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MATHEMATICS EXTENSION 1

HSC Exam* Questions by Topic

2023 - 2007

v2024

Licensed from NESA

Year 11 Course

Year 12 Course

Complete Papers

Question Difficulty

Easy

Mid-range

Difficult

Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2023 HSC)

Questions by Topic from …

- **2007 – 2023 HSCs (MX1: Mathematics Extension 1, M: Mathematics)**
- **NESA Sample HSC examination [SP]**
- **Additional NESA sample questions [SQ]**
- **NESA Topic Guidance [TG] – selected questions**

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Year 11: Functions
F1.1 Algebraic techniques

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

F1.1: Graphical relationships

Students:

- examine the relationship between the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$ and hence sketch the graphs (ACMSM099)
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y^2 = f(x)$ and $y = \sqrt{f(x)}$ and hence sketch the graphs
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = |f(x)|$ and $y = f(|x|)$ and hence sketch the graphs (ACMSM099)
- examine the relationship between the graphs of $y = f(x)$ and $y = g(x)$ and the graphs of $y = f(x) + g(x)$ and $y = f(x)g(x)$ and hence sketch the graphs
- apply knowledge of graphical relationships to solve problems in practical and abstract contexts **AAM**

Reference Sheet

23 MX 1 **8** The diagram shows the graph of a function. **1** **Solution**

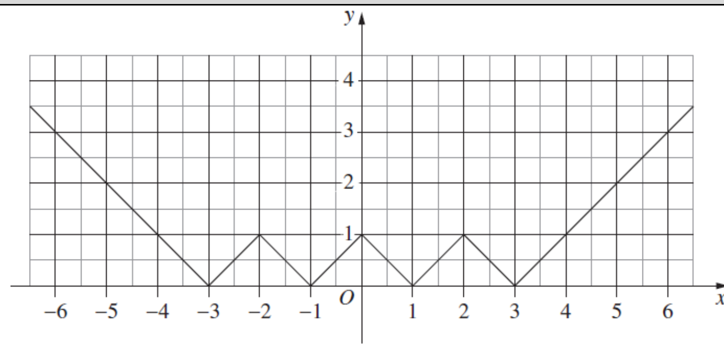
Which of the following is the equation of the function?

A. $y = |1 - ||x| - 2||$

B. $y = |2 - ||x| - 1||$

C. $y = |1 - |x - 2||$

D. $y = |2 - |x - 1||$



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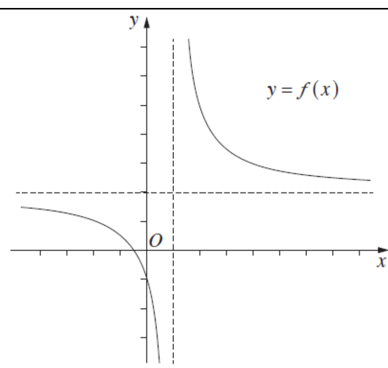
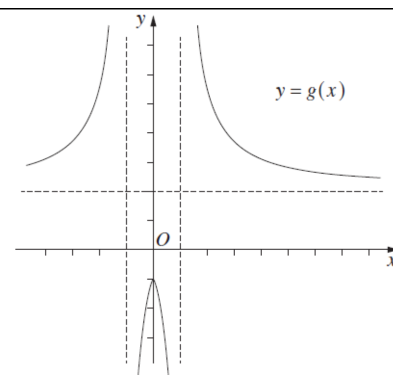
22 MX 1 **2** The graph of $f(x) = \dfrac{3}{x-1} + 2$ is shown. **1** **Solution**

The graph of $f(x)$ was transformed to get the graph of $g(x)$ as shown.



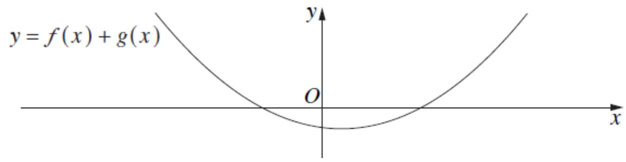


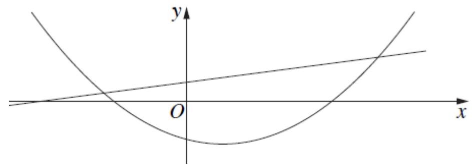
What transformation was applied?

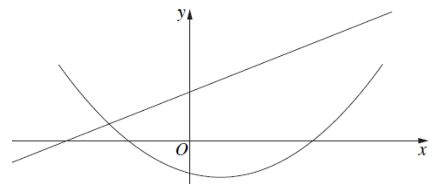
A. $g(x) = f(|x|)$ B. $g(x) = \sqrt{f(x)}$ C. $g(x) = -f(x)$ D. $g(x) = \dfrac{1}{f(x)}$

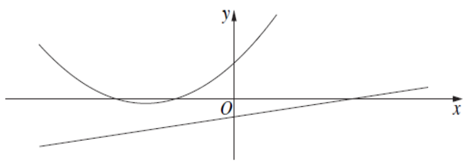
NESA 2022 Mathematics Extension 1 HSC Examination

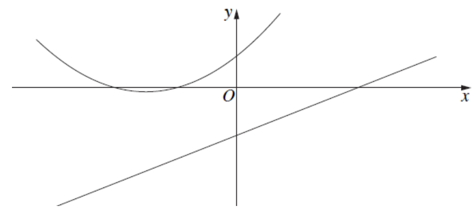
22 MX 1 **4** The diagram shows the graph of the sum of the functions $f(x)$ and $g(x)$.
Which of the following best represents the graphs of both $f(x)$ and $g(x)$? **1** **Solution**



A.


B.


C.


D.


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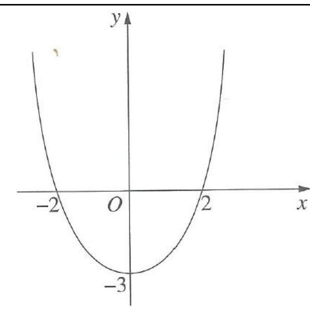
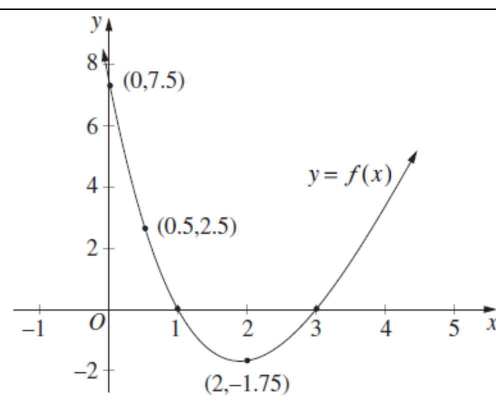
20 MX 1 **11 c** The diagram shows the graph of $y = f(x)$. **3** Solution



Sketch the graph of $y = \dfrac{1}{f(x)}$.

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **14 c** The diagram is a sketch of the graph of the function $y = f(x)$. Solution



(i) Sketch the graph of $y = \dfrac{1}{|f(x)|}$. **3**

Your sketch should show any asymptotes and intercepts, together with the location of the points, corresponding to the labelled points on the original sketch.

(ii) How any solutions does the equation $\dfrac{1}{|f(x)|} = x$ have? **1**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

TG **1** On the same number plane draw the graphs of $y = 3\sin 2x$ and $y = 1 + 3\sin 2x$, for $-2\pi \le x \le 2\pi$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2** On the same number plane draw the graphs of $y = 3\sin x$ and $y = 3\sin x + x$, for $0 < x \le 4$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3** On the same number plane draw the graphs of $y = x$ and $y = e^x$.
Hence, draw the graph of $y = xe^{-x}$ on the same number plane. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

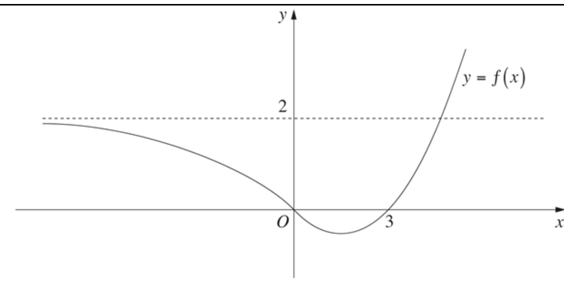
TG **4** The diagram shows the graph of $y = f(x)$. **Solution**

On separate sketches draw the graphs of:

(a) $y = |f(x)|$

(b) $y = \dfrac{1}{f(x)}$

(c) $y = \sqrt{f(x)}$



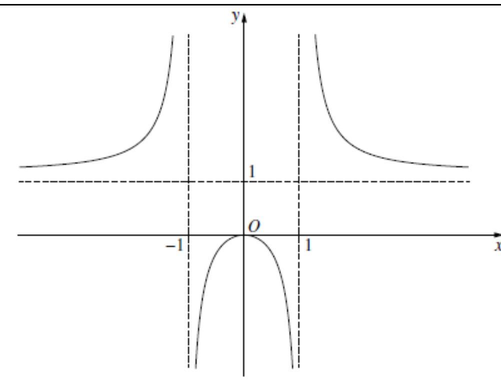
NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

19 MX 1 **4** The diagram shows the graph of $f(x)$. **1** **Solution**

Which equation best describes the graph?

A. $y = \dfrac{x}{x^2 - 1}$ B. $y = \dfrac{x^2}{x^2 - 1}$

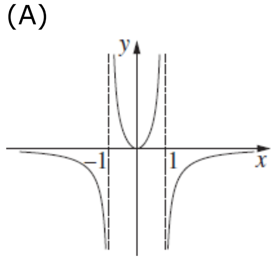
C. $y = \dfrac{x}{1 - x^2}$ D. $y = \dfrac{x^2}{1 - x^2}$



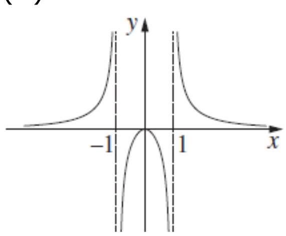
NESA 2019 Mathematics Extension 1 HSC Examination

17 MX 1 **5** Which graph best represents the function $y = \dfrac{2x^2}{1 - x^2}$? **1** **Solution**

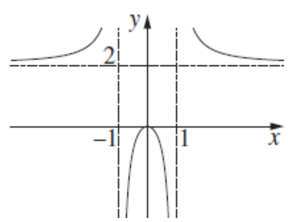
(A)



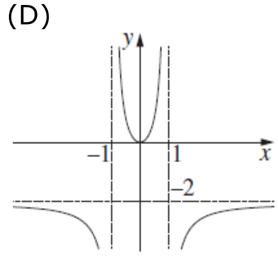
(B)



(C)



(D)



NESA 2017 Mathematics Extension 1 HSC Examination

12 MX 1 **13 b** **Solution**

(i) Find the horizontal asymptote of the graph $y = \dfrac{2x^2}{x^2 + 9}$. **1**

(ii) Without the use of calculus, sketch the graph $y = \dfrac{2x^2}{x^2 + 9}$, showing the asymptote found in part (i). **2**

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Year 11: Functions
F1.2 Inequalities

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Students:

- solve quadratic inequalities using both algebraic and graphical techniques
- solve inequalities involving rational expressions, including those with the unknown in the denominator
- solve absolute value inequalities of the form $|ax+b| \geq k$, $|ax+b| \leq k$, $|ax+b| < k$ and $|ax+b| > k$

Reference Sheet

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22 MX 1 **11 f**

Solve $\dfrac{x}{2-x} \geq 5$. **3**

Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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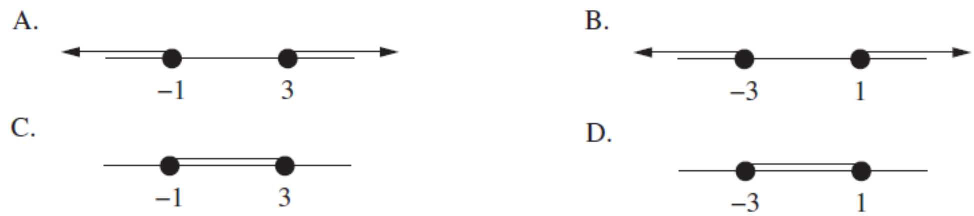
20 MX 1 **1**

Which diagram best represents the solution set of $x^2 - 2x - 3 \geq 0$? **1**

Solution



NESA 2020 Mathematics Extension 1 HSC Examination

TG **1**

Find the domain of $f(x) = \sqrt{9 - x^2}$

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2**

Solve:

(a) $x^2 - 5x \geq 0$
(b) $(x-4)^2 > 0$
(c) $m^2 - 2m - 3 \leq 0$
(d) $x^2 + 8x + 3 \leq 0$

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3**

Solve $\dfrac{2t+1}{t-2} > 1$.

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **4**

Solve $\dfrac{2}{|x+3|} < 1$.

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **5**

Find the range of values of k for which the expression $x^2 - 2x + (3 - 2k)$ is always positive.

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **6**

Explain why $2x^2 + 4x + 3 > 0$ for all x.

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

19 MX 1	**11 b**	For what values of x is $\dfrac{x}{x+1} < 2$? NESA 2019 Mathematics Extension 1 HSC Examination	**3**	Solution
18 MX 1	**11 e**	Consider the function $f(x) = \dfrac{1}{4x-1}$. (i) Find the domain of $f(x)$. (ii) For what values of x is $f(x) < 1$? NESA 2018 Mathematics Extension 1 HSC Examination	 **1** **2**	Solution
17 MX 1	**11 c**	Solve $\dfrac{2x}{x+1} > 1$. NESA 2017 Mathematics Extension 1 HSC Examination	**3**	Solution
16 MX 1	**11 e**	Solve $\dfrac{3}{2x+5} - x > 0$. NESA 2016 Mathematics Extension 1 HSC Examination	**3**	Solution
15 MX 1	**11 c**	Solve the inequality $\dfrac{4}{x+3} \geq 1$. NESA 2015 Mathematics Extension 1 HSC Examination	**3**	Solution
14 MX 1	**11 e**	Solve $\dfrac{x^2+5}{x} > 6$. NESA 2014 Mathematics Extension 1 HSC Examination	**3**	Solution
13 MX 1	**10**	Which inequality has the same solution as $\lvert x + 2\rvert + \lvert x - 3\rvert = 5$? (A) $\dfrac{5}{3-x} \geq 1$ (B) $\dfrac{1}{x-3} - \dfrac{1}{x+2} \leq 0$ (C) $x^2 - x - 6 \leq 0$ (D) $\lvert 2x - 1\rvert \geq 5$ NESA 2013 Mathematics Extension 1 HSC Examination	**1**	Solution
11 MX 1	**1c**	Solve $\dfrac{4-x}{x} < 1$. NESA 2011 Mathematics Extension 1 HSC Examination	**3**	Solution
10 MX 1	**1d**	Solve $\dfrac{3}{x+2} < 4$. NESA 2010 Mathematics Extension 1 HSC Examination	**3**	Solution
10 M	**2b**	Solve the inequality $x^2 - x - 12 < 0$. NESA 2010 Mathematics HSC Examination	**2**	Solution
09 MX 1	**1d**	Solve the inequality $\dfrac{x+3}{2x} > 1$. NESA 2009 Mathematics Extension 1 HSC Examination	**3**	Solution
07 MX 1	**3b**	(i) Find the vertical and horizontal asymptotes of the hyperbola $y = \dfrac{x-2}{x-4}$ and hence sketch the graph of $y = \dfrac{x-2}{x-4}$. (ii) Hence, or otherwise, find the values of x for which $\dfrac{x-2}{x-4} \leq 3$. NESA 2007 Mathematics Extension 1 HSC Examination	**3** **2**	Solution

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Year 11: Functions
F1.3 Inverse functions

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Students:

- define the inverse relation of a function $y = f(x)$ to be the relation obtained by reversing all the ordered pairs of the function
- examine and use the reflection property of the graph of a function and the graph of its inverse (ACMSM096)
  - understand why the graph of the inverse relation is obtained by reflecting the graph of the function in the line $y = x$
  - using the fact that this reflection exchanges horizontal and vertical lines, recognise that the horizontal line test can be used to determine whether the inverse relation of a function is again a function
- write the rule or rules for the inverse relation by exchanging x and y in the function rules, including any restrictions, and solve for y, if possible
- when the inverse relation is a function, use the notation $f^{-1}(x)$ and identify the relationships between the domains and ranges of $f(x)$ and $f^{-1}(x)$
- when the inverse relation is not a function, restrict the domain to obtain new functions that are one-to-one, and compare the effectiveness of different restrictions
- solve problems based on the relationship between a function and its inverse function using algebraic or graphical techniques **AAM**

Reference Sheet

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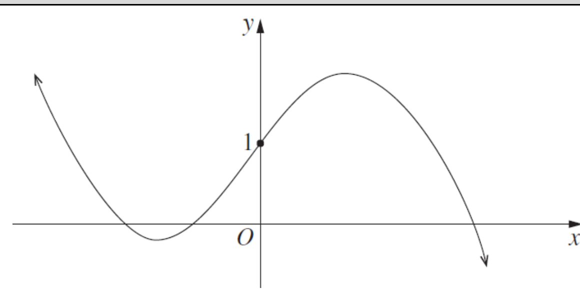
23 MX 1 **9** The graph of a cubic function, $y = f(x)$, is given below. **1** **Solution**



Which of the following functions has an inverse relation whose graph has more than 3 points with an x-coordinate of 1?

A. $y = \sqrt{f(x)}$ B. $y = \dfrac{1}{f(x)}$ C. $y = f(|x|)$ D. $y = |f(x)|$

NESA 2023 Mathematics Extension 1 HSC Examination

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21 MX 1 **12 d** A function is defined by $f(x) = 4 - \left(1 - \dfrac{x}{2}\right)^2$ for x in the domain $(-\infty, 2]$. **Solution**

(i) Sketch the graph of $y = f(x)$ showing the x- and y-intercepts. **2**

(ii) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(iii) Sketch the graph of $y = f^{-1}(x)$. **1**

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **2** Given $f(x) = 1 + \sqrt{x}$, what are the domain and range of $f^{-1}(x)$? **1** **Solution**

A. $x \geq 0,\ y \geq 0$ B. $x \geq 0,\ y \geq 1$ C. $x \geq 1,\ y \geq 0$ D. $x \geq 1,\ y \geq 1$

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **11 b** A function $f(x)$ is given by $x^2 + 4x + 7$. Solution

(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -2$. **2**

(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (b). **2**

(iv) Sketch the curve $y = f^{-1}(x)$. **2**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

TG **1** For each function, state the domain and range of $f^{-1}(x)$ and sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same axes: Solution

(a) $f(x) = x^3$

(b) $f(x) = 1 - 3x$

(c) $f(x) = x^3 + 5$

(d) $f(x) = \sqrt{x} - 3$, for $x > 0$

(e) $f(x) = (x - 1)^2 - 6$ for $x \geq 1$

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2** Functions f and g are defined by $f(x) = 4x + 5$ and $g(x) = 3 - 2x$. Solution

Find the inverse of the composite function $f \circ g$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3** The function f is defined by $f(x) = x^2 - 2x + 7$ with domain $x \leq k$. Solution
Given that f is a one-to-one function, find the greatest possible value of k and find the inverse function f^{-1}.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **4** Graph the inverse of $f(x) = \sqrt{x+1}$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

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19 MX 1 **10** The function $f(x) = -\sqrt{1+\sqrt{1+x}}$ has inverse $f^{-1}(x)$. **1** Solution

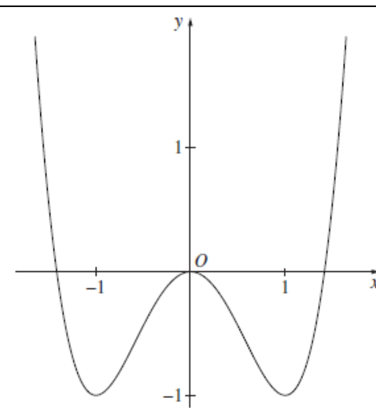
The graph of $y = f^{-1}(x)$ forms part of the curve $y = x^4 - 2x^2$.

The diagram shows the curve $y = x^4 - 2x^2$.

How many points do the graphs of $y = f(x)$ and $y = f^{-1}(x)$ have in common?

A. 1 B. 2

C. 3 D. 4



NESA 2019 Mathematics Extension 1 HSC Examination

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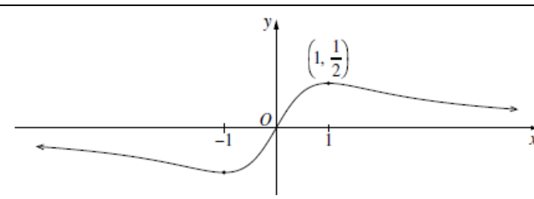
18 MX 1 **13 b** The diagram shows the graph $y = \dfrac{x}{x^2+1}$, for all real x. Solution



Consider the function $y = \dfrac{x}{x^2+1}$, for $x \geq 1$.

The function $f(x)$ has an inverse. (Do NOT prove this.)

(i) State the domain and range of $y = f^{-1}(x)$. **2**

(ii) Sketch the graph of $y = f^{-1}(x)$. **1**

(iii) Find an expression for $f^{-1}(x)$. **3**

NESA 2018 Mathematics Extension 1 HSC Examination

16 MX 1 **11 a** Find the inverse of the function $y = x^3 - 2$. **2** Solution

NESA 2016 Mathematics Extension 1 HSC Examination

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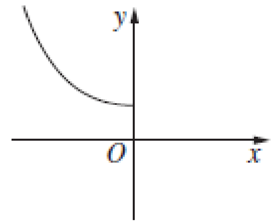
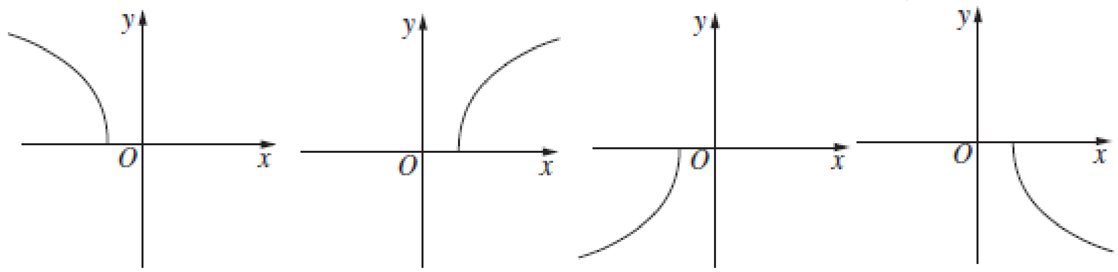
13 MX 1 **2** The diagram shows the graph $y = f(x)$. **1** Solution



Which diagram shows the graph of $y = f^{-1}(x)$?



(A) (B) (C) (D)

NESA 2013 Mathematics Extension 1 HSC Examination

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12 MX 1 **12 b** Let $f(x) = \sqrt{4x - 3}$ Solution

(i) Find the domain of $f(x)$. **1**

(ii) Find an expression for the inverse function $f^{-1}(x)$. **2**

(iii) Find the points where the graphs $y = f(x)$ and $y = x$ intersect. **1**

(iv) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ showing the information found in part (iii). **2**

NESA 2012 Mathematics Extension 1 HSC Examination

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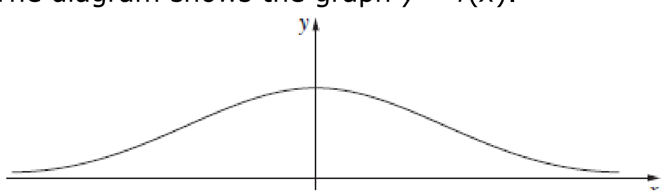
10 MX 1 **3b** Let $f(x) = e^{-x^2}$. The diagram shows the graph $y = f(x)$. Solution



(i) The graph has two points of inflexion. Find the x coordinate of these points. **3**

(ii) Explain why the domain of $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(iii) Find a formula for $f^{-1}(x)$ if the domain of f(x) is restricted to $x \geq 0$. **2**

(iv) State the domain of $f^{-1}(x)$. **1**

(v) Sketch the curve $y = f^{-1}(x)$. **1**

NESA 2010 Mathematics Extension 1 HSC Examination

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09 MX 1 **3a** Let $f(x) = \dfrac{3 + e^{2x}}{4}$ Solution

(i) Find the range of $f(x)$ **1**

(ii) Find the inverse function $f^{-1}(x)$. **2**

NESA 2009 Mathematics Extension 1 HSC Examination

08 MX 1 **5a** Let $f(x) = x - \frac{1}{2}x^2$ for $x \leq 1$. This function has an inverse, $f^{-1}(x)$. **Solution**

(i) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes. (Use the same scale on both axes.) **2**

(ii) Find an expression for $f^{-1}(x)$. **3**

(iii) Evaluate $f^{-1}(\frac{3}{8})$. **1**

NESA 2008 Mathematics Extension 1 HSC Examination

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07 MX 1 **6b** Consider the function $f(x) = e^x - e^{-x}$. **Solution**

(i) Show that $f(x)$ is increasing for all values of x. **1**

(ii) Show that the inverse function is given by **3**

$$f^{-1}(x) = \log_e\left(\frac{x + \sqrt{x^2 + 4}}{2}\right)$$

(iii) Hence, or otherwise, solve $e^x - e^{-x} = 5$. **1**
Give your answer correct to two decimal places.

NESA 2007 Mathematics Extension 1 HSC Examination

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Year 11: Functions

F1.4 Parametric form of functions or relations

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Students:

- understand the concept of parametric representation and examine lines, parabolas and circles expressed in parametric form
  - understand that linear and quadratic functions, and circles can be expressed in either parametric form or Cartesian form
  - convert linear and quadratic functions, and circles from parametric form to Cartesian form and vice versa
  - sketch linear and quadratic functions, and circles expressed in parametric form

Reference Sheet

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23 MX 1 **11 a** The parametric equations of a line are given below. **2**

$$x = 1 + 3t$$
$$y = 4t$$

Find the Cartesian equation of this line in the form $y = mx + c$.

Solution

NESA 2023 Mathematics Extension 1 HSC Examination

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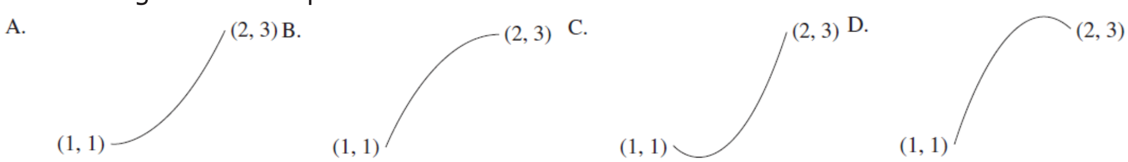
22 MX 1 **5** A curve is defined in parametric form by $x = 2 + t$ and $y = 3 - 2t^2$ for $-1 \le t \le 0$. **1**

Which diagram best represents this curve?



Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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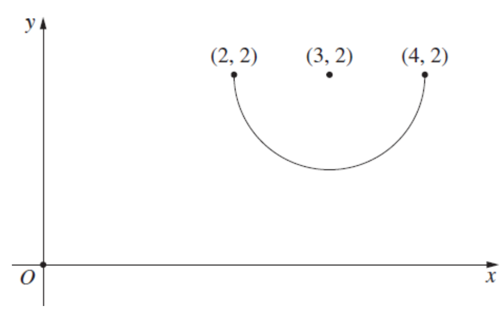
21 MX 1 **8** The diagram shows a semicircle. **1**

Which pair of parametric equations represents the semicircle shown?



A. $\begin{cases} x = 3 + \sin t \\ y = 2 + \cos t \end{cases}$ for $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

B. $\begin{cases} x = 3 + \cos t \\ y = 2 + \sin t \end{cases}$ for $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

C. $\begin{cases} x = 3 - \sin t \\ y = 2 - \cos t \end{cases}$ for $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

D. $\begin{cases} x = 3 - \cos t \\ y = 2 - \sin t \end{cases}$ for $-\dfrac{\pi}{2} \le t \le \dfrac{\pi}{2}$

Solution

NESA 2021 Mathematics Extension 1 HSC Examination

TG **1** The parametric equations of a curve are $x = 2\cos t$, $y = 2\sin t$ for $0 \le t \le 2\pi$.

What is the value of t at the point (0, 2)?

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2** Sketch the curve given by $x = t^2$, $y = \dfrac{1}{t}$ for $t > 0$. **Solution**

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3** Find Cartesian equations for curves with these parametric equations: **Solution**

(a) $x = 3t^2$, $y = 6t$

(b) $x = 1 - \dfrac{1}{t}$, $y = 1 + \dfrac{1}{t}$

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **4** Given the parametric equations $x = p + r\cos t$, $y = q + r\sin t$, eliminate the parameter t to obtain the Cartesian equation of the circle in the form **Solution**

$(x - p)^2 + (y - q)^2 = r^2$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

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Year 11: Functions
F2.1 Remainder and factor theorems

Back

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- define a general polynomial in one variable, x, of degree n with real coefficients to be the expression: $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, where $a_n \neq 0$
  - understand and use terminology relating to polynomials including degree, leading term, leading coefficient and constant term
- use division of polynomials to express $P(x)$ in the form $P(x) = A(x).Q(x) + R(x)$ where $\deg R(x) < \deg A(x)$ and $A(x)$ is a linear or quadratic divisor, $Q(x)$ the quotient and $R(x)$ the remainder
  - review the process of division with remainders for integers
  - describe the process of division using the terms: dividend, divisor, quotient, remainder
- prove and apply the factor theorem and the remainder theorem for polynomials and hence solve simple polynomial equations (ACMSM089, ACMSM091)

Reference Sheet

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23 MX 1 **11c** Consider the polynomial **3** Solution

$$P(x) = x^3 + ax^2 + bx - 12,$$

where a and b are real numbers.
It is given that $x + 1$ is a factor of $P(x)$ and that, when $P(x)$ is divided by $x - 2$, the remainder is -18.
Find a and b.

NESA 2023 Mathematics Extension 1 HSC Examination

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22 MX 1 **3** Let $P(x)$ be a polynomial of degree 5. **1** Solution
When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is $2x + 5$.
Which of the following is true about the degree of Q?

A. The degree must be 1. B. The degree could be 1.
C. The degree must be 2. D. The degree could be 2.

NESA 2022 Mathematics Extension 1 HSC Examination

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21 MX 1 **3** What is the remainder when $P(x) = -x^3 - 2x^2 - 3x + 8$ is divided by $x + 2$? **1** Solution

A. -14 B. -2 C. 2 D. 14

NESA 2021 Mathematics Extension 1 HSC Examination

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20 MX 1 **11 a** Let $P(x) = x^3 + 3x^2 - 13x + 6$ Solution

(i) Show that $P(2) = 0$. **1**

(ii) Hence, factor the polynomial $P(x)$ as $A(x)\,B(x)$, where $B(x)$ is a quadratic polynomial. **2**

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **6** / **19 MX 1** **7**

Let $P(x) = qx^3 + rx^2 + rx + q$ where q and r are constants, $q \neq 0$. **1**

One of the zeros of $P(x)$ is -1.

Given that α is a zero of $P(x)$, $\alpha \neq -1$, which of the following is also a zero?

A. $-\frac{1}{\alpha}$ B. $-\frac{q}{\alpha}$ C. $\frac{1}{\alpha}$ D. $\frac{q}{\alpha}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

Solution

TG **1**

Factorise the polynomial $x^3 - 4x^2 + x + 6$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

Solution

TG **2**

Perform the division $(x^3 + 4x + 2) \div (x^2 + 1)$ and express the result in the form:

(a) $P(x) = A(x)Q(x) + R(x)$

(b) $\frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}$

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

Solution

TG **3** / **04 MX 1** **3b**

Let $P(x) = (x + 1)(x - 3)Q(x) + a(x + 1) + b$, where $Q(x)$ is a polynomial and a and b are real numbers.

When $P(x)$ is divided by $(x + 1)$ the remainder is -11.

When $P(x)$ is divided by $(x - 3)$ the remainder is 1.

(i) What is the value of b? **1**

(ii) What is the remainder when $P(x)$ is divided by $(x + 1)(x - 3)$? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions
NESA 2004 Mathematics Extension 1 HSC Examination

Solution

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19 MX 1 **11 d**

Find the polynomial $Q(x)$ that satisfies $x^3 + 2x^2 - 3x - 7 = (x - 2)Q(x) + 3$. **2**

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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18 MX 1 **11 a**

Consider the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$.

(i) Show that $x = 1$ is a zero of P(x). **1**

(ii) Find the other roots. **2**

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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17 MX 1 **1**

Which polynomial is a factor of $x^3 - 5x^2 + 11x - 10$? **1**

(A) $x - 2$ (B) $x + 2$ (C) $11x - 10$ (D) $x^2 - 5x + 11$

NESA 2017 Mathematics Extension 1 HSC Examination

Solution

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16 MX 1 **2**

What is the remainder when $2x^3 - 10x^2 + 6x + 2$ is divided by $x - 2$? **1**

(A) -66 (B) -10

(C) $-x^3 + 5x^2 - 3x - 1$ (D) $x^3 - 5x^2 + 3x + 1$

NESA 2016 Mathematics Extension 1 HSC Examination

Solution

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15 MX 1 **1**

What is the remainder when $x^3 - 6x$ is divided by $x + 3$? **1**

(A) -9 (B) 9 (C) $x^2 - 2x$ (D) $x^2 - 3x + 3$

NESA 2015 Mathematics Extension 1 HSC Examination

Solution

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15 MX 1 **11 f**

Consider the polynomials $P(x) = x^3 - kx^2 + 5x + 12$ and $A(x) = x - 3$.

(i) Given that $P(x)$ is divisible by $A(x)$, show that $k = 6$. **1**

(ii) Find all the zeros of $P(x)$ when $k = 6$. **2**

NESA 2015 Mathematics Extension 1 HSC Examination

Solution

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14 MX 1 **9**

The remainder when the polynomial $P(x) = x^4 - 8x^3 - 7x^2 + 3$ is divided by $x^2 + x$ is $ax + 3$. What is the value of a? **1**

(A) -14 (B) -11 (C) -2 (D) 5

NESA 2014 Mathematics Extension 1 HSC Examination

Solution

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13 MX 1	**1**	The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor $x - 2$. What is the value of k? (A) 2 (B) 12 (C) 20 (D) 36 NESA 2013 Mathematics Extension 1 HSC Examination	**1**	Solution
12 MX 1	**8**	When the polynomial $P(x)$ is divided by $(x + 1)(x - 3)$, the remainder is $2x + 7$. What is the remainder when $P(x)$ is divided by $x - 3$? (A) 1 (B) 7 (C) 9 (D) 13 NESA 2012 Mathematics Extension 1 HSC Examination	**1**	Solution
11 MX 1	**2a**	Let $P(x) = x^3 - ax^2 + x$ be a polynomial, where a is a real number. When $P(x)$ is divided by $x - 3$ the remainder is 12. Find the remainder when $P(x)$ is divided by $x + 1$. NESA 2011 Mathematics Extension 1 HSC Examination	**3**	Solution
10 MX 1	**2c**	Let $P(x) = (x + 1)(x - 3)\,Q(x) + ax + b$ where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $x - 3$. When $P(x)$ is divided by $x + 1$ the remainder is 8. (i) Find the values of a and b. **2** (ii) Find the remainder when $P(x)$ is divided by $(x + 1)(x - 3)$. **1** NESA 2010 Mathematics Extension 1 HSC Examination		Solution
09 MX 1	**2a**	The polynomial $p(x) = x^3 - ax + b$ has a remainder of 2 when divided by $(x - 1)$ and a remainder of 5 when divided by $(x + 2)$. Find the values of a and b. NESA 2009 Mathematics Extension 1 HSC Examination	**3**	Solution
08 MX 1	**1a**	The polynomial x^3 is divided by $x + 3$. Calculate the remainder. NESA 2008 Mathematics Extension 1 HSC Examination	**2**	Solution
07 MX 1	**2c**	The polynomial $P(x) = x^2 + ax + b$ has a zero at $x = 2$. When $P(x)$ is divided by $x + 1$, the remainder is 18. Find the values of a and b. NESA 2007 Mathematics Extension 1 HSC Examination	**3**	Solution

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Year 11: Functions

F2.2 Sums and Products of roots of polynomials

Back

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- solve problems using the relationships between the roots and coefficients of quadratic, cubic and quartic equations **AAM**
  - consider quadratic, cubic and quartic equations, and derive formulae as appropriate for the sums and products of roots in terms of the coefficients
- determine the multiplicity of a root of a polynomial equation
  - prove that if a polynomial equation of the form $P(x) = 0$ has a root of multiplicity $r > 1$, then $P'(x) = 0$ has a root of multiplicity $r - 1$
- graph a variety of polynomials and investigate the link between the root of a polynomial equation and the zero on the graph of the related polynomial function
  - examine the sign change of the function and shape of the graph either side of roots of varying multiplicity

Reference Sheet

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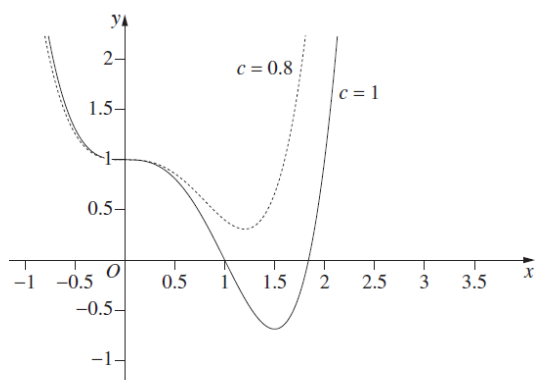
23 MX 1 **14 b** Solution

Consider the hyperbola $y = \dfrac{1}{x}$ and the circle $(x - c)^2 + y^2 = c^2$, where c is a constant.

(i) Show that the x-coordinates of any points of intersection of the hyperbola and circle are zeros of the polynomial $P(x) = x^4 - 2cx^3 + 1$. **1**

(ii) The graphs of $y = x^4 - 2cx^3 + 1$ for $c = 0.8$ and $c = 1$ are shown. **3**

By considering the given graphs, or otherwise, find the exact value of $c > 0$ such that the hyperbola $y = \dfrac{1}{x}$ and the circle $(x - c)^2 + y^2 = c^2$ intersect at only one point.



NESA 2023 Mathematics Extension 1 HSC Examination

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22 MX 1 **13 d** **3** Solution

The monic polynomial, P, has degree 3 and roots α, β, γ.
It is given that

$$\alpha^2 + \beta^2 + \gamma^2 = 85 \text{ and}$$
$$P'(\alpha) + P'(\beta) + P'(\gamma) = 87.$$

Find $\alpha\beta + \beta\gamma + \gamma\alpha$.

NESA 2022 Mathematics Extension 1 HSC Examination

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21 MX 1 **11 h** **2** Solution

The roots of $x^4 - 3x + 6 = 0$ are α , β , γ and δ .

What is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$?

NESA 2021 Mathematics Extension 1 HSC Examination

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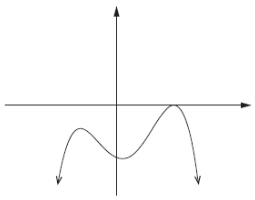
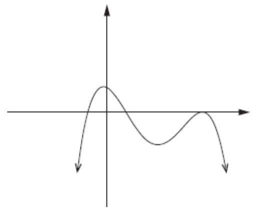
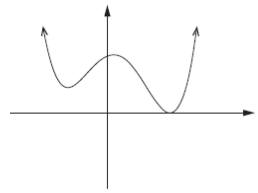
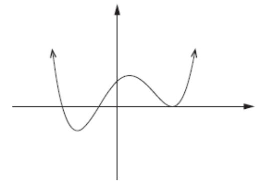
20 MX 1 **5** A monic polynomial $p(x)$ of degree 4 has one repeated zero of multiplicity 2 and is divisible by $x^2 + x + 1$. **1** Solution

Which of the following could be the graph of $p(x)$?

A.


B.


C.


D.


NESA 2020 Mathematics Extension 1 HSC Examination

TG **1** Sketch the graph of the polynomial function $f(x) = (x - 1)^3(x + 2)$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **2** Sketch the graph of $f(x) = x^3 - 3x^2 - 4x$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

TG **3** For the polynomial $(a - 1)x^5 + (b - 5)x^2 + 2x$, discuss the possible values of a and b if the polynomial is of degree: Solution

(a) 1
(b) 2
(c) 5

NESA Mathematics Extension 1 Year 11 Topic Guide: Functions

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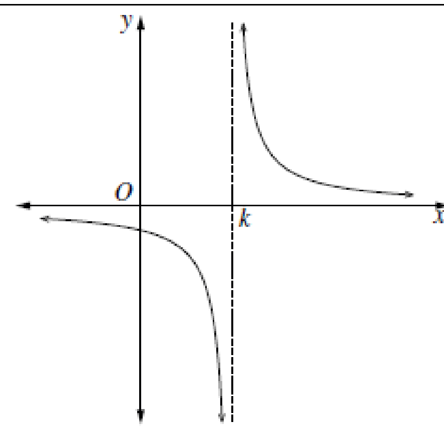
19 MX 1 **14 b** The diagram shows the graph of $y = \dfrac{1}{x-k}$, where k is a positive real number. Solution



(iii) By considering the graphs of $y = x^2$ and $y = \dfrac{1}{x-k}$, explain why the function $f(x) = x^3 - kx^2 - 1$ has exactly one real zero. **2**

Let this zero be α and let $x_1 = k$ be a first approximation to α.

