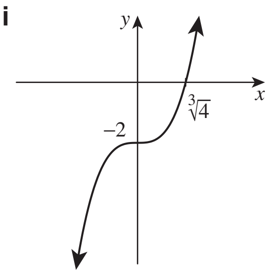
c the circle $x^2 + y^2 = 10$ translated left 1, up 1, $(x + 1)^2 + (y - 1)^2 = 10$

d the circle $x^2 + y^2 = 5$ translated right 2, down 1, $(x - 2)^2 + (y + 1)^2 = 5$

10 a



i



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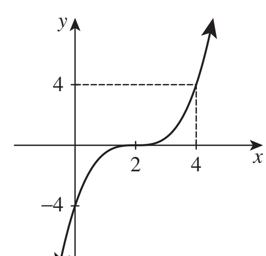

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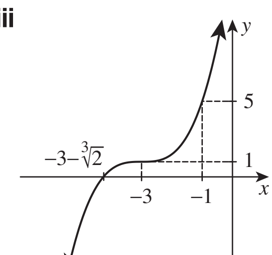

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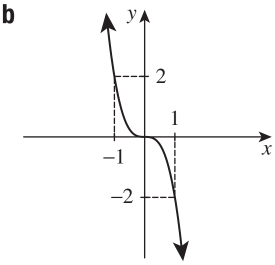
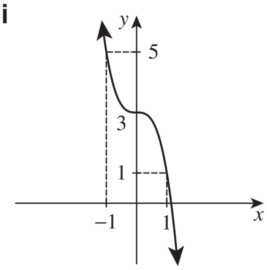
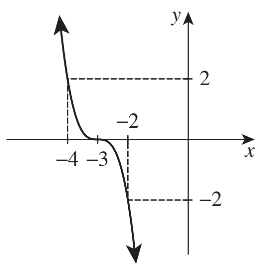
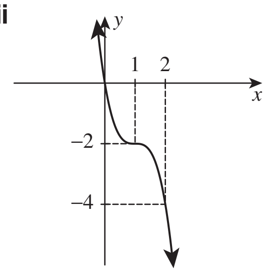
b 

i 

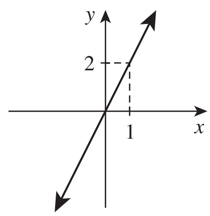
ii 

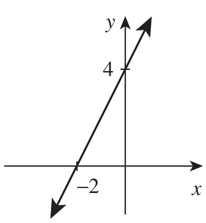
iii 

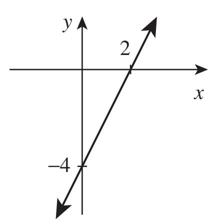
11 a From $y = 2x$:

i shift up 4 (or left 2)

ii shift down 4 (or right 2)



i 

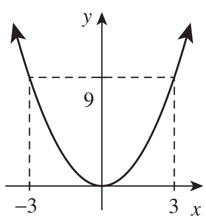
ii 

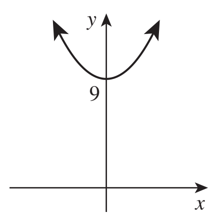
b From $y = x^2$:

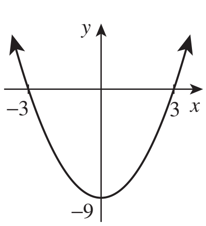
i shift up 9

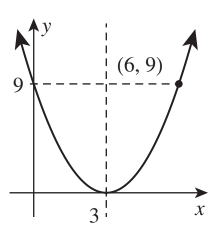
ii shift down 9

iii shift right 3



i 

ii 

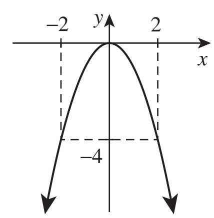
iii 

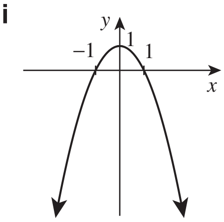
c From $y = -x^2$:

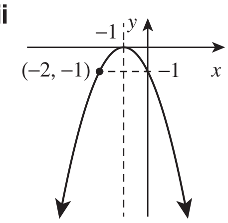
i shift up 1

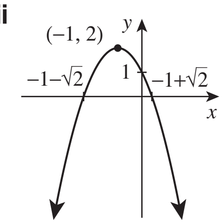
ii shift left 1

iii shift left 1 and up 2



i 

ii 

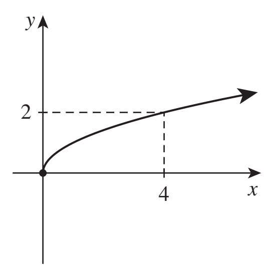
iii 

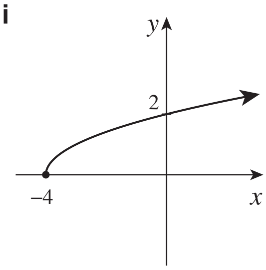
d From $y = \sqrt{x}$:

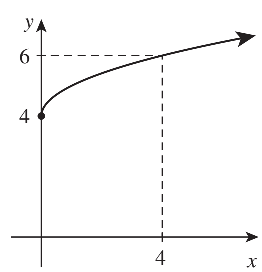
i shift left 4

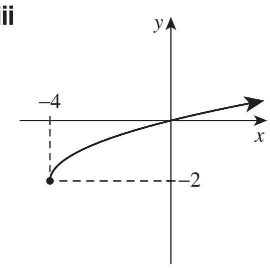
ii shift up 4

iii shift left 4 and down 2



i 

ii 

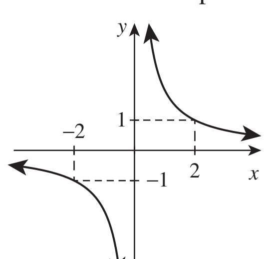
iii 

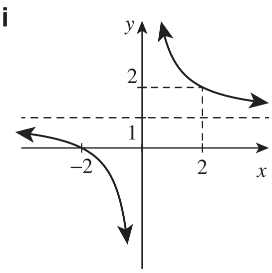
e From $y = \frac{2}{x}$:

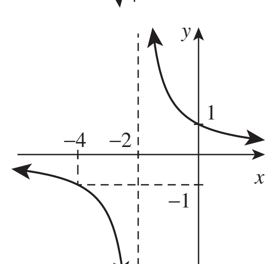
i shift up 1

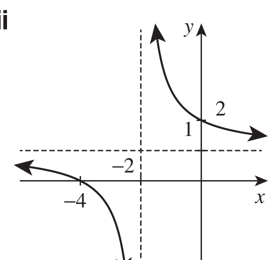
ii shift left 2

iii shift left 2 and up 1



i 

ii 

iii 

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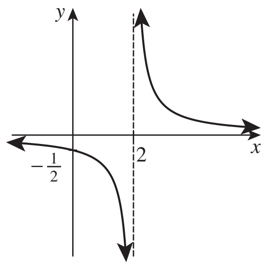
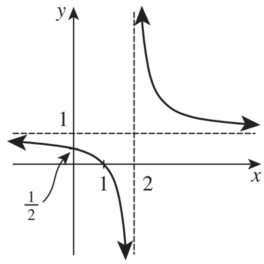
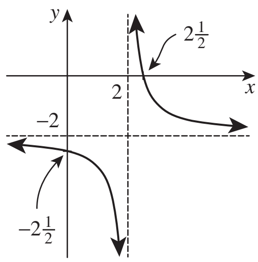
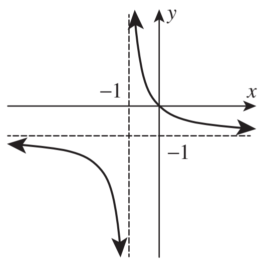
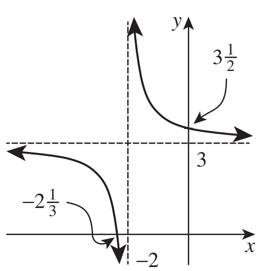
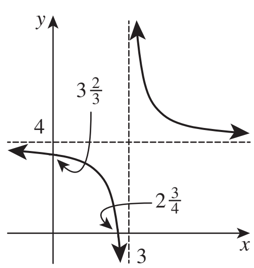
12 a 

b 

c 

d 

e 

f 

13 a the parabola $y = x^2$ translated right 2, $y = (x - 2)^2$

b the hyperbola $xy = 1$ translated right 2, $y = \dfrac{1}{x - 2}$

c the exponential $y = 3^x$ translated left 1, $y = 3^{x+1}$

d the circle $x^2 + y^2 = 4$ translated left 2, down 1, $(x + 2)^2 + (y + 1)^2 = 4$

e the hyperbola $xy = 1$ translated right 2, down 1, $y + 1 = \dfrac{1}{x - 2}$

f the parabola $y = x^2$ translated right 2, down 1, $y + 1 = (x - 2)^2$

g the circle $x^2 + y^2 = 5$ translated right 2, down 1, $(x - 2)^2 + (y + 1)^2 = 5$

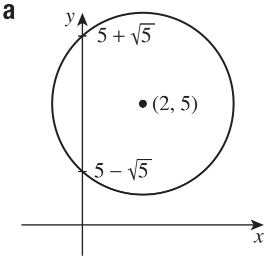
h the exponential $y = 4^x$ translated down 4, $y = 4^x - 4$

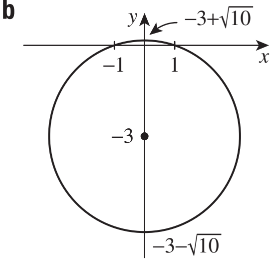
14 a $(x - 2)^2 + (y - 5)^2 = 9$, $r = 3$, centre $(2, 5)$, intercepts $(0, -\sqrt{5})$, $(0, \sqrt{5})$

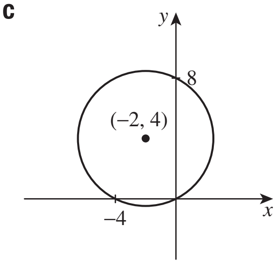
b $x^2 + (y + 3)^2 = 10$, $r = \sqrt{10}$, centre $(0, -3)$, intercepts $(0, -3-\sqrt{10})$, $(0, -3+\sqrt{10})$, $(-1, 0)$, $(1, 0)$

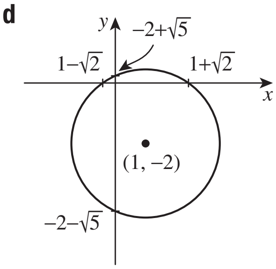
c $(x + 2)^2 + (y - 4)^2 = 20$, with $r = 2\sqrt{5}$, and centre $(-2, 4)$, intercepts, $(0, 0)$, $(0, 8)$, $(-4, 0)$

d $(x - 1)^2 + (y + 2)^2 = 6$, $r = \sqrt{6}$, centre $(1, -2)$, intercepts $(0, -2-\sqrt{5})$, $(0, -2 + \sqrt{5})$, $(1-\sqrt{2}, 0)$, $(1+\sqrt{2}, 0)$

a 

b 

c 

d 

15 a $x + 2y - 2 = 0$ **b** $x + 2y - 2 = 0$

c Both translations yield the same result.

16 a $(x - h)^2 + (y - k)^2 = r^2$

17 a $y - y_1 = m(x - x_1)$ is the line $y = mx$ shifted right by x_1 and up by y_1.

b Because only shifts are involved, the lines in part **a** are parallel. Thus parallel lines have the same gradient m.

18 a $y = f(x - a)$, $y - b = f(x - a)$

b $y - b = f(x)$, $y - b = f(x - a)$

c The final transformed function is the same in both cases. Thus, the order of shifts is irrelevant.

19 a $(1, 2)$, $(1, 4)$, $(-3, 2)$, $(-3, 4)$

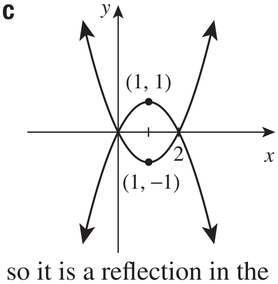
b It is a 4×2 rectangle.

c $C = (-1, 3)$, $r = \sqrt{5}$

d $(x + 1)^2 + (y - 3)^2 = 5$

Exercise 4B

1 b $y = x^2 - 2x$:
8, 3, 0, −1, 0, 3, 8
$y = 2x - x^2$:
−8, −3, 0, 1, 0, −3, −8
$y = x^2 - 2x$:
$V = (1, -1)$.
$y = 2x - x^2$: $V = (1, 1)$.

c 

d Here y is replaced with $-y$, so it is a reflection in the x-axis.

2 b $y = \dfrac{2}{x - 2}$: $-\frac{1}{3}$, $-\frac{2}{5}$, $-\frac{1}{2}$, $-\frac{2}{3}$, -1, -2, *, 2, 1

$y = -\dfrac{2}{x + 2}$: 1, 2, *, −2, −1, $-\frac{2}{3}$, $-\frac{1}{2}$, $-\frac{2}{5}$, $-\frac{1}{3}$

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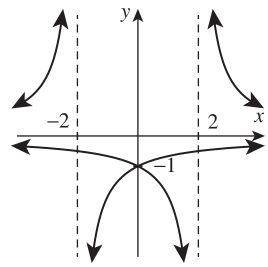
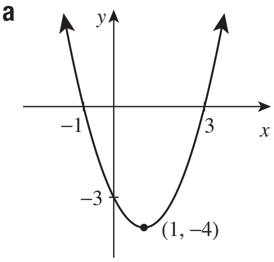
c $y = \dfrac{2}{x-2}$: $x = 2$.

$y = -\dfrac{2}{x+2}$: $x = -2$.

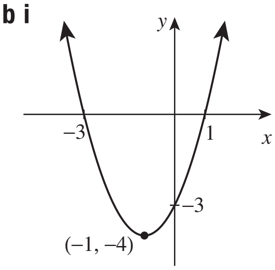
d Here x is replaced with $-x$, so it is a reflection in the y-axis.



3 a

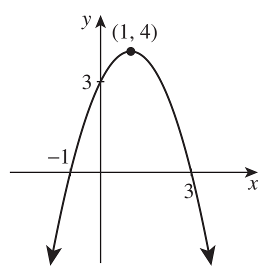


b i



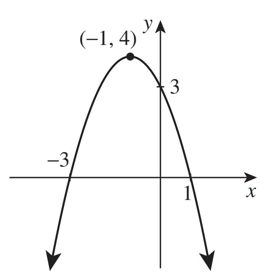
$y = x^2 + 2x - 3$

ii



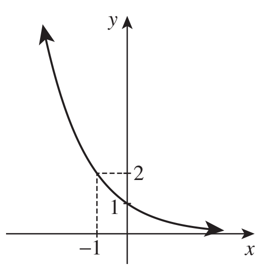
$y = -x^2 + 2x + 3$

iii

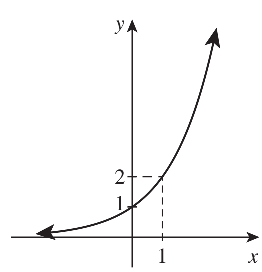


$y = -x^2 - 2x + 3$

4 a

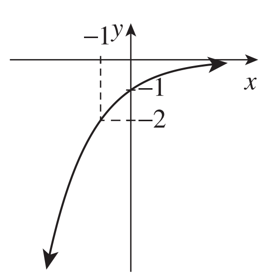


b i



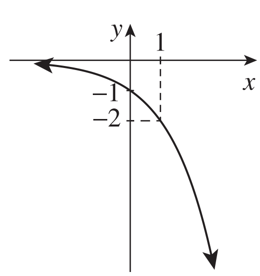
$y = 2^x$

ii



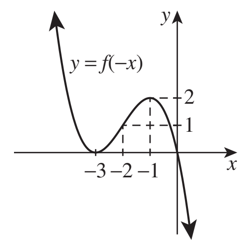
$y = -2^{-x}$

iii



$y = -2^x$

5 a i



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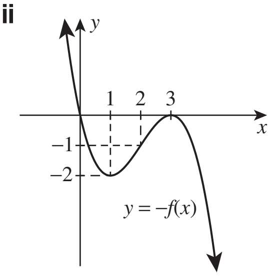

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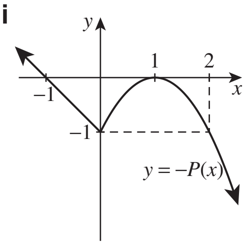

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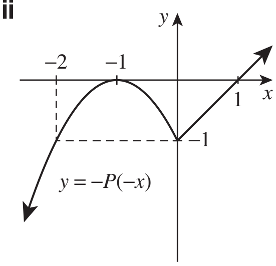
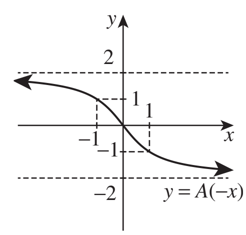


c i



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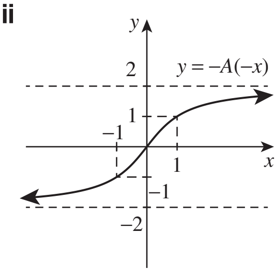
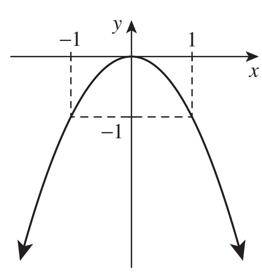
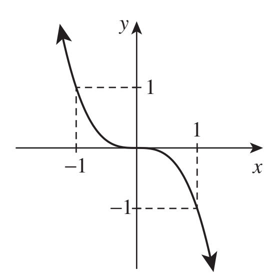


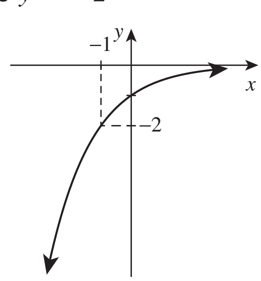
6 a $y = -x^2$



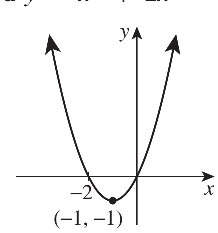
b $y = -x^3$



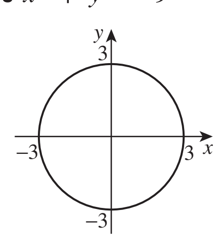
c $y = -2^{-x}$



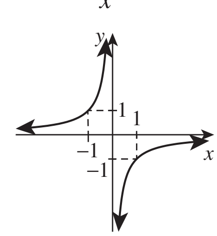
d $y = x^2 + 2x$



e $x^2 + y^2 = 9$



f $y = -\dfrac{1}{x}$



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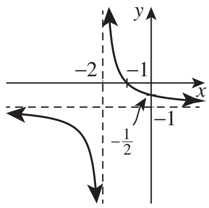
7 a



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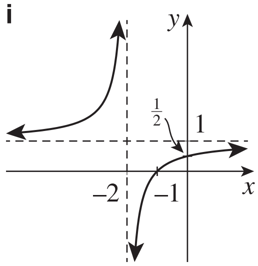
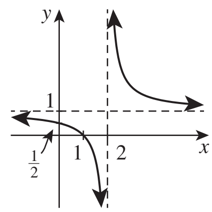


Reflect in the x-axis:

$y = 1 - \dfrac{1}{x + 2}$.

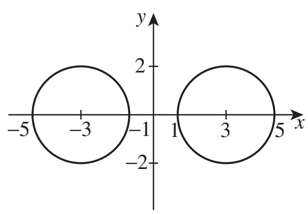
ii



Rotate by 180°:

$y = 1 - \dfrac{1}{2 - x}$.

8 a



b Reflect in the y-axis.

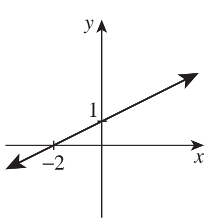
c You will need to use $(-x - 3)^2 = (x + 3)^2$.

d Shift left by 6 units.

e Replace x with $(x + 6)$.

9 a The circle is symmetric in both axes.

10 a From $y = \frac{1}{2}x + 1$:
i reflect in the y-axis
ii reflect in the x-axis
iii rotate by 180°



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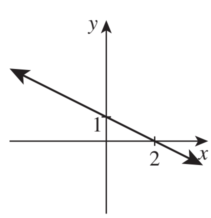

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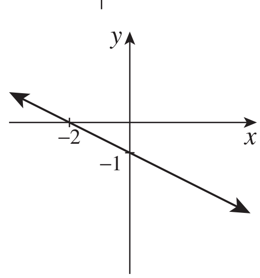

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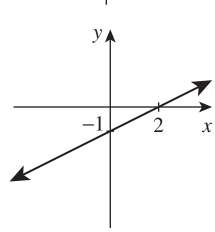
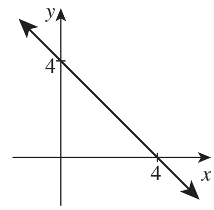


b From $y = 4 - x$:
i reflect in the x-axis
ii reflect in the y-axis
iii rotate by 180°



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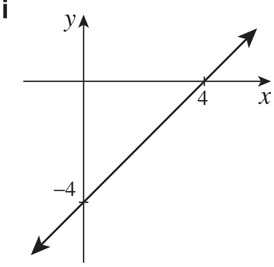

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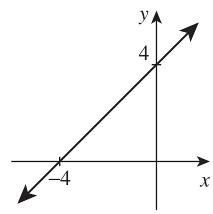

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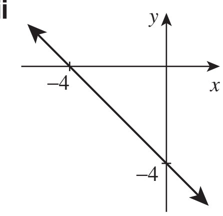
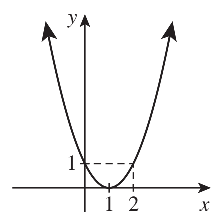


c From $y = (x - 1)^2$:
i rotate by 180°
ii reflect in the y-axis
iii reflect in the x-axis



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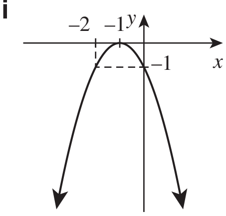

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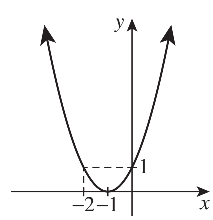

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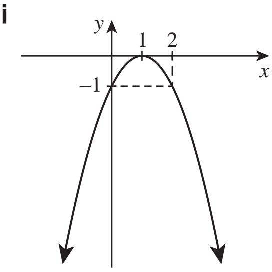
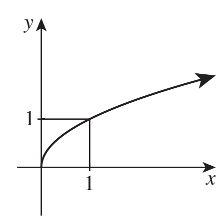
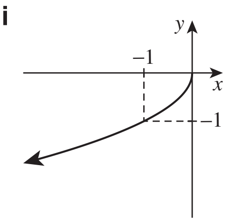


d From $y = \sqrt{x}$:
i rotate by 180°
ii reflect in the x-axis
iii reflect in the y-axis



i



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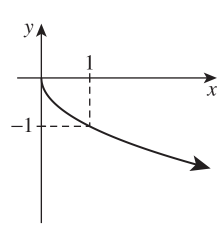

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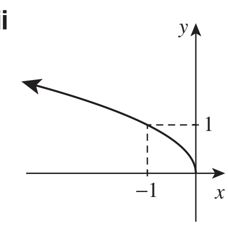

e From $y = 3^x$:
i reflect in the x-axis
ii rotate by 180°
iii reflect in the y-axis

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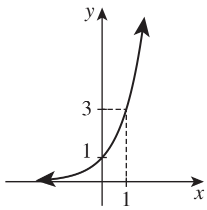

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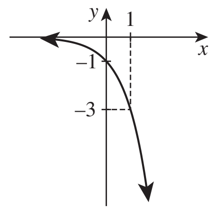

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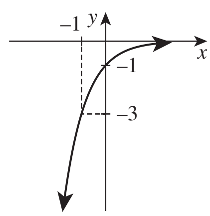

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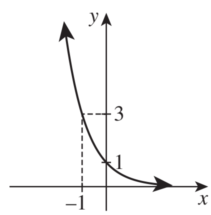
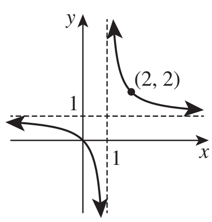


f From $y = 1 + \dfrac{1}{x - 1}$:

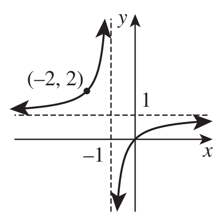
i reflect in the y-axis

ii rotate by 180°

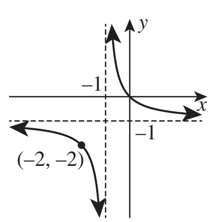
iii reflect in the x-axis



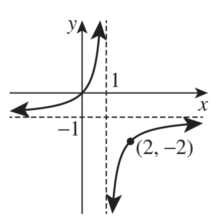
i



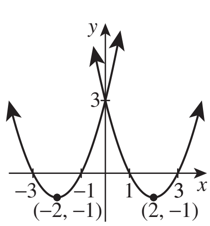
ii



iii



11 a



b Reflect in the y-axis.

c Shift left 4 units.

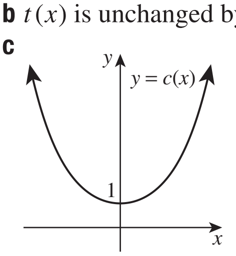
d $(x + 4)^2 - 4(x + 4) + 3 = x^2 + 4x + 3$

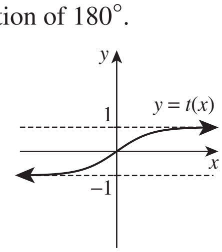
e part **b**, part **c**, part **f**

12 a $c(x)$ is the same when reflected in the y-axis.

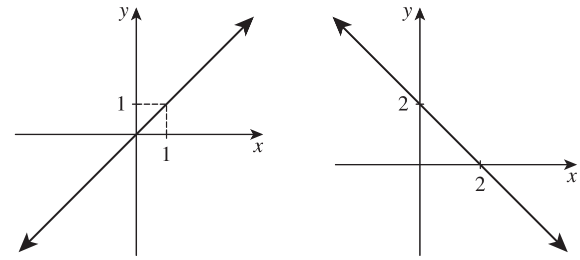
b $t(x)$ is unchanged by a rotation of 180°.

c

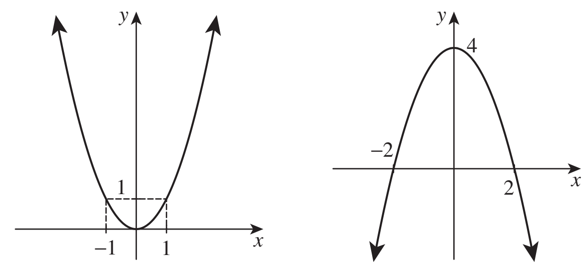




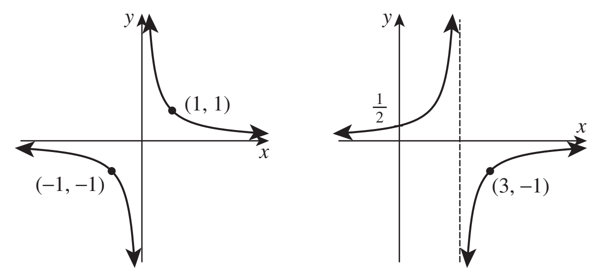
13 a Reflect in the y-axis then shift up 2.



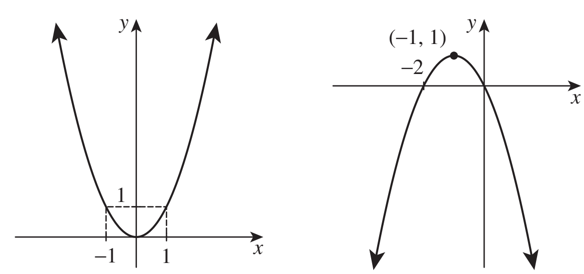
b Reflect in the x-axis then shift up 4.



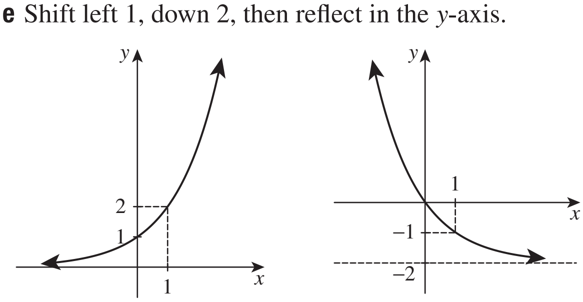
c Shift left 2 then reflect in the y-axis.



d Reflect in the x-axis then shift left 1, up 1.



e Shift left 1, down 2, then reflect in the y-axis.



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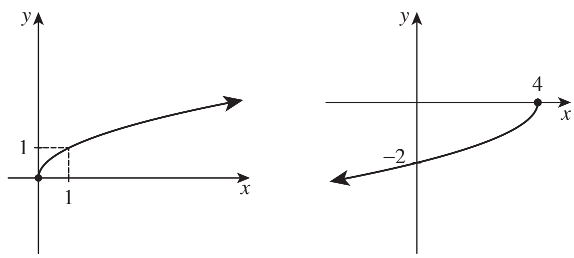
f Shift left 4, then reflect in both axes.



14 a i $y = (x - 2)^2$ **ii** $y = (x + 2)^2$

b i $y = (x + 1)^2$ **ii** $y = x^2$

c Yes: the answer depends on the order.

d The order is irrelevant when the shift is parallel with the axis of reflection.

15 a $y - a = f(x)$, $-y - a = f(x)$, $-y = f(x)$, $y = f(x)$

b $y = f(x - a)$, $-y = f(x - a)$, $-y = f(x - 2a)$, $y = f(x - 2a)$

c $x = f(y)$, $-x = f(y)$, $-y = f(x)$, $y = f(x)$

d $-x = f(y)$, $-y = f(-x)$, $x = f(-y)$, $y = f(x)$

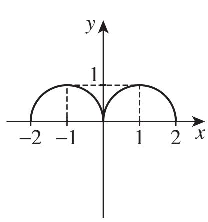
16 Shift left by a to get $y = f(x + a)$, then reflect in the y-axis to get $y = f(a - x)$, finally shift right by a to get $y = f(2a - x)$.

Exercise 4C

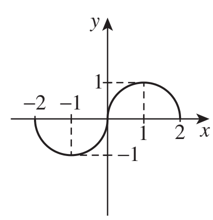
1 a even **b** neither **c** neither

d odd **e** odd **f** even

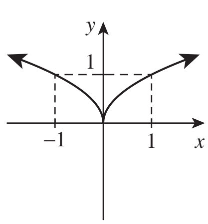
2 a i



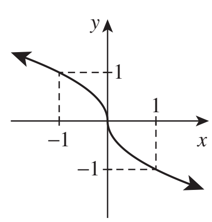
ii



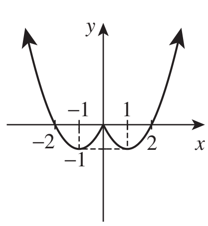
b i



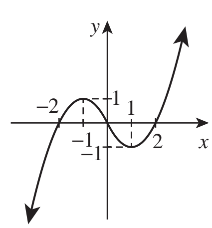
ii



c i



ii



3 a $f(-x) = x^4 - 2x^2 + 1$

b $f(-x) = f(x)$, so it is even.

4 a $g(-x) = -x^3 + 3x$

b $-g(x) = -(x^3 - 3x) = g(-x)$, so it is odd.

5 a $h(-x) = -x^3 + 3x^2 - 2$

b $-h(x) = -x^3 - 3x^2 + 2$. Because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$, it is neither.

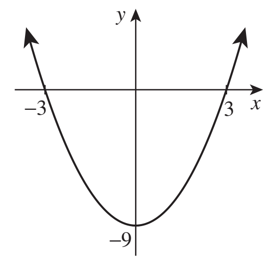
6 a even **b** neither **c** odd **d** even

e neither **f** odd **g** odd **h** neither

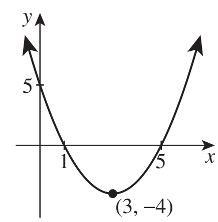
7 a . . . if all powers of x are odd.

b . . . if all powers of x are even.

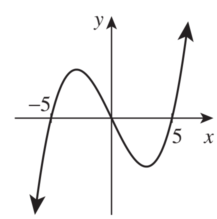
8 a $y = (x + 3)(x - 3)$



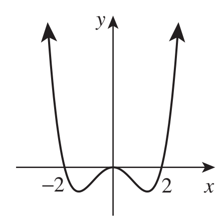
b $y = (x - 1)(x - 5)$



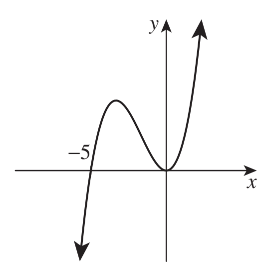
c $y = x(x - 5)(x + 5)$



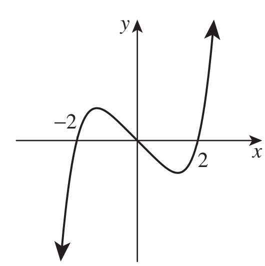
d $y = x^2(x - 2)(x + 2)$.



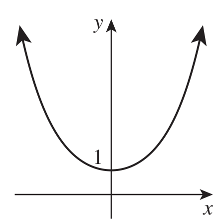
e $y = x^2(x + 5)$



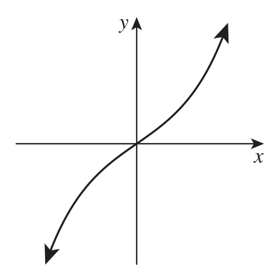
f $y = x(x - 2)(x + 2)(x^2 + 4)$



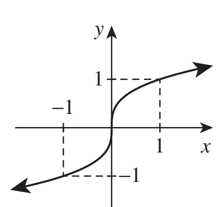
9 a even



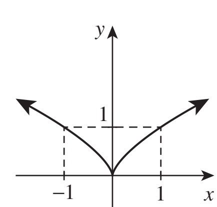
b odd



c odd



d even



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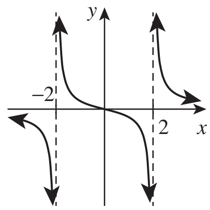
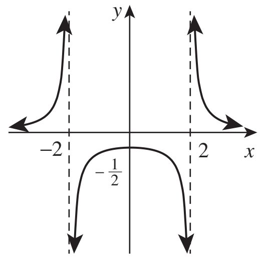
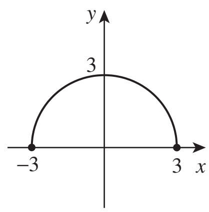
e odd



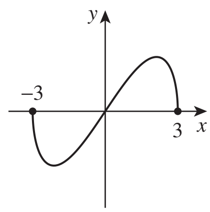
f even



g even



h odd



10 a neither **b** neither **c** even **d** even
e odd **f** even **g** odd **h** neither

11 a It is symmetric in the y-axis.
b It is symmetric in both axes.

13 a i even **ii** even **iii** odd
b i even **ii** odd
iii in general, neither

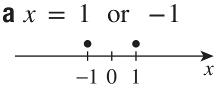
14 a Suppose $f(0) = c$. Then because $f(x)$ is odd, $f(0) = -f(0) = -c$. So $c = -c$, and hence $c = 0$.
b It is not defined at the origin (it is 1 for $x > 0$, and -1 for $x < 0$).

15 b i $g(x) = 1 + x^2$ and $h(x) = -2x$
ii $g(x) = \dfrac{2^x + 2^{-x}}{2}$ and $h(x) = \dfrac{2^x - 2^{-x}}{2}$
c In the first, $g(x)$ and $h(x)$ are not defined for all x in the natural domain of $f(x)$, specifically at $x = -1$. In the second, $x = 0$ is the only place at which $g(x)$ and $h(x)$ are defined.

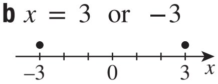
Exercise 4D

1 a 3 **b** 3 **c** 3 **d** 3
e 7 **f** 1 **g** 16 **h** −3

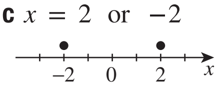
2 a $x = 1$ or -1



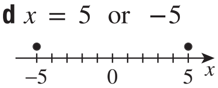
b $x = 3$ or -3



c $x = 2$ or -2

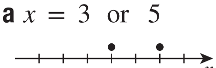


d $x = 5$ or -5

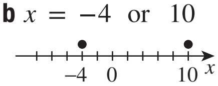


e no solutions **f** no solutions

3 a $x = 3$ or 5

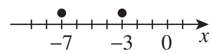


b $x = -4$ or 10

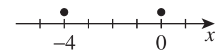


c no solutions **d** no solutions

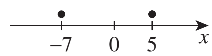
e $x = -3$ or -7



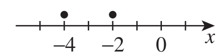
f $x = -4$ or 0



g $x = -7$ or 5

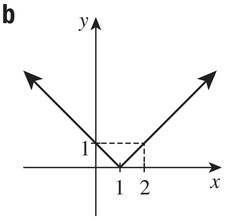


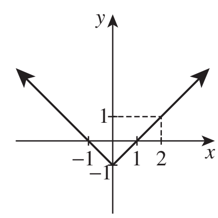
h $x = -4$ or -2



4 a For $|x - 1|$: 3, 2, 1, 0, 1, 2.
For $|x| - 1$: 1, 0, −1, 0, 1, 2.

b



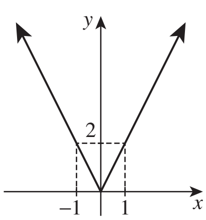


The two graphs overlap for $x \geq 1$.
c The first is $y = |x|$ shifted right 1 unit, the second is $y = |x|$ shifted down 1 unit.

5 a LHS = 2, RHS = −2
b LHS = 2, RHS = −2
c LHS = 0, RHS = 4
d LHS = 1, RHS = −1
e LHS = 3, RHS = 1
f LHS = 8, RHS = −8

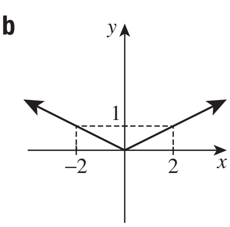
6 a false: $x = 0$ **b** true
c true **d** false: $x = -2$
e true **f** true
g false: $x = -2$ **h** true

7 a



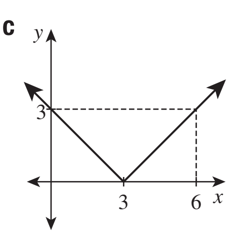
$y = \begin{cases} 2x, & \text{for } x \geq 0, \\ -2x, & \text{for } x < 0. \end{cases}$

b



$y = \begin{cases} \frac{1}{2}x, & \text{for } x \geq 0, \\ -\frac{1}{2}x, & \text{for } x < 0. \end{cases}$

c



shift right 3,
$y = \begin{cases} x - 3, & \text{for } x \geq 3, \\ 3 - x, & \text{for } x < 3. \end{cases}$

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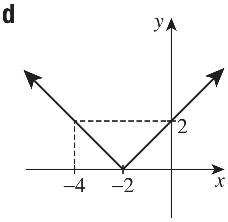
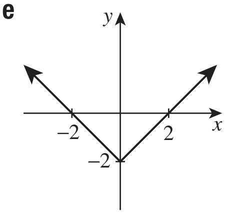
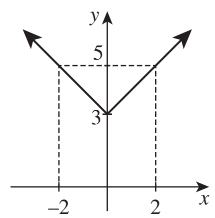
d 

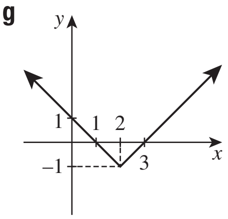
shift left 2,
$$y = \begin{cases} x + 2, & \text{for } x \ge -2, \\ -x - 2, & \text{for } x < -2. \end{cases}$$

e 

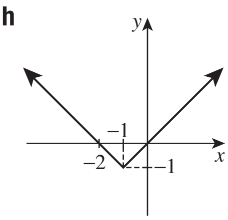
shift down 2,
$$y = \begin{cases} x - 2, & \text{for } x \ge 0, \\ -x - 2, & \text{for } x < 0. \end{cases}$$

f 

shift up 3,
$$y = \begin{cases} x + 3, & \text{for } x \ge 0, \\ 3 - x, & \text{for } x < 0. \end{cases}$$

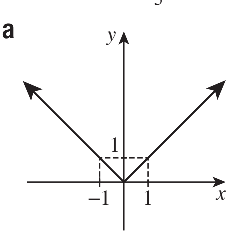
g 

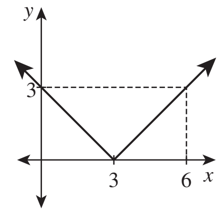
shift right 2, down 1,
$$y = \begin{cases} x - 3, & \text{for } x \ge 2, \\ 1 - x, & \text{for } x < 2. \end{cases}$$

h 

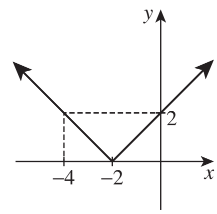
shift left 1, down 1,
$$y = \begin{cases} x, & \text{for } x \ge -1, \\ -x - 2, & \text{for } x < -1. \end{cases}$$

8 a $x = 5$ or -5
b $x = -2$ or 1
c $x = 6$ or -5
d no solution
e no solution
f $x = -\frac{2}{5}$
g $x = \frac{5}{3}$
h $x = \frac{1}{3}$ or 2
i $x = -2$ or $\frac{2}{5}$

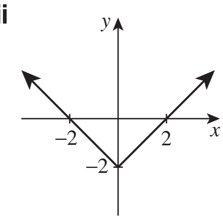
9 a 

b i 

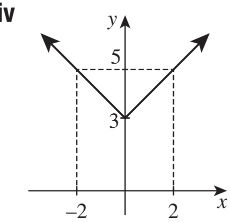
shift right 3,
$$y = \begin{cases} x - 3, & \text{for } x \ge 3, \\ 3 - x, & \text{for } x < 3. \end{cases}$$

ii 

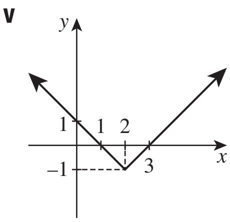
shift left 2,
$$y = \begin{cases} x + 2, & \text{for } x \ge -2, \\ -x - 2, & \text{for } x < -2. \end{cases}$$

iii 

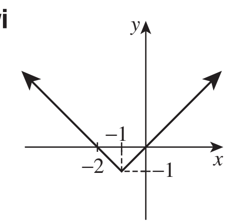
shift down 2,
$$y = \begin{cases} x - 2, & \text{for } x \ge 0, \\ -x - 2, & \text{for } x < 0. \end{cases}$$

iv 

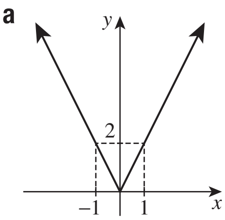
shift up 3,
$$y = \begin{cases} x + 3, & \text{for } x \ge 0, \\ 3 - x, & \text{for } x < 0. \end{cases}$$

v 

shift right 2, down 1,
$$y = \begin{cases} x - 3, & \text{for } x \ge 2, \\ 1 - x, & \text{for } x < 2. \end{cases}$$

vi 

shift left 1, down 1,
$$y = \begin{cases} x, & \text{for } x \ge -1, \\ -x - 2, & \text{for } x < -1. \end{cases}$$

10 a 

$$y = \begin{cases} 2x, & \text{for } x \ge 0, \\ -2x, & \text{for } x < 0. \end{cases}$$

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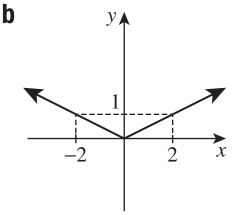
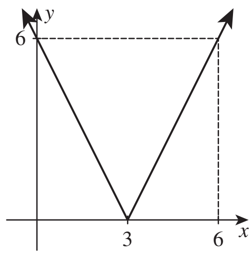
b



$$y = \begin{cases} \frac{1}{2}x, & \text{for } x \geq 0, \\ -\frac{1}{2}x, & \text{for } x < 0. \end{cases}$$

11 a



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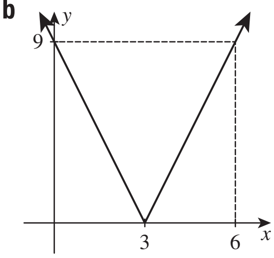

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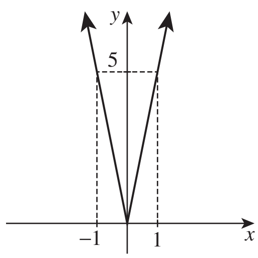
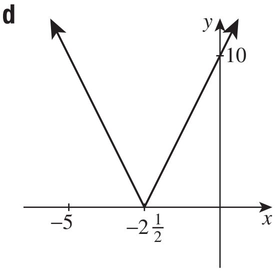
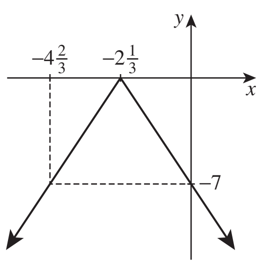


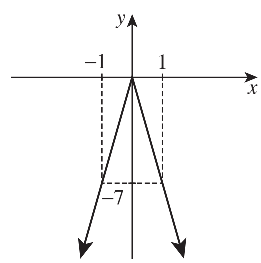
d



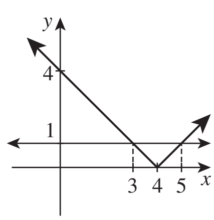
e



f

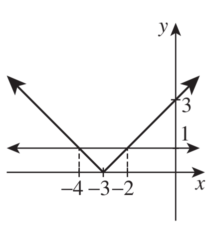


12 a i



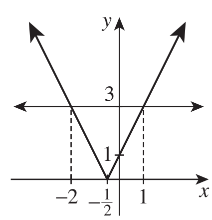
ii The x-coordinates of the points of intersection give: $x = 3$ or 5

b i



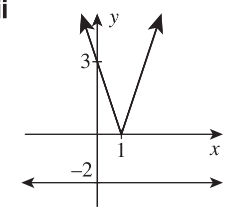
$x = -4$ or -2

ii



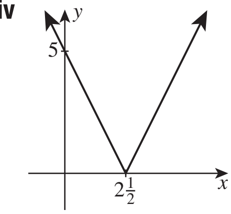
$x = -2$ or 1

iii



no solution

iv



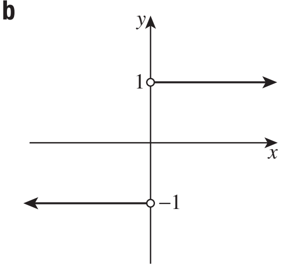
$x = 2\frac{1}{2}$

13 b The graph is symmetric in the y-axis.

14 a even **b** neither **c** odd **d** even

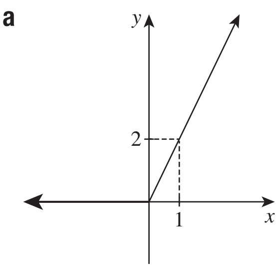
15 a $x = 0$

b



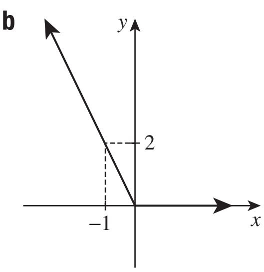
c $y = \begin{cases} 1, & \text{for } x > 0, \\ -1, & \text{for } x < 0. \end{cases}$

16 a



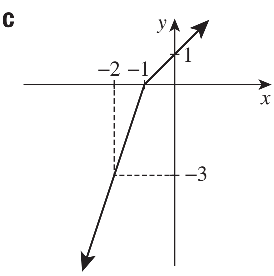
$$y = \begin{cases} 2x, & \text{for } x \geq 0, \\ 0, & \text{for } x < 0. \end{cases}$$

b



$$y = \begin{cases} 0, & \text{for } x \geq 0, \\ -2x, & \text{for } x < 0. \end{cases}$$

c



$$y = \begin{cases} x + 1, & \text{for } x \geq -1, \\ 3x + 3, & \text{for } x < -1. \end{cases}$$

Answers 4D

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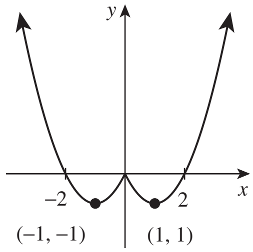
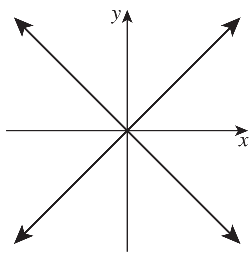
d



$$y = \begin{cases} x^2 - 2x, & \text{for } x \geq 0, \\ x^2 + 2x, & \text{for } x < 0. \end{cases}$$

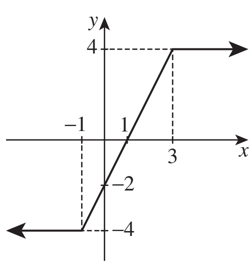
17 $f(|-x|) = f(|x|)$

18



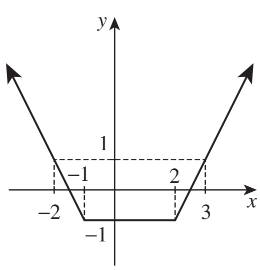
$y = x$ or $y = -x$

19 a



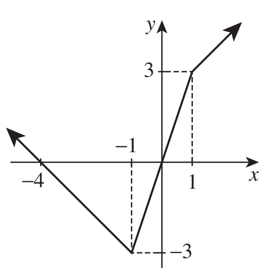
$$y = \begin{cases} -4, & \text{for } x < -1, \\ 2x - 2, & \text{for } -1 \leq x < 3, \\ 4, & \text{for } x \geq 3. \end{cases}$$

b



$$y = \begin{cases} -2x - 3, & \text{for } x < -1, \\ -1, & \text{for } -1 \leq x < 2, \\ 2x - 5, & \text{for } x \geq 2. \end{cases}$$

c



$$y = \begin{cases} -x - 4, & \text{for } x < -1, \\ 3x, & \text{for } -1 \leq x < 1, \\ x + 2, & \text{for } x \geq 1. \end{cases}$$

20 a $|\Delta AOB| = \frac{1}{2}\left|\frac{c^2}{ab}\right|$

b $|\Delta AOB| = \frac{1}{2}p\left|\frac{c}{ab}\right|\sqrt{a^2 + b^2}$

c $p = \dfrac{|c|}{\sqrt{a^2 + b^2}}$

d $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ **e** $\frac{|6 - 10 + 3|}{\sqrt{2^2 + (-5)^2}} = \frac{1}{\sqrt{29}}$

Exercise 4E

1 a i 4 **ii** 7 **iii** 3 **iv** −4

b i $x + 4$ **ii** $x + 6$ **c** $x = -4$

2 a $F(F(0)) = 0$, $F(F(7)) = 28$, $F\big(F(F(x))\big) = 8x$

$F(F(-3)) = -12$, $F\big(F(F(-11))\big) = -44$

b $F(F(x)) = 4x$, $F\big(F(F(x))\big) = 8x$

c $x = 8$

3 a $g(g(0)) = 0$, $g(g(4)) = 4$, $g(g(-2)) = -2$,

$g(g(-9)) = -9$

b $g(g(x)) = 2 - (2 - x) = x$

c $g\big(g(g(x))\big) = g(x)$

4 a $h(h(0)) = -20$, $h(h(5)) = 25$,

$h(h(-1)) = -29$, $h(h(-5)) = -65$

b $h(h(x)) = 9x - 20$, $h\big(h(h(x))\big) = 27x - 65$

5 a $f(g(7)) = 12$, $g(f(7)) = 13$, $f(f(7)) = 9$,

$g(g(7)) = 19$

b i $2x - 2$ **ii** $2x - 1$ **iii** $x + 2$ **iv** $4x - 9$

c Shift 1 unit to the left (or shift two units up).

d Shift 1 unit up (or shift $\frac{1}{2}$ left).

6 a $\ell(q(-1)) = -2$, $q(\ell(-1)) = 16$,

$\ell(\ell(-1)) = -7$, $q(q(-1)) = 1$

b i $x^2 - 3$ **ii** $(x - 3)^2$ **iii** $x - 6$ **iv** x^4

c i Domain: all real x, range: $y \geq -3$

ii Domain: all real x, range: $y \geq 0$

d It is shifted 3 units to the right.

e It is shifted 3 units down.

7 a $F(G(25)) = 20$, $G(F(25)) = 10$,

$F(F(25)) = 400$, $G(G(25)) = \sqrt{5}$

b $4\sqrt{x}$ **c** $\sqrt{4x} = 2\sqrt{x}$

e Domain: $x \geq 0$, range: $y \geq 0$

8 a $f\left(h\left(-\frac{1}{4}\right)\right) = 4$, $h\left(f\left(-\frac{1}{4}\right)\right) = 4$,

$f\left(f\left(-\frac{1}{4}\right)\right) = -\frac{1}{4}$, $h\left(h\left(-\frac{1}{4}\right)\right) = -\frac{1}{4}$

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b i Both sides equal $-\frac{1}{x}$, for all $x \neq 0$.

ii Both sides equal x, for all $x \neq 0$.

c Domain: $x \neq 0$, range $y \neq 0$

d It is reflected in the y-axis (or in the x-axis).

9 a $f\big(g(x)\big) = -5 - \sqrt{x}$.

Domain: $x \geq 0$, range: $y \leq -5$. Take the graph of $y = \sqrt{x}$, reflect it in the y-axis, then shift down 5.

b $f(x) = -5 - |x|$, which is negative for all x, so $\big(f(x)\big) = \sqrt{-5 - |x|}$ is never defined.

10 a $\big(f(-x)\big) = g\big(f(-x)\big) = -g\big(f(x)\big)$

b $\big(f(-x)\big) = g\big(f(-x)\big) = g\big(f(x)\big)$

c $g\big(f(-x)\big) = g\big(f(x)\big)$

11 a $g\big(f(x)\big) = 7$ for all x, $f\big(g(x)\big) = 4$ for all x

b $g\big(f(x)\big) = g(x)$, $f\big(g(x)\big) = g(x)$

12 a i Translation down a **ii** Translation right a

b i Reflection in the x-axis

ii Reflection in the y-axis

13 a $g\big(f(x)\big) = 10x + 15 + b$,

$f\big(g(x)\big) = 10x + 2b + 3$

b $b = 12$

14 a $g\big(f(x)\big) = 2ax + 3a + b$,

$f\big(g(x)\big) = 2ax + 2b + 3$

b First, $2a = 1$, so $a = \frac{1}{2}$. Secondly, $2b + 3 = 0$, so $b = -1\frac{1}{2}$.

15 a $f\big(g(0)\big) = -3$, $g\big(f(0)\big) = 3$, $f\big(g(-2)\big) = 3$, $g\big(f(-2)\big) = 1$

b i $x^2 + x - 3$ **ii** $x^2 - x - 3$

16 a All real y and $y \geq -1$.

b $x^2 + 2x + 1 = (x + 1)^2$, range: $y \geq 0$

c $x^2 + 4x + 3 = (x + 1)(x + 3)$, range: $y \geq -1$

d -1 and -3.

17 a $f(x)$ **b** $z(x)$, with domain D

c $z(x)$, with domain D

d If $f(0)$ exists, it is the function $f(x) = f(0)$ with domain $\mathbb{R}$. Otherwise it is the empty function, with domain the empty set.

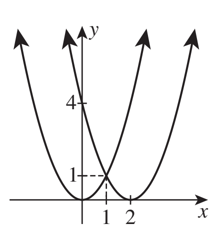
Chapter 4 review exercise

1a x^2: 4, 1, 0, 1, 4, 9, 16

$(x - 2)^2$: 16, 9, 4, 1, 0, 1, 4

b $y = x^2$, $V = (0, 0)$.

$y = (x - 2)^2$, $V = (2, 0)$.



c Here x is replaced by $(x - 2)$, so it is a shift right by 2 units.

2 a Replace x with $-x$.

b $y = x^2 - 2x$:

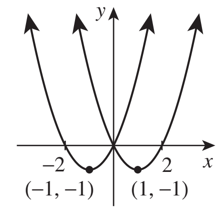
15, 8, 3, 0, −1, 0, 3

$y = x^2 + 2x$:

3, 0, −1, 0, 3, 8, 15

c $y = x^2 - 2x$: $(1, -1)$.

$y = x^2 + 2x$: $(-1, -1)$.



3 a 7 **b** 4 **c** 5 **d** 3 **e** −3 **f** 12

4 a $x = -5$ or 5 **b** $x = -6$ or 6

c $x = -2$ or 6 **d** $x = -5$ or -1

e $x = -1$ or 4 **f** $x = -1$ or $3\frac{2}{3}$

5 a Shift $y = x^2$ up by 5 units.

b Shift $y = x^2$ down by 1 unit.

c Shift $y = x^2$ right by 3 units.

d Shift $y = x^2$ left by 4 units and up by 7 units.

