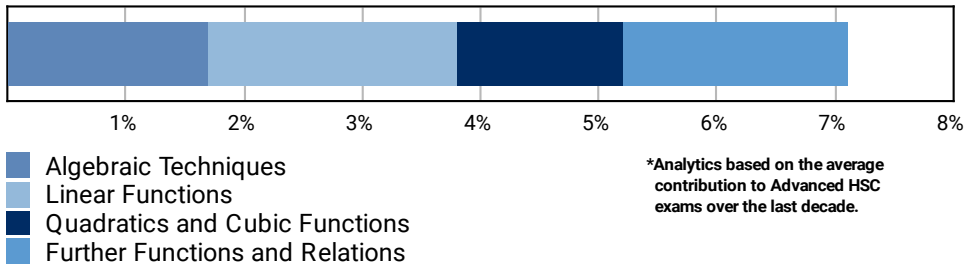
ADV: Functions (Adv), F1 Working with Functions (Adv)

Quadratics and Cubic Functions (Y11)
Further Functions and Relations (Y11)
Composite Functions (Y11)

Teacher: Troy McMurrich

Exam Equivalent Time: 180 minutes *(based on HSC allocation of 1.5 minutes approx. per mark)*

F1 Working With Functions



***Analytics based on the average contribution to Advanced HSC exams over the last decade.**

HISTORICAL CONTRIBUTION

- *F1 Working with Functions* is a Year 11 topic whose content represents the lowest of low hanging fruit in the new *Advanced* course.
- *F1 Working with Functions* includes new and a significant re-categorisation of old syllabus content. Any insights from past contributions in this topic area are unavoidably limited. However, with the information available, our analysis has it accounting for ~7.1%.
- We have split the topic into 5 categories for analysis purposes: *1-Algebraic Techniques, 2-Linear Functions, 3-Quadratics and Cubic Functions, 4-Composite Functions and 5-Further Functions and Relations.*
- This analysis looks at *Quadratics and Cubic Functions (1.4%).*

HSC ANALYSIS - What to expect and common pitfalls

- *Quadratic* factorisation has easily been the most common question style in this sub-topic, offering up easy marks in 5 exams within the last decade.
- Students have also been asked to solve quadratics using the general formula and to find the intersection of quadratic and linear equations.
- Cubic questions that fall within this sub-topic are rare, with the graphing of cubics typically requiring calculus - a question type covered within the *Calculus* topic.
- The graphic representation of an *odd function* was poorly answered in 2016 and should be reviewed.

Questions

1. Functions, 2ADV F1 2013 HSC 1 MC

   What are the solutions of $2x^2 - 5x - 1 = 0$?

   (A) $x = \dfrac{-5 \pm \sqrt{17}}{4}$

   (B) $x = \dfrac{5 \pm \sqrt{17}}{4}$

   (C) $x = \dfrac{-5 \pm \sqrt{33}}{4}$

   (D) $x = \dfrac{5 \pm \sqrt{33}}{4}$

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2. Functions, 2ADV F1 2014 HSC 6 MC

   Which expression is a factorisation of $8x^3 + 27$?

   (A) $(2x - 3)(4x^2 + 12x - 9)$

   (B) $(2x + 3)(4x^2 - 12x + 9)$

   (C) $(2x - 3)(4x^2 + 6x - 9)$

   (D) $(2x + 3)(4x^2 - 6x + 9)$

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3. Functions, 2ADV F1 2017 HSC 2 MC

   Which expression is equal to $3x^2 - x - 2$?

   (A) $(3x - 1)(x + 2)$

   (B) $(3x + 1)(x - 2)$

   (C) $(3x - 2)(x + 1)$

   (D) $(3x + 2)(x - 1)$

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4. Functions, 2ADV F1 2019 HSC 2 MC

What values of x satisfy $4 - 3x \leq 12$?

(A) $x \leq -\frac{16}{3}$

(B) $x \geq -\frac{16}{3}$

(C) $x \leq -\frac{8}{3}$

(D) $x \geq -\frac{8}{3}$

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5. Functions, 2ADV F1 SM-Bank 2 MC

Let $f(x)$ and $g(x)$ be functions such that $f(2) = 5$, $f(3) = 4$, $g(2) = 5$, $g(3) = 2$ and $g(4) = 1$.

The value of $f(g(3))$ is

A. 1

B. 2

C. 4

D. 5

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6. Functions, 2ADV F1 SM-Bank 5 MC

Let $g(x) = x^2 + 2x - 3$ and $f(x) = e^{2x+3}$.

Then $f(g(x))$ is given by

A. $e^{4x+6} + 2e^{2x+3} - 3$

B. $2x^2 + 4x - 6$

C. e^{2x^2+4x-3}

D. e^{2x^2+4x-6}

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7. Functions, 2ADV F1 SM-Bank 9 MC

If $f(x) = \frac{1}{2}e^{3x}$ and $g(x) = \log_e(2x) + 3$ then $g(f(x))$ is equal to

A. $3(x + 1)$

B. $e^{3x} + 3$

C. e^{8x+9}

D. $\log_e(3x) + 3$

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8. Functions, 2ADV F1 SM-Bank 13 MC

Which one of the following functions satisfies the functional equation $f(f(x)) = x$?

A. $f(x) = 2 - x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

---

9. Functions, 2ADV F1 2013 HSC 3 MC

Which inequality defines the domain of the function $f(x) = \frac{1}{\sqrt{x+3}}$?

(A) $x > -3$

(B) $x \geq -3$

(C) $x < -3$

(D) $x \leq -3$

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10. Functions, 2ADV F1 2020 HSC 1 MC

Which inequality gives the domain of $y = \sqrt{2x - 3}$?

A. $x < \frac{3}{2}$

B. $x > \frac{3}{2}$

C. $x \leq \frac{3}{2}$

D. $x \geq \frac{3}{2}$

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11. Functions, 2ADV F1 SM-Bank 8 MC

Let $f(x) = x^2$

Which one of the following is **not** true?

