Replacement questions
with content from the most up-to-date syllabus

Marks

Question 1 (2 marks)

(b) The weight of a person varies inversely as the square of the distance from the centre of the earth.

If the radius of the earth is 6400 km, how much would a 72-kg person weigh 100 km above the surface of the earth.

Leave your answer to the nearest kilogram. **2**

Question 3 (8 marks)

(a)* Lara deposited \$1200 at the end of each year in an annuity earning 3.5% per annum, compounded annually. She completed a table to show the amount in the account at the end of each year.

Year	Balance at beginning of year	Interest on balance	Annual deposit	Total at end of year
1	0.00	0.00	1200.00	1200.00
2	1200.00	42.00	1200.00	2442.00
3	2442.00		1200.00	**A**
4				
5				
6				
7				
8				**B**

(i) Find the value of **A**. **1**

(ii) Use a geometric series to find the value of **B**. **3**

(a)** The continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} ke^{-\frac{x}{2}} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

Marks

(i) Show that $k = \frac{1}{2}$. **2**

(ii) What is the probability that the random variable X has a value that lies between $\frac{1}{3}$ and $\frac{1}{2}$? Leave your answer in terms of e. **2**

Question 4 (8 marks)

(a) A set of data has a lower quartile (Q_L) of 12 and an upper quartile (Q_U) of 20.

What is the maximum possible range for this set of data if there are no outliers? **2**

(c)* The discrete random variable X has the following probability distribution.

x	0	1	2	3	8
$P(X = x)$	p	$\frac{2}{5}$	$\frac{3}{20}$	$\frac{1}{4}$	p

(i) Show that $p = \frac{1}{10}$. **1**

(ii) Let μ be the mean of X.

What is $P(X < \mu)$? **2**

(c)** For events A and B, $P(A|B) = \frac{2}{5}$ and $P(B|A) = \frac{1}{3}$.

Let $P(A \cap B) = p$.

(i) Show that $P(A) = 3p$. **1**

(ii) Find $P(A' \cap B')$ in terms of p. **2**

Question 6 (6 marks)

(b) (iv) By using the trapezoidal rule with five function values, estimate the distance travelled by the particle between $t = 0$ and $t = 8$. **3**

(c) A container, which holds 20 litres when full, is drained so that at time t seconds the volume of water V, in litres, in the container is given by $V = 20\left(1 - \frac{t}{60}\right)^2$, for $0 \le t \le 60$.

(i) After how many seconds was the cooler one-quarter full? **1**

(ii) At what rate was the water draining out when the cooler was one-quarter full? **2**

Marks

Question 8 (4 marks)

(b) Claudia wants to invest \$65 000 for a total of 6 years. She has three investment options.

Option A – simple interest is paid at the rate of 5% per annum.

Option B – compound interest is paid at a rate of 4.8% per annum, compounded quarterly.

Option C – compound interest is paid at a rate of 4.2% per annum, compounded monthly.

Determine Claudia's best investment option. Support your answer with calculations. **4**

Solutions to replacement questions

QUESTION 1

(b) Let W = weight and r = radius:

$$W = \frac{k}{r^2}$$

Substitute $W = 72, r = 6400$:

$$72 = \frac{k}{6400^2}$$

$$k = 2\,949\,120\,000$$

$$W = \frac{2\ 949\ 120\ 000}{r^2}$$

Substitute $r = 6500$:

$$W = \frac{2\ 949\ 120\ 000}{6500^2}$$

$$W = 69.801\,6568\ldots$$

$$= 70 \text{ (nearest whole)}$$

The person would weigh 70 kg.

QUESTION 3

(a)* (i) $A = 2442 \times 1.035 + 1200$

$= 3727.47$

(ii) $B = 1200 + 1200 \times 1.035 + 1200 \times 1.035^2 + \ldots + 1200 \times 1.035^7$

$= 1200(1 + 1.035 + 1.035^2 + \ldots + 1.035^7)$

Using $a = 1, r = 1.035, n = 8, S_n = \dfrac{a(r^n - 1)}{r - 1}$:

$$B = 1200 \times \frac{1(1.035^8 - 1)}{1.035 - 1}$$

$$= 10\,862.024\,12\ldots$$

$$= 10\,862.02 \text{ (2 dec. pl.)}$$

(a)** (i)

$$\int_0^\infty ke^{-\frac{x}{2}}\,dx = 1$$

$$k\left[-2e^{-\frac{x}{2}}\right]_0^\infty = 1$$

$$-2k\left[e^{-\infty} - e^0\right] = 1$$

$$-2k[0 - 1] = 1 \qquad (\text{as } e^{-\infty} = 0)$$

$$2k = 1$$

$$k = \frac{1}{2}$$

(ii) $\int_{\frac{1}{3}}^{\frac{1}{2}} \frac{1}{2} e^{-\frac{x}{2}}\,dx = \frac{1}{2}\int_{\frac{1}{3}}^{\frac{1}{2}} e^{-\frac{x}{2}}\,dx$

$$= \frac{1}{2}\left[-2e^{-\frac{x}{2}}\right]_{\frac{1}{3}}^{\frac{1}{2}}$$

$$= -\left[e^{-\frac{1}{4}} - e^{-\frac{1}{6}}\right]$$

$$= e^{-\frac{1}{6}} - e^{-\frac{1}{4}}$$

QUESTION 4

(a)
$$IQR = 20 - 12$$
$$= 8$$
$$Q_L - 1.5 \times IQR = 12 - 1.5(8)$$
$$= 0$$
$$Q_U + 1.5 \times IQR = 20 + 1.5(8)$$
$$= 32$$

$\therefore$ the maximum range of scores is 0 to 32, or a range of 32.

(c)* (i)
$$p + \frac{2}{5} + \frac{3}{20} + \frac{1}{4} + p = 1$$
$$2p + \frac{4}{5} = 1$$
$$2p = \frac{1}{5}$$
$$p = \frac{1}{10}$$

(ii) $\mu = 0\left(\frac{1}{10}\right) + 1\left(\frac{2}{5}\right) + 2\left(\frac{3}{20}\right) + 3\left(\frac{1}{4}\right) + 8\left(\frac{1}{10}\right)$

$$= 2\frac{1}{4}$$

$$P(X < \mu) = P\left(X < 2\frac{1}{4}\right)$$
$$= P(X = 0) + P(X = 1) + P(X = 2)$$
$$= \frac{1}{10} + \frac{2}{5} + \frac{3}{20}$$
$$= \frac{13}{20}$$

(c)** (i) $P(B|A) = \frac{1}{3}, P(A \cap B) = p$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$
$$\frac{1}{3} = \frac{p}{P(A)}$$
$$P(A) = 3p$$

(ii) $P(A|B) = \frac{2}{5}, P(A \cap B) = p$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{5} = \frac{p}{P(B)}$$

$$P(B) = \frac{5p}{2}$$

Now, $P(A' \cap B') = \mathrm{P}((A \cup B)')$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[3p + \frac{5p}{2} - p\right]$$

$$= 1 - \frac{9p}{2}$$

$$= \frac{2 - 9p}{2}$$

QUESTION 6

(b) (iv)

t	0	2	4	6	8
$v(t)$	20	50	70	80	60
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(b)$

$$\int_0^8 v(t)\,dt \approx \frac{b-a}{2n}\{f(a) + f(b) + 2[f(x_1) + \ldots + f(x_{n-1})]\}$$

$$= \frac{8-0}{2(4)}\{f(0) + f(8) + 2[f(2) + f(4) + f(6)]\}$$

$$= \frac{8}{8}\{20 + 60 + 2[50 + 70 + 80]\}$$

$$= 480$$

$\therefore$ distance is approximately 480 m.

(c) (i) As one quarter full $= 20 \div 4 = 5$ L,

$$5 = 20\left(1 - \frac{t}{60}\right)^2$$

$$\left(1 - \frac{t}{60}\right)^2 = \frac{1}{4}$$

$$1 - \frac{t}{60} = \pm\frac{1}{2}$$

$$\frac{t}{60} = \frac{1}{2} \text{ or } 1\frac{1}{2}$$

$$t = 30 \text{ or } 90$$

But, $0 \le t \le 60$, hence $t = 30$ seconds.

(ii) $V = 20\left(1-\frac{t}{60}\right)^2$

$$\frac{dV}{dt} = 40\left(1-\frac{t}{60}\right)\times -\frac{1}{60}$$

$$= -\frac{2}{3}\left(1-\frac{t}{60}\right)$$

Subs. $t = 30$:

$$\frac{dV}{dt} = -\frac{2}{3}\left(1-\frac{30}{60}\right)$$

$$= -\frac{1}{3}$$

$\therefore$ draining out at $\frac{1}{3}$ L/sec

QUESTION 8

(b) Option A: Interest $= 65\,000 \times 0.05 \times 6$

$= 19\,500$

Option B: 4.8% p.a. = 1.2% per quarter, 6 years = 24 quarters

$A = 65\,000(1.012)^{24}$

$= 86\,545.73224\ldots$

$= 86\,545.73$ (2 dec. pl.)

Interest $= 86\,545.73 - 65\,000$

$= 21\,545.73$

Option C: 4.2% p.a. = 0.35% per month, 6 years = 72 months

$A = 65\,000(1.0035)^{72}$

$= 83\,591.95607\ldots$

$= 83\,591.96$ (2 dec. pl.)

Interest $= 83\,591.96 - 65\,000$

$= 18\,591.96$

The best strategy is Option B with interest of \$21 545.73.

Replacement questions
with content from the most up-to-date syllabus

Marks

Question 3 (10 marks)

(b) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{x}{8}(x^2+2) & 0 \le x \le 2 \\ 0 & \textit{elsewhere} \end{cases}$$

Find, correct to 2 decimal places,

(i) the mean 2

(ii) the standard deviation. 3

(c) Simplify $\dfrac{x+2}{x} \div \dfrac{x^2-4}{2x}$. 2

(d) The line $2x + 3y - 6 = 0$ cuts the x-axis at A. The line $x - 2y + 4 = 0$ cuts the x-axis at B.

The lines $2x + 3y - 6 = 0$ and $x - 2y + 4 = 0$ intersect at C.

Find the area of triangle ABC. 3

Question 4 (7 marks)

(c) The heights of Year 12 girls are normally distributed with a mean of 163 cm and a standard deviation of 6 cm.

(i) What percentage of Year 12 girls have a height of more than 169 cm? 1

(ii) Calculate the z-score for a girl who has a height of 160 cm. 1

(iii) The probability that a Year 12 girl chosen at random has a height between 160 cm and 169 cm can be found by evaluating $\int_a^b f(x)\,dx$ where a and b are constants and where

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

is the normal probability density function with mean 0 and standard deviation 1.

By first finding the values of a and b, calculate an approximate value for this probability by using the trapezoidal rule with 4 function values. 3

Question 4 (continued) Marks

(iv) In a particular year there were 54 600 students enrolled in Year 12 in NSW. 52% of the students were girls. Using the results above, approximately how many girls had a height of less than 160 cm? **2**

Question 6 (7 marks)

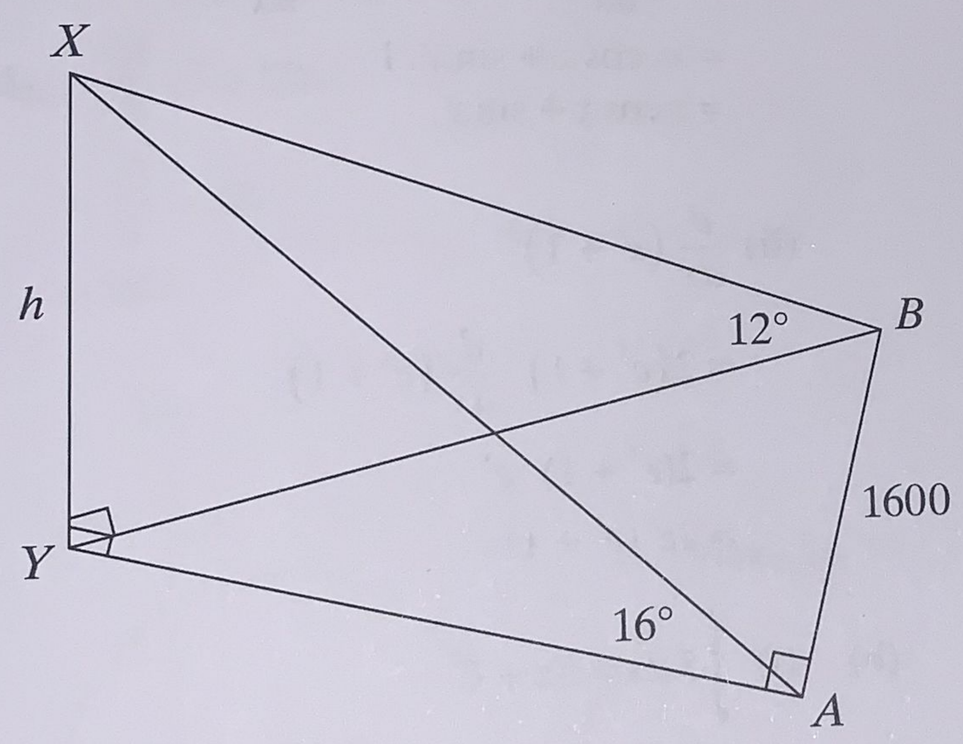
(a) The number of siblings, X, that students have at a large high school is a random variable with the following discrete probability distribution.

x	0	1	2	3
$P(X = x)$	0.3	0.45	0.15	p

(i) Show that $p = 0.1$. **1**

(ii) If two students are chosen at random, what is the probability that they have the same number of siblings? **2**

(b)



Declan walks 1600 metres due north along a road from point A to point B.

The point A is due east of a hill XY, where X is the top of the hill.

The point Y is directly below point X and is on the same horizontal plane as the road.

Let the height of the hill above point Y be h metres.

From point A, the angle of elevation to the top of the hill is 16°.

From point B, the angle of elevation to the top of the hill is 12°.

(i) Show that $BY = h \cot 12°$. **1**

(ii) Hence, find the value of h. **3**

Solutions to replacement questions

QUESTION 3

(b) (i) $\mu = \int_0^2 x \times \frac{x}{8}\left(x^2+2\right)dx$

$= \frac{1}{8}\int_0^2 \left(x^4+2x^2\right)dx$

$= \frac{1}{8}\left[\frac{x^5}{5}+\frac{2x^3}{3}\right]_0^2$

$= \frac{1}{8}\left[\frac{2^5}{5}+\frac{2(2)^3}{3}-0\right]$

$= 1.4666666\ldots$

$= 1.47$ (2 dec. pl.)

(ii) $E(X^2) = \int_0^2 x^2 \times \frac{x}{8}\left(x^2+2\right)dx$

$= \frac{1}{8}\int_0^2 \left(x^5+2x^3\right)dx$

$= \frac{1}{8}\left[\frac{x^6}{6}+\frac{x^4}{2}\right]_0^2$

$= \frac{1}{8}\left[\frac{2^6}{6}+\frac{2^4}{2}-0\right]$

$= 2.33333333\ldots$

$= 2.33$ (2 dec. pl.)

$\sigma^2 = E(X^2) - \mu^2$

$= 2.33 - 1.47^2$

$= 0.182222241\ldots$

$\sigma = 0.426874972\ldots$

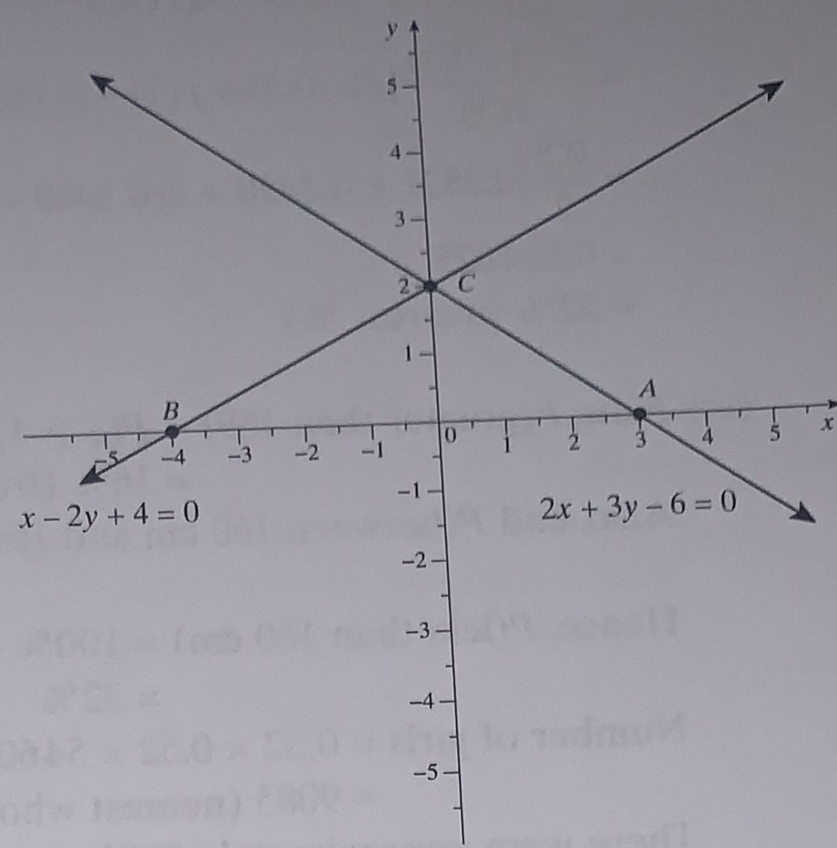
$= 0.43$ (2 dec. pl.)

(c) $\frac{x+2}{x} \div \frac{x^2-4}{2x} = \frac{x+2}{x} \times \frac{2x}{x^2-4}$

$= \frac{x+2}{x} \times \frac{2x}{(x-2)(x+2)}$

$= \frac{2}{x-2}$

(d)



Substitute $y = 0$ in $2x + 3y - 6 = 0$. $\therefore A(3, 0)$

Substitute $y = 0$ in $x - 2y + 4 = 0$. $\therefore B(-4, 0)$

Solving simultaneously, $C(0, 2)$.

Area $= \frac{1}{2} \times 7 \times 2$

$= 7$

$\therefore$ area is 7 units^2.

QUESTION 4

(c) (i) As $169 = 163 + 6 = \bar{x} + \sigma$, it has a z-score of 1.

Hence $z \geq 1$ is $\frac{100-68}{2}\% = 16\%$ of the Year 12 girls.

(ii) $z = \frac{x-\mu}{\sigma}$

$= \frac{160-163}{6}$

$= -0.5$

(iii) As 160 cm has a z-score of –0.5, and 169 cm has a z-score of 1, then $a = -0.5$ and $b = 1$.

Temp. (°C)	160 cm	163 cm	166 cm	169 cm
x	–0.5	0	0.5	1
$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	0.3521	0.3989	0.3521	0.2420
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(b)$

$$\text{Area} \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$$
$$= \frac{1-(-0.5)}{2(3)}\{f(-0.5)+f(1)+2[f(0)+f(0.5)]\}$$
$$= \frac{0.5}{2}(0.3521 + 0.2420 + 2(0.3989 + 0.3521))$$
$$= 0.524025$$
$$= 52\% \text{ (nearest \%)}$$

(iv) Now, $P(\text{greater than } 169) = P(z \geq 1)$
$= 16\%$ (from part (i))

Also, and $P(\text{between 160 cm and 169 cm}) = P(-0.5 \leq z \leq 1)$
$\approx 52\%$ (from (iii)).

Hence, $P(\text{less than 160 cm}) = 100\% - (16\% + 52\%)$
$= 32\%$

Number of girls $= 0.32 \times 0.52 \times 54600$
$= 9085$ (nearest whole)

There were approximately 9085 girls with a height of less than 160 cm.

QUESTION 6

(a) (i) $0.3 + 0.45 + 0.15 + p = 1$ (Sum of probabilities = 1)
$0.9 + p = 1$
$p = 0.1$

(ii) $P(\text{both have same number of siblings}) = P(\text{both } 0) + P(\text{both } 1) + P(\text{both } 2) + P(\text{both } 3)$
$= 0.3^2 + 0.45^2 + 0.15^2 + 0.1^2$
$= 0.325$

(b) (i) $\dfrac{h}{BY} = \tan 12°$

$\therefore \dfrac{BY}{h} = \cot 12°$

$BY = h \cot 12°$

(ii) Similarly, $AY = h \cot 16°$

Using Pythagoras' theorem for triangle BAY:

$$(h \cot 12°)^2 = (h \cot 16°)^2 + 1600^2$$
$$h^2 (\cot^2 12°) = h^2 (\cot^2 16°) + 1600^2$$
$$h^2 (\cot^2 12°) - h^2 (\cot^2 16°) = 1600^2$$
$$h^2 [\cot^2 12° - \cot^2 16°] = 1600^2$$
$$h^2 = \frac{1600^2}{\cot^2 12° - \cot^2 16°}$$
$$h = \frac{1600}{\sqrt{\cot^2 12° - \cot^2 16°}}$$
$$h = 506.6873518\ldots$$
$$= 507 \text{ (nearest whole)}$$

Replacement questions
with content from the most up-to-date syllabus

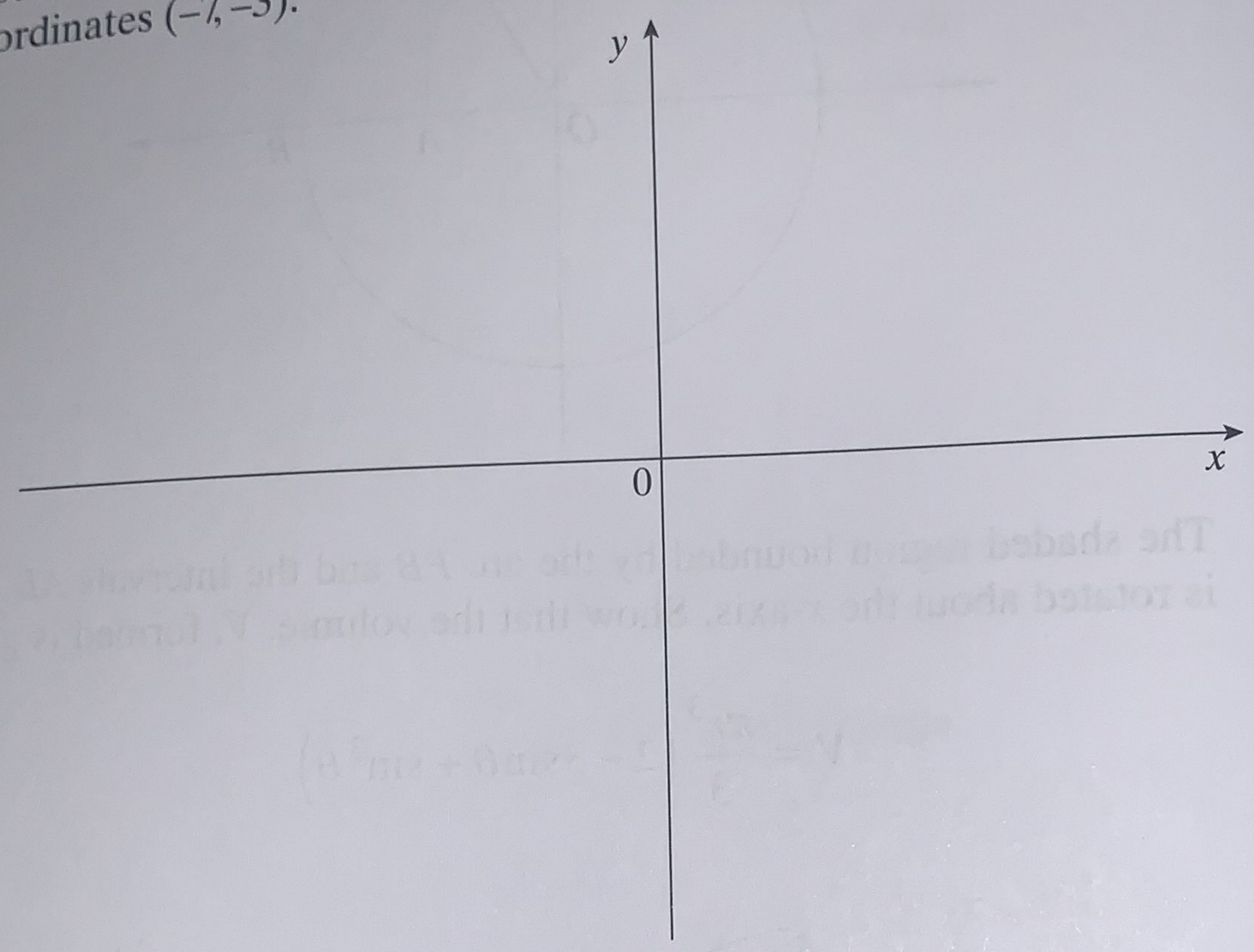
Marks

Question 3 (8 marks)

(a)* The coordinates of A, B and C are $(4, -6)$, $(-18, 0)$ and $(0, 6)$.

