

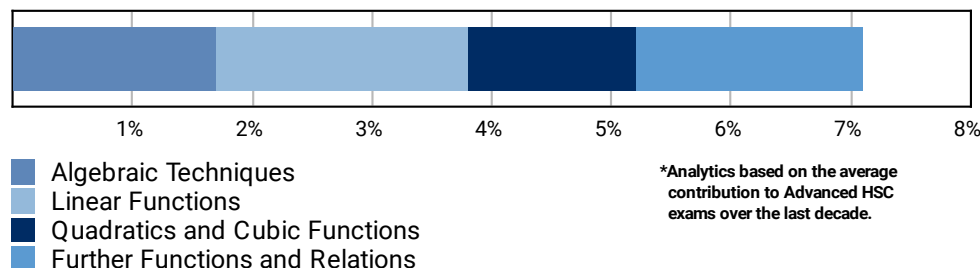
## ADV: Functions (Adv), F1 Working with Functions (Adv) Algebraic Techniques (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 75 minutes (based on HSC allocation of 1.5 minutes approx. per mark)



### F1 Working With Functions



### HISTORICAL CONTRIBUTION

- *F1 Working with Functions* is a Year 11 topic whose content represents the lowest of low hanging fruit in the new Advanced course.
- *F1 Working with Functions* has seen significant re-categorisation of content in the new syllabus. Our analysis has it accounting for approximately 7.1% per exam over the last decade.
- We have split the topic into 5 categories for analysis purposes: 1-*Algebraic Techniques*, 2-*Linear Functions*, 3-*Quadratics and Cubic Functions*, 4-*Composite Functions* and 5-*Further Functions and Relations*.
- This analysis looks at *Algebraic Techniques* (1.7%).

### HSC ANALYSIS - What to expect and common pitfalls

- *Algebraic Techniques* were regularly tested in the old *Mathematics* course, with surd calculations representing the dominant question type.
- Surds have historically been examined in 2 out of every 3 years (not examined in 2021 or 2020). Note that virtually all past questions have involved compound surds in the denominator which are no longer examinable. This fact is reflected in the database along with supplementary questions that look at examinable surd denominators (i.e. non-compound surds).
- Algebraic Fractions were examined in the 2021 Advanced paper and deserve attention, particularly as they represent an overlap between the new Advanced and Standard 2 content.
- Rounding questions can require students to know standard decimal place rounding, significant figures (2019) and scientific notation (2015).

### Questions

#### 1. Functions, 2ADV F1 2018 HSC 1 MC

What is the value of  $7^{-1.3}$  correct to two decimal places?

- (A) 0.07
- (B) 0.08
- (C) -12.54
- (D) -12.55

#### 2. Functions, 2ADV F1 2012 HSC 1 MC

What is **4.09784** correct to three significant figures?

- (A) **4.09**
- (B) **4.10**
- (C) **4.097**
- (D) **4.098**

#### 3. Functions, 2ADV F1 2014 HSC 1 MC

What is the value of  $\frac{\pi^2}{6}$ , correct to 3 significant figures?

- (A) **1.64**
- (B) **1.65**
- (C) **1.644**
- (D) **1.645**

#### 4. Functions, 2ADV F1 2015 HSC 1 MC

What is **0.005 233 59** written in scientific notation, correct to 4 significant figures?

- (A)  **$5.2336 \times 10^{-2}$**
- (B)  **$5.234 \times 10^{-2}$**
- (C)  **$5.2336 \times 10^{-3}$**
- (D)  **$5.234 \times 10^{-3}$**

5. Functions, 2ADV F1 2019 HSC 1 MC

What is the value of  $\pi^{10}$  to two significant figures?

- (A)  $9.36 \times 10^4$   
 (B)  $9.4 \times 10^4$   
 (C)  $9.36 \times 10^5$   
 (D)  $9.4 \times 10^5$

6. Functions, 2ADV F1 2005 HSC 1a

Evaluate  $\sqrt{\frac{275.4}{5.2 \times 3.9}}$  correct to two significant figures. (2 marks)

7. Functions, 2ADV F1 2007 HSC 1a

Evaluate  $\sqrt{\pi^2 + 5}$  correct to two decimal places. (2 marks)

8. Functions, 2ADV F1 SM-Bank 50

Rationalise the denominator of  $\frac{1}{4\sqrt{3}}$ . (2 marks)

9. Functions, 2ADV F1 SM-Bank 53

i. If  $\frac{1}{\sqrt[3]{7} + \pi} = (7 + \pi)^x$ , find  $x$ . (1 mark)

ii. Calculate the value of  $\frac{1}{\sqrt[3]{7} + \pi}$  to 3 significant figures. (1 mark)