A. $f(xy) = f(x)f(y)$

B. $f(x) - f(-x) = 0$

C. $f(2x) = 4f(x)$

D. $f(x - y) = f(x) - f(y)$

---

12. Functions, 2ADV F1 SM-Bank 12 MC

If $f(x - 1) = x^2 - 2x + 3$, then $f(x)$ is equal to

A. $x^2 - 2$

B. $x^2 + 2$

C. $x^2 - 2x + 4$

D. $x^2 - 4x + 6$

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13. Functions, 2ADV F1 SM-Bank 15 MC

If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then $f(x)$ could equal

A. $\dfrac{x^2}{2}$

B. $\sqrt{2x}$

C. $2x$

D. $x - 2$

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14. Functions, 2ADV F1 SM-Bank 21 MC

A circle with centre $(a, -2)$ and radius 5 units has equation

$x^2 - 6x + y^2 + 4y = b$ where a and b are real constants.

The values of a and b are respectively

A. −3 and 38

B. 3 and 12

C. −3 and −8

D. 3 and 18

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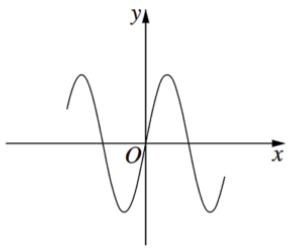
15. Functions, 2ADV F1 2016 HSC 4 MC

Which diagram shows the graph of an odd function?

(A)



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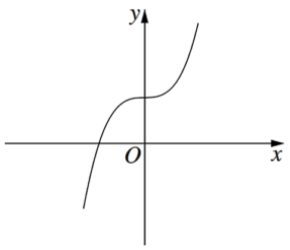

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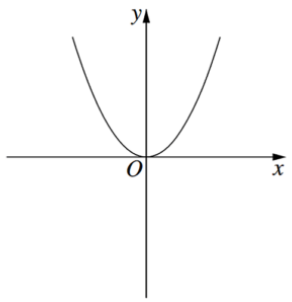
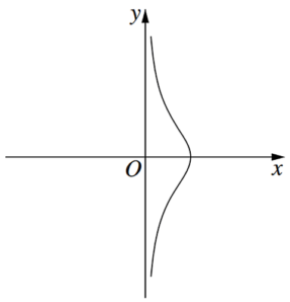


(D)



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16. Functions, 2ADV F1 SM-Bank 1 MC

Let $h(x) = \dfrac{1}{x - 1}$ for $-1 < h < 1$.

Which one of the following statements about h is **not** true?

A. $h(x)h(-x) = -h(x^2)$

B. $h(x) - h(0) = xh(x)$

C. $h(x) - h(-x) = 2xh(x^2)$

D. $(h(x))^2 = h(x^2)$

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17. Functions, 2ADV F1 SM-Bank 4 MC

The function $f(x)$ satisfies the functional equation $f(f(x)) = x$ for $\{x: \text{all } x,\ x \neq 1\}$.

The rule for the function is

A. $f(x) = x + 1$

B. $f(x) = x - 1$

C. $f(x) = \dfrac{x-1}{x+1}$

D. $f(x) = \dfrac{x+1}{x-1}$

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18. Functions, 2ADV F1 SM-Bank 6 MC

Let $f(x) = e^x + e^{-x}$.

$f(2u)$ is equal to

A. $f(u) + f(-u)$

B. $2f(u)$

C. $(f(u))^2 - 2$

D. $(f(u))^2 + 2$

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19. Functions, 2ADV F1 SM-Bank 14 MC

Let $g(x) = \log_2(x),\ \ x > 0$

Which one of the following equations is true for all positive real values of x?

A. $2g(8x) = g(x^2) + 8$

B. $2g(8x) = g(x^2) + 6$

C. $2g(8x) = (g(x) + 8)^2$

D. $2g(8x) = g(2x) + 6$

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20. Functions, 2ADV F1 2006 HSC 1b

Factorise $2x^2 + 5x - 3$. *(2 marks)*

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21. Functions, 2ADV F1 2007 HSC 1b

Solve $2x - 5 > -3$ and graph the solution on a number line. *(2 marks)*

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22. Functions, 2ADV F1 2007 HSC 1e

Factorise $2x^2 + 5x - 12$. *(2 marks)*

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23. Functions, 2ADV F1 2009 HSC 1c

Solve $|x + 1| = 5$. *(2 marks)*

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24. Functions, 2ADV F1 2010 HSC 1d

Solve $|2x + 3| = 9$. *(2 marks)*

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25. Functions, 2ADV F1 2016 HSC 11e

Find the points of intersection of $y = -5 - 4x$ and $y = 3 - 2x - x^2$. *(3 marks)*

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26. Functions, 2ADV F1 2006 HSC 1e

Solve $3 - 5x \leq 2$. *(2 marks)*

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27. Functions, 2ADV F1 2010 HSC 1a

Solve $x^2 = 4x$. *(2 marks)*

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28. Functions, 2ADV F1 2011 HSC 1b

Simplify $\dfrac{n^2 - 25}{n - 5}$. *(1 mark)*

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29. Functions, 2ADV F1 2011 HSC 1e

Solve $2 - 3x \leq 8$. *(2 marks)*

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30. Functions, 2ADV F1 2012 HSC 11a

Factorise $2x^2 - 7x + 3$ *(2 marks)*

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31. Functions, 2ADV F1 2014 HSC 11b

Factorise $3x^2 + x - 2$. *(2 marks)*

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32. Functions, 2ADV F1 2015 HSC 11b

Factorise fully $3x^2 - 27$. *(2 marks)*

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33. Functions, 2ADV F1 2017 HSC 11g

Solve $|3x-1|=2$. *(2 marks)*

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34. Functions, 2ADV F1 2018 HSC 11b

Solve $1-3x>10$. *(2 marks)*

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35. Functions, 2ADV F1 2008 HSC 1d

Solve $|4x-3|=7$. *(2 marks)*

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36. Functions, 2ADV F1 SM-Bank 10

