

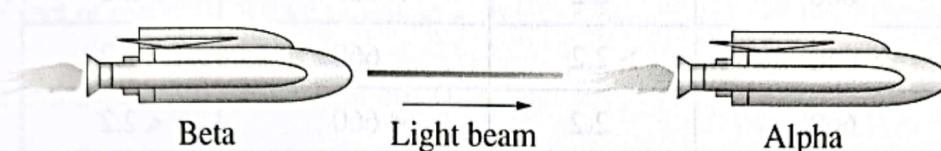
# Questions

## Module 7: The Nature of Light

### 7.4 Light and Special Relativity

Multiple-choice questions: 1 mark each

1. Two spaceships are both travelling at relativistic speeds. Spaceship Beta shines a light beam forward as shown.



What is the speed of the light beam according to an observer on spaceship Alpha?

- (A) The speed of the light beam is equal to  $c$ .
- (B) The speed of the light beam is less than  $c$ .
- (C) The speed of the light beam is greater than  $c$ .
- (D) More information is required about the relative speed of the spaceships.

2004 HSC Q5

2. Which of the following is a true statement about scientific theories, such as Einstein's theory of special relativity?

- (A) They are valid but unreliable ideas.
- (B) They are useful in making predictions.
- (C) They are concepts that lack an experimental basis.
- (D) They are ideas that can't be accepted until they have been tested.

2015 HSC Q6

3. During this Physics course, you will have analysed and evaluated the evidence confirming or denying Einstein's two postulates. What is a 'postulate'?

- (A) One of Einstein's thought experiments.
- (B) An observation of time dilation or contraction.
- (C) A suggestion assumed to be true and put forward for further reasoning or discussion.
- (D) A frame of reference that describes where an observation is being made from.

4. Muons are subatomic particles which at rest have a lifetime of 2.2 microseconds ( $\mu\text{s}$ ). When they are produced in Earth's upper atmosphere, they travel at  $0.9999 c$ .

Using classical physics, the distance travelled by a muon in its lifetime can be calculated as follows:

$$x = vt \\ = 660 \text{ m}$$

Which row of the table correctly summarises the behaviour of these muons?

Muon's reference frame		Earth's reference frame	
<i>Distance travelled</i> (m)	<i>Lifetime</i> ( $\mu\text{s}$ )	<i>Distance travelled</i> (m)	<i>Lifetime</i> ( $\mu\text{s}$ )
(A) 660	2.2	> 660	> 2.2
(B) > 660	> 2.2	660	2.2
(C) 660	2.2	< 660	< 2.2
(D) < 660	< 2.2	660	2.2

5. The rest length of a train is 200 m and the rest length of a railway platform is 160 m. The train rushes past the platform so fast that, when observed in the platform's frame of reference, the train and the platform are the same length.

How fast is the train moving?

- (A)  $0.60c$   
(B)  $0.75c$   
(C)  $0.80c$   
(D)  $1.25c$

6. A spaceship is travelling at a very high speed. What effects would be noted by a stationary observer?

- (A) Time runs slower on the spaceship and it contracts in length.  
(B) Time runs faster on the spaceship and it contracts in length.  
(C) Time runs slower on the spaceship and it increases in length.  
(D) Time runs faster on the spaceship and it increases in length.

7. An astronaut set out in a spaceship from Earth orbit to travel to a distant star in our galaxy. The spaceship travelled at a speed of  $0.8c$ . When the spaceship reached the star the on-board clock showed the astronaut that the journey took 10 years. An identical clock remained on Earth. What time in years had elapsed on this clock when seen from the astronaut's spaceship?

- (A) 6.0  
(B) 2.0  
(C) 1.2  
(D) 10.0

Adapted 2003 HSC Q5

8. An object of rest mass 8.0 kg moves at a speed of  $0.6c$  relative to an observer.

What is the observed mass of the object?

- (A) 6.4 kg  
(B) 10.0 kg  
(C) 12.5 kg  
(D) 13.4 kg

2004 HSC Q4

9. A spaceship sitting on its launch pad is measured to have a length  $L$ . This spaceship passes an outer planet at a speed of  $0.95c$ .

Which observations of the length of the spaceship are correct?

<i>Observer on the spaceship</i>	<i>Observer on the planet</i>
No change	Shorter than $L$
No change	Greater than $L$
Shorter than $L$	No change
Greater than $L$	No change

2007 HSC Q2

- 10.** A spaceship is travelling away from Earth at  $1.8 \times 10^8 \text{ m s}^{-1}$ . The time interval between consecutive ticks of a clock on board the spaceship is 0.50 s. Each time the clock ticks, a radio pulse is transmitted back to Earth.

What is the time interval between consecutive radio pulses as measured on Earth?

- (A) 0.40 s
- (B) 0.50 s
- (C) 0.63 s
- (D) 0.78 s

2008 HSC Q5

- 11.** A scientist at a particle accelerator laboratory observes the lifetime of a particular subatomic particle to be  $1.0 \times 10^{-6} \text{ s}$  when it is travelling at  $0.9999 c$ .

What would the lifetime of the particle be if it were stationary in the laboratory?

- (A)  $1.4 \times 10^{-8} \text{ s}$
- (B)  $4.5 \times 10^{-8} \text{ s}$
- (C)  $1.0 \times 10^{-6} \text{ s}$
- (D)  $7.1 \times 10^{-5} \text{ s}$

2010 HSC Q3

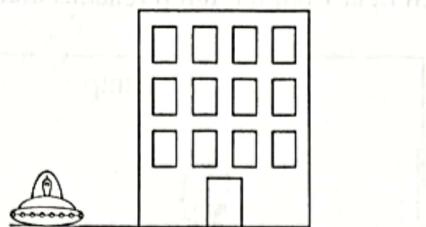
- 12.** A spaceship moves close to the speed of light, relative to a planet.

The rest-frame length of the spaceship can be determined by an observer who is

- (A) on the spaceship measuring the time taken for light to travel between two points on the planet.
- (B) on the planet measuring the time taken for light to travel from the front to the back of the spaceship.
- (C) on the spaceship measuring the time taken for light to travel from the front to the back of the spaceship.
- (D) on the planet measuring the difference in the arrival time of light from the front and the back of the spaceship.

2013 HSC Q19

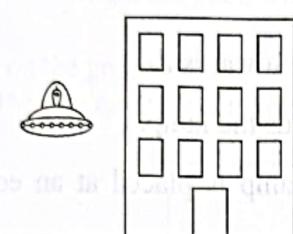
13. The diagram shows a stationary spacecraft next to a building, as seen by an observer across the street.