6 a $y = (x - 1)^2$ **b** $y = x^2 - 2$

c $y = (x + 1)^2 + 5$ **d** $y = (x - 4)^2 - 9$

7 a $C(0, 0)$, $r = 1$ **b** $C(-1, 0)$, $r = 2$

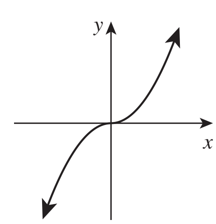
c $C(2, -3)$, $r = \sqrt{5}$ **d** $C(0, 4)$, $r = 8$

8 a $y = -x^3 + 2x + 1$ **b** $y = -x^2 + 3x + 4$

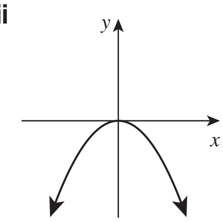
c $y = -2^{-x} - x$ **d** $y = \sqrt{9 - x^2}$

9 a neither **b** odd **c** even

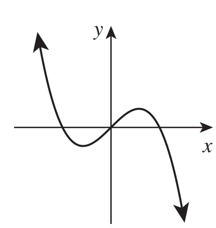
10 a i



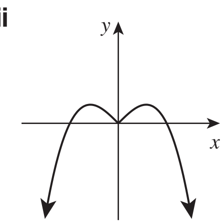
ii



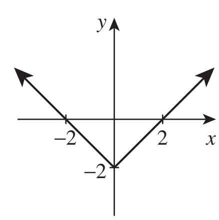
b i



ii



11 a i



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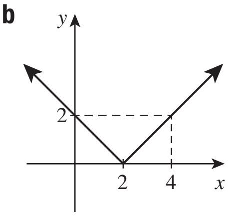

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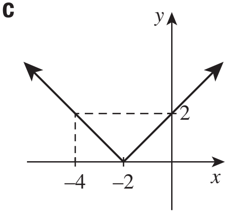
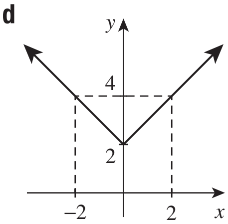
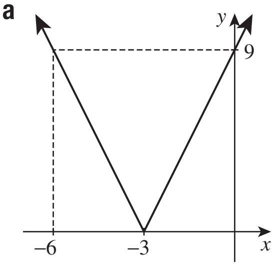
c



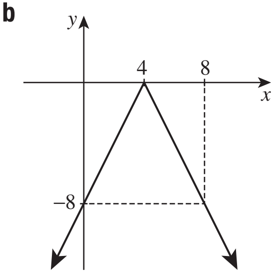
d



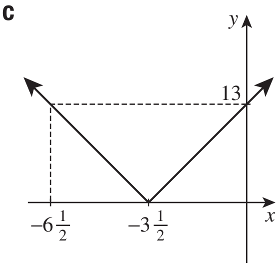
12 a



b



c



13 a 5 or -5 **b** 1 or -9 **c** no solutions

d 12 or -2 **e** 1 or -8 **f** 4 or $\frac{4}{3}$

g $-\frac{2}{7}$ **h** 5 or -5

14 a neither **b** even **c** odd **d** odd

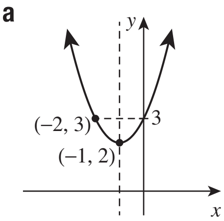
15 a $y = (x - 1)^2 + 4, V = (1, 4)$

b $y = (x + 2)^2 - 7, V = (-2, -7)$

c $y = 2(x + 2)^2 + 3, V = (-2, 3)$

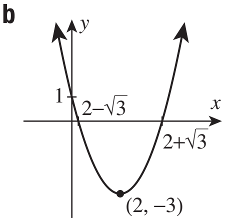
d $y = -(x - 3)^2 + 10, V = (3, 10)$

16 a



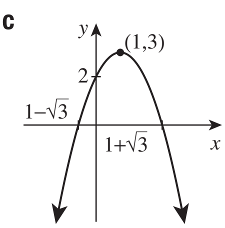
$y = (x + 1)^2 + 2$

b



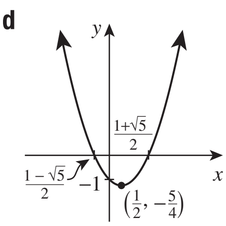
$y = (x - 2)^2 - 3$

c



$y = 3 - (x - 1)^2$

d



$y = \left(x - \frac{1}{2}\right)^2 - \frac{5}{4}$

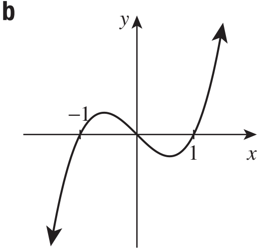
17 a $C(0, 1), r = 2$

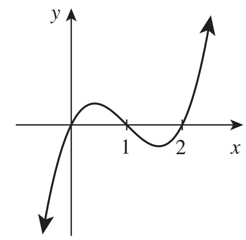
b $C(-3, 0), r = 1$

c $C(2, -3), r = 4$

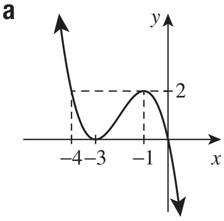
d $C(4, -7), r = 10$

18 b





19 a



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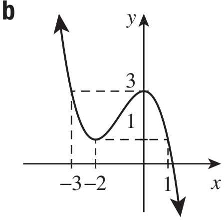

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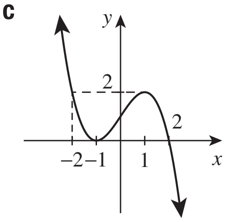

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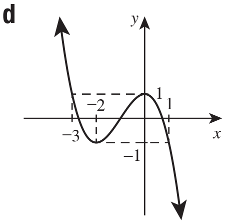
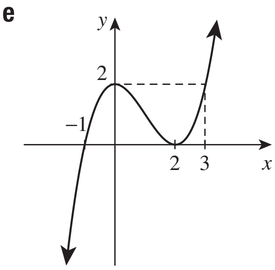
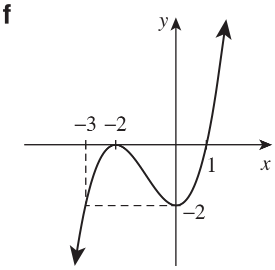


e



f



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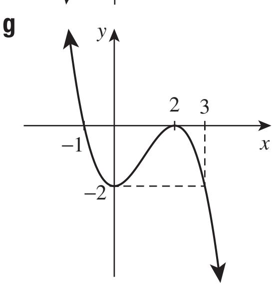
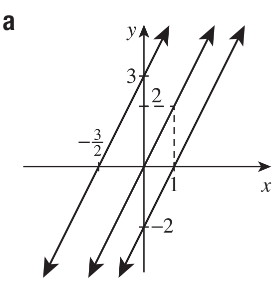


20 a



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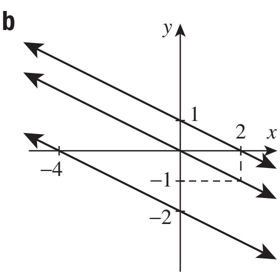

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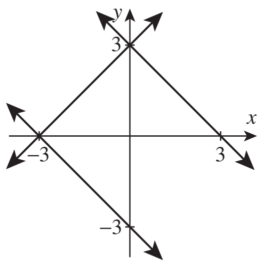
c


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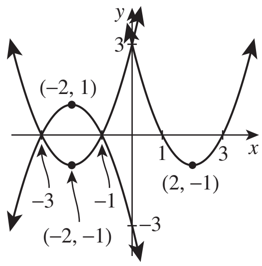

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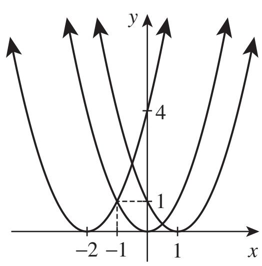

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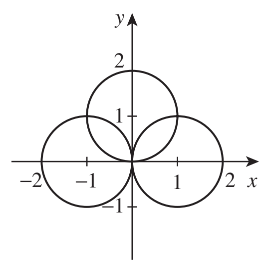

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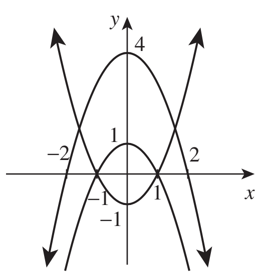

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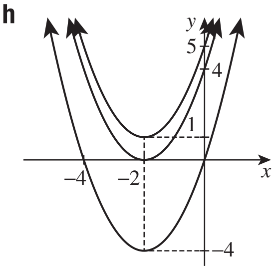

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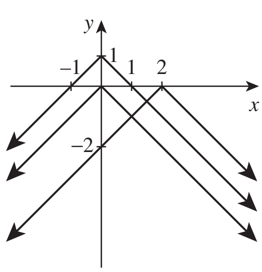

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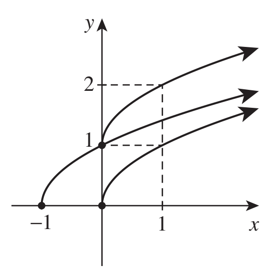

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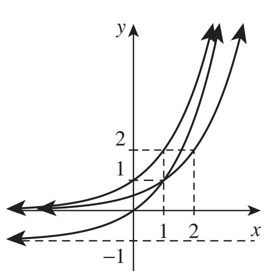

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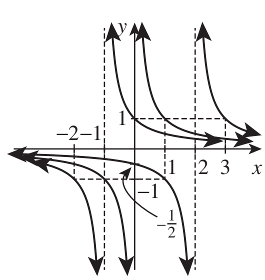

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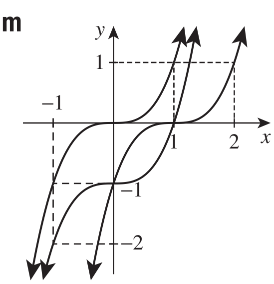

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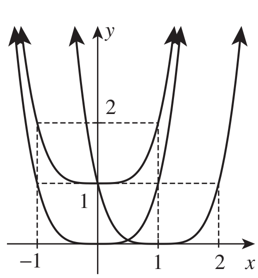
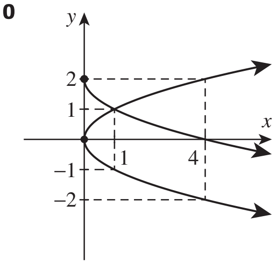


o


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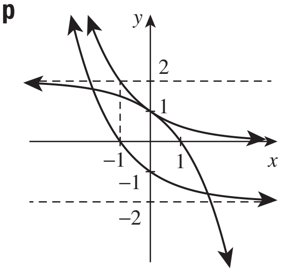
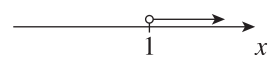
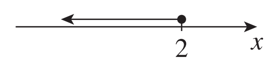


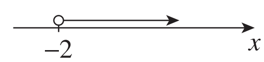
21 $g(x) = \frac{1}{2}\big(f(x) + f(-x)\big)$
$h(x) = \frac{1}{2}\big(f(x) - f(-x)\big)$

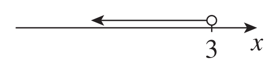
Chapter 5

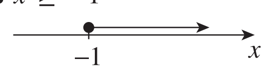
Exercise 5A

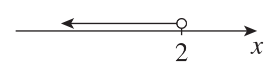
1 a $x > 1$


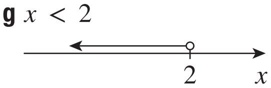
b $x \le 2$


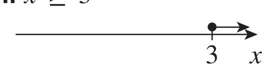
c $x > -2$


d $x < 3$


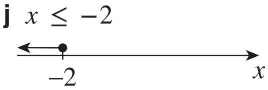
e $x \ge -1$


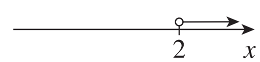
f $x < 2$


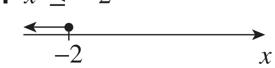
g $x < 2$


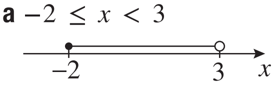
h $x \ge 3$


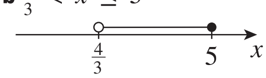
i $x \ge 3$

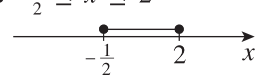

j $x \le -2$


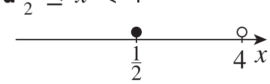
k $x > 2$


l $x \le -2$


2 a $-2 \le x < 3$


b $\frac{4}{3} < x \le 5$


c $-\frac{1}{2} \le x \le 2$


d $\frac{1}{2} \le x < 4$


3 a $x > 4$ **b** $x \le 2$
c $x < 2$ **d** $x \le -1$
e $-2 \le x < 1$ **f** $-6 \le x \le 15$

4 a i $0 < x < 4$ **ii** $x < 0$ or $x > 4$
b i $-1 \le x \le 3$ **ii** $x \le -1$ or $x \ge 3$
c i $x \le 0$ or $x \ge 2$ **ii** $0 < x < 2$

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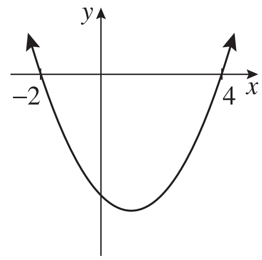
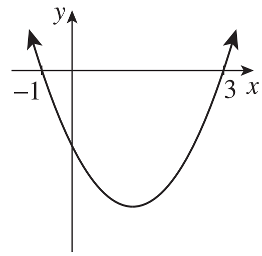
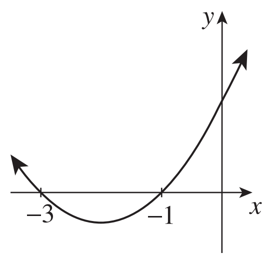
5 a $-2 < x < 4$



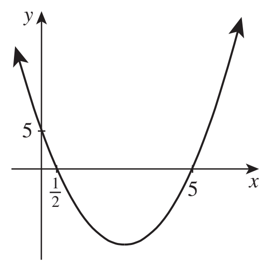
b $x < -1$ or $x > 3$



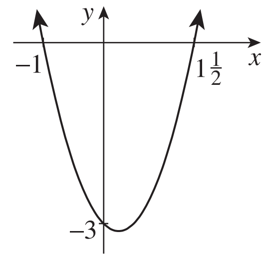
c $x \le -3$ or $x \ge -1$



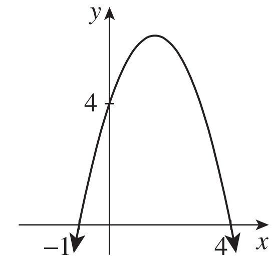
d $x < \frac{1}{2}$ or $x > 5$



e $-1 \le x \le 1\frac{1}{2}$



f $-1 < x < 4$

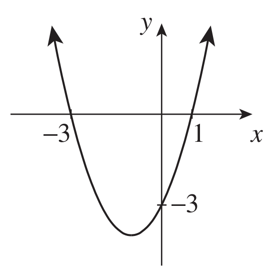


6 a $-3 < x < 3$ **b** $x < 0$ or $x > 6$
c $x \le -10$ or $x \ge 10$ **d** $-4 \le x \le 0$

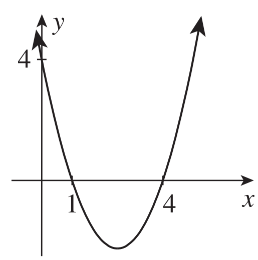
7 a $x = 7$ or -7 **b** $x = 0$
c $-2 \le x \le 2$ **d** $x < -5$ or $x > 5$
e $-\frac{1}{4} < x < \frac{1}{4}$ **f** $x \le -\frac{3}{2}$ or $x \ge \frac{3}{2}$

8 a $0 < x < 1$ **b** $x < 0$ or $x > 3$
c $0 < x \le \frac{1}{2}$ **d** $x \le -\frac{3}{4}$ or $x > 0$

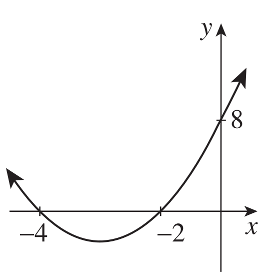
9 a $-3 < x < 1$



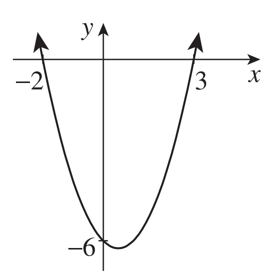
b $x \le 1$ or $x \ge 4$



c $x < -4$ or $x > -2$



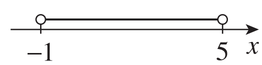
d $-2 \le x \le 3$



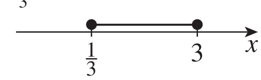
10 a $-1 < x < 3$ **b** $x \le 1$ or $x \ge 9$
c $x < -4$ or $x > 2$ **d** $-14 \le x \le -2$

11 a $x < -1$ or $x \ge 1$ **b** $3 < x < 5$
c $-4 < x \le -2\frac{1}{2}$ **d** $x < \frac{3}{2}$ or $x > 4$
e $1 < x < 3$ **f** $\frac{5}{3} < x \le 3$

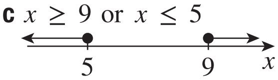
12 a $-1 < x < 5$



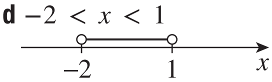
b $\frac{1}{3} \le x \le 3$



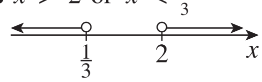
c $x \ge 9$ or $x \le 5$



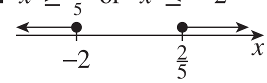
d $-2 < x < 1$



e $x > 2$ or $x < \frac{1}{3}$



f $x \ge \frac{2}{5}$ or $x \le -2$



13 a $x = 0$
b $x < 0$ or $x > 0$ (or simply $x \ne 0$)
c $x \le -5$ or $x \ge 5$
d $x < 0$ or $x > 25$
e No solution for x.
f $x = 1$

14 a $\frac{1}{2} < x \le 3$ **b** $-3 < x < -2$
c $x < 1$ or $x \ge 3$ **d** $x < -\frac{1}{7}$ or $x > 2$

15 a The first holds when x is positive, the second when x is negative.
b **i** $-2 < x < 2$ or $-10 < x < -6$
ii $3 \le x < 4\frac{1}{2}$ or $\frac{1}{2} < x \le 2$

16 a false: $x = 0$ **b** false: $x = \frac{1}{2}$ **c** true
d false: $x = \frac{1}{2}$ or $x = -2$
e false: $x = -1$ **f** true
g false: $x = -1$ **h** true

17 a No solutions **b** No solutions
c All real x **d** $x = \frac{5}{3}$

18 a An absolute value must be positive.
b $x > 1$

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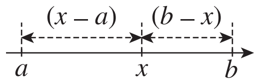
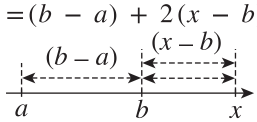
19 $-1 \le x < 0$ or $1 \le x \le 2$

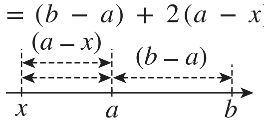
20 a $|x - a| + |x - b| = (x - a) + (b - x) < c$



b $|x - a| + |x - b| = (x - a) + (x - b)$
$= (b - a) + 2(x - b) < c$



c $|x - a| + |x - b| = (a - x) + (b - x)$
$= (b - a) + 2(a - x) < c$



d The result follows directly from parts **a**, **b** and **c**.

e $-3 < x < 7$

Exercise 5B

1 a $x \le 0$ or $1 \le x \le 2$

b $-2 < x < 0$ or $2 < x < 4$

c $0 < x < 3$ or $x > 3$

d $x = 0$ or $x \ge 4$

e $x = -3$ or $x = 3$

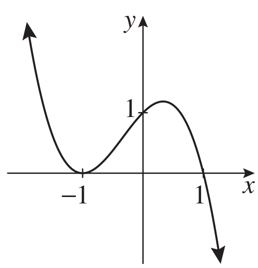
f $x = -3$ or $x \ge 0$

2 a

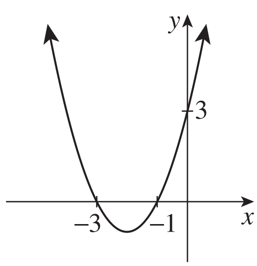
x	-2	-1	0	1	2
y	3	0	1	0	-9
sign	$+$	0	$+$	0	$-$

b Solution: $x \le 1$

c

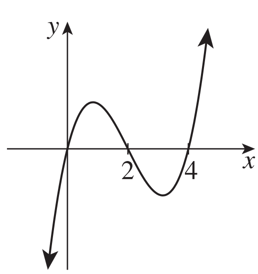


3 a



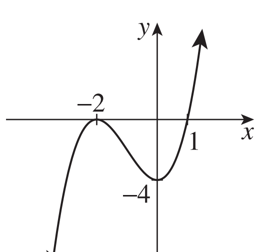
$-3 < x < -1$

b



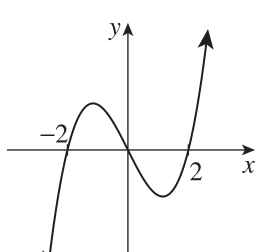
$0 \le x \le 2$ or $x \ge 4$

c



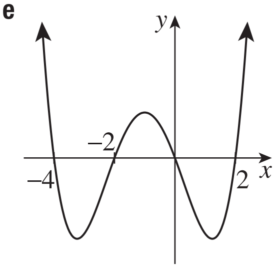
$x = -2$ or $x \ge 1$

d



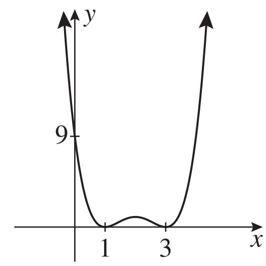
$x \le -2$ or $0 \le x \le 2$

e



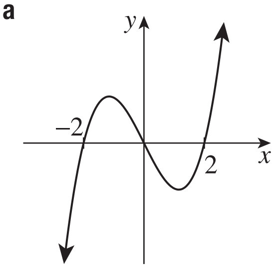
$x < -4$ or
$-2 < x < 0$ or $x > 2$

f



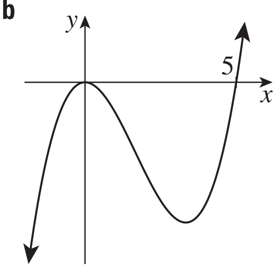
$x = 1$ or $x = 3$

4 a



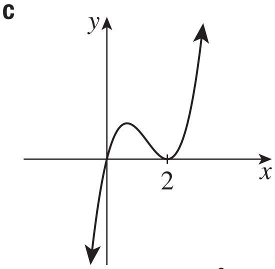
$f(x) = x(x - 2)(x + 2)$

b



$f(x) = x^2(x - 5)$

c



$f(x) = x(x - 2)^2$

5 a $-2 < x < 0$ or $x > 2$

b $x < 0$ or $0 < x < 5$

c $x \le 0$ or $x = 2$

6 a

x	-1	0	1	3	4
y	$-\frac{1}{4}$	0	$-\frac{1}{2}$	*	16
sign	$-$	0	$-$	*	$+$

b $x < 0$ or $0 < x < 3$

7 a $x < 1$ or $3 < x < 5$

b $x \ne 1$ and $x \ne 3$ (alternatively, $x < 1$ or $1 < x < 3$ or $x > 3$)

c $-2 < x \le 4$

d $-3 < x < 0$ or $x > 3$

e $-3 < x < -1$

f $x < 0$ or $0 < x < 5$

g $x \le 0$ or $x \ge 5$

h $-2 \le x < 0$ or $x \ge 2$

i $x < -3$ or $0 < x \le 2$

8 a i $y = x(x + 1)(x - 1)$, $x = -1, 0$ or 1

ii $y = (x - 2)(x - 1)(x + 1)$, $x = -1, 1$ or 2

iii $y = (x + 2)^2(x - 2)$, $x = -2$ or 2

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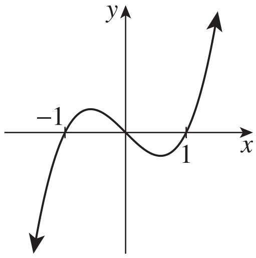
b **i**



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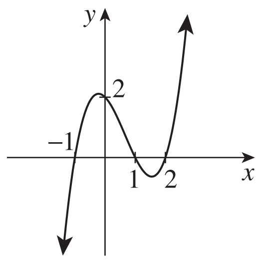

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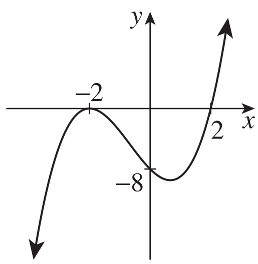


9 a zero for $x = 0$, undefined at $x = 3$, positive for $x < 0$ or $x > 3$, negative for $0 < x < 3$

b zero for $x = 4$, undefined at $x = -2$, positive for $x < -2$ or $x > 4$, negative for $-2 < x < 4$

c zero for $x = -3$, undefined at $x = -1$, positive for $x < -3$ or $x > -1$, negative for $-3 < x < -1$

10 a $x \le -4$ or $-3 < x \le 1$

b $-2 < x < -1\frac{1}{2}$ or $x > \frac{1}{2}$

c $-\frac{1}{2} \le x < 1\frac{1}{2}$ or $x \ge 2\frac{1}{2}$

Exercise 5C

1 In each case $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

a **i** Domain: $x \ne 1$. When $x = 0$, $y = -1$.

ii When $y = 1$, $x = 2$. When $y = -1$, $x = 0$.

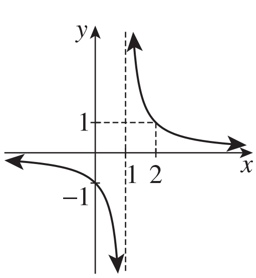
v Vertical asymptote: $x = 1$. As $x \to 1^+$, $y > 0$ so $y \to \infty$, and as $x \to 1^-$, $y < 0$ so $y \to -\infty$.

b **i** Domain: $x \ne 3$. When $x = 0$, $y = \frac{2}{3}$.

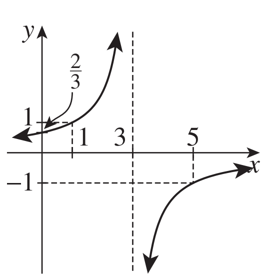
ii When $y = 1$, $x = 1$. When $y = -1$ at $x = 5$.

v Vertical asymptote: $x = 3$. As $x \to 3^+$, $y < 0$ so $y \to -\infty$, and as $x \to 3^-$, $y > 0$ so $y \to \infty$.

a



b



c **i** Domain: $x \ne -2$. When $x = 0$, $y = -1$.

ii When $y = 1$, $x = -4$. When $y = -1$, $x = 0$.

v Vertical asymptote: $x = -2$. As $x \to -2^+$, $y < 0$ so $y \to -\infty$, and as $x \to -2^-$, $y > 0$ so $y \to \infty$.

d **i** Domain: $x \ne -2\frac{1}{2}$. When $x = 0$, $y = 1$.

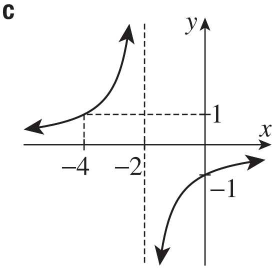
ii When $y = 1$, $x = 0$. When $y = -1$, $x = -5$.

v Vertical asymptote: $x = -2\frac{1}{2}$.

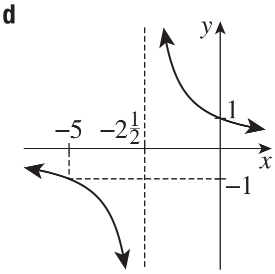
As $x \to -2\frac{1}{2}^+$, $y > 0$ so $y \to \infty$,

and as $x \to -2\frac{1}{2}^-$, $y < 0$ so $y \to -\infty$.

c



d



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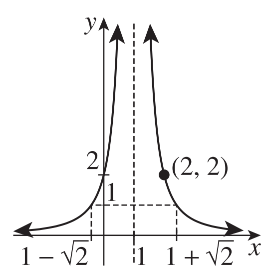


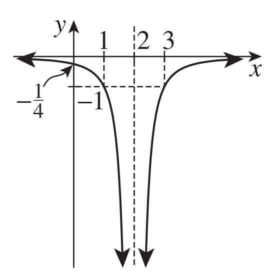
i Domain: $x \ne 1$.

ii When $y = 1$, $x = 1 + \sqrt{2}$ or $x = 1 - \sqrt{2}$.

iii Horizontal asymptote $y = 0$, $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

v Vertical asymptote $x = 1$. As $x \to 1^+$, $y > 0$ so $y \to \infty$ and as $x \to 1^-$, $y > 0$ so $y \to \infty$.

3



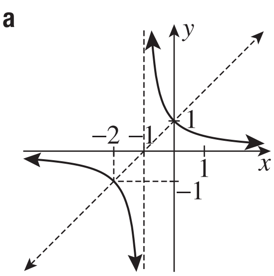
i Domain: $x \ne 2$.

ii When $y = -1$, $x = 1$ or 3.

iii Horizontal asymptote $y = 0$, $y \to 0$ as $x \to \infty$ and as $x \to -\infty$.

v Vertical asymptote $x = 2$. As $x \to 2^+$, $y < 0$ so $y \to -\infty$, and as $x \to 2^-$, $y < 0$ so $y \to -\infty$.

4 a



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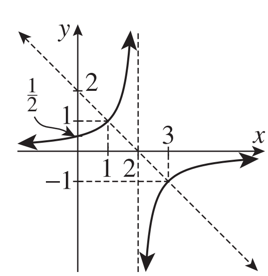

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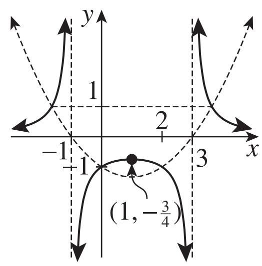
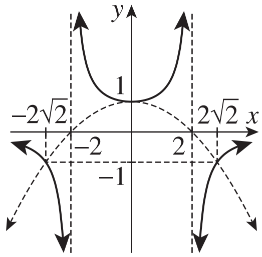
5 

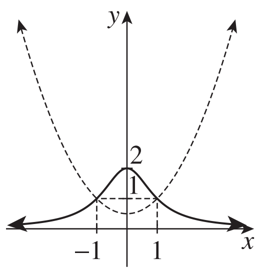
The curves also meet where $x = 1 - \sqrt{7}$ and $x = 1 + \sqrt{7}$.

c Range: $y \geq -\frac{4}{3}$.

e Range: $y \leq -\frac{3}{4}$ or $y > 0$.

6 a 

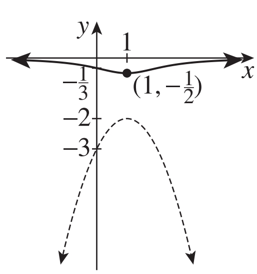
b Range: $y \leq 1$.

7 

b $\frac{1}{2}$

d As $x \to \infty$ or $x \to -\infty$, $y \to 0$.

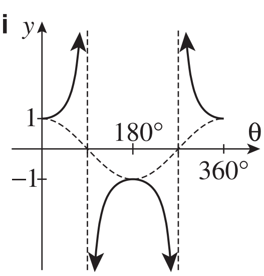
e 2

8 

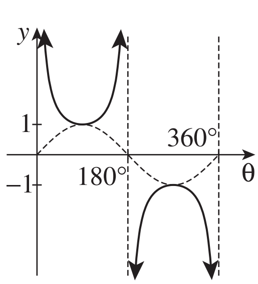
a -2

c As $x \to \infty$ or $x \to -\infty$, $y \to 0$.

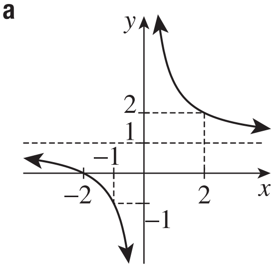
d $-\frac{1}{2}$

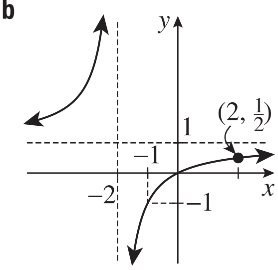
9 a i 

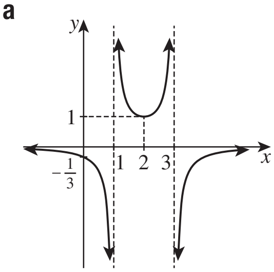
ii Domain: $0^\circ \leq \theta \leq 360^\circ$, except that $\theta \neq 90^\circ$ and $\theta \neq 270^\circ$.
Range: $y \leq -1$ or $y \geq 1$.

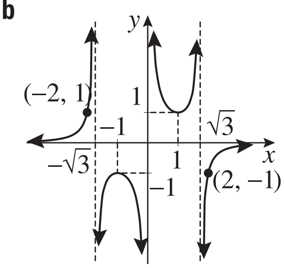
b i 

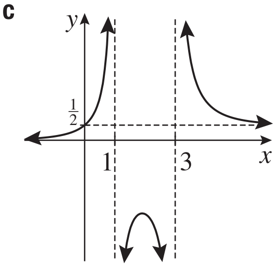
ii Domain: $0^\circ \leq \theta \leq 360^\circ$, except that $\theta \neq 0^\circ$, $\theta \neq 180^\circ$ and $\theta \neq 360^\circ$.
Range: $y \leq -1$ or $y \geq 1$.

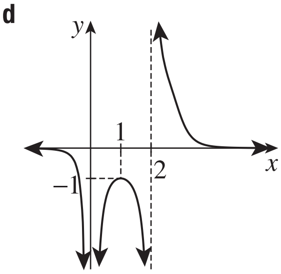
11 a 

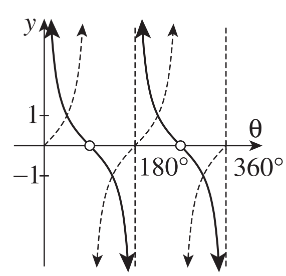
b 

12 a 

b 

c 

d 

13 

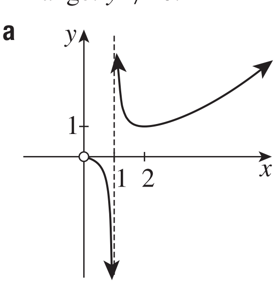
a Domain: $0^\circ \leq \theta \leq 360^\circ$ except that $\theta \neq 90^\circ$ and $\theta \neq 270^\circ$.

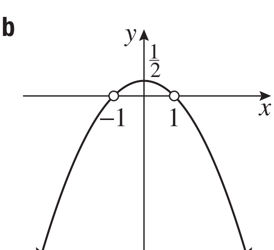
b $\tan\theta = 0$ at $\theta = 0^\circ$, $\theta = 180^\circ$ and $\theta = 360^\circ$.

c Domain: $0^\circ < \theta < 360^\circ$, except that $\theta \neq 90^\circ$, $\theta \neq 180^\circ$ and $\theta \neq 270^\circ$

d 0

f Range: $y \neq 0$.

14 a 

b 

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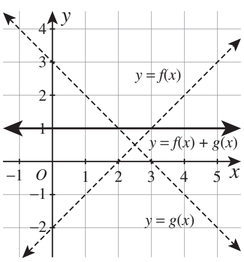
15 The problem is that zero does not have a reciprocal. For example, $y = -x^2$ has a maximum of 0 when $x = 0$, and $y = \dfrac{-1}{x^2}$ has an asymptote at $x = 0$, not a minimum. The statement should be, 'When one curve has a non-zero local maximum, the other curve has a non-zero local minimum.'

16 $y = x - 2$, for $x \neq 2$

Exercise 5D

1 a



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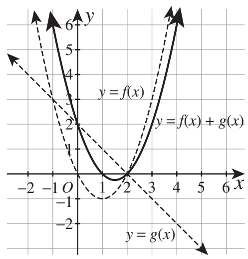

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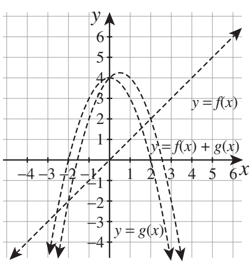
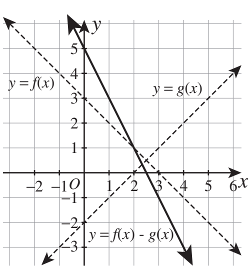


2 a



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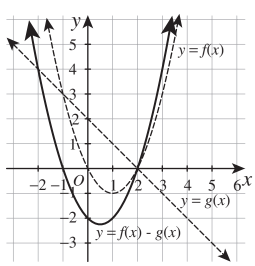

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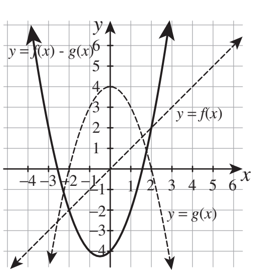

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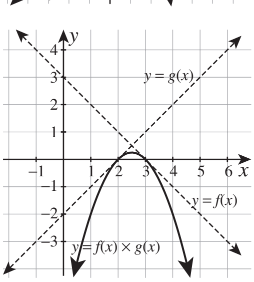

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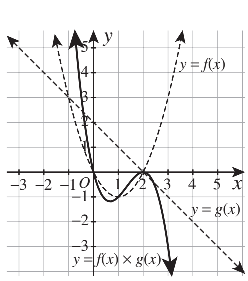

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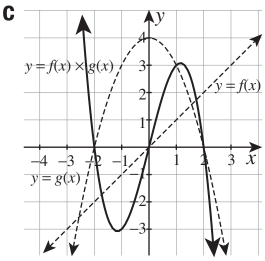

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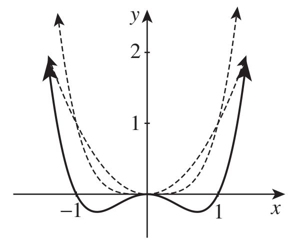

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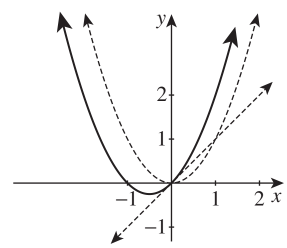

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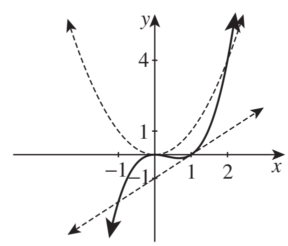

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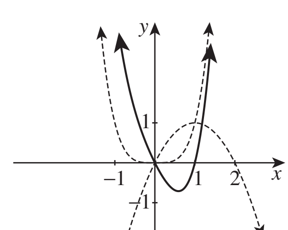
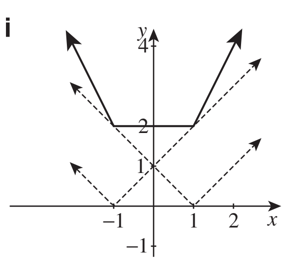


8 a i



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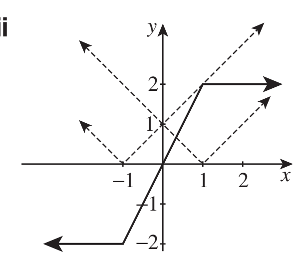
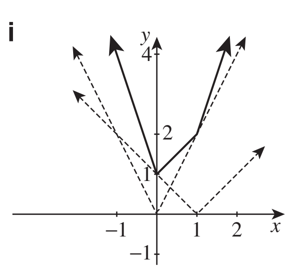


b i



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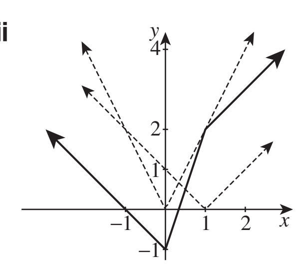

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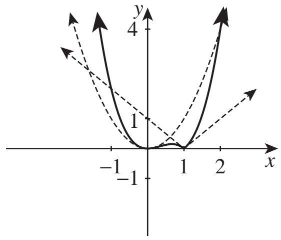
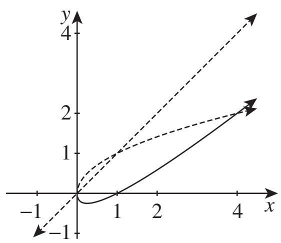
9 b

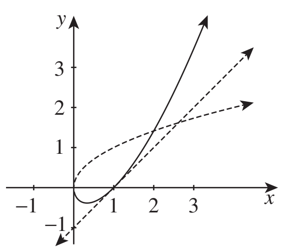


c Because $0 \le x^2 \le 1$ and $0 \le x - 1 \le 1$, the product will also lie between 0 and 1 inclusive.

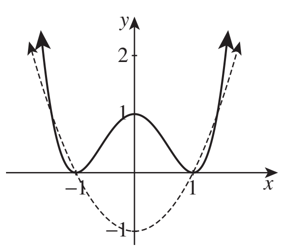
10 b



11



12 a



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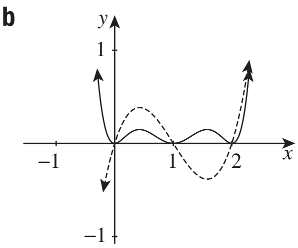

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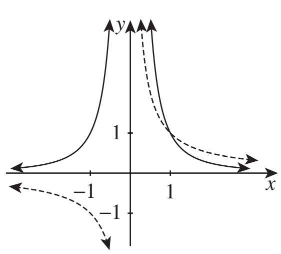

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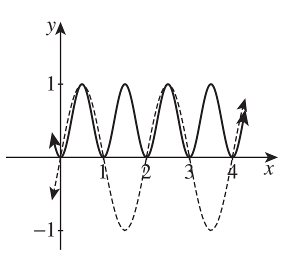
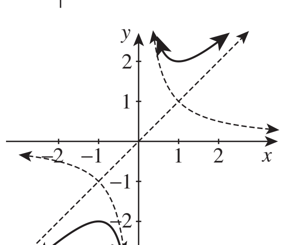


14 a



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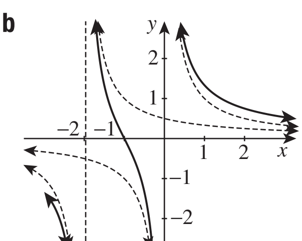

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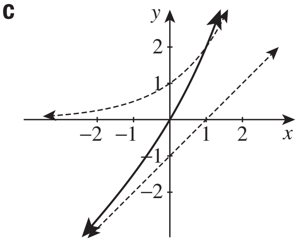

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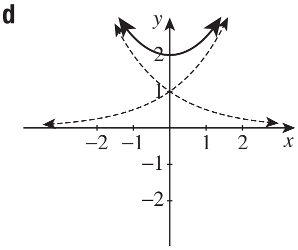

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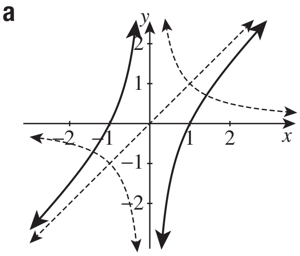

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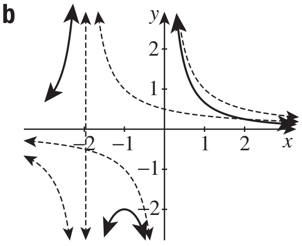

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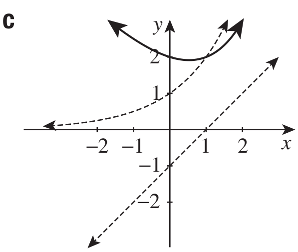

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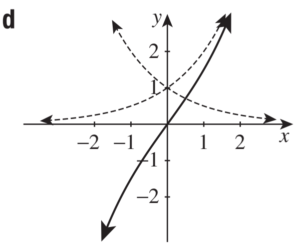


16 a

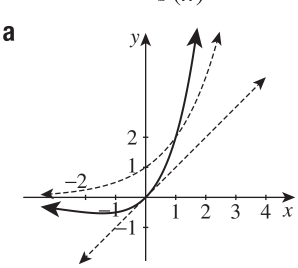
	$f(x)$ even, $g(x)$ even
$s(x)$	even
$d(x)$	even
$p(x)$	even

	$f(x)$ odd, $g(x)$ odd
$s(x)$	odd
$d(x)$	odd
$p(x)$	even

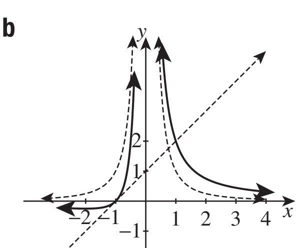
	$f(x)$ even, $g(x)$ odd
$s(x)$	neither
$d(x)$	neither
$p(x)$	odd

b
$$\begin{aligned} s(-x) &= f(-x) + g(-x) \\ &= -f(x) - g(x) \\ &= -\big(f(x) + g(x)\big) \\ &= -s(x) \end{aligned}$$

17 a



b



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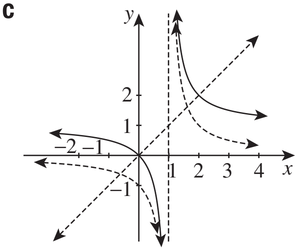
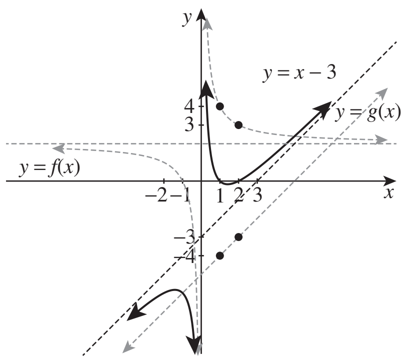
c



18 a $(1,-4)$, $(1,4)$ and $(2,-3)$, $(2,3)$

d

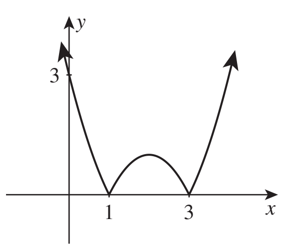


19 a As $x \to \infty$ and as $x \to -\infty$, $s(x) - (x+1) \to 0$.

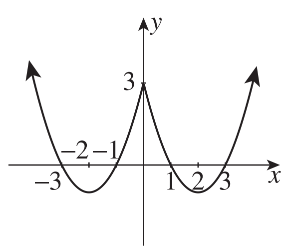
b $y = -x + 5$ **c** $y = 3x - 5$

Exercise 5E

1 a



b



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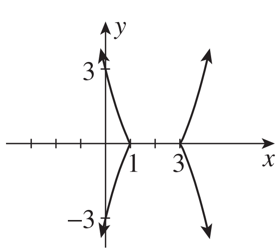
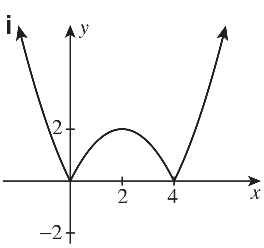


2 a i



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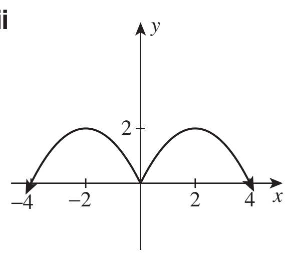

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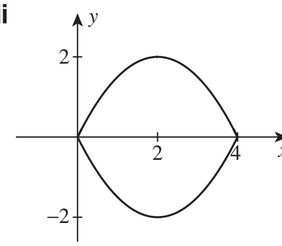
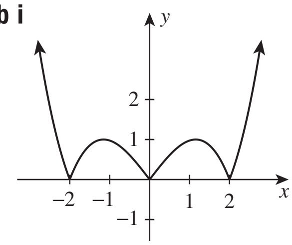


b i



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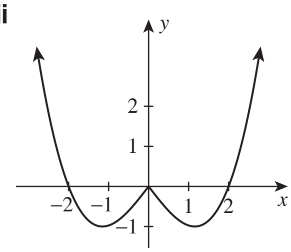

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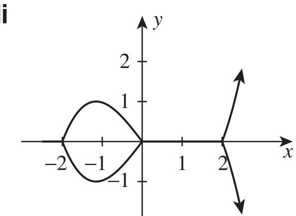
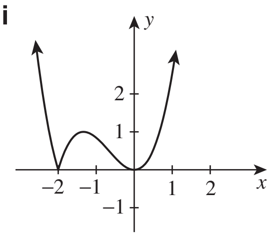


c i



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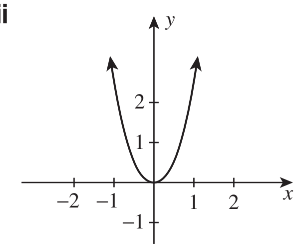

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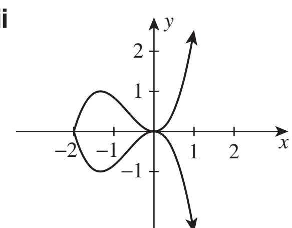
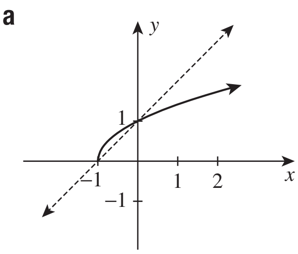


3 a



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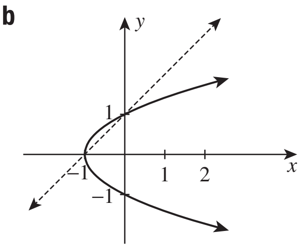
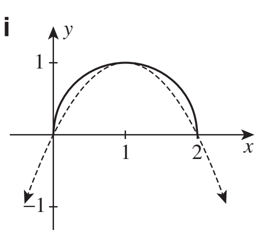


4 a i



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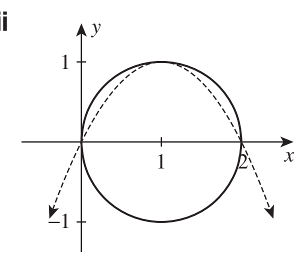

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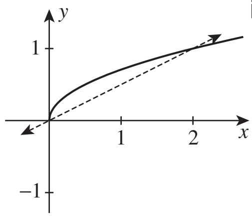
b i



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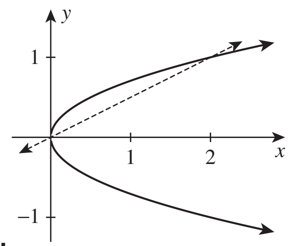
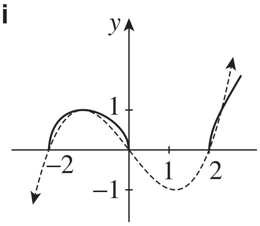


c i



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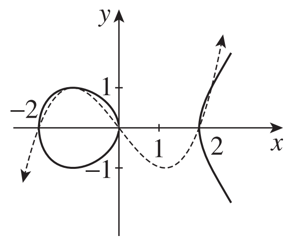
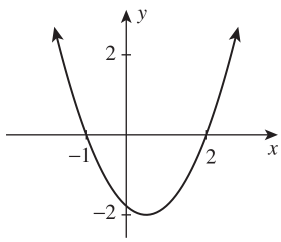
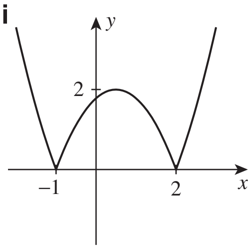


5 a



b i



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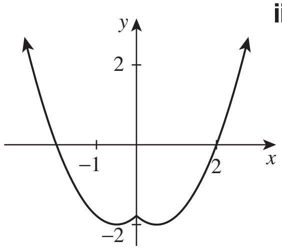

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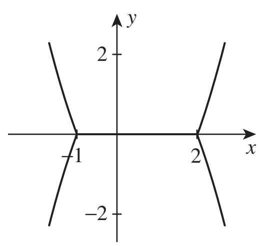
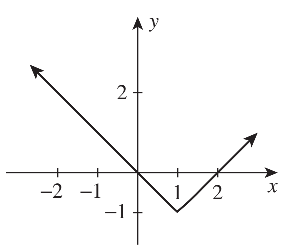
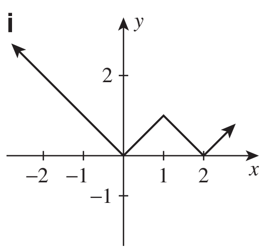


6 a



b i



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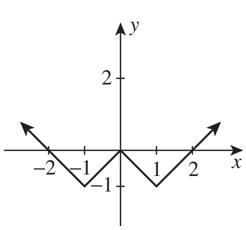

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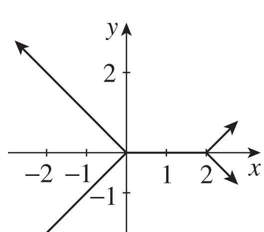
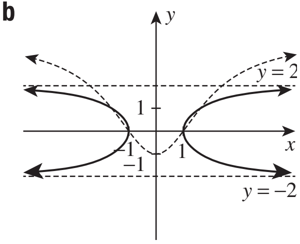
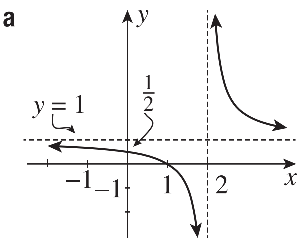


7 a As $x \to \pm\infty$, $\sqrt{f(x)} \to 2$, hence $y = 2$ will be the horizontal asymptote of the transformed graph.

b



8 a



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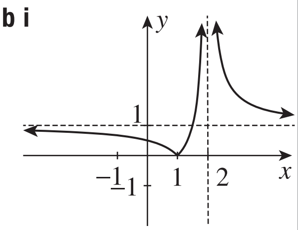

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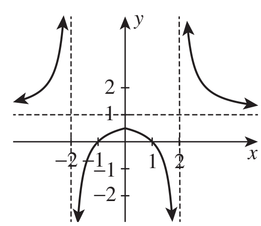

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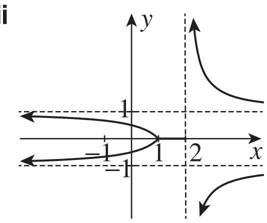
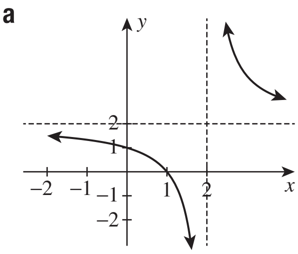
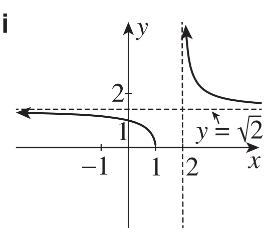


9 a



b i



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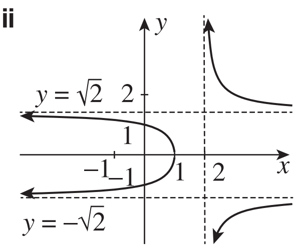
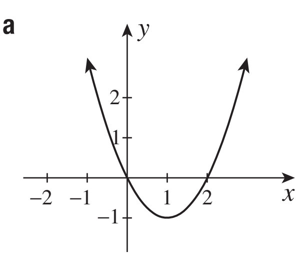


10 a



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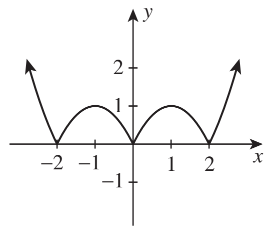
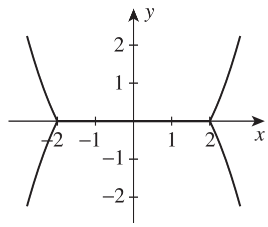
b i



ii



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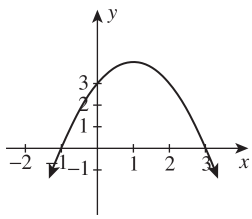
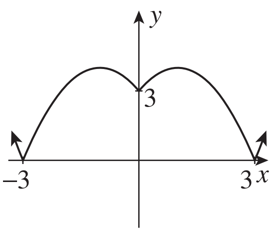


i



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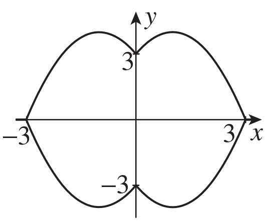


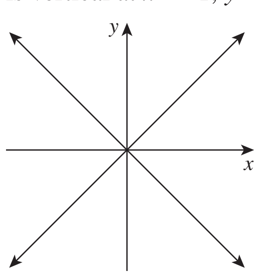
11a i The transformed graph is $y = \sqrt{x - 1}$, which is vertical at $x = 1$ (it is the graph $y = \sqrt{x}$ shifted 1 unit right).

ii The transformed graph is $y = |x - 1|$, which meets the axis at 45°.

iii The transformed graph is $y = (x - 1)^2$, which is horizontal at $x = 1$.

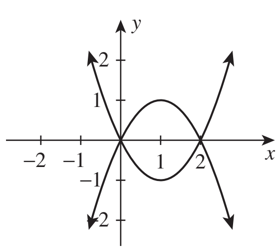
b When $f(x) < 1$, we know that $\sqrt{f(x)} > f(x)$, so that $y = \sqrt{f(x)}$ is always steeper than $y = f(x)$ at a zero of the original function. Because $y = \sqrt{x - 1}$ is vertical at $x = 1$, $y = \sqrt[4]{x - 1}$ must be also.

12

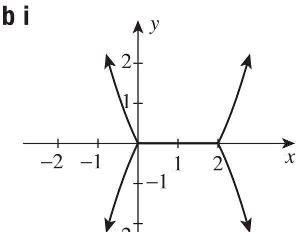


The two lines of the graph are inclined at 45° to the axes.

14 a



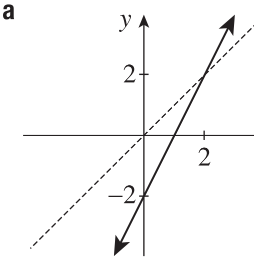
b i



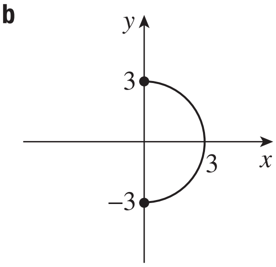
ii First, the parts of the original graph $y = f(x)$ below the x-axis were lost when sketching the function $|y| = f(x)$. Secondly, the parts of the graph of $|y| = f(x)$ below the x-axis will be lost in the steps of Box 14.

Exercise 5F

1 a



b



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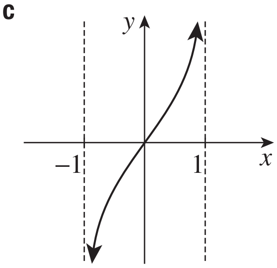
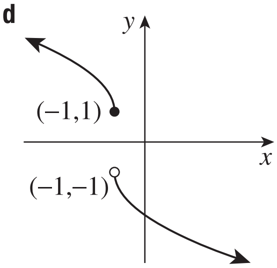
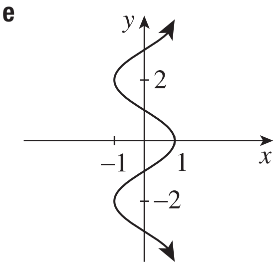


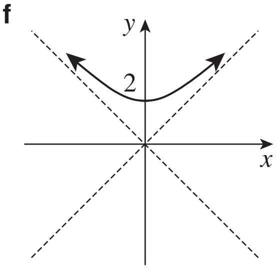
d



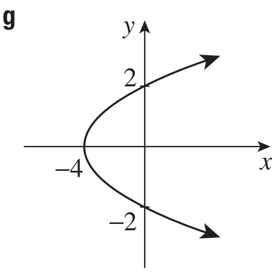
e



f



g



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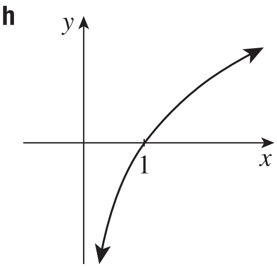

2 Original is a function: all except part **f**. Inverse is a function: part **a**, **c**, **d**, **f**, **h**

3 One-to-one: part **a**, **c**, **d**, **h**. Many-to-one: part **e**, **g**. One-to-many: part **b**, **f**

4 a $y = \dfrac{x + 2}{3}$ **b** $y = 2x - 2$ **c** $y = 6 - 2x$

d $y = x - 1$ **e** $y = -\frac{5}{2}x + 5$ **f** $x = 2$

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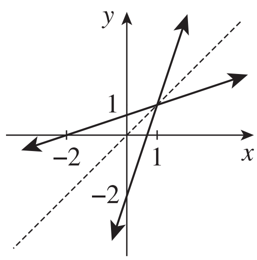
5 a



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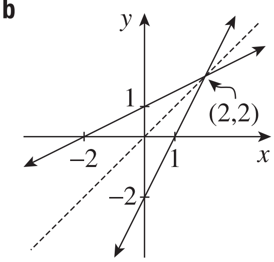

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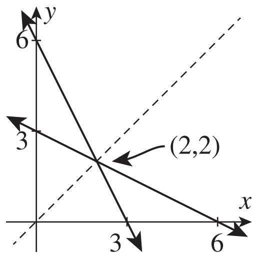

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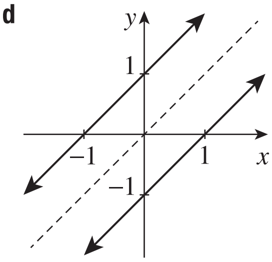

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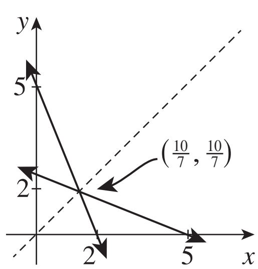

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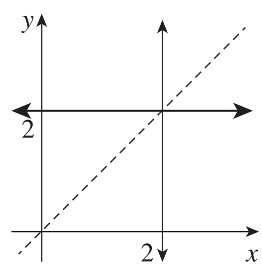


6 a i $y = \dfrac{1}{x-1}$ **ii** $y = \dfrac{1}{x} - 1$ **iii** $y = \dfrac{2x+2}{x-1}$

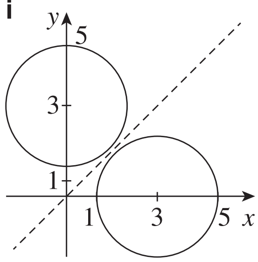
iv $y = \dfrac{2x}{3-x}$

b i For the function, domain: $x \neq 0$, range: $y \neq 1$. For the inverse function, domain: $x \neq 1$, range: $y \neq 0$.

iv For the function, domain: $x \neq -2$, range: $y \neq 3$. For the inverse function, domain: $x \neq 3$, range: $y \neq -2$.

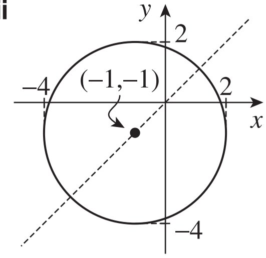
7 Each inverse is identical to the original function. Therefore the graph is symmetric about the line $y = x$.

8 a i



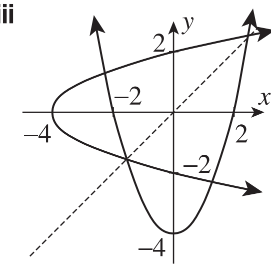
$x^2 + (y-3)^2 = 4$. Neither the original relation nor its inverse is a function.

ii



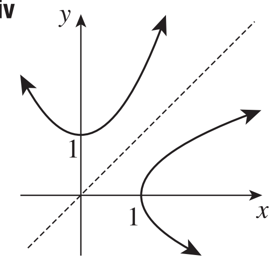
$(x+1)^2 + (y+1)^2 = 9$ The inverse relation is the same as the original relation, and is not a function.

iii



$x = y^2 - 4$. The original relation is a function, but its inverse is not.

iv

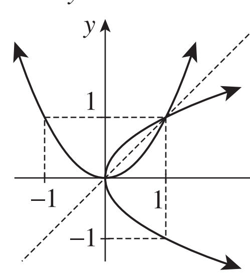


$x = y^2 + 1$. The original relation is a function, but its inverse is not.

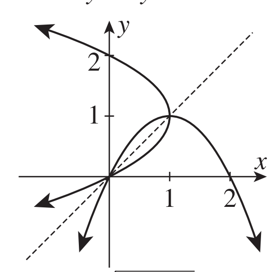
b i For the original, domain: $1 \leq x \leq 7$, range: $-2 \leq y \leq 2$. For the inverse, domain: $-2 \leq x \leq 2$, range: $1 \leq y \leq 7$.

iv For the function, domain: all real x, range: $y \geq 1$. For the inverse function, domain: $x \geq 1$, range: all real y.

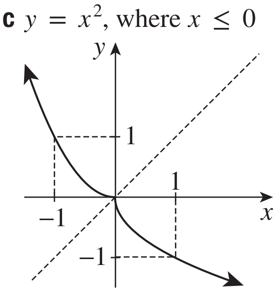
9 a $x = y^2$



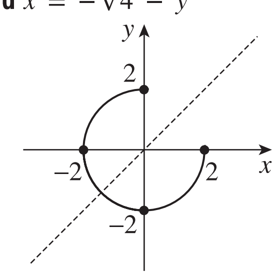
b $x = 2y - y^2$



c $y = x^2$, where $x \leq 0$



d $x = -\sqrt{4 - y^2}$



10 a Inverse: $x = 3y - 10$, where $y < 2$.

Hence $y = \dfrac{1}{6}(x + 10)$, where $x < -4$.

b Inverse: $x = 13 - 6y$, where $y \geq 3$.

Hence $y = \dfrac{1}{6}(13 - x)$, where $x \leq -5$.

c Inverse: $x = y^3 + 2$, where $y < 3$.
Hence $y = \sqrt[3]{x - 2}$, where $x < 29$.

d Inverse: $x = y^2 - 3$, where $y \geq -2$.
Hence $y^2 = 3 + x$, where $y \geq -2$, which is not a function because $x = -2$ corresponds to $y = 1$ and also to $y = -1$

12 a i One-to-one **ii** Many-to-one **iii** One-to-many

b Parts **i** and **iii**

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13 a One-to-one **b** One-to-many
c Many-to-one **d** Many-to-many

16 b No. Look at $y = x^2$, which is even. Its inverse is $x = y^2$, which is not even.

Exercise 5G

1 b They are inverse functions, that is, $g(x) = f^{-1}(x)$ and $f(x) = g^{-1}(x)$.

3 a Let $y = 2x + 5$.
The inverse is $x = 2y + 5$
$2y = x - 5$
$y = \tfrac{1}{2}(x - 5)$
so $f^{-1}(x) = \tfrac{1}{2}(x - 5)$

c i $y = \dfrac{1}{3}(4 - x)$ **ii** $f^{-1}(x) = \sqrt[3]{x + 2}$
iii $f^{-1}(x) = \dfrac{1}{x} + 5$

4 a It fails the horizontal line test, for example $f(1) = f(-1) = 1$, so the inverse is not a function.
b $f^{-1}(x) = x^2$, where $x \ge 0$.
c It fails the horizontal line test, for example $f(1) = f(-1) = 1$, so the inverse is not a function.
d $f^{-1}(x) = (x - 1)^{\frac{1}{3}}$
e It fails the horizontal line test, for example $f(1) = f(-1) = 8$, so the inverse is not a function.
f $f^{-1}(x) = \sqrt{9 - x}$
g It fails the horizontal line test, for example $f(1) = f(-1) = \dfrac{1}{3}$, so the inverse is not a function.
h $f^{-1}(x) = \dfrac{1 - 3x}{1 + x}$ **i** $f^{-1}(x) = -\sqrt{x}$
j $f^{-1}(x) = 1 + \sqrt{1 + x}$ **k** $f^{-1}(x) = 1 - \sqrt{1 + x}$
l $f^{-1}(x) = \dfrac{x + 1}{x - 1}$

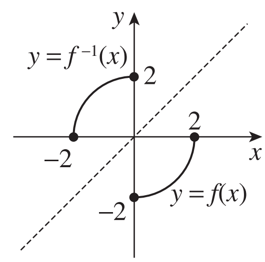
5 b The inverse of the first, $x = -y^2$, is not a function. The second is a natural restriction of the domain of the first so that its inverse $y = \sqrt{-x}$ is a function.

6 a gradient $= a$ **b** $x = ay + b$
c The equation can be solved for y when $a \ne 0$.
or The graph is a non-horizontal line when $a \ne 0$.
d $y = \dfrac{x}{a} - \dfrac{b}{a}$, gradient $= \dfrac{1}{a}$. A non-zero number and its reciprocal have the same sign.
e Reflection in $y = x$ exchanges the rise and run in every gradient construction.

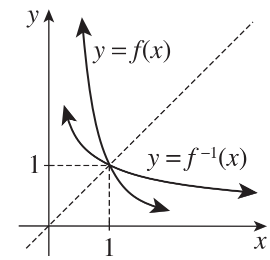
7 a Show that $h^{-1}\big(h(x)\big) = x$ and $h\big(h^{-1}(x)\big) = x$.
b $h^{-1}(x) = \dfrac{1}{x} + 3$
c $h(x) = g\big(f(x)\big)$, where $f(x) = x - 3$ and $g(x) = \dfrac{1}{x}$.

8 a $f^{-1}(x) = \tfrac{1}{3}x + \tfrac{2}{3}$, where $1 \le x \le 10$.
b $f^{-1}\big(f(x)\big)$ has domain $1 \le x \le 4$, and $f\big(f^{-1}(x)\big)$ has domain $1 \le x \le 10$.

