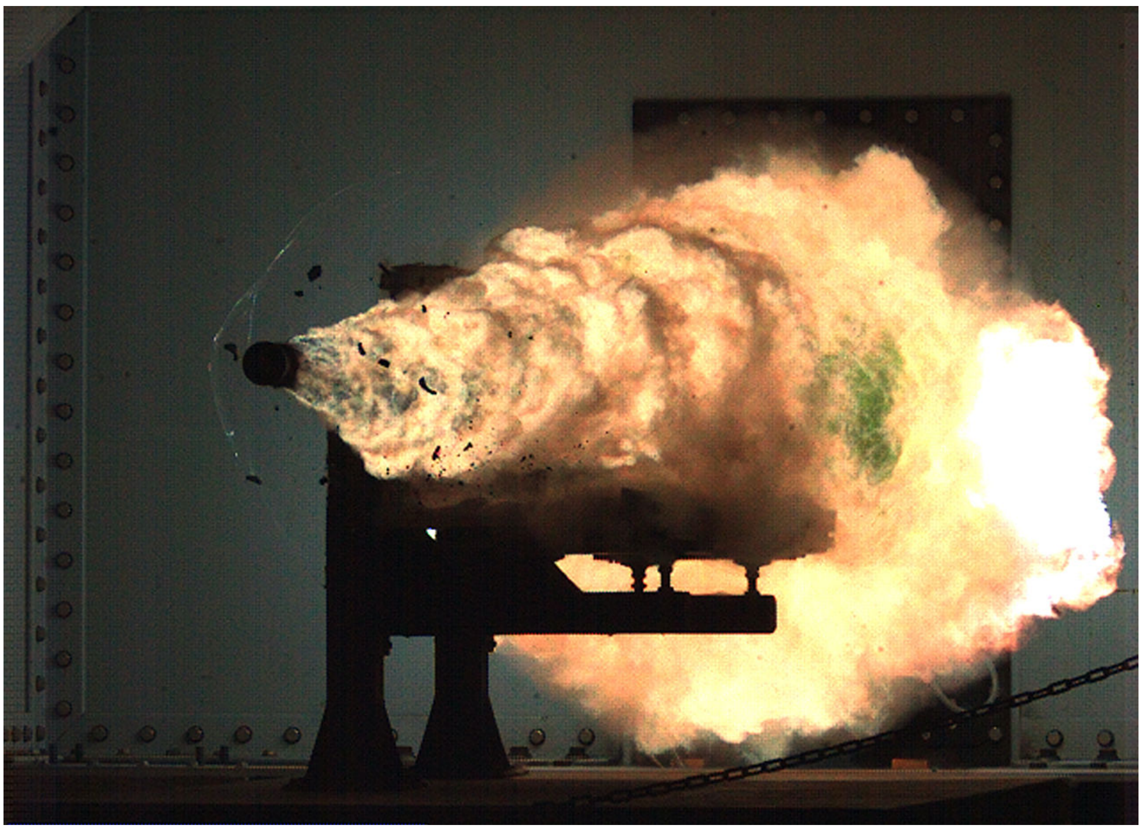
Module 6: Electromagnetism

Part 2: The motor effect



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Cover photo credit: Photograph taken from a high-speed video camera during a record-setting firing of an electromagnetic railgun (EMRG) at Naval Surface Warfare Center, Dahlgren, Va., on January 31, 2008, firing a 3.2 kg projectile at 10.64MJ (megajoules) with a muzzle velocity of 2520 meters per second. U.S. Navy Photograph (Released) `https://www.navy.mil/view_image.asp?id=54942`

Syllabus content: Electromagnetism

The motor effect

Inquiry question: Under what circumstances is a force produced on a current-carrying conductor in a magnetic field?

