

# MATHEMATICS ADVANCED

## HSC Exam\* Questions by Topic

### 2023 - 2007

v2024

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**Year 11 Course****Functions**[F1.1 Algebraic techniques](#)[F1.2 Introduction to functions](#)[F1.3 Linear, quadratic & cubic functions](#)[F1.4 Further functions & relations](#)**Trigonometric Functions**[T1.1 Trigonometry](#)[T1.2 Radians](#)[T2 Trigonometric functions & identities](#)**Calculus**[C1.1 Gradients of tangents](#)[C1.2 Difference quotients](#)[C1.3 The derivative function & its graph](#)[C1.4 Calculating with derivatives](#)**Exponential & Logarithmic Functions**[E1.1 Introducing logarithms](#)[E1.2 Logarithmic laws & applications](#)[E1.3 Exponential function & natural logs](#)[E1.4 Graphs & apps of exp & log functions](#)**Statistical Analysis**[S1.1 Probability & Venn diagrams](#)[S1.2 Discrete probability distributions](#)**Year 12 Course****Functions**[F2 Graphing techniques](#)**Trigonometric Functions**[T3 Trig functions and graphs](#)**Calculus**[C2.1 Diff of trig, exp & log fns](#)[C2.2 Rules of differentiation](#)[C3.1 The first & second derivs](#)[C3.2 Applications of the deriv](#)[C4.1 The anti-derivative](#)[C4.2 Areas & the definite integral](#)**Financial Mathematics**[M1.1 Modelling investments & loans](#)[M1.2 Arithmetic sequences & series](#)[M1.3 Geometric sequences & series](#)[M1.4 Financial apps of sequences & series](#)**Statistical Analysis**[S2.1 Data and summary statistics](#)[S2.2 Bivariate data analysis](#)[S3.1 Continuous random variables](#)[S3.2 The normal distribution](#)**Complete Papers**[2023 HSC](#)[2022 HSC](#)[2021 HSC](#)[2020 HSC](#)[2020 NESA Sample](#)**Question Difficulty**

Easy



Mid-range



Difficult

**Mathematics Advanced, Ext 1, Ext 2 Reference Sheet (2023 HSC)**

Questions by Topic from ...

- 2023 – 2020 Mathematics Advanced HSC and 2019 – 2007 Mathematics HSC
- NESA Sample Examination Paper [MA SP] and other examination questions [MA SQ]
- Selected NESA Topic Guidance questions [TG]
- Selected NESA Maths Stand 2 Sample exam questions [MS SQ] (common topics)
- Selected Qs from 2023 – 2007 Maths Extension 1 and 2023 – 2007 Maths Stand 2/General HSCs
- NESA's Mathematics Standard 2 Sample exam questions [MS SQ]

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# Year 11: Functions

## F1.1 Algebraic techniques



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

[Reference Sheet](#)

Students:

- use index laws and surds
- solve quadratic equations using the quadratic formula and by completing the square (ACMMM008)
- manipulate complex algebraic expressions involving algebraic fractions

**21 11 MA** Solve  $x + \frac{x-1}{2} = 9$ .

**2** [Solution](#)



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**TG 1** Simplify

[Solution](#)

(a)  $\left(\frac{x}{y}\right)^3 \div (x^2y^{-3})$



(b)  $x^{\frac{1}{3}} \times x^{\frac{5}{3}}$



(c)  $3\sqrt{2} + 5\sqrt{18}$

NESA Mathematics Advanced Year 11 Topic Guide: Functions

**TG 2** Express  $\frac{5\sqrt{3}}{4\sqrt{2}}$  with a rational denominator.

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Functions

**TG 3** If  $\frac{1}{\sqrt{(x+a)}} = (x+a)^m$ , write down the value of  $m$ .

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Functions

**TG 4** Simplify the following:

[Solution](#)

(a)  $\frac{x}{3} - \frac{x+1}{5}$



(b)  $\frac{1}{x} + \frac{1}{y}$



(c)  $\frac{x^2 + 3x + 2}{x + 2}$



(d)  $\frac{3m - 6m^2}{4} \times \frac{8m}{m^2 - 2m}$



(e)  $\frac{x+1}{x} - \frac{2x+1}{3x}$



NESA Mathematics Advanced Year 11 Topic Guide: Functions

<b>TG 5</b>	Working alone, worker A can complete a task in $a$ hours, and worker B can complete the same task in $b$ hours. This means that A can complete $\frac{1}{a}$ of the task in one hour.	<a href="#">Solution</a>
	(a) Write an algebraic expression for the fraction of the task that could be completed in one hour if A and B work together.	
	(b) What does the reciprocal of this fraction represent?	
	NENESA Mathematics Advanced SA Year 11 Topic Guide: Functions	
<b>TG 6</b>	A thin lens has focal length $p$ , while another thin lens has a focal length $q$ . The lenses are separated by a distance $d$ . Find their combined focal length, which is given by the reciprocal of $(\frac{1}{p} + \frac{1}{q} - \frac{d}{pq})$ .	<a href="#">Solution</a>
	NESA Mathematics Advanced Year 11 Topic Guide: Functions	
<b>TG 7</b>	The average cost per unit for the production of $x$ units is given as Average Cost = Total cost $\div x$ . A company producing a certain product finds that the average cost of production is given by $\frac{6000}{x} + 65 + 0.1x$ where $x$ is the number of units of the product.	<a href="#">Solution</a>
	(a) Write this expression as a single fraction. (b) Find an expression for the total cost for the production of $x$ units by the company.	 
	NESA Mathematics Advanced Year 11 Topic Guide: Functions	
<b>19 M 3</b>	What is the value of $p$ so that $\frac{a^2 a^{-3}}{\sqrt{a}} = a^p$ ?	<b>1</b> <a href="#">Solution</a>
	A. -3      B. $-\frac{3}{2}$ C. $-\frac{1}{2}$ D. 12	
	NESA 2019 Mathematics HSC Examination	
<b>18 M a 11</b>	Rationalise the denominator of $\frac{3}{3 + \sqrt{2}}$ .	<b>2</b> <a href="#">Solution</a>
	NESA 2018 Mathematics HSC Examination	
<b>17 M a 11</b>	Rationalise the denominator of $\frac{2}{\sqrt{5} - 1}$ .	<b>2</b> <a href="#">Solution</a>
	NESA 2017 Mathematics HSC Examination	
<b>15 M c 11</b>	Express $\frac{8}{2 + \sqrt{7}}$ with a rational denominator.	<b>2</b> <a href="#">Solution</a>
	NESA 2015 Mathematics HSC Examination	
<b>14 M a 11</b>	Rationalise the denominator of $\frac{1}{\sqrt{5} - 2}$ .	<b>2</b> <a href="#">Solution</a>
	NESA 2014 Mathematics HSC Examination	
<b>13 M 1 1</b>	What are the solutions of $2x^2 - 5x - 1 = 0$ ?	<b>1</b> <a href="#">Solution</a>
	(A) $x = \frac{-5 \pm \sqrt{17}}{4}$ (B) $x = \frac{5 \pm \sqrt{17}}{4}$ (C) $x = \frac{-5 \pm \sqrt{33}}{4}$ (D) $x = \frac{5 \pm \sqrt{33}}{4}$	
	NESA 2013 Mathematics HSC Examination	
<b>12 M 2 2</b>	Which of the following is equal to $\frac{1}{2\sqrt{5} - \sqrt{3}}$ ?	<b>1</b> <a href="#">Solution</a>
	(A) $\frac{2\sqrt{5} - \sqrt{3}}{7}$ (B) $\frac{2\sqrt{5} + \sqrt{3}}{7}$ (C) $\frac{2\sqrt{5} - \sqrt{3}}{17}$ (D) $\frac{2\sqrt{5} + \sqrt{3}}{17}$	
	NESA 2012 Mathematics HSC Examination	

**11 1b** M Simplify  $\frac{n^2 - 25}{n - 5}$ .

**1** [Solution](#)  


NESA 2011 Mathematics HSC Examination

**11 1f** M Rationalise the denominator of  $\frac{4}{\sqrt{5} - \sqrt{3}}$ . Give your answer in the simplest form.

**2** [Solution](#)  


NESA 2011 Mathematics HSC Examination

**11 9d** M (i) Rationalise the denominator in the expression  $\frac{1}{\sqrt{n} + \sqrt{n+1}}$ ,

**1** [Solution](#)  


where  $n$  is an integer and  $n \geq 1$ .

(ii) Using your result from part (i), or otherwise, find the value of the sum

**2** 

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{99} + \sqrt{100}}.$$

NESA 2011 Mathematics HSC Examination

**10 1a** M Solve  $x^2 = 4x$ .

**2** [Solution](#)  


NESA 2010 Mathematics HSC Examination

**10 1b** M Find integers  $a$  and  $b$  such that  $\frac{1}{\sqrt{5} - 2} = a + b\sqrt{5}$ .

**2** [Solution](#)  


NESA 2010 Mathematics HSC Examination

**09 1b** M Solve  $\frac{5x - 4}{x} = 2$ .

**2** [Solution](#)  


NESA 2009 Mathematics HSC Examination

**08 1c** M Simplify  $\frac{2}{n} - \frac{1}{n+1}$ .

**2** [Solution](#)  


NESA 2008 Mathematics HSC Examination

**08 1e** M Expand and simplify  $(\sqrt{3} - 1)(2\sqrt{3} + 5)$ .

**2** [Solution](#)  


NESA 2008 Mathematics HSC Examination

**07 1c** M Rationalise the denominator of  $\frac{1}{\sqrt{3} - 1}$ .

**2** [Solution](#)  


NESA 2007 Mathematics HSC Examination

## Year 11: Functions

### F1.2 Introduction to functions

 Back**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

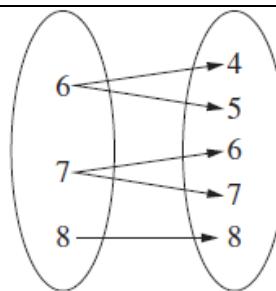
- define and use a function and a relation as mappings between sets, and as a rule or a formula that defines one variable quantity in terms of another
  - define a relation as any set of ordered pairs  $(x, y)$  of real numbers
  - understand the formal definition of a function as a set of ordered pairs  $(x, y)$  of real numbers such that no two ordered pairs have the same first component (or  $x$ -component)
- use function notation, domain and range, independent and dependent variables (ACMMM023) 
- understand and use interval notation as a way of representing domain and range, eg  $[4, \infty)$
- understand the concept of the graph of a function (ACMMM024)
- identify types of functions and relations on a given domain, using a variety of methods
  - know what is meant by one-to-one, one-to-many, many-to-one and many-to-many
  - use the vertical line test to identify a function
  - determine if a function is one-to-one (ACMSM094)
- define odd and even functions algebraically and recognise their geometric properties
- define the sum, difference, product and quotient of functions and consider their domains and ranges where possible
- define and use the composite function  $f(g(x))$  of functions  $f(x)$  and  $g(x)$  where appropriate
  - identify the domain and range of a composite function
- recognise that solving the equation  $f(x) = 0$  corresponds to finding the values of  $x$  for which the graph of  $y = f(x)$  cuts the  $x$ -axis (the  $x$ -intercepts)

[Reference Sheet](#)

<b>23</b>	<b>3</b>	What is the domain of $f(x) = \frac{1}{\sqrt{1-x}}$ ?	<b>1</b>	<a href="#">Solution</a>
MA				  
A. $x < 1$ B. $x \leq 1$ C. $x > 1$ D. $x \geq 1$				
NESA 2023 Mathematics Advanced HSC Examination				
<b>23</b>	<b>9</b>	Let $f(x)$ be any function with domain all real numbers.	<b>1</b>	<a href="#">Solution</a>
MA		Which of the following is an even function, regardless of the choice of $f(x)$ ?		  
A. $2f(x)$ B. $f(f(x))$ C. $f(-x)^2$ D. $f(x)f(-x)$				
NESA 2023 Mathematics Advanced HSC Examination				
<b>20</b>	<b>1</b>	Which inequality gives the domain of $\sqrt{2x-3}$ ?	<b>1</b>	<a href="#">Solution</a>
MA				  
A. $x < \frac{3}{2}$ B. $x > \frac{3}{2}$ C. $x \leq \frac{3}{2}$ D. $x \geq \frac{3}{2}$				
NESA 2020 Mathematics Advanced HSC Examination				

**MA**  
**SP****1**  
Band  
2-3

What type of relation is shown?

**1**[Solution](#)

A. Many-to-many

B. One-to-many

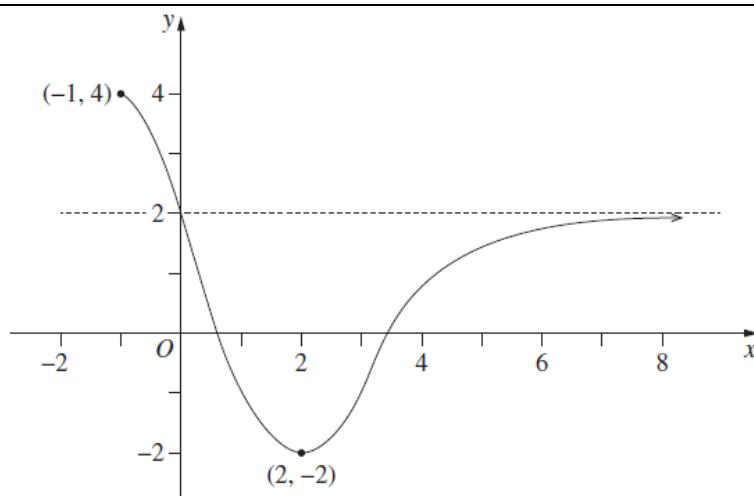
C. One-to-one

D. Many-to-one

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA**  
**SP****17**  
Band  
4-5Given the function  $f(x) = x^2 + 2$  and  $g(x) = \sqrt{x - 6}$ , sketch  $y = f(g(x))$  over its natural domain.**2**[Solution](#)

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA**  
**SP****28**  
Band  
3-4The graph of a function  $f(x)$  is shown.**2**[Solution](#)It has an asymptote at  $y = 2$ .Using interval notation, state the domain and range of  $f(x)$ .

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**TG**1 The function  $g(m) = m^2$  has the given domain  $1 \leq m \leq 7$ .  
What is the range?[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

**TG**

2 Sketch the graph of a relation that is not a function.

[Solution](#)

NENESA Mathematics Advanced SA Year 11 Topic Guide: Functions

**TG**

3 Find the domain and range of the functions:

[Solution](#)

- (a)  $f(x) = \sqrt{(x - 4)}$   
(b)  $g(t) = t^2 + 1$

NESA Mathematics Advanced Year 11 Topic Guide: Functions

**TG**4 Which of the following functions are even functions? What are odd functions?  
Which are neither even nor odd? Justify your answers.[Solution](#)

- (a)  $f(x) = 4x$   
(b)  $f(x) = 2x^7$   
(c)  $f(x) = x^3 + 5$   
(d)  $f(x) = x^3 + 3x^2$   
(e)  $f(x) = x^4 + 3x^2$   
(f)  $f(x) = \frac{6x^3}{x^2 + x^4}$   
(g)  $f(x) = 2^x$

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 5** For the functions listed below, use the horizontal line test to determine which are one-to-one functions.
- (a)  $f(x) = 4x$   
 (b)  $f(x) = 2x^7$   
 (c)  $f(x) = x^3 + 5$   
 (d)  $f(x) = x^3 + 3x^2$   
 (e)  $f(x) = x^4 + 3x^2$   
 (f)  $f(x) = \frac{6x^3}{x^2 + x^4}$   
 (g)  $f(x) = 2^x$



NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 6** A function has a domain of  $-2 \leq x \leq 2$ , and a range of  $1 \leq f(x) \leq 6$ .  
 It is also known that  $f(x)$  is an even function.  
 Draw a possible graph of  $y = f(x)$ .



NESA Mathematics Advanced Year 11 Topic Guide: Functions

- 17 M 11 h** Find the domain of the function  $f(x) = \sqrt{3 - x}$ .
- 16 M 4** Which diagram best shows the graph of an odd function?  
 (A)   
 (B)   
 (C)   
 (D)

**2** [Solution](#)

NESA 2017 Mathematics HSC Examination

- 13 M 3** Which inequality defines the domain of the function  $f(x) = \frac{1}{\sqrt{x+3}}$ ?  
 (A)  $x > -3$       (B)  $x \geq -3$       (C)  $x < -3$       (D)  $x \leq -3$

**1** [Solution](#)

NESA 2013 Mathematics HSC Examination

- 10 M 1g** Let  $f(x) = \sqrt{x-8}$ . What is the domain of  $f(x)$ ?

**1** [Solution](#)

NESA 2010 Mathematics HSC Examination

**Year 11: Functions****F1.3 Linear, quadratic and cubic functions**[Back](#)**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- model, analyse and solve problems involving linear functions AAM  

  - recognise that a direct variation relationship produces a straight-line graph
  - explain the geometrical significance of  $m$  and  $c$  in the equation  $f(x) = mx + c$
  - derive the equation of a straight line passing through a fixed point  $(x_1, y_1)$  and having a given gradient  $m$  using the formula  $y - y_1 = m(x - x_1)$
  - derive the equation of a straight line passing through two points  $(x_1, y_1)$  and  $(x_2, y_2)$  by first calculating its gradient  $m$  using the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$
  - understand and use the fact that parallel lines have the same gradient and that two lines with gradient  $m_1$  and  $m_2$  respectively are perpendicular if and only if  $m_1 m_2 = -1$
  - find the equations of straight lines, including parallel and perpendicular lines, given sufficient information (ACMMM004) 

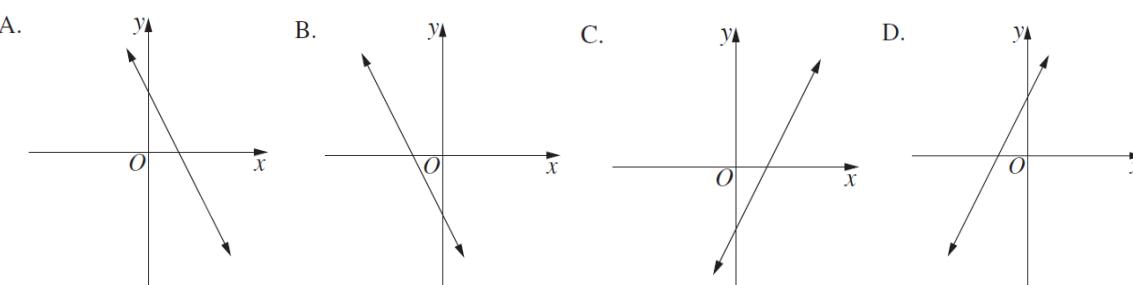
- model, analyse and solve problems involving quadratic functions AAM   

  - recognise features of the graph of a quadratic, including its parabolic nature, turning point, axis of symmetry and intercepts (ACMMM007)
  - find the vertex and intercepts of a quadratic graph by either factorising, completing the square or solving the quadratic equation as appropriate
  - understand the role of the discriminant in relation to the position of the graph 
  - find the equation of a quadratic given sufficient information (ACMMM009)

- solve practical problems involving a pair of simultaneous linear and/or quadratic functions algebraically and graphically, with or without the aid of technology; including determining and interpreting the break-even point of a simple business problem AAM  

  - understand that solving  $f(x) = k$  corresponds to finding the values of  $x$  for which the graph  $y = f(x)$  cuts the line  $y = k$

- recognise cubic functions of the form:  $f(x) = kx^3$ ,  $f(x) = k(x - b)^3 + c$  and  $f(x) = k(x - a)(x - b)(x - c)$ , where  $a$ ,  $b$ ,  $c$  and  $k$  are constants, from their equation and/or graph and identify important features of the graph 

**S  
T  
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N**[Reference Sheet](#)**22 1** Which of the following could be the graph of  $y = -2x + 2$ ?**MA****1** [Solution](#)**COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations**

- 22 4** Which of the following is the range of the function  $f(x) = x^2 - 1$ ?

- MA**
- $[-1, \infty)$
  - $(-\infty, 1]$
  - $[-1, 1]$
  - $(-\infty, \infty)$

**1** [Solution](#)

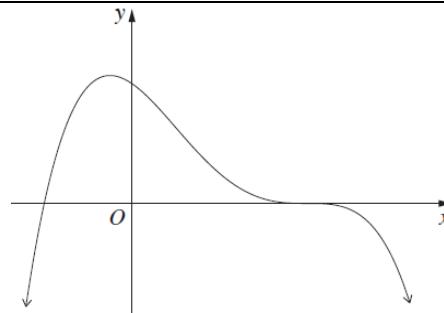


NESA 2022 Mathematics Advanced HSC Examination

- 21 8** The graph of  $y = f(x)$  is shown.

- MA** Which of the following could be the equation of this graph?

- $y = (1 - x)(2 + x)^3$
- $y = (x + 1)(x - 2)^3$
- $y = (x + 1)(2 - x)^3$
- $y = (x - 1)(2 + x)^3$



**1** [Solution](#)



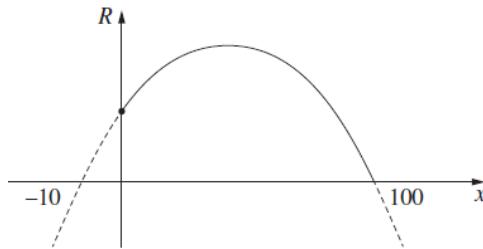
NESA 2021 Mathematics Advanced HSC Examination

- 21 35** A publisher sells a book for \$10. At this price, 5000 copies of the book will be sold and the revenue raised will be  $5000 \times 10 = \$50\,000$ . The publisher is considering increasing the price of the book. For every dollar the price of the book is increased, the publisher will sell 50 fewer copies of the book. If the publisher charges  $(10 + x)$  dollars for each book, a quadratic model for the revenue raised,  $R$ , from selling the books is  $R = -50x^2 + 4500x + 50\,000$ .

[Solution](#)

A graph of this quadratic model for revenue is shown. A dashed line is used for values of  $x$  which are not relevant to the practical context of this problem.

- By first finding a suitable value of  $x$ , find the price the publisher should charge for each book to maximise the revenue raised from sales of the book.
- Find the value of the intercept of the parabola with the vertical axis.



**2**



**1**



NESA 2021 Mathematics Standard 2 HSC Examination

- 21** **34** In a park the only animals are goannas and emus.  
**MS**  
**2**

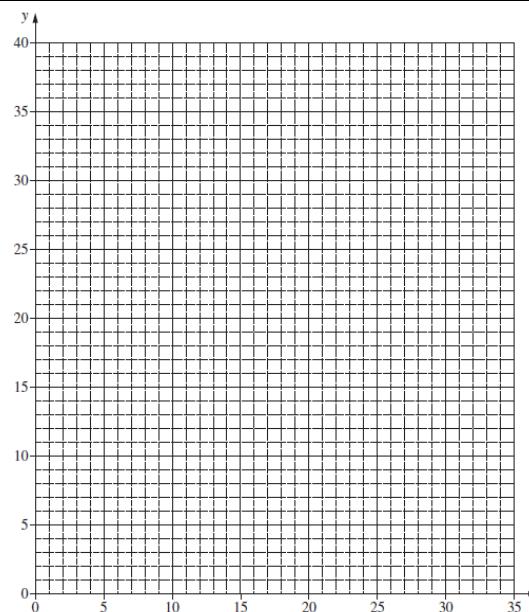
Let  $x$  be the number of goannas and let  $y$  be the number of emus.

The number of goannas plus the number of emus in the park is 31.  
Hence  $x + y = 31$ .

Each goanna has four legs and each emu has two legs.

In total the emus and goannas have 76 legs.

By writing another relevant equation and graphing both equations on the grid on the following page, find the number of goannas and the number of emus in the park.



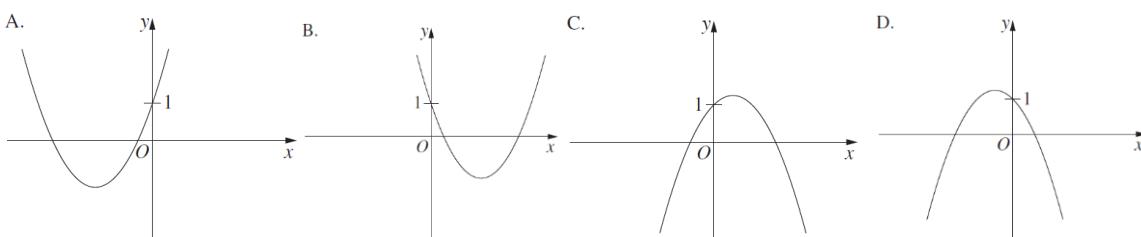
Number of goannas = .....

Number of emus = .....

NESA 2021 Mathematics Standard 2 HSC Examination

- 20** **5** Which of the following could represent the graph of  $y = -x^2 + bx + 1$ , where  $b > 0$ ? **MA**

**1** [Solution](#)



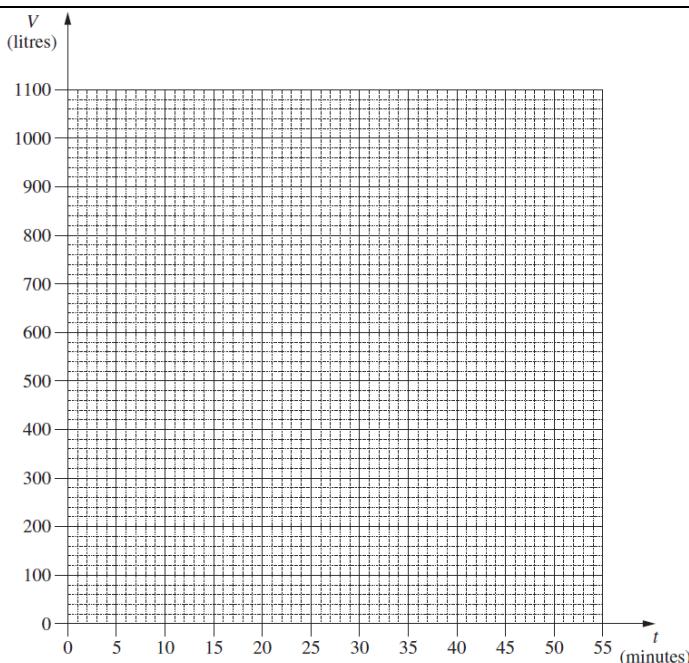
NESA 2020 Mathematics Advanced HSC Examination

- 20 11 MA** There are two tanks on a property, Tank A and Tank B. Initially, Tank A holds 1000 litres of water and Tank B is empty.