D is the midpoint of AB.

(i) Plot the points on a number plane, and show that D has coordinates $(-7, -3)$. 1



(ii) Show that the equation of the line AC is $3x + y - 6 = 0$. 2

(iii) Show that the line BC is perpendicular to the line AC. 1

(iv) Find the equation of the circle with diameter AB. 2

(a)** A set of data has a lower quartile (Q_L) of 24 and an upper quartile (Q_U) of 42.

What is the maximum possible range for this set of data if there are no outliers? 2

Marks

Question 5 (2 marks)

(b) (iii) Differentiate $\log_2(x^2 + 1)$. **2**

Question 8 (4 marks)

(a) The results (as percentages) of a large group of students in a Mathematics competition are represented by a random variable X and are normally distributed with a mean of 70 and a variance of 100. All results are expressed as whole percentages.

(i) Determine the z-score for a student who scored 66. **1**

(ii) Students who scored in the top 2.5% of the group receive a monetary prize.

What is the minimum mark required to receive the prize? **1**

(iii) Students who gained a mark with a z-score range of $0 \le z \le 1$ received a credit certificate.

Those students with a mark with a z-score range of $1 \le z \le 2$ received a distinction certificate.

If 816 credit certificates were awarded, how many distinction certificates were issued? **2**

Question 10 (12 marks)

(a) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} kx^2(2-x) & 0 \le x \le 2 \\ 0 & \textit{elsewhere} \end{cases}$$

(i) Show that $k = \dfrac{3}{4}$. **2**

(ii) Find $P(X \ge 1.5)$, correct to 2 decimal places. **2**

(iii) Find the value of $E(X)$. **2**

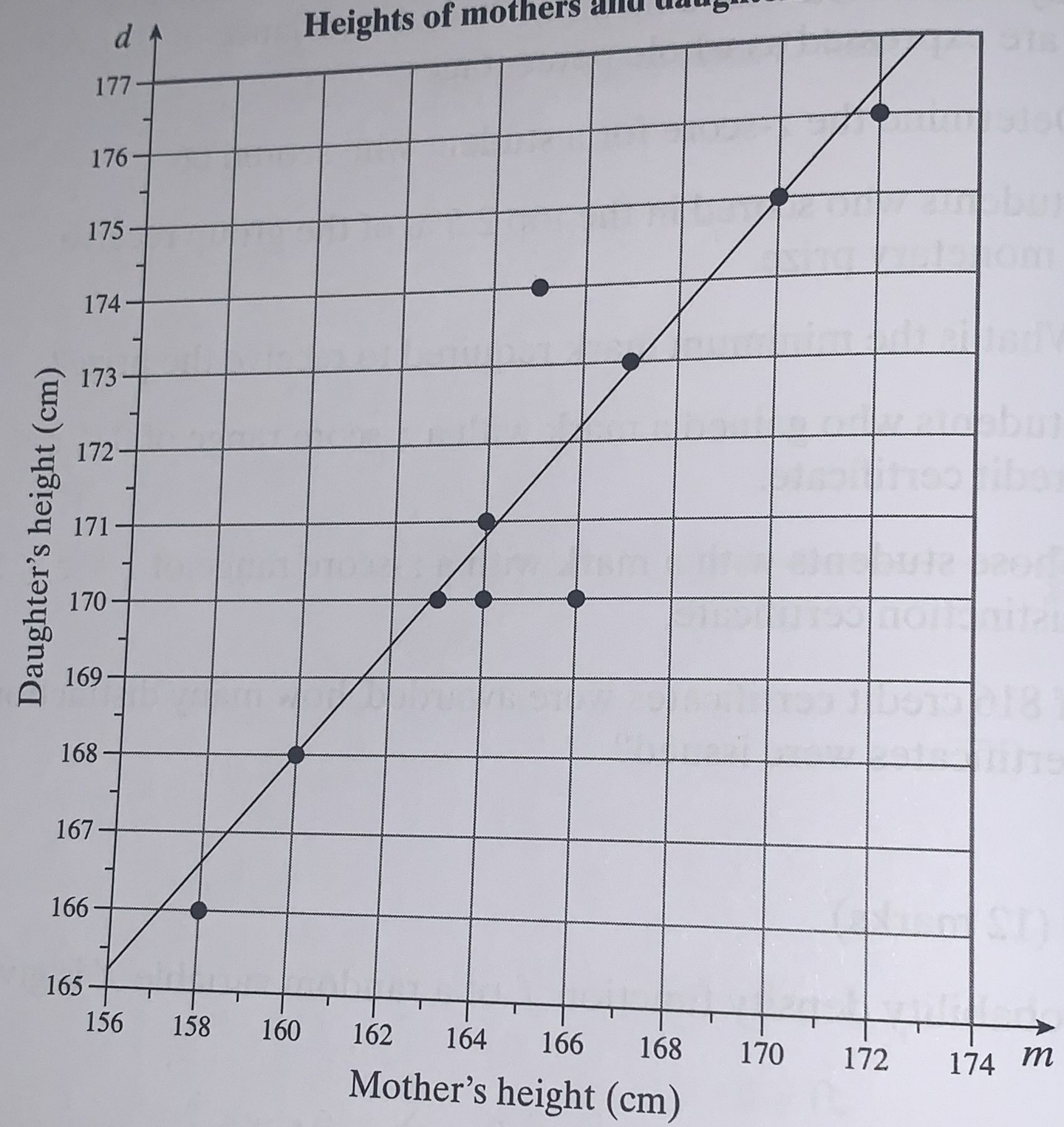
(b)* Use the trapezoidal rule with three subintervals to find an approximation for $\int_0^{\pi} x \sin^2 dx$, using three subintervals. **3**

Marks

(b)** The heights (cm) of 10 mothers (m) and their daughters (d) were recorded in the table below.

Mother's height in cm (m)	170	163	160	172	164	158	164	166	167	165
Daughter's height in cm (d)	175	170	168	176	170	166	171	170	173	174

The data was used to create a scatter plot and Melia constructed a regression line by eye, as shown below.



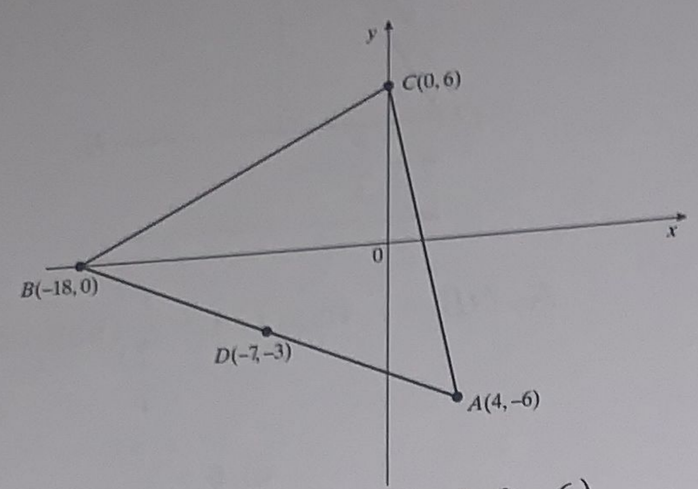
(i) Determine the equation of Melia's line of best fit shown. 2

(ii) Why would this model NOT be useful for predicting the height of the daughter of a woman of height 182 cm? 1

Solutions to replacement questions

QUESTION 3

(a)* (i)



Midpoint of $AB = \left(\frac{-18+4}{2}, \frac{0-6}{2}\right)$

$= (-7, -3)$

$\therefore D$ has coordinates $(-7, -3)$.

(1 mark)

(ii) gradient $AC = \frac{6+6}{0-4}$

$= -3$

$\therefore$ gradient $= -3$ and y-intercept $= 6$:

$y = -3x + 6$

$\therefore 3x + y - 6 = 0$

(2 marks)

(iii) gradient $BC = \frac{6-0}{0+18}$

$= \frac{1}{3}$

As $-3 \times \frac{1}{3} = -1$, then BC is perpendicular to AC.

(1 mark)

(iv) Length $AD = \sqrt{(-7-4)^2 + (-3+6)^2}$

$= \sqrt{130}$

Circle has centre $(-7, -3)$ and radius $\sqrt{130}$:

Equation is $(x + 7)^2 + (y + 3)^2 = 130$.

(2 marks)

(a)**

$IQR = 42 - 24$

$= 18$

$Q_L - 1.5 \times IQR = 24 - 1.5(18)$

$= -3$

$Q_U + 1.5 \times IQR = 42 + 1.5(18)$

$= 69$

$\therefore$ the maximum range of scores is -3 to 69, or a range of 72.

(2 marks)

QUESTION 5

(b) (iii) Firstly, $\log_2(x^2 + 1) = \frac{\ln(x^2+1)}{\ln 2}$.

$\therefore \frac{d}{dx}\left(\frac{\ln(x^2+1)}{\ln 2}\right) = \frac{2x}{(x^2+1)\ln 2}$

(2 marks)

QUESTION 8

(a) (i) $\mu = 70, \sigma = \sqrt{100} = 10$

$z = \frac{x-\mu}{\sigma}$

$= \frac{66-70}{10}$

$= -0.4$

(1 mark)

(ii) As $-2 \le z \le 2 = 95\%$ of scores, then $z \ge 2$ is top 2.5% of scores.

$2 = \frac{x-70}{10}$

$x - 70 = 20$

$x = 90$

Students with scores of 90 or above receive a monetary prize.

(1 mark)

(iii) $0 \le z \le 1 = \frac{68}{2} = 34\%$ of scores and

$1 \le z \le 2 = \frac{95-68}{2} = 13.5\%$

34% of scores $= 816$

13.5% of scores $= 816 \div 34 \times 13.5$

$= 324$

There were 324 distinction certificates awarded.

(2 marks)

QUESTION 10

(a) (i) $\int_0^2 \left(kx^2(2-x)\right)dx = 1$ (probability sums to 1)

$$k\int_0^2 \left(2x^2 - x^3\right)dx = 1$$

$$k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = 1$$

$$k\left(\frac{2(2)^3}{3} - \frac{2^4}{4} - 0\right) = 1$$

$$\frac{4k}{3} = 1$$

$$k = \frac{3}{4}$$

(2 marks)

(ii) $P(X \geq 1.5) = \frac{3}{4}\int_{1.5}^2 \left(2x^2 - x^3\right)dx$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_{1.5}^2$$

$$= \frac{3}{4}\left[\frac{2(2)^3}{3} - \frac{2^4}{4} - \left(\frac{2(1.5)^3}{3} - \frac{1.5^4}{4}\right)\right]$$

$$= 0.26171875\ldots$$

$$= 0.26 \text{ (2 dec. pl.)}$$

(2 marks)

(iii) $E(X) = \frac{3}{4}\int_0^2 \left(2x^3 - x^4\right)dx$

$$= \frac{3}{4}\left[\frac{x^4}{2} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{2^4}{2} - \frac{2^5}{5} - 0\right]$$

$$= 1.2$$

(2 marks)

(b)* Complete the table for $f(x) = x\sin^2 x$:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π
$f(x)$	0	$\frac{\pi}{4}$	$\frac{\pi}{4}$	0
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(b)$

$$\text{Area} \approx \frac{\pi - 0}{2(3)}\left[f(0) + f(\pi) + 2\{f(\tfrac{\pi}{3}) + f(\tfrac{2\pi}{3})\}\right]$$

$$= \frac{\pi}{6}\left[0 + 0 + 2(\tfrac{\pi}{4} + \tfrac{\pi}{2})\right]$$

$$= \frac{\pi^2}{4}$$

$$\therefore \int_0^\pi x\sin^2 x\,dx \approx \frac{\pi^2}{4}$$

(3 marks)

(b)** (i) The line passes through the points (160, 168) and (170, 175).

$$\text{Gradient} = \frac{175 - 168}{170 - 160}$$

$$= 0.7$$

The line has equation $d = 0.7m + c$

As the line passes through (160, 168), then

$168 = 0.7(160) + c$

$c = 56$

The equation is $d = 0.7m + 56$

(2 marks)

(ii) The model was only designed for mothers with a height between 158 cm and 172 cm. *(1 mark)*

Replacement questions

with content from the most up-to-date syllabus

Marks

Question 1 (4 marks)

(a) The table shows the future value of an annuity of $1 for varying interest rates and time periods. The contribution is made at the end of each period.

Table of future value interest factors						
	Interest rate per period					
Period	**0.50%**	**1.50%**	**2.00%**	**3.00%**	**4.00%**	**6.00%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0050	2.0150	2.0200	2.0300	2.0400	2.0600
3	3.0150	3.0452	3.0604	3.0909	3.1216	3.1836
4	4.0301	4.0909	4.1216	4.1836	4.2465	4.3746
5	5.0503	5.1523	5.2040	5.3091	5.4163	5.6371
6	6.0755	6.2296	6.3081	6.4684	6.6330	6.9753
7	7.1059	7.3230	7.4343	7.6625	7.8983	8.3938
8	8.1414	8.4328	8.5830	8.8923	9.2142	9.8975
9	9.1821	9.5593	9.7546	10.1591	10.5828	11.4913
10	10.2280	10.7027	10.9497	11.4639	12.0061	13.1808
11	11.2792	11.8633	12.1687	12.8078	13.4864	14.9716
12	12.3356	13.0412	13.4121	14.1920	15.0258	16.8699

Erica is planning to take a holiday in 3 years.

She needs $8000 for this holiday.

She is going to make regular quarterly payments into an account that earns interest at the rate of 2% per annum compounded quarterly.

What is the minimum investment, to the nearest $10, Erica needs to make each quarter in order to take this holiday? Support your answer with calculations. **2**

(e) Consider the following discrete probability distribution for the random variable X.

x	0	1	2	5
$P(X = x)$	p	$2p$	$3p$	$4p$

What is the mean of the distribution? **2**

Marks

Question 2 (3 marks)

(a) A scientist is studying the relationship between the pressure, measured in atm (atmospheres) and the volume of a gas, measured in litres. The following is a sample of the scientist's results.

Pressure P (atm)	Volume V (L)
2.4	5.2

The data can be modelled in the form $P = \dfrac{k}{V}$, where k is a constant.

(i) Using this data find the value of k. 1

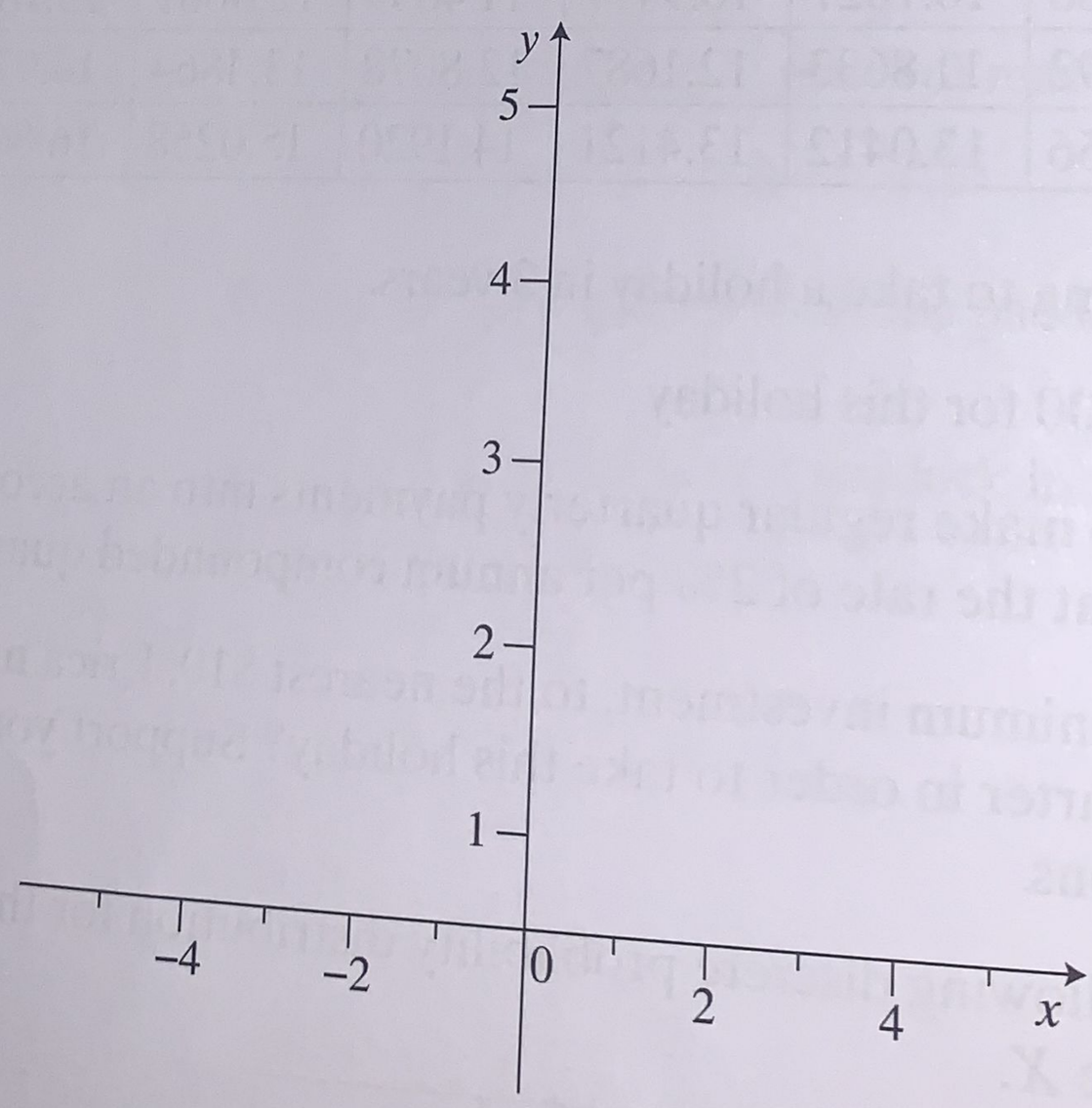
(ii) Predict the volume when the pressure of the gas is 3 atm. 2

Question 3 (8 marks)

(b) The function $y = e^x$ is translated right 2 units and up by 1 unit to give the function $f(x)$.

(i) Write down the equation of the function $f(x)$. 1

(ii) Sketch $y = f(x)$ on the number plane below. 1



(c) In Jerry's Plains, the maximum temperature for each day is recorded. The mean of these maximum temperatures during September over the past 150 years is 26 °C, and the standard deviation is 2.4 °C. You may assume that these maximum temperatures are normally distributed. Marks

(i) What percentage of September days have a maximum temperature less than 23.6 °C? **1**

(ii) Calculate the z-score for a maximum temperature of 22.4 °C. **1**

(iii) The probability that a September day chosen at random has a maximum temperature between 22.4 °C and 23.6 °C can be found by evaluating $\int_a^b f(x)\,dx$ where a and b are constants and where

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

is the normal probability density function with mean 0 and standard deviation 1.

By first finding the values of a and b, calculate an approximate value for this probability by using the trapezoidal rule with 3 function values. **3**

(iv) Using the results above, find the approximate probability that a September day chosen at random has a maximum temperature of 22.4 °C or less. **1**

Question 4 (2 marks)

(e) Solve $\log_4 x + \log_4 (x - 12) = 3$ **2**

Question 5 (3 marks)

(c) The table gives the speed v of a jogger at time t in minutes over a 20-minute period. The speed v is measured in metres per minute, in intervals of 5 minutes.

t	0	5	10	15	20
v	173	81	127	195	168

The distance covered by the jogger over the 20-minute period is given by

$$\int_0^{20} v\,dt.$$

Use the trapezoidal rule and the speed at each of the five time values to find the approximate distance the jogger covers in the 20-minute period. **3**

REPLACEMENT QUESTIONS

Marks

Question 6 (6 marks)

(a) Solve $2^{2x+3} - 4.2^x = 0$ **3**

(b) Find the coordinates of the point P on the line $x = -4$, such that the line joining P to $Q(1, -3)$ is perpendicular to the line $3x - 2y = 12$. **3**

Question 8 (3 marks)

(b) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{k}(x^2+5) & 0 \le x \le 3 \\ 0 & \textit{elsewhere} \end{cases}$$

(i) Show that $k = 24$. **1**

(ii) Find the mean. **2**

Question 9 (3 marks)

(a) Two events, A and B, are such that $P(A) = \frac{2}{7}$ and $P(B) = \frac{3}{5}$.

(i) If $P(A \cup B) = \frac{4}{5}$, find the value of $P(A' \cap B)$. **2**

(ii) When A and B are mutually exclusive, find the value of $P(A \cap B')$. **1**

REPLACEMENT QUESTIONS

Solutions to replacement questions

QUESTION 1

(a) 3 years = 12 quarters,
2% p.a. = 0.5% per quarter

Let investment = A

$8000 = A \times 12.3356$

$A = \dfrac{8000}{12.3356}$

$= 648.5294595\ldots$

$= 650$ (nearest ten)

Erica needs to invest \$650 each quarter.

(2 marks)

(e) $p + 2p + 3p + 4p = 1$

$10p = 1$

$p = \dfrac{1}{10}$

$E(X) = 0\left(\dfrac{1}{10}\right) + 1\left(\dfrac{2}{10}\right) + 2\left(\dfrac{3}{10}\right) + 5\left(\dfrac{4}{10}\right)$

$= 2.8$

(2 marks)

QUESTION 2

(a) (i) $P = \dfrac{k}{V}$

Substitute $P = 2.4$, $V = 5.2$:

$2.4 = \dfrac{k}{5.2}$

$k = 12.48$ *(1 mark)*

(ii) $P = \dfrac{12.48}{V}$

Substitute $P = 3$:

$3 = \dfrac{12.48}{V}$

$3V = 12.48$

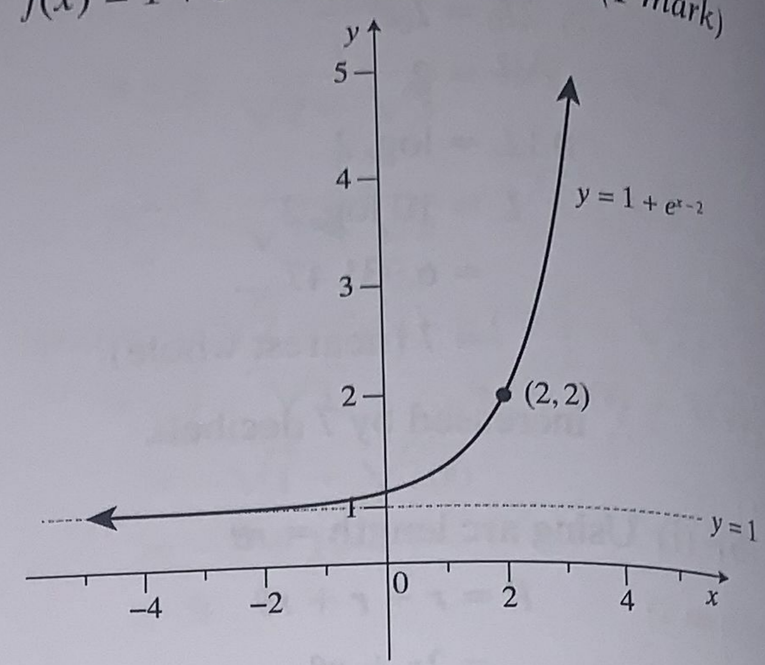
$V = 4.16$

The volume is 4.16 L. *(2 marks)*

QUESTION 3

(b) (i) $f(x) = 1 + e^{x-2}$ *(1 mark)*

(ii)



(1 mark)

(c) (i) As $23.6 = \bar{x} - \sigma$, it has a z-score of -1.
68% of population lie in $-1 \le z \le 1$.
100% − 68% = 32%.
Hence $z \le -1$ is $\dfrac{32}{2}$%, or 16% of population. *(1 mark)*

(ii) $z = \dfrac{x - \mu}{\sigma}$

$= \dfrac{22.4 - 26}{2.4}$

$= -1.5$ *(1 mark)*

(iii) As 22.4 has z-score of -1.5,
and 23.6 has a z-score of -1, then
$a = -1.5$ and $b = -1$.
Also, 23 will have a z-score of -1.25.

