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2023 BORED OF STUDIES TRIAL EXAMINATION

Physics

Marking Guidelines

Section I

Multiple-choice answer key

1	2	3	4	5	6	7	8	9	10
D	A	A	D	A	D	B	A	B	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	C	B	B	A	D	B	C	D

Multiple-choice explanations

Question	Ans	Explanation
1	D	Phototrons do not exist.
2	A	Planck's Law gives the spectral radiance of a blackbody as a function of temperature and wavelength. $$B = \frac{2hc^2}{\lambda^5} . \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$
3	A	Hadrons are particles made from quarks. Fermions are particles with half-integer spin. Baryons are particles made from three quarks (protons and neutrons) so all baryons are hadrons. Quarks have half-integer spin, so all baryons are also fermions. Bosons are particles with integer spin. Mesons are particles made from two quarks (half-integer spin), so all mesons have integer spin and are bosons. Leptons have half-integer spin, but are not made of quarks, so not all fermions are hadrons.
4	D	When the satellite is moved to a higher orbit, the radius increases. The gravitational potential energy is $$U = -\frac{GMm}{R}$$ The orbital velocity $v = \sqrt{\frac{GM}{R}}$ gives a kinetic energy of $$K = \frac{1}{2}\frac{GMm}{R} = -\frac{1}{2}U$$ Hence the total energy is given by $E = K + U = \frac{1}{2}U = -K$ The work done is the magnitude of the change in total energy which is the

		same as the absolute change in kinetic energy, giving option D.
5	A	The finite width double slit pattern is the convolution of a double slit interference pattern bounded by a single slit diffraction envelope. Removing the second slit produces just the diffraction envelope, hence decreasing the number of maxima seen.
6	D	The electric field halfway between the charges is the vector sum of the contributions from each. The magnitude of the contribution from a charge q and a distance $\frac{r}{2}$ is $$\lvert E_i \rvert = \frac{1}{4\pi\epsilon_0}.\frac{q_i}{\left(\frac{r}{2}\right)^2}$$ If the charges are the same sign, then the electric field acts in opposite directions and have the same magnitude so they cancel out. If the charges are opposite in sign, then the electric field strength is doubled. Hence $$\lvert E \rvert = \frac{1}{4\pi\epsilon_0}.\frac{\lvert q_1 - q_2 \rvert}{\left(\frac{r}{2}\right)^2}$$
7	B	$$E_{\text{total}} = mc^2 + (\gamma - 1)mc^2$$ $$E_{\text{total}} = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$
8	A	Only the surface temperature of a star can be determined from the absorption spectra, not the core temperature.
9	B	At the threshold frequency f $$K = 0 = hf - \phi$$ So, the work function is $\phi = hf$ At frequency *2f* $$K_{2f} = 2hf - hf = hf$$ At frequency *4f* $$K_{4f} = 4hf - hf = 3hf = 3K_{2f}$$ The kinetic energy of photoelectrons is three times greater and $K \propto v^2$ then $$v_{4f} = \sqrt{3}v_{2f}$$
10	B	In cluster Y, the more massive stars have reached the main sequence, while the lower mass stars are still in the pre-main sequence (T Tauri) stage. In cluster X, the more massive stars have either gone to supernova or expanded into red giants or super giants, leaving the lower mass stars in the main sequence.

		In cluster Z, even the medium mass stars have started forming red giants. There is also the formation of white dwarfs beginning.
11	B	The torque on the coil should be balanced at equilibrium. Since the magnetic field is radial, the torque is not dependent on the angle. $$nIAB - k\theta = 0$$ Solving this gives $$I = \frac{k\theta}{nAB} = \frac{0.034 \times 18 \times \frac{\pi}{180}}{100 \times 0.03^2 \times 0.54} = 0.22 \text{ A}$$
12	C	First, find the maximum height s_1 of the projectile if the acceleration does not change, which occurs when $$v_y^2 = 0 = u_y^2 - 2gs_1$$ So $$s_1 = \frac{u_y^2}{2g} = \frac{(50 \sin 30°)^2}{2 \times 9.8} = 31.89 \text{ m}$$ At half this height, the acceleration changes to $\frac{g}{2}$. At this point, we let $v_{\frac{1}{2},y}$ be the upward velocity of the projectile at this time. Then $$v_{\frac{1}{2},y} = \sqrt{u_y^2 - \frac{2gs_1}{2}} = \sqrt{(50 \sin 30°)^2 - \frac{2 \times 9.8 \times 31.89}{2}} = 17.68 \text{ ms}^{-1}$$ Now to find the new maximum height above $\frac{s_1}{2}$, set $$v_y^2 = 0 = v_{\frac{1}{2},y}^2 - \frac{2g}{2} s_2$$ $$s_2 = \frac{v_{\frac{1}{2},y}^2}{g} = \frac{17.68^2}{9.8} = 31.89 \text{ m}$$ And finally, the maximum height above the ground is $$h = \frac{s_1}{2} + s_2 = \frac{31.89}{2} + 31.89 = 48 \text{ m}$$ Hence, the answer is C.
13	C	First consider the ball as it rolls along the table. Relative to the table, the ball accelerates at 3.0 ms^{-2} to the right. The horizontal velocity as it reaches the edge is $$v^2 = 2as$$

