ADV: Trigonometry (Adv), T3 Trig Functions and Graphs (Adv)

Trig Graphs (Y12)
Trig Applications (Y12)

Teacher: Troy McMurrich

Exam Equivalent Time: 139.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

T3 Trig Functions and Graphs

Trig Graphs
Trig Applications

***Analytics based on the average contribution to the 2ADV HSC exams over the past decade.**

HISTORICAL CONTRIBUTION

- *T3 Trig Functions and Graphs* is a small contributor to past Mathematics exams, accounting for an average of 2.5% of past papers. We expect this to increase for reasons outlined below.
- This topic has been split into two sub-topics for analysis purposes: *1-Trig Graphs (1.0%), and 2-Trig Applications (1.5%).*
- This analysis looks at the sub-topic *Trig Graphs.*

HSC ANALYSIS - What to expect and common pitfalls

- *Trig Graphs* have been examined in each of the last 6 years, receiving a multiple choice question on 4 occasions and longer answer questions worth 2-3 marks in 2021 and 2017.
- We expect *Trig Graphs* to be examined more often going forward due to the new syllabus content looking at transformations. This has not been the case in 2020-21, but in our view, it has been clearly flagged by the inclusion of 2 separate trig graph questions in the NESA sample HSC exam. Our database, in response, has been significantly expanded in this area.
- We recommend close revision of *T3 EQ-Bank 3 and 5* which are informed by the question style and difficulty level of NESA's sample questions. Also, special attention should be given to *2013 HSC 6 MC* which was surprisingly poorly answered.
- Note that more than half of students answered the 2016 multiple choice question on a tan function graph's *period* incorrectly. Deserves attention.

Questions

1. Trigonometry, 2ADV T3 2020 HSC 6 MC

Which interval gives the range of the function $y = 5 + 2\cos 3x$?

A. $[2, 8]$

B. $[3, 7]$

C. $[4, 6]$

D. $[5, 9]$

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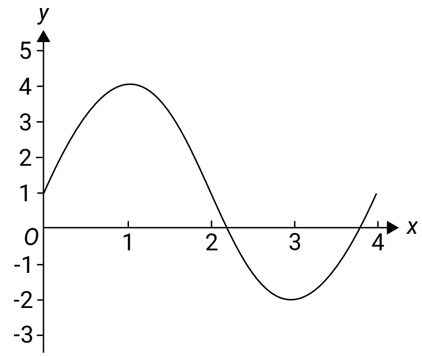
2. Trigonometry, 2ADV T3 SM-Bank 8 MC

The diagram below shows one cycle of a circular function.



The amplitude and period of this function are respectively

A. $3 \text{ and } 2$

B. $3 \text{ and } \dfrac{\pi}{2}$

C. $4 \text{ and } \dfrac{\pi}{4}$

D. $3 \text{ and } 4$

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3. Trigonometry, 2ADV T3 SM-Bank 1 MC

$f(x) = 2\sin(3x) - 3$

The period and range of this function are respectively

(A) $\text{period} = \dfrac{2\pi}{3}$ and $\text{range} = [-5, -1]$

(B) $\text{period} = \dfrac{2\pi}{3}$ and $\text{range} = [-2, 2]$

(C) $\text{period} = \dfrac{\pi}{3}$ and $\text{range} = [-1, 5]$

(D) $\text{period} = 3\pi$ and $\text{range} = [-1, 5]$

4. Trigonometry, 2ADV T3 SM-Bank 2 MC

Let $f(x) = 1 - 2\cos\left(\dfrac{\pi x}{2}\right)$.

The period and range of this function are respectively

(A) 4 and $[-2, 2]$

(B) 4 and $[-1, 3]$

(C) 1 and $[-1, 3]$

(D) 4π and $[-2, 2]$

5. Trigonometry, 2ADV T3 SM-Bank 3 MC

Let $f(x) = 5\sin(2x) - 1$.

The period and range of this function are respectively

(A) π and $[-1, 4]$

(B) 2π and $[-1, 5]$

(C) π and $[-6, 4]$

(D) 2π and $[-6, 4]$

6. Trigonometry, 2ADV T3 SM-Bank 18

The period of the function $f(x) = \tan\left(\dfrac{\pi x}{2}\right)$ is

A. 2

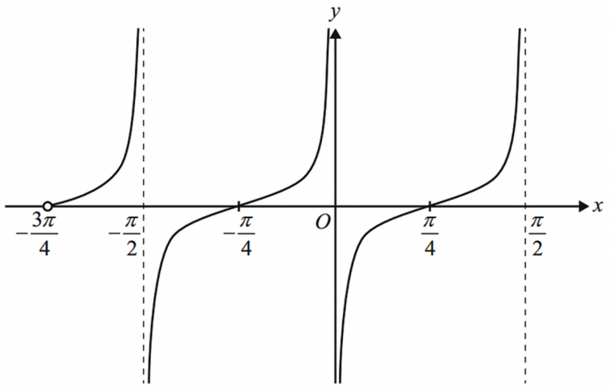
B. 4

C. 2π

D. 4π

7. Trigonometry, 2ADV T3 SM-Bank 4 MC

A section of the graph of $f(x)$ is shown below.



The equation of $f(x)$ could be

(A) $f(x) = \tan(x)$

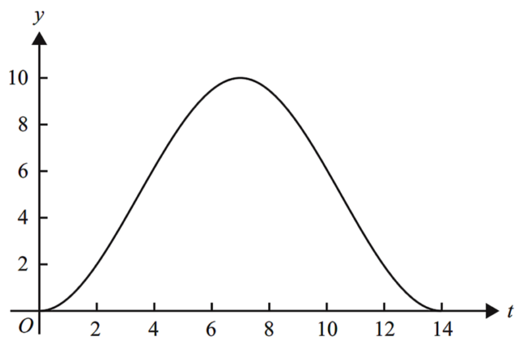
(B) $f(x) = \tan\left(x - \dfrac{\pi}{4}\right)$

(C) $f(x) = \tan\left(2\left(x - \dfrac{\pi}{4}\right)\right)$

(D) $f(x) = \tan\left(2\left(x - \dfrac{\pi}{2}\right)\right)$

8. Trigonometry, 2ADV T3 SM-Bank 5 MC

The UV index, y, for a summer day in Newcastle East is illustrated in the graph below, where t is the number of hours after 6 am.



