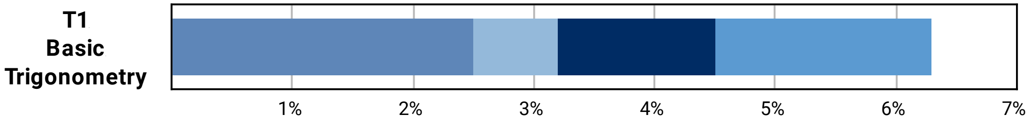
ADV: Trigonometry (Adv), T1 Trigonometry and Measure of Angles (Adv)

Trig Ratios, Sine and Cosine Rules (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 100.5 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

T1 Trigonometry and Measure of Angles



- Trig Ratios, Sine and Cosine Rules
- 3D Trigonometry
- Bearings
- Circular Measure

***Analytics based on the average contribution to the 2ADV HSC exams over the past decade.**

HISTORICAL CONTRIBUTION

- *T1 Trigonometry and Measure of Angles* is a mixture of content that previously belonged to the Standard 2, Mathematics and Ext1 courses. Our analysis has it accounting for an estimated 6.3% of past papers.
- This topic has been split into four sub-topics for analysis purposes: *1-Trig Ratios, Sine and Cosine Rules (2.5%), 2- 3D Trigonometry (0.7%), 3-Bearings (1.3%) and 4-Circular Measure (1.8%).*
- This analysis looks at the largest sub-topic *Trig Ratios, Sine and Cosine Rules.*

HSC ANALYSIS - What to expect and common pitfalls

- *Trig Ratios, Sine and Cosine Rules* is most commonly examined using non-right angled trigonometry, often involving "2-triangle" examples.
- It has been examined in 8 times in the last decade (most recently in 2021) in questions of varying difficulty, producing sub-50% mean marks on 3 occasions.
- This area presents a great opportunity for high scoring, with the 2021 exam allocating 5 very achievable marks over 2 questions.
- Using one exact trig ratio to find others for further calculations has caused problems in the past and should be reviewed *(see 2015 Adv 13a).*
- The specific syllabus mention of the "ambiguous case" warrants attention and *2021 Q18* along with *T1 EQ-Bank 2* should be reviewed.

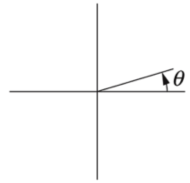
Questions

1. Trigonometry, 2ADV T1 2016 HSC 1 MC

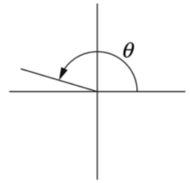
For the angle θ, $\sin\theta = \dfrac{7}{25}$ and $\cos\theta = -\dfrac{24}{25}$.

Which diagram best shows the angle θ?

(A)



(B)



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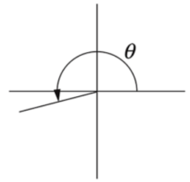
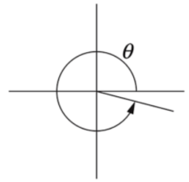


(D)



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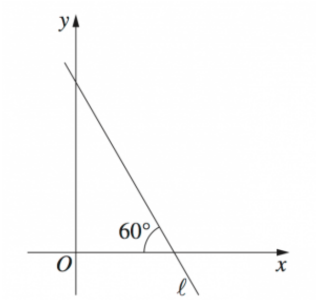
2. Trigonometry, 2ADV T1 2013 HSC 2 MC

The diagram shows the line l.



What is the slope of the line l?

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\dfrac{1}{\sqrt{3}}$

(D) $-\dfrac{1}{\sqrt{3}}$

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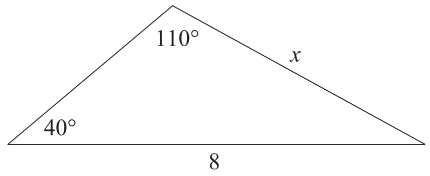
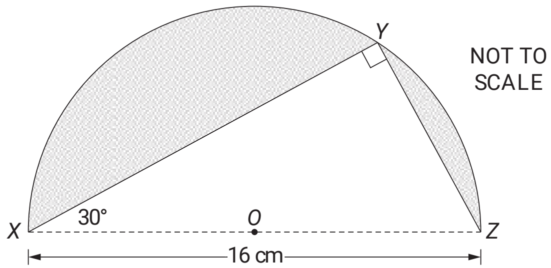
3. Trigonometry, 2ADV T1 2019 HSC 11a

Using the sine rule, find the value of x correct to one decimal place. ***(2 marks)***



4. Trigonometry, 2ADV T1 2021 HSC 12

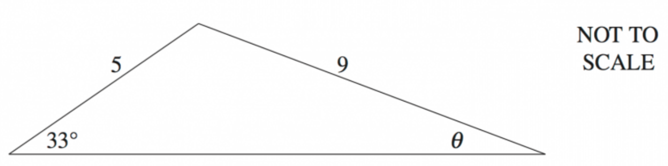
A right-angled triangle XYZ is cut out from a semicircle with centre O. The length of the diameter XZ is 16 cm and $\angle YXZ$ = 30°, as shown on the diagram.



