Name: ____________________________

Teacher's Name: ____________________

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Oakhill

COLLEGE

2024	YEAR 12 **MOCK** TRIAL PAPER

MATHEMATICS ADVANCED

General Instructions:	• Reading Time – 10 minutes • Working time – 3 hours • Write using black pen • NESA approved calculators may be used • A Reference sheet is provided at the back of this paper
Total Marks: 100	**Section I – 10 marks** (pages 1 to 4) • Attempt Questions 1 – 10 • Use the Multiple-choice answer sheet provided **Section II – 90 marks** (pages 5 to 28) • For Questions 16 - 32 show relevant mathematical reasoning and/or calculations • Answer each question in the space provided

Section I - 10 marks

Allow about 18 minutes for this section
Attempt Questions 1 – 10 using the multiple-choice answer sheet provided.

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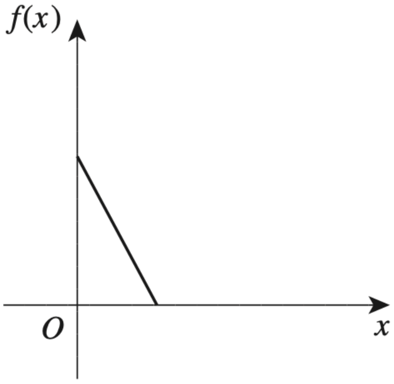
1 Which of the following graphs does NOT represent a probability density function?

A.



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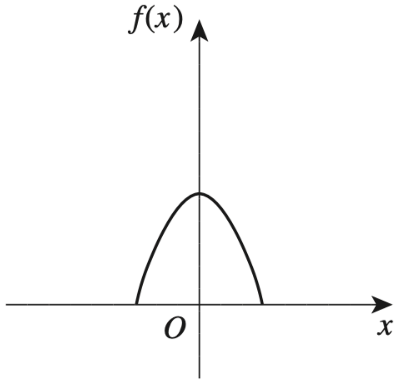

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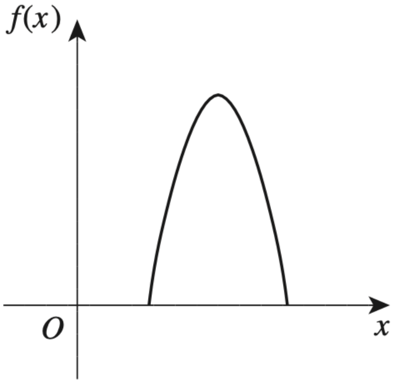

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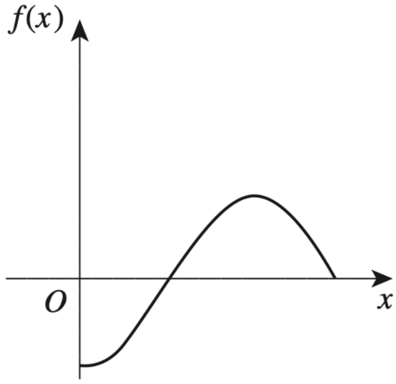


2 What is the derivative of $y = 3^{2x+1}$?

A. $y' = 2\ln 3 \cdot 3^{2x+1}$

B. $y' = \ln 3 \cdot 3^{2x+1}$

C. $y' = 2 \cdot 3^{2x+1}$

D. $y' = (2x+1) \cdot 3^{2x+1}$

3 The amount of water that Eleanor uses to wash her car is normally distributed with a mean of 50 litres and a standard deviation of 4 litres.
On what percentage of occasions would Eleanor expect to use between 42 litres and 46 litres of water to wash her car?

A. 13.5%

B. 27%

C. 34%

D. 68%

4 The table shows the accidents recorded on a motorway.

	Cause		
	Speeding	*Alcohol consumption*	*Driver fatigue*
Fatal	35	41	12
Non-fatal	75	149	55

What is the probability that an accident is fatal, given that alcohol consumption was NOT the cause?