		$$v = \sqrt{2 \times 3 \times 2.1} = 3.54965 \ldots \text{ ms}^{-1}$$ Now consider the time taken for the ball to fall 1 m vertically. $$h = \frac{1}{2}gt^2$$ $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1}{9.8}} = 0.45175 \ldots \text{ s}$$ Then the distance travelled by the ball horizontally during this time is $$x = vt + \frac{1}{2}at^2 = 3.54965 \times 0.45175 + \frac{1}{2} \times 3 \times 0.45175^2 = 1.9 \text{ m}$$
14	B	At the minimum uncertainty, $$\Delta x \Delta p = \frac{h}{4\pi}$$ Since the electron is bound to the atom $\Delta x = 1 \times 10^{-10}$ m then $$\Delta x m \Delta v = \frac{h}{4\pi}$$ and the uncertainty in the velocity is $$\Delta v = \frac{h}{4\pi m \Delta x} = \frac{6.626 \times 10^{-34}}{4\pi \times 9.109 \times 10^{-31} \times 1 \times 10^{-10}} = 5.8 \times 10^5 \text{ ms}^{-1}$$ To 1 significant figure, this gives the answer B.
15	B	The energy released in the reaction is the binding energy of the products less the binding energy of the reactants. Hence, $$\Delta E = 234 \times 7.6 + 4 \times 7.07 - 238 \times E_{\text{per nucleon U}} = 4.27 \text{ MeV}$$ $$E_{\text{per nucleon U}} = \frac{234 \times 7.6 + 4 \times 7.07 - 4.27}{238} = 7.573 \ldots \text{ MeV}$$ Converting to Joules, $$E_{\text{per nucleon U}} = 7.573 \ldots \times 1.602 \times 10^{-19} \times 10^6 = 1.21 \times 10^{-12} \text{ J}$$
16	A	A single helium nucleus produced by the proton-proton cycle involves the fusion of two pairs of hydrogen into deuterium. $$^1_1\text{H} + ^1_1\text{H} \rightarrow ^2_1\text{H} + ^0_1\text{e}^+ + \nu$$ To balance the above equation, the fusion must be accompanied by the emission of a positron, which always required the emission of a neutrino.

17	D	By Einstein's Special Theory of Relativity, every inertial reference frame must be correct, so the apparent frequency of must be the same for both. Note that answer C does not resolve the paradox. Hence, the equation for the classical doppler effect (which uses the velocity of the wave relative to some medium) cannot be valid for light since there is no medium. This can also be confirmed by noting that the derivation of the classical doppler effect does not consider relativistic effects.
18	B	First note that the triangle is equilateral, so the internal angles are all 60°. The magnitude of the force per unit length due to the top wire on the bottom left wire is $$\frac{F}{l}=\frac{\mu_0 I_{left} I_{top}}{2\pi r}=\frac{4\pi\times10^{-7}\times1.5\times2.0}{2\pi\times1}=6.0\times10^{-7}\ \text{Nm}^{-1}$$ This is attractive, so this is directed at 60° above the positive *x*-axis. The magnitude of the force per unit length due to the bottom right wire on the bottom left wire is $$\frac{F}{l}=\frac{\mu_0 I_{left} I_{top}}{2\pi r}=\frac{4\pi\times10^{-7}\times1.5\times2.7}{2\pi\times1}=8.1\times10^{-7}\ \text{Nm}^{-1}$$ This is repulsive, so this is directed in the negative *x* direction. Adding these forces as vectors gives a magnitude of $$\frac{\Sigma F}{l}=\sqrt{6^2+8.1^2-2\times6\times8.1\cos120°}\times10^{-7}=1.23\times10^{-6}\ \text{Nm}^{-1}$$
19	C	By trial and error, using the Rydberg formula $$\frac{1}{\lambda}=R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$ Gives $$f=cR\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$$ 2.3×10^{14} Hz corresponds to the transition $n=5\rightarrow4$ 6.2×10^{14} Hz corresponds to the transition $n=4\rightarrow2$ 3.1×10^{15} Hz corresponds to the transition $n=4\rightarrow1$ No transition corresponds to 1.1×10^{15} Hz
20	D	The half-life model of radioactivity is inherently probabilistic. For such a small sample size it is impossible to predict exactly how many nuclei remain at any time.

Section II

Question 21

Criteria	Marks
• Applies Kepler's Second law to provide a valid explanation	2
• Provides some relevant information	1

Sample answer:
Kepler's Second Law states that satellites will sweep equal areas in equal times. Since ellipses are symmetrical about their major axis, the area swept by the moon as it travels from the apogee to the perigee is equal to the area swept by the moon as it travels from the perigee to the apogee. Thus, by Kepler's Second Law, the times taken are equal.