a. Find the length of XY in centimetres, correct to two decimal places. ***(2 marks)***

b. Hence, find the area of the shaded region in square centimetres, correct to one decimal place. ***(3 marks)***

5. Trigonometry, 2ADV T1 2006 HSC 1d



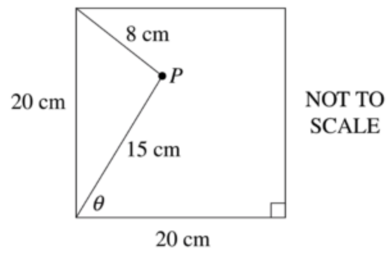
Find the value of θ in the diagram. Give your answer to the nearest degree. ***(2 marks)***

6. Trigonometry, 2ADV T1 2016 HSC 12c

Square tiles of side length 20 cm are being used to tile a bathroom.

The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.

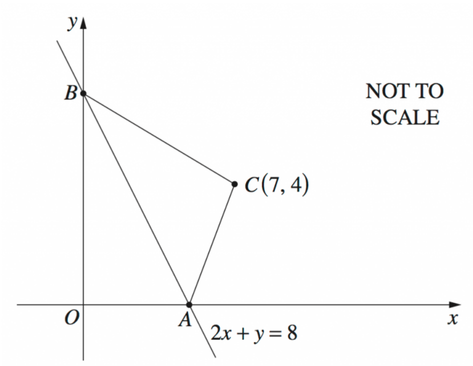
To locate the point P the tiler needs to know the size of the angle θ shown in the diagram.



Find the size of the angle θ to the nearest degree. ***(3 marks)***

7. Trigonometry, 2ADV T1 2012 HSC 13a

The diagram shows a triangle ABC. The line $2x + y = 8$ meets the x and y axes at the points A and B respectively. The point C has coordinates $(7, 4)$.

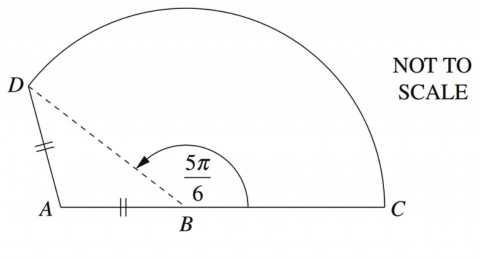


i. Calculate the distance AB. ***(2 marks)***

ii. It is known that $AC = 5$ and $BC = \sqrt{65}$ (Do NOT prove this)
Calculate the size of $\angle ABC$ to the nearest degree. ***(2 marks)***

iii. The point N lies on AB such that CN is perpendicular to AB.
Find the coordinates of N. ***(3 marks)***

8. Trigonometry, 2ADV T1 2006 HSC 4a



In the diagram, $ABCD$ represents a garden. The sector BCD has centre B and $\angle DBC = \dfrac{5\pi}{6}$

The points A, B and C lie on a straight line and $AB = AD = 3$ metres.

Copy or trace the diagram into your writing booklet.

i. Show that $\angle DAB = \dfrac{2\pi}{3}$. *(1 mark)*

ii. Find the length of BD. *(2 marks)*

iii. Find the area of the garden $ABCD$. *(2 marks)*

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9. Trigonometry, 2ADV T1 SM-Bank 2

Determine all possible dimensions for triangle ABC given $AB = 6.2\text{ cm}$, $\angle ABC = 35^\circ$ and $AC = 4.1$.

Give all dimensions correct to one decimal place. *(3 marks)*

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10. Trigonometry, 2ADV T1 2005 HSC 3b

The lengths of the sides of a triangle are 7 cm, 8 cm and 13 cm.

i. Find the size of the angle opposite the longest side. *(2 marks)*

ii. Find the area of the triangle. *(1 marks)*

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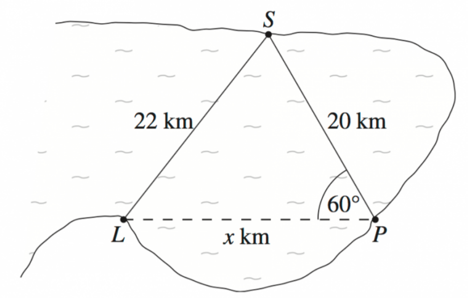
11. Trigonometry, 2ADV T1 2011 HSC 8a

In the diagram, the shop at S is 20 kilometres across the bay from the post office at P. The distance from the shop to the lighthouse at L is 22 kilometres and $\angle SPL$ is 60°.