The graph is most likely to be the graph of

(A) $y = 5 + 5\cos\left(\dfrac{\pi t}{7}\right)$

(B) $y = 5 - 5\cos\left(\dfrac{\pi t}{7}\right)$

(C) $y = 5 + 5\cos\left(\dfrac{\pi t}{14}\right)$

(D) $y = 5 - 5\cos\left(\dfrac{\pi t}{14}\right)$

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9. Trigonometry, 2ADV T3 SM-Bank 6 MC

The function with equation $f(x) = 4\tan\left(\dfrac{x}{3}\right)$ has period

(A) $\dfrac{2\pi}{3}$

(B) 6π

(C) 3

(D) 3π

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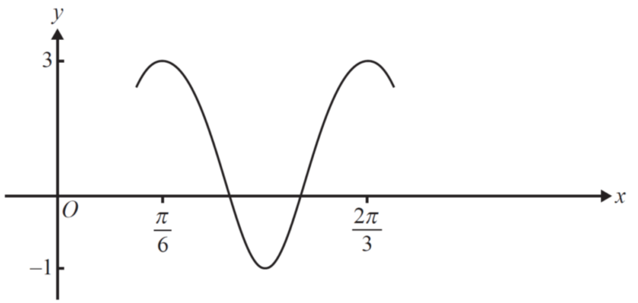
10. Trigonometry, 2ADV T3 SM-Bank 7 MC



The graph shown could have equation

(A) $y = 2\cos\left(x + \dfrac{\pi}{6}\right) + 1$

(B) $y = 2\cos 4\left(x - \dfrac{\pi}{6}\right) + 1$

(C) $y = 4\sin 2\left(x - \dfrac{\pi}{12}\right) - 1$

(D) $y = 3\cos\left(2x + \dfrac{\pi}{6}\right) - 1$

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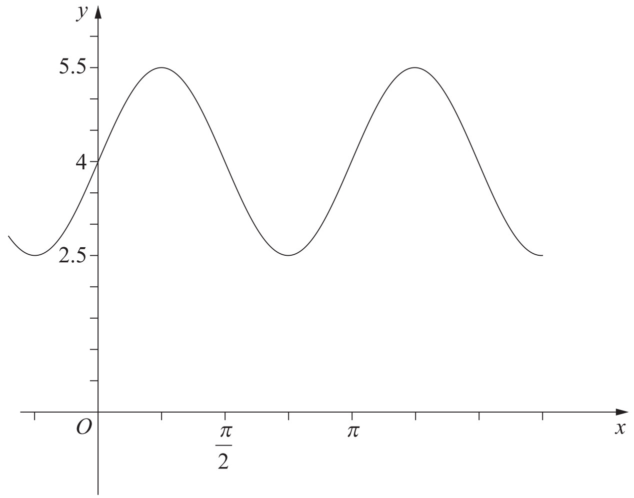
11. Trigonometry, 2ADV T3 2019 HSC 7 MC

The diagram shows part of the graph of $y = a\sin(bx) + 4$.



What are the values of a and b?

(A) $a = 3 \qquad b = \dfrac{1}{2}$

(B) $a = 3 \qquad b = 2$

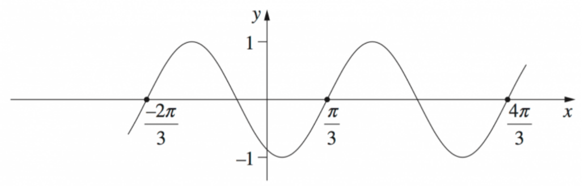
(C) $a = 1.5 \qquad b = \dfrac{1}{2}$

(D) $a = 1.5 \qquad b = 2$

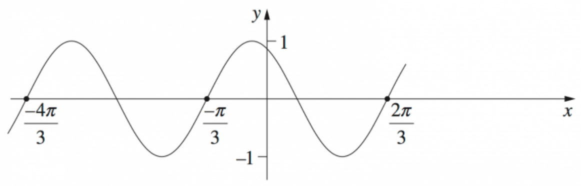
12. Trigonometry, 2ADV T3 2013 HSC 6 MC

Which diagram shows the graph $y = \sin\left(2x + \dfrac{\pi}{3}\right)$?

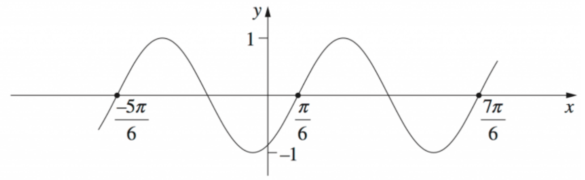
(A)



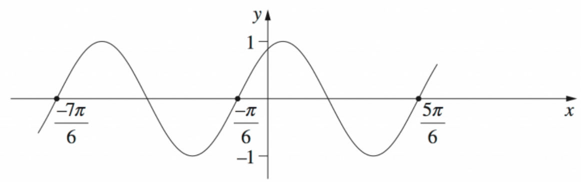
(B)



(C)



(D)



13. Trigonometry, 2ADV T3 2016 HSC 6 MC

What is the period of the function $f(x) = \tan(3x)$?

(A) $\dfrac{\pi}{3}$

(B) $\dfrac{2\pi}{3}$

(C) 3π

(D) 6π

14. Trigonometry, 2ADV T3 2018 HSC 10 MC

A trigonometric function $f(x)$ satisfies the condition

$$\int_0^{\pi} f(x)\,dx \neq \int_{\pi}^{2\pi} f(x)\,dx.$$

Which function could be $f(x)$?

(A) $f(x) = \sin(2x)$

(B) $f(x) = \cos(2x)$

(C) $f(x) = \sin\left(\dfrac{x}{2}\right)$

(D) $f(x) = \cos\left(\dfrac{x}{2}\right)$

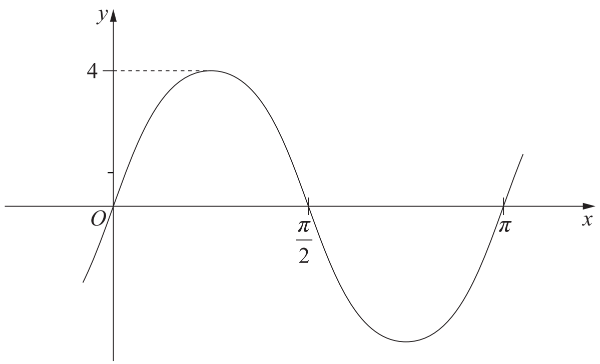
15. Trigonometry, 2ADV T3 EQ-Bank 5

The function $f(x) = \sin x$ is transformed into the function $g(x) = \dfrac{\sin(4x)}{3}$.

Describe in words how the amplitude and period have changed in this transformation. ***(2 marks)***

16. Trigonometry, 2ADV T3 2010 HSC 8c

The graph shown is $y = A\sin bx$.



i. Write down the value of A. ***(1 mark)***

ii. Find the value of b. ***(1 mark)***

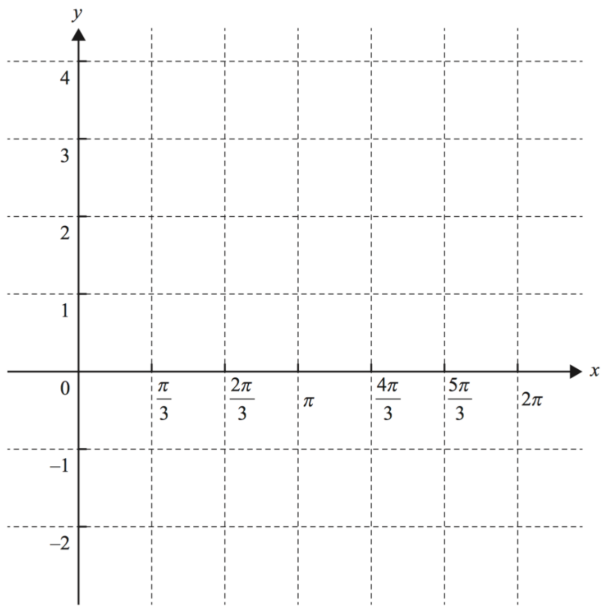
iii. Copy or trace the graph into your writing booklet.
On the same set of axes, draw the graph $y = 3\sin x + 1$ for $0 \leq x \leq \pi$. ***(2 marks)***