10. Functions, 2ADV F1 2004 HSC 1c

Solve  $\frac{x-5}{3} - \frac{x+1}{4} = 5$ . (2 marks)

11. Functions, 2ADV F1 2005 HSC 1d

Express  $\frac{(2x-3)}{2} - \frac{(x-1)}{5}$  as a single fraction in its simplest form. (2 marks)

12. Functions, 2ADV F1 2011 HSC 1a

Evaluate  $\sqrt[3]{\frac{651}{4\pi}}$  correct to four significant figures. (2 marks)

13. Functions, 2ADV F1 2013 HSC 11a

Evaluate  $\ln 3$  correct to three significant figures. (1 mark)

14. Functions, 2ADV F1 2008 HSC 1a

Evaluate  $2 \cos\left(\frac{\pi}{5}\right)$  correct to three significant figures. (2 marks)

15. Functions, 2ADV F1 2008 HSC 1e

Expand and simplify  $(\sqrt{3} - 1)(2\sqrt{3} + 5)$ . (2 marks)

16. Functions, 2ADV F1 2021 HSC 11

Solve  $x + \frac{x-1}{2} = 9$  (2 marks)

17. Functions, 2ADV F1 SM-Bank 46

Find  $a$  and  $b$  such that  $a, b$  are real numbers and

$$\frac{8 - \sqrt{27}}{2\sqrt{3}} = a + b\sqrt{3}. \quad (2 \text{ marks})$$

18. Functions, 2ADV F1 SM-Bank 47

Find  $a$  and  $b$  such that  $a, b$  are real numbers and

$$\frac{\sqrt{32} - 6}{3\sqrt{2}} = a + b\sqrt{2} \quad (2 \text{ marks})$$

19. Functions, 2ADV F1 SM-Bank 51

Find  $a$  and  $b$  such that  $a, b$  are real numbers and

$$\frac{\sqrt{3} - 2}{2\sqrt{3}} = a + b\sqrt{3}. \quad (2 \text{ marks})$$

## 20. Functions, 2ADV F1 SM-Bank 52

Find ***a*** and ***b*** such that ***a, b*** are real numbers and

$$\frac{6\sqrt{3} - \sqrt{5}}{2\sqrt{5}} = a + b\sqrt{15}. \quad (2 \text{ marks})$$

## 21. Functions, 2ADV F1 SM-Bank 54

Find the reciprocal of  $\frac{1}{a} + \frac{1}{b} - \frac{c}{ab}$ . *(2 marks)*

## 22. Functions, 2ADV F1 SM-Bank 56

Simplify  $\frac{9x^2}{x+3} \div \frac{3x}{x^2-9}$ . *(2 marks)*

## 23. Functions, 2ADV F1 SM-Bank 55

Simplify  $\frac{4p-12p^2}{3} \times \frac{6p}{3p^2-p}$ . *(2 marks)*

## 24. Functions, 2ADV F1 EQ-Bank 22

Worker A picks a bucket of blueberries in ***a*** hours. Worker B picks a bucket of blueberries in ***b*** hours.

- Write an algebraic expression for the fraction of a bucket of blueberries that could be picked in one hour if A and B worked together. *(2 marks)*
- What does the reciprocal of this fraction represent? *(1 mark)*

## 25. Functions, 2ADV F1 2004 HSC 1a

The radius of Mars is approximately **3 397 000 m**. Write this number in scientific notation, correct to two significant figures. *(2 marks)*

## 26. Functions, 2ADV F1 2004 HSC 1d

Find integers ***a*** and ***b*** such that  $(3 - \sqrt{2})^2 = a - b\sqrt{2}$ . *(2 marks)*

## 27. Functions, 2ADV F1 2008 HSC 1c

Simplify  $\frac{2}{n} - \frac{1}{n+1}$ . *(2 marks)*

## 28. Functions, 2ADV F1 EQ-Bank 21

Simplify  $\left(\frac{p}{q}\right)^3 \div (pq^{-2})$ . *(2 marks)*

## Worked Solutions

1. Functions, 2ADV F1 2018 HSC 1 MC

$$\begin{aligned}7^{-1.3} &= 0.0796\dots \\ &= 0.08 \text{ (2 d.p.)} \\ \Rightarrow B\end{aligned}$$

2. Functions, 2ADV F1 2012 HSC 1 MC

$$\begin{aligned}4.10 \\ \Rightarrow B\end{aligned}$$

3. Functions, 2ADV F1 2014 HSC 1 MC

$$\begin{aligned}\frac{\pi^2}{6} &= 1.6449\dots \\ &= 1.64 \text{ (3 sig. figures)} \\ \Rightarrow A\end{aligned}$$