Let $f(x)=x^2+1$ and $g(x)=2x+1$. Write down the rule of $f(g(x))$. *(1 mark)*

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37. Functions, 2ADV F1 SM-Bank 33

i. State the domain and range of $y=-\sqrt{12-x^2}$. *(2 marks)*

ii. Sketch the graph. *(1 mark)*

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38. Functions, 2ADV F1 SM-Bank 41

Find the values of x for which $|x+1|=5$. *(2 marks)*

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39. Functions, 2ADV F1 SM-Bank 42

Find the values of x for which $|x-3|=1$. *(2 marks)*

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40. Functions, 2ADV F1 SM-Bank 44

Solve $|x-2|=3$. *(2 marks)*

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41. Functions, 2ADV F1 SM-Bank 37

Find all values of x for which $|x-4|=\frac{x}{2}+7$. *(3 marks)*

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42. Functions, 2ADV F1 2016 HSC 11a

Sketch the graph of $(x-3)^2+(y+2)^2=4$. *(2 marks)*

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43. Functions, 2ADV F1 2019 MET1-N 2

Let $f(x)=-x^2+x+4$ and $g(x)=x^2-2$.

a. Find $g(f(3))$. *(2 marks)*

b. Express $f(g(x))$ in the form ax^4+bx^2+c, where a, b and c are non-zero integers. *(2 marks)*

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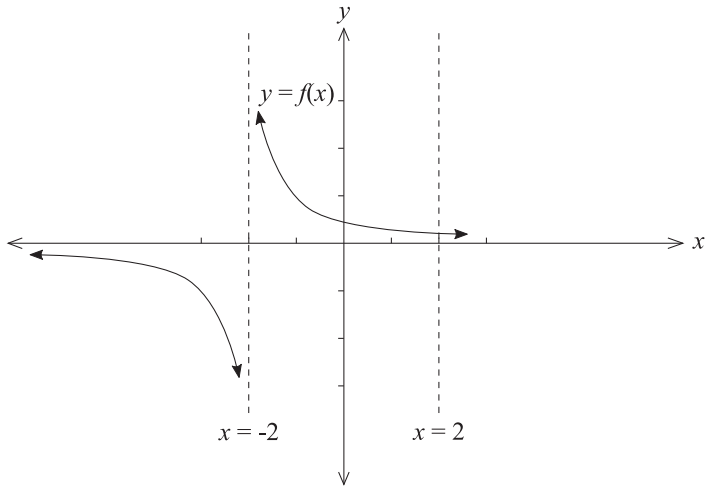
44. Functions, 2ADV F1 SM-Bank 36

Consider the function $f(x)=\frac{1}{x+2}$.



i. Sketch the graph $y=f(-x)$. *(2 marks)*

ii. On the same graph, sketch $y=-f(x)$. *(2 marks)*

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45. Functions, 2ADV F1 SM-Bank 30

Given $f(x)=\sqrt{x}$ and $g(x)=25-x^2$

i. Find $g(f(x))$. *(1 mark)*

ii. Find the domain and range of $f(g(x))$. *(2 marks)*

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46. Functions, 2ADV F1 2019 HSC 13e

i. Sketch the graph of $y=|x-1|$ for $-4\leq x\leq 4$. *(1 mark)*

ii. Using the sketch from part i, or otherwise, solve $|x-1|=2x+4$. *(2 marks)*

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47. Functions, 2ADV F1 EQ-Bank 11

Given the function $f(x) = \sqrt{3 - x}$ and $g(x) = x^2 - 2$, sketch $y = g(f(x))$ over its natural domain. ***(2 marks)***

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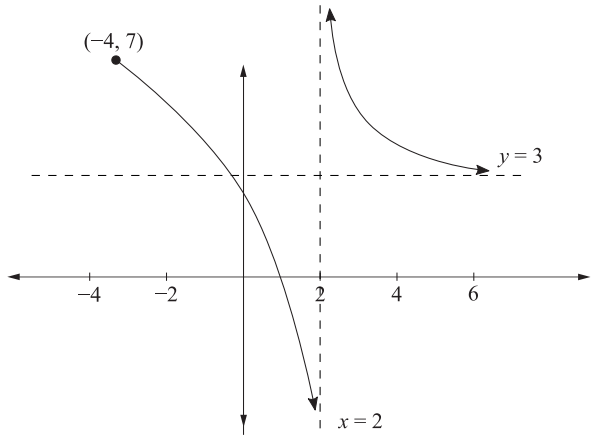
48. Functions, 2ADV F1 EQ-Bank 6

The graph of $f(x)$ is shown below. It has asymptotes at $y = 3$ and $x = 2$.



Using interval notation, state the domain and range of $f(x)$. ***(2 marks)***

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49. Functions, 2ADV F1 EQ-Bank 7

The current of an electrical circuit, measured in amps (*A*), varies inversely with its resistance, measured in ohms (*R*).

When the resistance of a circuit is 28 ohms, the current is 3 amps.

What is the current when the resistance is 8 ohms? ***(2 marks)***

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50. Functions, 2ADV F1 EQ-Bank 8

Jacques is a marine biologist and finds that the mass of a crab is directly proportional to the cube of the diameter of its shell.

If a crab with a shell diameter of 15 cm weighs 680 grams, what will be the diameter of a crab that weighs 1.1 kilograms? Give your answer to 1 decimal place. ***(2 marks)***

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51. Functions, 2ADV F1 SM-Bank 23

Find the values of k for which the expression $x^2 - 3x + (4 - 2k)$ is always positive. ***(3 marks)***

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52. Functions, 2ADV F1 EQ-Bank 26

Fuifui finds that for Giant moray eels, the mass of an eel is directly proportional to the cube of its length.

An eel of this species has a length of 25 cm and a mass of 4350 grams.

What is the expected length of a Giant moray eel with a mass of 6.2 kg? Give your answer to one decimal place. ***(3 marks)***

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53. Functions, 2ADV F1 EQ-Bank 27

The stopping distance of a car on a certain road, once the brakes are applied, is directly proportional to the square of the speed of the car when the brakes are first applied.