17. Trigonometry, 2ADV T3 SM-Bank 9

Let $f(x) = 2\cos(x) + 1$ for $0 \leq x \leq 2\pi$.

i. Solve the equation $2\cos(x) + 1 = 0$ for $0 \leq x \leq 2\pi$. ***(2 marks)***

ii. Sketch the graph of the function $f(x)$ on the axes below. Label the endpoints and local minimum point with their coordinates. ***(3 marks)***



18. Trigonometry, 2ADV T3 2006 HSC 7b

A function $f(x)$ is defined by $f(x) = 1 + 2\cos x$.

i. Show that the graph of $y = f(x)$ cuts the x-axis at $x = \dfrac{2\pi}{3}$. ***(1 mark)***

ii. Sketch the graph of $y = f(x)$ for $-\pi \leq x \leq \pi$ showing where the graph cuts each of the axes. ***(3 marks)***

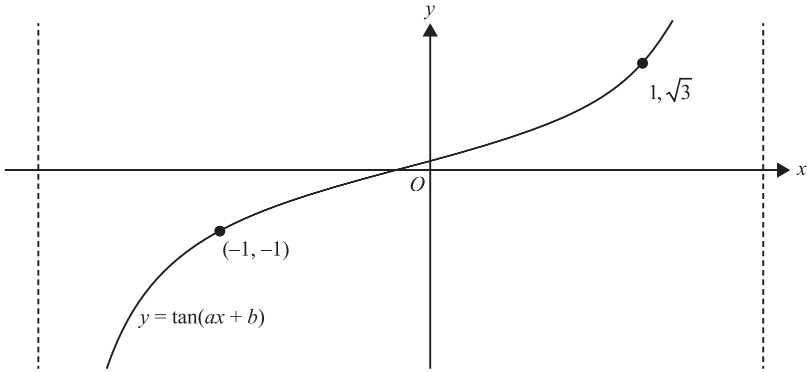
iii. Find the area under the curve $y = f(x)$ between $x = -\dfrac{\pi}{2}$ and $x = \dfrac{2\pi}{3}$. ***(3 marks)***

19. Trigonometry, 2ADV T3 2017 HSC 14a

Sketch the curve $y = 4 + 3\sin 2x$ for $0 \leq x \leq 2\pi$. ***(3 marks)***

20. Trigonometry, 2ADV T3 2010 MET1 3

Shown below is part of the graph of a period of the function of the form $y = \tan(ax + b)$.



Find the value of a and the value of b, where $a > 0$ and $0 < b < 1$. ***(3 marks)***

21. Trigonometry, 2ADV T3 2021 HSC 20

For what values of x, in the interval $0 \le x \le \frac{\pi}{4}$, does the line $y = 1$ intersect the graph of $y = 2\sin 4x$? ***(2 marks)***

22. Trigonometry, 2ADV T3 SM-Bank 12

State the range and period of the function

$$h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right).$$ ***(2 marks)***

23. Trigonometry, 2ADV T3 SM-Bank 13

On any given day, the depth of water in a river is modelled by the function

$$h(t) = 14 + 8\sin\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24$$

where h is the depth of water, in metres, and t is the time, in hours, after 6 am.

i. Find the minimum depth of the water in the river. ***(1 mark)***

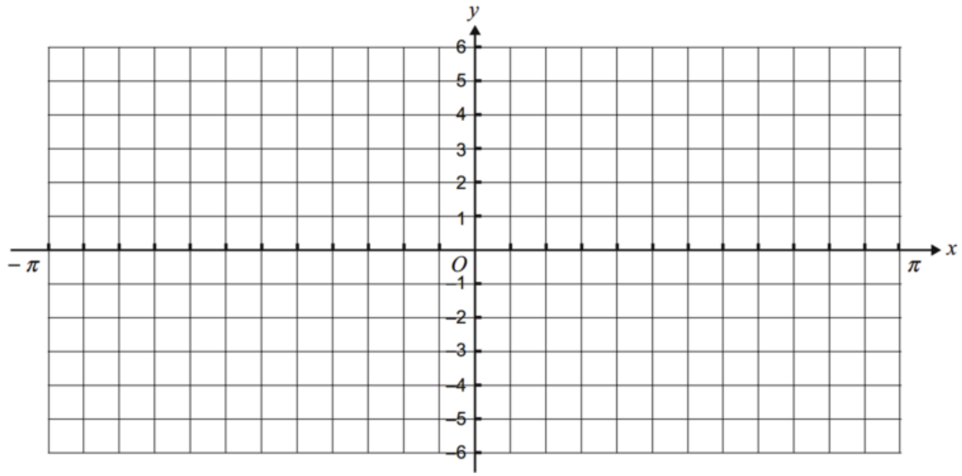
ii. Find the values of t for which $h(t) = 10$. ***(2 marks)***

24. Trigonometry, 2ADV T3 SM-Bank 14

For the function $f(x) = 5\cos\left(2\left(x + \frac{\pi}{3}\right)\right), \quad -\pi \le x \le \pi$

a. Write down the amplitude and period of the function ***(2 marks)***

b. Sketch the graph of the function $f(x)$ on the set of axes below. Label axes intercepts with their coordinates.
Label endpoints of the graph with their coordinates. ***(3 marks)***



25. Trigonometry, 2ADV T3 SM-Bank 15

The graphs of $y = \cos(x)$ and $y = a\sin(x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$.

i. Find the value of a. ***(2 marks)***

ii. Find the x-coordinate of the other point of intersection of the two graphs, given $0 \le x \le 2\pi$ ***(1 mark)***

26. Trigonometry, 2ADV T3 2018 HSC 15a

The length of daylight, $L(t)$, is defined as the number of hours from sunrise to sunset, and can be modelled by the equation

$$L(t) = 12 + 2\cos\left(\frac{2\pi t}{366}\right),$$

where t is the number of days after 21 December 2015, for $0 \le t \le 366$.

i. Find the length of daylight on 21 December 2015. ***(1 mark)***

ii. What is the shortest length of daylight? ***(1 mark)***

iii. What are the two values of t for which the length of daylight is 11? ***(2 marks)***

27. Trigonometry, 2ADV T3 SM-Bank 10

The population of wombats in a particular location varies according to the rule

$n(t) = 1200 + 400\cos\left(\dfrac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2018.