- (a) Tank A begins to lose water at a constant rate of 20 litres per minute.

The volume of water in Tank A is modelled by  $V = 1000 - 20t$  where  $V$  is the volume in litres and  $t$  is the time in minutes from when the tank begins to lose water.

On the grid below, draw the graph of this model and label it as Tank A.



1



- (b) Tank B remains empty until  $t = 15$  when water is added to it at a constant rate of 30 litres per minute.

By drawing a line on the grid on the previous page, or otherwise, find the value of  $t$  when the two tanks contain the same volume of water.

- (c) Using the graphs drawn, or otherwise, find the value of  $t$  (where  $t > 0$ ) when the total volume of water in the two tanks is 1000 litres.

2



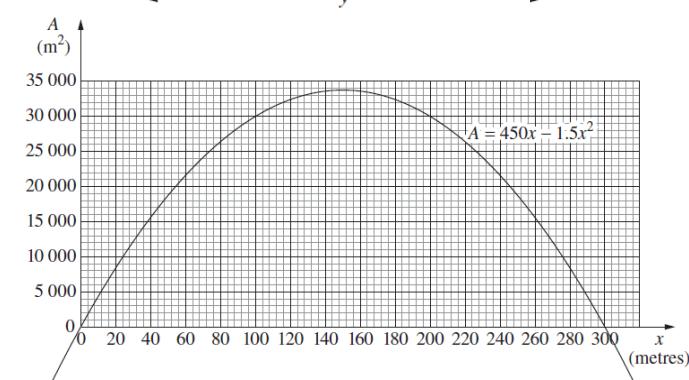
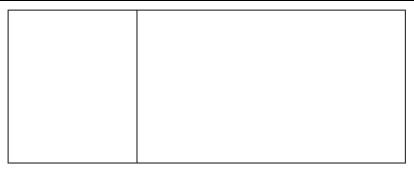
**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

- 20 19 MS 2** A fence is to be built around the outside of a rectangular paddock. An internal fence is also to be built.

The side lengths of the paddock are  $x$  metres and  $y$  metres, as shown in the diagram.

A total of 900 metres of fencing is to be used. Therefore  $3x + 2y = 900$ .

The area,  $A$ , in square metres, of the rectangular paddock is given by  $A = 450x - 1.5x^2$ . The graph of this equation is shown.

**Solution**

- (a) If the area of the paddock is  $30\ 000\ m^2$ , what is the largest possible value of  $x$ ?

1



- (b) Find the values of  $x$  and  $y$  so that the area of the paddock is as large as possible.

2



- (c) Using your values from part (b), find the largest possible area of the paddock.

1

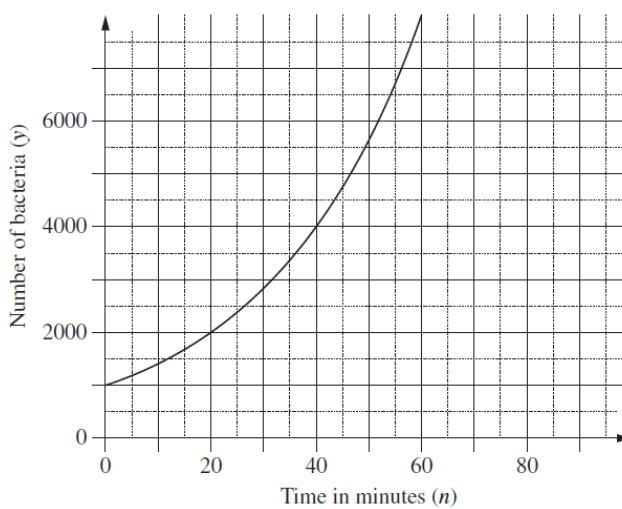


NESA 2020 Mathematics Standard 2 HSC Examination

- 20 33** The graph shows the number of bacteria,  $y$ , at time  $n$  minutes.  
**MS 2** Initially (when  $n = 0$ ) the number of bacteria is 1000.

- (a) Find the number of bacteria at 40 minutes.  
(b) The number of bacteria can be modelled by the equation  $y = A \times b^n$ , where  $A$  and  $b$  are constants.

Use the guess and check method to find, to two decimal places, an upper and lower estimate for the value of  $b$ . The upper and lower estimates must differ by 0.01.

**Solution****1****2**

- MA 3** What is the gradient of any line perpendicular to  $3x + 2y = 5$ ?

**SP Band 3-4**

- A.  $\frac{2}{3}$       B.  $-\frac{2}{3}$       C.  $\frac{3}{2}$       D.  $-\frac{3}{2}$

**1****Solution**

- MA 15** Two teams play a game. There are only two possible ways to score points: hitting the red target or hitting the blue target.

**SP Band 3-5**

Hitting the red target scores  $R$  points and hitting the blue target scores  $B$  points. The results of a game are shown.

By forming a pair of simultaneous equations, or otherwise, find the values of  $R$  and  $B$ .

NESA Mathematics Advanced Sample Examination Paper (2020)

Team	Number of red target hits	Number of blue target hits	Total score
Team 1	7	6	47
Team 2	5	11	47

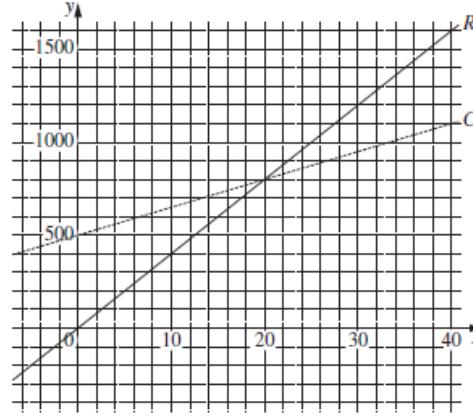
**3****Solution**

- MA 22** A small business makes and sells bird houses. Technology was used to draw straight-line graphs to represent the cost of making bird houses ( $C$ ) and the revenue from selling bird house ( $R$ ). The  $x$ -axis displays the number of bird houses and the  $y$ -axis displays the cost/revenue in dollars.

**SP Band 2-5**

- 19 36** (a) How many bird houses need to be sold to break even?  
(b) By first forming equations for cost ( $C$ ) and revenue ( $R$ ), determine how many bird houses need to be sold to earn a profit of \$1900.

NESA Mathematics Advanced Sample Examination Paper (2020)

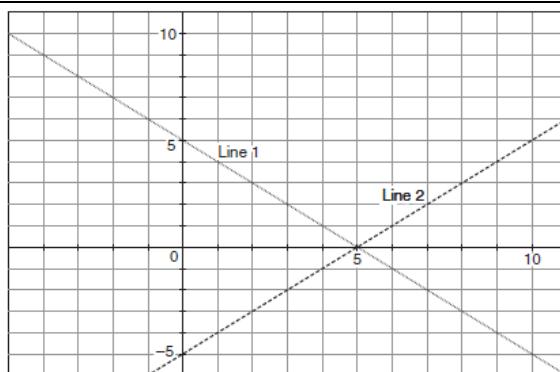
**Solution****1****3**

NESA Mathematics Advanced Sample Examination Paper (2020)

NESA 2019 Mathematics Standard 2 HSC Examination

**MA  
SQ  
2019**

- 1** Band 2-3 A computer application was used to draw the graphs of the equations  $x + y = 5$  and  $x - y = 5$ . Part of the screen is shown. Which row of the table correctly matches the equations with the lines drawn and identifies the solution when the equations are solved simultaneously?

**1**[Solution](#)

	$x + y = 5$	$x - y = 5$	<i>Solution</i>
A.	Line 1	Line 2	$x = 5, y = 0$
B.	Line 1	Line 2	$x = 5, y = 5$
C.	Line 2	Line 1	$x = 5, y = 0$
D.	Line 2	Line 1	$x = 5, y = 5$

NESA Mathematics Advanced Sample examination materials (2019)

- TG 1** The income from selling raffle tickets for Joan's club varies directly with the number of tickets sold. If Joan has collected \$75 from the sale of 30 tickets, how much would she collect from the sale of 50 tickets?

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 2** Find the equation of the line that has a gradient of 3 and passes through the point  $(2, -5)$  and hence determine if the point  $(5, 8)$  lies on the line?

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 3** Find the equation of the line that passes through the points  $(-2, 4)$  and  $(1, 12)$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 4** Find the equation of the line that is parallel to the line  $2x + 6y - 7 = 0$  and crosses the  $x$ -axis at  $x = 3$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 5** Sketch the graph of the parabola  $y = x^2 - 4x - 5$ , indicating the coordinates of the vertex and the intercepts on the axes.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 6** Find the maximum value of  $y$  given  $y = 8 + 2x - x^2$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 7** Show that the four points  $(0, 0)$ ,  $(2, 1)$ ,  $(3, -1)$ ,  $(1, -2)$  are the corners of a square.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 8** Given that  $A$ ,  $B$  and  $C$  are the points  $(-1, -2)$ ,  $(2, 5)$  and  $(4, 1)$  respectively, find  $D$  so that  $ABCD$  is a parallelogram.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 9** Find the coordinates of the point  $A$  on the line  $x = -3$ , such that the line joining  $A$  to  $B(3, 5)$  is perpendicular to the line  $2x + 5y = 12$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 10** Find the range of values of  $k$  for which the expression  $x^2 - 2x + (3 - 2k)$  is always positive.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 11** It is estimated that a certain river can sustain 80 000 fish if there is no pollution, and that for each tonne of pollution in the river, 2500 fewer fish can be supported. Assuming a linear relationship, write an equation linking the number of fish ( $y$ ) and the number of tonnes of pollution ( $x$ ) and represent this relationship graphically.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 12** Peter owns a publishing company and sells calendars for \$30 each. The cost of producing these calendars includes a set-up cost of \$5000 and additional costs of \$5 per calendar. What is his break-even point?



NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 13** Margaret is obtaining quotes for a celebration. Caterer A charges \$500 plus \$20 per guest, while Caterer B charges \$200 plus \$80 per guest. Represent these costs both algebraically and graphically and describe how the prices of the two companies compare.



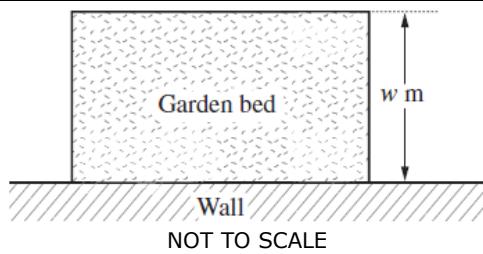
NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 14** Shalma uses a straight-line depreciation model of 9% of the original price per annum to depreciate the cost of furniture originally valued at \$4000. Construct a function to represent the value of the furniture after  $x$  years, and state the domain and range of the function.



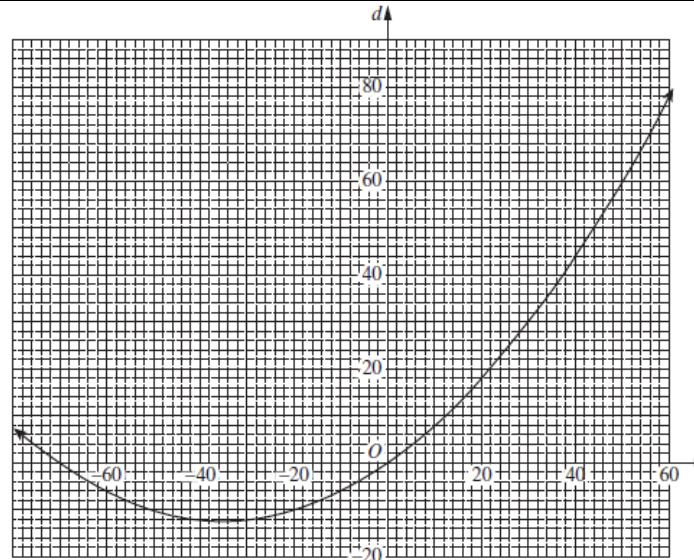
NESA Mathematics Advanced Year 11 Topic Guide: Functions

- MS AL** A gardener has created a rectangular garden bed with 24 metres of fencing, using a wall as one side of its bed. The plan for the garden bed is shown, where  $w$  metres is the width. Which equation gives the area,  $A$ , of the garden bed?  
**SQ 10**
- A.  $A = 12w - w^2$       B.  $A = 12w - 2w^2$   
 C.  $A = 24w - w^2$       D.  $A = 24w - 2w^2$

**1**

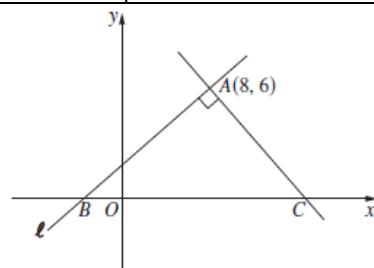
NESA Mathematics Standard 2 Sample examination materials

- MS AL** A student is investigating stopping distances for a car travelling at different speeds. To model this, the student uses the equation  $d = 0.01s^2 + 0.7s$ , where  $d$  is the stopping distance in metres and  $s$  is the speed of the car in km/h. The graph of this equation is drawn.  
**SQ 21**
- (a) The student knows that only part of this curve applies to the model for stopping distances. On the graph drawn, show or highlight clearly the part of this curve that applies for stopping distances.  
 (b) What is the difference between the stopping distances when travelling at a speed of 40 km/h and when travelling at a speed of 70 km/h?

**1****1**

NESA Mathematics Standard 2 Sample examination materials

- 19 12** The line  $\ell$ , with equation  $x - 2y + 4 = 0$ ,  
**M a** passes through the point  $A(8, 6)$  and intersects the  $x$ -axis at the point  $B$ . The line through  $A$  perpendicular to  $\ell$  intersects the  $x$ -axis at the point  $C$ .  
 (i) Find the equation of the line  $AC$ .  
 (ii) Find the area of triangle  $ABC$ .

**Solution****2**

NESA 2019 Mathematics HSC Examination

- 19 14** Last Saturday, Luke had 165 followers on social media. Rhys had 537 followers. On average, Luke gains another 3 followers per day and Rhys loses 2 followers per day.

**1** [Solution](#)



If  $x$  represents the number of days since last Saturday and  $y$  represents the number of followers, which pair of equations model this situation?

- |                         |                         |
|-------------------------|-------------------------|
| A. Luke: $y = 165x + 3$ | B. Luke: $y = 165 + 3x$ |
| Rhys: $y = 537x - 2$    | Rhys: $y = 537 - 2x$    |
| C. Luke: $y = 3x + 165$ | D. Luke: $y = 3 + 165x$ |
| Rhys: $y = 2x - 537$    | Rhys: $y = 2 - 537x$    |

NESA 2019 Mathematics Standard 2 HSC Examination

- 19 31** A rectangle has width  $w$  centimetres. The area of the rectangle,  $A$ , in square centimetres, is  $A = 2w^2 + 5w$ .

[Solution](#)

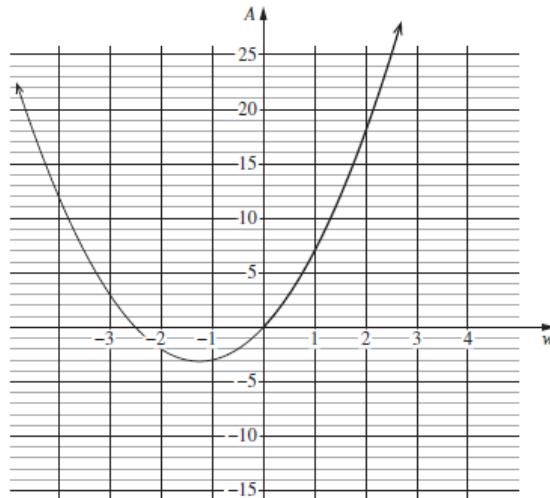
The graph of  $A = 2w^2 + 5w$  is shown.  
(a) Explain why, in this context, the model  $A = 2w^2 + 5w$  only makes sense for the bold section of the graph.

**1**



- (b) The area of the rectangle is  $18 \text{ cm}^2$ . Calculate the perimeter of the rectangle.

**2**



NESA 2019 Mathematics Standard 2 HSC Examination

- 19 34** The relationship between British pounds ( $p$ ) and Australian dollars ( $d$ ) on a particular day is shown in the graph.

[Solution](#)

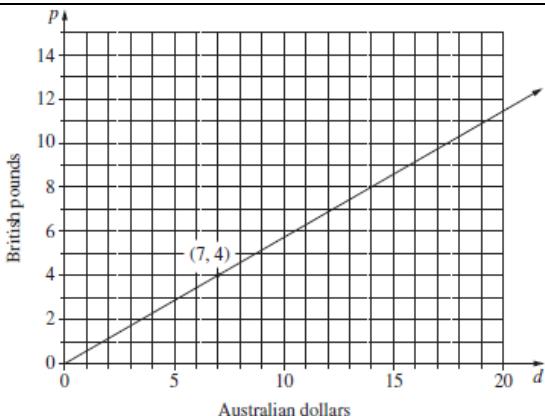
- (a) Write the direct variation equation British pounds to Australian dollars in the form  $p = md$ . Leave  $m$  as a fraction  
(b) The relationship between Japanese yen ( $y$ ) and Australian dollars on the same day is given by the equation  $y = 76d$ .

**1**



Convert 93 100 Japanese yen to British pounds.

**2**



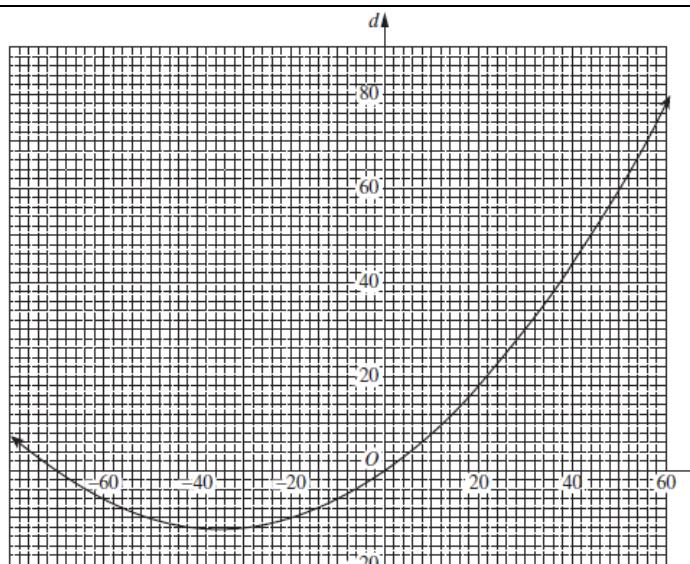
NESA 2019 Mathematics Standard 2 HSC Examination

- SQ** **AL** A student is investigating  
**MS** **21** stopping distances for a car  
**2** travelling at different speeds.  
 To model this, the student

- 09** **28** uses the equation  
**MG** **a**  $d = 0.01s^2 + 0.7s$ , where  
 $d$  is the stopping distance in  
 metres and  $s$  is the speed of  
 the car in km/h. The graph of  
 this equation is drawn.

(a) The student knows that  
 only part of this curve applies  
 to the model for stopping  
 distances. On the graph  
 drawn, show or highlight  
 clearly the part of this curve  
 that applies for stopping  
 distances.

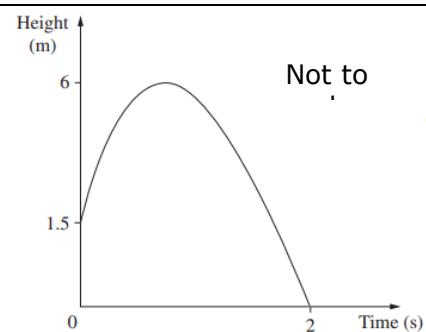
(b) What is the difference between the stopping distances when travelling at a  
 speed of 40 km/h and when travelling at a speed of 70 km/h?

[Solution](#)**1****1**

NESA Mathematics Standard 2 Sample examination materials

NESA 2009 Mathematics General HSC Examination

- SQ** **AL** Lucy throws a ball vertically into the air.  
**MS** **22** The graph shows the height of the ball above  
**2** the ground at time  $t$  seconds.  
 How far has the ball travelled by the time it  
 hits the ground?

**2**

NESA Mathematics Standard 2 Sample examination materials

- 18** **3** What is the  $x$ -intercept of the line  $x + 3y + 6 = 0$ ?  
**M** A.  $(-6, 0)$  B.  $(6, 0)$  C.  $(0, -2)$  D.  $(0, 2)$

NESA 2018 Mathematics HSC Examination

**1**

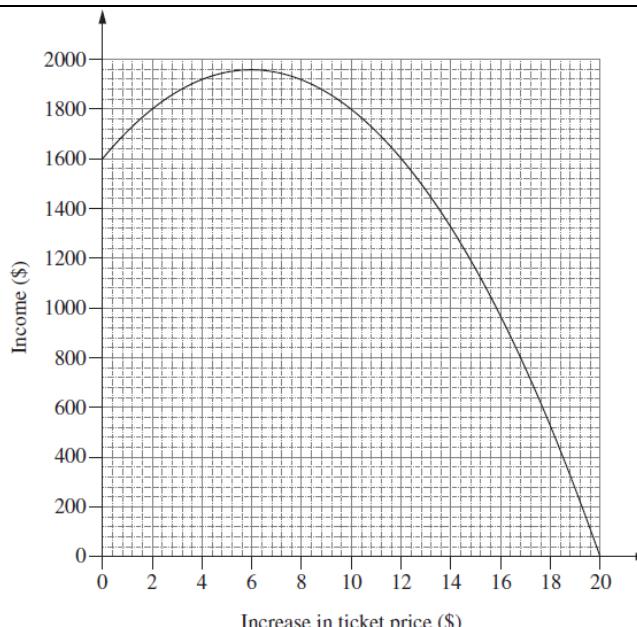
- 17** **1** What is the gradient of  $2x + 3y + 4 = 0$ ?  
**M** (A)  $-\frac{2}{3}$  (B)  $\frac{2}{3}$  (C)  $-\frac{3}{2}$  (D)  $\frac{3}{2}$

NESA 2017 Mathematics HSC Examination

**1**

- 17 28 MG e** A movie theatre has 200 seats. Each ticket currently costs \$8. The theatre owners are currently selling all 200 tickets for each session. They decide to increase the price of tickets to see if they can increase the income earned from each movie session. It is assumed that for one dollar increase in ticket price, there will be 10 fewer tickets sold. A graph showing the relationship between an increase in ticket prices and the income is shown.

- (i) What ticket price should be charged to maximise the income from a movie session?  
(ii) What is the number of tickets sold when the income is maximised?  
(iii) The cost to the theatre owners of running each session is \$500 plus \$2 per ticket sold. Calculate the profit earned by the theatre owners when the income from a session is maximized.



- 1   
1   
2

NESA 2017 Mathematics General HSC Examination

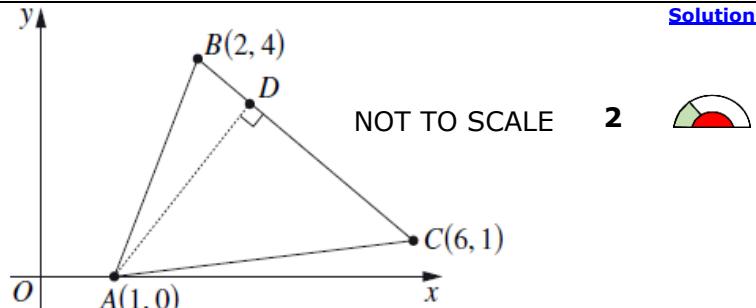
- 16 11 M e** Find the points of intersection of  $y = -5 - 4x$  and  $y = 3 - 2x - x^2$ .

NESA 2016 Mathematics HSC Examination

**3** [Solution](#)

- 16 12 M a** The diagram shows points  $A(1, 0)$ ,  $B(2, 4)$  and  $C(6, 1)$ . The point  $D$  lies on  $BC$  such that  $AD \perp BC$ .  
(i) Show that the equation of  $BC$  is  $3x + 4y - 22 = 0$ .

(ii) and (iii) not in Maths Adv course



NESA 2016 Mathematics HSC Examination

**2** [Solution](#)

- 15 2 M** What is the slope of the line with equation  $2x - 4y + 3 = 0$ ?

**1** [Solution](#)

- (A) -2      (B)  $-\frac{1}{2}$       (C)  $\frac{1}{2}$       (D) 2

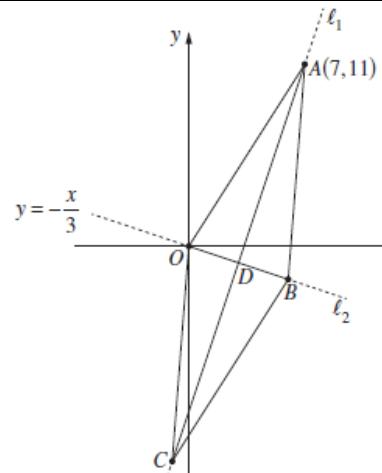
NESA 2015 Mathematics HSC Examination

- 15 12** The diagram shows the rhombus  $OABC$ . The  
**M b** diagonal from the point  $A(7, 11)$  to the point  $C$   
lies on the line  $\ell_1$ . The other diagonal, from the  
origin  $O$  to the point  $B$ , lies on the line  $\ell_2$

which has equation  $y = -\frac{x}{3}$ .