Question 22 (a)

Criteria	Marks
• Correctly calculates the component of weight force parallel to the surface	1

Sample answer:
$mg\sin\theta = 1000 \times 9.8 \times \sin 30° = 4900\ \text{N}$

Question 22 (b)

Criteria	Marks
• Correctly calculates magnitude of frictional force	3
• Shows steps or substantial working in calculating the frictional force	2
• Provides a step in calculating the frictional force	1

Sample answer:
Let the *x*-axis go *across* the ramp to the right and let the *y*-axis go *up* the ramp. The magnitude of the net force acting on the car is

$$\frac{mv^2}{r} = \frac{1000 \times 3^2}{5} = 1800\ \text{N}$$

The direction of the net force is towards the centre of the circle, thus

Net force in the *x*-direction is given by $\Sigma F_x = F_{\text{g},x} + F_{\text{fr},x}$ so

$$F_{\text{fr},x} = \Sigma F_x - F_{\text{g},x} = 1800\cos 45° - 0 = 1273\ \text{N}$$

Net force in the *y*-direction is given by $\Sigma F_y = F_{\text{g},y} + F_{\text{fr},y}$ so

$$F_{\text{fr},y} = \Sigma F_y - F_{\text{g},y} = -1800\sin 45° - (-4900) = 3627\ \text{N}$$

Hence, the magnitude of the friction is $F_{\text{fr}} = \sqrt{F_{\text{fr},x}^2 + F_{\text{fr},y}^2} = \sqrt{1273^2 + 3627^2} = 3844\ \text{N}$

Question 22 (c)

Criteria	Marks
• Provides correct calculation • Uses the work energy theorem	2
• Provides some relevant information	1

Sample answer:
Since the kinetic energy of the car does not change, applying the Work-Energy theorem gives the following result

$$\Delta K = W_{gravity} + W_{friction} = 0$$

Hence,

$$W_{friction} = -W_{gravity} = -4900 \times 10 = -49000 \text{ J}$$

Question 23

Criteria	Marks
• Provides a comprehensive explanation of how step-up transformers reduce energy loss • Provides a comprehensive explanation of at least TWO methods of improving the efficiency of transformers • Provides and integrates at least TWO appropriate equations	5
• Provides an explanation of how step-up transformers reduce energy loss • Provides an explanation of at least ONE method of improving the efficiency of transformers • Provides at least ONE appropriate equation	3-4
• Provides some relevant information	1-2

Sample answer:
To reduce energy loss, a step-up transformer is used to increase the voltage before long-distance transmission. In an ideal transformer, the power generated is constant. So, $P = IV$ means that increasing the voltage V decreases the current I. The power dissipated as heat in a wire with resistance R is $P = I^2R$, so decreasing the current reduces the power lost during transmission.

To reduce energy loss due to the transformer, a soft iron core is used to direct the magnetic flux from the primary to the secondary coil. This prevents flux leakage, where some magnetic flux does not contribute to the induced current in the secondary coil, hence dissipating the magnetic energy.

By Faraday's Law, the changing magnetic flux due to the AC voltage, will induce eddy currents in the iron core. The eddy currents heat up the iron core, converting the energy of the magnetic field into heat. By laminating the sheets perpendicular to the magnetic field with an insulating material, the size of these eddy currents is reduced and hence the amount of energy lost.

Question 24 (a)

Criteria	Marks
• Assesses the claim as false since the length is contracted • Justifies why time dilation cannot be used to analyse this situation	2
• Assesses the claim as false since the length is contracted OR • Justifies why time dilation cannot be used to analyse this situation	1

Sample answer:

The claim is false because neither X or Y experience proper time since the events occur at different positions in both reference frames. Instead, the distance from X to the sensor is length contracted in Y's reference frame, so Y sees the light pulse take a shorter time.

Question 24 (b)

Criteria	Marks
• Provides correct calculation for X's frame • Provides correct calculation for Y's frame	3
• Provides correct calculation for X's frame • Applies length contraction for Y's frame	2
• Provides correct calculation for X's frame	1

Sample answer:

In X's frame, light travels a distance $\frac{L}{2}$ at the speed of light *c*. The time according to X is

$$t_X = \frac{L}{2c} = \frac{35}{2 \times 3 \times 10^8} = 58 \text{ ns}$$

In Y's frame, the distance between the back of the train and the light pulse is length contracted and reduces at speed $c + v$. The contracted length of the whole train is

$$L' = L\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

The time according to Y is

$$t_Y = \frac{L\sqrt{1 - \left(\frac{v}{c}\right)^2}}{2(c + v)} = \frac{35 \times \sqrt{1 - 0.95^2}}{2(3 \times 10^8 + 0.95 \times 3 \times 10^8)} = 9.3 \text{ ns}$$

This supports the answer to part (a) since $t_Y < t_X$.

Question 25 (a)

Criteria	Marks
• Provides the correct lengths for all THREE sections	2
• Provides the correct length of at least ONE section	1

Sample answer:
The first section of the graph is freefall above the tube and can be ignored. The linear sections occur inside the metal sections. Copper is more conductive than aluminium and hence the gradient is shallower so the second section is copper. The third section is non-linear and must be wood. The final section is linear with a steeper gradient and is aluminium.

From the displacement on the graph, the lengths are:
- Cu 0.38 m
- Wood: 2.8 m
- Al: 3.6 m

Remark: A length of 4.2 m also accepted for the wood if students assumed the freefall above the tube was in wood.

Question 25 (b)

Criteria	Marks
• Provides a comprehensive explanation of the motion of the magnet in the metal with explicit reference to conservation of energy via Faraday and Lenz's Law • Provides a comprehensive comparison of the motion of the magnet between the three sections	4
• Applies conservation of energy • Provides some explanation of the motion of the magnet in the metal • Provides some comparison of the motion of the magnet between the three sections	2-3
• Provides some relevant information	1

Sample answer:
As the magnet falls, there is a change in flux through the pipe, inducing an EMF by Faraday's Law. In the wood section, current cannot flow, so the only force acting is gravity, which does work to convert the magnet's gravitational potential energy into kinetic energy.

