ADVANCED MATHEMATICS
Logs and Exponentials (Adv), E1 Logs and Exponentials (Adv)

Log/Index Laws and Equations (Y11)
Graphs and Applications (Y11)

Teacher: Cathyanne Horvat
Exam Equivalent Time: 60 minutes *(based on allocation of 1.5 minutes per mark)*

HISTORICAL CONTRIBUTION

- *E1 Logs and Exponentials* is a Year 11 topic covering significant amounts of gateway content that is applicable to multiple other Year 12 topic areas.
- It's contribution to other sub-topics is highly significant, including *Tangents, Areas Under Curves, Probability Density Functions, Rates of Change and L&E Calculus* just to name a few. Importantly, these are not covered in this analysis as they are included in other topic areas.
- *E1 Logs and Exponentials* has been split into two sub-categories for the purposes of analysis which are: *1-Index Laws and Equations and 2-L&E Graphs and Applications.*
- This analysis look at *L&E Graphs and Applications.*

HSC ANALYSIS - What to expect and common pitfalls

- *L&E Graphs and Applications* has attracted just the single multiple choice question (2021) in new syllabus exams to date (it is important to note that this *does not* include exponential graphs that have been major contributors to *Rates of Change* and other topic areas mentioned above).
- *2021 Adv 5 MC* required students to express the domain of a log function using set notation. The 52% mean mark flags an issue here and we recommend it is reviewed closely.
- NESA Topic Guidance makes it clear that past HSC questions in the Std2 course are relevant to the new Advanced course in this topic area. We have included a number of these questions in the database, which can be identified with "STD2 A4" in their title (all bands and mean marks are relevant to Std2).

Questions

1. Functions, 2ADV F1 SM-Bank 53

   i. If $\frac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^x$, find x. ***(1 mark)***

   ii. Calculate the value of $\frac{1}{\sqrt[3]{7+\pi}}$ to 3 significant figures. ***(1 mark)***

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2. L&E, 2ADV E1 SM-Bank 12

   Solve the equation $\log_e(3x+5) + \log_e(2) = 2$, for x. ***(2 marks)***

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3. L&E, 2ADV E1 SM-Bank 2

   The population of Indian Myna birds in a suburb can be described by the exponential function

   $$N = 35e^{0.07t}$$

   where t is the time in months.

   i. What will be the population after 2 years? ***(1 mark)***

   ii. Draw a graph of the population. ***(2 marks)***

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4. L&E, 2ADV E1 2005 HSC 5a

   Use the change of base formula to evaluate $\log_3 7$, correct to two decimal places. ***(1 mark)***

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5. Functions, 2ADV F2 SM-Bank 1

   i. Draw the graph $y = \ln x$. ***(1 mark)***

   ii. Explain how the above graph can be transformed to produce the graph

   $$y = 3\ln(x+2)$$

   and sketch the graph, clearly identifying all intercepts. ***(3 marks)***

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6. L&E, 2ADV E1 SM-Bank 14

   The spread of a highly contagious virus can be modelled by the function

   $$f(x) = \frac{8000}{1+1000e^{-0.12x}}$$

   Where x is the number of days after the first case of sickness due to the virus is diagnosed and $f(x)$ is the total number of people who are infected by the virus in the first x days.

   i. Calculate $f(0)$. ***(1 mark)***

   ii. Find the value of $f(365)$ and interpret it result. ***(2 marks)***

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7. L&E, 2ADV E1 SM-Bank 10

Solve the equation $2^{3x-3} = 8^{2-x}$ for x. *(2 marks)*

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8. L&E, 2ADV E1 2009 HSC 1f

Solve the equation $\ln x = 2$. Give you answer correct to four decimal places. *(2 marks)*

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9. L&E, 2ADV E1 2010 HSC 4d

Let $f(x) = 1 + e^x$.