A. $\frac{47}{367}$

B. $\frac{47}{177}$

C. $\frac{47}{88}$

D. $\frac{177}{367}$

5 What is the domain and range of a circle with the equation $x^2 + 4x + y^2 = 5$?

A. domain $= \left[-2-\sqrt{5},\ -2+\sqrt{5}\right]$; range $= \left[-\sqrt{5},\ \sqrt{5}\right]$

B. domain $= \left[2-\sqrt{5},\ 2+\sqrt{5}\right]$; range $= \left[-\sqrt{5},\ \sqrt{5}\right]$

C. domain $= [-1,\ 5]$; range $= [-3,\ 3]$

D. domain $= [-5,\ 1]$; range $= [-3,\ 3]$

6 What is the maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ and the value of x occurring in the domain $[0, 2\pi]$?

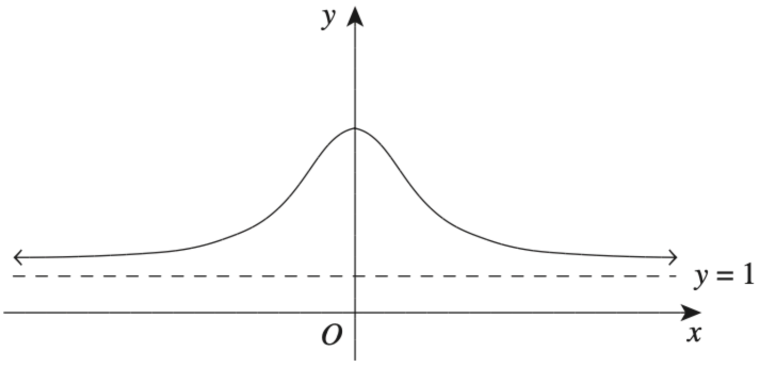
A. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 2 occurring at $x = \frac{\pi}{2}$.

B. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 3 occurring at $x = \frac{\pi}{2}$.

C. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 2 occurring at $x = \frac{3\pi}{2}$.

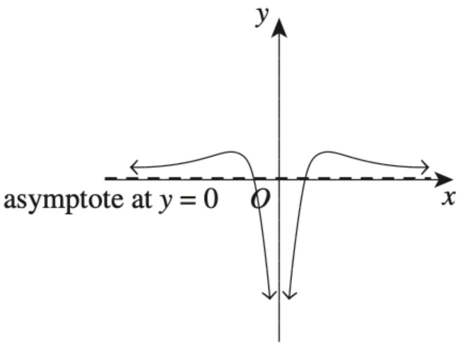
D. The maximum value of $y = 2\sin\left(\frac{x}{3}\right)+1$ is 3 occurring at $x = \frac{3\pi}{2}$.

7 The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$.

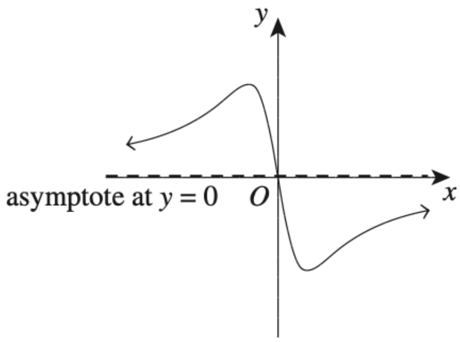


Which of the following could be the graph of $y = f'(x)$?

A.



B.



