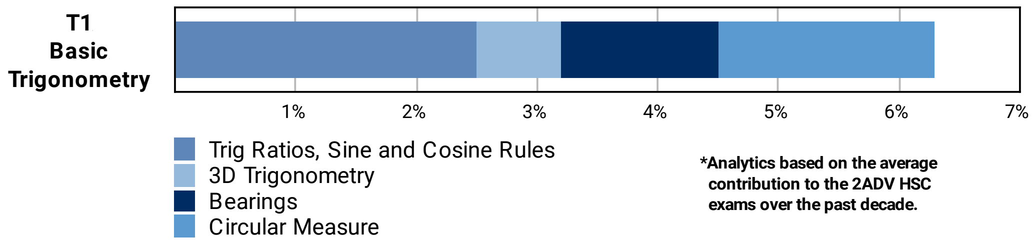
ADV: Trigonometry (Adv), T1 Trigonometry and Measure of Angles (Adv)

Bearings (Y11)
3D Trigonometry (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 100.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

T1 Trigonometry and Measure of Angles



HISTORICAL CONTRIBUTION

- *T1 Trigonometry and Measure of Angles* is a mixture of content that previously belonged to the Standard 2, Mathematics and Ext1 courses. Our analysis has it accounting for an estimated 6.3% of past papers.
- This topic has been split into four sub-topics for analysis purposes: *1-Trig Ratios, Sine and Cosine Rules (2.5%), and 2- 3D Trigonometry (0.7%), Bearings (1.3%) and Circular Measure (1.8%).*
- This analysis looks at the sub-topic *Bearings*.

HSC ANALYSIS - What to expect and common pitfalls

- *Bearings* is a prime topic area to test Advanced and Standard 2 students with common content. The 2020 exam confirmed this with a common question worth a substantial 5 marks.
- In our view, bearings' contribution to Advanced exams will be meaningfully higher than it has been historically, primarily due to the common content described above.
- Before 2020, bearings was most recently tested in the *Advanced exam* in 2018 and 2014. With good mark allocations on offer when it is asked, a revision focus is warranted here.
- A number of Std2 past HSC questions have been included in this database (identified by 2ADV* in the title).

Questions

1. Measurement, STD2 M6 SM-Bank 2 MC

Which of the following expresses S60°W as a true bearing?

A. 030°

B. 060°

C. 120°

D. 240°

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2. Measurement, STD2 M6 SM-Bank 3 MC

Which of the following expresses S65°W as a true bearing?

A. 065°

B. 155°

C. 245°

D. 295°

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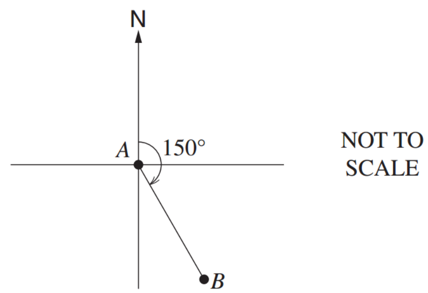
3. Trigonometry, 2ADV* T1 2010 HSC 10 MC

A plane flies on a bearing of 150° from A to B.



What is the bearing of A from B?

(A) 30°

(B) 150°

(C) 210°

(D) 330°

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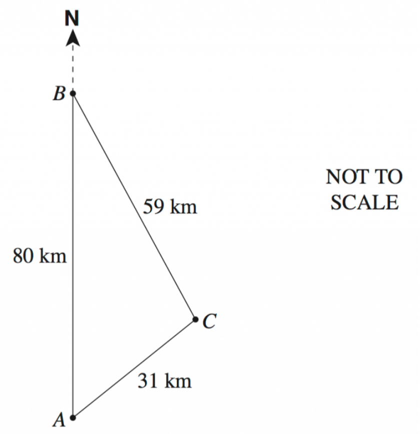
4. Trigonometry, 2ADV* T1 2012 HSC 20 MC

Town B is 80 km due north of Town A and 59 km from Town C.

Town A is 31 km from Town C.



What is the bearing of Town C from Town B?

(A) $019°$

(B) $122°$

(C) $161°$

(D) $341°$

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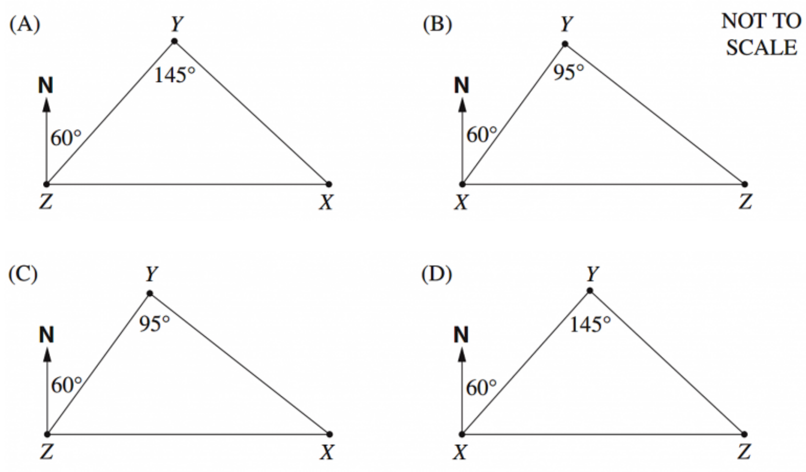
5. Trigonometry, 2ADV* T1 2014 HSC 23 MC

The following information is given about the locations of three towns X, Y and Z:

- X is due east of Z
- X is on a bearing of $145°$ from Y
- Y is on a bearing of $060°$ from Z.

Which diagram best represents this information?



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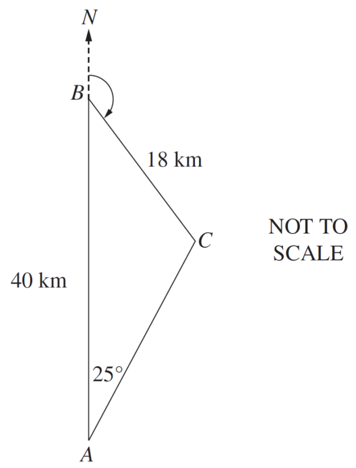
6. Trigonometry, 2ADV* T1 2016 HSC 25 MC

The diagram shows towns A, B and C. Town B is 40 km due north of town A. The distance from B to C is 18 km and the bearing of C from A is 025°. It is known that $\angle BCA$ is obtuse.



