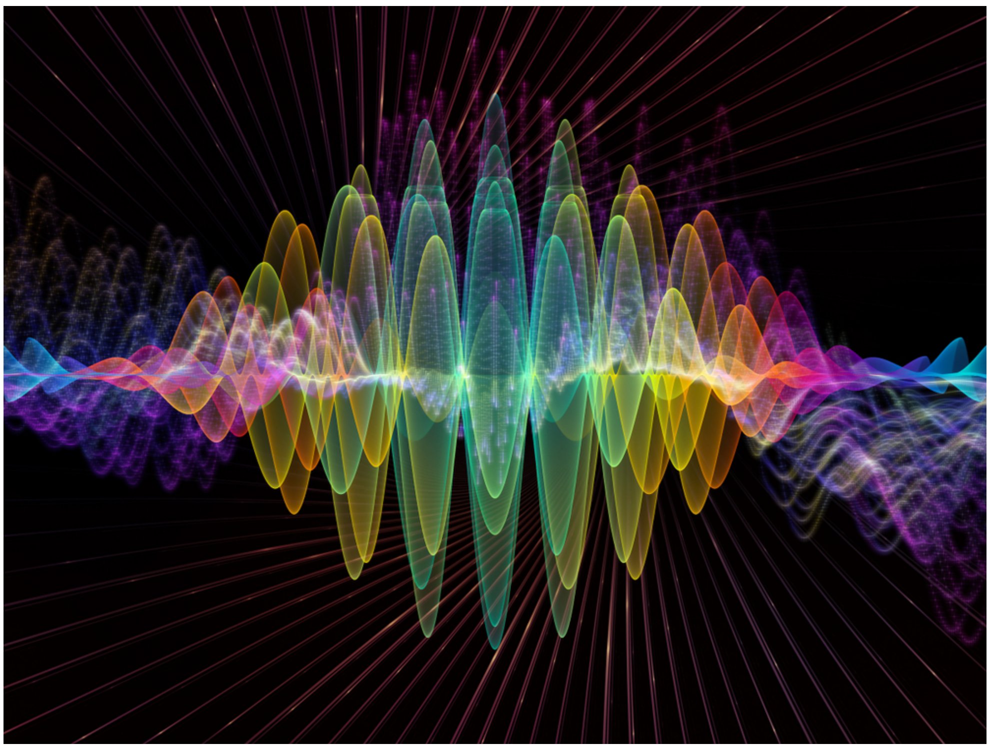
Module 7: The Nature of Light

Part 3: Light - Quantum Model



Tammy Humphrey

Contents

Cover photo credit: Shutterstock

Syllabus content: The Nature of Light

Light: Quantum Model

Inquiry question: What evidence supports the particle model of light and what are the implications of this evidence for the development of the quantum model of light?

Students:

- analyse the experimental evidence gathered about black body radiation, including Wien's Law related to Planck's contribution to a changed model of light (ACSPH137)
  - $\lambda_{max} = \frac{b}{T}$
- investigate the evidence from photoelectric effect investigations that demonstrated inconsistency with the wave model for light (ACSPH087, ACSPH123, ACSPH137)
- analyse the photoelectric effect $K_{max} = hf - \phi$ it occurs in metallic elements by applying the law of conservation of energy and the photon model of light (ACSPH119)

Introduction

In this section we will consider two physical phenomena in detail - black body radiation and the photoelectric effect. We are interested in these two relatively obscure areas of physics as this was where the first evidence began to emerge that classical physics - consisting of Newton's law of Motion and Universal Gravitation, Maxwell's laws of Electromagnetism and Classical thermodynamics - were not completely correct in their description of the universe.

Overview - What is Blackbody radiation?

In part 1 of this module we examined how Maxwell's equations predict the emission of electromagnetic radiation by any accelerating charge. Objects at finite temperature contain charges that are in constant motion, and so are constantly emitting electromagnetic radiation.

How much radiation is emitted and at what wavelengths depends primarily on the temperature of the object, but also how efficiency the object absorbs and emits EM radiation at different wavelengths. Objects that absorb radiation poorly at a certain wavelength also emit poorly at that wavelength.

It is usual to consider an ideal material (a 'black body'), that absorbs perfectly at all wavelengths:

> A **black body** is an idealised object that absorbs all radiation incident upon it.

Black bodies are also "perfect emitters" of radiation, in the sense that they emit the most thermal radiation that it is possible for an object of that temperature to emit at each wavelength. Our sun and other stars are almost perfect black bodies and many terrestrial materials can be treated approximately as black bodies.

The spectrum of wavelengths emitted by such an idealised object is called a "black body spectrum". It depends *only* on the temperature of the object and is shown in figure 4.

Figure 1: Glowing coals emitting light due to their high temperature

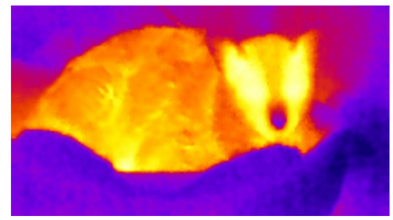


Figure 2: An image of my cat taken with an infrared camera.

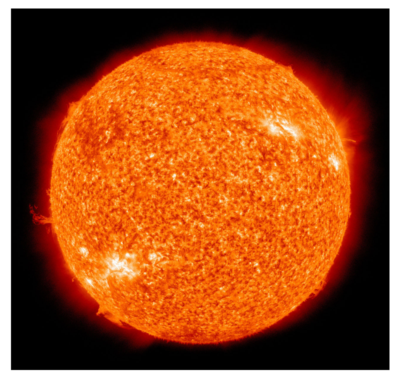


Figure 3: The Sun is a black body to an excellent approximation. By NASA/SDO (AIA)[Public Domain], `https://commons.wikimedia.org/w/index.php?curid=11348381`.

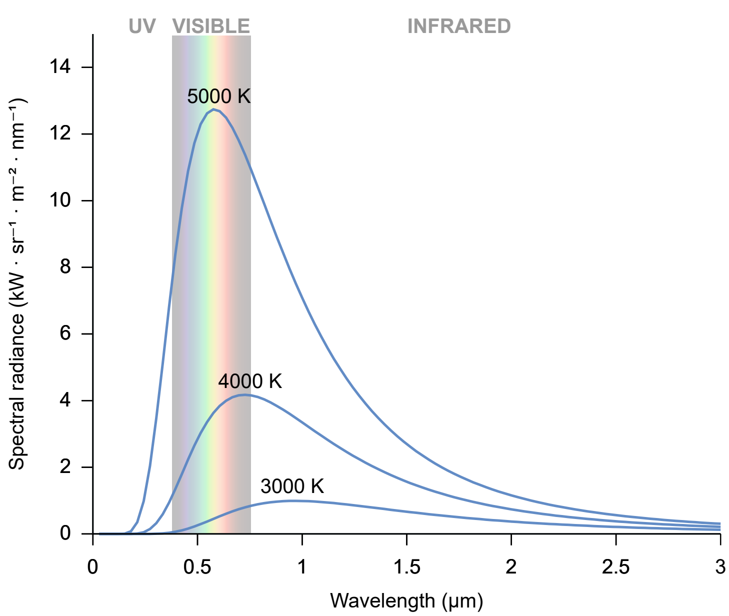


Figure 4: The spectrum of radiation emitted by black bodies of three different temperatures. Figure attribution: Based on work by Darth Kule [Public domain].

Wein's law

The spectrum of radiation emitted by a black body depends on temperature in two ways. One of these is Wein's law, describing the fact that the wavelength at which the body emits the most radiation (the 'peak' wavelength) is inversely proportional to temperature.