(ii) Show that using one application of Newton's method gives a second approximation $x_2 = k + \dfrac{1}{k^2}$. **2**

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(iii) Show that $x_1 < \alpha < x_2$. **3**

NESA 2019 Mathematics Extension 1 HSC Examination

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18 MX 1 **1** The polynomial $2x^3 + 6x^2 - 7x - 10$ has zeros α, β and γ. What is the value of $\alpha\beta\gamma(\alpha + \beta + \gamma)$? **1** Solution

A. −60 B. −15 C. 15 D. 60

NESA 2018 Mathematics Extension 1 HSC Examination

18 MX 1 **4** The diagram shows the graph of $y = a(x + b)(x + c)(x + d)^2$. **1** Solution

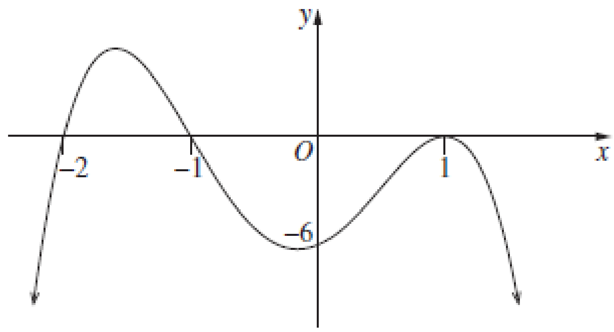
What are the possible values of a, b, c and d?

A. $a = -6$, $b = -2$, $c = -1$, $d = 1$

B. $a = -6$, $b = 2$, $c = 1$, $d = -1$

C. $a = -3$, $b = -2$, $c = -1$, $d = 1$

D. $a = -3$, $b = 2$, $c = 1$, $d = -1$



NESA 2018 Mathematics Extension 1 HSC Examination

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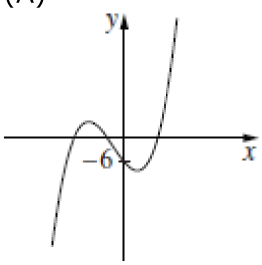
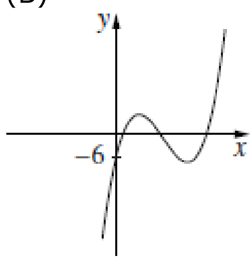
16 MX 1 **10** Consider the polynomial $p(x) = ax^3 + bx^2 + cx + d$ with a and b positive. Which graph could represent $p(x)$? **1** Solution

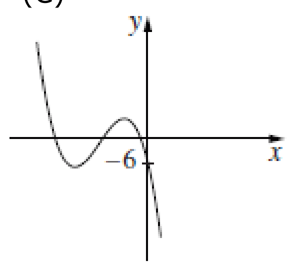
(A)



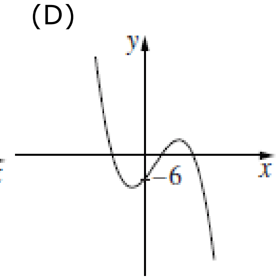
(B)



(C)



(D)



NESA 2016 Mathematics Extension 1 HSC Examination

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14 MX 1 **5** Which group of three numbers could be the roots of the polynomial equation $x^3 + ax^2 - 41x + 42 = 0$? **1** Solution

(A) 2, 3, 7 (B) 1, −6, 7 (C) −1, −2, 21 (D) −1, −3, −14

NESA 2014 Mathematics Extension 1 HSC Examination

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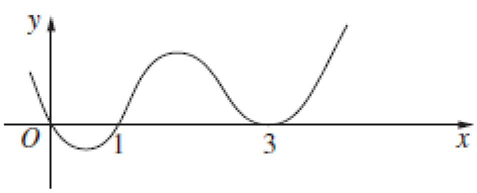
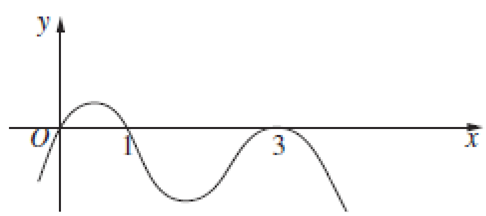
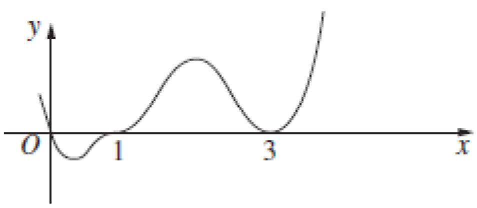
13 MX 1 **4** Which diagram best describe the graph of $y = x(1 - x)^3(3 - x)^2$? **1** Solution



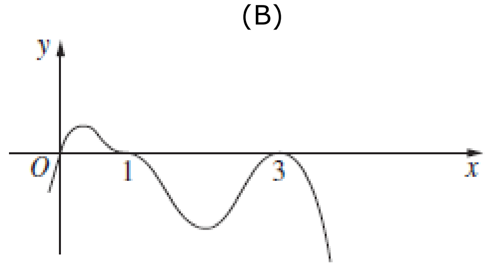
(A)



(B)



(C)



(D)

NESA 2013 Mathematics Extension 1 HSC Examination

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13 MX 1 **11 a** The polynomial equation $2x^3 - 3x^2 - 11x + 7 = 0$ has roots α, β and γ. **1** Solution

Find $\alpha\beta\gamma$.

NESA 2013 Mathematics Extension 1 HSC Examination

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08 MX 1 **2c** The polynomial $p(x)$ is given by $p(x) = ax^3 + 16x^2 + cx - 120$, where a and c are constants. The three zeros of $p(x)$ are −2, 3 and α. Find the value of α. **3** Solution

NESA 2008 Mathematics Extension 1 HSC Examination

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Year 11: Trigonometric Functions
T1 Inverse trigonometric functions

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- define and use the inverse trigonometric functions (ACMSM119)
  - understand and use the notation $\arcsin x$ and $\sin^{-1}x$ for the inverse function of $\sin x$ when $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (and similarly for $\cos x$ and $\tan x$) and understand when each notation might be appropriate to avoid confusion with the reciprocal functions
  - use the convention of restricting the domain of $\sin x$ to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, so the inverse function exists. The inverse of this restricted sine function is defined by: $y = \sin^{-1}x$, $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
  - use the convention of restricting the domain of $\cos x$ to $0 \le x \le \pi$, so the inverse function exists. The inverse of this restricted cosine function is defined by: $y = \cos^{-1}x$, $-1 \le x \le 1$ and $0 \le y \le \pi$
  - use the convention of restricting the domain of $\tan x$ to $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so the inverse function exists. The inverse of this restricted tangent function is defined by: $y = \tan^{-1}x$, x is a real number and $-\frac{\pi}{2} < y < \frac{\pi}{2}$
  - classify inverse trigonometric functions as odd, even or neither odd nor even
- sketch graphs of the inverse trigonometric functions
- use the relationships $\sin(\sin^{-1}x) = x$ and $\sin^{-1}(\sin x) = x$, $\cos(\cos^{-1}x) = x$ and $\cos^{-1}(\cos x) = x$, and $\tan(\tan^{-1}x) = x$ and $\tan^{-1}(\tan x) = x$ where appropriate, and state the values of x for which these relationships are valid
- prove and use the properties: $\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $\tan^{-1}(-x) = -\tan^{-1}x$ and $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$
- solve problems involving inverse trigonometric functions in a variety of abstract and practical situations **AAM**

Reference Sheet

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23 MX 1 **5** Which of the following is the value of $\sin^{-1}(\sin \alpha)$ given that $\pi < \alpha < \frac{3\pi}{2}$? **1** **Solution**

A. $\alpha - \pi$ B. $\pi - \alpha$ C. α D. $-\alpha$

NESA 2023 Mathematics Extension 1 HSC Examination

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23 MX 1 **7** Which statement is always true for real numbers a and b where $-1 \le a < b \le 1$? **1** **Solution**

A. $\sec a < \sec b$ B. $\sin^{-1} a < \sin^{-1} b$

C. $\arccos a < \arccos b$ D. $\cos^{-1} a + \sin^{-1} a < \cos^{-1} b + \sin^{-1} b$

NESA 2023 Mathematics Extension 1 HSC Examination

23 MX 1 **14 a** Solution

Let $f(x) = 2x + \ln x$, for $x > 0$.

(i) Explain why the inverse of $f(x)$ is a function. **1**

(ii) Let $g(x) = f^{-1}(x)$. By considering the value of $f(1)$, or otherwise, evaluate $g'(2)$. **2**

NESA 2023 Mathematics Extension 1 HSC Examination

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22 MX 1 **1** **1** Solution

It is given that $\cos\left(\dfrac{23\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

Which of the following is the value of $\cos^{-1}\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$?

A. $\dfrac{23\pi}{12}$ B. $\dfrac{11\pi}{12}$ C. $\dfrac{\pi}{12}$ D. $-\dfrac{11\pi}{12}$

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **13c** Solution

The function f is defined by $f(x) = \sin(x)$ for all real numbers x. Let g be the function defined on $[-1, 1]$ by $g(x) = \arcsin(x)$. **2**

Is g the inverse of f? Justify your answer.

NESA 2022 Mathematics Extension 1 HSC Examination

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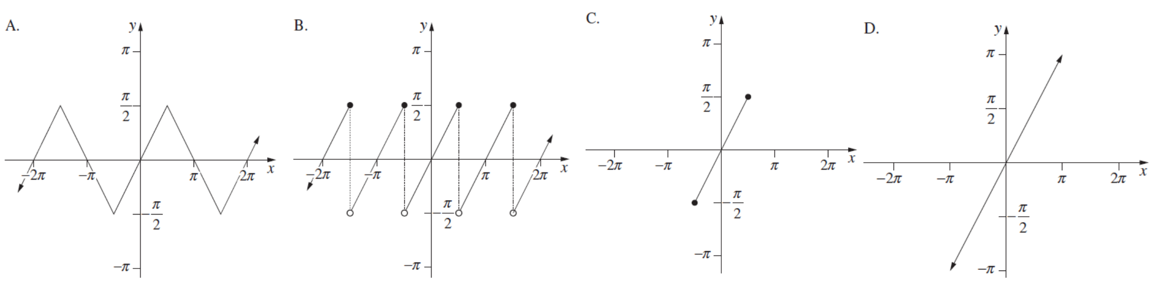
21 MX 1 **9** Which graph represents the function $y = \sin^{-1}(\sin x)$? **1** Solution



NESA 2021 Mathematics Extension 1 HSC Examination

SP MX 1 **10** The graph of the function $y = \sin^{-1}(x - 4)$ is transformed by being dilated horizontally with a scale factor of 2 and then translated to the right by 1. **1** Solution

What is the equation of the transformed graph?

A. $y = \sin^{-1}\left(\dfrac{x-9}{2}\right)$ B. $y = \sin^{-1}\left(\dfrac{x-10}{2}\right)$

C. $y = \sin^{-1}(2x - 6)$ D. $y = \sin^{-1}(2x - 5)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG **1** For each function, state the domain and range of the function and sketch its graph: Solution

(a) $f(x) = \sin^{-1}(x + 5)$

(b) $g(x) = 2\cos^{-1}x$

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

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TG **2** Evaluate α if $\alpha = \cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

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TG **3** Determine the exact value of $\sin^{-1}\left(\dfrac{1}{2}\right) - \sin^{-1}\left(-\dfrac{1}{2}\right) - \tan^{-1}(-\sqrt{3})$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

TG **4** Show that $\sin(\cos^{-1} p) = \sqrt{1 - p^2}$. [Solution](#)

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

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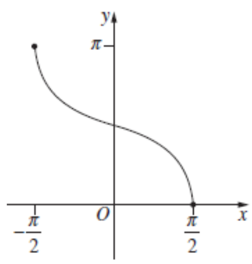
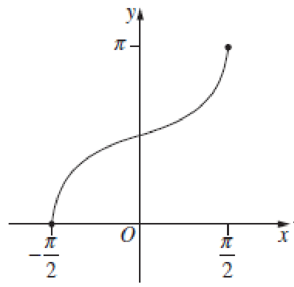
19 MX 1 **9** Which graph best represents $y = \cos^{-1}(-\sin x)$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$? **1** [Solution](#)

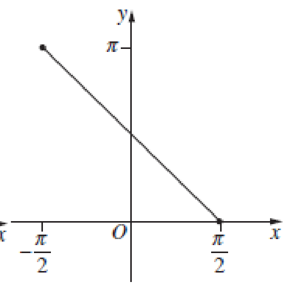
A.



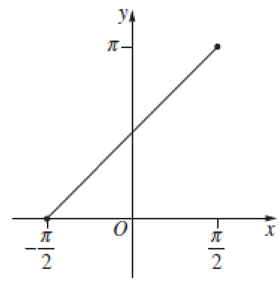
B.



C.



D.



NESA 2019 Mathematics Extension 1 HSC Examination

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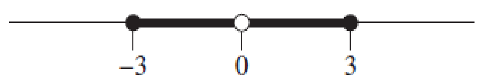
17 MX 1 **7** Which diagram represents the domain of the function $f(x) = \sin^{-1}\left(\frac{3}{x}\right)$? **1** [Solution](#)

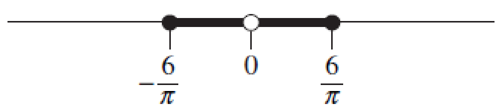
(A)

(B)



(C)

(D)



NESA 2017 Mathematics Extension 1 HSC Examination

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17 MX 1 **11 d** Sketch the graph of the function $y = 2\cos^{-1} x$. **2** [Solution](#)

NESA 2017 Mathematics Extension 1 HSC Examination

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15 MX 1 **6** What is the domain of the function $f(x) = \sin^{-1}(2x)$? **1** [Solution](#)

(A) $-\pi \le x \le \pi$

(B) $-2 \le x \le 2$

(C) $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

(D) $-\frac{1}{2} \le x \le \frac{1}{2}$

NESA 2015 Mathematics Extension 1 HSC Examination

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14 MX 1 **11 c** Sketch the graph $y = 6\tan^{-1} x$, clearly indicating the range. **2** [Solution](#)

NESA 2014 Mathematics Extension 1 HSC Examination

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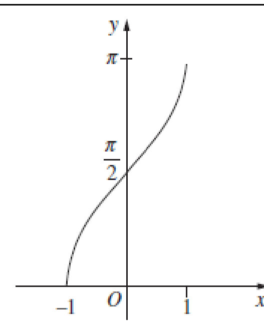
13 MX 1 **9** The diagram shows the graph of a function. **1** [Solution](#)



Which function does the graph represent?

(A) $y = \cos^{-1} x$

(B) $y = \frac{\pi}{2} + \sin^{-1} x$

(C) $y = -\cos^{-1} x$

(D) $y = -\frac{\pi}{2} - \sin^{-1} x$

NESA 2013 Mathematics Extension 1 HSC Examination

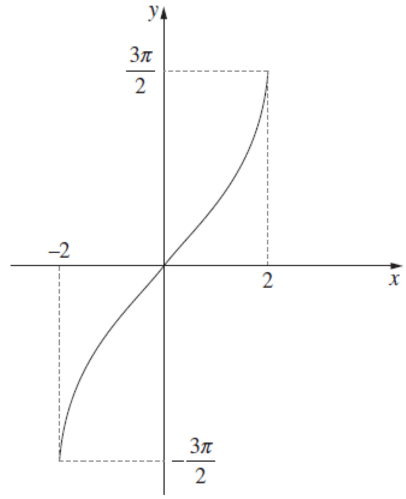
12 MX 1 **4** Which function best describes the following graph? **1** Solution

(A) $y = 3\sin^{-1} 2x$

(B) $y = \dfrac{3}{2}\sin^{-1} 2x$

(C) $y = 3\sin^{-1} \dfrac{x}{2}$

(D) $y = \dfrac{3}{2}\sin^{-1} \dfrac{x}{2}$



NESA 2012 Mathematics Extension 1 HSC Examination

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12 MX 1 **13 a** Write $\sin\left(2\cos^{-1}\left(\dfrac{2}{3}\right)\right)$ in the form $a\sqrt{b}$, where a and b are rational. **2** Solution

NESA 2012 Mathematics Extension 1 HSC Examination

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11 MX 1 **1e** Find the exact value of $\cos^{-1}\left(-\dfrac{1}{2}\right)$. **1** Solution

NESA 2011 Mathematics Extension 1 HSC Examination

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11 MX 1 **2d** Sketch the graph of the function $f(x) = 2\cos^{-1} x$. **2** Solution
Clearly indicate the domain and range of the function.

NESA 2011 Mathematics Extension 1 HSC Examination

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10 MX 1 **1b** Let $f(x) = \cos^{-1}\left(\dfrac{x}{2}\right)$. What is the domain of $f(x)$? **1** Solution

NESA 2010 Mathematics Extension 1 HSC Examination

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07 MX 1 **2b** Let $f(x) = 2\cos^{-1}x$. Solution

(i) Sketch the graph of $y = f(x)$, indicating clearly the coordinates of the endpoints of the graph. **2**

(ii) State the range of $f(x)$. **1**

NESA 2007 Mathematics Extension 1 HSC Examination

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07 MX 1 **5c** Find the exact values of x and y which satisfy the simultaneous equations **3** Solution

$$\sin^{-1} x + \frac{1}{2}\cos^{-1} y = \frac{\pi}{3} \quad \text{and}$$

$$3\sin^{-1} x - \frac{1}{2}\cos^{-1} y = \frac{2\pi}{3}.$$

NESA 2007 Mathematics Extension 1 HSC Examination

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Year 11: Trigonometric Functions
T2 Further trigonometric identities

Back

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https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- derive and use the sum and difference expansions for the trigonometric functions $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ (ACMSM044)
  - $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  - $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
  - $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- derive and use the double angle formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ (ACMSM044)
  - $\sin 2A = 2 \sin A \cos A$
  - $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
  - $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
- derive and use expressions for $\sin A$, $\cos A$ and $\tan A$ in terms of t where $t = \tan\frac{A}{2}$ (the t-formulae)
  - $\sin A = \frac{2t}{1+t^2}$
  - $\cos A = \frac{1-t^2}{1+t^2}$
  - $\tan A = \frac{2t}{1-t^2}$
- derive and use the formulae for trigonometric products as sums and differences for $\cos A \cos B$, $\sin A \sin B$, $\sin A \cos B$ and $\cos A \sin B$ (ACMSM047)
  - $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
  - $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
  - $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$
  - $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

Reference Sheet

TG 1 Find the exact value of $\tan 75°$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

TG 2 Find the exact value of $\cos\dfrac{\pi}{8}$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

TG 3 If $\cos\theta = -\dfrac{3}{5}$, and $0 < \theta < \pi$, determine the exact value of $\tan\theta$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

TG 4 By expanding the left-hand side, show that $\sin(5x + 4x) + \sin(5x - 4x) = 2\sin 5x \cos 4x$. Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Trigonometric functions

16 MX 1 **3**

Which expression is equivalent to $\dfrac{\tan 2x - \tan x}{1 + \tan 2x \tan x}$? **1** Solution

(A) $\tan x$ (B) $\tan 3x$ (C) $\dfrac{\tan 2x - 1}{1 + \tan 2x}$ (D) $\dfrac{\tan x}{1 + \tan 2x \tan x}$

NESA 2016 Mathematics Extension 1 HSC Examination

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13 MX 1 **8**

The angle θ satisfies $\sin\theta = \dfrac{5}{13}$ and $\dfrac{\pi}{2} < \theta < \pi$. What is the value of $\sin 2\theta$? **1** Solution

(A) $\dfrac{10}{13}$ (B) $-\dfrac{10}{13}$ (C) $\dfrac{120}{169}$ (D) $-\dfrac{120}{169}$

NESA 2013 Mathematics Extension 1 HSC Examination

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09 MX 1 **3c** Solution

(i) Prove that $\tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$ provided that $\cos 2\theta \neq 1$. **2**

(ii) Hence find the exact value of $\tan \dfrac{\pi}{8}$. **1**

NESA 2009 Mathematics Extension 1 HSC Examination

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Year 11: Calculus

C1.1 Rates of change with respect to time

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Students:

- describe the rate of change of a physical quantity with respect to time as a derivative
  - investigate examples where the rate of change of some aspect of a given object with respect to time can be modelled using derivatives **AAM**
  - use appropriate language to describe rates of change, for example 'at rest', 'initially', 'change of direction' and 'increasing at an increasing rate'
- find and interpret the derivative $\frac{dQ}{dt}$, given a function in the form $Q = f(t)$, for the amount of a physical quantity present at time t
- describe the rate of change with respect to time of the displacement of a particle moving along the x-axis as a derivative $\frac{dx}{dt}$ or $\dot{x}$
- describe the rate of change with respect to time of the velocity of a particle moving along the x-axis as a derivative $\frac{d^2x}{dt^2}$ or $\ddot{x}$

Reference Sheet

TG **1** A particle is moving along the x axis. **Solution**

Its velocity v at position x is given by $v = \sqrt{10t - t^2}$.

Find the acceleration of the particle at $t = 4$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **2** / **02 M** **7b** A cooler, which is initially full, is drained so that at time t seconds the volume of water V, in litres, is given by $V = 25(1 - \frac{t}{60})^2$ for $0 \le t \le 60$. **Solution**

(a) How much water was initially in the cooler? **1**

(b) After how many seconds was the cooler one-quarter full? **2**

(c) At what rate was the water draining out when the cooler was one-quarter full? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2002 Mathematics HSC Examination

18 M **12 d** The displacement of a particle moving along the x-axis is given by **Solution**

$x = \frac{t^3}{3} - 2t^2 + 3t$, where x is the displacement from the origin in metres and t is the time in seconds, for $t \ge 0$.

(i) What is the initial velocity of the particle? **1**

(ii) At which times is the particle stationary? **2**

(iii) Find the position of the particle when the acceleration is zero. **2**

NESA 2018 Mathematics HSC Examination

14 M **13 c** The displacement of a particle moving along the x-axis is given by $x = t - \dfrac{1}{1+t}$, where x is the displacement from the origin in metres, t is the time in seconds, and $t \geq 0$. **Solution**

(i) Show that the acceleration of the particle is always negative. **2**

(ii) What value does the velocity approach as t increases indefinitely? **1**

NESA 2014 Mathematics HSC Examination

13 M **10** A particle is moving along the x-axis. The displacement of the particle at time t seconds is x metres. At a certain time, $\dot{x} = -3 \text{ m s}^{-1}$ and $\ddot{x} = 2 \text{ m s}^{-2}$. **1** **Solution**

Which statement describes the motion of the particle at that time?

(A) The particle is moving to the right with increasing speed.
(B) The particle is moving to the left with increasing speed.
(C) The particle is moving to the right with decreasing speed.
(D) The particle is moving to the left with decreasing speed.

NESA 2013 Mathematics HSC Examination

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Year 11: Calculus

C1.2 Exponential growth and decay

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Students:

- construct, analyse and manipulate an exponential model of the form $N(t) = Ae^{kt}$ to solve a practical growth or decay problem in various contexts (for example population growth, radioactive decay or depreciation) **AAM**
  - establish the simple growth model, $\frac{dN}{dt} = kN$, where N is the size of the physical quantity, $N = N(t)$ at time t and k is the growth constant
  - verify (by substitution) that the function $N(t) = Ae^{kt}$ satisfies the relationship $\frac{dN}{dt} = kN$, with A being the initial value of N
  - sketch the curve $N(t) = Ae^{kt}$ for positive and negative values of k
  - recognise that this model states that the rate of change of a quantity varies directly with the size of the quantity at any instant
- establish the modified exponential model, $\frac{dN}{dt} = k(N - P)$, for dealing with problems such as 'Newton's Law of Cooling' or an ecosystem with a natural 'carrying capacity' **AAM**
  - verify (by substitution) that a solution to the differential equation $\frac{dN}{dt} = k(N - P)$ is $N(t) = P + Ae^{kt}$, for an arbitrary constant A, and P a fixed quantity, and that the solution is $N = P$ in the case when $A = 0$
  - sketch the curve $N(t) = P + Ae^{kt}$ for positive and negative values of k
  - note that whenever $k < 0$, the quantity N tends to the limit P as $t \to \infty$, irrespective of the initial conditions
  - recognise that this model states that the rate of change of a quantity varies directly with the difference in the size of the quantity and a fixed quantity at any instant
- solve problems involving situations that can be modelled using the exponential model or the modified exponential model and sketch graphs appropriate to such problems **AAM**

Reference Sheet

23 MX 1 **1** The temperature $T(t)$°C of an object at time t seconds is modelled using Newton's Law of Cooling, **1** **Solution**

$$T(t) = 15 + 4e^{-3t}$$

What is the initial temperature of the object?

A. −3 B. 4 C. 15 D. 19

NESA 2023 Mathematics Extension 1 HSC Examination

21 MX 1 **12 b**

A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After *t* minutes, the temperature of the water, in degrees Celsius, is *T*. Solution

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \text{ (Do NOT prove this.)}$$

where *k* is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C. **3**

By solving the differential equation, find the value of *t* when the temperature of the water reaches 20°C. Give your answer to the nearest minute.

(ii) Sketch the graph of *T* as a function of *t*. **1**

NESA 2021 Mathematics Extension 1 HSC Examination

TG **1**

The growth rate of a population of bacteria is 10% of the population. Solution

At $t = 0$, the population is 1.0×10^6.

Sketch the graph of population against time and determine the population after 3.5 hours, correct to four significant figures.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **2**

On an island, the population in 1960 was 1732, and in 1970 it was 1260. Solution

Find the annual growth rate to the nearest percent, assuming it is proportional to the population.

In how many years will the population be half of what it was in 1960?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG **3**

12 M **14 c**

Professor Smith has a colony of bacteria. Initially, there are 1000 bacteria. The number of bacteria, $N(t)$, after t minutes is given by $N(t) = 1000e^{kt}$. Solution

(i) After 20 minutes there are 2000 bacteria. **1**
Show that $k = 0.0347$ correct to four decimal places.

(ii) How many bacteria are there when $t = 120$? **1**

(iii) What is the rate of change of the number of bacteria per minute, when $t = 120$? **1**

(iv) How long does it take for the number of bacteria to increase from 1000 to 100 000? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2012 Mathematics HSC Examination

TG **4**

07 M **8a**

One model for the number of mobile phones in use worldwide is the exponential growth model, $N = Ae^{kt}$, where *N* is the estimate for the number of mobile phones in use (in millions), and *t* is the time in years after 1 January 2008. Solution

(i) It is estimated that at the start of 2009, when $t = 1$, there will be 1600 million mobile phones in use, while at the start of 2010, when $t = 2$, there will be 2600 million. Find *A* and *k*. **3**

(ii) According to the model, during which month and year will the number of mobile phones in use first exceed 4000 million? **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2007 Mathematics HSC Examination

TG 05 MX 1 **5 2d**

A salad, which is initially at a temperature of 25°C, is placed in a refrigerator that has a constant temperature of 3°C. The cooling rate of the salad is proportional to the difference between the temperature of the refrigerator and the temperature, T, of the salad. That is, T satisfies the equation $\frac{dT}{dt} = -k(T - 3)$, where t is the number of minutes after the salad is placed in the refrigerator.

(i) Show that $T = 3 + Ae^{-kt}$ satisfies this equation. **1**

(ii) The temperature of the salad is 11°C after 10 minutes. **3**
Find the temperature of the salad after 15 minutes.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2005 Mathematics Extension 1 HSC Examination

Solution

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19 MX 1 **12 d**

A refrigerator has a constant temperature of 3°C. A can of drink with temperature 30°C is placed in the refrigerator.
After being in the refrigerator for 15 minutes, the temperature of the can of drink is 28°C.
The change in the temperature of the can of drink can be modelled by $\frac{dT}{dt} = k(T - 3)$, where T is the temperature of the can of drink, t is the time in minutes after the can is placed in the refrigerator and k is a constant.

(i) Show that $T = 3 + Ae^{kt}$, where A is a constant, satisfies $\frac{dT}{dt} = k(T - 3)$. **1**

(ii) After 60 minutes, at what rate is the temperature of the can of drink changing? **3**

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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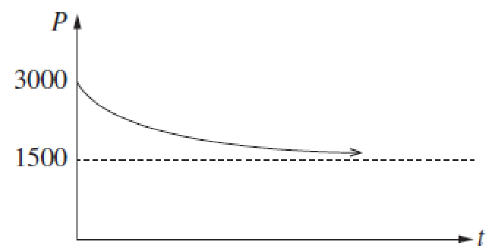
18 MX 1 **5**

The diagram shows the number of penguins, $P(t)$, on an island at time t. **1**
Which equation best represents this graph?



A. $P(t) = 1500 + 1500e^{-kt}$

B. $P(t) = 3000 - 1500e^{-kt}$

C. $P(t) = 3000 + 1500e^{-kt}$

D. $P(t) = 4500 - 1500e^{-kt}$

NESA 2018 Mathematics Extension 1 HSC Examination

Solution

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17 M **14 c**

Carbon-14 is a radioactive substance that decays over time. The amount of carbon-14 present in a kangaroo bone is given by $C(t) = Ae^{kt}$, where A and k are constants, and t is the number of years since the kangaroo died.

(i) Show that $C(t)$ satisfies $\frac{dC}{dt} = kC$. **1**

(ii) After 5730 years, half of the original amount of carbon-14 is present. **2**
Show that the value of k, correct to 2 significant figures, is -0.00012.

(iii) The amount of carbon-14 now present in a kangaroo bone is 90% of the original amount. Find the number of years since the kangaroo died. **2**
Give your answer correct to 2 significant figures.

NESA 2017 Mathematics HSC Examination

Solution

16 MX 1 **12 b** [Solution]

In a chemical reaction, a compound X is formed from a compound Y. The mass in grams of X and Y are $x(t)$ and $y(t)$ respectively, where t is the time in seconds after the start of the chemical reaction.
Throughout the reaction the sum of the two masses is 500 g.
At any time t, the rate at which the mass of compound X is increasing is proportional to the mass of compound Y.

At the start of the chemical reaction, $x = 0$ and $\dfrac{dx}{dt} = 2$.

(i) Show that $\dfrac{dx}{dt} = 0.004(500 - x)$. **3**

(ii) Show that $x = 500 - Ae^{-0.004t}$ satisfies the equation in part (i), and find the value of A. **2**

NESA 2016 Mathematics Extension 1 HSC Examination

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15 MX 1 **2** **1** [Solution]

Given that $N = 100 + 80e^{kt}$, which expression is equal to $\dfrac{dN}{dt}$?

(A) $k(100 - N)$ (B) $k(180 - N)$ (C) $k(N - 100)$ (D) $k(N - 180)$

NESA 2015 Mathematics Extension 1 HSC Examination

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15 M **15 a** [Solution]

The amount of caffeine, C, in the human body decreases according to the equation $\dfrac{dC}{dt} = -0.14C$, where C is measured in mg and t is the time in hours.

(i) Show that $C = Ae^{-0.14t}$ is a solution to $\dfrac{dC}{dt} = -0.14C$, where A is a constant. **1**

When $t = 0$, there are 130 mg of caffeine in Lee's body.

(ii) Find the value of A. **1**

(iii) What is the amount of caffeine in Lee's body after 7 hours? **1**

(iv) What is the time taken for the amount of caffeine in Lee's body to halve? **2**

NESA 2015 Mathematics HSC Examination

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14 MX 1 **12 f** **3** [Solution]

Milk taken out a refrigerator has a temperature of 2°C. It is placed in a room of constant temperature 23°C. After t minutes the temperature, T°C, of the milk is given by $T = A - Be^{-0.03t}$, where A and B are positive constants.
How long does it take for the milk to reach a temperature of 10°C?

NESA 2014 Mathematics Extension 1 HSC Examination

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14 M **13 b** [Solution]

A quantity of radioactive material decays according to the equation $\dfrac{dM}{dt} = -kM$,

where M is the mass of the material in kg, t is the time in years and k is a constant.

(i) Show that $M = Ae^{-kt}$ is a solution to the equation, where A is a constant. **1**

(ii) The time for half of the material to decay is 300 years. If the initial amount of material is 20 kg, find the amount remaining after 1000 years. **3**

NESA 2014 Mathematics HSC Examination

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13 MX 1 **12 c** **3** [Solution]

A cup of coffee with an initial temperature of 80°C is placed in a room with a constant temperature of 22°C.
The temperature, T°C, of the coffee after t minutes is given by $T = A + Be^{-kt}$, where A, B and k are positive constants.
The temperature of the coffee drops to 60°C after 10 minutes.
How long does it take for the temperature of the coffee to drop to 40°C?
Give your answer to the nearest minute.

NESA 2013 Mathematics Extension 1 HSC Examination

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13 M **16 b**

Trout and carp are types of fish. A lake contains a number of trout. At a certain time 10 carp are introduced into the lake and start eating the trout. As a consequence, the number of trout, N, decreases according to $N = 375 - e^{0.04t}$, where t is the time in months after the carp are introduced.

Solution

The population of carp, P, increases according to $\dfrac{dP}{dt} = 0.02P$.

(i) How many trout were in the lake when the carp were introduced? **1**

(ii) When will the population of trout be zero? **1**

(iii) Sketch the number of trout as a function of time. **1**

(iv) When is the rate of increase of carp equal to the rate of decrease of trout? **3**

(v) When is the number of carp equal to the number of trout? **2**

NESA 2013 Mathematics HSC Examination

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11 MX 1 **5b**

To test some forensic science students, an object has been left in a park. At 10 am the temperature of the object is measured to be 30°C. The temperature in the park is a constant 22°C. The object is moved immediately to a room where the temperature is a constant 5°C.

Solution

(i) The temperature of the object in the room can be modelled by the equation $T = 5 + 25e^{-kt}$, where T is the temperature of the object in degrees Celsius, t is the time in hours since the object was placed in the room and k is a constant. After one hour in the room the temperature of the object is 20°C. **2**

Show that $k = \ln\left(\dfrac{5}{3}\right)$.

(ii) In a similar manner, the temperature of the object in the park before it was discovered can be modelled by an equation of the form $T = A + Be^{-kt}$, with the same constant $k = \ln\left(\dfrac{5}{3}\right)$. Find the time of day when the object had a temperature of 37°C. **3**

NESA 2011 Mathematics Extension 1 HSC Examination

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10 MX 1 **2b**

The mass M of a whale is modelled by $M = 36 - 35.5e^{-kt}$, where M is measured in tonnes, t is the age of the whale in years and k is a positive constant.

Solution

(i) Show that the rate of growth of the mass of the whale is given by the differential equation $\dfrac{dM}{dt} = k(36 - M)$. **1**

(ii) When the whale is 10 years old its mass is 20 tonnes. Find the value of k, correct to three decimal places. **2**

(iii) According to this model, what is the limiting mass of the whale? **1**

NESA 2010 Mathematics Extension 1 HSC Examination

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10 M **8a**

Assume that the population, P, of cane toads in Australia has been growing at a rate proportional to P. That is, $\dfrac{dP}{dt} = kP$, where k is a positive constant. **4**

Solution

There were 102 cane toads brought to Australia from Hawaii in 1935.
Seventy-five years later, in 2010, it is estimated that there are 200 million cane toads in Australia.
If the population continues to grow at this rate, how many cane toads will there be in Australia in 2035?

NESA 2010 Mathematics HSC Examination

09 M **6b** Radium decays at a rate proportional to the amount of radium present. That is, if $Q(t)$ is the amount of radium present at time t, then $Q = Ae^{-kt}$, where k is a positive constant and A is the amount present at $t = 0$. It takes 1600 years for an amount of radium to reduce by half. **Solution**

(i) Find the value of k. **2**

(ii) A factory site is contaminated with radium. The amount of radium on the site is currently three times the safe level. How many years will it be before the amount of radium reaches the safe level? **2**

NESA 2009 Mathematics HSC Examination

08 MX 1 **4a** A turkey is taken from the refrigerator. Its temperature is 5°C when it is placed in an oven preheated to 190°C. Its temperature, T° C, after t hours in the oven satisfies the equation $\frac{dT}{dt} = -k(T - 190)$. **Solution**

(i) Show that $T = 190 - 185e^{-kt}$ satisfies both this equation and the initial condition. **2**

(ii) The turkey is placed into the oven at 9 am. At 10 am the turkey reaches a temperature of 29°C. The turkey will be cooked when it reaches a temperature of 80°C.
At what time (to the nearest minute) will it be cooked? **3**

NESA 2008 Mathematics Extension 1 HSC Examination

projectmaths

Year 11: Calculus
C1.3 Related rates of change

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- solve problems involving related rates of change as instances of the chain rule (ACMSM129) **AAM**
- develop models of contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, and to which the chain rule can be applied

Reference Sheet

21 MX 1 **11 e**

A spherical bubble is moving up through a liquid. **2** **Solution**

As it rises, the bubble gets bigger and its radius increases at the rate of 0.2 mm/s.

At what rate is the volume of the bubble increasing when its radius reaches 0.6 mm?

Express your answer in mm^3/s rounded to one decimal place.

NESA 2021 Mathematics Extension 1 HSC Examination

20 MX 1 **10**

The quantities P, Q and R are connected by the related rates, **1** **Solution**

$$\frac{dR}{dt} = -k^2$$

$$\frac{dP}{dt} = -l^2 \times \frac{dR}{dt}$$

$$\frac{dP}{dt} = m^2 \times \frac{dQ}{dt}$$

where k, l and m are non-zero constants.

Which of the following statements is true?

A P is increasing and Q is increasing   B P is increasing and Q is decreasing

C P is decreasing and Q is increasing   D P is decreasing and Q is decreasing

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **9**

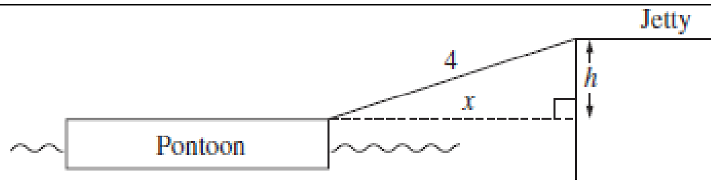
A stone drops into a pond, creating a circular ripple. **1** **Solution**
The radius of the ripple increases from 0 cm at a constant rate of 5 cm s^{-1}.
At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?

A. 25π cm^2 s^{-1} B. 30π cm^2 s^{-1} C. 150π cm^2 s^{-1} D. 225π cm^2 s^{-1}

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG 1 — **04 MX 1 3c** — Solution

A ferry wharf consists of a floating pontoon linked to a jetty by a 4 metre long walkway. Let h metres be the difference in height between the top of the pontoon and the top of the jetty and let x metres be the horizontal distance between the pontoon and the jetty.



(i) Find an expression for x in terms of h. **1**

(ii) When the top of the pontoon is 1 metre lower than the top of the jetty, the tide is rising at a rate of 0.3 metres per hour.
At what rate is the pontoon moving away from the jetty? **3**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2004 Mathematics Extension 1 HSC Examination

TG 2 — Solution

A spherical balloon is being deflated so that the radius decreases at a constant rate of 10 mm per second.
Calculate the rate of change of volume when the radius of the balloon is 100 mm.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG 3 — Solution

A spherical bubble is expanding so that its volume increases at the constant rate of 70 mm^3 per second.
What is the rate of increase of its surface area when the radius is 10 mm?

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

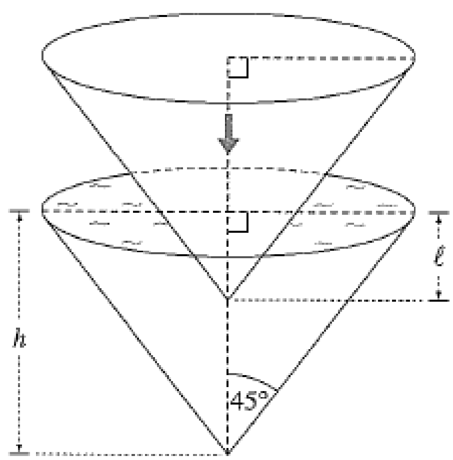
TG 4 — Solution

A hot air balloon is at a constant height of 160 metres above the ground, and moving parallel to the ground, at a speed of 20 metres per minute. Find the rate at which the balloon is moving away from an observer on the ground at the time when the distance from the observer to the balloon is 400 metres.

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus

TG 5 — **11 MX 1 7a** — Solution

The diagram shows two identical circular cones with a common vertical axis. Each cone has height h cm and semi-vertical angle 45°.
The lower cone is completely filled with water. The upper cone is lowered vertically into the water as shown in the diagram. The rate at which it its lowered is given by $\frac{d\ell}{dt} = 10$, where ℓ cm is the distance the upper cone has descended into the water after t seconds. As the upper cone is lowered, water spills from the lower cone. The volume of water remaining in the lower cone at time t is V cm^3.