Since Cu and Al are conductive metals, this EMF causes current to flow in the direction that produces a magnetic field opposing the original change in flux by Lenz's Law. This applies a resistive magnetic force with magnitude that increases with the rate of change of magnetic flux i.e., speed of the magnet, so it reaches a constant 'terminal' velocity quickly in the metals, producing the linear sections of the graph. The strength of the magnetic field is proportional to the induced current and hence conductivity of the metal. Since Cu is more conductive than Al, the terminal speed in Cu is lower and hence the gradient is shallower.

In the metals, the magnetic force does work to oppose gravity, converting kinetic energy into electrical energy and heat. By the Law of Conservation of Energy, energy cannot be created or destroyed, only transformed. Mathematically, this can be written

$$E_i = E_f$$
$$K_i + U_i = K_f + U_f + W$$

where *W* represents the work done by the magnetic force or the energy lost to heat. Since

$$U_f = K_i = 0$$
$$U_i = mgh$$
$$K_f = \frac{1}{2}mv^2$$
$$W = mgh - \tfrac{1}{2}mv^2$$

From the gradient of the final section of the graph, the speed at the bottom is $v = 2.0$ ms^{-1}. From the graph, $h = 8.0$ m which yields an energy lost to heat of

$$W = 0.5 \times 9.8 \times 8.0 - \frac{1}{2} \times 0.5 \times 2.0^2 = 38.2 \text{ J}$$

Question 26

Criteria	Marks
• Provides the correct description of unpolarised light • Provides a comprehensive explanation of an experiment to support claim • Provides the correct judgement of the statement	5
• Provides a description of unpolarised light • Provides a description of an experiment to support claim • Provides a consistent judgement of the statement	3-4
• Provides some relevant information	1-2

Sample answer:
While unpolarised light is comprised of *x* and *y* polarisations, the ratio of these two components is rapidly and randomly varying such that no overall polarisation can be observed. A constant mix of *x* and *y* polarisations would simply have a linear polarisation of the vectors sum of the two components.

To differentiate between a constant mix of *x* and *y* polarisations and unpolarised light, we propose an experiment.

1. Shine unpolarised light of a controlled intensity through a polariser with plane perpendicular to the direction of light.
2. Measure the intensity of light that passes through the polariser as the polariser rotates about an axis parallel to the direction of light with a photometer.

If the unpolarised light was comprised of a constant mix of *x* and *y* polarisations, the intensity is expected to vary with the angle that the polarisation axis makes with the vector sum of the *x* and *y* polarisations, by Malus's Law. However, in this experiment a constant intensity that is half of the original intensity is observed. This is because the variations are so rapid, that only the time-averaged intensity can be observed, which is always half of the original intensity. Hence, it can be concluded that the statement is partially valid.

Question 27 (a)

Criteria	Marks
• Provides the correct formula OR • Equivalent merit with some correct working	1

Sample answer:
The path length difference is $(AB + BC) - AC'$. Since

$$\sin\theta = \frac{d}{AB}$$

$$\tan\theta = \frac{d}{0.5AC} = \frac{2d}{AC}$$

Also, since $\Delta AC'C$ is a right-angle triangle then

$$AC' = AC\cos\theta = \frac{2d}{\tan\theta}.\cos\theta = \frac{2d}{\sin\theta}(\cos^2\theta)$$

Combining the equations, noting that $AB = BC$

$$(AB + BC) - AC' = \frac{2d}{\sin\theta}(1 - \cos^2\theta) = \frac{2d}{\sin\theta}.\sin^2\theta = 2d\sin\theta$$

For constructive interference, this path length is an integer multiple of the wavelength. Also note that the reflections are the same so the phase change can be ignored.

$2d\sin\theta = m\lambda$ for some positive integer m.

Remark: Any mathematically equivalent formula was accepted.

Question 27 (b)

Criteria	Marks
• Provides the correct lattice spacing	2
• Provides some relevant information	1

Sample answer:

Suppose the *n* and (*n* + 1)th order peaks are at 27° and 43°. Then

$$d = \frac{\lambda}{2} . \frac{n}{\sin 27^\circ} = \frac{\lambda}{2} . \frac{n+1}{\sin 43^\circ}$$

Solving for *n*

$$n = \frac{\frac{1}{\sin 27^\circ}}{\frac{1}{\sin 27^\circ} - \frac{1}{\sin 43^\circ}} = 2$$

Then substitute back in for *d*

$$d = \frac{n\lambda}{2 \sin 27^\circ} = \frac{2 \times 3.2 \times 10^{-9}}{2 \sin 27^\circ} = 7.0 \text{ nm}$$

Question 27 (c)

Criteria	Marks
• Provides the correct wavelength • Provides the correct expression for kinetic energy • Provides the correct voltage	3
• Provides the correct wavelength • Provides the correct expression for kinetic energy	2
• Provides some relevant working	1

Sample answer:
First, find the corresponding value of θ for ϕ.

$$\theta = 90 - \frac{\phi}{2}$$

Then since this is the first order maximum, $n = 1$ and so the wavelength of the electrons is

$$\lambda = 2d\sin\left(90° - \frac{\phi}{2}\right) = 2 \times 0.091 \times 10^{-9} \sin\left(90° - \frac{50°}{2}\right) = 1.6495 \times 10^{-10} \text{ m}$$