4. Functions, 2ADV F1 2015 HSC 1 MC

$$\begin{aligned}0.005\ 233\ 59 \\ &= 5.234 \times 10^{-3} \\ \Rightarrow D\end{aligned}$$

5. Functions, 2ADV F1 2019 HSC 1 MC

$$\begin{aligned}\pi^{10} &= 93648.04\dots \\ &= 9.4 \times 10^4 \\ \Rightarrow B\end{aligned}$$

6. Functions, 2ADV F1 2005 HSC 1a

$$\begin{aligned}\sqrt{\frac{275.4}{5.2 \times 3.9}} &= 3.685\dots \\ &= 3.7 \text{ (2 sig. figures)}\end{aligned}$$

7. Functions, 2ADV F1 2007 HSC 1a

$$\begin{aligned}\sqrt{\pi^2 + 5} &= 3.8561\dots \\ &= 3.86 \text{ (to 2 d.p.)}\end{aligned}$$

8. Functions, 2ADV F1 SM-Bank 50

$$\begin{aligned}\frac{1}{4\sqrt{3}} \times \frac{4\sqrt{3}}{4\sqrt{3}} &= \frac{4\sqrt{3}}{16 \times 3} \\ &= \frac{\sqrt{3}}{12}\end{aligned}$$

9. Functions, 2ADV F1 SM-Bank 53

$$\begin{aligned}\text{i. } \frac{1}{\sqrt[3]{7+\pi}} &= (7+\pi)^{-\frac{1}{3}} \\ \text{ii. } \frac{1}{\sqrt[3]{7+\pi}} &= 0.4619\dots \\ &= 0.462 \text{ (to 3 sig. fig.)}\end{aligned}$$

10. Functions, 2ADV F1 2004 HSC 1c

$$\begin{aligned}\frac{x-5}{3} - \frac{x+1}{4} &= 5 \\ 12\left(\frac{x-5}{3}\right) - 12\left(\frac{x+1}{4}\right) &= 12 \times 5 \\ 4x - 20 - 3x - 3 &= 60 \\ x - 23 &= 60 \\ \therefore x &= 83\end{aligned}$$

11. Functions, 2ADV F1 2005 HSC 1d

$$\begin{aligned}\frac{(2x-3)}{2} - \frac{(x-1)}{5} \\&= \frac{5(2x-3) - 2(x-1)}{10} \\&= \frac{10x-15-2x+2}{10} \\&= \frac{8x-13}{10}\end{aligned}$$

12. Functions, 2ADV F1 2011 HSC 1a

$$\begin{aligned}\sqrt[3]{\frac{651}{4\pi}} &= 3.72783... \\&= 3.728 \text{ (to 4 sig. figures)}\end{aligned}$$

**MARKER'S COMMENT:** Show answer to 5 or 6 decimals before rounding. Correct rounding of a wrong answer still receives half marks.

13. Functions, 2ADV F1 2013 HSC 11a

$$\begin{aligned}\ln 3 &= 1.09861... \\&= 1.10 \text{ (to 3 sig. figures)}\end{aligned}$$

14. Functions, 2ADV F1 2008 HSC 1a

$$\begin{aligned}2 \cos\left(\frac{\pi}{5}\right) &= 1.6180... \\&= 1.62 \text{ (3 sig)}\end{aligned}$$

15. Functions, 2ADV F1 2008 HSC 1e

$$\begin{aligned}(\sqrt{3}-1)(2\sqrt{3}+5) \\&= 2 \times 3 + 5\sqrt{3} - 2\sqrt{3} - 5 \\&= 1 + 3\sqrt{3}\end{aligned}$$

16. Functions, 2ADV F1 2021 HSC 11

$$\begin{aligned}x + \frac{x-1}{2} &= 9 \\2x + x - 1 &= 18 \\3x &= 19 \\x &= \frac{19}{3}\end{aligned}$$

17. Functions, 2ADV F1 SM-Bank 46

$$\begin{aligned}\frac{8-\sqrt{27}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} &= \frac{2\sqrt{3}(8-3\sqrt{3})}{(2\sqrt{3})^2} \\&= \frac{16\sqrt{3}-18}{12} \\&= -\frac{3}{2} + \frac{4}{3}\sqrt{3} \\\therefore a &= -\frac{3}{2}, b = \frac{4}{3}\end{aligned}$$