What is the bearing of C from B?

(A) 070°

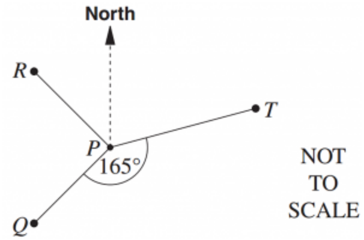
(B) 095°

(C) 110°

(D) 135°

7. Trigonometry, 2ADV* T1 2008 HSC 17 MC

The diagram shows the position of Q, R and T relative to P.



In the diagram,

Q is south-west of P

R is north-west of P

$\angle QPT$ is 165°

What is the bearing of T from P?

(A) 060°

(B) 075°

(C) 105°

(D) 120°

8. Measurement, STD2 M6 2019 HSC 4 MC

Which compass bearing is the same as a true bearing of 110°?

A. $\text{S}20^\circ\text{E}$

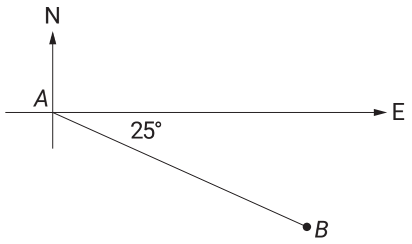
B. $\text{S}20^\circ\text{W}$

C. $\text{S}70^\circ\text{E}$

D. $\text{S}70^\circ\text{W}$

9. Measurement, STD2 M6 2021 HSC 14 MC

Consider the diagram below.



NOT TO SCALE

What is the true bearing of A from B?

- **A.** $025°$
- **B.** $065°$
- **C.** $115°$
- **D.** $295°$

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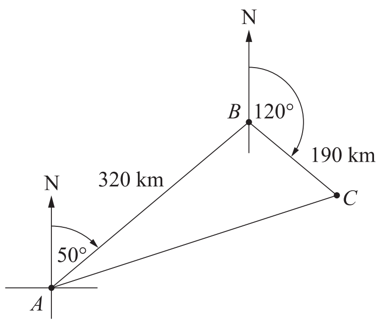
10. Trigonometry, 2ADV T1 2018 HSC 12a

A ship travels from Port *A* on a bearing of 050° for 320 km to Port *B*. It then travels on a bearing of 120° for 190 km to Port *C*.



NOT TO SCALE

i. What is the size of $\angle ABC$? ***(1 mark)***

ii. What is the distance from Port *A* to Port *C* ? Answer to the nearest 10 kilometres. ***(2 marks)***

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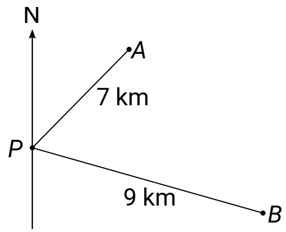
11. Trigonometry, 2ADV T1 2020 HSC 15

Mr Ali, Ms Brown and a group of students were camping at the site located at P. Mr Ali walked with some of the students on a bearing of 035° for 7 km to location A. Ms Brown, with the rest of the students, walked on a bearing of 100° for 9 km to location B.



NOT TO SCALE

a. Show that the angle APB is 65°. ***(1 mark)***

b. Find the distance AB. ***(2 marks)***

c. Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree. ***(2 marks)***

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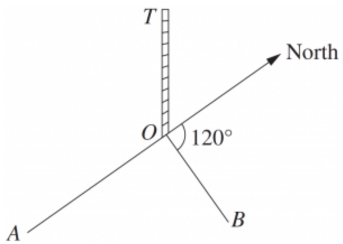
12. Trigonometry, 2ADV' T1 2008 HSC 6a

From a point A due south of a tower, the angle of elevation of the top of the tower T, is 23°. From another point B, on a bearing of 120° from the tower, the angle of elevation of T is 32°. The distance AB is 200 metres.



i. Copy or trace the diagram into your writing booklet, adding the given information to your diagram. ***(1 mark)***

ii. Hence find the height of the tower. Give your answer to the nearest metre. ***(3 marks)***

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13. Trigonometry, 2ADV' T1 2010 HSC 5a

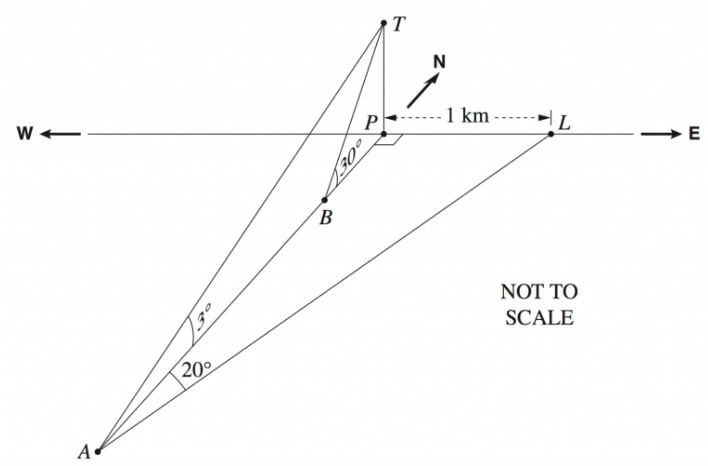
A boat is sailing due north from a point A towards a point P on the shore line.

The shore line runs from west to east.

In the diagram, T represents a tree on a cliff vertically above P, and L represents a landmark on the shore. The distance PL is 1 km.

From A the point L is on a bearing of 020°, and the angle of elevation to T is 3°.

After sailing for some time the boat reaches a point B, from which the angle of elevation to T is 30°.



i. Show that $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$. *(3 marks)*

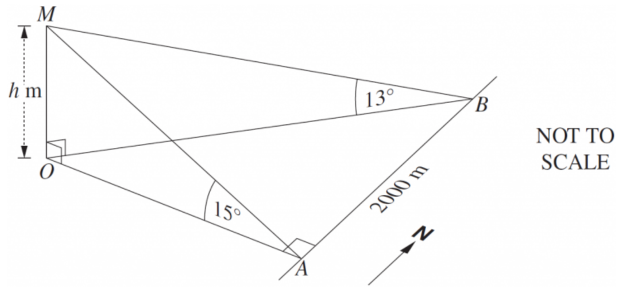
ii. Find the distance AB. Give your answer to 1 decimal place. *(1 mark)*

14. Trigonometry, 2ADV' T1 2015 HSC 12c

A person walks 2000 metres due north along a road from point A to point B. The point A is due east of a mountain OM, where M is the top of the mountain. The point O is directly below point M and is on the same horizontal plane as the road. The height of the mountain above point O is h metres.