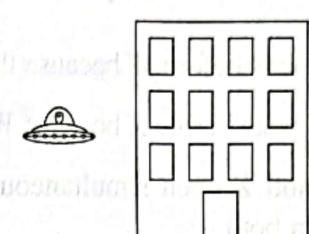
A short time later the spacecraft is observed to be travelling vertically upwards at  $0.8c$ , relative to the building.

Which diagram best represents the appearance of the moving spacecraft, as seen by the observer?

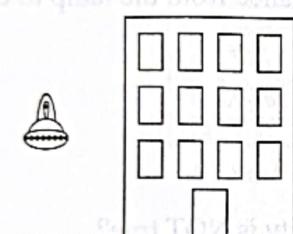
(A)



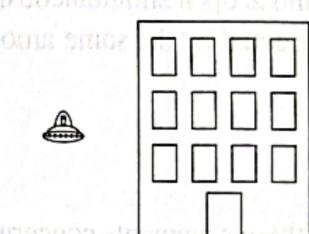
(B)



(C)



(D)



14. Astronauts travel at a velocity of  $0.9c$  to Alpha Centauri. Newtonian physics predicts that this journey would take 4.86 years.

How many years will the journey take in the frame of reference of the astronauts?

(A) 0.923

(B) 1.54

(C) 2.12

(D) 11.1

2015 HSC Q16

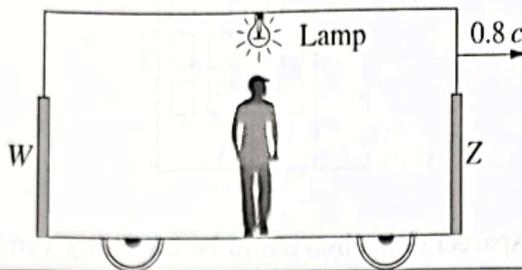
$v_2 = 0.1 \times 0.2$

$v_2 = 0.1 \times 1.8$

$v_2 = 0.1 \times 0.1$  (A)

$v_2 = 0.1 \times 5.4$  (B)

15. In a thought experiment, a train is moving at a constant speed of  $0.8c$ . A lamp is located at the midpoint of a carriage. There are doors W and Z at each end of the carriage which open automatically when light from the lamp reaches them.



The passenger standing at the midpoint of the carriage switches on the lamp.

Which statement best explains what the passenger observes about the doors?

- (A) Z opens before W because the lamp is moving towards Z.
- (B) W opens before Z because W is moving towards the lamp.
- (C) W and Z open simultaneously because the lamp is placed at an equal distance from both.
- (D) W and Z open simultaneously because the distance from the lamp to each door has contracted by the same amount.

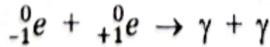
2016 HSC Q10

16. Which of these statements concerning special relativity is NOT true?

- (A) The measurement of time intervals, lengths and masses seem unchanged when viewed from within their own frame of reference.
- (B) The measurement of time intervals, lengths and masses is relative to the reference frame of the observer.
- (C) When viewed from a frame of reference that is moving, clocks tick more slowly and objects appear shorter.
- (D) When viewed from within their own frame of reference, a stationary clock will tick more slowly and objects will appear longer.

17. If a positron ( ${}_{+1}^0e$ ) and an electron ( ${}_{-1}^0e$ ) collide at low energies, they annihilate each other.

As a result, two gamma rays ( $\gamma$ ) are produced:

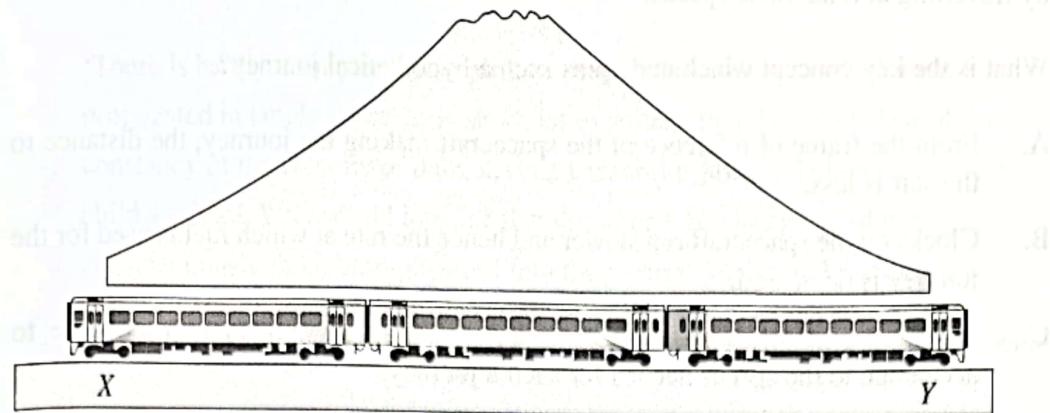


Using  $9.109 \times 10^{-31}$  kg as the rest mass of a positron (and electron), calculate the energy (in eV) produced during this annihilation.

- (A)  $1.64 \times 10^{-13}$  J
- (C)  $2.63 \times 10^{-32}$  eV
- (B)  $1.02 \times 10^6$  eV
- (D)  $5.1 \times 10^5$  eV

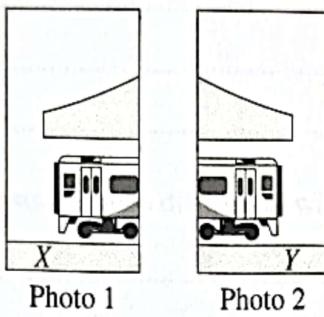
18.

When a train is at rest in a tunnel, the train is slightly longer than the tunnel.



In a thought experiment, the train is travelling from left to right fast enough relative to the tunnel that its length contracts and it fits inside the tunnel.

An observer on the ground sets up two cameras, at *X* and *Y*, to take photos at exactly the same time. The photos show that both ends of the train are inside the tunnel.



A passenger travelling on the train at its centre can see both ends of the tunnel and is later shown the photos.

From the point of view of the passenger, what is observed and what can be deduced about the photos?

- A. The tunnel's length contracts so the train does not fit, and photo 2 is taken before photo 1.
- B. The tunnel's length contracts so the train does not fit, and photos 1 and 2 are taken at the same time.
- C. The tunnel appears to expand due to length contraction of the train, allowing it to fit in the tunnel, and photo 1 is taken before photo 2.
- D. The tunnel appears to expand due to length contraction of the train, allowing it to fit in the tunnel, and photos 1 and 2 are taken at the same time.