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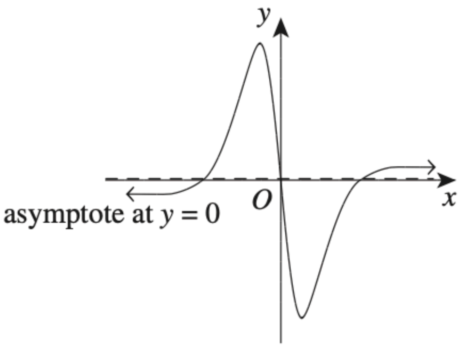

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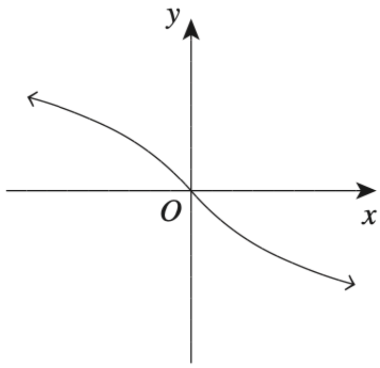


8 Given that $\int_2^6 f(x)\,dx = 3$, what is the value of $\int_4^6 f(2(x-3))\,dx$?

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 3

D. 8

9 Given that $f(x)$ is an even function and $g(x)$ is an odd function, which of the following statements is true?

A. $f(x)\times g(x)$ is an even function.

B. $f(x)+g(x)$ is an odd function.

C. $f\left[g(x)\right]$ is an even function.

D. $g\left[f(x)\right]$ is an odd function.

10 The discrete random variable X has the following probability distribution.

x	-1	0	1	a	$2a$
$P(X=x)$	$\frac{1}{10}$	a	b	b	$2b$

What is the range of possible values for the expected value, $E(X)$?

A. $-\frac{1}{10}\leq E(X)\leq 0$

B. $-\frac{1}{10}\leq E(X)\leq \frac{1}{8}$

C. $-\frac{1}{10}\leq E(X)\leq \frac{89}{320}$

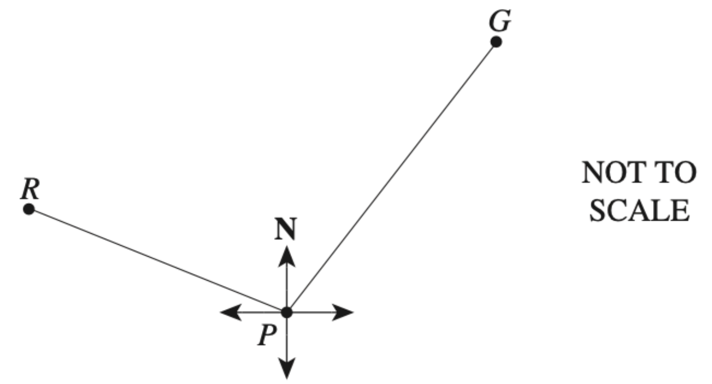
D. $0\leq E(X)\leq \frac{89}{320}$

End of Section I

Section II - 90 marks

Question 11 (3 marks)

Ringo and Greg start at location P and drive in different directions. Ringo drives at a speed of 70 km/h on a bearing of 285°, while Greg drives at a speed of 60 km/h on a bearing of 030°, as shown in the diagram.



(a) Show that $\angle GPR = 105°$. **1**

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(b) What will be the distance between Ringo and Greg after travelling for 2 hours? Give your answer correct to the nearest kilometre. **2**

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Question 12 (3 marks)

In a chess tournament, each player has a 'chess rating', which is a value from 1600 to 2000. **3**

Danny wants to know if there is a correlation between the number of years a player has spent playing chess and their chess rating.

Danny randomly surveyed 10 players at the tournament. The results are shown in the table.

	Years spent playing chess (x)	*Chess rating (y)*
Player 1	1	1650
Player 2	2	1550
Player 3	2	1600
Player 4	3	1500
Player 5	4	1900
Player 6	4	1650
Player 7	6	1700
Player 8	8	1950
Player 9	10	1950
Player 10	15	2000

Beth, a competitor in the tournament, has spent 5 years playing chess.

Use the equation of the least-squares regression line for this data set to determine Beth's chess rating. Give your answer correct to the nearest whole number.

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Question 13 (7 marks)

A tennis arena has a maximum capacity of 15 000 spectators. The number of spectators, N, and the ticket price in dollars, P, are modelled using the linear relationship $N = m \times P + c$, where m and c are constants.