From point A, the angle of elevation to the top of the mountain is 15°.

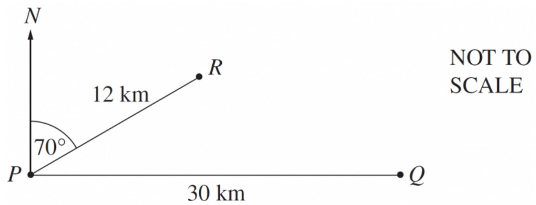
From point B, the angle of elevation to the top of the mountain is 13°.



i. Show that $OA = h\cot 15^\circ$. *(1 mark)*

ii. Hence, find the value of h. *(2 marks)*

15. Trigonometry, 2ADV T1 2004 HSC 3c



The diagram shows a point P which is 30 km due west of the point Q.

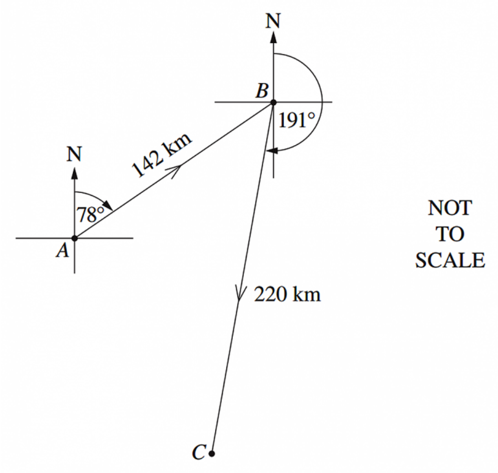
The point R is 12 km from P and has a bearing from P of 070°.

i. Find the distance of R from Q. *(2 marks)*

ii. Find the bearing of R from Q. *(2 marks)*

16. Trigonometry, 2ADV T1 2014 HSC 13d

Chris leaves island A in a boat and sails 142 km on a bearing of 078° to island B. Chris then sails on a bearing of 191° for 220 km to island C, as shown in the diagram.



i. Show that the distance from island C to island A is approximately 210 km. ***(2 marks)***

ii. Chris wants to sail from island C directly to island A. On what bearing should Chris sail? Give your answer correct to the nearest degree. ***(3 marks)***

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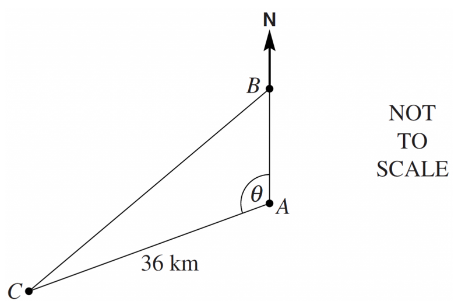
17. Trigonometry, 2ADV* T1 2005 HSC 27c



The bearing of C from A is 250° and the distance of C from A is 36 km.

i. Explain why θ is 110°. ***(1 mark)***

ii. If B is 15 km due north of A, calculate the distance of C from B, correct to the nearest kilometre. ***(3 marks)***

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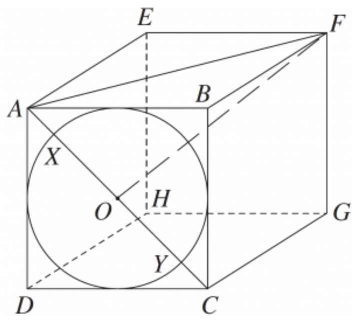
18. Trigonometry, 2ADV' T1 2004 HSC 3d



The length of each edge of the cube $ABCDEFGH$ is 2 metres. A circle is drawn on the face $ABCD$ so that it touches all four edges of the face. The centre of the circle is O and the diagonal AC meets the circle at X and Y.

i. Explain why $\angle FAC = 60^\circ$. ***(1 mark)***

ii. Show that $FO = \sqrt{6}$ metres. ***(1 mark)***

iii. Calculate the size of $\angle XFY$ to the nearest degree. ***(1 mark)***

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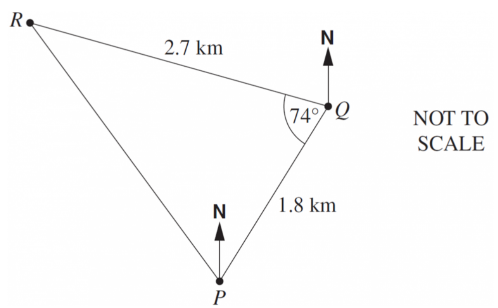
19. Trigonometry, 2ADV* T1 2009 HSC 27b

A yacht race follows the triangular course shown in the diagram. The course from P to Q is 1.8 km on a true bearing of 058°.

At Q the course changes direction. The course from Q to R is 2.7 km and $\angle PQR = 74^\circ$.



i. What is the bearing of R from Q? ***(1 mark)***

ii. What is the distance from R to P? ***(2 marks)***

iii. The area inside this triangular course is set as a 'no-go' zone for other boats while the race is on. What is the area of this 'no-go' zone? ***(1 mark)***

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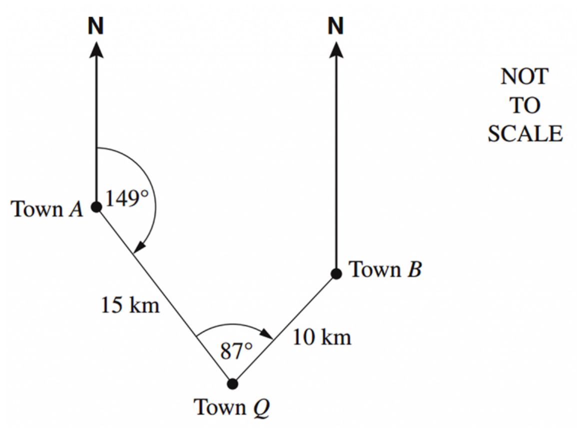
20. Trigonometry, 2ADV* T1 2007 26a

The diagram shows information about the locations of towns A, B and Q.