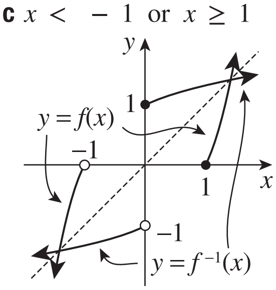
9 a $0 \le x \le 2$



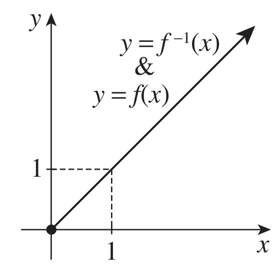
b $x > 0$



c $x < -1$ or $x \ge 1$



d $x \ge 0$



10 a $g\big(f(x)\big) = a\alpha x + b\alpha + \beta$. Put $a\alpha = 1$ and $b\alpha + \beta = 0$
b One example is $f(x) = x + 1$, $g(x) = 2x + 1$, $h(x) = \tfrac{1}{2}x - \tfrac{3}{2}$

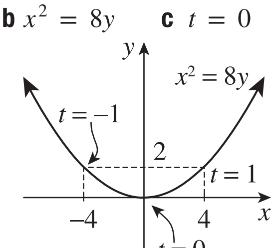
11 The empty function has no ordered pairs, so its inverse relation also has no ordered pairs, and is therefore the empty function. Thus the empty function is the inverse function of itself.

Exercise 5H

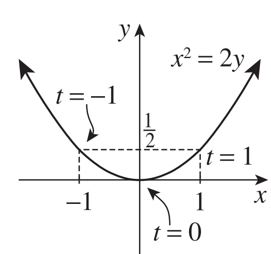
1 a

t	-6	-4	-2	-1	0	1	2	4	6
x	-24	-16	-8	-4	0	4	8	16	24
y	72	32	8	2	0	2	8	32	72

b $x^2 = 8y$ **c** $t = 0$



2 a $x^2 = 2y$ **b** $t = 0$



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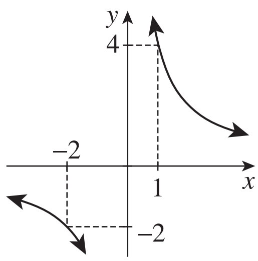
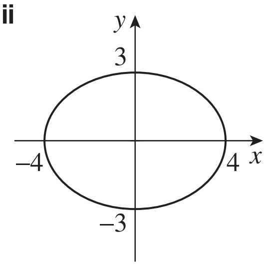
3 c As $p \to \infty$, $x \to \infty$ and $y \to 0$.
As $p \to -\infty$, $x \to -\infty$ and $y \to 0$.
As $p \to 0^+$, $x \to 0$ and $y \to \infty$.
As $p \to 0^-$ $x \to 0$ and $y \to -\infty$



4 b ii



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

5 a

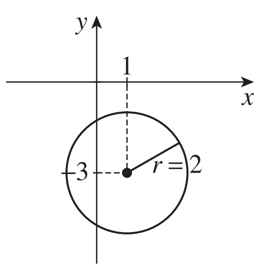
t	-2	-1	0	1	2
x	-4	-3	-2	-1	0
y	-5	-3	-1	-1	3

b When x increases by 1, y increases by 2, so it is a line with gradient 2.
c From the last column, when $x = 0$, $y = 3$.
d $y = 2x + 3$

6 a **i** $A = (-3, -5)$, $B = (-1, 1)$, gradient $= 3$
ii When $x = 0$, $t = 1\frac{1}{2}$, so $y = 4$
iii $y = 3x + 4$
b **i** $y = \frac{3}{2}x + \frac{5}{2}$ **ii** $y = \dfrac{cx}{a} + \dfrac{ad - bc}{a}$

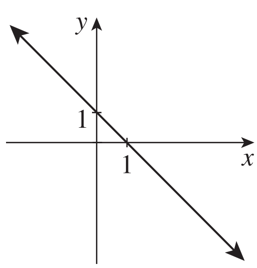
7 a $2x + y - 7 = 0$
b $4(y + 4)^2 - 9(x - 1)^2 = 36$
c $y = x^2 - 2$
d $x^2 + y^2 = 2$

8 b

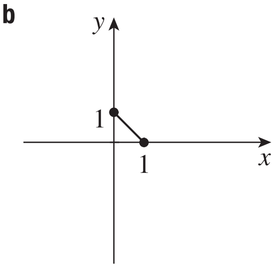


9 The point $(1, 0)$ is missing, because when $y = 0$, $t = 0$, so $x = -1$.

10 a



b



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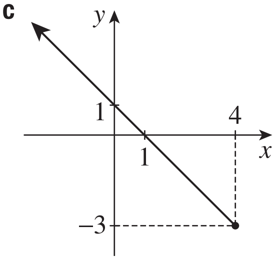


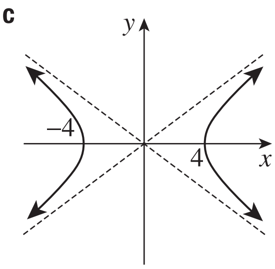
11 a $(x - 3)^2 + (y + 2)^2 = r^2$, circle with centre $(3, -2)$ and radius r
b $y = x\tan\theta - (3\tan\theta + 2)$, straight line with gradient $\tan\theta$

12 a The point $(0, 1)$ is missing, because when $x = 0$, $t = -\frac{1}{2}$, so $y = -1$. **b** One-to-one

13 a Without the variable z and the third equation, the curve would be a circle. Because of the third equation, as t increases, the height z of the curve in the third dimension increases, so the curve never meets back up with itself. Instead it describes a spiral heading upwards (and downwards as $t \to -\infty$), with the curve remaining distant 1 unit from the z-axis.
b One-to-one

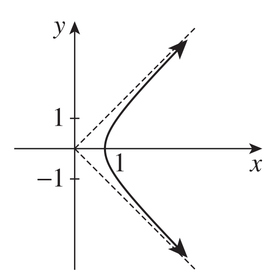
$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

14 c



d Many-to-one

15 Cartesian equation:
$x^2 - y^2 = 1$,
where $x > 0$.



16 a **i** Nothing **ii** Rotation of 180° about O
iii Reflection in the x-axis
iv Reflection in the y-axis

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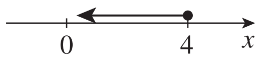
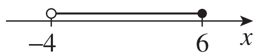
b They are inverse relations.
c The graph is all in the first quadrant.
d The graph is a subset of the line $y = x$.

Chapter 5 review exercise

1 a 

$x \le 4$

b 

$-4 < x \le 6$

c 

$x > -12.$

2 a $3 \le x \le 5$ **b** $x < 0$ or $x > 6$
c $x < -\frac{4}{3}$ or $x > 3$

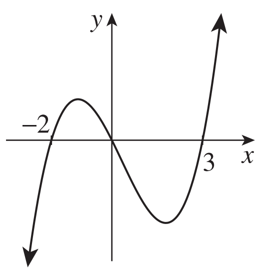
3 a $-3 < x < 3$ **b** $x \le -6$ or $x \ge 2$
c $-3 \le x \le 8$

4 a $0 < x < 5$ **b** $x < 3$ or $x \ge 6$
c $-2 \le x < -1$

5 a The zeroes are -2, 0 and 3.

x	-3	-2	-1	0	1	3	4
y	-18	0	4	0	-6	0	24
sign	$-$	0	$+$	0	$-$	0	$+$

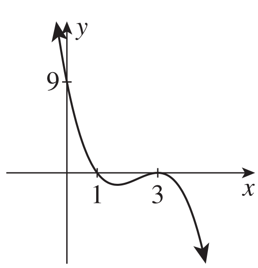
b $f(x)$ is positive for $-2 < x < 0$ and for $x > 3$, and negative for $x < -2$ and for $0 < x < 3$.
c $x \le -2$ or $0 \le x \le 3$
d



6 a Zeroes are 1 and 3.

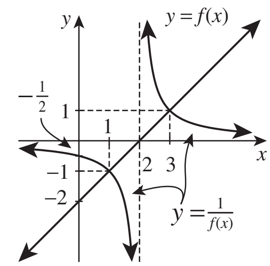
x	0	1	2	3	4
y	9	0	-1	0	-3
sign	$+$	0	$-$	0	$-$

b $x \le 1$ or $x = 3$
c

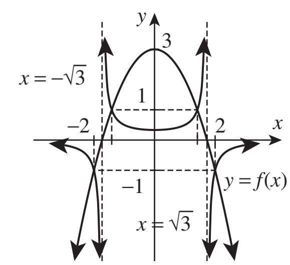


7 a $0 < x < 5$ **b** $x < 3$ or $x \ge 6$
c $-2 \le x < -1$

8

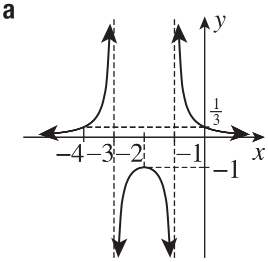


9



d As $x \to 2^-$,
$y \to -\infty$, and as
$x \to 2^+, y \to +\infty$.

10 a



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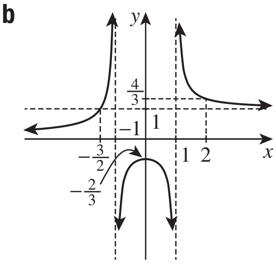
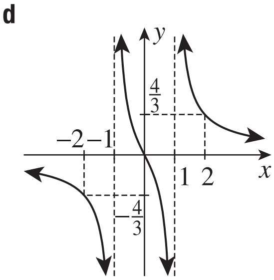


11 a i Vertical asymptote: $x = -1$.
ii Vertical asymptote: $x = 2$.
iii Vertical asymptotes: $x = 5$ and $x = -5$.
b Zero: $x = 0$, discontinuities: $x = -5$ and $x = 5$.

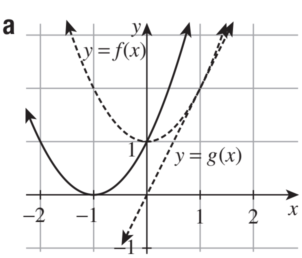
x	-6	-5	-1	0	1	5	6
y	$-\frac{24}{11}$	*	$\frac{1}{6}$	0	$-\frac{1}{6}$	*	$\frac{24}{11}$
sign	$-$	*	$+$	0	$-$	*	$+$

As $x \to (-5)^-$, $y \to -\infty$, and as $x \to (-5)^+$, $y \to \infty$. As $x \to 5^-$, $y \to -\infty$, and as $x \to 5^+$, $y \to \infty$.

12 d



13 a



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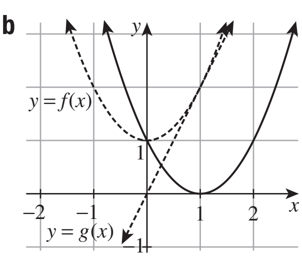

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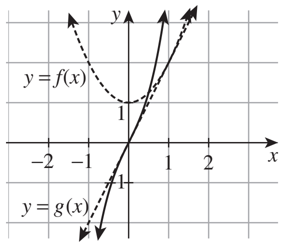
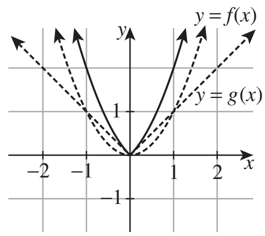
c



14 a i



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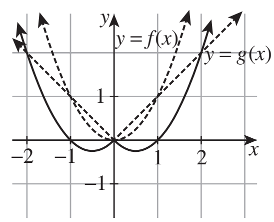
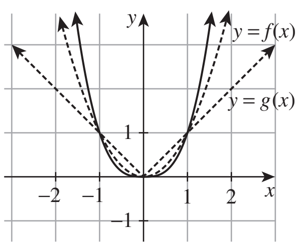
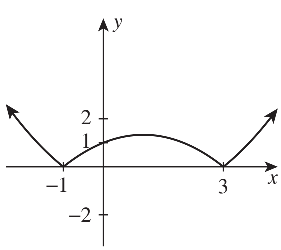


iii

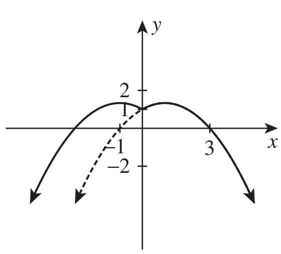


b The original graphs and your answers should be even.

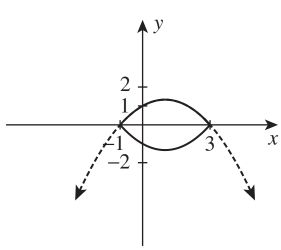
15 a



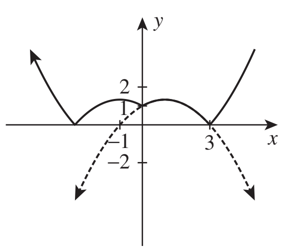
b



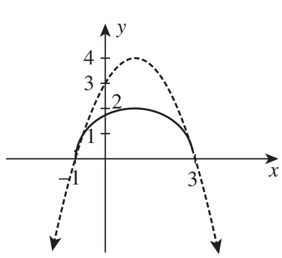
c



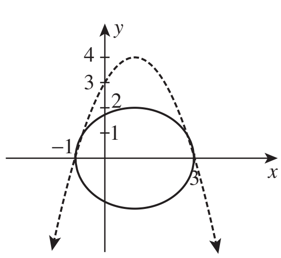
d



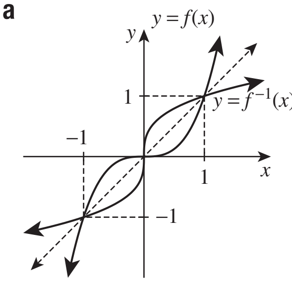
16 a



b

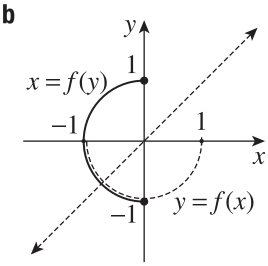


17 a



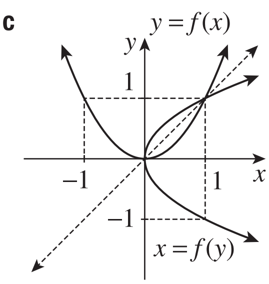
Inverse is a function

b



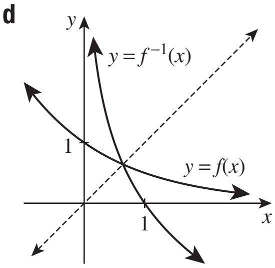
Inverse is not a function

c



Inverse is not a function

d



Inverse is a function

18 a $y = \frac{1}{3}(5 - x)$ **b** $y = \dfrac{5}{x} + 3$ **c** $y = \dfrac{3x}{x - 5}$

d $y = \sqrt[3]{x - 5}$

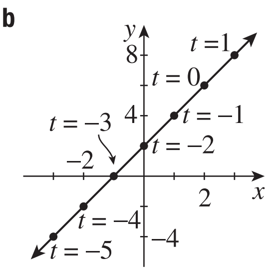
19 a $f^{-1}(x) = 2(x - 4)$ **b** $f^{-1}(x) = \sqrt[3]{x} - 2$

c $f^{-1}(x) = \dfrac{3}{x + 6}$

20 a

t	−5	−4	−3	−2	−1	0	1
x	−3	−2	−1	0	1	2	3
y	−4	−2	0	2	4	6	8

b



c $y = 2x + 2$

21 a

t	−6	−4	−2	−1	0	1	2	4	6
x	−3	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
y	9	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4	9

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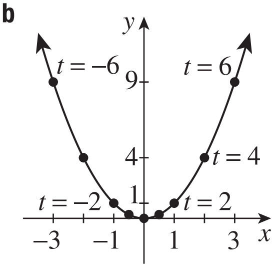
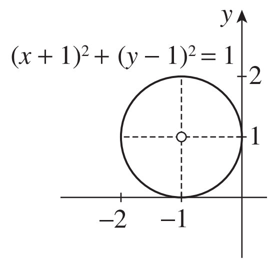
b



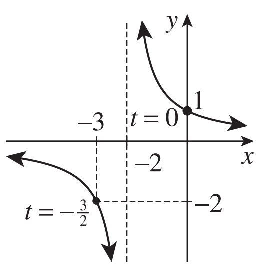
c $y = x^2$

22



a $(x + 1)^2 + (y - 1)^2 = 1$
b It is a circle with centre $(-1, 1)$ and radius 1 unit.

23



$y = \dfrac{2}{x + 2}$

Chapter 6

Exercise 6A

1 a $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{4}{5}$ **e** $\frac{3}{5}$ **f** $\frac{4}{3}$
2 a 0.4067 **b** 0.4848 **c** 0.7002 **d** 0.9986
e 0.0349 **f** 0.8387 **g** 0.0175 **h** 0.9986
3 a 1.5697 **b** 0.8443 **c** 4.9894 **d** 0.9571
e 0.6833 **f** 0.1016 **g** 0.0023 **h** 0.0166
4 a 76° **b** 46° **c** 12° **d** 27°
e No such angle — $\cos\ \theta$ cannot exceed 1.
f 39° **g** 60°
h No such angle — $\sin\ \theta$ cannot exceed 1.
5 a 41°25′ **b** 63°26′ **c** 5°44′ **d** 16°42′
e 46°29′ **f** 57°25′
6 a 13 **b** 19 **c** 23 **d** 88
7 a 53° **b** 41° **c** 67° **d** 59°
8 a $\frac{12}{13}$ **b** $\frac{5}{12}$ **c** $\frac{13}{12}$ **d** $\frac{5}{12}$ **e** $\frac{13}{12}$ **f** $\frac{13}{5}$
9 a 6 and 17
b i $\frac{15}{17}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$ **iv** $\frac{17}{8}$ **v** $\frac{5}{3}$ **vi** $\frac{15}{8}$
10 a $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{3}}$ **c** $\frac{1}{\sqrt{2}}$ **d** 2 **e** $\sqrt{2}$ **f** $\sqrt{3}$
11 a 19.2 **b** 21.6 **c** 30.3 **d** 8.3
12 a 29.78 **b** 10.14 **c** 16.46 **d** 29.71
13 a 36°2′ **b** 68°38′ **c** 34°44′ **d** 38°40′
e 54°19′ **f** 70°32′
14 b 3 **c** $\frac{1}{3}\sqrt{5}, \frac{2}{3}$
15 a i $\frac{1}{2}\sqrt{22}$ **ii** $\frac{3}{2}\sqrt{2}$
16 a 1 **b** $\frac{1}{2}$ **c** 4 **d** 1
18 a $\angle QPR = 90° - \theta$, so $\angle RPS = \theta$.
b $\dfrac{h}{a}$ and $\dfrac{b}{h}$
21 a 108°

Exercise 6B

1 2.65 metres
2 63°
3 55 km
4 038°T
5 13.2 metres
6 2.5 metres
7 77 km
8 23 metres
9 73°
10 21.3 metres
11 11°
12 a 46° **b** 101°T
13 b 67 km
14 a $\angle PQR = 360° - (200° + 70°) = 90°$ (using co-interior angles on parallel lines and the fact that a revolution is 360°)
b $110° + 39° = 149°$T
15 a 5.1 cm **b** 16 cm
c $PQ = 18 \sin\ 40°$, 63°25′
17 457 metres
18 a $y = x \tan\ 39°$ and $y + 7 = x \tan\ 64°$
19 a If $\angle RBQ = \alpha$, then $\angle RQB = 90° - \alpha$ (angle sum of ΔBQR) and so $\angle RQP = \alpha$ (complementary angles). Therefore $\angle QPR = 90° - \alpha$ (angle sum of ΔPQR) and so $\angle QPC = \alpha$ (complementary angles). Thus $\angle RBQ = \angle RQP = \angle QPC$.
22 a If $OA = OB = x$ and $OP = y$, then $AP - PB = (x + y) - (x - y) = 2y = 2 \times OP$.

Exercise 6C

1 a 15 cm **b** 17 cm **c** 28°
2 a i 90° **ii** 90° **iii** 90° **b i** $\sqrt{2}$ **ii** $\sqrt{3}$
c i 35° **ii** 35°
3 a i $2\sqrt{5}$ cm **ii** $2\sqrt{6}$ cm **b** 90° **c** 66°
4 a i 90° **ii** 90° **iii** 90° **b i** 2 cm **ii** $2\sqrt{2}$ cm
c i 72° **ii** 65°
5 a i 90° **ii** 90° **b** 27°
6 a $3\sqrt{2}$ cm **b** 43°
7 a $BQ = 30 \tan\ 72°$ **b** 145 m

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8 b 16 m **c** 21°

9 b 76 m **c** 14°

10 a 1 cm **b** $\sqrt{2}$ cm **c** $\sqrt{2}$ **d** $70^\circ 32'$

11 c $67^\circ 23'$

12 a $h \cot 55^\circ$

b It is the angle between south and east.

d 114 metres

13 b 13 metres

14 a $AT = h\operatorname{cosec} 55^\circ$, $BT = h\operatorname{cosec} 40^\circ$

b 90° **d** 52 metres

Exercise 6D

3 a -320° **b** -250° **c** -170°

d -70° **e** -300° **f** -220°

4 a 310° **b** 230° **c** 110°

d 10° **e** 280° **f** 170°

5 a $70^\circ, 430^\circ, -290^\circ, -650^\circ$

b $100^\circ, 460^\circ, -260^\circ, -620^\circ$

c $140^\circ, 500^\circ, -220^\circ, -580^\circ$

d $200^\circ, 560^\circ, -160^\circ, -520^\circ$

e $240^\circ, 600^\circ, -120^\circ, -480^\circ$

f $340^\circ, 700^\circ, -20^\circ, -380^\circ$

6 a $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\operatorname{cosec}\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

b $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\operatorname{cosec}\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

c $\sin\theta = -\frac{2}{\sqrt{5}}$, $\cos\theta = -\frac{1}{\sqrt{5}}$, $\tan\theta = 2$, $\operatorname{cosec}\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = \frac{1}{2}$

d $\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$, $\operatorname{cosec}\theta = -\frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = -\frac{12}{5}$

7 All six trigonometric functions are sketched in Section 6E.

8 a i 0.5 **ii** -0.5 **iii** 0.95 **iv** 0.95 **v** 0.59

vi 0.81 **vii** -0.89 **viii** 0.45 **ix** -0.81 **x** 0.59

b i $30^\circ, 150^\circ$ **ii** $120^\circ, 240^\circ$ **iii** $64^\circ, 116^\circ$

iv $53^\circ, 307^\circ$ **v** $53^\circ, 127^\circ$ **vi** $143^\circ, 217^\circ$

vii $204^\circ, 336^\circ$ **viii** $107^\circ, 253^\circ$

c $45^\circ, 225^\circ$

10 $\tan(\theta + 90^\circ) = \dfrac{\sqrt{1 - k^2}}{k}$

Exercise 6E

1 a + **b** + **c** – **d** – **e** + **f** – **g** –

h + **i** – **j** + **k** – **l** – **m** – **n** +

o + **p** –

2 a 10° **b** 30° **c** 50° **d** 20° **e** 80°

f 70° **g** 70° **h** 80° **i** 10° **j** 20°

3 a $-\tan 50^\circ$ **b** $\cos 50^\circ$ **c** $-\sin 40^\circ$ **d** $\tan 80^\circ$

e $-\cos 10^\circ$ **f** $-\sin 40^\circ$ **g** $-\cos 5^\circ$ **h** $\sin 55^\circ$

i $-\tan 35^\circ$ **j** $\sin 85^\circ$ **k** $-\cos 85^\circ$ **l** $\tan 25^\circ$

4 a 0 **b** -1 **c** 0

d 0 **e** 1 **f** 1

g -1 **h** undefined **i** 0

j 0 **k** undefined **l** 0

5 a $\frac{\sqrt{3}}{2}$ **b** $\frac{\sqrt{3}}{2}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $-\frac{\sqrt{3}}{2}$ **e** $\frac{1}{\sqrt{2}}$ **f** $-\frac{1}{\sqrt{2}}$

g $-\frac{1}{\sqrt{2}}$ **h** $\frac{1}{\sqrt{2}}$ **i** $\frac{1}{\sqrt{3}}$ **j** $-\frac{1}{\sqrt{3}}$ **k** $\frac{1}{\sqrt{3}}$ **l** $-\frac{1}{\sqrt{3}}$

6 a $-\frac{1}{2}$ **b** 1 **c** $-\frac{1}{2}$ **d** $\frac{1}{\sqrt{2}}$ **e** $\sqrt{3}$ **f** $-\frac{\sqrt{3}}{2}$

g -1 **h** $\frac{1}{2}$ **i** $-\frac{1}{\sqrt{2}}$ **j** $-\frac{\sqrt{3}}{2}$ **k** $-\frac{1}{2}$ **l** $-\sqrt{3}$

7 a 2 **b** $-\sqrt{2}$ **c** $-\frac{1}{\sqrt{3}}$ **d** $\sqrt{3}$ **e** $\frac{2}{\sqrt{3}}$ **f** $-\frac{2}{\sqrt{3}}$

8 a 1 **b** -1 **c** undefined

d undefined **e** 0 **f** undefined

9 a 60° **b** 20° **c** 30° **d** 60°

e 70° **f** 10° **g** 50° **h** 40°

10 a $\frac{1}{2}$ **b** $-\frac{\sqrt{3}}{2}$ **c** $\sqrt{3}$ **d** $\frac{1}{\sqrt{2}}$ **e** $-\frac{1}{\sqrt{3}}$ **f** $-\frac{1}{\sqrt{2}}$

g $\sqrt{3}$ **h** $-\frac{\sqrt{3}}{2}$ **i** $\frac{1}{\sqrt{2}}$ **j** $-\frac{1}{2}$ **k** $-\frac{1}{2}$ **l** 1

11 a 0.42 **b** -0.91 **c** 0.91

d -0.42 **e** 0.49 **f** 0.49

12 a -0.70 **b** -1.22 **c** -0.70

d -0.52 **e** 1.92 **f** -0.52

14 a $-\sin\theta$ **b** $\cos\theta$ **c** $-\tan\theta$ **d** $\sec\theta$

e $\sin\theta$ **f** $-\sin\theta$ **g** $-\cos\theta$ **h** $\tan\theta$

15 a $\left(2, 2\sqrt{3}\right)$ **b** $\left(-\sqrt{3}, 1\right)$

c $(1, -1)$ **d** $\left(-5, -5\sqrt{3}\right)$

16 a $53^\circ 8'$ **b** $138^\circ 11'$ **c** 300° **d** $213^\circ 41'$

17 All six graphs are many-to-one.

19 a $y = \sin\theta$ and $y = \cos\theta$ have range $-1 \le y \le 1$, $y = \tan\theta$ and $y = \cot\theta$ have range $\mathbb{R}$, $y = \sec\theta$ and $y = \operatorname{cosec}\theta$ have range $y \ge 1$ or $y \le -1$.

b $\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ and $\sec\theta$ have period 360°; $\tan\theta$ and $\cot\theta$ have period 180°.

c $\sin\theta$, $\operatorname{cosec}\theta$, $\tan\theta$ and $\cot\theta$ are odd; $\cos\theta$ and $\sec\theta$ are even.

d The graphs have point symmetry about every θ-intercept, and about every point where an asymptote crosses the θ axis.

e $\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ and $\sec\theta$ have line symmetry in every vertical line through a maximum or minimum; $\tan\theta$ and $\cot\theta$ have no axes of symmetry.

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Exercise 6F

1 a $\sin\theta = \frac{15}{17}, \cos\theta = \frac{8}{17}, \tan\theta = \frac{15}{8}$

b $\sin\theta = \frac{4}{5}, \cos\theta = -\frac{3}{5}, \tan\theta = -\frac{4}{3}$

c $\sin\theta = -\frac{7}{25}, \cos\theta = -\frac{24}{25}, \tan\theta = \frac{7}{24}$

d $\sin\theta = -\frac{21}{29}, \cos\theta = \frac{20}{29}, \tan\theta = -\frac{21}{20}$

2 a $y = 12, \sin\alpha = \frac{12}{13}, \cos\alpha = \frac{5}{13}, \tan\alpha = \frac{12}{5}$

b $r = 3, \sin\alpha = \frac{2}{3}, \cos\alpha = -\frac{\sqrt{5}}{3}, \tan\alpha = -\frac{2}{\sqrt{5}}$

c $x = -4, \sin\alpha = -\frac{3}{5}, \cos\alpha = -\frac{4}{5}, \tan\alpha = \frac{3}{4}$

d $y = -3, \sin\alpha = -\frac{3}{\sqrt{13}}, \cos\alpha = \frac{2}{\sqrt{13}}, \tan\alpha = -\frac{3}{2}$

3 a i $\sin\theta = -\frac{4}{5}$ **ii** $\tan\theta = -\frac{4}{3}$

b i $\sin\theta = \frac{5}{13}$ **ii** $\cos\theta = -\frac{12}{13}$

4 a $\cos\theta = -\frac{3}{4}$ and $\tan\theta = \frac{\sqrt{7}}{3}$, or $\cos\theta = \frac{3}{4}$ and $\tan\theta = -\frac{\sqrt{7}}{3}$

b $\sin\theta = \frac{\sqrt{15}}{4}$ and $\tan\theta = -\sqrt{15}$, or $\sin\theta = -\frac{\sqrt{15}}{4}$ and $\tan\theta = \sqrt{15}$

5 a $2\sqrt{2}$ **b** $-\frac{3}{4}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $\frac{3}{\sqrt{13}}$ **e** $\frac{9}{41}$ **f** $\frac{1}{2}$

6 a $\frac{1}{\sqrt{10}}$ or $-\frac{1}{\sqrt{10}}$ **b** $\frac{1}{\sqrt{5}}$ or $-\frac{1}{\sqrt{5}}$ **c** $\frac{4}{5}$ or $-\frac{4}{5}$

d $\frac{\sqrt{5}}{2}$ or $-\frac{\sqrt{5}}{2}$ **e** $\frac{12}{5}$ or $-\frac{12}{5}$ **f** $\frac{\sqrt{3}}{\sqrt{7}}$ or $-\frac{\sqrt{3}}{\sqrt{7}}$

7 a $-\frac{3}{4}$ **b** $-\frac{15}{17}$ **c** $-\frac{\sqrt{15}}{4}$

d $\frac{35}{37}$ **e** $-\frac{21}{20}$ **f** $\frac{\sqrt{11}}{6}$

8 a $\sqrt{2}$ or $-\sqrt{2}$ **b** $\frac{15}{8}$ or $-\frac{15}{8}$

c $\frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$ **d** $\frac{6}{5}$ or $-\frac{6}{5}$

9 a $-\frac{3}{\sqrt{5}}$ **b** $-\frac{41}{9}$ **c** $-\frac{15}{8}$ **d** $\frac{\sqrt{7}}{\sqrt{3}}$

10 a $\frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$ **b** $-\frac{3}{2\sqrt{10}} = -\frac{3\sqrt{10}}{20}$

c 1 **d** $\frac{12}{13}$

11 $\cos\theta = -\frac{\sqrt{q^2 - p^2}}{q}, \tan\theta = -\frac{p}{\sqrt{q^2 - p^2}}$

12 $\sin\alpha = \frac{k}{\sqrt{1 + k^2}}$ or $-\frac{k}{\sqrt{1 + k^2}}$,

$\sec\alpha = \sqrt{1 + k^2}$ or $-\sqrt{1 + k^2}$

13 b $\sin x = \frac{2t}{1 + t^2}, \tan x = \frac{2t}{1 - t^2}$

14 Note that $\tan\theta$ could be positive or negative.

Exercise 6G

2 a $\operatorname{cosec}\theta$ **b** $\cot\alpha$ **c** $\tan\beta$ **d** $\cot\phi$

3 a 1 **b** 1 **c** 1

5 a $\cos\theta$ **b** $\operatorname{cosec}\alpha$ **c** $\cot\beta$ **d** $\tan\phi$

6 a 1 **b** $\sin^2\beta$ **c** $\sec^2\phi$ **d** 1

7 a $\cos^2\beta$ **b** $\operatorname{cosec}^2\phi$ **c** $\cot^2 A$ **d** -1

8 a $\cos^2\theta$ **b** $\tan^2\beta$ **c** $\cot^2 A$ **d** 1

10 a $\cos^2\alpha$ **b** $\sin^2\alpha$ **c** $\sin A$ **d** $\cos A$

14 a $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **b** $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

c $(x - 2)^2 + (y - 1)^2 = 1$ **d** $x^2 + y^2 = 2$

15 a 2 **b** 0 **c** 1 **d** 0

18 a $y - x = 1$ **b** $x^2 + 2xy + 2y^2 = 5$

c $x^2 y = y + 2$

Exercise 6H

1 a $\theta = 60°$ or $120°$ **b** $\theta = 30°$ or $150°$

c $\theta = 45°$ or $225°$ **d** $\theta = 60°$ or $240°$

e $\theta = 135°$ or $225°$ **f** $\theta = 120°$ or $300°$

g $\theta = 210°$ or $330°$ **h** $\theta = 150°$ or $210°$

2 a $\theta = 90°$ **b** $\theta = 0°$ or $360°$

c $\theta = 90°$ or $270°$ **d** $\theta = 180°$

e $\theta = 0°$ or $180°$ or $360°$ **f** $\theta = 270°$

3 a $x \doteqdot 65°$ or $295°$ **b** $x \doteqdot 7°$ or $173°$

c $x \doteqdot 82°$ or $262°$ **d** $x \doteqdot 222°$ or $318°$

e $x \doteqdot 114°$ or $294°$ **f** $x \doteqdot 140°$ or $220°$

4 a $\alpha \doteqdot 5°44'$ or $174°16'$ **b** $\alpha \doteqdot 95°44'$ or $264°16'$

c $\alpha = 135°$ or $315°$ **d** $\alpha = 270°$

e no solutions **f** $\alpha = 120°$ or $240°$

g $\alpha = 150°$ or $330°$ **h** $\alpha \doteqdot 18°26'$ or $198°26'$

5 a $x \doteqdot -16°42'$ or $163°18'$ **b** $x = 90°$ or $-90°$

c $x = 45°$ or $-45°$

d $x \doteqdot -135°34'$ or $-44°26'$

6 a $\theta = 60°, 300°, 420°$ or $660°$

b $\theta = 90°, 270°, 450°$ or $630°$

c $\theta = 210°, 330°, 570°$ or $690°$

d $\theta = 22°30', 202°30', 382°30'$ or $562°30'$

7 a $x = 15°, 75°, 195°$ or $255°$

b $x = 30°, 120°, 210°$ or $300°$

c $x = 67°30', 112°30', 247°30'$ or $292°30'$

d $x = 135°$ or $315°$

8 a $\alpha = 75°$ or $255°$ **b** $\alpha = 210°$ or $270°$

c $\alpha = 300°$ **d** $\alpha = 210°$ or $300°$

9 a $\theta = 45°$ or $225°$ **b** $\theta = 135°$ or $315°$

c $\theta = 60°$ or $240°$ **d** $\theta = 150°$ or $330°$

10 a $\theta = 0°, 90°, 180°$ or $360°$

b $\theta = 60°, 90°, 270°$ or $300°$

c $\theta = 0°, 60°, 180°, 300°$ or $360°$

d $\theta = 30°, 150°$ or $270°$

e $\theta = 60°$ or $300°$, or $\theta \doteqdot 104°29'$ or $255°31'$

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f $\theta \doteqdot 70°32'$ or $289°28'$

g $\theta \doteqdot 23°35', 156°25', 221°49'$ or $318°11'$

h $\theta = 0°, 60°, 120°, 180°, 240°, 300°$ or $360°$

11 Show that $\sin\theta = \frac{1 \pm \sqrt{5}}{4}$,

then $\theta = 54°, 126°, 198°$ or $342°$.

Exercise 6I

1 a 8.2 **b** 4.4 **c** 4.9 **d** 1.9 **e** 9.2 **f** 3.5

2 a 14.72 **b** 46.61 **c** 5.53

3 a 49° **b** 53° **c** 43° **d** 20° **e** 29° **f** 42°

4 a 5 cm^2 **b** 19 cm^2 **c** 22 cm^2

5 a $b \doteqdot 10.80$ cm, $c \doteqdot 6.46$ cm

6 a $49°46'$ **b** $77°53'$ **c** 3.70 cm^2

7 42°, 138°

8 62°, 118°

9 a $69°2'$ or $110°58'$ **b** 16.0 cm or 11.0 cm

10 317 km

11 b 9 metres

12 a 32 **b** $\frac{5}{7}$

13 a 16 metres **b** 11.35 metres **c** 3.48 metres

14 a 30° or 150° **b** $17°27'$ or $162°33'$

c No solutions, because $\sin\theta = 1.2$ is impossible.

15 a $3\sqrt{6}$ **b** $3\sqrt{2}$ **c** $2\sqrt{6}$ **d** $6\sqrt{2}$

16 11.0 cm

17 a $\angle QSM = 36°$ (angle sum of ΔQRS) and $\angle PSM = 48°$ (angle sum of ΔPSM), so $\angle PSQ = 48° - 36° = 12°$. $\angle SPQ = 24°$. So $\angle PQS = 180° - 24° - 12° = 144°$ (angle sum of ΔPQS).

c 473 metres

18 a Adding the known angle β and the obtuse solution,

$$(180° - \alpha) + \beta = 180° - (\alpha - \beta),$$

so the third angle is $\alpha - \beta$. If $\alpha \le \beta$, then the third angle is zero or negative, which is impossible . If $\alpha > \beta$, then there is room for the third angle.

b Two angles add to more than 180°. It is an impossible triangle, because the longest side b should be opposite the largest angle $\angle B = 150°$.

20 a $8\sqrt{3}$ **b** 2:1

21 d $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

22 b $13°41'$

Exercise 6J

1 a 3.3 **b** 4.7 **c** 4.0 **d** 15.2 **e** 21.9 **f** 24.6

2 a 39° **b** 56° **c** 76° **d** 94° **e** 117° **f** 128°

3 a $\sqrt{13}$ **b** $\sqrt{7}$

4 a $\sqrt{10}$ **b** $\sqrt{21}$

5 a $44°25'$ **b** $101°32'$ **c** $\frac{7}{32}$

6 167 nautical miles

7 20°

8 $13°10'$, 120°

10 a 19 cm **b** $\frac{37}{38}$

11b 108 km **c** $\angle ACB \doteqdot 22°$, bearing $\doteqdot 138°$

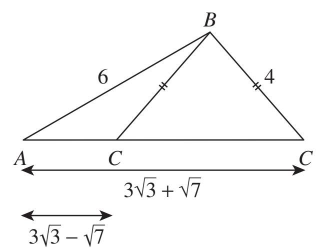
12 a $\angle DAP = \angle DPA = 60°$ (angle sum of isosceles triangle), so ΔADP is equilateral.
Hence $AP = 3$ cm.

b $3\sqrt{7}$ cm

13 a $x \cot 27°$

14 3 or 5

15c



16 120°

Exercise 6K

1 a 28.3 **b** 17.3 **c** 12.5

d 36.2 **e** 12.6 **f** 23.2

2 a 59° **b** 55° **c** 40° **d** 37° **e** 52° **f** 107°

3 a 26 cm **b** 28 cm **c** 52° **d** 62°

4b 28 metres

5 a $\angle ACP + 31° = 68°$ (exterior angle of ΔACP)

c 6 cm

6 a 11.6 cm **b** 49°

7 a $44°25'$ **b** 10 cm^2

8 b 36 cm

10 a PQ is inclined at 26° to a north–south line through Q, because of alternate angles on parallel lines.
Then $\angle PQR = 26° + 90°$.

b 112 nautical miles

11 a $46°59'$ or $133°1'$

b 66.4 metres or 52.7 metres

12 a $\angle PJK = \angle PBQ = 20°$ (corresponding angles on parallel lines),
but $\angle PJK = \angle PAJ + \angle APJ$ (exterior angle of triangle), so $\angle APJ = 20° - 5° = 15°$.

d 53 metres

13 a 38 tan 68° **b** 111 m

14 b 131 m

15 P_1 by 2.5 min

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16 a $34°35'$

b $\angle PDA = \angle ABP$ (base angles of isosceles ΔABD) and $\angle ABP = \angle PDC$ (alternate angles on parallel lines), so $\angle PDA = \angle PDC$ and $\angle PDC = \frac{1}{2}\angle ADC$.

c $65°35'$

17 50.4 metres

18 a $-\cos\theta$

20 a $y = h\cot\beta$

Chapter 6 review exercise

1 a 0.2924 **b** 0.9004 **c** 0.6211 **d** 0.9904

2 a $17°27'$ **b** $67°2'$ **c** $75°31'$ **d** $53°8'$

3 a 10.71 **b** 5.23 **c** 10.36 **d** 15.63

4 a $45°34'$ **b** $59°2'$ **c** $58°43'$ **d** $36°14'$

5 a $\sqrt{3}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** 1 **e** 2 **f** $\frac{2}{\sqrt{3}}$

6 6.25 metres

7 $65°$

8 b 114 km **c** 108°T

9 All six trigonometric graphs are drawn just before Exercise 6E.

10 a $-\cos 55°$ **b** $-\sin 48°$ **c** $\tan 64°$ **d** $\sin 7°$

11 a $\sqrt{3}$ **b** $-\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** $-\frac{1}{\sqrt{3}}$

12 a 0 **b** -1

c undefined **d** -1

13 a $y = 3, \sin\theta = \frac{3}{5}, \cos\theta = -\frac{4}{5}, \tan\theta = -\frac{3}{4}$

b $x = -2\sqrt{5}, \sin\theta = -\frac{\sqrt{5}}{5}, \cos\theta = -\frac{2\sqrt{5}}{5}, \tan\theta = \frac{1}{2}$

14 a $\sin\alpha = \frac{12}{13}, \cos\alpha = \frac{5}{13}$

b $\cos\beta = \frac{5}{7}, \tan\beta = \frac{2\sqrt{6}}{5}$

15 a $\sin\alpha = -\frac{9}{41}, \cos\alpha = \frac{40}{41}$

b $\cos\beta = -\frac{5}{7}, \tan\beta = -\frac{2\sqrt{6}}{5}$

16 a $\sec\theta$ **b** $\tan\theta$ **c** $\tan\theta$

d $\cos^2\theta$ **e** 1 **f** $\cot^2\theta$

18 a $x = 60°$ or $300°$ **b** $x = 90°$

c $x = 135°$ or $315°$ **d** $x = 90°$ or $270°$

e $x = 30°$ or $210°$ **f** $x = 0°, 180°$ or $360°$

g $x = 225°$ or $315°$ **h** $x = 150°$ or $210°$

i $x = 45°, 135°, 225°$ or $315°$

j $x = 30°, 150°, 210°$ or $330°$

k $x = 15°$ or $135°$

l $\tan x = -\sqrt{3}, x = 120°$ or $300°$

19 a $\sin\theta = 0$ or $-\frac{1}{2}, \theta = 0°, 180°, 210°, 330°$ or $360°$

b $\cos\theta = -1$ or $2, \theta = 180°$

c $\tan\theta = \frac{1}{2}$ or $-3, \theta = 26°34', 108°26', 206°34'$ or $288°26'$

20 a 8.5 **b** 10.4 **c** 7.6 **d** 8.9

21 a 27 cm^2 **b** 56 cm^2

22 a $57°55'$ **b** $48°33'$ **c** $24°29'$ **d** $150°26'$

23 28 cm^2

24 a $\frac{5\sqrt{3}}{3}$ cm **b** $30°$ or $150°$

25 b 48 metres

26 b 31.5 metres

27 b 316 nautical miles **c** 104°T

28 a $10\tan 77°$ **b** 45 m

29 a 9.85 metres **b** 5.30 metres **c** 12.52 metres

30 c 34 metres

31 a $\dfrac{86\sin 60°45'}{\sin 65°45'}$ **b** 66 metres

34 c 129 metres

35 a $BD = \sqrt{3}h, CD = h$

Chapter 7

Exercise 7A

1 a $(2, 7)$ **b** $(5, 6)$ **c** $(2, -2)$

d $\left(0, 3\frac{1}{2}\right)$ **e** $\left(-5\frac{1}{2}, -10\right)$ **f** $(4, 0)$

2 a 5 **b** 13 **c** 10

d $\sqrt{8} = 2\sqrt{2}$ **e** $\sqrt{80} = 4\sqrt{5}$ **f** 13

3 a $M(1, 5)$ **b** $PM = MQ = 5$

4 a $PQ = QR = \sqrt{17}, PR = \sqrt{50} = 5\sqrt{2}$

5 a $AB = 15, BC = 20$ and $AC = 25$

b LHS $= AB^2 + BC^2 = 15^2 + 20^2 = 625 =$ RHS

6 a $AB = \sqrt{58}, BC = \sqrt{72} = 6\sqrt{2}, CA = \sqrt{10}$

b AB: $\left(1\frac{1}{2}, 1\frac{1}{2}\right)$, BC: $(0, 1)$, CA: $\left(-1\frac{1}{2}, 4\frac{1}{2}\right)$

7 a 13 **b** $\sqrt{41}$ **c** $(5, -3)$

8 a $(1, 6)$ **b** $(1, 6)$

c The diagonals bisect each other.

d parallelogram

9 a All sides are $5\sqrt{2}$. **b** rhombus

10 a $XY = YZ = \sqrt{52} = 2\sqrt{13}$, $ZX = \sqrt{104} = 2\sqrt{26}$

b $XY^2 + YZ^2 = 104 = ZX^2$

c 26 square units

11 a Each point is $\sqrt{17}$ from the origin.

b $\sqrt{17}, 2\sqrt{17}, 2\pi\sqrt{17}, 17\pi$

12 a $(x - 5)^2 + (y + 2)^2 = 45$

b $(x + 2)^2 + (y - 2)^2 = 74$

13 $(5, 2)$

14 a $S(-5, -2)$ **b i** $P = (4, -14)$

ii $P = (-1, -17)$ **iii** $P = (7, -7)$

c $B = (0, 7)$ **d** $R = (12, -9)$

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15 a ABC is an equilateral triangle.
b PQR is a right triangle.
c DEF is none of these.
d XYZ is an isosceles triangle.

16 a Check the results using the distance formula — there are eight such points.
b $y = 4$ or 10
c $a = 1 + \sqrt{2}$ or $1 - \sqrt{2}$

17 a $(-2, 1)$ **b** $M = \left(4\frac{1}{2}, 1\frac{1}{2}\right)$

18 a $x = \frac{3}{2}a$, a vertical straight line through the midpoint of AB.
b $(x - 4a)^2 + y^2 = (2a)^2$, a circle with centre $(4a, 0)$ and radius $2a$.

Exercise 7B

1 a i 2 **ii** $\frac{3}{4}$ **iii** $-1\frac{1}{2}$
b i $-\frac{1}{2}$ **ii** $-\frac{4}{3}$ **iii** $\frac{2}{3}$

2 a $-1, 1$ **b** $2, -\frac{1}{2}$ **c** $\frac{1}{2}, -2$
d $-\frac{1}{2}, 2$ **e** $3, -\frac{1}{3}$ **f** $-\frac{7}{10}, \frac{10}{7}$

3 a vertical **b** horizontal **c** neither
d horizontal **e** neither **f** vertical

4 a 3 **b** $\frac{1}{2}$
c parallelogram

5 a $m_{AB} = m_{CD} = \frac{1}{2}, m_{BC} = m_{DA} = -\frac{1}{5}$.
b $m_{AB} = 2, m_{CD} = -3$

6 a 0.27 **b** -1.00 **c** 0.41 **d** 3.08

7 a 45° **b** 120° **c** 76° **d** 30°

8 a $m_{AB} = m_{CD} = -\frac{1}{2}, m_{BC} = m_{DA} = 2$
b $m_{AB} \times m_{BC} = -1$ **c** $AB = BC = 2\sqrt{5}$

9 In each case, show that each pair of opposite sides is parallel.
a Show also that two adjacent sides are equal.
b Show also that two adjacent sides are perpendicular.
c Show that it is both a rhombus and a rectangle.

10 a $-2, -\frac{7}{3}$, non-collinear **b** $\frac{2}{3}, \frac{2}{3}$, collinear

11 The gradients of AB, BC and CD are all $\frac{1}{3}$.

12 $m_{AB} = \frac{1}{2}, m_{BC} = -2$ and $m_{AC} = 0$, so $AB \perp BC$.

13 a $m_{PQ} = 4, m_{QR} = -\frac{1}{4}$ and $m_{PR} = -\frac{5}{3}$, so $PQ \perp QR$.
Area $= 8\frac{1}{2}$ square units
b $m_{XY} = \frac{7}{3}, m_{YZ} = \frac{2}{5}$ and $m_{XZ} = -\frac{5}{2}$, so $XZ \perp YZ$.
Area $= 14\frac{1}{2}$ square units

14 a -5 **b** 5

15 a $A(-2, 0), B(0, 6)\ m = 3, \alpha \doteqdot 72^\circ$
b $A(2, 0), B(0, 1), m = -\frac{1}{2}, \alpha \doteqdot 153^\circ$
c $A(-4, 0), B(0, -3), m = -\frac{3}{4}, \alpha \doteqdot 143^\circ$
d $A(3, 0), B(0, -2), m = \frac{2}{3}, \alpha \doteqdot 34^\circ$

16 a $P = (2, -1), Q = (-1, 4), R = (-3, 2), S = (0, -3)$
b $m_{PQ} = m_{RS} = -\frac{5}{3}$ and $m_{PS} = m_{QR} = 1$

17 a They all satisfy the equation, or they all lie 5 units from O.
b The centre $O(0, 0)$ lies on AB.
c $m_{AC} = \frac{1}{2}, m_{BC} = -2$

18 $a = -\frac{1}{2}$

19 $k = 2$ or -1

21 a They are collinear if and only if $\Delta = 0$, that is $a_1b_2 + a_2b_3 + a_3b_1 = a_2b_1 + a_3b_2 + a_1b_3$.

22 a $x = \dfrac{4p}{1 - p^2}$ **b** $x = p - \dfrac{1}{p}$

Exercise 7C

1 a not on the line
b on the line
c on the line

2 a $(4, 0)$ and $(0, 3)$
b $(1.5, 0)$ and $(0, -6)$
c $(8, 0)$ and $(0, -4)$

3 Check the points in your answer by substitution. $(0, 8)$, $(3, 7)$ and $(6, 6)$ will do.

4 a $x = 1, y = 2$
b $x = 0, y = -4$
c $x = 5, y = 0$

5 a $m = 4, b = -2$ **b** $m = \frac{1}{5}, b = -3$
c $m = -1, b = 2$ **d** $m = -\frac{5}{7}, b = 0$

6 a $y = -3x + 5$ **b** $y = -3x - \frac{2}{3}$ **c** $y = -3x$

7 a $y = 5x - 4$ **b** $y = -\frac{2}{3}x - 4$ **c** $y = -4$

8 a $x - y + 3 = 0$ **b** $2x + y - 5 = 0$
c $x - 5y - 5 = 0$ **d** $x + 2y - 6 = 0$

9 a $m = 1, b = 3$ **b** $m = -1, b = 2$
c $m = \frac{1}{3}, b = 0$ **d** $m = -\frac{3}{4}, b = \frac{5}{4}$

10 a $m = 1, \alpha = 45^\circ$ **b** $m = -1, \alpha = 135^\circ$
c $m = 2, \alpha \doteqdot 63^\circ 26'$ **d** $m = -\frac{3}{4}, \alpha \doteqdot 143^\circ 8'$

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11 The sketches required are clear from the intercepts.
a $A(3, 0), B(0, 5)$
b $A(-3, 0), B(0, 6)$
c $A(-4, 0), B\left(0, 2\frac{2}{5}\right)$

12 a $y = 2x + 4, 2x - y + 4 = 0$
b $y = -x, x + y = 0$
c $y = -\frac{1}{3}x - 4, x + 3y + 12 = 0$

13 a $y = -2x + 3, y = \frac{1}{2}x + 3$
b $y = \frac{5}{2}x + 3, y = -\frac{2}{5}x + 3$
c $y = -\frac{3}{4}x + 3, y = \frac{4}{3}x + 3$

14 a $-3, \frac{1}{2}, -3, \frac{1}{2}$, parallelogram
b $\frac{4}{3}, -\frac{3}{4}, \frac{4}{3}, -\frac{3}{4}$, rectangle

15 The gradients are $\frac{5}{7}, \frac{2}{5}$ and $-\frac{7}{5}$, so the first and last are perpendicular.
a $A(-3, 0), B(0, 3)$ **b** $A(2, 0), B(0, 2)$
c $A\left(2\frac{1}{2}, 0\right), B(0, -5)$ **d** $A(-6, 0), B(0, 2)$
e $A\left(1\frac{2}{3}, 0\right), B\left(0, 1\frac{1}{4}\right)$ **f** $A\left(1\frac{1}{3}, 0\right), B(0, -2)$

16 a $x = 3, x = 0, y = -7, y = -2$
b $y = 0, y = -4x + 12, y = 2x + 12$

17 a $x - y + 3 = 0$ **b** $-\sqrt{3}x + y + 1 = 0$
c $x - \sqrt{3}y - 2\sqrt{3} = 0$ **d** $x + y - 1 = 0$

18 a They are about 61° and 119°.
b It is isosceles. (The two interior angles with the x-axis are equal.)

19 a $k = -\frac{1}{3}$ **b** $k = 3$

21 $(x - a)^2 + (y - a)^2 = a^2$,
where $a = 2 - \sqrt{2}$ or $a = 2 + \sqrt{2}$,
$\left(x - \sqrt{2}\right)^2 + \left(y + \sqrt{2}\right)^2 = 2$,
$\left(x + \sqrt{2}\right)^2 + \left(y - \sqrt{2}\right)^2 = 2$

22 a From their gradients, two pairs of lines are parallel and two lines are perpendicular.
b The distance between the x-intercepts of one pair of lines must equal the distance between the y-intercepts of the other pair. Thus $k = 2$ or 4.

Exercise 7D

1 a $2x - y - 1 = 0$ **b** $x + y - 4 = 0$
c $5x + y = 0$ **d** $x + 3y - 8 = 0$
e $4x + 5y + 8 = 0$

2 a $y = 2x + 1$ **b** $y = -\frac{1}{2}x + 6$
c $y = \frac{1}{5}x - 8$ **d** $y = \frac{3}{7}x + 9$
e $y = \frac{5}{2}x + 10$

3 a 3 **b** $3x - y - 5 = 0$

4 a $2, 2x - y - 2 = 0$ **b** $-2, 2x + y - 1 = 0$
c $\frac{1}{3}, x - 3y + 13 = 0$ **d** $2, 2x - y + 2 = 0$
e $-\frac{1}{4}, x + 4y + 4 = 0$ **f** $1, x - y - 3 = 0$

5 a $-\frac{3}{2}$
b i $3x + 2y + 1 = 0$ **ii** $2x - 3y - 8 = 0$

6 a $2x - 3y + 2 = 0$ **b** $2x - 3y - 9 = 0$

7 a $4x - 3y - 8 = 0$ **b** $4x - 3y + 11 = 0$

8 a $M(3, -1)$
c i No, the first two intersect at $(-4, 7)$, which does not lie on the third.
ii They all meet at $(5, 4)$.

9 a $y = -2x + 5, y = \frac{1}{2}x + 6$
b $y = 2\frac{1}{2}x - 8\frac{1}{2}, y = -\frac{2}{5}x + 4\frac{1}{5}$
c $y = -1\frac{1}{3}x + 3, y = \frac{3}{4}x + 6\frac{1}{2}$

10 a $x - y - 1 = 0$
b $\sqrt{3}x + y + \sqrt{3} = 0$
c $x - y\sqrt{3} - 4 - 3\sqrt{3} = 0$
d $x + \sqrt{3}y + 2 + 5\sqrt{3} = 0$

11 a i $x - 3 = 0$ **ii** $y + 1 = 0$
b $3x + 2y - 6 = 0$
c i $x - y + 4 = 0$ **ii** $\sqrt{3}x + y - 4 = 0$
d $x\sqrt{3} + y + 6\sqrt{3} = 0$

12 $\ell_1 \parallel \ell_2$, and $\ell_3 \parallel \ell_4$, so there are two pairs of parallel sides. The vertices are $(-2, -1), (-4, -7), (1, -2), (3, 4)$.

13 $m_{BC} \times m_{AC} = -1$ so $BC \perp AC$.
AB: $y = x - 1$, BC: $y = \frac{1}{2}x + 2$,
AC: $y = 2 - 2x$

14 a $m_{AC} = \frac{2}{3}, \theta \doteqdot 34^\circ$
b $2x - 3y - 2 = 0$
c $D(4, 2)$
d $m_{AC} \times m_{BD} = \frac{2}{3} \times -\frac{3}{2} = -1$, hence they are perpendicular.
e isosceles
f area $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times \sqrt{52} \times \sqrt{52} = 26$
g $E(8, -4)$

15 a $4y = 3x + 12$ **b** $ML = MP = 5$
c $N(4, 6)$ **d** $x^2 + (y - 3)^2 = 25$

16 $k = 2\frac{1}{2}$

17 a $\mu = 4$ **b** $\mu = -9$

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18 $bx + ay = ab$

19 $bx + ay = 2ab$

21 c i 1 **ii** $\frac{1}{13}\sqrt{13}$

Exercise 7E

1 a i $1, -1$

ii The product of their gradients is -1.

b i $1, -1$

ii The product of their gradients is -1.

2 a i $M = (4, 5)$

ii $OM = PM = QM = \sqrt{41}$

iii OM, PM and QM are three radii of the circle.

b $M = (p, q)$, $OM = PM = QM = \sqrt{p^2 + q^2}$

3 a i $P(5, 2)$ and $Q(4, 1)$.

iv $AC = 2\sqrt{2}$ and $PQ = \sqrt{2}$

b $P(a + b, c)$, $Q(b, c)$, $y = c$ and so $Q(b, c)$ lies on $y = c$. Also, $AC = 2a$ and $PQ = a$ so $PQ = \frac{1}{2}AC$.

4 a $P = \left(\frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2)\right)$,

$Q = \left(\frac{1}{2}(b_1 + c_1), \frac{1}{2}(b_2 + c_2)\right)$,

$R = \left(\frac{1}{2}(c_1 + d_1), \frac{1}{2}(c_2 + d_2)\right)$,

$S = \left(\frac{1}{2}(d_1 + a_1), \frac{1}{2}(d_2 + a_2)\right)$

b Both midpoints are $\left(\frac{1}{4}(a_1 + b_1 + c_1 + d_1), \frac{1}{4}(a_2 + b_2 + c_2 + d_2)\right)$.

c Part **b** shows that its diagonals bisect each other, so (using Box 4) it is a parallelogram.

6 a $\frac{x}{3} + \frac{y}{4} = 1$ and $4y = 3x$, thus $C = \left(\frac{48}{25}, \frac{36}{25}\right)$.

b $OA = 3$, $AB = 5$, $OC = \frac{12}{5}$, $BC = \frac{16}{5}$, $AC = \frac{9}{5}$

7 a $AB = BC = CA = 2a$

b $AB = AD = 2a$

c $BD = 2a\sqrt{3}$

8 a AB and DC have gradient $\frac{b}{a}$; AD and BC have gradient $\frac{d}{c}$.

b Both the midpoints are $(a + c, b + d)$.

c The midpoints coincide.

9 a i $P = (1, 4)$, $Q = (-1, 0)$ and $R = (3, 2)$, BQ: $x - y + 1 = 0$, CR: $y - 2 = 0$, AP: $x = 1$

ii The medians intersect at $(1, 2)$.

b i $P(-3a, 3c - 3b)$, $Q(3a, 3c + 3b)$, $R(0, 0)$

ii The median passing through B is $3a(y + 6b) = (c + 3b)(x + 6a)$. The median passing through A is $-3a(y - 6b) = (c - 3b)(x - 6a)$.

iii The medians intersect at $(0, 2c)$.

10 a gradient $AB = 0$, gradient $BC = \dfrac{c}{b + a}$, gradient $CA = \dfrac{c}{b - a}$.

b perpendicular bisector of AB: $x = 0$, of BC: $c(c - y) = (b + a)(x - b + a)$, of AC: $c(c - y) = (b - a)(x - b - a)$

c They all meet at $\left(0, \dfrac{c^2 + b^2 - a^2}{c}\right)$.

d Any point on the perpendicular bisector of an interval is equidistant from the endpoints of that interval.

Chapter 7 review exercise

1 a $\left(8, 6\frac{1}{2}\right)$ **b** $-\frac{5}{12}$ **c** 13

2 a $AB = 5$, $BC = \sqrt{2}$, $CA = 5$

b isosceles

3 a $P(3, 7)$, $Q(6, 5)$, $R(3, -3)$, $S(0, -1)$

b PQ and RS have gradient $-\frac{2}{3}$, QR and SP have gradient $\frac{8}{3}$.

c parallelogram

4 a $C = (-1, 1)$, $r = \sqrt{45} = 3\sqrt{5}$

b $PC = \sqrt{53}$, no

5 a $m_{LM} = -2$, $m_{MN} = -\frac{8}{9}$, $m_{NL} = \frac{1}{2}$

b $m_{LM} \times m_{NL} = -1$

6 a -1 **b** $a = 8$

c $Q = (7, -4)$

d $d^2 = 16$, so $d = 4$ or -4.

7 a $2x + y - 5 = 0$ **b** $2x - 3y + 9 = 0$

c $x + 7y = 0$ **d** $3x + y + 8 = 0$

e $x\sqrt{3} - y - 2 = 0$

8 a $b = -\frac{7}{6}$, $m = \frac{5}{6}$, $\alpha \doteqdot 39°48'$

b $b = \frac{3}{4}$, $m = -1$, $\alpha = 135°$

9 a $8x - y - 24 = 0$ **b** $5x + 2y - 21 = 0$

10 a No; $m_{LM} = -\frac{1}{3}$ and $m_{MN} = -\frac{5}{12}$.

b Yes; they all pass through $(2, 5)$.

11 a Yes; the 2nd and 3rd lines have gradients $\frac{3}{2}$ and $-\frac{2}{3}$ and so are perpendicular.

b Trapezium; the 1st and 3rd lines are parallel.

12 a $A = (6, 0)$, $B = \left(0, 7\frac{1}{2}\right)$

b $22\frac{1}{2}$ square units

13 a $m_{AB} = -\frac{3}{4}$, $AB = 10$, $M = (6, 5)$

c $C = (15, 17)$ **d** $AC = BC = 5\sqrt{10}$

e $75\,\text{u}^2$

f $\sin\theta = \frac{3}{5}$, $\theta \doteqdot 36°52'$

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Chapter 8

Exercise 8A

1 a The factors are $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243$.
b Population in 2010 = 810000, population in 2020 = 2430000, so the decade was 2010–2020.