Let the distance PL be x kilometres.



i. Use the cosine rule to show that $x^2 - 20x - 84 = 0$. *(1 mark)*

ii. Hence, find the distance from the post office to the lighthouse. Give your answer correct to the nearest kilometre. *(2 mark)*

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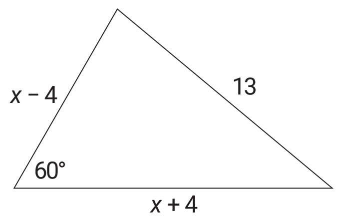
12. Trigonometry, 2ADV T1 2017 HSC 13a

Using the cosine rule, find the value of x in the following diagram. *(3 marks)*



NOT TO SCALE

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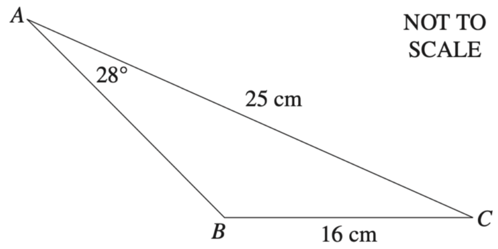
13. Trigonometry, 2ADV T1 2021 HSC 18

The diagram shows a triangle ABC where AC = 25 cm, BC = 16 cm, $\angle BAC$ = 28° and angle ABC is obtuse.



Find the size of the obtuse angle ABC correct to the nearest degree. ***(3 marks)***

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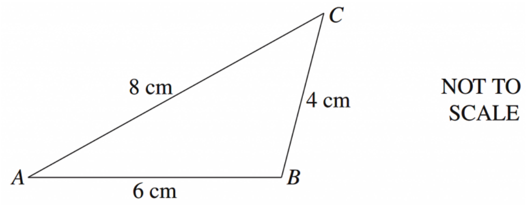
14. Trigonometry, 2ADV T1 2015 HSC 13a

The diagram shows ΔABC with sides $AB = 6$ cm, $BC = 4$ cm and $AC = 8$ cm.



i. Show that $\cos A = \dfrac{7}{8}$. ***(1 mark)***

ii. By finding the exact value of $\sin A$, determine the exact value of the area of ΔABC. ***(2 marks)***

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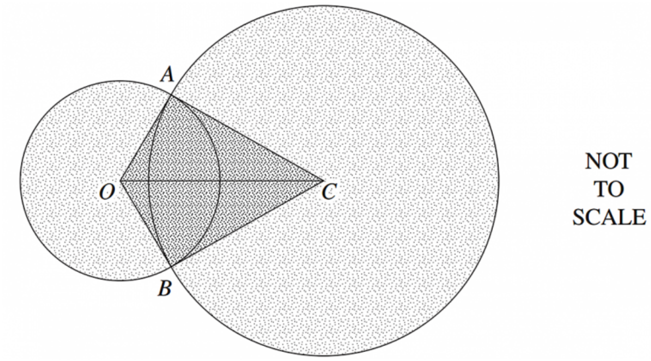
15. Trigonometry, 2ADV T1 2007 HSC 4c



An advertising logo is formed from two circles, which intersect as shown in the diagram.

The circles intersect at A and B and have centres at O and C.

The radius of the circle centred at O is 1 metre and the radius of the circle centred at C is $\sqrt{3}$ metres. The length of OC is 2 metres.

i. Use Pythagoras' theorem to show that $\angle OAC = \dfrac{\pi}{2}$. ***(1 mark)***

ii. Find $\angle ACO$ and $\angle AOC$. ***(2 marks)***

iii. Find the area of the quadrilateral $AOBC$. ***(1 mark)***

iv. Find the area of the major sector ACB. ***(1 mark)***

v. Find the total area of the logo (the sum of all the shaded areas). ***(2 marks)***

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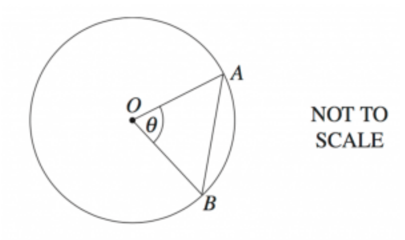
16. Trigonometry, 2ADV T1 2009 HSC 5c

The diagram shows a circle with centre O and radius 2 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



i. There are two possible values of θ for which the area of ΔAOB is $\sqrt{3}$ square centimetres. One value is $\dfrac{\pi}{3}$.

Find the other value. ***(2 marks)***

ii. Suppose that $\theta = \dfrac{\pi}{3}$.