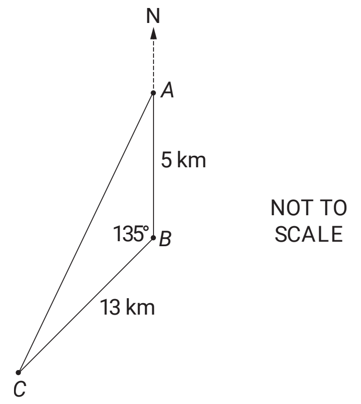
i. It takes Elina 2 hours and 48 minutes to walk directly from Town A to Town Q. Calculate her walking speed correct to the nearest km/h. ***(1 mark)***

ii. Elina decides, instead, to walk to Town B from Town A and then to Town Q. Find the distance from Town A to Town B. Give your answer to the nearest km. ***(2 marks)***

iii. Calculate the bearing of Town Q from Town B. ***(1 mark)***

21. Trigonometry, 2ADV* T1 2017 HSC 30c

The diagram shows the location of three schools. School A is 5 km due north of school B, school C is 13 km from school B and $\angle ABC$ is 135°.

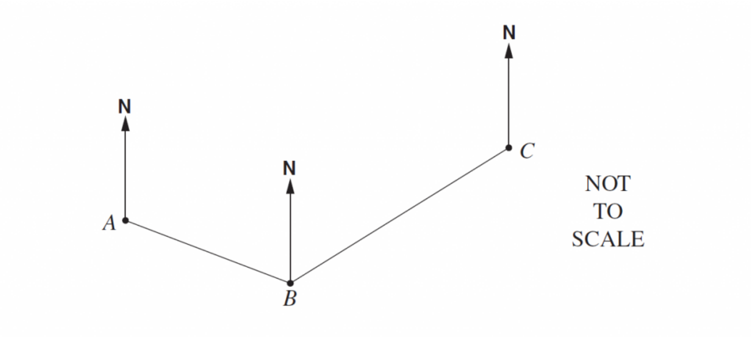


i. Calculate the shortest distance from school A to school C, to the nearest kilometre. ***(2 marks)***

ii. Determine the bearing of school C from school A, to the nearest degree. ***(3 marks)***

22. Trigonometry, 2ADV* T1 2011 HSC 24c

A ship sails 6 km from A to B on a bearing of 121°. It then sails 9 km to C. The size of angle ABC is 114°.



Copy the diagram into your writing booklet and show all the information on it.

i. What is the bearing of C from B? ***(1 mark)***

ii. Find the distance AC. Give your answer correct to the nearest kilometre. ***(2 marks)***

iii. What is the bearing of A from C? Give your answer correct to the nearest degree. ***(3 marks)***

23. Trigonometry, 2ADV T1 SM-Bank 1

A tower is built on flat ground.

Three tourists, A, B and C are observing the tower from ground level.

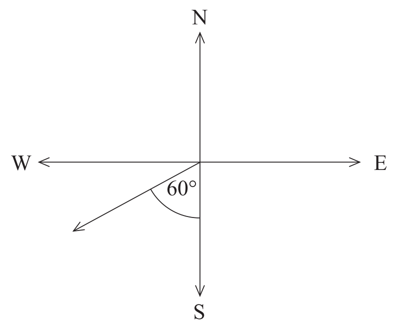
A is due north of the tower, C is due east and B is on the line of sight from A and C and between them.

The angles of elevation to the top of the tower from A, B and C are 26°, 28° and 30°, respectively.

What is the bearing of B from the tower? ***(4 marks)***

Worked Solutions

1. Measurement, STD2 M6 SM-Bank 2 MC



$$\begin{aligned} \text{S}60^\circ\text{W} &= 180 + 60 \\ &= 240^\circ \end{aligned}$$

$\Rightarrow D$

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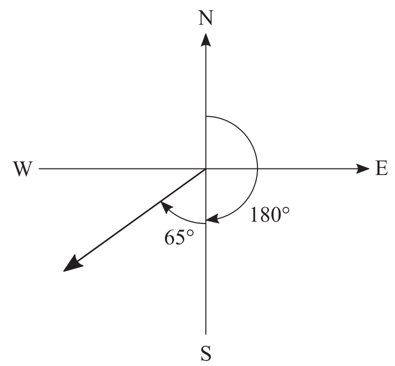
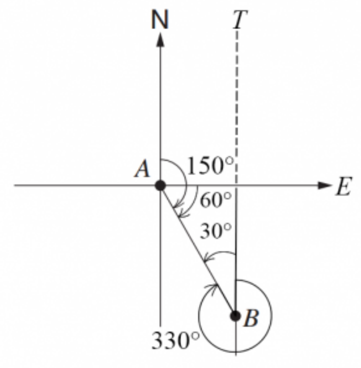
2. Measurement, STD2 M6 SM-Bank 3 MC



$$\begin{aligned} \text{True bearing} &= 180 + 65 \\ &= 245^\circ \end{aligned}$$

$\Rightarrow C$

3. Trigonometry, 2ADV* T1 2010 HSC 10 MC



$\angle TBA = 30°$ (angle sum of triangle)

$\therefore$ Bearing of A from B

$= 360 - 30$

$= 330°$

$\Rightarrow D$

4. Trigonometry, 2ADV* T1 2012 HSC 20 MC

Using the cosine rule:

$$\cos \angle B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{59^2 + 80^2 - 31^2}{2 \times 59 \times 80}$$

$$= 0.9449...$$

$$\angle B = 19° \text{ (nearest degree)}$$

$\therefore$ Bearing of Town C from B $= 180 - 19 = 161°$

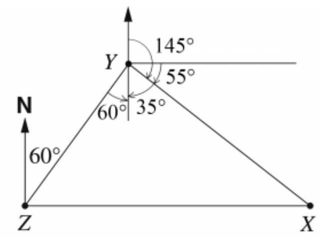
$\Rightarrow C$

5. Trigonometry, 2ADV* T1 2014 HSC 23 MC

Since X is due east of Z

$\Rightarrow$ Cannot be B or D

COMMENT: Drawing a parallel North/South line through Y makes this question ***much simpler*** to solve.