Show that $f(x) \times f(-x) = f(x) + f(-x)$. *(2 marks)*

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10. L&E, 2ADV E1 2004 HSC 6a

Solve the following equation for x:

$e^{2x} + 3e^x - 10 = 0$. *(2 marks)*

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11. Algebra, STD2 A4 2004 HSC 26a

i. The number of bacteria in a culture grows from 100 to 114 in one hour.
What is the percentage increase in the number of bacteria? *(1 mark)*

ii. The bacteria continue to grow according to the formula $n = 100(1.14)^t$, where n is the number of bacteria after t hours.
What is the number of bacteria after 15 hours? *(1 mark)*

Time in hours (t)	0	5	10	15
Number of bacteria (n)	100	193	371	?

iii. Use the values of n from $t = 0$ to $t = 15$ to draw a graph of $n = 100(1.14)^t$.
Use about half a page for your graph and mark a scale on each axis. *(4 marks)*

iv. Using your graph or otherwise, estimate the time in hours for the number of bacteria to reach 300. *(1 mark)*

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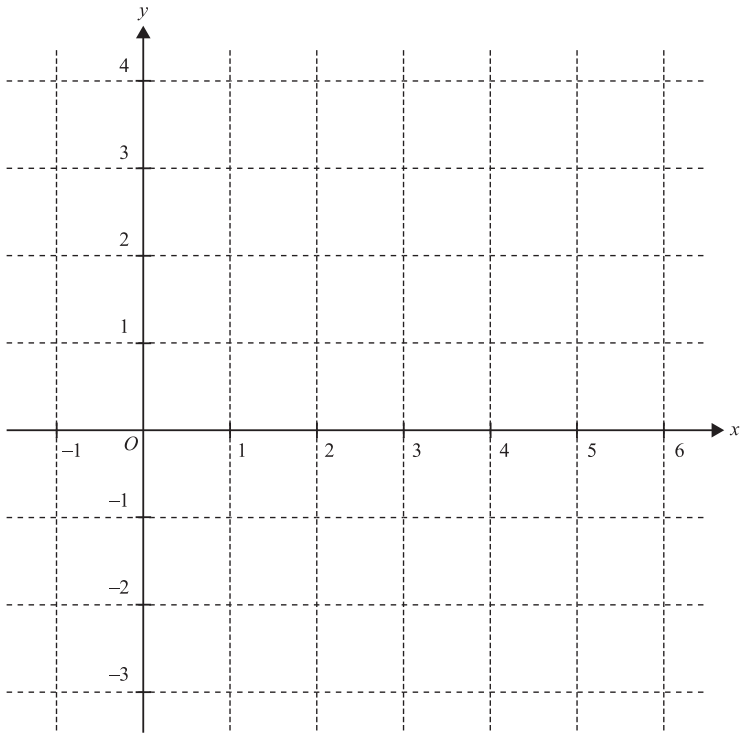
12. L&E, 2ADV E1 2019 MET1 4

Given the function $f(x) = \log_e(x - 3) + 2$,

a. State the domain and range of $f(x)$. *(1 mark)*

b. **i.** Find the equation of the tangent to the graph of $f(x)$ at $(4, 2)$. *(2 marks)*

ii. On the axes below, sketch the graph of the function $f(x)$, labelling any asymptote with its equation.
Also draw the tangent to the graph of $f(x)$ at $(4, 2)$. *(4 marks)*



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13. L&E, 2ADV E1 2019 HSC 15a