2 a 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096
b i 1, 3, 9, 27, 81, 243, 729
ii 1, 5, 25, 125, 625, 3125
iii 1, 6, 36, 216 **iv** 1, 7, 49, 343
c i 1, 4, 16, 64, 256, 1024, 4096
ii 1, 8, 64, 512, 4096

3 a 64 **b** $\frac{4}{9}$ **c** $\frac{8}{27}$
d $\frac{81}{10000}$ **e** $\frac{16}{49}$ **f** $\frac{5}{9}$
g 1 **h** $\frac{1}{5}$ **i** $\frac{1}{11}$
j $\frac{1}{36}$ **k** $\frac{1}{100}$ **l** $\frac{1}{27}$
m $\frac{1}{125}$ **n** $\frac{1}{32}$ **o** $\frac{1}{1000000}$

4 a 11 **b** $\frac{7}{2}$ or $3\frac{1}{2}$ **c** $\frac{2}{7}$
d $\frac{23}{10}$ or $2\frac{3}{10}$ **e** 10 **f** 100
g 50 **h** $\frac{1}{25}$ **i** 125
j 16 **k** 1000000 **l** $\frac{9}{4}$
m $\frac{16}{81}$ **n** $\frac{25}{4}$ **o** 1

5 a 2^{14} **b** a^{15} **c** $9^0 = 1$ **d** x^2
e $a^0 = 1$ **f** 8 **g** 7^5 **h** a^{-2}
i 2^{16} **j** 1 **k** x^{12} **l** y^{11}
m y^{-11} **n** x^{15} **o** x^{15} **p** z^{14}
q a^{-6} **r** a^{-6} **s** 5^{-28} **t** 2^{16}

6 a $9x^2$ **b** $125a^3$ **c** $64c^6$
d $81s^4t^4$ **e** $49x^2y^2z^2$ **f** $\dfrac{1}{x^5}$
g $\dfrac{9}{x^2}$ **h** $\dfrac{y^2}{25}$ **i** $\dfrac{49a^2}{25}$
j $\dfrac{27x^3}{8y^3}$

7 a $\dfrac{1}{9}$ **b** $\dfrac{1}{x}$ **c** $\dfrac{1}{b^2}$
d $-\dfrac{1}{a^4}$ **e** $\dfrac{1}{7x}$ **f** $\dfrac{7}{x}$
g $-\dfrac{9}{x}$ **h** $\dfrac{1}{9a^2}$ **i** $\dfrac{3}{a^2}$
j $\dfrac{4}{x^3}$

8 a $\frac{2}{3}$ **b** $\frac{3}{8}$ **c** $\frac{4}{25}$ **d** $\frac{27}{1000}$
e 5 **f** $\frac{4}{9}$ **g** $\frac{4}{25}$ **h** 400

9 a $3\,\text{km}^3$
b $(10^3 \times 10^3)^3 = 10^{18}$
c 3×10^{18}

10 a x^{-1} **b** $-x^{-2}$
c $-12x^{-1}$ **d** $9x^{-2}$
e $-x^{-3}$ **f** $12x^{-5}$
g $7x^{-3}$ **h** $-6x^{-1}$
i $\frac{1}{6}x^{-1}$ **j** $-\frac{1}{4}x^{-2}$

11 a $x = -1$ **b** $x = -3$ **c** $x = -1$ **d** $x = -1$
e $x = 0$ **f** $x = 2$ **g** $x = -1$ **h** $x = -2$
i $x = \frac{10}{13}$ or $-\frac{10}{13}$ **j** $x = 2$ or -2
k $x = \frac{1}{3}$ **l** $x = \frac{9}{8}$ or $-\frac{9}{8}$
m $x = 6$ **n** $x = 8$ **o** $x = -4$ **p** $x = 2$

12 a 2^{x+3} **b** 3^{x+1} **c** $\dfrac{1}{7^x}$ **d** $\dfrac{5^{2x}}{5^3}$
e 10^{6x} **f** $\dfrac{1}{5^{8x}}$ **g** 6^{14x} **h** $\dfrac{2^{3x}}{2^4}$

13 a x^6y^4 **b** $\dfrac{y}{x^2}$ **c** $\dfrac{21a^3}{x}$ **d** $\dfrac{1}{3st^2}$
e $\dfrac{7x}{y^2}$ **f** $\dfrac{5b^{10}}{4a^6}$ **g** $\dfrac{s^6}{y^9}$ **h** $\dfrac{c^2}{5d^3}$
i $27x^8y^{17}$ **j** $\dfrac{2a^7}{y^{15}}$ **k** $5s^5$ **l** $\dfrac{250x^8}{y^{12}}$

14 a $x^2 + 2 + \dfrac{1}{x^2}$ **b** $x^2 - 2 + \dfrac{1}{x^2}$ **c** $x^4 - 2 + \dfrac{1}{x^4}$

15 a $\dfrac{b - a}{ab}$ **b** $\dfrac{y}{y + 1}$ **c** $\dfrac{x^2y^2}{y^2 - x^2}$
d $\dfrac{ab}{b - a}$ **e** $\dfrac{x^3 - y^3}{x^3y^3}$ **f** $\dfrac{1}{a + 1}$

16 a 2^{6n} **b** 81 **c** 2^{3x}
d $2^{2x}3^{2x}$ (or 6^{2x}) **e** $5^{4n-4}2^{4n-5}$ **f** 2^x3^{1-x}

17 a 50×7^n **b** 26 **c** $124 \times 5^{n-3}$
d 7 **e** $7 \times 2^{2n-1}$ **f** 2^n

18 $\dfrac{3^n}{2}$ **b** $\dfrac{1}{3^x}$ **c** -2^n3^n

19 a Take the reciprocal, 5.97×10^{26}
b $5.73 \times 10^{-45}\,\text{m}^3$
c $2.9 \times 10^{17}\,\text{kg/m}^3$

Exercise 8B

1 a 6 **b** 4 **c** 10 **d** 125 **e** 9
f 8 **g** 81 **h** 16 **i** 32 **j** 8

2 a $\frac{1}{2}$ **b** $\frac{5}{7}$ **c** $\frac{3}{2}$ **d** $\frac{81}{16}$
e $\frac{4}{9}$ **f** $\frac{1}{128}$ **g** $\frac{27}{1000}$ **h** $\frac{1}{125}$

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e $\frac{1}{9}$ **f** $\frac{1}{27}$ **g** 2 **h** 5

i 8 **j** $\frac{27}{8}$ **k** $\frac{8}{27}$ **l** $\frac{4}{25}$

4 a x^6 **b** $x^{3\frac{1}{2}}$ **c** $15x^2y^{-1}$

d x **e** $x^{-4\frac{1}{2}}$ **f** $7a^{-1}b^{-1}$

g x^{-4} **h** x^6 **i** $27s^{-6}t^{7\frac{1}{2}}$

5 a $2^1 = 2$ **b** $2^0 = 1$ **c** $2^3 = 8$

d $3^{-1} = \frac{1}{3}$ **e** $25^{\frac{1}{2}} = 5$ **f** $7^0 = 1$

g $3^{-3} = \frac{1}{27}$ **h** $3^{-2} = \frac{1}{9}$ **i** $9^2 = 81$

6 a $x = \frac{1}{2}$ **b** $x = \frac{1}{2}$ **c** $x = \frac{1}{4}$

d $x = \frac{1}{6}$ **e** $x = \frac{1}{2}$ **f** $x = \frac{1}{3}$

7 a $\sqrt{x}$ **b** $\sqrt[3]{x}$

c $7\sqrt{x}$ **d** $\sqrt{7x}$

e $15\sqrt[4]{x}$ **f** $\sqrt{x^3}$ or $\left(\sqrt{x}\right)^3$

g $6\sqrt{x^5}$ or $6\left(\sqrt{x}\right)^5$ **h** $\sqrt[3]{x^4}$ or $\left(\sqrt[3]{x}\right)^4$

8 a $x^{\frac{1}{2}}$ **b** $3x^{\frac{1}{2}}$ **c** $(3x)^{\frac{1}{2}}$ **d** $12x^{\frac{1}{3}}$

e $9x^{\frac{1}{6}}$ **f** $x^{\frac{3}{2}}$ **g** $x^{\frac{9}{2}}$ **h** $25x^{\frac{6}{5}}$

9 a $x^{2\frac{1}{2}}$ **b** $x^{-2\frac{1}{2}}$ **c** $x^{3\frac{2}{3}}$ **d** $x^{\frac{1}{3}}$

10 a 5.765×10^6 **b** 1.261×10^1

c 8.244×10^{-1} **d** 7.943×10^{-3}

e 8.825×10^0 **f** 2.595×10^1

g 7.621×10^{-2} **h** 5.157×10^4

11 a $\$6000 \times (1.03)^0 = \$6000$

b $\$6000 \times (1.03)^1 = \$6180$

c i $\$6000 \times (1.03)^5 \doteqdot \$6960$

ii $\$6000 \times (1.03)^{\frac{1}{2}} \doteqdot \$6090$

iii $\$6000 \times (1.03)^{\frac{7}{2}} \doteqdot \$6650$

12 a 9 **b** 3 **c** $\frac{1}{20}$ **d** $\frac{3}{10}$

13 a $9xy^3$ **b** $35b$ **c** $3s^{\frac{1}{2}}$

d $x^{1\frac{1}{2}}y^{2\frac{1}{2}}$ **e** a **f** $a^{-1}b^2$

g $2xy^{-2}$ **h** p^2q^{-6} **i** x^7

14 a $x^{-\frac{1}{2}}$ **b** $12x^{-\frac{1}{2}}$ **c** $-5x^{-\frac{1}{2}}$ **d** $15x^{-\frac{1}{3}}$

e $-4x^{-\frac{2}{3}}$ **f** $x^{1\frac{1}{2}}$ **g** $5x^{-1\frac{1}{2}}$ **h** $8x^{2\frac{1}{2}}$

15 a $x = -\frac{1}{2}$ **b** $x = -\frac{1}{4}$

c $x = \frac{2}{3}$ **d** $x = -\frac{2}{3}$

e $x = \frac{3}{2}$ **f** $x = -\frac{3}{2}$

g $x = \frac{3}{4}$ **h** $x = -\frac{4}{3}$

i $x = -\frac{1}{2}$ **j** $x = -\frac{2}{3}$

16 a $x + 2 + x^{-1}$ **b** $x - 2 + x^{-1}$ **c** $x^5 - 2 + x^{-5}$

17 a $x^2 + 10 + \dfrac{25}{x^2}$ **b** $x^4 - 14 + \dfrac{49}{x^4}$ **c** $9x - 12 + \dfrac{4}{x}$

18 a $x = -\frac{1}{3}$ **b** $x = \frac{1}{4}$ **c** $-\frac{2}{3}$

d $x = \frac{5}{12}$ **e** $x = -4$ **f** $x = -2$

19 a $b = \frac{1}{343}$ **b** $\frac{1}{11}$ **c** $x = \frac{1}{81}$

20 a $x = 3$ and $y = 4$ **b** $x = 0$ and $y = -1$

c $x = -2$ and $y = \frac{1}{2}$

22 a $3^{\frac{1}{3}} > 2^{\frac{1}{2}}$ **b** $2^{\frac{1}{2}} > 5^{\frac{1}{5}}$ **c** $7^{\frac{3}{2}} < 20$ **d** $5^{\frac{1}{5}} < 3^{\frac{1}{3}}$

23 a $12 < 2^{\frac{11}{3}} < 13$ **b** $13 < 2^{\frac{15}{4}} < 14$

24 $\lim\limits_{x\to 0^+} 0^x = 0$ and $\lim\limits_{x\to 0} x^0 = 1$, so there is no sensible way to define 0^0.

Exercise 8C

1 a because $2^3 = 8$. **b** because $5^2 = 25$.

c because $10^3 = 1000$. **d** so $\log_7 49 = 2$.

e so $\log_3 81 = 4$. **f** so $\log_{10} 100\,000 = 5$.

2 a $\ldots x = a^y$ **b** $\ldots x = \log_a y$

3 a $10^x = 1000, x = 3$ **b** $10^x = 10, x = 1$

c $10^x = 1, x = 0$ **d** $10^x = \frac{1}{100}, x = -2$

e $3^x = 9, x = 2$ **f** $5^x = 125, x = 3$

g $2^x = 64, x = 6$ **h** $4^x = 64, x = 3$

i $8^x = 64, x = 2$ **j** $7^x = \frac{1}{7}, x = -1$

k $12^x = \frac{1}{12}, x = -1$ **l** $11^x = \frac{1}{121}, x = -2$

m $6^x = \frac{1}{36}, x = -2$ **n** $4^x = \frac{1}{64}, x = -3$

o $8^x = \frac{1}{64}, x = -2$ **p** $2^x = \frac{1}{64}, x = -6$

4 a $x = 7^2 = 49$ **b** $x = 5^3 = 125$

c $x = 2^5 = 32$ **d** $x = 100^3 = 1\,000\,000$

e $x = 7^1 = 7$ **f** $x = 11^0 = 1$

g $x = 13^{-1} = \frac{1}{13}$ **h** $x = 12^{-2} = \frac{1}{144}$

i $x = 5^{-3} = \frac{1}{125}$ **j** $x = 7^{-3} = \frac{1}{343}$

k $x = 2^{-5} = \frac{1}{32}$ **l** $x = 3^{-4} = \frac{1}{81}$

5 a $x^2 = 49, x = 7$ **b** $x^3 = 8, x = 2$

c $x^3 = 27, x = 3$ **d** $x^4 = 10\,000, x = 10$

e $x^2 = 10\,000, x = 100$ **f** $x^6 = 64, x = 2$

g $x^2 = 64, x = 8$ **h** $x^1 = 125, x = 125$

i $x^1 = 11, x = 11$ **j** $x^{-1} = \frac{1}{17}, x = 17$

k $x^{-1} = \frac{1}{6}, x = 6$ **l** $x^{-1} = \frac{1}{7}, x = 7$

m $x^{-2} = \frac{1}{9}, x = 3$ **n** $x^{-2} = \frac{1}{49}, x = 7$

o $x^{-3} = \frac{1}{8}, x = 2$ **p** $x^{-2} = \frac{1}{81}, x = 9$

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6 a 0.301 **b** 1.30 **c** 2.00
d 20.0 **e** 3.16 **f** 31.6
g 0.500 **h** 1.50 **i** −0.155
j −2.15 **k** 0.700 **l** 0.00708

7 a $a^x = a, x = 1$ **b** $x = a^1 = a$
c $x^1 = a, x = a$ **d** $a^x = \frac{1}{a}, x = -1$
e $x = a^{-1} = \frac{1}{a}$ **f** $x^{-1} = \frac{1}{a}, x = a$
g $a^x = 1, x = 0$ **h** $x = a^0 = 1$
i $x^0 = 1$, where x can be any positive number.

8 a 1 **b** −1 **c** 3 **d** −2
e −5 **f** $\frac{1}{2}$ **g** $-\frac{1}{2}$ **h** 0

9 a 1 & 2 **b** 0 & 1 **c** 3 & 4 **d** 5 & 6

10 a 2 & 3 **b** 1 & 2 **c** 0 & 1
d 9 & 10 **e** 3 & 4 **f** 0 & 1
g 3 & 4 **h** 4 & 5 **i** 2 & 3
j 1 & 2 **k** −1 & 0 **l** −2 & −1
m −2 & −1 **n** −1 & 0 **o** −2 & −1

11 a $7^x = \sqrt{7}, x = \frac{1}{2}$ **b** $11^x = \sqrt{11}, x = \frac{1}{2}$
c $x = 9^{\frac{1}{2}} = 3$ **d** $x = 144^{\frac{1}{2}} = 12$
e $x^{\frac{1}{2}} = 3, x = 9$ **f** $x^{\frac{1}{2}} = 13, x = 169$
g $6^x = \sqrt[3]{6}, x = \frac{1}{3}$ **h** $9^x = 3, x = \frac{1}{2}$
i $x = 64^{\frac{1}{3}} = 4$ **j** $x = 16^{\frac{1}{4}} = 2$
k $x^{\frac{1}{3}} = 2, x = 8$ **l** $x^{\frac{1}{6}} = 2, x = 64$
m $8^x = 2, x = \frac{1}{3}$ **n** $125^x = 5, x = \frac{1}{3}$
o $x = 7^{\frac{1}{2}}$ or $\sqrt{7}$ **p** $x = 7^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{7}}$
q $x^{-\frac{1}{2}} = \frac{1}{7}, x = 49$ **r** $x^{-\frac{1}{2}} = \frac{1}{20}, x = 400$
s $4^x = \frac{1}{2}, x = -\frac{1}{2}$ **t** $27^x = \frac{1}{3}, x = -\frac{1}{3}$
u $x = 121^{-\frac{1}{2}} = \frac{1}{11}$ **v** $x = 81^{-\frac{1}{4}} = \frac{1}{3}$
w $x^{-\frac{1}{4}} = \frac{1}{2}, x = 16$ **x** $x^{-\frac{1}{4}} = 2, x = \frac{1}{16}$

12 a $\log_{10}45 \doteqdot 1.7$ **b** $10^{1.7} \doteqdot 50$
c 5 significant figures. $10^{1.653} \doteqdot 44.98$ and $10^{1.6532} \doteqdot 45.00$

13 a i 100 **ii** 50 **iii** 10^{100}
iv 5×10^{99} **v** 1 **vi** 10^{98}
b 332 and 333

Exercise 8D

1 a $\log_6 36 = 2$ **b** $\log_5 25 = 2$
c $\log_{15} 15 = 1$ **d** $\log_{12} 144 = 2$
e $\log_{10} 1000 = 3$ **f** $\log_3 3 = 1$
g $\log_2 8 = 3$ **h** $\log_3 81 = 4$
i $\log_2 8 = 3$ **j** 1
k 2 **l** 0

2 a −2 **b** −3 **c** −2 **d** −2 **e** −2 **f** 1

3 a $3 \log_a 2$ **b** $4 \log_a 2$
c $6 \log_a 2$ **d** $-\log_a 2$
e $-3 \log_a 2$ **f** $-5 \log_a 2$
g $\frac{1}{2} \log_a 2$ **h** $-\frac{1}{2} \log_2 2$

4 a $2 \log_2 3$ **b** $2 \log_2 5$
c $1 + \log_2 3$ **d** $1 + \log_2 5$
e $1 + 2\log_2 3$ **f** $2 + \log_2 5$
g $1 - \log_2 3$ **h** $-1 + \log_2 5$

5 a 3.90 **b** 3.16 **c** 3.32 **d** 5.64
e 0.58 **f** −0.74 **g** −0.58 **h** 6.22

6 a 3 **b** 5 **c** 1.3 **d** n

7 a 100 **b** 7 **c** 3.6 **d** y

8 a 2 **b** 15 **c** −1 **d** 6

9 a $3 \log_a x$ **b** $-\log_a x$
c $\frac{1}{2} \log_a x$ **d** $-2 \log_a x$
e $-2 \log_a x$ **f** $2 \log_a x$
g $8 - 8 \log_a x$ **h** $\log_a x$

10 a $\log_a y + \log_a z$ **b** $\log_a z - \log_a y$
c $4 \log_a y$ **d** $-2 \log_a x$
e $\log_a x + 3 \log_a y$
f $2 \log_a x + \log_a y - 3 \log_a z$
g $\frac{1}{2} \log_a y$ **h** $\frac{1}{2} \log_a x + \frac{1}{2} \log_a z$

11 a 1.30 **b** −0.70 **c** 2.56 **d** 0.15
e 0.45 **f** −0.50 **g** 0.54 **h** −0.35

12 a $6x$ **b** $-x - y - z$
c $3y + 5$ **d** $2x + 2z - 1$
e $y - x$ **f** $x + 2y - 2z - 1$
g $-2z$ **h** $3x - y - z - 2$

13 a $10 = 3^{\log_3 10}$ **b** $3 = 10^{\log_{10} 3}$
c $0.1 = 2^{\log_2 0.1}$ **d** $2 = \log_{10} 100$
e $-4 = \log_3 3^{-4}$ or $\log_3 \frac{1}{81}$
f $\frac{1}{2} = \log_7 7^{\frac{1}{2}}$ or $\log_7 \sqrt{7}$

14 a $\log_{25} 5 = \frac{1}{2}$ **b** $\log_{81} \frac{1}{3} = -\frac{1}{4}$
c $\log_8 \frac{1}{32} = -\frac{5}{3}$ **d** $\log_{\frac{1}{32}} \frac{1}{2} = \frac{1}{5}$

15 a $\frac{1}{2}$ **b** 49 **c** 15 **d** x^n
e $\frac{1}{x}$ **f** $x \times 5^x$ **g** x^x **h** $x^{1/x}$

16 a $x + y = xy$ **b** $x = 1000y$
c $x = y^4$ **d** $x^2y^3 = z^4$
e $2^x = y$ **f** $x = yz^n$
g $64x^3 = y^2$ **h** $(2x + 1)^2 = (2x - 1)^3$

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17 Let $\log_2 3 = \frac{a}{b}$, where a and b are positive whole numbers.
Then $b \log_2 3 = a$
$\log_2 3^b = a$
$3^b = 2^a$.
This is impossible because 3^b is odd and 2^a is even.

Exercise 8E

2 a 2.807 **b** 4.700 **c** -3.837
d 7.694 **e** 0.4307 **f** 1.765
g 0.6131 **h** 0.2789 **i** -2.096
j -7.122 **k** 2.881 **l** 7.213
m 0.03323 **n** 578.0 **o** -687.3

3 a $x = \log_2 15 \doteqdot 3.907$ **b** $x = \log_2 5 \doteqdot 2.322$
c $x = \log_2 1.45 \doteqdot 0.5361$ **d** $x = \log_2 0.1 \doteqdot -3.322$
e $x = \log_2 0.0007 \doteqdot -10.48$ **f** $x = \log_3 10 \doteqdot 2.096$
g $x = \log_3 0.01 \doteqdot -4.192$ **h** $x = \log_5 10 \doteqdot 1.431$
i $x = \log_{12} 150 \doteqdot 2.016$ **j** $x = \log_8 \frac{7}{9} \doteqdot -0.1209$
k $x = \log_6 1.4 \doteqdot 0.1878$ **l** $x = \log_{30} 2 \doteqdot 0.2038$
m $x = \log_{0.7} 0.1 \doteqdot 6.456$ **o** $x = \log_{0.99} 0.01 \doteqdot 458.2$
n $x = \log_{0.98} 0.03 \doteqdot 173.6$

4 a $x > 5$ **b** $x \le 5$
c $x < 6$ **d** $x \ge 4$
e $x > 1$ **f** $x \le 0$
g $x < -1$ **h** $x \le -3$

5 a $x = 1$ or $x = \log_2 7$
b $x = 2$ ($3^x = -1$ has no solutions.)
c i $x = 2$ or $x = 0$ **ii** $x = 0$ or $x = \log_3 4$
iii $x = \log_3 5$ ($3^x = -4$ has no solutions.)
iv The quadratic has no solutions
v $x = 3$
vi $x = 2$ or $x = 0$

6 a $0 < x < 8$ **b** $x \ge 8$
c $x > 1000$ **d** $x \ge 10$
e $x > 1$ **f** $0 < x < 6$
g $0 < x \le 125$ **h** $x > 36$

7 a $x > \log_2 12 \doteqdot 3.58$ **b** $x < \log_2 100 \doteqdot 6.64$
c $x < \log_2 0.02 \doteqdot -5.64$ **d** $x > \log_2 0.1 \doteqdot -3.32$
e $x < \log_5 100 \doteqdot 2.86$ **f** $x < \log_3 0.007 \doteqdot -4.52$
g $x > \log_{1.2} 10 \doteqdot 12.6$ **h** $x > \log_{1.001} 100 \doteqdot 4610$

8 a After 1 year, the price is 1.05 times greater, after 2 years, it is $(1.05)^2$ times greater, and so on.
b $\log_{1.05} 1.5 \doteqdot 8.3$ years

9 a $\log_8 x = \dfrac{\log_2 x}{\log_2 8} = \dfrac{1}{3}\log_2 x$

b $\log_{a^n} x = \dfrac{\log_a x}{\log_a a^n} = \dfrac{1}{n}\log_a x$

10 a $x = 3$ **b** $x = 2$
c $x < 1$ **d** $x \le 9$
e $x = 0$ **f** $x = \frac{1}{5}$
g $x < 4.81$ **h** $x > -2.90$

11 a $x < 33.2$, 33 powers
b $x < 104.8$, 104 powers

12 a $10^2 < 300 < 10^3$ **b** $1 \le \log_{10} x < 2$
c 5 digits **d** 27.96, 28 digits
e $1000 \log_{10} 2 = 301.03$, 302 digits

14 a $x = 1$ or $x = \log_4 3 \doteqdot 0.792$
b $x = \log_{10} \frac{1+\sqrt{5}}{2} \doteqdot 0.209$. $\log_{10} \frac{1-\sqrt{5}}{2}$ does not exist because $\frac{1-\sqrt{5}}{2}$ is negative.
c $x = -1$ or $x = \log_{\frac{1}{5}} 2 \doteqdot -0.431$

15 a $\dfrac{\log_{10}47 + 4\log_{10}3}{\log_{10}3} \doteqdot 7.505$

b $\dfrac{-5\log_{10}2 - \log_{10}5}{\log_{10}2} \doteqdot -7.322$

c $\dfrac{\log_{10}6}{2\log_{10}5 - \log_{10}6} \doteqdot 1.256$

d $\dfrac{\log_{10}7 - \log_{10}6 + 3\log_{10}5}{\log_{10}5 + \log_{10}7} \doteqdot 1.401$

16 a $SD = \frac{1}{4}(2^{2x} - 2^{-2x})$, $S + D = 2^x$, $S - D = 2^{-x}$, $S^2 - D^2 = 1$
b $x = \log_2(S + \sqrt{S^2 - 1})$,
$x = \log_2(D + \sqrt{D^2 + 1})$

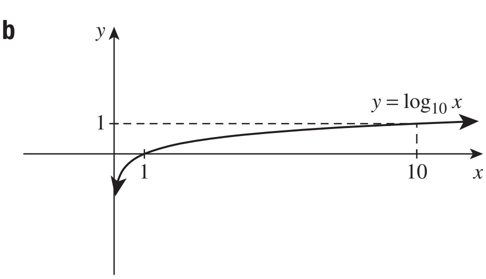
Exercise 8F

1 a

x	0.1	0.25	0.5	0.75	1	2
$\log_{10}x$	-1	-0.60	-0.30	-0.12	0	0.30

x	3	4	5	6	7	8	9	10
$\log_{10}x$	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1

b



2 a i

x	-3	-2	-1	0	1	2	3
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

ii

x	-3	-2	-1	0	1	2	3
2^{-x}	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

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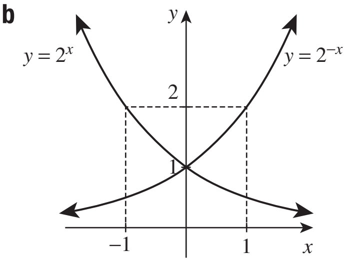
b



c The values of $y = 2^{-x}$ are the values of $y = 2^x$ in reverse order.

d The two graphs are reflections of each other in the y-axis, because x has been replaced with $-x$.

e For both, domain: all real x, range: $y > 0$

f For both, the asymptote is $y = 0$ (the x-axis).

g i 'As $x \to -\infty$, $2^x \to 0$.'

ii 'As $x \to \infty$, $2^x \to \infty$.'

h i 'As $x \to -\infty$, $2^{-x} \to \infty$.'

ii 'As $x \to \infty$, $2^{-x} \to 0$.'

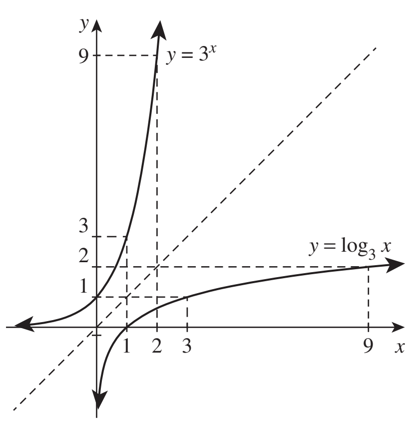
3 i

x	-2	-1	0	1	2
3^x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

ii

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$\log_3 x$	-2	-1	0	1	2

b



c The two rows have been exchanged.

d The two graphs are reflections of each other in the diagonal line $y = x$, because the two functions are inverses of each other.

e i domain: all real x, range: $y > 0$

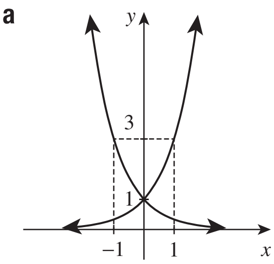
ii domain: $x > 0$, range: all real y

f i $y = 0$ (the x-axis) **ii** $x = 0$ (the y-axis)

g i 'As $x \to -\infty$, $3^x \to 0$'.

ii 'As $x \to 0^+$, $\log_3 x \to -\infty$'.

4 a



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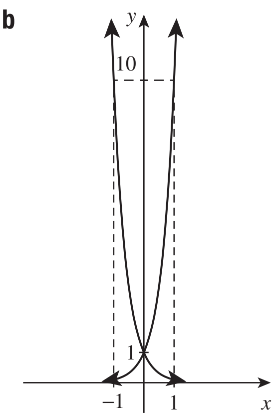

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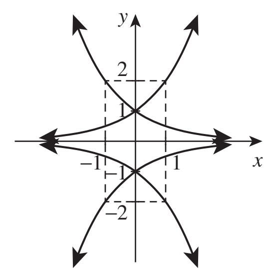

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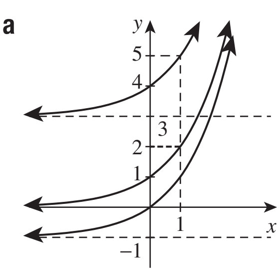

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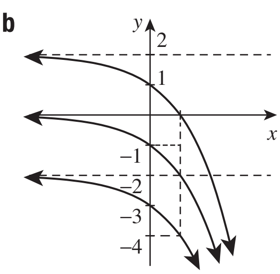

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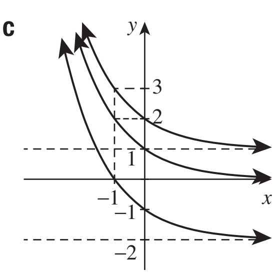

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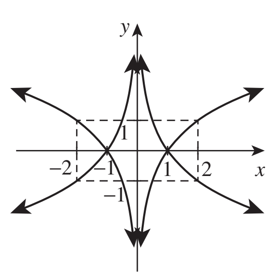
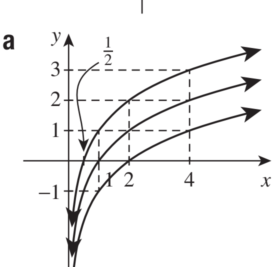


8 a



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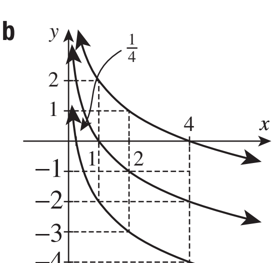

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9 a **i** 4 **ii** $\frac{1}{4}$ **iii** 2.83 **iv** 1.32 **v** 0.66

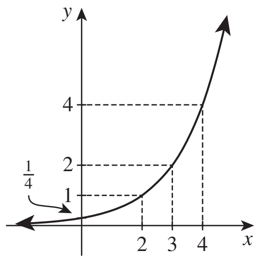
b **i** 1 **ii** 1.58 **iii** 0.26 **iv** -1.32

c **i** $0 \le x \le 2$ **ii** $0 \le x \le 1$

iii $0.58 \le x \le 1.58$ **iv** $-1 \le x \le 1$

d **i** 2 **ii** 1.58 **iii** 0.49 **iv** -0.32

10 a



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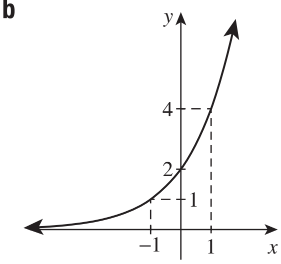

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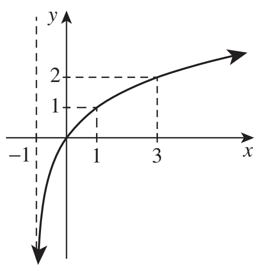

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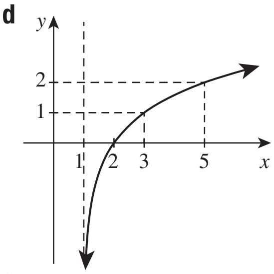

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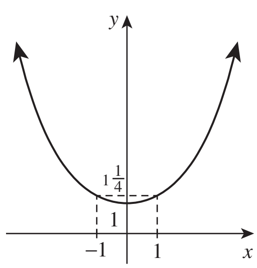

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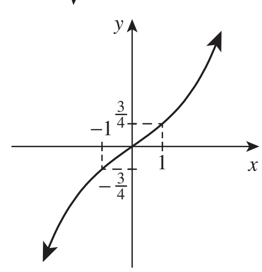
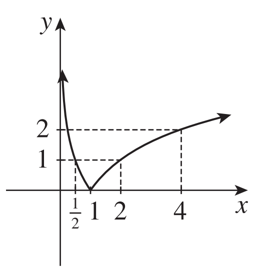


11 a



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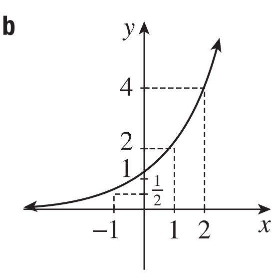

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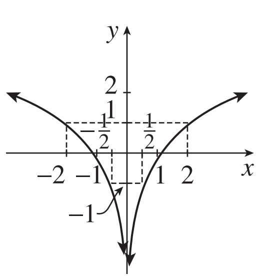

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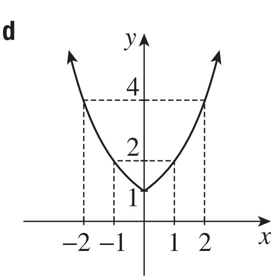

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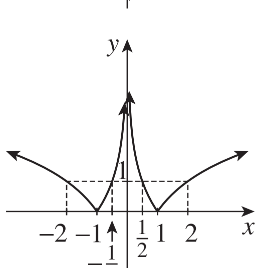

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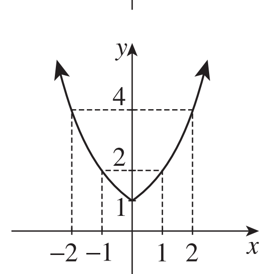
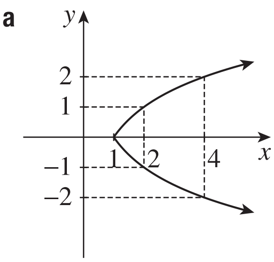


12 a



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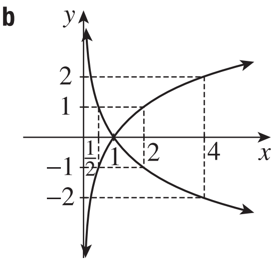

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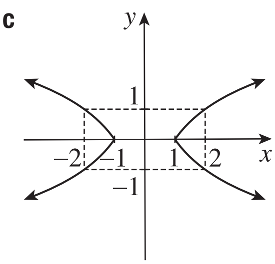

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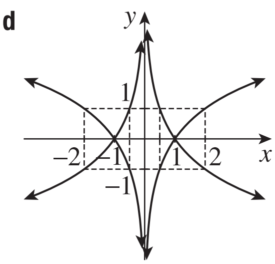
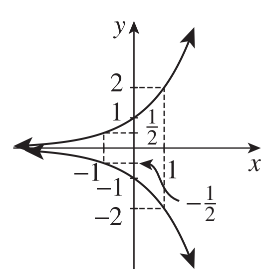
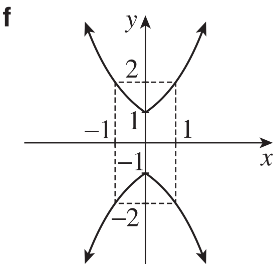


e v



f



Exercise 8G

1 a 5000, 2594

b $\frac{t}{2} = \log_{10}\frac{Q}{5}$, so $t = 2\log_{10}\frac{Q}{5}$

c 4, 3.419

2 a 60, $20\log_{10}12 = \dfrac{20\log_{10}12}{\log_{10}2} \doteqdot 71.70$

b $\frac{t}{20} = \log_2 2Q$, so $2Q = 2^{\frac{t}{20}}$, so $Q = \frac{1}{2} \times 2^{\frac{t}{20}}$

c 2, 2.378

3 a There are $\frac{n}{30}$ thirty-year intervals in n years.

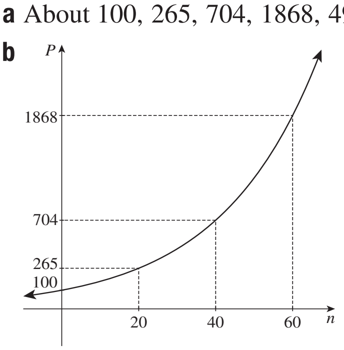
b **i** 24 000 000 **ii** 30 000 000

c **i** 120 years

ii $30\log_2 20 \doteqdot 130$ years

4 a About 100, 265, 704, 1868, 4956, 13 150

b



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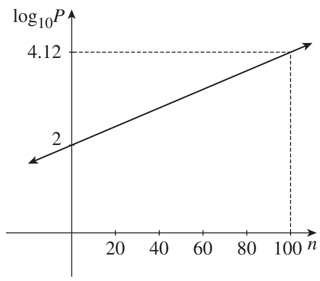
c The values are about 2, 2.42, 2.85, 3.27, 3.70, 4.12

d



e The new graph is a straight line, and $\log_{10}P$ is a linear function of n.

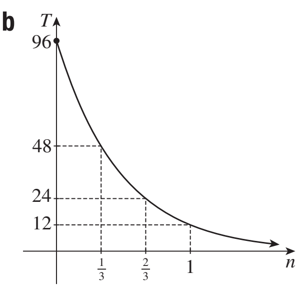
5 a $\frac{n}{2}$ is the number of 2-year periods.

b $D = 2^{20}D_0 \doteqdot 1050000D_0$

c $2^{\frac{n}{2}} = 10^7$, so $\frac{n}{2} = \log_2 10^7$,
so $n = 2\log_2 10^7 \doteqdot 47$ years, that is, in 2022.

6 a $3n$ is the number of 20-minutes periods in n hours.

b



c $96 \times \left(\frac{1}{2}\right)^6 = 1\frac{1}{2}\,^\circ\text{C}$

d $3n = \log_{\frac{1}{2}} \frac{T}{96}$, so $n = \frac{1}{3}\log_{\frac{1}{2}} \frac{T}{96}$. (Alternatively, $n = -\frac{1}{3}\log_2 \frac{T}{96}$.)

e $n = \frac{1}{3}\log_{\frac{1}{2}} \frac{1}{96} = 2.1949\ldots$ hours
$\doteqdot$ 2 hours and 12 minutes

7 a The mass halves every 700 000 000 years.

b When $n = 4$ billion, $\dfrac{n}{700000000} = \frac{40}{7}$, so
$M = M_0 \times \left(\frac{1}{2}\right)^{\frac{40}{7}} \doteqdot 1.9\%$ of M_0

c When $n = -4.5$ billion, $\dfrac{n}{700000000} = -\frac{45}{7}$, so
$M = M_0 \times \left(\frac{1}{2}\right)^{-\frac{45}{7}} \doteqdot 86\,M_0$

8 a 96 dB

b $I = I_r \times 10^{\frac{n}{10}}$, $3.16 \times 10^{-5}\ \text{W/m}^2$

c $10^{3.6} \doteqdot 3980$ times

d $70 - 10\log_{10}1600 \doteqdot 38$ dB

9 a $\dfrac{1}{1000}$ **b** $1000^{\frac{3}{2}} \doteqdot 32000$

c Ratio of shaking amplitudes is 10^5, ratio of energies released is about 3.2×10^7.

10 a $[\text{H}^+] = 10^{-\text{pH}}$ **b** About $10^{-7}\,\text{mol/L}$

c About $10^{-2}\,\text{mol/L}$, about 100 000 times more acidic than water

d About $7.94 \times 10^{-9}\,\text{mol/L}$, about 12.6 times more alkaline than water

11 a C 261.63 Hz, C♯ 277.18 Hz, E 329.63 Hz

b C 264 Hz, C♯275 Hz, E 330 Hz

c C 2.37 Hz, C♯2.18 Hz, E 0.37 Hz

d A′ would be $220 \times \left(\frac{5}{4}\right)^3 \doteqdot 429.69$ Hz, beating at 10.31 Hz with A′440

e Meantone:245.97 Hz, equal temperament: 246.94 Hz, beating at about 1 Hz

Chapter 8 review exercise

1 a 125 **b** 256 **c** 1000000000 **d** $\frac{1}{17}$
e $\frac{1}{81}$ **f** $\frac{1}{8}$ **g** $\frac{1}{81}$ **h** 1
i $\frac{8}{27}$ **j** $\frac{12}{7}$ **k** $\frac{36}{25}$ **l** 6
m 3 **n** 4 **o** 243 **p** $\frac{2}{7}$
q 1 **r** $\frac{5}{3}$ **s** $\frac{4}{9}$ **t** $\frac{1000}{27}$

2 a x^{-1} **b** $7x^{-2}$ **c** $-\frac{1}{2}x^{-1}$ **d** $x^{\frac{1}{2}}$
e $30x^{\frac{1}{2}}$ **f** $4x^{-\frac{1}{2}}$ **g** yx^{-1} **h** $2yx^{\frac{1}{2}}$

3 a x^{20} **b** $\dfrac{81}{a^{12}}$ **c** $5x^3$ **d** $\dfrac{2r}{t^2}$

4 a x^3y^3 **b** $60xy^3z^5$ **c** $18x^{-1}y^{-2}$
d $4a^3b^3c^{-1}$ **e** x^2y^{-2} **f** $2x^{-3}y$
g m^2n^{-1} **h** $72s^9t^3$ **i** $8x^3y^{-3}$

5 a 4 **b** 2 **c** −1 **d** −5
e 2 **f** 3 **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

6 a $2^x = 8, x = 3$ **b** $3^x = 9, x = 2$
c $10^x = 10000, x = 4$ **d** $5^x = \frac{1}{5}, x = -1$
e $7^x = \frac{1}{49}, x = -2$ **f** $13^x = 1, x = 0$
g $9^x = 3, x = \frac{1}{2}$ **h** $2^x = \sqrt{2}, x = \frac{1}{2}$
i $7^2 = x, x = 49$ **j** $11^{-1} = x, x = \frac{1}{11}$
k $16^{\frac{1}{2}} = x, x = 4$ **l** $27^{\frac{1}{3}} = x, x = 3$
m $x^2 = 36, x = 6$ **n** $x^3 = 1000, x = 10$
o $x^{-1} = \frac{1}{7}, x = 7$ **p** $x^{\frac{1}{2}} = 4, x = 16$

7 a 1 **b** 2 **c** 2 **d** −2 **e** 2 **f** 0

8 a $\log_a x + \log_a y + \log_a z$ **b** $\log_a x - \log_a y$
c $3\log_a x$ **d** $-2\log_a z$
e $2\log_a x + 5\log_a y$
f $2\log_a y - \log_a x - 2\log_a z$
g $\frac{1}{2}\log_a x$
h $\frac{1}{2}\log_a x + \frac{1}{2}\log_a y + \frac{1}{2}\log_a z$

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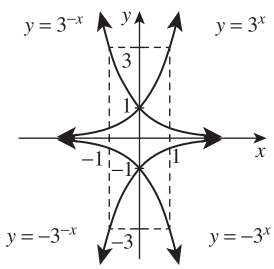
9 a 1 & 2 **b** 2 & 3 **c** 4 & 5 **d** 5 & 6
e -1 & 0 **f** -3 & -2 **g** -4 & -3 **h** -2 & -1

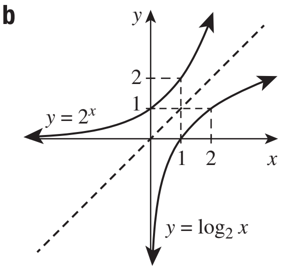
10 a 2.332 **b** -2.347 **c** 2.010 **d** 9.966
e -0.9551 **f** 69.66 **g** -3 **h** 687.3

11 a 3.459 **b** -4.644 **c** 3.010 **d** -0.3645
e 161.7 **f** -161.7 **g** 10.32 **h** 458.2

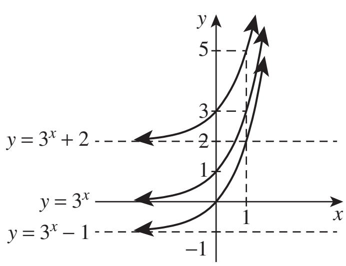
12 a



b



c



13 a There are $\frac{n}{4}$ four-hour periods in n hours.

b i 800 **ii** $100 \times 2^{3.25} \doteqdot 950$

c $\frac{n}{4} = \log_2 \frac{P}{100}$, so $n = 4 \log_2 \frac{P}{100}$.

d $4 \log_2 100\,000 \doteqdot 66$ hours

Chapter 9

Exercise 9A

1 The values of $f'(x)$ should be about $-4, -3, -2, -1, 0, 1, 2, 3, 4$. The graph of $y = f'(x)$ should approximate a line of gradient 2 through the origin; its exact equation is $f'(x) = 2x$.

2 Answers the same as Question 1

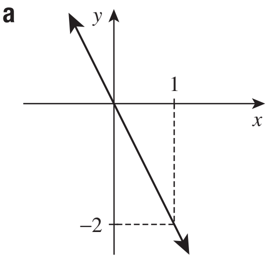
3 The values of $f'(x)$ should be about $1\frac{1}{2}, 0, -0.9, -1.2, -0.9, 0, 1\frac{1}{2}$. The graph of $y = f'(x)$ is a parabola crossing the x-axis at $x = -2$ and $x = 2$.

4 The eventual graph of $f'(x)$ is a parabola with its vertex at the origin. Depending on the software, you may be able to see that it is $y = 3x^2$.

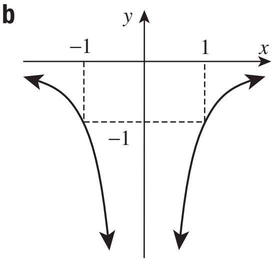
5 a 2 **b** -3 **c** $\frac{1}{2}$ **d** 0 **e** a
f $\frac{2}{3}$ **g** $-\frac{5}{4}$ **h** $-\frac{10}{3}$ **i** 0

6 a $\frac{7}{2}$ **b** 12 **c** 0

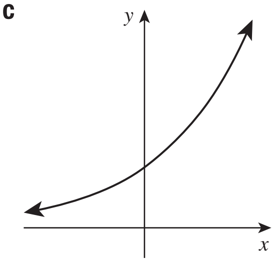
7 a



b



c



8 a $-\frac{4}{3}$ **b** $-\frac{3}{4}$ **c** 0 **d** $\frac{4}{3}$ **e** $\frac{3}{4}$

9 a $\dfrac{-x}{\sqrt{1 - x^2}}$ **b** $\dfrac{x}{\sqrt{1 - x^2}}$ **c** $\dfrac{-x}{\sqrt{4 - x^2}}$

10 a $\dfrac{-x}{\sqrt{9 - x^2}}$ **b** $\dfrac{x}{\sqrt{16 - x^2}}$

c $\dfrac{7 - x}{\sqrt{36 - (x - 7)^2}}$ **d** $\dfrac{x - 1}{\sqrt{2x - x^2}}$

Exercise 9B

3 c At A, $f'(1) = -2$.
d At B, $f'(3) = 2$; at C, $f'(2) = 0$.

4 a $\dfrac{f(x + h) - f(x)}{h} = 3$. Trivially this has limit 3 as $h \to 0$.

b $\dfrac{f(x + h) - f(x)}{h} = m$. Trivially this has limit m as $h \to 0$.

c $\dfrac{f(x + h) - f(x)}{h} = 0$. Trivially this has limit 0 as $h \to 0$.

5 a i $2x + h, 2x$ **ii** 4 **iii** $(0, 10)$
b i $2x + h + 6, 2x + 6$
ii 10
iii $(-3, -7)$
c i $4x + 2h - 20, 4x - 20$
ii -12
iii $(5, -50)$
d i $-8x - 4h, -8x$
ii -16
iii $(0, 9)$

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6 b **i** -10 **ii** 10 **iii** 0 **iv** -1 **v** $+1$

c $90°$

7 b **i** $(3, -6)$ **ii** $(2, -6)$ **iii** $(5, 0)$

iv $(0, 0)$ **v** $\left(2\frac{1}{2}, -6\frac{1}{4}\right)$

8 b At $(6, 0)$, $f'(6) = 5$. At $(1, 0)$, $f'(1) = -5$.

c $A = (0, 6)$, $m = f'(0) = -7$, $B = (7, 6)$.

d $\left(3\frac{1}{2}, -6\frac{1}{4}\right)$

9 b $x = -\dfrac{b}{2a}$

c It is the axis of symmetry of the parabola.

10 a ii $f'(x) = 3x^2$

b ii $f'(x) = 4x^3$

11 b $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-2x - h}{(x+h)^2 x^2}$

c As $x \to 0^+$ and as $x \to 0^-$, the gradient decreases without bound, so the tangents slope more and more steeply backwards. As $x \to \infty$ and as $x \to -\infty$, the gradient approaches zero, so the tangents become more and more horizontal.

12 c $f'(x) = \dfrac{1}{2\sqrt{x}}$

d As $x \to 0^+$, the gradient increases without bound, so the tangents slope more and more steeply.
As $x \to \infty$, the gradient approaches zero, so the tangents become more and more horizontal.

13 a $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-2x - h}{(x+h)^2 x^2}$

b $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{\sqrt{x(x+h)}\left(\sqrt{x} + \sqrt{x+h}\right)}$

14 The line is a tangent when the two points coincide, that is, when $m = 2a$, so the gradient of the tangent is twice the x-coordinate.

15 They meet at $x = \frac{1}{2}\left(m + \sqrt{m^2 + 4b}\right)$ and at $x = \frac{1}{2}\left(m - \sqrt{m^2 + 4b}\right)$. The line is a tangent when these coincide, that is, when $m^2 + 4b = 0$, in which case the tangent at $x = \frac{1}{2}m$ has gradient m, which is twice the x-coordinate.

Exercise 9C

1 a $7x^6$ **b** $45x^4$

c $2x^5$ **d** $6x - 5$

e $4x^3 + 3x^2 + 2x + 1$ **f** $-3 - 15x^2$

g $2x^5 - 2x^3 + 2x$ **h** $x^3 + x^2 + x + 1$

2 a $12 - 12x$ **b** $3x^2 + 1$

c $6x - 6x^2 - 16x^3$ **d** $2x + 2$

e $8x$ **f** $2x - 14$

g $4x^3 + 12x$ **h** $3x^2 - 28x + 49$

i $3x^2 - 10x + 3$

3 a $4ax^3 - 2bx$ **b** $2a^2x - 10a$

c $2kax$ **d** $\ell x^{\ell - 1}$

e $(5a + 1)x^{5a}$ **f** $3b^2 x^{3b-1}$

4 a $1, -1, 45°, 135°$ **b** $-1, 1, 135°, 45°$

c $-6, \frac{1}{6}$, about $99°28'$, $9°28'$

5 a $y = -6x + 14$, $x - 6y + 47 = 0$

b $y = 4x - 21$, $x + 4y - 18 = 0$

c $y = -8x + 15$, $x - 8y + 120 = 0$

d $y = -1$, $x = 4$

6 a $(2, 8)$ and $y = 8$

b $(2, 8)$ and $y = 8$, $(-2, 40)$ and $y = 40$

c None

d $(2a, 4a^2)$ and $y = 4a^2$

e $(0, 0)$ and $y = 0$, $(1, -1)$ and $y = -1$, $(-1, -1)$ and $y = -1$

f None, because $5x^4 + 1$ is always positive.

7 $f'(x) = 3x^2$, which is positive for $x \neq 0$ and zero for $x = 0$. The tangent crosses the curve at the origin.

8 $y = -2x + 5$, $y = 2x + 5$, $(0, 5)$

9 $2x + y = 16$, $A = (8, 0)$, $B = (0, 16)$, $AB = 8\sqrt{5}$, $|\Delta OAB| = 64$ square units

10 $y = -2x + 10$, $x - 2y + 15 = 0$, $A = (5, 0)$, $B = (-15, 0)$, $AB = 20$, $|\Delta AKB| = 80$ square units

11 $y = 3x - 2$, $x + 3y = 4$, $P = (0, -2)$, $Q = \left(0, 1\frac{1}{3}\right)$, $|\Delta QUP| = 1\frac{2}{3}$ square units

12 a $f'(9) = 14$, $f'(-5) = -14$

b The vertex is $(2, -49)$, where 2 is the mean of 9 and -5. The parabola has line symmetry in the vertical line through the vertex, and this symmetry exchanges the two x-intercepts and reflects a line with gradient m to a line with gradient $-m$.

13 $f'(x) = 3x^2 + a$, $x = \sqrt{\dfrac{-a}{3}}$ and $x = -\sqrt{\dfrac{-a}{3}}$, $a \le 0$ (but no restriction on b)

14 a $G'(t) = 3t^2 - 8t + 6$, $G'(3) = 9$

b $\ell'(h) = 20h^3$, $\ell'(2) = 160$

c **i** $2ak - a^2$ **ii** a^2

iii $-a^2$ **iv** $2a^2$

15 The tangent has gradient $2a - 6$, the normal has gradient $\dfrac{1}{6 - 2a}$.

a 4 **b** $2\frac{7}{8}$ **c** $2\frac{1}{2}$

d $3 - \frac{1}{2}\sqrt{3}$ **e** $3\frac{1}{3}$ **f** $2\frac{1}{4}$

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16 a -7 **b** 6 **c** 1
d 5 **e** 6 or -6

17 a The tangents are $y = 2ax - a^2$ and $y = 2bx - b^2$. They meet at $K = \left(\frac{1}{2}(a + b), ab\right)$.
b The y-coordinate ab of K is positive when a and b have the same sign, that is, when A and B are both on the right of the y-axis, or both on the left of the y-axis. The sketch of the parabola should make this result obvious.

18 a $b^2 > 3ac$ **b** $b^2 = 3ac$ **c** $b^2 < 3ac$

20 a $y = (2at + b)x - at^2 + c$. a and c must have the same sign, or $c = 0$ (b is arbitrary).
$y = (2\sqrt{ac} + b)x$ and $y = (-2\sqrt{ac} + b)x$
b Points of contact: $\left(\sqrt{\frac{c}{a}}, 2c + b\sqrt{\frac{c}{a}}\right)$ and $\left(-\sqrt{\frac{c}{a}}, 2c - b\sqrt{\frac{c}{a}}\right)$, whose midpoint is $(0, 2c)$.
c $2\sqrt{\frac{c^3}{a}}$ square units

Exercise 9D

1 a $3x^2 + 6x + 6, 3$ **b** $4x^3 + 2x + 8, 2$
c $0, 0$ **d** $4x - 5, -9$

2 a i $\frac{dy}{dx} = 6x^5 + 2, \frac{d^2y}{dx^2} = 30x^4, \frac{d^3y}{dx^3} = 120x^3$
ii $\frac{dy}{dx} = 10x - 5x^4, \frac{d^2y}{dx^2} = 10 - 20x^3,$
$\frac{d^3y}{dx^3} = -60x^2$
iii $\frac{dy}{dx} = 4, \frac{d^2y}{dx^2} = 0, \frac{d^3y}{dx^3} = 0$
b i $f'(x) = 30x^2 + 1, f''(x) = 60x, f'''(x) = 60,$
$f^{(4)}(x) = 0$
ii $f'(x) = 8x^3, f''(x) = 24x^2, f'''(x) = 48x,$
$f^{(4)}(x) = 48$
iii $f'(x) = 0, f''(x) = 0, f'''(x) = 0, f^{(4)}(x) = 0$
c i 6 times **ii** $n + 1$ times

3 a $(1, -4)$
b $(1, -3)$ and $(-1, 3)$
c none

4 a 1 **b** $0, 3, -3$ **c** none

5 a $\frac{dy}{dx} = 9x^2 - 9 = 9(x - 1)(x + 1)$, which is zero when $x = 1$ or $x = -1$
b $\frac{dy}{dx} = 5x^2 + 2x = x(5x + 2)$, which is zero when $x = 0$ or $x = -\frac{2}{5}$

6 a $y = -6x, y = \frac{1}{6}x$
b $y = 2x + 2, x + 2y + 1 = 0$
c $y = 0, x = 1$

7 a They all have derivative $3x^2 + 7$. First to second, shift down 10. First to third, shift down $7\frac{1}{2}$. First to fourth, shift up 96.
b The third has derivative $-2x^3 + 6x$. The other three have derivative $2x^3 + 6x$.

8 $63°26'$ at $(-1, 0)$, $116°34'$ at $(1, 0)$

9 a $\left(1, -6\frac{2}{3}\right), \left(-1, -7\frac{1}{3}\right)$
b $\left(-1, \frac{2}{3}\right)$ **c** $\left(-\frac{1}{2}\sqrt{3}, 1\frac{3}{4}\right)$

10 a $y = 2px + 9 - p^2$
b Substitute $(0, 0)$. At $(3, 18)$ the tangent is $y = 6x$, and at $(-3, 18)$ the tangent is $y = -6x$.

11 a $y = (2t - 10)x - t^2 + 9, t = 3$ and $y = -4x$, or $t = -3$ and $y = -16x$
b $y = (2t + 15)x - t^2 + 36, t = 6$ and $y = 27x$, or $t = -6$ and $y = 3x$

12 a $y = 2(t + 1)x - t^2 - 8$
b $(1, -5), (3, 7)$

13 a $y = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$
b $y = 2x^3 - 7x + C$
c $y = \frac{5}{4}x^4 + x^3 - 4x + C$
d $y = 2x^5 - 4x^3 - 24x + C$

14 a $b = 7, c = 0$ **b** $b = -2, c = -3$
c $b = -10, c = 25$ **d** $b = -1, c = -2$
e $b = -9, c = 14$ **f** $b = -\frac{17}{3}, c = 4$

15 The tangent is $y = x$.

16 At $(2, 1)$ the gradient is 2, which is perpendicular to $x + 2y = 4$; at $\left(-\frac{1}{2}, \frac{9}{4}\right)$ the gradient is -3.

17 a $y = 2ax - a^2, U = \left(\frac{1}{2}a, 0\right), V = (0, -a^2)$
b $T = (5, 25)$ or $(-5, 25)$

18 At $(1, -3)$ the tangent is $x + y + 2 = 0$, at $(-1, 3)$ the tangent is $x + y - 2 = 0$. The first tanget is the line given in the question.

19 a A and B are $(2, -4)$ and $(-3, 6)$.
b $M = \left(-\frac{1}{2}, 1\right)$, gradients of tangent at T and chord AB are both -2.

20 $\frac{dP}{dx} = 2tx + 3u, \frac{dP}{du} = 6tu + 3x,$
$\frac{dP}{dt} = x^2 + 3u^2 + 1$

22 b $y = x^2 - 6x$ and $y = \frac{25}{81}x^2 + \frac{2}{9}x$
c $y = x^2 - x - 6$

23 a $y - (at^3 + 2bt^2 + ct + d)$
$= (3at^2 + 2bt + c)(x - t)$

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b The condition for P to lie on the tangent at T is
$y_0 - (at^3 + bt^2 + ct + d)$
$= (3at^2 + 2bt + c)(x_0 - t)$.
This is a cubic in t, and every cubic has at least one solution. (Why?)

Exercise 9E

1 $\frac{du}{dx} = 2x, \frac{dy}{du} = 5u^4,$
$\frac{dy}{dx} = 5(x^2 + 9)^4 \times 2x = 10x(x^2 + 9)^4$

2 a $12(3x + 7)^3$
b $-28(5 - 4x)^6$
c $24x(x^2 + 1)^{11}$
d $-64x(7 - x^2)^3$
e $9(2x + 3)(x^2 + 3x + 1)^8$
f $-18(3x^2 + 1)(x^3 + x + 1)^5$

3 a $25(5x - 7)^4$
b $49(7x + 3)^6$
c $180(5x + 3)^3$
d $-21(4 - 3x)^6$
e $12\left(\frac{1}{2}x - 1\right)^3$
f $-\frac{8}{9}\left(5 - \frac{1}{3}x\right)^3$

4 $20(5x - 2)^3, 300(5x - 2)^2, 3000(5x - 2),$ $15\,000, 0, 0$

5 $2(x - 3)$

6 a $2\frac{1}{2}$ and 1 **b** 2 and $1\frac{1}{2}$

7 a $y = 20x - 19, x + 20y = 21$
b $y = 24x - 16, x + 24y = 193$

8 a $6(x - 5)^5, (5, 4)$
b $-14(x - 5), (5, 24)$
c $2a(x - h), (h, k)$
d $6x(x^2 - 1)^2, (0, -1), (1, 0), (-1, 0)$
e $8(x - 2)(x^2 - 4x)^3, (0, 0), (2, 256), (4, 0)$
f $10(x + 1)(2x + x^2)^4, (0, 0), (-2, 0), (-1, -1)$

9 a 4 or 8
b -17

10 a $a = \frac{1}{16}, b = 12$
b $a = \frac{1}{9}, b = -10$

11 a $P = \left(7\frac{1}{2}, 3\frac{1}{4}\right), Q = \left(6\frac{1}{2}, 3\frac{1}{4}\right)$
b area $= \frac{1}{2}PQ^2 = \frac{1}{2}$

12 c $y = -16, y = -32x$

13 a 4t, -4
b $\frac{b}{a}, \frac{b}{a}$
c $\frac{9}{4}t, -\frac{9}{4}$

14 a $y = \frac{1}{3}x + 15$ **b** $y = 3x - 4$

17 The second tangent is the first tangent reflected in the line $y = x$, which exchanges the rise and run and thus does not change the sign of the gradient. Alternatively, $g'(b) = \frac{1}{f'(a)}$ by the formula for differentiating inverse functions, so they have the same sign.