Not to scale

(i) Show that $V = \frac{\pi}{3}\left(h^3 - \ell^3\right)$. **1**

(ii) Find the rate at which V is changing with respect to time when $\ell = 2$. **2**

(iii) Find the rate at which V is changing with respect to time when the lower cone has lost $\frac{1}{8}$ of its water. Give your answer in terms of h. **2**

NESA Mathematics Extension 1 Year 11 Topic Guide: Calculus
NESA 2011 Mathematics Extension 1 HSC Examination

19 MX 1 **12 a**

Distance A is inversely proportional to distance B, such that $A = \dfrac{9}{B}$, where A and B are measured in metres. The two distances vary with respect to time. Distance B is increasing at a rate of 0.2 ms^{-1}. What is the value of $\dfrac{dA}{dt}$ when $A = 12$? **3** Solution

NESA 2019 Mathematics Extension 1 HSC Examination

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17 MX 1 **8**

A stone drops into a pond, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 5 cm s^{-1}. At what rate is the area of enclosed within the ripple increasing when the radius is 15 cm? **1** Solution

(A) 25π cm^2 s^{-1} (B) 30π cm^2 s^{-1} (C) 150π cm^2 s^{-1} (D) 225π cm^2 s^{-1}

NESA 2017 Mathematics Extension 1 HSC Examination

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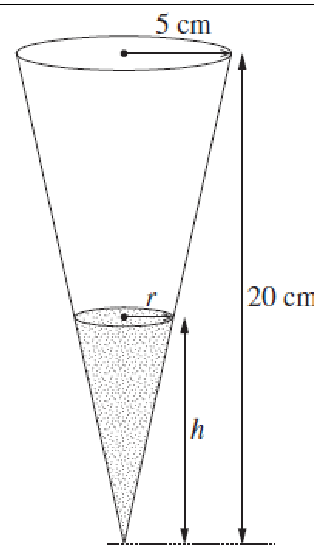
16 MX 1 **12 a** Solution

The diagram shows a conical soap dispenser of radius 5 cm and height 20 cm. At any time t seconds, the top surface of the soap in the container is a circle of radius r cm and its height is h cm.



The volume of the soap is given by $v = \dfrac{1}{3}\pi r^2 h$.

(i) Explain why $r = \dfrac{h}{4}$. **1**

(ii) Show that $\dfrac{dv}{dh} = \dfrac{\pi}{16}h^2$. **1**

The dispenser has a leak which causes soap to drip from the dispenser. The area of the circle formed by the top surface of the soap is decreasing at a constant rate of 0.04 cm^{2}s^{-1}.

(iii) Show that $\dfrac{dh}{dt} = \dfrac{-0.32}{\pi h}$. **2**

(iv) What is the rate of change of the volume of the soap, with respect to time, when $h = 10$? **2**

NESA 2016 Mathematics Extension 1 HSC Examination

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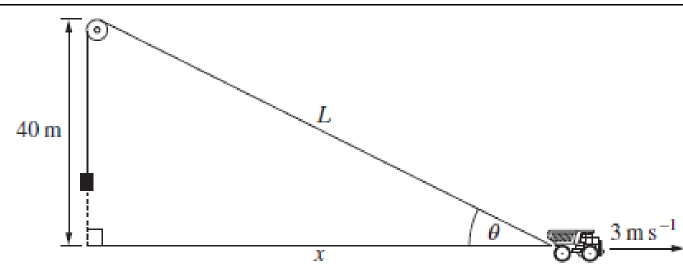
14 MX 1 **13 b** Solution

One end of a rope is attached to a truck and the other end to a weight. The rope passes over a small wheel located at a vertical distance of 40 m above the point where the rope is attached to the truck. The distance from the truck to the small wheel is L m and the horizontal distance between them is x m. The rope makes an angle θ with the horizontal at the point where it is attached to the truck. The truck moves left to right at a constant speed of 3 ms^{-1}, as shown in the diagram.



(i) Using Pythagoras' Theorem, or otherwise, show that $\dfrac{dL}{dx} = \cos\theta$. **2**

(ii) Show that $\dfrac{dL}{dt} = 3\cos\theta$. **1**

NESA 2014 Mathematics Extension 1 HSC Examination

13 MX 1 **13 a** A spherical raindrop of radius r metres loses water through evaporation at a rate that depends on its surface area. **Solution**

The rate of change of the volume V of the raindrop is given by $\frac{dV}{dt} = -10^{-4}A$,

where t is time in seconds and A is the surface area of the raindrop.
The surface area and the volume of the raindrop are given by $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ respectively.

(i) Show that $\frac{dr}{dt}$ is constant. **1**

(ii) How long does it take for a raindrop of volume 10^{-6} m^3 to completely evaporate? **2**

NESA 2013 Mathematics Extension 1 HSC Examination

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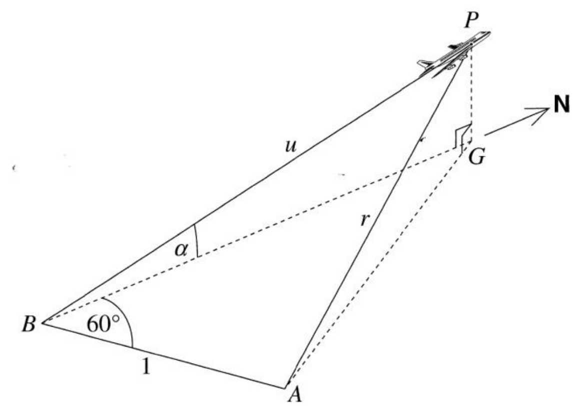
12 MX 1 **14 c** A plane P takes off from a point B. It flies due north at a constant angle α to the horizontal. An observer is located at A, 1 km from B, at a bearing 060° from B. Let u km be the distance from B to the plane and let r km be the distance from the observer to the plane. The point G is on the ground directly below the plane. **Solution**



(i) Show that $r = \sqrt{1 + u^2 - u\cos\alpha}$. **3**

(ii) The plane is travelling at a constant speed of 360 km/h. At what rate, in terms of α, is the distance of the plane from the observer changing 5 minutes after take-off? **2**

NESA 2012 Mathematics Extension 1 HSC Examination

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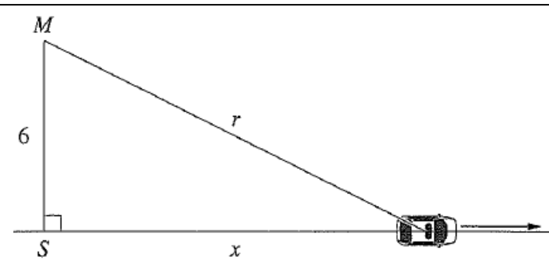
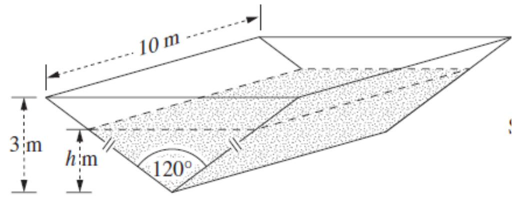
10 MX 1 **2d** A radio transmitter M is situated 6 km from a straight road. The closest point on the road to the transmitter is S. A car travelling away from S along the road at a speed of 100 km h^{-1}. The distance from the car to S is x km and from the car to M is r km. **3** **Solution**



Find an expression in terms of x for $\frac{dr}{dt}$, where t is time in hours.

NESA 2010 Mathematics Extension 1 HSC Examination

09 MX 1 **5b** The cross-section of a 10 metre long tank is an isosceles triangle, as shown in the diagram. The top of the tank is horizontal. When the tank is full, the depth of water is 3 m. The depth of water at time t days is h metres. **Solution**



(i) Find the volume, V, of water in the tank when the depth of water is h metres. **1**

(ii) Show that the area, A, of the top surface of the water is given by $A = 20\sqrt{3}\,h$ **1**

(iii) The rate of evaporation of the water is given by $\frac{dV}{dt} = -kA$, where k is a positive constant. Find the rate at which the depth of water is changing at time t. **2**

(iv)* It takes 100 days for the depth to fall from 3 m to 2 m. Find the time taken for the depth to fall from 2 m to 1 m. **1**
(* involves integration (y12))

NESA 2009 Mathematics Extension 1 HSC Examination

projectmaths

Year 11: Combinatorics
A1.1 Permutations and combinations

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

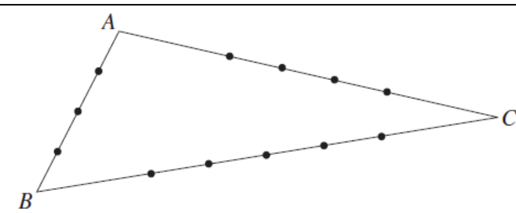
Students:

- list and count the number of ways an event can occur
- use the fundamental counting principle (also known as the multiplication principle)
- use factorial notation to describe and determine the number of ways n different items can be arranged in a line or a circle
  - solve problems involving cases where some items are not distinct (excluding arrangements in a circle)
- solve simple problems and prove results using the pigeonhole principle (ACMSM006)
  - understand that if there are n pigeonholes and $n+1$ pigeons to go into them, then at least one pigeonhole must hold 2 or more pigeons
  - generalise to: If n pigeons are sitting in k pigeonholes, where $n > k$, then there is at least one pigeonhole with at least $\frac{n}{k}$ pigeons in it
  - prove the pigeonhole principle
- understand and use permutations to solve problems (ACMSM001)
  - understand and use the notation ${}^nP_r$ and the formula ${}^nP_r = \frac{n!}{(n-r)!}$
- solve problems involving permutations and restrictions with or without repeated objects (ACMSM004)
- understand and use combinations to solve problems (ACMSM007)
  - understand and use the notations $\binom{n}{r}$ and ${}^nC_r$ and the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$ (ACMMM045, ACMSM008)
- solve practical problems involving permutations and combinations, including those involving simple probability situations **AAM**

Reference Sheet

23 MX 1	**10**	A group with 5 students and 3 teachers is to be arranged in a circle. In how many ways can this be done if no more than 2 students can sit together? A. 4! × 3! B. 5! × 3! C. 2! × 5! × 3! D. 2! × 2! × 2! × 3! NESA 2023 Mathematics Extension 1 HSC Examination	**1**	**Solution**
23 MX 1	**11 b**	In how many different ways can all the letters of the word CONDOBOLIN be arranged in a line? NESA 2023 Mathematics Extension 1 HSC Examination	**2**	**Solution**

22 MX 1 **7** The diagram shows triangle ABC with points chosen on each of the sides.
On side AB, 3 points are chosen.
On side AC, 4 points are chosen.
On side BC, 5 points are chosen.



1 **Solution**

How many triangles can be formed using the chosen points as vertices?

A. 60 B. 145 C. 205 D. 220

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **12 b** A sports association manages 13 junior teams. It decides to check the age of all players. Any team that has more than 3 players above the age limit will be penalised.
A total of 41 players are found to be above the age limit.
Will any team be penalised? Justify your answer. **2** **Solution**

NESA 2022 Mathematics Extension 1 HSC Examination

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21 MX 1 **10** The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only. **1** **Solution**

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A 236 B 237 C 238 D 239

NESA 2021 Mathematics Extension 1 HSC Examination

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21 MX 1 **11 d** A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women. In how many different ways can the committee be formed? **1** **Solution**

NESA 2021 Mathematics Extension 1 HSC Examination

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20 MX 1 **8** Out of 10 contestants, six are to be selected for the final round of a competition. Four of those six will be placed 1st, 2nd, 3rd and 4th. **1** **Solution**

In how many ways can this process be carried out?

A $\dfrac{10!}{6!4!}$ B $\dfrac{10!}{6!}$ C $\dfrac{10!}{4!2!}$ D $\dfrac{10!}{4!4!}$

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **12 c** To complete a course, a student must choose and pass exactly three topics. **2** **Solution**

There are eight topics from which to choose.

Last year 400 students completed the course.

Explain, using the pigeonhole principle, why at least eight students passed exactly the same three topics.

NESA 2020 Mathematics Extension 1 HSC Examination

20 MX 1 **14 a**

(i) Use the identity $(1 + x)^{2n} = (1 + x)^n(1 + x)^n$ to show that **2** Solution

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2 \text{ where } n \text{ is a positive integer.}$$

(ii) A club has $2n$ members, with n women and n men. **2**

A group consisting of an even number $(0, 2, 4, \ldots, 2n)$ of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

(iii) From the group chosen in part (ii), one of the men and one of the women are selected as leaders. **2**

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is $1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \ldots + n^2\binom{n}{n}^2$.

(iv) The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men. **2**

By considering this reversed process, and using part (ii), find a simple expression for the sum in part (iii).

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **7** Each of the students in an athletics team is randomly allocated their own locker from a row of 100 lockers. **1** Solution

What is the smallest number of students in the team that guarantees that two students are allocated consecutive lockers?

A. 26 B. 34 C. 50 D. 51

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **8** A team of 11 students is to be chosen from a group of 18 students. **1** Solution

Among the 18 students are 3 students who are left-handed.

What is the number of possible teams containing at least 1 student who is left-handed?

A. 19 448 B. 30 459 C. 31 824 D. 58 344

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2016 Mathematics Extension 1 HSC Examination

MX SQ 2019 **2** A bag contains 4 white, 4 red and 4 black marbles. Kim takes some marbles out of the bag without looking at them and places them on the table. **1** Solution

What is the smallest number of marbles that must be taken from the bag in order that at least three marbles of the same colour are on the table?

A. 3 B. 5 C. 7 D. 9

NESA Mathematics Extension 1 Sample examination materials (2019)

TG **1** How many numbers greater than 5000 can be formed with the digits 2, 3, 5, 7, 9 if no digit is repeated? Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG **2** In how many ways can the letters of EERIE be arranged in a line? Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

TG 3

(a) In how many ways can the numbers 1, 2, 3, 4, 5, 6 be arranged around a circle?
(b) How many of these arrangements have at least two even numbers together?

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG 4

How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order?

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG 5 / **10 MX 1 3a**

At the front of a building there are five garage doors. Two of the doors are to be painted red, one is to be painted green, one blue and one orange.

(a) How many possible arrangements are there for the colours on the doors? **1**

(b) How many possible arrangements are there for the colours on the doors if the two red doors are next to each other? **1**

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2010 Mathematics Extension 1 HSC Examination

TG 6

In horse racing, betting on the trifecta pays on the first three horses in correct order, while the quinella pays on the first two horses in either order.

(a) In a 12-horse race, determine the number of possible quinellas.

(b) The Melbourne cup is a 17-horse race.
Determine the number of possible trifectas.

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

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TG 7 / **07 MX 1 5b**

Mr and Mrs Roberts and their four children go to the theatre. **2**

They are randomly allocated six adjacent seats in a single row.

What is the probability that the four children are allocated seats next to each other?

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2007 Mathematics Extension 1 HSC Examination

TG 8 / **04 MX 1 2e**

A four-person team is to be chosen at random from nine women and seven men.

(i) In how many ways can this team be chosen? **1**

(ii) What is the probability that the team will consist of four women? **1**

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2004 Mathematics Extension 1 HSC Examination

TG 9 / **15 MX 1 14 c**

Two players *A* and *B* play a series of games against each other to get a prize. In any game, either of the players is equally likely to win. To begin with, the first player who wins a total of 5 games gets the prize.

(i) Explain why the probability of player *A* getting the prize in exactly 7 games is $\binom{6}{4}\left(\frac{1}{2}\right)^7$. **1**

(ii) Write an expression for the probability of player *A* getting the prize in at most 7 games. **1**

(iii) Suppose now that the prize is given to the first player to win a total of $(n + 1)$ games, where n is a positive integer. **2**
By considering the probability that *A* gets the prize, prove that

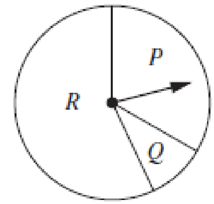
$$\binom{n}{n}2^n + \binom{n+1}{n}2^{n-1} + \binom{n+2}{n}2^{n-2} + \ldots + \binom{2n}{n}2^n = 2^{2n}.$$

Solution

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2015 Mathematics Extension 1 HSC Examination

TG 10
14 MX 1 14 b

Two players A and B play a game that consists of taking turns until a winner is determined. Each turn consists of spinning the arrow on a spinner once. The spinner has three sectors P, Q and R. The probabilities that the arrow stops in sectors P, Q and R are p, q and r respectively.
The rules of the game are as follows:



- If the arrow stops in sector P, then the player having the turn wins.
- If the arrow stops in sector Q, then the player having the turn loses and the other player wins
- If the arrow stops in sector R, then the other player takes a turn.

Player A takes the first turn.

(i) Show that the probability of player A winning on the first or second turn of the game is $(1 - r)(p + r)$. **2**

(ii) Show that the probability that player A eventually wins the game is $\dfrac{p+r}{1+r}$. **3**

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics
NESA 2014 Mathematics Extension 1 HSC Examination

Solution

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19 MX 1 8

In how many ways can all the letters of the word PARALLEL be placed in a line with the three Ls together? **1**

A. $\dfrac{6!}{2!}$ B. $\dfrac{6!}{2!3!}$ C. $\dfrac{8!}{2!}$ D. $\dfrac{8!}{2!3!}$

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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18 MX 1 8

Six men and six women are to be seated at a round table. In how many different ways can they be seated if men and women alternate? **1**

A. 5! 5! B. 5! 6! C. 2! 5! 5! D. 2! 5! 6!

NESA 2018 Mathematics Extension 1 HSC Examination

Solution

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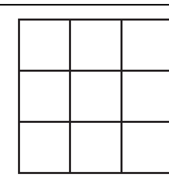
17 MX 1 10

Three squares are chosen at random from the 3×3 grid below, and a cross is placed in each chosen square. What is the probability that all three crosses lie in the same row, column or diagonal? **1**



(A) $\dfrac{1}{28}$ (B) $\dfrac{2}{21}$

(C) $\dfrac{1}{3}$ (D) $\dfrac{8}{9}$

NESA 2017 Mathematics Extension 1 HSC Examination

Solution

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15 MX 1 4

A rowing team consists of 8 rowers and a coxswain. The rowers are selected from 12 students in Year 10. The coxswain is selected from 4 students in Year 9. In how many ways could the team be selected? **1**

(A) ${}^{12}C_8 + {}^{4}C_1$ (B) ${}^{12}P_8 + {}^{4}P_1$ (C) ${}^{12}C_8 \times {}^{4}C_1$ (D) ${}^{12}P_8 \times {}^{4}P_1$

NESA 2015 Mathematics Extension 1 HSC Examination

Solution

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14 MX 1 8

In how many ways can 6 people from a group of 15 people be chosen and then arranged in a circle? **1**

(A) $\dfrac{14!}{8!}$ (B) $\dfrac{14!}{8! \times 6}$ (C) $\dfrac{15!}{9!}$ (D) $\dfrac{15!}{9! \times 6}$

NESA 2014 Mathematics Extension 1 HSC Examination

Solution

13 MX 1	**7**	A family of eight is seated randomly around a circular table. What is the probability that the two youngest members of the family sit together? (A) $\frac{6!\times 2!}{7!}$ (B) $\frac{6!}{7!\times 2!}$ (C) $\frac{6!\times 2!}{8!}$ (D) $\frac{6!}{8!\times 2!}$ NESA 2013 Mathematics Extension 1 HSC Examination	**1**	Solution
12 MX 1	**5**	How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order? (A) 5! (B) 6! (C) 2 × 5! (D) 2 × 6! NESA 2012 Mathematics Extension 1 HSC Examination	**1**	Solution
12 MX 1	**11 e**	In how many ways can a committee of 3 men and 4 women be selected from a group of 8 men and 10 women? NESA 2012 Mathematics Extension 1 HSC Examination	**1**	Solution
11 MX 1	**2e**	Alex's playlist consists of 40 different songs that can be arranged in any order. (i) How many arrangements are there for the 40 songs? **1** (ii) Alex decides that she wants to play her three favourite songs first, in any order. How many arrangements of the 40 songs are now possible? **1** NESA 2011 Mathematics Extension 1 HSC Examination		Solution
10 MX 1	**7c**	A box contains n identical red balls and n identical blue balls. A selection of r balls is made from the box, where $0 \le r \le n$. (i) Explain why the number of possible colour combinations is $r + 1$. **1** (ii) Another box contains n white balls labelled consecutively from 1 to n. A selection of $n - r$ balls is made from the box, where $0 \le r \le n$. **1** Explain why the number of different selections is $\binom{n}{r}$. (iii) The n red balls, the n blue balls and the n white labeled balls are all placed into one box, and a selection of n balls is made. Using Question 7 part (b)* or otherwise, show that the number of different selections is $(n + 2)2^{n-1}$. **3** NESA 2010 Mathematics Extension 1 HSC Examination		Solution
08 MX 1	**4b**	Barbara and John and six other people go through a doorway one at a time. (i) In how many ways can the eight people go through the doorway if John goes through the doorway after Barbara with no-one in between? **1** (ii) Find the number of ways in which the eight people can go through the doorway if John goes through the doorway after Barbara. **1** NESA 2008 Mathematics Extension 1 HSC Examination		Solution

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Year 11: Combinatorics

A1.2 Binomial expansions and Pascal's triangle

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Students:

- expand $(x + y)^n$ for small positive integers n (ACMMM046)
  - note the pattern formed by the coefficients of x in the expansion of $(1 + x)^n$ and recognise links to Pascal's triangle
  - recognise the numbers $\binom{n}{r}$ (also denoted ${}^nC_r$) as binomial coefficients (ACMMM047)
- derive and use simple identities associated with Pascal's triangle (ACMSM009)
  - establish combinatorial proofs of the Pascal's triangle relations ${}^nC_0 = 1$, ${}^nC_n = 1$; ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $1 \le r \le n-1$; and ${}^nC_r = {}^nC_{n-r}$

Reference Sheet

23 MX 1 **12 d**

It is known that ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for all integers such that $1 \le r \le n-1$. **2**

(Do NOT prove this.)

Find ONE possible set of values for p and q such that

$$ {}^{2022}C_{80} + {}^{2022}C_{81} + {}^{2023}C_{1943} = {}^pC_q $$

NESA 2023 Mathematics Extension 1 HSC Examination

Solution

22 MX 1 **11 c**

Find the coefficients of x^2 and x^3 in the expansion of $\left(1 - \frac{x}{2}\right)^8$. **2**

NESA 2022 Mathematics Extension 1 HSC Examination

Solution

21 MX 1 **11 b**

Expand and simplify $(2a - b)^4$. **2**

NESA 2021 Mathematics Extension 1 HSC Examination

Solution

TG **1**

Find the coefficient of x^4 in the expansion of $(x^2 - \frac{2}{x})^5$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

Solution

TG **2**

If $(1 - 2\sqrt{3})^6 = x + y\sqrt{3}$, evaluate x and y by using a binomial expansion.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

Solution

TG **3**

What is the probability of choosing three letters from the word PROBING and only just one letter being a vowel?

Assume the probability of each letter choice is equal.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

Solution

TG **4**

Find the term independent of x in the expansion of $(2x - \frac{1}{x^2})^{12}$.

NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics

Solution

TG **96 MX 1**	**5** **7a**	Using the fact that $(1 + x)^4(1 + x)^9 = (1 + x)^{13}$, show that $^4C_0\,^9C_4 + {}^4C_1\,^9C_3 + {}^4C_2\,^9C_2 + {}^4C_3\,^9C_1 + {}^4C_4\,^9C_0 = {}^{13}C_4$ NESA Mathematics Extension 1 Year 11 Topic Guide: Combinatorics NESA 1996 Mathematics Extension 1 HSC Examination	**2**	Solution
19 MX 1	**13 b**	In the expansion of $(5x + 2)^{20}$, the coefficients of x^k and x^{k+1} are equal. What is the value of k? NESA 2019 Mathematics Extension 1 HSC Examination	**3**	Solution
17 MX 1	**9**	When expanded, which expression has a non-zero constant term? (A) $\left(x + \frac{1}{x^2}\right)^7$ (B) $\left(x^2 + \frac{1}{x^3}\right)^7$ (C) $\left(x^3 + \frac{1}{x^4}\right)^7$ (D) $\left(x^4 + \frac{1}{x^5}\right)^7$ NESA 2017 Mathematics Extension 1 HSC Examination	**1**	Solution
15 MX 1	**13 b**	Consider the binomial expansion $(2x + \frac{1}{3x})^{18} = a_0x^{18} + a_1x^{16} + a_2x^{14} + \ldots$ where $a_0, a_1, a_2, \ldots$ are constants. (i) Find an expression for a_2. (ii) Find an expression for the term independent of x. NESA 2015 Mathematics Extension 1 HSC Examination	 **2** **2**	Solution
14 MX 1	**3**	What is the constant term in the binomial expansion of $\left(2x - \frac{5}{x^3}\right)^{12}$? (A) $\binom{12}{3} 2^9 5^3$ (B) $\binom{12}{9} 2^3 5^9$ (C) $-\binom{12}{3} 2^9 5^3$ (D) $-\binom{12}{9} 2^3 5^9$ NESA 2014 Mathematics Extension 1 HSC Examination	**1**	Solution
12 MX 1	**11 f**	(i) Use the binomial theorem to find an expression for the constant term in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{12}$. (ii) For what values of n does $\left(2x^3 - \frac{1}{x}\right)^n$ have a non-zero constant term? NESA 2012 Mathematics Extension 1 HSC Examination	**2** **1**	Solution
11 MX 1	**2c**	Find an expression for the coefficient of x^2 in the expansion of $\left(3x - \frac{4}{x}\right)^8$. NESA 2011 Mathematics Extension 1 HSC Examination	**2**	Solution
08 MX 1	**1d**	Find an expression for the coefficient of x^8y^4 in the expansion of $(2x + 3y)^{12}$. NESA 2008 Mathematics Extension 1 HSC Examination	**2**	Solution

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Year 12: Proof

P1 Proof by mathematical induction

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Students:

- understand the nature of inductive proof, including the 'initial statement' and the inductive step (ACMSM064)
- prove results using mathematical induction
  - prove results for sums, for example $1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n (ACMSM065)
  - prove divisibility results, for example $3^{2n} - 1$ is divisible by 8 for any positive integer n (ACMSM066)
- identify errors in false 'proofs by induction', such as cases where only one of the required two steps of a proof by induction is true, and understand that this means that the statement has not been proved
- recognise situations where proof by mathematical induction is not appropriate

Reference Sheet

23 MX 1	**12 b**	Use mathematical induction to prove that $$(1 \times 2) + (2 \times 2^2) + (3 \times 2^3) + \ldots + (n \times 2^n) = 2 + (n - 1)2^{n+1}$$ for all integers $n \geq 1$. NESA 2023 Mathematics Extension 1 HSC Examination	**3**	**Solution**
22 MX 1	**12 f**	Use mathematical induction to prove that $15^n + 6^{2n+1}$ is divisible by 7 for all integers $n \geq 0$. NESA 2022 Mathematics Extension 1 HSC Examination	**3**	**Solution**
21 MX 1	**12 c**	Use the principle of mathematical induction to prove $$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$ that for all integers $n \geq 1$. NESA 2020 Mathematics Extension 1 HSC Examination	**3**	**Solution**
20 MX 1	**12 a**	Use the principle of mathematical induction to show that for all integers $n \geq 1$, $$1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots + n(3n - 1) = n^2(n + 1).$$ NESA 2020 Mathematics Extension 1 HSC Examination	**3**	**Solution**
SP **12**	**12d** **12a**	Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$. NESA Mathematics Extension 1 Sample Examination Paper (2020) NESA 2012 Mathematics Extension 1 HSC Examination	**3**	**Solution**
TG **08 MX 1**	**1** **3b**	Use mathematical induction to prove that, for all integers $n \geq 1$, $$1 \times 3 + 2 \times 4 + 3 \times 5 + \ldots + n(n + 2) = \frac{n}{6}(n + 1)(2n + 7).$$ NESA Mathematics Extension 1 Year 12 Topic Guide: Proof NESA 2008 Mathematics Extension 1 HSC Examination	**3**	**Solution**

TG	**2**	Prove by mathematical induction that $3^{2n+4} - 2^{2n}$ is a multiple of 5. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
TG	**3**	Prove by mathematical induction that $n^3 + 2n$ is divisible by 3 for all positive integers n. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
TG	**4**	Show that the inductive step can be proven for the false proposition that $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n - 1)(n + 2)$ for integers $n \geq 1$, but the initial case does not hold true. NESA Mathematics Extension 1 Year 12 Topic Guide: Proof		**Solution**
19 MX 1	**14 a**	Prove by mathematical induction that, for all integers $n \geq 1$, $1(1!) + 2(2!) + 3(3!) + \ldots + n(n!) = (n + 1)! - 1$ NESA 2019 Mathematics Extension 1 HSC Examination	**3**	**Solution**
18 MX 1	**13 a**	Prove by mathematical induction that, for $n \geq 1$, $2 - 6 + 18 - 54 + \ldots + 2(-3)^{n-1} = \dfrac{1-(-3)^n}{2}$ NESA 2018 Mathematics Extension 1 HSC Examination	**3**	**Solution**
17 MX 1	**14 a**	Prove by mathematical induction that $8^{2n+1} + 6^{2n-1}$ is divisible by 7, for any integer $n \geq 1$. NESA 2017 Mathematics Extension 1 HSC Examination	**3**	**Solution**
16 MX 1	**14 a**	(i) Show that $4n^3 + 18n^2 + 23n + 9$ can be written as $(n + 1)(4n^2 + 14n + 9)$. (ii) Using the results in part (i), or otherwise, prove by mathematical induction that, for $n \geq 1$, $1 \times 3 + 3 \times 5 + 5 \times 7 + \ldots + (2n - 1)(2n + 1) = \frac{1}{3}n(4n^2 + 6n - 1)$. NESA 2016 Mathematics Extension 1 HSC Examination	**1** **3**	**Solution**
15 MX 1	**13 c**	Prove by mathematical induction that for all integers $n \geq 1$, $\dfrac{1}{2!} + \dfrac{2}{3!} + \dfrac{3}{4!} + \ldots + \dfrac{n}{(n+1)!} = 1 - \dfrac{1}{(n+1)!}$. NESA 2015 Mathematics Extension 1 HSC Examination	**3**	**Solution**
14 MX 1	**13 a**	Use mathematical induction to prove that $2^n + (-1)^{n+1}$ is divisible by 3 for all integers $n \geq 1$. NESA 2014 Mathematics Extension 1 HSC Examination	**3**	**Solution**
11 MX 1	**6a**	Use mathematical induction to prove that, for $n \geq 1$, $1 \times 5 + 2 \times 6 + 3 \times 7 + \ldots + n(n + 4) = \frac{1}{6}n(n + 1)(2n + 13)$. NESA 2011 Mathematics Extension 1 HSC Examination	**3**	**Solution**
10 MX 1	**7a**	Prove by induction that $47^n + 53 \times 147^{n-1}$ is divisible by 100 for all integers $n \geq 1$. NESA 2010 Mathematics Extension 1 HSC Examination	**3**	**Solution**
07 MX 1	**4b**	Use mathematical induction to prove that, $7^{2n-1} + 5$ is divisible by 12, for all integers $n \geq 1$. NESA 2007 Mathematics Extension 1 HSC Examination	**3**	**Solution**

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Year 12: Vectors
V1.1 Introduction to vectors

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Students:

- define a vector as a quantity having both magnitude and direction, and examine examples of vectors, including displacement and velocity (ACMSM010)
  – explain the distinction between a position vector and a displacement (relative) vector
- define and use a variety of notations and representations for vectors in two dimensions (ACMSM014)
  – use standard notations for vectors, for example: $\underset{\sim}{a}$, $\overrightarrow{AB}$ and $\mathbf{a}$
  – represent vectors graphically in two dimensions as directed line segments
  – define unit vectors as vectors of magnitude 1, and the standard two-dimensional perpendicular unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$
  – express and use vectors in two dimensions in a variety of forms, including component form, ordered pairs and column vector notation
- perform addition and subtraction of vectors and multiplication of a vector by a scalar algebraically and geometrically, and interpret these operations in geometric terms **AAM**
  – graphically represent a scalar multiple of a vector (ACMSM012)
  – use the triangle law and the parallelogram law to find the sum and difference of two vectors
  – define and use addition and subtraction of vectors in component form (ACMSM017)
  – define and use multiplication by a scalar of a vector in component form (ACMSM018)

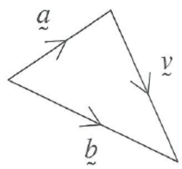
Reference Sheet

21 MX 1	**1**	Given $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, what is $\overrightarrow{PQ}$? A. $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ B. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ C. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ D. $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$ NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**
21 MX 1	**11 a**	Find $(\underset{\sim}{i} + 6\underset{\sim}{j}) + (2\underset{\sim}{i} - 7\underset{\sim}{j})$. NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**

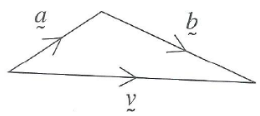
20 MX 1 **6** The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown. **1** Solution

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$?

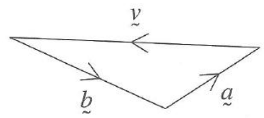
A.



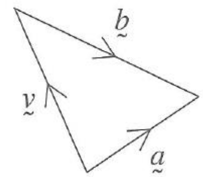
B.



C.

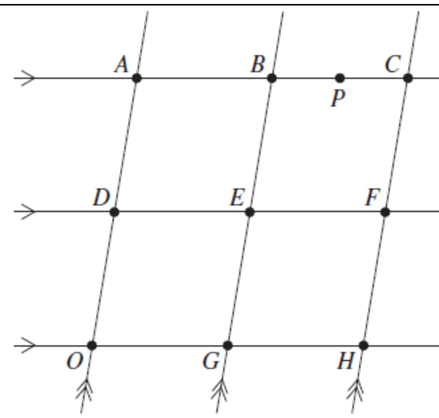


D.



NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **2** The diagram shows a grid of equally spaced lines. **1** Solution



The vector $\overrightarrow{OH} = \underset{\sim}{h}$ and the vector $\overrightarrow{OA} = \underset{\sim}{a}$.

The point P is halfway between B and C.

Which expression represents the vector $\overrightarrow{OP}$?

A. $-\frac{1}{2}\underset{\sim}{a} - \frac{1}{4}\underset{\sim}{h}$ B. $\frac{1}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{h}$

C. $\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{h}$ D. $\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{h}$

NESA Mathematics Extension 1 Sample HSC Examination (2020)

TG **1** Express each of the following vectors as column vectors, and illustrate your answers geometrically: Solution

(a) $\underset{\sim}{i} + 2\underset{\sim}{j}$

(b) $3\underset{\sim}{i}$

(c) $\underset{\sim}{j} - \underset{\sim}{i}$

(d) $4\underset{\sim}{i} - 3\underset{\sim}{j}$

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **2** If $\underset{\sim}{p} = 4\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{q} = 6\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{r} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$, find numbers s and t such that Solution

$s\underset{\sim}{p} + t\underset{\sim}{q} = \underset{\sim}{r}$.

Illustrate your answer geometrically.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **3** C is the point on AB produced such that $\overrightarrow{AB} = \overrightarrow{BC}$. Express $\underset{\sim}{c}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **4** If C is the point on AB such that $\overrightarrow{AC} = t\overrightarrow{AB}$, prove that $\underset{\sim}{c} = t\underset{\sim}{b} + (1 - t)\underset{\sim}{a}$. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

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Year 12: Vectors
V1.2 Further operations with vectors

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Students:

- define, calculate and use the magnitude of a vector in two dimensions and use the notation $|\underset{\sim}{u}|$ for the magnitude of a vector $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$
  - prove that the magnitude of a vector, $\underset{\sim}{u} = x\underset{\sim}{i} + y\underset{\sim}{j}$, can be found using: $|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^2 + y^2}$
  - identify the magnitude of a displacement vector $\overrightarrow{AB}$ as being the distance between the points A and B
  - convert a non-zero vector $\underset{\sim}{u}$ into a unit vector $\hat{\underset{\sim}{u}}$ by dividing by its length: $\hat{\underset{\sim}{u}} = \frac{\underset{\sim}{u}}{|\underset{\sim}{u}|}$
- define and use the direction of a vector in two dimensions
- define, calculate and use the scalar (dot) product of two vectors $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$ and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

  AAM
  - apply the scalar product, $\underset{\sim}{u} \cdot \underset{\sim}{v}$, to vectors expressed in component form, where $\underset{\sim}{u} \cdot \underset{\sim}{v} = x_1x_2 + y_1y_2$
  - use the expression for the scalar (dot) product, $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$ where θ is the angle between vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ to solve problems
  - demonstrate the equivalence, $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_1x_2 + y_1y_2$ and use this relationship to solve problems
  - establish and use the formula $\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$
  - calculate the angle between two vectors using the scalar (dot) product of two vectors in two dimensions
- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)
- define and use the projection of one vector onto another (ACMSM022)
- solve problems involving displacement, force and velocity involving vector concepts in two dimensions (ACMSM023) **AAM**
- prove geometric results and construct proofs involving vectors in two dimensions including to proving that: **AAM**
  - the diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
  - the midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
  - the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides (ACMSM041)

Reference Sheet

23 MX 1 **6**

Given the two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, let $\underset{\sim}{c}$ be the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **1** **Solution**

What is the projection of $10\underset{\sim}{a}$ onto $2\underset{\sim}{b}$?

A. $2\underset{\sim}{c}$ B. $5\underset{\sim}{c}$ C. $10\underset{\sim}{c}$ D. $20\underset{\sim}{c}$

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23 MX 1 **14 c** **3** **Solution**

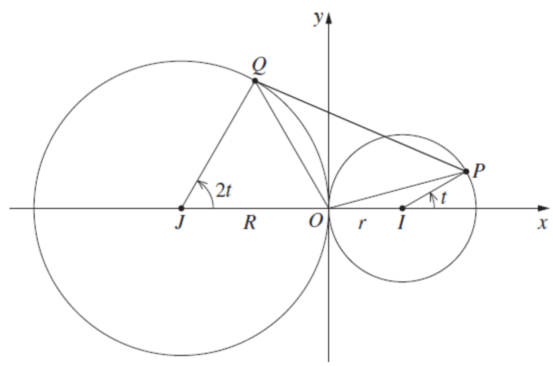
(i) Given a non-zero vector $\begin{pmatrix} p \\ q \end{pmatrix}$, it is known that the vector $\begin{pmatrix} q \\ -p \end{pmatrix}$ is perpendicular to $\begin{pmatrix} p \\ q \end{pmatrix}$ and has the same magnitude. (Do NOT prove this.)

Points A and B have position vectors $\overrightarrow{OA} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ respectively.

Using the given information, or otherwise, show that the area of triangle OAB is $\frac{1}{2}\left|a_1 b_2 - a_2 b_1\right|$.

(ii) The point P lies on the circle centred at $I(r, 0)$ with radius $r > 0$, such that $\overrightarrow{IP}$ makes an angle of t to the horizontal. **4**

The point Q lies on the circle centred at $J(-R, 0)$ with radius $R > 0$, such that $\overrightarrow{JQ}$ makes an angle of $2t$ to the horizontal.



Note that $\overrightarrow{OP} = \overrightarrow{OI} + \overrightarrow{IP}$ and $\overrightarrow{OQ} = \overrightarrow{OJ} + \overrightarrow{JQ}$.

Using part (i), or otherwise, find the values of t, where $-\pi \le t \le \pi$, that maximise the area of triangle OPQ.

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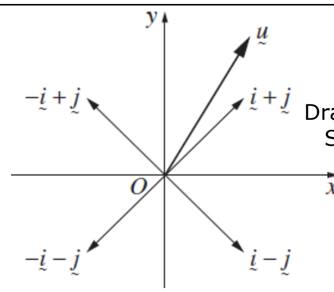
22 MX 1 **6** (1 mark) Solution

The following diagram shows the vector $\underset{\sim}{u}$ and the vectors $\underset{\sim}{i} + \underset{\sim}{j}, -\underset{\sim}{i} + \underset{\sim}{j}, -\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{i} - \underset{\sim}{j}$.

Which statement regarding this diagram could be true?



Drawn to Scale

A. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $1.1\underset{\sim}{i} + 1.8\underset{\sim}{j}$.

B. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $-0.4\underset{\sim}{i} + 0.4\underset{\sim}{j}$.

C. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $3.2\underset{\sim}{i} + 3.2\underset{\sim}{j}$.

D. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $0.5\underset{\sim}{i} - 0.5\underset{\sim}{j}$.

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **8** (1 mark) Solution

The angle between two unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is θ and $|\underset{\sim}{a} + \underset{\sim}{b}| < 1$.

Which of the following best describes the possible range of values of θ?

A. $0 \le \theta < \dfrac{\pi}{3}$ B. $0 \le \theta < \dfrac{2\pi}{3}$ C. $\dfrac{\pi}{3} < \theta \le \pi$ D. $\dfrac{2\pi}{3} < \theta \le \pi$

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **11 a** Solution

For the vectors $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j}$, evaluate each of the following.

(i) $\underset{\sim}{u} + 3\underset{\sim}{v}$ **1**

(ii) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

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22 MX 1 **11 d** (2 marks) Solution

The vectors $\underset{\sim}{u} = \begin{pmatrix} a \\ 2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} a-7 \\ 4a-1 \end{pmatrix}$ are perpendicular.