18. Functions, 2ADV F1 SM-Bank 47

$$\begin{aligned}\frac{\sqrt{32}-6}{3\sqrt{2}} \times \frac{3\sqrt{2}}{3\sqrt{2}} &= \frac{3\sqrt{2}(4\sqrt{2}-6)}{18} \\&= \frac{24-18\sqrt{2}}{18} \\&= \frac{4}{3} - \sqrt{2} \\\therefore a &= \frac{4}{3}, b = -1\end{aligned}$$

19. Functions, 2ADV F1 SM-Bank 51

$$\begin{aligned}\frac{\sqrt{3}-2}{2\sqrt{3}} &= \frac{\sqrt{3}-2}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} \\ &= \frac{2\sqrt{3}(\sqrt{3}-2)}{4 \times 3} \\ &= \frac{6-4\sqrt{3}}{12} \\ &= \frac{1}{2} - \frac{1}{3}\sqrt{3} \\ \therefore a &= \frac{1}{2}, b = -\frac{1}{3}\end{aligned}$$

20. Functions, 2ADV F1 SM-Bank 52

$$\begin{aligned}\frac{6\sqrt{3}-\sqrt{5}}{2\sqrt{5}} &= \frac{6\sqrt{3}-\sqrt{5}}{2\sqrt{5}} \times \frac{2\sqrt{5}}{2\sqrt{5}} \\ &= \frac{2\sqrt{5}(6\sqrt{3}-\sqrt{5})}{4 \times 5} \\ &= \frac{12\sqrt{15}-10}{20} \\ &= -\frac{1}{2} + \frac{3}{5}\sqrt{15} \\ \therefore a &= -\frac{1}{2}, b = \frac{3}{5}\end{aligned}$$

21. Functions, 2ADV F1 SM-Bank 54

$$\begin{aligned}\frac{1}{a} + \frac{1}{b} - \frac{c}{ab} &= \frac{b+a-c}{ab} \\ \therefore \text{Reciprocal} &= \frac{ab}{a+b-c}\end{aligned}$$

22. Functions, 2ADV F1 SM-Bank 56

$$\begin{aligned}\frac{9x^2}{x+3} \div \frac{3x}{x^2-9} &= \frac{9x^2}{x+3} \times \frac{x^2-9}{3x} \\ &= \frac{9x^2}{x+3} \times \frac{(x-3)(x+3)}{3x} \\ &= 3x(x-3)\end{aligned}$$

23. Functions, 2ADV F1 SM-Bank 55

$$\begin{aligned}\frac{4p-12p^2}{3} \times \frac{6p}{3p^2-p} &= \frac{4p(1-3p)}{3} \times \frac{6p}{p(3p-1)} \\ &= \frac{8p(1-3p)}{3p-1} \\ &= -8p\end{aligned}$$

24. Functions, 2ADV F1 EQ-Bank 22

i. In one hour:

Worker A picks  $\frac{1}{a}$  bucket.

Worker B picks  $\frac{1}{b}$  bucket.

$\therefore$  Fraction picked in 1 hour working together

$$\begin{aligned}&= \frac{1}{a} + \frac{1}{b} \\ &= \frac{a+b}{ab}\end{aligned}$$

ii. The reciprocal represents the number of hours it would take to fill one bucket, with A and B working together.

25. Functions, 2ADV F1 2004 HSC 1a

$$\begin{aligned}&3\,397\,000 \\ &= 3.397 \times 10^6 \\ &= 3.4 \times 10^6 \text{ m (2 sig figures)}\end{aligned}$$

**COMMENT:** Note that the question asks for "a fraction".

26. Functions, 2ADV F1 2004 HSC 1d

$$\begin{aligned}(3 - \sqrt{2})^2 &= 9 - 6\sqrt{2} + (\sqrt{2})^2 \\&= 9 - 6\sqrt{2} + 2 \\&= 11 - 6\sqrt{2}\end{aligned}$$

$$\therefore a = 11, b = 6$$

27. Functions, 2ADV F1 2008 HSC 1c

$$\begin{aligned}\frac{2}{n} - \frac{1}{n+1} \\&= \frac{2(n+1) - 1(n)}{n(n+1)} \\&= \frac{2n + 2 - n}{n(n+1)} \\&= \frac{n+2}{n(n+1)}\end{aligned}$$

28. Functions, 2ADV F1 EQ-Bank 21

$$\begin{aligned}\left(\frac{p}{q}\right)^3 \div (pq^{-2}) &= \frac{p^3}{q^3} \div \frac{p}{q^2} \\&= \frac{p^3}{q^3} \times \frac{q^2}{p} \\&= \frac{p^2}{q}\end{aligned}$$