2018 HSC Q16

19. A hypothetical journey to a distant star might be accomplished in an astronaut's life span by travelling at relativistic speeds.

What is the key concept which underpins such a hypothetical journey?

- A. From the frame of reference of the spacecraft making the journey, the distance to the star is less.
- B. Clocks on the spacecraft run slower and hence the rate at which fuel is used for the journey is decreased.
- C. Relativistic effects on the spacecraft reduce its mass, making it possible to accelerate to the speeds needed for such a journey.
- D. The relativistic increase in the mass of the fuel on board makes it possible to complete a longer journey with less fuel.

2018 HSC Q9



Two identical houses are shown. House 1 is on the left and House 2 is on the right. Both houses have a triangular roof, a chimney, and a small garden in front. The houses are identical in every way except for their position. This diagram is used to illustrate the concept of relativity, specifically length contraction, where the distance between the houses would appear shorter to an observer moving relative to the houses.

The diagram illustrates the concept of length contraction. If an observer moves past the houses at a significant fraction of the speed of light, the distance between them would appear smaller to the observer than it does to someone stationary relative to the houses. This is because the houses are moving away from each other at a constant velocity, and the distance between them is contracting.

The diagram also serves as a reminder that the speed of light is constant in all frames of reference. Even if the houses are moving towards or away from the observer, the speed of light remains constant. This is a fundamental principle of special relativity.

20. ‘There is hardly a simpler law in physics than that according to which light is propagated in empty space ... in short, let us assume that the simple law of the constancy of the velocity of light,  $c$ , (in a vacuum) is justifiably believed by the child at school. Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?’

From Einstein, *The Theory of Relativity* (1920)

- (a) What experimental evidence exists today to support the constancy of the velocity of light in a vacuum?

- (b) Explain why physicists had such difficulties with this ‘simple law’.

- (c) Describe two effects that are predicted as a consequence of accepting this ‘simple law’, but which are contrary to the predictions of classical mechanics.

21. The nearest galaxy to ours is the Large Magellanic Cloud, with its centre located  $1.70 \times 10^5$  light years from Earth. Assume you are in a spacecraft travelling at a speed of  $0.99999c$  toward the Large Magellanic Cloud.

- (a) In your frame of reference, what is the distance between Earth and the Large Magellanic Cloud?

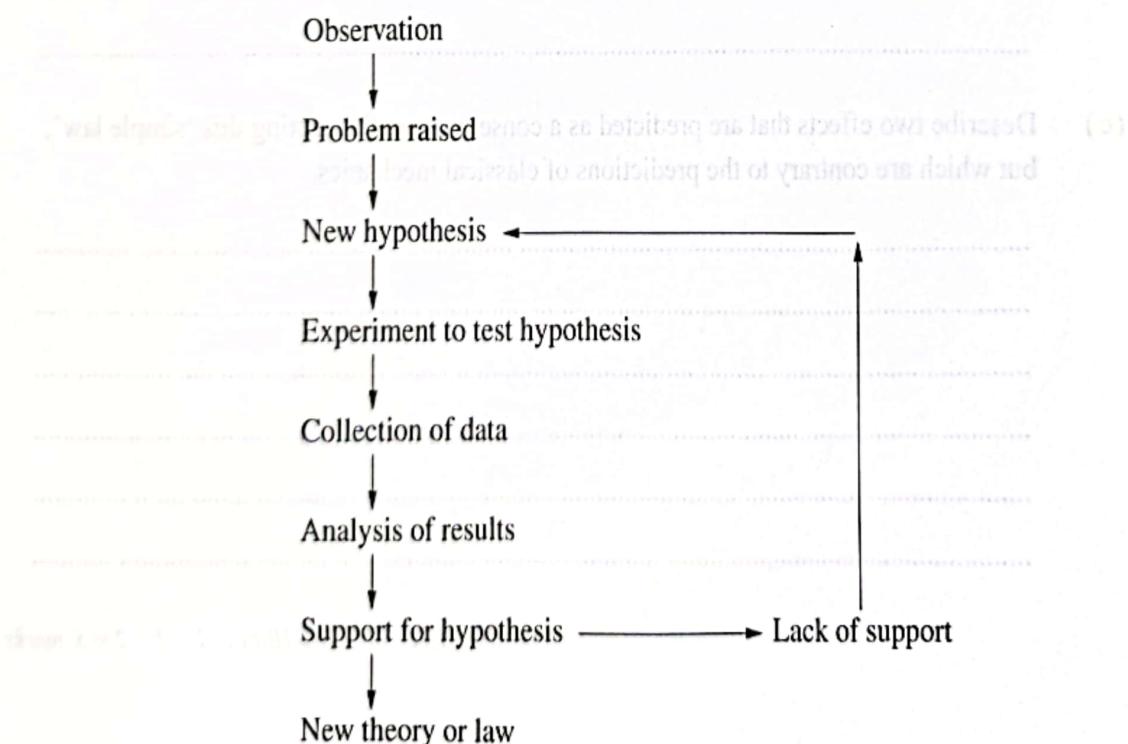
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- (b) In your frame of reference, how long will it take you to travel from Earth to the Large Magellanic Cloud?

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2009 HSC Q18 ... 2 + 2 = 4 marks

22. This flowchart represents one model of scientific method used to show the relationship between theory and the evidence supporting it.



Question 22 continues

Analyse Einstein's Theory of Special Relativity and the evidence supporting it as an application of this model of scientific method.

*End of Question 22*

2007 HSC Q19 ... 6 marks

23. Muons are very short-lived sub-atomic particles. Protons collide with each other. A beam of muons can be produced by very high-energy particle accelerators.

The high-speed muons produced for an experiment by the Fermilab accelerator are measured to have a lifetime of 5.0 microseconds. When these muons are brought to rest, their lifetime is measured to be 2.2 microseconds.

- (a) Name the effect demonstrated by these observations of the lifetimes of the muons.

(b) Calculate the velocity of the muons as they leave the accelerator.

24. Two observers,  $A$  and  $B$ , are travelling through space.  $B$  is receding from  $A$  with a speed of  $0.8c$ .  $A$  transmits a pulse of light and sees it travel towards  $B$  with a speed of  $c$ .

With what speed does  $B$  see the light pulse pass? Explain why.

Adapted 1991 HSC O Elective 1R(b) ... 2 marks

23. Muons are very short-lived sub-atomic particles that are created when energetic protons collide with each other. A beam of muons can be produced by very high-energy particle accelerators.