What are the possible values of a?

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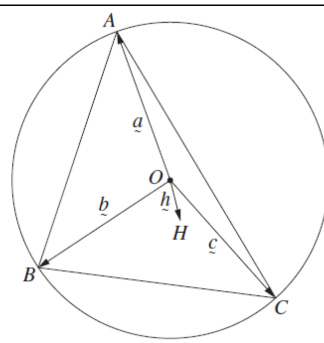
22 MX 1 **13 a** (3 marks) Solution

Three different points A, B and C are chosen on a circle centred at O.

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$.

Let $\underset{\sim}{h} = \underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ and let H be the point such that $\overrightarrow{OH} = \underset{\sim}{h}$, as shown in the diagram.

Show that $\overrightarrow{BH}$ and $\overrightarrow{CA}$ are perpendicular.



Not to scale

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **14 b** **3** **Solution**

The vectors $\vec{u}$ and $\vec{v}$ are not parallel. The vector $\vec{p}$ is the projection of $\vec{u}$ onto the vector $\vec{v}$.

The vector $\vec{p}$ is parallel to $\vec{v}$ so it can be written $\lambda_0 \vec{v}$ for some real number λ_0. (Do NOT prove this.)

Prove that $\left|\vec{u} - \lambda \vec{v}\right|$ is smallest when $\lambda = \lambda_0$ by showing that, for all real numbers λ, $\left|\vec{u} - \lambda_0 \vec{v}\right| \le \left|\vec{u} - \lambda \vec{v}\right|$.

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21 MX 1 **5** **1** **Solution**

For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is known that $\overrightarrow{OA} \cdot \overrightarrow{OB} < 0$.
Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or, $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $|\overrightarrow{OA}||\overrightarrow{OB}|$ is negative.

D. The points O, A and B are collinear.

NESA 2021 Mathematics Extension 1 HSC Examination

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21 MX 1 **14 a** **3** **Solution**

A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h.

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination?

NESA 2021 Mathematics Extension 1 HSC Examination

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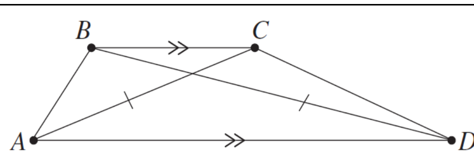
21 MX 1 **14 c** **Solution**

(i) For vector $\underset{\sim}{v}$, show that $\underset{\sim}{v} \cdot \underset{\sim}{v} = \left|\underset{\sim}{v}\right|^2$. **1**



NOT TO SCALE

(ii) In the trapezium $ABCD$, BC is parallel to AD and $|\overrightarrow{AC}| = |\overrightarrow{BD}|$. **3**

Let $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$.

Using part (i) or otherwise, show that $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1 - k)\left|\underset{\sim}{b}\right|^2 = 0$.

NESA 2021 Mathematics Extension 1 HSC Examination

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20 MX 1 **4** **1** **Solution**

Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$.

How far from the origin is she?

A. $\sqrt{77}$ B. $\sqrt{85}$ C. $2\sqrt{13} + \sqrt{5}$ D. $\sqrt{5} + \sqrt{7} + \sqrt{13}$

NESA 2020 Mathematics Extension 1 HSC Examination

20 MX 1 **9**

The projection of the vector $\begin{pmatrix}6\\7\end{pmatrix}$ onto the line $y = 2x$ is $\begin{pmatrix}4\\8\end{pmatrix}$. **1** Solution

The point (6, 7) is reflected in the line $y = 2x$ to a point A.

What is the position vector of the point A?

A $\begin{pmatrix}6\\12\end{pmatrix}$ B $\begin{pmatrix}2\\9\end{pmatrix}$ C $\begin{pmatrix}-6\\7\end{pmatrix}$ D $\begin{pmatrix}-2\\1\end{pmatrix}$

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **11 b**

For what value(s) of a are the vectors $\begin{pmatrix}a\\-1\end{pmatrix}$ and $\begin{pmatrix}2a-3\\2\end{pmatrix}$ perpendicular? **3** Solution

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **1**

What is the angle between the vectors $\begin{pmatrix}7\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\end{pmatrix}$? **1** Solution

A. $\cos^{-1}(0.6)$ B. $\cos^{-1}(0.06)$ C. $\cos^{-1}(-0.06)$ D. $\cos^{-1}(-0.6)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 b** Solution

A force described by the vector $\underset{\sim}{F} = \begin{pmatrix}2\\1\end{pmatrix}$ newtons is applied to an object lying on a line ℓ which is parallel to the vector $\begin{pmatrix}3\\4\end{pmatrix}$.

(i) Find the component of the force $\underset{\sim}{F}$ in the direction of the line ℓ. **2**

(ii) What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 c** Solution

The points A and B are fixed points in a plane and have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. The point P with position vector $\underset{\sim}{p}$ also lies in the plane and is chosen so that $\angle APB = 90°$.

(i) Explain why $(\underset{\sim}{a} - \underset{\sim}{p})\cdot(\underset{\sim}{b} - \underset{\sim}{p}) = 0$ **1**

(ii) Let $\underset{\sim}{m} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ denote the position vector of M, the midpoint of A and B. **3**

Using the properties of vectors, show that $|\underset{\sim}{p} - \underset{\sim}{m}|^2$ is independent of $\underset{\sim}{p}$ and find its value.

(iii) What does the result in part (ii) prove about the point P? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

MX SQ 2019 **15** Points P and A are on the number plane. **4** Solution

The vector is $\overrightarrow{PA}$ is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

Point B is chosen so that the area of $\triangle PAB$ is 10 square units and $|\overrightarrow{PB}| = 4\sqrt{5}$.

Find all possible vectors $\overrightarrow{PB}$.

NESA Mathematics Extension 1 Sample examination materials (2019)

TG **1** Let $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} - \underset{\sim}{j}$. Solution

(a) Calculate $\underset{\sim}{a} \cdot \underset{\sim}{b}$ and $\underset{\sim}{a} \cdot \underset{\sim}{c}$.

(b) Verify that $\underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) = \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{c}$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **2** Let $\underset{\sim}{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. Solution

Find the magnitude of $\underset{\sim}{a}$, and find a unit vector in the same direction as $\underset{\sim}{a}$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **3** Let A and B be points with position vectors $\underset{\sim}{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ respectively. Solution

(a) Draw a diagram showing the points O, A and B.

(b) Calculate the angle AOB

(i) by finding the tangents of the angles α and β between $\underset{\sim}{a}$ and the unit vector $\underset{\sim}{i}$, and $\underset{\sim}{b}$ and the unit vector $\underset{\sim}{j}$, and using the formula for $\tan(\alpha - \beta)$.

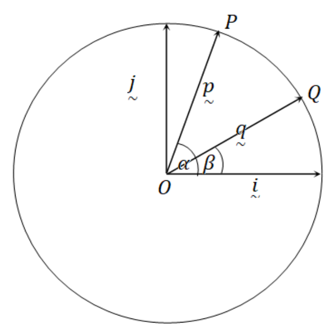
(ii) by using a method based on scalar products.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **4** Find the angle between the line joining (1, 2) and (3, −5) and the line joining (2, −3) to (1, 4). Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **5** Use vectors and the diagram below of the unit circle to derive the formula for the expansion of $\cos(\alpha - \beta)$ where $0 \le \beta \le \alpha \le \frac{\pi}{2}$. Solution



NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 6 [Solution]

In a triangle *ABC* denote the displacement vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ by $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively. Use vectors to prove that, in a triangle *ABC*, $a^2 = b^2 + c^2 - 2bc\cos A$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 7 [Solution]

Let *OABC* be a kite, with *OB* as its line of symmetry. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

a. Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

b. Using the fact that the lengths of *AB* and *CB* are equal, write an equation involving scalar products.

c. Use this equation to prove that the diagonals of a kite are perpendicular.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

projectmaths

Year 12: Vectors
V1.3 Projectile motion

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

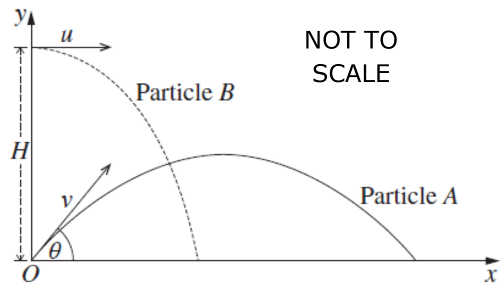
Students:

- understand the concept of projectile motion, and model and analyse a projectile's path assuming that:
  - the projectile is a point
  - the force due to air resistance is negligible
  - the only force acting on the projectile is the constant force due to gravity, assuming that the projectile is moving close to the Earth's surface
- model the motion of a projectile as a particle moving with constant acceleration due to gravity and derive the equations of motion of a projectile **AAM**
  - represent the motion of a projectile using vectors
  - recognise that the horizontal and vertical components of the motion of a projectile can be represented by horizontal and vertical vectors
  - derive the horizontal and vertical equations of motion of a projectile
  - understand and explain the limitations of this projectile model
- use equations for horizontal and vertical components of velocity and displacement to solve problems on projectiles
- apply calculus to the equations of motion to solve problems involving projectiles (ACMSM115) **AAM**

Reference Sheet

23 MX 1 **13 b**

Particle A is projected from the origin with initial speed v m s^{-1} at an angle θ with the horizontal plane. At the same time, particle B is projected horizontally with initial speed u m s^{-1} from a point that is H metres above the origin, as shown in the diagram.



Solution

The position vector of particle A, t seconds after it is projected, is given by

$$\mathbf{r}_A(t) = \begin{pmatrix} vt\cos\theta \\ vt\sin\theta - \frac{1}{2}gt^2 \end{pmatrix} \qquad \text{(Do NOT prove this.)}$$

The position vector of particle B, t seconds after it is projected, is given by

$$\mathbf{r}_B(t) = \begin{pmatrix} ut \\ H - \frac{1}{2}gt^2 \end{pmatrix} \qquad \text{(Do NOT prove this.)}$$

The angle θ is chosen so that $\tan\theta = 2$.
The two particles collide.

(i) By first showing that $\cos\theta = \dfrac{1}{\sqrt{5}}$, verify that $v = \sqrt{5}\,u$. **2**

(ii) Show that the particles collide at time $T = \dfrac{H}{2u}$. **1**

When the particles collide, their velocity vectors are perpendicular.

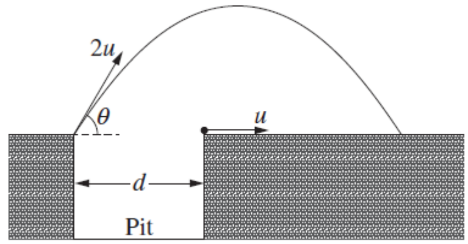
(iii) Show that $H = \dfrac{2u^2}{g}$. **3**

(iv) Prior to the collision, the trajectory of particle A was a parabola. (Do NOT prove this.)
Find the height of the vertex of that parabola above the horizontal plane.
Give your answer in terms of H. **2**

NESA 2023 Mathematics Extension 1 HSC Examination

22 MX 1 **14 c** A video game designer wants to include an obstacle in the game they are developing. The player will reach one side of a pit and must shoot a projectile to hit a target on the other side of the pit in order to be able to cross. However, the instant the player shoots, the target begins to move away from the player at a constant speed that is half the initial speed of the projectile shot by the player, as shown in the diagram below. **4** **Solution**

The initial distance between the player and the target is d, the initial speed of the projectile is $2u$ and it is launched at an angle of *q* to the horizontal. The acceleration due to gravity is g. The launch angle is the ONLY parameter that the player can change.



Taking the position of the player when the projectile is launched as the origin, the positions of the projectile and target at time t after the projectile is launched are as follows.

$$\vec{r}_P = \begin{pmatrix} 2ut\cos\theta \\ 2ut\sin\theta - \frac{g}{2}t^2 \end{pmatrix} \quad \text{Projectile}$$

$$\vec{r}_T = \begin{pmatrix} d + ut \\ 0 \end{pmatrix} \quad \text{Target}$$

Do NOT prove these

Show that, for the player to have a chance of hitting the target, d must be less than 37% of the maximum possible range of the projectile (to 2 significant figures)

NESA 2022 Mathematics Extension 1 HSC Examination

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21 MX 1 **13 b** When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is **4** Solution

$$\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (-5t^2 + Vt\sin\theta + h)\,\underset{\sim}{j}.$$ (Do NOT prove this.)

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

NESA 2021 Mathematics Extension 1 HSC Examination

SP MX 1 **11 a** A particle is fired from the origin O with initial velocity 18 ms^{-1} at an angle 60° to the horizontal. Solution

The equations of motion are $\frac{d^2x}{dt^2} = 0$ and $\frac{d^2y}{dt^2} = -10$

(i) Show that $x = 9t$. **1**

(ii) Show that $y = 9\sqrt{3}\,t - 5t^2$. **2**

(ii) Hence find the Cartesian equation for the trajectory of the particle. **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG **1** A ball is thrown in the air with speed 12 ms^{-1} at an angle of 70° to the horizontal. Solution

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

Using g = 9.8 ms^{-2}, find

(a) its position vector* after 1.5 seconds

(b) its velocity vector* after 1.5 seconds.

* projectmaths included the word 'vector'.

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **2** A golf ball is driven with a speed of 45 ms^{-1} at 37° to the horizontal across a horizontal fairway. The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.** Solution

Use g = 9.8 ms^{-2}.

(a) How high above the ground does the ball rise?

(b) How far away from the tee does it first land?

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **3** A famine relief aircraft, flying over horizontal ground at a height of 160 metres, drops a sack of food. The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.** Solution

Use g = 10 ms^{-2}.

(a) Calculate the time that the sack takes to fall.

(b) Calculate the vertical component of the velocity with which the sack hits the ground.

(c) If the speed of the aircraft is 70 ms^{-1}, at what distance before the target zone should the sack be released?

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **4** A projectile reaches its greatest height after two seconds, when it is 35 metres from its point of projection. Solution

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

Using g = 9.8 ms^{-2}, determine the initial velocity.

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **5** A particle is projected upwards at velocity u and an angle of θ to the horizontal from a point A. Find an expression for the horizontal distance away from point A at which the particle reaches its greatest height, in terms of u and θ. Solution

The position vector is given as $\underset{\sim}{r}(t) = ut\cos\theta\,\underset{\sim}{i} + (ut\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 6 In a cricket match a batsman hits the ball for six, and it is measured by the cameras after two seconds as moving at a horizontal velocity of 30 ms^{-1} and a vertical velocity of 10 ms^{-1} upwards. Calculate the initial velocity and angle of projection at which the ball was hit by the batter. Use g = 9.8 ms^{-2}. **Solution**

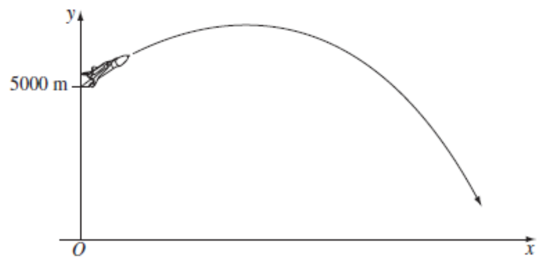
The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (Vt\sin\theta - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG 7
05 MX 1 6b

An experimental rocket is at a height of 5000 m, ascending with a velocity of $200\sqrt{2}$ m s^{-1} at an angle of 45° to the horizontal, when its engine stops. **Solution**



After this time, the equations of motion of the rocket are:

$$x = 200t$$

$$y = -4.9t^2 + 200t + 5000,$$

where t is measured in seconds after the engine stops. (Do NOT show this.)

(i) What is the maximum height the rocket will reach, and when will it reach this height? **2**

(ii) The pilot can only operate the ejection seat when the rocket is descending at an angle between 45° and 60° to the horizontal. What are the earliest and latest times that the pilot can operate the ejection seat? **3**

(iii) For the parachute to open safely, the pilot must eject when the speed of the rocket is no more than 350 m s^{-1}. What is the latest time at which the pilot can eject safely? **2**

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors
NESA 2005 Mathematics Extension 1 HSC Examination

TG 8 (a) Prove that the range on a horizontal plane of a particle projected upwards at an angle α to the plane with velocity V metres per second is $\dfrac{V^2\sin 2\alpha}{g}$ metres, where g metres per second per second is the acceleration due to gravity. **Solution**

(b) A garden sprinkler sprays water symmetrically about its vertical axis at a constant speed of V metres per second. The initial direction of the spray varies continuously between angles of 15° and 60° to the horizontal.

(i) Prove that, from a fixed position O on level ground, the sprinkler will wet the surface of an annular region with centre O and with internal and external radii $\dfrac{V^2}{2g}$ metres and $\dfrac{V^2}{g}$ metres respectively.

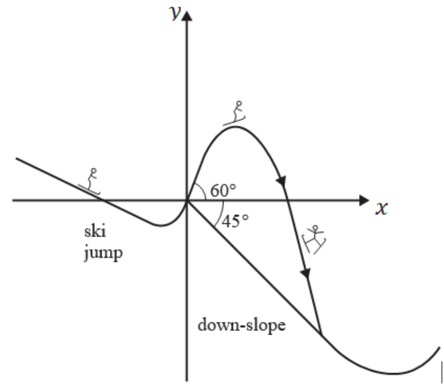
(ii) Deduce that, by locating the sprinkler appropriately relative to a rectangular garden bed of size 6 metres by 3 metres, the entire bed may be watered provided that $\dfrac{V^2}{2g} \geq 1 + \sqrt{7}$.

The position vector is given as $\underset{\sim}{r}(t) = Vt\cos\alpha\,\underset{\sim}{i} + (Vt\sin\alpha - \frac{1}{2}gt^2)\,\underset{\sim}{j}$.**

** This vector information was provided by projectmaths.

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

TG **9** A skier accelerates down a slope and then skis up a short ski jump (see diagram below). The skier leaves the jump at a speed of 12m/s and at an angle of 60° to the horizontal. The skier performs various gymnastic twists and lands on a straight line section of the 45° down-slope T seconds after leaving the jump. [Solution]



Let the origin O of a Cartesian coordinate system be at the point where the skier leaves the jump, with $\underset{\sim}{i}$ a unit vector in the positive x direction, and $\underset{\sim}{j}$ a unit vector in the positive y direction. Displacements are measured in metres, and time in seconds.

(a) Show that the initial velocity of the skier when leaving the jump is $6\underset{\sim}{i} + 6\sqrt{3}\,\underset{\sim}{j}$.

(b) The acceleration of the skier, t seconds after leaving the ski jump, is given by

$\ddot{\underset{\sim}{r}}(t) = -0.1t\,\underset{\sim}{i} - (g - 0.1t)\,\underset{\sim}{j}$, $0 \le t \le T$.

Show that the position vector of the skier, t seconds after leaving the jump, is given by $r(t) = (6t - \frac{1}{60}t^3)\underset{\sim}{i} + (6t\sqrt{3} - \frac{1}{2}gt^2 + \frac{1}{60}t^3)\underset{\sim}{j}$, $0 \le t \le T$.

(c) Show that $T = \frac{12}{g}(\sqrt{3} + 1)$.

(d) At what speed, in metres per second, does the skier land on the down-slope?

Give your answer correct to one decimal place.

(Source: Question 4, VCE Specialist Mathematics 2, 2005 © VCAA, reproduced by permission.)

NESA Mathematics Extension 1 Year 12 Topic Guide: Vectors

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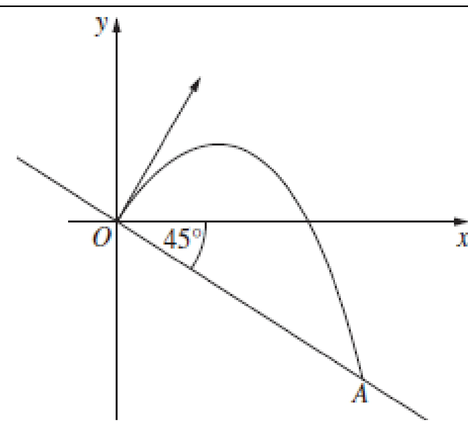
19 MX 1 **13 d** The point O is on a sloping plane that forms an angle of 45° to the horizontal. A particle is projected from the point O. The particle hits a point A on the sloping plane as shown in the diagram. [Solution]



The equation of the line OA is $y = -x$.

The equations of motion of the particle are

$x = 18t$ and $y = 18\sqrt{3}\,t - 5t^2$, where t is the time in seconds after projection.

Do NOT prove these equations.

(i) Find the distance OA between the point of projection and the point where the particle hits the sloping plane. **2**

(ii) What is the size of the acute angle that the path of the particle makes with the sloping plane as the particle hits the point A? **3**

NESA 2019 Mathematics Extension 1 HSC Examination

18 MX 1 **13 c** [**Solution**]

An object is projected from the origin with an initial velocity of V at an angle θ to the horizontal. The equations of motion of the object are

$x(t) = Vt\cos\theta$ and $y(t) = Vt\sin\theta - \dfrac{gt^2}{2}$. (Do NOT prove these.)

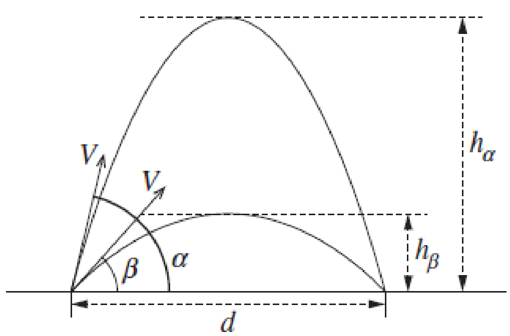
(i) Show that when the object is projected at an angle θ, the horizontal range is $\dfrac{V^2}{g}\sin 2\theta$. **2**

(ii) Show that when the object is projected at an angle $\dfrac{\pi}{2} - \theta$, the horizontal range is also $\dfrac{V^2}{g}\sin 2\theta$. **1**

(iii) The object is projected with initial velocity V to reach a horizontal distance d, which is less than the maximum possible horizontal range. There are two angles at which the object can be projected in order to travel that horizontal distance before landing. **3**

Let these angles be α and β where $\beta = \dfrac{\pi}{2} - \alpha$.

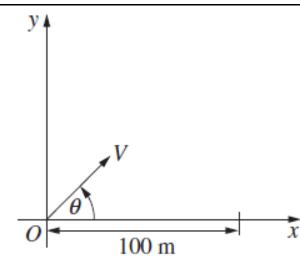
Let h_α be the maximum height reached by the object when projected at an angle α to the horizontal.
Let h_β be the maximum height reached by the object when projected at an angle β to the horizontal.
Show that the average of the two heights, $\dfrac{h_\alpha + h_\beta}{2}$, depends only on V and g.



NESA 2018 Mathematics Extension 1 HSC Examination

17 MX 1 **13 c** [**Solution**]

A golfer hits a golf ball with initial speed V ms^{-1} at an angle θ to the horizontal. The golf ball is hit from one side of a lake and must have a horizontal range of 100 m or more to avoid landing in the lake. Neglecting the effects of air resistance, the equations describing the motion of the ball are



$$x = Vt\cos\theta \text{ and } y = Vt\sin\theta - \frac{1}{2}gt^2,$$

where t is the time in seconds after the ball is hit and g is the acceleration due to gravity in ms^{-2}. Do NOT prove these equations.

(i) Show that the horizontal range of the golf ball is $\dfrac{V^2\sin 2\theta}{g}$ metres. **2**

(ii) Show that if $V^2 < 100g$ then the horizontal range of the ball is less than 100 m. **1**

It is now given that $V^2 = 200g$ and that the horizontal range of the ball is 100 m or more.

(iii) Show that $\dfrac{\pi}{12} \le \theta \le \dfrac{5\pi}{12}$. **2**

(iv) Find the greatest height the ball can achieve. **2**

NESA 2017 Mathematics Extension 1 HSC Examination

16 MX 1 **13 b**

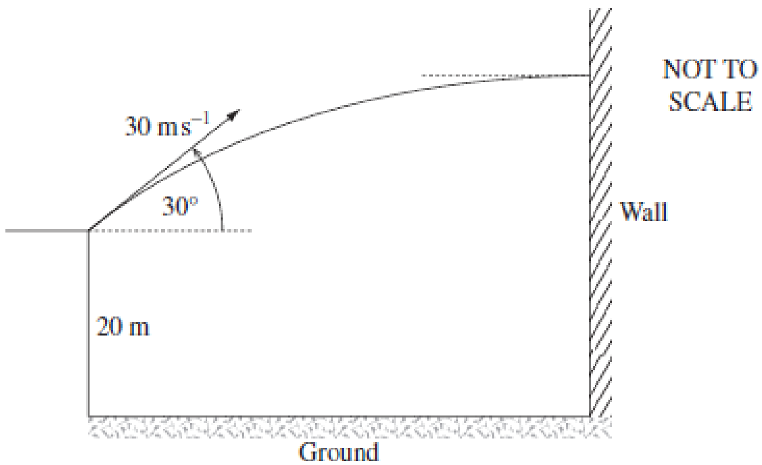
The trajectory of a projectile fired with speed u ms^{-1} at an angle θ to the horizontal is represented by the parametric equations $x = ut\cos\theta$ and $y = ut\sin\theta - 5t^2$, where t is the time in seconds. [Solution](#)

(i) Prove that the greatest height reached by the projectile is $\dfrac{u^2\sin^2\theta}{20}$. **2**

A ball is thrown from a point 20 m above the horizontal ground.

It is thrown with speed 30 ms^{-1} at an angle of 30° to the horizontal.

At its highest point the ball hits a wall, as shown in the diagram.



(ii) Show that the ball hits the wall at a height of $\dfrac{125}{4}$ m above the ground. **2**

The ball then rebounds horizontally from the wall with speed 10 ms^{-1}. You may assume that the acceleration due to gravity is 10 ms^{-1}.

(iii) How long does it take the ball to reach the ground after it rebounds from the wall? **2**

(iv) How far from the wall is the ball when it hits the ground? **1**

NESA 2016 Mathematics Extension 1 HSC Examination

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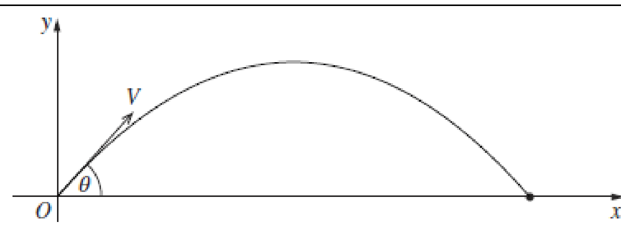
15 MX 1 **14 a**

A projectile is fired from the origin O with initial velocity V ms^{-1} at an angle θ to the horizontal. The equations of motion are given by $x = Vt\cos\theta$, $y = Vt\sin\theta - \dfrac{1}{2}gt^2$. (Do NOT prove this.) [Solution](#)



(i) Show that the horizontal range of the projectile is $\dfrac{V^2\sin 2\theta}{g}$. **2**

A particular projectile is fired so that $\theta = \dfrac{\pi}{3}$.

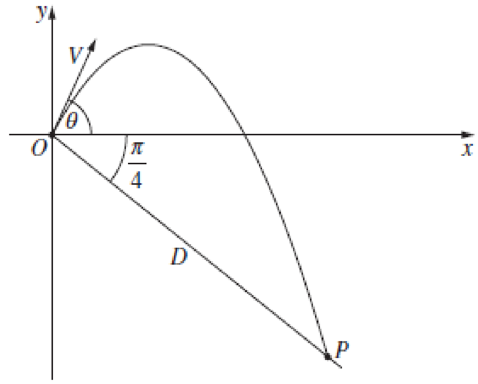
(ii) Find the angle that this projectile makes with the horizontal when $t = \dfrac{2V}{\sqrt{3}g}$. **2**

(iii) State whether this projectile is travelling upwards or downwards when $t = \dfrac{2V}{\sqrt{3}g}$. Justify your answer. **1**

NESA 2015 Mathematics Extension 1 HSC Examination

14 MX 1 **14 a** [Solution]

The take-off point O on a ski jump is located at the top of a downslope. The angle between the downslope and the horizontal is $\frac{\pi}{4}$. A skier takes off from O with velocity V ms^{-1} at an angle θ to the horizontal, where $0 \le \theta < \frac{\pi}{2}$.



The skier lands on the downslope at some point P, a distance D metres from O.
The flight path of the skier is given by

$$x = Vt\cos\theta, \qquad y = -\frac{1}{2}gt^2 + Vt\sin\theta,$$

(Do NOT prove this.) where t is the time in seconds after take-off.

(i) Show that the Cartesian equation of the flight path of the skier is given by **2**

$$y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta.$$

(ii) Show that $D = 2\sqrt{2}\,\frac{V^2}{g}\cos\theta\,(\cos\theta + \sin\theta)$. **3**

(iii) Show that $\frac{dD}{d\theta} = 2\sqrt{2}\,\frac{V^2}{g}(\cos 2\theta - \sin 2\theta)$. **2**

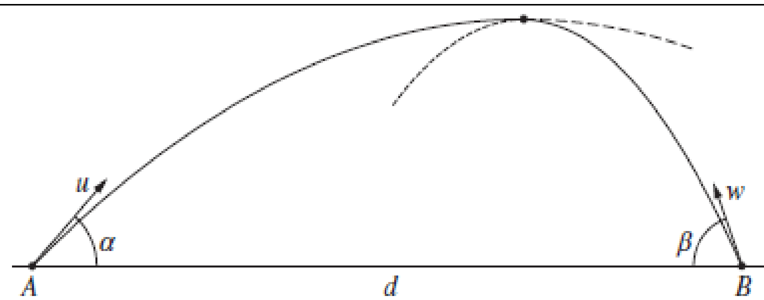
(iv) Show that D has a maximum value and find the value of θ for which this occurs. **3**

NESA 2014 Mathematics Extension 1 HSC Examination

13 MX 1 **13 c** [Solution]

Points A and B are located d metres apart on a horizontal plane.

A projectile is fired from A towards B with initial velocity u ms^{-1} at angle α to the horizontal.



At the same time, another projectile is fired from B towards A with initial velocity w ms^{-1} and angel β to the horizontal, as shown in the diagram.
The projectiles collide when they both reach their maximum height.
The equations of motion of a projectile fired from the origin with initial velocity V ms^{-1} at angle θ to the horizontal are

$$x = Vt\cos\theta \text{ and } y = Vt\sin\theta - \frac{g}{2}t^2. \text{ (Do NOT prove this.)}$$

(i) How long does the projectile fired from A take to reach its maximum height? **2**

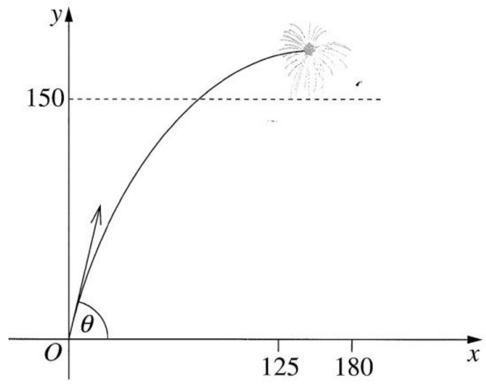
(ii) Show that $u\sin\alpha = w\sin\beta$ **1**

(iii) Show that $d = \frac{uw}{g}\sin(\alpha + \beta)$. **2**

NESA 2013 Mathematics Extension 1 HSC Examination

12 MX 1 **14 b** Solution

A firework is fired from O, on level ground, with velocity 70 metres per second at an inclination θ. The equations of motion of the firework are $x = 70t\cos\theta$ and $y = 70t\sin\theta - 4.9t^2$ (Do NOT prove this). The firework explodes when it reaches its maximum height.



(i) Show that the firework explodes at a height of $250\sin^2\theta$ metres. **2**

(ii) Show that the firework explodes at a horizontal distance of $250\sin 2\theta$ metres from O. **1**

(iii) For best viewing, the firework must explode at a horizontal distance between 125 m and 180 m from O, and at least 150 m above the ground. For what values of θ will this occur? **3**

NESA 2012 Mathematics Extension 1 HSC Examination

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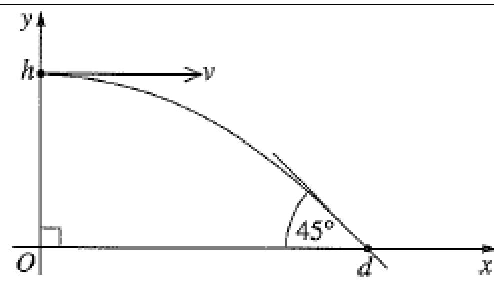
11 MX 1 **6b** Solution

The diagram shows the trajectory of a ball thrown horizontally, at speed v ms^{-1}, from the top of a tower h metres above ground level. The ball strikes the ground at an angle of 45°, d metres from the base of the tower, as shown in the diagram.



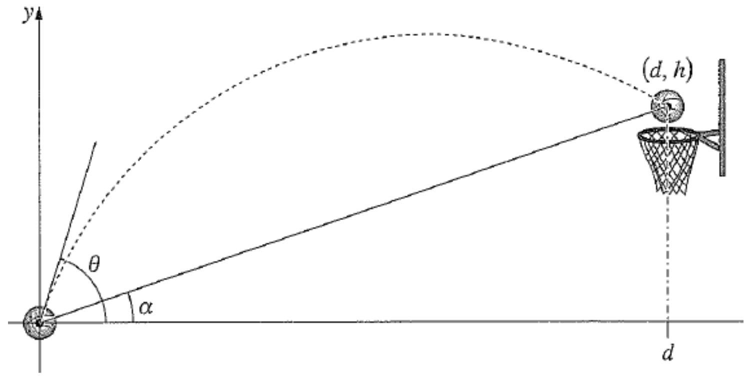
The equations describing the trajectory of the ball are $x = vt$ and $y = h - \frac{1}{2}gt^2$, (Do NOT prove this.) where g is the acceleration due to gravity, and t is time in seconds.

(i) Prove that the ball strikes the ground at time $t = \sqrt{\frac{2h}{g}}$ seconds. **1**

(ii) Hence, or otherwise, show that $d = 2h$. **2**

NESA 2011 Mathematics Extension 1 HSC Examination

10 MX 1 **6b** A basketball player throws a ball with an initial velocity v ms^{-1} at an angle θ to the horizontal. At the time the ball is released is at (0, 0), and the player is aiming for the point (d, h) as shown on the diagram. The line joining (0, 0) and (d, h) makes an angle α with the horizontal, where $0 < \alpha < \theta < \frac{\pi}{2}$. **Solution**



Assume that at any time t seconds after the ball is thrown its centre is at the point (x, y), where $x = vt\cos\theta$ and $y = vt\sin\theta - 5t^2$.
(You are NOT required to prove these equations.)

(i) If the centre of the ball passes through (d, h) show that **3**

$$v^2 = \frac{5d}{\cos\theta\sin\theta - \cos^2\theta\tan\alpha}.$$

(ii) (1) What happens to v as $\theta \to \alpha$. **1**

(2) What happens to v as $\theta \to \frac{\pi}{2}$. **1**

(iii) For a fixed value of a, let $F(\theta) = \cos\theta\sin\theta - \cos^2\theta\tan\alpha$. **2**
Show that $F'(\theta) = 0$ when $\tan 2\theta\tan\alpha = -1$.

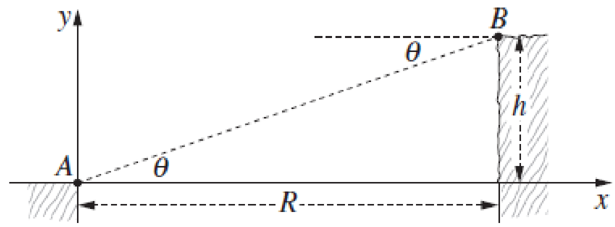
(iv) Using part (a) (ii) or otherwise show that $f'(\theta) = 0$ when $\theta = \frac{\alpha}{2} + \frac{\pi}{4}$. **1**

(v) Explain why v^2 is a minimum when $\theta = \frac{\alpha}{2} + \frac{\pi}{4}$. **2**

NESA 2010 Mathematics Extension 1 HSC Examination

09 MX 1 **6a** Two points, A and B, are on cliff tops on either side of a deep valley. Let h and R be the vertical and horizontal distances between A and B as shown in the diagram. The angle of elevation of B from A is θ, so that $\theta = \tan^{-1}\left(\frac{h}{R}\right)$.

Solution



At time $t = 0$, projectiles are fired simultaneously from A and B. The projectile from A is aimed at B, and has initial speed U at an angle θ above the horizontal. The projectile from B is aimed at A and has initial speed V at an angle θ below the horizontal. The equations for the motion of the projectile from A are

$$x_1 = Ut\cos\theta \text{ and } y_1 = Ut\sin\theta - \frac{1}{2}gt^2,$$

and the equations for the motion of the projectile from B are

$$x_2 = R - Vt\cos\theta \text{ and } y_2 = h - Vt\sin\theta - \frac{1}{2}gt^2,$$

(Do NOT prove these equations.)

(i) Let T be the time at which $x_1 = x_2$. **1**

Show that $T = \dfrac{R}{(U+V)\cos\theta}$.

(ii) Show that the projectiles collide. **2**

(iii) If the projectiles collide on the line $x = \lambda R$, where $0 < \lambda < 1$, **1**

show that $V = \left(\frac{1}{\lambda} - 1\right)U$.

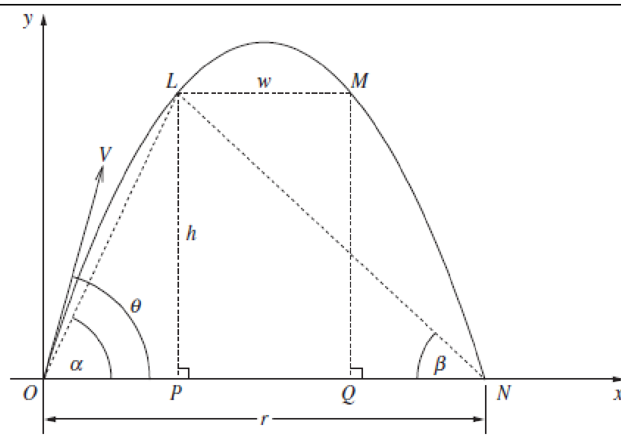
NESA 2009 Mathematics Extension 1 HSC Examination

08 MX 1 **7** A projectile is fired from O with velocity V at an angle of inclination θ across level ground. The projectile passes through the points L and M, which are both h metres above the ground, at times t_1 and t_2 respectively. The projectile returns to the ground at N. The equations of motion of the projectile are

$x = Vt\cos\theta$ and

$y = Vt\sin\theta - \frac{1}{2}gt^2$.

(Do NOT prove this.)



Solution

(a) Show that $t_1 + t_2 = \frac{2V}{g}\sin\theta$ AND $t_1 t_2 = \frac{2h}{g}$. **2**

Let $\angle LON = \alpha$ and $\angle LNO = \beta$. It can be shown that $\tan\alpha = \frac{h}{Vt_1\cos\theta}$ and

$\tan\beta = \frac{h}{Vt_2\cos\theta}$. (Do NOT prove this.)

(b) Show that $\tan\alpha + \tan\beta = \tan\theta$. **2**

(c) Show that $\tan\alpha\tan\beta = \frac{gh}{2V^2\cos^2\theta}$. **1**

Let $ON = r$ and $LM = w$.

(d) Show that $r = h(\cot\alpha + \cot\beta)$ and $w = h(\cot\alpha - \cot\beta)$. **2**

Let the gradient of the parabola at L be $\tan\phi$.

(e) Show that $\tan\phi = \tan\alpha - \tan\beta$. **3**

(f) Show that $\frac{w}{\tan\varphi} = \frac{r}{\tan\theta}$. **2**

NESA 2008 Mathematics Extension 1 HSC Examination

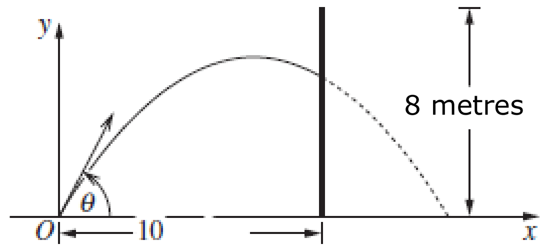
07 MX 1 **7b** A small paintball is fired from the origin with initial velocity 14 metres per second towards an eight-metre high barrier. The origin is at ground level, 10 metres from the base of the barrier. **Solution**

The equations of motion are

$x = 14t\cos\theta$

$y = 14t\sin\theta - 4.9t^2$

where θ is the angle to the horizontal at which the paintball is fired and t is the time in seconds. (Do NOT prove these equations of motion.)