Students:

- investigate qualitatively and quantitatively the interaction between a current-carrying conductor and a uniform magnetic field $F = lI_{\perp}B = lIB\sin\theta$ to establish: (ACSPH080, ACSPH081)
  - conditions under which the maximum force is produced
  - the relationship between the directions of the force, magnetic field strength and current
  - conditions under which no force is produced on the conductor
- conduct a quantitative investigation to demonstrate the interaction between two parallel current-carrying wires
- analyse the interaction between two parallel current-carrying wires $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$ and determine the relationship between the International System of Units (SI) definition of an ampere and Newton's Third Law of Motion (ACSPH081, ACSPH106)

The force on a current carrying conductor in a magnetic field

Review of the force on a charge moving in a magnetic field

In the previous part of this topic, we covered the force that acts on a charge moving in a magnetic field,

$$F = qvB\sin\theta \tag{1}$$

In this section, we will investigate what happens if that moving charge is part of a current flowing in a conductor. Consider the situation shown in figure 3 Using equation 1, we can write the magnitude

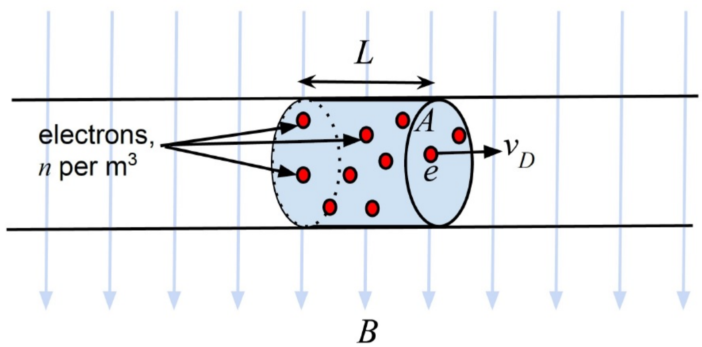


Figure 1: A small section of a conductor of length L and cross-sectional area A. It contains n free charges per unit volume, each moving with a drift velocity of v_D.

of the force acting on *each electron* moving in the conductor as

$$F = qv_D B\sin\theta \tag{2}$$

where q is the charge on an electron, v_D is the drift velocity of the electron due to the electric field in the wire that is driving the current, B is magnetic field strength, and θ is the angle between the direction the charges are flowing in the wire and the magnetic field.

The total number of free electrons in the volume of wire shown is given by $N = nAL$, so that means that the total force on the *wire* due to the force on all the electron in a volume $A \times L$ is given by

$$F = BnqAv_D L\sin\theta$$

The current flowing in the wire is defined as the total charge crossing a surface through the wire per unit time, and we will need to obtain an expression for current in terms of the variables we have in our equation.

Here's the bit that is slightly tricky: we note that the electrons in the wire move v_D meters every second. If we pick a particular cross-section in the wire, electrons from a distance $(v_D\text{ms}^{-1} \times 1\text{s})$ back in the wire will all cross this surface over the course of 1s. The total number of electrons crossing that surface in 1s is then $nAv_D \times 1\text{s}$ (the number of electrons in this volume of the wire)

The current (total charge crossing this surface per second) is therefore given by:

$$I = nqAv_D$$

Using this relationship, we can rewrite equation 2 for the force on the length of conductor in figure 3 as

$$F = BIL\sin\theta \tag{3}$$

The direction of the force on the wire is given by the right hand rule (for positive charges).

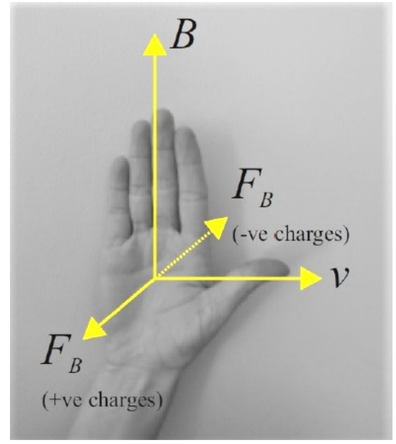
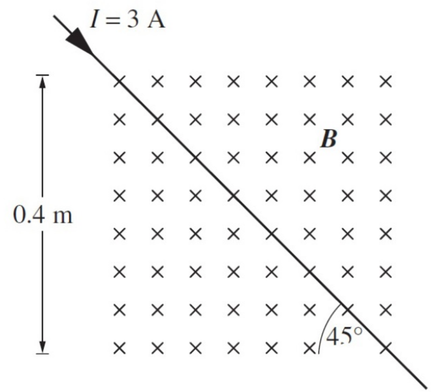


Figure 2: The direction of the force on a current carrying wire is given by the right hand rule.