A car travelling at 70 km/h takes 58.8 metres to stop.

How far does it take to stop if it is travelling at 105 km/h? ***(3 marks)***

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54. Functions, 2ADV F1 SM-Bank 32

Find the centre and radius of the circle with the equation

$$x^2 - 12x + y^2 + 2y - 12 = 0$$ ***(2 marks)***

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55. Functions, 2ADV F1 SM-Bank 31

Find the domain and range of $f(g(x))$ given

$f(x) = 2x^2 - 8x$ and $g(x) = x + 2$. ***(2 marks)***

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56. Functions, 2ADV F1 2010 HSC 1c

Write down the equation of the circle with centre (−1, 2) and radius 5. ***(1 mark)***

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57. Functions, 2ADV F1 2010 HSC 1g

Let $f(x) = \sqrt{x - 8}$. What is the domain of $f(x)$? ***(1 mark)***

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58. Functions, 2ADV F1 2017 HSC 11h

Find the domain of the function $f(x) = \sqrt{3 - x}$. ***(2 marks)***

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59. Functions, 2ADV F1 SM-Bank 3

Let $f(x) = \sqrt{x+1}$ for $x \geq 0$

i. State the range of $f(x)$. *(1 mark)*

ii. Let $g(x) = x^2 + 4x + 3$, where $x \leq c$ and $c \leq 0$.

Find the largest possible value of c such that the range of g(x) is a subset of the domain of $f(x)$. *(2 marks)*

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60. Functions, 2ADV F1 SM-Bank 7

Let $f(x) = \log_e(x)$ for $x > 0$, and $g(x) = x^2 + 1$ for all x.

i. Find $h(x)$, where $h(x) = f(g(x))$. *(1 mark)*

ii. State the domain and range of $h(x)$. *(2 marks)*

iii. Show that $h(x) + h(-x) = f\left((g(x))^2\right)$. *(2 marks)*

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61. Functions, 2ADV F1 2020 HSC 24

The circle of $x^2 - 6x + y^2 + 4y - 3 = 0$ is reflected in the x-axis.

Sketch the reflected circle, showing the coordinates of the centre and the radius. *(3 marks)*

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62. Functions, 2ADV F1 SM-Bank 11

Given $f(x) = \sqrt{x^2 - 9}$ and $g(x) = x + 5$

a. Find integers c and d such that $f(g(x)) = \sqrt{(x+c)(x+d)}$ *(2 marks)*

b. State the domain for which $f(g(x))$ is defined. *(2 marks)*

Worked Solutions

1. Functions, 2ADV F1 2013 HSC 1 MC

$2x^2 - 5x - 1 = 0$

$\text{Using } x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times (-1)}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25+8}}{4}$$

$$= \frac{5 \pm \sqrt{33}}{4}$$

$\Rightarrow D$

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2. Functions, 2ADV F1 2014 HSC 6 MC

$$8x^3 + 27$$
$$= (2x)^3 + 3^3$$
$$= (2x+3)(4x^2 - 6x + 9)$$

$\Rightarrow D$

COMMENT: Factorising a cubic is only examinable with scaffolding, as provided here by expanding the answer options.

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3. Functions, 2ADV F1 2017 HSC 2 MC

$$3x^2 - x - 2$$
$$= (3x+2)(x-1)$$

$\Rightarrow D$

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4. Functions, 2ADV F1 2019 HSC 2 MC

$$4 - 3x \leq 12$$
$$-3x \leq 8$$
$$x \geq -\frac{8}{3}$$

$\Rightarrow D$

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5. Functions, 2ADV F1 SM-Bank 2 MC

$$\begin{aligned} f(g(3)) &= f(2) \\ &= 5 \end{aligned}$$

$\Rightarrow D$

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6. Functions, 2ADV F1 SM-Bank 5 MC

By trial and error,

Consider: $f(x) = e^{2x^2+4x-3}$

$$\begin{aligned} f(g(x)) &= e^{2(x^2+2x-3)+3} \\ &= e^{2x^2+4x-3} \end{aligned}$$

$\Rightarrow C$

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7. Functions, 2ADV F1 SM-Bank 9 MC

$$\begin{aligned} g(f(x)) &= \log_e\left(2 \times \frac{1}{2}e^{3x}\right) + 3 \\ &= \log_e e^{3x} + 3 \\ &= 3x + 3 \\ &= 3(x+1) \end{aligned}$$

$\Rightarrow A$

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8. Functions, 2ADV F1 SM-Bank 13 MC

By trial and error,

Consider: $f(x) = 2 - x$

$$\begin{aligned} f(f(x)) &= 2 - (2 - x) \\ &= x \end{aligned}$$

$\Rightarrow A$

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9. Functions, 2ADV F1 2013 HSC 3 MC

Given $f(x) = \dfrac{1}{\sqrt{x+3}}$

We know $(x+3) > 0$

$$x > -3$$

$\therefore$ **The domain of** $f(x)$ **is** $f(x) > -3$

$\Rightarrow A$

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10. Functions, 2ADV F1 2020 HSC 1 MC

Domain exists when:

$$\begin{aligned} 2x - 3 &\geq 0 \\ 2x &\geq 3 \\ x &\geq \frac{3}{2} \end{aligned}$$

$\Rightarrow D$

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11. Functions, 2ADV F1 SM-Bank 8 MC

By trial and error,

Consider option D**:**

$$\begin{aligned} f(x-y) &= (x-y)^2 \\ &= x^2 - 2xy + y^2 \\ f(x) - f(y) &= x^2 - y^2 \end{aligned}$$

$\therefore f(x-y) \neq f(x) - f(y)$

$\Rightarrow D$

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12. Functions, 2ADV F1 SM-Bank 12 MC

Let $g(x) = f(x-1)$

$g(x+1) = f(x)$

$$\begin{aligned} g(x+1) &= (x+1)^2 - 2(x+1) + 3 \\ &= x^2 + 2x + 1 - 2x - 2 + 3 \\ &= x^2 + 2 \end{aligned}$$