i. Find the period and amplitude of the function n. *(2 marks)*

ii. Find the maximum and minimum populations of wombats in this location. *(2 marks)*

iii. Find $n(10)$. *(1 mark)*

iv. Over the 12 months from 1 March 2018, find the fraction of time when the population of wombats in this location was less than $n(10)$. *(2 marks)*

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28. Trigonometry, 2ADV T3 2013 HSC 13a

The population of a herd of wild horses is given by

$$P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$$

where t is time in months.

i. Find all times during the first 12 months when the population equals 375 horses. *(2 marks)*

ii. Sketch the graph of $P(t)$ for $0 \le t \le 12$. *(2 marks)*

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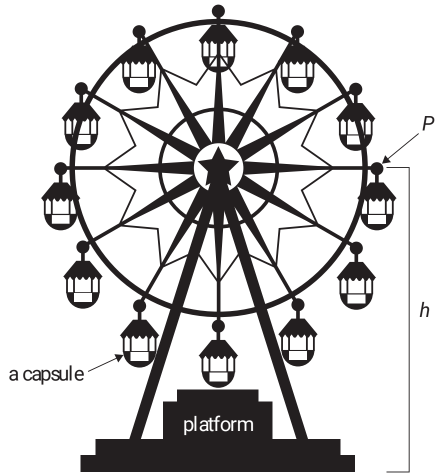
29. Trigonometry, 2ADV T3 SM-Bank 16

Sammy visits a giant Ferris wheel. Sammy enters a capsule on the Ferris wheel from a platform above the ground. The Ferris wheel is rotating anticlockwise. The capsule is attached to the Ferris wheel at point P. The height of P above the ground, h, is modelled by

$h(t) = 65 - 55\cos\left(\dfrac{\pi t}{15}\right)$, where t is the time in minutes after Sammy enters the capsule and h is measured in metres.

Sammy exits the capsule after one complete rotation of the Ferris wheel.



i. State the minimum and maximum heights of P above the ground. *(1 mark)*

ii. For how much time is Sammy in the capsule? *(1 mark)*

iii. Find the rate of change of h with respect to t and, hence, state the value of t at which the rate of change of h is at its maximum. *(2 marks)*

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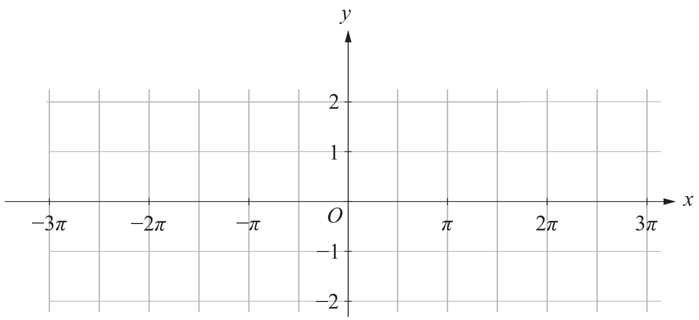
30. Trigonometry, 2ADV T3 EQ-Bank 3

By drawing graphs on the number plane, show how many solutions exist for the equation $\cos x = \left|\dfrac{x-\pi}{4}\right|$ in the domain $(-\infty, \infty)$ *(3 marks)*



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31. Trigonometry, 2ADV T3 2011 SPEC1 8

Find the coordinates of the points of intersection of the graph of the relation

$y = \operatorname{cosec}^2\left(\dfrac{\pi x}{6}\right)$ with the line $y = \dfrac{4}{3}$, for $0 < x < 12.$ *(3 marks)*

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32. Trigonometry, 2ADV T3 SM-Bank 8

$f(x) = 2\sin(2x)$ is defined in the domain $\left\{x \colon \dfrac{\pi}{8} \le x < \dfrac{\pi}{3}\right)$

What is the range of the function $f(x)$? *(2 marks)*

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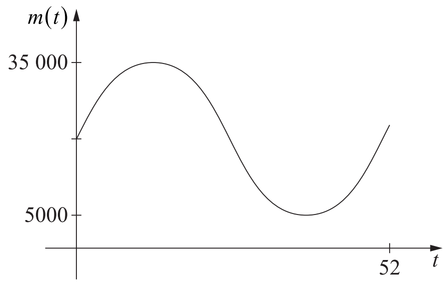
33. Trigonometry, 2ADV T3 2020 HSC 31

The population of mice on an isolated island can be modelled by the function.

$$m(t) = a\sin\left(\frac{\pi}{26}t\right) + b,$$

where t is the time in weeks and $0 \le t \le 52$. The population of mice reaches a maximum of 35 000 when $t = 13$ and a minimum of 5000 when $t = 39$. The graph of $m(t)$ is shown.



a. What are the values of a and b? *(2 marks)*

b. On the same island, the population of cats can be modelled by the function

$$c(t) = -80\cos\left(\frac{\pi}{26}(t-10)\right) + 120$$

Consider the graph of $m(t)$ and the graph of $c(t)$.
Find the values of t, $0 \le t \le 52$, for which both populations are increasing. *(3 marks)*

c. Find the rate of change of the mice population when the cat population reaches a maximum. *(2 marks)*

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34. Trigonometry, 2ADV T3 2009 HSC 7b

Between 5 am and 5 pm on 3 March 2009, the height, h, of the tide in a harbour was given by

$$h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right) \quad \text{for } 0 \le t \le 12$$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

i. What is the period of the function h? *(1 mark)*

ii. What was the value of h at low tide, and at what time did low tide occur? *(2 marks)*

iii. A ship is able to enter the harbour only if the height of the tide is at least 1.35 m.
Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour. *(3 marks)*

Worked Solutions

1. Trigonometry, 2ADV T3 2020 HSC 6 MC

$$-1 \le \cos 3x \le 1$$
$$-2 \le 2\cos 3x \le 2$$
$$3 \le 5 + 2\cos 3x \le 7$$
$$\therefore \text{ Range } [3, 7]$$
$$\Rightarrow B$$

2. Trigonometry, 2ADV T3 SM-Bank 8 MC

$$\text{Graph centres around } \ y = 1$$
$$\text{Amplitude } = 3$$
$$\text{Period:} = 4$$
$$\Rightarrow D$$

3. Trigonometry, 2ADV T3 SM-Bank 1 MC

$$\text{Range: } [-3-2, \ -3+2]$$
$$= [-5, \ -1]$$
$$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{3}$$
$$\Rightarrow \ A$$

4. Trigonometry, 2ADV T3 SM-Bank 2 MC

$$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{2}} = 4$$

$$\text{Amplitude} = 2$$
$$\text{Graph centre line (median): } \ y = 1.$$
$$\therefore \text{ Range} = [1-2, \ \ 1+2]$$
$$= [-1, 3]$$
$$\Rightarrow \ B$$

5. Trigonometry, 2ADV T3 SM-Bank 3 MC

$$\text{Period} = \frac{2\pi}{2} = \pi$$
$$\text{Range} = [-1-5, \ -1+5]$$
$$= [-6, 4]$$
$$\Rightarrow C$$

