Topic 1 - Probability and Venn Diagram

$P(A \cup B) = P(A) + P(B)$ ← mutually exclusive

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Non-mutually exclusive)

$P(A) = P(A \cap B) + P(A \cap \overline{B})$

Use a Lattice Diagram (table of all possible outcomes) for **Dice questions**

When independent,

$P(A \cap B) = P(A) \times P(B)$

$P(A|B) = P(A)$

When dependant,

$P(A \cap B) = P(A) \times P(B|A)$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

Topic 3 - Discrete probability distributions

$E(X)$ = sum of (each possible value × its probability)

$Var(X) = E(X^2) - [E(X)]^2$ or $= E(X^2) - \mu^2$

$\sigma = \sqrt{Var(x)}$

Topic 4 - Algebraic techniques

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^m \times x^n = x^{m+n}$, $x^m \div x^n = x^{m-n}$, $x^{\frac{n}{m}} = \sqrt[m]{x^n}$

Topic 6 - Linear, Quadratic and Cubic Functions

$y - y_1 = m(x - x_1)$ **-** point - gradient form

Perpendicular, $m_2 = -\frac{1}{m_1}$

Real, Rational root, $\Delta = 0$ or a perfect square

Axis of symmetry: $\frac{-b}{2a}$

Always **positive,** positive definite

Parallelogram if **ONE** of the following is true:

- Both pairs of opposite angles are equal
- Both pairs of opposite sides are equal
- Both pairs of opposite sides are parallel
- Diagonals bisect each other
- One pair of opposite sides are equal/parallel

Topic 6 - Linear, Quadratic and Cubic Functions

Rectangle if **ONE** of the following is true:

- All angles are 90^o
- Diagonals are equal AND bisect each other

Rhombus if **ONE** of the following is true:

- All sides are equal
- Diagonals bisect each other at 90^o

Square if **ONE** of the following is true:

- All sides are equal and one angle is 90^o
- All angles are 90^o and two adjacent sides are equal
- Diagonals are equal and bisect each other at 90^o

Topic 7 - Introduction to Functions

Vertical test fails, Horizontal test fails, **many-to-many**
Only vertical test fails, **one-to-many**
Only horizontal test fails, **many-to-one**
Passes both, **one-to-one**

$f(x) = f(-x)$, even
$f(-x) = -f(x)$, odd
$x \in [0, 2)$ is an example of new notation

Topic 8 - Further Functions and Relations

$Q \alpha \frac{1}{x}$, $Q = \frac{k}{x}$ (If Q varies inversely with x)

$|a| = a$, if $a \geq 0$, $= -a$, if $a < 0$

$|a - b| = a - b$, if $a > b$, $= 0$, if $a = b$,
$= b - a$ if $a < b$

$\sqrt{a^2} = |a|$, thus following the rules above

To graph, $|x| + |y| = 1$, find the sign of each variable in each quadrant, creating separate equations for each quadrant

Topic 12 - Applied Trigonometry

REMEMBER Angle of **elevation** (from ground) and angle of **depression** (from imaginary horizontal line down to the line)

$sin\,(A) = cos(90^o - A)$, and vice-versa
$tan\,(A) = cot(90^o - A)$, and vice-versa
$sec\,(A) = cosec(90^o - A)$, and vice-versa

Topic 13 - Trigonometric Equations and Radians

'All Stations To Central'

Period = interval which the function repeats at
Amplitude = half the distance from min to max value

In
$y = a\,sin(bx)$ and $y = a\,cos(bx)$**,** amplitude = $|a|$,
period = $2\pi\ (or\ 360^{o}) \div b$

Circle parts
Sector = 'slice' of the circle
Segment = straight line (called a secant) which splits the circle into a major segment and minor segment

Degrees to radians, multiply by $\frac{\pi}{180}$
Radians to degrees, multiply by $\frac{180}{\pi}$

Arc length
$= \frac{\theta}{2\pi} \times 2\pi r = r\theta$, where θ is in **radians**
$= \frac{\theta}{360} \times 2\pi r$ where θ is in **degrees**