(1) Find the area of sector AOB ***(1 mark)***

(2) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. ***(2 marks)***

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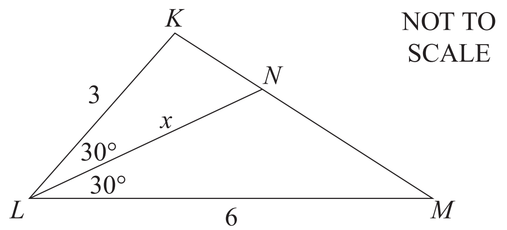
17. Trigonometry, 2ADV T1 2018 HSC 14a

In $\Delta KLM, KL$ has length 3, LM has length 6 and $\angle KLM$ is 60°. The point N is chosen on side KM so that LN bisects $\angle KLM$. The length LN is x.



i. Find the exact value of the area of ΔKLM. ***(1 mark)***

ii. Hence, or otherwise, find the exact value of x. ***(2 marks)***

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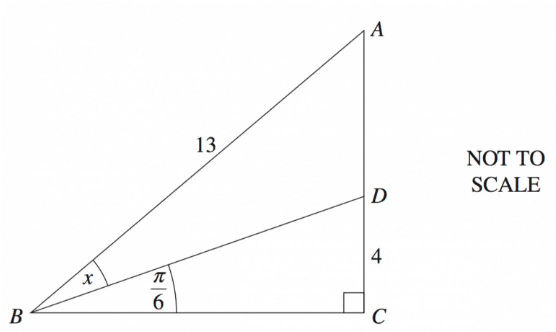
18. Trigonometry, 2ADV T1 2013 HSC 14c



The right-angled triangle ABC has hypotenuse $AB = 13$. The point D is on AC such that $DC = 4$, $\angle DBC = \dfrac{\pi}{6}$ and $\angle ABD = x$.

Using the sine rule, or otherwise, find the exact value of $\sin x$. ***(3 marks)***

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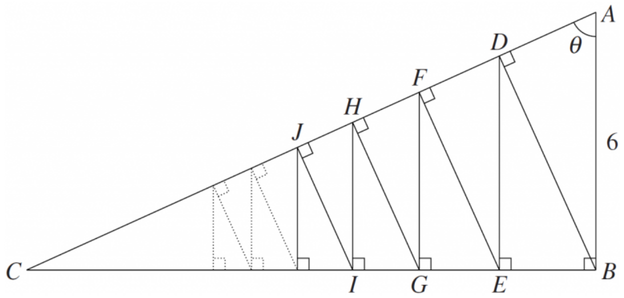
19. Trigonometry, 2ADV T1 2005 HSC 9b



The triangle ABC has a right angle at B, $\angle BAC = \theta$ and $AB = 6$. The line BD is drawn perpendicular to AC. The line DE is then drawn perpendicular to BC. This process continues indefinitely as shown in the diagram.

i. Find the length of the interval BD, and hence show that the length of the interval EF is $6\sin^3\theta$. ***(2 marks)***

ii. Show that the limiting sum

$$BD + EF + GH + \cdots$$

is given by $6\sec\theta\tan\theta$. ***(3 marks)***

Worked Solutions

Worked Solutions

1. Trigonometry, 2ADV T1 2016 HSC 1 MC

$$\text{Since } \sin\theta > 0 \text{ and } \cos\theta < 0,$$
$$\frac{\pi}{2} < \theta < \pi$$
$$\Rightarrow B$$

2. Trigonometry, 2ADV T1 2013 HSC 2 MC

Gradient is negative

(slopes from top left to bottom right)

$$\tan 60^\circ = \sqrt{3}$$
$$\therefore \text{ Gradient is } -\sqrt{3}$$
$$\Rightarrow B$$

3. Trigonometry, 2ADV T1 2019 HSC 11a

$$\begin{aligned}\frac{x}{\sin 40^\circ} &= \frac{8}{\sin 110^\circ}\\ x &= \frac{8 \times \sin 40^\circ}{\sin 110^\circ}\\ &= 5.47\\ &= 5.5 \text{ (1 d.p.)}\end{aligned}$$

4. Trigonometry, 2ADV T1 2021 HSC 12

a.
$$\begin{aligned}\cos 30^\circ &= \frac{XY}{16}\\ XY &= 16\cos 30^\circ\\ &= 13.8564\\ &= 13.86 \text{ cm (2 d.p.)}\end{aligned}$$

b.
$$\begin{aligned}\text{Area of semi-circle} &= \frac{1}{2} \times \pi r^2\\ &= \frac{1}{2}\pi \times 8^2\\ &= 100.531 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of } \Delta XYZ &= \frac{1}{2} ab\sin C\\ &= \frac{1}{2} \times 16 \times 13.856 \times \sin 30^\circ\\ &= 55.42 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{ Shaded Area} &= 100.531 - 55.42\\ &= 45.111\\ &= 45.1 \text{ cm}^2 \text{ (1 d.p.)}\end{aligned}$$

5. Trigonometry, 2ADV T1 2006 HSC 1d

Using the sine rule

$$\begin{aligned}\frac{\sin\theta}{5} &= \frac{\sin 33^\circ}{9}\\ \sin\theta &= \frac{5 \times \sin 33^\circ}{9}\\ &= 0.30257\ldots\\ \therefore \theta &= 17.612\ldots\\ &= 18^\circ \text{ (nearest degree)}\end{aligned}$$