$\Rightarrow \ B$

13. Functions, 2ADV F1 SM-Bank 15 MC

We need $f(2x) = 2\,f(x)$,

Consider C,

$f(x) = 2x$,

$$\begin{aligned} f(2x) &= 2(2x) \\ &= 2\,f(x) \end{aligned}$$

$\Rightarrow \ C$

14. Functions, 2ADV F1 SM-Bank 21 MC

$x^2 - 6x + y^2 + 4y = b$

Completing the squares:

$$\begin{aligned} x^2 - 6x + 3^2 - 9 + y^2 + 4y + 2^2 - 4 &= b \\ (x-3)^2 + (y+2)^2 - 13 &= b \\ (x-3)^2 + (y+2)^2 &= b + 13 \end{aligned}$$

$\therefore a = 3$

$\therefore b + 13 = 25 \ \Rightarrow \ b = 12$

$\Rightarrow B$

15. Functions, 2ADV F1 2016 HSC 4 MC

♦ Mean mark 38%.

Odd functions occur when:

$f(x) = -f(x)$

Graphically, this occurs when a function has symmetry when rotated 180° about the origin.

$\Rightarrow \ A$

16. Functions, 2ADV F1 SM-Bank 1 MC

By trial and error, consider option E:

♦ Mean mark 46%.

$$h(x) = \frac{1}{x-1}$$

$$(h(x))^2 = \frac{1}{(x-1)^2} = \frac{1}{x^2 - 2x + 1}$$

$$h(x^2) = \frac{1}{x^2 - 1} \neq (h(x))^2$$

$\Rightarrow D$

17. Functions, 2ADV F1 SM-Bank 4 MC

By trial and error:

♦ Mean mark 47%.

Consider $f(x) = \dfrac{x+1}{x-1}$

$$\begin{aligned} f(f(x)) &= \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} \\ &= \frac{x+1+x-1}{x+1-x+1} \\ &= x \end{aligned}$$

$\Rightarrow D$

18. Functions, 2ADV F1 SM-Bank 6 MC

By trial and error,

♦ Mean mark 44%.

Consider $(f(u))^2 - 2$:

$$f(2u) = e^{2u} + e^{-2u}$$

$$\begin{aligned} (f(u))^2 &= \left(e^u + e^{-u}\right)^2 \\ &= e^{2u} + 2 + e^{-2u} \end{aligned}$$

$\therefore f(2u) = (f(u))^2 - 2$

$\Rightarrow C$

19. Functions, 2ADV F1 SM-Bank 14 MC

♦♦ Mean mark 35%.

Consider Option B:

$$\begin{aligned}\text{LHS} &= 2g(8x)\\ &= 2\log_2(8x)\\ &= 2\log_2(8) + 2\log_2(x)\\ &= 2\log_2\left(2^3\right) + 2\log_2(x)\\ &= 6 + \log_2\left(x^2\right)\\ &= g\left(x^2\right) + 6\end{aligned}$$

$\Rightarrow B$

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20. Functions, 2ADV F1 2006 HSC 1b

$$\begin{aligned}&2x^2 + 5x - 3\\ &= (2x-1)(x+3)\end{aligned}$$

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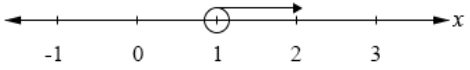
21. Functions, 2ADV F1 2007 HSC 1b

$$\begin{aligned}2x - 5 &> -3\\ 2x &> 2\\ x &> 1\end{aligned}$$



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22. Functions, 2ADV F1 2007 HSC 1e

$$\begin{aligned}&2x^2 + 5x - 12\\ &= (2x-3)(x+4)\end{aligned}$$

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23. Functions, 2ADV F1 2009 HSC 1c

$$|x+1| = 5$$

$$\begin{aligned}(x+1) = 5 \quad & -(x+1) = 5\\ x = 4 \quad & -x - 1 = 5\\ & x = -6\end{aligned}$$

$\therefore x = 4 \text{ or } -6$

---

24. Functions, 2ADV F1 2010 HSC 1d

$$|2x+3| = 9$$

$$\begin{aligned}2x + 3 = 9 \quad & -(2x+3) = 9\\ 2x = 6 \quad & -2x - 3 = 9\\ x = 3 \quad & -2x = 12\\ & x = -6\end{aligned}$$

$\therefore x = 3 \text{ or } -6$

---

25. Functions, 2ADV F1 2016 HSC 11e

$y = 3 - 2x - x^2$

Substitute $y = -5 - 4x$ into equation

$$-5 - 4x = 3 - 2x - x^2$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$\therefore x = 4 \text{ or } -2$

When $x = 4$, $y = -5 - 4(4) = -21$

When $x = -2$, $y = -5 - 4(-2) = 3$

$\therefore$ Intersection at $(4, -21)$ and $(-2, 3)$

---

26. Functions, 2ADV F1 2006 HSC 1e

$$\begin{aligned}3 - 5x &\leq 2\\ -5x &\leq -1\\ x &\geq \frac{1}{5}\end{aligned}$$

---

27. Functions, 2ADV F1 2010 HSC 1a

$$
\begin{aligned}
x^2 &= 4x \\
x^2 - 4x &= 0 \\
x(x-4) &= 0
\end{aligned}
$$

$\therefore\ x = 0 \text{ or } 4$

---

28. Functions, 2ADV F1 2011 HSC 1b

$$
\begin{aligned}
\frac{n^2 - 25}{n-5} &= \frac{(n-5)(n+5)}{n-5} \\
&= n + 5
\end{aligned}
$$

---

29. Functions, 2ADV F1 2011 HSC 1e

$$
\begin{aligned}
2 - 3x &\leq 8 \\
-3x &\leq 6 \\
x &\geq -\frac{6}{3} \\
x &\geq -2
\end{aligned}
$$

---

30. Functions, 2ADV F1 2012 HSC 11a

$$
\begin{aligned}
&2x^2 - 7x + 3 \\
&= (2x-1)(x-3)
\end{aligned}
$$

STRATEGY: Check your answer by expanding factors.