Temp (°C)	22.4	23	23.6
x	-1.5	-1.25	-1
$\dfrac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$	0.1295	0.1826	0.2420
	$f(a)$	$f(x_1)$	$f(b)$

(3 marks)

$$\text{Area} \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$$
$$= \frac{-1-(-1.5)}{2(2)}\{f(-1.5)+f(-1)+2f(-1.25)\}$$
$$= 0.125(0.1295 + 0.2420 + 2(0.1826))$$
$$= 0.0920875$$
$$= 9.2\% \text{ (nearest tenth of \%)}$$

(iv) As $P(X \le 23.6) = 16\%$ (from (i)),
and $P(22.4 \le X \le 23.6) \approx 9.2\%$ (from (iii)), then
$P(X \le 22.4) \approx 16\% - 9.2\% = 6.8\%$
The approximate probability is 6.8%.

(1 mark)

QUESTION 4

(e) $\log_4 x + \log_4 (x-12) = 3$

$$\log_4 [x(x-12)] = 3$$
$$x(x-12) = 4^3$$
$$x^2 - 12x - 64 = 0$$
$$(x-16)(x+4) = 0$$
$$x = 16 \text{ (as } x > 0)$$

(2 marks)

QUESTION 5

(c)

t	0	5	10	15	20
v	173	81	127	195	168
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(b)$

$$\therefore \text{Distance travelled} = \int_0^{20} v(t)\,dt \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$$
$$= \frac{20-0}{2(4)}\{f(0)+f(20)+2[f(5)+f(10)+f(15)]\}$$
$$= \frac{5}{2}\{173+168+2[81+127+195]\}$$
$$= 2867.5$$

$\therefore$ the jogger ran approximately 2867.5 m.

(3 marks)

QUESTION 6

(a) $2^{2x+3} - 2^2.2^x = 0$

$$2^{2x+3} - 2^{x+2} = 0$$
$$2^{x+2}(2^{x+1} - 1) = 0$$
$$2^{x+1} - 1 = 0 \text{ (as } 2^{x+2} \ne 0)$$
$$2^{x+1} = 1$$
$$2^{x+1} = 2^0$$
$$x = -1$$

Another method:

$$2^{2x+3} - 4.2^x = 0$$
$$8.2^{2x} - 4.2^x = 0$$
$$2.2^{2x} - 2^x = 0$$

Let $m = 2^x$:

$$2m^2 - m = 0$$
$$m(2m-1) = 0$$
$$m = 0, \frac{1}{2}$$
$$\therefore 2^x = 0,\ 2^x = \frac{1}{2}$$
$$\therefore x = -1 \text{ (as } 2^x \ne 0)$$

(3 marks)

(b) Gradient of $3x - 2y = 12$:

$$2y = 3x - 12$$
$$y = \frac{3x-12}{2}$$

$\therefore$ gradient $= \frac{3}{2}$

$\therefore$ gradient of perpendicular $= -\frac{2}{3}$

Now, find equation of PQ, using $m = -\frac{2}{3}$ and passing through $(1, -3)$:

$$y + 3 = -\frac{2}{3}(x - 1)$$

Substitute $x = -4$:

$$y + 3 = -\frac{2}{3}(-4 - 1)$$
$$y + 3 = -\frac{2}{3}(-5)$$
$$3y + 9 = 10$$
$$3y = 1$$
$$y = \frac{1}{3}$$

The coordinates of P are $(-4, \frac{1}{3})$.

(3 marks)

QUESTION 8

(b) (i) As probability sums to 1, need to solve

$$\int_0^3 \frac{1}{k}\left(x^2+5\right)\,dx = 1.$$

$$\frac{1}{k}\int_0^3 \left(x^2+5\right)\,dx = 1$$

$$\int_0^3 \left(x^2+5\right)\,dx = k$$

$$k = \left[\frac{x^3}{3} + 5x\right]_0^3$$
$$= \frac{3^3}{3} + 5(3) - 0$$
$$= 9 + 15$$
$$= 24$$

(1 mark)

(ii) $\mu = \int_0^3 x \times \frac{1}{24}\left(x^2+5\right)\,dx$

$$= \frac{1}{24}\int_0^3 \left(x^3+5x\right)dx$$
$$= \frac{1}{24}\left[\frac{x^4}{4} + \frac{5x^2}{2}\right]_0^3$$
$$= \frac{1}{24}\left[\frac{3^4}{4} + \frac{5(3)^2}{2} - 0\right]$$
$$= 1.781\,25$$

(2 marks)

QUESTION 9

(a) (i) $P(A') = \frac{5}{7}$ and $P(B) = \frac{3}{5}$

As $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

then, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

Also, $P(B) = P(A \cap B) + P(A' \cap B)$

Hence, $P(A' \cap B) = P(B) - P(A \cap B)$

$$P(A' \cap B) = P(B) - [P(A) + P(B) - P(A \cup B)]$$
$$= P(A \cup B) - P(A)$$
$$= \frac{4}{5} - \frac{2}{7}$$
$$= \frac{18}{35}$$

(2 marks)

(ii) If A and B are mutually exclusive then $P(A \cap B) = 0$.

$$P(A \cap B') = P(A)$$
$$= \frac{2}{7}$$

(1 mark)

Replacement questions
with content from the most up-to-date syllabus

Marks

1 Which of these is not a one-to-one function?

A. $f(x) = 2x^3$

B. $f(x) = 2^x + 1$

C. $f(x) = \sqrt{x-2}$

D. $f(x) = x^3 + 2x^2$ **1**

3 Which of these is the exact value of $\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)$?

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{2-\sqrt{2}}{2\sqrt{2}}$

C. $\frac{2-\sqrt{3}}{2\sqrt{2}}$

D. $\frac{\sqrt{3}-2}{2\sqrt{2}}$ **1**

5 Which of these is the anti-derivative of $2^x \ln 4$? **1**

A. $\frac{2^x}{2\ln 2} + c$

B. $2^{2x} + c$

C. $2^{x+1} + c$

D. $\frac{2^x}{3\ln 2} + c$

Marks

8 The lifetime of a particular make of lightbulb is normally distributed with mean 940 hours and standard deviation 65 hours.

Using the Cumulative Normal Distribution Table below, find the probability that a lightbulb of the same make chosen at random from a batch of 5000 lightbulbs has a lifetime of at least 1031 hours. **1**

z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.788	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9731
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.000

A. 8.08%

B. 8.86%

C. 8.92%

D. 9.16%

Question 11 (6 marks)

(a) Given that $f(x) = x^2 - 3$ and $g(x) = \dfrac{4x}{x+2}$.

(i) Write an expression for $g(f(x))$. **1**

(ii) Hence or otherwise find the value of $g(f(-2))$. **1**

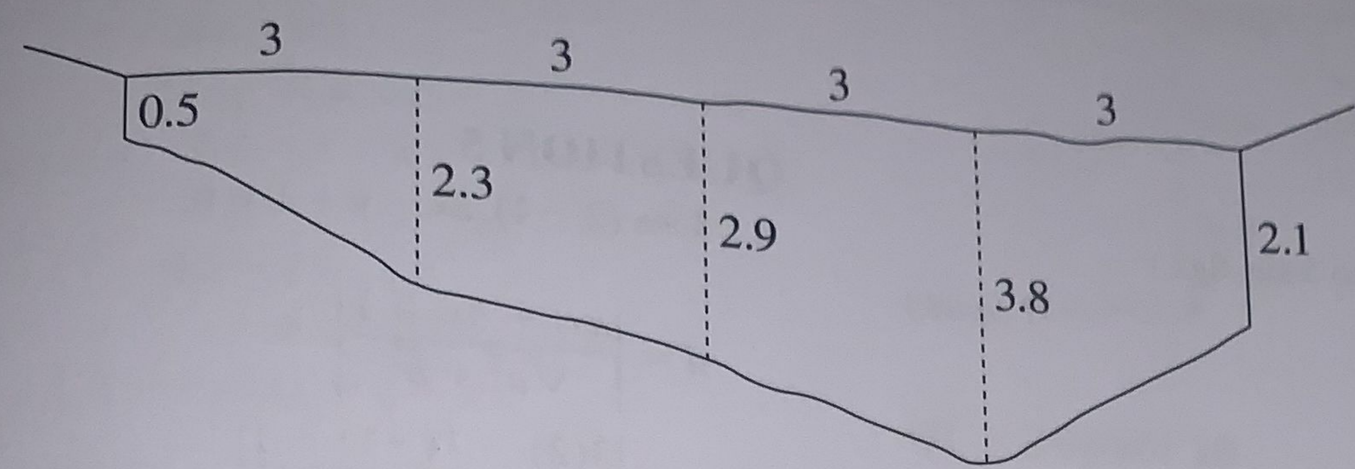
(b) The time to prepare a paddock for planting is inversely proportional to the number of people who are working. A large paddock can be prepared by 3 workers in 16 days. In order to finish the paddock sooner, the farmer plans to hire additional workers. How many workers are needed to finish the paddock in 6 days? **2**

(e) Find the value(s) of k for which the equation $2^{kx-8} = 4^{x^2}$ has exactly one solution. **2**

Question 12 (4 marks)

Marks

(d) At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3-metre intervals. The cross-section is shown.



(i) Use the trapezoidal rule with the five depth measurements to calculate the approximate area of the cross-section. **3**

(ii) The river flows at 0.4 metres per second.

Calculate the approximate volume of water flowing through the cross-section in 10 seconds. **1**

Question 14 (3 marks)

(b) Layla wants to invest \$30 000 for a total of 4 years. She has three investment options.

Option A—simple interest is paid at the rate of 5% per annum

Option B—compound interest is paid at a rate of 4.5% per annum, compounded annually

Option C—compound interest is paid at a rate of 4.2% per annum, compounded quarterly

Determine Layla's best investment option. Support your answer with calculations. **3**

Question 16 (4 marks)

(a) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \dfrac{x+2}{16} & 0 \le x \le 4 \\ 0 & \textit{elsewhere} \end{cases}$$

(i) Find the value of a such that $P(X \le a) = \frac{3}{8}$. **2**

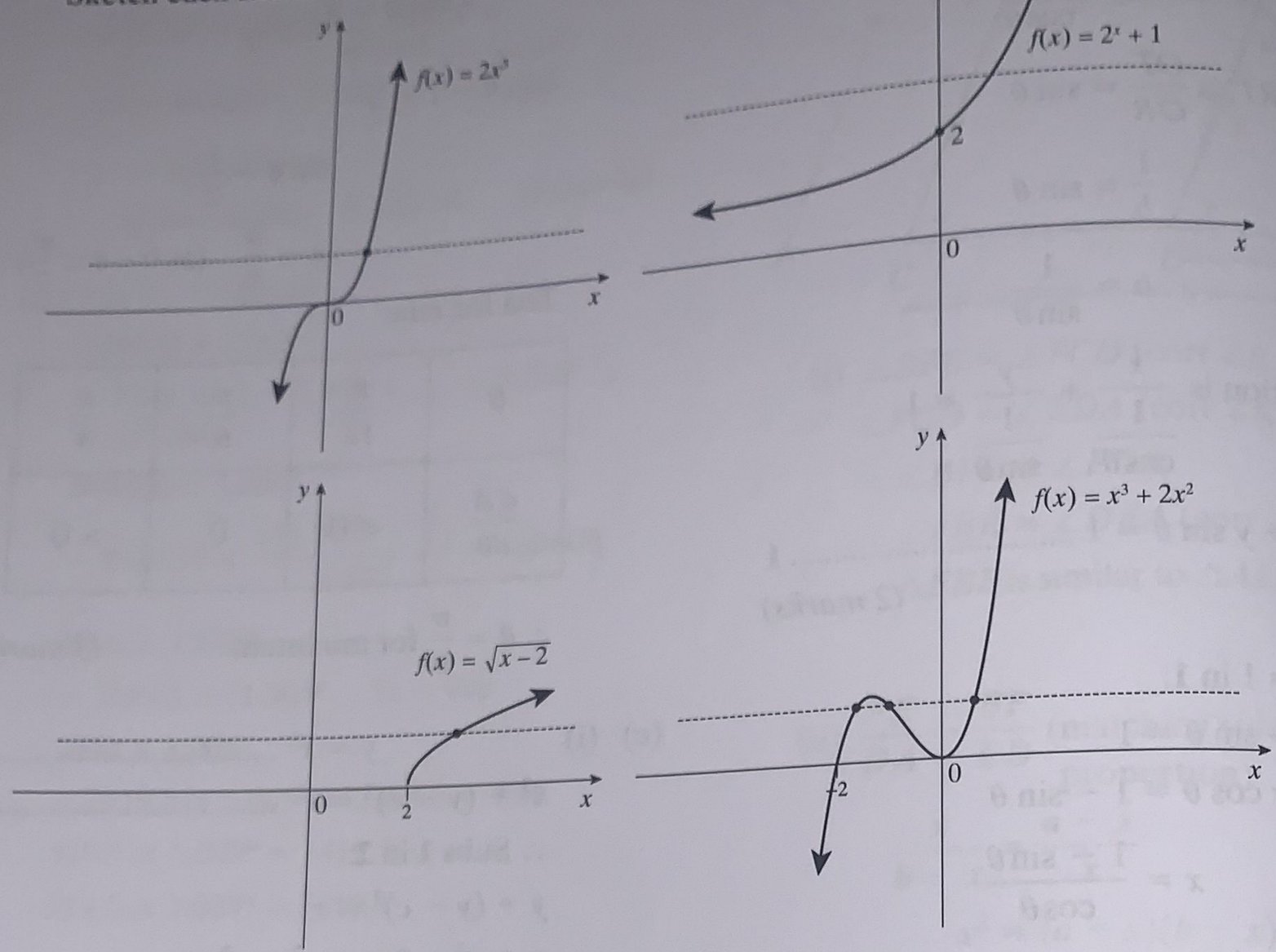
(ii) Find $E(X)$. **2**

REPLACEMENT QUESTIONS

Solutions to replacement questions

QUESTION 1

Sketch each function and use the horizontal line test.



A horizontal line can be drawn on $f(x) = x^3 + 2x^2$ and it cuts at more than one point.

Answer D

(1 mark)

QUESTION 3

$$\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right) = \frac{1}{\sqrt{2}} - \frac{1}{2}$$

$$= \frac{2-\sqrt{2}}{2\sqrt{2}}$$

Answer B

(1 mark)

QUESTION 5

Using $\int a^x dx = \dfrac{a^x}{\ln a} + c$,

$$\int 2^x \ln 4\, dx = \ln 4 \int 2^x dx$$

$$= 2\ln 2 \times \frac{2^x}{\ln 2} + c$$

$$= 2 \times 2^x + c$$

$$= 2^{x+1} + c$$

Answer C

(1 mark)

QUESTION 8

Let $x = 1031$, $\mu = 940$, $\sigma = 65$:

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{1031-940}{65}$$

$$= 1.4$$

The table shows $P(z < 1.4) = 0.9192$

Hence, $P(z > 1.4) = 1 - 0.9192$

$= 0.0808$

$\therefore$ the probability is 8.08%.

Answer A

(1 mark)

QUESTION 11

(a) (i) $g(x) = \dfrac{4x}{x+2}$

$$g(f(x)) = \frac{4(x^2-3)}{(x^2-3)+2}$$

$$= \frac{4x^2-12}{x^2-1}$$ *(1 mark)*

(ii) $g(f(-2)) = \dfrac{4(-2)^2-12}{(-2)^2-1}$

$$= \frac{4}{3}$$ *(1 mark)*

(b) Let T = time and w = workers:

$$T = \frac{k}{w}$$

Substitute $T = 16$, $w = 3$:

$$16 = \frac{k}{3}$$

$$k = 48$$

$$T = \frac{48}{w}$$

Substitute $T = 6$:

$$6 = \frac{48}{w}$$

$$6w = 48$$

$$= 8$$

The farmer would need 8 workers. *(2 marks)*

(e)

$$2^{kx-8} = 4^{x^2}$$

$$2^{kx-8} = 2^{2x^2}$$

$$kx - 8 = 2x^2$$

$$2x^2 - kx + 8 = 0$$

For equal roots, $\Delta = (-k)^2 - 4(2)(8) = 0$

$$k^2 - 64 = 0$$

$$k = \pm 8$$ *(2 marks)*

QUESTION 12

(d) (i) $A \approx \dfrac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$

$$= \frac{3}{2}\{0.5 + 2.1 + 2[2.3 + 2.9 + 3.8]\}$$

$$= 30.9$$

$\therefore$ area of approximately 30.9 m^2 *(3 marks)*

(ii) As $10 \times 0.4 = 4$,

Volume $= 30.9 \times 4$

$= 123.6$ *(1 mark)*

$\therefore$ volume is approximately 123.6 m^3

QUESTION 14

(b) Option A: Interest $= 30\,000 \times 0.05 \times 4$

$= 6000$

Option B: $A = 30\,000(1.045)^4$

$= 35\,775.55802\ldots$

$= 35\,775.56$ (nearest cent

Interest $= 35\,775.56 - 30\,000$

$= 5775.56$

Option C: 4.2% p.a. = 1.05% per quarter, 4 years = 16 quarters

$$A = 30\,000(1.0105)^{16}$$
$$= 35\,457.028\,82\ldots$$
$$= 35\,457.03 \text{ (2 dec. pl.)}$$

Interest $= 35\,457.03 - 30\,000$

$= 5457.03$

The best strategy is Option A with interest of \$6000.

(3 marks)

QUESTION 16

(a) (i) $\displaystyle\int_0^a \frac{1}{16}(x+2)\,dx = \frac{3}{8}$

$$\frac{1}{16}\left[\frac{x^2}{2}+2x\right]_0^a = \frac{3}{8}$$

$$\frac{a^2}{2}+2a-0=6$$

$$a^2+4a-12=0$$

$$(a-2)(x+6)=0 \qquad a=2 \text{ (as } a>0)$$

(2 marks)

(ii) $\displaystyle E(X) = \int_0^4 \frac{x}{16}(x+2)\,dx$

$$= \frac{1}{16}\int_0^4 \left(x^2+2x\right)dx$$

$$= \frac{1}{16}\left[\frac{x^3}{3}+x^2\right]_0^4$$

$$= \frac{1}{16}\left[\frac{4^3}{3}+4^2-0\right]$$

$$= 2\frac{1}{3}$$

(2 marks)

Replacement questions
with content from the most up-to-date syllabus

Marks

7 A and B are events of a sample space.

Given that $P(B|A) = \sqrt{p}$, $P(A) = p$ and $P(B) = p^2$, which of these is an expression for $P(A|B)$? 1

A. $p^{\frac{3}{2}}$

B. $\sqrt{p}$

C. $\frac{1}{p}$

D. $\frac{1}{\sqrt{p}}$

Question 11 (3 marks)

(g) The number of hours N that it takes for a block of ice to melt varies inversely with the temperature t. At 32 °C it takes 6 hours for a block of ice to melt.

(i) Find an equation for the graph in terms of N and t in the form $N = \frac{k}{t}$, where k is a constant. 1

(ii) How much longer will it take the same size block of ice to melt at 10 °C compared to 25 °C? 2

Question 12 (6 marks)

(b) In a game a six-sided dice is rolled twice.

The difference between the two results is recorded.

The following table is produced. Some of the results are missing.

		Second roll					
		1	**2**	**3**	**4**	**5**	**6**
First roll	**1**	0	1	2	3		5
	2	1	0	1	2	3	
	3	2	1			2	3
	4		2	1	0	1	
	5	4	3	2	1	0	1
	6	5	4	3	2		0

Question 12 (continued)

Marks

(i) Complete the table. **1**

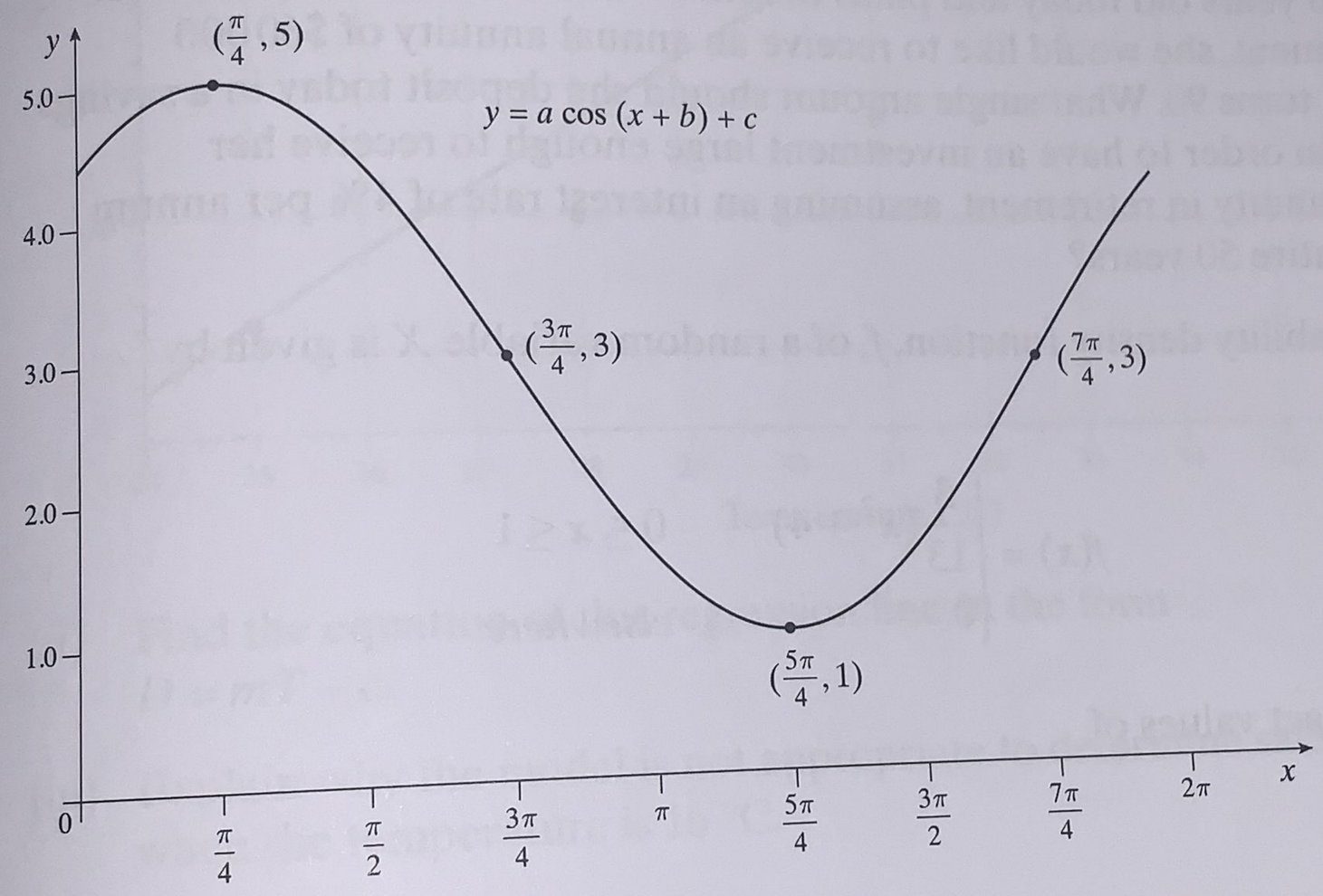
(ii) A probability distribution table is drawn to summarise the results. Complete the table. **2**

x	0	1	2	3	4	5
$P(X = x)$			$\frac{2}{9}$			$\frac{1}{18}$

(iii) Find the expected value, $E(X)$, and the standard deviation σ. **3**

Question 15 (7 marks)

(a) The graph of $y = a\cos(x + b) + c$, over the domain $0 \le x \le 2\pi$, has been drawn on the number plane below.



What are the values of a, b and c? **3**

(b) The market value of Kate's house has been increasing at a constant value of 2.5% per annum since she bought it at the start of 2007.

At the start of 2019 the house was valued at \$435 000.

(i) What was the value of the house at the start of 2007 when Kate bought it? **2**

Express your answer to the nearest dollar.

(ii) Daniel also bought a house at the start of 2007 for \$305 000.

The value of his house has been increasing at the rate of 3.2% per annum. **2**

In what year did the two houses have the same value?

Marks

Question 16 (12 marks)

(b)* The table shows the present value interest factors for various interest rates and periods of time.