6. Trigonometry, 2ADV T3 SM-Bank 18

$$n = \frac{\pi}{2}$$
$$\text{Period} = \frac{\pi}{n} = \frac{\pi}{\frac{\pi}{2}} = 2$$
$$\Rightarrow \ A$$

7. Trigonometry, 2ADV T3 SM-Bank 4 MC

$$\text{Period} = \frac{\pi}{2}$$
$$\Rightarrow \text{ must be } C \text{ or } D$$
$$\text{Shift } \ y = \tan(x) \ \text{ right } \ \frac{\pi}{4}.$$
$$\Rightarrow \ C$$

8. Trigonometry, 2ADV T3 SM-Bank 5 MC

$$\text{Centre line (median): } \ y = 5$$
$$\text{Amplitude} = 5$$
$$\text{Period: } \ 14 = \frac{2\pi}{n}$$
$$n = \frac{\pi}{7}$$

$$\therefore \text{ Graph: } \ y = 5 - 5\cos\left(\frac{\pi t}{7}\right)$$
$$\Rightarrow \ B$$

9. Trigonometry, 2ADV T3 SM-Bank 6 MC

$$\begin{aligned}\text{Period} &= \frac{\pi}{n}\\ &= \frac{\pi}{\frac{1}{3}}\\ &= 3\pi\end{aligned}$$

$\Rightarrow \ D$

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10. Trigonometry, 2ADV T3 SM-Bank 7 MC

$\text{Amplitude} = 2 \quad (\text{range from} -1 \text{ to } 3)$

$\text{Graph centre line (median):}\ \ y = 1$

$\therefore$ Eliminate C and D.

$\text{Period} = \dfrac{2\pi}{3} - \dfrac{\pi}{6} = \dfrac{\pi}{2} \ \ (\text{from graph})$

Consider option B,

$\text{Period} = \dfrac{2\pi}{n} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

$\Rightarrow B$

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11. Trigonometry, 2ADV T3 2019 HSC 7 MC

$a = \dfrac{1}{2}(5.5 - 2.5) = 1.5$

Since graph passes through $\left(\dfrac{\pi}{4}, 5.5\right)$:

$5.5 = 1.5\sin\left(b \times \dfrac{\pi}{4}\right) + 4$

$$\begin{aligned}\sin\left(b \times \frac{\pi}{4}\right) &= 1\\ b \times \frac{\pi}{4} &= \frac{\pi}{2}\\ \therefore b &= 2\end{aligned}$$

$\Rightarrow \ D$

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12. Trigonometry, 2ADV T3 2013 HSC 6 MC

♦♦ *Mean mark 34%*

$\text{At } x = 0,\ \ y = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ It cannot be A or C

Find x when $y = 0$,

$\sin\left(2x + \dfrac{\pi}{3}\right) = 0$

$$\begin{aligned}\therefore\ 2x + \frac{\pi}{3} &= 0 \quad (\sin 0 = 0)\\ 2x &= -\frac{\pi}{3}\\ x &= -\frac{\pi}{6}\end{aligned}$$

$\Rightarrow \ D$

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13. Trigonometry, 2ADV T3 2016 HSC 6 MC

♦ *Mean mark 42%.*

$$\begin{aligned}\text{Period} &= \frac{\pi}{n}\\ &= \frac{\pi}{3}\end{aligned}$$

$\Rightarrow \ A$

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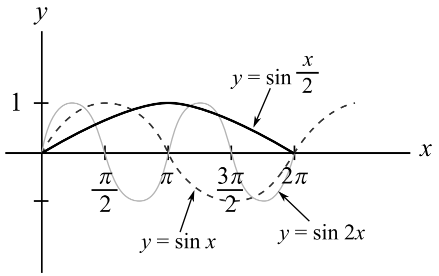
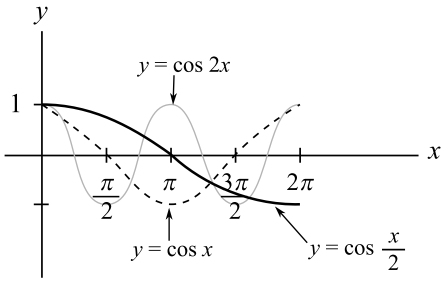
14. Trigonometry, 2ADV T3 2018 HSC 10 MC

Consider options A and C



Consider options B and D



When $y = \cos \dfrac{x}{2}$,

$\displaystyle\int_0^{\pi} f(x)\, dx \neq \int_{\pi}^{2\pi} f(x)\, dx$

$\Rightarrow D$

15. Trigonometry, 2ADV T3 EQ-Bank 5

$g(x) = \dfrac{1}{3}\sin(4x)$

$\Rightarrow$ The new amplitude is one third of the original amplitude.

Period $= \dfrac{2\pi}{n} \;\Rightarrow\; n = \dfrac{1}{4}$

$\Rightarrow$ The new period is one quarter of the original period.

16. Trigonometry, 2ADV T3 2010 HSC 8c

i. $A = 4$

ii. Since the graph passes through $\left(\dfrac{\pi}{4}, 4\right)$

Substituting into $y = 4\sin bx$

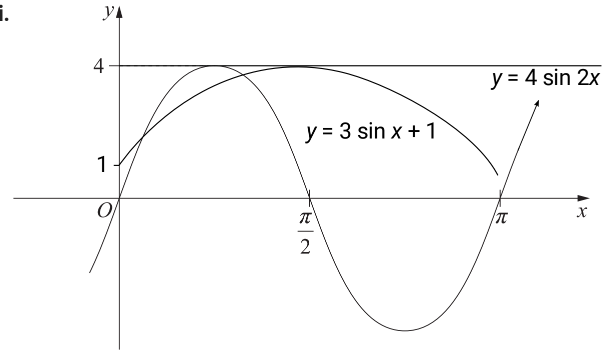
$$4\sin\left(b \times \frac{\pi}{4}\right) = 4$$

$$\sin\left(b \times \frac{\pi}{4}\right) = 1$$

$$b \times \frac{\pi}{4} = \frac{\pi}{2}$$

$$\therefore b = 2$$

iii.



MARKER'S COMMENT: Graphs are consistently drawn too small by many students. Aim to make your diagrams 1/3 to 1/2 of a page.

17. Trigonometry, 2ADV T3 SM-Bank 9

i. $2\cos(x) + 1 = 0$

$$\cos(x) = -\frac{1}{2}$$

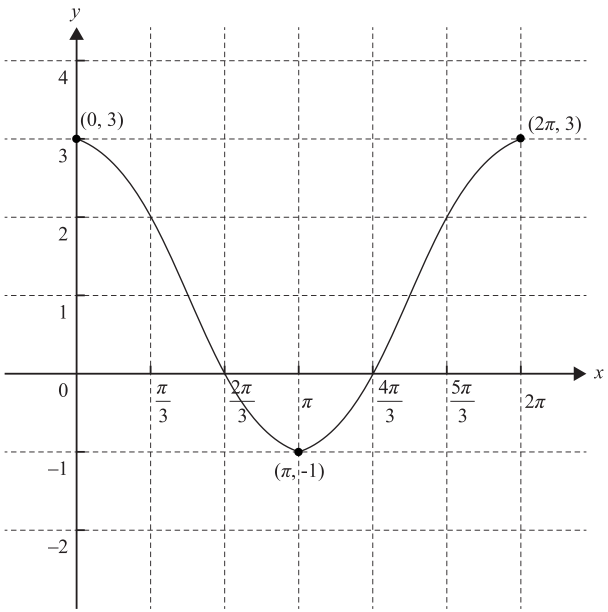
$$\Rightarrow \cos\frac{\pi}{3} = \frac{1}{2} \text{ and cos is negative}$$

in 2nd/3rd quadrant

$$\therefore x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}, \frac{4\pi}{3}$$

ii.