Wein's law:

$$\lambda_{max} = \frac{b}{T} \tag{1}$$

where $b = 2.898 \times 10^{-3}\text{m} \cdot \text{K}$ is Wein's displacement constant.

Observe in figure 4 that the **peak wavelength** moves to lower wavelengths with increasing temperature.

The Stephan-Boltzmann law

The other way in which temperature affects the black body spectrum is to dramatically change the total power emitted by the object, where the power is proportional to the area under the black body spectrum.

The Stephan-Boltzmann law:

$$P = A\sigma\varepsilon T^4 \tag{2}$$

where P is the power emitted, A the surface area, ε is the emissivity ($\varepsilon = 1$ for a black body, and $0 < \varepsilon < 1$ for objects that are not perfect black bodies) and T is the temperature of the object.

Observe in figure 4 that the **area under the curve** (which is proportional to the emitted power) increases very substantially as temperature increases due to the dependence on T^4.

While Wein's law is explicitly mentioned in the syllabus along with its equation, the Stephan-Bolzmann law is not. It is nevertheless a good idea to be familiar with this as we will use it later in module 8 when we are looking at the intensity of light from stars of different sizes and temperatures.

Data analysis using Wein's displacement law

We will take (simulated) data for wavelength at which the peak emission occurs for blackbodies of a range of temperatures.

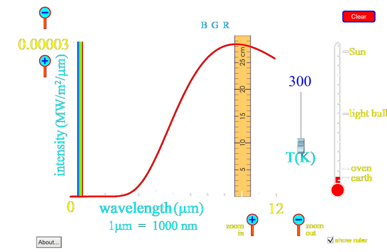


Figure 5: Screenshot from the PhET on Black body radiation

- Open the PhET simulation on blackbody radiation `https://phet.colorado.edu/sims/blackbody-spectrum/blackbody-spectrum_en.html` (or google PhET Blackbody - you will need flash to run this)
- Draw a table in the space to the right (or in your own document or book) with a column for "temperature" and "peak wavelength" (and a spare column). Be careful with your units.
- Choose 5 temperatures (for example 1000K, 2000K, 3000K, 4000K and 5000K) and zoom the graph appropriately so you can measure the wavelength at which the peak emission occurs for each temperature (you can use the "ruler" tool to guide your eye) and record these in your table.
- Decide what you should calculate in the third column in order to obtain a straight line graph that will test Wein's displacement law.
- Plot a graph in the space below (or in a spreadsheet program) to test this. Do your results support Wein's law?

Experimental black bodies

To study a black body radiation experimentally, scientists examine the radiation emerging from a small hole in a heated cavity (e.g. an oven). Such a hole can be considered a black body as any light entering the small hole becomes trapped in the cavity and is eventually absorbed rather than being reflected (so the cavity can be considered 'perfectly absorbing'). It is a little counter-intuitive that a hole in an object should be a perfect emitter. To help you feel convinced, figure 6 shows a small box made from galvanised steel. This box was heated to over 400°C (we can know this as the zinc coating had begun to melt) and observed with an near-infrared camera. Figure 7 shows that in the infrared, the holes in the box glow much brighter than the surface of the box, even though the surface is the same temperature (metal has a low emissivity ε, which means it is a poor approximation to a black body). A video of this demonstration is available here: `https://www.youtube.com/watch?v=-_xHPp-10NU`.

Figure 6: Cavities in a steel box shown in visible light

Figure 7: Cavities in a steel box heated to about 400° imaged with a near infrared camera.

Historical development - Black body radiation

Newton (theory - 1704)

By 1704 Newton had correctly guessed that heated objects emit due to thermal vibrations of particles - in his book Opticks[1] he says:

> Do not all fix'd Bodies, when heated beyond a certain degree, emit Light and shine; and is not this Emission perform'd by the vibrating motion of its parts? - Query 8

[1] Isaac Newton. *Opticks or, a Treatise of the reflexions, refractions, inflexions and colours of light . Also two treatises of the species and magnitude of curvilinear figures*. 1704. URL `https://gallica.bnf.fr/ark:/12148/bpt6k3362k`

Kirchoff (theory - 1860)

Around 1860 German physicist Gustav Kirchoff (also responsible for Kirchoff's laws in circuit theory) proved that the spectrum of radiation emitted by all objects which perfectly absorbed light would be identical, dependent only on temperature. He predicted that it would be very important, but experimentally challenging to determine what the wavelength dependence (shape) of the spectrum actually was.

Stephan and Boltzmann (theory - 1879/1884)

In 1879 Josef Stephan deduced that the power emitted by a black body was proportional to its temperature to the forth power ($P \propto T^4$, as discussed on the previous page) and in 1884 Ludwig Boltzmann derived this from theoretical considerations by using Maxwell's equations to relate the pressure exerted by light inside a cavity to its energy density.

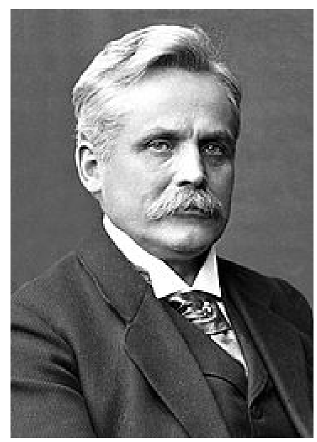


Figure 8: Wilhelm Wein [Public domain].

Wein (theory - 1893/1896)

In 1893 German physicist Wilhem Wein derived his displacement law (Wein's law, discussed on the previous page) and in 1896 he suggested that the shape of the black body spectrum was $I(f,T) \propto f^3 e^{-\frac{Cf}{T}}$.

Lummer and Pringsheim (experiment - 1899)

Otto Lummer and Ernst Pringsheim, working at Berlin's Imperial Institute of Physics and Technology performed careful experiments to measure the blackbody spectrum in the infrared in 1899[2]. Their work provided experimental support for Wein's displacement law ($_{max} \propto \frac{1}{T}$) and for the Stephan-Boltzmann law.

[2] Helge Kragh. *Quantum generations : a history of physics in the twentieth century*. Princeton University Press, 1999. ISBN 0691095523

Their experimental work also showed that Wein's proposal that the black body spectrum was proportional to $I(f,T) \propto f^3 e^{-\frac{Cf}{T}}$ did not accurately describe the spectrum at low frequencies (i.e. in the infrared).