Table of present value interest factors

	Interest rate per period						
Period	**0.50%**	**1.00%**	**1.25%**	**1.50%**	**2.00%**	**3.00%**	**4.00%**
20	18.9874	18.0456	17.5993	17.1686	16.3514	14.8775	13.5903
30	27.7941	25.8077	24.8889	24.0158	22.3965	19.6004	17.2920
40	36.1722	32.8347	31.3269	29.9158	27.3555	23.1148	19.7928
50	44.1428	39.1961	37.0129	34.9997	31.4236	25.7298	21.4822
120	90.0735	69.7005	61.9828	55.4985	45.3554	32.3730	24.7741
180	118.5035	83.3217	71.4496	62.0956	48.5844	33.1703	24.9785
240	139.5808	90.8194	75.9423	64.7957	49.5686	33.3057	24.9980

Mel is 40 years old today and plans to retire when she turns 60 years old. In retirement, she would like to receive an annual annuity of $60 000 until she turns 90. What single amount should she deposit today in a savings account in order to have an investment large enough to receive her annual annuity in retirement, assuming an interest rate of 4% per annum for the entire 50 years? 4

(b)** The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{3}{13}(x^2+4) & 0 \le x \le 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Find the exact values of

(i) $E(X)$ 2

(ii) $E(X^2)$. 2

(c) The maximum temperature, T, in °C, on eight randomly selected days is shown in the table below. The table also shows the number of drinks, D, purchased from a vending machine.

Maximum temperature, T	28	25	36	34	37	31	32	30
Number of drinks sold, D	20	16	48	34	46	24	40	32

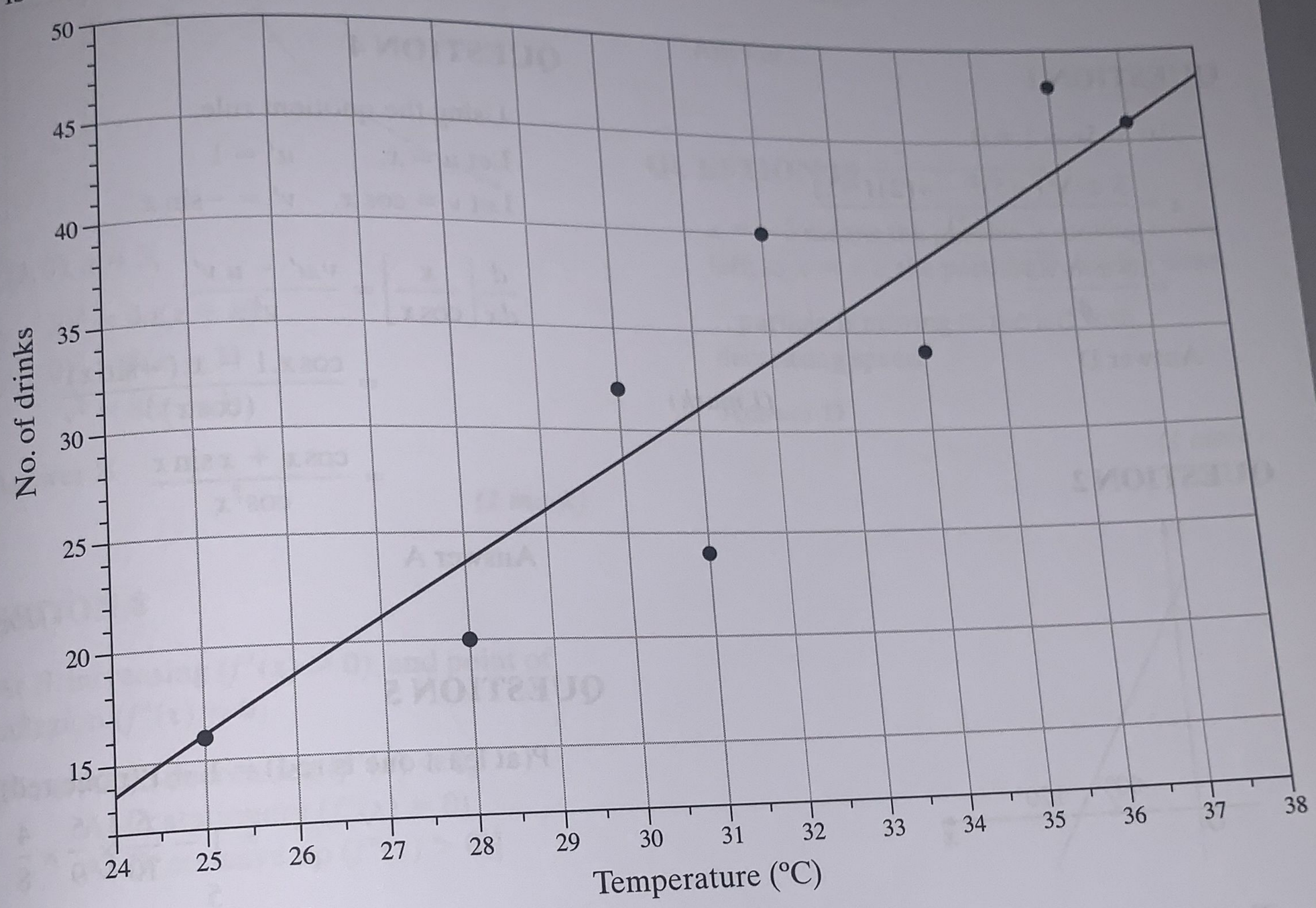
(i) Find the value of r, the correlation coefficient, correct to 4 decimal places. 1

Question 16 (continued)

Marks

The relationship between the variables can be modelled by the regression line $D = mT + c$.

The data is displayed on a scatterplot and a regression line, by eye, is drawn.



(ii) Find the equation of this regression line, in the form $D = mT + c$. **2**

(iii) Explain why the model is not appropriate to determine sales when the temperature is 16 °C. **1**

Solutions to replacement questions

QUESTION 7

As $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$

$$\sqrt{p} = \frac{P(B \cap A)}{p}$$

$$P(B \cap A) = p^{\frac{3}{2}}$$

As $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{p^{\frac{3}{2}}}{p^2}$$

$$= p^{-\frac{1}{2}}$$

$$= \frac{1}{\sqrt{p}}$$

Answer D *(1 mark)*

QUESTION 11

(g) (i) Substitute $N = 6, t = 32$:

$$6 = \frac{k}{32}$$

$$k = 192$$

$$\therefore N = \frac{192}{t}$$ *(1 mark)*

(ii) Substitute $t = 10$ and $t = 25$:

$$\text{Difference} = \frac{192}{10} - \frac{192}{25}$$

$$= 19.2 - 7.68$$

$$= 11.52$$

It will take 11.52 hours longer.

(2 marks)

QUESTION 12

(b) (i)

		Second roll					
		1	**2**	**3**	**4**	**5**	**6**
First roll	**1**	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

(1 mark)

(ii)

x	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

(2 marks)

(iii) $E(X) = 0\left(\frac{1}{6}\right) + 1\left(\frac{5}{18}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{9}\right) + 5\left(\frac{1}{18}\right)$

$$= 1\frac{17}{18}$$

$$E(X^2) = 0^2\left(\frac{1}{6}\right) + 1^2\left(\frac{5}{18}\right) + 2^2\left(\frac{2}{9}\right) + 3^2\left(\frac{1}{6}\right) + 4^2\left(\frac{1}{9}\right) + 5^2\left(\frac{1}{18}\right)$$

$$= 5\frac{5}{6}$$

As $E(X) = \mu = 1\frac{17}{18}$, then $\mu^2 = 3\frac{253}{324}$:

$$Var(X) = E(X^2) - \mu^2$$
$$= 5\frac{5}{6} - 3\frac{253}{324}$$
$$= 2.052\,469\,136\ldots$$
$$\sigma = 1.43 \text{ (2 dec. pl.)}$$

(3 marks)

QUESTION 15

(a) Amplitude $= \dfrac{5-1}{2} = 2$

$\therefore a = 2$

$\therefore y = 2\cos(x + b) + c$

As the centre of motion is at $y = 3$, then $c = 3$.

$\therefore y = 2\cos(x + b) + 3$

Now, compared to $y = \cos x$, the graph has been shifted to the right $\dfrac{\pi}{4}$ units.

$\therefore b = \dfrac{\pi}{4}$

The equation of the curve is $y = 2\cos\left(x - \dfrac{\pi}{4}\right) + 3$, and hence

$a = 2, b = \dfrac{\pi}{4}$ and $c = 3$.

(3 marks)

(b) (i) Using $P = 435\,000, r = 0.025, n = 12$:

$$P = A(1 + r)^n$$
$$435\,000 = A \times 1.025^{12}$$
$$A = \frac{435\,000}{1.025^{12}}$$
$$= 323\,446.81$$
$$= 323\,447 \text{ (nearest whole)}$$

Kate paid \$323 447 for the house.

(2 marks)

(ii) Let $323\,447(1.025)^n = 305\,000(1.032)^n$

$$\frac{1.025^n}{1.032^n} = \frac{305\,000}{323\,447}$$
$$\left(\frac{1.025}{1.032}\right)^n = \frac{305\,000}{323\,447}$$
$$n \ln\left(\frac{1.025}{1.032}\right) = \ln\left(\frac{305\,000}{323\,447}\right)$$
$$n = \ln\left(\frac{305\,000}{323\,447}\right) \div \ln\left(\frac{1.025}{1.032}\right)$$
$$= 8.628\,125\,312\ldots$$

As $2007 + 8.628\ldots = 2015.628\ldots$, the two houses had the same value in 2015.

(3 marks)

QUESTION 16

(b)* From 60 years old to 90 years old is 30 years. Also, using 4% from the table:

$$PV = 60\,000 \times 17.2920$$
$$= 1\,037\,520$$

$\therefore$ Mel needs \$1 037 520 in her savings account when she turns 60.

Using $A = 1\,037\,520, r = 0.04$ and $n = 20$:

$$A = P(1 + r)^n$$
$$1\,037\,520 = P(1.04)^{20}$$
$$P = \frac{1037\,520}{1.04^{20}}$$
$$= 473\,510.5844\ldots$$
$$= 473\,511 \text{ (nearest whole)}$$

$\therefore$ Mel needs to deposit \$473 511 in her savings account today.

(4 marks)

(b)** (i) $E(X) = \displaystyle\int_0^1 x \times \frac{3}{13}\left(x^2 + 4\right) dx$

$$= \frac{3}{13}\int_0^1 (x^3 + 4x)$$
$$= \frac{3}{13}\left[\frac{x^4}{4} + 2x^2\right]_0^1$$
$$= \frac{3}{13}\left[\frac{1^4}{4} + 2(1)^2 - 0\right]$$
$$= \frac{3}{13} \times \frac{9}{4}$$
$$= \frac{27}{52}$$

(2 marks)

(ii) $E(X^2) = \displaystyle\int_0^1 x^2 \times \frac{3}{13}(x^2 + 4)\, dx$

$$= \frac{3}{13}\int_0^1 (x^3 + 4x)\, dx$$
$$= \frac{3}{13}\left[\frac{x^5}{5} + \frac{4x^3}{3}\right]_0^1$$
$$= \frac{3}{13}\left[\frac{1^5}{5} + \frac{4(1)^3}{3} - 0\right]$$
$$= \frac{3}{13} \times \frac{23}{15}$$
$$= \frac{23}{65}$$

(2 marks)

(c) (i) $r = 0.919\,549\,645\ldots$
$= 0.9195$ (4 dec. pl.)

(1 mark)

(ii) The line passes through the points (25, 16) and (37, 46).

$$\text{Gradient} = \frac{46-16}{37-25}$$
$$= \frac{5}{2}$$

The line has equation $D = 2.5T + c$
As the line passes through (25, 16), then

$16 = 2.5(25) + c$
$16 = 62.5 + c$
$c = -46.5$

The equation is $D = 2.5T - 46.5$

(2 marks)

(iii) The line is based on temperatures from 25 to 37 and 16 is beyond the bounds of the data points to which it is fitted. So, it is not valid to draw conclusions from it. *(1 mark)*

Replacement questions

with content from the most up-to-date syllabus

Marks

1 If X is a random variable such that $P(X > 2) = c$ and $P(X > 5) = d$, then what is $P(X \le 2 | X \le 5)$? 1

A. $\dfrac{c}{d}$

B. $\dfrac{1-d}{1-c}$

C. $\dfrac{1-c}{1-d}$

D. $\dfrac{3-c}{d}$

Question 11 (2 marks)

(b) The time it takes to fly from Chicago to London varies inversely with the speed of the plane.

If the trip takes 9 hours at an average speed of 800 km/h, how long would it take at an average speed of 850 km/h?

Express your answer to the nearest minute. 2

Question 12 (6 marks)

(b) A pizza shop estimates that the time X, in hours, to deliver a pizza from when it is ordered is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{2}{5}(3-x) & 0 \le x \le 1 \\ 0 & \textit{elsewhere} \end{cases}$$

(i) What is the probability of the pizza being delivered within 30 minutes of being ordered? 1

(ii) Calculate the mean delivery time. 2

(iii) Find the standard deviation of the delivery time to the nearest minute. 3

Question 13 (4 marks)

Marks

(b) Let $f(x) = \dfrac{x}{x-1}$.

(i) Show $\dfrac{x}{x-1} = 1 + \dfrac{1}{x-1}$. **2**

(ii) Hence, or otherwise, on the same number plane, draw sketches of

$y = \dfrac{x}{x-1}$ and $y = \dfrac{1}{x-1}$. **2**

Question 14 (6 marks)

(b) Find the coordinates of the point P on the line $x = -1$, such that the line joining P to $Q(5, -3)$ is perpendicular to the line $3x - 2y = 6$. **3**

(c) A table of future value interest factors is used to compare annuities of \$1.

Table of future value interest factors

Period	Interest rate per period					
	0.50%	1.50%	2.00%	3.00%	4.00%	6.00%
4	4.0301	4.0909	4.1216	4.1836	4.2465	4.3746
8	8.1414	8.4328	8.5830	8.8923	9.2142	9.8975
16	16.6142	17.9324	18.6393	20.1569	21.8245	25.6725
32	34.6086	40.6883	44.2270	52.5028	62.7015	90.8898
48	54.0978	69.5652	79.3535	104.4084	139.2632	256.5645
96	122.8285	211.7202	284.6467	535.8502	1054.2960	4462.6505

Lucy and Archie were offered interest of 6% p.a. for 4 years.

Lucy invested \$1500 each quarter compounded quarterly.

Archie invested \$500 every month compounded monthly.

Which person earned the greater amount of interest? Support your answer with calculations. **3**

Marks

Question 15 (7 marks)

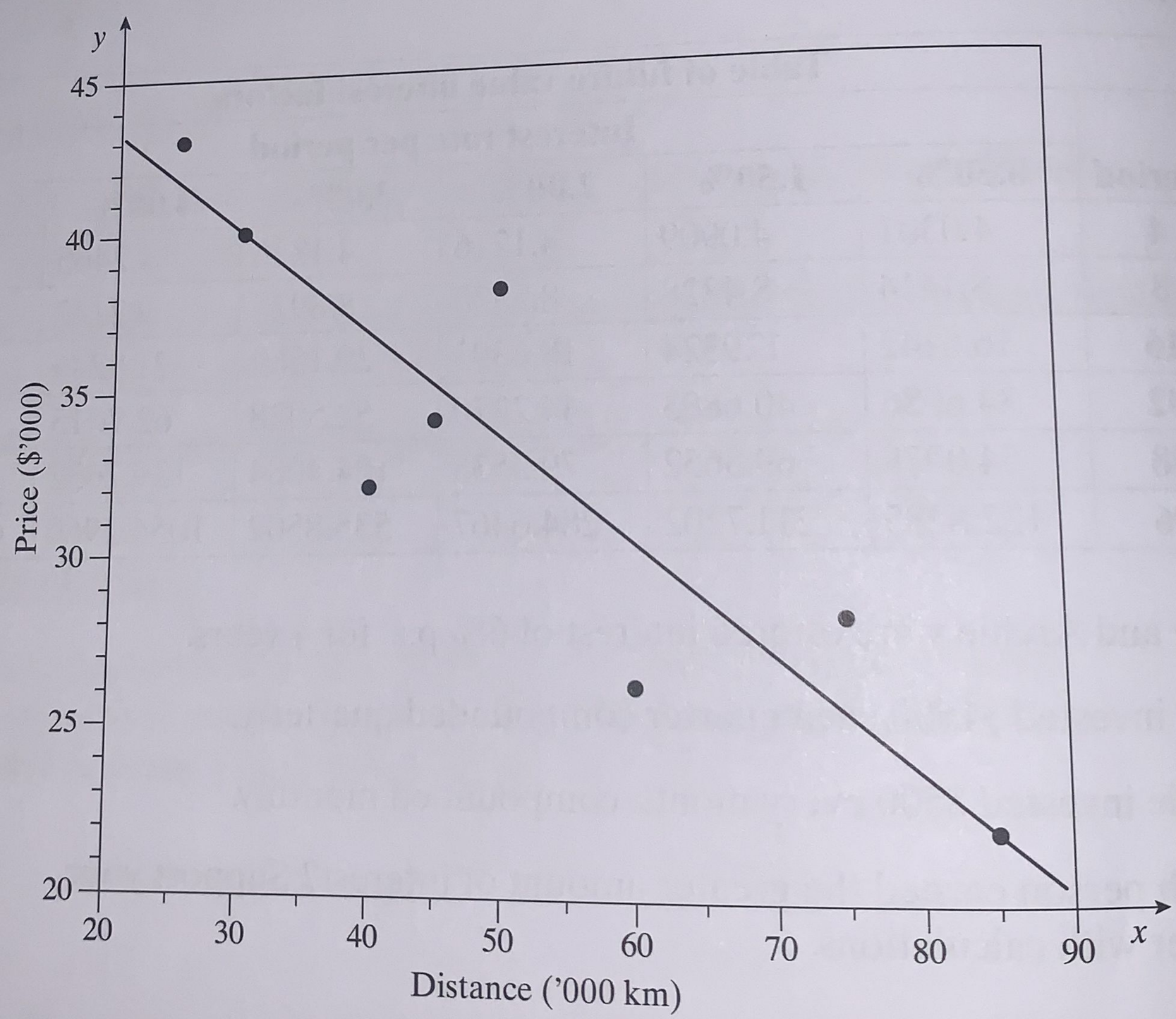
(b) Jim is a plumber and wants to buy a second-hand work truck.

The following table shows the price (y) in dollars of eight trucks, and the distance (x) in kilometres they had travelled.

Distance travelled (x km)	25 000	60 000	85 000	40 000	50 000	75 000	30 000	45 000
Price (y dollars)	43 000	26 000	22 000	32 000	38 000	28 000	40 000	34 000

(i) Comment on the association between x and y with reference to the value of Pearson's correlation coefficient. 2

(ii) The data is recorded on a scatterplot and Jim has drawn a regression line by eye.



The relationship between x and y can be modelled by the regression equation $y = mx + c$.

Find the values of m and c for Jim's regression line. Give your answers correct to 2 decimal places. 2

REPLACEMENT QUESTIONS

Marks

(iii) Use Jim's regression equation to estimate the price of a vehicle that has travelled 55 000 kilometres. **1**

(iv) At the start of 2020, Jim decides to buy the truck that had travelled 55 000 kilometres. The value of the truck decreases by 25% each year. During which year will the truck be valued at less than \$5000? **2**

Question 16 (3 marks)

(a) Use the trapezoidal rule with five function values to show that $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \, dx \approx \frac{\pi}{6}\left[\frac{4+3\sqrt{3}}{\sqrt{3}}\right]$. **3**

Solutions to replacement questions

QUESTION 1

As $P(X > 2) = c$, then $P(X \le 2) = 1 - c$, and similarly, $P(X \le 5) = 1 - d$.

$$P(X \le 2 | X \le 5) = \frac{P(X \le 2 \cap X \le 5)}{P(X \le 5)}$$
$$= \frac{P(X \le 2)}{P(X \le 5)}$$
$$= \frac{1-c}{1-d}$$

(1 mark)

Answer C

QUESTION 11

(b) Let T = time and s = speed:

$$T = \frac{k}{s}$$

Substitute $T = 9, s = 800$:

$$9 = \frac{k}{800}$$
$$k = 7200$$
$$T = \frac{7200}{s}$$

Substitute $s = 850$:

$$T = \frac{7200}{850}$$
$$= 8.470588235\ldots$$
$$= 8 \text{ h } 28 \text{ m (nearest minute)}$$

The time would be about 8 hours 28 minutes.

(2 marks)

QUESTION 12

(b) (i) $\displaystyle\int_0^{0.5}\left(\frac{2}{5}(3-x)\right)dx = \frac{2}{5}\int_0^{0.5}(3-x)dx$

$$= \frac{2}{5}\left[3x - \frac{x^2}{2}\right]_0^{0.5}$$
$$= \frac{2}{5}\left(3(0.5) - \frac{(0.5)^2}{2}\right)$$
$$= 0.55$$

(1 mark)

(ii) $\displaystyle E(X) = \frac{2}{5}\int_0^1 x(3-x)\,dx$

$$= \frac{2}{5}\int_0^1 (3x - x^2)\,dx$$
$$= \frac{2}{5}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= \frac{2}{5}\left(\frac{3(1)^2}{2} - \frac{1^3}{3} - 0\right)$$
$$= 0.466666666\ldots$$
$$= 0.467 \text{ (3 dec. pl.)}$$

$\therefore$ the mean time is 0.467 hours or 28 minutes.

(2 marks)

(iii) $\displaystyle E(X^2) = \frac{2}{5}\int_0^1 x^2(3-x)\,dx$

$$= \frac{2}{5}\int_0^1 (3x^2 - x^3)\,dx$$
$$= \frac{2}{5}\left[x^3 - \frac{x^4}{4}\right]_0^1$$
$$= \frac{2}{5}\left(1^3 - \frac{1^4}{4} - 0\right)$$
$$= 0.3$$
$$Var(X) = E(X^2) - [E(X)]^2$$
$$= 0.3 - 0.467^2$$
$$= 0.082222222\ldots$$
$$\sigma = 0.286744176\ldots$$
$$= 0.29 \text{ (2 dec. pl.)}$$

$\therefore$ the standard deviation is 0.29 hours or 17 minutes (to the nearest min.).

(3 marks)

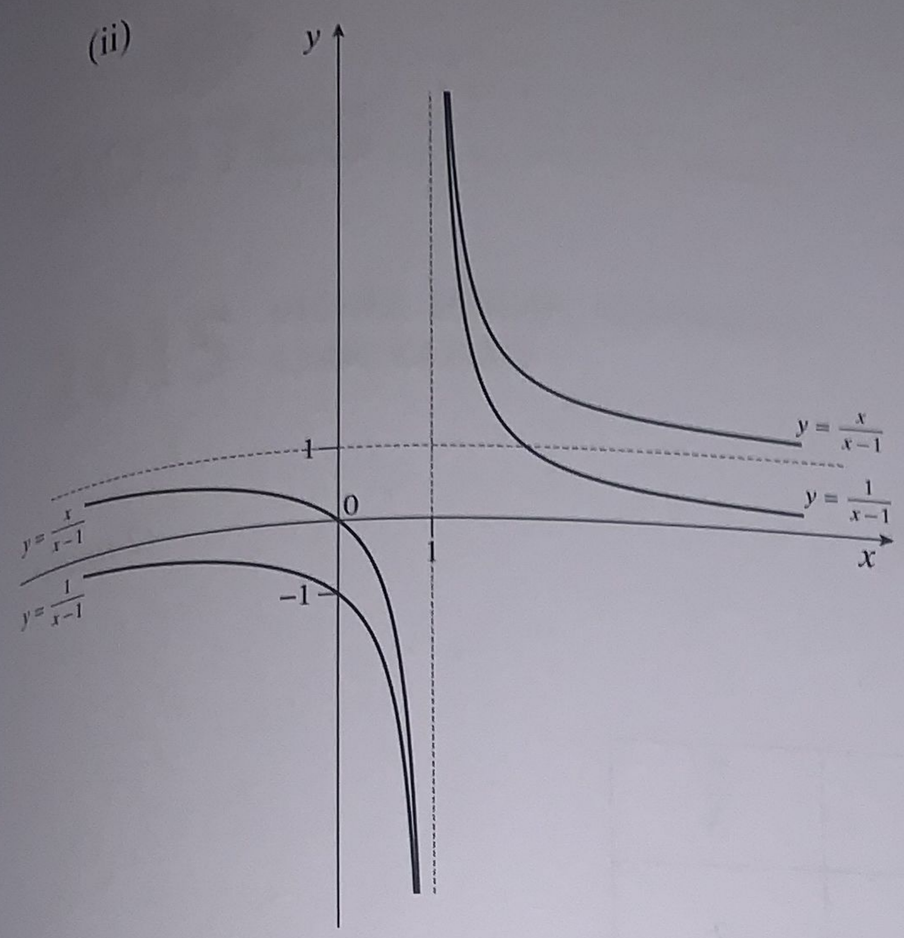
QUESTION 13

(b) (i) $\displaystyle \text{RHS} = 1 + \frac{1}{x-1}$

$$= \frac{x-1+1}{x-1}$$
$$= \frac{x}{x-1}$$
$$= \text{LHS}$$
$$\therefore \frac{x}{x-1} = 1 + \frac{1}{x-1}$$

(2 marks)

(ii)



(2 marks)

QUESTION 14

(b) For $3x - 2y = 6$

$$2y = 3x - 6$$

$$y = \frac{3x-6}{2}$$

$\therefore$ gradient $= \frac{3}{2}$

$\therefore$ gradient of perpendicular is $-\frac{2}{3}$.