18. Trigonometry, 2ADV T3 2006 HSC 7b

i. $f(x) = 1 + 2\cos x$

$f(x)$ cuts the x-axis when $f(x) = 0$

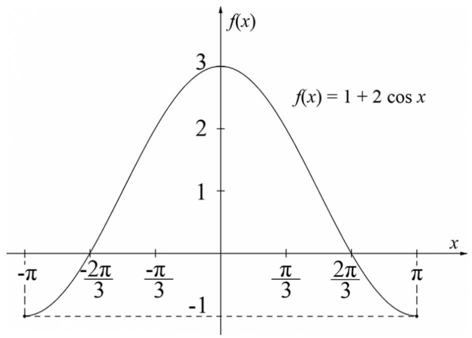
$$1 + 2\cos x = 0$$

$$2\cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$\therefore x = \frac{2\pi}{3} \ldots \text{as required}$$

ii.



iii. $\text{Area} = \displaystyle\int_{-\frac{\pi}{2}}^{\frac{2\pi}{3}} 1 + 2\cos x \ dx$

$$= [x + 2\sin x]_{-\frac{\pi}{2}}^{\frac{2\pi}{3}}$$

$$= \left[\left(\frac{2\pi}{3} + 2\sin\frac{2\pi}{3}\right) - \left(\frac{-\pi}{2} + 2\sin\frac{-\pi}{2}\right)\right]$$

$$= \left(\frac{2\pi}{3} + 2 \times \frac{\sqrt{3}}{2}\right) - \left(\frac{-\pi}{2} + 2(-1)\right)$$

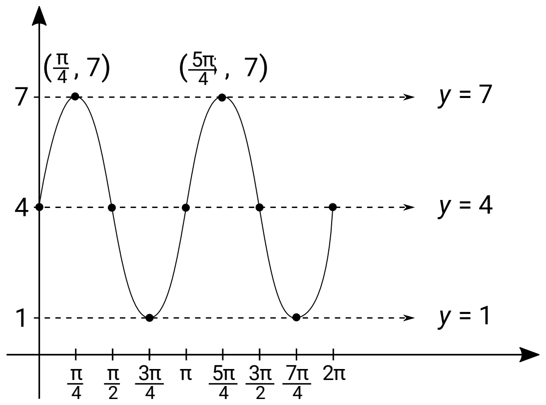
$$= \frac{2\pi}{3} + \sqrt{3} + \frac{\pi}{2} + 2$$

$$= \left(\frac{7\pi}{6} + \sqrt{3} + 2\right) \text{u}^2$$

19. Trigonometry, 2ADV T3 2017 HSC 14a

$y = 4 + 3\sin 2x$

$\Rightarrow$ Amplitude of 3 about $y = 4$



20. Trigonometry, 2ADV T3 2010 MET1 3

$y = \tan(ax + b)$

Substitute $(1, \sqrt{3}), (-1, -1)$ into equation:

$\tan(a + b) = \sqrt{3}$

$\tan(b - a) = -1$

$a + b = \dfrac{\pi}{3} \ldots (1)$

$b - 1 = -\dfrac{\pi}{4} \ldots (2)$

Add $(1) + (2)$:

$2b = \dfrac{\pi}{3} - \dfrac{\pi}{4}$

$b = \dfrac{\pi}{24}$

Substitute into (1):

$a + \dfrac{\pi}{24} = \dfrac{\pi}{3}$

$a = \dfrac{7\pi}{24}$

21. Trigonometry, 2ADV T3 2021 HSC 20

Find x such that:

$2\sin 4x = 1$

$\sin 4x = \dfrac{1}{2}$

$4x = \sin^{-1}\dfrac{1}{2}$

$4x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \ldots$

$\therefore x = \dfrac{\pi}{24}, \dfrac{5\pi}{24} \quad \left(0 \leq x \leq \dfrac{\pi}{4}\right)$

22. Trigonometry, 2ADV T3 SM-Bank 12

$-1 \leq \cos\left(\dfrac{\pi x}{2}\right) \leq 1$

$-3 \leq 3\cos\left(\dfrac{\pi x}{2}\right) \leq 3$

$1 \leq 4 + 3\cos\left(\dfrac{\pi x}{2}\right) \leq 7$

$\therefore$ Range: $1 \leq y \leq 7$

Period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

23. Trigonometry, 2ADV T3 SM-Bank 13

i. h_{min} occurs when $\sin\left(\dfrac{\pi t}{12}\right) = -1$

$$\therefore h_{\text{min}} = 14 - 8$$
$$= 6 \text{ m}$$

MARKER'S COMMENT: Students who used calculus to find the minimum were less successful.

ii. $14 + 8\sin\left(\dfrac{\pi}{12}t\right) = 10$

$$\sin\left(\frac{\pi}{12}t\right) = -\frac{1}{2}$$

Solve in general:

$$\frac{\pi}{12}t = \frac{7\pi}{6} + 2\pi n \quad \text{or} \quad \frac{\pi}{12}t = \frac{11t}{6} + 2\pi n,$$

$$t = 14 + 24n \qquad\qquad t = 22 + 24n$$

Substitute integer values for n,

$$\therefore t = 14 \text{ or } 22, \quad (0 \le t \le 24)$$

24. Trigonometry, 2ADV T3 SM-Bank 14

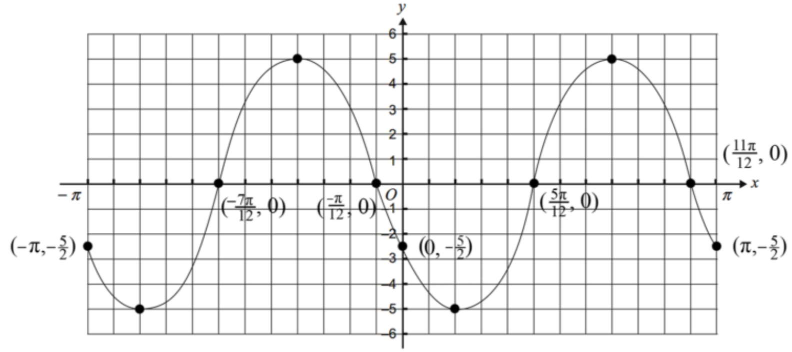
a. Amplitude $= 5$

$$\text{Period} = \frac{2\pi}{2} = \pi$$

b. Shift $y = 5\cos(2x)$ left $\dfrac{\pi}{3}$ units.