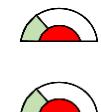
- Show that the equation of the line  $\ell_1$  is  $y = 3x - 10$ .
- The lines  $\ell_1$  and  $\ell_2$  intersect at the point  $D$ . Find the coordinates of  $D$ .

Not to scale



[Solution](#)

2  
2



NESA 2015 Mathematics HSC Examination

- 15 12** For what values of  $k$  does the quadratic equation  $x^2 - 8x + k = 0$  have real roots?

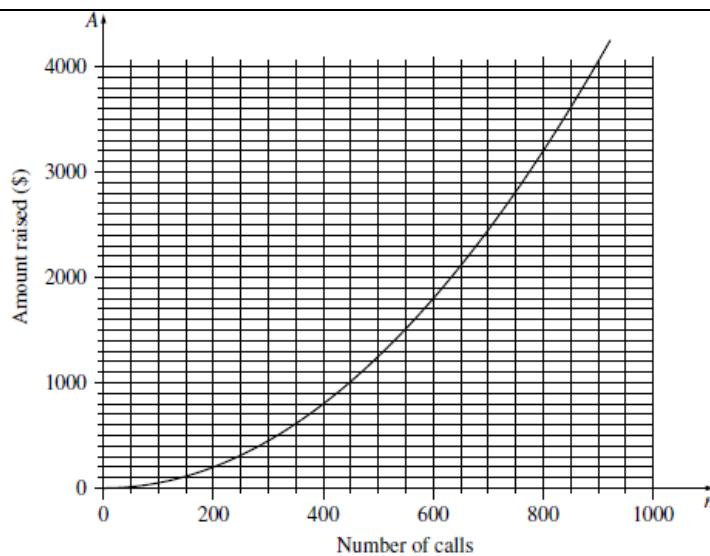
NESA 2015 Mathematics HSC Examination

2

[Solution](#)



- 15 28f** A charity seeks to raise money by telephoning people at random from a call centre and asking them to donate. Over the years, this charity has found that the amount of money raised ( $\$A$ ) is related to the number of telephone calls made ( $n$ ). A graph of this relationship is shown. It costs the charity \$2100 per week to run the call centre. It also costs an average of 50 cents per telephone call.



- Write an equation to represent the total cost,  $C$ , of running the call centre for a week in which  $n$  phone calls are made.
- By graphing this equation on the axes above, determine the number of phone calls the charity needs to make in order to break even?

NESA 2015 Mathematics General HSC Examination

1

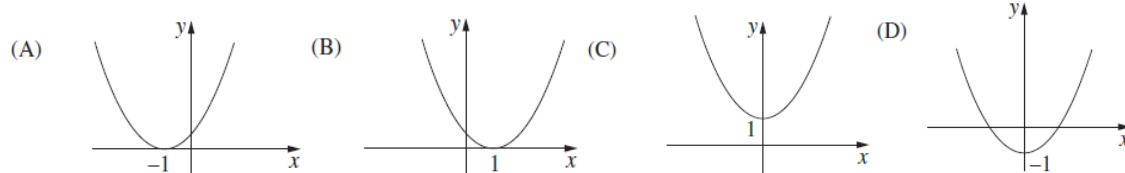


2



- 14 2** Which graph best represents  $y = (x - 1)^2$ ?

**M**



1



- 14 5** Which equation represents the line perpendicular to  $2x - 3y = 8$ , passing through the point  $(2, 0)$ ?

- (A)  $3x + 2y = 4$     (B)  $3x + 2y = 6$     (C)  $3x - 2y = -4$     (D)  $3x - 2y = 6$

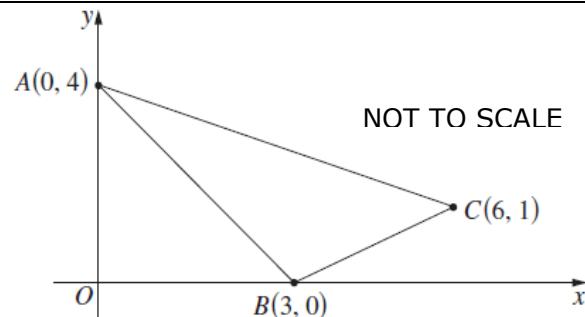
1



NESA 2014 Mathematics HSC Examination

- 14 12** The points  $A(0, 4)$ ,  $B(3, 0)$  and  $C(6, 1)$  form a triangle, as shown in the diagram.

- (i) Show that the equation of  $AC$  is  $x + 2y - 8 = 0$ .  
 (ii) and (iii) not in Maths Adv course

[Solution](#)

2

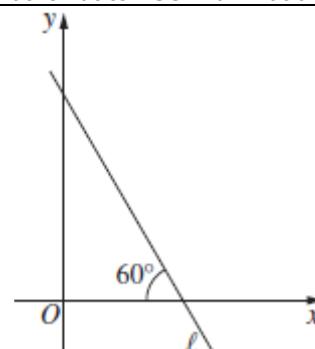


- 13 2** The diagram shows the line  $\ell$ . What is the slope of the line  $\ell$ ?

- (A)  $\sqrt{3}$   
 (B)  $-\sqrt{3}$   
 (C)  $\frac{1}{\sqrt{3}}$   
 (D)  $-\frac{1}{\sqrt{3}}$

NES 2014 Mathematics HSC Examination

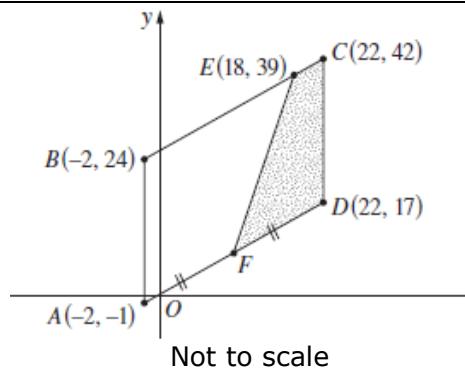
1

[Solution](#)

NES 2013 Mathematics HSC Examination

- 13 12** The points  $A(-2, -1)$ ,  $B(-2, 24)$ ,  $C(22, 42)$  and  $D(22, 17)$  form a parallelogram as shown. The point  $E(18, 39)$  lies on  $BC$ . The point  $F$  is the midpoint of  $AD$ .

- (i) Show that the equation of the line through  $A$  and  $D$  is  $3x - 4y + 2 = 0$ .  
 (ii) – (iv) not in Maths Adv course



2



Not to scale

NES 2013 Mathematics HSC Examination

**13 30** Wind turbines are used to generate power.

[Solution](#)

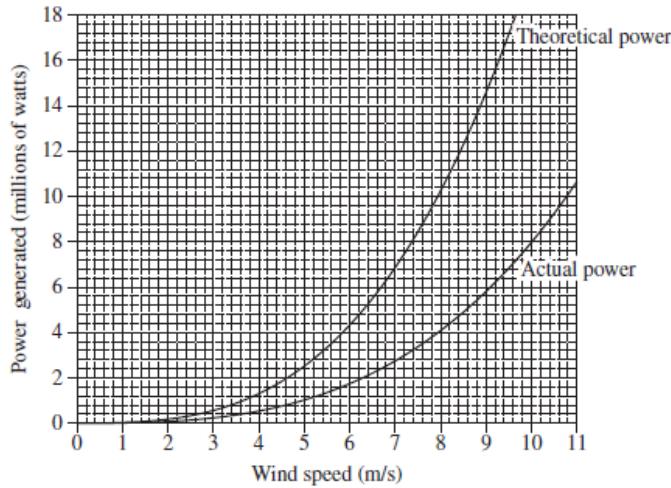
**MG a** In theory, the power that could be generated by a wind turbine is modelled using the equation  $T = 20 000w^3$

where  $T$  is the theoretical power generated, in watts  
 $w$  is the speed of the wind, in metres per second.

- (i) Using this equation, what is the theoretical power generated by a wind turbine of the wind speed is 7.3 m/s? **1**
- (ii) In practice, the actual power generated by a wind turbine is only 40% of the theoretical power. If  $A$  is the actual power generated, in watts, write an equation for  $A$  in terms of  $w$ . **1**

The graph shows both the theoretical power generated and the actual power generated by a particular wind turbine.

- (iii) Using the graph, or otherwise, find the difference between the theoretical power and the actual power generated when the wind speed is 9 m/s. **1**
- (iv) A particular farm requires at least 4.4 million watts of actual power to be self-sufficient. What is the minimum wind speed required for the farm to be self-sufficient? **1**
- (v) A more accurate formula to calculate the power ( $P$ ) generated by a wind turbine is  $P = 0.61 \times \pi \times r^2 \times w^3$  where  $r$  is the length of each blade, in metres  
 $w$  is the speed of the wind, in metres per second. **3**



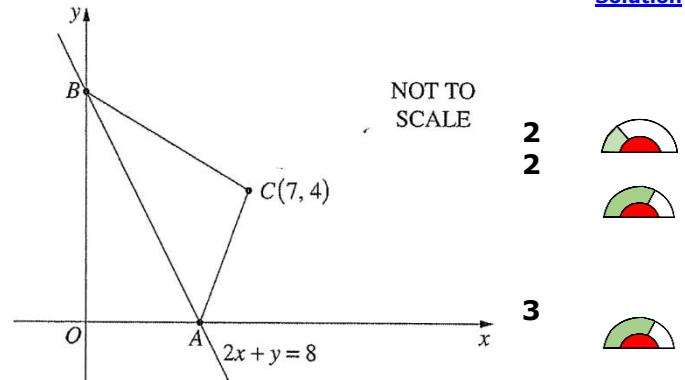
**12 13** The diagram shows a triangle  $ABC$ . The line

[Solution](#)

**M a**  $2x + y = 8$  meets the  $x$  and  $y$  axes at the points  $A$  and  $B$  respectively. The point  $C$  has coordinates  $(7, 4)$ .

- (i) Calculate the distance  $AB$ . **2**
- (ii) It is known that  $AC = 5$  and  $BC = \sqrt{65}$ . (Do NOT prove this.) Calculate the size of  $\angle ABC$  to the nearest degree. **2**
- (iii) The point  $N$  lies on  $AB$  such that  $CN$  is perpendicular to  $AB$ . Find the coordinates of  $N$ . **3**

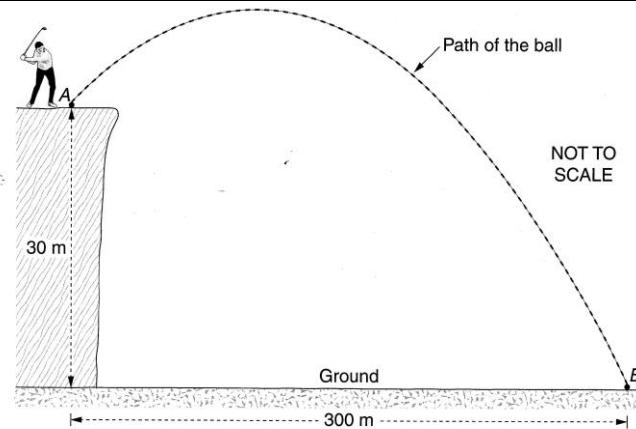
NESA 2013 Mathematics General HSC Examination



NESA 2012 Mathematics HSC Examination

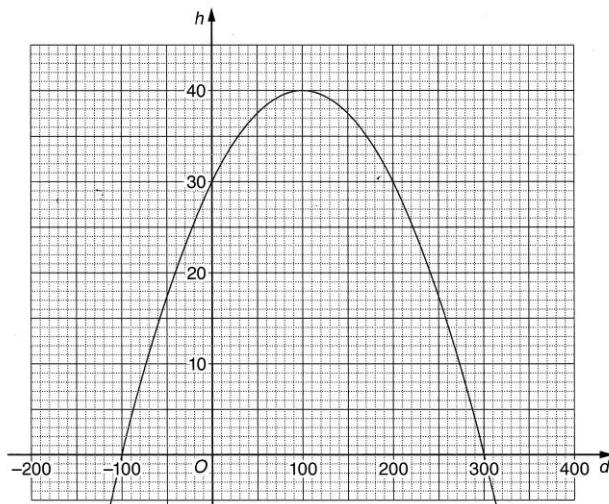
- 12 30 MG b** A golf ball is hit from point  $A$  to point  $B$ , which is on the ground as shown. Point  $A$  is 30 metre above the ground and the horizontal distance from point  $A$  to point  $B$  is 300 m. The path of the ball is modelled using the equation

$$h = 30 + 0.2d - 0.001d^2$$
 where  $h$  is the height of the golf ball above the ground in metres, and  $d$  is the horizontal distance of the golf ball from point  $A$  in metres.



The graph of this equation is drawn opposite.

- What is the maximum height the ball reaches above the ground?
- There are two occasions when the golf ball is at a height of 35 metres. What horizontal distance does the ball travel in the period between these two occasions?
- What is the height of the ball above the ground when it still has to travel a horizontal distance of 50 metres to hit the ground at point  $B$ ?
- Only part of the graph applies to this model. Find all values of  $d$  that are not suitable to use with this model, and explain why these values are not suitable.



**1**

**1**

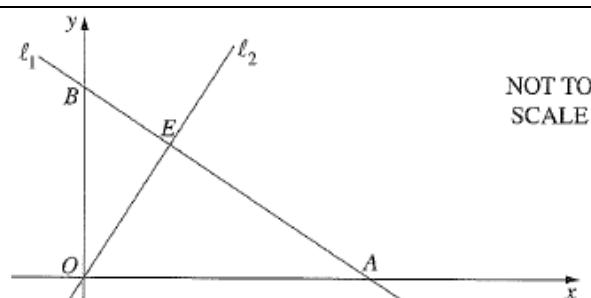
**1**

**2**

- 11 3c M** The diagram shows a line  $\ell_1$ , with equation  $3x + 4y - 12 = 0$ , which intersects the  $y$ -axis at  $B$ . A second line  $\ell_2$ , with equation  $4x - 3y = 0$ , passes through the origin  $O$  and intersects  $\ell_1$  at  $E$ .

- Show that the co-ordinates of  $B$  are  $(0, 3)$
- Show that  $\ell_1$  is perpendicular to  $\ell_2$ .

(iii) – (v) Not in Maths Adv course



**Solution**

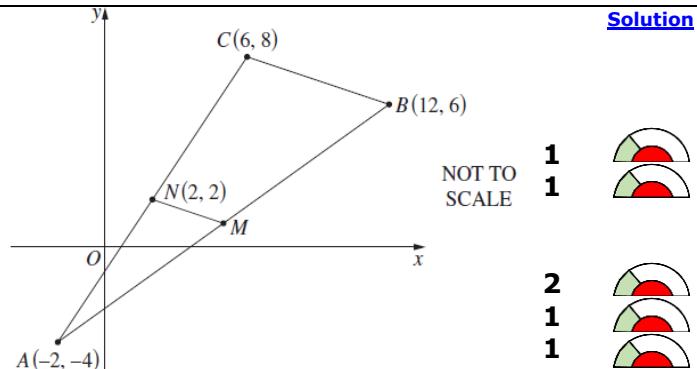
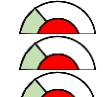
**1**

**2**

- 10 3a** In the diagram,  $A$ ,  $B$  and  $C$  are the points  $(-2, -4)$ ,  $(12, 6)$  and  $(6, 8)$  respectively.

The point  $N(2, 2)$  is the midpoint of  $AC$ . The point  $M$  is the midpoint of  $AB$ .

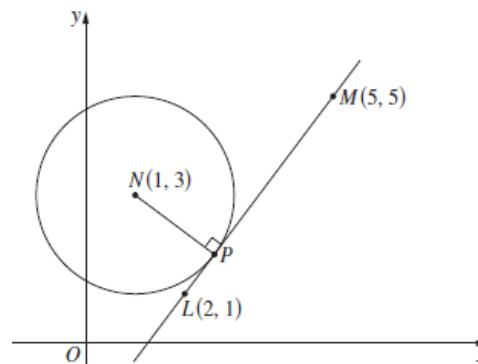
- Find the coordinates of  $M$ .
  - Find the gradient of  $BC$ .
  - Prove that  $\triangle ABC$  is similar to  $\triangle AMN$ .
- (iii) Not in Maths Adv course
- Find the equation of  $MN$ .
  - Find the exact length of  $BC$ .
  - Given that the area of  $\triangle ABC$  is 44 square units, find the perpendicular distance from  $A$  to  $BC$ .

[Solution](#)1  
12  
1  
1

- 09 3b** The circle in the diagram has centre  $N$ . The line  $LM$  is tangent to the circle at  $P$ .

- Find the equation of  $LM$  in the form  $ax + by + c = 0$ .
- (ii), (iii) Not in Maths Adv course

NES 2010 Mathematics HSC Examination

[Solution](#)

2



- 09 4b** Find the values of  $k$  for which the quadratic equation  $x^2 - (k + 4)x + (k + 7) = 0$  has equal roots.

3

[Solution](#)

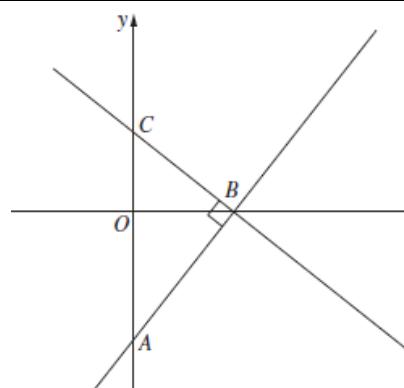
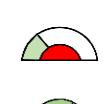
- 09 5a** In the diagram, the points  $A$  and  $C$  lie on the  $y$ -axis and the point  $B$  lies on the  $x$ -axis. The line  $AB$  has equation  $y = \sqrt{3}x - 3$ .

The line  $BC$  is perpendicular to  $AB$ .

- Find the equation of the line  $BC$ .
- Find the area of the triangle  $ABC$ .

NES 2009 Mathematics HSC Examination

NES 2009 Mathematics HSC Examination

[Solution](#)2  
2

- 08 2b** Let  $M$  be the midpoint of  $(-1, 4)$  and  $(5, 8)$ .

2

[Solution](#)

Find the equation of the line through  $M$  with gradient  $-\frac{1}{2}$ .

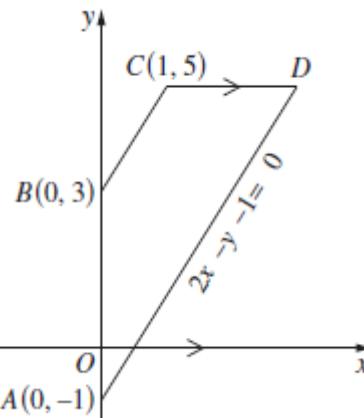
NES 2009 Mathematics HSC Examination

NES 2008 Mathematics HSC Examination



- 08 3a** In the diagram  $ABCD$  is a quadrilateral. The equation of the line  $AD$  is  $2x - y - 1 = 0$ .

- M**
- Show that  $ABCD$  is a trapezium by showing  $BC$  is parallel to  $AD$ .
  - The line  $CD$  is parallel to the  $x$ -axis. Find the co-ordinates of  $D$ .
  - (v) Not in Maths Adv course

[Solution](#)

2



1



- 07 1f** Find the equation of the line that passes through the point  $(-1, 3)$  and is perpendicular to  $2x + y + 4 = 0$ .

2

[Solution](#)

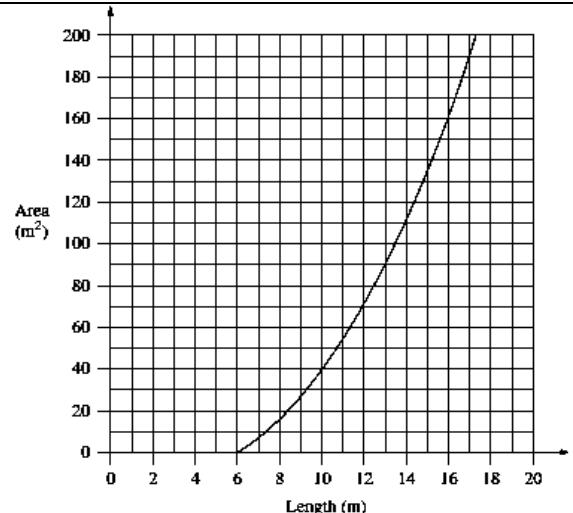
- 07 27 MG a** A rectangular playing surface is to be constructed so that the length is 6 metres more than the width.

- Give an example of a length and width that would be possible for this playing surface.
- Write an equation for the area ( $A$ ) of the playing surface in terms of its length ( $l$ ). A graph comparing the area of the playing surface to its length is shown.
- Why are lengths of 0 metres to 6 metres impossible?
- What would be the dimensions of the playing surface if it had an area of  $135 \text{ m}^2$ ?

Company A constructs playing surfaces

- Draw a graph to represent the cost of using Company A to construct all playing surface sizes up to and including  $200 \text{ m}^2$ . Use the horizontal axis to represent the area and the vertical axis to represent the cost.
- Company B charges a rate of \$360 per square metre regardless of size. Which company would charge less to construct a playing surface with an area of  $135 \text{ m}^2$ ? Justify your answer with suitable calculations

NES 2007 Mathematics HSC Examination

[Solution](#)

1



1



1



2



#### Company A charges

Size of playing surface	Charges
Up to and including $150 \text{ m}^2$	\$50 000
Greater than $150 \text{ m}^2$	\$50 000 plus a rate of \$300 per square metre for the area in excess of $150 \text{ m}^2$

2



1



NES 2007 Mathematics General HSC Examination

**Year 11: Functions****F1.4 Further functions and relations** Back**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

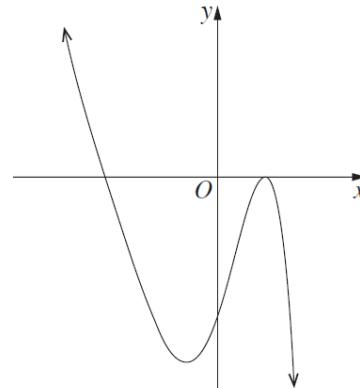
Students:

- define a real polynomial  $P(x)$  as the expression  $a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$  where  $n = 0, 1, 2, \dots$  and  $a_0, a_1, a_2, \dots, a_n$  are real numbers
- identify the coefficients and the degree of a polynomial (ACMMM015)
- identify the shape and features of graphs of polynomial functions of any degree in factored form and sketch their graphs 
- recognise that functions of the form  $f(x) = \frac{k}{x}$  represent inverse variation, identify the hyperbolic shape of their graphs and identify their asymptotes AAM 
- define the absolute value  $|x|$  of a real number  $x$  as the distance of the number from the origin on a number line without regard to its sign
- use and apply the notation  $|x|$  for the absolute value of the real number  $x$  and the graph of  $y = |x|$  (ACMSM098)
  - recognise the shape and features of the graph of  $y = |ax + b|$  and hence sketch the graph
- solve simple absolute value equations of the form  $|ax + b| = k$  both algebraically and graphically 
- given the graph of  $y = f(x)$ , sketch  $y = -f(x)$  and  $y = f(-x)$  and  $y = -f(-x)$  using reflections in the  $x$  and  $y$ -axes
- recognise features of the graphs of  $x^2 + y^2 = r^2$  and  $(x - a)^2 + (y - b)^2 = r^2$ , including their circular shapes, their centres and their radii (ACMMM020) 
  - derive the equation of a circle, centre the origin, by considering Pythagoras' theorem and recognise that a circle is not a function
  - transform equations of the form  $x^2 + y^2 + ax + by + c = 0$  into the form  $(x - a)^2 + (y - b)^2 = r^2$ , by completing the square
  - sketch circles given their equations and find the equation of a circle from its graph
  - recognise that  $y = \sqrt{r^2 - x^2}$  and  $y = -\sqrt{r^2 - x^2}$  are functions, identify the semicircular shape of their graphs and sketch them

**S  
T****Reference  
Sheet****23 4** The graph of a polynomial is shown.**MA**

Which row of the table is correct for this polynomial?

	Equation	Value of $b$	Value of $c$
A.	$y = -(x - b)(x - c)^2$	$b > 0$	$c < 0$
B.	$y = -(x - b)(x - c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x - b)(x - c)$	$b > 0$	$c < 0$
D.	$y = -x(x - b)(x - c)$	$b < 0$	$c > 0$

**1** **Solution**

NESA 2023 Mathematics Advanced HSC Examination

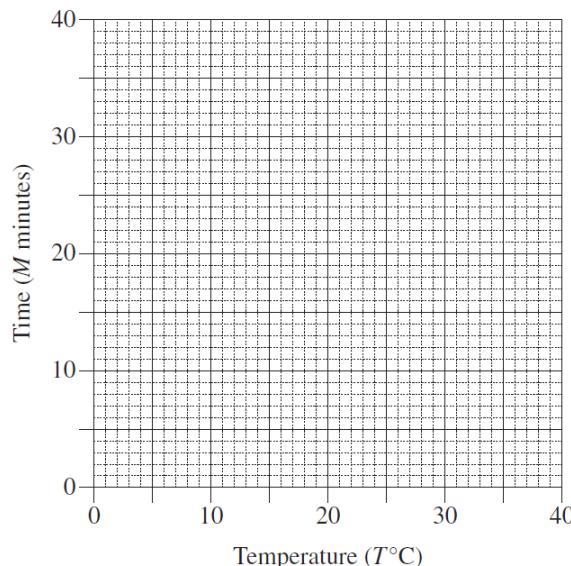
- 22 12 MA** A student believes that the time it takes for an ice cube to melt ( $M$  minutes) varies inversely with the room temperature ( $T^\circ\text{C}$ ).