The diagram shows we can find

$\angle ZYX = 60 + 35° = 95°$

Using alternate angles $(60°)$ and the $145°$ bearing of X from Y

$\Rightarrow C$

6. Trigonometry, 2ADV* T1 2016 HSC 25 MC

Using the sine rule,

$$\frac{\sin\angle BCA}{40} = \frac{\sin 25°}{18}$$

$$\sin\angle BCA = \frac{40 \times \sin 25°}{18}$$

$$= 0.939...$$

$$\angle BCA = 180 - 69.9 \quad (\angle BCA > 90°)$$

$$= 110.1^{\circ}$$

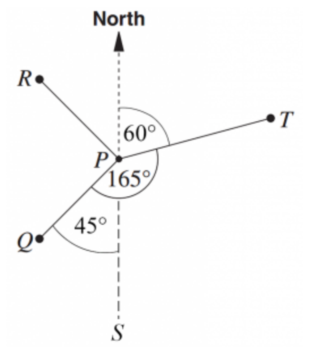
$\therefore$ Bearing of C from B

$= 110.1 + 25$ (external angle of triangle)

$= 135.1$

$\Rightarrow D$

7. Trigonometry, 2ADV* T1 2008 HSC 17 MC



$\angle QPS = 45°$ (Q is south west of P)

$\angle TPS = 165 - 45 = 120°$

$\therefore\ \angle NPT = 60°$ (180° in straight line)

$\Rightarrow\ A$

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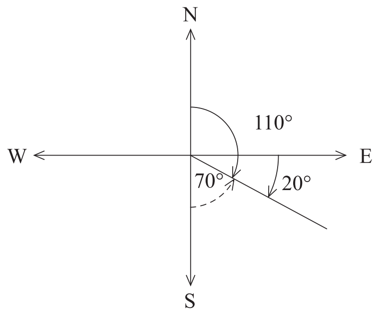
8. Measurement, STD2 M6 2019 HSC 4 MC

♦ *Mean mark 47%.*



$110^{\circ} = \text{S}70^{\circ}\text{E}$

$\Rightarrow C$

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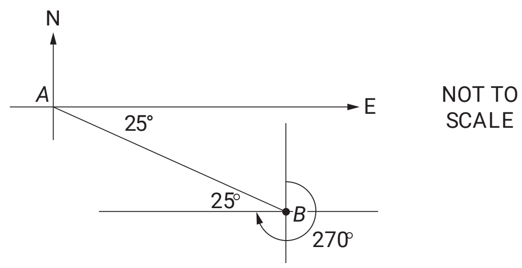
9. Measurement, STD2 M6 2021 HSC 14 MC

♦♦ *Mean mark 28%.*



$$\begin{aligned}\text{Bearing (A from B)} &= 270 + 25 \\ &= 295°\end{aligned}$$

$\Rightarrow D$

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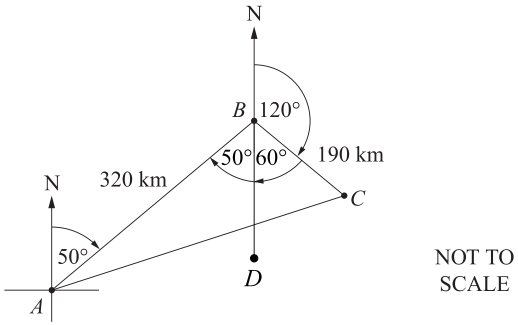
10. Trigonometry, 2ADV T1 2018 HSC 12a

i.



Let D be south of B

$\angle ABD = 50^\circ$ (alternate angles)

$\angle DBC = 60^\circ$ $(180^\circ$ in straight line)

$$\therefore \angle ABC = 50 + 60$$
$$= 110^\circ$$

ii. Using the cosine rule:

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos\angle ABC$$
$$= 320^2 + 190^2 - 2 \times 320 \times 190 \times \cos 110^\circ$$
$$= 180\ 089.64\ldots$$
$$\therefore AC = 424.36\ldots$$
$$= 420 \text{ km (nearest 10 km)}$$

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11. Trigonometry, 2ADV T1 2020 HSC 15

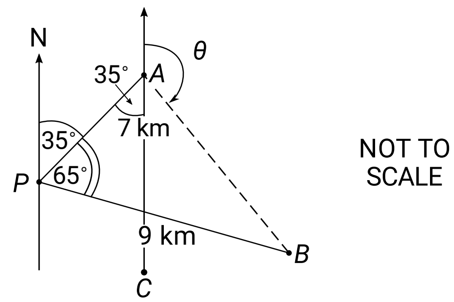
a. $\angle APB = 100 - 35$

$= 65^\circ$

b. Using cosine rule:

$$AB^2 = AP^2 + PB^2 - 2 \times AP \times PB\cos 65^\circ$$
$$= 49 + 81 - 2 \times 7 \times 9\cos 65^\circ$$
$$= 76.750\ldots$$
$$\therefore AB = 8.760\ldots$$
$$= 8.76 \text{ km (to 2 d.p.)}$$

c.



$\angle PAC = 35^\circ$ (alternate)

Using cosine rule, find $\angle PAB$:

$$\cos\angle PAB = \frac{7^2 + 8.76 - 9^2}{2 \times 7 \times 8.76}$$
$$= 0.3647\ldots$$
$$\therefore \angle PAB = 68.61\ldots^\circ$$
$$= 69^\circ \text{ (nearest degree)}$$

$\therefore$ Bearing of B from A (θ)

$$= 180 - (69 - 35)$$
$$= 146^\circ$$

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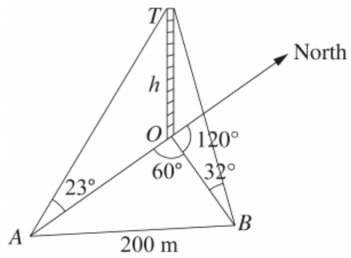
12. Trigonometry, 2ADV' T1 2008 HSC 6a

i.