According to the arena's records, the following is known.

- When the price of a ticket is \$50, the attendance averages 12 500 spectators.
- When the price of a ticket is \$35, the attendance averages 14 000 spectators.

(a) By developing and solving a pair of simultaneous equations, show that the number of spectators as a function of ticket price can be expressed as $N = -100P + 17500$. **3**

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Question 13 (continued)

The revenue generated from an event, R, is calculated using $R = N \times P$.

(b) What is the maximum revenue generated from this event? **2**

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(c) The manager of this event wants the tennis arena to be filled with 15 000 spectators. **2**
Show that this will result in a revenue loss of \$390 625.

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Question 14 (3 marks)

Find the equation of the tangent to the curve $y = \dfrac{1}{4x+1}$ at the point (0, 1). **3**

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Question 15 (4 marks)

(a) Differentiate $y = \dfrac{1}{2}\ln\left(x^2\right)$ with respect to x. **2**

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(b) Differentiate $y = \dfrac{e^x}{\sin x}$ with respect to x. **2**

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Question 16 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} x + \sin x \, dx$. **2**

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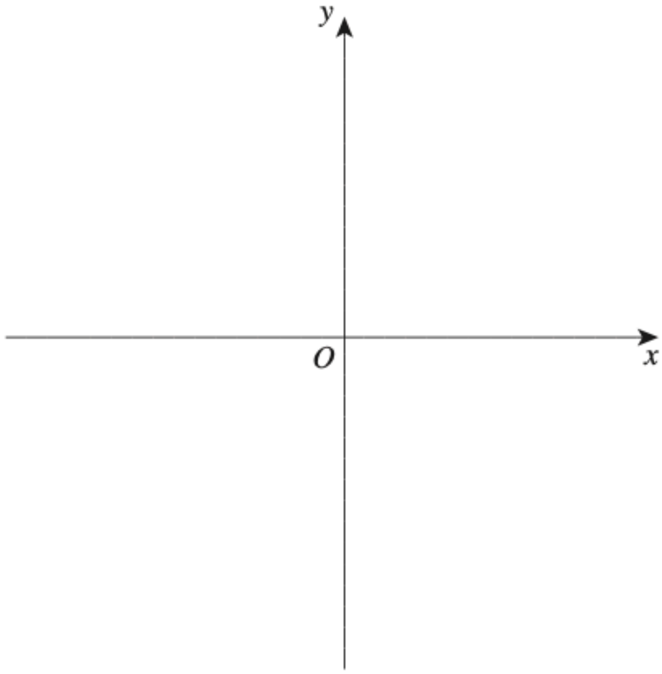
Question 17 (2 marks)

The function $y = f(x)$ is continuous for all values of x. The following is known of the function's properties. **2**

- When $x < -1$, $f'(x) > 0$ and $f''(x) < 0$.
- When $x > -1$, $f'(x) > 0$ and $f''(x) > 0$.
- $f(-1) = f'(-1) = f''(-1) = 0$

On the axes below, sketch the graph of $y = f(x)$, labelling any x-intercepts.



Question 18 (3 marks)

Solve the equation $\log_{10}(x-21) = 2 - \log_{10}x$. **3**

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Question 19 (4 marks)

(a) Express $y = 2x^2 - 12x + 23$ in the form $y = 2(x-c)^2 + d$ **2**

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(b) The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 12x + 23$ by the transformations: **2**

- A vertical dilation with scale factor k followed by
- A horizontal translation of p units followed by
- A vertical translation of q units.

Write down the values of k, p and q.

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Question 20 (4 marks)

Shirley uses her car every day. Sometimes, she needs a few attempts to start her car. Let the discrete random variable X represent the number of attempts needed.

The probability function for the random variable X is given by

$$P(X=x)=\begin{cases} a & x=1 \\ \frac{1}{2}P(X=x-1) & x=2,\ 3,\ 4,\ 5 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that $a=\frac{16}{31}$. **2**

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(b) The probability distribution table for the discrete random variable X is shown. **2**

x	1	2	3	4	5
$P(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

Shirley claims that after an entire year, she needs an average of two attempts to successfully start her car.