The student observes that at a room temperature of  $15^\circ\text{C}$  it takes 12 minutes for an ice cube to melt.

[Solution](#)

- (a) Find the equation relating  $M$  and  $T$ .  
 (b) By first completing this table of values, graph the relationship between temperature and time from  $T = 5^\circ\text{C}$  to  $T = 30^\circ\text{C}$ .

$T$	5	15	30
$M$			



COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

- MA 6 SP Band 4-5** What are the values of  $x$  for which  $|3 - 4x| = 4$ ?

[Solution](#)

- A.  $x = -\frac{7}{4}$  and  $x = -\frac{1}{4}$       B.  $x = -\frac{7}{4}$  and  $x = \frac{1}{4}$   
 C.  $x = \frac{7}{4}$  and  $x = -\frac{1}{4}$       D.  $x = \frac{7}{4}$  and  $x = \frac{1}{4}$

NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 25 SP Band 3-5** A circle is given by the equation  $x^2 + y^2 + 4x - 10y = -16$ .

[Solution](#)

Find the centre and radius of this circle.

NESA Mathematics Advanced Sample Examination Paper (2020)

- TG 1** Solve:

[Solution](#)

- (a)  $|5x| = 12$   
 (b) Solve  $|5x + 1| = 6$   
 (c) Find the values of  $t$  for which  $|t| \leq 2$  and plot the solution on a number line.

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 2** Sketch the graph of the function  $f(x) = (x - 1)^3(x + 2)$ , identifying all important features.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 3** Sketch the graph of  $y = \frac{1}{x - 3}$ , identifying any asymptotes and any intercepts on the axes.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 4** Fred has made an electrical circuit in which the current varies inversely with the resistance.

[Solution](#)

When the resistance is 10 units, the current is 8.6 units.  
 What is the current when the resistance is 12 units?

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 5** Hussein finds that for a certain species of fish the mass of a fish is directly proportional to the cube of the length of the fish.



For this species, if a fish of length 25 cm has a mass of 870 g, what is the length of a fish that has a mass of 1.3 kg?

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 6** The damage caused by a moving car when it hits an object is called the 'collision impact' and is proportional to the square of the speed of the car.



What happens to the collision impact when the speed of a car is:

- doubled?
- reduced by one-third?

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 7** Under certain conditions, the stopping distance of a car once the brakes are applied is directly proportional to the square of the speed at which the car is going when the brakes are first applied. If a car travelling at 60 km/h takes 43.2 metres to stop, how far does it take to stop if it is travelling at:



- 30 km/h?
- 67 km/h?

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 8** In her assignment, Pat is representing the populations of cities using circles. The area of each circle is directly proportional to the population of the city it represents.



- If a city of population 20 000 is represented by a circle of radius 1.5 cm, what would be the radius of the circle representing:
  - City A, population 10 000?
  - City B, population 36 500?
- Would Pat be able to use a circle to represent a city of population 4 500 000? Explain your answer.

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 9** Use a table of values to assist in explaining why the function  $f(x) = \frac{x}{|x|}$  has only two values in its range.



NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 10** Sketch the graph of:



- $f(x) = \frac{4}{x} + 3$  and  $g(x) = 1 + f(x)$
- $f(x) = |2x - 3|$

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- TG 11** Simplify the expression:



- $|x| + 3x$ , given  $x < 0$
- $|x - 1| + 3$ , given  $x > 1$

NESA Mathematics Advanced Year 11 Topic Guide: Functions

- 19 13** (i) Sketch the graph of  $y = |x - 1|$  for  $-4 \leq x \leq 4$ .

1 [Solution](#)

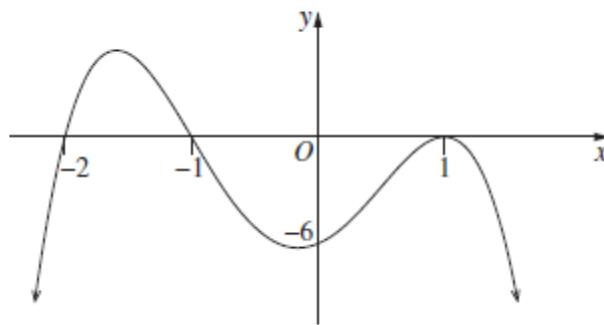
- M e** (ii) Using the sketch from part (i), or otherwise, solve  $|x - 1| = 2x + 4$ .

2 

NESA 2019 Mathematics HSC Examination

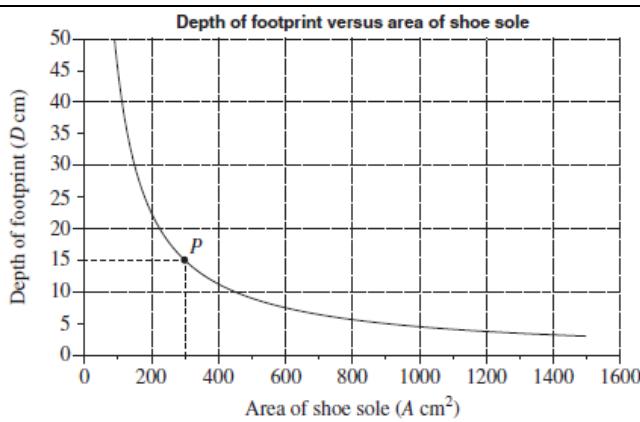
- 18 4** The diagram shows the graph of  $y = a(x + b)(x + c)(x + d)^2$ .  
**MX 1** What are the possible values of  $a, b, c$  and  $d$ ?

- A.  $a = -6, b = -2, c = -1, d = 1$   
B.  $a = -6, b = 2, c = 1, d = -1$   
C.  $a = -3, b = -2, c = -1, d = 1$   
D.  $a = -3, b = 2, c = 1, d = -1$

**1** [Solution](#)

NESA 2018 Mathematics Extension 1 HSC Examination

- 18 29** When people walk in snow, the depth ( $D$  cm) of each footprint depends on both the area ( $A$   $\text{cm}^2$ ) of the shoe sole and the weight of the person. The graph shows the relationship between the area of the shoe sole and the depth of the footprint in snow, for a group of people of the same weight.
- (i) The graph is a hyperbola because  $D$  is inversely proportional to  $A$ . The point  $P$  lies on the hyperbola. Find the equation relating  $D$  and  $A$ .
- (ii) A man from this group walks in snow and the depth of his footprint is 4 cm. Use your equation from part (i) to calculate the area of his shoe size.

[Solution](#)**2**

NESA 2018 Mathematics General 2 HSC Examination

- 17 11** Solve  $|3x - 1| = 2$ .  
**M g**

NESA 2017 Mathematics HSC Examination

**2** [Solution](#)

- 17 30** The cost of a jewellery box varies directly with the cube of its height.  
**MG b** A jewellery box with a height of 10 cm costs \$50. Calculate the cost of a jewellery box with a height of 12 cm.

NESA 2017 Mathematics General HSC Examination

**2** [Solution](#)

- 16 11** Sketch the graph of  $(x - 3)^2 + (y + 2)^2 = 4$ .  
**M a**
- 16 18** The value of  $E$  varies directly with the square of  $S$ . It is known that  $E = 20$  when  $S = 10$ . What is the value of  $E$  when  $S = 40$ ?  
(A) 40      (B) 80      (C) 320      (D) 400

NESA 2016 Mathematics HSC Examination

**2** [Solution](#)

NESA 2016 Mathematics General HSC Examination

**1** [Solution](#)



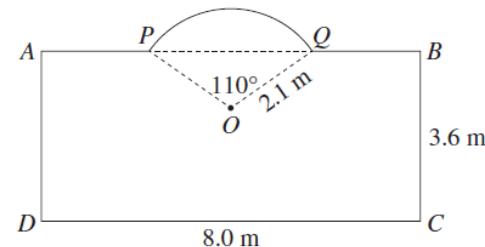


**Year 11: Trigonometric Functions****T.1 Trigonometry**[Back](#)**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>**Students:**

- use the sine, cosine and tangent ratios to solve problems involving right-angled triangles where angles are measured in degrees, or degrees and minutes
- establish and use the sine rule, cosine rule and the area of a triangle formula for solving problems where angles are measured in degrees, or degrees and minutes **AAM**
- find angles and sides involving the ambiguous case of the sine rule
  - use technology and/or geometric construction to investigate the ambiguous case of the sine rule when finding an angle, and the condition for it to arise
- solve problems involving the use of trigonometry in two and three dimensions **AAM**
  - interpret information about a two or three-dimensional context given in diagrammatic or written form and construct diagrams where required
- solve practical problems involving Pythagoras' theorem and the trigonometry of triangles, which may involve the ambiguous case, including finding and using angles of elevation and depression and the use of true bearings and compass bearings in navigation **AAM**

**STANDA****STANDARD**[Reference Sheet](#)

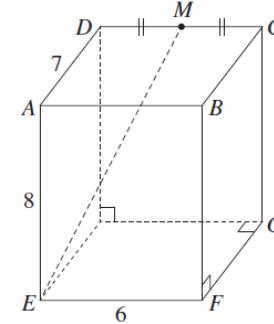
- 23 16 MA** The diagram shows a shape  $APQBCD$ . The shape consists of a rectangle  $ABCD$  with an arc  $PQ$  on side  $AB$  and with side lengths  $BC = 3.6 \text{ m}$  and  $CD = 8.0 \text{ m}$ .  
The arc  $PQ$  is an arc of a circle with centre  $O$  and radius  $2.1 \text{ m}$  and  $\angle POQ = 110^\circ$ .  
What is the perimeter of the shape  $APQBCD$ ?  
Give your answer correct to one decimal place.



NOT TO SCALE

**Solution****4****COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations**

- 23 22 MA** In the rectangular prism shown,  
 $AD = 7 \text{ cm}$ ,  $AE = 8 \text{ cm}$ ,  $EF = 6 \text{ cm}$ .  
Point  $M$  is the midpoint of  $CD$ .  
Find  $\angle AEM$ , to the nearest degree.

**3 Solution****3**

NESA 2023 Mathematics Advanced HSC Examination

- 23 MS 2** **27** The diagram shows the location of three places  $X$ ,  $Y$  and  $C$ .

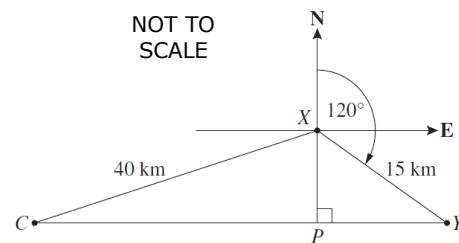
$Y$  is on a bearing of  $120^\circ$  and 15 km from  $X$ .

$C$  is 40 km from  $X$  and lies due west of  $Y$ .

$P$  lies on the line joining  $C$  and  $Y$  and is due south of  $X$ .

(a) Find the distance from  $X$  to  $P$ .

(b) What is the bearing of  $C$  from  $X$ , to the nearest degree?



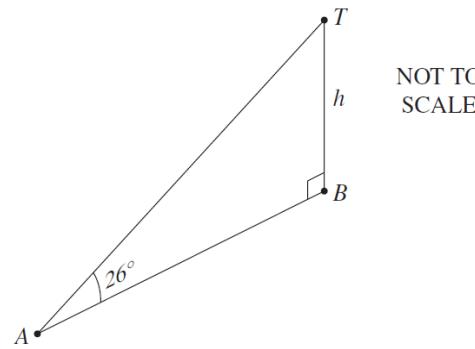
NESA 2023 Mathematics Standard 2 HSC Examination

- 22 MA 3** A tower  $BT$  has height  $h$  metres.

From point  $A$ , the angle of elevation to the top of the tower is  $26^\circ$  as shown.

Which of the following is the correct expression for the length of  $AB$ ?

- A.  $h \tan 26^\circ$
- B.  $h \cot 26^\circ$
- C.  $h \sin 26^\circ$
- D.  $h \cosec 26^\circ$

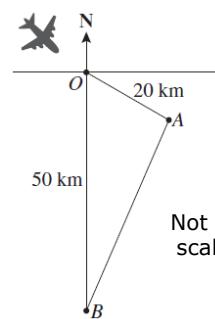


NESA 2022 Mathematics Advanced HSC Examination

- 22 MS 2** **33** The diagram shows an aeroplane that was flying towards an airport at  $A$  on a bearing of  $135^\circ$ T. When it was at point  $O$ ,

20 km away from the airport at  $A$ , the flight course was changed. The aeroplane landed at an airport at  $B$  directly south of  $O$ . The distance from  $O$  to  $B$  is 50 km.

- (a) Show that the distance between the airport at  $A$  and the airport at  $B$  is 38.5 km, correct to 1 decimal place.
- (b) Use the sine rule to find the angle  $OBA$  to the nearest degree.
- (c) What is the bearing of the airport at  $B$  from the airport at  $A$ ?



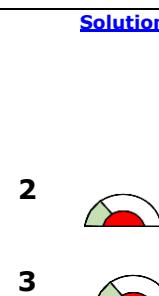
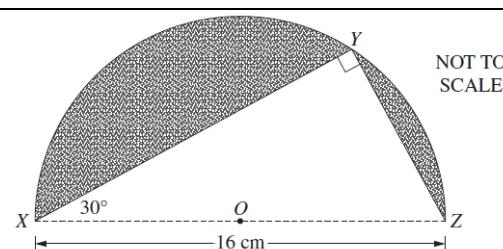
NESA 2022 Mathematics Standard 2 HSC Examination

- 21 MA 12** A right-angled triangle  $XYZ$  is cut out from a semicircle with centre  $O$ .

The length of the diameter  $XZ$  is 16 cm and  $\angle YXZ = 30^\circ$ , as shown on the diagram.

(a) Find the length of  $XY$  in cm, correct to two decimal places.

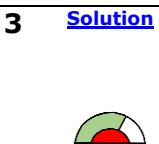
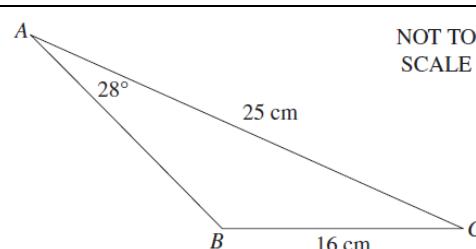
(b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place.



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 MA 18** The diagram shows a triangle  $ABC$  where  $AC = 25$  cm,  $BC = 16$  cm,  $\angle BAC = 28^\circ$  and angle  $ABC$  is obtuse.

Find the size of the obtuse angle  $ABC$  correct to the nearest degree.



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 MS 2** **39** The diagram shows a compass radial survey of the field ABCD.

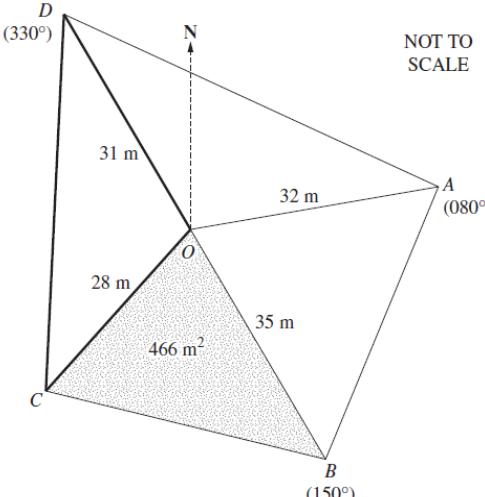
(a) Triangle COB has an area of  $466 \text{ m}^2$ .

Find the size of acute angle  $COB$ , correct to the nearest degree.

(b) A farmer wants to put a fence around the triangle DOC.

Find the length of fencing required.

Give your answer in metres correct to one decimal place.

[Solution](#)**2****3**

NESA 2021 Mathematics Standard 2 HSC Examination

- 20 MA** **15** Mr Ali, Ms Brown and a group of students were camping at the site located at  $P$ .

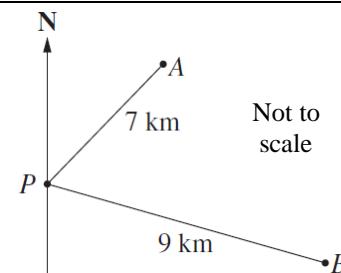
Mr Ali walked with some of the students on a bearing of  $035^\circ$  for 7 km to location  $A$ .

Ms Brown, with the rest of the students, walked on a bearing of  $100^\circ$  for 9 km to location  $B$ .

(a) Show that the angle  $APB$  is  $65^\circ$ .

(b) Find the distance  $AB$ .

(c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree.

[Solution](#)**1****2****2**

COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

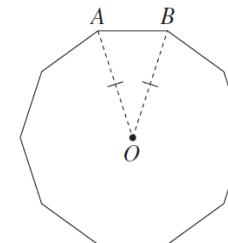
- 20 MA** **22** The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal).

The decagon has centre  $O$ .

The perimeter of the shape is 80 cm.

By considering triangle  $OAB$ , calculate the area of the ten-sided shape.

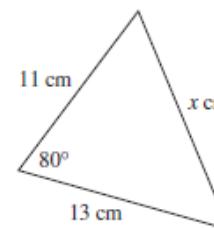
Give your answer in square centimetres, correct to one decimal place.

**4**

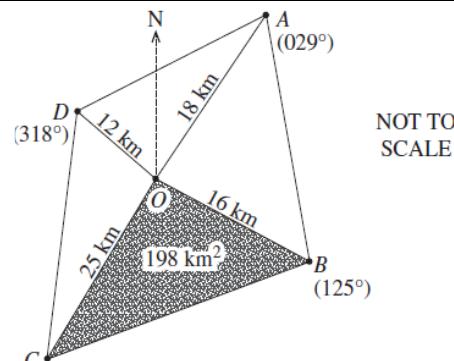
COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

- MA SP** **12 Band 2-4** The diagram shows a triangle with sides of length  $x \text{ cm}$ , 11 cm and 13 cm and an angle of  $80^\circ$ .

- 19 MS 2** **17** Use the cosine rule to calculate the value of  $x$ , correct to two significant figures.

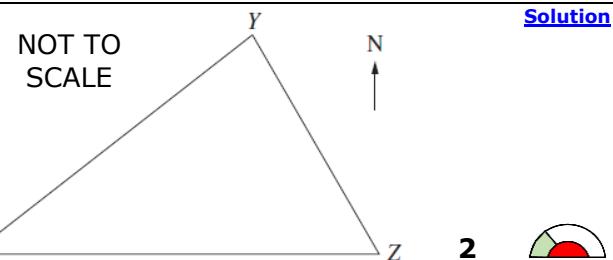
**3**[Solution](#)NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

- MA 21 Band 3-5** The diagram shows the distances of four towns  $A, B, C$  and  $D$  from point  $O$ .  
 The true bearings of towns  $A, B$  and  $D$  from point  $O$  are also shown.  
 The area of the acute-angled triangle  $BOC$  is  $198 \text{ cm}^2$ .  
 Calculate the true bearing of town  $C$  from point  $O$ , correct to the nearest degree.



**3** [Solution](#)

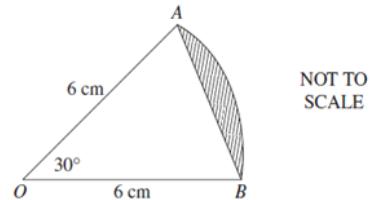
- MA 12 Band 2-5** The diagram shows the three towns  $X, Y$  and  $Z$ . Town  $Z$  is due east of Town  $X$ . The bearing of Town  $Y$  from Town  $X$  is N $39^\circ$ E and the bearing of Town  $Z$  from Town  $Y$  is S $51^\circ$ E. The distance between Town  $X$  and Town  $Y$  is 1330 km.  
 A plane flies between the three towns.  
 (a) Mark the given information on the diagram and explain why  $\angle XYZ = 90^\circ$ .  
 (b) Find the distance between Town  $X$  and Town  $Z$  to the nearest kilometre.  
 (c) The plane is going to fly from Town  $Y$  to Town  $X$ , stopping at Town  $Z$  on the way. Leaving Town  $Y$ , the pilot incorrectly sets the bearing of Town  $Z$  to S $50^\circ$ E. The pilot flies for 1650 km before realising the mistake, then changes course and flies directly to Town  $X$  without going to Town  $Z$ . Which is closer to Town  $X$ : Town  $Z$  or the point where the pilot changes course? Justify your answer.



[Solution](#)

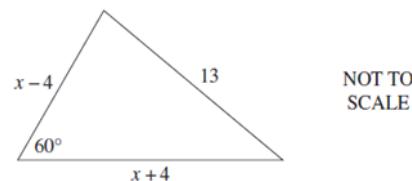
**2**   
**3**

- TG 1** In the diagram,  $OAB$  is a sector of the circle with centre  $O$  and radius 6 cm, where  $\angle AOB = 30^\circ$ . Determine the exact value of the area of the triangle  $OAB$ .



[Solution](#)

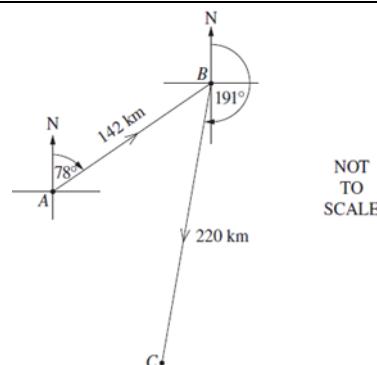
- TG 2** Find the value of  $x$  in the following diagram.



[Solution](#)

**17 13**  
**M a**

- TG 3** Chris leaves island  $A$  in a boat and sails 142 km on a bearing of  $078^\circ$  to island  $B$ .
- 14 13** Chris then sails on a bearing of  $191^\circ$  for 220 km to island  $C$ , as shown in the diagram.
- (a) Show that the distance from island  $C$  to island  $A$  is approximately 210 km.
- (b) Chris wants to sail from island  $C$  directly to island  $A$ . On what bearing should Chris sail? Give your answer correct to the nearest degree.

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions  
NESA 2014 Mathematics HSC Examination

- TG 4** Determine the possible dimensions for triangle  $ABC$  given  $AB = 5.4 \text{ cm}$ ,  $\angle BAC = 32^\circ$  and  $BC = 3 \text{ cm}$ .

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

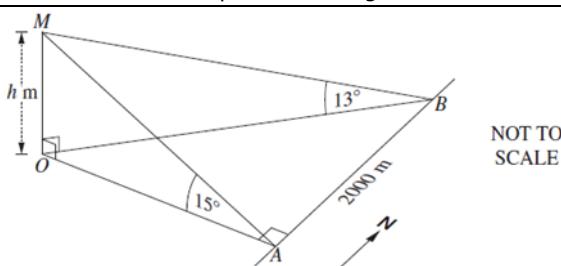


- TG 5** A person walks 2000 metres due north along a road from point  $A$  to point  $B$ . The point  $A$  is due east of a mountain  $OM$ , where  $M$  is the top of the mountain. The point  $O$  is directly below point  $M$  and is on the same horizontal plane as the road. The height of the mountain above point  $O$  is  $h$  metres.

From point  $A$ , the angle of elevation to the top of the mountain is  $15^\circ$ . From point  $B$ , the angle of elevation to the top of the mountain is  $13^\circ$ .

Determine the height of the mountain

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions



- TG 6** The Eiffel Tower is located in Paris, a city built on a flat floodplain. Three tourists  $A$ ,  $B$  and  $C$  are observing the Eiffel Tower from the ground.  $A$  is due north of the tower,  $C$  is due east of the tower, and  $B$  is on the line-of-sight from  $A$  to  $C$  and between them. The angles of elevation to the top of the Eiffel Tower from  $A$ ,  $B$  and  $C$  are  $26^\circ$ ,  $28^\circ$  and  $30^\circ$ , respectively. Determine the bearing of  $B$  from the Eiffel Tower.

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions



- MS ME** Which of the following expresses  $S20^\circ W$  as a true bearing?

- SQ 4** A.  $020^\circ$       B.  $070^\circ$       C.  $160^\circ$       D.  $200^\circ$

**1** [Solution](#)



- MS ME** Abbey walks 2 km due west from home to a coffee shop.

**1** [Solution](#)

- SQ 7** She then walks on a bearing of  $148^\circ$  to school, which is due south of her home. How far south, to the nearest 0.1 km, is Abbey from home?



- A. 1.1 km      B. 1.2 km      C. 3.2 km      D. 3.8 km

NESA Mathematics Standard 2 Sample examination materials

- MS ME** Paul travels from  $A$  to  $B$  on a bearing of  $150^\circ$ .

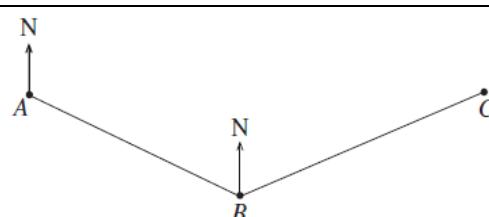
**1** [Solution](#)



He then turns and walks to  $C$  on a bearing of  $055^\circ$ .

What is the size of  $\angle ABC$ ?

- A.  $85^\circ$       B.  $90^\circ$       C.  $95^\circ$       D.  $115^\circ$



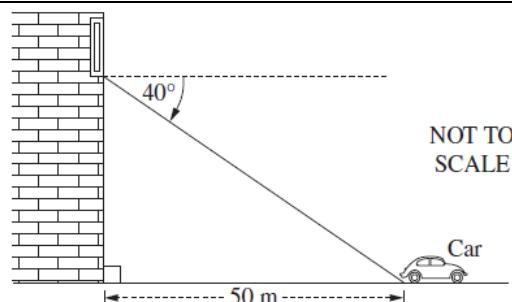
NOT TO SCALE  
NESA Mathematics Standard 2 Sample examination materials

- MS ME** The angle of depression from a window to a car on the ground is  $40^\circ$ .  
**SQ 9**

The car is 50 metres from the base of the building.

How high above the ground is the window, correct to the nearest metre?