6. Trigonometry, 2ADV T1 2016 HSC 12c

$\alpha + \theta = 90$

Using the cosine rule,

$$\cos\alpha = \frac{20^2 + 15^2 - 8^2}{2 \times 20 \times 15}$$
$$= 0.935$$
$$\alpha = 20.7\ldots^\circ$$

$$\therefore \theta = 90 - 20.7\ldots$$
$$= 69.22\ldots$$
$$= 69^\circ \text{ (nearest degree)}$$

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7. Trigonometry, 2ADV T1 2012 HSC 13a

i. Find distance AB:

Find A, $y = 0$

$2x + 0 = 8$

$x = 4 \Rightarrow A(4, 0)$

Find B, $x = 0$

$0 + y = 8 \quad \Rightarrow B(0, 8)$

Using Pythagoras:

$$AB^2 = OB^2 + OA^2$$
$$= 8^2 + 4^2$$
$$= 80$$
$$\therefore\ AB = \sqrt{80}$$
$$= 4\sqrt{5} \text{ units}$$

ii. Find $\angle ABC$:

Using cosine rule

$$\cos\angle ABC = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$
$$= \frac{(4\sqrt{5})^2 + (\sqrt{65})^2 - 5^2}{2 \times 4\sqrt{5} \times \sqrt{65}}$$
$$= \frac{80 + 65 - 25}{8 \times \sqrt{325}}$$
$$= \frac{120}{40\sqrt{13}}$$
$$= \frac{3}{\sqrt{13}}$$
$$= 0.83205\ldots$$
$$\therefore \angle ABC = 33.690\ldots$$
$$= 34^\circ \quad \text{(nearest degree)}$$

iii. Find N:

AB is $2x + y = 8$

$\Rightarrow$ Gradient $AB = -2$

$\therefore$ Gradient of $CN = ½$ $(m_1 m_2 = -1$ for $\perp$ lines)

Equation of CN, $m = ½$ through $(7, 4)$

$$y - 4 = ½\,(x - 7)$$
$$2y - 8 = x - 7$$
$$x - 2y + 1 = 0$$

MARKER'S COMMENT: Many students could not find the correct equation on CN because they took its gradient to be the reciprocal of AB and not the ***negative*** reciprocal.

N is intersection of AB and CN

$2x + y - 8 = 0$ ... (1)

$x - 2y + 1 = 0$ ... (2)

Multiply $(1) \times 2$

$4x + 2y - 16 = 0$ ... (3)

Add $(2) + (3)$

$$5x - 15 = 0$$
$$x = 3$$

Substitute $x = 3$ into (1)

$2(3) + y - 8 = 0 \Rightarrow y = 2$

$\therefore N(3, 2)$

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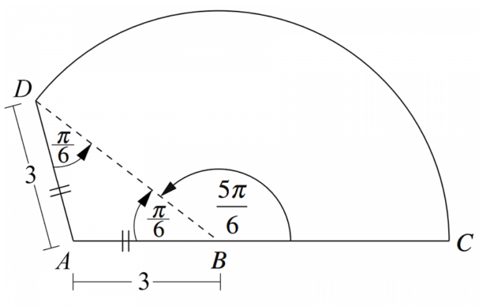
8. Trigonometry, 2ADV T1 2006 HSC 4a

i.



Show $\angle DAB = \dfrac{2\pi}{3}$

$\angle DBA = \pi - \dfrac{5\pi}{6}$ (π radians in straight angle ABC)

$= \dfrac{\pi}{6}$ radians

$\therefore \angle BDA = \dfrac{\pi}{6}$ radians (base angles of isosceles ΔADB)

$\therefore \angle DAB = \pi - \left(\dfrac{\pi}{6} + \dfrac{\pi}{6}\right)$ (angle sum of ΔADB)

$= \dfrac{2\pi}{3}$ radians ... as required

ii. Using the cosine rule:

$$\begin{aligned} BD^2 &= AD^2 + AB^2 - 2 \times AD \times AB \times \cos\frac{2\pi}{3} \\ &= 9 + 9 - (2 \times 3 \times 3 \times -0.5) \\ &= 27 \\ \therefore BD &= \sqrt{27} \\ &= 3\sqrt{3}\ \text{m} \end{aligned}$$

iii. Area of $\Delta ADB = \dfrac{1}{2} ab \sin C$

$$= \frac{1}{2} \times 3 \times 3 \times \sin\frac{2\pi}{3}$$
$$= \frac{9}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{9\sqrt{3}}{4} \text{ m}^2$$