(i) Show that the equation of trajectory of the paintball is **2**

$$y = mx - \left(\frac{1+m^2}{40}\right)x^2, \text{ where } m = \tan\theta.$$

(ii) Show that the paintball hits the barrier at height h metres when $m = 2 \pm \sqrt{3 - 0.4h}$. Hence determine the maximum value of h. **2**

(iii) There is a large hole in the barrier. The bottom of the hole is 3.9 metres above the ground and the top of the hole is 5.9 metres above the ground. The paintball passes through the hole if m is in one of two intervals. One interval is $2.8 \le m \le 3.2$. Find the other interval. **2**

(iv) Show that, if the paintball passes through the hole, the range is $\dfrac{40m}{1+m^2}$ metres. Hence find the widths of the two intervals in which the paintball can land at ground level on the other side of the barrier. **3**

NESA 2007 Mathematics Extension 1 HSC Examination

projectmaths

Year 12: Trigonometric Functions
T3 Trigonometric equations

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Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- convert expressions of the form $a\cos x + b\sin x$ to $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ and apply these to solve equations of the form $a\cos x + b\sin x = c$, sketch graphs and solve related problems (ACMSM048)
- solve trigonometric equations requiring factorising and/or the application of compound angle, double angle formulae or the t-formulae
- prove and apply other trigonometric identities, for example $\cos 3x = 4\cos^3 x - 3\cos x$ (ACMSM049)
- solve trigonometric equations and interpret solutions in context using technology or otherwise

Reference Sheet

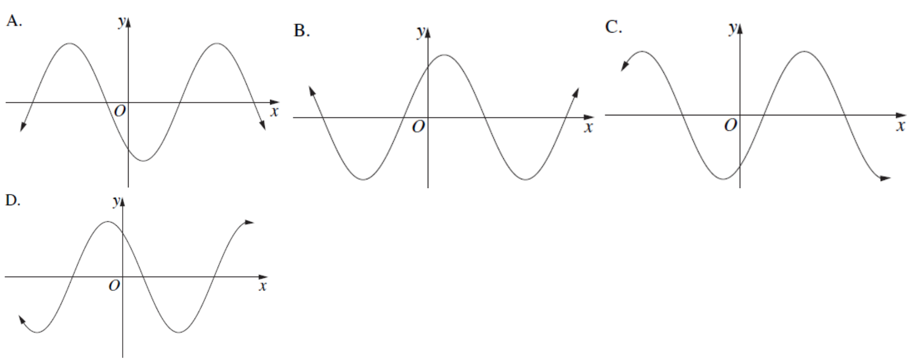
23 MX 1 **11 e** Solve $\cos\theta + \sin\theta = 1$ for $0 \le \theta \le 2\pi$. **3** **Solution**

NESA 2023 Mathematics Extension 1 HSC Examination

22 MX 1 **11 e** Express $\sqrt{3}\sin(x) - 3\cos(x)$ in the form $R\sin(x + \alpha)$. **3** **Solution**

NESA 2022 Mathematics Extension 1 HSC Examination

21 MX 1 **7** Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive? **1** **Solution**



NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1 **11 g** By factorizing, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$. **3** **Solution**

NESA 2021 Mathematics Extension 1 HSC Examination

21 MX 1	**13 d**	(i) The numbers A, B and C are related by the equations $A = B - d$ and $C = B + d$, where d is a constant. Show that $\dfrac{\sin A + \sin C}{\cos A + \cos C} = \tan B$. (ii) Hence, or otherwise, solve $\dfrac{\sin\frac{5\theta}{7} + \sin\frac{6\theta}{7}}{\cos\frac{5\theta}{7} + \cos\frac{6\theta}{7}} = \sqrt{3}$, for $0 \le \theta \le 2\pi$. NESA 2021 Mathematics Extension 1 HSC Examination	**2** **2**	Solution
20 MX 1	**11 d**	By expressing $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x + \alpha)$, solve $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$, for $0 \le x \le 2\pi$. NESA 2020 Mathematics Extension 1 HSC Examination	**4**	Solution
20 MX 1	**14 b**	(i) Show that $\sin^3\theta - \dfrac{3}{4}\sin\theta + \dfrac{\sin(3\theta)}{4} = 0$ (ii) By letting $x = 4\sin\theta$ in the cubic equation $x^3 - 12x + 8 = 0$, show that $\sin 3\theta = \dfrac{1}{2}$. (iii) Prove that $\sin^2\dfrac{\pi}{18} + \sin^2\dfrac{5\pi}{18} + \sin^2\dfrac{25\pi}{18} = \dfrac{3}{2}$. NESA 2020 Mathematics Extension 1 HSC Examination	**2** **2** **3**	Solution
SP MX 1	**3**	Given that $\cos\theta - 2\sin\theta + 2 = 0$, which of the following shows the two possible values for $\tan\dfrac{\theta}{2}$? A. −3 or −1 B. −3 or 1 C. −1 or 3 D. 1 or 3 NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)	**1**	Solution
SP MX 1	**13 b**	A device playing a signal given by $x = \sqrt{2}\sin t + \cos t$ produces distortion whenever $\lvert x\rvert \ge 1.5$. For what fraction of the time will the device produce distortion if the signal is played continuously? NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)	**4**	Solution
SP MX 1	**13 c**	(i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. (ii) Hence find expressions for the exact values of the solutions to the equation $8x^3 - 6x = 1$. NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)	**3** **4**	Solution
TG 19 MX 1	**1 6**	It is given that $\sin x = \dfrac{1}{4}$, where $\dfrac{\pi}{2} < x < \pi$. What is the value of $\sin 2x$? A. $-\dfrac{7}{8}$ B. $-\dfrac{\sqrt{15}}{8}$ C. $\dfrac{\sqrt{15}}{8}$ D. $\dfrac{7}{8}$ NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions NESA 2019 Mathematics Extension 1 HSC Examination	**1**	Solution
TG	**2**	Find all angles θ for which $\sin 2\theta = \cos\theta$. NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions		Solution

TG **3** If $0 \le x \le 2\pi$ find all values of x for which $8\cos x + 15\sin x = 1$. [Solution]

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions

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TG **4** (a) Prove that $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$. [Solution]

(b) Use the relationship from part (a) to solve the equation $\sin^2 2\alpha - \sin^2\alpha = \sin 3\alpha$ for $0 \le \alpha \le 2\pi$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions

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TG 03 MX 1 **5 2e** [Solution]

(a) Express $\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where α is in radians. **2**

(b) Hence, or otherwise, sketch the graph of $y = \cos x - \sin x$ for $0 \le x \le 2\pi$. **2**

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions
NESA 2003 Mathematics Extension 1 HSC Examination

TG 07 MX 1 **6 2a** [Solution]

By using the substitution $t = \tan\dfrac{\theta}{2}$, or otherwise, show that $\dfrac{1-\cos\theta}{\sin\theta} = \tan\dfrac{\theta}{2}$. **2**

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions
NESA 2007 Mathematics Extension 1 HSC Examination

TG **7** [Solution]

(a) Show that $\sin x + \sin 3x = 2\sin 2x\cos x$.

(b) Hence or otherwise find all the solutions of $\sin x + \sin 2x + \sin 3x = 0$ for $0 \le \alpha \le 2\pi$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions

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TG **8** [Solution]

α and β are two acute angles.

(a) Show that $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta} = \tan\left(\dfrac{\alpha+\beta}{2}\right)$.

(b) If $3\sin\alpha - 4\cos\alpha = 4\cos\beta - 3\sin\beta$, find the value of $\tan(\alpha+\beta)$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions

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TG **9** [Solution]

The current flowing in a particular electrical circuit may be modelled by the function $f(t) = 3\sin 0.01t + 4\cos 0.01t$.

(a) Express this function in the form $f(t) = A\sin(at+b)$.*

(b) Sketch the graph of $f(t)$.

(c) Find the time at which the current first attains its maximum value.

* Projectmaths: NESA has $f(t) = A\sin(at - b)$

NESA Mathematics Extension 1 Year 12 Topic Guide: Trigonometric Functions

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19 MX 1 **12 b** [Solution]

A particle is moving along the x-axis in simple harmonic motion*. The position of the particle is given by

$$x = \sqrt{2}\cos 3t + \sqrt{6}\cos 3t, \text{ for } t \ge 0.$$

(i) Write x in the form $R\cos(3t-\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. **2**

(ii) Find the two values for x where the particles come to rest. **1**

(iii) When is the first time that the speed of the particle is equal to half of its maximum speed? **2**

(Projectmaths: Simple Harmonic Motion is not in MX1 course, but this solution is completed within the scope of the syllabus)

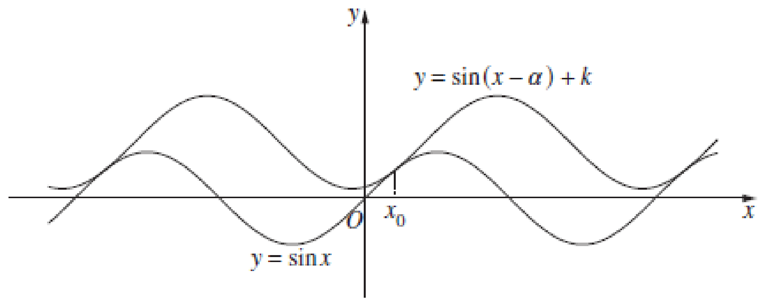
NESA 2019 Mathematics Extension 1 HSC Examination

19 MX 1 **14 c**

The diagram shows the two curves $y = \sin x$ and $y = \sin(x - \alpha) + k$, where $0 < \alpha < \pi$ and $k > 0$.

The two curves have a common tangent at x_0 where $0 < x_0 < \frac{\pi}{2}$.

Solution



(i) Explain why $\cos x_0 = \cos(x_0 - \alpha)$ **1**

(ii) Show that $\sin x_0 = -\sin(x_0 - \alpha)$. **2**

(iii) Hence, or otherwise, find k in terms of α. **2**

NESA 2019 Mathematics Extension 1 HSC Examination

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18 MX 1 **11 c**

Write $\sqrt{3}\sin x + \cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. **2**

Solution

NESA 2018 Mathematics Extension 1 HSC Examination

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17 MX 1 **4**

What is the value of $\tan\alpha$ when the expression $2\sin x - \cos x$ is written in the form $\sqrt{5}\sin(x - \alpha)$? **1**

Solution

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

NESA 2017 Mathematics Extension 1 HSC Examination

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16 MX 1 **7**

The displacement x of a particle at time t is given by $x = 5\sin 4t + 12\cos 4t$. **1**
What is the maximum velocity of the particle?

Solution

(A) 13 (B) 28 (C) 52 (D) 68

NESA 2016 Mathematics Extension 1 HSC Examination

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15 MX 1 **10**

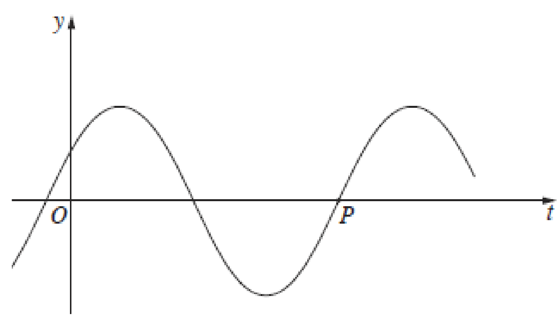
The graph of the function $y = \cos(2t - \frac{\pi}{3})$ is shown. **1**

Solution

What are the coordinates of the point P?

(A) $(\frac{5\pi}{12}, 0)$ (B) $(\frac{2\pi}{3}, 0)$

(C) $(\frac{11\pi}{12}, 0)$ (D) $(\frac{7\pi}{6}, 0)$



NESA 2015 Mathematics Extension 1 HSC Examination

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15 MX 1 **11 d**

Express $5\cos x - 12\sin x$ in the form $A\cos(x + \alpha)$, where $0 \le \alpha \le \frac{\pi}{2}$. **2**

Solution

NESA 2015 Mathematics Extension 1 HSC Examination

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14 MX 1 **2** Which expression is equal to $\cos x - \sin x$? **1** Solution

(A) $\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right)$ (B) $\sqrt{2}\cos\left(x - \dfrac{\pi}{4}\right)$

(C) $2\cos\left(x + \dfrac{\pi}{4}\right)$ (D) $2\cos\left(x + \dfrac{\pi}{4}\right)$

NESA 2014 Mathematics Extension 1 HSC Examination

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13 MX 1 **12 a** Solution

(i) Write $\sqrt{3}\cos x - \sin x$ in the form $2\cos(x + \alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$. **1**

(ii) Hence, or otherwise, solve $\sqrt{3}\cos x = 1 + \sin x$, where $0 < \alpha < 2\pi$. **2**

NESA 2013 Mathematics Extension 1 HSC Examination

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10 MX 1 **4b** Solution

(i) Express $2\cos\theta + 2\cos(\theta + \dfrac{\pi}{3})$ in the form $R\cos(\theta + \alpha)$, **3**

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(ii) Hence, or otherwise, solve $2\cos\theta + 2\cos(\theta + \dfrac{\pi}{3}) = 3$, **2**

for $0 < \theta < 2\pi$.

NESA 2010 Mathematics Extension 1 HSC Examination

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09 MX 1 **2b** Solution

(i) Express $3\sin x + 4\cos x$ in the form $A\sin(x + \alpha)$ **2**
where $0 \leq \alpha \leq 2\pi$.

(ii) Hence, or otherwise, solve $3\sin x + 4\cos x = 5$ for $0 \leq \alpha \leq 2\pi$. **2**
Give your answer, or answers, correct to two decimal places.

NESA 2009 Mathematics Extension 1 HSC Examination

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08 MX 1 **6b** It can be shown that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ for all values of θ. **3** Solution
(Do NOT prove this.)
Use this result to solve $\sin 3\theta + \sin 2\theta = \sin\theta$ for $0 \leq \theta \leq 2\pi$.

NESA 2008 Mathematics Extension 1 HSC Examination

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projectmaths

Year 12: Calculus
C2 Further calculus skills

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Students:

- find and evaluate indefinite and definite integrals using the method of integration by substitution, using a given substitution
  - change an integrand into an appropriate form using algebra
- prove and use the identities $\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$ and $\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$ to solve problems
- solve problems involving $\int \sin^2 nx\, dx$ and $\int \cos^2 nx\, dx$
- find derivatives of inverse functions by using the relationship $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$
- solve problems involving the derivatives of inverse trigonometric functions
- integrate expressions of the form $\frac{1}{\sqrt{a^2 - x^2}}$ or $\frac{a}{a^2 + x^2}$ (ACMSM121)

Reference Sheet

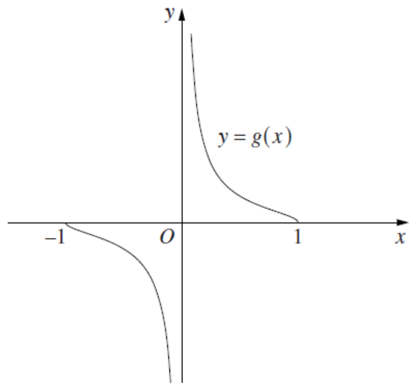
23 MX 1	**11 d**	Find $\int \frac{1}{\sqrt{4 - 9x^2}}\, dx$. NESA 2023 Mathematics Extension 1 HSC Examination	**2**	**Solution**
23 MX 1	**12 a**	Evaluate $\int_3^4 (x+2)\sqrt{x-3}\, dx$ using the substitution $u = x - 3$. NESA 2023 Mathematics Extension 1 HSC Examination	**3**	**Solution**
22 MX 1	**9**	A given function $f(x)$ has an inverse $f^{-1}(x)$. The derivatives of $f(x)$ and $f^{-1}(x)$ exist for all real numbers x. The graphs $y = f(x)$ and $y = f^{-1}(x)$ have at least one point of intersection. Which statement is true for all points of intersection of these graphs? A. All points of intersection lie on the line $y = x$. B. None of the points of intersection lie on the line $y = x$. C. At no point of intersection are the tangents to the graphs parallel. D. At no point of intersection are the tangents to the graphs perpendicular. NESA 2022 Mathematics Extension 1 HSC Examination	**1**	**Solution**
22 MX 1	**11 b**	Find the exact value of $\int_0^1 \frac{x}{\sqrt{x^2 + 4}}\, dx$ using the substitution $u = x^2 + 4$. NESA 2022 Mathematics Extension 1 HSC Examination	**3**	**Solution**

22 MX 1	**12 c**	Find the equation of the tangent to the curve $y = x\arctan(x)$ at the point with coordinates $(1, \frac{\pi}{4})$. Give your answer in the form $y = mx + c$. NESA 2022 Mathematics Extension 1 HSC Examination	**3**	**Solution**
21 MX 1	**2**	Which of the following integrals is equivalent to $\int \sin^2 3x \; dx$? A. $\int \frac{1+\cos 6x}{2}\,dx$ B. $\int \frac{1-\cos 6x}{2}\,dx$ C. $\int \frac{1+\sin 6x}{2}\,dx$ D. $\int \frac{1-\sin 6x}{2}\,dx$ NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**
21 MX 1	**11 c**	Use the substitution $u = x + 1$ to find $\int x\sqrt{x+1}\;dx$. NESA 2021 Mathematics Extension 1 HSC Examination	**3**	**Solution**
21 MX 1	**11f**	Evaluate $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. NESA 2021 Mathematics Extension 1 HSC Examination	**2**	**Solution**
21 MX 1	**14 e**	The polynomial $g(x) = x^3 + 4x - 2$ passes through the point (1, 3). Find the gradient of the tangent to $f(x) = xg^{-1}(x)$ at the point where $x = 3$. NESA 2021 Mathematics Extension 1 HSC Examination	**2**	**Solution**
20 MX 1	**3**	Which of the following is an anti-derivative of $\frac{1}{4x^2+1}$? A. $2\tan^{-1}\left(\frac{x}{2}\right) + c$ B. $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$ C. $2\tan^{-1}(2x) + c$ D. $\frac{1}{2}\tan^{-1}(2x) + c$ NESA 2020 Mathematics Extension 1 HSC Examination	**1**	**Solution**
20 MX 1	**12 d**	Find $\int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \; dx$. NESA 2020 Mathematics Extension 1 HSC Examination	**3**	**Solution**
20 MX 1	**13 a**	(i) Find $\frac{d}{dx}\left(\sin^3\theta\right)$. (ii) Use the substitution $x = \tan\theta$ to evaluate $\int_0^1 \frac{x^2}{\left(1+x^2\right)^{\frac{5}{2}}}\,dx$. NESA 2020 Mathematics Extension 1 HSC Examination	**1** **4**	**Solution**

20 MX 1 **13 c** Solution

Suppose $f(x) = \tan(\cos^{-1}(x))$ and $g(x) = \dfrac{\sqrt{1-x^2}}{x}$.

The graph of $y = g(x)$ is given.



(i) Show that $f'(x) = g'(x)$. **4**

(ii) Using part (i), or otherwise, show that $f(x) = g(x)$. **3**

NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **4** / **19 MX 1** **3** **1** Solution

What is the derivative of $\tan^{-1}\dfrac{x}{2}$?

A. $\dfrac{1}{2(4+x^2)}$ B. $\dfrac{1}{4+x^2}$ C. $\dfrac{2}{4+x^2}$ D. $\dfrac{4}{4+x^2}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

SP MX 1 **13 a** **3** Solution

Use the substitution $x = \sin^2\theta$ to determine $\displaystyle\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\, dx$.

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

TG **1** Solution

Use the substitution $u = x^2 + 1$ to determine $\displaystyle\int x\sqrt{1+x^2}\, dx$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **2** Solution

Use the substitution $u = 1 + t$ to determine $\displaystyle\int_0^1 \frac{t}{\sqrt{1+t}}\, dt$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **3** Solution

Use the substitution $u = x^2$ to determine $\displaystyle\int \frac{x}{1+x^4}\, dx$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **4** Solution

Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 2x\, dx$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **5** Solution

Use the substitution $u = \sin x$ to find the value of $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 x \cos x\, dx$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **6** Solution

Use the substitution $u = \sin x$ to evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 x \cos^3 x\, dx$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 7

Evaluate $4\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \sin^2 4x\, dx$.

Solution

TG 8

If $\dfrac{d^2y}{dx^2} = 2\cos^2 x$ and when $x = \dfrac{\pi}{2}$, $\dfrac{dy}{dx} = 0$, $y = 0$, then find y in terms of x.

Solution

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TG 9

Differentiate $\sin^{-1}x + \cos^{-1}x$, and hence show that $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$.

Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 10

If $y = \cos^{-1}x + \cos^{-1}(-x)$, find $\dfrac{dy}{dx}$ and show that $y = \pi$ for all x in the domain.

Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 11

Evaluate the following:

Solution

(a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{4 - x^2}}$

(b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{3 - x^2}}$

(c) $\displaystyle\int_0^{\frac{1}{2}} \frac{dx}{1 + 4x^2}$

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 12

(a) Prove that $\dfrac{d}{dx}(x\sin^{-1}x) = \sin^{-1}x + \dfrac{x}{\sqrt{1 - x^2}}$.

Solution

(b) Hence use the substitution $u = 1 - x^2$ to show that $\displaystyle\int_0^{\frac{1}{2}} \sin^{-1} x\, dx = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

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19 MX 1 **11 e**

Find $\displaystyle\int 2\sin^2 4x\, dx$. **2**

Solution

NESA 2019 Mathematics Extension 1 HSC Examination

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19 MX 1 **13 a**

Use the substitution $u = \cos^2 x$ to evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{4 + \cos^2 x}\, dx$. **3**

Solution

NESA 2019 Mathematics Extension 1 HSC Examination

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18 MX 1 **11f**

Evaluate $\displaystyle\int_{-3}^{0} \frac{x}{\sqrt{1 - x}}\, dx$, using the substitution $u = 1 - x$. **3**

Solution

NESA 2018 Mathematics Extension 1 HSC Examination

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18 MX 1 **12 a**

Find $\int \cos^2 3x \, dx$. **2** [Solution]

NESA 2018 Mathematics Extension 1 HSC Examination

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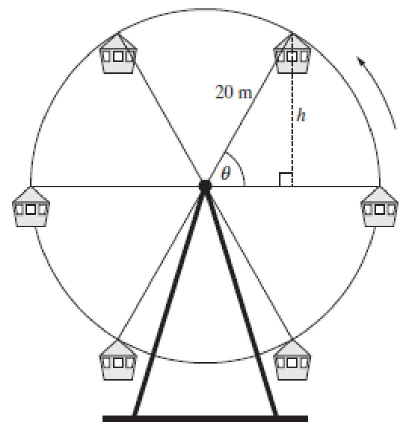
18 MX 1 **12 b** [Solution]

A ferris wheel has a radius of 20 metres and is rotating at a rate of 1.5 radians per minute. The top of a carriage is h metres above the horizontal diameter of the ferris wheel. The angle of elevation of the top of the carriage from the centre of the ferris wheel is θ.



(i) Show that $\dfrac{dh}{d\theta} = 20 \cos \theta$. **1**

(ii) At what speed is the top of the carriage rising when it is 15 metres higher than the horizontal diameter of the ferris wheel? Give your answer correct to one decimal place. **2**

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18 MX 1 **12 c** [Solution]

Let $f(x) = \sin^{-1}x + \cos^{-1}x$.

(i) Show that $f'(x) = 0$. **1**

(ii) Hence, or otherwise, prove that $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$. **1**

(iii) Hence, sketch $f(x) = \sin^{-1}x + \cos^{-1}x$. **1**

NESA 2018 Mathematics Extension 1 HSC Examination

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17 MX 1 **11 b**

Differentiate $\tan^{-1}(x^3)$. **2** [Solution]

NESA 2017 Mathematics Extension 1 HSC Examination

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17 MX 1 **11 e**

Evaluate $\displaystyle\int_0^3 \frac{x}{\sqrt{x+1}} \, dx$, using the substitution $x = u^2 - 1$. **3** [Solution]

NESA 2017 Mathematics Extension 1 HSC Examination

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17 MX 1 **11f**

Find $\int \sin^2 x \cos x \, dx$. **1** [Solution]

NESA 2017 Mathematics Extension 1 HSC Examination

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17 MX 1 **14 c** [Solution]

The concentration of a drug in a body is $F(t)$, where t is the time in hours after the drug is taken. Initially the concentration of the drug is zero. The rate of change of concentration of the drug is given by $F'(t) = 50e^{-0.5t} - 0.4F(t)$.

(i) By differentiating the product $F(t)e^{0.4t}$, show that $\dfrac{d}{dt}[F(t)e^{0.4t}] = 50e^{-0.1t}$. **2**

(ii) Hence or otherwise, show that $F(t) = 500(e^{-0.4t} - e^{-0.5t})$. **2**

(iii) The concentration of the drug increases to a maximum. **2**

For what value of t does this maximum occur?

NESA 2017 Mathematics Extension 1 HSC Examination

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16 MX 1 **5**

Which expression is equal to $\int \sin^2 2x\,dx$? **1** [Solution]

(A) $\frac{1}{2}\left(x - \frac{1}{4}\sin 4x\right) + c$ (B) $\frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) + c$

(C) $\frac{\sin^3 2x}{6} + c$ (D) $\frac{-\cos^3 2x}{6} + c$

NESA 2016 Mathematics Extension 1 HSC Examination

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16 MX 1 **11 b**

Use the substitution $u = x - 4$ to find $\int x\sqrt{x-4}\,dx$. **3** [Solution]

NESA 2016 Mathematics Extension 1 HSC Examination

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16 MX 1 **11 c**

Differentiate $3\tan^{-1}(2x)$. **2** [Solution]

NESA 2016 Mathematics Extension 1 HSC Examination

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15 MX 1 **7**

What is the value of k such that $\int_0^k \frac{1}{\sqrt{4-x^2}}\,dx = \frac{\pi}{3}$? **1** [Solution]

(A) 1 (B) $\sqrt{3}$ (C) 2 (D) $2\sqrt{3}$

NESA 2015 Mathematics Extension 1 HSC Examination

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15 MX 1 **11 a**

Find $\int \sin^2 x\,dx$. **2** [Solution]

NESA 2015 Mathematics Extension 1 HSC Examination

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15 MX 1 **11 e**

Use the substitution $u = 2x - 1$ to evaluate $\int_1^2 \frac{x}{(2x-1)^2}\,dx$. **3** [Solution]

NESA 2015 Mathematics Extension 1 HSC Examination

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15 MX 1 **13 d**

Let $f(x) = \cos^{-1}(x) + \cos^{-1}(-x)$, where $-1 \le x \le 1$. [Solution]

(i) By considering the derivative of $f(x)$, prove that $f(x)$ is constant. **2**

(ii) Hence deduce that $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$. **1**

NESA 2015 Mathematics Extension 1 HSC Examination

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14 MX 1 **6**

What is the derivative of $3\sin^{-1}\frac{x}{2}$? **1** [Solution]

(A) $\frac{6}{\sqrt{4-x^2}}$ (B) $\frac{3}{\sqrt{4-x^2}}$ (C) $\frac{3}{2\sqrt{4-x^2}}$ (D) $\frac{3}{4\sqrt{4-x^2}}$

NESA 2014 Mathematics Extension 1 HSC Examination

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14 MX 1 **11 d**

Evaluate $\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$ using the substitution $x = u^2 + 1$. **3** [Solution]

NESA 2014 Mathematics Extension 1 HSC Examination

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13 MX 1 **5**

Which integral is obtained when the substitution $u = 1 + 2x$ is applied **1** [Solution]

to $\int x\sqrt{1+2x}\,dx$?

(A) $\frac{1}{4}\int (u-1)\sqrt{u}\,du$ (B) $\frac{1}{2}\int (u-1)\sqrt{u}\,du$ (C) $\int (u-1)\sqrt{u}\,du$ (D) $2\int (u-1)\sqrt{u}\,du$

NESA 2013 Mathematics Extension 1 HSC Examination

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13 MX 1 **11 b**

Find $\displaystyle\int \frac{1}{\sqrt{49-4x^2}}\,dx$. **2** Solution

NESA 2013 Mathematics Extension 1 HSC Examination

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13 MX 1 **11 f**

Use the substitution $u = e^{3x}$ to evaluate $\displaystyle\int_0^{\frac{1}{3}} \frac{e^{3x}}{e^{6x}+1}\,dx$. **3** Solution

NESA 2013 Mathematics Extension 1 HSC Examination

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13 MX 1 **11 g**

Differentiate $x^2 \sin^{-1} 5x$. **2** Solution

NESA 2013 Mathematics Extension 1 HSC Examination

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12 MX 1 **7**

Which expression is equal to $\displaystyle\int \sin^2 3x\,dx$? **1** Solution

(A) $\dfrac{1}{2}\left(x - \dfrac{1}{3}\sin 3x\right) + C$ (B) $\dfrac{1}{2}\left(x + \dfrac{1}{3}\sin 3x\right) + C$

(C) $\dfrac{1}{2}\left(x - \dfrac{1}{6}\sin 6x\right) + C$ (D) $\dfrac{1}{2}\left(x + \dfrac{1}{6}\sin 6x\right) + C$

NESA 2012 Mathematics Extension 1 HSC Examination

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12 MX 1 **9**

What is the derivative of $\cos^{-1}(3x)$? **1** Solution

(A) $\dfrac{1}{3\sqrt{1-9x^2}}$ (B) $\dfrac{-1}{3\sqrt{1-9x^2}}$ (C) $\dfrac{3}{\sqrt{1-9x^2}}$ (D) $\dfrac{-3}{\sqrt{1-9x^2}}$

NESA 2012 Mathematics Extension 1 HSC Examination

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12 MX 1 **11 a**

Evaluate $\displaystyle\int_0^3 \frac{1}{9+x^2}\,dx$. **3** Solution

NESA 2012 Mathematics Extension 1 HSC Examination

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12 MX 1 **11 d**

Use the substitution $u = 2 - x$ to evaluate $\displaystyle\int_1^2 x(2-x)^5\,dx$. **3** Solution

NESA 2012 Mathematics Extension 1 HSC Examination

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11 MX 1 **1d**

Using the substitution $u = \sqrt{x}$, evaluate $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$. **3** Solution

NESA 2011 Mathematics Extension 1 HSC Examination

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10 MX 1 **1e**

Use the substitution $u = 1 - x$ to evaluate $\displaystyle\int_0^1 x\sqrt{1-x}\,dx$. **3** Solution

NESA 2010 Mathematics Extension 1 HSC Examination

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10 MX 1 **2a**

The derivative of a function $f(x)$ is given by $f'(x) = \sin^2 x$. **2** Solution

Find $f(x)$, given that $f(0) = 2$.

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10 MX 1 **5b** Solution

Let $f(x) = \tan^{-1} x + \tan^{-1}\left(\dfrac{1}{x}\right)$ for $x \neq 0$.

(i) By differentiating $f(x)$, or otherwise, show that $f(x) = \dfrac{\pi}{2}$ for $x > 0$. **3**

(ii) Given that $f(x)$ is an odd function, sketch the graph of $y = f(x)$. **1**

NESA 2010 Mathematics Extension 1 HSC Examination

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09 MX 1	**1f**	Use the substitution $u = x^3 + 1$ to evaluate $\int_0^2 x^2 e^{x^3+1}\, dx$. NESA 2009 Mathematics Extension 1 HSC Examination	**3**	Solution
08 MX 1	**1b**	Differentiate $\cos^{-1}(3x)$ with respect to x. NESA 2008 Mathematics Extension 1 HSC Examination	**2**	Solution
08 MX 1	**1c**	Evaluate $\int_{-1}^{1} \frac{1}{\sqrt{4-x^2}}\, dx$. NESA 2008 Mathematics Extension 1 HSC Examination	**2**	Solution
08 MX 1	**1e**	Evaluate $\int_0^{\frac{\pi}{4}} \cos\theta \sin^2\theta\, d\theta$. NESA 2008 Mathematics Extension 1 HSC Examination	**2**	Solution
08 MX 1	**2a**	Use the substitution $u = \log_e x$ to evaluate $\int_e^{e^2} \frac{1}{x(\log_e x)^2}\, dx$. NESA 2008 Mathematics Extension 1 HSC Examination	**3**	Solution
07 MX 1	**1c**	Differentiate $\tan^{-1}(x^4)$ with respect to x. NESA 2007 Mathematics Extension 1 HSC Examination	**2**	Solution
07 MX 1	**1e**	Use the substitution $u = 25 - x^2$ to evaluate $\int_3^4 \frac{2x}{\sqrt{25-x^2}}\, dx$. NESA 2007 Mathematics Extension 1 HSC Examination	**3**	Solution

projectmaths

Year 12: Calculus
C3.1 Further area and volume of solids

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Students:

- calculate area of regions between curves determined by functions (ACMSM124)
- sketch, with and without the use of technology, the graph of a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis **AAM**
- calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis, with and without the use of technology (ACMSM125) **AAM**
- determine the volumes of solids of revolution that are formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts **AAM**

Reference Sheet

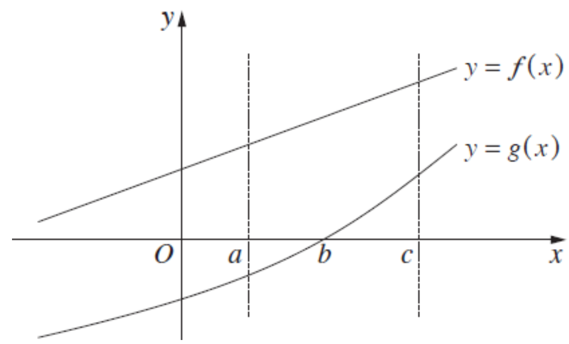
23 MX 1 **4** The diagram shows the graphs of the functions $f(x)$ and $g(x)$. **1** **Solution**

It is known that

$$\int_a^c f(x)\,dx = 10$$

$$\int_a^b g(x)\,dx = -2$$

$$\int_b^c g(x)\,dx = 3$$



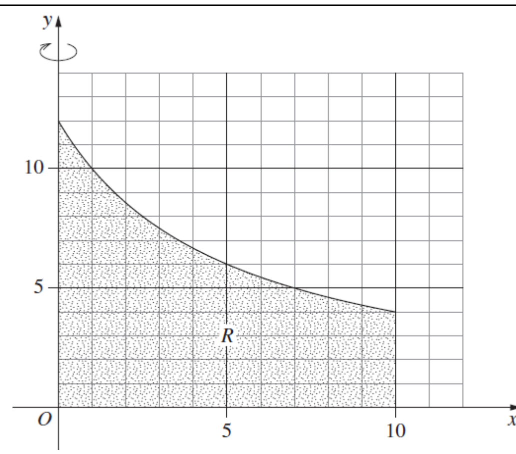
What is the area between the curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = c$?

A. 5 B. 7 C. 9 D. 11

NESA 2023 Mathematics Extension 1 HSC Examination

23 MX 1 **12 e** The region, R, bounded by the hyperbola $y = \dfrac{60}{x+5}$, the line $x = 10$ and the coordinate axes is shown. **4** **Solution**

Find the volume of the solid of revolution formed when the region R is rotated about the y-axis. Leave your answer in exact form.

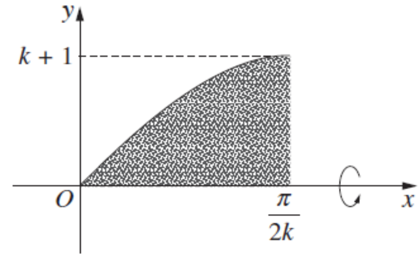


NESA 2023 Mathematics Extension 1 HSC Examination

22 MX 1 **13 b** **3** **Solution**

A solid of revolution is to be found by rotating the region bounded by the x-axis and the curve $y = (k + 1)\sin(kx)$, where $k > 0$, between $x = 0$ and $x = \dfrac{\pi}{2k}$ about the x-axis.

Find the value of k for which the volume is π^2.



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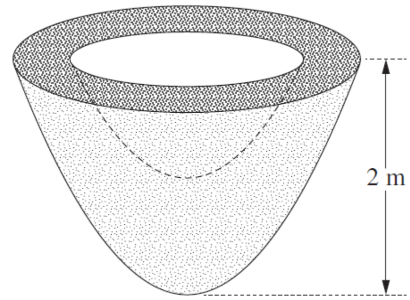
21 MX 1 **13 a** **3** **Solution**

A 2-metre-high sculpture is to be made out of concrete.

The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.

Find the volume of concrete needed to make the sculpture.



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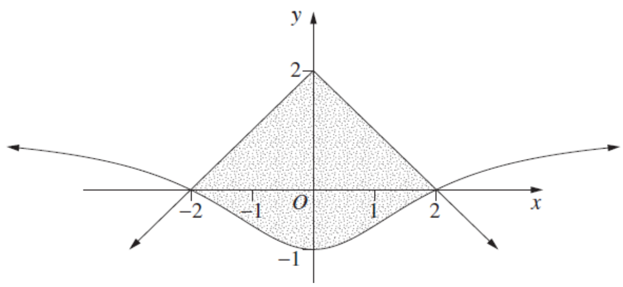
21 MX 1 **13 c** **3** **Solution**

The region enclosed by

$y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is

shaded in the diagram.

Find the exact value of the area of the shaded region.



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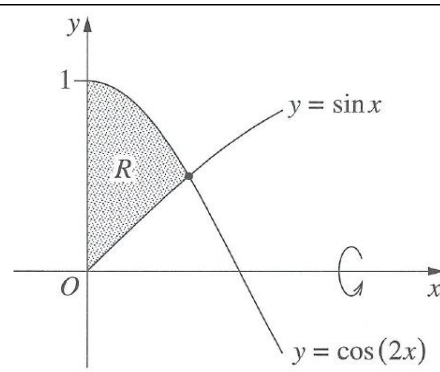
20 MX 1 **13 b** **4** **Solution**

The region R is bounded by the y-axis, the graph of $y = \cos(2x)$ and the graph of $y = \sin x$, as shown in the diagram.

Find the volume of the solid of revolution formed when the region R is rotated about the x-axis.



NESA 2020 Mathematics Extension 1 HSC Examination

SP MX 1 **14 a**

(i) Sketch the graph of $y = x\cos x$ for $-\pi \le x \le \pi$ and hence explain why **3** Solution

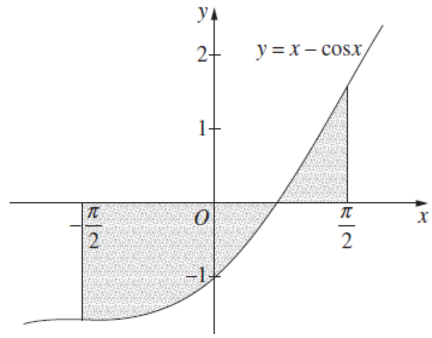
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx = 0.$$

(ii) Consider the volume of the solid of revolution produced by rotating about the x-axis the shaded region between the graph of **3**

$y = x - \cos x$, the x-axis and the lines

$x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$.

Using the results of part (i), or otherwise, find the volume of the solid.



NESA Mathematics Extension 1 Sample Examination Paper (2020)

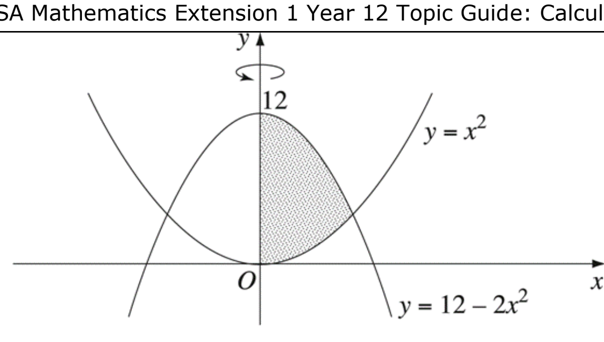
TG **1** Sketch the region bounded by the curve $y = x^2$ and the lines $y = 4$ and $y = 9$. Evaluate the area of this region. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **2** The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram. Solution

(a) Find the points of intersection of the two curves.

(b) The shaded region between the curves and the y-axis is rotated about the y-axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed.



NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **3** The region bounded by the curve $y = (x - 1)(3 - x)$ and the x-axis is rotated about the line $x = 3$ to form a solid. Solution

When the region is rotated, the horizontal line segment at height y sweeps out an annulus.

(a) Find the area of the annulus as a function of y.

(b) Find the volume of the solid.

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **4** The region enclosed by the curve $y = 4\sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x-axis.
Find the volume of the solid of revolution. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **5** A curved funnel has a shape formed by rotating part of the parabola $y = 2\sqrt{x}$ about the y-axis, where x and y are given in cm.
The funnel is 4 cm deep.
Find the volume of liquid which the funnel will hold if it is sealed at the bottom. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 6

(a) Sketch the region bounded by the curve $y = \sin x + \cos x$ and the coordinate axes in the first quadrant, taking the upper limit of x as $\frac{3\pi}{4}$. Show the intercepts on the axes, and calculate the area of the region.

(b) Find the volume of the solid formed if the region is rotated about the x-axis to form a solid of revolution.

[Solution]

NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

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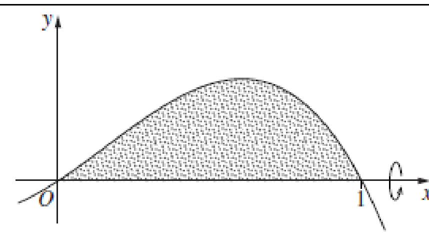
19 M 13 d — 3 marks

The diagram shows the region bounded by the curve $y = x - x^3$, and the x-axis between $x = 0$ and $x = 1$.
The region is rotated about the x-axis to form a solid.
Find the exact value of the volume of the solid formed.