18 a At P, $x = h + \frac{1}{2}m$. At Q, $x = h - \frac{1}{2}m$.
b $\frac{1}{4}m(m^2 + 1)$

19 b Both distances are $a(\alpha - h)^2$.
c $\alpha = \sqrt{h^2 + \frac{k}{a}}$ or $-\sqrt{h^2 + \frac{k}{a}}$

Exercise 9F

1 a $-x^{-2}$ **b** $-5x^{-6}$
c $-3x^{-2}$ **d** $-10x^{-3}$
e $4x^{-4}$ **f** $-4x^{-3} - 4x^{-9}$

2 a $f(x) = x^{-1}, f'(x) = -x^{-2} = -\frac{1}{x^2}$
b $f(x) = x^{-2}, f'(x) = -2x^{-3} = -\frac{2}{x^3}$
c $f(x) = x^{-4}, f'(x) = -4x^{-5} = -\frac{4}{x^5}$
d $f(x) = 3x^{-1}, f'(x) = -3x^{-2} = -\frac{3}{x^2}$

3 a $-42(7x - 5)^{-7}$ **b** $-5(3 + 5x)^{-2}$
c $-8(2x - 1)^{-2}$ **d** $-\frac{105}{4}(5x + 6)^{-8}$

4 $-x^{-2}, 2x^{-3}, -6x^{-4}, 24x^{-5}, -120x^{-6}$

5 a $(1, 1)$ and $(-1, -1)$
b $\left(1, \frac{1}{2}\right)$

6 a $y = -\frac{4}{3}x + \frac{7}{3}, 3x - 4y + 1 = 0$
b $y = -9x + 6, x - 9y - 28 = 0$

7 a $-\frac{3}{x^2}$ **b** $-\frac{1}{3x^2}$
c $\frac{7}{3x^2}$ **d** $-\frac{a}{x^2}$

8 a $2x - 2x^{-3}$ **b** 0 **c** $1 - \frac{1}{x^2}$

9 a $\frac{-2x}{(1 + x^2)^2}, (0, 1)$
b $\frac{3(4x^3 - 4x)}{(x^4 - 2x^2 - 1)^2}, (0, 3)$ and $\left(1, \frac{1}{2}\right)$

10 a $f'(x) = -x^{-2}, f''(x) = 2x^{-3}, f'''(x) = -6x^{-4},$ $f^{(4)}(x) = 24x^{-5}, f^{(5)}(x) = -120x^{-6}$
b $f'(1) = -1, f''(1) = 2, f'''(1) = -6,$ $f^{(4)}(1) = 24, f^{(5)}(1) = -120$
c Start with -1, then multiply by $-n$ to get each next term.
d Same as before, except that all the terms are negative.

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11 $a = -5$ or $a = -7$

12 a $x + y(b - 4)^2 = 2b - 4$

b i $x + 4y = 0$ **ii** $x + y = 6$

13 a They both have gradient -3.

b At M: $y = -3x + 12$. At N: $y = -\frac{1}{3}x + 4$. They intersect at $(3, 3)$.

c Part **a** follows from the curve's odd symmetry in the origin — the point $M(2, 6)$ and its tangent corresponds to $T(-2, -6)$ and its tangent — a rotation of 180° maps any line to a parallel line (going in the other direction). Part **b** follows from the curve's line symmetry in $y = x$.

14 a $cx + t^2y = 2ct$, $A = (2t, 0)$, $B = \left(0, 2\frac{c}{t}\right)$

b $2|c|$

15 b $\dfrac{dy}{dx} = -\dfrac{c^2}{x^2} = -\dfrac{1}{t^2}$

c tangent: $y = -\dfrac{x}{t^2} + \dfrac{2c}{t}$,

$A(2ct, 0)$, $AT^2 = OT^2 = (ct)^2 + \left(\dfrac{c}{t}\right)^2$.

Exercise 9G

1 a $-\frac{1}{2}x^{-1\frac{1}{2}}$ **b** $\frac{3}{2}x^{\frac{1}{2}}$ **c** $4x^{-\frac{1}{3}}$

d $-4x^{-1\frac{1}{3}}$ **e** $x^{-\frac{3}{4}} - 2x^{-\frac{5}{4}}$ **f** $\frac{49}{3}x^{1\frac{1}{3}}$

2 a $y = x^{\frac{1}{2}}$, $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

b $y = x^2\sqrt{x} = x^2 \times x^{\frac{1}{2}} = x^{2\frac{1}{2}} = x^{\frac{5}{2}}$,

$\dfrac{dy}{dx} = \frac{5}{2}x^{1\frac{1}{2}} = \frac{5}{2}x\sqrt{x}$

c i $y = x\sqrt{x} = x^1 \times x^{\frac{1}{2}} = x^{1\frac{1}{2}} = x^{\frac{3}{2}}$,

$\dfrac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

ii $y = x^{-\frac{1}{2}}$, $\dfrac{dy}{dx} = -\frac{1}{2}x^{-1\frac{1}{2}} = -\dfrac{1}{2x\sqrt{x}}$

iii $y = \dfrac{1}{x^1 \times x^{\frac{1}{2}}} = x^{-1\frac{1}{2}} = x^{-\frac{3}{2}}$,

$\dfrac{dy}{dx} = -\frac{3}{2}x^{-2\frac{1}{2}} = -\dfrac{3}{2x^2\sqrt{x}}$

3 $\frac{1}{2}x^{-\frac{1}{2}}$, $\frac{1}{4}x^{-\frac{3}{2}}$, $\frac{3}{8}x^{-\frac{5}{2}}$, $-\frac{15}{16}x^{-\frac{7}{2}}$, $\frac{105}{32}x^{-\frac{9}{2}}$

4 a $\frac{8}{3}(7 + 2x)^{\frac{1}{3}}$ **b** $\frac{1}{2}(x + 4)^{-\frac{1}{2}}$

c $-\frac{3}{2}(5 - 3x)^{-\frac{1}{2}}$ **d** $45(2 - 5x)^{-3\frac{1}{4}}$

5 a $\left(\frac{1}{4}, -\frac{1}{2}\right)$ **b** none

6 a Tangent: $y = \frac{1}{4}x + 1$, Normal: $y = -4x + 18$

b Tangent: $y = -\frac{1}{4}x + 3$, Normal: $y = 4x - 14$

7 a $\dfrac{3}{2\sqrt{x}}$ **b** $\dfrac{5}{\sqrt{x}}$ **c** $\dfrac{7}{2\sqrt{x}}$ **d** $\dfrac{\sqrt{7}}{2\sqrt{x}}$

8 a $(3, 2\sqrt{3})$ **b** $(1, 3)$ and $(-1, -3)$

9 a $1 + 3x^{-2} - 16x^{-3}$ **b** $1 + 3x^{-\frac{1}{2}}$

c $\frac{3}{2}x^{-\frac{1}{2}}$

10 a $\dfrac{x - 1}{\sqrt{x^2 - 2x + 5}}$, $(1, 2)$

b $\dfrac{x - 1}{\sqrt{x^2 - 2x}}$, none ($x = 1$ is outside the domain)

c $\dfrac{7x}{\sqrt{x^2 + 1}}$, $(0, 7)$ **d** $\dfrac{-1}{2\sqrt{x}(1 + \sqrt{x})^2}$, none

12 $a = 5$

13 a $12x + 5y = 169$, $y = \frac{5}{12}x$

b The intercepts are $\left(\frac{169}{12}, 0\right)$, $\left(0, \frac{169}{5}\right)$, the area is $\frac{169^2}{120}$.

c $\frac{13^3}{60} + \frac{169}{12} + \frac{169}{5} = \frac{169}{2}$

14 a $4x + 3y = 25$, $4x + 5y = 25$, they intersect at $\left(6\frac{1}{4}, 0\right)$

b $\lambda x_0 x + y\sqrt{25 - x_0^2} = 25\lambda$, $T = \left(\dfrac{25}{x_0}, 0\right)$,
$OM \times OT = 25 = OA^2$

15 c i $y\dfrac{dy}{dx} = \frac{1}{2}a^2$ **ii** $y\dfrac{dy}{dx} = na^2x^{2n-1}$

16 a $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dv} \times \dfrac{dv}{dx}$, $\dfrac{x}{(1 + \sqrt{1 - x^2})^2\sqrt{1 - x^2}}$

b $\dfrac{dy}{dx} = \dfrac{dy}{du_1} \times \dfrac{du_1}{du_2} \times \cdots \times \dfrac{du_{n-1}}{dx}$

17 a $-\frac{1}{2}x^{-\frac{3}{2}}$, $\frac{1 \times 3}{2^2}x^{-\frac{5}{2}}$, $-\frac{1 \times 3 \times 5}{2^3}x^{-\frac{7}{2}}$, $\frac{1 \times 3 \times 5 \times 7}{2^4}x^{-\frac{9}{2}}$

b $(-1)^n \times \frac{1 \times 3 \times 5 \times \cdots \times (2n - 1)}{2^n}x^{-\frac{2n+1}{2}}$

Exercise 9H

1 Let $u = 5x$
and $v = (x - 2)^4$.

Then $\dfrac{du}{dx} = 5$

and $\dfrac{dv}{dx} = 4(x - 2)^3$.

Let $y = 5x(x - 2)^4$.

$$
\begin{aligned}
\text{Then } \frac{dy}{dx} &= v\frac{du}{dx} + u\frac{dv}{dx} \\
&= (x - 2)^4 \times 5 + 5x \times 4(x - 2)^3 \\
&= 5(x - 2)^4 + 20x(x - 2)^3 \\
&= 5(x - 2)^3\big((x - 2) + 4x\big) \\
&= 5(x - 2)^3(5x - 2).
\end{aligned}
$$

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2 a $2x^2(2x - 3)$ **b** $4x - 9$ **c** $4x^3$

3 a $3(3 - 2x)^4(1 - 4x), 1\frac{1}{2}, \frac{1}{4}$
b $x^2(x + 1)^3(7x + 3), 0, -1, -\frac{3}{7}$
c $x^4(1 - x)^6(5 - 12x), 0, 1, \frac{5}{12}$
d $(x - 2)^2(4x - 5), 2, \frac{5}{4}$
e $2(x + 1)^2(x + 2)^3(7x + 10), -1, -2, -\frac{10}{7}$
f $6(2x - 3)^3(2x + 3)^4(6x - 1), 1\frac{1}{2}, -1\frac{1}{2}, \frac{1}{6}$

4 a $y = x, y = -x$
b $y = 2x - 1, x + 2y = 3$

5 a $(x^2 + 1)^4(11x^2 + 1)$
b $2\pi x^2(1 - x^2)^3(3 - 11x^2)$
c $-2(x^2 + x + 1)^2(7x^2 + 4x + 1)$

6 a $y' = 8x(x^2 - 1)^3, y'' = 8(x^2 - 1)^2(7x^2 - 1)$
b 8, 7 and 6
c $x = 1$ and $x = -1$

7 $10x^3(x^2 - 10)^2(x^2 - 4)$, $(0, 0)$, $(\sqrt{10}, 0)$, $(-\sqrt{10}, 0)$, $(2, -3456)$, $(-2, -3456)$

8 $y' = x^2(1 - x)^4(3 - 8x)$

9 a $\dfrac{3(3x + 2)}{\sqrt{x + 1}}, -\dfrac{2}{3}$ **b** $\dfrac{4(3x - 1)}{\sqrt{1 - 2x}}, \dfrac{1}{3}$
c $\dfrac{10x(5x - 2)}{\sqrt{2x - 1}}$, 0 and $\frac{2}{5}$

10 a $-1 \le x \le 1$ **b** $\dfrac{1 - 2x^2}{\sqrt{1 - x^2}}$
c $\left(\sqrt{\frac{1}{2}}, \frac{1}{2}\right)$ and $\left(-\sqrt{\frac{1}{2}}, -\frac{1}{2}\right)$
d $y = x, y = -x$

11 a $y' = a(2x - \alpha - \beta)$
b $y'(\alpha) = a(\alpha - \beta)\ y'(\beta) = a(\beta - \alpha)$,
$M = \left(\frac{1}{2}(\alpha + \beta), -\frac{1}{2}a(\alpha - \beta)^2\right)$
c $V = \left(\frac{1}{2}(\alpha + \beta), -\frac{1}{4}a(\alpha - \beta)^2\right)$

12 $f'(x) = (x - a)^{n-1}\big(nq(x) + (x - a)q'(x)\big)$.
The x-axis is a tangent to the curve at $x = a$.

13 a $P = \left(\dfrac{r}{r + s}, \dfrac{r^r s^s}{(r + s)^{r+s}}\right)$.
b When $r = s$, $P = \left(\frac{1}{2}, 2^{-2r}\right)$.

14 $y' = u'vw + uv'w + uvw'$
a $2x^4(x - 1)^3(x - 2)^2(3x - 5)(2x - 1)$, 0, 1, 2, $\frac{1}{2}$ and $\frac{5}{3}$
b $\dfrac{(x - 2)^3(11x^2 - x - 2)}{\sqrt{2x + 1}}, 2, \frac{1}{22}(1 + \sqrt{89})$,
$\frac{1}{22}(1 - \sqrt{89})$

15 $y' = u_1{}'u_2 \cdots u_n + u_1u_2{}' \cdots u_n + \cdots + u_1u_2 \cdots u_n{}'$

Exercise 9I

1 Let $u = 2x + 3$
and $v = 3x + 2$.
Then $u' = 2$
and $v' = 3$.

Let $y = \dfrac{2x + 3}{3x + 2}$.

$$
\begin{aligned}
\text{Then } \frac{dy}{dx} &= \frac{vu' - uv'}{v^2} \\
&= \frac{(3x + 2) \times 2 - (2x + 3) \times 3}{(3x + 2)^2} \\
&= \frac{6x + 4 - 6x - 9}{(3x + 2)^2} \\
&= \frac{-5}{(3x + 2)^2}.
\end{aligned}
$$

2 a $\dfrac{-2}{(x - 1)^2}$, none **b** $\dfrac{4}{(x + 2)^2}$, none
c $\dfrac{-13}{(x + 5)^2}$, none **d** $\dfrac{x(2 - x)}{(1 - x)^2}$, 0, 2
e $\dfrac{4x}{(x^2 + 1)^2}$, 0 **f** $\dfrac{1 + x^2}{(1 - x^2)^2}$, none
g $\dfrac{6x^2}{(x^3 + 2)^2}, x = 0$ **h** $\dfrac{10x}{(x^2 - 4)^2}, x = 0$

3 $\dfrac{-3}{(3x - 2)^2}$

4 a $y' = \dfrac{5}{(5 - 3x)^2}$, $y = 5x - 12$, $78°41'$,
$x + 5y + 8 = 0$, $168°41'$
b $y' = \dfrac{x^2 - 2x + 4}{(x - 1)^2}$, $4x - 3y = 4$, $53°8'$,
$3x + 4y = 28$, $143°8'$

5 a $\dfrac{m^2 - b^2}{(bx + m)^2}$ **b** $\dfrac{2x(a - b)}{(x^2 - b)^2}$ **c** $\dfrac{6nx^{n-1}}{(x^n + 3)^2}$

6 a $\dfrac{c^2 + 2c}{(c + 1)^2} = -3, c = -\frac{1}{2}$ or $-1\frac{1}{2}$
b $\dfrac{12k}{(9 - k)^2} = 1, k = 3$ or 27

7 a $y' = \dfrac{\alpha - \beta}{(x - \beta)^2}$
b The denominator is positive, being a square, so the sign of y' is the sign of $\alpha - \beta$.
c When $\alpha = \beta$, the curve is the horizontal line $y = 1$, and $y' = 0$ (except that y is undefined at $x = \beta$).

8 $\dfrac{20}{(5 - 2x)^2}$

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9 a $\dfrac{dy}{dx} = \dfrac{-(t+1)^2}{(t-1)^2}$, $T = \left(\frac{2}{3}, 2\right)$, $3x - 27y + 52 = 0$

b $y = \dfrac{x}{2x-1}$, $\dfrac{dy}{dx} = \dfrac{-1}{(2x-1)^2}$, $\frac{1}{9}$

10 a $\dfrac{1}{2\sqrt{x}(\sqrt{x}+2)^2}$, none

b $\dfrac{x+5}{2(x+1)^{\frac{3}{2}}}$, none ($x = -5$ is outside the domain.)

11 a $f'(x) = \dfrac{-\sqrt{2}}{\sqrt{x}(\sqrt{x}-\sqrt{2})^2}$, $f'(8) = -\frac{1}{4}$

b 3

12 a domain: $x \neq -1$, range: $y \neq 1$

c $I = (-1, 0)$, $G = (1, 0)$

d ii Substitute $(c, 0)$, then $c + a^2 = 0$, so $a = \sqrt{-c}$ or $-\sqrt{-c}$. For $-1 < c < 0$, they are both on the right-hand branch. For $c < -1$, they are on different branches.

14 a i $y' = \dfrac{2}{(x+1)^2}$, $y'' = \dfrac{-4}{(x+1)^3}$

ii $y' = \dfrac{-3}{(x-1)^2}$, $y'' = \dfrac{6}{(x-1)^3}$

iii $y' = \dfrac{x^2-2x}{(x-1)^2}$, $y'' = \dfrac{2}{(x-1)^3}$

15 a $12(3x-7)^3$ **b** $\dfrac{x^2+2}{x^2}$

c $8x$ **d** $\dfrac{-2x}{(x-3)^2(x+3)^2}$

e $4(1-x)(4-x)^2$ **f** $\dfrac{-6}{(3+x)^2}$

g $20x^3(x^2+1)^4(x+1)^4(x-1)^4$

h $\dfrac{1}{2(2-x)^{\frac{3}{2}}}$ **i** $6x^2(x^3+5)$

j $\dfrac{3x^2+x-1}{4x\sqrt{x}}$ **k** $\frac{2}{3}x(5x^3-2)$

l $\dfrac{5}{(x+5)^2}$ **m** $\frac{1}{2}\sqrt{x}(3+5x)$

n $\dfrac{2(x-1)(x+1)(x^2+1)}{x^3}$ **o** $x^2(x-1)^7(11x-3)$

p $\dfrac{(x+1)(x-1)}{x^2}$

16 b i $54, \frac{3}{2}, 9\sqrt{37}, \frac{3}{2}\sqrt{37}$ **ii** $\frac{1}{2}, 8, \frac{1}{2}\sqrt{17}, 2\sqrt{17}$

Exercise 9J

1 a $\dfrac{dQ}{dt} = 3t^2 - 20t$

b When $t = 2$, $Q = -32$, $\dfrac{dQ}{dt} = -28$

2 a $\dfrac{dQ}{dt} = 2t + 6$

b When $t = 2$, $Q = 16$, $\dfrac{dQ}{dt} = 10$

c i $t = -3$ **ii** $t > -3$ **iii** $t < -3$

3 a 7 and 15 **b** $\frac{15-7}{3-1} = 4$ **c** $\frac{7-15}{7-5} = -4$

4 a 180 mL **b** When $t = 0$, $V = 0$.

c 300 mL **d** 60 mL/s

e The derivative is a constant function.

5 a 80 000 litres **b** 35 000 litres

c 20 min **d** 2000 litres/min

6 a $\dfrac{dM}{dt} = 10 - 2t$

b $M = 24$ kg, $\dfrac{dM}{dt} = 2$ kg/s

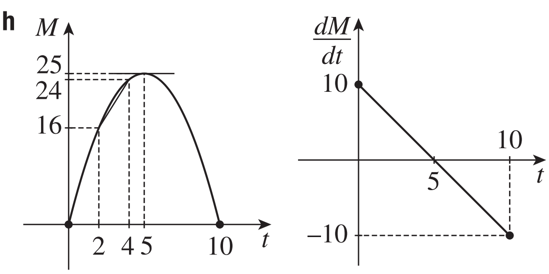
c $M = 16$ kg, average rate $= \frac{24-16}{4-2} = 4$ kg/s

d 0 seconds and 10 seconds

e 10 seconds **f** 5 seconds

g 5 seconds and 5 seconds

h



7 a \$2 **b** \$5.60

c $\dfrac{dP}{dt} = -0.8t + 4$, \$2.40 per annum

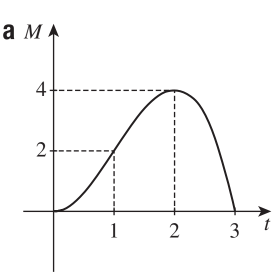
d $t = 5$, the start of 1975

e The price was increasing before then, and decreasing afterwards.

f $\dfrac{dP}{dt}$ is linear with negative gradient -0.8.

g At the start of 1980.

8 a



b $t = 2$

c $\dfrac{M}{t} = 6t - 3t^2$, $t = 1$ **d** $t = 1$

9 The scheme appears to have worked initially and the level of pollution decreased, but the rate at which the pollution decreased gradually slowed down and was almost zero in 2000. A new scheme would have been required to remove the remaining pollution.

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10 a The graph is stationary on 1 July and 1 January.

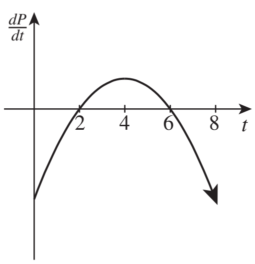
b It is maximum on 1 July and on 1 January. The price is locally minimum 1 March, but globally the graph has no minimum.

c It is increasing from 1 March to 1 July. It is decreasing from 1 January to 1 March and after 1 July.

d on 1 May

e from 1 March to 1 May

f



11 a $A = \pi r^2 = \pi\left(\frac{t}{3}\right)^2 = \frac{\pi}{9}t^2$ **b** $\frac{dA}{dt} = \frac{2\pi}{9}t$

c When $A = 5$, $t = \sqrt{\frac{45}{\pi}} \doteqdot 3.785$ seconds and $\frac{dA}{dt} = \frac{2\pi}{9}\sqrt{\frac{45}{\pi}} \doteqdot 2.642\,\text{km}^2/\text{s}$

12 a When $t = 0$, $h = 80$, so the building is 80 metres tall.

b When $h = 0$, $t = 4$, so it takes 4 seconds.

c $v = -10t$

d When $t = 4$, $v = -40$, so the stone hits the ground at 40 m/s.

e 10 m/s^2 downwards

13 a Yes. $\frac{dv}{dt} = -\frac{1}{2}$, meaning his velocity decreased at a constant rate of $\frac{1}{2}$m/s every second, just as he said.

b Yes. $\frac{dx}{dt} = -\frac{1}{2}t + 50$, which is what the truck's speed monitor recorded.

c Yes. $\frac{dy}{dt} = -\frac{1}{2}t + 50$, which also agrees with the truck's speed monitor.

d When $t = 0$, $x = 0$ and $y = 450$, so the truck was 450 metres ahead.

e Solving $-\frac{1}{2}t + 50 = 0$ gives 100 seconds. When $t = 100$, $x = 2500$ m or 2.5 km.

f When $t = 0$, $v = 50$ m/s, which is 180 km/h. He was in court for speeding.

14 a i Area $= h^2\text{cm}^2$ **ii** Volume $= 3000\,h^2\text{cm}^3$

b i $h = 3\sqrt{t}$, $\frac{dh}{dt} = \frac{3}{2\sqrt{t}}$

ii $h = 15\,\text{cm}$, $\frac{dh}{dt} = \frac{3}{10}$ cm/s

iii 100 seconds, $\frac{3}{20}$ cm/s

Exercise 9K

1 a Zeroes: none, discontinuities:, $x = 6$

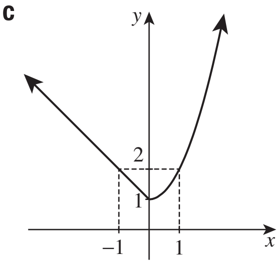
b Zeroes: $x = 0$, discontinuities:, $x = 1$, $x = 3$, $x = 5$.

c Zeroes: $x = 0$, $x = -1$, discontinuities:, $x = -2$, $x = -3$

2 a $f(0) = 1$. First table: 3, 2, 1. Second table: 1, 2, 5

b Yes

c

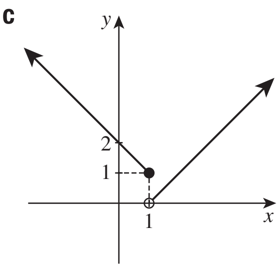


domain: all real x, range: $y \geq 1$

3 a $f(1) = 1$. First table: 3, 2, 1. Second table: 0, 1, 2

b No

c



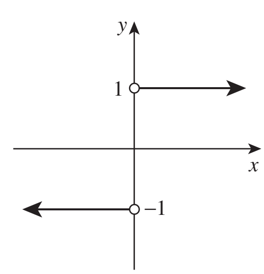
d domain: all real x, range: $y > 0$

4 a $f(x) = \dfrac{1}{x(x-5)}$, Zeroes: none, discontinuities: $x = 0$, $x = 5$

b $f(x) = \dfrac{x}{(x-2)(x-3)}$, Zeroes: $x = 0$, discontinuities: $x = 2$, $x = 3$

c $f(x) = \dfrac{(x-4)(x+4)}{(x-3)(x+3)}$, Zeroes: $x = 4$, $x = -4$, discontinuities: $x = -3$, $x = 3$

5



The table of values should make it clear that

$$y = \begin{cases} 1, & \text{for } x > 0, \\ -1, & \text{for } x < 0, \\ \text{undefined}, & \text{for } x = 0. \end{cases}$$

The curve is not continuous at $x = 0$ — it is not even defined there. domain: $x \neq 0$, range: $y = 1$ or -1

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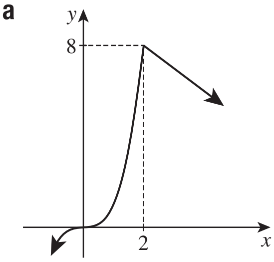
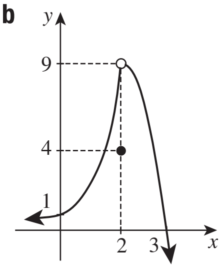
6 a



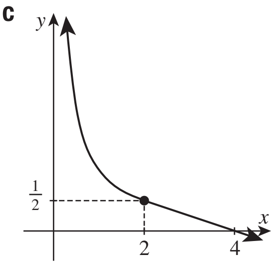
b



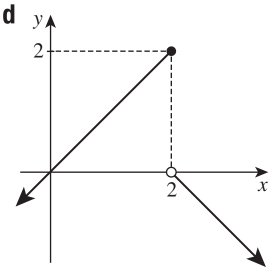
a $f(2) = 8$. When $x = 2$, $x^3 = 8$ and $10 - x = 8$. Thus $f(x)$ is continuous at $x = 2$. Domain: all real x, range: $y \le 8$

b $f(2) = 4$. When $x = 2$, $3^x = 9$ and $13 - x^2 = 9$. Thus $f(x)$ is not continuous at $x = 2$. domain: all real x, range: $y < 9$

c



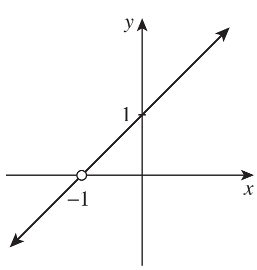
d



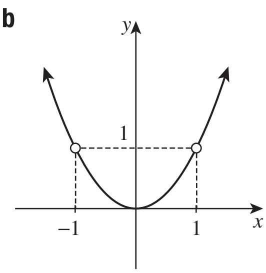
c $f(2) = \frac{1}{2}$. When $x = 2$, $\dfrac{1}{x} = \frac{1}{2}$ and $1 - \frac{1}{4}x = \frac{1}{2}$. Thus $f(x)$ is continuous at $x = 2$. domain: $x > 0$, range: all real y

d $f(2) = 2$. When $x = 2$, $x = 2$, but $2 - x = 0$. Thus $f(x)$ is not continuous at $x = 2$. domain: all real x, range: $y \le 2$

7 a



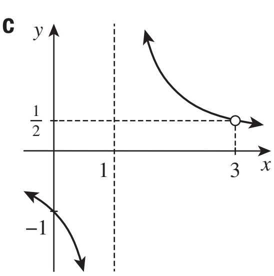
b



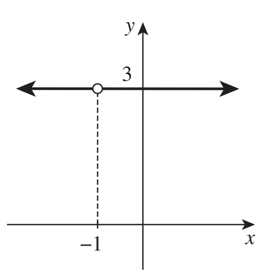
a $y = x + 1$, where $x \ne -1$, domain: $x \ne -1$, range: $y \ne 0$

b $y = x^2$, where $x \ne -1$ or 1, domain: $x \ne -1$ or 1, range: $y \ge 0$, $y \ne 1$

c



d



c $y = \dfrac{1}{x-1}$, where $x \ne 1$ or 3, domain: $x \ne 1$ or 3, range: $y \ne 0$ or $\frac{1}{2}$

d $y = 3$, where $x \ne -1$, domain: $x \ne -1$, range: $y = 3$

8 a $a = 5$ **b** $a = -2$

9 a zeroes: none, discontinuities: $360n°$, where $n \in \mathbb{Z}$

b zeroes: $135° + 180n°$, where $n \in \mathbb{Z}$, discontinuities: $45° + 180n°$, where $n \in \mathbb{Z}$

c zeroes: $45° + 180n°$, where $n \in \mathbb{Z}$, discontinuities: $135° + 180n°$, where $n \in \mathbb{Z}$

Exercise 9L

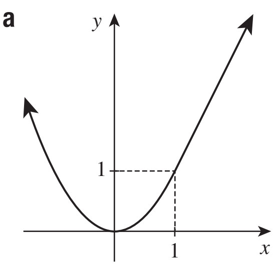
1 a continuous and differentiable at $x = 0$, neither at $x = 2$

b continuous and differentiable at $x = 0$, continuous but not differentiable at $x = 2$

c neither at $x = 0$, continuous and differentiable at $x = 2$

d neither at $x = 0$, continuous but not differentiable at $x = 2$

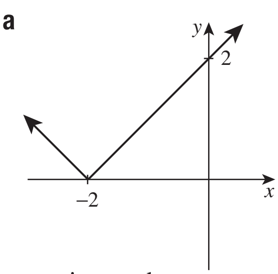
2 a



b $f(1) = 1$, $x^2 = 1$ when $x = 1$, $2x - 1 = 1$ when $x = 1$

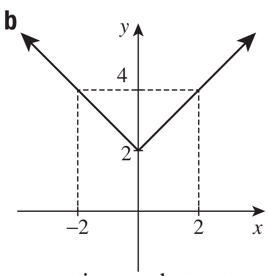
d $2x = 2$ when $x = 1$, and $2 = 2$ when $x = 1$. The tangent at $x = 1$ has gradient 2, so $f'(1) = 2$.

3 a



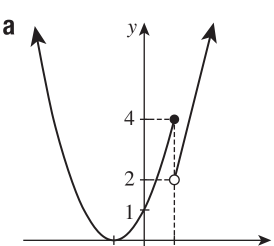
continuous but not differentiable at $x = -2$

b



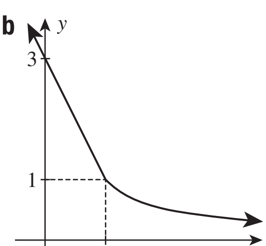
continuous but not differentiable at $x = 0$

4 a



not even continuous at $x = 1$

b



continuous but not differentiable at $x = 1$

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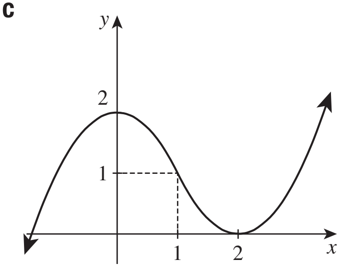
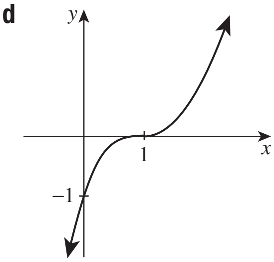
c



differentiable at $x = 1$,
$f'(1) = -2$

d



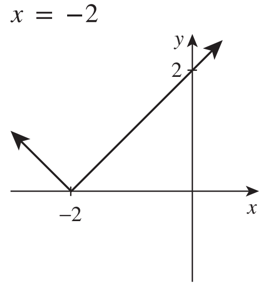
differentiable at $x = 1$,
$f'(1) = 0$

5 a Differentiable at $x = 0$. x^2 is never negative, so $|x^2| = x^2$ for all x.

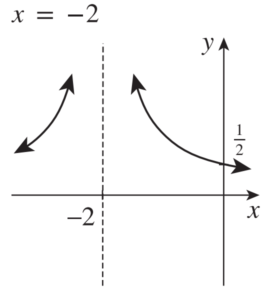
b Differentiable at $x = 0$. x^3 is flat at $x = 0$, so $|x^3|$ is also flat at $x = 0$.

c Continuous, but not differentiable, at $x = 0$. The graph of $y = \sqrt{x}$ becomes vertical near $x = 0$, producing a cusp.

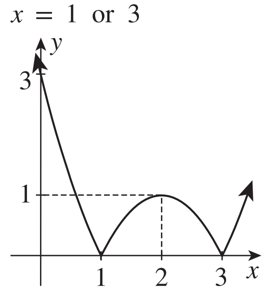
6 a not differentiable at $x = -2$



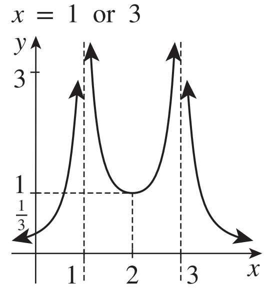
b not continuous at $x = -2$



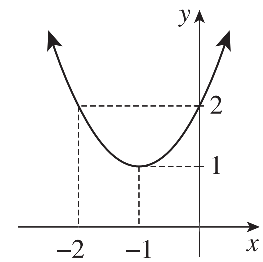
c not differentiable at $x = 1$ or 3



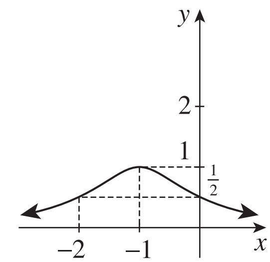
d not continuous at $x = 1$ or 3



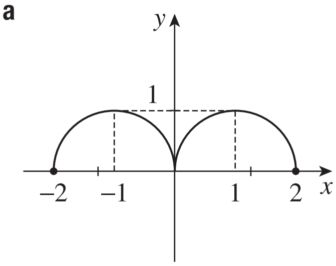
e differentiable everywhere



f differentiable everywhere

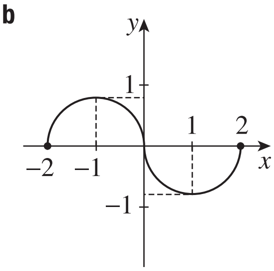


7 a



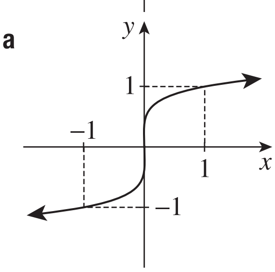
There is a cusp at the origin because the curve becomes infinitely steep on both sides. It slopes downwards on the left and upwards on the right.

b



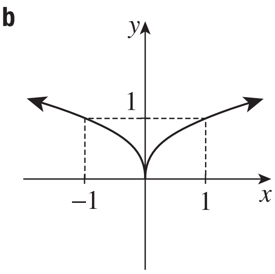
There is a vertical tangent at $x = 0$.

8 a



$f'(x) = \frac{1}{5}x^{-\frac{4}{5}}$. There is a vertical tangent at $(0, 0)$.

b



$f'(x) = \frac{2}{5}x^{-\frac{3}{5}}$. There is a cusp at $(0, 0)$.

9 a $4,\ (5, -5)$ **b** $3,\ (1, -8)$ **c** $-\frac{1}{4},\ \left(2, \frac{1}{2}\right)$

d 1. There are none, because all the tangents have negative gradients.

e 0. There are none, because the tangents have gradient 1 for $x > 0$ and gradient -1 for $x < 0$.

f $\alpha + \beta,\ \left(\frac{1}{2}(\alpha + \beta), \frac{1}{4}(\alpha + \beta)^2\right)$

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10 a q must be odd.

b $p \geq 0$ (When $p = 0$ it is reasonable to take $f(0) = 1$ and ignore the problem of 0^0, because $\lim\limits_{x \to 0} x^0 = 1$; thus when $p = 0$ the function is $y = 1$.)

c no conditions on p and q

d $p \geq 0$ and q is odd.

e $p \geq 0$ ($p = 0$ requires the qualification above.)

f $p \geq q$ and q is odd.

g $0 < p < q$ and q is odd and p is odd.

h $0 < p < q$ and q is odd and p is even.

Chapter 9 review exercise

1 a $2x + 5$ **b** $-2x$ **c** $6x - 2$

2 a $3x^2 - 4x + 3$ **b** $6x^5 - 16x^3$

c $9x^2 - 30x^4$ **d** $2x + 1$

e $-12x + 7$ **f** $-6x^{-3} + 2x^{-2}$

g $12x^2 + 12x^{-4}$ **h** $\frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-1\frac{1}{2}}$

i $x^{-2} - 2x^{-3}$

3 a $f'(x) = 4x^3 + 3x^2 + 2x + 1$, $f''(x) = 12x^2 + 6x + 2$

b $f'(x) = -10x^{-3}$, $f''(x) = 30x^{-4}$

c $f'(x) = -4x^{-\frac{3}{2}}$, $f''(x) = 6x^{-\frac{5}{2}}$

4 a $y = x^3 + 4x + C$

b $y = 7x - 6x^2 - 4x^3 + C$

c $y = 4x^5 - 4x^3 + 4x + C$

5 a $-\dfrac{3}{x^2}$ **b** $-\dfrac{1}{3x^3}$ **c** $\dfrac{7}{2\sqrt{x}}$

d $\dfrac{6}{\sqrt{x}}$ **e** $-\frac{9}{2}\sqrt{x}$ **f** $-\dfrac{3}{x\sqrt{x}}$

6 a $6x - 2$ **b** $x - \frac{1}{2}$

c $10x + \dfrac{7}{x^2}$ **d** $-\dfrac{2}{x^2} - \dfrac{2}{x^3}$

e $\dfrac{2}{\sqrt{x}}$ **f** $3\sqrt{x} + \dfrac{3}{2\sqrt{x}}$

7 a $9(3x + 7)^2$ **b** $-4(5 - 2x)$

c $-\dfrac{5}{(5x - 1)^2}$ **d** $\dfrac{14}{(2 - 7x)^3}$

e $\dfrac{5}{2\sqrt{5x + 1}}$ **f** $\dfrac{1}{2(1 - x)^{\frac{3}{2}}}$

8 a $42x(7x^2 - 1)^2$ **b** $-15x^2(1 + x^3)^{-6}$

c $8(1 - 2x)(1 + x - x^2)^7$

d $-6x(x^2 - 1)^{-4}$ **e** $-\dfrac{x}{\sqrt{9 - x^2}}$

f $\dfrac{x}{(9 - x^2)^{\frac{3}{2}}}$

9 a $x^8(x + 1)^6(16x + 9)$ **b** $\dfrac{x(2 - x)}{(1 - x)^2}$

c $2x(4x^2 + 1)^3(20x^2 + 1)$ **d** $\dfrac{12}{(2x + 3)^2}$

e $(9x - 1)(x + 1)^4(x - 1)^3$ **f** $\dfrac{(x - 5)(x + 1)}{(x - 2)^2}$

10 $\dfrac{dy}{dx} = 2x + 3$

a 3, 71°34′ **b** 1, 45° **c** −1, 135°

11 a tangent: $y = -3x$, normal: $3y = x$

b tangent: $y = -2$, normal: $x = 1$

c $(1, -2)$ and $(-1, 2)$ **d** $(2, 2)$ and $(-2, -2)$

12 a $y = -x - 4$, $y = x - 8$

b $A(-4, 0)$, $B(8, 0)$

c $AB = 12$, $|\Delta ABP| = 36$ square units

13 The tangent is $y = x$

14 a $\left(1, -6\frac{2}{3}\right)$, $\left(-1, -7\frac{1}{3}\right)$ **b** $\left(-1, \frac{2}{3}\right)$

15 At $(1, -3)$ the tangent is $\ell: x + y + 2 = 0$, at $(-1, 3)$ the tangent is $x + y - 2 = 0$.

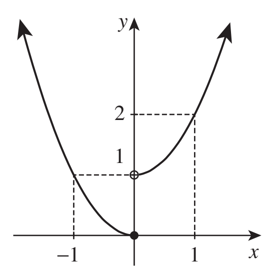
16 a $2\frac{1}{2}$ and 1 **b** 2 and $1\frac{1}{2}$

17 a $V = \frac{4}{3}\pi \times \left(\dfrac{t}{3}\right)^3 = \frac{4\pi}{81}t^3$ **b** $\dfrac{dV}{dt} = \frac{4\pi}{27}t^2$

c $V \doteqdot \frac{4\pi}{81} \times 0.001 \doteqdot 0.00016\,\text{km}^3$, $\dfrac{dV}{dt} \doteqdot \frac{4\pi}{27} \times 0.01 \doteqdot 0.0047\,\text{km}^3/\text{s}$

d $t^2 = \frac{27}{4\pi}$, $t \doteqdot 1.5\,\text{s}$

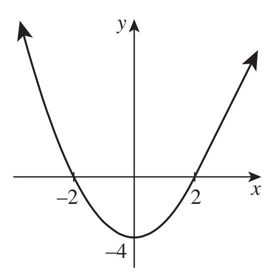
18



a $f(0) = 0$, $x^2 = 0$ when $x = 0$, $x^2 + 1 = 1$ when $x = 0$, so it is not continuous at $x = 0$.

b domain: all real x, range: $y \geq 0$

19



a $f(0) = 2$, $x^2 - 4 = 0$ when $x = 2$, $4x - 8 = 0$ when $x = 0$, so it is continuous at $x = 2$.

b $f'(2) = 4$ when $x < 2$ (substitute into $2x$), $f'(2) = 4$ when $x > 2$ (substitute into 4), so it is differentiable at $x = 2$, with $f'(2) = 4$

c domain: all real x, range: $y \geq 4$

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Chapter 10

Exercise 10A

1 a yes **b** no **c** no **d** no
e yes **f** no **g** yes **h** yes
i no **j** yes **k** yes **l** no

2 a 3, 4, $4x^3$, -11, not monic
b 3, -6, $-6x^3$, 10, not monic
c 0, 2, 2, 2, not monic
d 12, 1, x^{12}, 0, monic
e 3, 1, x^3, 0, monic
f 5, -1, $-x^5$, 0, not monic
g no degree, no leading coefficient, no leading term, 0, not monic
h 2, -3, $-3x^2$, 0, not monic
i 6, -4, $-4x^6$, -5, not monic

3 a $x^2 + 2x + 3$ **b** $x^2 + 2x + 3$
c $-x^2 + 8x + 1$ **d** $x^2 - 8x - 1$
e $5x^3 - 13x^2 - x + 2$ **f** $5x^3 - 13x^2 - x + 2$

5 a $x(x - 10)(x + 2)$, 0, 10, -2
b $x^2(2x + 1)(x - 1)$, 0, 1, $-\frac{1}{2}$
c $(x - 3)(x + 3)(x^2 + 9)$, 3, -3
d $(x - 3)(x + 3)(x^2 + 4)$, 3, -3

6 a 9, 8, -27 **b** 14, 120, 24

7 a i $p + q$
ii the maximum of p and q
b $P(x)Q(x)$ still has degree $p + p = 2p$, but $P(x) + Q(x)$ may have degree less than p (if the leading terms cancel out), or it could be the zero polynomial.
c $x^2 + 2$ and $-x^2 + 3$. Do not choose two opposite polynomials, such as $x^2 + 1$ and $-x^2 - 1$, because their sum is the zero polynomial, which does not have a degree.

8 $x + 1$

9 a $a = 3, b = -4$ and $c = 1$
b $a = 2$ and $b = 3$
c $a = 1, b = 2$ and $c = 1$
d $a = 1, b = 2$ and $c = -1$

10 c A polynomial is even if and only if the coefficients of the odd powers of x are zero. A polynomial is odd if and only if the coefficients of the even powers of x are zero.

11 a True. If $P(x)$ is even, then the terms are of the form $a_n x^{2n}$, where $n \geq 0$ is an integer. Therefore $P'(x)$ has terms of the form $2na_n x^{2n-1}$, so all powers of x will be odd.
b False. For example, $Q(x) = x^3 + 1$ is not odd but $Q'(x) = 3x^2$ is even.
c True. If $R(x)$ is odd, then the terms are of the form $a_n x^{2n+1}$, where $n \geq 0$ is an integer. Therefore $R'(x)$ has terms of the form $(2n + 1)a_n x^{2n}$, so all powers of x will be even.
d True. As $S'(x)$ is odd, it has no constant term, and all powers of x are odd. Therefore all the terms in $S(x)$ will have even powers.

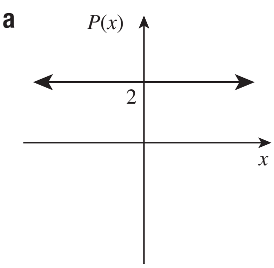
12 a $\sqrt{2}$ and $-\sqrt{2}$ **b** 1 and -1 **c** $\sqrt{3}$ and $-\sqrt{3}$
d Their discriminants are all negative.

13 a n **b** $n + 1$

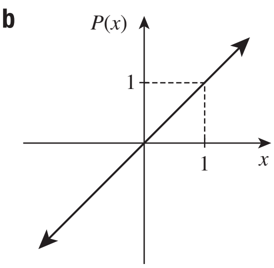
14 b The kth derivative of $a_k x^k$ is $k!a_k$ which is a constant. Substituting $x = 0$ proves that the coefficients of x^k are equal, since the common factor $k!$ cancels.

Exercise 10B

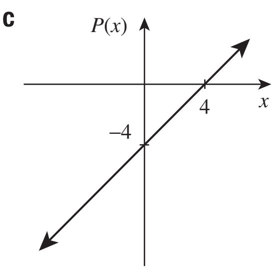
1 a



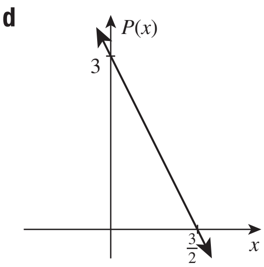
b



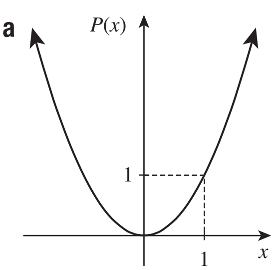
c



d



2 a



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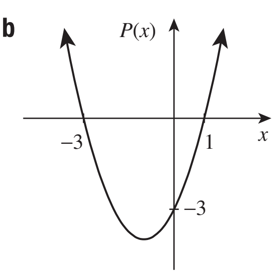
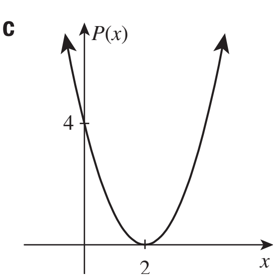
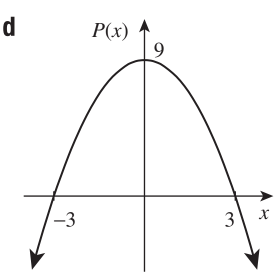


c



d



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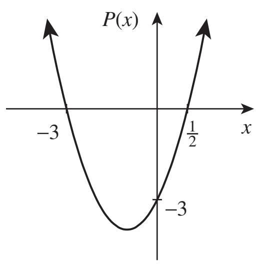
e



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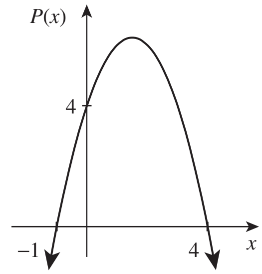
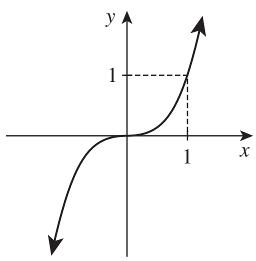


3 a



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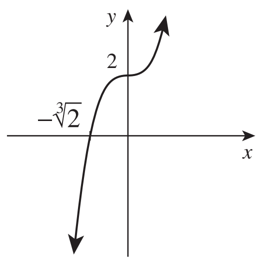

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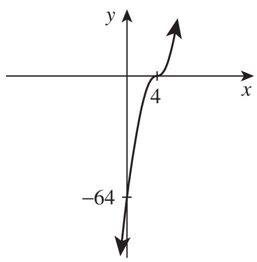

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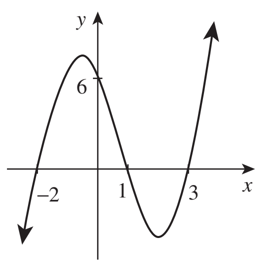

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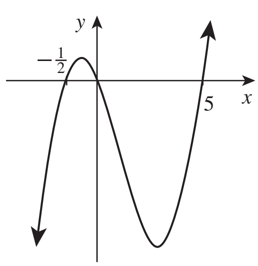

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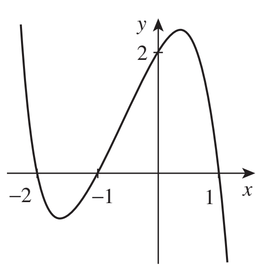

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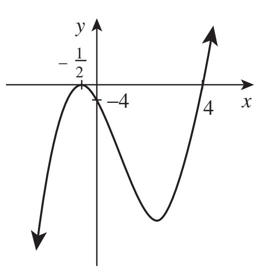

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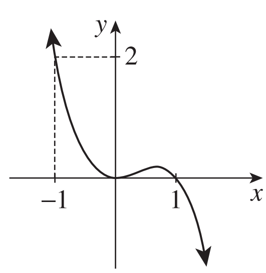
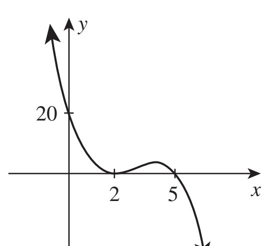
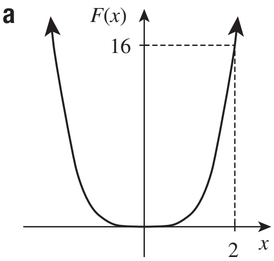


i



Parts **a**, **b** and **c** are one-to-one, the others are all many-to-one.

4 a



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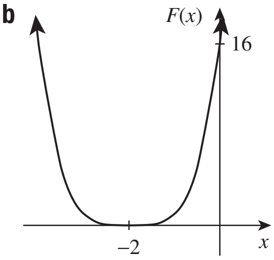
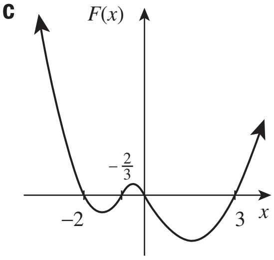


c



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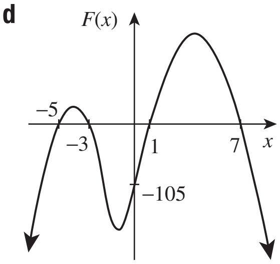

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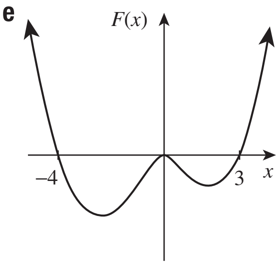

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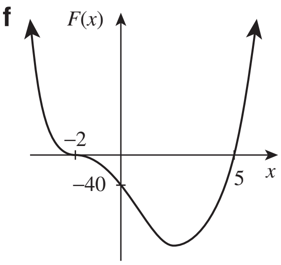

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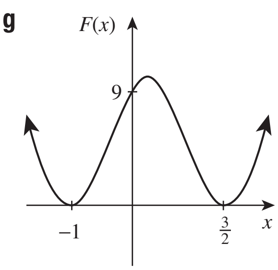

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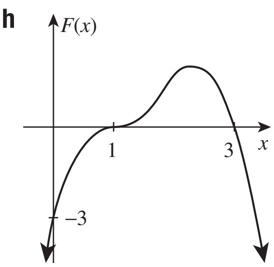

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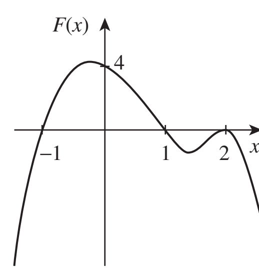

5 a There are two zeroes, one between 0 and 1, and one between 2 and 3.

b There are three zeroes, one between -2 and -1, one between -1 and 0, and one between 1 and 2.

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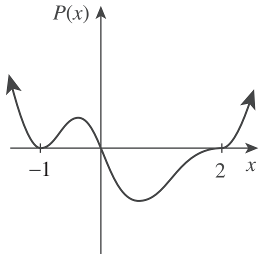
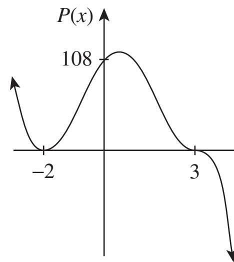
6 a



b



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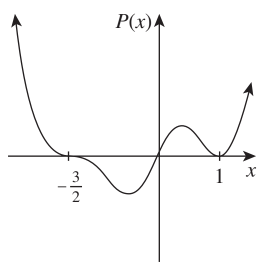

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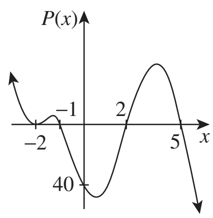
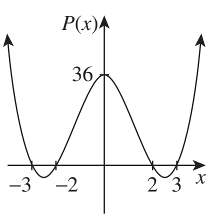


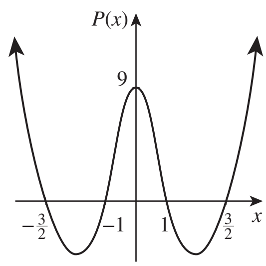
7 a $x > 2$ or $x < 0$, $x \neq -1$
b $x \leq 3$
c $x \leq -\frac{3}{2}$ or $x \geq 0$
d $x > 5$ or $-1 < x < 2$

8 a $(x + 3)(x - 3)(x + 2)(x - 2)$
b $(2x - 3)(2x + 3)(x + 1)(x - 1)$

a

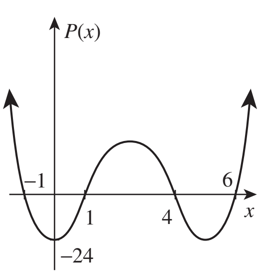


b

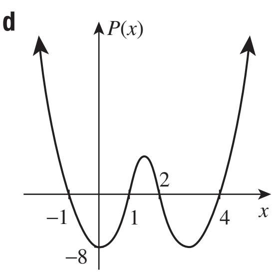


c $(x - 6)(x + 1)(x - 4)(x - 1)$
d $(x - 4)(x + 1)(x - 1)(x - 2)$

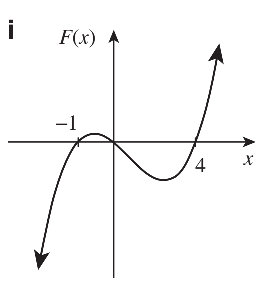
c



d



9 a i



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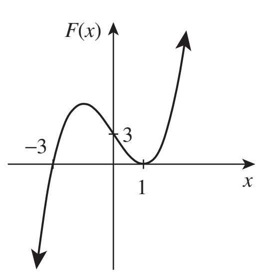

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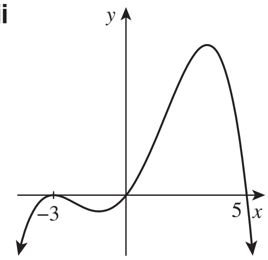

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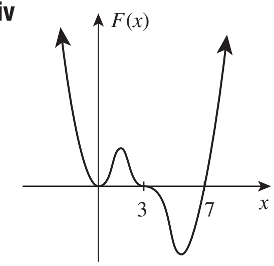
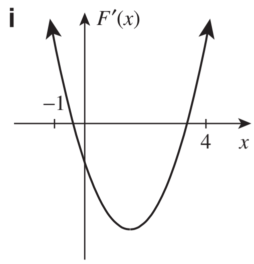


b i



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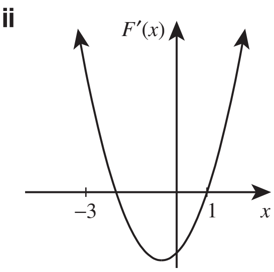

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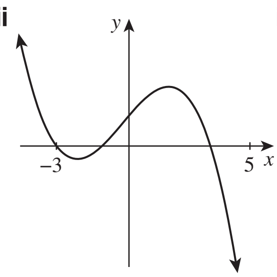

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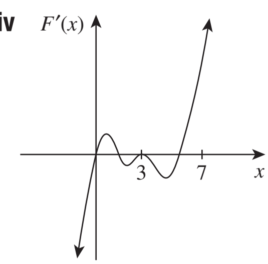

10 a $P(x) = (x^2 - 3)^2 + (x + 2)^2$
b None, because $P(x) > 0$ for all x, so its graph lies above the x-axis.

11 The graphs always intersect at $(0, 1)$ and at $(-1, 0)$. If m and n are both even, they also intersect at $(-2, 1)$, and if m and n are both odd, they also intersect at $(-2, -1)$.

Exercise 10C

1 a $63 = 5 \times 12 + 3$ **b** $125 = 8 \times 15 + 5$
c $324 = 11 \times 29 + 5$ **d** $1857 = 23 \times 80 + 17$

2 a $x^2 - 4x + 1 = (x + 1)(x - 5) + 6$
b $x^2 - 6x + 5 = (x - 5)(x - 1)$
c $x^3 - x^2 - 17x + 24$
$= (x - 4)(x^2 + 3x - 5) + 4$
d $2x^3 - 10x^2 + 15x - 14$
$= (x - 3)(2x^2 - 4x + 3) - 5$
e $4x^3 - 4x^2 + 7x + 14$
$= (2x + 1)(2x^2 - 3x + 5) + 9$
f $x^4 + x^3 - x^2 - 5x - 3$
$= (x - 1)(x^3 + 2x^2 + x - 4) - 7$
g $6x^4 - 5x^3 + 9x^2 - 8x + 2$
$= (2x - 1)(3x^3 - x^2 + 4x - 2)$
h $10x^4 - x^3 + 3x^2 - 3x - 2$
$= (5x + 2)(2x^3 - x^2 + x - 1)$

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3 a $\dfrac{x^2-4x+1}{x+1}=x-5+\dfrac{6}{x+1}$

b $\dfrac{x^2-6x+5}{x-5}=x-1$

c $\dfrac{x^3-x^2-17x+24}{x-4}=x^2+3x-5+\dfrac{4}{x-4}$

d $\dfrac{2x^3-10x^2+15x-14}{x-3}=2x^2-4x+3-\dfrac{5}{x-3}$

4 a $x^3+x^2-7x+6=(x^2+3x-1)(x-2)+4$

b $x^3-4x^2-2x+3=(x^2-5x+3)(x+1)$

c $x^4-3x^3+x^2-7x+3$
$=(x^2-4x+2)(x^2+x+3)+(3x-3)$

d $2x^5-5x^4+12x^3-10x^2+7x+9$
$=(x^2-x+2)(2x^3-3x^2+5x+1)$
$+(7-2x)$

5 a 0, 1 or 2

b $D(x)$ has degree 3 or higher.

6 a $x^3-5x+3=(x-2)(x^2+2x-1)+1$

b $2x^3+x^2-11=(x+1)(2x^2-x+1)-12$

c x^3-3x^2+5x-4
$=(x^2+2)(x-3)+(3x+2)$,
$\dfrac{x^3-3x^2+5x-4}{x^2+2}=x-3+\dfrac{3x+2}{x^2+2}$

d $2x^4-5x^2+x-2$
$=(x^2+3x-1)(2x^2-6x+15)+(13-50x)$

e $2x^3-3=(2x-4)(x^2+2x+4)+13$

f $x^5+3x^4-2x^2-3$
$=(x^2+1)(x^3+3x^2-x-5)+(x+2)$,
$\dfrac{x^5+3x^4-2x^2-3}{x^2+1}$
$=x^3+3x^2-x-5+\dfrac{x+2}{x^2+1}$

7 a $P(x)=(x-3)(x+1)(x+4)$

b $x>3$ or $-4<x<-1$

8 a $(x-2)(x+1)(2x-1)(x+3)$

b $-3\le x\le -1$ or $\dfrac{1}{2}\le x\le 2$

9 a quotient: x^2-3x+5, remainder: $12-13x$

b $a=8$ and $b=-5$

10 a $x^4-x^3+x^2-x+1$
$=(x^2+4)(x^2-x-3)+(3x+13)$

b $c=-4$ and $d=-12$

11 $m=41$ and $n=-14$

12 a $Q(x)=x^2-2x-4$ and $R(x)=25$

13 b $k=$ 19, 25, 34, 59 or 184

Exercise 10D

1 a 3 b 25 c -15 d -3 e 111 f -41

2 a yes b no c no d yes e no f yes

3 a $k=4$ b $m=-\frac{1}{2}$ c $p=-14$ d $a=-1$

4 a $(x-2)(x+1)(x+3)$

b $(x-1)(x-3)(x+7)$

c $(x+1)^2(3-x)$

d $(x-1)(x+2)(x+3)(x-5)$

5 a $-1, -4$ or 2 b 3 or -2

c $2, -\frac{2}{3}$ or $-\frac{1}{2}$ d $-2, \frac{1}{4}\left(-3\pm\sqrt{17}\right)$

6 b $P(x)=(x-3)(x+1)(x-6)$

7 a $P(x)=(x-3)(2x+1)(x+2)$

8 a $a=4$ and $b=11$ b $a=2$ and $b=-9$

9 a $\frac{29}{8}$ b $\frac{97}{8}$ c $\frac{95}{27}$

10 a $(2x-1)(x+3)(x-2)$

b $(3x+2)(2x+1)(x-1)$

11 $x+1$ is a factor when n is odd.