Area of sector BCD

$$= \frac{\frac{5\pi}{6}}{2\pi} \times \pi r^2$$

$$= \frac{5\pi}{12} \times \left(3\sqrt{3}\right)^2$$

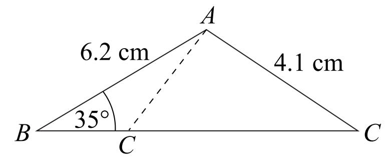
$$= \frac{45\pi}{4} \text{ m}^2$$

$\therefore$ Area of garden $ABCD$

$$= \frac{9\sqrt{3}}{4} + \frac{45\pi}{4}$$

$$= \frac{9\sqrt{3} + 45\pi}{4} \text{ m}^2$$

9. Trigonometry, 2ADV T1 SM-Bank 2



Using the sine rule:

$$\frac{\sin\angle ACB}{6.2} = \frac{\sin 35^\circ}{4.1}$$

$$\sin\angle ACB = \frac{6.2 \times \sin 35^\circ}{4.1}$$

$$= 0.8673\ldots$$

$$\angle ACB = 60.15\ldots^\circ \text{ or } 119.84\ldots^\circ$$

If $\angle ACB = 60.15^\circ$,

$\angle BAC = 180 - (35 + 60.15) = 84.85^\circ$

$$\frac{BC}{\sin 84.85} = \frac{4.1}{\sin 35^\circ}$$

$$BC = 7.11...$$

$$= 7.1 \text{ cm}$$

If $\angle ACB = 119.85^\circ$,

$\angle BAC = 180 - (35 + 119.85) = 25.15^\circ$

$$\frac{BC}{\sin 25.15} = \frac{4.1}{\sin 35^\circ}$$

$$BC = 3.03\ldots$$

$$= 3.0 \text{ cm}$$

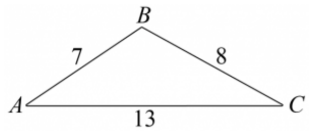
$\therefore$ Possible dimensions are:

7.1 cm, 6.2 cm, 4.1 cm or

3.0 cm, 6.2 cm, 4.1 cm.

10. Trigonometry, 2ADV T1 2005 HSC 3b

i.



$\angle ABC$ is opposite the longest side

Using the cosine rule

$$\cos \angle ABC = \frac{7^2+8^2-13^2}{2\times 7\times 8}$$
$$= -\frac{1}{2}$$

Since $\cos 60^\circ = \dfrac{1}{2}$ and cos is negative

in 2nd quadrant,

$$\angle ABC = 180-60$$
$$= 120^\circ$$

ii. Using the sine rule

$$\text{Area } \Delta ABC = \frac{1}{2}\, ab \sin C$$
$$= \frac{1}{2}\times 7\times 8 \sin 120^\circ$$
$$= 28\times \frac{\sqrt{3}}{2}$$
$$= 14\sqrt{3}\ \text{cm}^2$$

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11. Trigonometry, 2ADV T1 2011 HSC 8a

i. Using the cosine rule

$$\cos 60^\circ = \frac{x^2+SP^2-SL^2}{2\times x\times 20}$$
$$\frac{1}{2} = \frac{x^2+20^2-22^2}{40x}$$
$$20x = x^2-84$$

$$\therefore x^2-20x-84=0 \quad \text{... as required}$$

ii. Find LP:

$$x^2-20x-84=0$$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{20\pm\sqrt{20^2-4\times 1\times(-84)}}{2}$$
$$= \frac{20\pm\sqrt{736}}{2}$$
$$= 23.546... \quad (x>0)$$
$$= 24 \text{ km} \ \ (\text{nearest km})$$

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12. Trigonometry, 2ADV T1 2017 HSC 13a

Cosine Rule:

$$c^2 = a^2+b^2-2ab\cos C$$
$$13^2 = (x-4)^2+(x+4)^2-2(x-4)(x+4)\cos 60^\circ$$
$$169 = x^2-8x+16+x^2+8x+16-\left(x^2-16\right)$$
$$169 = x^2+48$$
$$x^2 = 121$$
$$\therefore x = 11, \quad (x\neq -11)$$

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13. Trigonometry, 2ADV T1 2021 HSC 18

Using the sine rule:

$$\frac{\sin\theta}{25} = \frac{\sin 28^\circ}{16}$$

$$\sin\theta = \frac{25 \times \sin 28^\circ}{16}$$

$$\sin\theta = 0.73355$$

$$\theta = 47^\circ$$

$$\therefore \angle ABC = 180 - 47$$

$$= 133^\circ$$

14. Trigonometry, 2ADV T1 2015 HSC 13a

i. Show $\cos A = \dfrac{7}{8}$

Using the cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

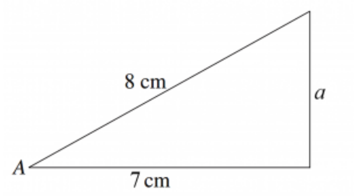
$$= \frac{8^2 + 6^2 - 4^2}{2 \times 8 \times 6}$$

$$= \frac{64 + 36 - 16}{96}$$

$$= \frac{84}{96}$$

$$= \frac{7}{8} \ \ldots \text{ as required}$$

ii.