Solve $e^{2\ln x} = x + 6$ *(2 marks)*

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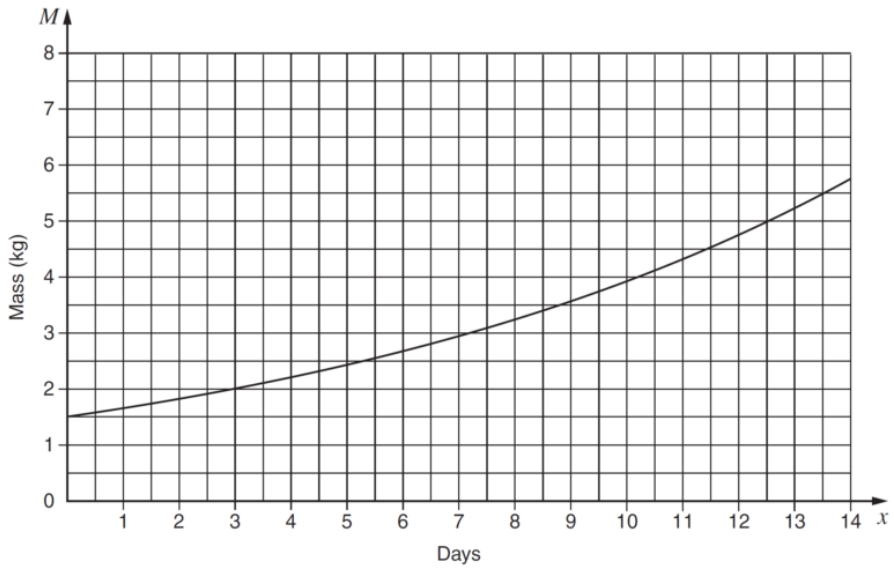
14. Algebra, STD2 A4 2016 HSC 29b

The mass M kg of a baby pig at age x days is given by $M = A(1.1)^x$ where A is a constant. The graph of this equation is shown.



i. What is the value of A? *(1 mark)*

ii. What is the daily growth rate of the pig's mass? Write your answer as a percentage. *(1 mark)*

Worked Solutions

1. Functions, 2ADV F1 SM-Bank 53

i. $\dfrac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^{-\frac{1}{3}}$

ii. $\dfrac{1}{\sqrt[3]{7+\pi}} = 0.4619\ldots$

$= 0.462$ (to 3 sig. fig.)

2. L&E, 2ADV E1 SM-Bank 12

Simplify using log laws:

$\log_e(6x+10) = 2$

$6x + 10 = e^2$

$\therefore x = \dfrac{e^2 - 10}{6}$

3. L&E, 2ADV E1 SM-Bank 2

i. $N = 35e^{0.07t}$

Find N when $t = 24$:

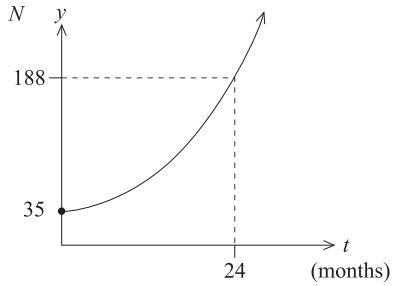
$N = 35e^{0.07 \times 24}$

$= 35e^{1.68}$

$= 187.79\ldots$

$= 188$ birds

ii.



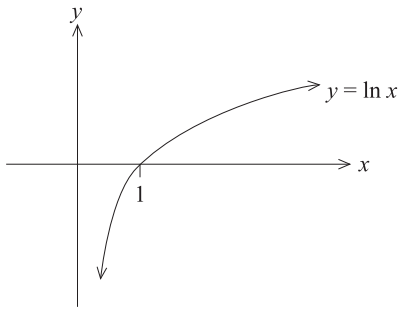
Worked Solutions

4. L&E, 2ADV E1 2005 HSC 5a

$$\begin{aligned}\log_3 7 &= \frac{\log_{10} 7}{\log_{10} 3} \\ &= 1.771\ldots \\ &= 1.77 \quad \text{(to 2 d.p.)}\end{aligned}$$

5. Functions, 2ADV F2 SM-Bank 1

i.

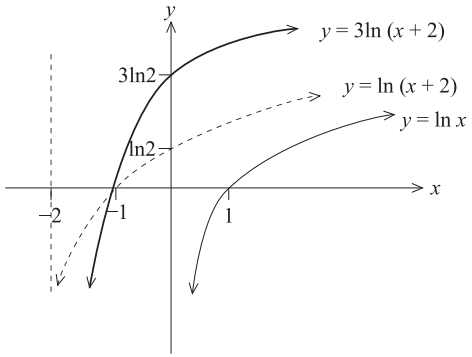


ii. Transforming $y = \ln x \Rightarrow y = \ln(x+2)$

$y = \ln x \Rightarrow$ shift 2 units to left.