12 a $P(x)=-x^3+16x$ b $p=2$ or $p=3$

13 a $P(x)=(x-1)(x+3)Q(x)+(2x+5)$

b 7 c -1

14 a The divisor has degree 2, so the remainder is zero or has degree 1 or 0.

b $a=-1$ and $b=3$

15 $3-2x$

16 a $2\frac{1}{2}$ b $a=-2$ and $b=-7$

c 8

17 b $P(x)=(2x+1)(2x-1)(x-1)(2x^2+x+3)$

18 a $a^2+b^2+c^2-ab-bc-ca$

b $(a-b)(b-c)(c-a)(a+b+c)$

Exercise 10E

1 a $(x+1)(x-3)(x-4)$

b $x(x+2)(x-3)(x-1)$

c $(3x-1)(2x+1)(x-1)$

2 a $(x-2)(x+3)(x+1)(x-5)$

3 a $P(x)=(x-1)(x+1)(x-3)(2x+1)$

b $P(x)=(x-1)(x-2)(x+2)(2x-3)$

c $P(x)=(2x-5)(3x-2)(x+1)(x-2)$

d $P(x)=(x-2)(x-3)(3x-1)^2$

4 a $a=2, b=\frac{1}{3}$ and $c=\frac{5}{2}$

b $a=-1, b=3, c=\frac{1}{2}$ and $d=\frac{5}{4}$

5 a $a=3, b=-16$ and $c=27$

b $a=2, b=-2, c=-7$ and $d=-7$

c $(x+1)^3-(x+1)^2-4(x+1)+5$

d $a=3, b=-2$ and $c=1$

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6 a $P(x) = (x-2)^2(x+5)$
b $P(x) = (x-1)(x+3)(2x-7)$
8 m
9 There must be a stationary point between each of the consecutive zeroes, where the curve turns around and returns to the x-axis.
10 $a = b = c = 0, d = k$
11 $x^5 - 4x^4 + x^3 + 4x^2 - 2x$
12 a The curves are tangent at $x = 3$ and cross at $x = -1$.
b The curves are tangent at $x = 2$ and cross at $x = 3$.
c The curves cross at $x = -5$, $x = -2$ and $x = 3$.
d The curves are tangent to one another and cross at $x = 1$, and cross at $x = -2$.
e The curves cross at the origin, and cross and are tangent to each other at $x = -1$.
13 $x^2 - 1$
14 -23
15 a $0, 1, 2, \ldots, n$ **c** 1

Exercise 10F

1 a 4 **b** 2 **c** 8 **d** 2 **e** 14
f 12 **g** 6 **h** 24 **i** $\frac{17}{2}$
2 a -2 **b** -11 **c** 12 **d** $-\frac{11}{12}$
e $-\frac{1}{6}$ **f** 0 **g** -132 **h** 26
i $\frac{13}{72}$
The roots are -1, -4 and 3.
3 a 5 **b** 2 **c** 4 **d** -3
e $-\frac{4}{3}$ **f** $-\frac{2}{3}$ **g** $-\frac{5}{3}$ **h** 21
4 a $-\frac{5}{2}$ **b** -2 **c** $\frac{41}{4}$ **d** $\frac{1}{2}\sqrt{57}$
5 a The other zero is $\frac{1}{2}$.
b The other factor is $(x - 4)$.
6 d The discriminant of the quadratic is negative.
e once
7 a 3 **b** $-\frac{1}{2}$
c -3, 1. Hence 1 is a double zero.
d $\frac{2}{3}$, 2
8 a $a = 3$ and $b = -24$, $(x-3)(x+4)(x+2)$
b $a = -1$ and $b = 3$, zeroes are $5, -4, \sqrt{3}, -\sqrt{3}$.
10 a $\frac{1}{3}, -4$ and 4 **b** $6, \frac{1}{2}$ and -4
c -3 (double root) and 6 **d** $4, \frac{1}{2}$ and 2
11 a $a = -12$ and the roots are -2, 2 and -3.
b $a = -5$ and the roots are 4, $\frac{1}{4}$ and -3.
12 $-1, -2, 2$ and 4
14 a i $\frac{1}{2}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{4}$

17 b $\alpha^2 + \beta^2 + \gamma^2 = -\frac{11}{4} < 0$ which is impossible if α, β and γ are all real numbers, because squares are never negative.
18 12
19 0, because 1 is one of the roots, so 0 is one of the factors of the expression.
20 a i $b = 1$ and $c = -2$
ii $b = 4$ and $c = 4$

Exercise 10G

1 a i 3 is at least a double zero of $P(x)$
b $3, 3, -2$
c $P(x) = (x-3)^2(x+2)$
2 a ii -1 is at least a triple zero of $P(x)$
b $-1, -1, -1, -5$
c $P(x) = (x+1)^3(x+5)$
3 a -3 and 3 **b** 3 **c** -6
4 a $\dfrac{5}{2}$ and -5 **b** -5 **c** 10
5 a -2
b $\frac{3}{2}$, $P(x) = (x+2)^2(2x-3)$
6 a $\frac{1}{2}$
b 2, $P(x) = (2x-1)^3(x-2)$
7 b $x = 3, 2 + \sqrt{3}$ or $2 - \sqrt{3}$
8 a $k = 27$ or -5
b When $k = 27$, $P(x) = (x-3)^2(x+3)$ and when $k = -5$, $P(x) = (x+1)^2(x-5)$.
9 $a = 1, b = -3, c = 2$
10 a -3 **b** $c = -54$
c $P(x) = (x+3)^3(x-2)$
11 a $b = -5$ and $c = 8$
b $x = \frac{1}{2}\left(3 - \sqrt{5}\right)$ or $\frac{1}{2}\left(3 + \sqrt{5}\right)$
14 Hint: consider $P(x) - P'(x)$
15 b ii $m < 0$
iii $x = -\sqrt{-\dfrac{m}{2}}$ or $\sqrt{-\dfrac{m}{2}}$

Exercise 10H

1 b The equation is $(x-4)^2 = 0$, so $x = 4$ is a double root, and the line is a tangent at $T(4, -8)$.
2 b i $\alpha + \alpha = 4$ **ii** $b = -4$.
iii $y = -4 - 2x$, $T = (2, -8)$.
3 b $\alpha + \beta = 4$, $M = (2, 3)$
4 b The roots are 1, 1 and 3.
c The line is a tangent at $(1, 2)$ because $x = 1$ is a double root of the equation. The other point is $(3, 0)$.

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5 b i $\alpha + \alpha + 0 = 5$

ii $m = -\frac{1}{4}$.

iii $y = -\frac{1}{4}x, T = \left(\frac{5}{2}, -\frac{5}{8}\right)$.

6 b $\alpha + \beta + 2 = 5, M = \left(\frac{3}{2}, -\frac{1}{2}\right)$

d $\sqrt{26}$

7 c The line intersects the curve at $x = -1$ and is tangent to the curve at $x = \alpha$. $\alpha = 2, P = (2, 5), m = 4$

8 a $y = mx - mp + p^3$

c $x = -\frac{1}{2}p$, so M lies on $x = -\frac{1}{2}p$.

9 a $\alpha = 1$ and $m = 2$

b $y + 3 = m(x + 2)$

c $y = 2x + 1$

10 a $y = (x + 1)(x - 2)(x - 5)(x + 2)$

b Because the line ℓ is tangent to the curve at A and B.

c $\alpha + \beta = 2, \alpha^2 + \beta^2 + 4\alpha\beta = -9,$
$2\alpha^2\beta + 2\alpha\beta^2 = m - 16, \alpha^2\beta^2 = 20 - b$

d $m = -10, b = -22\frac{1}{4}, y = -10x - 22\frac{1}{4}$

11 a $k = \frac{1}{4}, \left(\frac{\sqrt{2}}{2}, \frac{1}{4}\right)$ and $\left(-\frac{\sqrt{2}}{2}, \frac{1}{4}\right)$

b $k = 0$ and $T(0, 0)$ or $k = \frac{4}{27}$ and $T\left(\frac{2}{3}, \frac{8}{27}\right)$

c $k = \frac{5}{4}, \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ and $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

13 c $M = \left(\frac{\lambda + 2}{2\lambda}, -\frac{\lambda + 2}{2\lambda}\right)$, locus: $y = -x$

d $\lambda = 2(\sqrt{2} + 1)$

e $\lambda < -2(\sqrt{2} - 1)$ or $\lambda > 2(\sqrt{2} + 1)$, but $\lambda \neq -1$

Chapter 10 review exercise

1 a 3 b 2 c $2x^3$ d -11

2 a 3 b 9

3 -35

4 b $1 < x < 3$

6 a $Q(x) = 2x^2 + 13x + 35, R(x) = 110$

b $2x^3 + 7x^2 - 4x + 5$
$= (x - 3)(2x^2 + 13x + 35) + 110$

7 a -17 b -27

8 b $P(x) = (x - 2)(x - 3)(x + 5)$

9 $k = -1$

10 $b = -3$ and $c = -11$

11 $h = 5$ and $k = -9$

12 a The divisor has degree 2, so the remainder has degree 1 or 0.

b $a = -6$ and $b = 4$

13 a 6 b -4 c -24 d $-\frac{3}{2}$ e -13 f 44

14 a -10 b 5 c 20 d $\frac{1}{4}$ e $-\frac{1}{2}$

f -2 g 100 h 90 i $\frac{9}{40}$

15 a $\alpha = -9$ b $d = -189$ c $c = -57$

16 a $\gamma = \frac{1}{2}$ b $-\frac{2}{3}$ and 3

18 $x = -\frac{1}{3}, 1$ or $\frac{7}{3}$

19 $\frac{1}{4}, \frac{1}{2}$ and 1

20 a -2

b 5, $P(x) = (x + 2)^2(x - 5)$

21 a $\frac{5}{6}$ and 1 b 1 c $\frac{2}{3}$

22 a $k = 28$ or -80

b When $k = 28, P(x) = (x - 2)^2(x + 7)$ and when $k = -80, P(x) = (x + 4)^2(x - 5)$.

23 a $x^3 - 3x^2 - 9x - 5 = 0$

b The line is tangent to the curve at $x = -1$ and intersects the curve at $x = \alpha$. So -1 is a double root and α is a single root.

c $B = (5, 50)$

Chapter 11

Exercise 11A

1 e All the ratios are about 0.7.

f $\frac{dy}{dx} \doteqdot 0.7y$

x	-2	-1	0	1	2
height y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
gradient $\frac{dy}{dx}$	0.17	0.35	0.69	1.39	2.77
$\frac{\text{gradient}}{\text{height}}$	0.69	0.69	0.69	0.69	0.69

2 b Both are equal to 1.

c

height y	$\frac{1}{2}$	1	2	3
gradient $\frac{dy}{dx}$	$\frac{1}{2}$	1	2	3
$\frac{\text{gradient}}{\text{height}}$	1	1	1	1

d They are all equal to 1. e $\frac{dy}{dx} = y$.

3 c The values are: 0.14, 0.37, 1, 2.72.

d The x-intercept is always 1 unit to the left of the point of contact.

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4 a **i** AB has gradient 1

ii The curve is concave up, so the chord is steeper than the tangent.

b **i** CA has gradient 1

ii The curve is concave up, so the chord is not as steep as the tangent.

c As the base increases, the gradient at the y-intercept increases. With $y = 2^x$, the gradient at the y-intercept is less than 1, and with $y = 4^x$, the gradient at the y-intercept is greater than 1. Hence the base e for which the y-intercept is exactly 1 is between 2 and 4.

6 The values get closer and closer to the limit $\log_e 2 \doteqdot 0.69315$

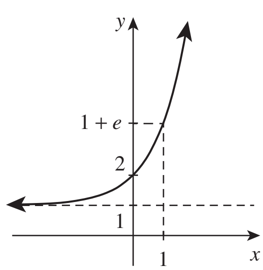
Exercise 11B

1 a 7.3891 **b** 22 026.4658 **c** 1.0000 **d** 2.7183
e 0.3679 **f** 0.1353 **g** 1.6487 **h** 0.6065
i 0.9990 **j** 0.0025

2 a $e^{-1} \doteqdot 0.3679$ **b** $e^{-4} \doteqdot 0.01832$
c $e^{\frac{1}{3}} \doteqdot 1.396$ **d** $e^{-\frac{1}{2}} \doteqdot 0.6065$
e 2.061×10^{-9} **f** 1.069×10^{13}

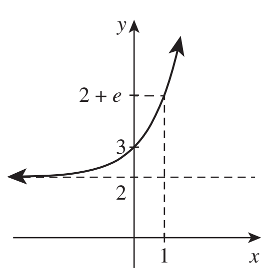
3 a $5e^2 \doteqdot 36.95$ **b** $\frac{1}{64}e^6 \doteqdot 6.304$
c $7e^{\frac{1}{2}} \doteqdot 11.54$ **d** $\frac{3}{5}e^{\frac{1}{2}} \doteqdot 0.9892$
e $4e^{-1} \doteqdot 1.472$ **f** $\frac{5}{7}e^{-4} \doteqdot 0.01308$

4 a



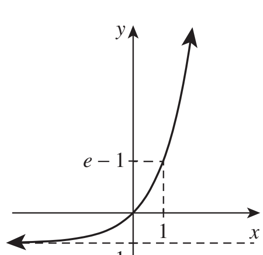
Shift up 1 unit, asymptote: $y = 1$, range: $y > 1$

b



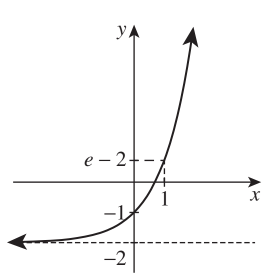
Shift up 2 units, asymptote: $y = 2$, range: $y > 2$

c



Shift down 1 unit, asymptote: $y = -1$, range: $y > -1$

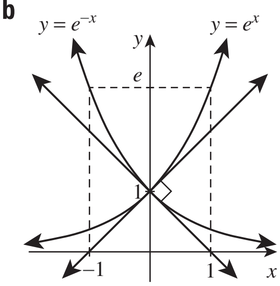
d



Shift down 2 units, asymptote: $y = -2$, range: $y > -2$

5 a For e^x, 0.14, 0.37, 1.00, 2.72, 7.39.
For $y = e^{-x}$, 7.39, 2.72, 1.00, 0.37, 0.14

b

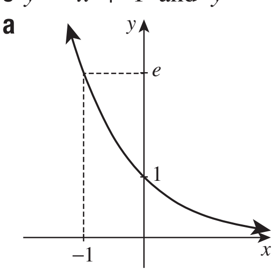


c Reflection in the y-axis.

d The graph of $y = e^{-x}$ is the reflection of $y = e^x$ in the y-axis, so its gradient at the y-intercept is -1. Hence the two tangents are perpendicular because the product of their gradients is -1 (or because $45° + 45° = 90°$).

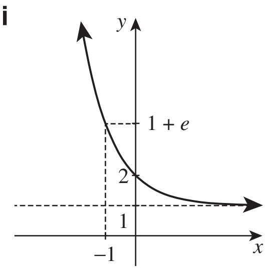
e $y = x + 1$ and $y = -x + 1$

6 a



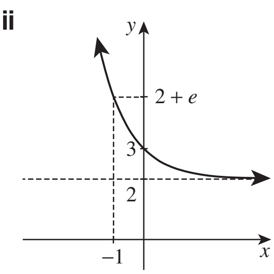
Asymptote: $y = 0$, range: $y > 0$

b **i**



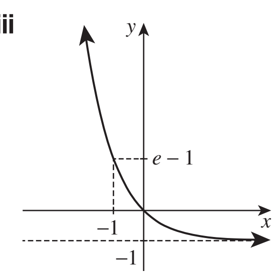
Shift up 1 unit, asymptote: $y = 1$, range: $y > 1$

ii



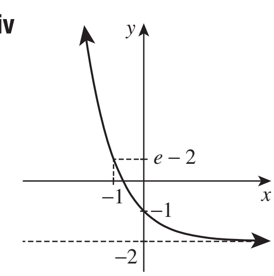
Shift up 2 units, asymptote: $y = 2$, range: $y > 2$

iii



Shift down 1 unit, asymptote: $y = -1$, range: $y > -1$

iv



Shift down 2 units, asymptote: $y = -2$, range: $y > -2$

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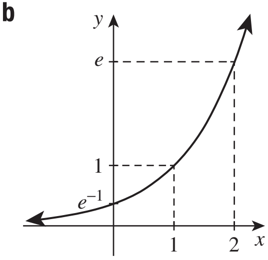
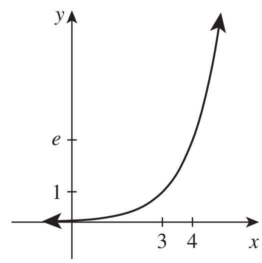
7 a Shift right 1 unit

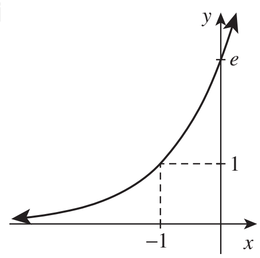
b



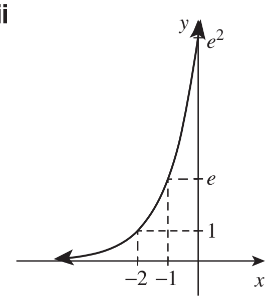
c i



ii



iii



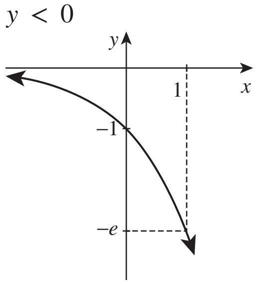
8 a $1, e, e^2$

b grad $AB = e - 1$, AB: $y - 1 = (e - 1)x$

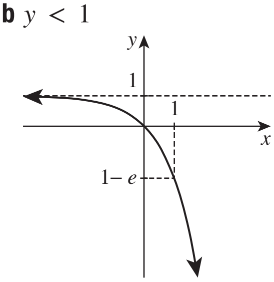
c grad $BC = e(e - 1)$,
BC: $y - e = e(e - 1)(x - 1)$

d grad $PQ = e^a(e - 1)$,
PQ: $y - e^a = e^a(e - 1)(x - a)$

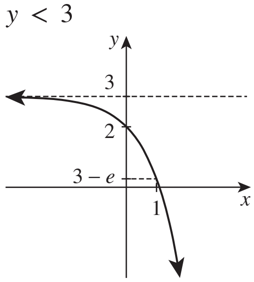
9 a $y < 0$



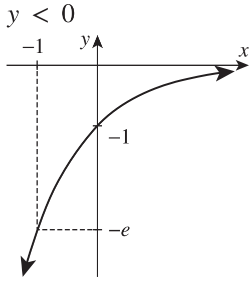
b $y < 1$



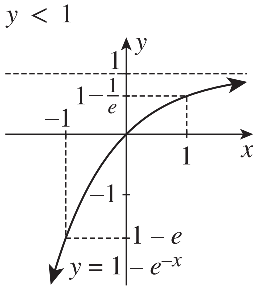
c $y < 3$

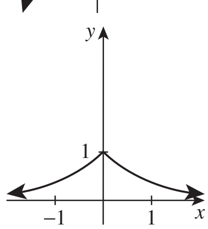


d $y < 0$



e $y < 1$





f $0 < y \le 1$

Exercise 11C

1 a $2e^{2x}$ **b** $7e^{7x}$ **c** $-e^{-x}$ **d** $-5e^{5x}$

e $\frac{1}{2}e^{\frac{x}{2}}$ **f** $2e^{\frac{1}{3}x}$ **g** $-\frac{1}{3}e^{-\frac{x}{3}}$ **h** $e^{\frac{x}{5}}$

2 a $f'(x) = e^{x+2}$ **b** $f'(x) = e^{x-3}$

c $f'(x) = 5e^{5x+1}$ **d** $f'(x) = 2e^{2x-1}$

e $f'(x) = -4e^{-4x+1}$ **f** $f'(x) = -3e^{-3x+4}$

g $f'(x) = -3e^{-3x-6}$ **h** $f'(x) = e^{\frac{x}{2}} + 4$

3 a $e^x - e^{-x}$ **b** $2e^{2x} + 3e^{-3x}$

c $e^{2x} + e^{3x}$ **d** $e^{4x} + e^{5x}$

e $\dfrac{e^x + e^{-x}}{2}$ **f** $\dfrac{e^x - e^{-x}}{3}$

4 a $y' = 2e^{2x}$

b When $x = 0$, $y' = 2$. When $x = 4$, $y' = 2e^8$.

5 a $f'(x) = -e^{-x+3}$

b When $x = 0$, $f'(x) = -e^3$.

c When $x = 4$, $f'(x) = -e^{-1}$.

6 a $y' = 3e^{3x}$, $y'(2) = 3e^6 \doteqdot 1210.29$

b $y' = -2e^{-2x}$, $y'(2) = -2e^{-4} \doteqdot -0.04$

c $y' = \frac{3}{2}e^{\frac{3x}{2}}$, $y'(2) = \frac{3}{2}e^3 \doteqdot 30.13$

7 a $-e^{-x}, e^{-x}, -e^{-x}, e^{-x}$. Successive derivatives alternate in sign. More precisely,

$$f^{(n)}(x) = \begin{cases} e^{-x}, & \text{if } n \text{ is even,} \\ -e^{-x}, & \text{if } n \text{ is odd.} \end{cases}$$

b $2e^{2x}, 4e^{2x}, 8e^{2x}, 16e^{2x}$. Each derivative is twice the previous one. More precisely,
$f^{(n)}(x) = 2^n e^{2x}$.

c e^x, e^x, e^x, e^x All derivatives are the same, and are equal to the original function.

d $y' = e^x + 2x + 1$, $y'' = e^x + 2$,
y''' and all subsequent derivatives are e^x.

8 a $5e^{5x} + 7e^{7x}$ **b** $4e^{4x+2} + 8e^{5+8x}$

c $-4e^{-x} - 12e^{-3x}$ **d** $-12e^{-2x-3} + 42e^{5-6x}$

e $10x - 4 + 3e^{-x}$ **f** $\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}x^{-\frac{1}{2}}$

9 a $y' = \frac{1}{2}\sqrt{e^x}$ **b** $y' = \frac{1}{3}\sqrt[3]{e^x}$

c $y' = -\dfrac{1}{2\sqrt{e^x}}$ **d** $y' = -\dfrac{1}{3\sqrt[3]{e^x}}$

10 a $y' = ae^{ax}$ **b** $y' = -ke^{-kx}$

c $y' = Ake^{kx}$ **d** $y' = -B\ell e^{-\ell x}$

e $y' = pe^{px+q}$ **f** $y' = pCe^{px+q}$

g $y' = \dfrac{pe^{px} - qe^{-qx}}{r}$ **h** $e^{ax} - e^{-px}$

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Exercise 11D

1 a 1 **b** $y = x + 1$

2 a e **b** $y = ex$

3 a $\dfrac{1}{e}$ **b** $y = \dfrac{1}{e}(x + 2)$

4 a $A = \left(\frac{1}{2}, 1\right)$ **b** $y' = 2e^{2x-1}$ **c** $y = 2x$

5 a $y' = e^x$, which is always positive.
b $y' = -e^{-x}$, which is always negative.

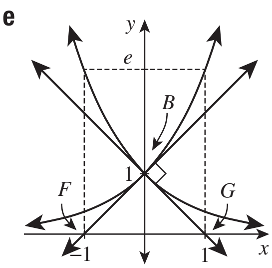
6 a $e - 1$
b $\dfrac{dy}{dx} = e^x$. When $x = 1$, $\dfrac{dy}{dx} = e$.
c $y = ex - 1$ **d i** never **ii** all real x **iii** never

7 a $y' = 1 - e^x$ **b** $1 - e$
c $y = (1 - e)x$. When $x = 0$, $y = 0$.
d i $x = 0$ **ii** $x < 0$ **iii** $x > 0$

8 a $R = \left(-\frac{1}{3}, 1\right)$ **b** $y' = 3e^{3x+1}$
c $-\frac{1}{3}$ **d** $3x + 9y - 8 = 0$.

9 a $-e$ **b** $\dfrac{1}{e}$
c $x - ey + e^2 + 1 = 0$
d $x = -e^2 - 1, y = e + e^{-1}$
e $\frac{1}{2}(e^3 + 2e + e^{-1})$

10 a 1
b $y = x + 1$
c -1
d $y = -x + 1$
e



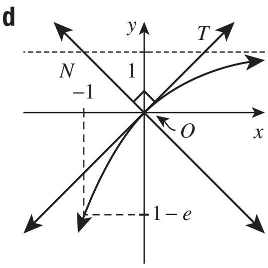
f isosceles right-angled triangle, 1 square unit

11 a 1, $45°$ **b** e, $69°48'$
c e^{-2}, $7°42'$ **d** e^5, $89°37'$

12 $A = (1, e^{-2}), B = (2, 1), y' = 2e^{2x-4}$
a $y' = 2e^{-2}$ **b** $y' = 2$ **c** $1 - e^{-2}$

13 a $y = e^t(x - t + 1)$

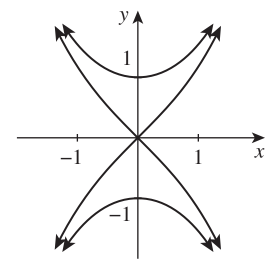
14 b $y = -x$
c $y = 1$
d



e 1 square unit

15 a $y = ex$. When $x = 0$, $y = 0$.
b $x + ey = 1 + e^2$, $A = (1 + e^2, 0)$,
$B = \left(0, \frac{1 + e^2}{e}\right)$
c area $\triangle OPA = \dfrac{e}{2}(e^2 + 1)$,
area $\triangle OPB = \dfrac{1}{2e}(e^2 + 1)$
d $AP : PB = e^2 : 1$ because both triangles can be regarded as having perpendicular height OP.

16 For $y = \cosh x$, $y' = 0$ when $x = 0$. For $y = \sinh x$, $y' = 1$ when $x = 0$. The only point of intersection is the origin.



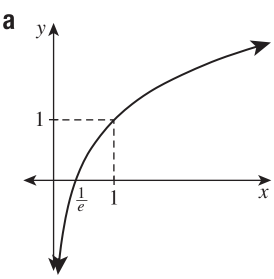
Exercise 11E

1 a 0 **b** 0.6931 **c** 1.0986 **d** 2.0794
e -0.6931 **f** -1.0986 **g** -2.0794 **h** -2.3026

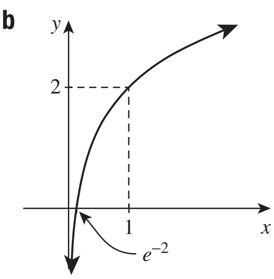
4 a $e^x = 1, x = 0$ **b** $e^x = e, x = 1$
c $e^x = e^2, x = 2$ **d** $e^x = \dfrac{1}{e}, x = -1$
e $e^x = \dfrac{1}{e^2}, x = -2$ **f** $e^x = \sqrt{e}, x = \frac{1}{2}$

5 a $2 \log_e e = 2$ **b** $5 \log_e e = 5$
c $200 \log_e e = 200$ **d** $-6 \log_e e = -6$
e $\log_e e^{-6} = -6 \log_e e = -6$
f $-\log_e e = -1$
g $\log_e e^{-1} = -\log_e e = -1$
h $\frac{1}{2} \log_e e = \frac{1}{2}$ **i** $\frac{1}{2} \log_e e = \frac{1}{2}$
j $\log_e e^{-\frac{1}{2}} = -\frac{1}{2} \log_e e = -\frac{1}{2}$

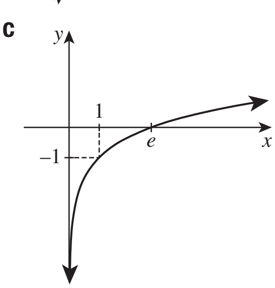
6 a



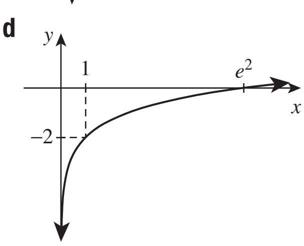
b



c



d



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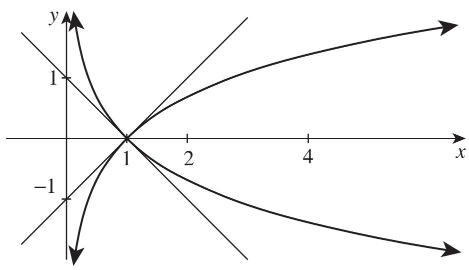
7 b



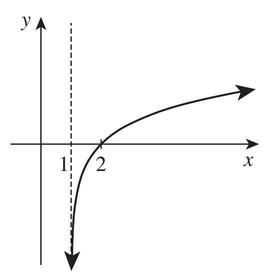
c The graph of $y = -\log_e x$ is obtained by reflecting the first in the x-axis. Hence its tangent has gradient -1, and the two are perpendicular.

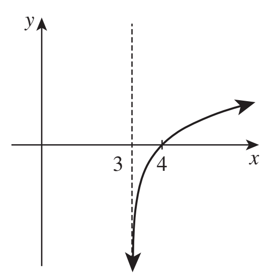
8 a e **b** $-\dfrac{1}{e}$ **c** 6 **d** $\frac{1}{2}$ **e** $2e$

f 0 **g** e **h** 1 **i** 0

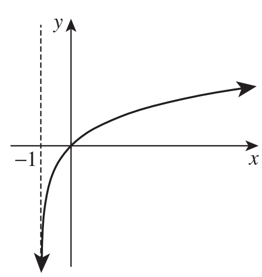
9 a $\log_e 6$ **b** $\log_e 4$ **c** $\log_e 4$ **d** $\log_e 27$

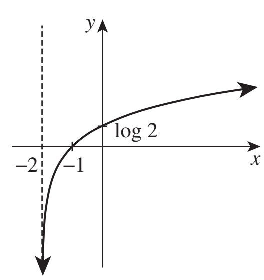
10 a $x > 1$ **b** $x > 3$



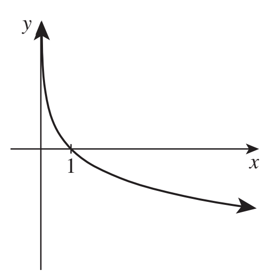


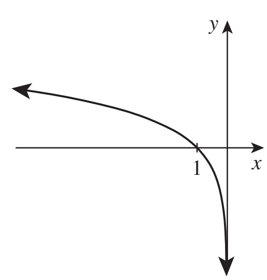
c $x > -1$ **d** $x > -2$





e $x > 0$ **f** $x < 0$





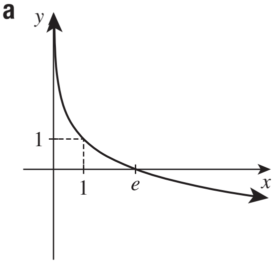
11 a $\log_e \dfrac{a}{b} = \log_e a - \log_e b$ and

$-\log_e \dfrac{b}{a} = -\log_e b + \log_e a$

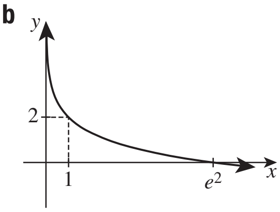
b $\log_{\frac{1}{e}} x = \dfrac{\log_e x}{\log_e \frac{1}{e}} = \dfrac{\log_e x}{-1} = -\log_e x$

c Using part **b**, $\log_{\frac{1}{e}} x^{-1} = -\log_e x^{-1} = +\log_e x$

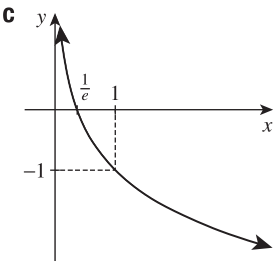
12 a



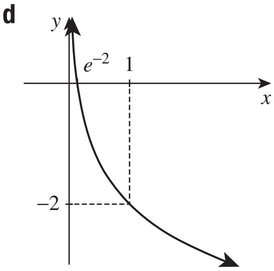
b



c



d



Exercise 11F

1 a $\dfrac{dQ}{dt} = 900e^{3t}$ **b** $Q = 300e^6 \doteqdot 121\,000.$

$\dfrac{dQ}{dt} = 900e^6 \doteqdot 363\,100$ **c** 60 360

2 a $\dfrac{dQ}{dt} = -20\,000e^{-2t}$ **b** $Q = 10\,000e^{-8} \doteqdot 3.355$

$\dfrac{dQ}{dt} = -20\,000e^{-8} \doteqdot -6.709$ **c** -2499

3 a $\dfrac{dQ}{dt} = 10e^{2t}$

b Put $1000 = 5e^{2t}$, $t = \frac{1}{2}\log_e 200 \doteqdot 2.649$

c Put $1000 = 10e^{2t}$, $t = \frac{1}{2}\log_e 100 \doteqdot 2.303$

4 a $P = 2000e^{1.5} \doteqdot 8963$ individuals

b $\dfrac{dP}{dt} = 600e^{0.3t}$

c $\dfrac{dP}{dt} = 600e^{1.5} \doteqdot 2689$ individuals per hour

d 1393 individuals per hour

5 a $C = 2000e^{-4} \doteqdot 36.63$ **b** $\dfrac{dC}{dt} = -4000e^{-2t}$

c $\dfrac{dC}{dt} = -4000e^{-4} \doteqdot -73.26$ per year

d -981.7 per year

6 a $t = 25\log_e 2 \doteqdot 17.33$ years **b** $\dfrac{dP}{dt} = 6e^{0.04t}$

c $t = 25\log_e 50 \doteqdot 97.80$ years

7 a $\dfrac{dP}{dt} = 400e^{0.4t}$ **b** $P = 1000e^2 \doteqdot 7400$

cats, $\dfrac{dP}{dt} = 400e^2 \doteqdot 3000$ cats per year

c $t = \frac{5}{2}\log_e 20 \doteqdot 7.5$ years

d $t = \frac{5}{2}\log_e 50 \doteqdot 9.8$ years

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8 a $t = -10\log_e\left(\frac{1}{2}\right) = 10\log_e 2 \doteqdot 6.931$ years

b $\frac{dM}{dt} = -\frac{1}{10}M_0 e^{-0.1t}$

c $(1 - e^{-0.1}) \times 100\% \doteqdot 9.516\%$

d When $\frac{dM}{dt} = -\frac{1}{100}M_0$,

$t = -10\log_e\left(\frac{1}{10}\right) = 10\log_e 10 \doteqdot 23.03$ years

9 a $\frac{dQ}{dt} = e^t$, which is always positive, so Q is increasing. Also $\frac{dQ}{dt}$ is increasing, so Q is increasing at an increasing rate.

b $\frac{dQ}{dt} = -e^{-t}$, which is always negative, so Q is decreasing. Also $\frac{dQ}{dt}$ is increasing, so the rate of change of Q is increasing, thus Q is decreasing at a decreasing rate. (The language here is not entirely satisfactory — more on this in Year 12.)

c i A and k both positive or both negative.

ii One positive and one negative.

d If $A = 0$, Q is the zero function. If $k = 0$, Q is the constant function $Q = A$.

11 a $\lambda = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $\lambda = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

b When $b^2 - 4ac < 0$ **c** $y = Ae^{2x}$ and $y = Ae^{5x}$

Exercise 11G

1 a $\frac{\pi}{2}$ **b** $\frac{\pi}{4}$ **c** $\frac{\pi}{6}$ **d** $\frac{\pi}{3}$ **e** $\frac{2\pi}{3}$ **f** $\frac{5\pi}{6}$
g $\frac{3\pi}{4}$ **h** $\frac{5\pi}{4}$ **i** 2π **j** $\frac{5\pi}{3}$ **k** $\frac{3\pi}{2}$ **l** $\frac{7\pi}{6}$

2 a 180° **b** 360° **c** 720° **d** 90° **e** 60°
f 45° **g** 120° **h** 150° **i** 135° **j** 270°
k 240° **l** 315° **m** 330°

3 a 0.84 **b** −0.42 **c** −0.14 **d** 0.64
e 0.33 **f** −0.69

4 a 1.274 **b** 0.244 **c** 2.932 **d** 0.377
e 1.663 **f** 3.686

5 a 114°35′ **b** 17°11′ **c** 82°30′ **d** 7°3′
e 183°16′ **f** 323°36′

6 a $\frac{1}{2}$ **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{\sqrt{3}}{2}$ **d** $\sqrt{3}$
e 1 **f** $\frac{1}{2}$ **g** $\sqrt{2}$ **h** $\frac{1}{\sqrt{3}}$

7 a $\frac{\pi}{9}$ **b** $\frac{\pi}{8}$ **c** $\frac{\pi}{5}$ **d** $\frac{5\pi}{9}$ **e** $\frac{5\pi}{8}$ **f** $\frac{7\pi}{5}$

8 a 15° **b** 72° **c** 400° **d** 247.5°
e 306° **f** 276°

9 a $\frac{\pi}{3}$ **b** $\frac{5\pi}{6}$

10 $\frac{4\pi}{9}$

11 a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{2}$ **c** $-\frac{\sqrt{3}}{2}$ **d** $\sqrt{3}$
e −1 **f** $\frac{1}{2}$ **g** $-\frac{1}{\sqrt{2}}$ **h** $\frac{1}{\sqrt{3}}$

12 a Hour hand: 30° or $\frac{\pi}{6}$ radians, minute hand: 360° or 2π radians.

b i 60° or $\frac{\pi}{3}$ radians **ii** $22\frac{1}{2}°$ or $\frac{\pi}{8}$ radians
iii 105° or $\frac{7\pi}{12}$ radians **iv** $172\frac{1}{2}°$ or $\frac{23\pi}{24}$ radians

13 a 0.733 **b** 0.349 **c** 63 334.508

14 a 0.283 **b** 0.819

15 a $k\pi$ is never an integer when k is an integer, except when $k = 0$.

b $n = 22$ **c** $\sin 22 \doteqdot \sin 7\pi = 0$

Exercise 11H

1 a $\frac{\pi}{4}$ **b** $\frac{\pi}{6}$ **c** $\frac{\pi}{4}$ **d** $\frac{\pi}{6}$ **e** $\frac{\pi}{3}$ **f** $\frac{\pi}{3}$

2 a $x \doteqdot 1.249$ **b** $x \doteqdot 0.927$ **c** $x \doteqdot 1.159$
d $x \doteqdot 0.236$ **e** $x \doteqdot 0.161$ **f** $x \doteqdot 1.561$

3 a $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ **b** $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$
c $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ **d** $x = \frac{\pi}{2}$
e $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ **f** $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$
g $x = \pi$ **h** $x = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$

4 a $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ **b** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
c $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ or $\frac{5\pi}{3}$ **d** $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

5 a $u^2 - u = 0$ **b** $u = 0$ or $u = 1$
c $\theta = 0, \frac{\pi}{2}, \frac{3\pi}{2}$ or 2π

6 a $u^2 - u - 2 = 0$ **b** $u = -1$ or $u = 2$
c $\theta = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, or $\theta \doteqdot 1.11$ or 4.25

7 a $\theta = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ or 2π

b $\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$ or 2π

c $\theta = \frac{\pi}{2}$

d $\theta \doteqdot 1.11, 1.89, 4.25$ or 5.03

e $\theta = \frac{\pi}{3}, \pi$ or $\frac{5\pi}{3}$ **f** $\theta = \frac{\pi}{2}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$
g $\theta \doteqdot 0.34$ or 2.80 **h** $\theta \doteqdot 1.91$ or 4.37

8 a $\theta = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{5\pi}{3}$

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b $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$, or $x \doteqdot 1.25$ or 4.39

c $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, or $x \doteqdot 0.25$ or 2.89

d $x \doteqdot 0.84$ or 5.44

9 a $\alpha = \frac{\pi}{2}$, or $\alpha \doteqdot 3.48$ or 5.94

b $\alpha \doteqdot 1.11, 2.82, 4.25$ or 5.96

10 a $x = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ or 2π

b $x \doteqdot 1.11, 1.25, 4.25$ or 4.39

11 a $x = -\frac{11\pi}{12}, -\frac{7\pi}{12}, \frac{\pi}{12}$ or $\frac{5\pi}{12}$

b $x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}$ or π **c** $x = -\frac{\pi}{2}$ or $\frac{\pi}{2}$

d $x = -\pi$ or $\frac{\pi}{2}$ or π **e** $x = -\frac{3\pi}{4}$

f $x = -\frac{2\pi}{3}$ or $\frac{\pi}{3}$

13 $\theta = \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}$ or $\frac{9\pi}{5}$

Exercise 11I

1 a 12 cm **b** 3 cm **c** 2π cm **d** $\frac{3\pi}{2}$ cm

2 a 32 cm^2 **b** 96 cm^2 **c** 8π cm^2 **d** 12π cm^2

3 4 cm

4 1.5 radians

5 a 2.4 cm **b** 4.4 cm

6 8727 m^2

7 a 8π cm **b** 16π cm^2

8 84°

9 11.6 cm

10 a 6π cm^2 **b** $9\sqrt{3}$ cm^2 **c** $3(2\pi - 3\sqrt{3})$ cm^2

12 15 cm^2

13 a $4(\pi + 2)$ cm **b** 8π cm^2

14 a $\frac{2\pi}{3}$ cm **b** $\frac{2\pi}{3}$ cm^2 **c** 2π cm

d $\sqrt{3}$ cm^2, $2(\pi - \sqrt{3})$ cm^2

15 $\frac{4}{3}(4\pi - 3\sqrt{3})$ cm^2

16 c $3\sqrt{55}\,\pi$ cm^3 **d** 24π cm^2

17 a 720 metres **b** 2.4 radians (about 137°31′)

c 559.22 metres **d** 317°31′T

18 a By Pythagoras, $h^2 = r^2 - \left(\frac{r}{2}\right)^2 = \frac{3}{4}r^2$. By the area formula, $A = \frac{1}{2}r^2\sin 60° = \frac{1}{2}r^2 \times \frac{\sqrt{3}}{2}$.

b i Partition the hexagon into six equilateral triangles. An interval is the shortest distance between two points, so
$6r$ = perimeter < circumference = $2\pi r$, so
$3 < \pi$.

ii Each equilateral triangle of the outer hexagon has height r, and hence by part **a** has side length $s = \frac{2r}{\sqrt{3}}$ and area $\frac{1}{4} \times \frac{4r^2}{3} \times \sqrt{3} = \frac{1}{3}r^2\sqrt{3}$. The circle lies inside the outer hexagon,

so πr^2 = area of circle < area of outer hexagon

$= 6 \times \frac{1}{3}r^2\sqrt{3} = 2r^2\sqrt{3}$, so $\pi < 2\sqrt{3}$.

19 2.54 cm^2

20 36 seconds

Exercise 11J

2 a All six graphs are many-to-one.

b i $\pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi$

ii $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

iii $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}, \frac{21\pi}{2}$

iv There are no solutions.

3 a $x = \frac{\pi}{2}, x = -\frac{\pi}{2}, x = \frac{3\pi}{2}, x = -\frac{3\pi}{2}$,
$x = \frac{5\pi}{2}, x = -\frac{5\pi}{2}, \ldots$

b $y = \operatorname{cosec} x$, the reciprocal of $y = \sin x$.

c Neither graph has any line symmetries.

4 a $x = 0, x = \pi, x = -\pi, x = 2\pi, x = -2\pi, \ldots$

b Line symmetry in the y-axis $x = 0$

c $y = \sec x$, the reciprocal of $y = \cos x$.

5 a $(0, 0), (\pi, 0), (-\pi, 0), (2\pi, 0), (-2\pi, 0), \ldots$

b Point symmetry in the origin $(0, 0)$

c $y = \operatorname{cosec} x$, the reciprocal of $y = \sin x$.

6 a $\left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right), \left(-\frac{3\pi}{2}, 0\right), \ldots$

b $y = \sec x$, the reciprocal of $y = \cos x$.

7 a $(0, 0), \left(\frac{\pi}{2}, 0\right), \left(-\frac{\pi}{2}, 0\right), (\pi, 0), (-\pi, 0)$,
$\left(\frac{3\pi}{2}, 0\right), \left(-\frac{3\pi}{2}, 0\right), \ldots$

b Both functions are odd, because both have point symmetry in the origin. Neither is even, because neither have line symmetry in the y-axis.

8 a Translations left or right by multiples of 2π.

b $y = \cos x$, $y = \operatorname{cosec} x$ and $y = \sec x$.

c $y = \tan x$ and $y = \cot x$ can each be mapped onto themselves by translations left or right by multiples of π.

d $y = \sin x$, $y = \cos x$, $y = \operatorname{cosec} x$, $y = \sec$ each has period 2π. $y = \tan x$, $y = \cot x$ each has period π.

9 a $x = \frac{\pi}{4}, x = -\frac{3\pi}{4}, x = \frac{5\pi}{4}, x = -\frac{7\pi}{4}, \ldots$

b $y = \operatorname{cosec} x$ and $y = \sec x$

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c $x = \frac{\pi}{4}, x = -\frac{\pi}{4}, x = \frac{3\pi}{4}, x = -\frac{3\pi}{4}, x = \frac{5\pi}{4},$

$x = -\frac{5\pi}{4}, \ldots$

10 a Translations left $\frac{\pi}{2}, \frac{5\pi}{2}, \ldots$, and right $\frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

b $y = \sin(x - \theta)$ is $y = \sin x$ shifted right by θ, so

$\sin(x - \theta) = \cos x$ for $\theta = \frac{3\pi}{2}, -\frac{\pi}{2}, \frac{7\pi}{2}, -\frac{5\pi}{2}, \frac{11\pi}{2}, -\frac{9\pi}{2}, \ldots$

c There are none.

11 There are none.

12 a $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(-\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}\right), \left(\frac{5\pi}{4}, \frac{1}{\sqrt{2}}\right), \left(-\frac{7\pi}{4}, -\frac{1}{\sqrt{2}}\right), \ldots$

b $\sin x = \cos x$, so $\tan x = 1$.

13 a $(0, 0), (\pi, 0), (-\pi, 0), (2\pi, 0), (-2\pi, 0), \ldots$

b $\sin x = \dfrac{\sin x}{\cos x}$, so $\sin x \cos x = \sin x$,

so $\sin x(\cos x - 1) = 0$, so $\sin x = 0$ or $\cos x = 1$.

14 a Roughly 0.7 (radians).

15 a They touch each other at their maxima and minima.

b $y = \cos x$ and $y = \sec x$.

c $y = \sin x$ & $y = \sec x$, $y = \cos x$ & $y = \operatorname{cosec} x$, $y = \tan x$ & $y = \sec x$, $y = \cot x$ & $y = \operatorname{cosec} x$

16 a

$$\cos x = \frac{\sin x}{\cos x}$$

$$\boxed{\times \cos x} \quad \cos^2 x = \sin x \text{ and } \cos x \neq 0$$

$$1 - \sin^2 x = \sin x$$

$$\sin^2 x + \sin x - 1 = 0$$

$$\Delta = 1 + 4 = 5$$

$$\sin x = \frac{-1 + \sqrt{5}}{2}$$

giving solutions in the first and second quadrants.

$\left(\frac{-1 - \sqrt{5}}{2} < -1,\right.$

so $\sin x = \frac{-1 - \sqrt{5}}{2}$ has no solutions.$\left.\right)$

b

$$\frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\boxed{\times \cos x} \quad \sin x = 1 \text{ and } \cos x \neq 0$$

There are no solutions,
because if $\sin x = 1$, then $\cos x = 0$.

Chapter 11 review exercise

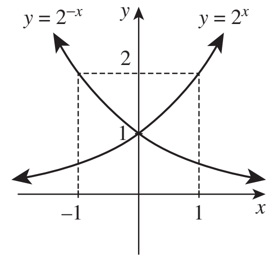
1 a 3^9 **b** 3^{12} **c** 3^5 **d** 6^5

2 a $\frac{1}{5}$ **b** $\frac{1}{100}$ **c** $\frac{1}{x^3}$ **d** $\frac{1}{3^x}$

3 a 3 **b** 3 **c** 4 **d** $\frac{1}{4}$ **e** $\frac{1}{9}$ **f** $\frac{1}{1000}$

4 a 2^{3x} **b** 2^{4x} **c** 2^{6x} **d** 10^x **e** 2^{2x+3} **f** 2^{2x-1}

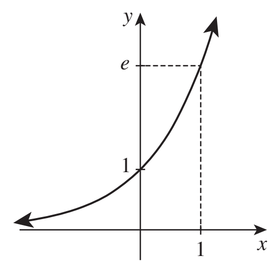
5 Each graph is reflected onto the other graph in the line $x = 0$.



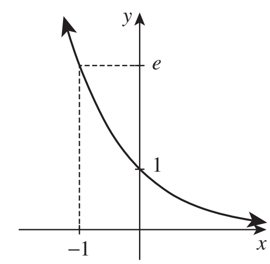
6 a 2.718 **b** 54.60 **c** 0.1353 **d** 4.482

7 a e^{5x} **b** e^{6x} **c** e^{-4x} **d** e^{9x}

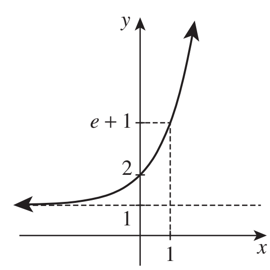
8 a $y > 0$



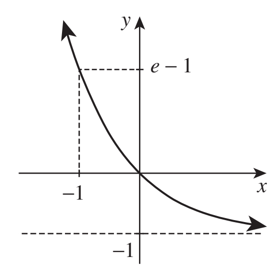
b $y > 0$



c $y > 1$



d $y > -1$



9 a e^x **b** $3e^{3x}$ **c** e^{x+3} **d** $2e^{2x+3}$ **e** $-e^{-x}$ **f** $-3e^{-3x}$ **g** $-2e^{3-2x}$ **h** $6e^{2x+5}$ **i** $2e^{\frac{1}{2}x}$ **j** $3x^2e^{x^3}$ **k** $(2x - 3)e^{x^2-3x}$ **l** $4e^{6x-5}$

10 a $5e^{5x}$ **b** $4e^{4x}$ **c** $-3e^{-3x}$ **d** $-6e^{-6x}$

11 2

12 $y = e^2x - e^2$, x-intercept 1, y-intercept $-e^2$.

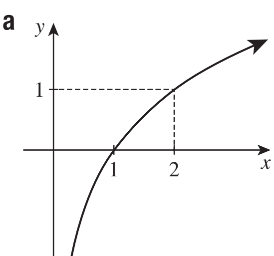
13 a 1.4314 **b** −0.3010 **c** 0.6931 **d** 2.6391

14 a 1.1761 **b** 0.4771 **c** 1.9459 **d** −1.0986

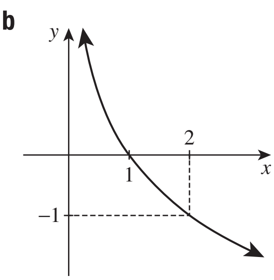
15 a 5 **b** $-\frac{1}{4}$ **c** 3 **d** $\frac{1}{5}$

16 a e **b** 3 **c** −1 **d** e

17 a



b



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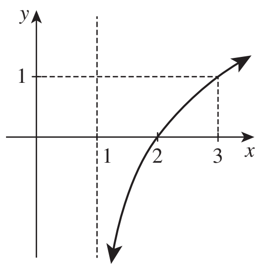
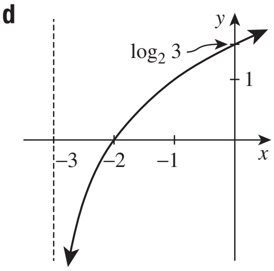
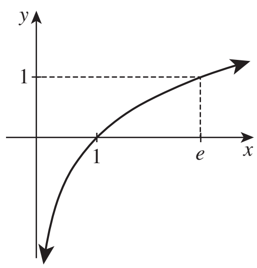
c



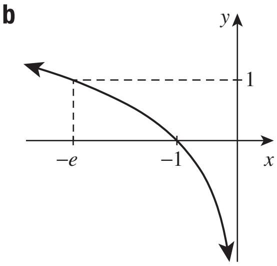
d



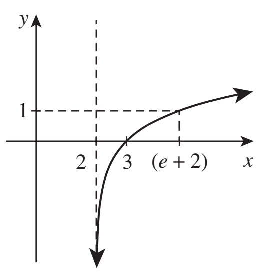
18 a



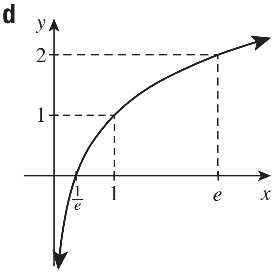
b



c



d



19 a $\dfrac{dP}{dt} = -\frac{1}{100}P_0e^{-0.01t}$

b $\dfrac{dP}{dt} = -\frac{1}{100}P_0e^{-0.45} = -0.0064\,P_0$ lizards per year.

c $P = P_0e^{-0.45} \doteqdot 64\%$ of the original population.

d $e^{-0.01t} = \frac{1}{10}$, so $t = 100\log_e 10 \doteqdot 230$ years

20 a π **b** $\frac{\pi}{9}$ **c** $\frac{4\pi}{3}$ **d** $\frac{7\pi}{4}$

21 a 30° **b** 108° **c** 540° **d** 300°

22 a $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{\sqrt{3}}$

23 a $x = \frac{\pi}{4}$ or $\frac{7\pi}{4}$ **b** $x = \frac{2\pi}{3}$ or $\frac{5\pi}{3}$

24 a $\sin\theta = 0$ or $-\frac{1}{2}$, $\theta = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ or 2π

b $\cos\theta = -1$ or 2, $\theta = \pi$ ($\cos\theta = 2$ has no solutions.)

c $\tan\theta = \frac{1}{2}$ and $\theta \doteqdot 0.46$ or 3.61,

or $\tan\theta = -3$ and $\theta \doteqdot 1.89$ or 5.03

25 a 3π cm **b** 12π cm^2

26 $16(\pi - 2) \doteqdot 18.3\,\text{cm}^2$

27 148°58′

28 6.77 cm

29 a 1.2661 radians **b** 49.2 cm

30 a $y = \sin x$ and $y = \cos x$ both have amplitude 1.

b $y = \sin x$, $y = \cos x$, $y = \operatorname{cosec} x$ and $y = \sec x$ all have period 2π, $y = \tan x$ and $y = \cot x$ both have period π.

c $y = \sin x$, $y = \tan x$, $y = \operatorname{cosec} x$ and $y = \cot x$ are all odd, $y = \cos x$, and $y = \sec x$ are both even.

31 a $\theta = \frac{3\pi}{2}$ **b** $\theta = \frac{\pi}{2}$ **c** $x = \frac{\pi}{4}$

Chapter 12

Exercise 12A

1 a $\frac{1}{20}$ **b** $\frac{19}{20}$

2 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** 1 **d** 0

3 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{1}{3}$

4 a $\frac{5}{12}$ **b** $\frac{7}{12}$ **c** 0

5 a $\frac{4}{9}$ **b** $\frac{5}{9}$ **c** $\frac{11}{18}$

6 a $\frac{4}{9}$ **b** $\frac{5}{9}$ **c** $\frac{11}{18}$ **d** $\frac{7}{18}$ **e** $\frac{1}{3}$ **f** $\frac{1}{6}$

7 a $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

8 a $\frac{1}{26}$ **b** $\frac{5}{26}$ **c** $\frac{21}{26}$ **d** 0 **e** $\frac{3}{26}$ **f** $\frac{5}{26}$

9 78%

10 a $\frac{4}{7}$ **b** 32

11 a 8 **b** $\frac{14}{15}$

12 a 10 sixes

b i $\frac{18}{60} = 30\%$

ii The experiment suggest a probability of about 30%.

iii The theoretical probability suggests that for an unbiased die, we would expect to get a six on one-sixth of the throws, that is, 10 times. The large number of sixes turning up suggests that this die is biased.

13 a $\frac{100}{400} = \frac{1}{4} = 25\%$ **b** $\frac{8}{20} = \frac{2}{5} = 40\%$

c We would expect him to get chicken one-quarter of the time, that is, on 5 occasions. He may have got more chicken sandwiches because of the way the canteen makes or sells the sandwiches, for example making the chicken sandwiches early and placing them at the front of the display, or making more Vegemite sandwiches as they sell out. Possibly also the sample is too small and the result would approach $\frac{1}{4}$ if the experiment were continued over a longer time. The experimental probability is only an estimate, and in fact it is possible he may have got no chicken sandwiches over the twenty days.

14 a $\frac{1}{20}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{2}{5}$

f $\frac{1}{5}$ **g** $\frac{1}{4}$ **h** 0 **i** 1

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15 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{1}{52}$
e $\frac{1}{4}$ **f** $\frac{3}{13}$ **g** $\frac{1}{2}$ **h** $\frac{1}{13}$
i $\frac{3}{13}$ (counting an ace as a one)

16 a $\frac{1}{15}$ **b** $\frac{7}{150}$ **c** $\frac{1}{2}$ **d** $\frac{4}{25}$ **e** $\frac{1}{75}$ **f** $\frac{17}{50}$

17 a $\frac{1}{5}$ **b** $\frac{3}{40}$ **c** $\frac{9}{20}$ **d** $\frac{7}{100}$ **e** $\frac{7}{50}$ **f** $\frac{1}{200}$

18 a $\frac{3}{4}$ **b** $\frac{1}{4}$

19 187 or 188

20 a The argument is invalid, because on any one day the two outcomes are not equally likely. The argument really can't be corrected.

b The argument is invalid. One team may be significantly better than the other, the game may be played in conditions that suit one particular team, and so on. Even when the teams are evenly matched, the high-scoring nature of the game makes a draw an unlikely event. The three outcomes are not equally likely. The argument really can't be corrected.

c The argument is invalid, because we would presume that Peter has some knowledge of the subject, and is therefore more likely to choose one answer than another. The argument would be valid if the questions were answered at random.

d The argument is only valid if there are equal numbers of red, white and black beads, otherwise the three outcomes are not equally likely.

e The argument is missing, but the conclusion is correct. Exactly one of the four players will win his semi-final and then lose the final. Our man is as likely to pick this player as he is to pick any of the other three players.

21 a $\frac{2}{9}$ **b** $\frac{\pi}{18}$

22 a $\frac{7}{8}$ **b** $\frac{\pi + 2}{9}$

Exercise 12B

1 a HH, HT, TH, TT
b i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{4}$

2 a H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6
b i $\frac{1}{4}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{4}$

3 a TO, OT, OE, EO, ET, TE
b i $\frac{1}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$

4 a AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC
b i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{6}$ **v** $\frac{1}{4}$ **vi** $\frac{3}{4}$

5 a 23, 32, 28, 82, 29, 92, 38, 83, 39, 93, 89, 98
b i $\frac{1}{12}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{6}$ **v** $\frac{1}{4}$ **vi** 0

6 a The captain is listed first and the vice-captain second: AB, AC, AD, AE, BC, BD, BE, CD, CE, DE, BA, CA, DA, EA, CB, DB, EB, DC, EC, ED
b i $\frac{1}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{5}$ **iv** $\frac{1}{5}$

7 HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** $\frac{1}{2}$

8 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66
a $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{36}$ **d** $\frac{1}{6}$ **e** $\frac{1}{6}$ **f** $\frac{1}{4}$
g $\frac{11}{36}$ **h** $\frac{4}{9}$ **i** $\frac{5}{36}$ **j** $\frac{1}{6}$

9 a i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **b i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$

10 a $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{11}{16}$ **d** $\frac{5}{16}$ **e** $\frac{3}{8}$ **f** $\frac{5}{16}$

11 a $\frac{2}{5}$ **b** $\frac{3}{5}$ **c** $\frac{1}{5}$

12 a 24 **b i** $\frac{2}{3}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{12}$ **iv** $\frac{1}{6}$

13 a $\frac{1}{2^n}$ **b** $1 - 2^{1-n}$
c $\frac{1}{2}$ That is, half the time there will be more tails than heads.

14 $\frac{1}{4}$

15 $\frac{1}{4}$. The experiment is the same as asking the probability that the first card is a diamond.

Exercise 12C

1 a $\{1, 3, 5, 7, 9\}$ **b** $\{6, 12, 18, 24, 30, 36\}$
c $\{1, 2, 3, 4, 5, 6\}$ **d** $\{1, 2, 4, 5, 10, 20\}$

2 a $A \cup B = \{1, 3, 5, 7\}$, $A \cap B = \{3, 5\}$
b $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A \cap B = \{4, 9\}$
c $A \cup B = \{h, o, b, a, r, t, i, c, e, n\}$, $A \cap B = \{h, o, b\}$
d $A \cup B = \{j, a, c, k, e, m\}$, $A \cap B = \{a\}$
e $A \cup B = \{1, 2, 3, 5, 7, 9\}$, $A \cap B = \{3, 5, 7\}$

3 a false **b** true **c** false
d false **e** true **f** true

4 a 3 **b** 2 **c** $\{1, 3, 4, 5\}$ **d** 4
e $\{3\}$ **f** 1 **g** $\{2, 4\}$ **h** $\{1, 2, 5\}$

5 a students who study both Japanese and History
b students who study either Japanese or History or both

6 a students at Clarence High School who do not have blue eyes
b students at Clarence High School who do not have blond hair
c students at Clarence High School who have blue eyes or blond hair or both
d students at Clarence High School who have blue eyes and blond hair

7 a $\varnothing$, $\{a\}$
b $\varnothing$, $\{a\}$, $\{b\}$, $\{a, b\}$
c $\varnothing$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$
d $\varnothing$

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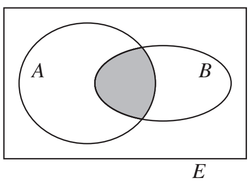
8 a true **b** false **c** true **d** false **e** true

9 a $\{2, 4, 5, 6, 8, 9\}$ **b** $\{1, 2, 3, 5, 8, 10\}$
c $\{7\}$ **d** $\{1, 2, 3, 4, 5, 6, 8, 9, 10\}$
e $\{1, 3, 4, 6, 7, 9, 10\}$ **f** $\{2, 5, 8\}$

10 a $\{2, 4, 5, 7, 9, 10\}$ **b** $\{1, 2, 5, 8, 9\}$
c $\{1, 2, 4, 5, 7, 8, 9, 10\}$ **d** $\{2, 5, 9\}$
e $\{1, 3, 4, 6, 7, 8, 10\}$ **f** $\{2, 5, 9\}$
g $\{3, 6\}$ **h** $\{1, 2, 4, 5, 7, 8, 9, 10\}$

11 a


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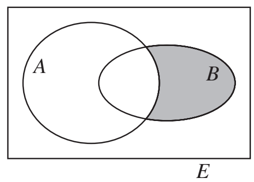

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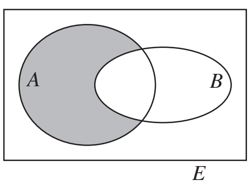

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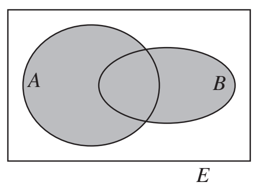

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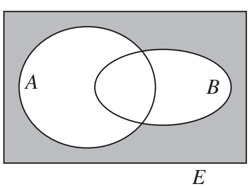

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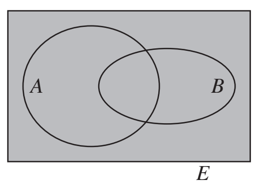


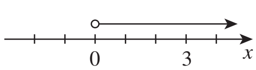
12 a true **b** true

13 a Q **b** P

14 a III **b** I **c** II **d** IV

15 a infinite **b** finite, 10 members
c finite, 0 members **d** infinite
e finite, 18 members **f** infinite
g finite, 6 members **h** finite, 14 members

16 a false **b** true **c** true
d false **e** true **f** false

17 a i


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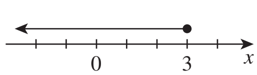

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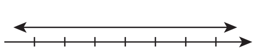

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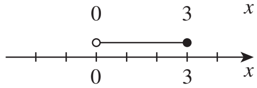
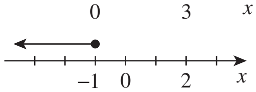


b i


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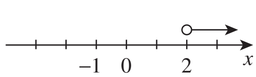

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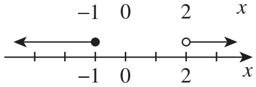

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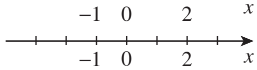
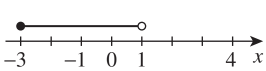


c i


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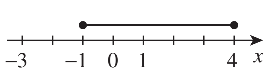

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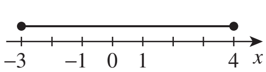

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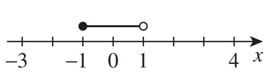
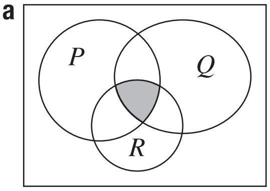
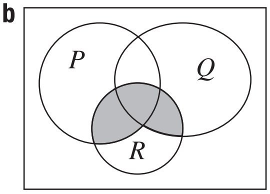
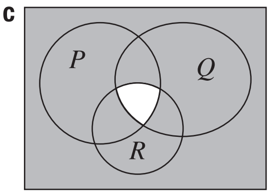


18 a $|A \cap B|$ is subtracted so that it is not counted twice.
b 5 **c** LHS $= 7$, RHS $= 5 + 6 - 4 = 7$

19 a 10 **b** 22 **c** 12

20 a


b


c


21 4

22 $\frac{7}{43}$

23 a $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
b 207
c $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$.
It is possible to draw a Venn diagram with four sets, but only if the fourth set is represented not by a circle, but by a complicated loop — the final diagram must have 16 regions.