[Solution]

NESA 2019 Mathematics HSC Examination

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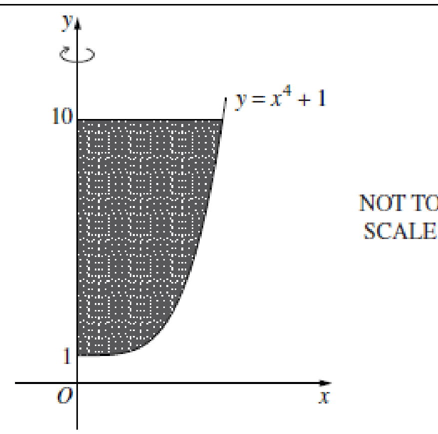
18 M 14 b — 3 marks

The shaded region shown in the diagram is bounded by the curve $y = x^4 + 1$, then y-axis and the line $y = 10$.
Find the volume of the solid of revolution formed when the shaded region is rotated about the y-axis.
.



[Solution]

NESA 2018 Mathematics HSC Examination

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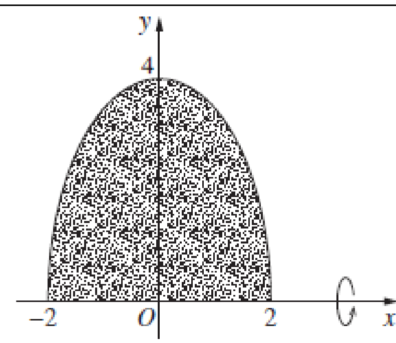
17 M 12 b — 3 marks

The diagram shows the region bounded by $y = \sqrt{16 - 4x^2}$ and the x-axis.
The region is rotated about the x-axis to form a solid.
Find the exact volume of the solid formed.



[Solution]

NESA 2017 Mathematics HSC Examination

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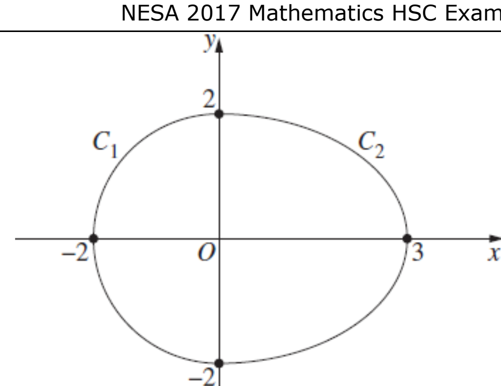
16 M 15 a — 4 marks

The diagram shows two curves C_1 and C_2.
The curve C_1 is the semicircle $x^2 + y^2 = 4$, $-2 \leq x \leq 2$. The curve C_2 has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1,\ 0 \leq x \leq 3.$$

An egg is modelled by rotating the curves about the x-axis to form a solid of revolution.
Find the exact value of the volume of the solid of revolution.

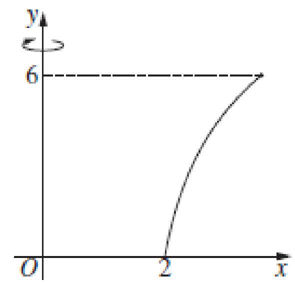


[Solution]

NESA 2016 Mathematics HSC Examination

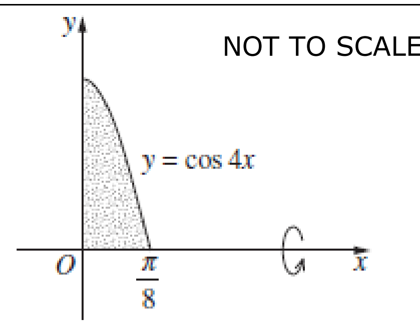
15 M	**16 b**	A bowl is formed by rotating the curve $y = 8\log_e(x - 1)$ about the y-axis for $0 \leq y \leq 6$. Find the volume of the bowl. Give your answer correct to 1 decimal place.	**3**	**Solution**

Not to scale



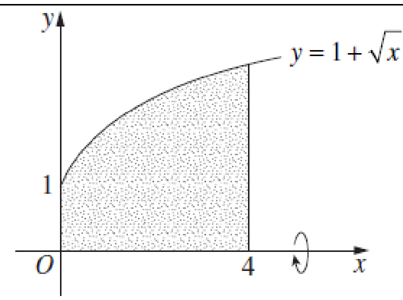
NESA 2015 Mathematics HSC Examination

14 MX 1	**12 b**	The region bounded by $y = \cos 4x$ and the x-axis, between $x = 0$ and $x = \frac{\pi}{8}$, is rotated about the x-axis to form a solid. Find the volume of the solid.	**3**	**Solution**



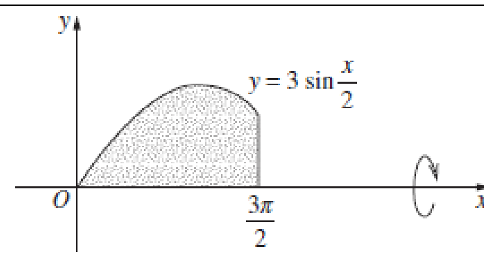
NESA 2014 Mathematics Extension 1 HSC Examination

14 M	**14 c**	The region bounded by the curve $y = 1 + \sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x–axis to form a solid. Find the volume of the solid.	**3**	**Solution**



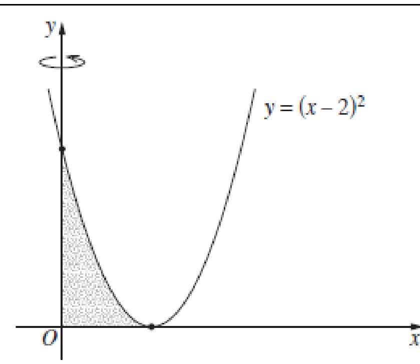
NESA 2014 Mathematics HSC Examination

13 MX 1	**12 b**	The region bounded by the graph $y = 3\sin\frac{x}{2}$ and the x-axis between $x = 0$ and $x = \frac{3\pi}{2}$ is rotated about the x-axis to form a solid. Find the exact volume of the solid.	**3**	**Solution**



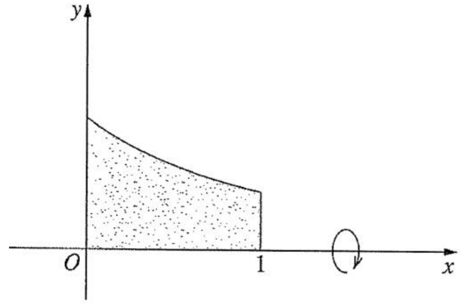
NESA 2013 Mathematics Extension 1 HSC Examination

13 M	**15 b**	The region bounded by the x-axis, the y-axis and the parabola $y = (x - 2)^2$ is rotated about the y-axis to form a solid. Find the volume of the solid.	**4**	**Solution**



NESA 2013 Mathematics HSC Examination

12 M **14 b** The diagram shows the region bounded by $y = \dfrac{3}{(x+2)^2}$, the x-axis, the y-axis, and the line $x = 1$.
The region is rotated about the x-axis to form a solid.
Find the volume of the solid. **3** [Solution]

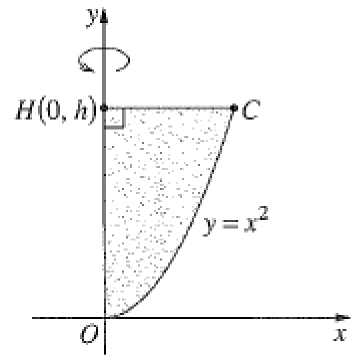


NESA 2012 Mathematics HSC Examination

11 M **8b** The diagram shows the region enclosed by the parabola $y = x^2$, the y-axis and the line $y = h$, where $h > 0$. This region is rotated about the y-axis to form a solid called a paraboloid. The point C is the intersection of $y = x^2$ and $y = h$. The point H has coordinates $(0, h)$. [Solution]

(i) Find the exact volume of the paraboloid in terms of h. **2**

(ii) A cylinder has radius HC and height h. What is the ratio of the volume of the paraboloid to the volume of the cylinder? **1**

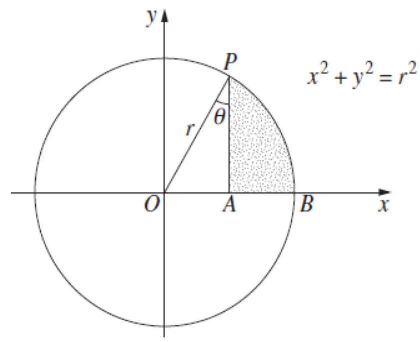


NESA 2011 Mathematics HSC Examination

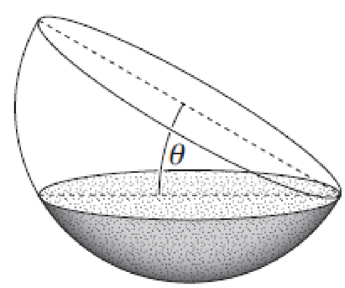
10 M **10 b** The circle $x^2 + y^2 = r^2$ has radius r and centre O. The circle meets the positive x-axis at B. The point A is on the interval OB. A vertical line through A meets the circle at P. Let $\theta = \angle OPA$. [Solution]

(i) The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x-axis. Show that the volume, V, formed is given by **3**

$$V = \frac{\pi r^3}{3}(2 - 3\sin\theta + \sin^3\theta).$$



(ii) A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.



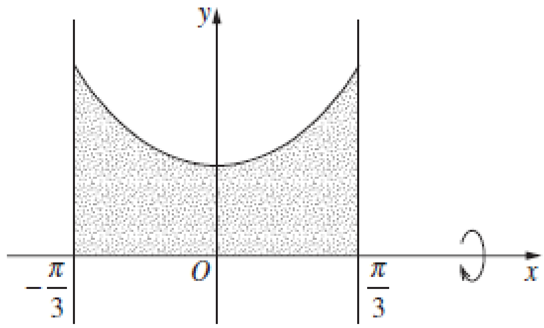
(1) Find θ so that the depth of water remaining is one half of the original depth. **1**

(2) What fraction of the original volume is left in the container? **2**

NESA 2010 Mathematics HSC Examination

09 M **6a** The diagram shows the region bounded by the curve $y = \sec x$, the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, and the x-axis.

The region is rotated about the x-axis.
Find the volume of the solid of revolution formed. **2** [Solution]



NESA 2009 Mathematics HSC Examination

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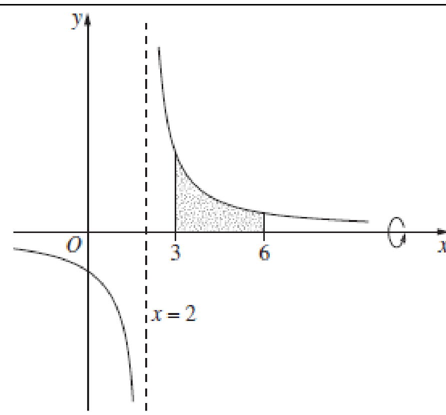
08 M **6c** The graph of $y = \dfrac{5}{x-2}$ is shown. The shaded region in the diagram is bounded by the curve $y = \dfrac{5}{x-2}$, the x-axis, and the lines $x = 3$ and $x = 6$.
Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. **3** [Solution]



NESA 2008 Mathematics HSC Examination

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07 MX 1 **3a** Find the volume of the solid of revolution formed when the region bounded by the curve $y = \dfrac{1}{\sqrt{9 + x^2}}$, the x-axis, the y-axis and the line $x = 3$, is rotated about the x-axis. **3** [Solution]

NESA 2007 Mathematics Extension 1 HSC Examination

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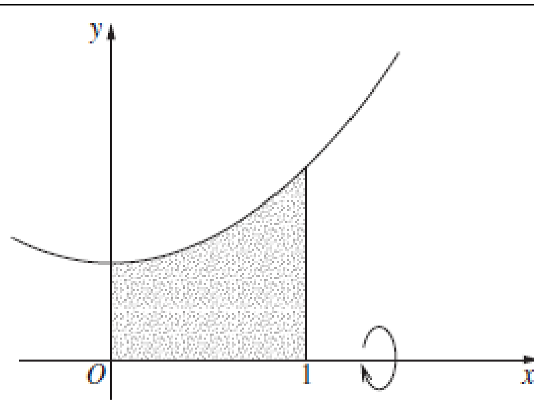
07 M **9a** In the shaded region in the diagram is bounded by the curve $y = x^2 + 1$, the x-axis, and the lines $x = 0$ and $x = 1$.
Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis. **3** [Solution]



NESA 2007 Mathematics HSC Examination

projectmaths

Year 12: Calculus
C3.2 Differential equations

Back

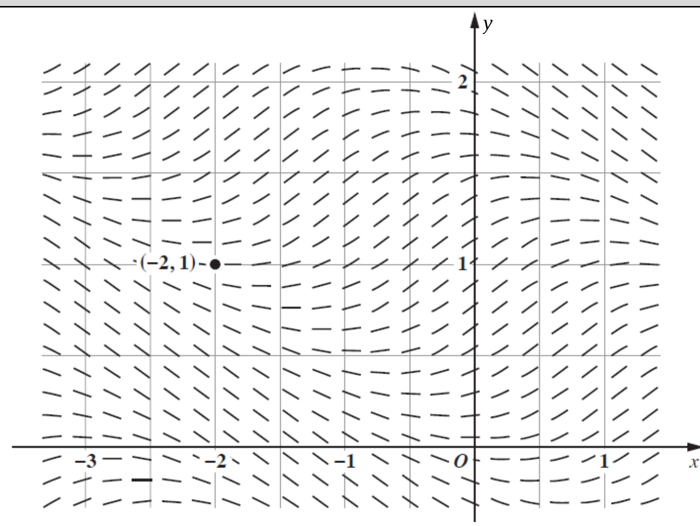
Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- recognise that an equation involving a derivative is called a differential equation
- recognise that solutions to differential equations are functions and that these solutions may not be unique
- sketch the graph of a particular solution given a direction field and initial conditions
  - form a direction field (slope field) from simple first-order differential equations
  - recognise the shape of a direction field from several alternatives given the form of a differential equation, and vice versa
  - sketch several possible solution curves on a given direction field
- solve simple first-order differential equations (ACMSM130)
  - solve differential equations of the form $\frac{dy}{dx} = f(x)$
  - solve differential equations of the form $\frac{dy}{dx} = g(y)$
  - solve differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables
- recognise the features of a first-order linear differential equation and that exponential growth and decay models are first-order linear differential equations, with known solutions
- model and solve differential equations including to the logistic equation that will arise in situations where rates are involved, for example in chemistry, biology and economics (ACMSM132) **AAM**

Reference Sheet

23 MX 1 **3** The diagram shows the direction field of a differential equation. A particular solution to the differential equation passes through $(-2, 1)$. Where does the solution that passes through $(-2, 1)$ cross the y-axis? **1** **Solution**



A. $y = 1.12$
B. $y = 1.34$
C. $y = 1.56$
D. $y = 1.78$

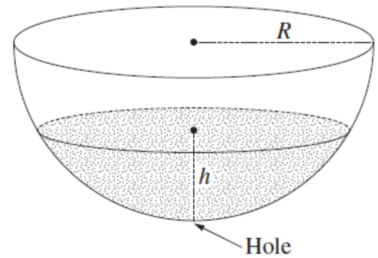
NESA 2023 Mathematics Extension 1 HSC Examination

23 MX 1 **13 a** A hemispherical water tank has radius R cm.

The tank has a hole at the bottom which allows water to drain out.

Initially the tank is empty. Water is poured into the tank at a constant rate of $2kR$ cm^3 s^{-1}, where k is a positive constant.

After t seconds, the height of the water in the tank is h cm, as shown in the diagram, and the volume of water in the tank is V cm^3.



Solution

It is known that $V = \pi\left(Rh^2 - \dfrac{h^3}{3}\right)$. (Do NOT prove this.)

While water flows into the tank and also drains out of the bottom, the rate of change of the volume of water in the tank is given by $\dfrac{dV}{dt} = k(2R - h)$.

(i) Show that $\dfrac{dh}{dt} = \dfrac{k}{\pi h}$. **2**

(ii) Show that the tank is full of water after $T = \dfrac{\pi R^2}{2k}$ seconds. **2**

(iii) The instant the tank is full, water stops flowing into the tank, but it continues to drain out of the hole at the bottom as before. **3**

Show that the tank takes 3 times as long to empty as it did to fill.

NESA 2023 Mathematics Extension 1 HSC Examination

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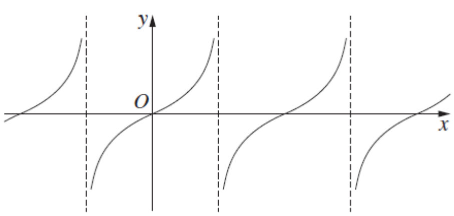
22 MX 1 **10** Which of the following could be the graph of a solution to the differential equation **1**

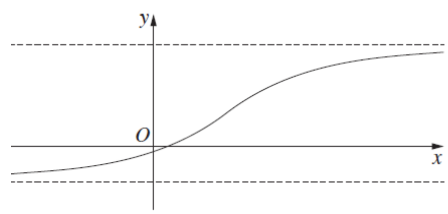
$$\frac{dy}{dx} = \sin y + 1?$$

Solution

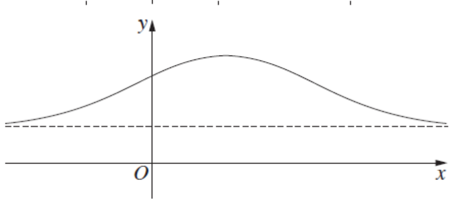
A.



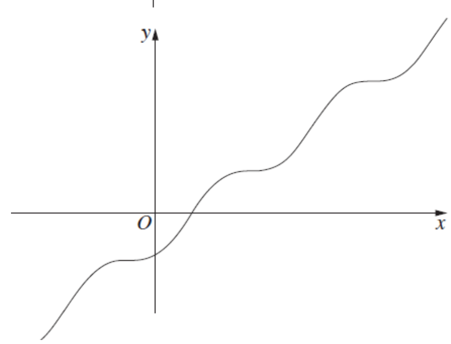
B.



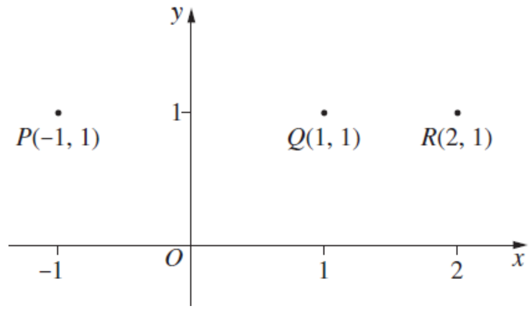
C.



D.



NESA 2022 Mathematics Extension 1 HSC Examination

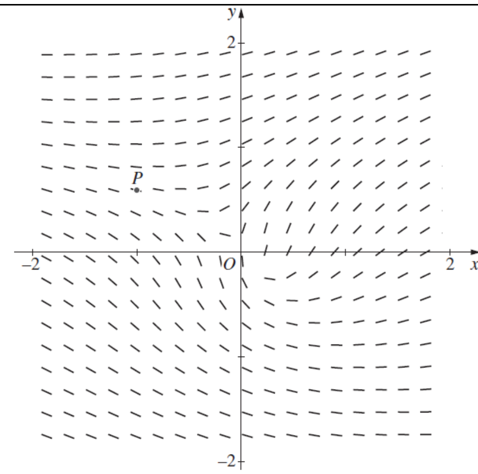
22 MX 1	**12 a**	A direction field is to be drawn for the differential equation $$\frac{dy}{dx} = \frac{x - 2y}{x^2 + y^2}$$ Clearly draw the correct slopes of the direction field at the points P, Q and R shown below.  NESA 2022 Mathematics Extension 1 HSC Examination	**2**	**Solution**
22 MX 1	**12 d**	In a room with temperature 12°C, coffee is poured into a cup. The temperature of the coffee when it is poured into the cup is 92°C, and it is far too hot to drink. The temperature, T, in degrees Celsius, of the coffee, t minutes after it is made, can be modelled using the differential equation $\frac{dT}{dt} = k(T - T_1)$, where k is the constant of proportionality and T_1 is a constant. (i) It takes 5 minutes for the coffee to cool to a temperature of 76°C. Using separation of variables, solve the given differential equation to show that $T = 12 + 80e^{\frac{t}{5}\ln\left(\frac{4}{5}\right)}$ (ii) The optimal drinking temperature for a hot beverage is 57°C. Find the value of t when the coffee reaches this temperature, giving your answer to the nearest minute. NESA 2022 Mathematics Extension 1 HSC Examination	(i) **3** (ii) **1**	**Solution**
22 MX 1	**14 a**	Find the particular solution to the differential equation $(x - 2)\frac{dy}{dx} = xy$ that passes through the point $(0, 1)$. NESA 2022 Mathematics Extension 1 HSC Examination	**4**	**Solution**
21 MX 1	**4**	Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$. Which of the following equations best represents this relationship between x and y ? A. $y^2 = x^2 + c$ B. $y^2 = \frac{x^2}{2} + c$ C. $y = x\ln\lvert y\rvert + c$ D. $y = \frac{x^2}{2}\ln\lvert y\rvert + c$ NESA 2021 Mathematics Extension 1 HSC Examination	**1**	**Solution**

21 MX 1 **12 a**

The direction field for a differential equation is given on page 1 of the Question 12 Writing Booklet.

The graph of a particular solution to the differential equation passes through the point P.

Sketch the graph of the particular solution that passes through the point P.



1 Solution

NESA 2021 Mathematics Extension 1 HSC Examination

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21 MX 1 **14 b**

In a certain country, the population of deer was estimated in 1980 to be 150 000. The population growth is given by the logistic equation $\dfrac{dP}{dt} = 0.1P\left(\dfrac{C-P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity.

In the year 2000, the population of deer was estimated to be 600 000.

Use the fact that $\dfrac{C}{P(C-P)} = \dfrac{1}{P} + \dfrac{1}{C-P}$ to show that the carrying capacity is approximately 1 130 000.

4 Solution

NESA 2021 Mathematics Extension 1 HSC Examination

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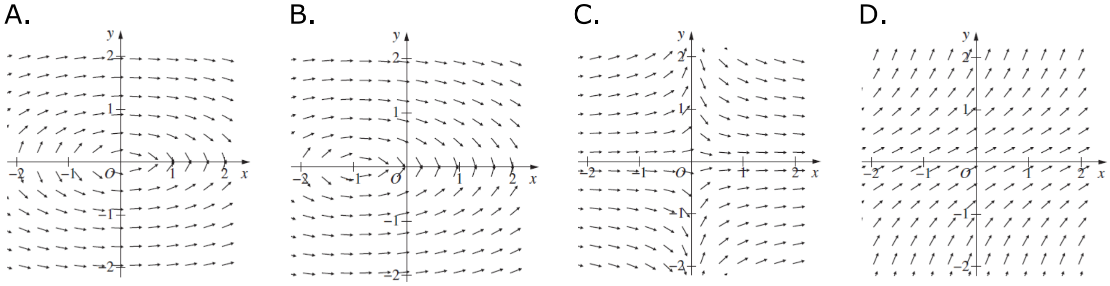
20 MX 1 **7**

Which of the following best represents the direction field for the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{4y}$?

A. B. C. D.



1 Solution

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **11 e**

Solve $\dfrac{dy}{dx} = e^{2y}$, finding x as a function of y.

2 Solution

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **12 e**

Find the curve which satisfies the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{y}$ and passes through the point (1, 0).

3 Solution

NESA 2020 Mathematics Extension 1 HSC Examination

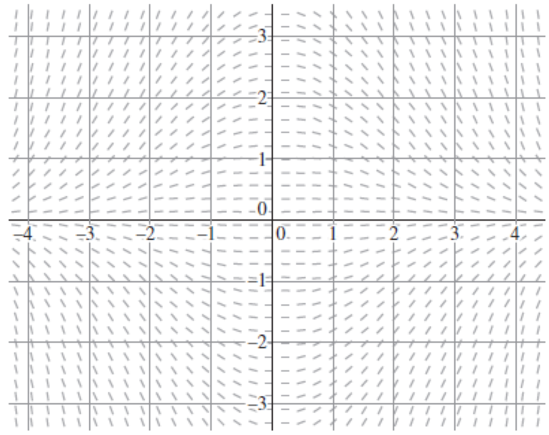
SP MX 1 **5** The slope field for a first order differential equation is shown. Which of the following could be the differential equation represented? **1** [Solution]

A. $\dfrac{dy}{dx} = \dfrac{x}{3y}$

B. $\dfrac{dy}{dx} = -\dfrac{x}{3y}$

C. $\dfrac{dy}{dx} = \dfrac{xy}{3}$

D. $\dfrac{dy}{dx} = -\dfrac{xy}{3}$

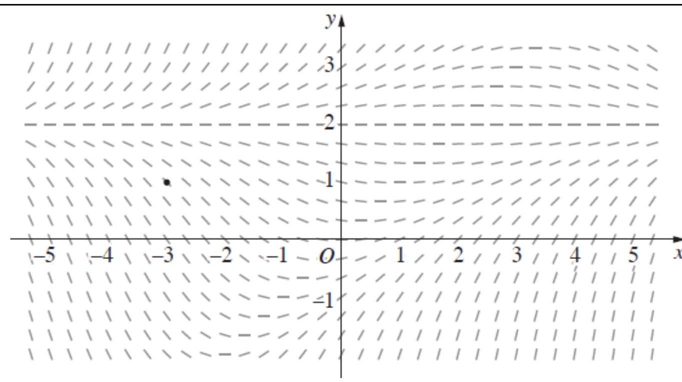


NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **11 c** The trajectories of particles in a fluid are described by the differential equation [Solution]

$$\frac{dy}{dx} = \frac{1}{4}(y - 2)(y - x).$$

The slope field for the differential equation is sketched.



(i) Identify any solutions of the form $y = k$, where k is a constant. **1**

(ii) Draw a sketch of the trajectory of a particle in the fluid which passes through the point $(-3, 1)$ and describe the trajectory as $x \to \pm\infty$. **3**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **14 b** The population of foxes on an island is modelled by the logistic equation [Solution]

$\dfrac{dy}{dt} = y(1 - y)$, where y is the fraction of the island's carrying capacity reached after t years.

At time $t = 0$, the population of foxes is estimated to be one-quarter of the island's carrying capacity.

(i) Use the substitution $y = \dfrac{1}{1 - w}$ to transform the logistic equation to $\dfrac{dw}{dt} = -w$. **2**

(ii) Using the solution of $\dfrac{dw}{dt} = -w$, find the solution of the logistic equation for y satisfying the initial conditions. **2**

(iii) How long will it take for the fox population to reach three-quarters of the island's carrying capacity? **2**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

TG 1 If a product which has just been launched is judged by the market to be of poor quality, sales will decline as people try the product but do not continue to buy it. [Solution]

For a certain product, the rate of weekly sales is modelled by

$S'(t) = \dfrac{400}{(t+1)^3} - \dfrac{200}{(t+1)^2}$, where S is the number of sales in millions and t is the number of weeks since the launch of the product.

(a) Find the function that describes the weekly sales.

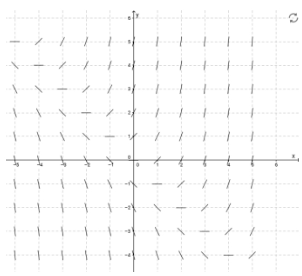
(b) Find the number of sales for the first week and for the tenth week.

(c) Comment on your results in the context of the given information.

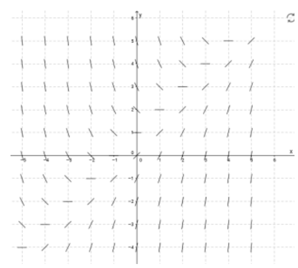
NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 2 Which of the following direction fields best represents the differential equation $\dfrac{dy}{dx} = x - y$? [Solution]

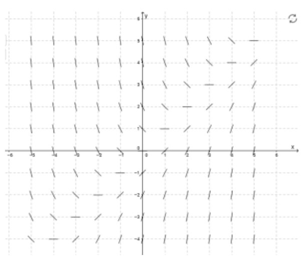
(A)



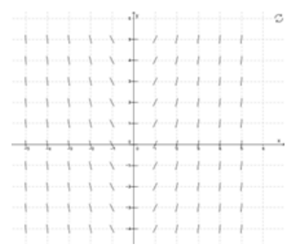
(B)



(C)



(D)



NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 3 Gardeners are concerned about the spread of a particular pest. All specimens detected so far lie within a circular region with radius 25 km and it is suggested that the increase of the radius r km might be modelled by a differential equation [Solution]

$\dfrac{dr}{dt} = \dfrac{1}{6}\sqrt{r}$, where t denotes the time in months. What does this model predict for the radius of the region affected by the pest after t months?

DET produced NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG 4 Water is slowly leaking from a tank. The depth of the water after t hours is h metres and the variables are related by the equation $\dfrac{dh}{dt} = -ae^{-0.1t}$. Initially the depth of water is 6 metres and after 2 hours it has fallen to 5 metres. [Solution]

At what depth will the level eventually settle?

DET produced NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

TG **5** When a ball is dropped from the roof of a tall building the greatest speed that it can reach is u ms^{-1}. One model for its speed v ms^{-1} when it has fallen a distance x m is given by the differential equation $\frac{dv}{dx} = c\frac{u^2 - v^2}{v}$ where c is a positive constant. **Solution**

Find an expression for v in terms of x.

DET produced NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

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TG **6** The number of cane toads in a colony after t months can be modelled by **Solution**

$C(t) = \frac{40000}{1 + 3e^{-t}}$.

(a) Sketch the function $C(t) = \frac{40000}{1 + 3e^{-t}}$.

(b) What is the initial cane toad population?

(c) What is the population after 2 months, correct to 3 significant figures?

(d) When does the population reach 30 000?

(e) Show that $C'(t) = \frac{120000e^t}{(e^t + 3)^2}$.

(f) Find the maximum growth rate of the cane toad population.

DET produced NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

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TG **7** The population of a town is decreasing at a rate proportional to the square root of the population at that time. **Solution**

(a) Write a differential equation to describe the situation.

(b) If the population was initially 2500 and decreased to 2025 after 10 years, find the population after 20 years.

DET produced NESA Mathematics Extension 1 Year 12 Topic Guide: Calculus

projectmaths

Year 12: Statistical analysis
C3.2 Bernoulli and binomial distribution

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- use a Bernoulli random variable as a model for two-outcome situations (ACMMM143)
  - identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)
- use Bernoulli random variables and their associated probabilities to solve practical problems (ACMMM146) **AAM**
  - understand and apply the formulae for the mean, $E(X) = \bar{x} = p$, and variance, $\mathrm{Var}(X) = p(1-p)$, of the Bernoulli distribution with parameter p, and X defined as the number of successes (ACMMM145)
- understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial (ACMMM147)
  - calculate the expected frequencies of the various possible outcomes from a series of Bernoulli trials
- use binomial distributions and their associated probabilities to solve practical problems (ACMMM150) **AAM**
  - identify contexts suitable for modelling by binomial random variables (ACMMM148)
  - identify the binomial parameter p as the probability of success
  - understand and use the notation $X \sim \mathrm{Bin}(n, p)$ to indicate that the random variable X is distributed binomially with parameters n and p
  - apply the formulae for probabilities $P(X = r) = {}^nC_r p^r (1-p)^{n-r}$ associated with the binomial distribution with parameters n and p and understand the meaning of ${}^nC_r$ as the number of ways in which an outcome with r successes can occur
  - understand and apply the formulae for the mean, $E(X) = \bar{x} = np$, and the variance, $\mathrm{Var}(X) = np(1-p)$, of a binomial distribution with parameters n and p

Reference Sheet

23 MX 1	**12 c**	A gym has 9 pieces of equipment: 5 treadmills and 4 rowing machines. On average, each treadmill is used 65% of the time and each rowing machine is used 40% of the time. (i) Find an expression for the probability that, at a particular time, exactly 3 of the 5 treadmills are in use. (ii) Find an expression for the probability that, at a particular time, exactly 3 of the 5 treadmills are in use and no rowing machines are in use. NESA 2023 Mathematics Extension 1 HSC Examination	 **2** **1**	**Solution**

22 MX 1 **12 e**

A game consists of randomly selecting 4 balls from a bag. After each ball is selected it is replaced in the bag. The bag contains 3 red balls and 7 green balls. For each red ball selected, 10 points are earned and for each green ball selected, 5 points are deducted. For instance, if a player picks 3 red balls and 1 green ball, the score will be $3 \times 10 - 1 \times 5 = 25$ points.
What is the expected score in the game? **2** **Solution**

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **14 d**

You may use the information here to answer this question. **4** **Solution**

An airline company that has empty seats on a flight is not maximising its profit.
An airline company has found that there is a probability of 5% that a passenger books a flight but misses it. The management of the airline company decides to allow for overbooking, which means selling more tickets than the number of seats available on each flight.
To protect their reputation, management makes the decision that no more than 1% of their flights should have more passengers showing up for the flight than available seats.
Given management's decision and using a suitable approximation, find the maximum number of tickets that can be sold for a flight which has 350 seats.

NESA 2022 Mathematics Extension 1 HSC Examination

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21 MX 1 **6**

The random variable X represents the number of successes in 10 independent Bernoulli trials. The probability of success is $p = 0.9$ in each trial. **1** **Solution**

Let $r = P(X \geq 1)$.

Which of the following describes the value of r?

A. $r > 0.9$ B. $r = 0.9$ C. $0.1 < r < 0.9$ D. $r \leq 0.1$

NESA 2021 Mathematics Extension 1 HSC Examination

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20 MX 1 **12 b** **Solution**

When a particular biased coin is tossed, the probability of obtaining a head is $\frac{3}{5}$.

This coin is tossed 100 times.

Let X be the random variable representing the number of heads obtained. This random variable will have a binomial distribution.

(i) Find the expected value, $E(X)$. **1**

(ii) By finding the variance, $\text{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

(iii) By using a normal distribution, find the approximate probability that X is between 55 and 65. **1**

NESA 2020 Mathematics Extension 1 HSC Examination

MX SQ 2019 **1**

An experiment consisted of tossing a biased coin five times and recording the number of heads obtained. **1** **Solution**
This experiment was repeated 250 times and the results are shown in the table.
Based on these results, what is the probability that the coin shows heads when tossed?

Number of heads	*Frequency*
0	10
1	44
2	81
3	75
4	34
5	6

A. 0.4 B. 0.47
C. 0.49 D. 0.5

Projectmaths: Slight change to NESA question

NESA Mathematics Extension 1 Sample examination materials (2019)

MX SQ 2019 **3** When a standard 6-sided die is thrown, the probability that it shows a multiple of 3 is $\frac{1}{3}$. If 12 standard dice are thrown, the number X, of times a multiple of 3 is showing has a binomial distribution. **1** Solution

What is the standard deviation of X, correct to 4 decimal places?

A. 0.2222 B. 0.4714 C. 1.6330 D. 2.6667

NESA Mathematics Extension 1 Sample examination materials (2019)

TG **1** (a) In a Bernoulli trial, $0 < p < 1$. Explain why p cannot take the values $p = 0$ or $p = 1$. Solution

(b) How many distinct values can X take in n Bernoulli trials?

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **2** A biased coin is tossed where $P(\text{Head}) = \frac{1}{3}$. If you obtain Heads you win a prize of \$1. Solution

Could this situation be modelled as a Bernoulli random variable? Explain how.

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **3** Four cards are marked A, B, C and D. A gambler bets that on randomly drawing one of the four cards, he will get a card D. Define this as a Bernoulli random variable, and calculate the mean and variance of the variable. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **4** A six-sided dice is tossed 21 times and the number of times neither a three nor a four occurs is recorded. If this is to be seen as a binomial experiment, what do we mean by a trial, a success and a sequence of trials? Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **5** Students are completing a quiz consisting of seven multiple-choice questions, with a pass mark of four. Each question has five possible answers. Unfortunately, one of the students did not study for the quiz and is randomly guessing the answer to each question. How many questions should this student expect to answer correctly, assuming that each question has only one correct answer? Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **6** It is known that 2% of the bolts produced by a machine are faulty. Solution

What is the probability that in a random sample of four bolts:

(a) no bolts are defective

(b) precisely one bolt is defective

(c) at most, two bolts are defective?

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **7** On average, batters in a certain cricket team make a scoring shot on every third ball. Estimate how many six-ball overs with precisely two scoring shots would occur in 1000 overs of batting by the team. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

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TG **8** A manufacturer makes earbuds that have a probability of 0·02 of being defective. Quality control officers test random samples of 50 earbuds each hour and reject the earbuds made in that hour if at least three earbuds are defective. Find the probability that the earbuds made in any hour will be rejected. Answer to two significant figures. Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

19 MX 1 **11 f**

Prizewinning symbols are printed on 5% of ice-cream sticks. The ice creams are randomly packed into boxes of 8.

(i) What is the probability that a box contains no prize-winning symbols? **1**

(ii) What is the probability that a box contains at least 2 prize-winning symbols? **2**

NESA 2019 Mathematics Extension 1 HSC Examination

Solution

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18 MX 1 **12 d**

A group of 12 people sets off on a trek. The probability that a person finishes the trek within 8 hours is 0.75. Find an expression for the probability that at least 10 people from the group complete the trek within 8 hours. **2**

NESA 2018 Mathematics Extension 1 HSC Examination

Solution

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17 MX 1 **11 g**

The probability that a particular type of seedling produces red flowers is $\frac{1}{5}$.

Eight of these seedlings are planted.

(i) Write an expression for the probability that exactly three of the eight seedlings produce red flowers. **1**

(ii) Write an expression for the probability that none of the eight seedlings produces red flowers. **1**

(iii) Write an expression for the probability that at least one of the eight seedlings produces red flowers. **1**

NESA 2017 Mathematics Extension 1 HSC Examination

Solution

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16 MX 1 **11 f**

A darts player calculates that when she aims for the bullseye the probability of her hitting the bullseye is $\frac{3}{5}$ with each throw.

(i) Find the probability that she hits the bullseye with exactly one of her first three throws. **1**

(ii) Find the probability that she hits the bullseye with at least two of her first six throws. **2**

NESA 2016 Mathematics Extension 1 HSC Examination

Solution

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14 MX 1 **11 b**

The probability that it rains on any particular day during the 30 days of November is 0.1. Write an expression for the probability that it rains on fewer than 3 days in November. **2**

NESA 2014 Mathematics Extension 1 HSC Examination

Solution

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13 MX 1 **11 c**

An examination has 10 multiple-choice questions, each with 4 options. In each question, only one option is correct. For each question a student choose one option at random. Write an expression for the probability that the student chooses the correct option for exactly 7 questions. **2**

NESA 2013 Mathematics Extension 1 HSC Examination

Solution

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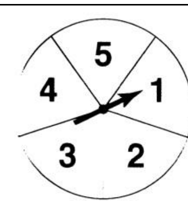
12 MX 1 **12 c**

Kim and Mel play a simple game using a spinner marked with the numbers 1, 2, 3, 4 and 5. The game consists of each player spinning the spinner once. Each of the five numbers is equally likely to occur. The player who obtains the higher number wins the game. If both players obtain the same number, the result is a draw.



(i) Kim and Mel play one game. What is the probability that Kim wins the game? **1**

(ii) Kim and Mel play six games. What is the probability that Kim wins exactly three games? **2**

NESA 2012 Mathematics Extension 1 HSC Examination

Solution

11 MX 1 **6c**

A game is played by throwing darts at a target. A player can choose to throw two or three darts. Darcy plays two games. In Game 1, he chooses to throw two darts, and wins if he hits the target at least once. In Game 2, he chooses to throw three darts, and wins if he hits the target at least twice. The probability that Darcy hits the target on any throw is p, where $0 < p < 1$.

Solution

(i) Show that the probability that Darcy wins Game 1 is $2p - p^2$. **1**

(ii) Show that the probability that Darcy wins Game 2 is $3p^2 - 2p^3$. **1**

(iii) Prove that Darcy is more likely to win Game 1 than Game 2. **2**

(iv) Find the value of p for which Darcy is twice as likely to win Game 1 as he is to win Game 2. **2**

NESA 2011 Mathematics Extension 1 HSC Examination

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10 MX 1 **1f**

Five ordinary six-sided dice are thrown. What is the probability that exactly two of the dice land showing a four? Leave your answer in unsimplified form. **1**

Solution

NESA 2010 Mathematics Extension 1 HSC Examination

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09 MX 1 **4a**

A test consists of five multiple-choice questions. Each question has four alternative answers. For each question only one of the alternative answers is correct. Huong randomly selects an answer to each of the five questions.

Solution

(i) What is the probability that Huong selects three correct and two incorrect answers? **2**

(ii) What is the probability that Huong selects three or more correct answers? **2**

(iii) What is the probability that Huong selects at least one incorrect answer? **1**

NESA 2009 Mathematics Extension 1 HSC Examination

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07 MX 1 **4a**

In a large city, 10% of the population has green eyes.