Area of Sector $= \frac{\theta r^2}{2}$**,** where θ is in **radians**
$= \frac{\theta}{360} \times \pi r^2$, where θ is in **degrees**

Area of a Segment $= \frac{\theta r^2}{2} - \frac{1}{2} r^2 sin(\theta)$, where θ is in **radians** (Ie: Area of sector - triangle made by line with centre of the circle)

Topic 14 - Trigonometric Functions and Identities

$1 + cot^2\theta = cosec^2\theta$

$tan^2\theta + 1 = sec^2\theta$

Topic 18 - Calculus

$f(x)$ isn't differentiable at points of **discontinuity**

Secant - line passing through two points on a curve

Differentiate from first principles:
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Normal = line perpendicular to the **tangent**
$m_{normal} = -\frac{1}{m_{tangent}}$

Topic 18 - Calculus

If $x = displacement,$
when $x > 0$, particle is right of the origin
when $x < 0$, particle is left of the origin
(same thing for velocity, but particle is described as moving)

- When acceleration and velocity are **same direction**, particle is speeding up
- When acceleration and velocity are **different directions**, particle is slowing down

Topic 19 - Exponentials and Logs

e is euler's number, which is where $f'(0) = 1$ in the equation $y = a^x$ (a is euler's number)

$f(x) = e^x$, $f'(x) = e^x$

Richter scale formula: $M = log_{10}(\frac{A}{A_0})$
(A = amplitude of the wave, A_0 is the reference value that corresponds to a zero-level earthquake)

Decibel formula:
$L = 10log_{10}\frac{P_2}{P_1}$, where P_2 **and** P_1 are two sounds

PH formula:
$pH = -log_{10}[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in moles per litre

Logarithms:
$y = a^x \quad \Leftrightarrow \quad log_a y = x$

$log_a x + log_a y = log_a xy$

$log_a x - log_a y = log_a \frac{x}{y}$

$log_a x^p = p\ log_a x$

$log_a x = \frac{log_b x}{log_b a}$

$y = log_a x$ **and** $y = a^x$ **are inverse functions**

YR 12

Topic 2 - Graphing Techniques

Consider functions of the form:
$y = k\,f[a(x + b)] + c$

- **Vertical dilation** with **scale factor** k (k stretches or compresses the function vertically
- c **translates** the graph **vertically**
- **Horizontal dilation** with scale factor of $\frac{1}{a}$ ($\frac{1}{a}$ stretches or compresses the function horizontally)
- b **translates** the graph **left** or **right**

ORDER OF TRANSFORMATIONS

1. Horizontal dilation with a factor $\frac{1}{a}$
2. Horizontal translation b units left/right
3. Vertical dilation with scale factor k
4. Vertical translation c units up/down

Topic 3A - Trigonometric Functions and Graphs

Use booklet to see how to do questions where a variable changes in a trigonometric pattern (wave tides, height on ferris wheels, etc)

Topic 4 - Data and Summary Statistics

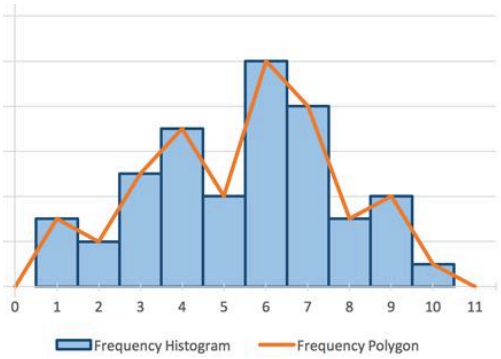
Categorical data (can be grouped into categories**)**:

- Nominal data: no special order (type of pet)
- Ordinal data: Order in it (1st, 2nd or small,medium)

Quantitative data (can be counted or measured):

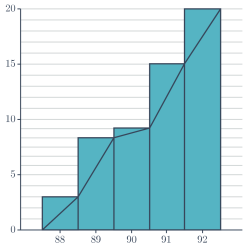
- Discrete data: countable number of numerical values (shoe size)
- Continuous data: measurements - infinite number of possibilities (height, weight)