The kinetic energy of the electrons is provided by the work done due to the accelerating voltage and can be related to the electron momentum

$$K = eV = \frac{p^2}{2m}$$

By De Broglie's matter-wave equivalence electron wavelength and momentum can be related by

$$p = \frac{h}{\lambda}$$

Substituting and solving for the voltage

$$V = \frac{h^2}{2me\lambda^2} = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times (1.6495 \times 10^{-10})^2} = 55 \text{ V}$$

Question 27 (d)

Criteria	Marks
• Comprehensively explains how the Davisson-Germer experiment contributed to an improvement on the Bohr model of the atom	5
• Explains how the Davisson-Germer experiment contributed to an improvement on the Bohr model of the atom	4
• Explains some aspects of how the Davisson-Germer experiment contributed to an improvement on the Bohr model of the atom	3
• Provides details of how the Davisson-Germer experiment contributed to an improvement on the Bohr model of the atom	2
• Provides some relevant information	1

Sample answer:
In the Bohr model, the atom contains a small dense positive nucleus orbited by electrons held by electrostatic attraction. The orbits of these electrons are governed by three postulates.

1. Electrons orbit the nucleus in a circle in certain discrete energy levels and radii.
2. A transition between energy levels is accompanied by the emission or absorption of a photon with frequency corresponding to the energy difference between the levels.
3. The angular momentum of electrons is quantised.

The Bohr model successfully explained the discrete lines in the emission spectra of hydrogen and single-electron systems – each line corresponds to a specific transition between energy levels. However, the Bohr model could not explain how electrons could orbit without radiating energy, since they must continuously be accelerating. It also could not explain why angular momentum would be quantised.

In the Davisson-Germer experiment, electrons were shown to undergo diffraction, which is a property of waves. This supports De Broglie's matter wave theory, where particles display wave properties and vice-versa. This provides an improvement to the Bohr model by explaining how and why these discrete energy levels exist. Electrons form standing waves around the nucleus, which requires an integer number of wavelengths to fill the circumference of the orbit. Hence, the nth energy level corresponds to n wavelengths inside the circumference, from which the third postulate can be derived. The energy of a standing waves is constant, so the existence of these states can be explained.

Question 28 (a)

Criteria	Marks
• Provides the correct escape velocity	1

Sample answer:
Using the formula for escape velocity

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 4.10 \times 10^{30}}{11.0 \times 10^{3}}} = 2.23 \times 10^{8} \text{ ms}^{-1}$$

Remark: Note that taking relativity into account actually returns the classical expression, but this is beyond the scope of the course.

Question 28 (b)

Criteria	Marks
• Provides the correct nuclear equation • Provides the correct energy per kg	2
• Provides some relevant working	1

Sample answer:
The fission of uranium is given by

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{92}_{36}\text{Kr} + {}^{141}_{56}\text{Ba} + 3{}^{1}_{0}\text{n}$$

Alternatively, the net equation is

$$^{235}_{92}\text{U} \rightarrow {}^{92}_{36}\text{Kr} + {}^{141}_{56}\text{Ba} + 2{}^{1}_{0}\text{n}$$

The fission of one uranium nuclei results is a mass difference of

$$\Delta m = 235.043925 - 140.914411 - 91.926156 - 2 \times 1.008665$$

$$\Delta m = 0.186028 \text{ amu}$$

This releases energy $E = 0.186028 \times 1.661 \times 10^{-27} \times (3 \times 10^{8})^{2} = 2.781 \times 10^{-11}$ J per nuclei of Uranium-235.

The mass of one nuclei of Uranium-235 is

$$m_{\text{nuclei}} = 235.043925 \times 1.661 \times 10^{-27} = 3.9041 \times 10^{-25} \text{ kg}$$

Then the energy released per kg of uranium is

$$E_{per\ kg} = \frac{E}{m_{\text{nuclei}}} = \frac{2.781 \times 10^{-11}}{3.9041 \times 10^{-25}} = 7.12 \times 10^{13} \text{ Jkg}^{-1}$$

Question 28 (c)

Criteria	Marks
• Correctly calculates percentage of the initial total mass of the rocket was the Uranium-235 fuel	4
• Provides substantial working to calculate percentage of the initial total mass of the rocket was the Uranium-235 fuel	3
• Calculates one relevant quantity	2
• Provides one correct step in calculation	1

Sample answer:
If the initial mass is m, including Δm of fuel, then by conservation of energy, the increase in mechanical energy must have come from the fuel.

$$E_i - E_f = E_{fuel}$$

$$\left(\frac{1}{2}mv^2 - \frac{GMm}{R}\right) - 0 = \Delta mc^2$$

Solving this gives the initial percentage by mass of fuel in the rocket.

$$\frac{\Delta m}{m} = \frac{1}{c^2}\left(\frac{GM}{R} - \frac{1}{2}v^2\right)$$

$$\frac{\Delta m}{m} = \frac{1}{(3 \times 10^8)^2}\left(\frac{6.67 \times 10^{-11} \times 4.10 \times 10^{30}}{11.0 \times 10^3} - \frac{1}{2}(0.743 \times 3 \times 10^8)^2\right) = 2.078 \times 10^{-4}$$