Line passing through $Q(5, -3)$ with $m = -\frac{2}{3}$:

$$y + 3 = -\frac{2}{3}(x - 5)$$

Substitute $x = -1$:

$$y + 3 = -\frac{2}{3}(-1 - 5)$$

$$y + 3 = 4$$

$$y = 1$$

$\therefore P(-1, 1)$

(3 marks)

(c) Lucy: 6% p.a. = 1.5% per quarter,
4 years = 16 quarters

Future value $= 1500 \times 17.9324$

$= 26\,898.60$

Interest $= 26\,898.60 - 1500 \times 16$

$= 2898.60$

Archie: 6% p.a. = 0.5% per month,
4 years = 48 months

Future value $= 500 \times 54.0978$

$= 27\,048.90$

Interest $= 27\,048.90 - 500 \times 48$

$= 3048.90$

Difference $= 3048.90 - 2898.60$

$= 150.30$

Archie earned \$150.30 more interest than Lucy.

(3 marks)

QUESTION 15

(b) (i) Pearson's coefficient

$r = -0.900\,631\,491\ldots$

$= -0.9006$ (4 dec. pl.)

The association between distance and price is strong and negative.

(2 marks)

(ii) The line passes through the points $(30, 40)$ and $(85, 22)$.

Gradient $m = \frac{22-40}{85-30}$

$= -0.327\,272\,72\ldots$

$= -0.33$ (2 dec. pl.)

The line has equation $y = -0.33x + c$

Substitute $(30, 40)$:

$40 = -0.33(30) + c$

$c = 49.818\,1818\ldots$

$= 49.82$ (2 dec. pl.)

$\therefore m = -0.33$ and $c = 49.82$

(2 marks)

(iii) $y = -0.33x + 49.82$

Let $x = 55$:

$y = -0.33(55) + 49.82$

$= 31.67$

The price would be \$31 670.

(1 mark)

(iv)

$$5000 = 31\,670(0.75)^n$$

$$0.75^n = \frac{5000}{31\,670}$$

$$n\ln 0.75 = \ln\frac{5000}{31\,670}$$

$$n = \frac{\ln\frac{5000}{31\,670}}{\ln 0.75}$$

$$n = 6.416569279\ldots$$

During 2026 the truck will be worth less than \$5000.

(2 marks)

QUESTION 16

(a)

x	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$f(x)$	2	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	2
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(b)$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x\,dx \approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$$

$$= \frac{\frac{\pi}{3}-\left(-\frac{\pi}{3}\right)}{2(4)}\left\{2+2+2\left[\frac{2}{\sqrt{3}}+1+\frac{2}{\sqrt{3}}\right]\right\}$$

$$= \frac{\pi}{12}\left\{4+2\left[\frac{4+\sqrt{3}}{\sqrt{3}}\right]\right\}$$

$$= \frac{\pi}{6}\left[2+\frac{4+\sqrt{3}}{\sqrt{3}}\right]$$

$$= \frac{\pi}{6}\left[\frac{2\sqrt{3}+4+\sqrt{3}}{\sqrt{3}}\right]$$

$$= \frac{\pi}{6}\left[\frac{4+3\sqrt{3}}{\sqrt{3}}\right]$$

(3 marks)

Replacement questions

with content from the most up-to-date syllabus

Marks

1 Which of these is the derivative of $\log_2(x+1)$?

A. $\dfrac{2}{x+1}$

B. $\dfrac{1}{2(x+1)}$

C. $\dfrac{1}{(x+1)\ln 2}$

D. $\dfrac{1}{\ln(2x+1)}$ 1

Question 11 (3 marks)

(a) Find the maximum value of y given $y = 5 + 4x - 2x^2$. 1

(b) A and B are events of a sample space S.

$P(A \cap B) = \dfrac{2}{5}$ and $P(A \cap B') = \dfrac{3}{8}$

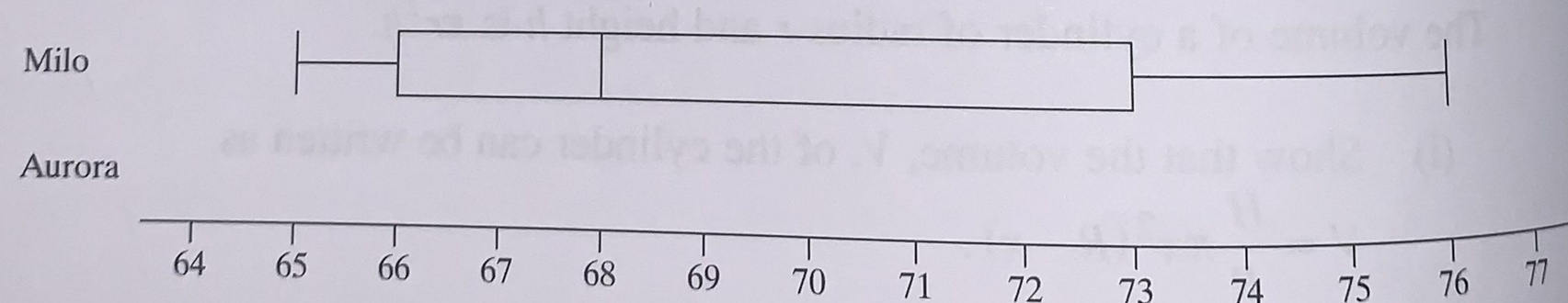
What is the value of $P(B'|A)$? 2

Question 12 (5 marks)

(e) Milo and Aurora each sat twelve class tests. Milo's results on the tests are displayed in the box-and-whisker plot.

(i) Aurora's 5-number summary for the tests is 65, 68, 70, 72, 75.

Draw a box-and-whisker plot to display Aurora's results below that of Milo's results. 1



(ii) What percentage of Milo's results were below 68? 1

(iii) Aurora claims that her results were better than Milo's. Is she correct?

Justify your answer by referring to the summary statistics and the skewness of the distributions. 3

Question 13 (2 marks)

Marks

(b) (ii) On the decibel scale, the loudness L of a sound of intensity S is given by $L = 10\log_{10}\left(\frac{S}{S_0}\right)$, where S_0 is a reference intensity used for comparisons. How many times more intense is the sound in the front row of a concert of a heavy metal band (110 decibels) than a normal conversation (60 decibels)? **2**

Question 15 (10 marks)

(a) A group of 20 000 students sat a state-wide examination.

The results of the examination were normally distributed.

In the examination, Freya achieved a score of 81, which corresponded to a z-score of 1.4.

In the same examination, Cooper achieved a score of 63, which corresponded to a z-score of –2.2.

(i) Using this information, form two equations and solve them simultaneously to find the values of the mean and the standard deviation. **3**

(ii) Hence, find the number of students who scored at least 69 in the examination. **2**

(b) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{3}{32}x(k-x) & 0 \le x \le k \\ 0 & \textit{elsewhere} \end{cases}$$

(i) Show that $k = 4$. **2**

(ii) Write down the value of $E(X)$. **1**

(iii) Calculate $Var(X)$. **2**

Marks

Question 16 (3 marks)

(b) A table of future values for an annuity of $1 is shown.

Table of future value interest factors

	Interest rate per period							
Period	**0.50%**	**1.00%**	**1.50%**	**2.00%**	**3.00%**	**4.00%**	**5.00%**	**6.00%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0050	2.0100	2.0150	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0150	3.0301	3.0452	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0301	4.0604	4.0909	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0503	5.1010	5.1523	5.2040	5.3091	5.4163	5.5256	5.6371
6	6.0755	6.1520	6.2296	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.1059	7.2135	7.3230	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.1414	8.2857	8.4328	8.5830	8.8923	9.2142	9.5491	9.8975

(i) Lincoln contributes $6000 at the end of each quarter into his superannuation account which gains interest at 6% p.a. compounded quarterly for two years.

What is the total amount of interest earned? 2

(ii) Mia makes six-monthly contributions into her superannuation account.

What contribution would provide a future value of $67 581.48 after 4 years at 6% p.a. with interest compounded six-monthly? 1

Solutions to replacement questions

QUESTION 1

$$\frac{d}{dx}(\log_2(x+1)) = \frac{d}{dx}\left(\frac{\ln(x+1)}{\ln 2}\right)$$
$$= \frac{d}{dx}\left(\frac{1}{\ln 2}\ln(x+1)\right)$$
$$= \frac{1}{\ln 2}\cdot\frac{1}{x+1}$$
$$= \frac{1}{(x+1)\ln 2}$$

Answer C *(1 mark)*

QUESTION 11

(a) Using $x = -\frac{b}{2a}$:

$$x = -\frac{4}{2(-2)}$$
$$= 1$$
$$y(1) = 5 + 4(1) - 2(1)^2$$
$$= 7$$

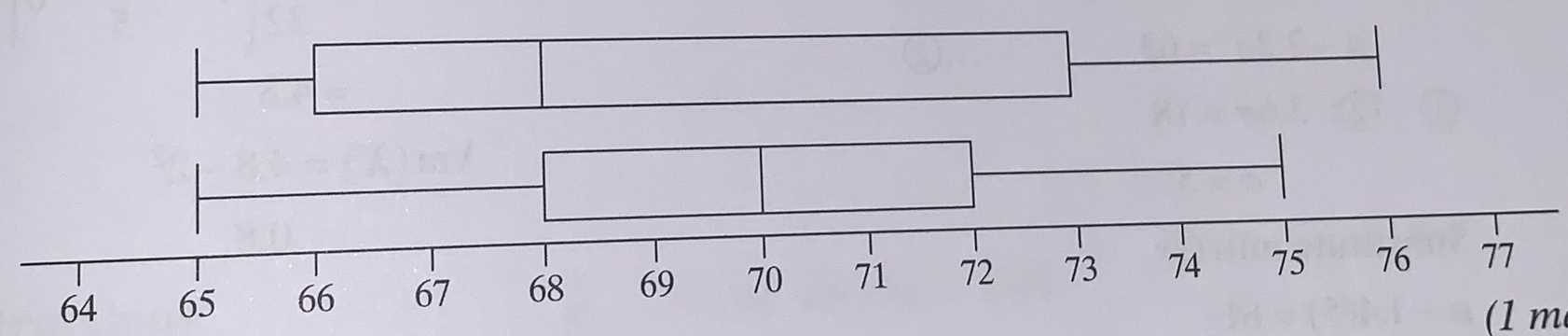
Maximum value of 7. *(1 mark)*

(b) $$P(B'|A) = \frac{P(B'\cap A)}{P(A)}$$
$$= \frac{P(B'\cap A)}{P(B'\cap A) + P(B\cap A)}$$
$$= \frac{\frac{3}{8}}{\frac{3}{8}+\frac{2}{5}}$$
$$= \frac{15}{31}$$

(2 marks)

QUESTION 12

(e) (i)



(1 mark)

(ii) 50% of Milo's scores are below 68. *(1 mark)*

(iii) Yes, Aurora's scores were better:

- Aurora's median (70) is higher than Milo's median (68)
- 75% of Aurora's scores are above 68 while only 50% of Milo's scores are above 68
- Aurora's scores are symmetrical whereas Milo's scores are positively skewed.

(3 marks)

QUESTION 13

(b) (ii) Difference in decibels $= 110 - 60$
$= 50$

$$L = 10\log_{10}\left(\frac{S}{S_0}\right)$$

$$50 = 10\log_{10}\left(\frac{S}{S_0}\right)$$

$$\log_{10}\left(\frac{S}{S_0}\right) = 5$$

$$\frac{S}{S_0} = 10^5$$

$$= 100000$$

It is 100000 times more intense.

QUESTION 15

(a) (i) Using $z = \dfrac{x-\mu}{\sigma}$:

Freya: $1.4 = \dfrac{81-\mu}{\sigma}$

$1.4\sigma = 81 - \mu$

$\mu + 1.4\sigma = 81$ ...①

Cooper: $-2.2 = \dfrac{63-\mu}{\sigma}$

$-2.2\sigma = 63 - \mu$

$\mu - 2.2\sigma = 63$ ...②

①−②: $3.6\sigma = 18$

$\sigma = 5$

Substitute into ①:

$\mu + 1.4(5) = 81$

$\mu + 7 = 81$

$\mu = 74$

Mean (μ) = 74 and standard deviation (σ) = 5. *(3 marks)*

(ii) As $69 = 74 - 5 = \mu - \sigma$, then 69 has a z-score of -1.

Hence $P(z \geq -1) = \dfrac{68\%}{2} + 50\%$

$= 84\%$

Number of students $= 0.84 \times 20000$

$= 16800$

(2 marks)

(b) (i) $\displaystyle\int_0^k \frac{3}{32}\left(kx - x^2\right)dx = 1$

$$\frac{3}{32}\left[\frac{kx^2}{2} - \frac{x^3}{3}\right]_0^k = 1$$

$$\frac{k(k)^2}{2} - \frac{k^3}{3} = \frac{32}{3}$$

$$3k^3 - 2k^3 = 64$$

$$k^3 = 64$$

$$k = 4$$

(2 marks)

(ii) As $f(x) = \dfrac{3}{32}\left(4x - x^2\right)$ is a parabola, then due to symmetry,

$E(X) = \dfrac{0+4}{2} = 2.$ *(1 mark)*

(iii) $E(X^2) = \displaystyle\int_0^4 \left(x^2 \times \frac{3}{32}\left(4x - x^2\right)\right)dx$

$$= \frac{3}{32}\int_0^4 (4x^3 - x^4)\,dx$$

$$= \frac{3}{32}\left[x^4 - \frac{x^5}{5}\right]_0^4$$

$$= \frac{3}{32}\left(4^4 - \frac{4^5}{5} - 0\right)$$

$$= 4.8$$

$\text{Var}(X) = 4.8 - 2^2$

$= 0.8$ *(2 marks)*

QUESTION 16

(b) (i) 1.5% per quarter, for 8 quarters:

Total contribution $= \$6000 \times 8$

$= \$48000$

Future value $= \$6000 \times 8.4328$

$= \$50596.80$

Amount of interest

$= \$50596.80 - \$48000$

$= \$2596.80$

Lincoln has earned interest of \$2596.80. *(2 marks)*

(ii) 3% per half-year, for 8 half-years:

Contribution $= \$67581.48 \div 8.8923$

$= \$7600$

Mia's contribution is \$7600 every six months. *(1 mark)*

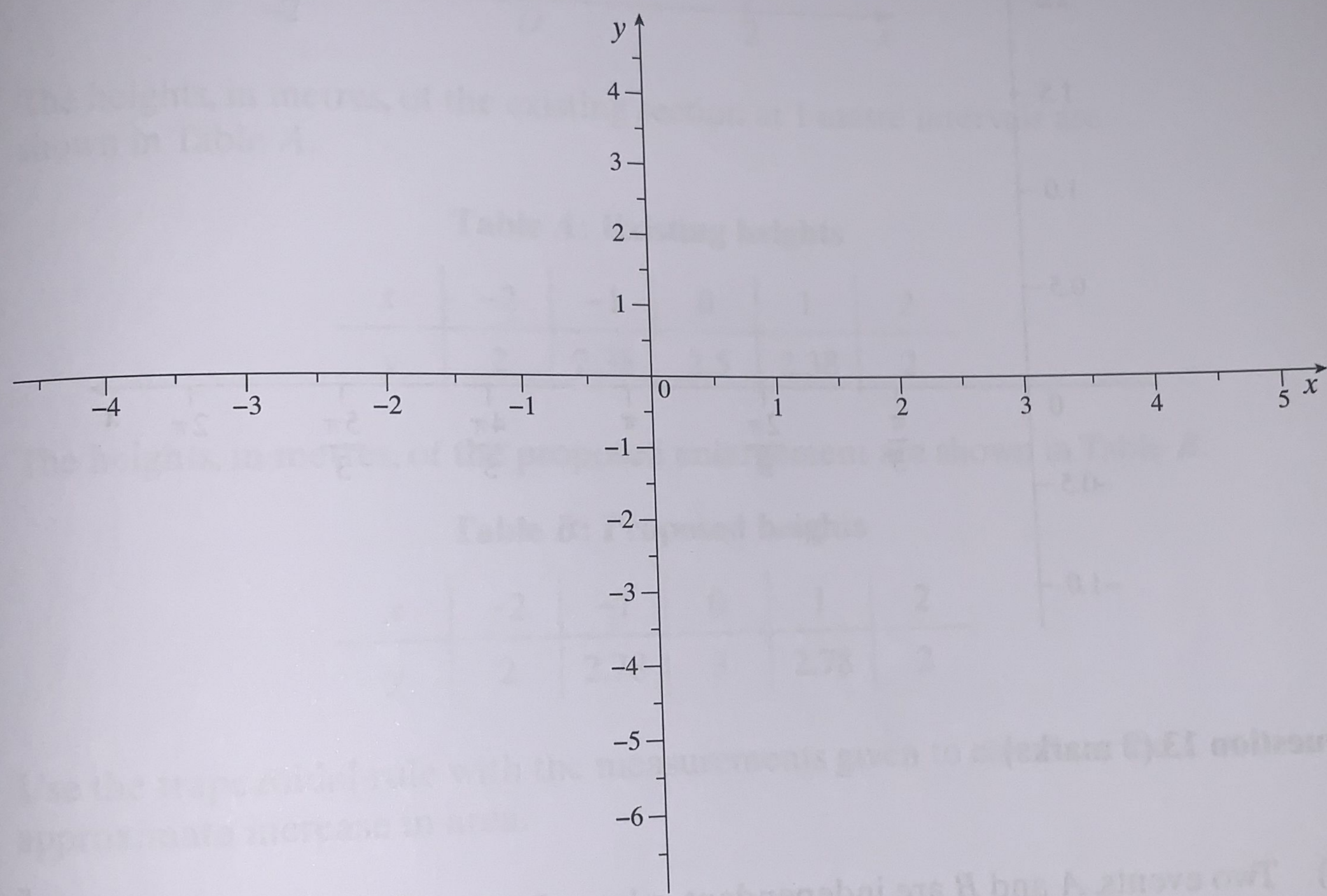
Replacement questions
with content from the most up-to-date syllabus

Question 11 (2 marks) Marks

(c) Find the range of values of k for which the expression $x^2 - 3x + (k + 2)$ is always positive. **2**

Question 12 (9 marks)

(a)



Line l cuts the axes at the points $(-2, 0)$ and $(0, 4)$.

The line m passes through $A(0, -6)$ and is parallel to line l.

The point $B(3, 0)$ lies on the line m.

Line n also passes through B and is perpendicular to line m.

The point C is the point of intersection of lines l and n.

(i) What are the coordinates of C? **3**

(ii) A circle is drawn with centre B and passing through C.

The circle cuts line n again at D.

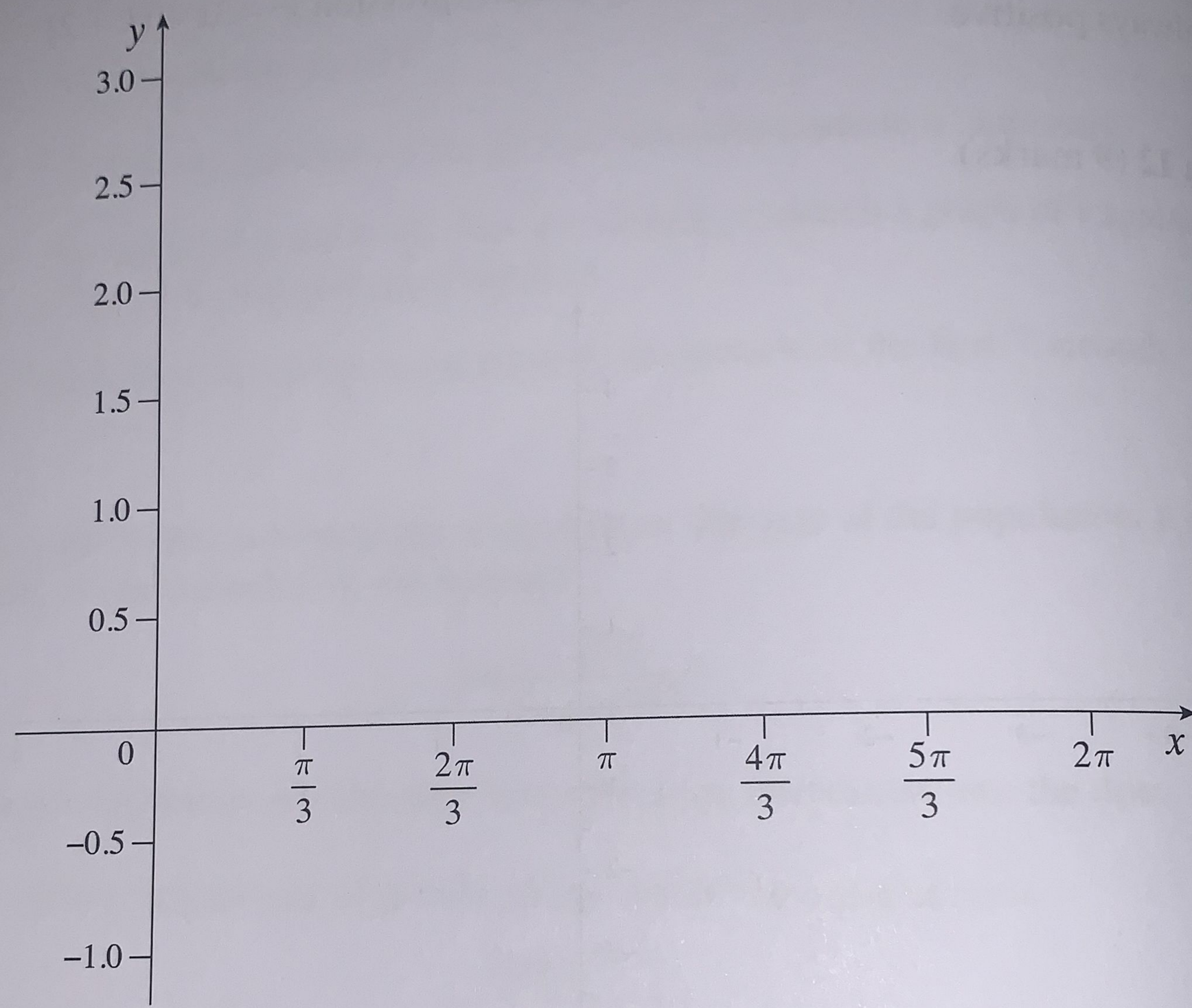
Solve a pair of simultaneous equations to find the coordinates of D. **3**

Marks

Question 12 (continued)

(b) (i) Write down the range of $f(x) = 2\cos x$. 1

(ii) Sketch $y = f\left(x + \frac{\pi}{3}\right) + 1$, for $0 \le x \le 2\pi$, on the number plane below. 2



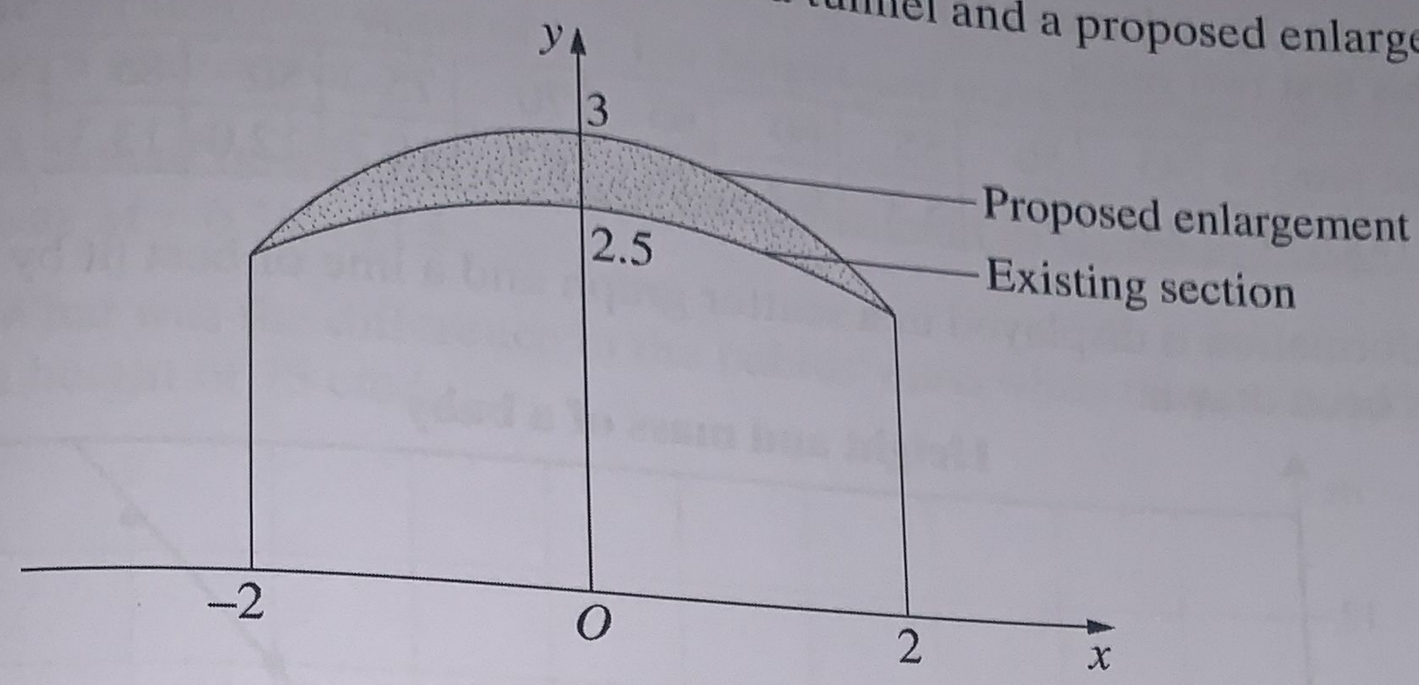
Question 13 (3 marks)

(b) Two events, A and B are independent, where $P(B) = 2P(A)$ and $P(A \cup B) = \frac{5}{8}$. What is the value of $P(A)$? 3

Question 14 (3 marks)

Marks

(a) The diagram shows the cross-section of a tunnel and a proposed enlargement.