Solution

(i) What is the probability that two randomly chosen people both have green eyes? **1**

(ii) What is the probability that exactly two of a group of 20 randomly chosen people have green eyes? Give your answer correct to three decimal places. **1**

(iii) What is the probability that more than two of a group of 20 randomly chosen people have green eyes? Give your answer correct to two decimal places. **2**

NESA 2007 Mathematics Extension 1 HSC Examination

projectmaths

Year 12: Statistical analysis
C3.2 Normal approximation for the sample proportion

Back

Syllabus: updated November 2019. Latest version @
https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-extension-1-2017

Students:

- use appropriate graphs to explore the behaviour of the sample proportion on collected or supplied data **AAM**
  - understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples (ACMMM174)
- explore the behaviour of the sample proportion using simulated data **AAM**
  - examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)
- understand and use the normal approximation to the distribution of the sample proportion and its limitations **AAM**

Reference Sheet

23 MX 1 **2** A standard six-sided die is rolled 12 times. **1** **Solution**

Let $\hat{p}$ be the proportion of the rolls with an outcome of 2.

Which of the following expressions is the probability that at least 9 of the rolls have an outcome of 2?

A. $P(\hat{p} \geq \frac{3}{4})$ B. $P(\hat{p} \geq \frac{1}{6})$ C. $P(\hat{p} \leq \frac{3}{4})$ D. $P(\hat{p} \leq \frac{1}{6})$

NESA 2023 Mathematics Extension 1 HSC Examination

23 MX 1 **11 f** A recent census found that 30% of Australians were born overseas. **Solution**

A sample of 900 randomly selected Australians was surveyed.

Let $\hat{p}$ be the sample proportion of surveyed people who were born overseas.

A normal distribution is to be used to approximate $P(\hat{p} \leq 0.31)$.

(i) Show that the variance of the random variable $\hat{p}$ is $\frac{7}{30\ 000}$. **2**

(ii) Use the standard normal distribution and the information on page 16 (click here) to approximate $P(\hat{p} \leq 0.31)$, giving your answer correct to two decimal places. **2**

NESA 2023 Mathematics Extension 1 HSC Examination

22 MX 1 **13 e**

You may use the information here to answer this question.

A chocolate factory sells 150-gram chocolate bars. There has been a complaint that the bars actually weigh less than 150 grams, so a team of inspectors was sent to the factory to check. They randomly selected 16 bars, weighed them and noted that 8 bars weighed less than 150 grams.

The factory manager claims 80% of the chocolate bars produced by the factory weigh 150 grams or more.

(i) The inspectors used the normal approximation to the binomial distribution to calculate the probability, $\mathcal{P}$, of having at least 8 bars weighing less than 150 grams in a random sample of 16, assuming the factory manager's claim is correct. Calculate the value of $\mathcal{P}$. **2**

(ii) The factory manager disagrees with the method used by the inspectors as described in part (i). **1**
Explain why the method used by the inspectors might not be valid.

NESA 2022 Mathematics Extension 1 HSC Examination

Solution

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21 MX 1 **14 d**

At a certain factory, the proportion of faulty items produced by a machine is $p = \frac{3}{500}$, which is considered to be acceptable. **3**

To confirm that the machine is working to this standard, a sample of size n is taken and the sample proportion $\hat{p}$ is calculated.

It is assumed that $\hat{p}$ is approximately normally distributed with $\mu = p$ and $\sigma^2 = \frac{p(1-p)}{n}$.

Production by this machine will be shut down if $\hat{p} \geq \frac{4}{500}$.

The sample size is to be chosen so that the chance of shutting down the machine unnecessarily is less than 2.5%.

Find the approximate sample size required, giving your answer to the nearest thousand.

NESA 2021 Mathematics Extension 1 HSC Examination

Solution

SP MX 1 **12 a**

A recent census showed that 20% of the adults in a city eat out regularly.

(i) A survey of 100 adults in this city is to be conducted to find the proportion who eat out regularly. **2**
Show that the mean and standard deviation for the distribution of sample proportions of such surveys are 0.2 and 0.04 respectivley.

(ii) Use the extract shown from a table giving values of $P(Z < z)$, where z has a standard normal distribution, to estimate the probability that a survey of 100 adults will find that at most 15 of those surveyed eat out regularly. **2**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

Solution

TG 1 State whether the following binomial distributions can or cannot be reasonably approximated by a normal distribution. **Solution**

Write a brief calculation to justify your conclusion in each case:

(a) Bin(50, 0.2)

(b) Bin(60, 0.1)

(c) Bin(70, 0.01)

(d) Bin(30, 0.7)

(e) Bin(40, 0.9)

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 2 A manufacturer of jam jars knows that 8% of the jars produced are defective. He supplies jars in cartons containing 12 jars. He supplies cartons of jars in crates of 60 cartons. In each case, making clear the distribution that you are using, calculate the probability that: **Solution**

(a) a carton contains exactly two defective jars.
(b) a carton contains at least one defective jar.
(c) a crate contains between 39 and 44 (inclusive) cartons with at least one defective jar.

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
…	…	…	…	…	…	…	…	…	…	…
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 3 A fair coin is tossed 18 times. Use the binomial distribution to find the probability of obtaining 14 Heads. Then use the normal distribution to find the probability of obtaining 14 Heads, and to find the probability of obtaining 14 or more Heads. **Solution**

Show that the approximation is valid. Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 4 Suppose that 45% of all HSC students exercise at least four days each week. If a random sample of 50 students is taken, what is the probability that at least 80% of them exercise at least four days per week? **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 5 It is known that 24% of HSC students do not have a driver licence. In a random sample of 16* HSC students, what is the probability that half of them will not have a driver licence? * NESA has 15 … but cannot use 'half of 15' … projectmaths **Solution**

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 6 A computer simulation is designed to draw random samples of size n from a large dataset. The proportion of the population that exhibits a certain characteristic is $p = 0.25$.

If $\hat{p}$ represents the sample proportion exhibiting the characteristic under investigation, find the largest* sample size that should be used so that the standard deviation of $\hat{p}$ is at least 0.01. * NESA has 'smallest' … projectmaths

Solution

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 7 Find the probability of obtaining 4, 5, 6 or 7 Heads when a fair coin is tossed 12 times

(a) using the binomial theorem.

(b) using a normal approximation to the binomial distribution.

Solution

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
…	…	…	…	…	…	…	…	…	…	…
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 8 It is given that 40% of voters support the Stats Party. One hundred and fifty voters are selected at random. Use a suitable approximation to find the probability that more than 55 of the 150 voters support the Stats Party. Projectmaths has provided this prob table extract:

Solution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

TG 9 It is estimated that approximately 45% of Australian people will experience a mental illness in their lifetime. If a random sample of 120 mature adults were surveyed, what is the probability of 50 or more having experienced a mental illness?

Solution

Projectmaths has provided this probability table extract:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389

NESA Mathematics Extension 1 Year 12 Topic Guide: Statistical Analysis

NSW GOVERNMENT

NSW Education Standards Authority

2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

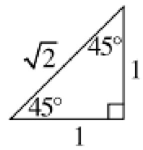
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

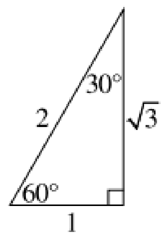
$A = \dfrac{1}{2}ab\sin C$



$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$



$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

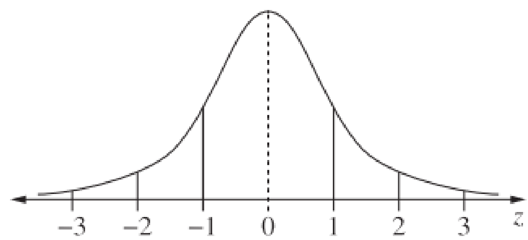
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n - r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x} p^x (1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

2023 HSC Paper

Back

23 MX 1 **1** The temperature $T(t)$°C of an object at time t seconds is modelled using Newton's Law of Cooling, **1** Solution

$$T(t) = 15 + 4e^{-3t}$$

What is the initial temperature of the object?

A. -3 B. 4 C. 15 D. 19

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23 MX 1 **2** A standard six-sided die is rolled 12 times. **1** Solution

Let $\hat{p}$ be the proportion of the rolls with an outcome of 2.

Which of the following expressions is the probability that at least 9 of the rolls have an outcome of 2?

A. $P(\hat{p} \geq \frac{3}{4})$ B. $P(\hat{p} \geq \frac{1}{6})$ C. $P(\hat{p} \leq \frac{3}{4})$ D. $P(\hat{p} \leq \frac{1}{6})$

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23 MX 1 **3** The diagram shows the direction field of a differential equation. A particular solution to the differential equation passes through $(-2, 1)$. **1** Solution

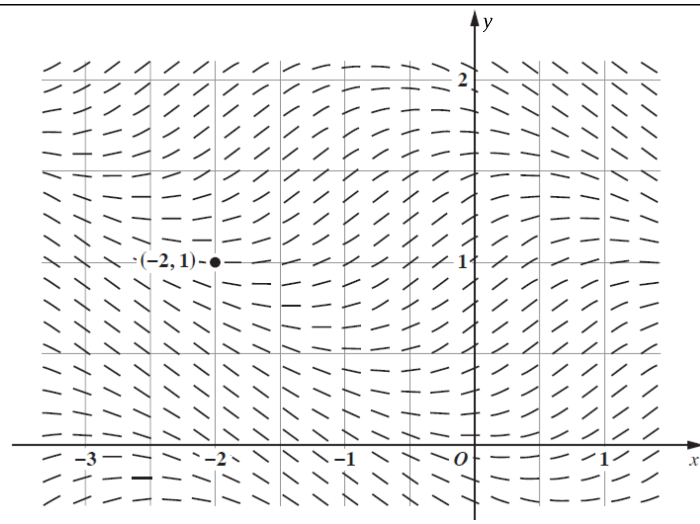
Where does the solution that passes through $(-2, 1)$ cross the y-axis?

A. $y = 1.12$

B. $y = 1.34$

C. $y = 1.56$

D. $y = 1.78$



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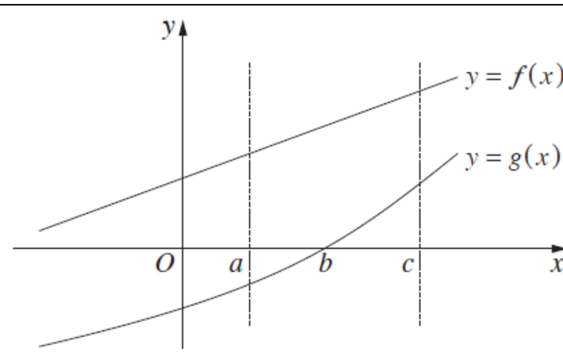
23 MX 1 **4** The diagram shows the graphs of the functions $f(x)$ and $g(x)$. **1** Solution

It is known that

$$\int_a^c f(x)\,dx = 10$$

$$\int_a^b g(x)\,dx = -2$$

$$\int_b^c g(x)\,dx = 3$$



What is the area between the curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = c$?

A. 5 B. 7 C. 9 D. 11

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23 MX 1 **5**

Which of the following is the value of $\sin^{-1}(\sin \alpha)$ given that $\pi < \alpha < \dfrac{3\pi}{2}$? **1** **Solution**

A. $\alpha - \pi$ B. $\pi - \alpha$ C. α D. $-\alpha$

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23 MX 1 **6**

Given the two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, let $\underset{\sim}{c}$ be the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **1** **Solution**

What is the projection of $10\underset{\sim}{a}$ onto $2\underset{\sim}{b}$?

A. $2\underset{\sim}{c}$ B. $5\underset{\sim}{c}$ C. $10\underset{\sim}{c}$ D. $20\underset{\sim}{c}$

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23 MX 1 **7**

Which statement is always true for real numbers a and b where $-1 \le a < b \le 1$? **1** **Solution**

A. $\sec a < \sec b$ B. $\sin^{-1} a < \sin^{-1} b$

C. $\arccos a < \arccos b$ D. $\cos^{-1} a + \sin^{-1} a < \cos^{-1} b + \sin^{-1} b$

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23 MX 1 **8**

The diagram shows the graph of a function. **1** **Solution**

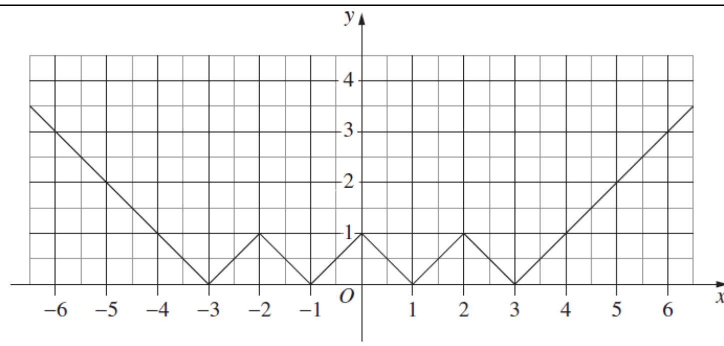
Which of the following is the equation of the function?

A. $y = |1 - ||x| - 2||$

B. $y = |2 - ||x| - 1||$

C. $y = |1 - |x - 2||$

D. $y = |2 - |x - 1||$



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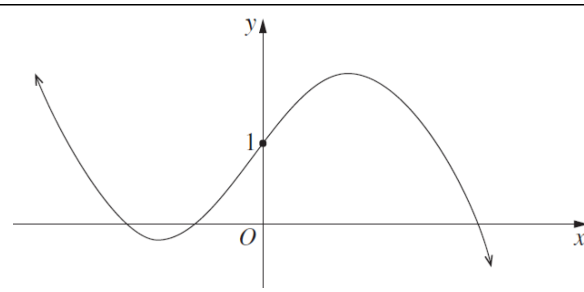
23 MX 1 **9**

The graph of a cubic function, $y = f(x)$, is given below. **1** **Solution**



Which of the following functions has an inverse relation whose graph has more than 3 points with an x-coordinate of 1?

A. $y = \sqrt{f(x)}$ B. $y = \dfrac{1}{f(x)}$ C. $y = f(|x|)$ D. $y = |f(x)|$

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23 MX 1 **10**

A group with 5 students and 3 teachers is to be arranged in a circle. **1** **Solution**

In how many ways can this be done if no more than 2 students can sit together?

A. $4! \times 3!$ B. $5! \times 3!$ C. $2! \times 5! \times 3!$ D. $2! \times 2! \times 2! \times 3!$

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23 MX 1 **11 a** The parametric equations of a line are given below. **2** **Solution**

$$x = 1 + 3t$$
$$y = 4t$$

Find the Cartesian equation of this line in the form $y = mx + c$.

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23 MX 1 **11 b** In how many different ways can all the letters of the word CONDOBOLIN be arranged in a line? **2** **Solution**

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23 MX 1 **11c** Consider the polynomial **3** **Solution**

$$P(x) = x^3 + ax^2 + bx - 12,$$

where a and b are real numbers.

It is given that $x + 1$ is a factor of $P(x)$ and that, when $P(x)$ is divided by $x - 2$, the remainder is -18.

Find a and b.

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23 MX 1 **11 d** Find $\int \frac{1}{\sqrt{4 - 9x^2}}\, dx$. **2** **Solution**

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23 MX 1 **11 e** Solve $\cos\theta + \sin\theta = 1$ for $0 \le \theta \le 2\pi$. **3** **Solution**

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23 MX 1 **11 f** A recent census found that 30% of Australians were born overseas. **Solution**

A sample of 900 randomly selected Australians was surveyed.

Let $\hat{p}$ be the sample proportion of surveyed people who were born overseas.

A normal distribution is to be used to approximate $P(\hat{p} \le 0.31)$.

(i) Show that the variance of the random variable $\hat{p}$ is $\frac{7}{30\ 000}$. **2**

(ii) Use the standard normal distribution and the information on page 16 (click here) to approximate $P(\hat{p} \le 0.31)$, giving your answer correct to two decimal places. **2**

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23 MX 1 **12 a** Evaluate $\int_3^4 (x+2)\sqrt{x-3}\ dx$ using the substitution $u = x - 3$. **3** **Solution**

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23 MX 1 **12 b** **3** Solution

Use mathematical induction to prove that

$$(1 \times 2) + (2 \times 2^2) + (3 \times 2^3) + \ldots + (n \times 2^n) = 2 + (n - 1)2^{n+1}$$

for all integers $n \geq 1$.

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23 MX 1 **12 c** Solution

A gym has 9 pieces of equipment: 5 treadmills and 4 rowing machines.
On average, each treadmill is used 65% of the time and each rowing machine is used 40% of the time.

(i) Find an expression for the probability that, at a particular time, exactly 3 of the 5 treadmills are in use. **2**

(ii) Find an expression for the probability that, at a particular time, exactly 3 of the 5 treadmills are in use and no rowing machines are in use. **1**

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23 MX 1 **12 d** **2** Solution

It is known that ${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ for all integers such that $1 \leq r \leq n - 1$.

(Do NOT prove this.)

Find ONE possible set of values for p and q such that

$${}^{2022}C_{80} + {}^{2022}C_{81} + {}^{2023}C_{1943} = {}^{p}C_{q}$$

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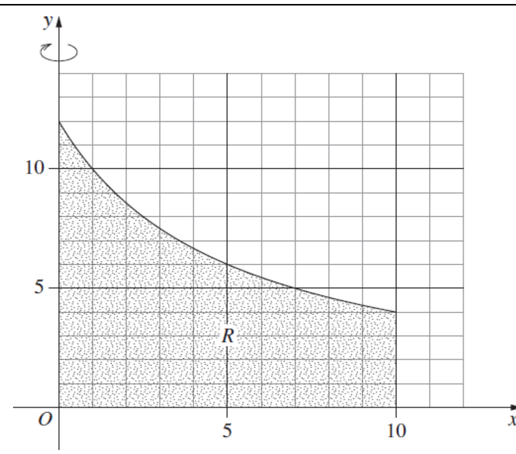
23 MX 1 **12 e** **4** Solution

The region, R, bounded by the hyperbola $y = \dfrac{60}{x+5}$, the line $x = 10$ and the coordinate axes is shown.

Find the volume of the solid of revolution formed when the region R is rotated about the y-axis. Leave your answer in exact form.



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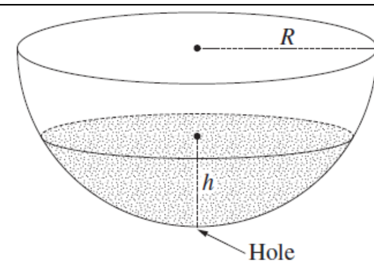
23 MX 1 **13 a**

Solution

A hemispherical water tank has radius R cm.
The tank has a hole at the bottom which allows water to drain out.
Initially the tank is empty. Water is poured into the tank at a constant rate of $2kR$ cm^3 s^{-1}, where k is a positive constant.
After t seconds, the height of the water in the tank is h cm, as shown in the diagram, and the volume of water in the tank is V cm^3.



It is known that $V = \pi\left(Rh^2 - \dfrac{h^3}{3}\right)$. (Do NOT prove this.)

While water flows into the tank and also drains out of the bottom, the rate of change of the volume of water in the tank is given by $\dfrac{dV}{dt} = k(2R - h)$.

(i) Show that $\dfrac{dh}{dt} = \dfrac{k}{\pi h}$. **2**

(ii) Show that the tank is full of water after $T = \dfrac{\pi R^2}{2k}$ seconds. **2**

(iii) The instant the tank is full, water stops flowing into the tank, but it continues to drain out of the hole at the bottom as before.
Show that the tank takes 3 times as long to empty as it did to fill. **3**

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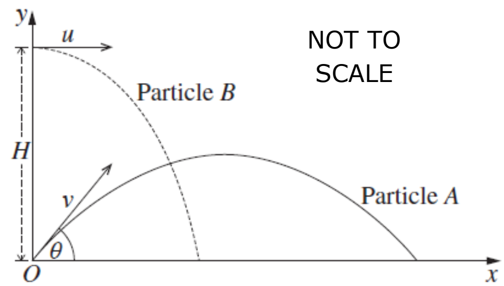
23 MX 1 **13 b** Particle A is projected from the origin with initial speed v m s^{-1} at an angle θ with the horizontal plane. At the same time, particle B is projected horizontally with initial speed u m s^{-1} from a point that is H metres above the origin, as shown in the diagram.

Solution



The position vector of particle A, t seconds after it is projected, is given by

$$\mathbf{r}_A(t) = \begin{pmatrix} vt\cos\theta \\ vt\sin\theta - \frac{1}{2}gt^2 \end{pmatrix} \qquad \text{(Do NOT prove this.)}$$

The position vector of particle B, t seconds after it is projected, is given by

$$\mathbf{r}_B(t) = \begin{pmatrix} ut \\ H - \frac{1}{2}gt^2 \end{pmatrix} \qquad \text{(Do NOT prove this.)}$$

The angle θ is chosen so that $\tan\theta = 2$.
The two particles collide.

(i) By first showing that $\cos\theta = \dfrac{1}{\sqrt{5}}$, verify that $v = \sqrt{5}\,u$. **2**

(ii) Show that the particles collide at time $T = \dfrac{H}{2u}$. **1**

When the particles collide, their velocity vectors are perpendicular.

(iii) Show that $H = \dfrac{2u^2}{g}$. **3**

(iv) Prior to the collision, the trajectory of particle A was a parabola. (Do NOT prove this.) **2**
Find the height of the vertex of that parabola above the horizontal plane.
Give your answer in terms of H.

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23 MX 1 **14 a** Let $f(x) = 2x + \ln x$, for $x > 0$.

Solution

(i) Explain why the inverse of $f(x)$ is a function. **1**

(ii) Let $g(x) = f^{-1}(x)$. By considering the value of $f(1)$, or otherwise, evaluate $g'(2)$. **2**

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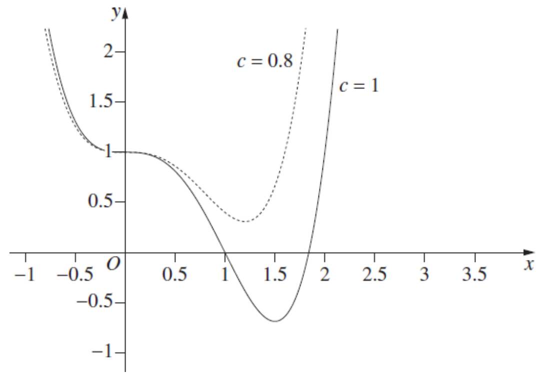
23 MX 1 **14 b** Consider the hyperbola $y = \dfrac{1}{x}$ and the circle $(x - c)^2 + y^2 = c^2$, where c is a constant. **Solution**

(i) Show that the x-coordinates of any points of intersection of the hyperbola and circle are zeros of the polynomial $P(x) = x^4 - 2cx^3 + 1$. **1**

(ii) The graphs of $y = x^4 - 2cx^3 + 1$ for $c = 0.8$ and $c = 1$ are shown. By considering the given graphs, or otherwise, find the exact value of $c > 0$ such that the hyperbola $y = \dfrac{1}{x}$ and the circle $(x - c)^2 + y^2 = c^2$ intersect at only one point. **3**



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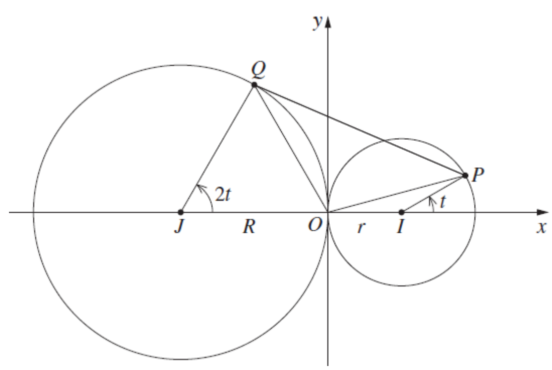
23 MX 1 **14 c** (i) Given a non-zero vector $\begin{pmatrix} p \\ q \end{pmatrix}$, it is known that the vector $\begin{pmatrix} q \\ -p \end{pmatrix}$ is perpendicular to $\begin{pmatrix} p \\ q \end{pmatrix}$ and has the same magnitude. (Do NOT prove this.) **3** **Solution**

Points A and B have position vectors $\overrightarrow{OA} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ respectively.

Using the given information, or otherwise, show that the area of triangle OAB is $\dfrac{1}{2}\left|a_1 b_2 - a_2 b_1\right|$.

(ii) The point P lies on the circle centred at $I(r, 0)$ with radius $r > 0$, such that $\overrightarrow{IP}$ makes an angle of t to the horizontal.
The point Q lies on the circle centred at $J(-R, 0)$ with radius $R > 0$, such that $\overrightarrow{JQ}$ makes an angle of $2t$ to the horizontal. **4**



Note that $\overrightarrow{OP} = \overrightarrow{OI} + \overrightarrow{IP}$ and $\overrightarrow{OQ} = \overrightarrow{OJ} + \overrightarrow{JQ}$.
Using part (i), or otherwise, find the values of t, where $-\pi \le t \le \pi$, that maximise the area of triangle OPQ.

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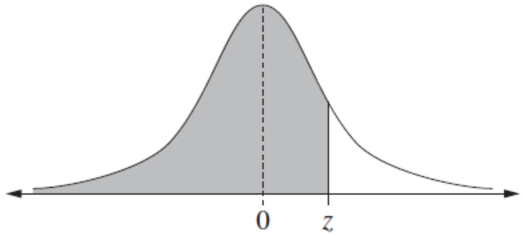
Back to question 11f

Table of values $P(Z \le z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

2022 HSC Paper

Back

22 MX 1 **1**

It is given that $\cos\left(\dfrac{23\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

Which of the following is the value of $\cos^{-1}\left(\dfrac{\sqrt{6}+\sqrt{2}}{4}\right)$?

A. $\dfrac{23\pi}{12}$ B. $\dfrac{11\pi}{12}$ C. $\dfrac{\pi}{12}$ D. $-\dfrac{11\pi}{12}$

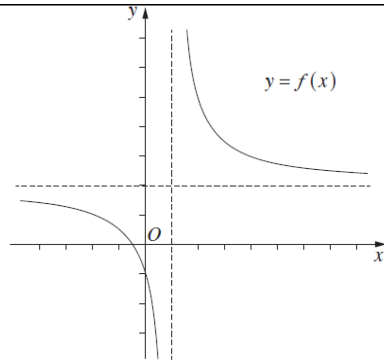
1 Solution

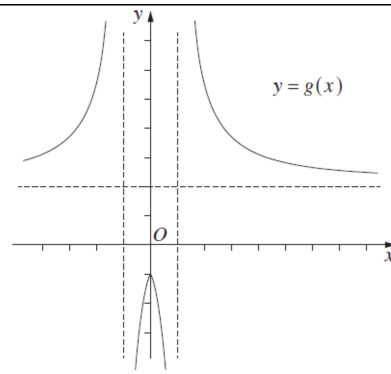
NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **2**

The graph of $f(x) = \dfrac{3}{x-1} + 2$ is shown.

The graph of $f(x)$ was transformed to get the graph of $g(x)$ as shown.





What transformation was applied?

A. $g(x) = f(|x|)$ B. $g(x) = \sqrt{f(x)}$ C. $g(x) = -f(x)$ D. $g(x) = \dfrac{1}{f(x)}$

1 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **3**

Let $P(x)$ be a polynomial of degree 5.

When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is $2x + 5$.

Which of the following is true about the degree of Q?

A. The degree must be 1. B. The degree could be 1.

C. The degree must be 2. D. The degree could be 2.

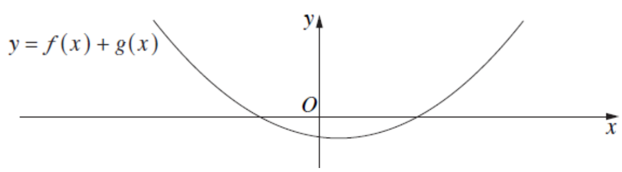
1 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

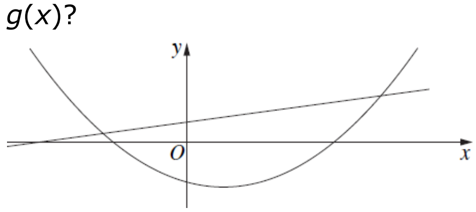
22 MX 1 **4**

The diagram shows the graph of the sum of the functions $f(x)$ and $g(x)$.

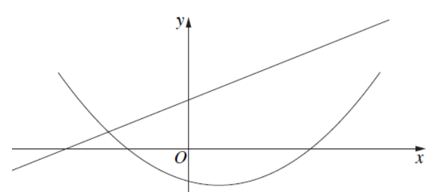
Which of the following best represents the graphs of both $f(x)$ and $g(x)$?



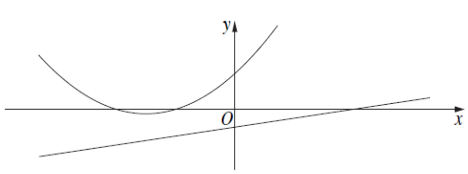
A.



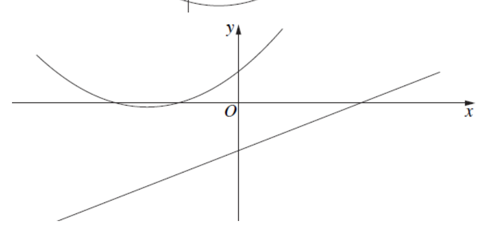
B.



C.



D.

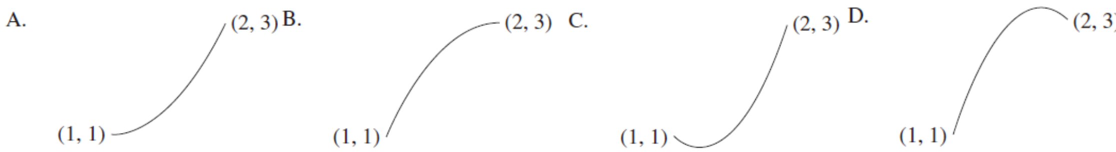


1 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 | **5** A curve is defined in parametric form by $x = 2 + t$ and $y = 3 - 2t^2$ for $-1 \le t \le 0$. **1** **Solution**

Which diagram best represents this curve?



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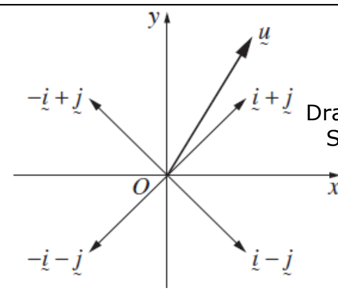
22 MX 1 | **6** The following diagram shows the vector $\underset{\sim}{u}$ and the vectors $\underset{\sim}{i} + \underset{\sim}{j}$, $-\underset{\sim}{i} + \underset{\sim}{j}$, $-\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{i} - \underset{\sim}{j}$. **1** **Solution**



Drawn to Scale

Which statement regarding this diagram could be true?

A. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $1.1\underset{\sim}{i} + 1.8\underset{\sim}{j}$.

B. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} + \underset{\sim}{j}$ is the vector $-0.4\underset{\sim}{i} + 0.4\underset{\sim}{j}$.

C. The projection of $\underset{\sim}{u}$ onto $-\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $3.2\underset{\sim}{i} + 3.2\underset{\sim}{j}$.

D. The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{i} - \underset{\sim}{j}$ is the vector $0.5\underset{\sim}{i} - 0.5\underset{\sim}{j}$.

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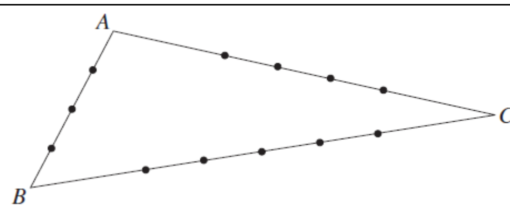
22 MX 1 | **7** The diagram shows triangle *ABC* with points chosen on each of the sides. **1** **Solution**

On side *AB*, 3 points are chosen.
On side *AC*, 4 points are chosen.
On side *BC*, 5 points are chosen.



How many triangles can be formed using the chosen points as vertices?

A. 60 B. 145 C. 205 D. 220

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22 MX 1 | **8** The angle between two unit vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is θ and $|\underset{\sim}{a} + \underset{\sim}{b}| < 1$. **1** **Solution**

Which of the following best describes the possible range of values of θ?

A. $0 \le \theta < \frac{\pi}{3}$ B. $0 \le \theta < \frac{2\pi}{3}$ C. $\frac{\pi}{3} < \theta \le \pi$ D. $\frac{2\pi}{3} < \theta \le \pi$

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22 MX 1 | **9** A given function $f(x)$ has an inverse $f^{-1}(x)$. **1** **Solution**

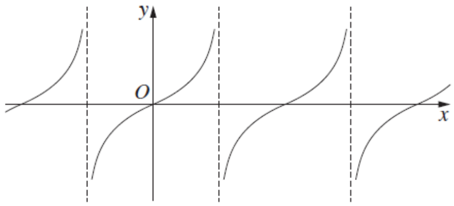
The derivatives of $f(x)$ and $f^{-1}(x)$ exist for all real numbers x.
The graphs $y = f(x)$ and $y = f^{-1}(x)$ have at least one point of intersection.
Which statement is true for all points of intersection of these graphs?

A. All points of intersection lie on the line $y = x$.
B. None of the points of intersection lie on the line $y = x$.
C. At no point of intersection are the tangents to the graphs parallel.
D. At no point of intersection are the tangents to the graphs perpendicular.

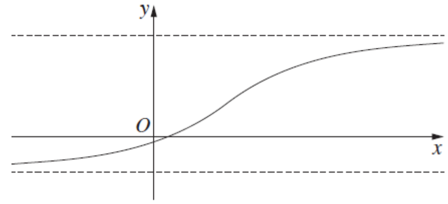
22 MX 1 **10** Which of the following could be the graph of a solution to the differential equation

$$\frac{dy}{dx} = \sin y + 1?$$

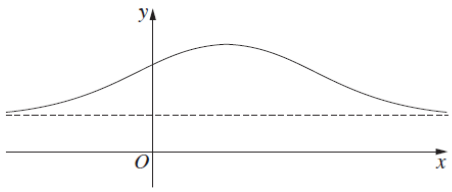
A.



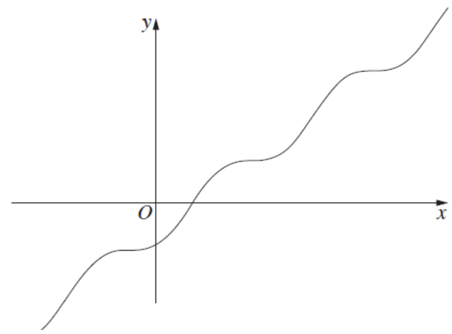
B.



C.



D.



NESA 2022 Mathematics Extension 1 HSC Examination — **1** — **Solution**

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22 MX 1 **11 a** For the vectors $\underset{\sim}{u} = \underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j}$, evaluate each of the following. **Solution**

(i) $\underset{\sim}{u} + 3\underset{\sim}{v}$ **1**

(ii) $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **11 b** Find the exact value of $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+4}}\,dx$ using the substitution $u = x^2 + 4$. **3** **Solution**

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **11 c** Find the coefficients of x^2 and x^3 in the expansion of $\left(1 - \dfrac{x}{2}\right)^8$. **2** **Solution**

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **11 d** The vectors $\underset{\sim}{u} = \begin{pmatrix} a \\ 2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} a-7 \\ 4a-1 \end{pmatrix}$ are perpendicular. **2** **Solution**

What are the possible values of a?

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **11 e**

Express $\sqrt{3}\sin(x) - 3\cos(x)$ in the form $R\sin(x + \alpha)$. **3**

Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **11 f**

Solve $\dfrac{x}{2 - x} \geq 5$. **3**

Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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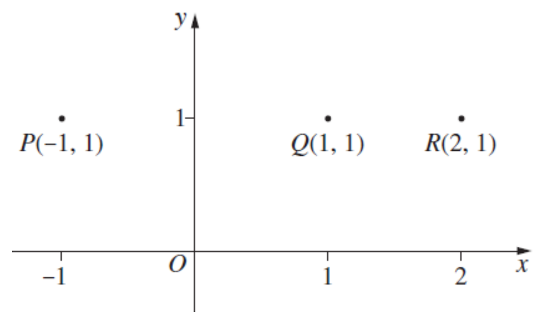
22 MX 1 **12 a**

A direction field is to be drawn for the differential equation

$$\frac{dy}{dx} = \frac{x - 2y}{x^2 + y^2}$$

Clearly draw the correct slopes of the direction field at the points P, Q and R shown below. **2**

Solution



NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **12 b**

A sports association manages 13 junior teams. It decides to check the age of all players. Any team that has more than 3 players above the age limit will be penalised. **2**

A total of 41 players are found to be above the age limit.

Will any team be penalised? Justify your answer.

Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **12 c**

Find the equation of the tangent to the curve $y = x\arctan(x)$ at the point with coordinates $(1, \frac{\pi}{4})$. Give your answer in the form $y = mx + c$. **3**

Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **12 d**

In a room with temperature 12°C, coffee is poured into a cup. The temperature of the coffee when it is poured into the cup is 92°C, and it is far too hot to drink.

The temperature, T, in degrees Celsius, of the coffee, t minutes after it is made, can be modelled using the differential equation $\dfrac{dT}{dt} = k(T - T_1)$, where k is the constant of proportionality and T_1 is a constant.

Solution

(i) It takes 5 minutes for the coffee to cool to a temperature of 76°C. **3**

Using separation of variables, solve the given differential equation to show that $T = 12 + 80e^{\frac{t}{5}\ln\left(\frac{4}{5}\right)}$

(ii) The optimal drinking temperature for a hot beverage is 57°C. **1**

Find the value of t when the coffee reaches this temperature, giving your answer to the nearest minute.

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22 MX 1 **12 e**

A game consists of randomly selecting 4 balls from a bag. After each ball is selected it is replaced in the bag. The bag contains 3 red balls and 7 green balls. For each red ball selected, 10 points are earned and for each green ball selected, 5 points are deducted. For instance, if a player picks 3 red balls and 1 green ball, the score will be $3 \times 10 - 1 \times 5 = 25$ points.
What is the expected score in the game?

2 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **12 f**

Use mathematical induction to prove that $15^n + 6^{2n+1}$ is divisible by 7 for all integers $n \geq 0$.

3 Solution

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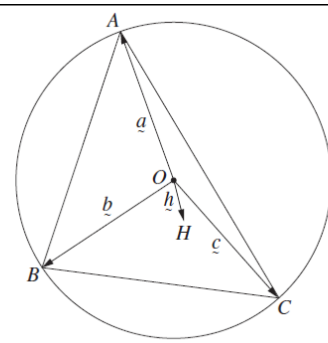
22 MX 1 **13 a**

Three different points A, B and C are chosen on a circle centred at O.

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$.

Let $\underset{\sim}{h} = \underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ and let H be the point such that $\overrightarrow{OH} = \underset{\sim}{h}$, as shown in the diagram.

Show that $\overrightarrow{BH}$ and $\overrightarrow{CA}$ are perpendicular.



Not to scale

3 Solution

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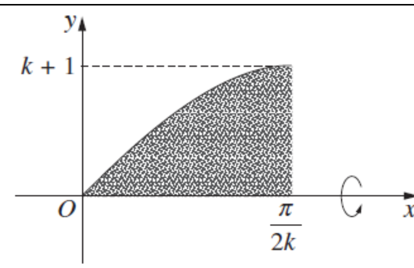
22 MX 1 **13 b**

A solid of revolution is to be found by rotating the region bounded by the x-axis and the curve $y = (k + 1)\sin(kx)$, where $k > 0$, between $x = 0$ and $x = \dfrac{\pi}{2k}$ about the x-axis.

Find the value of k for which the volume is π^2.



3 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

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22 MX 1 **13 c**

The function f is defined by $f(x) = \sin(x)$ for all real numbers x. Let g be the function defined on $[-1, 1]$ by $g(x) = \arcsin(x)$.
Is g the inverse of f? Justify your answer.

2 Solution

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22 MX 1 **13 d**

The monic polynomial, P, has degree 3 and roots α, β, γ.
It is given that

$$\alpha^2 + \beta^2 + \gamma^2 = 85 \text{ and}$$
$$P'(\alpha) + P'(\beta) + P'(\gamma) = 87.$$

Find $\alpha\beta + \beta\gamma + \gamma\alpha$.

3 Solution

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **13 e** Solution

You may use the information here to answer this question.

A chocolate factory sells 150-gram chocolate bars. There has been a complaint that the bars actually weigh less than 150 grams, so a team of inspectors was sent to the factory to check. They randomly selected 16 bars, weighed them and noted that 8 bars weighed less than 150 grams.

The factory manager claims 80% of the chocolate bars produced by the factory weigh 150 grams or more.

(i) The inspectors used the normal approximation to the binomial distribution to calculate the probability, $\mathcal{P}$, of having at least 8 bars weighing less than 150 grams in a random sample of 16, assuming the factory manager's claim is correct. Calculate the value of $\mathcal{P}$. **2**

(ii) The factory manager disagrees with the method used by the inspectors as described in part (i). **1**
Explain why the method used by the inspectors might not be valid.