Calculate the expected value and comment on the accuracy of Shirley's claim.

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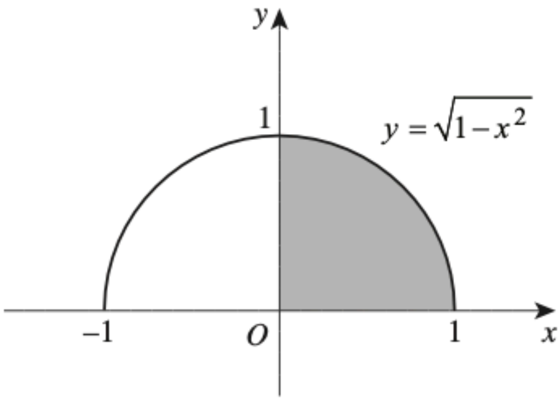
Question 21 (4 marks)

The graph of $y = \sqrt{1-x^2}$ is shown.



The table gives the values of $\sqrt{1-x^2}$ for x.

x	0	0.25	0.5	0.75	1
$\sqrt{1-x^2}$	1	0.968		0.661	0

(a) Complete the table above. **1**

(b) Hence, use the trapezoidal rule to find an approximation of the shaded area. **2**

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(c) How could a better approximation of the shaded area be obtained using the trapezoidal rule? **1**

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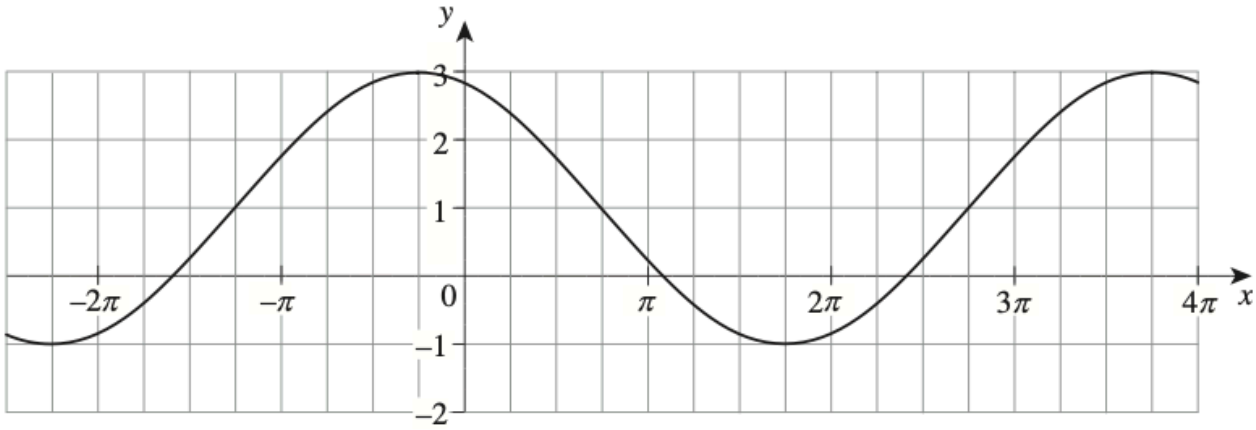
Question 22 (3 marks)

The graph is a function of the form $y = k\cos\left(a(x+b)\right)+c$.



Find the equation of the function. **3**

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Question 23 (3 marks)

Alison received her scores for a class test in each of two subjects. The class's scores for each test were normally distributed.

The table shows the subjects and Alison's scores as well as the mean and standard deviation of the class scores on each test.

Subject	*Alison's score*	*Mean*	*Standard deviation*
English	85	60	22
Mathematics	75	52	15

(a) Calculate Alison's standardised z-scores for both tests. Give your answers correct to two decimal places. **2**

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(b) Alison is unsure if she performed better in the English test or the Mathematics test. **1**

Relative to the rest of the class, in which subject has Alison achieved a better result? Justify your answer with reference to the standardised z-scores calculated in part (a).