Transforming $y = \ln(x+2)$ to $y = 3\ln(x+2)$

$\Rightarrow$ increase each y value by a factor of 3



6. L&E, 2ADV E1 SM-Bank 14

i. $$\begin{aligned}f(0) &= \frac{8000}{1 + 1000e^0} \\ &= \frac{8000}{1001} \\ &= 7.99\ldots\end{aligned}$$

ii. $$\begin{aligned}f(365) &= \frac{8000}{1 + 1000e^{-0.12 \times 365}} \\ &= \frac{8000}{1 + 1000e^{-43.8}} \\ &\approx 8000\end{aligned}$$

After 1 year, the model predicts the total number of people infected by the virus is 8000.

7. L&E, 2ADV E1 SM-Bank 10

$$\begin{aligned}2^{3x-3} &= 2^{3(2-x)} \\ 3x - 3 &= 6 - 3x \\ 6x &= 9 \\ \therefore x &= \frac{3}{2}\end{aligned}$$

8. L&E, 2ADV E1 2009 HSC 1f

$$\begin{aligned}\ln x &= 2 \\ \log_e x &= 2 \\ x &= e^2 \\ &= 7.38905\ldots \\ &= 7.3891 \quad \text{(to 4 d.p.)}\end{aligned}$$

MARKER'S COMMENT: Students are reminded to write answers to more decimal places than required before rounding up.

9. L&E, 2ADV E1 2010 HSC 4d

$$
\begin{aligned}
&f(x) \times f(-x) \\
&\quad = (1+e^{x})(1+e^{-x}) \\
&\quad = 1+e^{-x}+e^{x}+e^{x}e^{-x} \\
&\quad = e^{x}+e^{-x}+2
\end{aligned}
$$

MARKER'S COMMENT: A common error in this question was not to realise that $e^{x}e^{-x}=e^{0}=1$.

$$
\begin{aligned}
&f(x)+f(-x) \\
&\quad = 1+e^{x}+1+e^{-x} \\
&\quad = e^{x}+e^{-x}+2 \\
&\quad = f(x) \times f(-x) \quad \ldots \text{ as required}
\end{aligned}
$$

10. L&E, 2ADV E1 2004 HSC 6a

$$e^{2x}+3e^{x}-10=0$$

$$\therefore (e^{x})^{2}+3e^{x}-10=0$$

Let $X=e^{x}$,

$$\therefore X^{2}+3X-10=0$$

$$(X+5)(X-2)=0$$

$$\therefore X=2 \text{ or } -5$$

If $X=2$

$$e^{x}=2$$

$$\ln e^{x}=\ln 2$$

$$x=\ln 2$$

If $X=-5$

$e^{x}=-5$ (no solution)

$$\therefore x=\ln 2$$

11. Algebra, STD2 A4 2004 HSC 26a

i. Percentage increase

$$
\begin{aligned}
&= \frac{114-100}{100} \times 100 \\
&= 14\%
\end{aligned}
$$

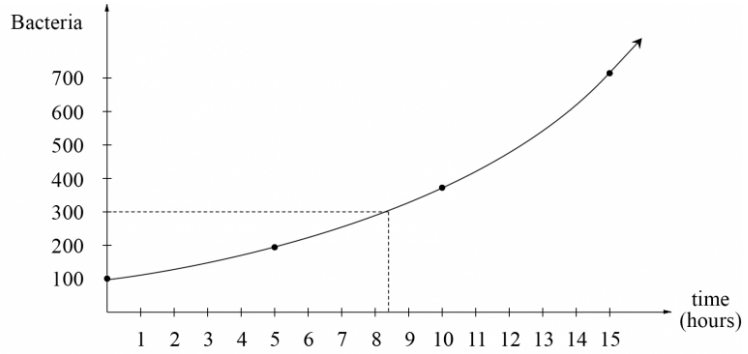
COMMENT: Common ADV/STD2 content in new syllabus.