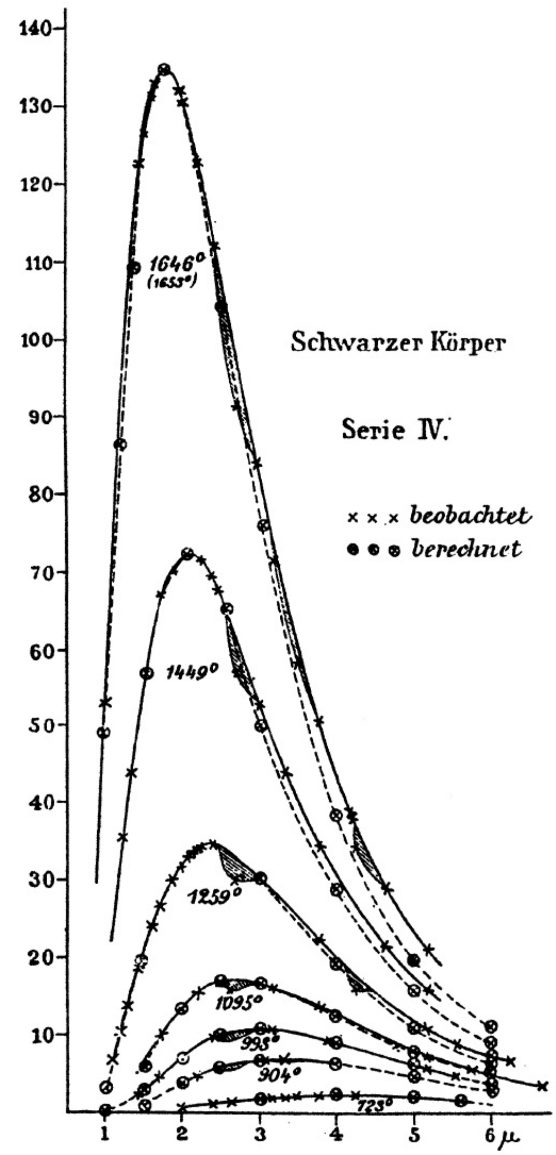


Figure 9: The experimental results of Lummer and Pringsheim in 1899. For higher temperatures the experimental curve is above that predicted by Wein's formula for the black body spectrum. From: Kragh "Quantum Generations"

Rubens and Kurlbaum (experiment - 1900)

Heinrich Rubens and Ferdinand Kurlbaum performed experimental work on the blackbody spectrum at the University of Berlin in 1900 at wavelengths even further in the infrared than Lummer and Pringsheim[3]. Their results provided further experimental proof that Wein's suggested mathematical description of the black body spectrum was incorrect. In particular, their experiments demonstrated that at a fixed wavelength, the spectrum was linearly proportional to the temperature T.

Figure 10: Heinrich Rubens

[3] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL https://link.aps.org/doi/10.1103/RevModPhys.51.863

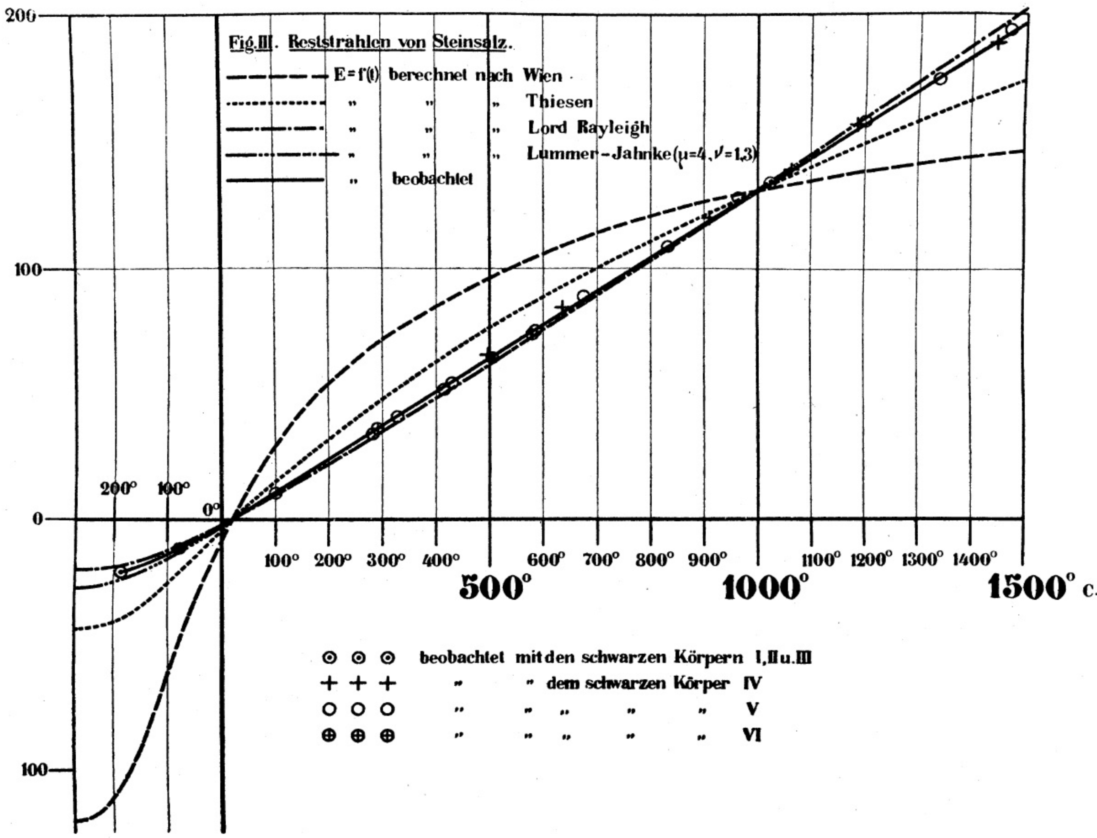


Figure 11: The experimental results of Rubens and Kurlbaum, showing radiation density for a wavelength of 51.2μm, as a function of temperature. The experimental results ("beobachtet") are linear, and a number of predictions made by theorists, including Wein and Lord Rayleigh (but not yet Planck) are shown. From A. Pais "Einstein and the quantum theory" Rev. Mod. Phys. 51, 863 (1979) Fulltext available at: http://ursula.chem.yale.edu/~batista/classes/vvv/RevModPhys.51.863.pdf

Lord Rayleigh (theory - 1900)

In June 1900 Lord Rayleigh (John William Strutt) published a paper in which he used equipartition of energy (the concept in classical thermodynamics that energy is evenly distribution among all particles in a distribution - in this case frequencies of light) to propose that the for low frequencies the black body spectrum is proportional to f^2T. In order to suppress the catastrophic behaviour of this function at large frequencies, he introduced an additional *ad-hoc* term, so that his expression became $I = C_1 f^2 T e^{-\frac{C_2 f}{T}}$, where C_1 and C_2 are constants, f is frequency and T the temperature [4].

Figure 11 shows that Rayleigh's expression deviates from the experimental results obtained by Rubens and Kurlbaum in 1900.

Planck (theory - 1900)

Max Planck was a theorist, also working at the University of Berlin in 1900, who had been thinking about and working on the problem of the black body spectrum for a number of years. After Rubens mentions his results to Planck (at dinner one Sunday afternoon! [5]), Planck is immediately able to use this experimental result to write down a mathematical description of the black body spectrum that simplifies to Wein's law at high frequencies, but is proportional to temperature (for a fixed wavelength) at low frequencies.

$$I(f,T) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{\frac{hf}{kT}} - 1}$$

In the limit $hf >> kT$, Planck's formula[6] reduces to Wein's expression for the black body spectrum at high frequencies. In this case the '-1' in the denominator is very small compared to the exponential term, so that

$$I(f,T) \approx \frac{8\pi h f^3}{c^3} e^{-\frac{hf}{kT}}$$

In the limit that $hf << kT$, then using a Taylor series expansion of $e^{\frac{hf}{kT}} \approx 1 + \frac{hf}{kT}$ it can be shown that Planck's formula reduces to that derived by Lord Rayleigh using equipartition (without the additional ad-hoc term).

$$I(f,T) \approx \frac{8\pi}{c^3} f^2 kT$$

Due to its excellent agreement with experimental results, Planck's empirical formula was immediately accepted as the correct mathematical description of black body spectra.