Exercise 12D

1 a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{3}$ **d** 0
e 1 **f** 0 **g** $\frac{1}{6}$ **h** $\frac{2}{3}$

2 a $\frac{1}{13}$ **b** $\frac{1}{13}$ **c** $\frac{2}{13}$ **d** 0 **e** $\frac{11}{13}$ **f** $\frac{1}{2}$
g $\frac{3}{13}$ **h** $\frac{3}{26}$ **i** $\frac{8}{13}$ **j** $\frac{5}{13}$

3 a $A = \{\text{HH}\}, B = \{\text{HT, TH}\}, P(A \text{ or } B) = \frac{3}{4}$, $P(A) = \frac{1}{4}, P(B) = \frac{2}{4}$
b $A = \{\text{RS}\}, B = \{\text{RT, ST}\}, P(A \text{ or } B) = \frac{3}{3}$, $P(A) = \frac{1}{3}, P(B) = \frac{2}{3}$

4 a no **b i** $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{5}{6}$

5 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{3}{4}$ **e** $\frac{1}{4}$
f $\frac{1}{6}$ **g** $\frac{1}{6}$ **h** $\frac{1}{36}$ **i** $\frac{11}{36}$ **j** $\frac{25}{36}$

6 a i $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{2}$ **v** $\frac{1}{2}$
b i $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{5}$ **iv** 0 **v** 1
c i $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{2}{3}$ **iv** $\frac{1}{3}$ **v** $\frac{5}{6}$

7 a $\frac{7}{15}$ **b** 0 **c** $\frac{3}{5}$ **d** $\frac{5}{7}$

8 a i no **ii** $\frac{1}{2}, \frac{1}{4}, \frac{3}{20}, \frac{3}{5}$ **b i** no **ii** $\frac{1}{2}, \frac{3}{10}, \frac{3}{20}, \frac{13}{20}$
c i yes **ii** $\frac{1}{4}, \frac{9}{20}, 0, \frac{7}{10}$

9 a $\frac{9}{25}$ **b** $\frac{7}{50}$ **c** $\frac{17}{50}$

10 a 10 **b i** $\frac{4}{21}$ **ii** $\frac{1}{3}$

11 $\frac{1}{4}$

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12 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{11}{100}$ **d** $\frac{39}{100}$

13 a $\frac{7}{12}$ **b** $\frac{13}{60}$ **c** $\frac{3}{10}$ **d** $\frac{7}{60}$

Exercise 12E

1 a $\frac{1}{24}$ **b** $\frac{1}{28}$ **c** $\frac{1}{12}$ **d** $\frac{1}{96}$ **e** $\frac{1}{42}$ **f** $\frac{1}{336}$

2 a $\frac{1}{12}$ **b** $\frac{1}{12}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3}$

3 a $\frac{1}{25}$ **b** $\frac{2}{25}$ **c** $\frac{3}{25}$ **d** $\frac{3}{25}$ **e** $\frac{4}{25}$ **f** $\frac{2}{25}$ **g** $\frac{1}{25}$

4 a $\frac{15}{49}$ **b** $\frac{8}{49}$ **c** $\frac{6}{49}$

5 a $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{10}$ **d** $\frac{3}{10}$

6 a $\frac{1}{36}$ **b** $\frac{1}{12}$ **c** $\frac{1}{36}$ **d** $\frac{1}{9}$ **e** $\frac{1}{6}$

7 a $\frac{1}{7}$ **b** $\frac{180}{1331}$

8 a i $\frac{13}{204}$ **ii** $\frac{1}{17}$ **iii** $\frac{4}{663}$ **iv** $\frac{1}{2652}$

b $\frac{1}{16}, \frac{1}{16}, \frac{1}{169}, \frac{1}{2704}$

9 a i $\frac{2}{3}$ **ii** $\frac{1}{3}$ **b i** $\frac{8}{27}$ **ii** $\frac{1}{27}$ **iii** $\frac{4}{27}$

10 a The argument is invalid, because the events ‘liking classical music’ and ‘playing a classical instrument’ are not independent. One would expect that most of those playing a classical instrument would like classical music, whereas a smaller proportion of those not playing a classical instrument would like classical music. The probability that a student does both cannot be discovered from the given data — one would have to go back and do another survey.

b The argument is invalid, because the events ‘being prime’ and ‘being odd’ are not independent — two out of the three odd numbers less than 7 are prime, but only one out of the three such even numbers is prime. The correct argument is that the odd prime numbers amongst the numbers 1, 2, 3, 4, 5 and 6 are 3 and 5, hence the probability that the die shows an odd prime number is $\frac{2}{6} = \frac{1}{3}$.

c The teams in the competition may not be of equal ability, and factors such as home-ground advantage may also affect the outcome of a game, hence assigning a probability of $\frac{1}{2}$ to winning each of the seven games is unjustified. Also, the outcomes of successive games are not independent — the confidence gained after winning a game may improve a team’s chances in the next one, a loss may adversely affect their chances, or a team may receive injuries in one game leading to a depleted team in the next. The argument really can’t be corrected.

d This argument is valid. The coin is normal, not biased, and tossed coins do not remember their previous history, so the next toss is completely unaffected by the previous string of heads.

11 The chance that at least one of them will shoot a basket is $1 - P$ (they both miss). The boy missing and the girl missing are independent events. The correct answer is 0.895.

12 a $\frac{1}{36}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$

d $\frac{1}{36}$ **e** $\frac{1}{36}$ **f** $\frac{1}{18}$

g $\frac{1}{12}$ **h** $\frac{1}{12}$ **i** $\frac{1}{6}$

13 HHH, HHM, HMH, MHH, HMM, MHM, MMH, MMM

a $P(\text{HHH}) = 0.9^3 = 0.729$ **b** 0.001

c $P(\text{HMM}) = 0.9 \times 0.1^2 = 0.009$

d $P(\text{HMM}) + P(\text{MHM}) + P(\text{MMH}) = 3 \times 0.009 = 0.027$

e 0.081 **f** 0.243

14 a $P(\text{CCCCC}) = \left(\frac{1}{5}\right)^5 = \frac{1}{3125}$ **b** $\frac{1024}{3125}$

c $\frac{16}{3125}$ **d** $\frac{256}{3125}$ **e** $\frac{256}{625}$ **f** $\frac{4}{625}$

15 a $\frac{1}{46656}$ **b** $\frac{5}{7776}$

16 a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

17 a $\frac{3}{64}$ **b** $\frac{17}{64}$ **c** $\frac{5}{17}$

18 a i $\frac{3}{4}$ **ii** $\frac{31}{32}$ **iii** $\frac{1023}{1024}$

b $1 - \dfrac{1}{2^n} = \dfrac{2^n - 1}{2^n}$ **c** 14

19 a $\frac{9}{25}$ **b** 11

20 a $\frac{1}{12960000}$ **b** 233

21 a $\frac{1}{9}$

b $\frac{1}{9}$. Retell as ‘Nick begins by picking out two socks for the last morning and setting them aside’.

c $\frac{1}{9}$. Retell as ‘Nick begins by picking out two socks for the third morning and setting them aside’.

d $\frac{1}{63}$ **e** $\frac{1}{9 \times 7 \times 5 \times 3}$ **f** zero

22 a In each part, retell the process of selection as ‘First choose a court for Kia, then choose one of the remaining 11 positions for Abhishek’.

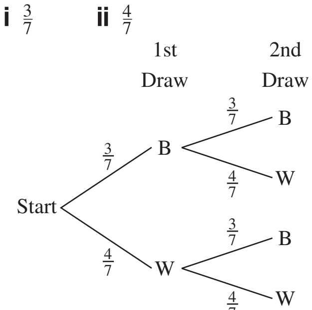
a $\dfrac{3}{11}$ **b** $\dfrac{1}{11}$ **c** $\dfrac{4}{33}$

Exercise 12F

1 a i $\frac{9}{49}$ **ii** $\frac{12}{49}$ **iii** $\frac{12}{49}$ **iv** $\frac{16}{49}$

b i $\frac{25}{49}$ **ii** $\frac{24}{49}$

c i $\frac{3}{7}$ **ii** $\frac{4}{7}$



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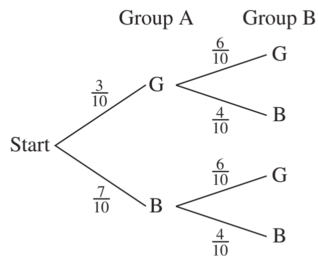
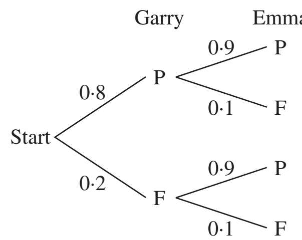
2 a i 90.25% **ii** 4.75% **iii** 4.75% **iv** 0.25%
b 99.75%
3 a i $\frac{6}{25}$ **ii** $\frac{9}{25}$ **iii** $\frac{4}{25}$ **iv** $\frac{6}{25}$
b i $\frac{12}{25}$ **ii** $\frac{13}{25}$
4 a i $\frac{9}{50}$ **ii** $\frac{3}{25}$ **iii** $\frac{21}{50}$ **iv** $\frac{7}{25}$
b i $\frac{23}{50}$ **ii** $\frac{27}{50}$



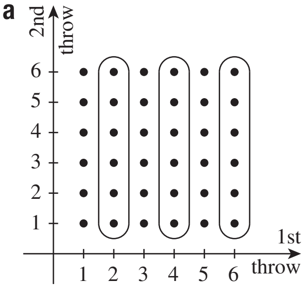
5 a 8% **b** 18% **c** 26% **d** 28%



6 a $\frac{9}{25}$ **b** $\frac{21}{25}$
7 4.96%
8 a 0.01 **b** 0.23
9 0.35
10 $\frac{4}{7}$
11 a $\frac{21}{3980}$ **b** $\frac{144}{995}$
12 a $\frac{3}{10}$ **b** $\frac{7}{24}$ **c** $\frac{21}{40}$
13 a $\frac{1}{11}$ **b** $\frac{14}{33}$ **c** $\frac{10}{33}$ **d** $\frac{19}{33}$
14 a $\frac{5}{6}$ **b** $\frac{5}{12}$ **c** $\frac{1}{6}$
15 a $\frac{4}{9}$ **b** $\frac{65}{81}$ **c** 4
16 The term 'large school' is code for saying that the probabilities do not change for the second choice because the sample space hardly changes.
a 0.28 **b** 0.50
17 a $\frac{1}{25}$ **b** $\frac{3}{5}$
18 a $\frac{1}{20}$ **b** $\frac{57}{8000}$
19 a 31.52% **b** 80.48%
20 a i $\frac{5}{33}$ **ii** $\frac{5}{22}$ **iii** $\frac{19}{33}$ **iv** $\frac{1}{4}$ **v** $\frac{19}{66}$ **vi** $\frac{47}{66}$
b $\frac{25}{144}, \frac{5}{24}, \frac{5}{9}, \frac{1}{4}, \frac{25}{72}, \frac{47}{72}$
21 a $\frac{1}{36}$ **b** $\frac{1}{46656}$ **c** $\frac{11}{36}$
22 a $\frac{1}{216}$ **b** $\frac{5}{72}$ **c** $\frac{5}{12}$ **d** $\frac{5}{9}$
23 $\frac{1}{3}$
24 a $\frac{1}{25}$ **b** $\frac{3}{25}$ **c** $\frac{6}{25}$ **d** $\frac{19}{25}$

Exercise 12G

1 a



b $\frac{1}{18}$ **c** $\frac{4}{9}$ **d** $\frac{1}{9}$
2 a $\frac{340}{1000} = \frac{17}{50} = 0.34$ **b** $\frac{190}{420} = \frac{19}{42} \doteqdot 0.45$
c $\frac{130}{340} = \frac{13}{34} \doteqdot 0.38$ **d** $\frac{20}{130} = \frac{2}{13} \doteqdot 0.15$
3 a Totals in last column: 56, 137, 193. Totals in last row: 124, 69, 193.
b i $\frac{42}{193} \doteqdot 0.22$ **ii** $\frac{29}{124} \doteqdot 0.23$
iii $\frac{29}{56} \doteqdot 0.52$ **iv** $\frac{95}{137} \doteqdot 0.69$
4 a $\frac{1}{16}$ **b** HH, HD, HC, HS; $\frac{1}{4}$
c HH, HD, HC, HS, DH, CH, SH; $\frac{1}{7}$
d HH, HD, HC, HS, DH, DD, DC, DS; $\frac{1}{8}$
5 a

	1	2	3	4	5	6
HH	3	4	5	6	7	8
HT	2	3	4	5	6	7
TH	2	3	4	5	6	7
TT	1	2	3	4	5	6

b $\frac{1}{24}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$
6 a $\frac{5}{7}$ **b** $\frac{3}{8}$ **c** $\frac{16}{19}$
7 a $P(A \cap B) = 0.24$ **b** $P(A \cap B) = 0.15$
c $P(A|B) = 0.4$ **d** $P(A|B) = 0.7$
8 a dependent **b** independent
c dependent **d** independent
e impossible—$P(A \cap B)$ cannot be bigger than $P(A)$ or $P(B)$.
f independent
9 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b The cases 1 + 4, 2 + 3, and 4 + 1 make up the reduced sample space.
i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** 1

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10 a i 0.1 **ii** $\frac{1}{3}$ **iii** $\frac{1}{4}$
b $\frac{3}{7}$ **c** $\frac{1}{2}$ **d** $\frac{5}{9}$

11 $\frac{4}{11}$

12 $\frac{5}{8}$ or 62.5%

13 a $\frac{1}{2}$ **b** $\frac{1}{3}$

14 a BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG
b $\frac{4}{7}$ **c** $\frac{2}{3}$

15 a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{11}{153}$

16 Draw up a 6×6 sample space and mark the points that are in A, in S and in M.
First, $P(A) = P(S) = \frac{1}{2}$ and $P(M) = \frac{1}{4}$.
a A and S are independent because $P(S|A) = \frac{1}{2} = P(S)$. A and M are not independent because $P(M|A) = \frac{1}{2}$, but $P(M) = \frac{1}{4}$. S and M are not independent because $P(M|S) = 0$, but $P(M) = \frac{1}{4}$.
b $P(A \cap S) = \frac{1}{4} = P(A) \times P(S)$, so A and S are independent. $P(A \cap M) = \frac{1}{4}$, but $P(A) \times P(M) = \frac{1}{8}$, so A and M are not independent. $P(S \cap M) = 0$, but $P(S) \times P(M) = \frac{1}{8}$, so A and M are not independent.

17 a $P(A \cup B) = 0.76$ **b** $P(A \cup B) = 0.72$

18 a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{1}{5}$

19 $\frac{7}{15}$

20 $\frac{9}{23}$

21 $\frac{3}{7}$

22 a 5.75% **b** 4.95% **c** 86% **d** 0.21%
e It is most important that the number of false negatives is low — that almost all patients with the disease are picked up. False positives are worrying for the patient, but further tests should determine that they do not have the disease.

23 a
$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{P(B \cap A)}{P(B)} \\ &= \frac{P(B|A)P(A)}{P(B)} \\ &= \frac{P(B|A)}{P(B)} \times P(A) \end{aligned}$$

24 If B is independent of A then,
$$\begin{aligned} P(A|B) &= \frac{P(B|A)}{P(B)} \times P(A) \\ &= \frac{P(B)}{P(B)} \times P(A) \\ &= P(A) \end{aligned}$$
which states that A is dependent of B.

25 Suppose first that the contestant changes her choice. If her original choice was correct, she loses, otherwise she wins, so her chance of winning is $\frac{2}{3}$. Suppose now that the contestant does not change her choice. If her original choice was correct, she wins, otherwise she loses, so her chance of winning is $\frac{1}{3}$. Thus the strategy of changing will double her chance of winning.

26 Let $G1$ be, 'A girl is born on a Sunday', let $B1$ be, 'A boy is born on a Sunday', let $G2$ be, 'A girl is born on a Monday', …, giving 14 equally likely events at the birth of every child.

In this particular family, there are two children, giving $14^2 = 196$ equally likely possible outcomes for the two successive births in this family.

Draw up the 2×2 sample space, showing at least all the entries in the row indexed by $G2$ and the column indexed by $G2$.

Let A be, 'At least one child is a girl born on a Monday.' There are 27 favourable outcomes for A.

Let B be, 'Both children are girls.' There are 13 favourable outcomes for the event $A \cap B$.
Hence $P(B|A) = |A \cap B|/|A| = \frac{13}{27}$.

Chapter 12 review exercise

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

2 a $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** 0 **e** 1 **f** $\frac{3}{10}$

3 a $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{13}$ **d** $\frac{1}{52}$ **e** $\frac{1}{2}$ **f** $\frac{12}{13}$

4 37%

5 a $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

6 a $\frac{1}{36}$ **b** $\frac{1}{9}$ **c** $\frac{1}{6}$ **d** $\frac{11}{36}$
e $\frac{4}{9}$ **f** $\frac{1}{9}$ **g** $\frac{1}{6}$ **h** $\frac{11}{36}$

7 a $\frac{17}{60}$ **b** $\frac{19}{60}$ **c** $\frac{1}{6}$

8 a w **b i** $\frac{1}{2}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{5}{6}$

9 a $\frac{1}{12}$ **b** $\frac{1}{5}$ **c** $\frac{3}{20}$ **d** $\frac{1}{20}$

10 a i $\frac{13}{204}$ **ii** $\frac{1}{17}$ **iii** $\frac{4}{663}$ **iv** $\frac{1}{2652}$
b i $\frac{1}{16}$ **ii** $\frac{1}{16}$ **iii** $\frac{1}{169}$ **iv** $\frac{1}{2704}$

11 a 14% **b** 24% **c** 38% **d** 6%

12 a $\frac{2}{21}$ **b** $\frac{11}{21}$ **c** $\frac{10}{21}$ **d** $\frac{2}{7}$

13 a $\frac{19}{12475}$ **b** $\frac{979}{12475}$

14 a Independent **b** Dependent
c Independent, with $P(A \cap B) = 0.18$

15 $\frac{3}{11}$

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16 a $\frac{1}{5}$

b $\frac{1}{5}$. The answer is independent of the day of the week.

c $\frac{1}{120}$

d 0. There cannot be only 1 day where the short and tie do not match.

Chapter 13

Exercise 13A

1 a Numeric, discrete **b** Numeric, continuous

c Categorical **d** Numeric, continuous

e Categorical **f** Categorical

g On a standard scale of shoes sizes, this is numeric and discrete. The length of a person's foot would be a numeric, continuous distribution.

h Numeric, discrete. Reported ATAR scores are between 30 and 99.95 in steps of 0.05. There are around 1400 different scores awarded.

2 a

Outcome	HH	HT	TH	TT
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Uniform distribution (and categorical).

b

Outcome	2 heads	1 head & 1 tail	2 tails
Probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

3 a

Outcome	red	green
Probability	$\frac{4}{7}$	$\frac{3}{7}$

b

Outcome	J	K	L	O
Probability	0.06	0.08	0.04	0.82

c

Outcome	P	A	R	M	T
Probability	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$

d

Outcome	1	2	3	4
Probability	$\frac{9}{1000}$	$\frac{90}{1000}$	$\frac{900}{1000}$	$\frac{1}{1000}$

e

Outcome	even	prime	neither
Probability	$\frac{5}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

4 a Let X be the number of letters in a randomly-chosen word.

Outcome x	3	4	6
Probability $P(X = x)$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{1}{8}$

b Let X be the number of heads recorded when 2 coins are thrown.

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

c Let X be the digits recorded from the first 12 digits of $\sqrt{2}$

x	1	2	3	4	5	6	7
$P(X = x)$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

d Let X be the number selected.

x	1	2	3	4	5
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(Note that the answer is the same if the sets are amalgamated. Why?)

5 a {T}, {F1}, {F2}, {T, F1}, {T, F2}, {F1, F2}, {T, F1, F2}

b

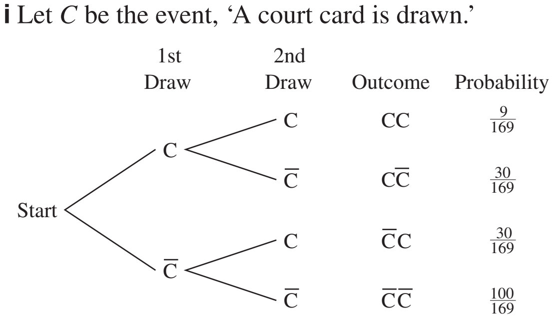
x	5	10	15	20
$P(X = x)$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$

6 a Yes **b** No **c** Yes **d** Yes **e** No **f** Yes

7 a 0.2 **b** 0.6 **c** 0.75 **d** 0 **e** 0.6

f 0.85 **g** 0.9 **h** 0.7 **i** 0.45

8 a i Let C be the event, 'A court card is drawn.'



ii

x	0	1	2
$P(X = x)$	$\frac{100}{169}$	$\frac{60}{169}$	$\frac{9}{169}$

b i The eight outcomes EEE, EEO, EOE, EOO, OEE, OEO, OOE, OOO each have probability $\frac{1}{8}$.

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ii

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

c GGG has probability $\frac{8}{125}$, GGB, GBG, BGG each have probability $\frac{12}{125}$, GBB, BGB, BBG each have probability $\frac{18}{125}$, BBB has probability $\frac{27}{125}$.

x	0	1	2	3
$P(X = x)$	$\frac{8}{125}$	$\frac{36}{125}$	$\frac{54}{125}$	$\frac{27}{125}$

d Let S be the event, 'A wallaby from Snake Ridge was selected'. SSS has probability 0.027, $\overline{S}SS$, $S\overline{S}S$, $SS\overline{S}$ each have probability 0.063, $\overline{S}\,\overline{S}S$, $\overline{S}S\overline{S}$, $S\overline{S}\,\overline{S}$ each have probability 0.147, $\overline{S}\,\overline{S}\,\overline{S}$ has probability 0.343.

x	0	1	2	3
$P(X = x)$	0.343	0.441	0.189	0.027

9 a $a = \frac{1}{25}$ **b** $a = \frac{1}{14}$ **c** $a = \frac{1}{27}$
d $a = \frac{1}{10}$ **e** $a = 1$

10 a i EE and OO each have probability $\frac{1}{5}$, EO and OE each have probability $\frac{3}{10}$.

ii

x	0	1	2
$P(X = x)$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$

b BB has probability $\frac{2}{5}$, BG and GB each have probability $\frac{4}{15}$, GG has probability $\frac{1}{15}$.

x	0	1	2
$P(X = x)$	$\frac{2}{5}$	$\frac{8}{15}$	$\frac{1}{15}$

c i EE has probability $\frac{3}{10}$, ER, RE, ET, TE each have probability $\frac{3}{20}$, RT and TR each have probability $\frac{1}{20}$.

ii

x	0	1	2
$P(X = x)$	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{10}$

11

x	22	44	55	24 or 42
$P(X = x)$	$\frac{1}{9}$	$\frac{1}{4}$	$\frac{1}{36}$	$\frac{1}{3}$
	25 or 52	45 or 54		
	$\frac{1}{9}$	$\frac{1}{6}$		

12 a

Outcome	RR	RG	GR	GG
Probability	$\frac{16}{49}$	$\frac{12}{49}$	$\frac{12}{49}$	$\frac{9}{49}$

b

Outcome	RR	RG	GR	GG
Probability	$\frac{12}{42}$	$\frac{12}{42}$	$\frac{12}{42}$	$\frac{6}{42}$

c

Outcome	HH	DD	SS	CC
Probability	$\frac{1}{17}$	$\frac{1}{17}$	$\frac{1}{17}$	$\frac{1}{17}$

HS or SH	HC or CH	HD or DH
$\frac{13}{102}$	$\frac{13}{102}$	$\frac{13}{102}$
SC or CS	SD or DS	CD or DC
$\frac{13}{102}$	$\frac{13}{102}$	$\frac{13}{102}$

13a There is no guarantee that their results will be identical, though you would expect more *trials* (repeats of the experiment) would bring your results closer to each other and to the theoretical probabilities.

b Theoretical results: $P(X = 0) = 0.3$, $P(X = 1) = 0.6$, $P(X = 2) = 0.1$

c It might be easier to perform the experiment with coloured balls or tokens. Running the experiment in pairs with a nominated recorder also helps. The paper pieces need to be indistinguishable and well mixed in the bag. You could increase the number of trials or combine the class results.

14 EEE and OOO each have probability $\frac{1}{20}$, the other six possible outcomes each have probability $\frac{3}{20}$,

x	0	1	2	3
$P(X = x)$	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$

15 a The condition that the sum of the probabilities is 1 gives $a = \frac{1}{4}$ or $a = 1$. But $a = 1$ gives probabilities outside the interval $0 \le p \le 1$, and the only valid answer is $a = \frac{1}{4}$.

b $a = 1$ or $\frac{7}{6}$ (both are valid)

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16 a Let X be the sum of the numbers on the three cards. This question is best done by asking what card is *discarded*.

x	20	21	22
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

b

x	20	21	22
$P(X = x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

Exercise 13B

1 a

x	0	1	2	3	Sum
$p(x)$	0.4	0.1	0.2	0.3	1
$x\,p(x)$	0	0.1	0.4	0.9	1.4

Hence $E(X) = 1.4$.

b

x	2	4	6	8	Sum
$p(x)$	0.1	0.4	0.4	0.1	1
$x\,p(x)$	0.2	1.6	2.4	0.8	5

Hence $E(X) = 5$.

c

x	-50	-20	0	30	100	Sum
$p(x)$	0.1	0.35	0.4	0.1	0.05	1
$xp(x)$	-5	-7	0	3	5	-4

Hence $E(X) = -4$.

2 a

x	-40	0	30	60	Sum
$p(x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1
$x\,p(x)$	-20	0	5	10	-5

b Expected value $= -5$

c The average cost to the player per game is 5 cents.

d $100 \times (-5) = -500$ cents. Thus the player expects to lose 500 cents and the casino expects to make 500 cents profit. This is an expected average value, not guaranteed.

4 a

x	2	4	6	8	10	Sum
p_i	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	1
$x_i p_i$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{8}{5}$	$\frac{10}{5}$	6

So $E(X) = 6$.

b

x	-3	1	2	5	6	Sum
p_i	0.1	0.3	0.2	0.3	0.1	1
$x_i p_i$	-0.3	0.3	0.4	1.5	0.6	2.5

So $E(X) = 2.5$.

5 a

x	1.50	2.10	2.40	Sum
$p(x)$	$\frac{5}{12}$	$\frac{4}{12}$	$\frac{3}{12}$	1
$x\,p(x)$	0.625	0.7	0.60	1.925

The expected value is \$1.925.

b If 100 purchases are made at random the expected cost is \$192.50.

6 a $E(X) = 3$

b i $E(Y) = 6$ **ii** Yes

c i $E(Z) = 4$ **ii** Yes

7 a 15 **b** 10 **c** $\frac{5}{2}$ **d** 3 **e** 0 **f** 18

8

x	0	1	2	3	Sum
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1
$x\,p(x)$	0	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{3}{8}$	$\frac{12}{8}$

The expected value is $1\frac{1}{2}$, as might be expected from the symmetry of the table of probabilities.

9

x	0	1	2	Sum
$p(x)$	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$	1
$x\,p(x)$	0	$\frac{13}{34}$	$\frac{2}{17}$	$\frac{17}{34}$

The expected value is $\frac{1}{2}$.

10 d

x	0	1	2	3	4	5	Sum
$p(x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	1
$x\,p(x)$	0	$\frac{10}{36}$	$\frac{16}{36}$	$\frac{18}{36}$	$\frac{16}{36}$	$\frac{10}{36}$	$\frac{70}{36}$

Hence $E(X) = \frac{35}{18}$.

f In any dice experiment, it is important to check the randomness of your dice rolls. This can depend on your rolling technique. Try throwing a die 12 times and see if every outcome is equally likely. Does each outcome seem independent of the last?

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11 a $\frac{3}{15}, \frac{3}{15}, \frac{3}{15}, \frac{2}{15}, \frac{2}{15}, \frac{2}{15}$

b -12, so the bank expects to make 12 cents each game, on average.

12 a $P(\text{Orange}) = \frac{1}{6}, P(\text{Strawberry}) = \frac{2}{6}, P(\text{Apple}) = \frac{3}{6}$

b

outcome	OOO	SSS	AAA	Other	Sum
x	$11k$	$2k$	k	0	—
$p(x)$	$\frac{1}{216}$	$\frac{8}{216}$	$\frac{27}{216}$	$\frac{180}{216}$	1

c The payout will be \$44 and their profit would be \$43, accounting for the \$1 entry fee.

13 b $\mu = 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} + 4 \times \frac{1}{16} + 5 \times \frac{1}{32} + 6 \times \frac{1}{64} + \cdots \quad (1)$

Doubling,

$2\mu = 1 \times 1 + 2 \times \frac{1}{2} + 3 \times \frac{1}{4} + 4 \times \frac{1}{8} + 5 \times \frac{1}{16} + 6 \times \frac{1}{32} + \cdots \quad (2)$

Subtracting (1) from (2),

$\mu = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \quad (3)$

Doubling,

$2\mu = 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \quad (4)$

Subtracting (3) from (4), $\mu = 2$.

c On average, we would expect to get a head on the second throw. You could test this by recording how many throws it takes over say 50 trials and averaging the results.

14 $E(X) = 2 \times \frac{1}{2} + 4 \times \frac{1}{4} + 8 \times \frac{1}{8} + 16 \times \frac{1}{16} + \cdots$
$= 1 + 1 + 1 + 1 + \cdots$

The expected value 'increases without bound', that is, $E(X) \to \infty$ as the game continues. This suggests that there is no reasonable price the casino could put on this game and expect to break even. There are various issues with this scenario in real life. Casinos would not provide a game which had no upper limit to the payout. Patrons would also be unwilling to pay a large price for a game with such low apparent probabilities for the later stages of the game. The calculation of a simple expected value may not be the best way to analyse this game.

Exercise 13C

1 a

x	1	2	3	4	Sum
$p(x)$	0.3	0.5	0.1	0.1	1
$xp(x)$	0.3	1	0.3	0.4	2
$(x-\mu)^2$	1	0	1	4	—
$(x-\mu)^2 p(x)$	0.3	0	0.1	0.4	0.8

$\mu = 2$, $\text{Var}(X) = 0.8$, $\sigma = \sqrt{0.8} \doteqdot 0.89$

2 a

x	1	2	3	4	Sum
$p(x)$	0.3	0.5	0.1	0.1	1
$xp(x)$	0.3	0.1	0.3	0.4	2
x^2	1	4	9	16	—
$x^2p(x)$	0.3	2	0.9	1.6	4.8

b $\text{Var}(X) = 4.8 - 2^2 = 0.8$.

3 a $E(X) = 2$, $\text{Var}(X) = 2$

b $E(X) = 3$, $\text{Var}(X) = 1$

c $E(X) = 0$, $\text{Var}(X) = 2.6$

d $E(X) = 2.8$, $\text{Var}(X) = 1.36$

4 a i $E(Y) = 2$, $\text{Var}(Y) = 1$, $\sigma = 1$

ii $E(Z) = 2$, $\text{Var}(Z) = 4$, $\sigma = 2$

iii $E(V) = 1$, $\text{Var}(V) = 0.8$, $\sigma \doteqdot 0.89$

iv $E(W) = 3$, $\text{Var}(W) = 0.8$, $\sigma \doteqdot 0.89$

b i Both sets of data are centred around 2 and the expected value of each is unsurprisingly 2. The second data set is more spread out — in fact in moving from Y to Z the distances from the mean to each data point have been doubled and the standard deviation is doubled.

ii The data has been 'flipped over', but is no more spread out than before — the variance is unchanged. You may notice that $W = 4 - V$.

5 $E(X) = 2$, $\text{Var}(X) = 0$.

6 a $E(J) = 1.55$, $\text{Var}(J) = 2.05$, $E(L) = 1.4$, $\text{Var}(L) = 0.84$.

b Over the season John might be expected to score more baskets, because his expected value is higher.

c Liam is the more consistent player, with the lower variance. Coaches may prefer a more consistent player, particularly if it is more important to score *some* goals, rather than the maximum number. This may also be a sign that John needs to work on the consistency of his game.

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7 a Each outcome has probability $\frac{1}{3}$. This is a uniform distribution.

b $\mathrm{E}(X) = 2$ **c** $\mathrm{Var}(X) = \frac{2}{3}$

8 a Two standard deviations

b One and a half standard deviations below the mean.

c The English score was more standard deviations below the mean than the Mathematics result, so it may be considered less impressive.

9 a Visual Arts is 1 standard deviation below the mean, Music is 1.75 standard deviations below the mean, hence the Visual Arts score is better.

b Earth Science is 2 standard deviations above the mean, Biology is 1.5 standard deviations above the mean, hence the Earth Science score is more impressive.

c Chinese is 2 standard deviations above the mean, Sanskrit is also 2 standard deviations above the mean, hence the scores are equally impressive.

10 a $\mathrm{E}(X) = 3.3, \sigma = 1.45$

b 8 appears to be a long way from 3.3 and well removed from the rest of the data.

c 8 is 3.2 standard deviations above the mean and thus would be an outlier by this definition.

d $\mathrm{E}(X) = 3.15, \sigma = 1.06$.

e The mean and standard deviation have changed significantly, especially the standard deviation.

f Outliers are interesting values in any distribution and should be a flag to investigate more closely. Were results recorded correctly? Was there an error in the experiment, for example, did Jasmine use a more powerful bow with greater range, or maybe she used a new set of arrows with better fletching? It may, however, be that Jasmine is inconsistent, occasionally getting much better results, but often getting fairly poor results — in this case the large standard deviation is warranted as a measure of this distribution. Over 20 trials, a probability of 0.05 only represents one set of 10 shots, so a larger set of results may give a better picture of her long-term accuracy and reduce the impact of one strong result amongst many other weaker scores.

11 $k = \frac{1}{10}, \mathrm{E}(X) = 3, \sigma = 1$

12 a $\dfrac{1}{n}$ **b** $\dfrac{n+1}{2}$ **c** $\frac{1}{12}(n^2 - 1)$

13 a Because $Z = X + a$,

$$\begin{aligned}\mathrm{E}(Z) &= \sum zP(Z = z)\\ &= \sum (x + a)\, P(X + a = x + a)\\ &= \sum (x + a)\, P(X = x)\\ &= \sum xP(X = x) + \sum aP(X = x)\\ &= \sum xP(X = x) + a\sum P(X = x)\\ &= \mu + a,\end{aligned}$$

because $\sum P(X = x) = 1$.

b Because $Z = kX$,

$$\begin{aligned}\mathrm{E}(Z) &= \sum zP(Z = z)\\ &= \sum (kx)\, P(kX = kx)\\ &= \sum (kx)\, P(X = x),\\ &= k \times \sum xP(X = x)\\ &= k\mu.\end{aligned}$$

14 a The mean of Z is $\mu + a$, by the previous question. Hence

$$\begin{aligned}\mathrm{Var}(Z) &= \mathrm{E}\big((Z - (\mu + a))^2\big)\\ &= \mathrm{E}\big((Z - a - \mu)^2\big)\\ &= \mathrm{E}\big((X - \mu)^2\big)\\ &= \mathrm{Var}(X)\end{aligned}$$

Hence the standard deviation of the new distribution remains σ. This is to be expected, because the distribution is no more spread out than previously.

b The mean of Z is $k\mu$, by the previous question. Hence

$$\begin{aligned}\mathrm{Var}(Z) &= \mathrm{E}\big((Z - k\mu)^2\big)\\ &= \mathrm{E}\big((kX - k\mu)^2\big)\\ &= k^2 \times \mathrm{E}\big((X - \mu)^2\big)\\ &= k^2\mathrm{Var}(X)\end{aligned}$$

Hence the standard deviation of the new distribution is $\sqrt{k^2\sigma^2} = k\sigma$.

Exercise 13D

1 a

x	0	1	2	3	Sum
$p(x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{8}{27}$	$\frac{1}{27}$	
$xp(x)$	0	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$	1
$x^2p(x)$	0	$\frac{12}{27}$	$\frac{24}{27}$	$\frac{9}{27}$	$1\frac{2}{3}$

$\mu = 1, \sigma^2 = 1\frac{2}{3} - 1^2 = \frac{2}{3}, \sigma \doteqdot 0.82$

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b

x	0	1	2	3	Sum
f	33	47	16	4	100
f_r	0.33	0.47	0.16	0.04	1
xf_r	0	0.47	0.32	0.12	0.91
x^2f_r	0	0.47	0.64	0.36	1.47

$\bar{x} = 0.91, s^2 = 1.47 - (0.91)^2 = 0.6419,$
$s \doteqdot 0.80$

c The sample results are a little below what is predicted by the theoretical probabilities.

2 a $\mu = 7, \sigma^2 = \frac{35}{6}, \sigma \doteqdot 2.42$

4 c

x	0	1	2	3	4	5	Sum
$p(x)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	1
$xp(x)$	0	$\frac{10}{36}$	$\frac{16}{36}$	$\frac{18}{36}$	$\frac{16}{36}$	$\frac{10}{36}$	$\frac{70}{36}$
$x^2p(x)$	0	$\frac{10}{36}$	$\frac{32}{36}$	$\frac{54}{36}$	$\frac{64}{36}$	$\frac{50}{36}$	$\frac{210}{36}$

$\mu \doteqdot 1.94, \sigma^2 = \frac{210}{36} - \left(\frac{70}{36}\right)^2, \sigma \doteqdot 1.43$

7 a $\mu = 3.441, \sigma \doteqdot 2.46$

12 a Later people taking part in the experiment will be influenced by earlier guesses, particularly if the previous guesses have been measured for accuracy. Perhaps students could record their estimate, or draw their estimated shape, at the same time and before any measuring occurs. Perhaps students go into a separate room for the experiment.

14 a $m - k$ is the number of serial numbers not yet discovered in the range from 1 to m. If these serial numbers are spread between the k gaps, the average size of the gap (number of undiscovered serials) is $\frac{m-k}{k}$.

b The gap of $\frac{m-k}{k}$ integers should extend past m to $m + \frac{m-k}{k}$. Using this estimate the last serial will be:

$$\begin{aligned} N &= m + \frac{m-k}{k} \\ &= m + \frac{m}{k} - 1 \end{aligned}$$

Chapter 13 review exercise

1a Numeric, continuous
b Numeric, discrete
c Numeric, discrete (and infinite)
d Categorical

2 a Yes **a** No **b** No

3 The probabilities are not all positive, do not sum to 1, and are not all less than 1.

4 a $E(X) = 1.4$ **b** $E(X) = -0.8$

5 a $E(X) = 27.22$
b His expected cost is $\$27.22 \times 52 = \$1415.56$.

6 a $E(X) = 2$, $Var(X) = 1$, $\sigma = 1$.
b $E(X) = 5.1$, $Var(X) = 0.69$, $\sigma \doteqdot 0.83$.

7 a $E(X) = 2$, $E(X^2) = 5$, $Var(X) = 1$
b $E(X) = 5.1$, $E(X^2) = 26.70$, $Var(X) = 0.69$

8 a $E(X) = 1.9$, $Var(X) = 0.49$, $\sigma = 0.7$
b $E(X) = 2$, $Var(X) = 2.6$, $\sigma \doteqdot 1.61$

9 Expected value is a measure of central tendency — it measures the centre of the data set. It may also be thought of as a weighted mean (weighted by the probabilities of the distribution). If the experiment is carried out experimentally a large number of times we would expect that the average of the outcomes would approach the expected value.

10 The standard deviation is the square root of the variance. Both measure the spread of the data, so that a distribution with a larger standard deviation is more spread out than a distribution with a smaller standard deviation. Both are zero if the distribution only takes one value — that is, if it is not spread out at all. If the distribution is stretched (multiplied) by a constant k the standard deviation also increases by a factor k.

11 a $12, 8, 2\sqrt{2}$ **b** $11, 2, \sqrt{2}$ **c** $17, 18, 3\sqrt{2}$

12 a

x	5	6	7	8	9
$p(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$E(X) = 7$, $Var(X) = 2$, $\sigma = \sqrt{2}$.

Chapter 14

Exercise 14A

1 a 8! **b** 4! **c** 3! **d** 101! **e** 20!
2 a 6 **b** 120 **c** 1 **d** 15
e 45 **f** 35 **g** 220 **h** 70
3 a 5040 **b** 3 628 800 **c** 1 **d** 15 120
e 6720 **f** 2520 **g** 5005 **h** 13 860
4 a $6x^5$ **b** $30x^4$ **c** $120x^3$ **d** $360x^2$
e $720x$ **f** 720 **g** 0

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5 a n **b** $n!$
c 1 **d** $n(n+1)$
e $(n+1)(n+2)$ **f** $\dfrac{1}{n(n-1)}$
g $\dfrac{n-2}{n}$ **h** $\dfrac{(n-1)!}{n+1}$

6 a $7 \times 7!$
b $n \times n!$
c $57 \times 6!$
d $(n^2+n+1) \times (n-1)!$
e $9^2 \times 7!$
f $(n+1)^2 \times (n-1)!$

7 a $\dfrac{1+n}{n!}$ **b** $\dfrac{n}{(n+1)!}$ **c** $\dfrac{1-n-n^2}{(n+1)!}$

8 a i nx^{n-1} **ii** $n(n-1)x^{n-2}$ **iii** $n!$
iv $n(n-1)(n-2)\cdots(n-k+1)x^{n-k}$
$= \dfrac{n!}{(n-k)!}x^{n-k}$
b i $-1! \times x^{-2}$
ii $2! \times x^{-3}$
iii $-5! \times x^{-6}$
iv $(-1)^n \times n! \times x^{-(n+1)}$

9 a $(n+1)! - 1$

10 a i 2^8 **ii** 10^2
b i 2^{97} **ii** 5^{24} **iii** 7^{16} **iv** 13^7

12 a $\frac{1}{2}, \frac{1}{3}, \frac{1}{8}, \frac{1}{30}, \frac{1}{144}$ **b** $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}, \frac{719}{720}$
c $S_n = 1 - \dfrac{1}{(n+1)!}$. The limit is 1.
d The sum can be written as
$$\left(\frac{1}{1!} - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \left(\frac{1}{3!} - \frac{1}{4!}\right) + \cdots + \left(\frac{1}{n!} - \frac{1}{(n+1)!}\right).$$

13 a $2^{15} \times 15!$ **b** $\dfrac{30!}{2^{15} \times 15!}$ or $\dfrac{29!}{2^{14} \times 14!}$
c $\dfrac{2^{30} \times (15!)^2}{30!}$

14 0.14%

Exercise 14B

1 There are 6: DOG, DGO, ODG, OGD, GOD, GDO
2 FEG, FGE, FEH, FHE, FEI, FIE, FGH, FHG, FGI, FIG, FHI, FIH
3 a 360 **b** 720
4 a 120 **b** 625
5 60, 36
6 5040
7 a ${}^{10}P_3 = 720$ **b** ${}^{5}P_3 = 60$ **c** ${}^{100}P_3 = 970\,200$
8 a $9^3 = 729$ **b** $100^3 = 1\,000\,000$ **c** $2^{10} = 1024$
9 a 40 320 **b** 336
10 a 12 **b** 864
11 720 **12** 48
13 a 10^7 **b** 5×10^6 **c** 5^7 **d** 32 000
14 a 10 000 **b** 5040 **c** 625 **d** 1000
15 a 3024 **b** 336 **c** 1344 **d** 336 **e** 1008
16 a 6561 **b** 729 **c** 2916 **d** 729 **e** 2187
17 a 6 760 000 **b** 3 276 000 **c** 26 000 **d** 48
18 a 720 **b** 120
c 24 **d** 360 (half of them)
19 1728 **20** 24
21 a 120 **b** 24 **c** 95
22 a i 64 **ii** 32
b i 340 **ii** 170
23 a 96 **b** 36 **c** 24
24 a 3 **b** 3
25 a 30 000
b $9 \times 9 \times 9 \times 9 \times 3 = 19\,683$ (Choose the last digit so that the sum is a multiple of 3.)
c $9 \times 9 \times 9 \times 9 \times 5 = 26\,244$
d $4 \times 9 \times 9 \times 9 \times 3 = 8748$ (Choose the last digit first and the first digit last.)

Exercise 14C

1 a $5! \times 2! = 240$ **b** $2! \times 2! = 4$
c $3! \times 2! = 12$ **d** $5! \times 3! = 720$
2 a $4! \times 2! = 48$ **b** $4! = 24$
3 $3! \times 2! \times 2! = 24$
4 622 080
5 a $2 \times 3 \times 3 \times 2 \times 1 + 3 \times 4! = 36 + 72 = 108$
b $5! - 2 \times 3! = 120 - 12 = 108$
6 If the father drives, there are $2 \times 2 \times 1$ ways to arrange the seating. If the mother drives, there are $1 \times 2 \times 1$ ways to arrange the seating. Thus there are 6 ways in total.
7 Number of three-digit numbers $= 3 \times 2 \times 1 = 6$.
Number of two-digit numbers $= 2 \times 2 = 4$.
The total number of numbers is 10.
8 a 144 **b** 120
c 144 **d** 2520 (half of the total)
9 a 720 **b** 720 **c** 4320
10 a 24 **b** 240
11 2046
12 a 1152 **b** 1152
13 a 720 **b** 120 **c** 1680
d 4200 **e** 960 **f** 480

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14 a 5040 **b** 4320 **c** 720 **d** 144
e 720 **f** 960 **g** 1440
15 a 7^7 **b** 6×7^6 **c** 7^6
d $3^4 \times 4^3 + 4^4 \times 3^3 = 7 \times 12^3$
16 a i 3 628 800 **ii** 725 760
iii 725 760 **iv** 2 257 920
b i $2(n-1)!$ **ii** $2(n-1)!$
iii $(n-2)(n-3)(n-2)!$
17 8640
18 a 40 320 **b** 20 160 **c** 17 280
19 a 5040 **b** 20 160
20 a 5^5 ways **b** $5! = 120$ ways
c $5 \times 4^3 = 320$ ways
21 a 133 **b** 104 **c** 29 **d** 56
22 a $D(1) = 0, D(2) = 1, D(3) = 2, D(4) = 9$
c $D(5) = 44, D(6) = 265, D(7) = 1854,$
$D(8) = 14\,833$

Exercise 14D

1 a 3 **b** 12 **c** 120
d 6720 **e** 10 080 **f** 90 720
g 4 989 600 **h** 45 360 **i** 25 740
2 60
3 a 6 **b** 15 **c** 20
4 a 40 320 **b** 8 **c** 56 **d** 560
5 a 56 **b** 20
6 a 56 **b** 5
7 a 60 **b** 24
c 36 **d** 30 (half of them)
8 a i 180 **ii** 60 **iii** 120 **iv** 24
b 40
9 a 90 720 **b** 720
c 720 **d** 45 360 (half of them)
10 2 721 600
11 a 1024 **b** 256 **c** 45 **d** 252
e 56 **f** 512 **g** 8 **h** 70
12 a 60 **b** 60
13 a 120 **b** 60
14 a 453 600 **b** 90 720 **c** 5040 **d** 10 080
e 80 640 **f** 282 240 **g** 15 120
15 a 3 628 800 **b** 4
16 a 2520 **b** 720
c i 600 **ii** 480 **iii** 360 **iv** 240 **v** 120
d 840. Insert the letters U, M, T and R successively into the word EGE. Alternatively, the answer is one third of all arrangements.
e 210 **f** 420
17 1 995 840
18 864. The problem can be done by applying the inclusion–exclusion principle from the Extension section of Exercise 12C, or by considering separately the various different patterns.
19 a 4!
b Each is $3! = 6$, so subtract $4 \times 3! = 24$.
c 2! permutations leave A and B unmoved, and there are ${}^4C_2$ pairs of letters, so add $6 \times 2! = 12$.
d $D(4) = 1 \times 4! - 4 \times 3! + 6 \times 2!$
$- 4 \times 1! + 1 \times 0! = 9$
g The ratio of the number of permutations of n distinct letters to the number of derangements of them converges to e as $n \to \infty$. Thus, for example, if a long queue is formed at random and then rearranged into alphabetical order, the probability that no-one remains in his or her original position is $\frac{1}{e}$.

Exercise 14E

1 There are ${}^5C_2 = 10$ possible combinations:
PQ, PR, PS, PT, QR, QS, QT, RS, RT and ST.
2 a 21 **b** 35 **c** 15 **d** 126
3 a i 45 **ii** 45
b ${}^{10}C_2 = {}^{10}C_8$, and in general ${}^nC_r = {}^nC_{n-r}$.
4 a 44 352 **b** 34 650
5 a 70 **b** 36 **c** 16 **d** 1 **e** 69
6 a 126 **b** 45 **c** 51 **d** 75
7 a 2002 **b** 56 **c** 6 **d** 840
e 420 **f** 1316 **g** 715 **h** 1287
8 a 70 **b** 5 **c** 35
9 a 792 **b** 462 **c** 120 **d** 210 **e** 420
10 a i 252
ii 126. The number cannot begin with a zero.
b In each part, once the five numbers have been selected, they can only be arranged in one way.
11 13 860
12 a 1 745 944 200 **b** 413 513 100
13 a 45 **b** 120 **c** 36 **d** 8
14 a 10 **b** 110
15 a 65 780 **b** 1287 **c** 48
d 22 308 **e** 288 **f** 3744
16 a i ${}^6C_1 + {}^6C_2 = 21$
ii ${}^5C_2 = 10$ (choose the two people to go in the same group as Laura)
b i 4 **ii** 3
c i 92 **ii** 35
17 a 2 **b** 5 **c** 35 **d** ${}^nC_2 - n$
18 a 220 **b** 9240
c i 2772 **ii** 6468

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19 **a** 1024 **b** 968 **c** 466 **d** 247
20 **a** 16 **b** 20 **c** 12 **d** 8 **e** 5
21 **a** 252 **b** 126
22 **a** 315 **b** 210
23 **a** 12 **b** 49 **c** 120
d $(a + 1)(b + 1)(c + 1)$
24 **a** 30 **b** 24
25 **a i** 210 **ii** 90 **iii** 126 **iv** 126
26 5151
27 1360
28 **a** 3 **b** 315
c i 155 925 **ii** 10 800
29 **b** 0 (undefined), 2 (3), 30 (3), 864 (2.917), 39480 (2.872), 2631600 (2.844)
30 **a i** ${}^{a}C_0$ **ii** ${}^{a+1}C_1$ **iii** ${}^{a+2}C_2$ **iv** ${}^{a+3}C_3$ **v** ${}^{a+r}C_r$
b Add them up.
c $a + 1$ 0s and b 1s, total length $a + b + 1$.
d Go to the last 0 in the string, and remove it and any 1s that follow it. What remains is a string with a 0s and at most b 1s. When the process in part **c** is applied to the truncated string, the original string returns.
e The one-to-one correspondence in parts **c** and **d** show this.
f This follows from parts **b** and **e**.

Exercise 14F

1 **a** 84 **b** $\frac{5}{42}$
2 **a** $\frac{1}{210}$ **b** $\frac{2}{5}$ **c** $\frac{3}{5}$ **d** $\frac{4}{15}$
3 **a** $\frac{1}{13}$ **b** $\frac{46}{455}$ **c** $\frac{3}{91}$ **d** $\frac{3}{13}$
4 **a** $\frac{8}{429}$ **b** $\frac{1}{143}$ **c** $\frac{140}{429}$ **d** $\frac{421}{429}$ **e** $\frac{2}{11}$ **f** $\frac{1}{3}$
5 **a** $\frac{1}{22\,100}$ **b** $\frac{1}{5525}$
c $\frac{11}{850}$ **d** $\frac{22}{425}$
e $\frac{11}{1105}$ **f** $\frac{13}{34}$
g $\frac{16}{5525}$ **h** $\frac{6}{5525}$
i $\frac{741}{1700}$ **j** $\frac{64}{425}$
6 **a** $\frac{3}{70\,304}$ **b** $\frac{1}{2197}$
c $\frac{1}{64}$ **d** $\frac{1}{16}$
e $\frac{27}{2197}$ **f** $\frac{3}{8}$
g $\frac{6}{2197}$ **h** $\frac{3}{2197}$
i $\frac{27}{64}$ **j** $\frac{5}{32}$
7 **a** $\frac{1}{10}$ **b** $\frac{1}{10}$ **c** $\frac{1}{3}$
8 **a** $\frac{1}{10}$ **b** $\frac{2}{5}$
9 **a** $\frac{1}{15}$ **b** $\frac{2}{3}$
10 **a** $\frac{1}{42}$ **b** $\frac{2}{7}$ **c** $\frac{2}{7}$ **d** $\frac{1}{35}$ **e** $\frac{1}{7}$
11 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{5}$ **d** $\frac{1}{60}$ **e** $\frac{2}{3}$
12 **a** $\frac{1}{7}$ **b** $\frac{2}{7}$ **c** $\frac{1}{7}$ **d** $\frac{2}{7}$
13 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{2}{15}$ **d** $\frac{1}{5}$
14 **a** $\frac{1}{26}$ **b** $\frac{5}{13}$ **c** $\frac{15}{26}$ **d** $\frac{1}{26}$
15 **a** 0.403 **b** 0.597 **c** 0.001 74 **d** 0.291
16 **a** $\frac{1}{22}$ **b** $\frac{125}{1728}$ **c** $\frac{5}{144}$
17 $\frac{10}{31}$ **b** $\frac{15}{31}$ **c** $\frac{6}{31}$
18 **a** $\frac{1}{60}$ **b** $\frac{3}{5}$ **c** $\frac{1}{5}$ **d** $\frac{2}{5}$ **e** $\frac{1}{20}$
f $\frac{3}{5}$ **g** $\frac{3}{10}$ **h** $\frac{9}{10}$ **i** $\frac{1}{10}$ **j** $\frac{3}{5}$
19 **a** $\frac{3}{8}$ **b** $\frac{1}{2}$ **c** $\frac{21}{32}$ **d** $\frac{3}{32}$ **e** $\frac{17}{64}$
20 **a** $\frac{281}{462}$ **b** 8
21 **a** $\frac{1}{27\,417}$ **b** $\frac{28}{703}$
22 In each part, the sample space has ${}^{52}C_5$ members.
a $\frac{352}{833}$. Choose the value of the pair in 13 ways, then choose the cards in the pair in ${}^{4}C_2 = 6$ ways, then choose the three values of the three remaining cards in ${}^{12}C_3$ ways, then choose the suits of those three cards in 4^3 ways.
b $\frac{198}{4165}$. Choose the values of the two pairs in ${}^{13}C_2$ ways, then choose the suits of the cards in the two pairs in ${}^{4}C_2 \times {}^{4}C_2$ ways, then choose the remaining card in 44 ways.
c $\frac{88}{4165}$ **d** $\frac{1}{4165}$ **e** $\frac{6}{4165}$
f $\frac{128}{32\,487}$. Choose the lowest card in 10 ways, then choose the suits of the five cards in 4^5 ways.
g $\frac{33}{16\,660}$ **h** $\frac{1}{649\,740}$
23 **a** $\frac{1}{125}$ **b** $\frac{4}{125}$ **c** $\frac{16}{125}$ **d** $\frac{108}{125}$
24 **a** $\frac{48}{125}$ **b** $\dfrac{n^2(n - 1)^2(n - 2)!}{2n^n}$
25 **a** 0.0082 **b** $1 - \dfrac{{}^{365}P_n}{365^n}$
d 23 **e** 41
26 **a** $\frac{1}{25}$ **b** $\frac{3}{25}$ **c** $\frac{19}{25}$
27 **b** $\frac{1}{8}$ **c** 2^{1-n}

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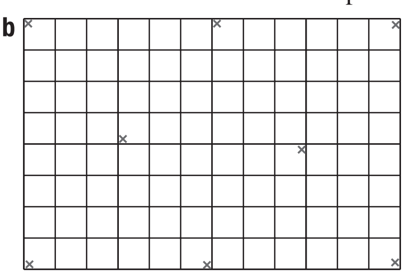
Exercise 14G

1 a i 120 **ii** 24
b i 3 628 800 **ii** 362 880
2 a 10 080 **b** 1440
3 a 24 **b** 6 **c** 4 **d** 12 **e** 4
4 a 5040 **b** 144 **c** 576
d 1440 **e** 3600 **f** 240
5 a $\frac{3}{10}$ **b** $\frac{1}{5}$ **c** $\frac{1}{10}$ **d** $\frac{9}{10}$
6 a 5040 **b** 576 **c** 144
d 2304 **e** 1440 **f** 3600
7 a $\frac{1}{12}$ **b** $\frac{1}{9}$
8 a $(n-1)!$ **b** $2 \times (n-2)!$
c $(n-3) \times (n-2)!$ **d** $6 \times (n-3)!$
9 a 39 916 800 **b** 165
10 145 152
11 a 288 **b** $\frac{1}{4}$
12 $\dfrac{n!\,(n+1)!}{(2n)!}$
13 a 60 **b** 181 440 **c** 9

Exercise 14H

1 a 7
b There is no guarantee — it is 'possible' (but unlikely) that 6 never turns up, even in 1 000 000 throws, or in any number of throws.
2 Pigeonhole the numbers as either odd or even. Because there are three numbers, at least one pigeonhole must have 2 numbers in it.
3 $16 \div 5 = 3$ remainder 1, so at least one location must have 4 eggs (maybe more).
4 a 4 **b** 7
c $3(n-1)+1 = 3n-2$
5 Imagine that the pigeonholes are labelled with the numbers 1 to 6, and on each throw a token (pigeon) is put in the corresponding pigeonhole. Because $13 \div 6 = 2$ remainder 1, then when filling the 6 pigeonholes following each draw, at least one pigeonhole has three tokens.
6 Less than ten.
7 a 10
b Four — all might use the same item.
8 Divide the grid into 49 one-metre squares. Because $100 \div 49 = 2$ remainder 2, there must be a one-metre square covering at least three points.
9 The possible totals are 2, 3, 4, 5 and 6. After 6 throws one of these sums must have occured at least twice.
10 $100 \div 12 = 8$ remainder 4, so there must be at least one birth month shared by at least nine people in the group.
11 The pigeonholes correspond to the four suits. As $10 \div 4 = 2$ remainder 2, so one suit must occur three times.
12 Yes, because $567 \div 23 = 24$ remainder 15.
13 a Zero — they might all be in another group.
b $19 \div 3 = 6$ remainder 1.
c All 19.
14 a Divide the board into sixteen 2×2 squares, and place a king in each square. If each king is in a corresponding place, the arrangement is permissible, so 16 kings can be arranged as required. If, however, a second king is placed into any square, the two kings in that square will be adjacent, so 16 is the maximum.
b 8 — lay them along the main diagonal.
15 a Divide the field into 4×3 rectangles — two rows and four columns of them. There must be a rectangle containing two cows which, by Pythagoras' theorem, must be closer than 5 metres apart.
b

16 The remainder on division by 3 is either 0, 1 or 2. If 100 numbers are placed in these three pigeonholes, at least one pigeonhole must contain 34 entries, because $100 \div 3 = 33$ remainder 1.
17 Consider the powers of 2 from 2^1 up to 2^{2020} and pigeonhole them by their remainders on division by 2019. The remainder must be a number from 0 to 2018. By the pigeonhole principle, there must be two powers that leave the same remainder, thus their difference is a multiple of 2019.
18 Consider the 13 pigeonholes $\{1, 51\}, \{3, 49\}, \ldots, \{25, 27\}$. When the 14 odd numbers are distributed amongst these pigeonholes at least one must have two members; the two members of this pigeonhole add to 52.
19 Pair the numbers to form 50 pigeonholes labelled $\{1, 199\}, \{3, 197\}, \ldots, \{99, 101\}$. Given 51 odd numbers less than 200, two must fall in the same pigeonhole and add to 200.

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20 Draw an equilateral triangle in the plane with side length 1 unit. Then two of the three vertices must be the same colour (three pigeons (vertices) must lie in the same pigeonhole (colour)). This theorem may be generalised to three colours but it is an open problem for the case of four, five and six colours. The result is false for seven colours.

21 a $41 \div 10 = 4$ remainder 1

b Arrange the 10 pigeonholes as $A, B, C \ldots, J$ in descending order $|A| \geq |B| \geq |C| \geq \cdots \geq |J|$, where $|X|$ is the number of pigeons in pigeonhole X. Then $|A| \geq 5$ by part **a**.
If $|B| \geq 4$, then $|A| + |B| \geq 9$, as required, so suppose that $|B| \leq 3$. Then also $|C| \leq 3, |D| \leq 3, \ldots, |J| \leq 3$, so $|B| + |C| + \cdots + |J| \leq 3 \times 9 = 27$, in which case $|A| \geq 14$, so $|A| + |B| \geq 14$.

22 a There must be more than $26 \times 26 = 676$ students.

b Half the school share at most $8 \times 26 = 208$ addresses, thus it is known that the school has at least 417 students

c We need to know when one of the 26×26 pigeonholes has more than 11 members. This must happen when 7437 addresses are assigned (but may happen sooner), thus in the thirty-third year of this scheme's operation, because $1200 + 32 \times 200 > 7437$.

23 The pigeonholes are the number of friends, 0–5. We need to reduce the number of categories if we are going to apply the pigeonhole principle usefully. If there is someone with no friends at the table, then there is no one with five friends. In this case there are five pigeonholes to place the six people and at least one pigeonhole has two of the people, that is, they have the same number of friends.
If there is someone with five friends at the table, then there is no one with no friends. As before, we have six people and five pigeonholes and thus two people must have the same number of friends.

24 a The number of rods is ${}^6C_2 = 15$.

b The number of triangles is ${}^6C_3 = 20$.

c Choose any vertex O. Five rods are joined to O, so there must be at least three rods OA, OB and OC of the same colour. If any one of the rods AB, BC or CA has that same colour, then that rod and O form a mono-coloured triangle. If all the rods have the other colour, then ABC is a mono-coloured triangle.