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22 MX 1 **14 a** **4** Solution

Find the particular solution to the differential equation $(x - 2)\dfrac{dy}{dx} = xy$ that passes through the point $(0, 1)$.

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22 MX 1 **14 b** **3** Solution

The vectors $\vec{u}$ and $\vec{v}$ are not parallel. The vector $\vec{p}$ is the projection of $\vec{u}$ onto the vector $\vec{v}$.

The vector $\vec{p}$ is parallel to $\vec{v}$ so it can be written $\lambda_0\vec{v}$ for some real number λ_0. (Do NOT prove this.)

Prove that $\left|\vec{u} - \lambda\vec{v}\right|$ is smallest when $\lambda = \lambda_0$ by showing that, for all real numbers λ, $\left|\vec{u} - \lambda_0\vec{v}\right| \le \left|\vec{u} - \lambda\vec{v}\right|$.

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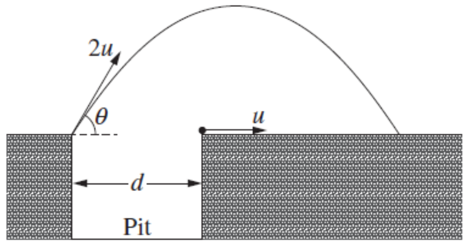
22 MX 1 **14 c** **4** **Solution**

A video game designer wants to include an obstacle in the game they are developing. The player will reach one side of a pit and must shoot a projectile to hit a target on the other side of the pit in order to be able to cross. However, the instant the player shoots, the target begins to move away from the player at a constant speed that is half the initial speed of the projectile shot by the player, as shown in the diagram below.

The initial distance between the player and the target is *d*, the initial speed of the projectile is 2*u* and it is launched at an angle of *q* to the horizontal. The acceleration due to gravity is *g*. The launch angle is the ONLY parameter that the player can change.



Taking the position of the player when the projectile is launched as the origin, the positions of the projectile and target at time *t* after the projectile is launched are as follows.

$$\vec{r}_P = \begin{pmatrix} 2ut\cos\theta \\ 2ut\sin\theta - \frac{g}{2}t^2 \end{pmatrix} \quad \text{Projectile}$$

$$\vec{r}_T = \begin{pmatrix} d+ut \\ 0 \end{pmatrix} \quad \text{Target}$$

Do NOT prove these

Show that, for the player to have a chance of hitting the target, *d* must be less than 37% of the maximum possible range of the projectile (to 2 significant figures)

NESA 2022 Mathematics Extension 1 HSC Examination

22 MX 1 **14 d** **4** **Solution**

You may use the information here to answer this question.

An airline company that has empty seats on a flight is not maximising its profit.

An airline company has found that there is a probability of 5% that a passenger books a flight but misses it. The management of the airline company decides to allow for overbooking, which means selling more tickets than the number of seats available on each flight.

To protect their reputation, management makes the decision that no more than 1% of their flights should have more passengers showing up for the flight than available seats.

Given management's decision and using a suitable approximation, find the maximum number of tickets that can be sold for a flight which has 350 seats.

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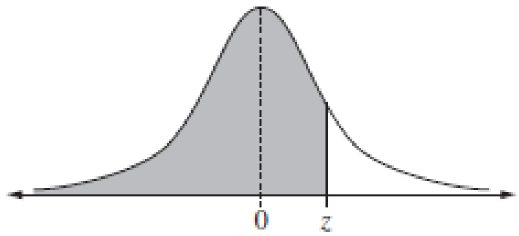
You may use the information below to answer Question 13 (e) and Question 14 (d).

Table of values $P(Z \le z)$ for the normal distribution $N(0, 1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

Back to Question 13e

Back to Question 14d

Back to Question 13e

Back to Question 14d

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Back

21 MX 1 **1**

Given $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, what is $\overrightarrow{PQ}$? **1**

Solution

A. $\begin{pmatrix} 1 \\ -6 \end{pmatrix}$ B. $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ C. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ D. $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

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21 MX 1 **2**

Solution

Which of the following integrals is equivalent to $\int \sin^2 3x \, dx$? **1**

A. $\int \frac{1+\cos 6x}{2} \, dx$ B. $\int \frac{1-\cos 6x}{2} \, dx$ C. $\int \frac{1+\sin 6x}{2} \, dx$ D. $\int \frac{1-\sin 6x}{2} \, dx$

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21 MX 1 **3**

What is the remainder when $P(x) = -x^3 - 2x^2 - 3x + 8$ is divided by $x + 2$? **1**

Solution

A. -14 B. -2 C. 2 D. 14

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21 MX 1 **4**

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$. **1**

Solution

Which of the following equations best represents this relationship between x and y?

A. $y^2 = x^2 + c$ B. $y^2 = \frac{x^2}{2} + c$

C. $y = x \ln|y| + c$ D. $y = \frac{x^2}{2} \ln|y| + c$

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21 MX 1 **5**

For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is known that $\overrightarrow{OA} \cdot \overrightarrow{OB} < 0$. **1**

Solution

Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or, $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $|\overrightarrow{OA}||\overrightarrow{OB}|$ is negative.

D. The points O, A and B are collinear.

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21 MX 1 **6**

The random variable X represents the number of successes in 10 independent Bernoulli trials. The probability of success is $p = 0.9$ in each trial. **1**

Solution

Let $r = P(X \geq 1)$.

Which of the following describes the value of r?

A. $r > 0.9$ B. $r = 0.9$ C. $0.1 < r < 0.9$ D. $r \leq 0.1$

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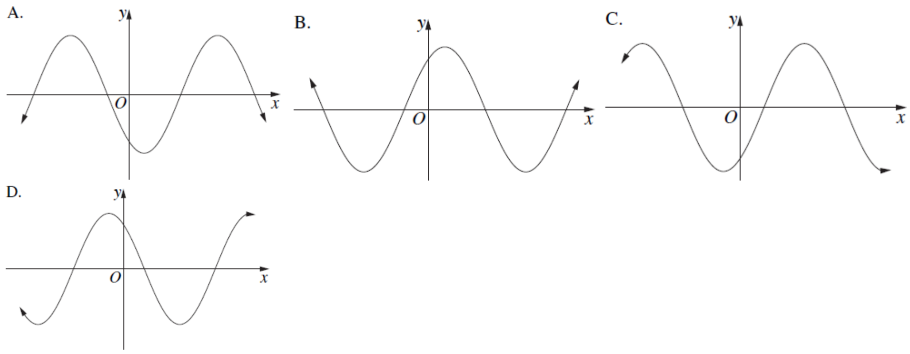
21 MX 1 **7** Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive? **1** Solution



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21 MX 1 **8** The diagram shows a semicircle. **1** Solution

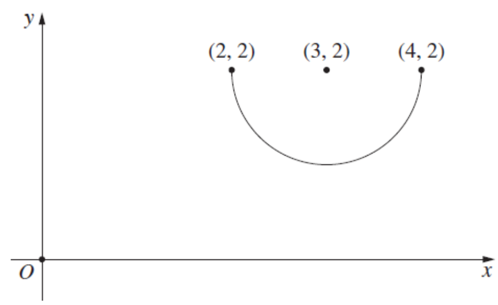
Which pair of parametric equations represents the semicircle shown?

A. $\begin{cases} x = 3 + \sin t \\ y = 2 + \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

B. $\begin{cases} x = 3 + \cos t \\ y = 2 + \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

C. $\begin{cases} x = 3 - \sin t \\ y = 2 - \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

D. $\begin{cases} x = 3 - \cos t \\ y = 2 - \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



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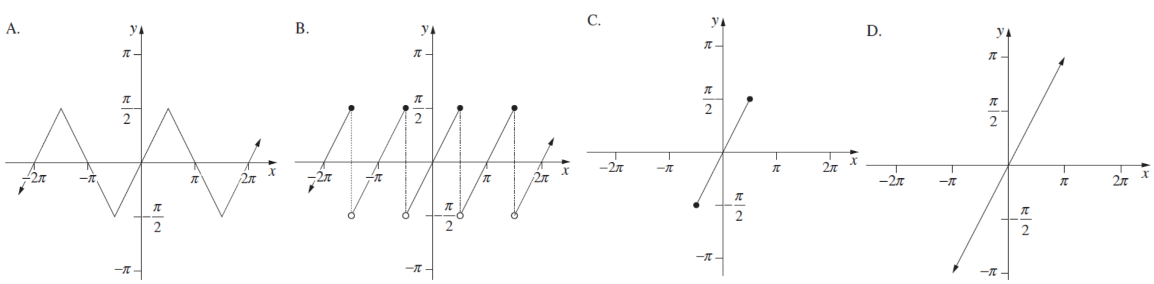
21 MX 1 **9** Which graph represents the function $y = \sin^{-1}(\sin x)$? **1** Solution



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21 MX 1 **10** The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only. **1** Solution

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A 236 B 237 C 238 D 239

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21 MX 1 **11 a**

Find $(\underset{\sim}{i} + 6\underset{\sim}{j}) + (2\underset{\sim}{i} - 7\underset{\sim}{j})$. **1** Solution

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21 MX 1 **11 b**

Expand and simplify $(2a - b)^4$. **2** Solution

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21 MX 1 **11 c**

Use the substitution $u = x + 1$ to find $\int x\sqrt{x+1}\,dx$. **3** Solution

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21 MX 1 **11 d**

A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women. In how many different ways can the committee be formed? **1** Solution

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21 MX 1 **11 e**

A spherical bubble is moving up through a liquid. **2** Solution

As it rises, the bubble gets bigger and its radius increases at the rate of 0.2 mm/s.

At what rate is the volume of the bubble increasing when its radius reaches 0.6 mm?

Express your answer in mm^3/s rounded to one decimal place.

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21 MX 1 **11f**

Evaluate $\displaystyle\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\,dx$. **2** Solution

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21 MX 1 **11 g**

By factorizing, or otherwise, solve $2\sin^3 x + 2\sin^2 x - \sin x - 1 = 0$ for $0 \le x \le 2\pi$. **3** Solution

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21 MX 1 **11 h**

The roots of $x^4 - 3x + 6 = 0$ are α, β, γ and δ. **2** Solution

What is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta}$?

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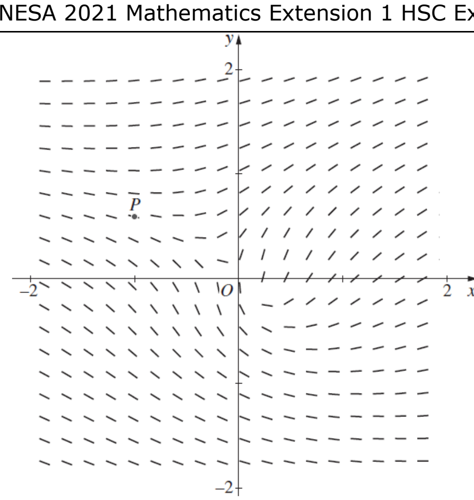
21 MX 1 **12 a**

The direction field for a differential equation is given on page 1 of the Question 12 Writing Booklet. **1** Solution

The graph of a particular solution to the differential equation passes through the point P.

Sketch the graph of the particular solution that passes through the point P.



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21 MX 1 **12 b**

A bottle of water, with temperature 5°C, is placed on a table in a room. The temperature of the room remains constant at 25°C. After t minutes, the temperature of the water, in degrees Celsius, is T. **Solution**

The temperature of the water can be modelled using the differential equation

$$\frac{dT}{dt} = k(T - 25) \text{ (Do NOT prove this.)}$$

where k is the growth constant.

(i) After 8 minutes, the temperature of the water is 10°C. **3**

By solving the differential equation, find the value of t when the temperature of the water reaches 20°C. Give your answer to the nearest minute.

(ii) Sketch the graph of T as a function of t. **1**

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21 MX 1 **12 c**

Use the principle of mathematical induction to prove **3** **Solution**

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

that for all integers $n \geq 1$.

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21 MX 1 **12 d**

A function is defined by $f(x) = 4 - \left(1 - \frac{x}{2}\right)^2$ for x in the domain $(-\infty, 2]$. **Solution**

(i) Sketch the graph of $y = f(x)$ showing the x- and y-intercepts. **2**

(ii) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(iii) Sketch the graph of $y = f^{-1}(x)$. **1**

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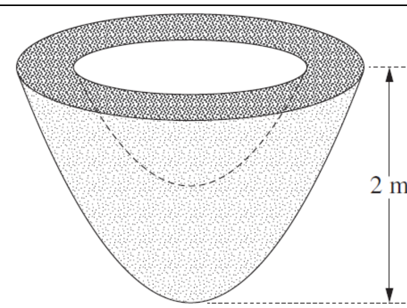
21 MX 1 **13 a**

A 2-metre-high sculpture is to be made out of concrete. **3** **Solution**

The sculpture is formed by rotating the region between $y = x^2$, $y = x^2 + 1$ and $y = 2$ around the y-axis.

Find the volume of concrete needed to make the sculpture.



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21 MX 1 **13 b**

When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is **4** **Solution**

$$\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + (-5t^2 + Vt\sin\theta + h)\,\underset{\sim}{j}. \text{ (Do NOT prove this.)}$$

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial velocity of the ball is 12 m/s at an angle of 30° to the horizontal.

Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

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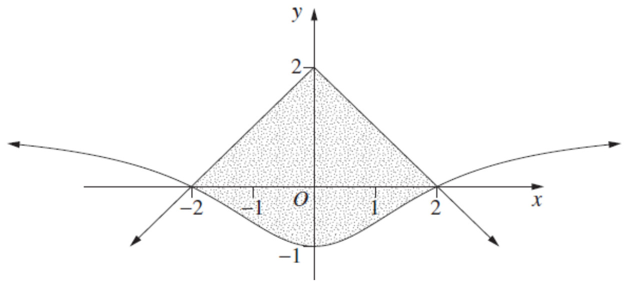
21 MX 1 **13 c** **3** Solution

The region enclosed by

$y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is

shaded in the diagram.

Find the exact value of the area of the shaded region.



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21 MX 1 **13 d** Solution

(i) The numbers A, B and C are related by the equations $A = B - d$ and $C = B + d$, where d is a constant. **2**

Show that $\dfrac{\sin A + \sin C}{\cos A + \cos C} = \tan B$.

(ii) Hence, or otherwise, solve $\dfrac{\sin\dfrac{5\theta}{7} + \sin\dfrac{6\theta}{7}}{\cos\dfrac{5\theta}{7} + \cos\dfrac{6\theta}{7}} = \sqrt{3}$, for $0 \le \theta \le 2\pi$. **2**

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21 MX 1 **14 a** **3** Solution

A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h.

On what constant bearing, to the nearest degree, should the direction of the plane be set in order to reach the destination?

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21 MX 1 **14 b** **4** Solution

In a certain country, the population of deer was estimated in 1980 to be 150 000.

The population growth is given by the logistic equation $\dfrac{dP}{dt} = 0.1P\left(\dfrac{C - P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity.

In the year 2000, the population of deer was estimated to be 600 000.

Use the fact that $\dfrac{C}{P(C-P)} = \dfrac{1}{P} + \dfrac{1}{C-P}$ to show that the carrying capacity is approximately 1 130 000.

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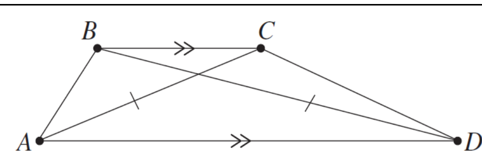
21 MX 1 **14 c** Solution

(i) For vector $\underset{\sim}{v}$, show that $\underset{\sim}{v} \cdot \underset{\sim}{v} = \left|\underset{\sim}{v}\right|^2$. **1**



NOT TO SCALE

(ii) In the trapezium $ABCD$, BC is parallel to AD and $|\overrightarrow{AC}| = |\overrightarrow{BD}|$. **3**

Let $\underset{\sim}{a} = \overrightarrow{AB}$, $\underset{\sim}{b} = \overrightarrow{BC}$ and $\overrightarrow{AD} = k\overrightarrow{BC}$, where $k > 0$.

Using part (i) or otherwise, show that $2\underset{\sim}{a} \cdot \underset{\sim}{b} + (1 - k)\left|\underset{\sim}{b}\right|^2 = 0$.

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21 MX 1 **14 d** At a certain factory, the proportion of faulty items produced by a machine is **3** Solution

$p = \dfrac{3}{500}$, which is considered to be acceptable.

To confirm that the machine is working to this standard, a sample of size n is taken and the sample proportion $\hat{p}$ is calculated.

It is assumed that $\hat{p}$ is approximately normally distributed with $\mu = p$ and $\sigma^2 = \dfrac{p(1-p)}{n}$.

Production by this machine will be shut down if $\hat{p} \geq \dfrac{4}{500}$.

The sample size is to be chosen so that the chance of shutting down the machine unnecessarily is less than 2.5%.

Find the approximate sample size required, giving your answer to the nearest thousand.

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21 MX 1 **14 e** The polynomial $g(x) = x^3 + 4x - 2$ passes through the point $(1, 3)$. **2** Solution

Find the gradient of the tangent to $f(x) = xg^{-1}(x)$ at the point where $x = 3$.

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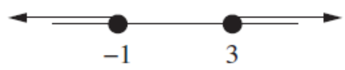
20 MX 1 1 Which diagram best represents the solution set of $x^2 - 2x - 3 \geq 0$? **1** **Solution**

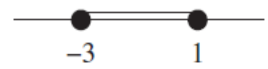
A.



B.

C.

D.



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20 MX 1 2 Given $f(x) = 1 + \sqrt{x}$, what are the domain and range of $f^{-1}(x)$? **1** **Solution**

A. $x \geq 0,\ y \geq 0$ B. $x \geq 0,\ y \geq 1$ C. $x \geq 1,\ y \geq 0$ D. $x \geq 1,\ y \geq 1$

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20 MX 1 3 Which of the following is an anti-derivative of $\dfrac{1}{4x^2+1}$? **1** **Solution**

A. $2\tan^{-1}\left(\dfrac{x}{2}\right) + c$ B. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{x}{2}\right) + c$ C. $2\tan^{-1}(2x) + c$ D. $\dfrac{1}{2}\tan^{-1}(2x) + c$

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20 MX 1 4 Maria starts at the origin and walks along all of the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$, then walks along all of the vector $3\underset{\sim}{i} - 2\underset{\sim}{j}$ and finally along all of the vector $4\underset{\sim}{i} - 3\underset{\sim}{j}$. **1** **Solution**

How far from the origin is she?

A. $\sqrt{77}$ B. $\sqrt{85}$ C. $2\sqrt{13} + \sqrt{5}$ D. $\sqrt{5} + \sqrt{7} + \sqrt{13}$

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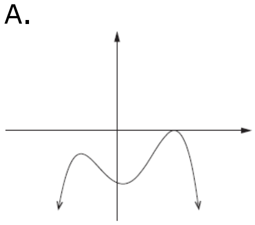
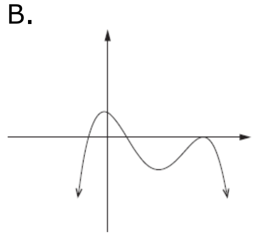
20 MX 1 5 A monic polynomial $p(x)$ of degree 4 has one repeated zero of multiplicity 2 and is divisible by $x^2 + x + 1$. **1** **Solution**

Which of the following could be the graph of $p(x)$?

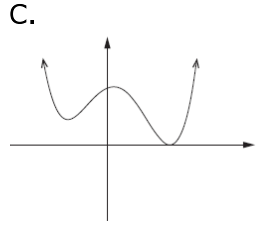
A.



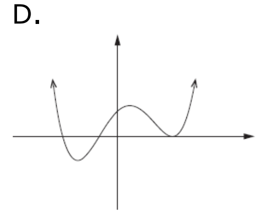
B.



C.



D.



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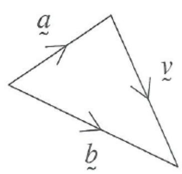
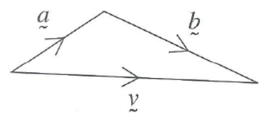
20 MX 1 **6** The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are shown. **1** [Solution]

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{a} - \underset{\sim}{b}$?

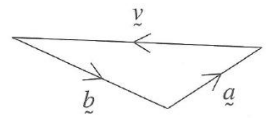
A.



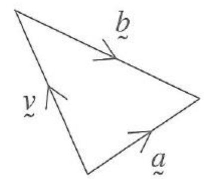
B.



C.



D.



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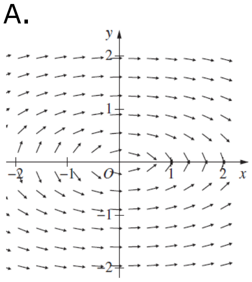
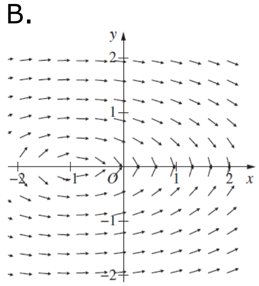
20 MX 1 **7** Which of the following best represents the direction field for the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{4y}$? **1** [Solution]

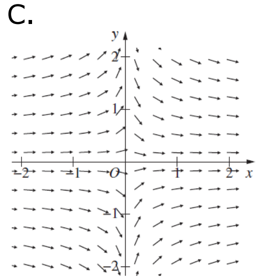
A.



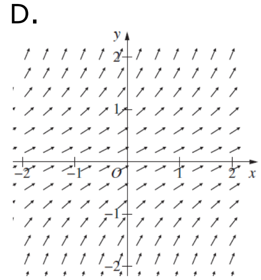
B.



C.



D.



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20 MX 1 **8** Out of 10 contestants, six are to be selected for the final round of a competition. Four of those six will be placed 1^{st}, 2^{nd}, 3^{rd} and 4^{th}. **1** [Solution]

In how many ways can this process be carried out?

A $\dfrac{10!}{6!4!}$ B $\dfrac{10!}{6!}$ C $\dfrac{10!}{4!2!}$ D $\dfrac{10!}{4!4!}$

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20 MX 1 **9** The projection of the vector $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ onto the line $y = 2x$ is $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$. **1** [Solution]

The point (6, 7) is reflected in the line $y = 2x$ to a point A.

What is the position vector of the point A?

A $\begin{pmatrix} 6 \\ 12 \end{pmatrix}$ B $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$ C $\begin{pmatrix} -6 \\ 7 \end{pmatrix}$ D $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

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20 MX 1 **10** The quantities P, Q and R are connected by the related rates, **1** Solution

$$\frac{dR}{dt} = -k^2$$

$$\frac{dP}{dt} = -l^2 \times \frac{dR}{dt}$$

$$\frac{dP}{dt} = m^2 \times \frac{dQ}{dt}$$

where k, l and m are non-zero constants.

Which of the following statements is true?

A P is increasing and Q is increasing

B P is increasing and Q is decreasing

C P is decreasing and Q is increasing

D P is decreasing and Q is decreasing

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20 MX 1 **11 a** Let $P(x) = x^3 + 3x^2 - 13x + 6$ Solution

(i) Show that $P(2) = 0$. **1**

(ii) Hence, factor the polynomial $P(x)$ as $A(x)\,B(x)$, where $B(x)$ is a quadratic polynomial. **2**

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20 MX 1 **11 b** For what value(s) of a are the vectors $\begin{pmatrix} a \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? **3** Solution

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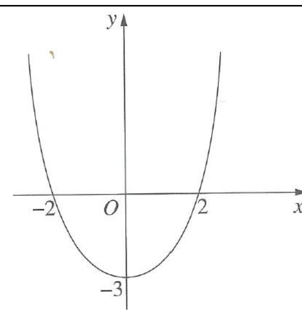
20 MX 1 **11 c** The diagram shows the graph of $y = f(x)$. **3** Solution



Sketch the graph of $y = \dfrac{1}{f(x)}$.

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20 MX 1 **11 d** By expressing $\sqrt{3}\sin x + 3\cos x$ in the form $A\sin(x + \alpha)$, **4** Solution

solve $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$, for $0 \le x \le 2\pi$.

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20 MX 1 **11 e** Solve $\dfrac{dy}{dx} = e^{2y}$, finding x as a function of y. **2** Solution

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20 MX 1 **12 a** Use the principle of mathematical induction to show that for all integers $n \ge 1$, **3** Solution

$1 \times 2 + 2 \times 5 + 3 \times 8 + \ldots + n(3n - 1) = n^2(n + 1)$.

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20 MX 1 **12 b** Solution

When a particular biased coin is tossed, the probability of obtaining a head is $\frac{3}{5}$.

This coin is tossed 100 times.

Let X be the random variable representing the number of heads obtained. This random variable will have a binomial distribution.

(i) Find the expected value, $E(X)$. **1**

(ii) By finding the variance, $\text{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

(iii) By using a normal distribution, find the approximate probability that X is between 55 and 65. **1**

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20 MX 1 **12 c** Solution

To complete a course, a student must choose and pass exactly three topics. **2**

There are eight topics from which to choose.

Last year 400 students completed the course.

Explain, using the pigeonhole principle, why at least eight students passed exactly the same three topics.

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20 MX 1 **12 d** Solution

Find $\displaystyle\int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \; dx$. **3**

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **12 e** Solution

Find the curve which satisfies the differential equation $\dfrac{dy}{dx} = -\dfrac{x}{y}$ and passes through the point $(1, 0)$. **3**

NESA 2020 Mathematics Extension 1 HSC Examination

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20 MX 1 **13 a** Solution

(i) Find $\dfrac{d}{dx}\left(\sin^3\theta\right)$. **1**

(ii) Use the substitution $x = \tan\theta$ to evaluate $\displaystyle\int_0^1 \frac{x^2}{\left(1+x^2\right)^{\frac{5}{2}}}\,dx$. **4**

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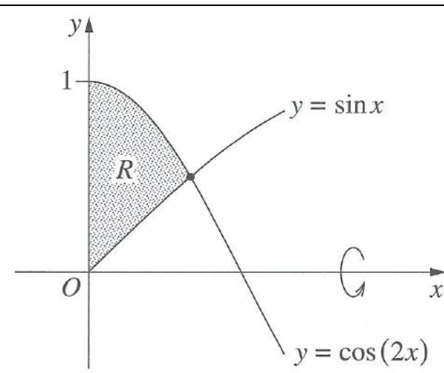
20 MX 1 **13 b** Solution

The region R is bounded by the y-axis, the graph of $y = \cos(2x)$ and the graph of $y = \sin x$, as shown in the diagram. **4**

Find the volume of the solid of revolution formed when the region R is rotated about the x-axis.



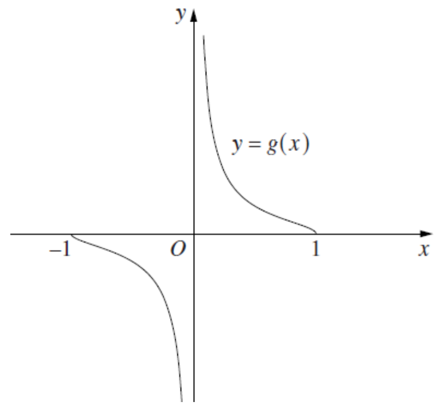
NESA 2020 Mathematics Extension 1 HSC Examination

20 MX 1 **13 c**

Solution

Suppose $f(x) = \tan(\cos^{-1}(x))$ and $g(x) = \dfrac{\sqrt{1-x^2}}{x}$.

The graph of $y = g(x)$ is given.



(i) Show that $f'(x) = g'(x)$. **4**

(ii) Using part (i), or otherwise, show that $f(x) = g(x)$. **3**

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20 MX 1 **14 a**

Solution

(i) Use the identity $(1 + x)^{2n} = (1 + x)^n(1 + x)^n$ to show that **2**

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2$$ where n is a positive integer.

(ii) A club has $2n$ members, with n women and n men. **2**

A group consisting of an even number $(0, 2, 4, \ldots, 2n)$ of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

(iii) From the group chosen in part (ii), one of the men and one of the women are selected as leaders. **2**

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is $1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \ldots + n^2\binom{n}{n}^2$.

(iv) The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men. **2**

By considering this reversed process, and using part (ii), find a simple expression for the sum in part (iii).

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20 MX 1 **14 b**

Solution

(i) Show that $\sin^3\theta - \dfrac{3}{4}\sin\theta + \dfrac{\sin(3\theta)}{4} = 0$ **2**

(ii) By letting $x = 4\sin\theta$ in the cubic equation $x^3 - 12x + 8 = 0$, **2**

show that $\sin 3\theta = \dfrac{1}{2}$.

(iii) Prove that $\sin^2\dfrac{\pi}{18} + \sin^2\dfrac{5\pi}{18} + \sin^2\dfrac{25\pi}{18} = \dfrac{3}{2}$. **3**

NESA 2020 Mathematics Extension 1 HSC Examination

2020 NESA Sample Paper

Back

SP MX 1 **1** What is the angle between the vectors $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$? **1** Solution

A. $\cos^{-1}(0.6)$ B. $\cos^{-1}(0.06)$ C. $\cos^{-1}(-0.06)$ D. $\cos^{-1}(-0.6)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **2** The diagram shows a grid of equally spaced lines. **1** Solution

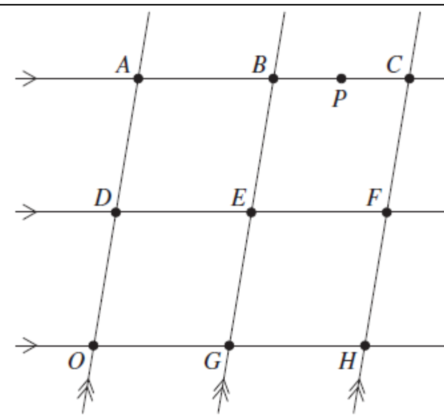
The vector $\overrightarrow{OH} = \underset{\sim}{h}$ and the vector $\overrightarrow{OA} = \underset{\sim}{a}$.

The point P is halfway between B and C.

Which expression represents the vector $\overrightarrow{OP}$?

A. $-\frac{1}{2}\underset{\sim}{a} - \frac{1}{4}\underset{\sim}{h}$ B. $\frac{1}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{h}$

C. $\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{h}$ D. $\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{h}$



NESA Mathematics Extension 1 Sample HSC Examination (2020)

SP MX 1 **3** Given that $\cos\theta - 2\sin\theta + 2 = 0$, which of the following shows the two possible values for $\tan\frac{\theta}{2}$? **1** Solution

A. -3 or -1 B. -3 or 1 C. -1 or 3 D. 1 or 3

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **4** What is the derivative of $\tan^{-1}\frac{x}{2}$? **1** Solution

19 MX 1 **3**

A. $\dfrac{1}{2(4+x^2)}$ B. $\dfrac{1}{4+x^2}$ C. $\dfrac{2}{4+x^2}$ D. $\dfrac{4}{4+x^2}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

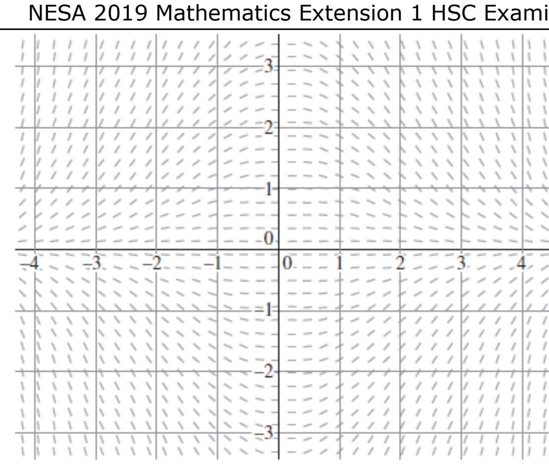
SP MX 1 **5** The slope field for a first order differential equation is shown. Which of the following could be the differential equation represented? **1** Solution

A. $\dfrac{dy}{dx} = \dfrac{x}{3y}$

B. $\dfrac{dy}{dx} = -\dfrac{x}{3y}$

C. $\dfrac{dy}{dx} = \dfrac{xy}{3}$

D. $\dfrac{dy}{dx} = -\dfrac{xy}{3}$



NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 6 / **19 MX 1** 7

Let $P(x) = qx^3 + rx^2 + rx + q$ where q and r are constants, $q \neq 0$. **1** Solution

One of the zeros of $P(x)$ is -1.

Given that α is a zero of $P(x)$, $\alpha \neq -1$, which of the following is also a zero?

A. $-\dfrac{1}{\alpha}$ B. $-\dfrac{q}{\alpha}$ C. $\dfrac{1}{\alpha}$ D. $\dfrac{q}{\alpha}$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2019 Mathematics Extension 1 HSC Examination

SP MX 1 7

Each of the students in an athletics team is randomly allocated their own locker from a row of 100 lockers. **1** Solution

What is the smallest number of students in the team that guarantees that two students are allocated consecutive lockers?

A. 26 B. 34 C. 50 D. 51

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 8

A team of 11 students is to be chosen from a group of 18 students. **1** Solution

Among the 18 students are 3 students who are left-handed.

What is the number of possible teams containing at least 1 student who is left-handed?

A. 19 448 B. 30 459 C. 31 824 D. 58 344

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)
NESA 2016 Mathematics Extension 1 HSC Examination

SP MX 1 9

A stone drops into a pond, creating a circular ripple. **1** Solution

The radius of the ripple increases from 0 cm at a constant rate of 5 cm s^{-1}.

At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?

A. 25π cm^2 s^{-1} B. 30π cm^2 s^{-1} C. 150π cm^2 s^{-1} D. 225π cm^2 s^{-1}

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 10

The graph of the function $y = \sin^{-1}(x - 4)$ is transformed by being dilated horizontally with a scale factor of 2 and then translated to the right by 1. **1** Solution

What is the equation of the transformed graph?

A. $y = \sin^{-1}\left(\dfrac{x-9}{2}\right)$ B. $y = \sin^{-1}\left(\dfrac{x-10}{2}\right)$

C. $y = \sin^{-1}(2x - 6)$ D. $y = \sin^{-1}(2x - 5)$

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 11 a

A particle is fired from the origin O with initial velocity 18 ms^{-1} at an angle 60° to the horizontal. Solution

The equations of motion are $\dfrac{d^2x}{dt^2} = 0$ and $\dfrac{d^2y}{dt^2} = -10$

(i) Show that $x = 9t$. **1**

(ii) Show that $y = 9\sqrt{3}\,t - 5t^2$. **2**

(ii) Hence find the Cartesian equation for the trajectory of the particle. **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 11 b

A function $f(x)$ is given by $x^2 + 4x + 7$. Solution

(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -2$. **2**

(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (b). **2**

(iv) Sketch the curve $y = f^{-1}(x)$. **2**

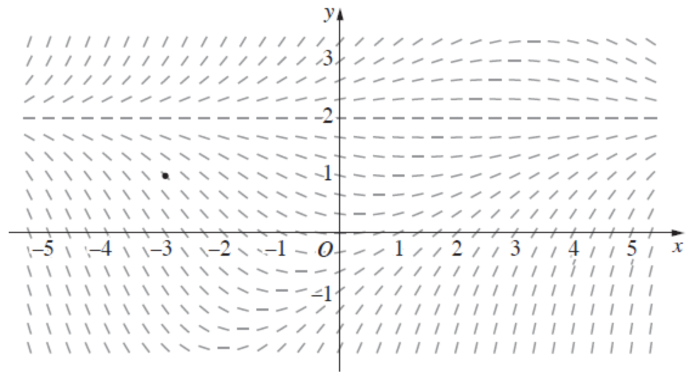
NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **11 c** Solution

The trajectories of particles in a fluid are described by the differential equation

$$\frac{dy}{dx} = \frac{1}{4}(y - 2)(y - x).$$

The slope field for the differential equation is sketched.



(i) Identify any solutions of the form $y = k$, where k is a constant. **1**

(ii) Draw a sketch of the trajectory of a particle in the fluid which passes through the point $(-3, 1)$ and describe the trajectory as $x \to \pm\infty$. **3**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **12 a** Solution

A recent census showed that 20% of the adults in a city eat out regularly.

(i) A survey of 100 adults in this city is to be conducted to find the proportion who eat out regularly. **2**
Show that the mean and standard deviation for the distribution of sample proportions of such surveys are 0.2 and 0.04 respectivley.

(ii) Use the extract shown from a table giving values of $P(Z < z)$, where z has a standard normal distribution, to estimate the probability that a survey of 100 adults will find that at most 15 of those surveyed eat out regularly. **2**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 b** Solution

A force described by the vector $\underset{\sim}{F} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ newtons is applied to an object lying on a line ℓ which is parallel to the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(i) Find the component of the force $\underset{\sim}{F}$ in the direction of the line ℓ. **2**

(ii) What is the component of the force $\underset{\sim}{F}$ in the direction perpendicular to the line? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **12 c** The points A and B are fixed points in a plane and have position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. The point P with position vector $\underset{\sim}{p}$ also lies in the plane and is chosen so that $\angle APB = 90^\circ$. **Solution**

(i) Explain why $(\underset{\sim}{a} - \underset{\sim}{p}) \cdot (\underset{\sim}{b} - \underset{\sim}{p}) = 0$ **1**

(ii) Let $\underset{\sim}{m} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$ denote the position vector of M, the midpoint of A and B. Using the properties of vectors, show that $|\underset{\sim}{p} - \underset{\sim}{m}|^2$ is independent of $\underset{\sim}{p}$ and find its value. **3**

(iii) What does the result in part (ii) prove about the point P? **1**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

S 12 **12d** Use mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for $n \geq 1$. **3** **Solution**

12 12a

NESA Mathematics Extension 1 Sample Examination Paper (2020)

NESA 2012 Mathematics Extension 1 HSC Examination

SP MX 1 **13 a** Use the substitution $x = \sin^2\theta$ to determine $\displaystyle\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\,dx$. **3** **Solution**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **13 b** A device playing a signal given by $x = \sqrt{2}\sin t + \cos t$ produces distortion whenever $|x| \geq 1.5$.
For what fraction of the time will the device produce distortion if the signal is played continuously? **4** **Solution**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **13 c** **Solution**

(i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **3**

(ii) Hence find expressions for the exact values of the solutions to the equation $8x^3 - 6x = 1$. **4**

NESA Mathematics Extension 1 Sample HSC Examination Paper (2020)

SP MX 1 **14 a**

(i) Sketch the graph of $y = x\cos x$ for $-\pi \le x \le \pi$ and hence explain why **3** Solution

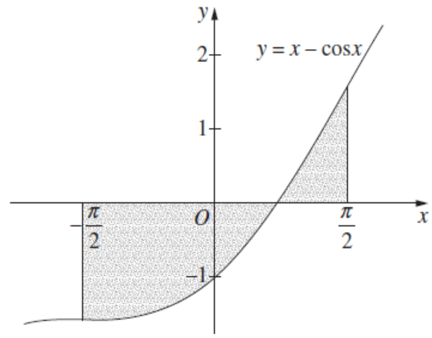
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx = 0.$$

(ii) Consider the volume of the solid of revolution produced by rotating about the x-axis the shaded region between the graph of **3**

$y = x - \cos x$, the x-axis and the lines

$x = -\dfrac{\pi}{2}$ and $x = -\dfrac{\pi}{2}$.

Using the results of part (a), or otherwise, find the volume of the solid.



NESA Mathematics Extension 1 Sample Examination Paper (2020)

SP MX 1 **14 b**

The population of foxes on an island is modelled by the logistic equation Solution

$\dfrac{dy}{dt} = y(1 - y)$, where y is the fraction of the island's carrying capacity reached

after t years.

At time $t = 0$, the population of foxes is estimated to be one-quarter of the island's carrying capacity.

(i) Use the substitution $y = \dfrac{1}{1-w}$ to transform the logistic equation to $\dfrac{dw}{dt} = -w$. **2**

(ii) Using the solution of $\dfrac{dw}{dt} = -w$, find the solution of the logistic equation for **2**

y satisfying the initial conditions.

(iii) How long will it take for the fox population to reach three-quarters of the island's carrying capacity? **2**

NESA Mathematics Extension 1 Sample Examination Paper (2020)

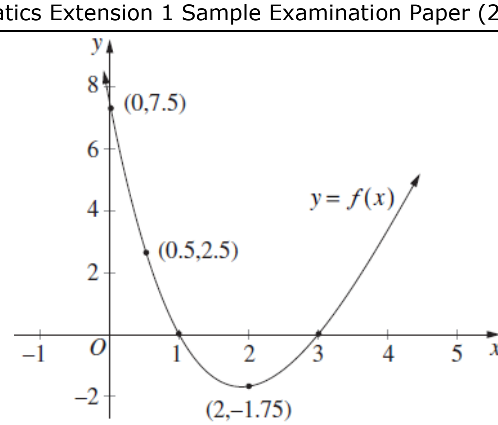
SP MX 1 **14 c**

The diagram is a sketch of the graph of the function $y = f(x)$. Solution

(i) Sketch the graph of $y = \dfrac{1}{|f(x)|}$. **3**

Your sketch should show any asymptotes and intercepts, together with the location of the points, corresponding to the labelled points on the original sketch.

(ii) How any solutions does the equation $\dfrac{1}{|f(x)|} = x$ have? **1**



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