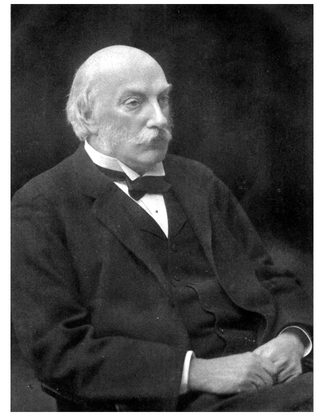


Figure 12: Lord Rayleigh (John William Strutt). Public Domain, `https://commons.wikimedia.org/w/index.php?curid=405463`

[4] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL `https://link.aps.org/doi/10.1103/RevModPhys.51.863`

Figure 13: Max Planck at the time he received his Nobel Prize in 1918. Image credit: AB Lagrelius amp; Westphal. The American Institute of Physics also credits the photo [1] to AB Lagrelius amp; Westphal, which is the Swedish company used by the Nobel Foundation for most photos of its book series Les Prix Nobel. [Public domain]

[5] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL `https://link.aps.org/doi/10.1103/RevModPhys.51.863`

[6] Note: You do not have to know this formula!

Planck immediately began an attempt to derive this empirical result from theoretical considerations. He used considerations of entropy to analyse how electromagnetic radiation described by Maxwell's equations is emitted and absorbed by "charged oscillators" (modelling electric charges in the walls of a black body cavity) [7]. By December 1900 he had succeeded, but found that he needed to make an unusual assumption in order to arrive at the correct result.

[7] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL `https://link.aps.org/doi/10.1103/RevModPhys.51.863`

Electromagnetic radiation is emitted and absorbed by matter in "quanta" with energy

$$E = hf$$

where h is a constant, now known as Planck's constant.

This assumption was without precedence at the time, and initially Planck believed it might be just a mathematical "trick" that could be eliminated from the derivation once physicists understood the physics of blackbodies more thoroughly.

Over the next 10 years Planck continued to work on his derivation of black body radiation. He gradually convinced himself that his assumption that light is emitted and absorbed by matter in packets of energy equal to $E = hf$ was *essential* to obtain the correct result for the black body spectrum - quantisation was indeed of fundamental significance in physics, laying the foundations for the field of quantum mechanics.

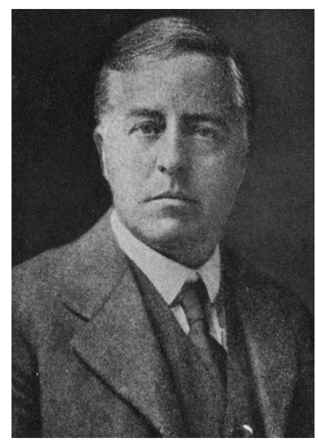


Figure 14: Sir James Hopwood Jeans. [Public Domain], `https://commons.wikimedia.org/w/index.php?curid=11002056`

Rayleigh and Jean (theory - 1905)

In May 1905 Lord Rayleigh submits for publication a further paper on black body radiation, in which he derived the proportionality constant. His derivation was out by a factor of 8, and this was pointed out by James Jeans in another paper, submitted in June 1905.

The ultraviolet catastrophe

The Rayleigh-Jeans derivation (which is also independently derived by Einstein in his paper submitted in June 1905) assumes two results from classical thermodynamics:

- Equipartition of energy (that energy is distributed evenly amongst available states)
- The number of available states available for electromagnetic radiation in a cavity is proportional to f^2.

The implication of these two results are that the number of states that can be occupied by electromagnetic radiation is unbounded at high frequencies, and that any available energy should be evenly distributed amongst these states, resulting in a spectrum that tends to infinity at high frequencies (see figure 16). It is clear that this prediction is not in agreement with the true black body spectrum. Rayleigh and Jeans initially had different ideas about the origin of the problem. Rayleigh's belief was "that we have to admit the failure of the law of equipartition of energy in these extreme cases" (i.e. at high frequencies) [8]. Jeans, on the other hand, believed that the law of equipartition is correct, but "the supposition that the energy of the aether is in equilibrium with that of matter is utterly erroneous in the case of aether vibrations of short wavelength under experimental conditions" [9].

This debate as to whether the problem with the classical derivation could be attributed to a failure of equipartition or a failure of equilibrium between matter and radiation continued for a number of years after 1905.

Hendrik Lorentz pointed out in a lecture in Rome in 1908 that something was seriously amiss with classical physics when it came to describing black body radiation, as classical physics clearly predicts the Rayleigh-Jeans law.

In 1911, Paul Ehrenfest dubbed the problem the "ultraviolet catastrophe" (in the sense that classical physics had suffered a catastrophic setback in predicting such a completely wrong spectrum for black body radiation).

[8] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL `https://link.aps.org/doi/10.1103/RevModPhys.51.863`

[9] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL `https://link.aps.org/doi/10.1103/RevModPhys.51.863`

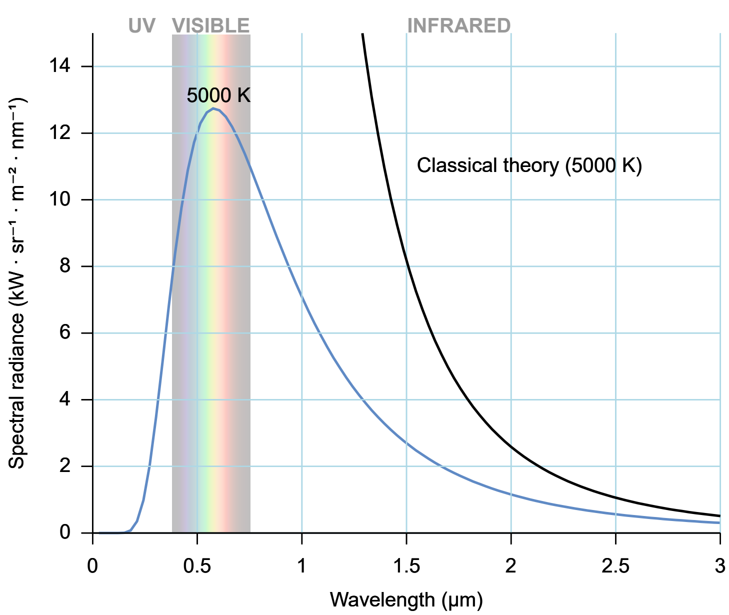


Figure 15: Planck's black body spectrum (in agreement with experimental results) and the curve predicted by classical physics (the "ultraviolet catastrophe" - spectacularly disagreeing with experimental results at small wavelengths)

How quantisation solves the ultraviolet catastrophe

- The **average** thermal energy available to a charge in an object with temperature T is $\approx kT$ where k is a fundamental constant known as Boltzmann's constant.
- If light can only be emitted or absorbed in "packets" with energy $E = hf$, then an oscillating charge can only emit a quanta of frequency f if $hf \lesssim kT$.
- If $hf \gg kT$, then charges in that object will almost never have sufficient energy to emit a quanta with that frequency, and so the intensity of emission at high frequencies (where high means $hf \gg kT$) tends to zero.

Some numbers: For an object at a temperature of 300K a quanta of infrared light has an energy of about $hf \approx kT$, a quanta of visible light has an energy of about $hf \approx 100kT$ and a quanta of UV light has an energy of about $hf \approx 1000kT$.