Chapter 14 review exercise

1 $8! = 40320$

2 a 72 **b** $n(n + 1)$ **c** $k \times k!$

3 a 792 **b** 6 **c** 1140

4 ${}^7P_4 = 840$

5 $26^3 \times 10^4 = 175\,760\,000$

6 a $4! \times 3! = 144$ **b** $5! = 120$ **c** $3! \times 4! = 144$

d $7! \div 2 = 2520$ (half of the total)

7 a $10! = 3\,628\,800$

b $2 \times 5! \times 5! = 28\,800$

c $9! \times 2! = 725\,760$

8 $\dfrac{8!}{3! \times 2!} = 3360$

9 ${}^6P_3 + {}^6P_4 = 480$

10 $\dfrac{10!}{6! \times 4!} = 210$

11 a ${}^{16}C_7 = 11\,440$

b ${}^{10}C_7 = 120$

c 0 (there are no such committees of 7)

d ${}^6C_2 \times {}^{10}C_5 = 3780$

e ${}^{10}C_4 \times {}^6C_3 = 4200$

f By adding cases: 9360

g ${}^{15}C_6 = 5005$

h The complement of part **g**, that is, 6435

i The complement of the case where both are members, that is, 9438.

12 a ${}^8C_5 = 56$ **b** ${}^8C_4 = 70$ **c** 35

13 Choose the four men members, then choose the two women members, $\dfrac{{}^4C_4 \times {}^3C_2}{{}^7C_6} = \frac{3}{7}$. Alternatively, only one of the seven is not on the committee, and there will be a majority of men, when this person is a women. Thus the probability is $\frac{3}{7}$.

14 a $\frac{1}{22\,100}$ **b** $\frac{1}{5525}$ **c** $\frac{11}{850}$ **d** $\frac{22}{425}$

e $\frac{13}{34}$ **f** $\frac{16}{5525}$ **g** $\frac{6}{5525}$ **h** $\frac{64}{425}$

15 a 120 **b** 12

c 36 **d** 48

e $120 - 48 = 72$ **f** 24

16 $17 \div 6 = 2$ remainder 5, so there must be a monkey who receives at least three peanuts.

17 Place the 1500 pigeons in pigeonholes labelled by the 366 possible days of a year. $1500 \div 366 = 4$ remainder 36, so there is at least one day shared by at least 5 people.

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Chapter 15

Exercise 15A

2 a $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$
b $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$
c $1 + 9x + 36x^2 + 84x^3 + 126x^4 + 126x^5 + 84x^6 + 36x^7 + 9x^8 + x^9$
d $1 - 9x + 36x^2 - 84x^3 + 126x^4 - 126x^5 + 84x^6 - 36x^7 + 9x^8 - x^9$
e $1 + 5c + 10c^2 + 10c^3 + 5c^4 + c^5$
f $1 + 8y + 24y^2 + 32y^3 + 16y^4$
g $1 + \frac{7x}{3} + \frac{7x^2}{3} + \frac{35x^3}{27} + \frac{35x^4}{81} + \frac{7x^5}{81} + \frac{7x^6}{729} + \frac{1x^7}{2187}$
h $1 - 9z + 27z^2 - 27z^3$
i $1 - \frac{8}{x} + \frac{28}{x^2} - \frac{56}{x^3} + \frac{70}{x^4} - \frac{56}{x^5} + \frac{28}{x^6} - \frac{8}{x^7} + \frac{1}{x^8}$
j $1 + \frac{10}{x} + \frac{40}{x^2} + \frac{80}{x^3} + \frac{80}{x^4} + \frac{32}{x^5}$
k $1 + \frac{5y}{x} + \frac{10y^2}{x^2} + \frac{10y^3}{x^3} + \frac{5y^4}{x^4} + \frac{y^5}{x^5}$
l $1 + \frac{12x}{y} + \frac{54x^2}{y^2} + \frac{108x^3}{y^3} + \frac{81x^4}{y^4}$

4 a i $55x^2$ **ii** $165x^8$
b i $-35x^3$ **ii** $-21x^5$
c i $240x^4$ **ii** $192x^5$
d i $-\frac{12}{x}$ **ii** $\frac{54}{x^2}$

6 a $\big(1 + (x - 1)\big)^3 = x^3$
b $\big(1 - (x + 1)\big)^6 = (-x)^6 = x^6$

7 21

8 a $a = 76, b = 44$ **b** $a = 16, b = -8$

10 a 1.018 14 **b** 0.815 37

11 a i $1 + 4x + 6x^2 + \cdots$ **ii** -14
b i $1 + 10x + 40x^2 + 80x^3 + \cdots$
ii 40
c i $1 - 12x + 54x^2 - 108x^3 + \cdots$
ii -228

12 a $x = 0$ or $\frac{1}{2}$ **b** $x = 0, 1$ or 5

13 a -12 **b** 0 **c** 380 **d** $-\frac{5}{3}$

14 a 97 **b** $1\frac{10}{27}$

15 a i $15x^2$ **ii** $20x^3$
iii $3 : 4x$ **iv** $135, 540, 1 : 4$
b i $\frac{224}{81x^5}$ **ii** $\frac{448}{729x^6}$
iii $9x : 2$ **iv** $\frac{7}{81}, \frac{7}{729}, 9 : 1$

16 a $k = 5$ **b** $k = -2$

18 1.0634

19 $(1 + x + y)^0 = 1$, $(1 + x + y)^1 = 1 + x + y$,
$(1 + x + y)^2 = 1 + 2x + 2y + 2xy + x^2 + y^2$,
$(1 + x + y)^3 = 1 + 3x + 3y + 6xy + 3x^2 + 3y^2 + 3x^2y + 3xy^2 + x^3 + y^3$,
$(1 + x + y)^4 = 1 + 4x + 4y + 12xy + 6x^2 + 6y^2 + 6x^2y^2 + 12x^2y + 12xy^2 + 4x^3 + 4y^3 + 4x^3y + 4xy^3 + x^4 + y^4$

The coefficients form a triangular pyramid, with 1s on the edges, and each face a copy of Pascal's triangle.

Exercise 15B

1 a $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
b $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
c $r^6 - 6r^5s + 15r^4s^2 - 20r^3s^3 + 15r^2s^4 - 6rs^5 + s^6$
d $p^{10} + 10p^9q + 45p^8q^2 + 120p^7q^3 + 210p^6q^4 + 252p^5q^5 + 210p^4q^6 + 120p^3q^7 + 45p^2q^8 + 10pq^9 + q^{10}$
e $a^9 - 9a^8b + 36a^7b^2 - 84a^6b^3 + 126a^5b^4 - 126a^4b^5 + 84a^3b^6 - 36a^2b^7 + 9ab^8 - b^9$
f $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$
g $p^7 - 14p^6q + 84p^5q^2 - 280p^4q^3 + 560p^3q^4 - 672p^2q^5 + 448pq^6 - 128q^7$
h $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$
i $a^3 - \frac{3a^2b}{2} + \frac{3ab^2}{4} - \frac{1b^3}{8}$
j $\frac{1r^5}{32} + \frac{5r^4s}{48} + \frac{5r^3s^2}{36} + \frac{5r^2s^3}{54} + \frac{5rs^4}{162} + \frac{1s^5}{243}$
k $x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

2 a $1 + 4x^2 + 6x^4 + 4x^6 + x^8$
b $1 - 9x^2 + 27x^4 - 27x^6$
c $x^{12} + 12x^{10}y^3 + 60x^8y^6 + 160x^6y^9 + 240x^4y^{12} + 192x^2y^{15} + 64y^{18}$
d $x^9 - 9x^7 + 36x^5 - 84x^3 + 126x - \frac{126}{x} + \frac{84}{x^3} - \frac{36}{x^5} + \frac{9}{x^7} - \frac{1}{x^9}$
e $x^3\sqrt{x} + 7x^3\sqrt{y} + 21x^2y\sqrt{x} + 35x^2y\sqrt{y} + 35xy^2\sqrt{x} + 21xy^2\sqrt{y} + 7y^3\sqrt{x} + y^3\sqrt{y}$
f $\frac{32}{x^5} + \frac{240}{x^2} + 720x + 1080x^4 + 810x^7 + 243x^{10}$

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3 a $\big(y + (x - y)\big)^5 = x^5$
b $\big(a - (a - b)\big)^4 = b^4$
c $\big(x + (2y - x)\big)^3 = (2y)^3 = 8y^3$
d $\big((x + y) - (x - y)\big)^6 = (2y)^6 = 64y^6$

4 a i $1024 + 1280x + 640x^2 + 160x^3 + \cdots$
ii -160
b i $1 - 12x + 60x^2 - 160x^3 + 240x^4 - \cdots$
ii 720
c i $2187 - 5103y + 5103y^2 - 2835y^3 + 945y^4 - \cdots$
ii 11 718

5 a $2x^6 + 30x^4y^2 + 30x^2y^4 + 2y^6$

6 a 540 **b** 48 **c** -960 **d** -8

7 a i $x^3 + 3x^2h + 3xh^2 + h^3$
ii $3x^2h + 3xh^2 + h^3$
iii $3x^2$ **b** $5x^4$

8 b 466 **c** 42

9 $\frac{7}{2}$

10 a 1.104 08 **b** 0.903 92

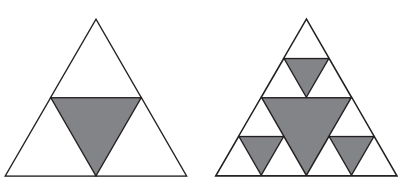
11 a i $\left(x^3 + \dfrac{1}{x^3}\right) + 3\left(x + \dfrac{1}{x}\right)$
ii $\left(x^5 + \dfrac{1}{x^5}\right) + 5\left(x^3 + \dfrac{1}{x^3}\right) + 10\left(x + \dfrac{1}{x}\right)$
iii $\left(x^7 + \dfrac{1}{x^7}\right) + 7\left(x^5 + \dfrac{1}{x^5}\right) + 21\left(x^3 + \dfrac{1}{x^3}\right) + 35\left(x + \dfrac{1}{x}\right)$
b i 2 **ii** 2 **iii** 2

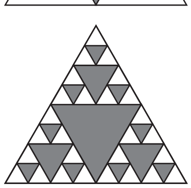
12 $a = 3$ or $a = -3$

14 a $x^6 + 6x^4 + 15x^2 + 20 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6}$
b $A = -6, B = 9$ and $C = -2$.

15 a $x^3 + y^3 + z^3 + 6xyz + 3x^2y + 3xy^2 + 3xz^2 + 3x^2z + 3y^2z + 3yz^2$
b 19

16 a The limiting figure for this process is called the Sierpinski Gasket. It is one of the classic regular fractals.





b Sierpinski's triangle is formed.

Exercise 15C

1 a 4 **b** 20 **c** 9 **d** 35

2 a 10 **b** 1 **c** 11 **d** 210

3 a 3003 **b** 1287 **c** 792 **d** 22 **e** 429 **f** 1292

4 a ${}^4C_0 = 1, {}^4C_1 = 4, {}^4C_2 = 6, {}^4C_3 = 4, {}^4C_4 = 1$
b i 16 **ii** 0

5 a 32 **b** 32 **c** 20 **d** 252

6 a i 56 **ii** 35
b 5 **c** $n = 4$ or $n = 8$

7 b i 6 **ii** 10

8 a $8568x^5$ **b** $2\,217\,093\,120x^9$
c $-19208x^3$ **d** 189

9 a $672x^2$ **b** $\dfrac{1001x^9y^5}{16}$ **c** $-\dfrac{33x^{10}y^2}{1024}$ **d** $190a^2b^9$

10 a $x = \frac{11}{2}$ **b** $x = -\frac{7}{3}$

11 a $5x^2 : 39$ **b** $5 : 2$ **c** $18\,304 : 1$

12 a i 1 **ii** n
iii $\frac{1}{2}n(n - 1)$ **iv** $\frac{1}{6}n(n - 1)(n - 2)$
b i 16 **ii** 9 **iii** 4
iv 6 **v** 4 **vi** 7

13 a $a = 2$ and $n = 14$ **b** $a = -\frac{1}{3}$ and $n = 10$

14 a $n = 14$ **b** $n = 13$

15 ${}^{40}C_{20} \doteqdot 1.378 \times 10^{11}$

17 a ${}^nC_0\,x^n + {}^nC_1\,x^{n-1}h + {}^nC_2\,x^{n-2}h^2 + \cdots + {}^nC_n\,h^n$
b nx^{n-1}

18 The second member is ${}^nC_1 = n$, so suppose that n is prime. Then n is coprime to every number less than n, so is coprime to $r!$ and $(n - r)!$ for all whole numbers $r = 2, 3, \ldots, r - 1$.

19 a 3 points, 3 segments, 1 triangle
b 4 points, 6 segments, 4 triangles, 1 quadrilateral
c 5 points, 10 segments, 10 triangles, 5 quadrilaterals, 1 pentagon
d 21

20 b i $1 + x + x^2 + x^3 + \cdots$
ii $1 + 2x + 3x^2 + 4x^3 + \cdots$
iii $1 - 2x + 3x^2 - 4x^3 + \cdots$
iv $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots$

c Using part **b i**,

$$\begin{aligned}\text{LHS} &= \frac{d}{dx}\left(\frac{1}{1 - x}\right)\\ &= \frac{d}{dx}(1 + x + x^2 + x^3 + \cdots)\\ &= 1 + 2x + 3x^2 + \cdots\\ &= \text{RHS by part } \mathbf{b\ ii}.\end{aligned}$$

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Exercise 15D

1 a i $1 + 4 + 6 + 4 + 1 = 16 = 2^4$.

ii The sum $1 + 6 + 1$ of the first, third and fifth terms on the row equals the sum $4 + 4$ of the second and fourth terms.

iii The sum of the first, third and fifth terms on the row is half the sum of the whole row.

b i $4(1 + x)^3 = {}^4C_1 + 2\,{}^4C_2 x + 3\,{}^4C_3 x^2 + 4\,{}^4C_4 x^3$

ii $1 \times 4 + 2 \times 6 + 3 \times 4 + 4 \times 1 = 32 = 4 \times 2^3$.

iii $1 \times 4 - 2 \times 6 + 3 \times 4 - 4 \times 1 = 0$.

4 a There are 10. **b** There are 10.

c C, D and E.

d Given 5 letters, choose 2. Those that are left form a set of 3. Thus for every set of 2, there is a corresponding set of 3. Thus ${}^5C_2 = {}^5C_3$.

e Given n people, choose r. Those that are left form a set of $n - r$. Thus for every set of r, there is a corresponding set of $n - r$. Thus ${}^nC_r = {}^nC_{n-r}$.

5 a To form a subset of S, take each of A, B, C and D in turn and decide whether it is in or out. Thus the total number of subsets of S is $2 \times 2 \times 2 \times 2 = 2^4$.

b The LHS is the sum of the numbers of 0-member, 1-member, 2-member, 3-member and 4-member subsets, and so is also the number of all subsets.

c Generalise the previous argument to an n-member set.

6 a i There are four of them.

b i There are six of them. **ii** Omit E from each set.

c Let T be a 3-letter subset of S. If T does not contain E, then T is one of the the 3-letter subsets of U. If Tdoes contain E, then remove E, and the remaining 2-letter subset pairs with one of the 2-letter subsets of U.

d Generalise the previous argument.

7 a Using the addition property,

$$\text{LHS} = \frac{{}^nC_{r-1}}{{}^{n+1}C_r} + \frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} \text{ and } \text{RHS} = \frac{2 \times {}^nC_r}{{}^{n+1}C_{r+1}}.$$

Now use the formula for ${}^nC_r$.

8 a As explained in Question 5, the LHS counts all the subsets of S. We can also count the subsets by choosing whether each element in turn goes into a subset of not, giving 2^n subsets.

b i First, every subset of S either contains A or does not contain A. Secondly, a subset containing A is paired up with the unique subset obtained by removing A from the subset.

ii Adding A to a subset without A changes the number of members from odd to even or from even to odd.

iii The LHS is the total number of even-order subsets, and the RHS is the total number of odd-order subsets. We have seen that they are paired with each other, so the LHS and RHS are equal.

9 a i
$$\begin{aligned}{}^7C_3 &= {}^6C_3 + {}^6C_2 = {}^5C_3 + {}^5C_2 + {}^6C_2\\ &= {}^4C_3 + {}^4C_2 + {}^5C_2 + {}^6C_2\\ &= {}^3C_3 + {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2,\end{aligned}$$
and ${}^3C_3 = {}^2C_2 = 1$

b i There are ${}^2C_2 = 1$ subsets with highest element 3. There are ${}^3C_2 = 3$ subsets with highest element 4. There are ${}^4C_2 = 6$ subsets with highest element 5. There are ${}^5C_2 = 10$ subsets with highest element 6. There are ${}^6C_2 = 15$ subsets with highest element 7. This makes 353-member subsets.

10 a A common denominator for the two fractions is required:

$$\begin{aligned}\text{LHS} &= \frac{n!}{r! \times (n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)!}\\ &= \frac{n!}{r \times (r-1)! \times (n-r)!}\\ &\quad + \frac{n!}{(r-1)! \times (n-r+1) \times (n-r)!}\\ &= \frac{(n-r+1) \times n! + r \times n!}{r \times (r-1)! \times (n-r+1) \times (n-r)!}\\ &= \frac{\big((n-r+1) + r\big) \times n!}{r! \times (n-r+1)!}\\ &= \frac{(n+1) \times n!}{r! \times (n-r+1)!}\\ &= \frac{(n+1)!}{r! \times (n-r+1)!}\\ &= \text{RHS}.\end{aligned}$$

b This is the addition property of Pascal's triangle.

12 a Regarding this as an $(a + b)$-letter word with a identical As and b identical Bs, the number of permutations is $\dfrac{(a+b)!}{a!b!} = {}^{a+b}C_a = {}^{a+b}C_b$

b ${}^{2n}C_n$

c i ${}^nC_2$ **ii** ${}^nC_{n-2} = {}^nC_2$

d Consider a $2n$-letter binary word with n As and n Bs, split into two equal halves, namely the first and second half. Consider the $n + 1$ cases where $0, 1, 2, 3, \ldots, n$ As fall in the first half and the rest fall in the second half. Using the arguments of part **b** and **c**, we have

$$\begin{aligned}{}^{2n}C_n &= ({}^{2n}C_0)({}^{2n}C_{2n}) + ({}^{2n}C_1)({}^{2n}C_{2n-1})\\ &\quad + ({}^{2n}C_2)({}^{2n}C_{2n-2}) + \cdots + ({}^{2n}C_{2n})({}^{2n}C_0)\\ &= ({}^{2n}C_0)({}^{2n}C_0) + ({}^{2n}C_1)({}^{2n}C_1)\\ &\quad + ({}^{2n}C_2)({}^{2n}C_2) + \cdots + ({}^{2n}C_{2n})({}^{2n}C_{2n})\end{aligned}$$

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Exercise 15E

1 a ${}^{13}C_k\, x^k$ **b** ${}^{7}C_k\, 2^k\, x^k$

c ${}^{12}C_k\, 5^{12-k}\, 7^k\, x^k$

d ${}^{9}C_k\, 2^{9-k}\,(-1)^k\, x^{9-k}\, y^k$

e ${}^{5}C_k\, x^{5-k}\, 2^k\, x^{-k} = {}^{5}C_k\, 2^k\, x^{5-2k}$

f ${}^{8}C_k\,(6x)^{8-k}\,(-2)^k\, x^{-k} = {}^{8}C_k\, 3^{8-k}\,(-1)^k\, 2^8\, x^{8-2k}$

2 b **i** 126 **ii** 36 **iii** 84

3 b **i** ${}^{10}C_4 \times 2^6 \times 3^4 = 2^7 \times 3^5 \times 5 \times 7$

ii ${}^{10}C_7 \times 2^3 \times 3^7 = 2^6 \times 3^8 \times 5$

iii ${}^{10}C_6 \times 2^4 \times 3^6 = 2^5 \times 3^7 \times 5 \times 7$

4 b **i** ${}^{15}C_2 \times 5^2 \times 2^{-13}$

ii $-{}^{15}C_7 \times 5^7 \times 2^{-8}$

iii ${}^{15}C_{10} \times 5^{10} \times 2^{-5}$

5 a ${}^{8}C_4 \times 3^4 = 5670$

b $-{}^{12}C_9 \times 2^3 = -1760$

c ${}^{10}C_8 \times 5^2 \times 2^8 = 288\,000$

d ${}^{6}C_4 \times a^2 \times \left(\frac{1}{2}\right)^4 = \frac{15}{16}a^2$

6 a -672 **b** $-112\,266$ **c** $\frac{969}{2}$

d 21 875 **e** $\frac{40}{49}$

7 a 3640 **b** -385 **c** 10 920 **d** -1241

8 a 6 **b** 45 **c** 84

9 a $a = -24,\ b = 158$ **b** $n = 13,\ 286$

10 ${}^{3n}C_n\,(= {}^{3n}C_{2n})$

11 a ${}^{12}C_r\,(-1)^r\, a^{12-r}\, b^r x^r$ **b** $\frac{5}{8}$

12 a $1 - 4x + 10x^2 - 16x^3 + 19x^4 - \cdots$

b i $\dfrac{9 - 9n}{2}$

ii $\dfrac{-9n(n-1)(2n-1)}{2}$

Chapter 15 review exercise

1 The answer is in the theory of 15A.

2 a $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

b $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

c $1 - 9x + 27x^2 - 27x^3$

d $1 - 4xy + 6x^2y^2 - 4x^3y^3 + x^4y^4$

3 a $1 + 35x + 490x^2 + \cdots$

b $(1 - 5x)(1 + 35x + 490x^2 + \cdots)$

$= (490 - 175)x^2 + \cdots.$

The coefficient of x^2 is 315.

4 a $1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7$

b $(1 + 0.02)^7 = 1 + 7 \times 0.2 + 21 \times 0.0004 + \cdots$

$= 1 + 0.14 +$ much smaller terms

The first decimal place will be 1.

5 a $81 + 216x + 216x^2 + 96x^3 + 16x^4$

b $125 - 75x + 15x^2 - x^3$

c $32x^5 + 320x^4y + 1280x^3y^2 + 2560x^2y^3 + 2560xy^4 + 1024y^5$

d $x^4 - 4x^2 + 6 - \dfrac{4}{x^2} + \dfrac{1}{x^4}$

6 a 10 **b** 35 **c** 56 **d** 2

7 a 45 **b** 792 **c** 84

d 40 **e** $\frac{84}{5}$ **f** $\frac{286}{9}$

8 a 1 (first entry in a row)

b 0 (reversibility of rows)

c $2^5 = 32$ (sum of a row)

d 0 (addition formula)

9 $n = 8$

10 $(1 - 3x)(1 + 5x)^{14}$

$= (1 - 3x)(\cdots + {}^{14}C_7\,(5x)^7 + {}^{14}C_8\,(5x)^8 + \cdots)$

The coefficient of x^8 will be

${}^{14}C_8 \times 5^8 - 3 \times {}^{14}C_7 \times 5^7 = 368\,671\,875.$

11 a $(1 + x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots.$

b Substitute $x = 2$:

$(1 + 2)^n = \binom{n}{0} + \binom{n}{1} \times 2 + \binom{n}{2} \times 4 + \binom{n}{3} \times 8 + \cdots.$

c When $n = 4$,

$$\begin{aligned}\text{LHS} &= 1 + 4 \times 2 + 6 \times 4 + 4 \times 8 + 1 \times 16\\ &= 1 + 8 + 24 + 32 + 16\\ &= 81\\ &= 3^4.\end{aligned}$$

12 b This important result is proven in the theory of Exercise 15D. It tells us that the sum of the row in Pascal's triangle indexed by n, that is, containing the coefficients of $(1 + x)^n$, is 2^n.

c The list of all subsets of a set of n objects may be partitioned into sets of sizes $0, 1, 2, \ldots, n$. The sum of the number of sets of each type will be the total number of all subsets, which is 2^n.

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Chapter 16

Exercise 16A

1 $\dfrac{dy}{dt} = (3x^2 + 1)\dfrac{dx}{dt}$

a 65 **b** $-\frac{3}{14}$

2 a 240π cm²/s

b $\frac{1}{12\pi}$ cm/s

3 a $\frac{2}{\pi}$ cm/s **b** $\frac{5}{\pi}$ cm **c** $\frac{50}{9\pi}$ cm/s

4 a 840 cm²/s, $6\sqrt{2}$ cm/s **b** 1200 cm²/s, $6\sqrt{2}$ cm/s

5 b i 1350 cm³/min, 180 cm²/min, 6 cm/min

ii 600 cm³/min, 120 cm²/min, 6 cm/min

c $10\sqrt{2}$ cm

6 a $90\,000\pi$ mm³/min

b The rate is constant at 6π mm/min.

7 a $\dfrac{dA}{dt} = \frac{1}{2}s\sqrt{3}\,\dfrac{ds}{dt}$, $\dfrac{dh}{dt} = \frac{1}{2}\sqrt{3}\dfrac{ds}{dt}$

b $\frac{9}{5}\sqrt{3}$ cm²/s, $\frac{3}{20}\sqrt{3}$ cm/s

8 a i $\frac{1}{24\pi}$ cm/s **ii** $1\frac{1}{2}$ cm³/s

b $\frac{32\,000\pi}{3}$ cm³

9 $\dfrac{dV}{dt} = 3\pi h^2\dfrac{dh}{dt}$

a i $\frac{1}{160\pi}$ m/min **ii** $\frac{3}{160\pi}$ m/min

b $\dfrac{dA}{dt} = 6\pi h\dfrac{dh}{dt}$, $\frac{3}{20}$ m²/min, $\frac{3}{20}\sqrt{3}$ m²/min

c $\frac{96\pi}{25}$ m³/min

10 a $V = \frac{4\pi}{3}h^3$ **b** $\frac{1}{32\pi}$ m/s

11 a When it is a square, $\ell = x\sqrt{2}$.

b Put $\dfrac{d\ell}{dt} = \dfrac{1}{2}\dfrac{dx}{dt}$, then $\ell = 2x$, so $x = \frac{4}{\sqrt{3}}$ m and $\ell = \frac{8}{\sqrt{3}}$ m.

c It follows from part **a** because ℓ is always greater than x. Alternatively, $\dfrac{d\ell}{dt}\Big/\dfrac{dx}{dt} = \dfrac{x}{\sqrt{16 + x^2}}$ is always less than 1 because the denominator is greater than the numerator.

f $\dfrac{d\ell}{dt}\Big/\dfrac{dx}{dt} = \dfrac{x}{\sqrt{16 + x^2}} \to 1$ as $x \to \infty$.

12 $\frac{1}{125\pi}$ cm/s, $\frac{4}{5}$ cm²/s, $\frac{4}{5}\sqrt{2}$ cm²/s

13 b 0.096 m³/s, $\frac{3}{125}\left(\sqrt{17} + 1\right)$ m²/s

14 $\dfrac{dy}{dt} = \dfrac{-x}{\sqrt{169 - x^2}}\dfrac{dx}{dt}$

a $\frac{5}{12}$ cm/s **b** 12 mm/s

15 b $\frac{1}{6}$ cm/s

16 b $\frac{1}{24}$ cm/min, $83\frac{1}{3}$ cm²/min

c 8 litre/min, 200 cm²/min

17 a When $r = 10$ cm, $\dfrac{dr}{dt} = \frac{1}{10\pi}$ cm/s.

b i $r = \left(\frac{30t}{\pi}\right)^{\frac{1}{3}}$ and $\dfrac{dr}{dt} = \frac{1}{3} \times \left(\frac{30}{\pi}\right)^{\frac{1}{3}}t^{-\frac{2}{3}}$

ii When $r = 10$, $t = \frac{100\pi}{3}$ s and $\dfrac{dr}{dt} = \frac{1}{10\pi}$ cm/s.

18 $4\sqrt{6}$ cm²/s

19 0.246 cm/s. Some exact forms are

$$\frac{3}{(27 + 9\sqrt{3})^{\frac{2}{3}}} \text{ and } \frac{1}{(3 + 3\sqrt{3})^{\frac{2}{3}}}.$$

Exercise 16B

1 a $\dfrac{dQ}{dt} = 1200e^{3t} = 3Q$. When $t = 0$, $Q = 400e^0 = 400$.

b i $Q = 400e^6 \doteqdot 1.614 \times 10^5$, $\dfrac{dQ}{dt} = 1200e^6 \doteqdot 4.841 \times 10^5$

ii $t = \frac{1}{3}\log_e 50 \doteqdot 1.304$ **iii** $t = \frac{1}{3}\log_e \frac{50}{3} \doteqdot 0.9378$

2 a $\dfrac{dQ}{dt} = -2000e^{-2t} = -2Q$. When $t = 0$, $Q = 1000e^0 = 1000$.

b i $Q = 1000e^{-0.8} \doteqdot 449$, $\dfrac{dQ}{dt} = -2000e^{-0.8} \doteqdot -899$

ii $t = \frac{1}{2}\log_e 25 \doteqdot 1.609$ **iii** $t = \frac{1}{2}\log_e 50 \doteqdot 1.956$

3 b 1350 **c** 135 per hour **d** 23 hours

4 c 6.30 grams, 1.46 grams per minute

d 6 minutes 58 seconds

e 20 g, $20e^{-k} \doteqdot 15.87$ g, $20e^{-2k} \doteqdot 12.60$ g, $20e^{-3k} = 10$ g, the common ratio is $e^{-k} = 2^{-\frac{1}{3}} \doteqdot 0.7937$

5 b $-\frac{1}{5}\log_e \frac{7}{10}$

c 10290

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d At $t \doteqdot 8.8$, that is, some time in the fourth year from now.

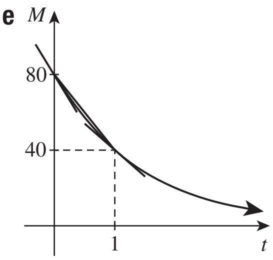
6 a 80 g, 40 g, 20 g, 10 g

b 40 g, 20 g, 10 g. During each hour, the average mass loss is 50%.

c $M_0 = 80, k = \log_e 2 \doteqdot 0.693$

d 55.45 g/h, 27.73 g/h, 13.86 g/h, 6.93 g/h

e



7 b 30

c i 26 **ii** $\frac{1}{5}\log_e \frac{15}{13}\left(\text{or} -\frac{1}{5}\log_e \frac{13}{15}\right)$

8 b $L = \frac{1}{2}$

9 c 25

d $\frac{k}{A} = \frac{1}{2}\log_e \frac{5}{3}\left(\text{or } -\frac{1}{2}\log_e \frac{3}{5}\right)$

e 6 hours 18 minutes

10 b $C_0 = 20000, k = \frac{1}{5}\log_e \frac{9}{8} \doteqdot 0.024$

c 64 946 ppm

d i 330 metres from the cylinder

ii If it had been rounded down, then the concentration would be above the safe level.

11 a ii $k = \frac{1}{12}\log_e \frac{122}{105}$ **b ii** $\ell = \frac{1}{12}\log_e \frac{217}{100}$

c At $t = \dfrac{\log_e \frac{525}{100}}{\ell - k} \doteqdot 31.85$, that is, in the 32nd month.

d $\ell C = \ell \times 100 \times e^{32\ell} \doteqdot 51$ cents per month

12 a $y(3) = A_0 e^{3k} = A_0\left(e^k\right)^3$ and we know that $e^k = \dfrac{3}{4}$.

b $y(3) = \frac{27}{64}A_0$

13 a $B = \dfrac{2N_0^2}{N_c}$ and $C = \left(\dfrac{N_0}{N_c}\right)^2$

b $\dfrac{B}{C} = 2N_c$

Exercise 16C

1 a ii 12 000, $P \to \infty$ as $t \to \infty$

b ii 12 000, $P \to 10000$ as $t \to \infty$

c ii 8000, $P \to 10000$ as $t \to \infty$

2 b $A = 1000, k = \frac{1}{3}\log_e 6$

c 67 420 bugs **d** 10.4 weeks

3 b $B = 970000, k = -\frac{1}{10}\log_e \frac{47}{97} = \frac{1}{10}\log_e \frac{97}{47}$

c 158 000 flies **d** 73 days

4 b $T_e = 20, A = 70$

c $k = \frac{1}{6}\log_e \frac{7}{3}\left[\text{Alternatively, } k = -\frac{1}{6}\log_e \frac{3}{7}.\right]$

d 13 minutes 47 seconds

5 a $A = 34$ **b** $\frac{1}{45}\log_e 2\left(\text{or } -\frac{1}{45}\log_e \frac{1}{2}\right)$ **c** 16.5°C

6 a $1 - e^{-\frac{1}{16}t}$ is always positive for $t > 0$. The body is falling.

b It is the acceleration of the body.

c -160 m/s **d** $16\log_e \frac{8}{7} \doteqdot 2.14$ s

7 a As $t \to \infty, P \to B$. As $t \to -\infty, P \to \infty$.

b As $t \to \infty, P \to \infty$. As $t \to -\infty, P \to B$.

8 a The average level is 15 cm, so $x - 15$ is the difference between the left level and the average level. We write $x - 15$ so that k is positive.

b Substituting $t = 0$ shows that $A = 15$.

c 15 cm **d** $k = \frac{1}{5}\log_e \frac{5}{3}$

9 b $-\dfrac{V}{R}$ **c** $I \to \dfrac{V}{R}$ **d** 4.62×10^{-4} s

10 b $M \to a$ as $t \to \infty$ **c** $k = \dfrac{1}{120}\log_e 100$

d 2 minutes 45 seconds

11 a $2w$ g/min **b** $\dfrac{Q}{1000}$ g/L **c** $\dfrac{Qw}{1000}$ g/min

f -2000 **g** $Q \to 2000$

h $w = \frac{1000}{345}\log_e 2 \doteqdot 2$ L/min

13 b $A = 1000, I = 9000$ and $k = \dfrac{1}{7}\log_e 3$

c 36 000

14 Adam's coffee is cooler.

Chapter 16 review exercise

1 a 600 mm^2/h.

b $3\sqrt{2}$ mm/h (The rate is constant.)

2 a $h = r\sqrt{3}$ **b** $\dfrac{V}{t} = \pi r^2\sqrt{3}\dfrac{r}{t}$

c $\ell = 2r$ **d** $\dfrac{A}{t} = 4\pi r\dfrac{r}{t}$ **f** $\frac{5}{\sqrt{3}}$ m^2/s

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3 b $k = \frac{1}{5}\log_e \frac{13}{8}$, $80000e^{18k} \doteqdot 459000$

c $t = \frac{1}{k}\log_e 12.5$, year 2036

4 b $k = \frac{\log_e 2}{30.2}$ **c** 10.07% **d** about 201 years

5 a The temperature is dropping, but $T - E$ is positive.

c i $A = 500$, $k = -\frac{1}{6}\log_e 2$, $T = 500e^{15k} \doteqdot 88°C$

ii $A = 460$, $k = -\frac{1}{6}\log_e \frac{46}{21}$,
$T = 40 + 460e^{15k} \doteqdot 105°\text{C}$

6 a The population is growing, and $P - M$ is negative.

b As $t \to \infty$, $P \to M - 0 = M$.

c i $A = 9500$, $k = \frac{1}{10}\log_e \frac{19}{16}$

ii $p = 10000 - 9500e^{-20k} \doteqdot 3260$

iii $t = \frac{1}{k}\log_e \frac{19}{4} \doteqdot 91$ years, year 2100.

Chapter 17

Exercise 17A

1 a $3 \le y \le 5$

b domain: $3 \le x \le 5$, range: $0 \le y \le 2$

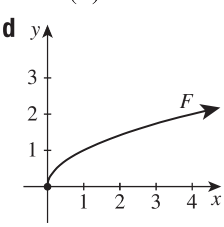
c $f^{-1}(x) = x - 3$

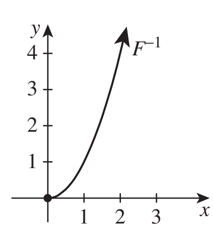
2 a $0 \le y \le 2$

b domain: $0 \le x \le 2$, range: $0 \le y \le 4$

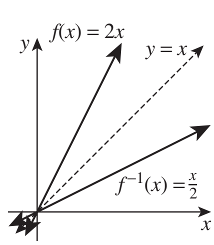
c $F^{-1}(x) = x^2$

d

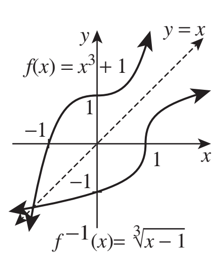




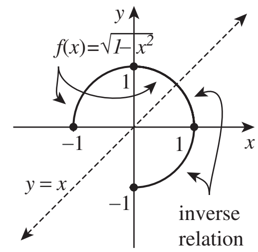
3 a The inverse is a function.
$f^{-1}(x) = \frac{1}{2}x$



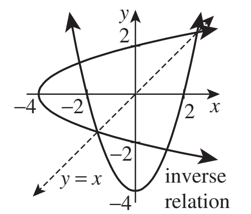
b The inverse is a function.
$f^{-1}(x) = \sqrt[3]{x - 1}$



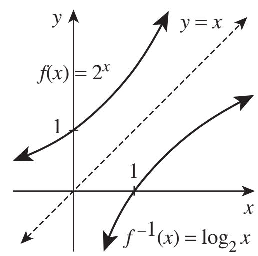
c The inverse is not a function.



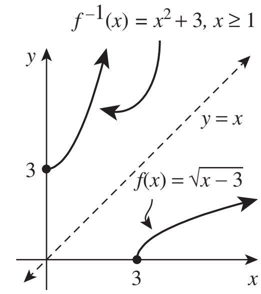
d The inverse is not a function.
$f(x) = x^2 - 4$



e The inverse is a function.
$f^{-1}(x) = \log_2 x$



f The inverse is a function.
$f^{-1}(x) = x^2 + 3, x \ge 0$



4 a Both x.

b They are inverse functions.

5 a $g^{-1}(x) = \sqrt{x}$, domain: $x \ge 0$, range: $y \ge 0$

b $g^{-1}(x) = -\sqrt{x - 2}$, domain: $x \ge 2$, range: $y \le 0$

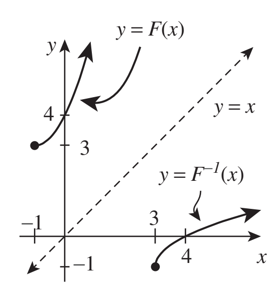
c $g^{-1}(x) = \sqrt{4 - x^2}$, $-2 \le x \le 0$,
domain: $-2 \le x \le 0$, range: $0 \le y \le 2$

6 a $3x^2$ **b** $\frac{1}{3}(y + 1)^{-\frac{2}{3}}$

7 $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, $\frac{dx}{dy} = 2y$

8 b They are both one-to-one.

c $F^{-1}(x) = -1 + \sqrt{x - 3}$,
domain: $x \ge 3$,
range: $y \ge -1$

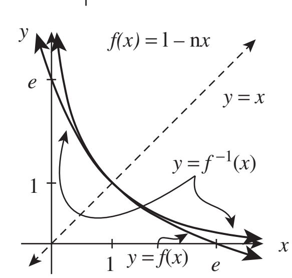


9 a $x = e$

b Reflect $y = \ln x$ in the x-axis, then shift it one unit up.

d $f^{-1}(x) = e^{1-x}$, domain: all real x, range: $y > 0$

e Both are decreasing.



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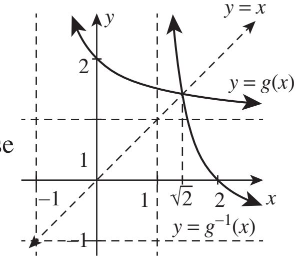
10 b One-to-one.

c $g^{-1}(x) = \dfrac{2 - x}{x - 1}$, for $x > 1$

d $x = \sqrt{2}$. It works because the graphs meet on the line of symmetry $y = x$.



11 a $y = \sqrt[3]{-x}$

b $(-1, 1)$, $(0, 0)$ and $(1, -1)$

12 a No. The graph of the inverse is a vertical line, which is not a function.

13 a Shift two units left and four units down.

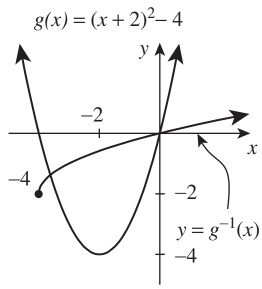
b x-intercepts: -4, 0, vertex: $(-2, -4)$.

c Many-to-one.

d $x \geq -2$

e $x \geq -4$, increasing

f $g^{-1}(x) = -2 + \sqrt{x + 4}$

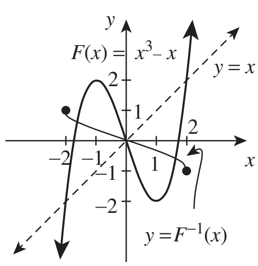


14 a x-intercepts: $0, \sqrt{3}, -\sqrt{3}$, stationary points: $(-1, 2)$, $(1, -2)$

b Many-to-one.

c $-1 \leq x \leq 1$

d $-2 \leq x \leq 2$



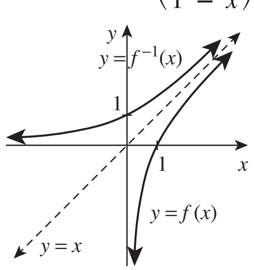
15 a all real x

b $f'(x) > 0$ for all x.

c Because $f(x)$ is always increasing, the graph of $f(x)$ passes the horizontal line test.

$f^{-1}(x) = \ln\left(\dfrac{x}{1 - x}\right)$

16



17 a Suppose that (a, b) lies on the graph of the inverse relation. Then (b, a) lies on the graph of the relation. Because the relation is odd, $(-b, -a)$ lies on the graph of the relation. Hence $(-a, -b)$ lies on the graph of the inverse relation.

b Let $y = f^{-1}(-x)$. Then $-x = f(y)$, so $x = -f(y) = f(-y)$ because f is odd. Hence $-y = f^{-1}(x)$, $y = -f^{-1}(x)$. This proves that $f^{-1}(-x) = -f^{-1}(x)$.

c Functions whose domain is $x = 0$ alone, because if $f(a) = b$, then $f(-a) = b$, so the graph fails the horizontal line test unless $a = -a$, that is, unless $a = 0$.

18 b From part **a** we see, for example, that $g\left(\frac{1}{2}\right) = g(2)$, so the inverse is not a function.

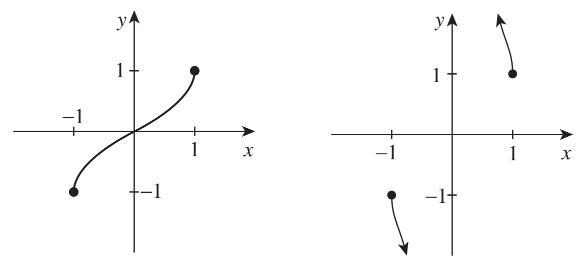
c **i** $-1 \leq x \leq 1$

ii $g^{-1}(x) = \dfrac{1 - \sqrt{1 - x^2}}{x}$

d domain: $x \leq -1$ or $x \geq 1$,

$g^{-1}(x) = \dfrac{1 + \sqrt{1 - x^2}}{x}$

e Because of the result in part **a**.



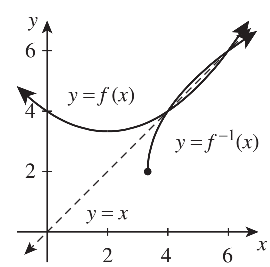
19 a vertex: $\left(2, \frac{10}{3}\right)$, y-intercept: 4

b $x \geq 2$

c $x \geq \frac{10}{3}$

d The easy way is to solve $y = f(x)$ simultaneously with $y = x$. They intersect at $(4, 4)$ and $(6, 6)$.

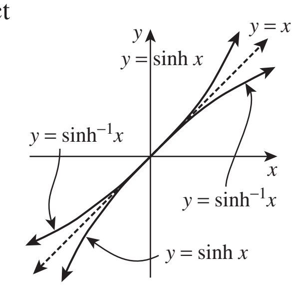
e $4 - N$



20 a all real x

b 0

d $\frac{1}{2}(e^x + e^{-x})$, which is positive for all real x.



e To $y = \frac{1}{2}e^x$ on the right. To $y = -\frac{1}{2}e^{-x}$ on the left.

f $\sinh x$ is a one-to-one function.

Exercise 17B

1 a 1.16 **b** 0.64 **c** 1.32

d 1.67 **e** 1.98 **f** 2.42

2 a 0 **b** $\frac{\pi}{6}$ **c** 0 **d** $\frac{\pi}{4}$

e $-\frac{\pi}{2}$ **f** $\frac{\pi}{2}$ **g** 0 **h** $-\frac{\pi}{4}$

i $-\frac{\pi}{3}$ **j** $\frac{3\pi}{4}$ **k** $-\frac{\pi}{6}$ **l** π

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3 a 1.447 **b** 1.694 **c** 0.730
d -0.730 **e** 1.373 **f** -1.373

4 a $\frac{\pi}{2}$ **b** 1 **c** 1
d $\frac{\pi}{6}$ **e** $\frac{1}{2}$ **f** $\frac{3\pi}{4}$
g $-\frac{\pi}{6}$ **h** 0 **i** $\frac{\pi}{3}$

5 a $-\frac{\pi}{3}$ **b** $\frac{\pi}{4}$ **c** $-\frac{\pi}{6}$
d $\frac{3\pi}{4}$ **e** $-\frac{\pi}{2}$ **f** $\frac{\pi}{3}$

7 a i $\frac{4}{5}$ **ii** $\frac{5}{12}$ **iii** $\frac{1}{3}\sqrt{5}$
iv $\frac{8}{17}$ **v** $\frac{3}{10}\sqrt{10}$ **vi** $-\frac{1}{3}\sqrt{7}$

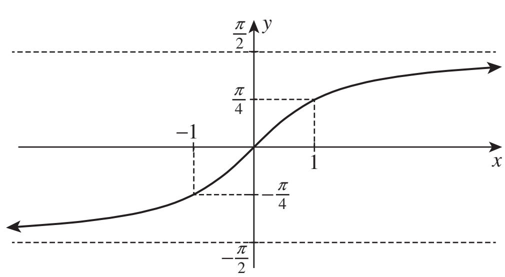
12 c $-\frac{\pi}{2}$

13 a $0 \le y < \frac{\pi}{2}$ **b** $0 < y \le \frac{\pi}{4}$

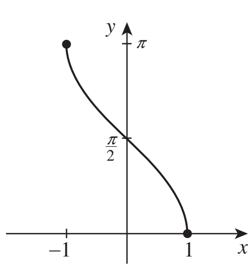
14 a 2 is within the range of the inverse cosine function, which is $0 \le y \le \pi$. However, 2 is outside the range of the inverse sine function, which is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.
b It is because the sine curve is symmetrical about $x = \frac{\pi}{2}$.
c $\pi - 2$

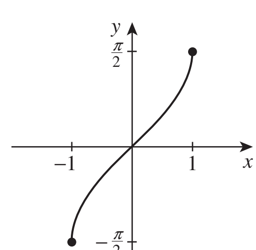
Exercise 17C

1 a domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd

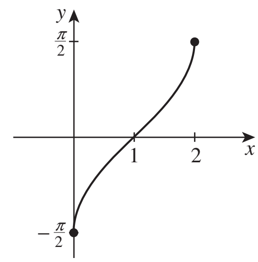


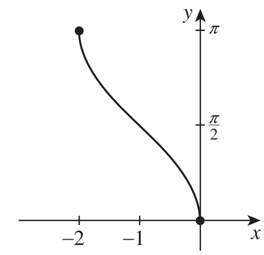
b domain: $-1 \le x \le 1$, range: $0 \le y \le \pi$, neither
c domain: $-1 \le x \le 1$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd



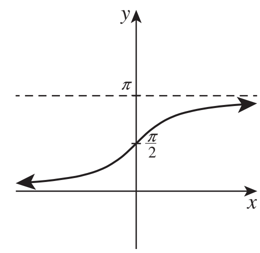


2 a domain: $0 \le x \le 2$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, neither
b domain: $-2 \le x \le 0$, range: $0 \le y \le \pi$, neither

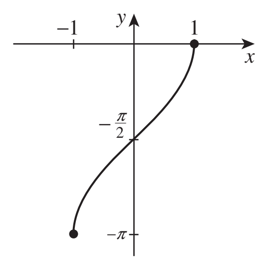


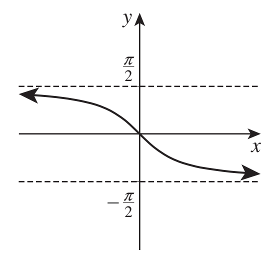


c domain: all real x, range: $0 < y < \pi$, neither

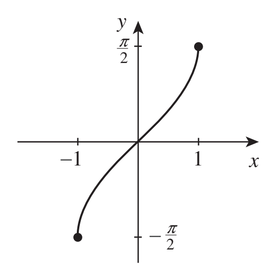


3 a domain: $-1 \le x \le 1$, range: $-\pi \le y \le 0$, neither
b domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd

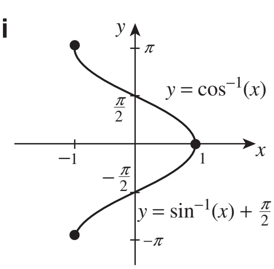




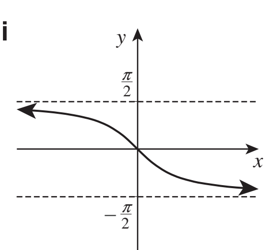
c domain: $-1 \le x \le 1$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd



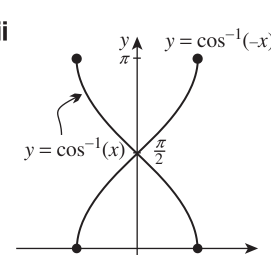
4 a i



b i



ii



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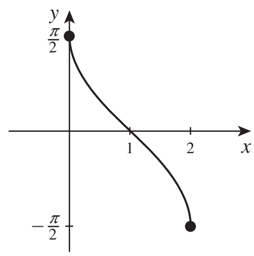
5 a $0 \le x \le 2$

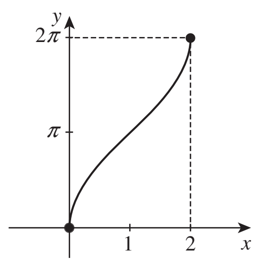
b $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

c $y = \frac{\pi}{2}, 0, -\frac{\pi}{2}$

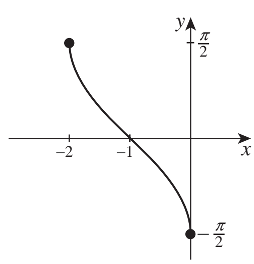
d $x = \frac{1}{2}$



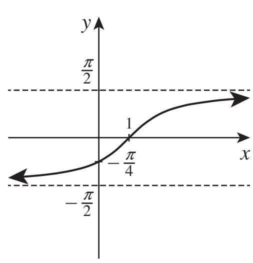
6 a domain: $0 \le x \le 2$, range: $0 \le y \le \pi$



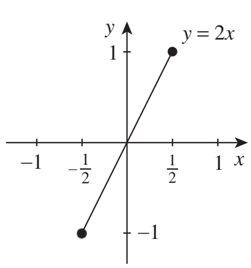
b domain: $-2 \le x \le 0$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



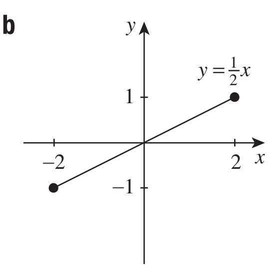
c domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



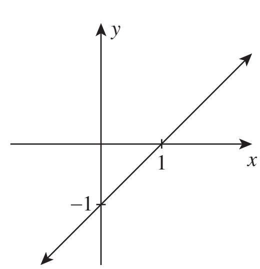
7 a



b



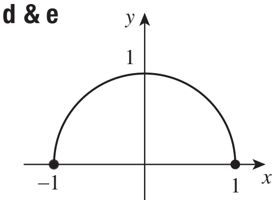
c The first two are odd, and the third is neither even nor odd.



8 a $-1 \le x \le 1$, even

b $0 \le \cos^{-1} x \le \pi$, so $\sin(\cos^{-1} x) \ge 0$.

d & e

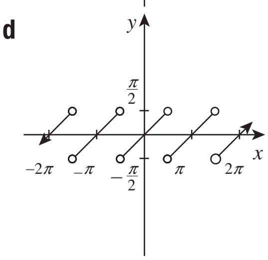


9 a domain: all real x, $x \ne \dfrac{(2n+1)\pi}{2}$, where n is an integer, range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$, odd

b x

c π

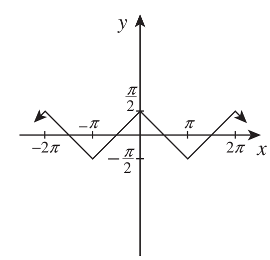
d



10 $\sin^{-1}(\cos x)$

$= \frac{\pi}{2} - \cos^{-1}(\cos x)$,

so we reflect in the x-axis and then shift $\frac{\pi}{2}$ units up. It is even.

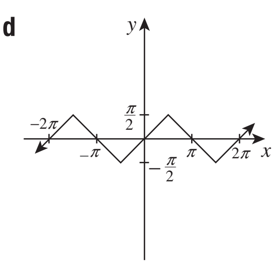


11 a domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, period: 2π, odd

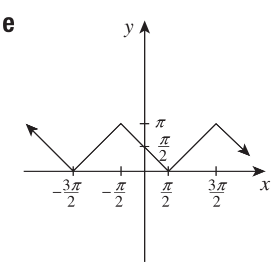
b x

c $\cos^{-1} \sin x = \frac{\pi}{2} - \sin^{-1} \sin x$, so we reflect in the x-axis and then shift $\frac{\pi}{2}$ units up.

d



e



Exercise 17D

1 a $\sin x \cos y - \cos x \sin y$

b $\cos 2A \cos 3B - \sin 2A \sin 3B$

c $\sin 3\alpha \cos 5\beta + \cos 3\alpha \sin 5\beta$

d $\cos\theta \cos\dfrac{\phi}{2} + \sin\theta \sin\dfrac{\phi}{2}$

e $\dfrac{\tan A + \tan 2B}{1 - \tan A \tan 2B}$

f $\dfrac{\tan 3\alpha - \tan 4\beta}{1 + \tan 3\alpha \tan 4\beta}$

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2 a $\cos(x + y)$ **b** $\sin(3\alpha + 2\beta)$ **c** $\tan 20°$
d $\sin 3A$ **e** $\cos 50°$ **f** $\tan(\alpha + 10°)$

5 a 1 **b** $\frac{63}{65}$ **c** $\frac{\sqrt{5}(1 + 2\sqrt{3})}{12}$

7 b i $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ **ii** $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ **iii** $2 + \sqrt{3}$

9 a $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ **b** $\frac{1 - \sqrt{3}}{2\sqrt{2}}$ **c** $\sqrt{3} - 2$

10 a 1 **b** $\frac{56}{65}$
c $\frac{56}{33}$ **d** $\frac{10\sqrt{3} - \sqrt{5}}{20}$

11 b $x = \frac{1}{2}$ (note that $x \neq -1$)

12 a $x = \frac{1}{3}$ **b** $x = \frac{1}{3}$ or 1

13 b $\frac{\sqrt{3}}{2}$

16 a $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

18 $x = -\frac{3}{2}$ or $\frac{1}{3}$

Exercise 17E

1 a $\sin 2x$ **b** $\cos 2\theta$ **c** $\tan 2\alpha$
d $\sin 40°$ **e** $\cos 100°$ **f** $\tan 140°$
g $\sin 6\theta$ **h** $\cos 4A$ **i** $\tan 8x$

3 a $\frac{7}{25}$ **b** $\frac{1}{9}$ **c** $\frac{120}{169}$ **d** $\frac{4}{3}$

4 $-\frac{3\sqrt{7}}{8}$

8 a $-\frac{7}{9}$ **b** $\frac{12\sqrt{13}}{49}$ **c** $\frac{4}{3}$

9 b $\sqrt{2} - 1$

10 a $y = 2x^2 - 8x + 7$ **b** $y = \dfrac{2x - 2}{2x - x^2}$

11 a $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

b $\tan\theta = \dfrac{h}{30}$, $\tan 2\theta = \dfrac{h}{10}$

e $\angle AWB = \theta$ using the exterior angle $\angle WBC$ of ΔABW. Hence ΔABW is isosceles with $BW = BA = 20$ m, because the base angles are equal. Now use Pythagoras' theorem in ΔBCW.

16 a $\frac{1}{2}\sqrt{2}$

Exercise 17F

1 a $\dfrac{2t}{1 + t^2}$ **b** $\dfrac{1 - t^2}{1 + t^2}$ **c** $\dfrac{2t}{1 - t^2}$

d $\dfrac{1 + t^2}{1 - t^2}$ **e** $\dfrac{2t^2}{1 + t^2}$ **f** t

2 a $\dfrac{1 - t^2}{1 + t^2}$ **b** $\dfrac{(1 - t)^2}{1 + t^2}$ **c** $\dfrac{1 + t}{1 - t}$

3 a $\tan 20°$ **b** $\sin 20°$ **c** $\cos 20°$
d $\sin 4x$ **e** $\tan 4x$ **f** $\cos 4x$

4 a $\tan 30° = \frac{1}{\sqrt{3}}$ **b** $\sin 30° = \frac{1}{2}$

c $\cos 150° = -\frac{\sqrt{3}}{2}$ **d** $\sin 225° = -\frac{1}{\sqrt{2}}$

e $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ **f** $\tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$

6 c $x = 0, \frac{3\pi}{2}, 2\pi$

7 b $A = 0, \frac{2\pi}{3}, 2\pi$

8 a ii $-\sqrt{2} - 1 = \tan 112\frac{1}{2}°$,

because $\tan 225° = \tan 45° = 1$.

10 a $-\frac{3}{4}$ **b** $-\frac{3}{5}$ **c** $\frac{4}{5}$ **d** $3 + \sqrt{10}$

11 a i $\cos\theta = 2\cos^2\frac{1}{2}\theta - 1$

b ii $\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$

Exercise 17G

1 b i $\cos 50° + \cos 20°$ **ii** $\sin 80° - \sin 16°$
iii $\frac{1}{2}(\cos 27° - \cos 71°)$ **iv** $\frac{1}{2}(\sin 86° - \sin 36°)$
v $\sin 4\alpha + \sin 2\alpha$ **vi** $\frac{1}{2}(\cos 3\theta + \cos\theta)$
vii $\cos 2y - \cos 2x$ **viii** $\frac{1}{2}(\sin 4A + \sin 2B)$

7 b ii $\frac{1}{2}$

8 $\sin 3x = \frac{1}{2}$, $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

12 a $\cos 3x = 0$ or $\sin x = 0$, $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$, $0, \pi, 2\pi$

b $\sin 5x = 0$ or $\cos 2x = 0$, $x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi$, $\frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}, 2\pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

c $\cos 4x = 0$ or $\cos x = 0$, $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}$, $\frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, \frac{\pi}{2}, \frac{3\pi}{2}$

d $\sin\dfrac{5x}{2} = 0$ or $\sin\dfrac{x}{2} = 0$, $x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

Chapter 17 review exercise

1 a $-1 \le f(x) \le 4$
b $f^{-1}(x) = 2x - 2$
c $-1 \le x \le 4$, $-4 \le f^{-1}(x) \le 6$
d $y = x$

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2 a $x > -1$, all real y

b $F^{-1}(x) = e^x - 1$

c All real x, $y > -1$

e Both functions are increasing.

3 b $x \le 2$

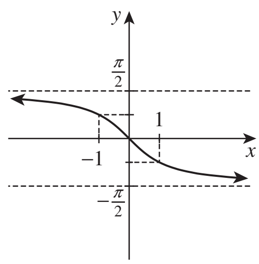
c $Q^{-1}(x) = 2 - \sqrt{x}$, because $Q^{-1}(x) \le 2$.

4 a $\frac{\pi}{2}$ **b** $\frac{\pi}{6}$ **c** $\frac{\pi}{3}$ **d** $-\frac{\pi}{4}$ **e** $\frac{2\pi}{3}$ **f** $-\frac{\pi}{6}$

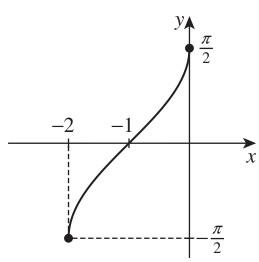
5 a 1 **b** $\frac{1}{\sqrt{2}}$ **c** $\frac{1}{2}$ **d** $-\frac{\pi}{6}$ **e** $\frac{2\pi}{3}$ **f** $-\frac{\pi}{3}$

6 a $\frac{2\sqrt{2}}{3}$ **b** $\frac{3}{4}$

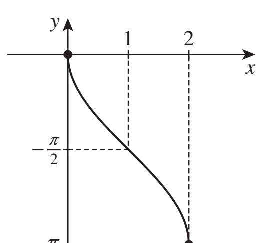
7 a All real x, $-\frac{\pi}{2} < y < \frac{\pi}{2}$



b $-2 \le x \le 0$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



c $0 \le x \le 2$, $-\pi \le y \le 0$



8 a $\cos 2\theta$ **b** $\sin 40°$ **c** $\tan 50°$

d $\cos 70°$ **e** $\sin 6\alpha$ **f** $\frac{1}{\tan\theta} = \cot\theta$

9 a $\sin 4\theta$ **b** $\cos x$ **c** $\cos 6\alpha$

d $\tan 70°$ **e** $\cos 50°$ **f** $\tan 8x$

10 a $\frac{4}{5}$ **b** $\frac{7}{25}$ **c** $-\frac{16}{65}$ **d** $\frac{120}{169}$ **e** $\frac{24}{7}$ **f** $\frac{33}{56}$

12 a $\sin 30° = \frac{1}{2}$ **b** $\cos 30° = \frac{\sqrt{3}}{2}$

c $\tan 135° = -1$ **d** $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

e $\frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{4}$ **f** $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

15 b $\frac{\pi}{4}$

18 a $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$ or $\frac{11\pi}{6}$

b $x = \frac{\pi}{6}, \frac{5\pi}{6}$ or $\frac{3\pi}{2}$

c $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

d $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$ or 2π

19 a $\dfrac{2t}{1+t^2}$ **b** $\dfrac{1-t^2}{1+t^2}$ **c** $\dfrac{2t}{1-t^2}$

d $\dfrac{1+t^2}{1-t^2}$ **e** $\dfrac{2}{1+t^2}$ **f** t

20 a $\tan 135° = -1$

b $\sin 135° = \frac{1}{\sqrt{2}}$

c $\cos 135° = -\frac{1}{\sqrt{2}}$

22 b ii $-2 + \sqrt{3} = \tan 165°$, because $\tan 330° = -\tan 30° = -\frac{1}{\sqrt{3}}$.

23 a $\cos 3A + \cos A$

b $\sin 8A - \sin 2A$

c $\sin(5A + B) + \sin(A + 3B)$

d $\cos(A + 2B) - \cos(3A + 8B)$

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