---

31. Functions, 2ADV F1 2014 HSC 11b

$$
\begin{aligned}
&3x^2 + x - 2 \\
&= (3x-2)(x+1)
\end{aligned}
$$

---

32. Functions, 2ADV F1 2015 HSC 11b

$$
\begin{aligned}
3x^2 - 27 &= 3\left(x^2 - 9\right) \\
&= 3(x+3)(x-3)
\end{aligned}
$$

---

33. Functions, 2ADV F1 2017 HSC 11g

$$
\begin{aligned}
&|3x - 1| = 2 \\
&3x - 1 = 2 \quad \text{or} \quad -(3x-1) = 2 \\
&\quad x = 1 \qquad\qquad -3x = 1 \\
&\qquad\qquad\qquad x = -\frac{1}{3}
\end{aligned}
$$

$\therefore\ x = 1 \text{ or } -\frac{1}{3}$

MARKER'S COMMENT: Note that both conditions must be satisfied! Dealing with negative signs and division for inequalities produced many errors.

---

34. Functions, 2ADV F1 2018 HSC 11b

$$
\begin{aligned}
1 - 3x &> 10 \\
-3x &> 9 \\
x &< -3
\end{aligned}
$$

---

35. Functions, 2ADV F1 2008 HSC 1d

$$
\begin{aligned}
&|\,4x - 3\,| = 7 \\
&4x - 3 = 7 \quad -(4x-3) = 7 \\
&\quad 4x = 10 \quad -4x + 3 = 7 \\
&\quad x = \frac{5}{2} \qquad -4x = 4 \\
&\qquad\qquad x = -1
\end{aligned}
$$

$\therefore x = \frac{5}{2} \text{ or } -1$

---

36. Functions, 2ADV F1 SM-Bank 10

$$
\begin{aligned}
f(g(x)) &= f(2x+1) \\
&= (2x+1)^2 + 1
\end{aligned}
$$

---

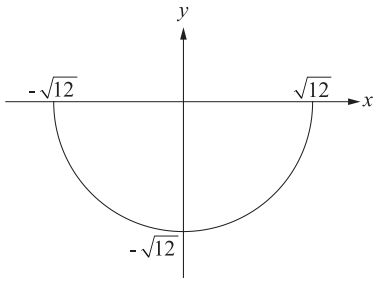
37. Functions, 2ADV F1 SM-Bank 33

i. $y = -\sqrt{12 - x^2}$

Domain: $-\sqrt{12} \leq x \leq \sqrt{12}$

Range: $-\sqrt{12} \leq y \leq 0$

ii.



38. Functions, 2ADV F1 SM-Bank 41

$|x + 1| = 5$

$(x + 1)^2 = 5^2$

$x^2 + 2x + 1 = 25$

$x^2 + 2x - 24 = 0$

$(x + 6)(x - 4) = 0$

$\therefore x = 4 \text{ or } -6$

39. Functions, 2ADV F1 SM-Bank 42

$|x - 3| = 1$

Method 1

$(x - 3)^2 = 1$

$x^2 - 6x + 9 = 1$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$\therefore x = 2 \text{ or } 4$

Method 2

$(x - 3) = 1$ $\quad -(x - 3) = 1$

$x = 4$ $\quad -x + 3 = 1$

$x = -2$

40. Functions, 2ADV F1 SM-Bank 44

$|x - 2| = 3$

$(x - 2)^2 = 3^2$

$(x^2 - 4x + 4) = 9$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$\therefore x = -1 \text{ or } 5$

41. Functions, 2ADV F1 SM-Bank 37

$x - 4 = \frac{x}{2} + 7$ $\quad \text{or} \quad$ $-(x - 4) = \frac{x}{2} + 7$

$2x - 8 = x + 14$ $\quad -2x + 8 = x + 14$

$x = 22$ $\quad 3x = -6$

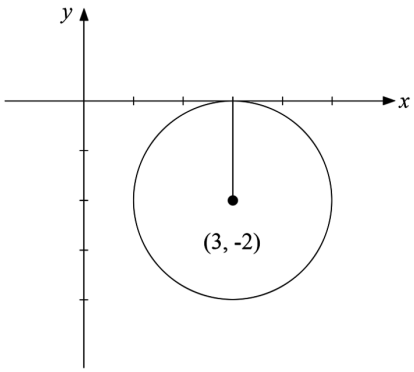
$x = -2$

$\therefore x = 22 \text{ or } -2$

42. Functions, 2ADV F1 2016 HSC 11a

$(x-3)^2+(y+2)^2=4$ is a circle,

centre $(3,\ -2)$, radius 2.