But the mass difference per kg of Uranium-235 is $\frac{0.186028}{235.043925} = 7.9146 \times 10^{-4}$ kg

Hence, the percentage of Uranium-235 is

$$\%\text{Uranium} = \frac{2.078 \times 10^{-4}}{7.9146 \times 10^{-4}} = 26.3\%$$

Question 29

Criteria	Marks
• Provides a comprehensive analysis of at least THREE relevant factors BOTH qualitatively and quantitatively • Provides the correct final steepness	7
• Provides a thorough analysis of at least THREE relevant factors BOTH qualitatively and quantitatively	5-6
• Provides a sound analysis of at least TWO relevant factors BOTH qualitatively and quantitatively OR • Provides a sound analysis of at least THREE relevant factors either qualitatively OR quantitatively only	3-4
• Provides some relevant information	1-2

Sample answer:

1. Friction

The static friction between the wheels and the surface provides the upwards force, so must be greater than the component of the weight force down the hill.

$$F_{g,\,\text{parallel}} < F_{\text{static friction}}$$
$$mg\sin\theta < \mu_s mg\cos\theta$$
$$\tan\theta < \mu_s$$
$$\theta < \tan^{-1}(0.62) = 31.8°$$

2. Max torque

The force produced by the motor must be greater than the component of the weight force down the hill.

$$mg\sin\theta < F_{\text{motor}} = \frac{\tau_{\text{motor}}}{r}$$
$$mg\sin\theta < \frac{nIAB}{r}$$
$$\sin\theta < \frac{nVAB}{rmgR}$$
$$\theta < \sin^{-1}\left(\frac{500 \times 20 \times 12 \times 10^{-4} \times 0.023}{0.11 \times 3.8 \times 9.8 \times 0.13}\right) = 31.2°$$

3. Centre of mass

The centre of mass cannot be directly above the back wheel, else the net torque around the back wheel will tip the cart over.

$$\theta + \tan^{-1}\frac{2h}{d} < 90°$$
$$\theta < 90° - \tan^{-1}\left(\frac{2 \times 0.95}{1.1}\right) = 30.1°$$

Overall, steepness is limited to $\theta < 30.1°$.

Question 30 (a)

Criteria	Marks
• Provides correct calculations of the ratio between the orbital radii	2
• Provides some relevant information	1

Sample answer:

Since Earth takes about 365 days to orbit around the Sun, Earth's angular velocity ω_e is given by

$$\frac{2\pi}{\omega_\mathrm{e}} = 365$$

$$\omega_\mathrm{e} = \frac{2\pi}{365} = 1.72 \times 10^{-2}\ \mathrm{d}^{-1}$$

The angular velocity of the Earth relative to Jupiter is ω_e - ω_j. Since Earth takes 400 days to orbit the Sun relative to Jupiter, this gives

$$\frac{2\pi}{\omega_\mathrm{e} - \omega_j} = 400$$

As a result

$$\omega_\mathrm{e} - \omega_j = \frac{2\pi}{400}$$

$$\frac{2\pi}{365} - \omega_j = \frac{2\pi}{400}$$

$$\omega_j = \frac{2\pi}{365} - \frac{2\pi}{400} = 1.51 \times 10^{-3}\ \mathrm{d}^{-1}$$

Applying Kepler's third law

$$\left(\frac{r_j}{r_e}\right)^3 = \left(\frac{T_j}{T_e}\right)^2$$

$$\frac{r_j}{r_e} = \left(\frac{T_j}{T_e}\right)^{\frac{2}{3}} = \left(\frac{\omega_e}{\omega_j}\right)^{\frac{2}{3}} = \left(\frac{1.72 \times 10^{-2}}{1.51 \times 10^{-3}}\right)^{\frac{2}{3}} = 5.07$$

Question 30 (b)

Criteria	Marks
• Identifies and explains how the speed of light is the main factor • Uses the Doppler Effect or otherwise to explain the variation • Correctly describes the variation of the duration of Io's eclipses	2
• Mentions the speed of light	1

Sample answer:

The apparent immersion and emergence of Io can be thought of as signals which travel at the speed of light. The immersion signal and emergence signal can be thought of as two successive wavefronts. The Doppler effect can be applied to this scenario.

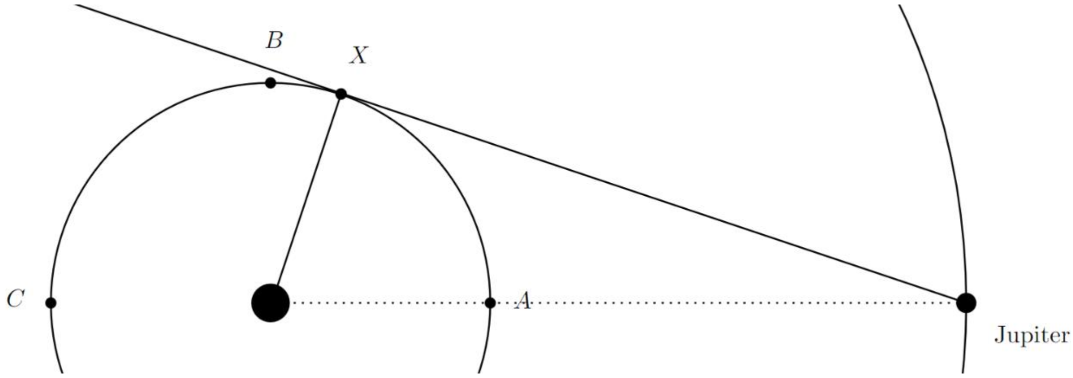
Starting from Point A, as the distance from Earth to Io increases, the Doppler effect causes the period between the immersion and emergence signals to increase. The maximum duration occurs near point B (travelling away from Io the fastest), and the minimum duration occurs near point D (travelling towards Io the fastest)