The heights, in metres, of the existing section at 1-metre intervals are shown in Table A.

Table A: Existing heights

x	−2	−1	0	1	2
y	2	2.38	2.5	2.38	2

The heights, in metres, of the proposed enlargement are shown in Table B.

Table B: Proposed heights

x	−2	−1	0	1	2
y	2	2.78	3	2.78	2

Use the trapezoidal rule with the measurements given to calculate the approximate increase in area. **3**

Question 15 (10 marks)

(a) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{3}{7}(x^2+2) & 0 \le x \le 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Find, correct to 2 decimal places:

(i) $P(X \le 0.8)$. **2**

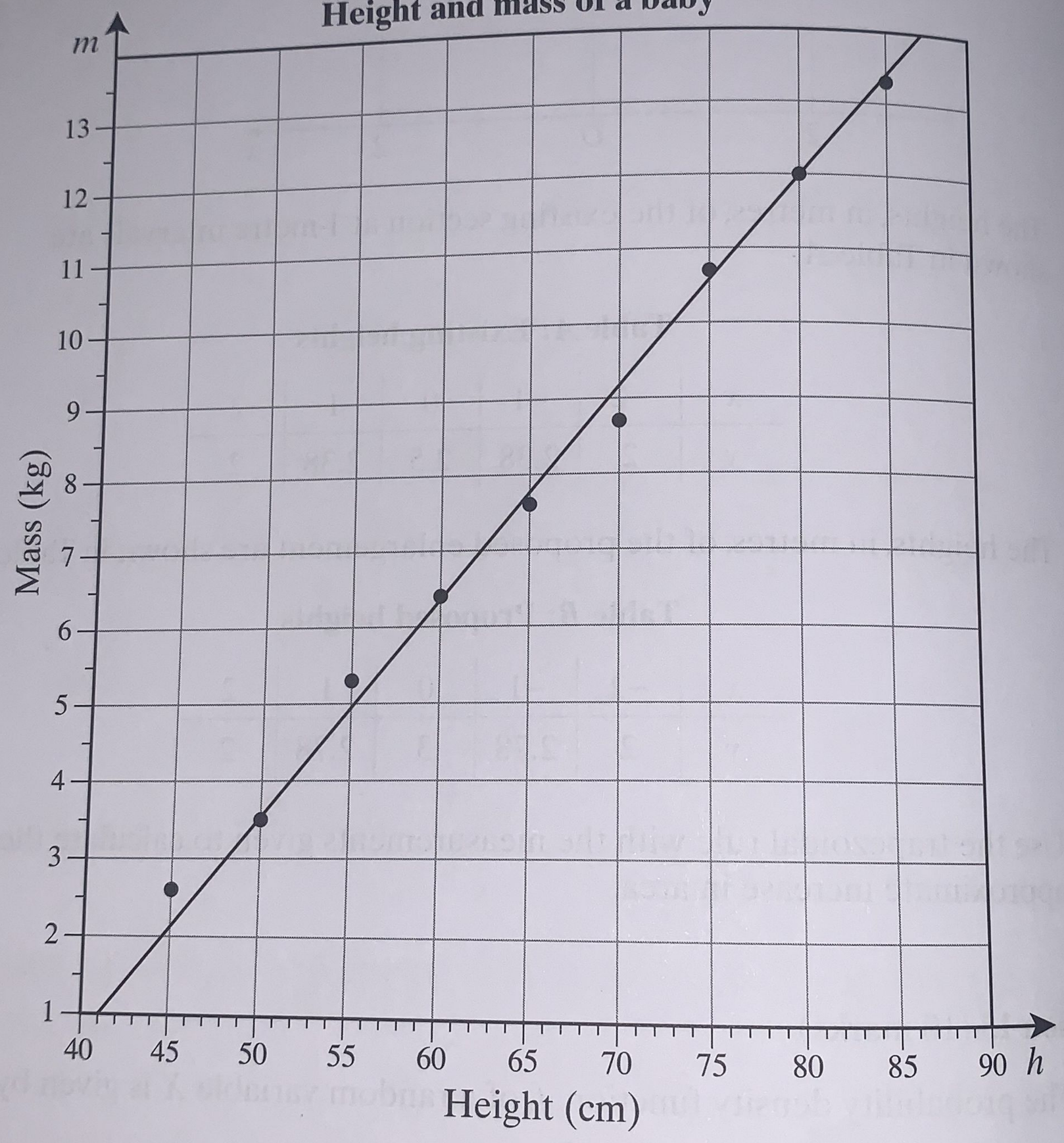
(ii) the mean. **2**

Question 15 (continued) Marks

(c) The height (h) and mass (M) of Baby Logan are measured and recorded over his first two years.

Height (cm), h	45	50	55	60	65	70	75	80	85
Mass (kg), M	2.6	3.5	5.3	6.4	7.6	8.7	10.7	12.0	13.3

This information is displayed in a scatter graph and a line of best fit by eye has been drawn.



(i) Calculate Pearson's coefficient (r), correct to 3 decimal places. 1

(ii) Hence, comment on the correlation between Logan's height and mass. 1

(iii) Explain why the equation of this line of best fit is approximately $M = 0.28h - 10.4$. 2

(iv) Based on the line of best fit, what is Logan's height when he has a mass of 12.5 kg? Leave your answer correct to 1 decimal place. 1

Question 15 (continued)

Marks

(v) Logan's cousin Ella is born five weeks later.

In her first two years of life her height and mass were also plotted on a scatterplot.

A line of best fit for her data was drawn and its equation was $M = 0.3h - 11.5$.

What was the difference in the babies' mass when they both had a height of 75 cm? **1**

REPLACEMENT QUESTION

Solutions to replacement questions

QUESTION 11

(c) $\Delta = b^2 - 4ac$

$= (-3)^2 - 4(1)(k+2) < 0$

$9 - 4k - 8 < 0$

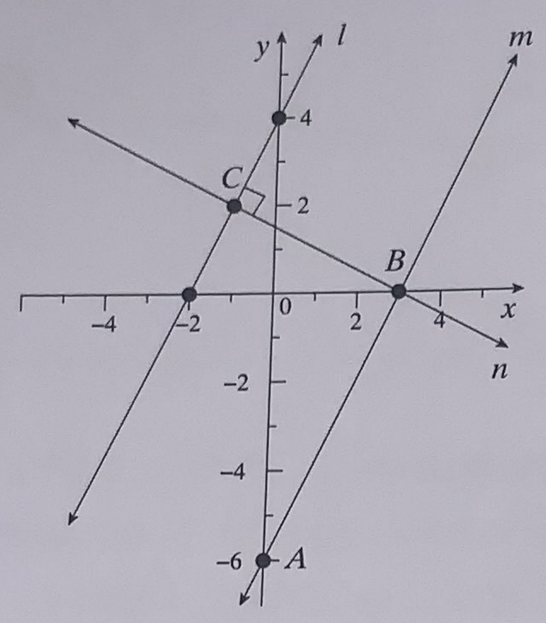
$1 - 4k < 0$

$4k > 1$

$k > \frac{1}{4}$

As $a > 0$ and $\Delta < 0$, then $k > \frac{1}{4}$. *(2 marks)*

QUESTION 12



(a) (i) Line l: gradient $= \frac{4}{2} = 2$ and

y-intercept $= 4$

$\therefore y = 2x + 4$

Line m: gradient $= 2$ and

y-intercept $= -6$

$\therefore y = 2x - 6$

Line n: gradient $= -\frac{1}{2}$ and passes through $(3, 0)$

$y - 0 = -\frac{1}{2}(x - 3)$

$2y = -x + 3$

$x + 2y - 3 = 0$

Coordinates of C is point of intersection of $y = 2x + 4$ and $x + 2y - 3 = 0$.

$\therefore x + 2(2x + 4) - 3 = 0$

$x + 4x + 8 - 3 = 0$

$5x + 5 = 0$

$5x = -5$

$x = -1$

Subs. in $y = 2x + 4$:

$y = 2(-1) + 4$

$= 2 \quad \therefore C(-1, 2)$ *(3 marks)*

(ii) Length $BC = \sqrt{((3-(-1))^2 + (0-2)^2}$

$= \sqrt{20}$

Circle with centre $(3, 0)$ and radius $\sqrt{20}$ is $(x-3)^2 + y^2 = 20$.

Coordinates of D is point of intersection of $x + 2y - 3 = 0$ and $(x-3)^2 + y^2 = 20$.

Substitute $x = 3 - 2y$ in $(x-3)^2 + y^2 = 20$:

$(3 - 2y - 3)^2 + y^2 = 20$

$4y^2 + y^2 = 20$

$5y^2 = 20$

$y = \pm 2$

For point D, substitute $y = -2$ in $x + 2y - 3 = 0$:

$x + 2(-2) - 3 = 0$

$x = 7 \quad \therefore D(7, -2)$

(3 marks)

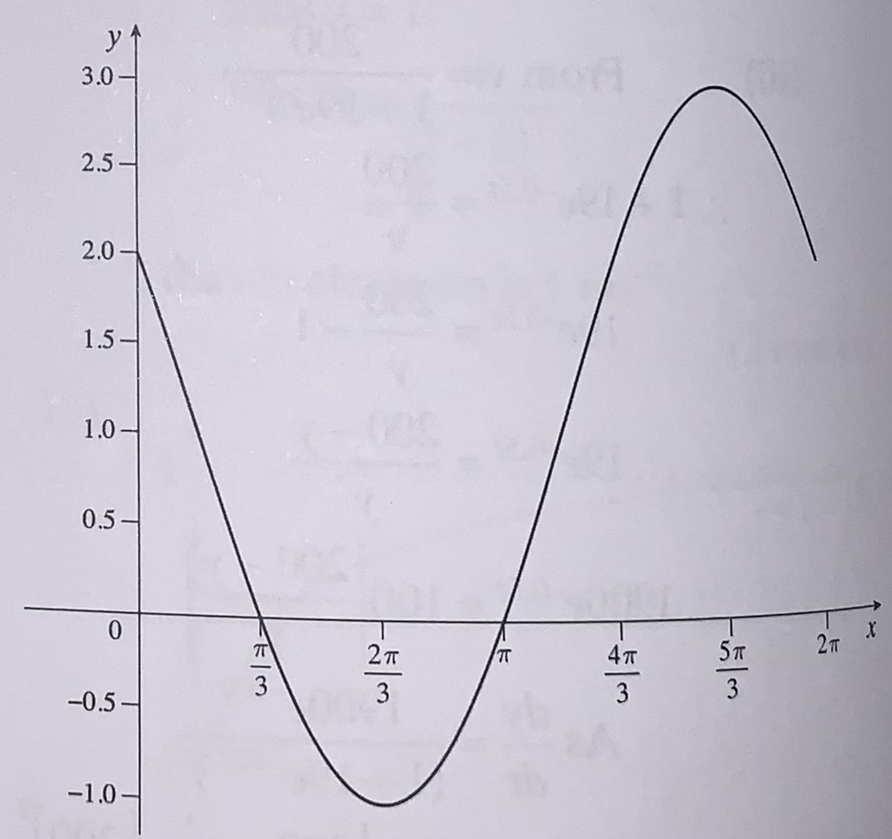
(b) (i) For $f(x) = 2\cos x$,

range $= -2 \le f(x) \le 2$. *(1 mark)*

(ii) $f\left(x + \frac{\pi}{3}\right) + 1 = 2\cos\left(x + \frac{\pi}{3}\right) + 1$.

The new range will be $-1 \le f(x) \le 3$.

To sketch $y = 2\cos\left(x + \frac{\pi}{3}\right) + 1$, substitute values for x.



(2 marks)

QUESTION 13

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= P(A) + P(B) - P(A).P(B)$$

$$\frac{5}{8} = P(A) + 2P(A) - P(A).2P(A)$$

$$\frac{5}{8} = 3P(A) - 2[P(A)]^2$$

$$2[P(A)]^2 - 3P(A) + \frac{5}{8} = 0$$

Let $m = P(A)$:

$$2m^2 - 3m + \frac{5}{8} = 0$$

$$16m^2 - 24m + 5 = 0$$

$$(4m-5)(4m-1) = 0$$

$$m = \frac{5}{4}, \frac{1}{4}$$

$$\therefore P(A) = \frac{1}{4} \text{ (as } 0 \le P(A) \le 1)$$

(3 marks)

QUESTION 14

(a) Form a table using the differences in the y-values:

x	–2	–1	0	1	2
Difference in y	0	0.4	0.5	0.4	0
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(b)$

Using trapezoidal rule:

$$\text{Increase} \approx \frac{b-a}{2n}\{f(a) + f(b) + 2[f(x_1) + \ldots + f(x_{n-1})]\}$$

$$= \frac{2-(-2)}{2(4)}\{0 + 0 + 2[0.4 + 0.5 + 0.4]\}$$

$$= \frac{1}{2}(2.6)$$

$$= 1.3 \qquad \therefore \text{ the increase is } 1.3 \text{ m}^2$$

(3 marks)

QUESTION 15

(a) (i) $\int_0^{0.8} \frac{3}{7}(x^2+2)\,dx = \frac{3}{7}\int_0^{0.8}(x^2+2)\,dx$

$= \frac{3}{7}\left[\frac{x^3}{3}+2x\right]_0^{0.8}$

$= \frac{3}{7}\left(\frac{0.8^3}{3}+2(0.8)-0\right)$

$= 0.758\,857\,142\ldots$

$= 0.76$ (2 dec. pl.)

(2 marks)

(ii) $\mu = \int_0^1 x \times \frac{3}{7}(x^2+2)dx$

$= \frac{3}{7}\int_0^1 (x^3+2x)dx$

$= \frac{3}{7}\left[\frac{x^4}{4}+x^2\right]_0^1$

$= \frac{3}{7}\left[\frac{1^4}{4}+1^2-0\right]$

$= 0.535\,714\,285\ldots$

$= 0.54$ (2 dec. pl.)

(2 marks)

(c) (i) $r = 0.997\,940\,765\ldots$

$= 0.998$ (3 dec. pl.)

(1 mark)

(ii) As 0.998 is close to 1, there is a very strong positive correlation between the height and mass of this baby.

(1 mark)

(iii) The line passes through the points (50, 3.5) and (80, 12).

Gradient $= \frac{12-3.5}{80-50}$

$= 0.283\,33\ldots$

$= 0.28$ (2 dec. pl.)

The line has equation $M = 0.28h + c$

As the line passes through (80, 12), then

$12 = 0.28(80) + c$

$12 = 22.4 + c$

$c = -10.4$

The equation is $M = 0.28h - 10.4$

(2 marks)

(iv) Let $M = 12.5$:

$12.5 = 0.28h - 10.4$

$0.28h = 22.9$

$h = 81.785\,714\,29\ldots$

$= 81.8$ cm

Logan's height is about 81.8 cm.

(1 mark)

(v) Logan: $M = 0.28(75) - 10.4$

$= 10.6$

Ella: $M = 0.3(75) - 11.5$

$= 11$

Difference $= 11 - 10.6$

$= 0.4$

Ella's mass is 0.4 kg more than Logan at a height of 75 cm.

(1 mark)

Replacement questions
with content from the most up-to-date syllabus

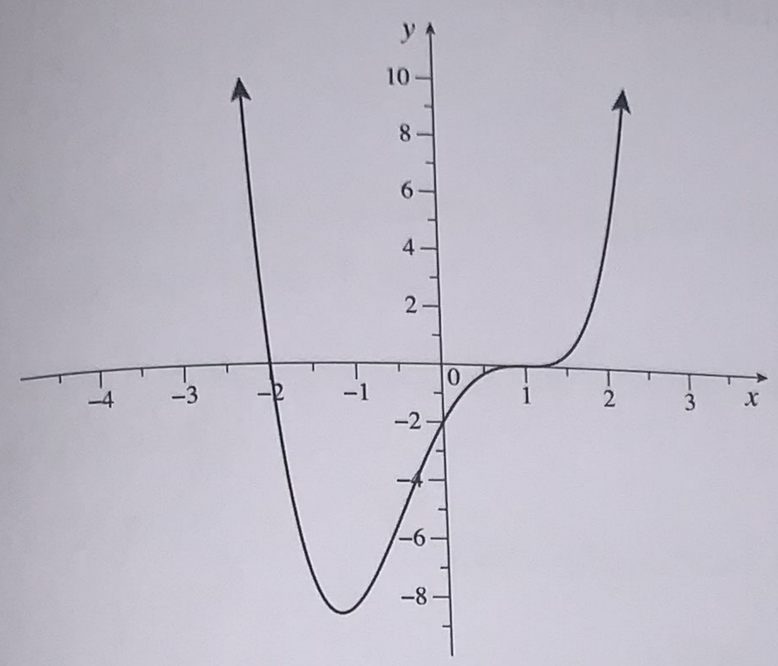
3

Marks

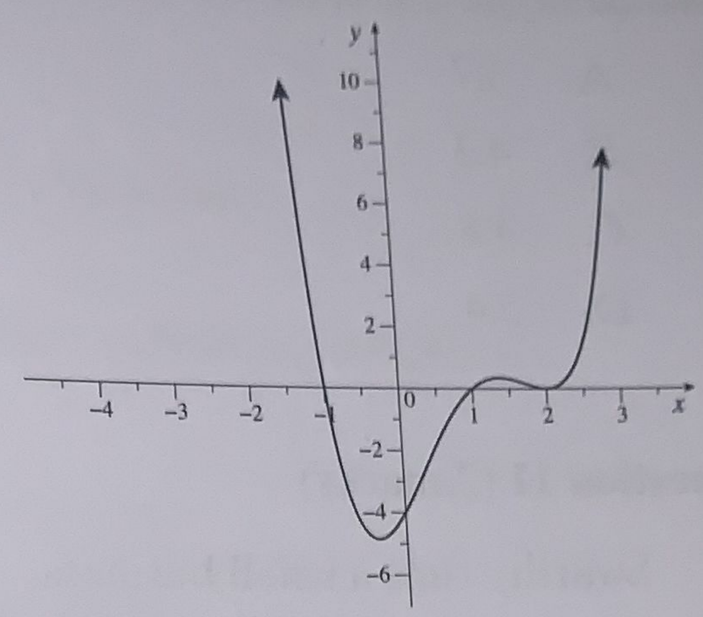
2 The function $y = (1 - x)(x + 1)(x - 2)^2$ is to be graphed.

Which of these is the graph of the function? **1**

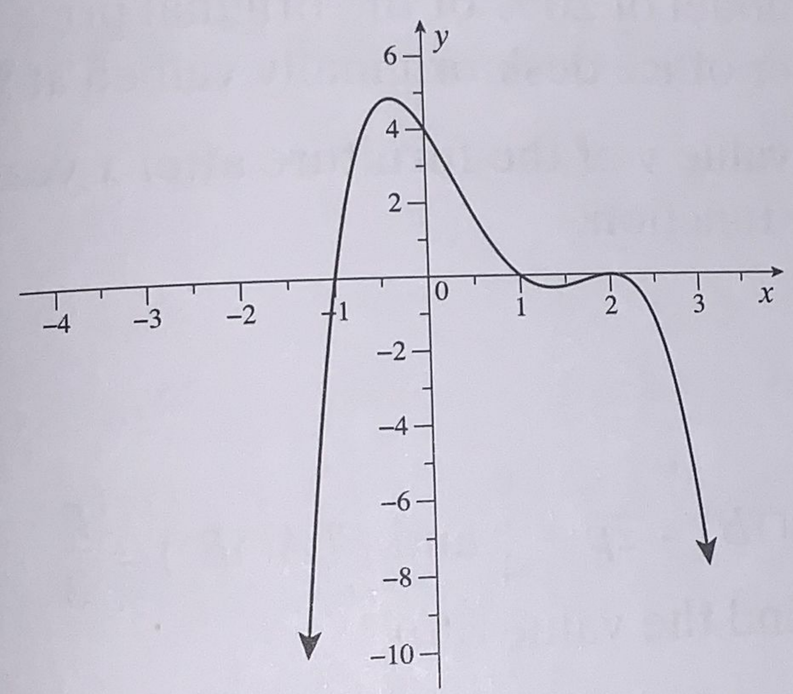
A.



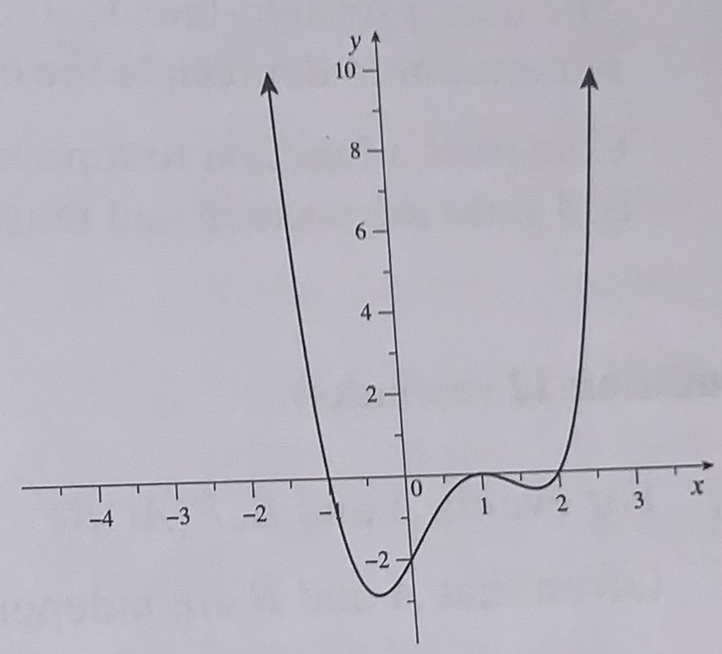
B.



C.



D.



3

2

6 On any given day, the number X of phone messages that Ella receives as she commutes to and from work is a random variable with a probability distribution given in the table below.

x	0	1	2	3	4	5
$P(X = x)$	0.2	0.3	0.2	0.15	0.1	0.05

1

What is the mean of X?

A. 0.3

B. 1.5

C. 1.8

D. 2.5

Marks

Question 8 (2 marks)

The pH value of a solution is given by the formula $pH = -\log_{10}[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in moles per litre.