ii. Find $OT = h$

Using the cosine rule in ΔAOB:

$$200^2 = OA^2 + OB^2 - 2 \cdot OA \cdot OB \cdot \cos 60 \ \ldots \ (*)$$

$$\text{In } \Delta OAT, \tan 23^\circ = \frac{h}{OA}$$

$$\Rightarrow OA = \frac{h}{\tan 23^\circ} \ \ldots \ (1)$$

$$\text{In } \Delta OBT, \ \tan 32^\circ = \frac{h}{OB}$$

$$\Rightarrow OB = \frac{h}{\tan 32^\circ} \ \ldots \ (2)$$

Substitute (1) and (2) into (*):

$$\begin{aligned}
200^2 &= \frac{h^2}{\tan^2 23^\circ} + \frac{h^2}{\tan^2 32^\circ} - 2 \cdot \frac{h}{\tan 23^\circ} \cdot \frac{h}{\tan 32^\circ} \cdot \frac{1}{2} \\
&= h^2\left(\frac{1}{\tan^2 23^\circ} + \frac{1}{\tan^2 32^\circ} + \frac{1}{\tan 23^\circ \cdot \tan 32^\circ}\right) \\
&= h^2(4.340\ldots) \\
h^2 &= \frac{40\ 000}{4.340\ldots} \\
&= 9214.55\ldots \\
\therefore h &= 95.99\ldots \\
&= 96 \text{ m} \quad (\text{to nearest m})
\end{aligned}$$

13. Trigonometry, 2ADV' T1 2010 HSC 5a

i. Show $BP = \dfrac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$

In ΔATP

$$\tan 3^\circ = \frac{TP}{AP}$$

$$\Rightarrow AP = \frac{TP}{\tan 3}$$

In ΔAPL:

$$\tan 20^\circ = \frac{1}{AP}$$

$$\Rightarrow AP = \frac{1}{\tan 20}$$

$$\therefore \frac{TP}{\tan 3} = \frac{1}{\tan 20}$$

$$TP = \frac{\tan 3^\circ}{\tan 20^\circ} \quad \text{... (1)}$$

In ΔBTP:

$$\tan 30^\circ = \frac{TP}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{TP}{BP}$$

$$BP = \sqrt{3} \times TP \quad \text{(using (1) above)}$$

$$= \frac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ} \quad \text{... as required}$$

ii. $AB = AP - BP$

$$AP = \frac{1}{\tan 20^\circ} \quad \text{(from part (i))}$$

$$\therefore AB = \frac{1}{\tan 20^\circ} - \frac{\sqrt{3}\tan 3^\circ}{\tan 20^\circ}$$

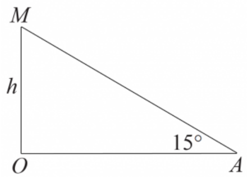
$$= \frac{1 - \sqrt{3}\tan 3}{\tan 20^\circ}$$

$$= 2.4980\ldots$$

$$= 2.5 \text{ km (to 1 d.p.)}$$

14. Trigonometry, 2ADV’ T1 2015 HSC 12c

i. Show $OA = h\cot 15^\circ$



In ΔMOA,

$$\tan 15^\circ = \frac{h}{OA}$$

$$OA = \frac{h}{\tan 15^\circ}$$

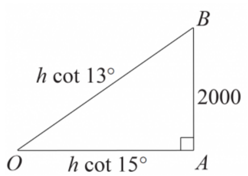
$$= h\cot 15^\circ \quad \ldots\text{as required}$$

ii. In ΔMOB

$$\tan 13^\circ = \frac{h}{OB}$$

$$OB = \frac{h}{\tan 13^\circ}$$

$$= h\cot 13^\circ$$



In ΔAOB:

$$OA^2 + AB^2 = OB^2$$

$$OB^2 - OA^2 = AB^2$$

$$(h\cot 13^\circ)^2 - (h\cot 15^\circ)^2 = 2000^2$$

$$h^2\left[(\cot^2 13^\circ - \cot^2 15^\circ)\right] = 2000^2$$

$$h^2 = \frac{2000^2}{\cot^2 13^\circ - \cot^2 15^\circ}$$

$\therefore h$

$$= \sqrt{\frac{2000^2}{\cot^2 13^\circ - \cot^2 15^\circ}}$$

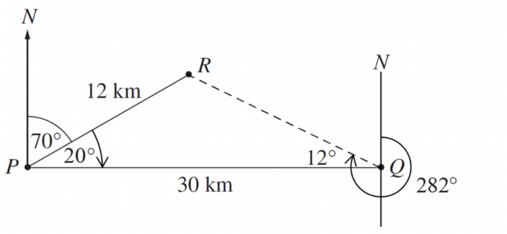
$$= 909.704\ldots$$

$$= 910\text{ m} \ \ (\text{nearest metre})$$

---

15. Trigonometry, 2ADV T1 2004 HSC 3c

i. Join RQ to form ΔRPQ



$\angle RPQ = 90 - 70 = 20^\circ$

Using the cosine rule:

$$RQ^2 = PR^2 + PQ^2 - 2\times PR\times PQ\times\cos\angle RPQ$$

$$= 12^2 + 30^2 - 2\times 12\times 30\times\cos 20^\circ$$

$$= 367.421\ldots$$

$$\therefore\ RQ = 19.168\ldots$$

$$= 19.2\text{ km} \ \ (1\text{ d.p.})$$

ii. Using sine rule:

$$\frac{\sin\angle RQP}{12} = \frac{\sin 20^\circ}{19.168\ldots}$$

$$\sin\angle RQP = \frac{12\times\sin 20^\circ}{19.168\ldots}$$

$$= 0.214\ldots$$

$$\angle RQP = 12.36\ldots^\circ$$

$$= 12^\circ \quad (\text{nearest degree})$$

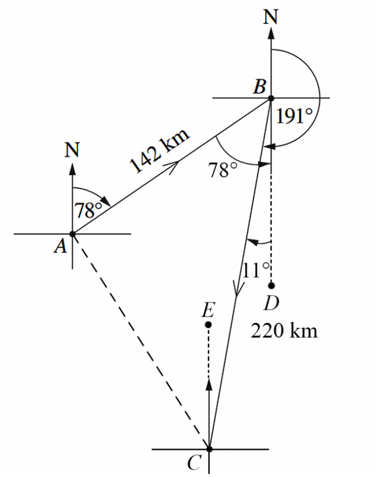
$\therefore$ Bearing of R from Q

$$= 270 + 12$$

$$= 282^\circ$$

16. Trigonometry, 2ADV T1 2014 HSC 13d

i.