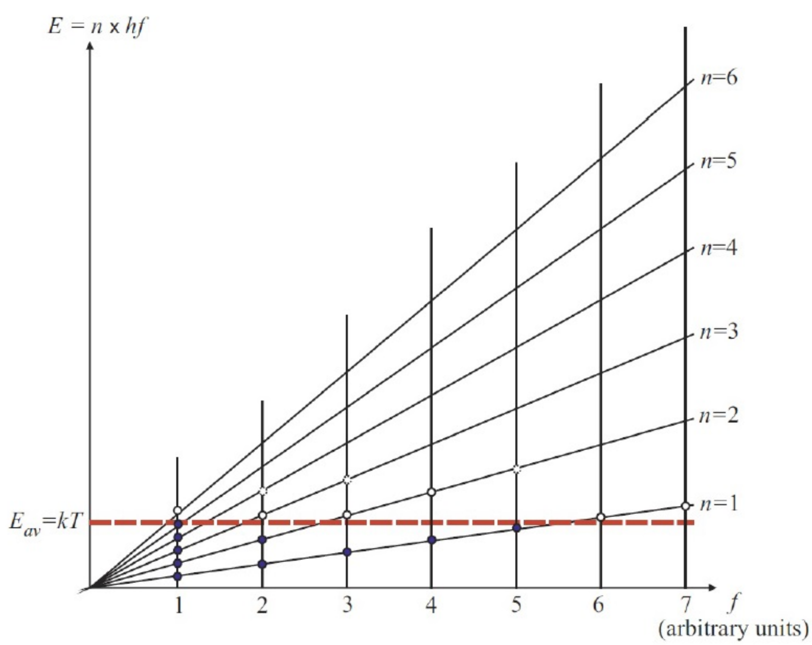


Figure 16: A diagram designed to help explain how quantisation circumvents the ultraviolet catastrophe. The vertical axis is energy and the horizontal axis is proportional to frequency. If the average thermal energy of charges in a particular object is given by the dotted line, then charges in this object almost always have sufficient energy to emit quanta (or multiple quanta, $n > 1$) of light with frequency $f = kT/h$, which are here indicated as frequencies equal to 1, 2, 3, 4, or 5 (in arbitrary units), but rarely have enough energy to emit frequencies where $f > kT/h$, which here would be frequencies 6 or 7 (and higher).

Summary - Black body radiation

Definition:

A black body is an idealised object which absorbs all radiation incident upon it.

Evidence for quantisation of the emission and absorption of light by matter:

Experimentalists including Lummer and Pringsheim in 1899 and Rubens and Kurlbaum in 1900 carefully measured the spectra of blackbodies in infrared wavelengths. This work provided convincing experimental evidence that black body spectra:

- obey Wein's displacement law that the wavelength of maximum emission is inversely proportional to wavelength $\lambda_{max} \propto \frac{1}{T}$
- obey the Stephan-Boltzmann law, that the power emitted by a black body is proportional to T^4.
- are described by the formula derived by Planck, which required the assumption that EM radiation is emitted and absorbed by matter in quanta with energy $E = hf$

Classical physics, in the form of a derivation by Rayleigh and Jeans in 1905, predicted a spectrum that diverged at high frequencies (the "ultraviolet catastrophe"). This prediction does not agree with observations that heated objects do not emit infinite amounts of radiation at high frequencies.

The failure of classical physics in this situation and the success of Planck's theory based on quantisation laid the foundations for the development of quantum mechanics.

The photoelectric effect - Overview

The photoelectric effect can be defined as:
The emission of electrons from a metallic surface in response to the absorption of light.

Figure 17: Heinrich Hertz. Robert Krewaldt [Public domain]. `https://upload.wikimedia.org/wikipedia/commons/1/1e/Wilhelm_Hallwachs.jpg`

Historical development - the Photoelectric effect

Hertz's discovery of the photoelectric effect - 1887

In the first part of this module ("The electromagnetic spectrum") we introduced Heinrich Hertz's confirmation of Maxwell's prediction of the existence of electromagnetic waves with his experiment to produce and detect radio waves. As the detection of radio waves involved observing a very small spark Hertz explains that:

> "I occasionally enclosed the spark B in a dark case so as to more easily make the observations; and in so doing I observed that the maximum spark length became decidedly smaller in the case than it was before... ...A phenomenon so remarkable called for closer investigation."

After undertaking substantial experimental work to ascertain the origin of the effect, Hertz concluded:

> "According to the results of our experiments, ultra-violet light has the property of increasing the sparking distance of an induction coil..."

Hertz had discovered that ultraviolet light has the property of allowing electrons (although Hertz does not yet know them as such) to readily escape a metal surface. When Hertz's receiver was in a dark case, the UV light (produced by sparking across the inductor producing the radiowaves) was no longer falling on his detector, and so no longer assisted electrons to move across a small gap (i.e. to 'spark').

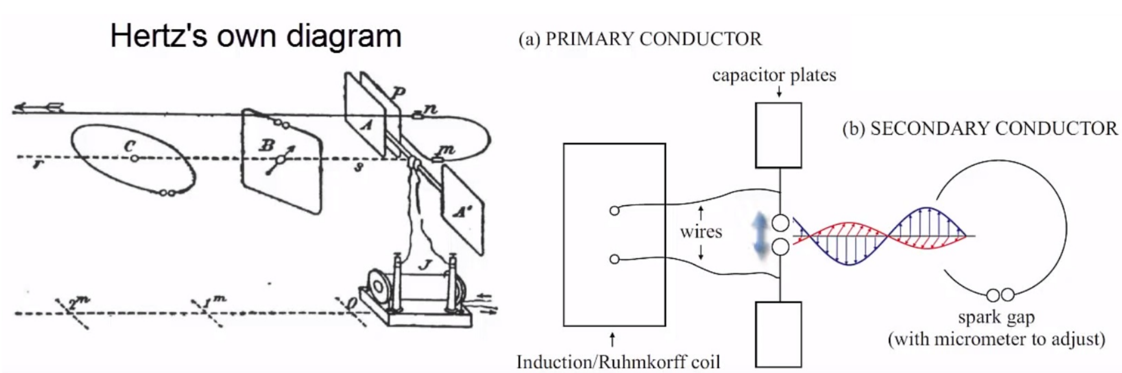


Figure 18: (left) Hertz's own diagram of his experimental setup for producing and detecting radio waves. (Right) A simplified, top-down view diagram.

Hallwachs' investigation using charged electroscopes - 1888

Wilhelm Hallwachs was an assistant of Heinrich Hertz during his efforts to produce and detect electromagnetic radiation. Hallwachs continued to try to clarify and understand the conditions under which the photoelectric effect occurred:

> "I have endeavored to obtain related phenomena which would occur under simpler conditions, in order to make the explanation of the phenomena easier. Success was obtained by investigating the action of the electric light on electrically charged bodies."

Hallwachs established that a neutral conductor becomes positively charged as a result of the photoectric effect[10].

Figure 19: Wilhelm Hallwachs. [Public Domain]. https://commons.wikimedia.org/wiki/File:Wilhelm_Hallwachs.jpg

[10] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL https://link.aps.org/doi/10.1103/RevModPhys.51.863

J.J. Thomson establishes that electrons are emitted (1899)

Thomson was able to measure the charge to mass ratio of the emissions which occurred when light was absorbed by a metallic surface and demonstrated that the charge emitted from the surface was in fact electrons. We will explore Thomson's discovery of the electron in detail in Module 8.

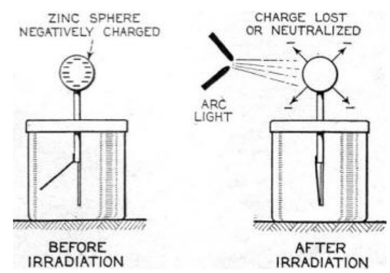


Figure 20: Hallwachs' experiments on the photoelectric effect using charged electroscopes

The photoelectric effect according to classical physics

The classical picture for the photoelectric effect was that light waves were absorbed gradually by electrons in atoms, the electric field of the EM waves causing them to oscillate back and forth. If they absorbed sufficient energy they could "break free" and be emitted from the surface.