Tylah tested a quantity of apple juice and determined it had a $H^+ = 3 \times 10^{-4}$.

Sean tested a quantity of ammonia and determined it had a $H^+ = 1.3 \times 10^{-9}$.

Which of these is closest to the difference in their pH? 1

A. 1.7

B. 4.3

C. 4.8

D. 5.4

Question 11 (2 marks)

(f) Swarthy runs a small business.

She uses a straight-line depreciation model of 20% of the original price per annum to depreciate the cost of her office desk originally valued at \$1850.

Construct a function to represent the value y of the furniture after x years, and state the domain and range of the function. 2

Question 12 (6 marks)

(b) For events A and B, $P(A \cap B) = p$, $P(A' \cap B) = 2p - \frac{1}{4}$ and $P(A \cap B') = \frac{p}{3}$.

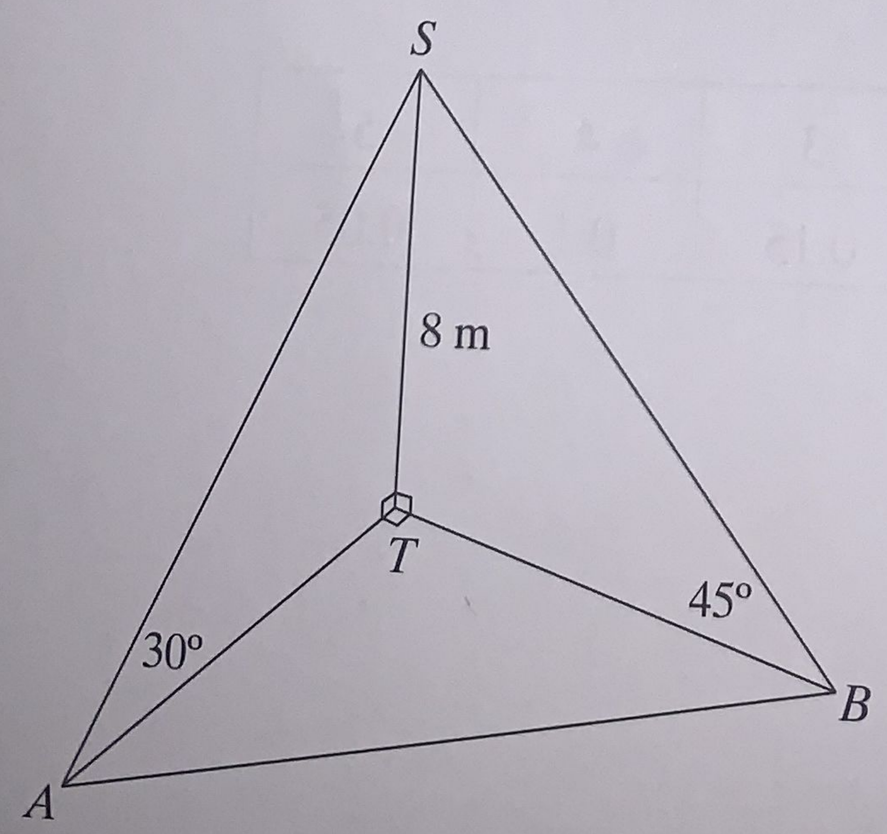
Given that A and B are independent, find the value of p. 3

(d) An 8-metre-high vertical flagpole ST is situated on flat ground.

The angle of elevation from A to the top of the tower S is 30°.

The angle of elevation from B to the top of the tower S is 45°.

B is south of the tower, and A is west of the tower.



Find the distance from A to B. 3

REPLACEMENT QUESTIONS

Question 14 (9 marks)

Marks

(b) (i) Find the value of $\int_0^{\pi} \sin x \, dx$. **1**

(ii) Using the trapezoidal rule with seven function values, find an approximation to the integral $\int_0^{\pi} \sin x \, dx$, leaving your answer in terms of π and $\sqrt{3}$. **2**

(iii) Using parts (i) and (ii), explain why $\dfrac{12}{2+\sqrt{3}}$ is a poor approximation for π. **1**

(c) The table below details the present value interest factors to compare annuities of \$1.

Table of present value interest factors

Period	Interest rate per period							
	0.45%	**0.50%**	**0.75%**	**1.00%**	**1.25%**	**1.50%**	**1.75%**	**2.00%**
60	52.4796	51.7256	48.1734	44.9550	42.0346	39.3803	36.9640	34.7609
120	92.5656	90.0735	78.9417	69.7005	61.9828	55.4985	50.0171	45.3554
180	123.1851	118.5035	98.5934	83.3217	71.4496	62.0956	54.6265	48.5844
240	146.5735	139.5808	111.1450	90.8194	75.9423	64.7957	56.2543	49.5686
300	164.4385	155.2069	119.1616	94.9466	78.0743	65.9009	56.8291	49.8685
360	178.0846	166.7916	124.2819	97.2183	79.0861	66.3532	57.0320	49.9599

Morgan has two choices for a loan of \$120 000. Loan A can be taken over a period of 20 years at 6% p.a. compounded monthly or loan B is for a period of 30 years at 5.4% p.a. compounded monthly.

(i) Which loan has the higher monthly instalment, and by how much? **3**

(ii) What is the difference in the interest paid on each of the loans? **2**

Marks

Question 15 (3 marks)

(a) Mae's class sits two Ancient History tests.

In the first test, the mean for the class was 74.2 and the standard deviation was 4.5.

In this test Mae scored 85.

(i) Find Mae's z-score in the first test. 1

(ii) In the second Ancient History test, the mean for the class was 78.6 and the standard deviation was 3.5. Calculate the mark that Mae needed to obtain in the second test to ensure that her performance relative to the class was maintained. 2

Question 16 (5 marks)

(c) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{3}{16}(x^2+5) & 0 \le x \le 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Find

(i) the mean 2

(ii) the standard deviation, correct to two decimal places. 3

Solutions to replacement questions

QUESTION 2

The function has zeros (x-intercepts) at $x = -1, x = 1$ and $x = 2$ (twice).

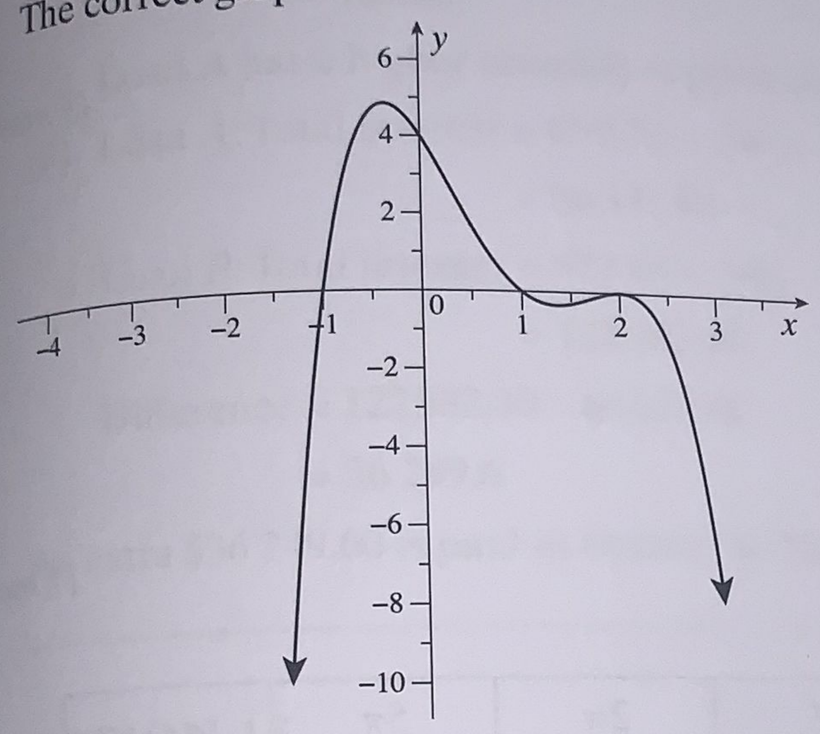
Substitute $x = 0$:

$y(0) = (1 - 0)(0 + 1)(0 - 2)^2$

$= 4$

The curve passes through $(0, 4)$.

The correct graph is



Answer C *(1 mark)*

QUESTION 6

$E(X) = 0(0.2) + 1(0.3) + 2(0.2) + 3(0.15) + 4(0.1) + 5(0.05)$

$= 1.8$

Answer C *(1 mark)*

QUESTION 8

(a) Apple juice: $pH = -\log_{10}[3 \times 10^{-4}]$

$= 3.522\,878\,745\ldots$

$= 3.5$ (1 dec. pl.)

Ammonia: $pH = -\log_{10}[1.3 \times 10^{-9}]$

$= 8.886\,095\,6648\ldots$

$= 8.9$ (1 dec. pl.)

Difference $= 8.9 - 3.5$

$= 5.4$

Answer D *(2 marks)*

QUESTION 11

(f) Depreciation $= 0.2 \times 1850$

$= 370$

$\therefore y = 1850 - 370x$

As $\frac{1850}{370} = 5$, then domain is $0 \le x \le 5$, and range is $0 \le y \le 1850$.

Function is $y = 1850 - 370x$, with domain $0 \le x \le 5$ and range $0 \le y \le 1850$.

(2 marks)

QUESTION 12

(b) $P(A \cap B) = p$, $P(A' \cap B) = 2p - \frac{1}{4}$,

$P(A \cap B') = \frac{p}{3}$.

$P(A) = P(A \cap B) + P(A \cap B')$

$= p + \frac{p}{3}$

$= \frac{4p}{3}$

$P(B) = P(B \cap A) + P(B \cap A')$

$= p + 2p - \frac{1}{4}$

$= 3p - \frac{1}{4}$

If A and B are independent then

$P(A).P(B) = P(A \cap B)$

$\frac{4p}{3}(3p - \frac{1}{4}) = p$

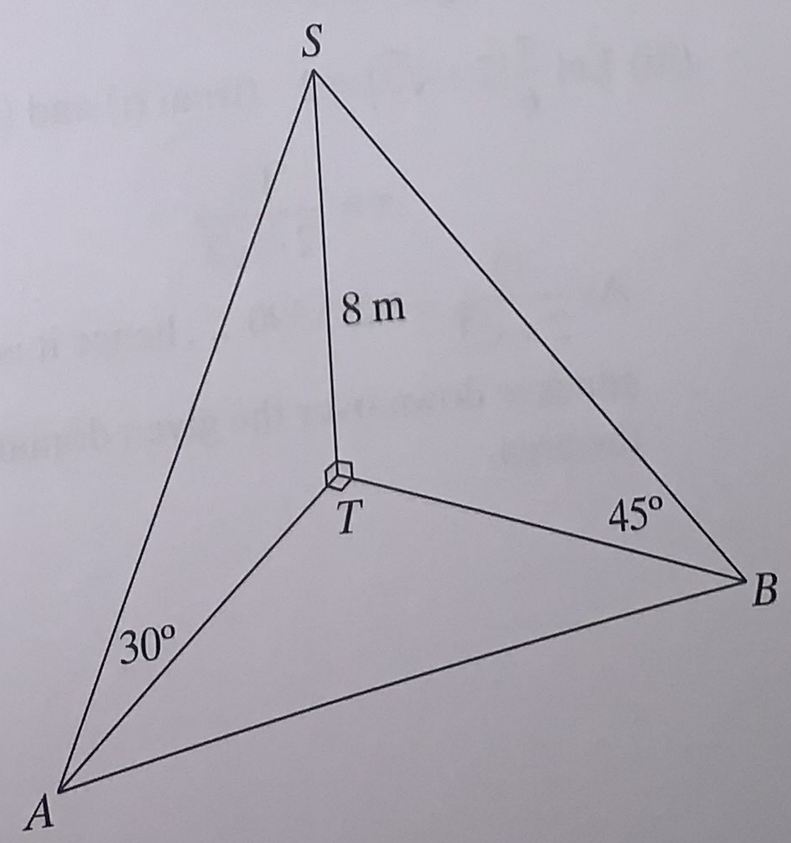
$12p^2 - p = 3p$

$12p^2 - 4p = 0$

$4p(3p - 1) = 0$

$p = \frac{1}{3}$ (as $p > \frac{1}{8}$) *(3 marks)*

(d)



$\angle AST = 60^\circ$

$\frac{AT}{8} = \tan 60^\circ$

$AT = 8\tan 60^\circ$

$= 8\sqrt{3}$

As ΔSBT is isosceles, $BT = 8$.

Let $AB = x$.

$x^2 = 8^2 + (8\sqrt{3})^2$ (by Pythagoras)

$= 64 + 192$

$= 256$

$x = 16$

The distance is 16 metres. *(3 marks)*

QUESTION 14

(b) (i) $\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi}$

$= -(\cos \pi - \cos 0)$

$= -(-1 - 1)$

$= 2$ *(1 mark)*

(ii)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$f(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0
	$f(a)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$	$f(x_5)$	$f(b)$

Using $\approx \frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+\ldots+f(x_{n-1})]\}$

$= \frac{\pi - 0}{2(6)}\left\{f(0)+f(\pi)+2\left[f\left(\frac{\pi}{6}\right)+f\left(\frac{\pi}{3}\right)+f\left(\frac{\pi}{2}\right)+f\left(\frac{2\pi}{3}\right)+f\left(\frac{5\pi}{6}\right)\right]\right\}$

$= \frac{\pi}{12}\left\{0+0+2\left[\frac{1}{2}+\frac{\sqrt{3}}{2}+1+\frac{\sqrt{3}}{2}+\frac{1}{2}\right]\right\}$

$= \frac{\pi}{12}(4+2\sqrt{3})$

$= \frac{\pi}{6}(2+\sqrt{3})$ *(2 marks)*

(iii) Let $\frac{\pi}{6}(2+\sqrt{3}) = 2$ (from (i) and (iii))

$\pi = \frac{12}{2+\sqrt{3}}$

As $\frac{12}{2+\sqrt{3}} = 3.215390\ldots$, hence it is a poor approximation. The graph of $y = \sin x$ is concave down over the given domain and so the trapezoidal rule will underestimate the area. *(1 mark)*

(c) (i) Loan A: 6% p.a. = 0.5% per month, 20 years = 240 months

Instalment $= 120\,000 \div 139.5808$

$= 859.717\,0958\ldots$

$= 859.72$

Loan B: 5.4% p.a. = 0.45% per month, 30 years = 360 months

Instalment $= 120\,000 \div 178.0846$

$= 673.837\,0415\ldots$

$= 673.84$

Difference $= 859.72 - 673.84$

$= 185.88$

Loan A has a higher monthly repayment of \$185.88

(ii) Loan A: Total interest $= 859.72 \times 240 - 120\,000$

$= 86\,332.80$ *(3 marks)*

Loan B: Total interest $= 673.84 \times 360 - 120\,000$

$= 122\,582.40$

Difference $= 122\,582.40 - 86\,332.80$

$= 36\,249.6$

An extra \$36 249.60 is paid in interest in Loan B. *(2 marks)*

QUESTION 15

(a) $z = \dfrac{x-\mu}{\sigma}$

$= \dfrac{x-78.6}{3.5}$

$= 2.4$ *(1 mark)*

(b) Mae's result in the second test needs to have a z-score of 2.4.

$2.4 = \dfrac{85-78.6}{3.5}$

$x - 78.6 = 8.4$

$x = 87$

Mae needs a mark of 87. *(2 marks)*

QUESTION 16

(c) (i) $\mu = \displaystyle\int_0^1 x \times \frac{3}{16}(x^2+5)\,dx$

$= \dfrac{3}{16}\displaystyle\int_0^1 (x^3+5x)\,dx$

$= \dfrac{3}{16}\left[\dfrac{x^4}{4} + \dfrac{5x^2}{2}\right]_0^1$

$= \dfrac{3}{16}\left[\dfrac{1^4}{4} + \dfrac{5(1)^2}{2} - 0\right]$

$= 0.515\,625$ *(2 marks)*

(ii) $E(X^2) = \displaystyle\int_0^1 x^2 \times \frac{3}{16}(x^2+5)\,dx$

$= \dfrac{3}{16}\displaystyle\int_0^1 (x^4+5x^2)\,dx$

$= \dfrac{3}{16}\left[\dfrac{x^5}{5} + \dfrac{5x^3}{3}\right]_0^1$

$= \dfrac{3}{16}\left[\dfrac{1^5}{5} + \dfrac{5(1)^3}{3} - 0\right]$

$= 0.35$

$\sigma^2 = E(X^2) - \mu^2$

$= 0.35 - 0.515\,625^2$

$= 0.084\,130\,859\ldots$

$\sigma = 0.290\,0532$

$= 0.29$ (2 dec. pl.) *(3 marks)*

Replacement questions

with content from the most up-to-date syllabus

Marks

1 Let f and g be two functions such that $f(3) = 2, f(5) = 3, g(2) = 3, g(4) = 5$.

What is the value of $f(g(4))$? **1**

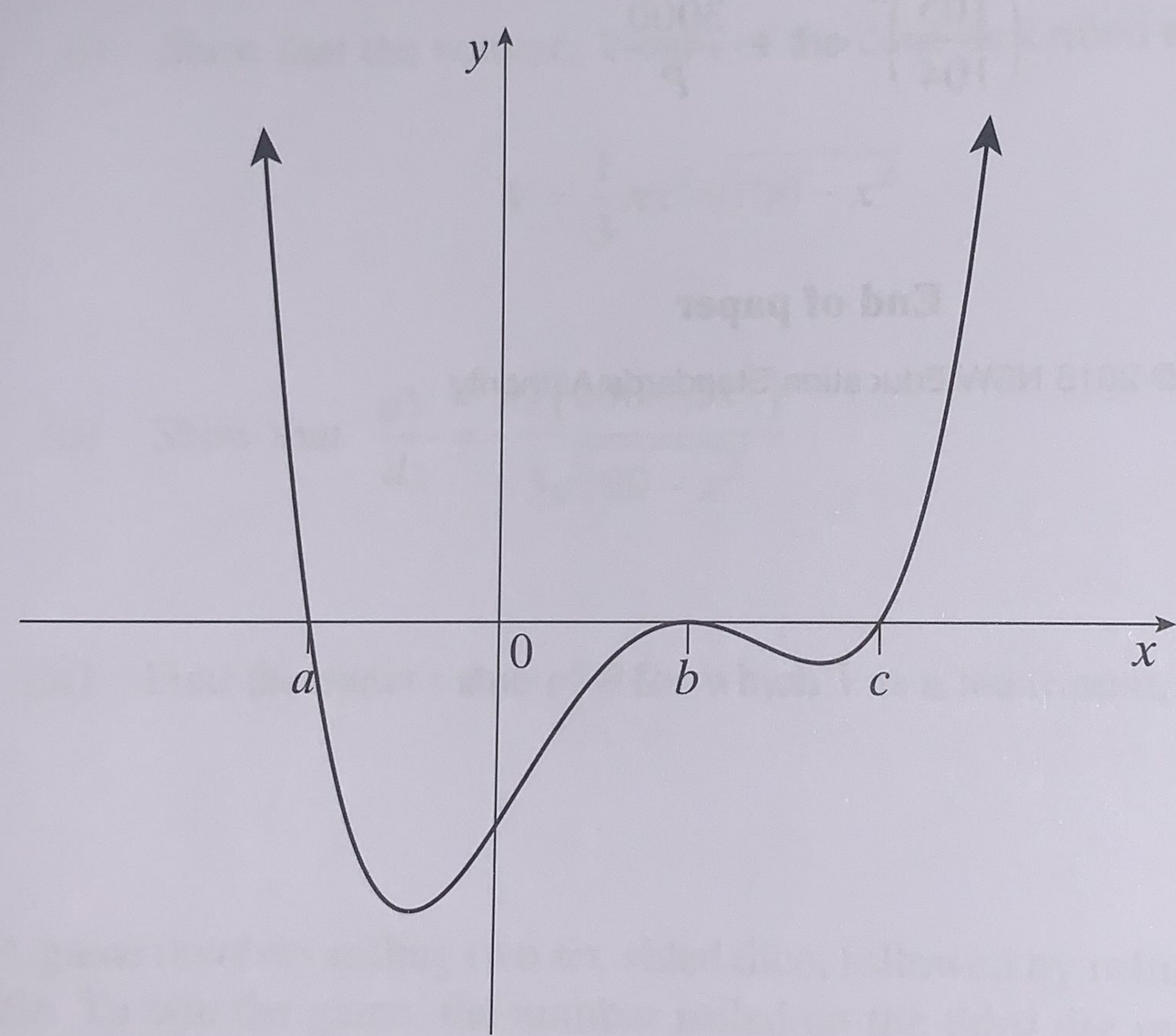
A. 2

B. 3

C. 4

D. 5

2 **1**



The rule for a function with the graph above could be

A. $y = -2(x-a)(x-b)^2(x-c)$

B. $y = 2(x-a)(x-b)^2(x-c)$

C. $y = 2(x+a)(x-b)^2(x-c)$

D. $y = 2(x-a)(x-b)(x-c)$

Marks

7 For the random variable, X, it is known that $E(X) = 2.6$.
If the expected value $E(X^2) = 8.4$, which of these is closest to the standard deviation σ? **1**

A. 1.28

B. 2.41

C. 7.99

D. 8.24

8 On the Richter scale, the magnitude R of an earthquake of intensity I is given by the formula $R = \log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a reference intensity used for comparisons.

The 1989 Newcastle Earthquake measured a magnitude of 5.6 on the Richter scale.

Which of these is closest to $\left(\frac{I}{I_0}\right)$, which is the number of times this earthquake is more intense than the reference intensity? **1**

A. 400 times

B. 4000 times

C. 40 000 times

D. 400 000 times

Question 11 (2 marks)

(b) The volume V of gas varies inversely to the pressure p.

The volume of a gas is 160 cm^3 under pressure of 36 kg/cm^2. **2**

What will be its volume under pressure of 48 kg/cm^2?

Marks

Question 12 (4 marks)

(c) The probability distribution of a discrete random variable, X, is given by the table below.

x	0	1	2	3
$P(X = x)$	0.1	$0.6p^2$	0.3	$1 - p$

(i) Show that $p = \frac{2}{3}$ or $p = 1$. **2**

(ii) Let $p = \frac{2}{3}$.

Find correct to 2 decimal places,

(α) $E(X)$ **1**

(β) $P(X \le E(X))$. **1**

Question 13 (4 marks)

(b) Suppose a train arrives shortly after 2 pm each day, and that the number of minutes after 2:00 pm that the train arrives can be modelled as a continuous random variable with probability density function, f, given by

$$f(x) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & \textit{elsewhere} \end{cases}$$

Find the mean and standard deviation of the number of minutes after 2:00 pm that the train arrives. **4**

Question 14 (3 marks)

(b) For events A and B from a sample space, $P(A|B) = \frac{1}{4}$ and $P(B|A) = \frac{1}{3}$.

Let $P(A \cap B) = p$.

(i) Find $P(B)$ in terms of p. **1**

(ii) Find $P(B' \cap A')$ in terms of p. **2**

REPLACEMENT QUESTIONS

Question 15 (3 marks)

Marks

(c) (ii) There will be 60 000 students sitting an examination. If their examination results form a normal distribution, how many students would be expected to score a result between 1 and 3 standard deviations above the mean? **2**

(c) (iv) A table of future values for an annuity of $1 is shown.

Table of future value interest factors

Period	Interest rate per period							
	0.50%	**1.00%**	**1.50%**	**2.00%**	**3.00%**	**4.00%**	**5.00%**	**6.00%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0050	2.0100	2.0150	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0150	3.0301	3.0452	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0301	4.0604	4.0909	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0503	5.1010	5.1523	5.2040	5.3091	5.4163	5.5256	5.6371
6	6.0755	6.1520	6.2296	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.1059	7.2135	7.3230	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.1414	8.2857	8.4328	8.5830	8.8923	9.2142	9.5491	9.8975

Ian is paid an annual salary of $108 000 and he contributes 9.5% of his salary into a superannuation account at the end of each year. At the contribution stage his fund manager charges $2120 each year for fees. If the remainder of the money earns interest of 6% p.a. what will be the balance in his fund after 7 years? Give your answer to the nearest dollar. **2**

Solutions to replacement questions

QUESTION 1

$g(4) = 5$

$f(g(4)) = f(5) = 3$

Answer B *(1 mark)*

QUESTION 2

The function has zeros (x-intercepts) at $x = a, x = b$ (twice) and $x = c$.

Also, for $x > c, y > 0$.