♦ *Mean mark 40%.*

$$a^2 + 7^2 = 8^2$$

$$a^2 + 49 = 64$$

$$a^2 = 15$$

$$a = \sqrt{15}$$

$$\therefore \sin A = \frac{\sqrt{15}}{8}$$

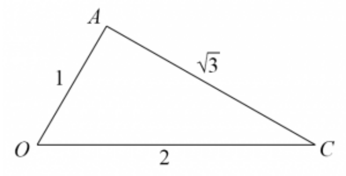
$$\therefore \text{ Area } \Delta ABC = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{15}}{8}$$

$$= 3\sqrt{15} \text{ cm}^2$$

15. Trigonometry, 2ADV T1 2007 HSC 4c

i.



In ΔAOC

$$\begin{aligned} AO^2 + AC^2 &= 1^2 + \sqrt{3}^2 \\ &= 1 + 3 \\ &= 4 \\ &= OC^2 \end{aligned}$$

$\therefore \Delta AOC$ is right-angled and $\angle OAC = \dfrac{\pi}{2}$

ii. $\sin \angle ACO = \dfrac{1}{2}$

$\therefore \angle ACO = \dfrac{\pi}{6}$

$\sin \angle AOC = \dfrac{\sqrt{3}}{2}$

$\therefore \angle AOC = \dfrac{\pi}{3}$

iii. Area $AOBC$

$$\begin{aligned} &= 2 \times \text{Area } \Delta AOC \\ &= 2 \times \frac{1}{2} \times b \times h \\ &= 2 \times \frac{1}{2} \times 1 \times \sqrt{3} \\ &= \sqrt{3} \ \text{m}^2 \end{aligned}$$

iv. $\angle ACB = \dfrac{\pi}{6} + \dfrac{\pi}{6} = \dfrac{\pi}{3}$

$$\begin{aligned} \therefore \angle ACB \text{ (reflex)} &= 2\pi - \frac{\pi}{3} \\ &= \frac{5\pi}{3} \end{aligned}$$

Area of major sector ACB

$$= \frac{\theta}{2\pi} \times \pi r^2$$
$$= \frac{\frac{5\pi}{3}}{2\pi} \times \pi\left(\sqrt{3}\right)^2$$
$$= \frac{5\pi}{6} \times 3$$
$$= \frac{5\pi}{2} \text{ m}^2$$

v. $\angle AOB = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

$$\therefore \angle AOB \text{ (reflex)} = 2\pi - \frac{2\pi}{3}$$
$$= \frac{4\pi}{3}$$

Area of major sector AOB

$$= \frac{\frac{4\pi}{3}}{2\pi} \times \pi \times 1^2$$
$$= \frac{2\pi}{3} \text{ m}^2$$

$\therefore$ Total area of the logo

$$= \frac{5\pi}{2} + \frac{2\pi}{3} + \text{Area } AOBC$$
$$= \frac{15\pi + 4\pi}{6} + \sqrt{3}$$
$$= \left(\frac{19\pi + 6\sqrt{3}}{6}\right) \text{ m}^2$$

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16. Trigonometry, 2ADV T1 2009 HSC 5c

i. Area $\Delta AOB = \frac{1}{2} ab \sin\theta$

$$= \frac{1}{2} \times 2 \times 2 \times \sin\theta$$
$$= 2\sin\theta$$

$$2\sin\theta = \sqrt{3} \quad \text{(given)}$$
$$\sin\theta = \frac{\sqrt{3}}{2}$$
$$\therefore \theta = \frac{\pi}{3},\ \pi - \frac{\pi}{3}$$
$$= \frac{\pi}{3},\ \frac{2\pi}{3}$$

$\therefore$ The other value of θ is $\frac{2\pi}{3}$ radians

ii. (1) Area of sector $AOB = \pi r^2 \times \frac{\theta}{2\pi}$

$$= \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$$
$$= \frac{2\pi}{3} \text{ cm}^2$$

ii. (2) Using the cosine rule:

$$AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \times \cos\theta$$
$$= 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos\left(\frac{\pi}{3}\right)$$
$$= 4 + 4 - 4$$
$$= 4$$
$$\therefore AB = 2$$

$$\text{Arc } AB = 2\pi r \times \frac{\theta}{2\pi}$$
$$= r\theta$$

$$= \frac{2\pi}{3} \text{ cm}$$

$$\therefore \text{ Perimeter} = \left(2 + \frac{2\pi}{3}\right) \text{ cm}$$

17. Trigonometry, 2ADV T1 2018 HSC 14a

i. Using sine rule:

$$\text{Area } \Delta KLM = \frac{1}{2} \times 3 \times 6 \times \sin 60^\circ$$

$$= \frac{9\sqrt{3}}{2} \text{ u}^2$$

ii. Area ΔKLN + Area ΔNLM = Area ΔKLM

$$\frac{1}{2} \times 3 \times x \times \sin 30^\circ + \frac{1}{2} \times x \times 6 \times \sin 30^\circ = \frac{9\sqrt{3}}{2}$$

$$\frac{3}{4}x + \frac{3}{2}x = \frac{9\sqrt{3}}{2}$$

$$\frac{9}{4}x = \frac{9\sqrt{3}}{2}$$

$$\therefore x = \frac{9\sqrt{3}}{2} \times \frac{4}{9}$$

$$= 2\sqrt{3}$$

♦ *Mean mark 37%.*

18. Trigonometry, 2ADV T1 2013 HSC 14c

Find $\angle ADB$

$$\angle ADB = \frac{\pi}{6} + \frac{\pi}{2} \quad \text{(exterior angle of } \Delta BDC)$$

$$= \frac{2\pi}{3} \text{ radians}$$

Find AD

$$\tan\left(\frac{\pi}{6}\right) = \frac{4}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{4}{BC}$$

$$BC = 4\sqrt{3}$$

Using Pythagoras:

$$AC^2 + BC^2 = AB^2$$

$$AC^2 + \left(4\sqrt{3}\right)^2 = 13^2$$

$$AC^2 = 169 - 48$$

$$= 121$$

$$\Rightarrow AC = 11$$

$$\therefore AD = AC - DC$$

$$= 11 - 4$$

$$= 7$$

Using sine rule:

$$\frac{AB}{\sin\angle BDA} = \frac{AD}{\sin x}$$

$$\frac{13}{\sin\left(\frac{2\pi}{3}\right)} = \frac{7}{\sin x}$$

$$13 \times \sin x = 7 \times \sin\left(\frac{2\pi}{3}\right)$$

$$\sin x = \frac{7}{13} \times \sin\left(\frac{2\pi}{3}\right)$$

$$= \frac{7}{13} \times \frac{\sqrt{3}}{2}$$

♦ *Mean mark 36%.*
STRATEGY TIP: The hint to use the sine rule should flag to students that they will be dealing in non-right angled trig (i.e. ΔABD) and to direct their energies at initially finding $\angle ADB$ and AD.

$$= \frac{7\sqrt{3}}{26}$$

$\therefore$ The exact value of $\sin x = \dfrac{7\sqrt{3}}{26}$.

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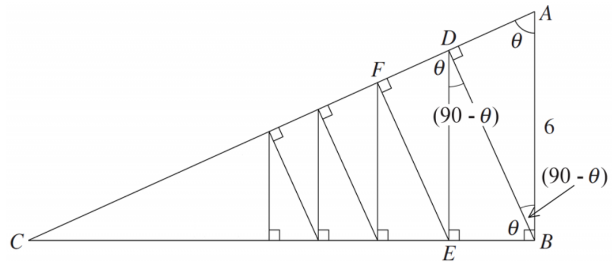
19. Trigonometry, 2ADV T1 2005 HSC 9b

i.



Show $EF = 6\sin^3\theta$

In ΔADB

$$\sin\theta = \frac{DB}{6}$$

$$DB = 6\sin\theta$$

$$\angle ABD = 90 - \theta \quad \text{(angle sum of } \Delta ADB\text{)}$$

$$\therefore \angle DBE = \theta \quad (\angle ABE \text{ is a right angle})$$

In ΔBDE:

$$\sin\theta = \frac{DE}{DB}$$

$$= \frac{DE}{6\sin\theta}$$

$$DE = 6\sin^2\theta$$

$$\angle BDE = 90 - \theta \quad \text{(angle sum of } \Delta DBE\text{)}$$

$$\angle EDF = \theta \quad (\angle FDB \text{ is a right angle})$$

In ΔDEF:

$$\sin\theta = \frac{EF}{DE}$$

$$= \frac{EF}{6\sin^2\theta}$$

$$\therefore EF = 6\sin^3\theta \ \ldots\text{as required}$$

ii. Show $BD + EF + GH \ldots$

has limiting sum $= 6\sec\theta\tan\theta$

$$\underbrace{6\sin\theta + 6\sin^3\theta + \ldots}_{\text{GP where } a=6\sin\theta,\ r=\sin^2\theta}$$

Since $0 < \theta < 90^\circ$

$$-1 < \sin\theta < 1$$

$$0 < \sin^2\theta < 1$$

$$\therefore |\,r\,| < 1$$

$$\begin{aligned}
\therefore S_\infty &= \frac{a}{1-r} \\
&= \frac{6\sin\theta}{1-\sin^2\theta} \\
&= \frac{6\sin\theta}{\cos^2\theta} \\
&= 6 \times \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta} \\
&= 6\sec\theta\tan\theta \ \ldots\text{as required.}
\end{aligned}$$