According to classical physics:

- Light with greater intensity (i.e. brighter light) would impart more kinetic energy to the photoelectrons as the amplitude of the electric field of the EM wave is larger (as the energy carried by an EM wave is proportional to the square of the amplitude of the electric field, a fact we used in our derivation of Malus' law)
- There is no expectation of a proportional relationship between the frequency of the EM radiation and the kinetic energy of photoelectrons (nor of any associated minimum frequency required for emission of electrons)
- that a time delay would be expected between light hitting the surface and the emission of electrons, due to the time taken for electrons to absorb sufficient energy to escape the surface.

Figure 21: J.J. Thomson. [Public Domain], https://commons.wikimedia.org/w/index.php?curid=2969861

Lenard shows that intensity does not affect the kinetic energy of photo-electrons (1902)

Figure 22: Philip Lenard. [Public Domain], https://commons.wikimedia.org/w/index.php?curid=2899386

In 1902 Philip Lenard used a carbon arc light source to investigate the photoelectric effect. The intensity of Lenard's source could be varied by a factor of 1000. Contrary to what would be expected from classical physics, Lenard found that the energy of the emitted photo-electrons [11]:

> "Showed not the least dependence on intensity"

Lenard also determined experimentally that the emission of photo-electrons was *simultaneous* with illumination of the surface.

[11] A. Pais. Einstein and the quantum theory. *Reviews of Modern Physics*, 51 (4):863–914, 10 1979. ISSN 0034-6861. DOI: 10.1103/RevModPhys.51.863. URL https://link.aps.org/doi/10.1103/RevModPhys.51.863

Experimental measurement of the maximum KE of photoelectrons

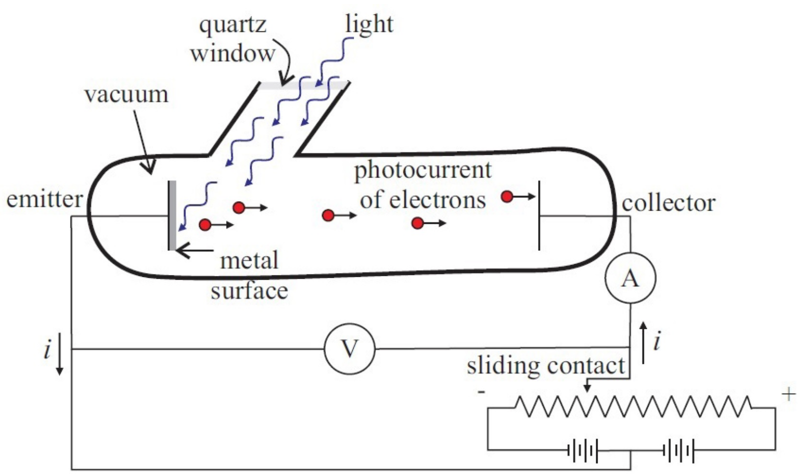


Figure 23: A schematic of an experimental setup for studying the photoelectric effect.

In figure 23 light of a particular wavelength passes through a quartz window [12] so that it falls on a metal surface. Photoelectrons emitted by the surface (called the 'cathode') can be accelerated towards an anode if a *forward bias* is applied. Usually however, a *back voltage* is applied such that the anode is more negative than the metal surface. In this case the electrons are *decelerated* as they travel towards the anode.

[12] Quartz is used rather than glass as glass blocks UV light

As mechanical energy is conserved for the electrons travelling through the evacuated tube, kinetic energy is converted into electrical potential energy as they approach the anode, so that $eV + \Delta KE = 0$.

For small back voltages the electrons may have sufficient energy to be able to travel across the gap to the anode, in which case the reduction in their kinetic energy is less than their total kinetic energy, so they arrive with energy to spare. To make a measurement of the maximum kinetic energy of the photoelectrons, the voltage is increased up to the "stopping voltage" - the voltage which *just* prevents electrons from arriving at the anode.

In this case

$$eV_{stop} = KE_{max}$$

where e is the charge on an electron, V_{stop} is the stopping voltage and KE_{max} is the maximum kinetic energy of the photoelectrons.

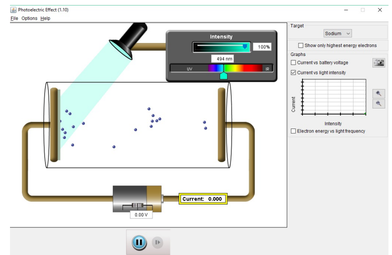


Figure 24: Screenshot from the PhET on the photoelectric effect

Photoectric effect - PhET simulation

We will use the Phet similation of the photoelectric effect to explore this in class: `https://phet.colorado.edu/en/simulation/photoelectric` (or google "PhET photoelectric effect").

Speed (kinetic energy) versus wavelength (qualitative):

- Choose Sodium as the metal surface on the right drop-down menu, set the wavelength to the infrared (IR) and set the intensity slider to 100%. Leave the voltage at 0V.
- Slowly drag the wavelength slider to the left, from IR towards red, then to green. Continue to slowly drag the wavelength towards the UV.

What do you observe when you reach green? What changes as you move to the UV?

Photocurrent versus intensity:

- Leaving the wavelength slider in the UV, click the checkbox to show current versus light intensity.
- Slowly change the intensity of the light from 100%.

What effect does changing the intensity have on the current?

Sketch a graph (in the space on the right) of how the current varies with light intensity.

Speed (kinetic energy) versus light intensity (qualitative):

- Very carefully examine the speed of the electron emerging from the Sodium surface as you change the light intensity from about 20% to 100%.

Does the speed appear to change in response to intensity?

Speed (kinetic energy) versus wavelength (quantitative):

- Now tick the checkbox to show the graph of maximum energy of the electrons versus frequency.
- Slide the wavelength slider back and forth for sodium. Carefully sketch the graph you obtain in the space on the right.
- Note down the threshold frequency and calculate the gradient.
- Repeat this process for Zinc and Platinum (select from right hand drop-down menu) and include data for these metals on your graph.

What is different for the three metals and what remains the same?

As discussed earlier, in a real experiment, the way in which the kinetic energy of the electrons is measured is to gradually increase the 'back voltage' until the current drops to zero. At this point, where electrons *just* don't make it to the anode, the change in potential energy eV is equal to the maximum kinetic energy of electrons KE_{max}.

- Reset the metal to Sodium.
- Tick the checkbox to show a graph of current versus battery voltage.
- Set the wavelength slider to about 250nm
- Begin to increase the back voltage (i.e. make the voltage more negative) until electrons *just* can't quite make it across and the photocurrent drops to zero. Note down this value of voltage.

Use this value to calculate the maximum kinetic energy of the electrons leaving Sodium as a result of absorbing light with a wavelength of 250nm.

Einstein postulates light quanta (1905)

Einstein and Black body radiation

In 1905, Einstein also performed the same derivation of the black body spectrum predicted by classical physics as Rayleigh and Jeans, noting the unphysical predictions obtained. He also noted that Planck's derivation contained inconsistencies.

Einstein then re-derived Planck's formula for black body radiation using a *different* assumption to Planck, that

> Light *itself* is quantised, with individual quanta having energy $E = hf$

The motivation for this hypothesis was Einstein's insight that Planck's equation for the black body spectrum had the same *form* as the equation for the distribution of energies of particles in a box (e.g. gas particles in a particular volume) *if* we can make the assumption that in this case each light quanta behaves as a particle with energy $E = hf$.