Example 1.

A current-carrying conductor passes through a square region of magnetic field, magnitude 0.5 T, as shown in the diagram. The magnetic field is directed into the page.



What is the magnitude of the magnetic force on the conductor?

(A) 0.170 N

(B) 0.424 N

(C) 0.600 N

(D) 0.849 N

Figure 3: Example from 2006 HSC Q7

The force between two parallel current carrying wires

Revision: The magnetic field produced by a current carrying wire

In year 11 we discussed the fact that moving charges produce a magnetic field equal to:

$$B = \frac{\mu_0 I}{2\pi r} \tag{4}$$

where $\mu_0 = 1.257 \times 10^{-6}\text{mkgs}^{-2}\text{A}^{-2}$ is the 'permeability of free space' (which is related to the speed of light), I is the current and r is the perpendicular distance from the axis of the current carrying wire.

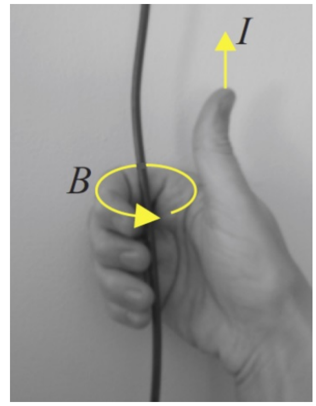


Figure 4: The right-hand "grip" rule is used to determine the direction of the magnetic field produced by a current.

Putting it all together - a current carrying wire in the B field from another current carrying wire

In the previous section of this topic, we have introduced the idea that magnetic fields exert forces on moving charges, and derived an expression for the force acting on a current carrying wire in a magnetic field (equation 3):

$$F = lIB\sin\theta$$

When current carrying wires are parallel to each other, one wire *makes* a magnetic field around it, which then exerts a force on the other wire. So the force experienced by the second wire is:

$$F = l\frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} \tag{5}$$

Here $\sin\theta = 1$ as the magnetic field produced by wire 1 is perpendicular to the direction of the current in wire 2.

Similarly, the second current carrying wire also produces a magnetic field which will exert a force on the first wire. By Newton's 2nd law, these two forces must be equal and opposite.

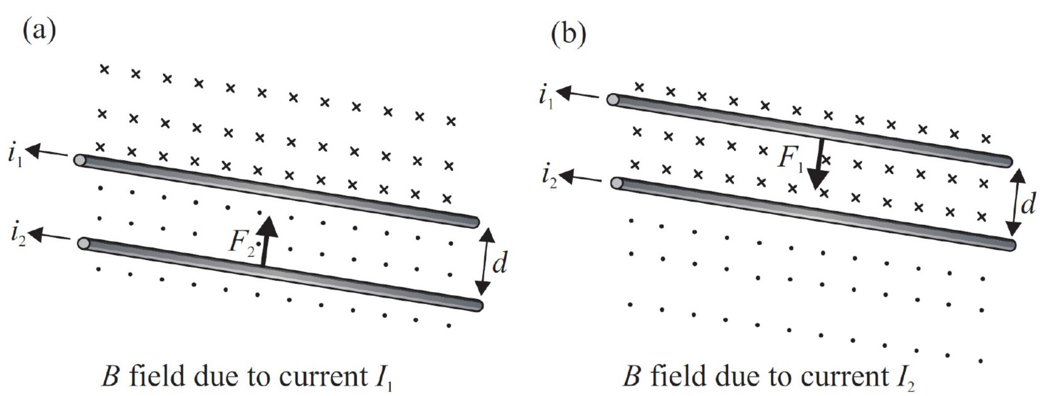


Figure 5: The force between wires carrying currents in the same direction is attractive.