Histogram (columns) and **Frequency Polygon (line graph)**



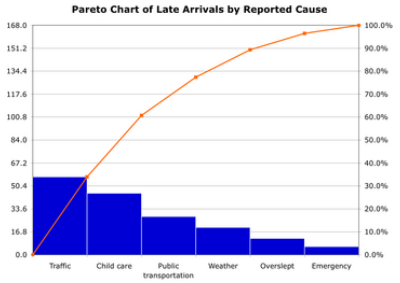
Topic 4 - Data and Summary Statistics

Ogive



Connects class centres (middle of each group of data). Eg. one column may represent data from 40-49 so will be displayed on the Ogive as a class centre of 44.5

Pareto chart (see booklet pg 8 on how to make one)



The **pareto principle** states that roughly 80% of the effects come from 20% of the causes

Measures of central tendency:

- Mean (average,
- $\bar{x}$ (sample mean) or μ (population mean))
- Mode (score that occurs most often
- Median (middle score after scores are arranged in order

Measures of Spread

- Range (highest score - lowest score)
- Interquartile range ($Q_3 - Q_1$) (IQR)

Quartiles is when data is split into 4 equal parts, with the separators being Q_1. Q_2 (median), and Q_3

Standard deviation measures the spread of data about the mean (the lower, the more consistent results are)

Box plots show the minimum value, Q_1, median (Q_2), Q_3 and maximum value (also shows the IQR)

Topic 4 - Data and Summary Statistics

A score is an **outlier** if it is above $Q_3 + 1.5 \times IQR$, or below $Q_1 - 1.5 \times IQR$

Data shapes/curves

- Symmetrical shape
- Smoothness
- Unimodal (one peak)
- Bimodal (two peaks)
- Multimodal (many peaks)

Skewness

- No skew (Symmetric)
- Positively skewed (more data on the left side)
- Negative skewed (more data on the right side)

Topic 5 - Bivariate Data Analysis

Scatterplot - a graph with dots representing values

- Independent variable on $x - axis$
- Dependant variable on $y - axis$

Correlation (how to describe)

- Linear or nonlinear
- Positive/Negative (gradient is + or -)
- Strength of association

Strength of association:

- Strong: dots follow a clear stream/pattern
- Moderate: more scatter in the plot and pattern is less clear
- Weak: pattern is even less clear

Person's Correlation Coefficient - Calculator

$0.75 \leq r < 1 \rightarrow$ strong positive correlation
$0.5 \leq r < 0.75 \rightarrow$ moderate positive correlation
$0.25 \leq r < 0.5 \rightarrow$ weak positive correlation
$r = 1 \rightarrow$ perfect positive correlation
$-0.25 < r < 0.25 \rightarrow$ no correlation
$-0.5 < r \leq 0.25 \rightarrow$ weak negative correlation
$-0.75 < r \leq 0.5 \rightarrow$ moderate negative correlation
$-1 < r \leq 0.75 \rightarrow$ strong negative correlation

Topic 5 - Bivariate Data Analysis

Least Squares Regression Line - Calculator

Mathematically best 'line of best fit' for data

$y = mx + c$, where:

$m = r\frac{s_y}{s_x}$

$c = \bar{y} - m\bar{x}$

s_x and s_y is the standard deviation of x and y

$\bar{x}$ and $\bar{y}$ is the mean of x and y

Interpolation - Make a prediction within data points
Extrapolation - Make a prediction outside data points

Topic 6 - Differentiation of Trig, Exponential and logarithmic function → Rules for Differentiation

$\lim_{x \to 0} \frac{sin(x)}{x} = 1$ $\lim_{x \to 0} \frac{cos(x)-1}{x} = 0$

$\lim_{x \to 0} \frac{tan(x)}{x} = 1$

Reference sheet!

Topic 9 - The first + second derivatives → Applications of the derivative

$y' > 0$ for all x, Increasing function
$y' < 0$ for all x, Decreasing function

Test for a **change** in **gradient sign/concavity** for stationary points or points of inflection

Test Endpoints for optimisation

Points of inflection include:

- an **oblique** point of inflection ($y' \neq 0$ at the point),
- a **horizontal** point of inflection (y' and $y'' = 0$ at the point),
- a **vertical** point of inflection (y' and y'' are undefined at the point)

(concavity must always change)