Period $= \pi$

Endpoints are $\left(-\pi, -\dfrac{5}{2}\right)$ and $\left(\pi, -\dfrac{5}{2}\right)$



25. Trigonometry, 2ADV T3 SM-Bank 15

i. Intersection occurs when $x=\dfrac{\pi}{3}$,

$$\begin{aligned} a\sin\left(\frac{\pi}{3}\right) &= \cos\left(\frac{\pi}{3}\right) \\ \tan\left(\frac{\pi}{3}\right) &= \frac{1}{a} \\ \sqrt{3} &= \frac{1}{a} \\ \therefore a &= \frac{1}{\sqrt{3}} \end{aligned}$$

ii. $\tan(x) = \sqrt{3}$

$$\begin{aligned} x &= \frac{\pi}{3}, \frac{4\pi}{3}, 2\pi+\frac{\pi}{3}, \ldots \\ \therefore x &= \frac{4\pi}{3} \quad (0 \le x \le 2\pi) \end{aligned}$$

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26. Trigonometry, 2ADV T3 2018 HSC 15a

i. $L(t) = 12 + 2\cos\left(\dfrac{2\pi t}{366}\right)$

On 21 Dec 2015 $\Rightarrow t = 0$

$$\begin{aligned} \therefore L(0) &= 12 + 2\cos 0 \\ &= 14 \text{ hours} \end{aligned}$$

ii. Shortest length of daylight occurs when

$$\cos\left(\frac{2\pi t}{366}\right) = -1$$

♦ *Mean mark 43%.*

$$\begin{aligned} \therefore \text{Shortest length} &= 12 + 2(-1) \\ &= 10 \text{ hours} \end{aligned}$$

iii. Find t such that $L(t) = 11$:

$$11 = 12 + 2\cos\left(\frac{2\pi t}{366}\right)$$

$$\cos\left(\frac{2\pi t}{366}\right) = -\frac{1}{2}$$

$$\begin{aligned} \frac{2\pi t}{366} &= \frac{2\pi}{3} & \text{or} \quad \frac{2\pi t}{366} &= \frac{4\pi}{3} \\ t &= \frac{366}{3} & t &= \frac{366\times 2}{3} \\ &= 122 & &= 244 \end{aligned}$$

$\therefore t = 122$ or 244

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27. Trigonometry, 2ADV T3 SM-Bank 10

i. Period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{\frac{\pi}{3}} = 6$ months

Amplitude $= 400$

ii. Max: $1200 + 400 = 1600$ wombats

Min: $1200 - 400 = 800$ wombats

iii. $$n(10) = 1200 + 400\cos\left(\frac{10\pi}{3}\right)$$
$$= 1200 + 400\cos\left(\frac{2\pi}{3}\right)$$
$$= 1200 - 400 \times \frac{1}{2}$$
$$= 1000 \text{ wombats}$$

iv. Find t when $n(t) = 1000$

$$1000 = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$$
$$\cos\left(\frac{\pi t}{3}\right) = -\frac{1}{2}$$
$$\frac{\pi t}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \dots$$
$$t = 2, 4, 8, 10$$

Since $n(0) = 1600$,

$\Rightarrow n(t)$ drops below 1000 between $t = 2$ and $t = 4$,

and between $t = 8$ and $t = 10$.

$$\therefore \text{ Fraction } = \frac{2+2}{12} = \frac{1}{3} \text{ year}$$

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28. Trigonometry, 2ADV T3 2013 HSC 13a

i. $P(t) = 400 + 50\cos\left(\frac{\pi}{6}t\right)$

Need to find t when $P(t) = 375$

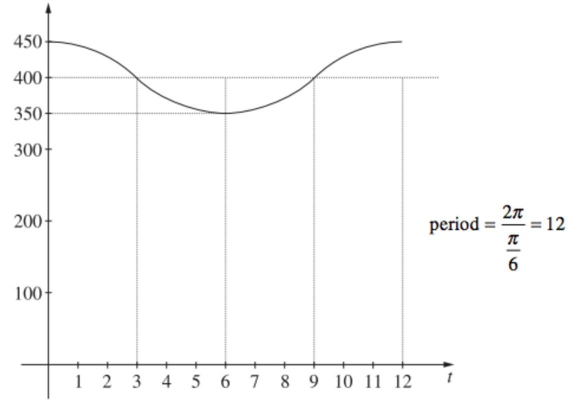
$$375 = 400 + 50\cos\left(\frac{\pi}{6}t\right)$$
$$50\cos\left(\frac{\pi}{6}t\right) = -25$$
$$\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2}$$

Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$, and cos is negative in $2^{\text{nd}}/3^{\text{rd}}$ quadrants:

$$\Rightarrow \frac{\pi}{6}t = \left(\pi - \frac{\pi}{3}\right), \left(\pi + \frac{\pi}{3}\right), \left(3\pi - \frac{\pi}{3}\right)$$
$$= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \dots$$
$$\therefore t = 4, 8, 16, \dots$$

$\therefore$ In the 1st 12 months, $P(t) = 375$ when $t = 4$ months and 8 months.

ii.



♦ Mean mark 39%

29. Trigonometry, 2ADV T3 SM-Bank 16

i. $h_{\text{min}} = 65 - 55 = 10\text{ m}$ $\quad h_{\text{max}} = 65 + 55 = 120\text{ m}$

ii. Period $= \dfrac{2\pi}{\frac{\pi}{15}} = 30$ min

iii. $h'(t) = 65 - 55\cos\left(\dfrac{\pi t}{15}\right)$

$$h'(t) = \frac{\pi}{15} \times 55\sin\left(\frac{\pi}{15}t\right) = \frac{11\pi}{3}\sin\left(\frac{\pi}{15}t\right)$$

Since $\sin\left(\dfrac{\pi}{15}t\right)_{\text{max}} = \sin\left(\dfrac{\pi}{2}\right)$,

$\therefore h'(t)_{\text{max}}$ occurs when

$$\frac{\pi t}{15} = \frac{\pi}{2}$$

$$\therefore t = \frac{\pi}{2} \times \frac{15}{\pi} = \frac{15}{2} \text{ minutes } (0 \leq t \leq 30)$$

30. Trigonometry, 2ADV T3 EQ-Bank 3

Sketch:

$y = \cos x$

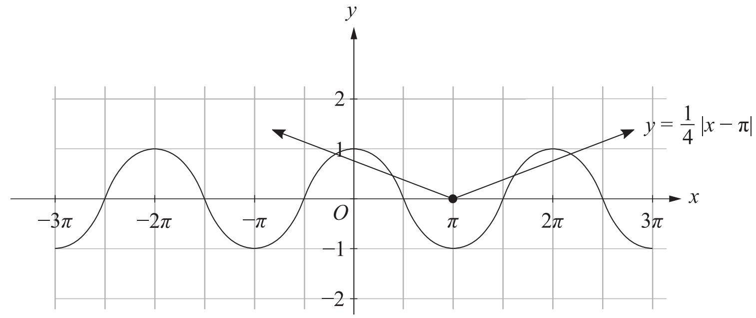
$y = \left|\dfrac{x-\pi}{4}\right|$

Translate π units to the right:

$y = |x| \Rightarrow y = |x - \pi|$

Multiply by $\dfrac{1}{4}$:

$$y = |x - \pi| \Rightarrow y = \frac{1}{4}|x - \pi| = \left|\frac{x-\pi}{4}\right|$$



$\therefore$ There are 4 solutions.