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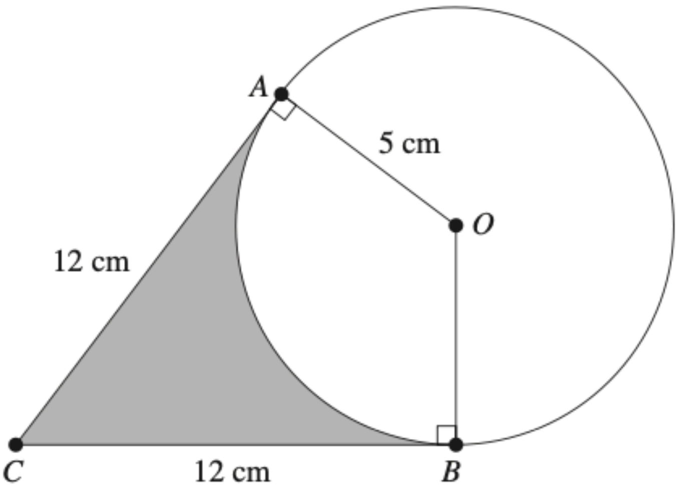
Question 24 (4 marks)

The diagram shows a circle with centre O and radius 5 cm. Tangents are drawn from the points A and B, which meet at point C.



NOT TO SCALE

(a) Show that $\angle AOB = 2.4$ radians correct to two significant figures. **2**

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(b) Hence, find the area of the shaded region. **2**

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Question 25 (5 marks)

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

(a) Draw a Venn diagram showing this information. **1**

(b) A student choosing to study Chemistry is event 1. A student choosing to study Physics is event 2. **2**

If a student is chosen at random, determine whether these events are independent of each other.

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Question 25 continues

(c) The entire year group is given the same survey. The results indicate that: **2**

- the probability that a student chose to study Chemistry is $\frac{1}{3}$
- the probability that a student chose to study Physics is $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is $\frac{3}{7}$.

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

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End of Question 25

Question 26 (6 marks)

A particle is moving in a straight line with a velocity given by

$$\frac{dx}{dt} = 8\cos\left(2t - \frac{\pi}{2}\right),$$

where x is the displacement from the origin in metres and t is time measured in seconds.

The particle is initially at rest 4 m to the right of the origin.

(a) Show that the displacement of the particle is given by $x = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$. **2**

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(b) Show that the particle comes to rest again when it is 12 m to the right of the origin and $t = \frac{\pi}{2}$ s. **2**

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(c) By finding a function for acceleration, describe the motion of the particle after $\frac{\pi}{2}$ s and before it next comes to rest. **2**

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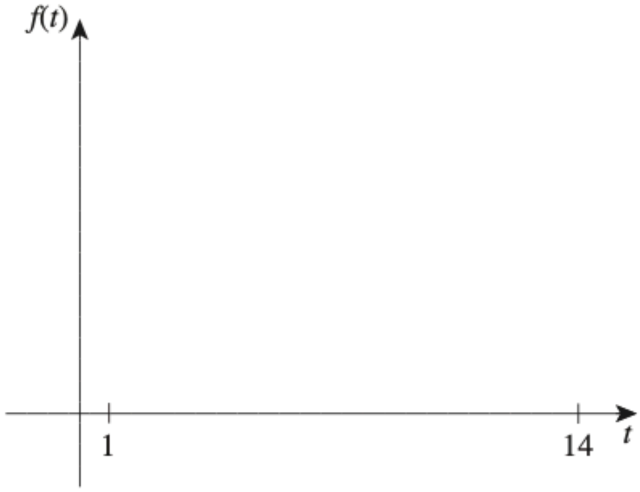
Question 27 (6 marks)

A continuous random variable, T, represents the time taken in days to show symptoms after contracting a virus. It has the following probability density function.