- A. 32 m  
 B. 38 m  
 C. 42 m  
 D. 48 m



**1** [Solution](#)



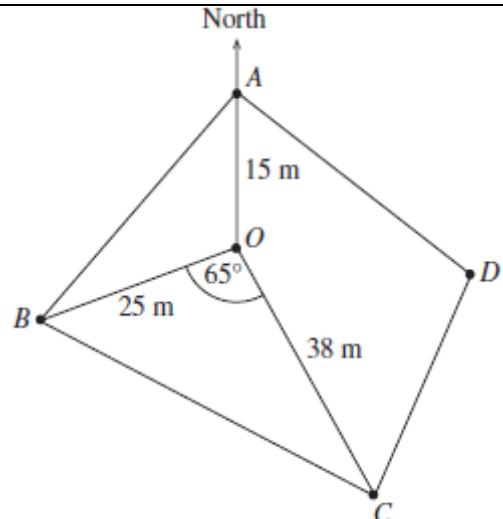
- MS ME** The diagram shows the radial survey of a piece of land.  
**SQ 24**

(a)  $B$  is south west of  $O$ .

What is the true bearing of  $C$  from  $O$ ?

(b) What is the area of angle of  $\triangle AOB$ , to the nearest  $m^2$ ?

NOT TO SCALE



[Solution](#)

**2**



**3**



- MS ME** Lisa owns a piece of land as shown in the diagram.

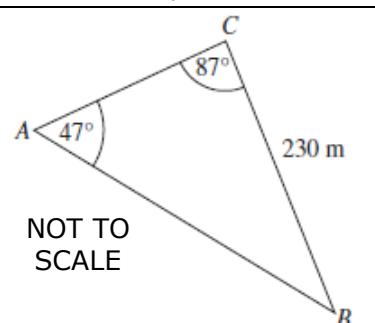
**SQ 25** The length of  $BC$  is 230 metres.

The size of angle  $BCA$  is  $87^\circ$  and of angle  $BAC$  is  $47^\circ$ .

Lisa wants to build a fence along  $AC$ .

Fencing can be purchased in metre lengths at a cost of \$65 per metre.

Calculate the cost of the fencing required.

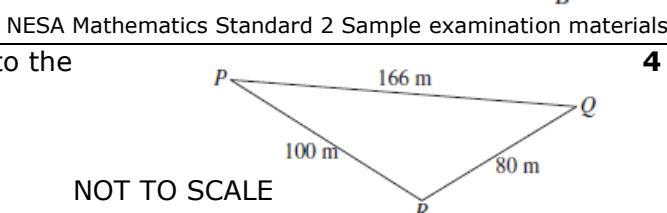


**4** [Solution](#)



- MS ME** Find the area of triangle  $PQR$ , correct to the

**SQ 26** nearest square metre.



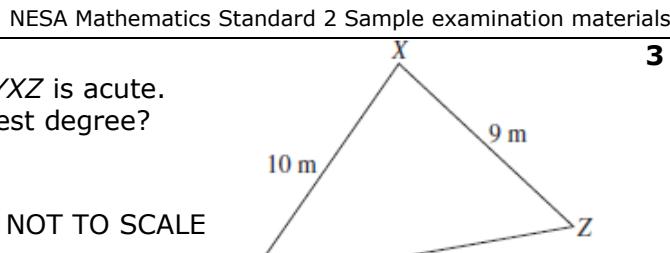
**4**



- MS ME** The diagram shows triangle  $XYZ$ .

**SQ 27** The area of the triangle  $43 \text{ m}^2$  and  $\angle YXZ$  is acute.

What is the size of  $\angle YXZ$ , to the nearest degree?



**3**

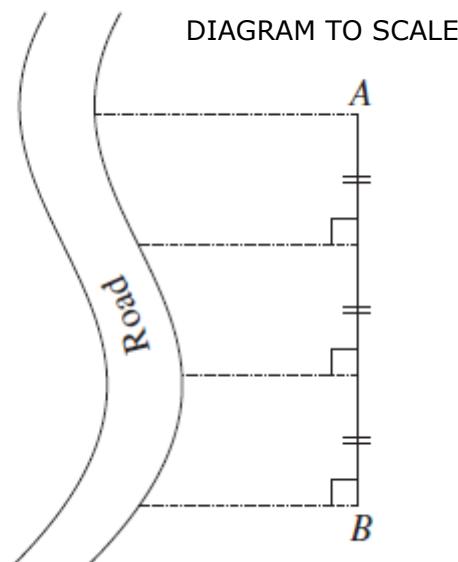


NESA Mathematics Standard 2 Sample examination materials

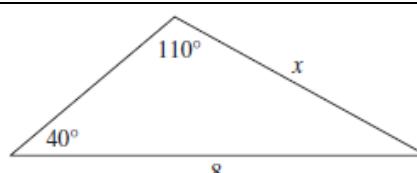
- MS ME** The scale diagram shows the aerial view of a block of land bounded on one side by a road. The length of the block,  $AB$ , is known to be 45 metres.

Calculate the approximate area of the block of land, using three applications of the trapezoidal rule.

[A note to students from projectmaths: Use a ruler to measure  $AB$  as 4.5 cm]

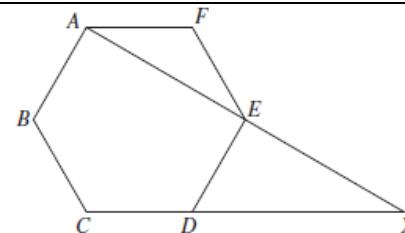
**3**[Solution](#)

- 19 11** Using the sine rule, find the value of  $x$  correct to one decimal place.

**2**[Solution](#)

NESA 2019 Mathematics HSC Examination

- 19 14** The regular hexagon  $ABCDEF$  has sides of length 1.  
**M c** The diagonal  $AE$  and the side  $CD$  are produced to meet at the point  $X$ .  
 Copy or trace the diagram into your writing booklet. Find the exact length of the line segment  $EX$ , justifying your answer.

**3**[Solution](#)

NESA 2019 Mathematics HSC Examination

- 19 MS 4** Which compass bearing is the same as a true bearing of  $110^\circ$ ?  
 A. S $20^\circ$ E      B. S $20^\circ$ W      C. S $70^\circ$ E      D. S $70^\circ$ W

**1**[Solution](#)

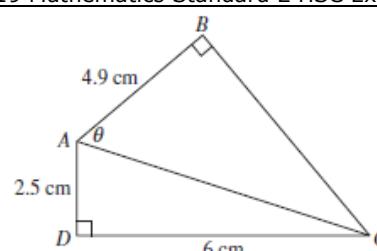
- 19 MS 12** An owl is 7 metres above ground level, in a tree. The owl sees a mouse on the ground at an angle of depression of  $32^\circ$ . How far must the owl fly in a straight line to catch the mouse, assuming the mouse does not move?  
 A. 3.7 m      B. 5.9 m      C. 8.3 m      D. 13.2 m

**1**[Solution](#)

- 19 MS 22** Two right-angled triangles,  $ABC$  and  $ADC$ , are shown.

**3**[Solution](#)

Calculate the size of angle  $\theta$ , correct to the nearest minute.



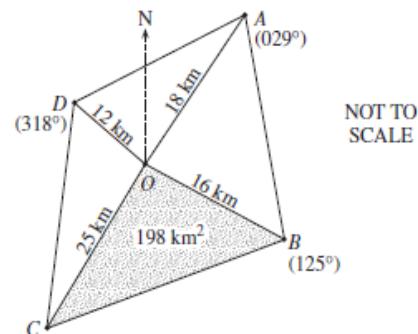
NOT TO SCALE

NESA 2019 Mathematics Standard 2 HSC Examination

- 19 MS 2** **35** A compass radial survey shows the positions of four towns  $A$ ,  $B$ ,  $C$  and  $D$  relative to the point  $O$ .

The area of the triangle  $BOC$  is  $198 \text{ km}^2$ .

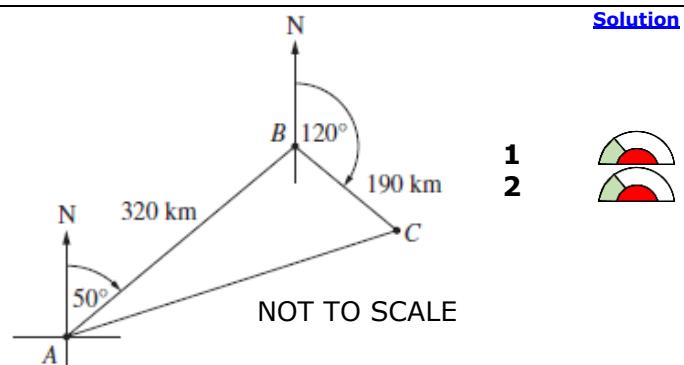
Calculate the bearing of town  $C$  from point  $O$ , correct to the nearest degree.



**3** [Solution](#)



- 18 M 12** A ship travels from Port  $A$  on a bearing of  $050^\circ$  for  $320 \text{ km}$  to Port  $B$ . It then travels on a bearing of  $120^\circ$  from  $190 \text{ km}$  to Port  $C$ .
- What is the size of  $\angle ABC$ ?
  - What is the distance from Port  $A$  to Port  $C$ ?
- Answer to the nearest kilometre.

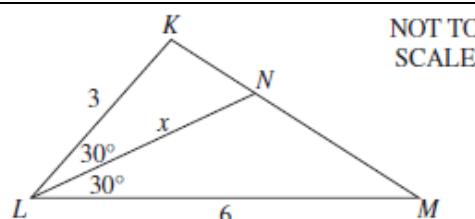


[Solution](#)

**1  
2**



- 18 M 14** In  $\triangle KLM$ ,  $KL$  has length  $3$ ,  $LM$  has length  $6$  and  $\angle KLM = 60^\circ$ . The point  $N$  is chosen on side  $KM$  so that  $LN$  bisects  $\angle KLM$ . The length  $LN$  is  $x$ .
- Find the exact value of the area of  $\triangle KLM$ .
  - Hence, or otherwise, find the exact value of  $x$ .

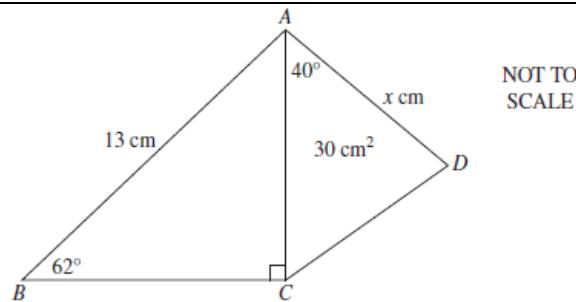


[Solution](#)

**1  
2**



- 18 MG 30** The diagram shows two triangles. Triangle  $ABC$  is right-angled, with  $AB = 13 \text{ cm}$  and  $\angle ABC = 62^\circ$ . In triangle  $ACD$ ,  $AD = x \text{ cm}$  and  $\angle DAC = 40^\circ$ . The area of triangle  $ACD$  is  $30 \text{ cm}^2$ . What is the value of  $x$ , correct to one decimal place?

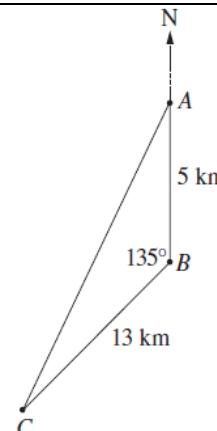


**3**



- 17 30 MG 2** The diagram shows the location of three schools. School A is 5 km due north of school B, school C is 13 km from school B and  $\angle ABC$  is  $135^\circ$ .

- (i) Calculate the shortest distance from school A to school C, to the nearest kilometre.  
(ii) Determine the bearing of school C from school A, to the nearest degree.



NOT TO SCALE

2



3

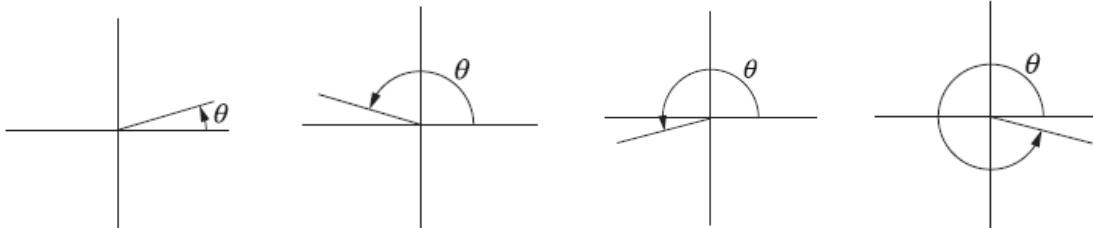


NESA 2017 Mathematics General 2 HSC Examination

- 16 M 1** For the angle  $\theta$ ,  $\sin \theta = \frac{7}{25}$  and  $\cos \theta = -\frac{24}{25}$ .

Which diagram best shows the angle  $\theta$ ?

- (A) (B) (C) (D)



NESA 2016 Mathematics HSC Examination

1

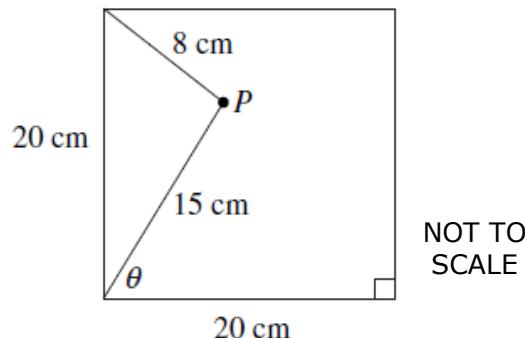
[Solution](#)

- 16 M 12** Square tiles of side length 20 cm are being used to tile a bathroom.

The tiler needs to drill a hole in one of the tiles at a point P which is 8 cm from one corner and 15 cm from an adjacent corner.

To locate the point P the tiler needs to know the size of the angle  $\theta$  shown in the diagram.

Find the size of the angle  $\theta$  to the nearest degree.



NOT TO SCALE

3

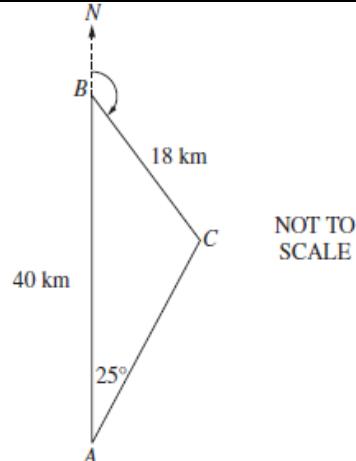
[Solution](#)

NESA 2016 Mathematics HSC Examination

- 16 MG 2** The diagram shows towns A, B and C. Town B is 40 km due north of town A. The distance from B to C is 18 km and the bearing of C from A is  $025^\circ$ . It is known that  $\angle BCA$  is obtuse.

What is the bearing of C from B?

- (A)  $070^\circ$   
(B)  $095^\circ$   
(C)  $110^\circ$   
(D)  $135^\circ$



NOT TO SCALE

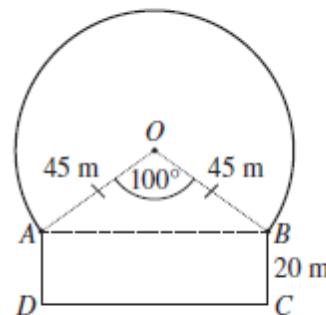
1

[Solution](#)

NESA 2016 Mathematics General 2 HSC Examination

- 16 MG 2 30 c** A school playground consists of part of a circle, with centre  $O$ , and a rectangle as shown in the diagram. The radius  $OB$  of the circle is 45 m, the width  $BC$  of the rectangle is 20 m and  $\angle AOB$  is  $100^\circ$ .

What is the area of the whole playground, correct to the nearest square metre?

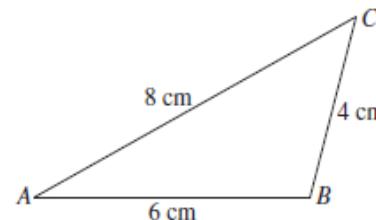


**5** [Solution](#)

NESA 2016 Mathematics General 2 HSC Examination

- 15 M 13** The diagram shows  $\triangle ABC$  with sides  $AB = 6 \text{ cm}$ ,  $BC = 4 \text{ cm}$  and  $AC = 8 \text{ cm}$ .

- (i) Show that  $\cos A = \frac{7}{8}$ .  
(ii) By finding the exact value of  $\sin A$ , determine the exact value of the area of  $\triangle ABC$ .



**1**   
**2**

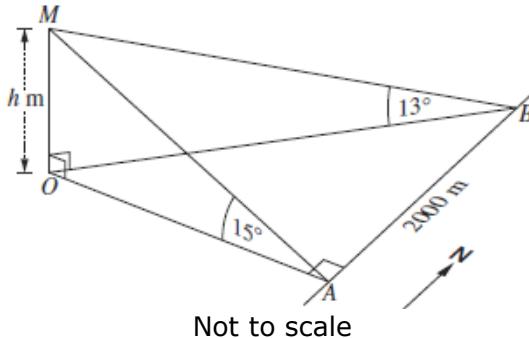
Not to scale

NESA 2015 Mathematics HSC Examination

[Solution](#)

- 15 MX 1 12** A person walks 2000 metres due north along a road from point  $A$  to point  $B$ . The point  $A$  is due east of a mountain  $OM$ , where  $M$  is the top of the mountain. The point  $O$  is directly below point  $M$  and is on the same horizontal plane as the road. The height of the mountain above point  $O$  is  $h$  metres. From point  $A$ , the angle of elevation to the top of the mountain is  $15^\circ$ . From point  $B$ , the angle of elevation to the top of the mountain is  $13^\circ$ .

- (i) Show that  $OA = h \cot 15^\circ$ .  
(ii) Hence, find the value of  $h$ .



**1**   
**2**

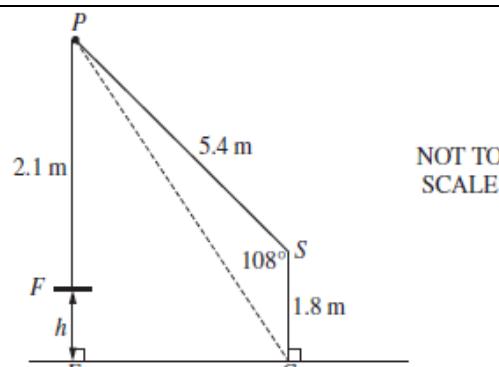
Not to scale

NESA 2015 Mathematics Extension 1 HSC Examination

[Solution](#)

- 15 MG 2 30 e** From point  $S$ , which is 1.8 m above the ground, a pulley at  $P$  is used to lift a flat object  $F$ . The length  $SP$  and  $PF$  are 5.4 m and 2.1 m respectively. The angle  $PSC$  is  $108^\circ$ .

- (i) Show that the length  $PC$  is 6.197 m, correct to 3 decimal places.  
(ii) Calculate  $h$ , the height of the object above the ground.

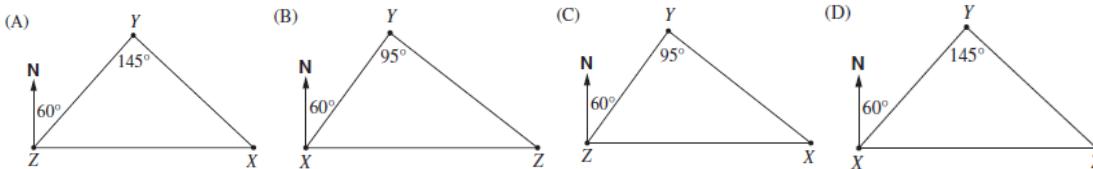


**1**   
**4**

NESA 2015 Mathematics General 2 HSC Examination

- 14 23** MG 2 The following information is given about the locations of three towns  $X$ ,  $Y$  and  $Z$ :
- $X$  is due east of  $Z$
  - $X$  is on a bearing of  $145^\circ$  from  $Y$
  - $Y$  is on a bearing of  $060^\circ$  from  $Z$

Which diagram best represents this information?



(Diagrams not to scale)

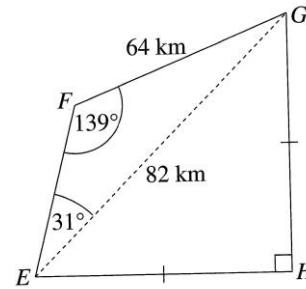
NESA 2014 Mathematics General 2 HSC Examination

[Solution](#)



- 12 29** GM c Raj cycles around a course. The course starts at  $E$ , passes through  $F$ ,  $G$  and  $H$  and finishes at  $E$ . The distances  $EH$  and  $GH$  are equal.

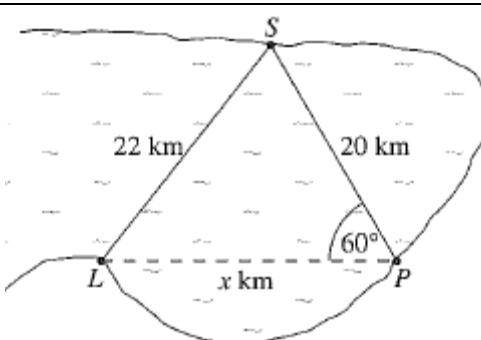
- What is the length of  $EF$ , to the nearest kilometre?
- What is the total distance that Raj cycles, to the nearest kilometre?



[Solution](#)



- 11 8a** M In the diagram, the shop at  $S$  is 20 kilometres across the bay from the post office at  $P$ . The distance from the shop to the lighthouse at  $L$  is 22 kilometres and  $\angle SPL$  is  $60^\circ$ . Let the distance  $PL$  be  $x$  kilometres.
- Use the cosine rule to show that  $x^2 - 20x - 84 = 0$ .
  - Hence, find the distance from the post office to the lighthouse. Give your answer correct to the nearest kilometre.

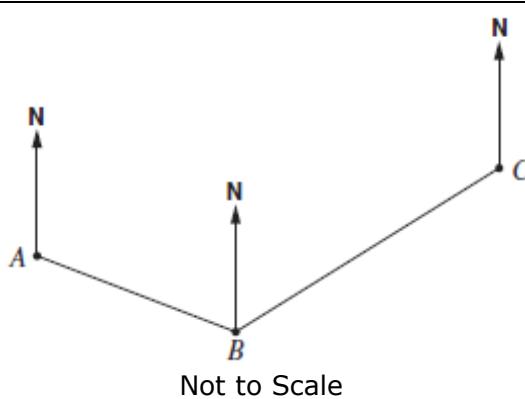


[Solution](#)



NESA 2011 Mathematics HSC Examination

- 11 24** GM c A ship sails 6 km from  $A$  to  $B$  on a bearing of  $121^\circ$ . It then sails 9 km to  $C$ . The size of angle  $ABC$  is  $114^\circ$ . Copy the diagram into your writing booklet and show all the information on it.
- What is the bearing of  $C$  from  $B$ ?
  - Find the distance  $AC$ . Give your answer correct to the nearest kilometre.
  - What is the bearing of  $A$  from  $C$ ? Give your answer correct to the nearest degree.

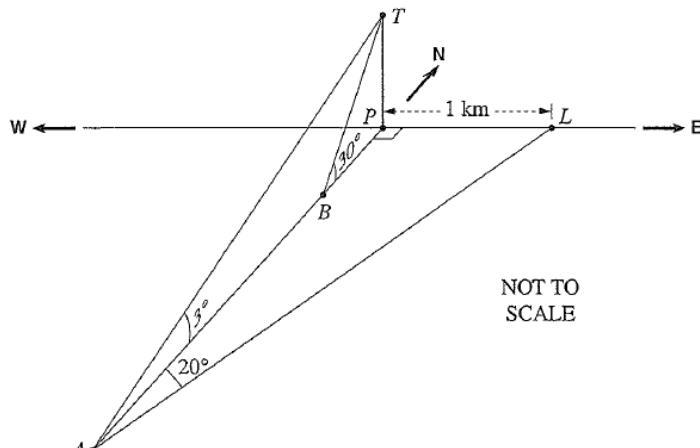


[Solution](#)



NESA 2011 General Mathematics HSC Examination

- 10 MX 1** **5a** A boat is sailing due north from a point  $A$  towards a point  $P$  on the shore line. The shore line runs from west to east. In the diagram,  $T$ , represents a tree on a cliff vertically above  $P$ , and  $L$  represents a landmark on the shore. The distance  $PL$  is 1 km.



From  $A$  the point  $L$  is on a bearing of  $020^\circ$ , and the angle of elevation to  $T$  is  $3^\circ$ . After sailing for some time the boat reaches a point  $B$ , from which the angle of elevation to  $T$  is  $30^\circ$ .

- (i) Show that  $BP = \frac{\sqrt{3} \tan 3^\circ}{\tan 20^\circ}$ .
- (ii) Find the distance  $AB$ .

**3**  
**1**

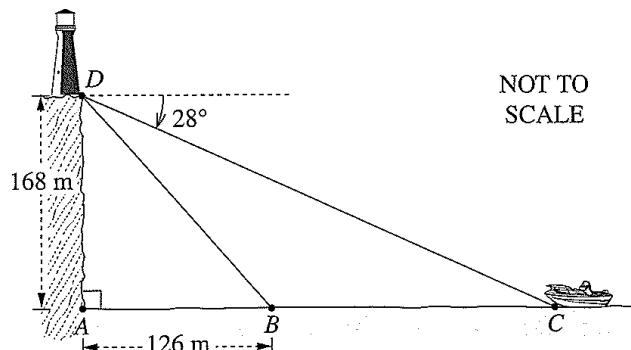


NESA 2010 Mathematics Extension 1 HSC Examination

[Solution](#)

- 10 GM d** The base of a lighthouse,  $D$ , is at the top of a cliff 168 metres above sea level. The angle of depression from  $D$  to a boat at  $C$  is  $28^\circ$ . The boat heads towards the base of the cliff,  $A$ , and stops at  $B$ . The distance  $AB$  is 126 metres.