The high-speed muons produced for an experiment by the Fermilab accelerator are measured to have a lifetime of 5.0 microseconds. When these muons are brought to rest, their lifetime is measured to be 2.2 microseconds.

- (a) Name the effect demonstrated by these observations of the lifetimes of the muons.

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- (b) Calculate the velocity of the muons as they leave the accelerator.

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2001 HSC Q16 ... 1 + 3 = 4 marks

24. Two observers, *A* and *B*, are travelling through space. *B* is receding from *A* with a speed of  $0.8c$ . *A* transmits a pulse of light and sees it travel towards *B* with a speed of  $c$ .

With what speed does *B* see the light pulse pass? Explain why.

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Adapted 1991 HSC Q Elective 1R(b) ... 2 marks

25.

In one of Einstein's famous thought experiments, a passenger travels on a train that passes through a station at 60% of the speed of light. According to the passenger, the length of the train carriage is 22 m from front to rear.

- (a) A light in the train carriage is switched on. Compare the velocity of the light beam as seen by the passenger on the train and a rail worker standing on the station platform.

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- (b) Calculate the length of the carriage as observed by the rail worker on the station platform.

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2002 HSC Q19 ... 1 + 3 = 4 marks

26. A proton has a rest mass of  $1.673 \times 10^{-27}$  kg. In the Large Hadron Collider, a particle accelerator at CERN, a proton can be made to move at a speed of 99.9% of the speed of light.

- (a) What is the relativistically corrected mass of such a proton?

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- (b) If a particle has a relativistic mass that is three times its rest mass, calculate the speed at which it is moving.

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... 2 + 2 = 4 marks

27. (a) Outline ONE piece of evidence supporting Einstein's theory of relativity.

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28. A sequence of fusion reactions occurs in the core of the Sun, releasing enormous amounts of energy. These fusion processes release about 62 MeV of energy each second.

- (a) How much energy is released each second (in joules)?

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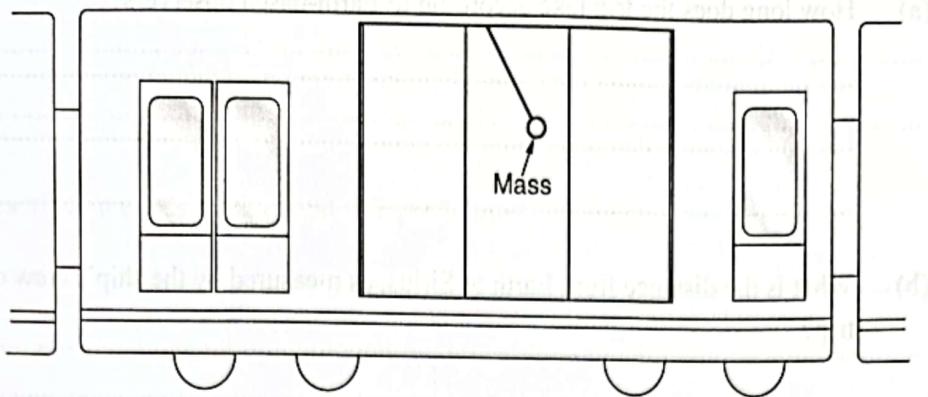
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29. A train is travelling on a straight horizontal track. A student on the train attaches a mass on a string to the ceiling of the train. The student observes that the mass remains stationary in the position shown.



- (a) Why does the mass hang with the string at an angle to the vertical?

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- (b) The string then breaks and the mass falls.

Indicate the path of the mass on the diagram above. Explain why the mass has taken this path.

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2010 HSC Q23 ... 2 + 3 = 5 marks

30. If an electron is travelling at  $0.85c$ , calculate the magnitude of its momentum (in  $\text{kg m s}^{-1}$ ) relative to an observer.

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... 2 marks

31. Sirius A, the brightest star in the sky after our Sun, is 8.7 light years from Earth. Suppose a space ship travels from Earth to Sirius A at a constant velocity of  $2.0 \times 10^8 \text{ m s}^{-1}$ .

- (a) How long does the trip take according to Earth-based observers?

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- (b) What is the distance from Earth to Sirius, as measured by the ship's crew during the trip?

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- (c) How long does the trip take according to the ship's crew? (The ship's clocks are synchronised with the clocks on Earth prior to departure.)

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1992 HSC Q Elective IR(b) ... 2 + 2 + 2 = 6 marks

32. A star radiates energy at the rate of  $4.0 \times 10^{26} \text{ W}$ . Calculate the corresponding mass loss.

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1991 HSC Q Elective IR(d) ... 2 marks

33. Einstein's 1905 theory of special relativity made several predictions that could not be verified for many years.

- (a) State ONE such prediction.

Slow | rapid |

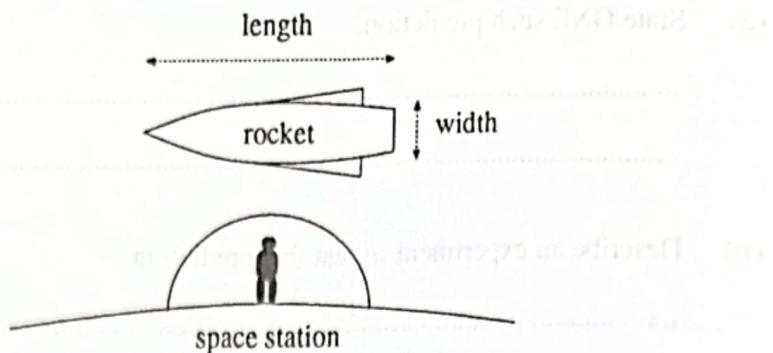
- (b) Describe an experiment to test this prediction.

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- (c) Explain how technological advances since 1905 have made it possible to carry out this experiment.

2005 HSC Q17 ... 1 + 2 + 3 = 6 marks

34. You are in a space station when a very fast rocket passes by at a speed of  $0.76c$ , as shown in the diagram.



Relative to you, the length of the rocket is 38 m and its width is 3.0 m.

What is its length and width when it eventually lands at its destination on the space station?

1993 HSC Q Elective 1R(c) ... 2 marks

35. Einstein's relativistic momentum equation is:  $p_v = \frac{m_0 v}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

Use this equation to explain why there is a limitation on the maximum velocity of a particle.

... 3 marks

36. In the Large Hadron Collider (LHC), protons travel in a circular path at a speed greater than  $0.9999 c$ .

Discuss the application of special relativity to the protons in the LHC.