Figure 25: Einstein at the time he was working at the patent office and publishing his groundbreaking papers. Taken by Lucien Chavan, a friend of Einstein's when he was living in Berne. [Public domain].`https://upload.wikimedia.org/wikipedia/commons/a/a0/Einstein_patentoffice.jpg`

Einstein and the Photoelectric effect

Once Einstein had made his light quantum hypothesis, that light itself is quantised, he predicted that with regard to the photoelectric effect:

- Quanta with energy $E = hf$ penetrate the metal surface and each quanta of light gives all (or none) of its energy to a single electron.
- The kinetic energy of the photoelectrons leaving the surface is the difference between the energy of a quanta of light and the energy needed to escape the lattice, denoted Φ.

That is, Einstein predicted that the maximum energy of photoelectrons should be given by

$$KE_{max} = hf - \Phi \tag{3}$$

where hf is the energy of incident light quanta and Φ is called the 'work function' of the metal. The work function is the minimum amount of energy required to extract an electron from the surface and varies for different metals.

The implications of Einstein's proposal are that:

- The maximum energy of photoelectrons should be independent of the intensity of light, as had already been experimentally observed by Lenard.
- There is a minimum frequency required to remove electrons from a given metal (the 'threshold frequency'), $f_{th} = \Phi/h$.
- For light with $f > f_{th}$, the photocurrent of electrons should be proportional to how many light quanta arrive at the surface per second, that is, the intensity of light.
- There should be no time delay between light being incident on the surface and the emission of photoelectrons (as had been observed experimentally by Lenard).

Questions on the photoelectric effect

Question 1. Using the information you obtained about sodium in the PhET simulation, calculate the work function of sodium and check your answer using information from the internet.

Question 2. What is the significance of the gradient that you calculated in the PhET simulation?

Question 3.

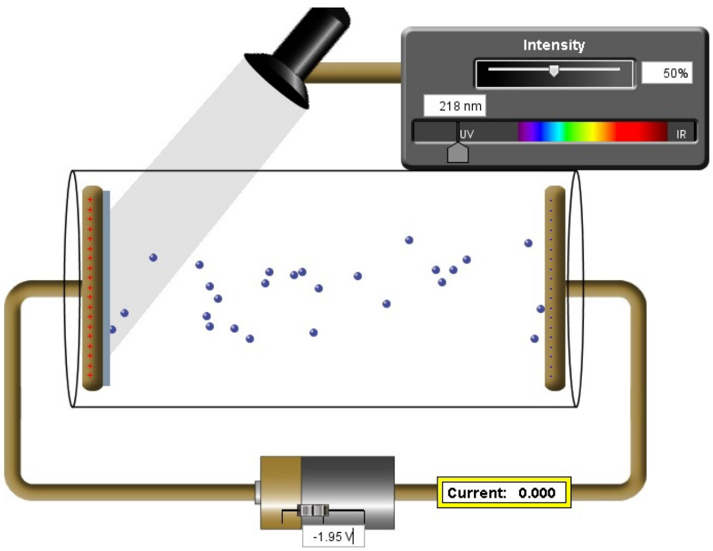


Figure 26: PhET photoelectric effect screenshot for the "unknown" (????) metal surface.

(a) Use the information in the screenshot above to calculate the workfunction of the "unknown" metal on the PhET simulation.

(b) Use the internet to find a list of the workfunctions of common metals and use it to identify the "unknown" metal.

Resistance to the idea of light quanta

For many years there was no support from Planck or other physicists for Einstein's interpretation of light as independent quanta. The reason for this resistance was that the physics community had already accepted the compelling evidence that came from Hertz's experiments and Young's double slit experiment that light was a wave, and physicists were confident that all there was to know about light was contained in Maxwell's equations.

For example, Planck states in a letter to Einstein in 1907 that:

> "I am not seeking the meaning of the quantum of action (light quantum) in the vacuum, but rather in places where absorption and emission occur, and [I] assume that what happens in the vacuum is rigorously described by Maxwell's equations."

As another example, when nominating Einstein for admission to the prestigious Prussian academy of science in 1913, Planck and Walther Nernst (among others) said:

> In sum, one can say that there is hardly one among the problems in which modern physics is so rich to which Einstein has not made a significant contribution. That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light quanta, cannot really be held too much against him, for it is not possible to introduce really new ideas even in the most exact science without sometimes taking risk."

Experimental evidence supporting Einstein's predictions regarding the photoelectric effect began to emerge with Millikan's careful photoelectric experiments in 1916, however his theory was still not widely accepted.

Millikan's experiments on the photoelectric effect (1916)

In 1916 Robert Millikan published a paper[13] [14] detailing his extensive experimental tests on the photoelectric effect. In his own words:

> Einstein's photoelectric equation has been subjected to very searching tests and it appears in every case to predict exactly the observed results.

However, Millikan also says:

> Despite then the apparently complete success of the Einstein equation, the physical theory of which it was designed to be the symbolic expression is found so untenable that Einstein himself, I believe, no longer holds to it.

In the introduction to his paper he describes the origins of the scientific opposition to Einstein's proposal:

> It was in 1905 that Einstein made the first coupling of photo effects and with any form of quantum theory by bringing forward the bold, not to say the reckless, hypothesis of an electro-magnetic light corpuscle of energy hf, which energy was transferred upon absorption to an electron. This hypothesis may well be called reckless first because an electromagnetic disturbance which remains localized in space seems a violation of the very conception of an electromagnetic disturbance, and second because it flies in the face of the thoroughly established facts of interference. The hypothesis was apparently made solely because it furnished a ready explanation of one of the most remarkable facts brought to light by recent investigations, viz., that the energy with which an electron is thrown out of a metal by ultra-violet light or X-rays is independent of the intensity of the light while it depends on its frequency.

This opinion was widely shared by leading scientists. In describing these experimental results in 1918, Rutherford says:

> "There is at present no physical explanation of this remarkable connection between energy and frequency."

Figure 27: Robert Millikan. [Public Domain], `https://commons.wikimedia.org/w/index.php?curid=41222283`

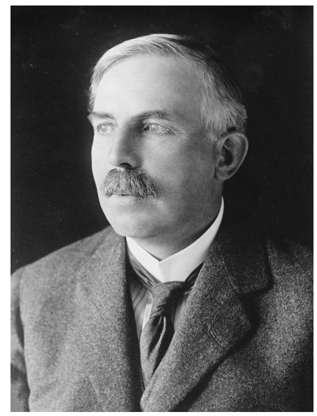


Figure 28: Ernest Rutherford.[Public Domain], `https://commons.wikimedia.org/w/index.php?curid=35928470`

[13] R. A. Millikan. A Direct Photoelectric Determination of Planck's " h ". *Physical Review*, 7(3):355–388, 3 1916. ISSN 0031-899X. DOI: 10.1103/PhysRev.7.355. URL `https://link.aps.org/doi/10.1103/PhysRev.7.355`

[14] Fulltext here: `https://journals.aps.org/pr/pdf/10.1103/PhysRev.7.355`

Einstein is awarded a Nobel prize for the photoelectric effect

The tide began to turn in the early 1920's. Einstein received his nobel prize in 1921, for his "services to theoretical physics, and especially for his discovery of the law of the photoelectric effect".

Einstein's own views continued to evolve after 1905. In 1909 he said:

> "It is my opinion that the next phase in the development of theoretical physics will bring us a theory of light that can be interpreted as a kind of fusion of the wave and emission theory."

In 1916 and 1917 Einstein developed a new, statistically based description of the emission and absorption of light, which has formed the basis for our understanding of many important phenomena involving the interaction of light and matter during the 20th century, such as lasers.