ii. $n=100(1.14)^{t}$

When $t=15$,

$$
\begin{aligned}
n &= 100(1.14)^{15} \\
&= 713.793\ldots \\
&= 714 \quad \text{(nearest whole)}
\end{aligned}
$$

iii.



iv. Using the graph

The number of bacteria reaches 300 after approximately 8.4 hours.

12. L&E, 2ADV E1 2019 MET1 4

a. **Domain:** $x > 3$

Range: $y \in R$

b.i. $g(x) = \log_e(x-3) + 2$

$g'(x) = \dfrac{1}{x-3}$

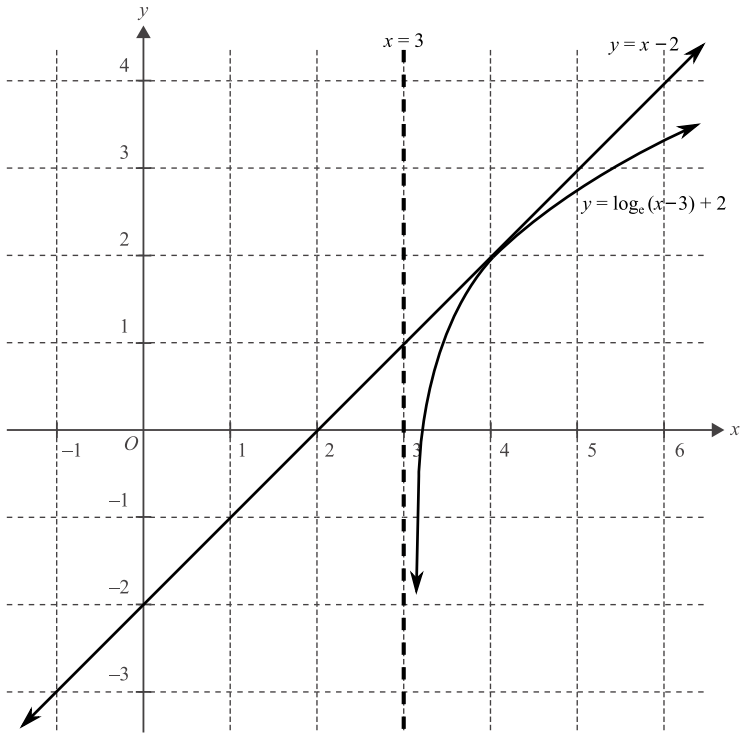
$g'(4) = 1$

Equation of tangent, $m = 1$ **through** $(4, 2)$**:**

$y-2 = 1(x-4)$

$y = x-2$

b.ii.



13. L&E, 2ADV E1 2019 HSC 15a

$$
\begin{aligned}
e^{2\ln x} &= x + 6 \\
\ln e^{2\ln x} &= \ln(x+6) \\
2\ln x &= \ln(x+6) \\
\ln x^2 &= \ln(x+6) \\
x^2 &= x + 6 \\
x^2 - x - 6 &= 0 \\
(x-3)(x+2) &= 0
\end{aligned}
$$

$\therefore x = 3 \quad (x > 0)$

♦ *Mean mark 47%.*

14. Algebra, STD2 A4 2016 HSC 29b

i. **When** $x = 0$**,**

$1.5 = A(1.1)^0$

$\therefore A = 1.5$ kg

♦ *Mean mark 48%.*
COMMENT: Common ADV/STD2 content in new syllabus.

ii. **Daily growth rate**

$= 0.1$

$= 10\%$

♦♦♦ *Mean mark part (ii) 6%.*
MARKER'S COMMENT: Interpretation of the exponential was very poorly understood.