Find $\angle ABC$

Let D be south of B

$\therefore\ \angle CBD = 191 - 180 = 11^\circ$

$\angle DBA = 78^\circ$ (alternate)

$\angle ABC = 78 - 11$

$= 67^\circ$

Using cosine rule:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos\angle ABC \\ &= 142^2 + 220^2 - 2 \times 142 \times 220 \times \cos 67^\circ \\ &= 44\,151.119... \end{aligned}$$

$$\begin{aligned} \therefore\ AC &= 210.121... \\ &\approx 210 \text{ km} \quad \text{... as required} \end{aligned}$$

ii. Find $\angle ACB$

Using sine rule:

$$\frac{\sin\angle ACB}{142} = \frac{\sin\angle ABC}{210}$$

$$\begin{aligned} \sin\angle ACB &= \frac{142 \times \sin 67^\circ}{210} \\ &= 0.6224... \end{aligned}$$

$\angle ACB = 38.494...$

$= 38^\circ$ (nearest degree)

Let E be due North of C

$\angle ECB = 11^\circ$ (alternate to $\angle CBD$)

$\therefore\ \angle ECA = 38 - 11$

$= 27^\circ$

$\therefore$ Bearing of A from C

$= 360 - 27$

$= 333^\circ$

---

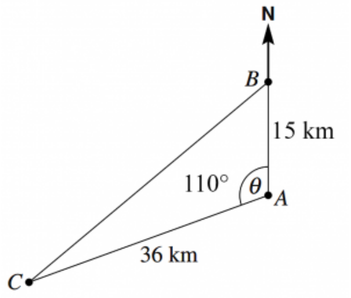
17. Trigonometry, 2ADV* T1 2005 HSC 27c

i. There is 360° about point A

$\therefore\ \theta + 250^\circ = 360^\circ$

$\theta = 110^\circ$

ii.



$$a^2 = b^2 + c^2 - 2ab\cos A$$

$$CB^2 = 36^2 + 15^2 - 2 \times 36 \times 15 \times \cos 110^\circ$$

$$= 1296 + 225 - (-369.38\ldots)$$

$$= 1890.38\ldots$$

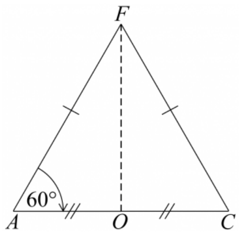
$$\therefore CB = 43.47\ldots$$

$$= 43 \text{ km (nearest km)}$$

---

18. Trigonometry, 2ADV' T1 2004 HSC 3d

i.



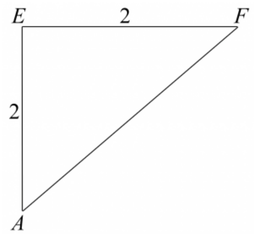
Since FA, AC and FC are all diagonals of sides of a cube,

$FA = AC = FC$

$\therefore \Delta FAC$ is equilateral

$\therefore \angle FAC = 60^\circ$

ii.



In ΔAEF

$$AF^2 = EF^2 + EA^2$$

$$= 2^2 + 2^2$$

$$= 8$$

$$AF = \sqrt{8}$$

$$= 2\sqrt{2}$$

In ΔAFO

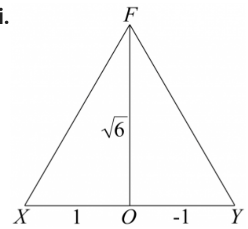
$$\sin 60^\circ = \frac{FO}{AF}$$

$$\frac{\sqrt{3}}{2} = \frac{FO}{2\sqrt{2}}$$

$$FO = \frac{\sqrt{3}}{2} \times 2\sqrt{2}$$

$= \sqrt{6}$ metres . . . as required.

iii.



XY is the diameter of a circle AND the width of the cube.

$\therefore XY = 2$

$\therefore OX = OY = 1$

$$\tan \angle OFX = \frac{1}{\sqrt{6}}$$

$$\angle OFX = 22.207\ldots^\circ$$

$$\therefore \angle XFY = 2 \times 22.407\ldots$$
$$= 44.415\ldots$$
$$= 44^\circ \text{ (nearest degree)}$$

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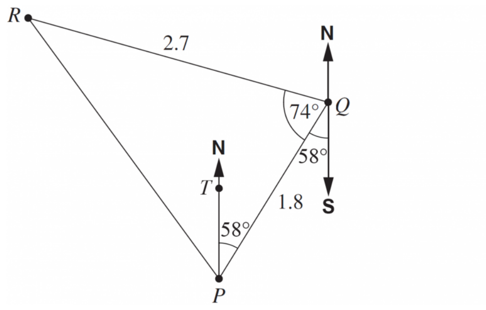
19. Trigonometry, 2ADV* T1 2009 HSC 27b

i.



$\angle PQS = 58^\circ$ (alternate to $\angle TPQ$)

Bearing of R from Q

$$= 180^\circ + 58^\circ + 74^\circ$$
$$= 312^\circ$$

TIP: Draw North-South parallel lines through relevant points to help calculate angles as shown in the Worked Solutions.

(ii) Using cosine rule:

$$RP^2 = RQ^2 + PQ^2 - 2 \times RQ \times PQ \times \cos \angle RQP$$
$$= 2.7^2 + 1.8^2 - 2 \times 2.7 \times 1.8 \times \cos 74^\circ$$
$$= 7.29 + 3.24 - 2.679...$$
$$= 7.851...$$
$$\therefore RP = \sqrt{7.851...}$$
$$= 2.8019...$$
$$\approx 2.8 \text{ km (1 d.p.)}$$

(iii) Using $A = \dfrac{1}{2}ab\sin C$

$$A = \frac{1}{2} \times 2.7 \times 1.8 \times \sin 74^\circ$$
$$= 2.3358...$$
$$= 2.3 \text{ km}^2$$

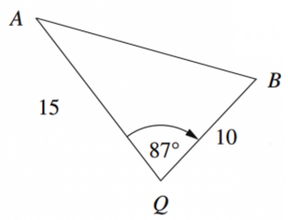
$\therefore$ No-go zone is 2.3 km²

20. Trigonometry, 2ADV* T1 2007 26a

i. 2 hrs 48 mins = 168 mins

$$\text{Speed } (A \text{ to } Q) = \frac{15}{168}$$
$$= 0.0892... \text{ km/min}$$

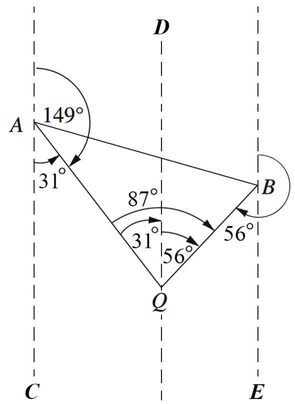
$$\text{Speed (in km/hr)} = 0.0892... \times 60$$
$$= 5.357... \text{ km/hr}$$
$$= 5 \text{ km/hr (nearest km/hr)}$$

ii. 