- (i) What is the angle of depression from  $D$  to  $B$ , correct to the nearest minute?
- (ii) How far did the boat travel from  $C$  to  $B$ , correct to the nearest metre?



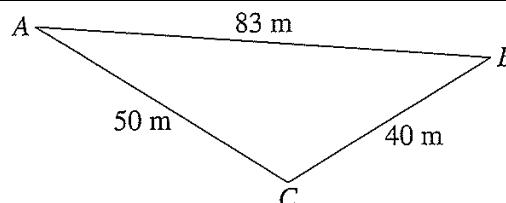
**3**  
**2**



NESA 2010 General Mathematics HSC Examination

**3**  
[Solution](#)

- 10 GM d** Find the area of triangle  $ABC$ , correct to the nearest square metre.



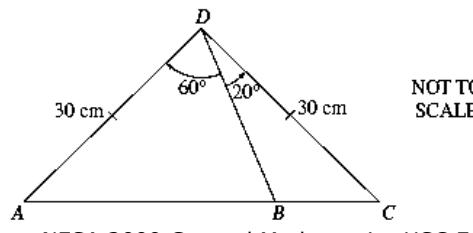
NESA 2010 General Mathematics HSC Examination

- 09 GM 22** In the diagram,  $AD$  and  $DC$  are equal to 30 cm.

What is the length of  $AB$  to the nearest centimetre?

- (A) 28 cm      (B) 31 cm  
 (C) 34 cm      (D) 39 cm

**1**  
[Solution](#)



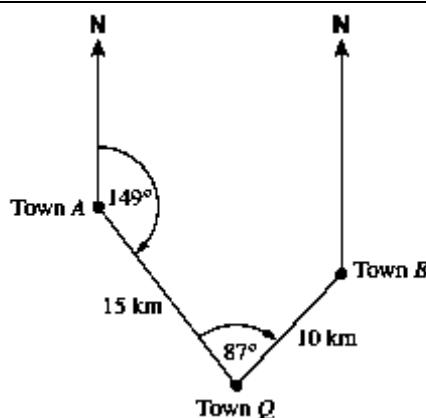
NESA 2009 General Mathematics HSC Examination



**07 26** The diagram shows information about

**GM a** the locations of towns *A*, *B* and *Q*.

- (i) It takes Elina 2 hours and 48 minutes to walk directly from Town *A* to Town *Q*. Calculate her walking speed correct to the nearest km/h.
- (ii) Elina decides, instead, to walk to Town *B* from Town *A* and then to Town *Q*. Find the distance from Town *A* to Town *B*. Give your answer to the nearest km.
- (iii) Calculate the bearing of Town *Q* from Town *B*.



[Solution](#)

NOT  
TO  
SCALE  
**1**



**2**



**1**



NESA 2007 General Mathematics HSC Examination

# Year 11: Trigonometric Functions

## T1.2 Radians



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- understand the unit circle definition of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  and periodicity using degrees (ACMMM029)
  - sketch the trigonometric functions in degrees for  $0^\circ \leq x \leq 360^\circ$
- define and use radian measure and understand its relationship with degree measure (ACMMM032)
  - convert between the two measures, using the fact that  $360^\circ = 2\pi$  radians
  - recognise and use the exact values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  in both degrees and radians for integer multiples of  $\frac{\pi}{6}$  and  $\frac{\pi}{4}$  (ACMMM035)
- understand the unit circle definition of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  and periodicity using radians (ACMMM034)
- solve problems involving trigonometric ratios of angles of any magnitude in both degrees and radians
- recognise the graphs of  $y = \sin x$ ,  $y = \cos x$  and  $y = \tan x$  and sketch on extended domains in degrees and radians (ACMMM036)
- derive the formula for arc length,  $l = r\theta$  and for the area of a sector of a circle,  $A = \frac{1}{2}r^2\theta$
- solve problems involving sector areas, arc lengths and combinations of either areas or lengths

[Reference Sheet](#)

**TG 1** Solve, if  $0 \leq x \leq 2\pi$ ,

- $\sqrt{3} \sin x = \cos x$
- $2 \cos 2x = 0$

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 2** Find the exact values of:

- $\cos \frac{4}{3}\pi$
- $\sin \frac{5}{6}\pi$
- $\tan (-45^\circ)$

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 3** Convert  $\frac{3\pi}{5}$  radians to degrees.  
**95 1c**

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions  
 NESA 1995 Mathematics HSC Examination

**TG 4** Find the exact value of  $\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)$ .

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 5** Solve  $2\sin x = 1$  for  $-2\pi \leq x \leq 2\pi$ .

[Solution](#)



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 6** Find the perimeter and the area of the segment cut off by a chord  $PQ$  of length 8 cm in a circle centre  $O$  and radius 6 cm.

[Solution](#)



Give your answers correct to 3 significant figures.

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

- TG 7** A chord of a circle which subtends an angle of  $\theta$  at the centre of the circle cuts off a segment equal in area to  $\frac{1}{3}$  of the area of the whole circle.

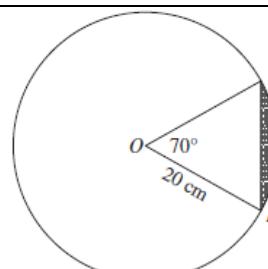
(a) Show that  $\theta - \sin \theta = \frac{2\pi}{3}$ .

(b) Verify that  $\theta = 2.61$  radians, correct to 2 decimal places.

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions



- 19**   **13** The diagram shows a circle with centre  $O$  and radius 20 cm.  
**M**   **b** The points  $A$  and  $B$  lie on the circle such that  $\angle AOB = 70^\circ$ .  
Find the perimeter of the shaded segment, giving your answer correct to one decimal place

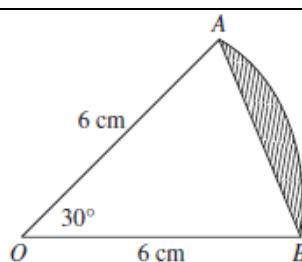


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### **3** Solution

**17** **11** In the diagram,  $OAB$  is a sector with centre  $O$   
**M** **e** and radius 6 cm, where  $\angle AOB = 30^\circ$ .

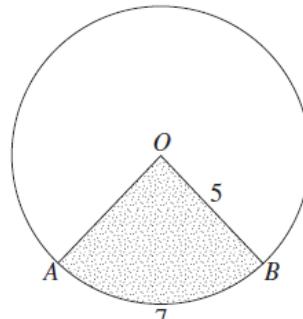
- (i) Find the exact value of the area of the triangle  $OAB$ .  
(ii) Find the exact value of the area of the shaded segment.



NESA 2017 Mathematics HSC Examination

**16**   **7** The circle centred at  $O$  has radius 5. Arc  $AB$  has length 7 as shown in the diagram.  
**M** What is the area of the shaded sector  $OAB$ ?

- (A)  $\frac{35}{2}$   
 (B)  $\frac{35}{2}\pi$   
 (C)  $\frac{125}{14}$   
 (D)  $\frac{125}{14}\pi$



# 1 Solution

NESA 2016 Mathematics H

1

- 15** Find the solutions of  $2 \sin \theta = 1$  for  $0 \leq \theta \leq 2\pi$ .

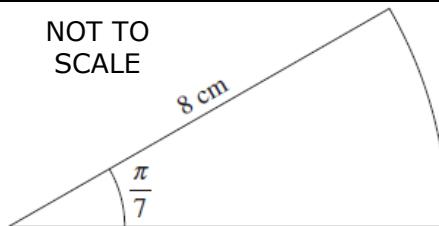
NESA 2016 Mathematics HSC Examination

**14**   **11** The angle of a sector in a circle of radius 8 cm is  $\frac{\pi}{3}$  radians, as shown in the diagram.

NOT TO  
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2

Find the exact value of the perimeter of the sector.

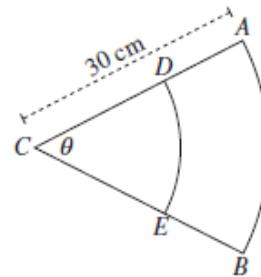


NESA 2014 Mathematics HSC Examination

- 13 13** The region  $ABC$  is a sector of a circle with radius 30 cm, centred at  $C$ . The angle of the sector is  $\theta$ . The arc  $DE$  lies on a circle also centred at  $C$ , as shown in the diagram.

The arc  $DE$  divides the sector  $ABC$  into two regions of equal area.

Find the exact length of the interval  $CD$ .



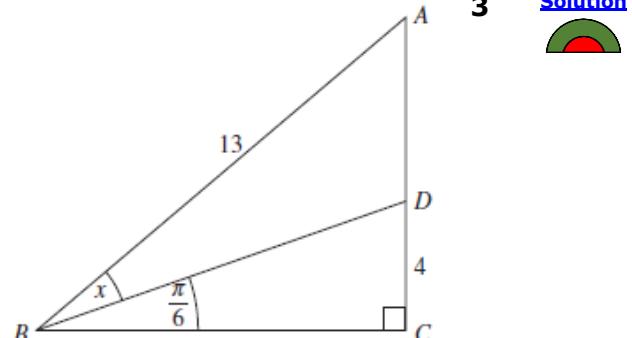
**2** [Solution](#)

- 13 14** The right-angled triangle  $ABC$  has hypotenuse  $AB = 13$ .

The point  $D$  is on  $AC$  such that  $DC = 4$ ,

$$\angle DBC = \frac{\pi}{6} \text{ and } \angle DBA = x.$$

Using the sine rule, or otherwise, find the exact value of  $\sin x$ .



Not to scale

NESA 2013 Mathematics HSC Examination

- 12 6** What are the solutions of  $\sqrt{3} \tan x = -1$  for  $0 \leq x \leq 2\pi$ ?

- M** (A)  $\frac{2\pi}{3}$  and  $\frac{4\pi}{3}$       (B)  $\frac{2\pi}{3}$  and  $\frac{5\pi}{3}$       (C)  $\frac{5\pi}{6}$  and  $\frac{7\pi}{6}$       (D)  $\frac{5\pi}{6}$  and  $\frac{11\pi}{6}$

**1** [Solution](#)



- 12 11** The area of a sector of a circle of radius 6 cm is  $50 \text{ cm}^2$ .

- M f** Find the length of the arc of the sector.

**2** [Solution](#)



- 11 2b** Find the exact values of  $x$  such that  $2 \sin x = -\sqrt{3}$ , where  $0 \leq x \leq 2\pi$ .

- M** The diagram shows a circle with centre  $O$  and radius 5 cm.  
The length of the arc  $PQ$  is 9 cm.  
Lines drawn perpendicular to  $OP$  and  $OQ$  at  $P$  and  $Q$  respectively meet at  $T$ .

**2** [Solution](#)



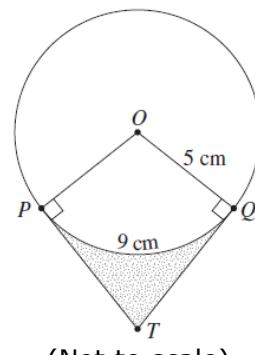
NESA 2011 Mathematics HSC Examination

- 10 6b** The diagram shows a circle with centre  $O$  and radius 5 cm.

The length of the arc  $PQ$  is 9 cm.

Lines drawn perpendicular to  $OP$  and  $OQ$  at  $P$  and  $Q$  respectively meet at  $T$ .

- Find  $\angle POQ$  in radians.
- Prove that  $\triangle OPT$  is congruent to  $\triangle OQT$ . *(delete)*
- Find the length of  $PT$ .
- Find the area of the shaded region.



**1** [Solution](#)



**1** [Solution](#)



(Not to scale)

NESA 2010 Mathematics HSC Examination

- 09 1e** Find the exact value of  $\theta$  such that  $2 \cos \theta = 1$ , where  $0 \leq \theta \leq \frac{\pi}{2}$ .

**2** [Solution](#)



NESA 2009 Mathematics HSC Examination

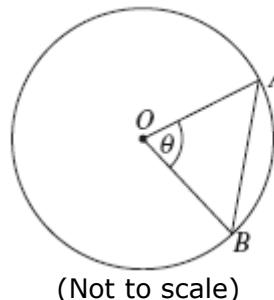
- 09 5c** The diagram shows a circle with centre  $O$  and radius 2 centimetres. The points  $A$  and  $B$  lie on the circumference of the circle and  $\angle AOB = \theta$ .

(i) There are two possible values of  $\theta$  for which the area of  $\triangle AOB$  is  $\sqrt{3}$  square centimetres.

One value is  $\frac{\pi}{3}$ . Find the other value.

(ii) Suppose that  $\theta = \frac{\pi}{3}$ .

- (1) Find the area of the sector  $AOB$ .
- (2) Find the exact length of the perimeter of the minor segment bounded by the chord  $AB$  and the arc  $AB$ .



2



1



2



NESAA 2009 Mathematics HSC Examination

- 08 1a** Evaluate  $2 \cos \frac{\pi}{5}$  correct to three significant figures.

2

[Solution](#)

NESAA 2008 Mathematics HSC Examination

- 08 7b** The diagram shows a sector with radius  $r$  and angle  $\theta$ ,

where  $0 \leq \theta \leq 2\pi$ .

The arc length is  $\frac{10\pi}{3}$ .

(i) Show that  $r \geq \frac{5}{3}$ .

(ii) Calculate the area of the sector when  $r = 4$ .



2



2



NESAA 2008 Mathematics HSC Examination

- 07 4a** Solve  $\sqrt{2} \sin x = 1$  for  $0 \leq x \leq 2\pi$ .

2

[Solution](#)

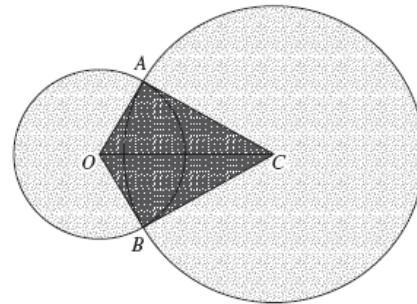
NESAA 2007 Mathematics HSC Examination

- 07 4c** An advertising logo is formed from two circles, which intersect as shown in the diagram. The circles intersect at  $A$  and  $B$  and have centres at  $O$  and  $C$ .

The radius of the circle centred at  $O$  is

1 metre and the radius of the circle centred at  $C$

is  $\sqrt{3}$  metres. The length of  $OC$  is 2 metres.



Not to scale

(i) Use Pythagoras' theorem to show that  $\angle OAC = \frac{\pi}{2}$ .

1



(ii) Find  $\angle ACO$  and  $\angle AOC$ .

2



(iii) Find the area of the quadrilateral  $AOBC$ .

1



(iv) Find the area of the major sector  $ACB$ .

1



(v) Find the total area of the logo (the sum of all the shaded areas).

2



NESAA 2007 Mathematics HSC Examination

# Year 11: Trigonometric Functions

## T1.3 Trigonometric functions and identities

**Syllabus:** updated November 2019. Latest version @<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- define the reciprocal trigonometric functions,  $y = \operatorname{cosec} x$ ,  $y = \sec x$  and  $y = \cot x$ 
  - $\operatorname{cosec} A = \frac{1}{\sin A}$ ,  $\sin A \neq 0$
  - $\sec A = \frac{1}{\cos A}$ ,  $\cos A \neq 0$
  - $\cot A = \frac{\cos A}{\sin A}$ ,  $\sin A \neq 0$
- sketch the graphs of reciprocal trigonometric functions in both radians and degrees
- prove and apply the Pythagorean identities  $\cos^2 x + \sin^2 x = 1$ ,  $1 + \tan^2 x = \sec^2 x$  and  $1 + \cot^2 x = \operatorname{cosec}^2 x$  (ACMSM046)
  - know the difference between an equation and an identity
- use  $\tan x = \frac{\sin x}{\cos x}$  provided that  $\cos x \neq 0$
- prove trigonometric identities
- evaluate trigonometric expressions using angles of any magnitude and complementary angle results
- simplify trigonometric expressions and solve trigonometric equations, including those that reduce to quadratic equations

[Reference Sheet](#)**21 1** Which of the following is equivalent to  $\sin^2 5x$ ?**1** [Solution](#)**MA** A.  $1 + \cos^2 5x$       B.  $1 - \cos^2 5x$       C.  $-1 + \cos^2 5x$       D.  $-1 - \cos^2 5x$ 

NESA 2021 Mathematics Advanced HSC Examination

**20 19** Prove that  $\sec \theta - \cos \theta = \sin \theta \tan \theta$ **2** [Solution](#)**MA**

NESA 2020 Mathematics Advanced HSC Examination

**TG 1** Find the exact values of:[Solution](#)

(a)  $\sec \frac{2}{3}\pi$



(b)  $\operatorname{cosec} \frac{5}{6}\pi$

(c)  $\cot(-\frac{2}{3}\pi)$

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 2** On the same set of axes, sketch  $y = \sin x$  and  $y = \cot x$  for  $0 \leq x \leq 2\pi$ .[Solution](#)Find the values of  $x$  for which  $\sin x = \cot x$ .

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 3** Show that:[Solution](#)

(a)  $\tan \theta + \cot \theta = \sec \theta \operatorname{cosec} \theta$



(b)  $\frac{1 + \cos \theta}{\sin^2 \theta} = \frac{1}{1 - \cos \theta}$



NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 4** If  $\tan A = p$ , express in terms of  $p$ :  $\tan(180^\circ + A) + \tan(360^\circ + A)$ .[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 5** Solve  $2\sin^2x - 3\sin x - 2 = 0$  for  $0 \leq x \leq 2\pi$ .

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

[Solution](#)

**TG 6** (a) Prove that  $\sec^2 x + \sec x \tan x = \frac{1 + \sin x}{\cos^2 x}$ .

(b) Hence prove that  $\sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$ .

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

[Solution](#)

**TG 7** Express  $5\cot^2 x - 2\operatorname{cosec} x + 2$  in terms of  $\operatorname{cosec} x$  and hence solve the equation

$5\cot^2 x - 2\operatorname{cosec} x + 2 = 0$  for  $0 \leq x \leq 2\pi$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**TG 8** Given that  $\cot \alpha = -\frac{15}{8}$ , and that  $\frac{1}{2}\pi < \alpha < \pi$ . Find the values of:

[Solution](#)

(a)  $\operatorname{cosec} \alpha$

(b)  $\cos \alpha$

NESA Mathematics Advanced Year 11 Topic Guide: Trigonometric functions

**19 13** Solve  $2\sin x \cos x = \sin x$  for  $0 \leq x \leq 2\pi$ .

**3**[Solution](#)

**M a**

NESA 2019 Mathematics HSC Examination

**17 7** Which expression is equivalent to  $\tan \theta + \cot \theta$ ?

**1**[Solution](#)

**M** (A)  $\operatorname{cosec} \theta + \sec \theta$  (B)  $\sec \theta \operatorname{cosec} \theta$ ? (C) 2

(D) 1

NESA 2017 Mathematics HSC Examination

**14 7** How many solutions of the equation  $(\sin x - 1)(\tan x + 2) = 0$  lie between 0 and  $2\pi$ ?

**1**[Solution](#)

**M** (A) 1 (B) 2 (C) 3 (D) 4

NESA 2014 Mathematics HSC Examination

**14 15** Find all solutions of  $2\sin^2 x + \cos x - 2 = 0$ , where  $0 \leq x \leq 2\pi$ .

**3**[Solution](#)

**M a**

NESA 2014 Mathematics HSC Examination

**Year 11: Calculus****C1.1 Gradients of tangents** Back**Syllabus:** updated November 2019. Latest version @<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- distinguish between continuous and discontinuous functions, identifying key elements which distinguish each type of function
  - sketch graphs of functions that are continuous and compare them with graphs of functions that have discontinuities
  - describe continuity informally, and identify continuous functions from their graphs
- describe the gradient of a secant drawn through two nearby points on the graph of a continuous function as an approximation of the gradient of the tangent to the graph at those points, which improves in accuracy as the distance between the two points decreases
- examine and use the relationship between the angle of inclination of a line or tangent,  $\theta$ , with the positive  $x$ -axis, and the gradient,  $m$ , of that line or tangent, and establish that  $\tan \theta = m$  AAM

**Reference Sheet****TG 1** Given the function  $y = \frac{1}{x-3}$ , for what values of  $x$  is the function[Solution](#)

- (a) continuous?  
(b) discontinuous?

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**TG 2**[Solution](#)

A function is defined as follows:  $f(x) = \begin{cases} 4 - x^2 & \text{for } x < 0 \\ ax + b & \text{for } 0 \leq x \leq 2 \\ (x - 2)^2 & \text{for } x > 2 \end{cases}$

Find the values of  $a$  and  $b$  which would ensure that  $f(x)$  is continuous.

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**TG 3** Find the equation of the line that is inclined at  $30^\circ$  to the positive  $x$ -axis and passes through the point  $(0, 2)$ .[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

# Year 11: Calculus

## C1.2 Difference quotients



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- describe the behaviour of a function and its tangent at a point, using language including increasing, decreasing, constant, stationary, increasing at an increasing rate AAM
- interpret and use the difference quotient  $\frac{f(x+h)-f(x)}{h}$  as the average rate of change of  $f(x)$  or the gradient of a chord or secant of the graph  $y = f(x)$
- interpret the meaning of the gradient of a function in a variety of contexts, for example on distance-time or velocity-time graphs

[Reference Sheet](#)

- TG 1** For each of the following functions determine the values of  $x$  for which the function is increasing and when the function is decreasing.

[Solution](#)



$$(a) f(x) = 4x$$

$$(b) f(x) = 2x^7$$

$$(c) f(x) = x^3 + 5$$

$$(d) f(x) = x^4 + 3$$

$$(e) f(x) = \frac{1}{x^2 + 4}$$

$$(f) f(x) = \frac{2}{x - 3}$$

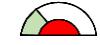
$$(g) f(x) = |x|$$

$$(h) f(x) = \frac{|x|}{x}$$

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

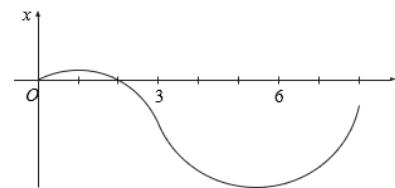
- TG 2** A particle  $P$  moves along a straight line for 8 seconds, starting at the fixed point  $S$  at time  $t = 0$ .

[Solution](#)



At time  $t$  seconds,  $P$  is  $x(t)$  metres to the right of  $S$ . The graph of  $x(t)$  is shown in the diagram.

- At approximately what times is the velocity of the particle equal to 0?
- At approximately what time is the particle moving with the greatest velocity?



NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**Year 11: Calculus****C1.3 The derivative function and its graph****Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>**Students:**

- examine the behaviour of the difference quotient  $\frac{f(x+h)-f(x)}{h}$  as  $h \rightarrow 0$  as an informal introduction to the concept of a limit (ACMMM081)
- interpret the derivative as the gradient of the tangent to the graph of  $y = f(x)$  at a point  $x$  (ACMMM085)
- estimate numerically the value of the derivative at a point, for simple power functions (ACMMM086)
- define the derivative  $f'(x)$  from first principles, as  $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$  and use the notation for the derivative:  $\frac{dy}{dx} = f'(x) = y'$ , where  $y = f(x)$
- use first principles to find the derivative of simple polynomials, up to and including degree 3
- understand the concept of the derivative as a function (ACMMM089)
- sketch the derivative function (or gradient function) for a given graph of a function, without the use of algebraic techniques and in a variety of contexts including motion in a straight line
- establish that  $f'(x) = 0$  at a stationary point,  $f'(x) > 0$  when the function is increasing and  $f'(x) < 0$  when it is decreasing, to form a framework for sketching the derivative function
- identify families of curves with the same derivative function (ACMMM121)
- use technology to plot functions and their gradient functions
- interpret and use the derivative at a point as the instantaneous rate of change of a function at that point **AAM**
- examine examples of variable rates of change of non-linear functions (ACMMM087)

[Reference Sheet](#)

**21 9** Let  $h(x) = f(g(x))$  where the function  $f(x)$  is an odd function and the function  $g(x)$  is an even function.

**1** [Solution](#)



The tangent to  $y = h(x)$  at  $x = k$ , where  $k > 0$ , has the equation  $y = mx + c$ .

What is the equation of the tangent to  $y = h(x)$  at  $x = -k$ ?

- A.  $y = mx + c$       B.  $y = -mx + c$       C.  $y = mx - c$       D.  $y = -mx - c$

NESA 2021 Mathematics Advanced HSC Examination

**21 31** By considering the equation of the tangent to  $y = x^2 - 1$  at the point  $(a, a^2 - 1)$ , find the equations of the two tangents to  $y = x^2 - 1$  which pass through  $(3, -8)$ .

**4** [Solution](#)



NESA 2021 Mathematics Advanced HSC Examination

**MA 11** Find the equation of the tangent to the curve  $f(x) = x^3 + 1$  at the point  $(1, 2)$ .

**2** [Solution](#)



NESA Mathematics Advanced Sample Examination Paper (2020)

**MA 20** A student was asked to differentiate  $f(x) = x^2 + 4x$  from first principles.

**2** [Solution](#)



The student began the solution as shown below.

Complete the solution.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

NESA Mathematics Advanced Sample Examination Paper (2020)

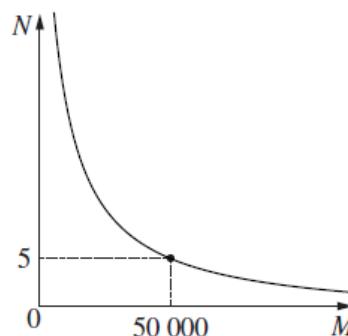
MA  
SQ  
2019

- 14** The number of years  $N$  required to carry out a project varies inversely with how much money \$ $M$  is spent on it. If \$50 000 is spent on the project, it takes 5 years to complete, as shown in the graph.