At this time he developed an understanding of light quanta as true particles (which later came to be known as *photons*) which carried momentum hf/c rather than just as a quanta of energy.

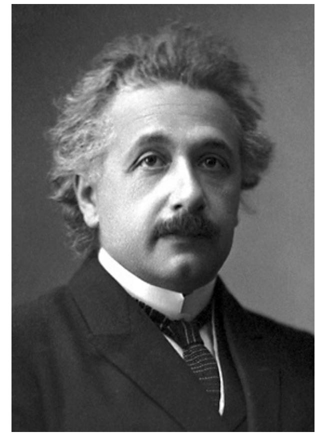


Figure 29: Einstein. Official 1921 Nobel Prize in Physics photograph [Public domain]. `https://commons.wikimedia.org/wiki/File:Albert_Einstein_(Nobel).png`

Compton (1923)

In 1923 Arthur Compton demonstrated that in the collision between an electron and a photon, the photon behaves as a particle with regard to conservation of momentum and energy.

This experiment was viewed as highly significant and the concept of a photon as a particle become widely accepted after this.

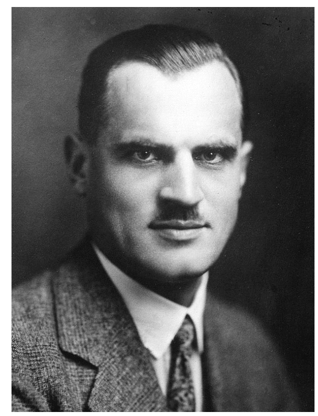


Figure 30: Arthur Compton, [Public Domain], `https://commons.wikimedia.org/w/index.php?curid=41219273`

Summary - the Photoelectric effect

Definition:

The photoelectric effect is the emission of electrons from a metal surface in response to the absorption of light.

The wave model predicts that for the photoelectric effect:

- Light with greater intensity (i.e. brighter light) would impart more kinetic energy to the photoelectrons
- There is no expectation that frequency of the EM radiation is proportional to the kinetic energy of photoelectrons, and no expectation that a minimum frequency would be required for the emission of electrons
- that a time delay would be expected between light hitting the surface and the emission of electrons, as an electron absorbs sufficient light energy to escape.

Lenard and Millikan conducted experimental investigations of the photoelectric effect with results that were inconsistent with the wave model for light, finding that:

- The maximum energy of photoelectrons is independent of the intensity of light
- There is a minimum frequency required to remove electrons from a given metal (the 'threshold frequency'), $f_{th} = \Phi/h$.
- For light with $f > f_{th}$, the photocurrent of electrons is proportional to how many light quanta arrive at the surface per second (the intensity of light).
- There is no time delay between light being incident on the surface and the emission of photoelectrons.

The implications of this evidence was that it provided support for the quantum model of light proposed by Einstein, in which:

- Light quanta with energy $E = hf$ penetrate the metal surface and each quanta gives all (or none) of its energy to a single electron.
- The kinetic energy of the photoelectrons leaving the surface is the difference between the energy of a quanta of light and the energy needed to escape the lattice,

$$KE_{max} = hf - \Phi$$

where hf is the energy of incident light quanta and Φ is called the 'work function' of the metal.

Practical activities - photoelectric effect

Practical 1 - Using LEDs

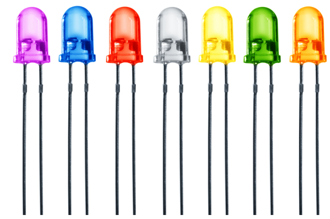


Figure 31: The colour (frequency) of light emitted by an LED depends on the energy of the "bandgap" of the semiconductor it is made of.

Light emitting diodes (LEDs) are a type of semiconductor device that produce light with an energy $E = hf$ that corresponds to an internal energy level for electrons in the material (the "bandgap"). Different semiconductor materials have different bandgaps, and so emit light with different frequencies.

In order for an LED to emit light, electrons with energy eV must be supplied (where V is the applied voltage), where this energy is *larger* than the bandgap. An LED is *just* at the point of emitting light when the energy of electrons equals the energy of the bandgap, and the current through the LED is *just* greater than zero.

$$eV = hf$$

By measuring the voltage at which an LED *just* begins to conduct ($I \approx 2\mu$A) and graphing this against the frequency of light emitted by the LED, we can estimate Planck's constant from the gradient of the graph.

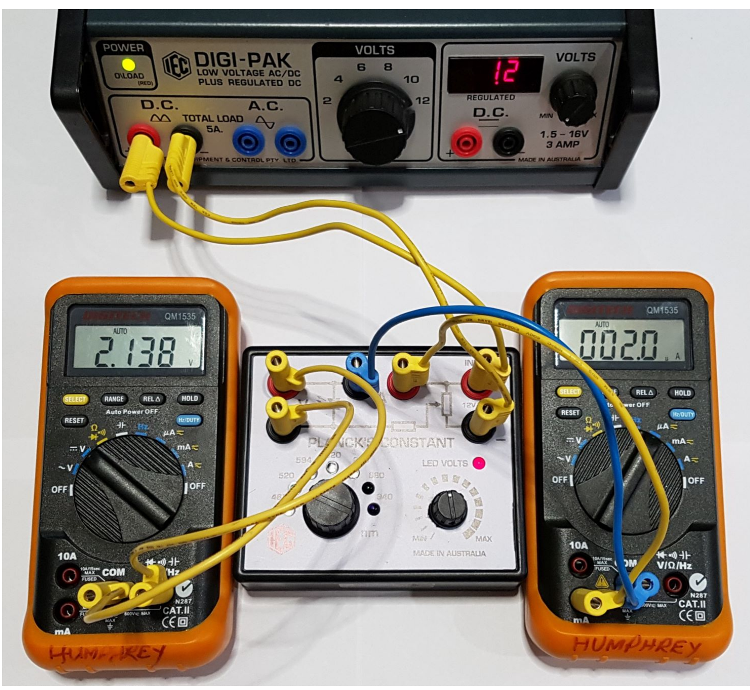


Figure 32: Setup for LED photoelectric prac

- Set up the equipment as shown in figure 32, using a power supply set to $12V$ and a multimeter set on 'V' to measure the voltage across each LED, and another multimeter set to 'μA' to measure the current through the LED. For each measurement, ensure you begin with "LED Volts" set to zero, so that you do not drive excess current through any of the LEDs (particularly those with long wavelengths and small bandgaps - red and IR).

- Select an LED and slowly increase the voltage across the LED until the current just reaches 2μA (i.e. the voltage at which it *just* begins to conduct).
- Record the wavelength and voltage across the LED in a table. Leave an empty column in your table in the space on the right.
- Consider what to graph so that you will obtain a straight line, and calculate and enter this in your empty column.
- Draw a graph and calculate a value of Planck's constant from your graph.

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Sample data for practical activities

Wein's displacement law (using PhET)

$T(\mathrm{K})$	$\lambda_{max}(\times 10^{-6}\mathrm{m})$	$\frac{1}{T}(\times 10^{-4}K^{-1})$
1020	2.8	9.80
2010	1.4	4.98
3000	1.0	3.33
4000	0.7	2.50
5000	0.6	2.00

LED photoelectric experiment

$\lambda(\times 10^{-9}\mathrm{m})$	Voltage (V)	$f(\times 10^{14}\mathrm{Hz})$
465	2.299	6.45
520	2.138	5.77
594	1.590	5.05
620	1.536	4.84
660	1.369	4.55
880	0.947	3.41
940	0.796	3.19