If the wires are carrying currents in opposite directions, then they repel each other, as shown in figure 6.

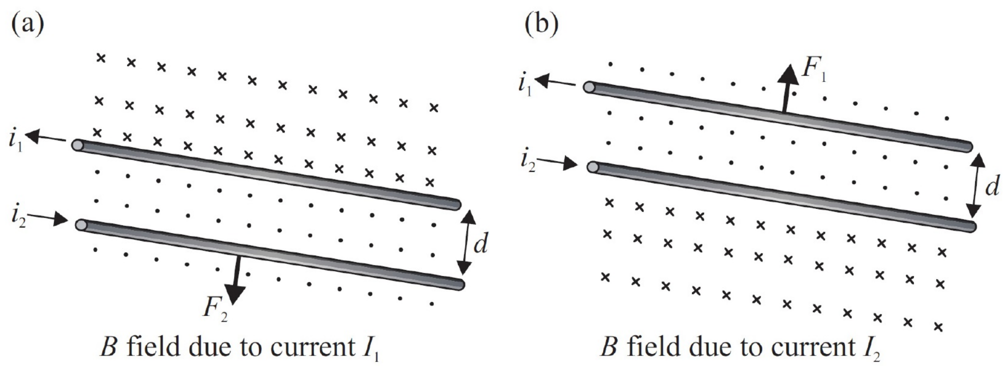
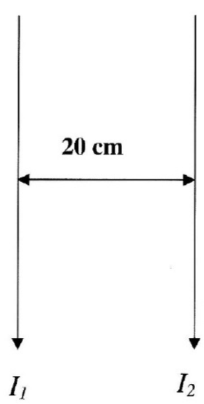


Figure 6: The force between wires carrying currents in the opposite direction is repulsive.

Example 2.

The following diagram shows two current-carrying wires separated by a distance of 20 cm. Current $I_1 = 2A$ and current $I_2 = 5A$.



The force per metre between the two wires is closest to:

(A) 1×10^{-5} N
(B) 1×10^{-8} N
(C) 5×10^{-5} N
(D) 2 N

The definition of the Ampere

The International System of Units (SI) definition of an ampere was previously:

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per meter of length.

This definition was problematic because force was still defined in terms of the kilogram, which was itself defined as the mass of a particular object - the international prototype of the kilogram.

In 2019 this definition was changed to:

> The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602176634 \times 10^{-19}$C. Then $1\text{C} = 1\text{A.s}$, where the second is defined in terms of the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom.

A couple of videos on this change: From NPL: `https://www.youtube.com/watch?v=thfSSEvBDVY` From NIST: `https://www.nist.gov/news-events/news/2016/08/counting-down-new-ampere`

In the syllabus, you are required to "determine the relationship between the International System of Units (SI) definition of an ampere and Newton"s Third Law of Motion".

This appears to be requiring you to note that if the force on one of the wires is 2×10^{-7}N, then by Newton's third law the force on the other wire is equal in magnitude and opposite in direction.

Worked Examples

Worked Example 1. The trick to this question is to notice that the angle between the direction of the current and the field is 90°. Then

$$F = BIL$$

where the length of wire in the field is given by $L = \sqrt{2 \times (0.4\text{m})^2} = 0.57\text{m}$, $B = 0.5\text{T}$ and $I = 3\text{A}$. So

$$F = 0.5\text{T} \times 3\text{A} \times 0.57\text{m} = 0.849\text{N}$$

Worked Example 2.

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

where $r = 0.2\text{m}$, $\mu_0 = 1.257 \times 10^{-6}\text{mkgs}^{-2}\text{A}^{-2}$, $I_1 = 2\text{A}$, $I_2 = 5\text{A}$, so

$$F = l\frac{1.257 \times 10^{-6}\text{mkgs}^{-2}\text{A}^{-2}}{2\pi}\frac{2\text{A} \times 5\text{A}}{0.2\text{m}} = 1 \times 10^{-5}\text{Nm}^{-1}(1s.f.)$$