Suppose such a particle is produced 9.0 km above the Earth's surface and travels towards the Earth at a velocity of  $0.998c$ , where  $c$  is the velocity of light.

The lifetime of such a particle is  $2.00 \times 10^{-6}$  s (proper time).

- (a) How far does a particle with velocity  $0.998c$  travel in  $2.00 \times 10^{-6}$  s?

- (b) Explain why this particle actually reaches the Earth's surface before it decays.

## 7.4 Light and Special Relativity

Multiple choice: 1 mark each

1. A
2. B
3. C
4. A
5. A
6. A
7. C
8. B
9. A
10. C
11. A
12. C
13. B
14. C
15. C
16. D
17. B
18. A
19. A

**Explanations:**

1. A Einstein stated that the speed of light is the same to all observers regardless of any relative motion, so the observer on the spaceship Alpha will record the speed of light as  $c$ , the same speed as it left the observer on spaceship Beta, as in (A).
2. B A scientific theory is an explanation of some aspect of nature that is supported by a vast body of evidence. A theory can be used to make predictions, as in (B), and may be tested and result in the theory being proved or disproved. As with Einstein's theories, the technology may not be available at the time to test the theory's predictions until long after the theory was proposed. So (D) is incorrect. Both (A) and (C) are incorrect, as they apply to some theories, but not all theories.
3. C A postulate is a suggestion that is assumed to be true and put forward for further reasoning or discussion, e.g. Einstein's postulate about the relativity of light. So (C) is the answer. It is not a thought experiment, observation or frame of reference, so (A), (B) and (D) are incorrect.
4. A In a muon's frame of reference, it is 'at rest' – so the distance it travels and its lifetime will be unchanged. However, from Earth's reference frame, a muon is travelling at a speed close to light – so Einstein's Laws of Relativity indicate that time will pass more slowly when viewed from Earth. So a muon's lifetime will be longer ( $>2.2\ \mu s$ ) and it will travel further ( $>660\ m$ ). So (A) is the answer.
5. A 
$$l_v = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$
$$160 = 200 \sqrt{1 - \frac{v^2}{c^2}}$$
 Rearranging:  $\frac{160}{200} = \sqrt{1 - \frac{v^2}{c^2}}$ 
$$0.8^2 = 1 - \frac{v^2}{c^2}$$
$$\therefore \frac{v^2}{c^2} = 1 - 0.8^2 = 1 - 0.64 = 0.36 \text{ and } \frac{v}{c} = \sqrt{0.36} = 0.60 \dots \text{as in (A).}$$

6. A A consequence of Einstein's theory of relativity is that values for time, length and mass are relative and not absolute when there is relative motion between the observed event and any observers (i.e. values are not fixed for a given event). As a result, the same event will be measured to have different values for observers who are moving relative to each other. To any observer, time on a vehicle moving relative to the observer runs *slower* than for the occupants, and the length of the vehicle itself will be *shorter* for observers, i.e. it will contract in length. So (A) is the answer.

[Note: In responses (A)–(D), the word 'it' grammatically refers to 'time', however since none of the answers were correct this way, 'it' needs to be taken to refer to the word 'spaceship', rather than to the word 'time'.]

7. C\* The astronaut's journey takes 10 years on the spaceship's clock. He has travelled at  $v = 0.8c$ , so his spaceship has travelled 8 light years distance from Earth to the star. So light from Earth will take 8 years to reach the astronaut.

Observers remaining on Earth will see 10 years has elapsed on their identical clock. However, when the astronaut sees the clock on Earth, this light has taken 8 years to reach him and so will be the light that left Earth 8 years earlier, when he was 2 years into his journey.

Earth is moving relative to the astronaut. So, when he sees the clock on Earth, the time he sees will be less than 2 years, as time moves more slowly on a moving clock.

This time dilation is given by:  $t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

where:  $t = 2$  years (time elapsed as measured on Earth)

$t_0 = ?$  (time elapsed on clock on Earth as seen by astronaut on spaceship),

$$V = 0.8c$$

$$\therefore t = t_0 \times \sqrt{\left(1 - \frac{v^2}{c^2}\right)} = 2.0 \times \sqrt{\left(1 - \frac{(0.8c)^2}{c^2}\right)}$$

$$= 2.0 \times \sqrt{(1 - 0.64)} = 2.0 \times 0.6$$

$$= 1.2 \text{ years} \dots \text{as in (C).}$$

[\* Note: The wording and answers for this question have been amended since the HSC, so that there is now a correct answer – as the original question in the 2003 HSC did not have a correct answer.]

$$8. B m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{8.0}{\sqrt{\left(1 - \frac{(0.6c)^2}{c^2}\right)}} = \frac{8.0}{\sqrt{1 - 0.36}} \\ = \frac{8.0}{\sqrt{0.64}} = \frac{8.0}{0.8} = 10.0 \text{ kg}$$

So (B) is the answer.

9. A The observer on the spaceship is not in motion relative to the spaceship, so that observer sees no change in length. The observer on the planet sees the spaceship travelling at high velocity and therefore sees a contraction in the length of the spaceship, i.e. it appears shorter than  $L$ . So (A) is the answer.

10. C Time passes more slowly in a rapidly moving frame of reference, as described by:

$$t_v = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{0.50}{\sqrt{1 - \frac{(1.8 \times 10^8)^2}{(3.00 \times 10^8)^2}}} = \frac{0.50}{\sqrt{1 - 0.6^2}} = \frac{0.50}{\sqrt{1 - 0.36}} = \frac{0.50}{\sqrt{0.64}} = \frac{0.50}{0.8} = 0.63 \text{ s, as in (C).}$$

11. A Transposing the given equation to make  $t_0$  the subject:

$$t_0 = t_v \sqrt{1 - \frac{v^2}{c^2}} = 1.0 \times 10^{-6} \sqrt{1 - \left(\frac{0.9999c}{c}\right)^2}$$

$$\therefore t_0 = 1.0 \times 10^{-6} \sqrt{1 - 0.9999^2} = 1.0 \times 10^{-6} \sqrt{1 - 0.99980001}$$

$$= 1.0 \times 10^{-6} \sqrt{1.9999 \times 10^{-4}} = 1.0 \times 10^{-6} \times 0.014 = 1.4 \times 10^{-8} \text{ s, as in (A).}$$

12. C To measure the rest-frame length of the spaceship, the observer must be at rest relative to the spaceship and must measure the distance from the front to the back of the spaceship by measuring the time that light takes to travel over the distance. So (C) is the answer.