43. Functions, 2ADV F1 2019 MET1-N 2

a. $f(3) = -3^2+3+4$

$= -2$

$g(f(3)) = g(-2)$

$= (-2)^2-2$

$= 2$

b. $f(g(x)) = -\left(x^2-2\right)^2+\left(x^2-2\right)+4$

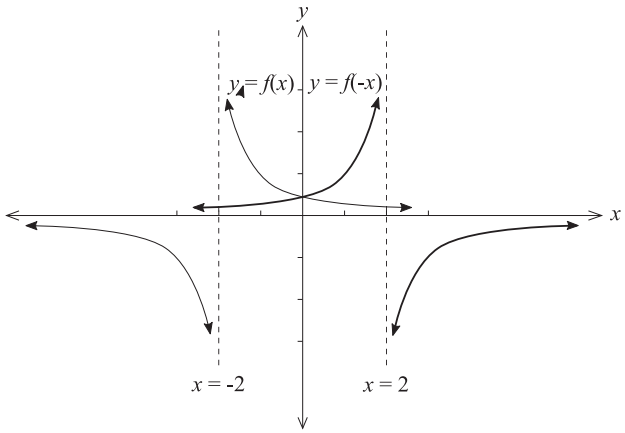
$= -\left(x^4-4x^2+4\right)+x^2+2$

$= -x^4+5x^2-2$

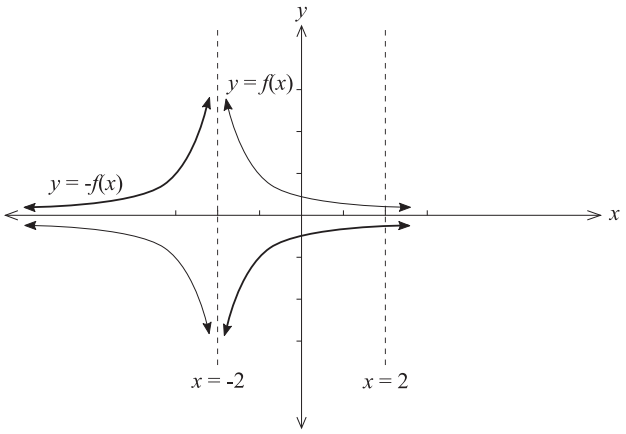
44. Functions, 2ADV F1 SM-Bank 36

i. Sketch $y=\dfrac{1}{x+2}$

$y=f(-x) \Rightarrow$ reflect $y=\dfrac{1}{x+2}$ in the y-axis.



ii. $y=-f(x) \Rightarrow$ reflect $y=\dfrac{1}{x+2}$ in the x-axis.



45. Functions, 2ADV F1 SM-Bank 30

i. $g(f(x)) = 25 - (f(x))^2$

$= 25 - (\sqrt{x})^2$

$= 25 - x$

ii. $f(g(x)) = \sqrt{g(x)}$

$= \sqrt{25 - x^2}$

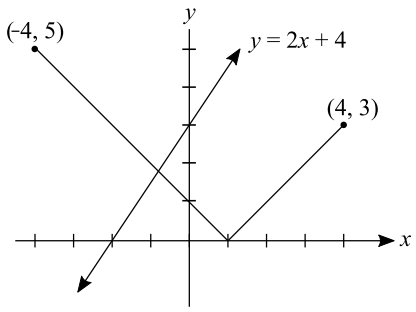
$\therefore$ **Domain:** $-5 \leq x \leq 5$

$\therefore$ **Range:** $0 \leq y \leq 5$

---

46. Functions, 2ADV F1 2019 HSC 13e

(i)



ii. **By inspection, intersection when** $x = -1$

Test:

$|-1-1| = -2+4$

$2 = 2$

$\therefore$ **Intersection at** $(-1, 2)$

---

47. Functions, 2ADV F1 EQ-Bank 11

$g(x) = x^2 - 2, \quad f(x) = \sqrt{3-x}$

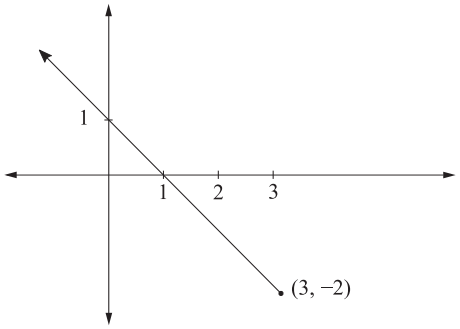
$g(f(x)) = \left(\sqrt{3-x}\right)^2 - 2$

$= 3 - x - 2$

$= 1 - x$

Since $f(x) = \sqrt{3-x}$,

$\Rightarrow$ **Domain:** $x \leq 3$



---

48. Functions, 2ADV F1 EQ-Bank 6

Domain: $[-4, 2) \cup (2, \infty)$

Range: $(-\infty, \infty)$

---

49. Functions, 2ADV F1 EQ-Bank 7

$$A \propto \frac{1}{R}$$

$$A = \frac{k}{R}$$

When $A = 3,\ R = 28$

$$3 = \frac{k}{28}$$

$$k = 84$$

Find A *when* $R = 8$:

$$A = \frac{84}{8}$$

$$= 10.5$$

---

50. Functions, 2ADV F1 EQ-Bank 8

$$M \propto d^3$$

$$M = kd^3$$

When $M = 680,\ d = 15$

$$680 = k \times 15^3$$

$$k = 0.201481...$$

Find d when $M = 1100$:

$$1100 = 0.20148... \times d^3$$

$$d = \sqrt[3]{\frac{1100}{0.20148...}}$$

$$= 17.608...$$

$$= 17.6 \text{ cm } \text{(to 1 d.p.)}$$

---

51. Functions, 2ADV F1 SM-Bank 23

$x^2 - 3x + (4 - 2k) > 0$

$x^2 - 3x + (4 - 2k) = 0$ is a concave up parabola

$\Rightarrow$ Always positive (no roots) if $\Delta < 0$

$$b^2 - 4ac < 0$$

$$(-3)^2 - 4 \cdot 1 \cdot (4 - 2k) < 0$$

$$9 - 16 + 8k < 0$$

$$8k < 7$$

$$k < \frac{7}{8}$$

---

52. Functions, 2ADV F1 EQ-Bank 26

$\text{Mass} \propto \text{length}^3$

$m = kl^3$

Find k:

$$4350 = k \times 25^3$$

$$k = \frac{4350}{25^3}$$

$$= 0.2784$$

Find l when $m = 6200$:

$$6200 = 0.2784 \times l^3$$

$$l^3 = \frac{6200}{0.2784}$$

$$\therefore l = 28.13...$$

$$= 28.1 \text{ cm } \text{(to 1 d.p.)}$$

---

53. Functions, 2ADV F1 EQ-Bank 27

Let $d =$ stopping distance

$d \propto s^2$

$d = ks^2$

Find k,

$58.8 = k \times 70^2$

$k = \dfrac{58.8}{70^2}$

$= 0.012$

Find d when $s = 105$:

$d = 0.012 \times 105^2$

$= 132.3$ metres

---

54. Functions, 2ADV F1 SM-Bank 32

$$x^2 - 12x + y^2 + 2y - 12 = 0$$

$$(x-6)^2 + (y+1)^2 - 36 - 1 - 12 = 0$$

$$(x-6)^2 + (y+1)^2 = 49$$

$\therefore$ Centre $(6, -1)$

$\therefore$ Radius $= 7$

---

55. Functions, 2ADV F1 SM-Bank 31

$$\begin{aligned} f(g(x)) &= 2(x+2)^2 - 8(x+2) \\ &= 2\left(x^2 + 4x + 4\right) - 8x - 16 \\ &= 2x^2 + 8x + 8 - 8x - 16 \\ &= 2\left(x^2 - 4\right) \end{aligned}$$

$\therefore$ Domain: all x

$\therefore$ Range: $-8 \leq y < \infty$

---

56. Functions, 2ADV F1 2010 HSC 1c

Circle with centre $(-1, 2)$, $r = 5$

$(x+1)^2 + (y-2)^2 = 25$

MARKER'S COMMENT: Expanding this equation is not necessary!