$$f(t)=\begin{cases}\dfrac{k}{2t-1} & \text{for } 1\le t\le 14\\ 0 & \text{for } 0\le t<1 \text{ or } t>14\end{cases}$$

(a) Show that $k=\dfrac{2}{\ln 27}$. **2**

(b) On the axes below, sketch the graph of $y=f(t)$. **2**



Question 27 continues

Question 27 (continued)

(c) After how many days will a person have a 75% chance of showing symptoms after they have contracted the virus? **2**

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End of Question 27

Question 28 (6 marks)

The mass of two substances, A and B, are decaying exponentially according to

$$M_A = 200e^{-0.05t}$$
$$M_B = 400 \times 3^{-0.12t},$$

where M is the mass in grams and t is the time in minutes.

(a) Find the time taken for substance A to decrease to half of its original value. Give your answer correct to the nearest minute. **2**

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(b) At what instance do both particles decay at the same rate? Give your answer correct to the nearest second. **4**

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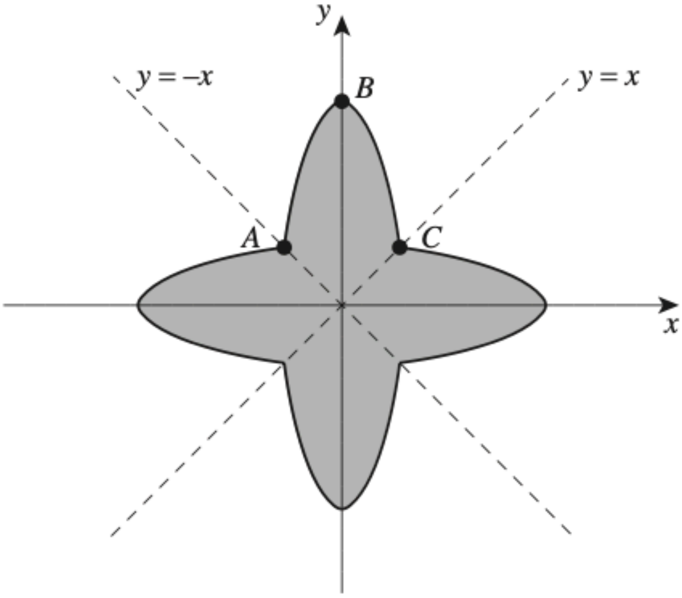
Question 29 (4 marks)

A company's logo is designed using the curve ABC, as shown by the shaded region in the diagram. **4**
The curve ABC shows part of the function $y = 4 - 3x^2$. This curve is reflected along the x-axis and the lines $y = x$ and $y = -x$ to form the rest of the logo.



Calculate the area of the company's logo.

Question 30 (4 marks)

A city is suffering through a drought and adopts a plan to import water from another city. It is given that the volume of water imported in the 1st year since the start of the plan is $1.5\times10^7\ m^3$ and in subsequent years, the volume of water imported each year is 10% less than the volume of water imported in the previous year.

(a) Find how much water was imported in the 3rd year. **1**

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(b) Find the total volume of water imported in the first 10 years. **2**

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(c) Explain why the total volume of water imported since the start of the plan will never exceed $1.6\times10^8\ m^3$. **1**

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Question 31 (8 marks)

(a) (i) Show that the equation $-\frac{1}{2}\sin 4x + 4\cos 4x = 0$ can be written as $\tan 4x = 8$. **1**