13. B If the spacecraft is moving vertically upwards at  $0.8c$  (i.e. at 80% the speed of light), the length of the spacecraft will appear to the observer to be contracted in the direction of this motion, i.e. vertically. It will be unchanged in the horizontal direction as there is no horizontal velocity component. So (B) is the answer.

(A) is incorrect as no contraction is shown in any direction. (C) is incorrect as contraction is shown in the horizontal direction. (D) is incorrect as contraction is shown in both vertical and horizontal directions.

14. C The astronauts travel at  $0.9c$ . The time for the journey, as observed by the astronauts, is given by:

$$t = t_0 \sqrt{1 - \frac{v^2}{c^2}} = 4.86 \times \sqrt{1 - \frac{0.9c^2}{c^2}} = 4.86 \times \sqrt{1 - 0.9^2} = 4.86 \times \sqrt{1 - 0.81}$$

$$\therefore t = 4.86 \times \sqrt{0.19} = 2.12 \text{ years} \dots \text{as in (C).}$$

15. C The train carriage and its passenger are in an inertial frame of reference. Within the carriage, the passenger is mid-way between the two doors that are equidistant from the passenger. Any motion of the train is irrelevant to the carriage and its passenger. Hence the light will travel to  $W$  and  $Z$  in the same time period. So (C) is the answer. Since all observations are made from the reference frame of the moving carriage, both (A) and (B) are incorrect. Only observers in another frame of reference can observe length contraction, so (D) is incorrect.

16. D When viewed from within their own frame of reference, all lengths and all clocks seem unchanged. Hence statement (D) is NOT true, so (D) is the answer. All other statements (A), (B) and (C) are true, and therefore incorrect answers for this question.

$$\begin{aligned}
 17. \quad B \quad E &= mc^2 \\
 &= 2 \times (9.109 \times 10^{-31}) \times (3.00 \times 10^8)^2 \\
 &= 1.63962 \times 10^{-13} \text{ J} \\
 &= \frac{1.63962 \times 10^{-13}}{1.602 \times 10^{-19}} \text{ eV} \quad [since] \\
 &= 1.02 \times 10^6 \text{ eV}
 \end{aligned}$$

So, the energy produced during this annihilation is  $1.02 \times 10^6$  eV ... as in (B)

18. A The passenger on the train observes the surrounds moving rapidly, whereas the observer on the ground sees the train moving rapidly. The train's passenger sees the *tunnel* contracted in length, while the observer on the ground sees the *train* as contracted in length. Therefore, from the passenger's point of view (frame of reference), the tunnel has contracted in length and not the train. So Photo 2 is taken before Photo 1 ... as in (A).

19. A As a result of the near light speed motion of the spacecraft, the distance to the destination star is contracted. This makes the distance to the star less, as in (A). In the frame of reference of the astronaut, the clock does not change its running speed, so (B) is incorrect. Both (C) and (D) are incorrect interpretations of relativity.

## Short-answer questions

20. (a) The Michelson-Morley experiment found that the speed of light was always the same i.e. constant, as it was not affected by the orbital motion of the Earth as it passed through the postulated aether.

[Note: Improved experiments since the Michelson-Morley experiment, have continued to show the same result.]

- (b) Waves such as light were thought to require a medium to travel through. Classical physics and common sense indicated that wave motion was affected by the movement of the source and the observer.
- (c) 1. The speed of light ( $c$ ) is constant regardless of the speed of either the observer or the source.  
2. It allowed the equivalence of mass and energy, as expressed by  $E = mc^2$ , to be predicted.

21. (a) 
$$l_v = l_o \sqrt{1 - \frac{v^2}{c^2}} = l_o \sqrt{1 - \left(\frac{v}{c}\right)^2} = 1.70 \times 10^5 \sqrt{1 - \left(\frac{0.99999c}{c}\right)^2}$$

$$\therefore l_v = 1.70 \times 10^5 \sqrt{1 - 0.99999^2} = 1.70 \times 10^5 \sqrt{1 - 0.999800001} \\ = 760.2612 \text{ ly}$$

i.e. distance between Earth and Large Magellanic Cloud is 760 ly

(b) Time taken  $= \frac{\text{distance}}{\text{speed}} = \frac{760.2612 \text{ c years}}{0.99999c}$   
 $= 760.2688 = 760.27 \text{ years}$

i.e. it will take 760.27 years

22. (a) The deflection of light by the Sun's gravity (Newton's law of gravitation) has been firmly established.

- (b) The bending of light by the Sun's gravity can be calculated using the formula  $\theta = 4GM/c^2R$ .

- (c) The deflection of light by the Sun's gravity is measured by the angle  $\theta$  between the path of the light ray as it would have travelled if there were no Sun, and the path of the light ray as it actually travelled in the presence of the Sun's gravitational field.

- (d) The speed of light is measured by the time taken for the light to travel from the Sun to the Earth.

- (e) The speed of light is measured by the time taken for the light to travel from the Sun to the Earth.

- (f) The speed of light is measured by the time taken for the light to travel from the Sun to the Earth.

- (g) The speed of light is measured by the time taken for the light to travel from the Sun to the Earth.

- (h) The speed of light is measured by the time taken for the light to travel from the Sun to the Earth.

22. For his Theory of Special Relativity Einstein proposed that all the laws of physics were identical in all inertial reference frames, and that all observers see light travelling at the same speed,  $c$ , independent of the speed of the source or observer. These hypotheses were to overcome inconsistencies in the electromagnetic theory. Einstein did not conduct experiments or make observations in this area, but instead used 'thought experiments', such as the the train traveller with an observer at the side of the track in which he determined that both must see light travelling at the same speed, as time passes differently for each observer. From such 'experiments', he 'visualised' interactions as they would occur in nature. By analysing these, he made predictions regarding time dilation, length contraction, relativistic mass, etc.

As technology improved during the 20th century, Einstein's predictions became testable and were supported, e.g. flying atomic clocks demonstrated time dilation, as did the extended life times of mesons in cosmic rays as they travel at near light speed through the atmosphere; accurate measurements of the masses of nuclides and the energy released in transmutations have verified that  $E = mc^2$ ; and mass increase in accordance with relativity has been verified using particle accelerators. Hence, no new hypotheses have been needed – and the stages of the scientific model shown have been completed, even if decades later.