(a) Find an equation for the graph in terms of

$M$  and  $N$  in the form  $N = \frac{k}{M}$ , where  $k$  is a constant.

- (b) The project needs to be carried out in one year or less. What is the least amount of money that could be spent in order to achieve this timeline?  
 (c) At a point on the graph the number of years required to carry out the project is decreasing at the rate of one year per \$1000 spent. Find the value of  $M$  for this point, correct to the nearest \$1000.

[Solution](#)**1****1****3**

NESA Mathematics Advanced Sample examination materials (2019)

- TG 1** Use differentiation by first principles to verify that if  $y = x^2 + 5x$ , the gradient function is  $y' = 2x + 5$ .

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

- TG 2** Given the equation  $y = x^2 + 3x$ , using first principles determine  $y'$  and hence find the equation of the tangent to the curve at  $(1, 4)$ .

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

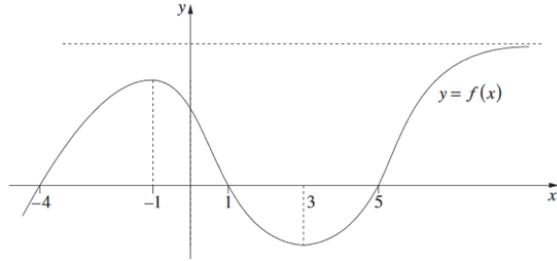
- TG 3** Use differentiation by first principles to verify that if  $y = 3x^2 - x$ , the gradient function is  $y' = 6x - 1$ .

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

- TG 4** The diagram shows the graph of a function  $y = f(x)$ .



- (a) For which values of  $x$  is the derivative,  $f'(x)$ , negative?  
 (b) What happens to  $f'(x)$ , for large values of  $x$ ?  
 (c) Sketch the graph  $y = f'(x)$ .

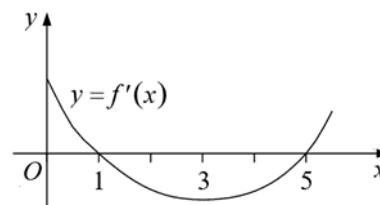


NESA Mathematics Advanced Year 11 Topic Guide: Calculus

- TG 5** The diagram shows the graph of the gradient function of the curve.



For what value of  $x$  does  $f(x)$  have a local minimum? Justify your answer.

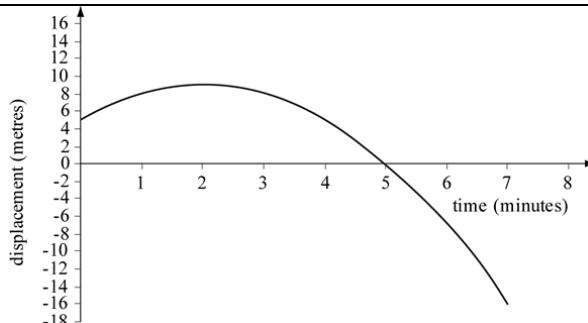


NESA Mathematics Advanced Year 11 Topic Guide: Calculus

- TG 6** The graph shows the displacement of a particle that is moving in a horizontal line.



- (a) Describe in words the motion of the particle during the first seven minutes.  
 (b) Sketch the graph of the velocity of the particle as a function of time.



NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**Year 11: Calculus****C1.4 Calculating with derivatives** Back**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- use the formula  $\frac{d}{dx}(x^n) = nx^{n-1}$  for all real values of  $n$  
- differentiate a constant multiple of a function and the sum or difference of two functions 
- understand and use the product, quotient and chain rules to differentiate functions of the form  $f(x)g(x)$ ,  $\frac{f(x)}{g(x)}$  and  $f(g(x))$  where  $f(x)$  and  $g(x)$  are functions
  - apply the product rule: If  $h(x) = f(x)g(x)$  then  $h'(x) = f(x)g'(x) + f'(x)g(x)$ , or if  $u$  and  $v$  are both functions of  $x$  then  $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
  - apply the quotient rule: If  $h(x) = \frac{f(x)}{g(x)}$  then  $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ , or if  $u$  and  $v$  are both functions of  $x$  then  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
  - apply the chain rule: If  $h(x) = f(g(x))$  then  $h'(x) = f'(g(x))g'(x)$ , or if  $y$  is a function of  $u$  and  $u$  is a function of  $x$  then  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- calculate derivatives of power functions to solve problems, including finding an instantaneous rate of change of a function in both real life and abstract situations **AAM**
- use the derivative in a variety of contexts, including to finding the equation of a tangent or normal to a graph of a power function at a given point **AAM**
- determine the velocity of a particle given its displacement from a point as a function of time
- determine the acceleration of a particle given its velocity at a point as a function of time

 Reference Sheet

**23 MA 7** It is given that  $y = f(g(x))$ , where  $f(1) = 3$ ,  $f'(1) = -4$ ,  $g(5) = 1$  and  $g'(5) = 2$ .  
What is the value of  $y'$  at  $x = 5$ ?

- A. -8      B. -4      C. 3      D. 6

**1**  Solution

NESA 2023 Mathematics Advanced HSC Examination

**23 MA 14** Find the equation of the tangent to the curve  $y = (2x + 1)^3$  at the point  $(0, 1)$ .

**3**  Solution

NESA 2023 Mathematics Advanced HSC Examination

**22 MA 5** Let  $h(x) = \frac{f(x)}{g(x)}$ , where

$$\begin{array}{ll} f(1) = 2 & f'(1) = 4 \\ g(1) = 8 & g'(1) = 12 \end{array}$$

What is the gradient of the tangent to the graph of  $y = h(x)$  at  $x = 1$ ?

- A. -8      B. 8      C.  $-\frac{1}{8}$       D.  $\frac{1}{8}$

**1**  Solution

NESA 2022 Mathematics Advanced HSC Examination

**TG 1** Find the derivative of:[Solution](#)

- (a)  $f(x) = x^7$   
 (b)  $f(t) = \sqrt{t}$   
 (c)  $g(x) = \frac{1}{x^3}$

**TG 2** Differentiate:[Solution](#)

- (a)  $f(x) = 5x^3 - 3x^2 + 8x + 2$   
 (b)  $f(x) = \sqrt{x^2 + 1}$   
 (c)  $f(x) = \frac{x+1}{x-1}$   
 (d)  $f(x) = \frac{1}{x^2 - 2x + 2}$   
 (e)  $f(x) = x^2(x+3)^{10}$

**TG 3** The line  $y = mx + c$  is a tangent to the curve  $y = x^3 - 3x + 1$  at the point  $(-2, -1)$ .  
Find the values of  $m$  and  $c$ .[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**TG 4** Find the equations of the tangents to the curve  $y = x^2 - 4x$  at the points where the curve cuts the  $x$ -axis.  
Where do the tangent lines meet?[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**TG 5** The displacement  $x$  metres from the origin at time  $t$  seconds of a particle travelling in a straight line is given by  $x = t^3 - 9t$ , where  $t \geq 0$ .  
(a) Find the velocity at time  $t$  seconds.  
(b) Calculate the velocity when  $t = 2$ .  
(c) Find the time when the particle is stationary.[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Calculus

**19 11** M c Differentiate  $\frac{2x+1}{x+5}$ .**2**[Solution](#)**19 14** M d The equation of the tangent to the curve  $y = x^3 + ax^2 + bx + 4$  at the point where  $x = 2$  is  $y = x - 4$ . Find the values of  $a$  and  $b$ .**3**[Solution](#)**18 12** M d The displacement of a particle moving along the  $x$ -axis is given by[Solution](#)

$$x = \frac{t^3}{3} - 2t^2 + 3t, \text{ where } x \text{ is the displacement from the origin in metres and } t \text{ is}$$

the time in seconds, for  $t \geq 0$ .

- (i) What is the initial velocity of the particle?  
 (ii) At which times is the particle stationary?  
 (iii) Find the position of the particle when the acceleration is zero.

**1****2****2**

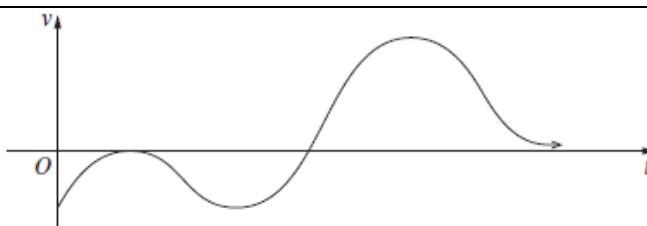
NESA 2018 Mathematics HSC Examination

- 17 10** A particle is moving along a straight line.

The graph shows the velocity,  $v$ , of the particle for time  $t \geq 0$ .

How many times does the particle change direction?

- (A) 1      (B) 2  
(C) 3      (D) 4



**1** [Solution](#)



- 17 12** Find the equation of the tangent to the curve  $y = x^2 + 4x - 7$  at the point  $(1, -2)$ .

**2** [Solution](#)



NESA 2017 Mathematics HSC Examination

- 16 11** Differentiate  $\frac{x+2}{3x-4}$ .

**2** [Solution](#)



NESA 2016 Mathematics HSC Examination

- 15 12** Find  $f'(x)$ , where  $f(x) = \frac{x^2 + 3}{x - 1}$ .

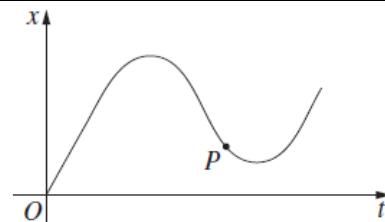
**2** [Solution](#)



NESA 2015 Mathematics HSC Examination

- 14 9** The graph shows the displacement  $x$  of a particle moving along a straight line as a function of time  $t$ .  
Which statement describes the motion of the particle at the point  $P$ ?

**1** [Solution](#)



- (A) The velocity is negative and the acceleration is positive.  
(B) The velocity is negative and the acceleration is negative.  
(C) The velocity is positive and the acceleration is positive.  
(D) The velocity is positive and the acceleration is negative.

NESA 2014 Mathematics HSC Examination

- 14 11** Differentiate  $\frac{x^3}{x+1}$ .

**2** [Solution](#)



NESA 2014 Mathematics HSC Examination

- 14 13** The displacement of a particle moving along the  $x$ -axis is given by  $x = t - \frac{1}{1+t}$ , where  $x$  is the displacement from the origin in metres,  $t$  is the time in seconds, and  $t \geq 0$ .

[Solution](#)

- (i) Show that the acceleration of the particle is always negative.  
(ii) What value does the velocity approach as  $t$  increases indefinitely?

**2**

**1**



NESA 2014 Mathematics HSC Examination

- 12 11** Find the equation of the tangent to the curve  $y = x^2$  at the point where  $x = 3$ .

**2**



NESA 2012 Mathematics HSC Examination

- 11 2c** Find the equation of the tangent to the curve  $y = (2x+1)^4$  at the point where  $x = -1$ .

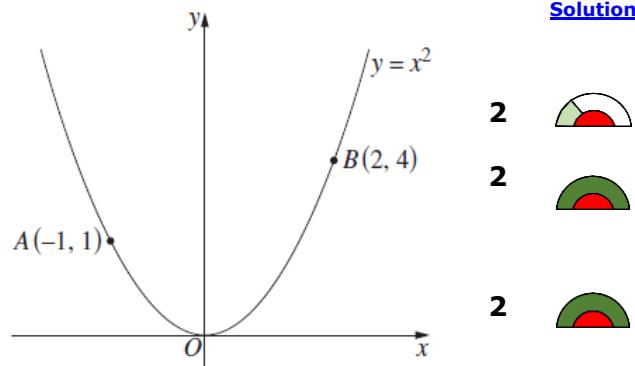
**3**



NESA 2011 Mathematics HSC Examination

- 10 7b** The parabola shown in the diagram is the graph  $y = x^2$ . The points  $A(-1, 1)$  and  $B(2, 4)$  are on the parabola.

- Find the equation of the tangent to the parabola at  $A$ .
- Let  $M$  be the midpoint of  $AB$ . There is a point  $C$  on the parabola such that the tangent at  $C$  is parallel to  $AB$ . Show that the line  $MC$  is vertical.
- The tangent at  $A$  meets the line  $MC$  at  $T$ . Show that the line  $BT$  is a tangent to the parabola.



2  
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NESAA 2010 Mathematics HSC Examination

- 09 1d** Find the gradient of the tangent to the curve  $y = x^4 - 3x$  at the point  $(1, -2)$ .

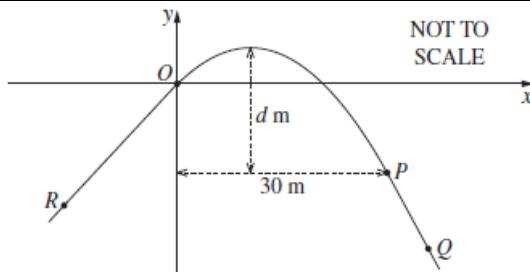
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NESAA 2009 Mathematics HSC Examination

- 09 6c** The diagram illustrates the design for part of a roller-coaster track. The section  $RO$  is a straight line with slope 1.2 and the section  $PQ$  is a straight line with slope -1.8. The section  $OP$  is a parabola  $y = ax^2 + bx$ . The horizontal distance from the  $y$ -axis to  $P$  is 30 m. In order that the ride is smooth, the straight sections must

be tangent to the parabola at  $O$  and at  $P$ .

- Find the values of  $a$  and  $b$  so that the ride is smooth.
- Find the distance  $d$ , from the vertex of the parabola to the horizontal line through  $P$ , as shown on the diagram.



3  
2

NESAA 2009 Mathematics HSC Examination

- 08 2a** Differentiate with respect to  $x$ : (i)  $(x^2 + 3)^9$

2

NESAA 2008 Mathematics HSC Examination

# Year 11: Exponential and Logarithmic Functions

## E1.1 Introducing logarithms



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- define logarithms as indices:  $y = a^x$  is equivalent to  $x = \log_a y$ , and explain why this definition only makes sense when  $a > 0, a \neq 1$
- recognise and sketch the graphs of  $y = ka^x$ ,  $y = ka^{-x}$  where  $k$  is a constant, and  $y = \log_a x$
- recognise and use the inverse relationship between logarithms and exponentials
  - understand and use the fact that  $\log_a a^x = x$  for all real  $x$ , and  $a^{\log_a x} = x$  for all  $x > 0$

**S**

Reference Sheet

**TG 1** Convert the following to exponential form:  $\log_{10} 10\ 000 = 4$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

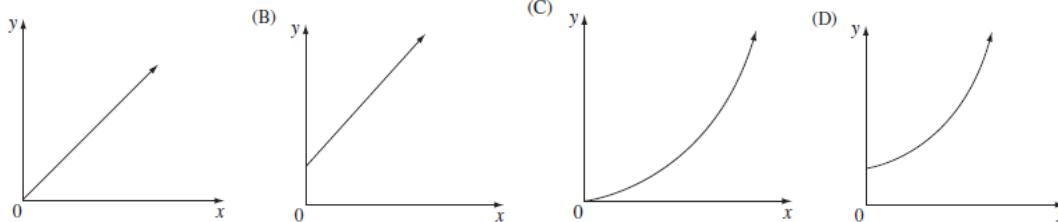
**TG 2** Convert the following to logarithmic form:  $5^2 = 25$ .

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

**11 MG 6** Which of the following graphs best describes the equation  $y = a^x$ , where  $a$  is a positive number greater than 1?

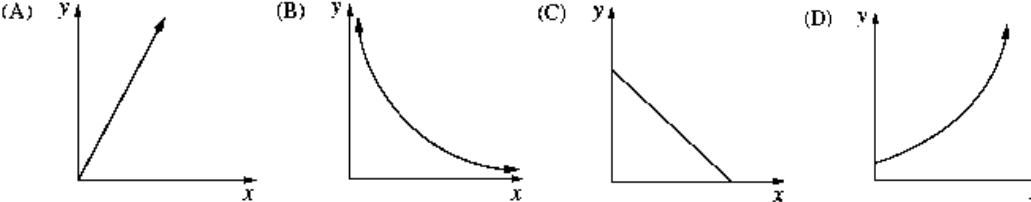
**1** [Solution](#)



NESA 2011 Mathematics General HSC Examination

**08 MG 4** Which graph best represents  $y = 3^x$ ?

**1** [Solution](#)



NESA 2008 Mathematics General HSC Examination

# Year 11: Exponential and Logarithmic Functions

## E1.2 Logarithmic Laws and Applications



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- derive the logarithmic laws from the index laws and use the algebraic properties of logarithms to simplify and evaluate logarithmic expressions  
 $\log_a m + \log_a n = \log_a(mn)$ ,  $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$ ,  $\log_a(m^n) = n \log_a m$ ,  
 $\log_a a = 1$ ,  $\log_a 1 = 0$ ,  $\log_a \frac{1}{x} = -\log_a x$
- consider different number bases and prove and use the change of base law  $\log_a x = \frac{\log_b x}{\log_b a}$  **AAM**  
💡
- interpret and use logarithmic scales, for example decibels in acoustics, different seismic scales for earthquake magnitude, octaves in music or pH in chemistry (ACMMM154) **AAM**
- solve algebraic, graphical and numerical problems involving logarithms in a variety of practical and abstract contexts, including applications from financial, scientific, medical and industrial contexts **AAM** 💡

[Reference Sheet](#)

**23 8 MA** What is the solution of the equation  $\log_a x^3 = b$ , where  $a$  and  $b$  are positive constants?

**1** [Solution](#)

- A.  $x = \frac{a}{b^3}$       B.  $x = \frac{b}{a^3}$       C.  $x = \frac{b^a}{3}$       D.  $x = \frac{a^b}{3}$

NESA 2023 Mathematics Advanced HSC Examination

**TG 1** Evaluate the following, without the use of a calculator:

[Solution](#)

- (a)  $\log_2 32$   
 (b)  $\log_4 0.25$   
 (c)  $\log_3 \frac{1}{81}$   
 (d)  $\log_a a^4$

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

**TG 2** Solve:

[Solution](#)

- (a)  $5^x = 7$   
 (b)  $\log_5 x = -3$   
 (c)  $\log_x 128 = 3.5$   
 (d)  $\log_{10}(x+4) - \log_{10}(x-5) = 1$

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

**TG 3** On the Richter scale, the magnitude  $R$  of an earthquake of intensity  $I$  is given by the

[Solution](#)

formula  $R = \log_{10}\left(\frac{I}{I_0}\right)$ , where  $I_0$  is a reference intensity used for comparisons.

- (a) Find  $R$  for an earthquake that is 4.3 million times more intense than the reference intensity.  
 (b) An earthquake measured 8.5 on the Richter scale. How many times more intense is this than the reference intensity?

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

- TG 4** On the decibel scale, the loudness  $L$  of a sound of intensity  $S$  is given by

[Solution](#)

$$L = 10 \log_{10} \left( \frac{S}{S_0} \right), \text{ where } S_0 \text{ is a reference intensity used for comparisons.}$$

- (a) A sound that causes pain in humans is about  $10^{14}$  times more intense than  $S_0$ .  
Find  $L$  for a sound of this intensity.  
(b) How many times more intense is the sound of a heated argument (about 67 decibels) than the sound of a quiet room (about 31 decibels)?



NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

- TG 5** The pH value of a solution is given by the formula  $\text{pH} = -\log_{10}[\text{H}^+]$ , where  $[\text{H}^+]$  is the concentration of hydrogen ions in moles per litre.

[Solution](#)

- (a) Find pH values for each of the following:



- (i) blood ( $[\text{H}^+] = 3.98 \times 10^{-8}$ )  
(ii) beer ( $[\text{H}^+] = 6.3 \times 10^{-5}$ )

- (b) Find the concentration of hydrogen ions in moles per litre for the following:  
(i) eggs ( $\text{pH} = 7.8$ )  
(ii) water ( $\text{pH} = 7.0$ )



NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

- 19 5** M Which of the following is equal to  $\frac{\log_2 9}{\log_2 3}$ ?

**1** [Solution](#)

- A. 2      B. 3      C.  $\log_2 3$       D.  $\log_2 6$

NESA 2019 Mathematics HSC Examination



- 18 11** MX b Solve  $\log_2 5 + \log_2(x - 2) = 3$ .

**2** [Solution](#)

NESA 2018 Mathematics Extension 1 HSC Examination

- 17 2** MX 1 It is given that  $\log_a 8 = 1.893$ , correct to 3 decimal places.

**1** [Solution](#)What is the value of  $\log_a 4$  correct to 2 decimal places?

- (A) 0.95      (B) 1.26      (C) 1.53      (D) 2.84

NESA 2017 Mathematics Extension 1 HSC Examination

- 16 10** M Which expression is equivalent to  $4 + \log_2 x$ ?

**1** [Solution](#)

- (A)  $\log_2(2x)$       (B)  $\log_2(16 + x)$       (C)  $4\log_2(2x)$       (D)  $\log_2(16x)$

NESA 2016 Mathematics Extension 1 HSC Examination



- 14 3** M What is the solution to the equation  $\log_2(x - 1) = 8$ ?

**1** [Solution](#)

- (A) 4      (B) 17      (C) 65      (D) 257

NESA 2014 Mathematics HSC Examination

- 11 1c** M Solve  $2^{2x+1} = 32$ .

**2** [Solution](#)

NESA 2011 Mathematics HSC Examination

# Year 11: Exponential and Logarithmic Functions

## E1.3 Exponential Function and Natural Logarithms



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- establish and use the formula  $\frac{d(e^x)}{dx} = e^x$  (ACMMM100)
  - using technology, sketch and explore the gradient function of exponential functions and determine that there is a unique number  $e \approx 2.71828182845$ , for which  $\frac{d(e^x)}{dx} = e^x$  where  $e$  is called Euler's number
- apply the differentiation rules to functions involving the exponential function,  $f(x) = ke^{ax}$ , where  $k$  and  $a$  are constants
- work with natural logarithms in a variety of practical and abstract contexts AAM
  - define the natural logarithm  $\ln x = \log_e x$  from the exponential function  $f(x) = e^x$  (ACMMM159)
  - recognise and use the inverse relationship of the functions  $y = e^x$  and  $y = \ln x$  (ACMMM160)
  - use the natural logarithm and the relationships  $e^{\ln x} = x$  where  $x > 0$ , and  $\ln(e^x) = x$  for all real  $x$  in both algebraic and practical contexts
  - use the logarithmic laws to simplify and evaluate natural logarithmic expressions and solve equations

[Reference Sheet](#)

**TG 1** If  $a = e^x$ , simplify  $\log_e a^2$ .

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions



**TG 2** Solve the equation:  $2 \ln x = \ln(5 + 4x)$ .

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions



**TG 3** Differentiate  $f(x) = 5e^x$  and hence find the gradient of the function when  $x = 2$ .

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

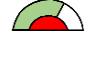


**19 15** Solve  $e^{2 \ln x} = x + 6$ .

**2** [Solution](#)

**M a**

NESA 2019 Mathematics HSC Examination

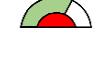


**17 5** It is given that  $\ln a = \ln b - \ln c$ , where  $a, b, c > 0$ . Which statement is true?

**1** [Solution](#)

**M** (A)  $a = b - c$       (B)  $a = \frac{b}{c}$       (C)  $\ln a = \frac{b}{c}$       (D)  $\ln a = \frac{\ln b}{\ln c}$

NESA 2017 Mathematics HSC Examination



**17 13**  $\frac{1}{M c}$  By letting  $m = t^{\frac{1}{3}}$ , or otherwise, solve  $t^{\frac{2}{3}} + t^{\frac{1}{3}} - 6 = 0$ .

**2** [Solution](#)

NESA 2017 Mathematics HSC Examination



**14 15** The line  $y = mx$  is a tangent to the curve  $y = e^{2x}$  at a point  $P$ .

[Solution](#)

**M c** (i) Sketch the line and the curve on one diagram.  
 (ii) Find the coordinates of  $P$ .  
 (iii) Find the value of  $m$ .