---

57. Functions, 2ADV F1 2010 HSC 1g

$f(x) = \sqrt{x - 8}$

Domain where

$(x - 8) \geq 0$

$x \geq 8$

♦ *Mean mark 49%.*
MARKER'S COMMENT: $x > 8$ was a common incorrect answer.

---

58. Functions, 2ADV F1 2017 HSC 11h

Solution 1

Domain of $f(x) = \sqrt{3 - x}$

$3 - x \geq 0$

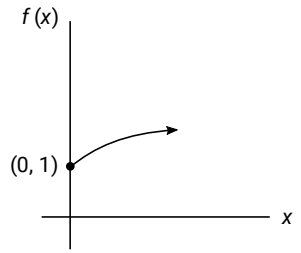
$x \leq 3$

Note domain can also be expressed as: $(-\infty, 3]$

---

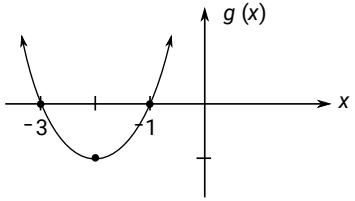
59. Functions, 2ADV F1 SM-Bank 3

i. **Sketch of $f(x)$:**



$\therefore$ **Range:** $y \geq 1$

ii. **Sketch** $g(x) = (x+1)(x+3)$



Domain of $f(x)$**:** $x \geq 0$

Find domain of $g(x)$ **such that range** $g(x)$**:** $y \geq 0$

Graphically, this occurs when $g(x)$ **has domain:**

$x \leq -3$ **and** $x \geq -1$

$\therefore c = -3$

---

60. Functions, 2ADV F1 SM-Bank 7

i. $h(x) = f\left(x^2+1\right)$

$= \log_e\left(x^2+1\right)$

ii. **Domain** (h) $=$ **Domain** (g)**: all** x

$\Rightarrow x^2+1 \geq 1$

$\Rightarrow \log_e\left(x^2+1\right) \geq 0$

$\therefore$ **Range** $h(x)$**:** $h \geq 0$

♦♦ *Mean mark part (a)(ii) 30%.*

iii. $\text{LHS} = h(x) + h(-x)$

$= \log_e\left(x^2 \_ 1\right) + \log_e\left((-x)^2+1\right)$

$= \log_e\left(x^2+1\right) + \log_e\left(x^2+1\right)$

$= 2\log_e\left(x^2+1\right)$

$\text{RHS} = f\left(\left(x^2+1\right)^2\right)$

$= 2\log_e\left(x^2+1\right)$

$\therefore h(x) + h(-x) = f\left(\left(g(x)\right)^2\right)$ ... as required

MARKER'S COMMENT: Many students were unsure of how to present their working in this question. Note the layout in the solution.

61. Functions, 2ADV F1 2020 HSC 24

$$x^2 - 6x + y^2 + 4y - 3 = 0$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 - 16 = 0$$

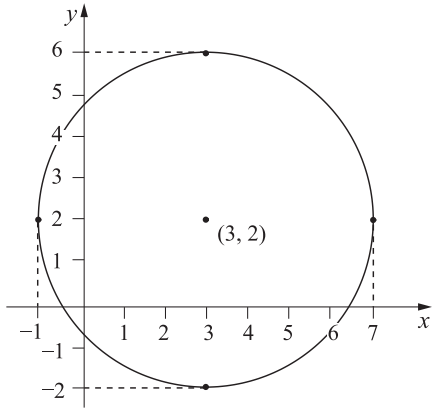
$$(x-3)^2 + (y+2)^2 = 16$$

$\Rightarrow$ **Original circle has centre $(3, -2)$, radius $= 4$**

Reflect in x-axis:

Centre $(3, -2) \to (3, 2)$

♦ *Mean mark 48%.*



62. Functions, 2ADV F1 SM-Bank 11

a. $f(g(x)) = \sqrt{(x+5)^2 - 9}$

$$= \sqrt{x^2 + 10x + 16}$$

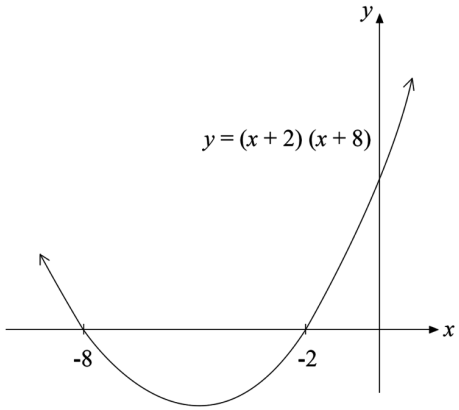
$$= \sqrt{(x+2)(x+8)}$$

$\therefore c = 2, d = 8$ or $c = 8, d = 2$

b. **Find x such that:**

$(x+2)(x+8) \geq 0$

♦♦♦ *Mean mark 13%.*
MARKER'S COMMENT: "Very poorly answered" with a common response of $-3 \leq x \leq 3$ that ignored the information from part (a).



$(x+2)(x+8) \geq 0$ **when**

$x \leq -8$ or $x \geq -2$

$\therefore$ **Domain:** $x \leq -8$ **and** $x \geq -2$