23. (a) Time dilation effect.

$$(b) t = \frac{t_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad [\text{Note: this is from Formulae sheet}]$$

$$5.0 \times 10^{-6} = \frac{2.2 \times 10^6}{\sqrt{\left(1 - \frac{v^2}{(3.00 \times 10^8)^2}\right)}}$$

$$\sqrt{1 - \frac{v^2}{(3 \times 10^8)^2}} = \frac{2.2 \times 10^6}{5.0 \times 10^6}$$

$$1 - \frac{v^2}{(3 \times 10^8)^2} = \left(\frac{2.2 \times 10^6}{5.0 \times 10^6}\right)^2 = 0.1936 \quad [\text{Note: by squaring both sides.}]$$

$$\frac{v^2}{(3 \times 10^8)^2} = 1 - 0.1936 = 0.8064$$

$$v = \sqrt{0.8064 \times (3 \times 10^8)^2}$$

$$= 2.694 \times 10^8 = 2.7 \times 10^8 \text{ m s}^{-1}$$

24. Both observers,  $A$  and  $B$ , will see the light pulse travelling at  $c$ . This is because the speed of light ( $c$ ) is the same for all frames of reference or observers moving at a constant velocity to one another.

25. (a) According to Einstein's second postulate, the velocity of light is a constant for all observers, and is independent of their frame of reference. Therefore, both the passenger and the rail worker observe the velocity of light as the same value, i.e. as  $3.00 \times 10^8 \text{ m s}^{-1}$ .

(b) 
$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$
 [Note: This length contraction formula is on the Data Sheet.]

$l_0 = 22$  m, and  $v = 0.6 c$

$$\therefore l = l_0 \sqrt{1 - \frac{(0.6c)^2}{c^2}} = 22 \times \sqrt{1 - 0.6^2} = 22 \times 0.8 = 17.6 \text{ m}$$

i.e. the carriage appears to be only 17.6 m in length to the rail worker.

26. (a)  $m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$

$$\begin{aligned} \text{So, } m &= \frac{1.673 \times 10^{-27}}{\sqrt{\left(1 - \frac{(0.999 \times c)^2}{c^2}\right)}} = \frac{1.673 \times 10^{-27}}{\sqrt{1 - 0.999^2}} \\ &= \frac{1.673 \times 10^{-27}}{\sqrt{1.999 \times 10^{-3}}} \\ &= 3.742 \times 10^{-26} \text{ kg} \end{aligned}$$

(b)  $m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$  where  $\frac{m}{m_0} = 3$

$$\text{Rearranging this: } \frac{m_0}{m} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\left(\frac{m_0}{m}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0}{m}\right)^2$$

$$\text{So, } v = c \sqrt{\left(1 - \left(\frac{m_0}{m}\right)^2\right)} = c \sqrt{1 - \left(\frac{1}{3}\right)^2} = 0.9428 c$$

$\therefore$  the speed,  $v = 2.83 \times 10^8 \text{ m s}^{-1}$

27. (a) The deflection of light by the Sun was able to be seen during a solar eclipse, thus showing that gravity bent light.

[Note: Other evidence includes: • the flying of atomic clocks to determine the existence of time dilation, • the energy yield from converted mass in nuclear reactions, • the observed increase in the mass of particles accelerated to near-light speed, in devices such as particle accelerators, • the dilated lifetimes of high velocity mesons penetrating the Earth's atmosphere.]

- (b) An hypothesis is developed to test the theory. The hypothesis needs to be tested by a valid experimental procedure. The data that is collected needs to be analysed to see if it supports the hypothesis. The experiment needs to be repeated several times to ensure reliability. For a theory to be validated, the results must explain all observations and other scientists then need to be able to obtain the same results with further testing.

$$(c) l = l_0 \sqrt{1 - \frac{v^2}{c^2}} = 40 \times 10^{-2} \sqrt{1 - \left(\frac{3.0 \times 10^7}{3.00 \times 10^8}\right)^2}$$

$$= 40 \times 10^{-2} \sqrt{1 - 0.1^2} = 0.398 \text{ m} = 39.8 \text{ cm}$$

$\therefore$  apparent distance is 39.8 cm.

28. (a)  $62 \text{ MeV s}^{-1} = (62 \times 10^6) \times (1.602 \times 10^{-19})$  [since  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J s}^{-1}$ ]

$$= 9.9324 \times 10^{-12}$$

$$= 9.93 \times 10^{-12} \text{ J s}^{-1}$$

(b)  $E = mc^2$

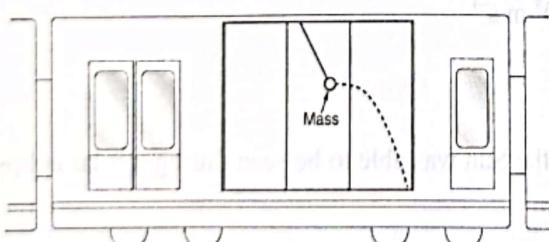
$$\text{So, } \Delta m = \frac{E}{c^2}$$

$$= \frac{9.9324 \times 10^{-12}}{(3.00 \times 10^8)^2} = 1.104 \times 10^{-28}$$

$$\therefore \text{mass loss} = 1.10 \times 10^{-28} \text{ kg s}^{-1}$$

29. (a) The train is changing its velocity and accelerating to the left, so it is a non-inertial frame. The mass is attempting to maintain its previous velocity.

(b)



In the frame of an outside observer, the mass will have the same horizontal velocity it had when the string broke. So, it will move right relative to the train and downwards due to gravity. The mass will travel further to the right relative to the carriage than relative to Earth because the train is accelerating to the left.

[Note: (1) The train is accelerating to the left, but no data is given to indicate in what direction the train is actually moving. It could be moving to either the left or the right.]

(2) In the frame of the train, the mass will travel in a straight line along the extension of the string towards the ground, as the train is still accelerating under the mass.



30.  $p_v = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\text{So, } p_v = \frac{9.109 \times 10^{-31} \times 0.85 \times 3.00 \times 10^8}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}} = \frac{9.109 \times 10^{-31} \times 0.85 \times 3.00 \times 10^8}{\sqrt{1 - (0.85)^2}}$$

$$= 4.409 \times 10^{-22}$$

$$\therefore p_v = 4.4 \times 10^{-22} \text{ kg m s}^{-1}$$

31. (a) Velocity of ship,  $v = 2.0 \times 10^8 \text{ m s}^{-1} = \frac{2}{3} c$

$$\text{Time taken (according to Earth-based observers)} = \frac{s}{v} \quad (\text{using Newtonian relativity})$$

$$= \frac{8.7}{2.0 \times 10^8} = \frac{8.7}{2 \times 10^8} = \frac{8.7}{2} \times 10^{-9} \text{ years}$$

$$= 4.35 \times 10^{-9} \text{ years} = 13.05 \text{ years}$$

(b) Distance from Earth to Sirius,  $l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$

$$= 8.7 \sqrt{\left(1 - \left(\frac{2.0 \times 10^8}{3.0 \times 10^8}\right)^2\right)}$$

$$= 6.48 \text{ light years}$$

(c) Length of trip according to crew is  $t_0$

$$\text{Since } t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and noting that the time experienced by the crew is } t_0 \text{ years}$$

$$\therefore t_0 = t \sqrt{1 - \frac{v^2}{c^2}} = 13.05 \sqrt{\left(1 - \left(\frac{2.0 \times 10^8}{3.0 \times 10^8}\right)^2\right)} = 9.73 \text{ years}$$

$\therefore$  The trip takes 9.73 years according to the ship's crew.