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(ii) Show that the exact value to the first two solutions to the equation $\tan 4x = 8$ in the domain $[0, \pi]$ are **2**

$$x_1 = \frac{1}{4}\tan^{-1} 8$$

$$\text{and } x_2 = \frac{1}{4}\left(\pi + \tan^{-1} 8\right).$$

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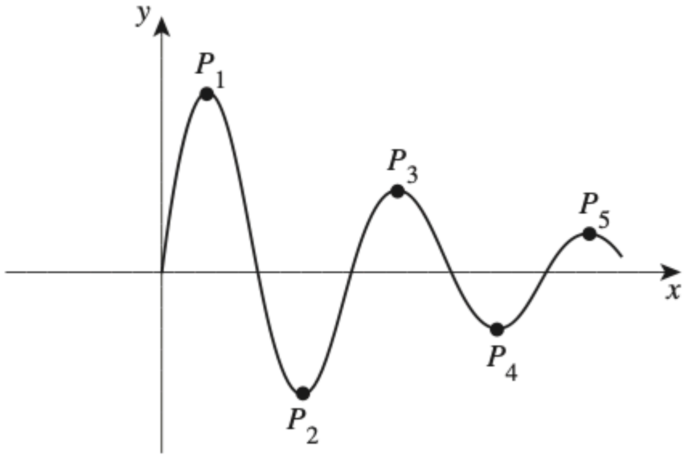
Question 31 continues

(b) The graph of $y = 10e^{-\frac{1}{2}x} \sin 4x$ in the domain $[0, \infty)$ is shown.



The stationary points are labelled $P_1, P_2, P_3, \ldots, P_n$. Let X_n and Y_n be the x- and y-coordinates of P_n respectively. This means the stationary points are $P_1(X_1, Y_1), P_2(X_2, Y_2), \ldots, P_n(X_n, Y_n)$.

(i) Write down the x-coordinates of P_1 and P_2. **2**

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Question 31 continues

Question 31 (continued)

(ii) It is known that the y-coordinates of the stationary points $Y_1, Y_2, Y_3, \ldots, Y_n$ form a geometric sequence. **3**

Find the exact value of the common ratio for this geometric sequence. Express your answer in simplest form.

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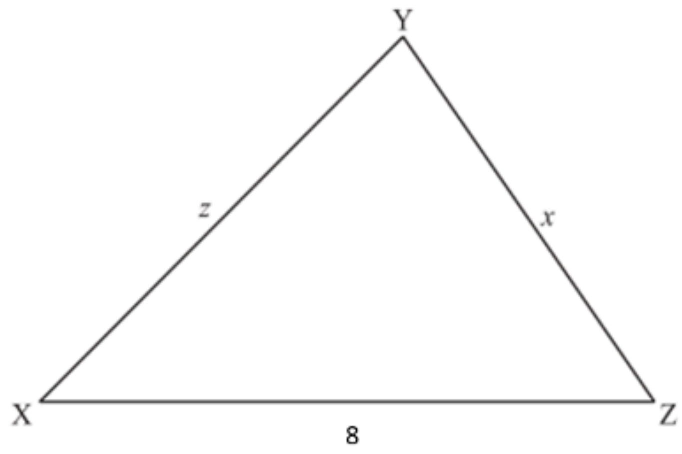
Question 32 (2 marks)

The triangle XYZ has $XZ = 8$, $YZ = x$, $XY = z$ as shown below.

The perimeter of the triangle XYZ is 22.



Show that $\cos Z = \dfrac{7x - 33}{4x}$.

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END OF PAPER

Name: ______________________________

Teacher's Name: _____________________

Oakhill
COLLEGE

2024 YEAR 12 MOCK TRIAL EXAM

MATHEMATICS ADVANCED

Section I – Multiple Choice

1	A ⬭	B ⬭	C ⬭	D ⬭
2	A ⬭	B ⬭	C ⬭	D ⬭
3	A ⬭	B ⬭	C ⬭	D ⬭
4	A ⬭	B ⬭	C ⬭	D ⬭
5	A ⬭	B ⬭	C ⬭	D ⬭
6	A ⬭	B ⬭	C ⬭	D ⬭
7	A ⬭	B ⬭	C ⬭	D ⬭
8	A ⬭	B ⬭	C ⬭	D ⬭
9	A ⬭	B ⬭	C ⬭	D ⬭
10	A ⬭	B ⬭	C ⬭	D ⬭