Using cosine rule

$$AB^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \times \cos 87^\circ$$
$$= 309.299...$$
$$AB = 17.586...$$
$$= 18 \text{ km (nearest km)}$$

$\therefore$ The distance from Town A to Town B is 18 km.

iii. 

$\angle CAQ = 31^\circ$ (straight angle at A)

$\angle AQD = 31^\circ$ (alternate angle $AC \parallel DQ$)

$\angle DQB = 87 - 31 = 56^\circ$

$\angle QBE = 56^\circ$ (alternate angle $DQ \parallel BE$)

$\therefore$ Bearing of Q from B

$= 180 + 56$

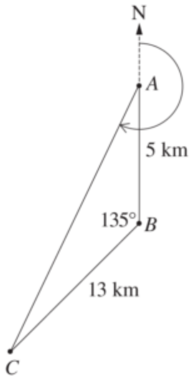
$= 236^\circ$

21. Trigonometry, 2ADV* T1 2017 HSC 30c

(i) Using cosine rule:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2 \times AB \times BC \times \cos 135^\circ \\ &= 5^2 + 13^2 - 2 \times 5 \times 13 \times \cos 135^\circ \\ &= 285.923\ldots \\ \therefore AC &= 16.909\ldots \\ &= 17 \text{ km (nearest km)} \end{aligned}$$

(ii)



Using sine rule, find $\angle BAC$:

$$\frac{\sin\angle BAC}{13} = \frac{\sin 135^\circ}{17}$$

$$\begin{aligned} \sin\angle BAC &= \frac{13 \times \sin 135^\circ}{17} \\ &= 0.5407\ldots \\ \angle BAC &= 32.7^\circ \end{aligned}$$

♦♦ *Mean mark part (ii) 31%.*

$\therefore$ Bearing of C from A

$$\begin{aligned} &= 180 + 32.7 \\ &= 212.7^\circ \\ &= 213^\circ \end{aligned}$$

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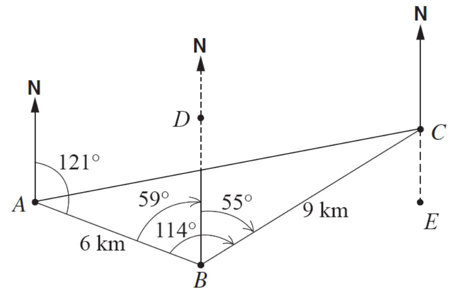
22. Trigonometry, 2ADV* T1 2011 HSC 24c

i.



STRATEGY: This deserves repeating again: *Draw North-South parallel lines through major points to make the angle calculations easier!*

Let point D be due North of point B

$$\begin{aligned} \angle ABD &= 180 - 121 \text{ (cointerior with } \angle A) \\ &= 59^\circ \\ \angle DBC &= 114 - 59 \\ &= 55^\circ \end{aligned}$$

$\therefore$ Bearing of C from B is 055°

ii. Using cosine rule:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2 \times AB \times BC \times \cos\angle ABC \\ &= 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 114^\circ \\ &= 160.9275... \\ \therefore AC &= 12.685... \quad \text{(Noting } AC > 0) \\ &= 13 \text{ km (nearest km)} \end{aligned}$$

iii. Need to find $\angle ACB$ (see diagram)

$$\begin{aligned} \cos\angle ACB &= \frac{AC^2 + BC^2 - AB^2}{2 \times AC \times BC} \\ &= \frac{(12.685...)^2 + 9^2 - 6^2}{2 \times (12.685..) \times 9} \\ &= 0.9018... \\ \angle ACB &= 25.6^\circ \text{ (to 1 d.p.)} \end{aligned}$$

MARKER'S COMMENT: The best responses showed clear working on the diagram.

From diagram,

$\angle BCE = 55^\circ$ (alternate angle, $DB \parallel CE$)

$\therefore$ Bearing of A from C

$= 180 + 55 + 25.6$

$= 260.6$

$= 261°$ (nearest degree)

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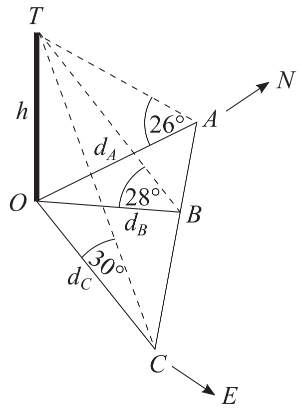
23. Trigonometry, 2ADV T1 SM-Bank 1



Let $h =$ height of tower

In ΔOAT:

$$\tan 26° = \frac{h}{d_A}$$

$$d_A = \frac{h}{\tan 26°}$$

Similarly,

$$d_B = \frac{h}{\tan 28°}$$

$$d_C = \frac{h}{\tan 30°}$$

In ΔOAC:

$$\tan\angle OAC = \frac{d_C}{d_A} = \frac{\frac{h}{\tan 30°}}{\frac{h}{\tan 26°}} = \frac{\tan 26°}{\tan 30°} = 0.8447\ldots$$

$$\angle OAC = 40.19°$$

Using sine rule in ΔOAB:

$$\frac{\sin\angle ABO}{d_A} = \frac{\sin\angle OAC}{d_B}$$

$$\begin{aligned}\sin\angle ABO &= \sin 40.2^\circ \times \frac{\tan 28^\circ}{\tan 26^\circ}\\ &= 0.7035\ldots\\ \angle ABO &= 44.71^\circ \text{ or } 135.29^\circ\end{aligned}$$

$$\begin{aligned}\text{Since } \ \angle OCA &= \tan^{-1}\left(\frac{\tan 30}{\tan 26}\right)\\ &= 49.8^\circ\end{aligned}$$

$\Rightarrow \angle OBC = 44.71^\circ$
(otherwise angle sum $\Delta OBC > 180^\circ$)

$$\begin{aligned}\angle AOB &= 180 - (40.19 + 135.29)\\ &= 4.52\end{aligned}$$

$\therefore$ Bearing of B from tower is 005°.