32. Since  $E = mc^2$

$$\text{So mass, } m = \frac{E}{c^2} = \frac{4.0 \times 10^{26} \text{ J s}^{-1}}{9 \times 10^{16} \text{ m}^2 \text{ s}^{-2}}$$

$$= 4.4 \times 10^9 \text{ kg s}^{-1}$$

$$\therefore \text{mass loss} = 4.4 \times 10^9 \text{ kg s}^{-1}$$

To work out the units:

$$E = mc^2 \text{ (in J)}$$

$$\text{So, } \text{J} = \text{kg} \cdot (\text{m s}^{-1})^2$$

$$\text{Also: } \text{W} = \text{J s}^{-1}$$

Two alternative answers are provided for Q33:

33. (a) The time recorded by a stationary observer for a fast moving object would be longer than the time recorded by an observer who was in a moving frame of reference.
- (b) Muons are produced in the upper atmosphere by incoming cosmic rays. They can also be created on Earth by a particle accelerator and studied at rest. Measuring the time taken for a muon to decay in each of these situations is a test for time dilation. A muon travelling through the atmosphere has a lifetime of about  $16\ \mu\text{s}$  in Earth's reference frame, whereas a muon at rest has a proper lifetime of about  $2.2\ \mu\text{s}$ . This dilated lifetime due to its relativistic speed supports Einstein's prediction.
- (c) One technological advance that made this experiment possible was the development of high-speed accelerators that could be used to bombard nuclei and as a result of high-energy particle collisions they could produce many different sub-atomic particles such as muons. Detectors were another advance as these were sensitive enough to use with sub-atomic particles. Muons are ionising radiation that can be detected and measured using sub-atomic particle detectors such as bubble chambers, scintillation detectors, or charge-coupled devices (CCDs). Timing devices that can make highly accurate measurements of time were also another advance. Time could be measured down to microseconds by using an atomic clock, e.g. a cesium atomic clock, which uses the oscillation frequencies within an atom to keep track of passing time.

*Alternative answer for 33:*

33. (a) The equivalence of energy and mass, which is given by  $E = mc^2$ . This established that energy can be converted into mass and vice versa.
- (b) An experiment to prove this involves  $\gamma$ -ray emission from radioactive sulfur and silicon atoms. The mass of the particles can be measured before and after  $\gamma$ -ray emission using a Penning Trap and then the energy can be calculated. The wavelength of each emitted  $\gamma$ -ray can be measured using a high-precision spectrometer and then since  $f = 1/\lambda$  and  $E = hf$ , the energy of these  $\gamma$ -rays can be calculated. The energy calculated from the difference in mass will be found to be equivalent to the energy of the  $\gamma$ -rays, thus proving that  $E = mc^2$ .

[Note: This experiment was done in 2006. The scientists who measured the mass were at MIT in Boston, while the scientists who determined the wavelengths of the  $\gamma$ -rays were at US National Institute of Standards & Technology (NIST). They found that  $E = mc^2$  to an accuracy of 1 in 50,000 (99.998% accuracy).]

- (c) Technological advances have led to more sophisticated measuring devices and spectrophotometers that can make the accurate measurements required by this experiment, e.g. to measure the mass of particles before and after  $\gamma$ -ray emission a very high-precision mass-measuring apparatus called a Penning Trap needs to be used; and a high-precision  $\gamma$ -ray spectrometer needs to be used to measure the wavelength of each emitted  $\gamma$ -ray and thus determine the energy of each  $\gamma$ -ray.

34. The length of the rocket is contracted due to the relative motion of the rocket and the observer. There is no relative motion in the direction of the width, so this will remain unaffected at 3 m.

$$l = l_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\text{So, } 38 = l_0 \sqrt{\left(1 - \frac{(0.76c)^2}{c^2}\right)}$$

$$\text{Hence } l_0 = \frac{38}{\sqrt{\left(1 - \frac{(0.76c)^2}{c^2}\right)}} = 58.468 = 58.5 \text{ m}$$

∴ length of rocket is 58.5 m and the width is 3 m.

35. If  $v = c$ , the denominator in this equation would be zero and the momentum would be infinite. So to accelerate an object up to  $v = c$  would thus require infinite energy, and so is not possible.

If  $v > c$ , the factor  $\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$  would be the square root of a negative number, which is an imaginary number. So lengths, times and mass would not be real.

Hence ordinary objects cannot equal or exceed the speed of light.

36. Mass dilation is a consequence of the theory of Special Relativity. This means that as the velocity of a moving object increases, its mass increases. This phenomenon has been demonstrated in particle accelerators, such as the LHC. As the velocity of the protons increases to nearly  $c$  in the LHC, their mass is observed to increase markedly, as represented by:

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}. \text{ So an increasing amount of energy is needed to accelerate the protons any further. This also means that a proton can approach } c, \text{ but never actually reach } c, \text{ as the value of } m_v \text{ becomes infinite when } v = c.$$

[Note: As  $v$  approaches  $c$ , the proton also experiences time differently. It means that time passes many times more slowly for rapidly moving protons than it does for stationary observers.]

37. (a) Distance particle travels =  $0.998 \times 3.00 \times 10^8 \times 2.00 \times 10^{-6}$   
 $= 5.88 \times 10^{-6}$   
 $= 599 \text{ m}$

$$(b) l = l_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\text{So, } l = 9000 \times \sqrt{\left(1 - \frac{(0.998c)^2}{c^2}\right)} = 9000 \times \sqrt{\left(1 - 0.998^2\right)}$$

$$= 568.925 = 569 \text{ m}$$

The particle will travel 569 m (in the frame of reference of the particle).

This is less than distance travelled (= 599 m) in the particle's lifetime = 599 m.

∴ the particle will reach Earth before it decays.