Question 30 (c)

Criteria	Marks
• Relates the deviation of the duration of Io's eclipse to Earth's recessional velocity relative to Jupiter AND • Explains how the graph of Earth's recessional velocity over time deviates from a perfect sine curve	3
• Relates the deviation of the duration of Io's eclipse to Earth's recessional velocity relative to Jupiter OR • Explains how the graph of Earth's recessional velocity over time deviates from a perfect sine curve	2
• Provides some relevant information	1

Sample answer:

Using reasoning from part (b), the faster Earth is receding from Io, the longer the duration and vice versa. The point of maximum deviation thus should be when Earth is receding from Io the fastest. However, this doesn't occur precisely at point B, instead this occurs at X.



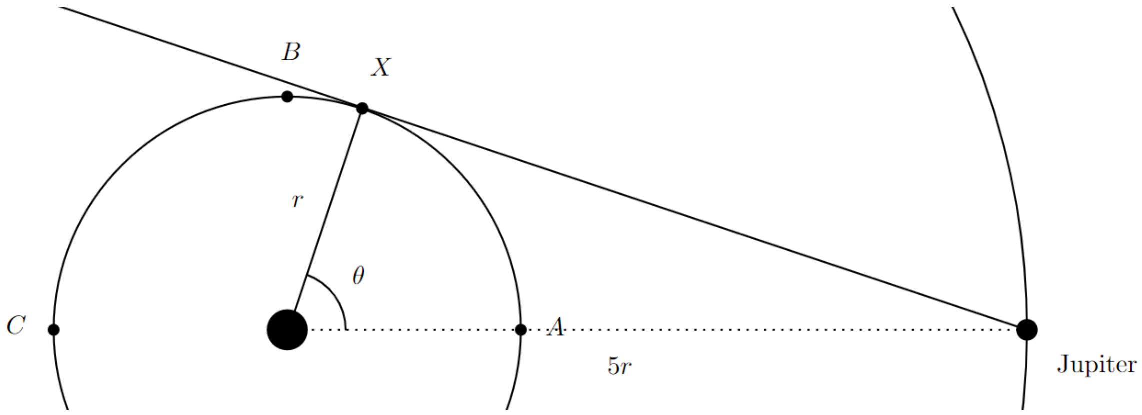
It occurs a little bit earlier, when Earth's tangential velocity vector is pointing parallel to the line between Jupiter and Earth. Similarly, the point of minimum deviation occurs when Earth is travelling towards Io the fastest. This will occur a little bit after point D.

This causes the deviation vs time graph to appear skewed, where the point of maximum deviation occurs slightly before a quarter of a synodic year, and the point of minimum deviation occurs slightly after three quarters of a synodic year.

Question 30 (d)

Criteria	Marks
• Calculates the correct time at which maximum deviation occurs	2
• Provides some relevant information	1

Sample answer:



The maximum deviation occurs when Earth is at point X as per the diagram above. Note, that the tangent line makes a right angle with the radius at X. Since

$$\cos\theta = \frac{1}{5}$$

$$t = \frac{\theta}{\omega} = \frac{\cos^{-1}\frac{1}{5}}{1.72 \times 10^{-2}} = 79.6 \text{ days}$$

Question 31 (a)

Criteria	Marks
• Explains how the speed can be determined by measuring the electric and magnetic fields when the beam is undeflected	1

Sample answer:
The magnetic field and electric field can be adjusted such that the beam passes through undeflected. The electric and magnetic force must be balanced, so the speed can be measured as the ratio of the electric and magnetic field strengths.

$$qvB = qE$$

$$v = \frac{E}{B}$$

Question 31 (b)

Criteria	Marks
• All entries in the table are correct	2
• At least one entry in the table is correct OR • All entries are correct except for a unit conversion error	1

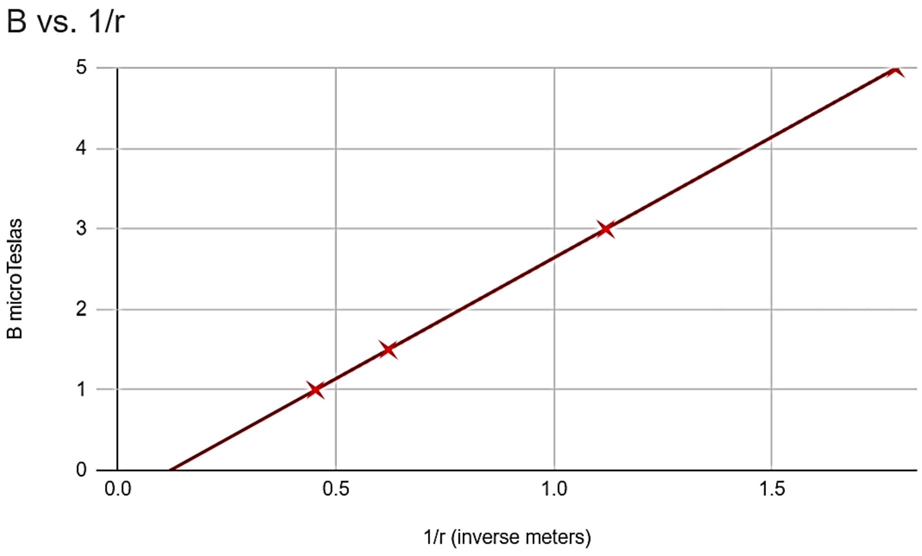
Sample answer:

Field Strength *B* (μT)	**Vertical deflection *d* (m)**	$\frac{1}{r}$ **(m^{-1})**
1.0	1.667×10^{-3}	0.4532
1.5	2.502×10^{-3}	0.6200
3.0	5.013×10^{-3}	1.110
5.0	8.392×10^{-3}	1.780

Question 31 (c)

Criteria	Marks
• Provides a graph with ALL required elements including an appropriate title, correct units and axes labels, accurately plotted points, and an appropriate line of best fit • Provides the correct mass to charge ratio from the gradient	4
• Provides a graph with SOME required elements • Provides a mass to charge ratio from the gradient	2-3
• Provides some relevant information	1

Sample answer:



This graph has gradient 3.01×10^{-6} Tm. The magnetic force provides the centripetal force, so

$$qvB = \frac{mv^2}{r}$$

$$B = \frac{mv}{q} \times \frac{1}{r}$$

$$\frac{mv}{q} = 3.01 \times 10^{-6}$$

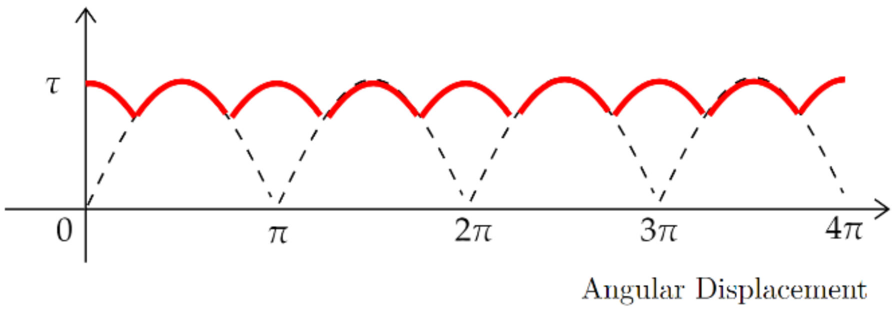
$$\frac{m \times 10^5}{q} = 3.01 \times 10^{-6}$$

$$\frac{m}{q} = 3.01 \times 10^{-11}\ \text{kgC}^{-1}$$

Question 32 (a)

Criteria	Marks
• Draws the correct curve	1

Sample answer:



Question 32 (b)

Criteria	Marks
• Explains the formula for the angular speed	1

Sample answer:
The terminal angular speed will occur when the net torque on the motor is 0. Since there is no load, the current must also be 0. Hence,

$$V_s - \epsilon_b = 0$$

$$V_s = \mu\omega$$

$$\mu = \frac{V_s}{\omega}$$

Question 32 (c)

Criteria	Marks
• Correctly calculates the initial acceleration of the bicycle	4
• Provides substantial working to calculate the initial acceleration of the bicycle	3
• Calculates one relevant quantity	2
• Provides one correct step in calculation	1

Sample answer:
Initially, there is no back EMF since the motor is not moving.

$$I = \frac{V_s}{P}$$

The net torque on the gear with radius R must be 0 since its mass is negligible.

$$R(T_2 - T_1) = \lambda I$$

The net torque on the wheel must also be 0.

$$F_{\mathrm{fr}} R_w = r_2(T_2 - T_1) = \frac{r_2 \lambda I}{R}$$

The force that accelerates the bicycle is the friction force.

$$F_{\mathrm{fr}} = ma$$

Hence the acceleration is

$$a = \frac{r_2 \lambda V_2}{R R_w m P}$$

Question 32 (d)

Criteria	Marks
• Provides the correct value of tension	1

Sample answer:
When the bicycle is put in reverse, the torque due to the top and bottom chains reverse direction, so

$$R(T_1 - T_2) = \lambda I$$

$$T_2 = T_1 - \frac{\lambda V_s}{PR}$$

Question 32 (e)

Criteria	Marks
• Provides the correct graph with ALL appropriate labels • Provides a comprehensive explanation for the dotted points	4
• Provides a graph with SOME appropriate labels • Provides a thorough explanation for the dotted points	3
• Provides a graph • Provides some description for the dotted points	2
• Provides some relevant information	1

Sample answer:

Since the cyclist moves at terminal velocity, the bike wheels maintain their angular velocities ω. At time t_1, the radius of the wheel gear decreases suddenly, so the speed of the chain decreases by $v = r\omega$. The velocity of the chain is equal to the speed of the outer edge of the wheel gear, which is also equal to the speed of the outer edge of the motor gear, so the angular velocity of the motor decreases.

The back EMF is proportional to the angular velocity of the motor, so the back-emf drops suddenly at time t_1. This causes the net voltage to become positive, so the motor starts providing torque. The bicycle wheels experience angular acceleration and rotate faster, causing the back EMF to increase again, approaching the supply voltage, where net torque returns to zero.