The rule for the function is

$y = 2(x - a)(x - b)^2(x - c)$

Answer B *(1 mark)*

QUESTION 4

$E(X) = \mu = 2.6$

Hence, $\mu^2 = 6.76$

$$\begin{aligned} E(X^2) &= 8.4 \\ Var(X) &= E(X^2) - \mu^2 \\ &= 8.4 - 6.76 \\ &= 1.64 \\ \sigma &= \sqrt{1.64} \\ &= 1.28 \text{ (2 dec. pl.)} \end{aligned}$$ *(1 mark)*

Answer A

QUESTION 8

$$\begin{aligned} 5.6 &= \log_{10}\left(\frac{I}{I_0}\right), \\ \frac{I}{I_0} &= 10^{5.6} \\ &= 398\,107.1706\ldots \\ &= 400\,000 \text{ (nearest hundred thousand)} \end{aligned}$$

Answer D *(1 mark)*

QUESTION 11

(b) $V = \dfrac{k}{p}$

Substitute $V = 160, p = 36$:

$$\begin{aligned} 160 &= \frac{k}{36} \\ k &= 5760 \\ V &= \frac{5760}{p} \end{aligned}$$

Substitute $p = 48$:

$$\begin{aligned} V &= \frac{5760}{48} \\ &= 120 \end{aligned}$$

The volume is 120 cm^3. *(2 marks)*

QUESTION 12

(c) (i)
$$\begin{aligned} 0.1 + 0.6p^2 + 0.3 + 1 - p &= 1 \\ 0.6p^2 - p + 0.4 &= 0 \\ 3p^2 - 5p + 2 &= 0 \\ (3p - 2)(p - 1) &= 0 \\ p &= \frac{2}{3}, 1 \end{aligned}$$ *(2 marks)*

(ii) (α)
$$\begin{aligned} E(X) &= 0(0.1) + 1(0.6)\left(\frac{2}{3}\right)^2 + 2(0.3) + 3\left(1 - \frac{2}{3}\right) \\ &= 1.866\,66\ldots \\ &= 1.87 \text{ (2 dec. pl.)} \end{aligned}$$ *(1 mark)*

(β)
$$\begin{aligned} P(X \le 1.87) &= P(X = 0) + P(X = 1) \\ &= 0.1 + (0.6)\left(\frac{2}{3}\right)^2 \\ &= 0.366\,66\ldots \\ &= 0.37 \text{ (2 dec. pl.)} \end{aligned}$$ *(1 mark)*

QUESTION 13

(b) $\mu = E(X)$

$$= \int_0^1 x \times f(x)\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx$$

$$= \left[x^2 - \frac{2x^3}{3}\right]_0^1$$

$$= 1^2 - \frac{2(1)^3}{3} - 0$$

$$= \frac{1}{3}$$

$$E(X^2) = \int_0^1 x^2 \times f(x)\,dx$$

$$= \int_0^1 (2x^2 - 2x^3)\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{2}\right]_0^1$$

$$= \frac{2(1)^3}{3} - \frac{1^4}{2} - 0$$

$$= \frac{1}{6}$$

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{1}{6} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{18}$$

$$\sigma = 0.23570226\ldots$$

$$= 0.24 \text{ (2 dec. pl.)}$$

The mean is $\frac{1}{3}$ minute after 2:00 pm with a standard deviation of 0.24 minutes.

(4 marks)

QUESTION 14

(b) (i) $P(A|B) = \frac{1}{4}, P(A \cap B) = p$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{4} = \frac{p}{P(B)}$$

$$P(B) = 4p$$

(1 mark)

(ii) $P(B|A) = \frac{1}{3}, P(A \cap B) = p$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\frac{1}{3} = \frac{p}{P(A)}$$

$$P(A) = 3p$$

Now,

$$P(B' \cap A') = P((B \cup A)')$$

$$= 1 - P(B \cup A)$$

$$= 1 - [P(B) + P(A) - P(B \cap A)]$$

$$= 1 - [4p + 3p - p]$$

$$= 1 - 6p$$

(2 marks)

QUESTION 15

(c) (ii) $\therefore 1 \le z \le 3$

Percentage for $1 \le z \le 3 = \frac{99.7 - 68}{2}$

$= 15.85\%$

Number of students $= 0.1585 \times 60000$

$= 9510$

It is expected 9510 student results will be between 1 and 3 standard deviations above the mean.

(2 marks)

(c) (iv) Contribution $= 0.095 \times \$108000 - \$2120$

$= \$8140$

Balance $= \$8140 \times 8.3938$

$= \$68325.532$

$= \$68326$ (nearest whole)

The balance in his fund is \$68326.

(2 marks)

Replacement questions
with content from the most up-to-date syllabus

Marks

1 Which of these is the antiderivative of $\dfrac{3x^3 - 2x^2}{x}$? **1**

A. $x^4 - x^3 + C$

B. $x^3 - x^2 + C$

C. $\dfrac{x^4}{4} - \dfrac{x^3}{3} + C$

D. $\dfrac{x^2 - x}{2} + C$

2 On the decibel scale, the loudness L of a sound of intensity S is given by

$L = 10 \log_{10}\left(\dfrac{S}{S_0}\right)$, where S_0 is a reference intensity used for comparisons.

How much louder is a sound of 47 decibels than a sound of 27 decibels? **1**

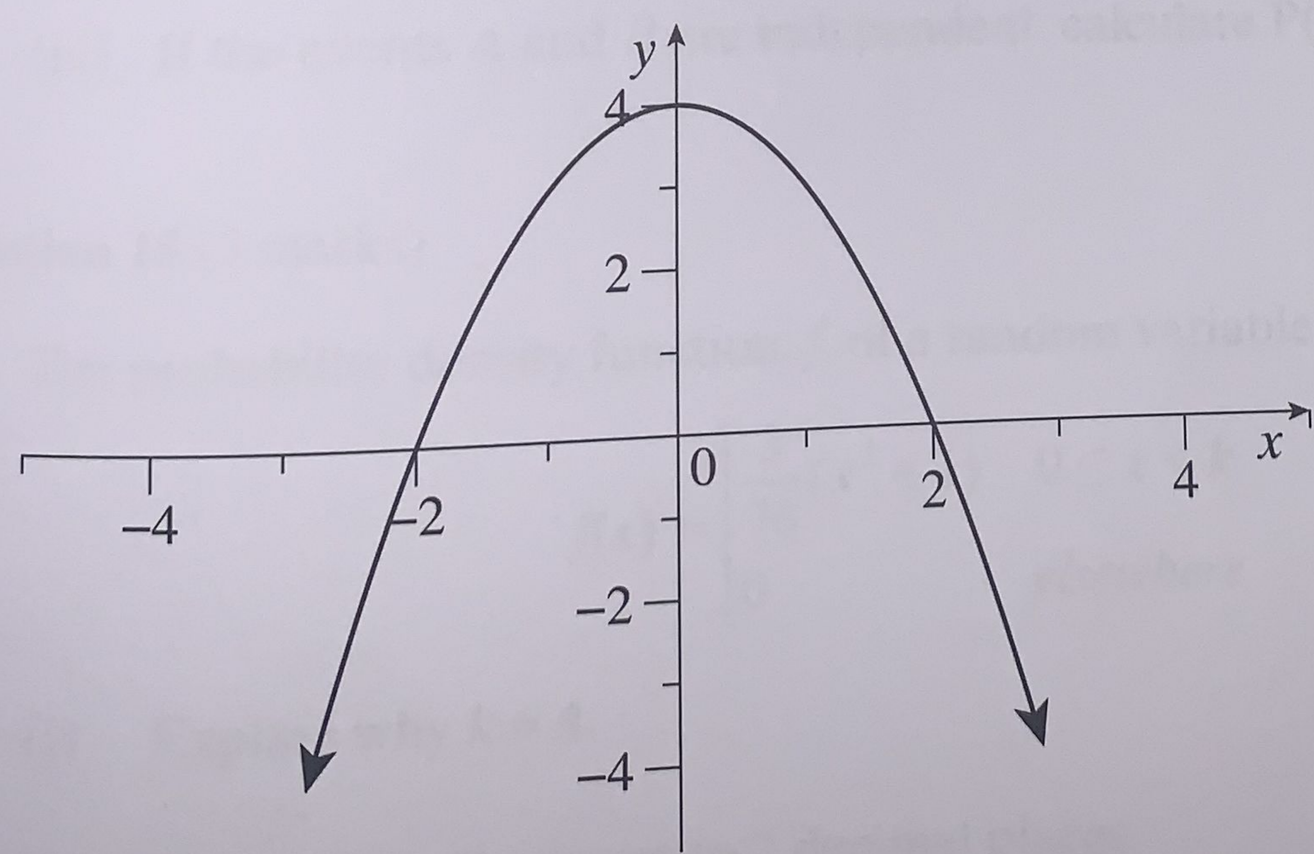
A. 2 times

B. 20 times

C. 100 times

D. 200 times

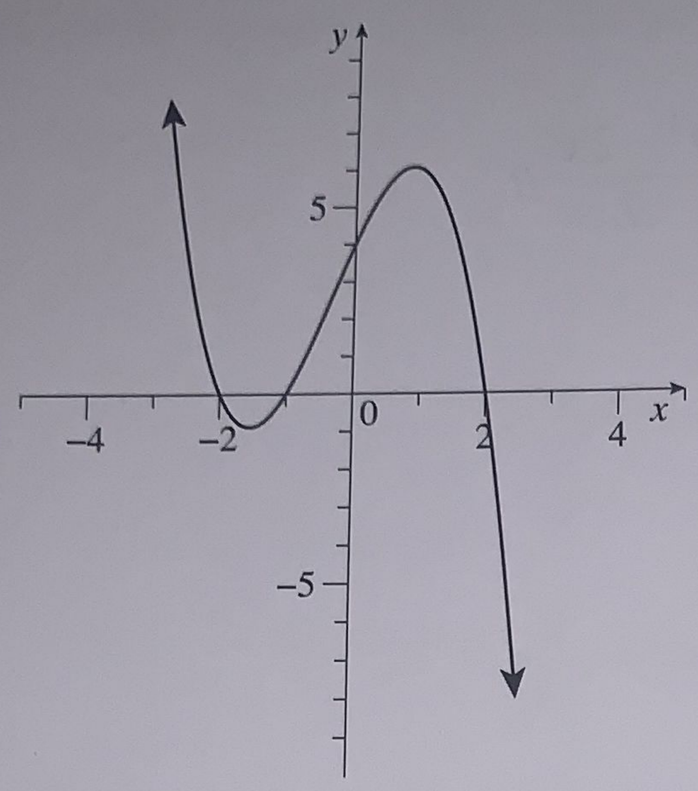
4 The graph of $y = f(x)$ is shown below.



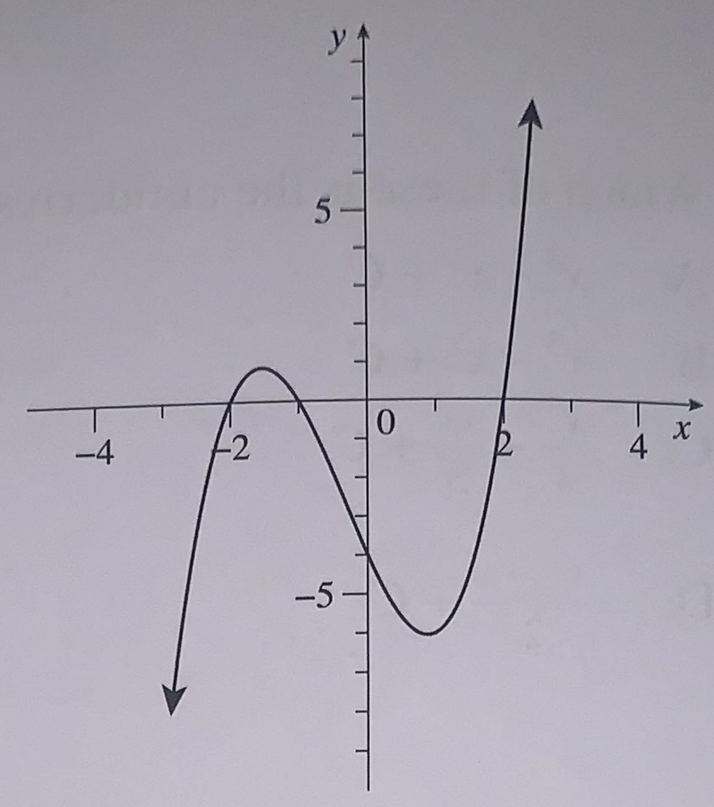
Marks

Which of the following is the graph of $y = -f(x + 1)$? **1**

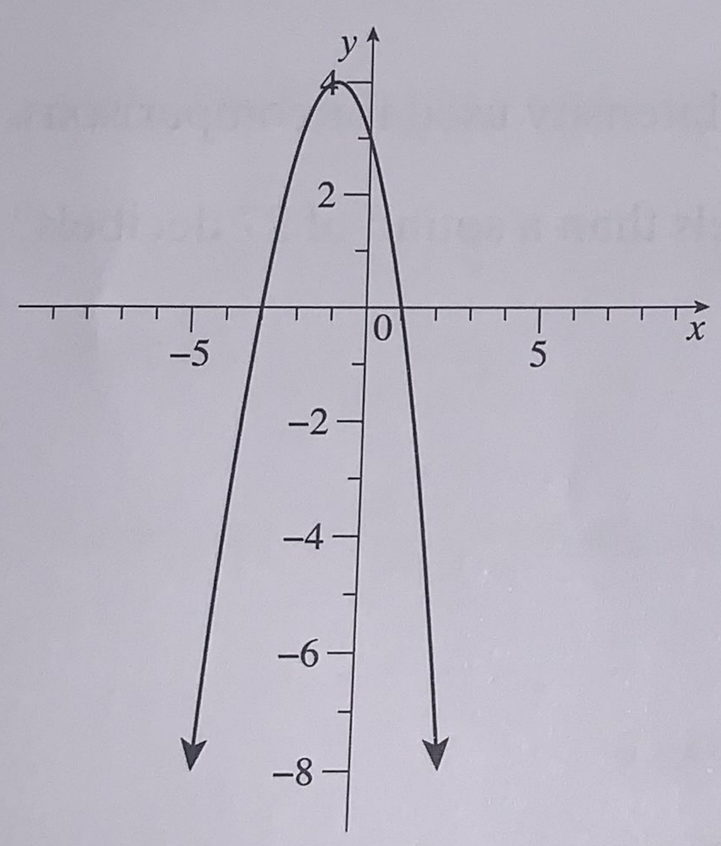
A.



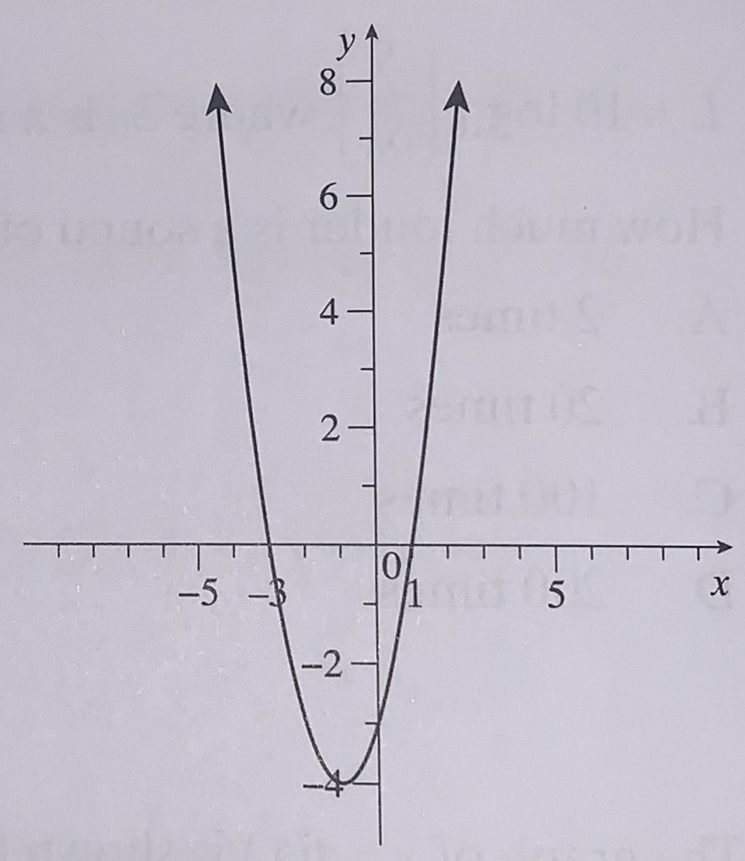
B.



C.



D.



Question 11 (2 marks)

Marks

(g) Logan's dentist determined the number of cavities in his patient's mouth each year is inversely proportional to the total number of seconds spent brushing during each session.

Logan developed 6 cavities during the year in which he spent 20 seconds brushing each time.

How many annual cavities will Logan develop if he increases his brushing time to a minute each time? **2**

Question 13 (3 marks)

(d) The discrete random variable X can only take the values 0, 1, 2, 3, or 4.

The probability distribution is described by the following statements:

- $P(X = 0) = P(X = 2) = a$
- $P(X = 1) = P(X = 3) = b$
- $P(X = 4) = 2a + b$
- $b = 2a$

Show that $Var(X) = 2.04$ **3**

Question 14 (3 marks)

(c) For events A and B from a sample space, $P(A|B) = \frac{2}{3}$ and $P(B) = \frac{1}{4}$.

(i) Calculate $P(A \cap B)$. **1**

(ii) Calculate $P(A' \cap B)$. **1**

(iii) If the events A and B are independent, calculate $P(A \cup B)$. **1**

Question 15 (3 marks)

(b) The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} \frac{3}{76}(x^2+1) & 0 \le x \le k \\ 0 & elsewhere \end{cases}$$

(i) Explain why $k = 4$. **2**

(ii) Find $P(X \ge 2)$ correct to 2 decimal places. **1**

Marks

Question 16 (3 marks)

(b) Elliot deposited \$5000 at the end of each year for 10 years in an annuity earning 2.15% per annum, compounded annually.

He completed a table to show the amount in the account at the end of each year.

Part of the table has been shaded.

Year	Balance at beginning of year	Interest on balance	Annual deposit	Total at end of year
1	0.00	0.00	5000.00	5000.00
2	5000.00	107.50	5000.00	10 107.50
3	10 107.50	217.31	5000.00	15 324.81
4	15 324.81	329.48	5000.00	20 654.29
5				A
6				
7				
8				
9				
10				B

(i) Find the value of A. **1**

(ii) Use the sum of a geometric series to find the value of B. **2**

REPLACEMENT QUESTIONS

Solutions to replacement questions

QUESTION 1

$$\frac{3x^3 - 2x^2}{x} = 3x^2 - 2x$$

$$\int (3x - 2x)\,dx = x^3 - x^2 + c$$

Answer B *(1 mark)*

QUESTION 2

Difference in decibels $= 47 - 27$

$= 20$

$$20 = 10\log_{10}\left(\frac{S}{S_0}\right)$$

$$\log 10\left(\frac{S}{S_0}\right) = 2$$

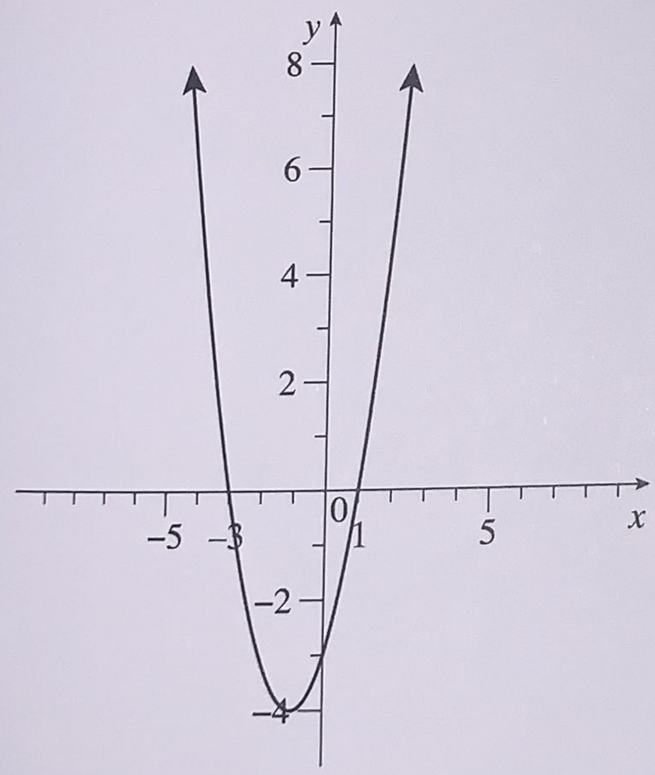
$$\frac{S}{S_0} = 10^2$$

$$= 100$$

The sound is 100 times louder.

Answer C *(1 mark)*

QUESTION 4



Answer D *(1 mark)*

QUESTION 11

(g) Let C = cavities and t = time in seconds:

$$C = \frac{k}{t}$$

Substitute $C = 6, t = 20$:

$$6 = \frac{k}{20}$$

$$k = 120$$

$$C = \frac{120}{t}$$

Substitute $t = 60$:

$$C = \frac{120}{60}$$

$$C = 2$$

Logan would have 2 cavities. *(2 marks)*

QUESTION 13

(d) The sum of the probabilites = 1

$$2a + 2b + 2a + b = 1$$

$$4a + 3b = 1$$

Substitute $b = 2a$ into $4a + 3b = 1$:

$$4a + 3(2a) = 1$$

$$10a = 1$$

$a = 0.1$, and hence $b = 0.2$.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.1	0.2	0.4

$$E(X) = 0(0.1) + 1(0.2) + 2(0.1) + 3(0.2) + 4(0.4)$$

$$= 2.6$$

$$\therefore \quad \mu^2 = 2.6^2$$

$$= 6.76$$

$$E(X^2) = 0^2(0.1) + 1^2(0.2) + 2^2(0.1) + 3^2(0.2) + 4^2(0.4)$$

$$= 8.8$$

$$\text{Var}(X) = 8.8 - 6.76$$

$$= 2.04$$

(3 marks)

QUESTION 14

(c) (i) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$\frac{2}{3} = \frac{P(A \cap B)}{\frac{1}{4}}$$

$$P(A \cap B) = \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{1}{6}$$

(1 mark)

(ii) $P(A \cap B) + P(A' \cap B) = P(B)$

$$\frac{1}{6} + P(A' \cap B) = \frac{1}{4}$$

$$P(A' \cap B) = \frac{1}{4} - \frac{1}{6}$$

$$= \frac{1}{12}$$

(1 mark)

(iii) If A and B are independent, then $P(A|B) = P(A) = \dfrac{2}{3}$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{3} + \frac{1}{4} - \frac{1}{6}$$

$$= \frac{3}{4}$$

(1 mark)

QUESTION 15

(b) (i) $\displaystyle\int_0^k \frac{3}{76}(x^2+1)\,dx = 1$ (probabilities sum to 1)

$$\frac{3}{76}\int_0^k (x^2+1)\,dx = 1$$

$$\left[\frac{x^3}{3} + x\right]_0^k = \frac{76}{3}$$

$$\frac{k^3}{3} + k - 0 = \frac{76}{3}$$

$$k^3 + 3k - 76 = 0$$

Substitute $k = 4$:

$$4^3 + 3(4) - 76 = 64 + 12 - 76$$

$$= 0$$

$\therefore k = 4$ is a solution.

(2 marks)

(ii) $\displaystyle\frac{3}{76}\int_2^4 (x^2+x)\,dx = \frac{3}{76}\left[\frac{x^3}{3} + x\right]_2^4$

$$= \frac{3}{76}\left[\frac{4^3}{3} + 4 - \left(\frac{2^3}{3} + 2\right)\right]$$

$$= 0.815\,789\,473\ldots$$

$$= 0.82 \text{ (2 dec. pl.)}$$

(1 mark)

QUESTION 16

(b) (i) $A = 20\,654.29 \times 1.0215 + 5000$

$= 26\,098.36$ (2 dec. pl.) *(1 mark)*

(ii) $B = 5000 + 5000 \times 1.0215 + 5000 \times 1.0215^2 + \ldots + 5000 \times 1.0215^9$

$= 5000(1 + 1.0215 + 1.0215^2 + \ldots + 1.0215^9)$

Using $a = 1, r = 1.0215, n = 10, S_n = \dfrac{a(r^n - 1)}{r - 1}$:

$$B = 5000 \times \frac{1(1.0215^{10} - 1)}{1.0215 - 1}$$

$= 55\,125.559\,41\ldots$

$= 55\,125.56$ (2 dec. pl.) *(2 marks)*