31. Trigonometry, 2ADV T3 2011 SPEC1 8

Intersection occurs when:

$$\text{cosec}^2\left(\frac{\pi x}{6}\right) = \frac{4}{3}$$

$$\text{cosec}\left(\frac{\pi x}{6}\right) = \pm\frac{2}{\sqrt{3}}$$

$$\sin\left(\frac{\pi x}{6}\right) = \pm\frac{\sqrt{3}}{2}$$

Given: $0 < x < 12 \ \Rightarrow\ 0 < \frac{\pi x}{6} < 2\pi$

$$\frac{\pi x}{6} = \frac{\pi}{3}, \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = 2, 4, 8, 10$$

$\Rightarrow y = \frac{4}{3}$ for each

$\therefore$ Intersection at: $\left(2, \frac{4}{3}\right), \left(4, \frac{4}{3}\right), \left(8, \frac{4}{3}\right), \left(10, \frac{4}{3}\right)$

32. Trigonometry, 2ADV T3 SM-Bank 8

♦ *Mean mark 45%.*

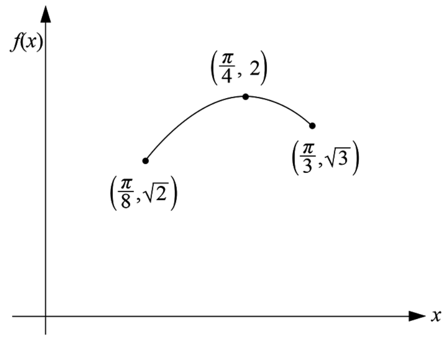
$\sin(2x)_{\text{max}}$ occurs when $x = \frac{\pi}{4}$ (within domain)

$\Rightarrow f(x)_{\text{max}} = 2\sin\left(\frac{\pi}{2}\right) = 2$

Checking endpoints:

When $x = \frac{\pi}{8} \ \Rightarrow\ y = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$

When $x = \frac{\pi}{3} \ \Rightarrow\ y = 2\sin\left(\frac{2\pi}{3}\right) = \sqrt{3}$



$\therefore$ Range $= \left[\sqrt{2}, 2\right]$,

33. Trigonometry, 2ADV T3 2020 HSC 31

a. $b = \dfrac{35\ 000 + 5000}{2}$

$= 20\ 000$

$a =$ amplitude of sin graph

$= 35\ 000 - 20\ 000$

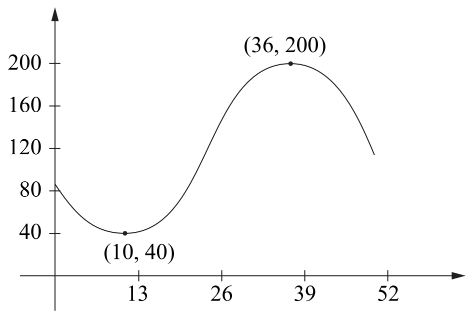
$= 15\ 000$

b. By inspection of the $m(t)$ graph

$m'(t) > 0$ when $0 \leq t < 13$ and $39 < t \leq 52$

♦♦ Mean mark part (b) 30%.

Sketch $c(t)$:



Minimum $(\cos 0)$ when $t = 10$

Maximum $(\cos \pi)$ when $t = 36$

$\therefore c'(t) > 0$ when $10 < t < 36$

$\therefore$ Both populations are increasing when $10 < t < 13$

c. $c(t)$ maximum when $t = 36$

♦♦♦ Mean mark part (c) 27%.

$$m(t) = 15\ 000 \sin\left(\frac{\pi}{26}t\right) + 20\ 000$$

$$m'(t) = \frac{15\ 000\pi}{26}\cos\left(\frac{\pi}{26}t\right)$$

$$m'(36) = \frac{15\ 000\pi}{26} \cdot \cos\left(\frac{36\pi}{26}\right)$$

$$= -642.7$$

$\therefore$ Mice population is decreasing at 643 mice per week.

34. Trigonometry, 2ADV T3 2009 HSC 7b

i. $h = 1 + 0.7\sin\left(\frac{\pi}{6}t\right)$ for $0 \leq t \leq 12$

$$T = \frac{2\pi}{n} \text{ where } n = \frac{\pi}{6}$$

$$= 2\pi \times \frac{6}{\pi}$$

$$= 12 \text{ hours}$$

$\therefore$ The period of h is 12 hours.

ii. Find h at low tide

$\Rightarrow h$ will be a minimum when

$$\sin\left(\frac{\pi}{6}t\right) = -1$$

IMPORTANT: Using $\sin x = -1$ for a minimum here is very effective and time efficient. This property of trig functions is ***often very useful*** in harder questions.

$$\therefore h_{\text{min}} = 1 + 0.7(-1)$$

$$= 0.3 \text{ metres}$$

Since $\sin x = -1$ when $x = \dfrac{3\pi}{2}$

$$\frac{\pi}{6}t = \frac{3\pi}{2}$$

$$t = \frac{3\pi}{2} \times \frac{6}{\pi}$$

$$= 9 \text{ hours}$$

$\therefore$ Low tide occurs at 2pm (5 am + 9 hours)

iii. Find t when $h \geq 1.35$

$$1 + 0.7\sin\left(\frac{\pi}{6}t\right) \geq 1.35$$

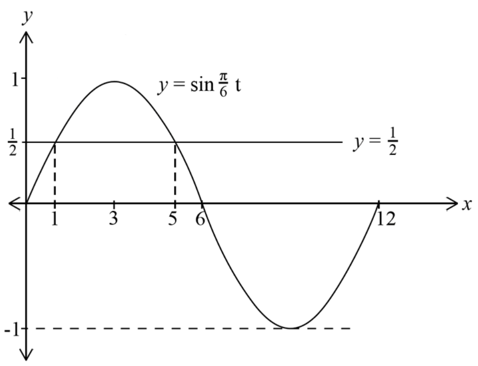
$$0.7\sin\left(\frac{\pi}{6}t\right) \geq 0.35$$

$$\sin\left(\frac{\pi}{6}t\right) \geq \frac{1}{2}$$

$\sin\left(\frac{\pi}{6}t\right) = \dfrac{1}{2}$ when

$$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \text{ etc ...}$$

$$t = 1,\ 5 \qquad (0 \le t \le 12)$$



From the graph,

$$\sin\left(\frac{\pi}{6}t\right) \ge \frac{1}{2} \quad \text{when } 1 \le t \le 5$$

$\therefore$ Ship can enter the harbour between 6 am and 10 am.