**1** [Solution](#)

**3** [Solution](#)

**1** [Solution](#)

NESA 2014 Mathematics HSC Examination



**13 11** Evaluate  $\ln 3$  correct to three significant figures.

**1** [Solution](#)

**M a**

NESA 2013 Mathematics HSC Examination



**12 7** Let  $a = e^x$ .

**1** [Solution](#)

**M** Which expression is equal to  $\log_e(a^2)$ ?

(A)  $e^{2x}$       (B)  $e^{x^2}$       (C)  $2x$       (D)  $x^2$

NESA 2012 Mathematics HSC Examination



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**11 2d** Find the derivative of  $y = x^2e^x$  with respect to  $x$ .  
**M**

**2**

NESA 2011 Mathematics HSC Examination

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**10 4d** Let  $f(x) = 1 + e^x$ . Show that  $f(x) \times f(-x) = f(x) + f(-x)$ .  
**M**

**2**

NESA 2010 Mathematics HSC Examination

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**10 1c** Solve  $\ln(x + 6) = 2\ln x$ .  
**MX**  
**1**

**3**

NESA 2010 Mathematics Extension 1 HSC Examination

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**07 6a** Solve the following equation for  $x$ :  $2e^{2x} - e^x = 0$ .  
**M**

**2**

NESA 2007 Mathematics HSC Examination

# Year 11: Exponential and Logarithmic Functions

## E1.4 Graphs and Applications of Exponential and Logarithmic Functions



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- solve equations involving indices using logarithms (ACMMM155)
- graph an exponential function of the form  $y = a^x$  for  $a > 0$  and its transformations  $y = ka^x + c$  and  $y = ka^{x+b}$  where  $k$ ,  $b$  and  $c$  are constants
- interpret the meaning of the intercepts of an exponential graph and explain the circumstances in which these do not exist
- establish and use the algebraic properties of exponential functions to simplify and solve problems (ACMMM064)
- solve problems involving exponential functions in a variety of practical and abstract contexts, using technology, and algebraically in simple cases (ACMMM067) AAM
- graph a logarithmic function  $y = \log_a x$  for  $a > 0$  and its transformations  $y = k \log_a x + c$ , using technology or otherwise, where  $k$  and  $c$  are constants
- recognise that the graphs of  $y = a^x$  and  $y = \log_a x$  are reflections in the line  $y = x$
- model situations and solve simple equations involving logarithmic or exponential functions algebraically and graphically AAM
- identify contexts suitable for modelling by exponential and logarithmic functions and use these functions to solve practical problems (ACMMM066, ACMMM158) AAM

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Reference Sheet

**21 3** Which of the following represents the domain of the function  $f(x) = \ln(1 - x)$ ?

**1** [Solution](#)

- MA**      A.  $[1, \infty)$       B.  $(1, \infty)$       C.  $(-\infty, 1]$       D.  $(-\infty, 1)$

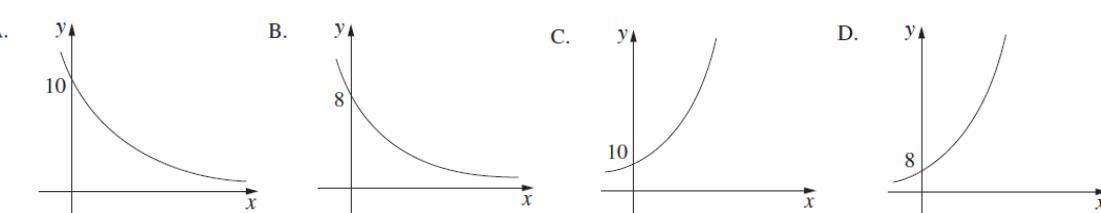
NESA 2021 Mathematics Advanced HSC Examination



**21 5** Which of the following best represents the graph of  $y = 10(0.8)^x$ ?

**1** [Solution](#)

**MA**



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**TG 1** Solve the following:

[Solution](#)

- $7^{3x} = 49^{2x-3}$
- $4^x - 12(2)^x = -32$
- $2e^{2x} - e^x = 0$
- $\log_e x - \frac{3}{\log_e x} = 2$



NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

**TG 2** The spread of a highly contagious virus can be modelled by the function

$$f(x) = \frac{4000}{1 + 1000e^{-0.07x}}$$



where  $x$  is the number of days after the first case of sickness due to the virus has been diagnosed and  $f(x)$  is the total number of people who are infected by the virus in the first  $x$  days.

Find and interpret the meaning of  $f(0)$ ,  $f(14)$  and  $f(365)$ .

NESA Mathematics Advanced Year 11 Topic Guide: Exponential and logarithmic functions

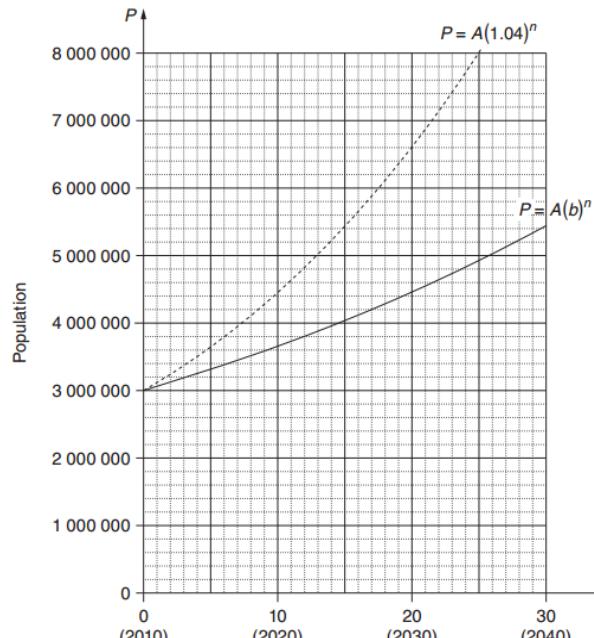
**TG 3** In 2010, the city of Thagoras modelled the predicted population of the city using the equation  $P = A(1.04)^n$ . That

**12 30 MG c** year, the city introduced a policy to slow its population growth.

The new predicted population was modelled using the equation  $P = A(b)^n$ .

In both equations,  $P$  is the predicted population and  $n$  is the number of years after 2010. The graph shows the two predicted populations.

- Use the graph to find the predicted population of Thagoras in 2030 if the population policy had NOT been introduced.
- In each of the two equations given, the value of  $A$  is 3 000 000. What does  $A$  represent?
- The guess-and-check method is to be used to find the value of  $b$ , in  $P = A(b)^n$ .
  - Explain, with or without calculations, why 1.05 is not a suitable first estimate for  $b$ .
  - With  $n = 20$  and  $P = 4\ 460\ 000$ , use the guess-and-check method and the equation  $P = A(b)^n$  to estimate the value of  $b$  to two decimal places. Show at least TWO estimate values for  $b$ , including calculations and conclusions.
  - The city of Thagoras was aiming to have a population under 7 000 000 in 2050. Does the model indicate that the city will achieve this aim? Justify your answer with suitable calculations.



..... Predicted population if the policy had not been introduced  
— Predicted population with the policy introduced



**Solution**

**19 1 MX 1** What is the domain of the function  $f(x) = \ln(4 - x)$ ?

- A.  $x < 4$       B.  $x \leq 4$       C.  $x > 4$       D.  $x \geq 4$

**1** **Solution**

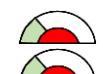


**18 13 M c** The population of a country grew exponentially between 1910 and 2010. This

population can be modelled by the equation  $P(t) = 92e^{kt}$ , where  $P(t)$  is the population of the country in millions,  $t$  is the time in years after 1910 and  $k$  is a positive constant. The population of the country in 1960 was 184 million.

- Show that the value of  $k$  is 0.0139, correct to 4 decimal places.
- Assuming that this model continues to be valid after 2010, estimate the population of the country in 2020 to the nearest million.

**Solution**



NESA 2019 Mathematics Extension 1 HSC Examination  
NESA 2012 Mathematics General HSC Examination

NESA 2018 Mathematics HSC Examination

**16 13** A radioactive isotope of Curium has a half-life of 163 days.[Solution](#)**M c** Initially there are 10 mg of Curium in a container.The mass  $M(t)$  in milligrams of Curium, after  $t$  days, is given by  $M(t) = Ae^{-kt}$ , where  $A$  and  $k$  are constants.(i) State the value of  $A$ .

1

(ii) Given that after 163 days only 5 mg of Curium remain, find the value of  $k$ .

2

NESA 2016 Mathematics HSC Examination

**16 29** The mass  $M$  kg of**MG b** a baby pig at age  $x$  days is given by  $M = A(1.1)^x$  where  $A$  is a constant.

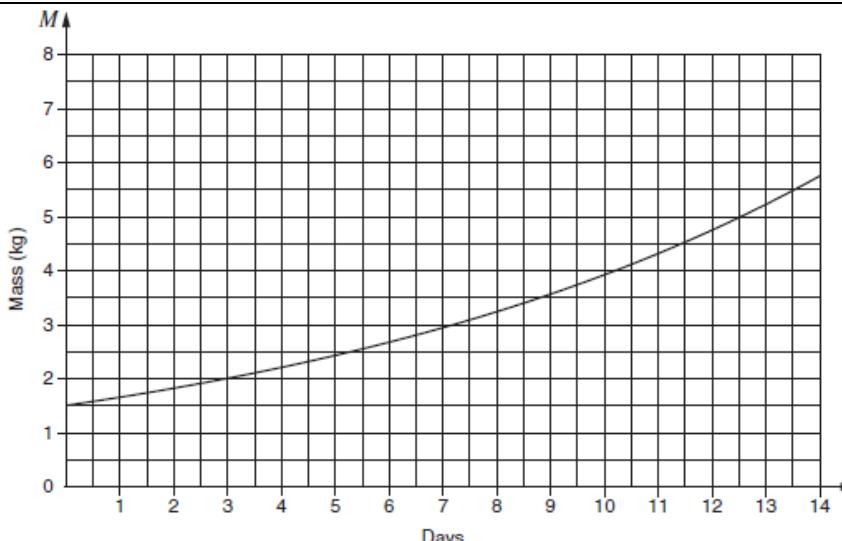
The graph of this equation is shown.

(i) What is the value of  $A$ ?

1

(ii) What is the daily growth rate of the pig's mass. Write your answer as a percentage.

1



NESA 2016 Mathematics General HSC Examination

**13 9** What is the solution of  $5^x = 4$ ?

1

- M** (A)  $x = \frac{\log_e 4}{5}$       (B)  $x = \frac{4}{\log_e 5}$       (C)  $x = \frac{\log_e 4}{\log_e 5}$       (D)  $x = \log_e \left(\frac{4}{5}\right)$



NESA 2013 Mathematics HSC Examination

**12 14** Professor Smith has a colony of bacteria. Initially, there are 1000 bacteria. The[Solution](#)**M c** number of bacteria,  $N(t)$ , after  $t$  minutes is given by  $N(t) = 1000e^{kt}$ .

(i) After 20 minutes there are 2000 bacteria.

1

Show that  $k = 0.0347$  correct to four decimal places.(ii) How many bacteria are there when  $t = 120$ ?

1

(iii) What is the rate of change of the number of bacteria per minute, when  $t = 120$ ?

1

(iv) How long does it take for the number of bacteria to increase from 1000 to 100 000?

2

NESA 2012 Mathematics HSC Examination

**11 10** The intensity  $I$ , measured in watt/m<sup>2</sup>, of a sound is given by  $I = 10^{-12} \times e^{0.1L}$ , where[Solution](#)**M a**  $L$  is the loudness of the sound in decibels.

(i) If the loudness of a sound at a concert is 110 decibels, find the intensity of the sound. Give your answer in scientific notation.

1

(ii) Ear damage occurs if the intensity of a sound is greater than  $8.1 \times 10^{-9}$  watt/m<sup>2</sup>. What is the maximum loudness of a sound so that no ear damage occurs?

2

(iii) By how much will the loudness of a sound have increased if its intensity has doubled?

2

NESA 2011 Mathematics HSC Examination

**11 1f** What is the range of the function  $f(x) = \ln(x^2 + e)$ ?

1



NESA 2011 Mathematics Extension 1 HSC Examination

**09 1f** Solve the equation  $\ln x = 2$ . Give your answer correct to four decimal places.

**2** [Solution](#)

**09 1b** Let  $f(x) = \ln(x - 3)$ . What is the domain of  $f(x)$ ?  
**MX 1**

**1** [Solution](#)

**08 7a** Solve  $\log_e x - \frac{3}{\log_e x} = 2$ .

**3** [Solution](#)

**08 25a** The number of penguins,  $P$ , after  $t$  years in a new colony can be found using the following formula  $P = a \times 2^t$ .

[Solution](#)

- (i) If there are 24 penguins after two years, find the value of  $a$ .
- (ii) How many years will it take for the number of penguins to first exceed 1500?

**2** 

NESAA 2009 Mathematics HSC Examination

NESAA 2009 Mathematics Extension 1 HSC Examination

NESAA 2008 Mathematics HSC Examination

NESAA 2008 Mathematics General HSC Examination

# Year 11: Statistical Analysis

## S.1 Probability and Venn Diagrams



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- understand and use the concepts and language associated with theoretical probability, relative frequency and the probability scale
- solve problems involving simulations or trials of experiments in a variety of contexts AAM
  - identify factors that could complicate the simulation of real-world events (ACMEM153)
  - use relative frequencies obtained from data as point estimates of probabilities (ACMMM055)
- use arrays and tree diagrams to determine the outcomes and probabilities for multi-stage experiments (ACMEM156) AAM
  - use everyday occurrences to illustrate set descriptions and representations of events and set operations (ACMMM051)
- establish and use the rules:  $P(\bar{A}) = 1 - P(A)$  and  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  (ACMMM054) AAM
  - understand the notion of conditional probability and recognise and use language that indicates conditionality (ACMMM056)
  - use the notation  $P(A|B)$  and the formula  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ,  $P(B) \neq 0$  for conditional probability (ACMMM057) AAM
- understand the notion of independence of an event  $A$  from an event  $B$ , as defined by  $P(A|B) = P(A)$  (ACMMM058)
- use the multiplication law  $P(A \cap B) = P(A)P(B)$  for independent events  $A$  and  $B$  and recognise the symmetry of independence in simple probability situations (ACMMM059)

STANDARD 2

ST

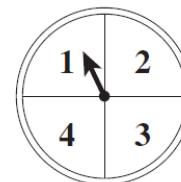
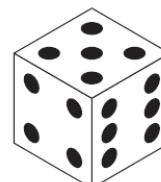
[Reference Sheet](#)

- 23 MA 2** A game involves throwing a die and spinning a spinner.

The sum of the two numbers obtained is the score. The table of scores below is partially completed.

What is the probability of getting a score of 7 or more?

- |                   |                   |
|-------------------|-------------------|
| A. $\frac{1}{6}$  | B. $\frac{1}{4}$  |
| C. $\frac{5}{18}$ | D. $\frac{5}{12}$ |



		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

**1** [Solution](#)



COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

- 23 31** Four Year 12 students want to organise a graduation party. All four students have the same probability,  $P(F)$ , of being available next Friday. All four students have the same probability,  $P(S)$ , of being available next Saturday.

It is given that  $P(F) = \frac{3}{10}$ ,  $P(S|F) = \frac{1}{3}$ , and  $P(F|S) = \frac{1}{8}$ .

Kim is one of the four students.

- (a) Is Kim's availability next Friday independent from his availability next Saturday? **1**
- Justify your answer.

- (b) Show that the probability that Kim is available next Saturday is  $\frac{4}{5}$ . **2**

- (c) What is the probability that at least one of the four students is NOT available next Saturday? **2**



NESA 2023 Mathematics Advanced HSC Examination

- 22 9** Liam is playing two games. He is equally likely to win each game.

**MA** The probability that Liam will win at least one of the games is 80%.

Which of the following is closest to the probability that Liam will win both games?

- A. 31%      B. 40%      C. 55%      D. 64%



NESA 2022 Mathematics Advanced HSC Examination

- 22 15** In a bag there are 3 six-sided dice.

**MA** Two of the dice have faces marked 1, 2, 3, 4, 5, 6.

The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

- (a) What is the probability that the die shows a 5? **1**

- (b) Given that the die shows a 5, what is the probability that it is the special die? **1**



NESA 2022 Mathematics Advanced HSC Examination

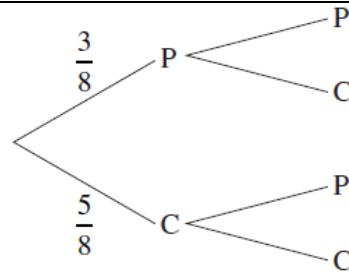
- 21 6** There are 8 chocolates in a box.

**MA** Three have peppermint centres (P) and five have caramel centres (C). Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.

A partially completed probability tree is shown.

What is the probability that Kim and Sam choose chocolates with different centres?

- A.  $\frac{15}{64}$       B.  $\frac{15}{56}$       C.  $\frac{15}{32}$       D.  $\frac{15}{28}$



**1** [Solution](#)



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**20 14** History and Geography are two subjects students may decide to study.[Solution](#)**MA** For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

(a) A student is chosen at random. By using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography.

**2**

(b) A student is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History?

**1**

(c) Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History?

**2**

NESA 2020 Mathematics Advanced HSC Examination

**20 15** The top of a rectangular table is divided into 8 equal sections as shown.**1**[Solution](#)

**MS 2** A standard die with faces labelled 1 to 6 is rolled onto the table.

1	2	3	4
5	6	7	8

The die is equally likely to land in any of the 8 sections of the table. If the die does not land entirely in one section of the table, it is rolled again.

A score is calculated by multiplying the value shown on the top face of the die by the number shown in the section of the table where the die lands.

What is the probability of getting a score of 6?

A  $\frac{1}{48}$

B  $\frac{1}{12}$

C  $\frac{1}{8}$

D  $\frac{1}{6}$

NESA 2020 Mathematics Standard 2 HSC Examination

**MA 13** A credit card requires a four-figure personal identification number (PIN) for purchases. The figures are chosen from the digits 0, 1, 2, 3, ..., 9. Repetition is allowed and the PIN can start with any of the 10 digits. The credit card is lost and the finder tries to guess the PIN by entering four digits.

[Solution](#)

- (a) What is the probability that the four digits entered are the correct PIN?  
 (b) What is the probability that the finder will guess at least one digit in its correct order?

**1  
1**

NESA Mathematics Advanced Sample Examination Paper (2020)

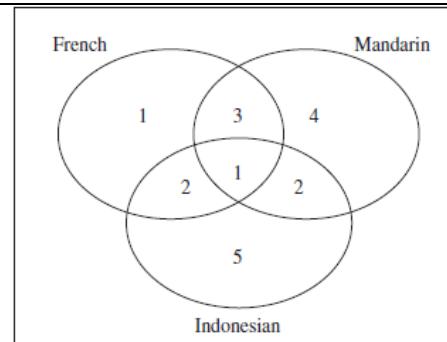
**MA 5** In a mixed language class, students study French, Mandarin and Indonesian. The number of students who study each language are shown in the Venn diagram.

**1**[Solution](#)

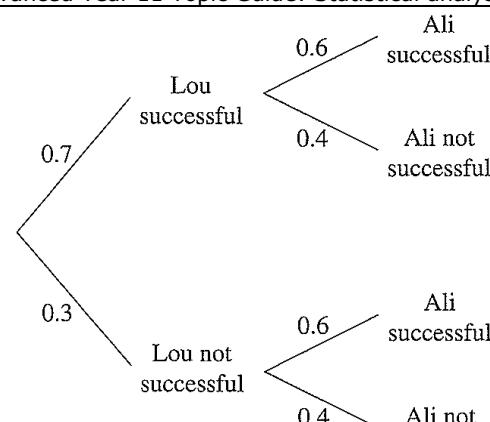
A student who studies Indonesian is selected at random.

What is the probability that he/she also studies French?

- A.  $\frac{1}{6}$       B.  $\frac{3}{10}$       C.  $\frac{3}{5}$       D.  $\frac{7}{10}$



NESA Mathematics Advanced Sample examination materials (2019)

<b>MA SQ 2019</b>	<b>16</b> <b>Band 2-5</b>	<p>A survey found that in a large population approximately 20% of people are left-handed.</p> <p>(a) Three people are selected at random. Find the probability that at least one of them is left-handed.</p> <p>(b) What is the smallest number of people that would need to be selected to have a greater than 99% chance that at least one of them is left-handed?</p>	<span style="float: right;"><a href="#">Solution</a></span> <span style="float: right;">2</span> 
		NESMA Mathematics Advanced Sample examination materials (2019)	
<b>TG</b>	<b>1</b>	<p>Six girls' names and five boys' names are placed in a hat. Two names are drawn without replacement. What is the probability that a girl's and a boy's name are chosen?</p>	<span style="float: right;"><a href="#">Solution</a></span> 
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	
<b>TG</b>	<b>2</b>	<p>In a raffle, 30 tickets are sold and there are two prizes. John buys five tickets. What is the probability that John wins at least one prize?</p>	<span style="float: right;"><a href="#">Solution</a></span> 
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	
<b>TG</b>	<b>3</b>	<p>In Australia, approximately 9% of the population has the blood type O negative. If three people are chosen at random from the population, find the probability that:</p> <p>(a) none has O negative blood (b) at least one has O negative blood.</p>	<span style="float: right;"><a href="#">Solution</a></span> 
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	
<b>TG</b>	<b>4</b>	<p>The manager of a team notices that the team has a probability of <math>\frac{2}{3}</math> of winning the game if it is raining and if it is dry, the probability of the team winning is <math>\frac{1}{5}</math>. The probability that it will rain on a day when they play is <math>\frac{1}{4}</math>.</p> <p>(a) Find the probability that they will not win. (b) Given that the team has won a game, calculate the probability that it rained on the day of the match.</p>	<span style="float: right;"><a href="#">Solution</a></span> 
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	
<b>TG</b>	<b>5</b>	<p>Lou and Ali are on a fitness program for one month. The probability that Lou will finish the program successfully is 0.7 while the probability that Ali will finish successfully is 0.6.</p> <p>10 MG</p> <p>The probability tree diagram shows this information.</p> <p>What is the probability that only one of Lou and Ali will be successful?</p>	
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	<span style="float: right;"><a href="#">Solution</a></span> 
<b>TG</b>	<b>20</b>	<p>NESA 2010 General Mathematics HSC Examination</p>	
<b>TG</b>	<b>6</b>	<p>A bag contains two red balls, one black ball, and one white ball. Andrew selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden.</p> <p>(a) Draw a tree diagram to show all the possible outcomes. (b) Find the probability that both the selected balls are red. (c) Find the probability that at least one of the selected balls is red. (d) Andrew drops one of the selected balls and we can see that it is red. What is the probability that the ball that is still hidden is also red?</p>	<span style="float: right;"><a href="#">Solution</a></span> 
		NESMA Mathematics Advanced Year 11 Topic Guide: Statistical analysis	

- 19 6** A game is played by tossing an ordinary 6-sided die and an ordinary coin at the same time. The game is won if the uppermost face of the die shows an even number or the uppermost face of the coin shows a tail (or both). What is the probability of winning this game?

A.  $\frac{1}{4}$

B.  $\frac{1}{2}$

C.  $\frac{3}{4}$

D. 1

**1** [Solution](#)

- 19 11** A bag contains 5 green beads and 7 purple beads. Two beads are selected at random, without replacement. What is the probability that the two beads are the same colour?

**2** [Solution](#)

- 19 15** The probability that a person chosen at random has red hair is 0.02.

[Solution](#)

- M d** (i) Two people are chosen at random.

**2**

What is the probability that at least ONE has red hair?

- (ii) What is the smallest number of people that can be chosen at random so that the probability that at least ONE has red hair is greater than 0.4

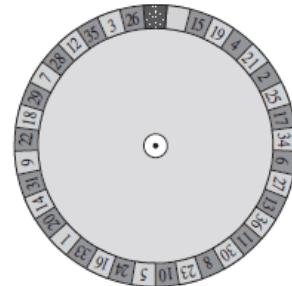
**2**

NESA 2019 Mathematics HSC Examination

- 19 20** A roulette wheel has the numbers 0, 1, 2, ..., 36 where each of the 37 numbers is equally likely to be spun.

**2**

If the wheel is spun 18 500 times, calculate the expected frequency of spinning the number 8.



NESA 2019 Mathematics Standard 2 HSC Examination

- 19 25** A bowl of fruit contains 17 apples of which 9 are red and 8 are green.  
**MS 2** Dennis takes one apple at random and eats it. Margaret also takes an apple at random and eats it.

**3** [Solution](#)

By drawing a probability tree diagram, or otherwise, find the probability that Dennis and Margaret eat apples of the same colour.

NESA 2019 Mathematics Standard 2 HSC Examination

- 18 6** A runner has four different pairs of shoes. If two shoes are selected at random, what is the probability that they will be a matching pair?

**1**

A.  $\frac{1}{56}$       B.  $\frac{1}{16}$       C.  $\frac{1}{7}$       D.  $\frac{1}{4}$

NESA 2018 Mathematics HSC Examination

- 18 14** Two machines, *A* and *B*, produce pens. It is known that 10% of the pens produced by machine *A* are faulty and that 5% of the pens produced by machine *B* are faulty.

[Solution](#)

- (i) One pen is chosen at random from each machine. What is the probability that at least one of the pens is faulty?

**1**

- (ii) A coin is tossed to select one of the two machines. Two pens are chosen at random from the selected machine. What is the probability that neither pen is faulty?

**2**

NESA 2018 Mathematics HSC Examination

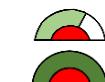
**18 16** A game involves rolling two six-sided dice, followed by rolling a third six-sided die. [Solution](#)

**M b** To win the game, the number rolled on the third die must lie between the two numbers rolled previously. For example, if the first two dice show 1 and 4, the game can only be won by rolling a 2 or 3 with the third die.

(i) What is the probability that a player has no chance of winning before rolling the third die? **2**

(ii) What is the probability that a player wins the game? **2**

NESA 2018 Mathematics HSC Examination

[Solution](#)**2****2**

**18 MG 26a** Jeremy rolled a biased 6-sided die a number of times. He recorded the results in a table. What is the relative frequency of rolling a 3?

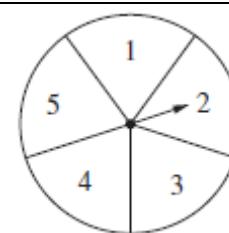
Number	1	2	3	4	5	6
Frequency	23	19	48	20	21	19

**1**

NESA 2018 Mathematics General 2 HSC Examination

**17 12** A spinner is marked with the numbers 1, 2, 3, 4 and 5. When **M e** it is spun, each of the five numbers is equally likely to occur.

The spinner is spun three times.

**1**

(i) What is the probability that an even number occurs on the first spin? **1**

(ii) What is the probability that an even number occurs on at least one of the three spins? **1**

(iii) What is the probability that an even number occurs on the first spin and odd numbers occur on the second and third spins? **1**

(iv) What is the probability that an even number occurs on exactly one of the three spins? **1**

NESA 2017 Mathematics HSC Examination

**17 24** A deck of 52 playing cards contain 12 picture cards. Two cards are drawn at random and placed on a table. What is the probability, correct to four decimal places, that exactly one picture card is on the table? **1**

- (A) 0.0498      (B) 0.1810      (C) 0.3550      (D) 0.3620



NESA 2017 Mathematics General 2 HSC Examination

**16 2** In a raffle, 30 tickets are sold and there is one prize to be won. **M**

**1**

What is the probability that someone buying 6 tickets wins the prize?

- (A)  $\frac{1}{30}$       (B)  $\frac{1}{6}$       (C)  $\frac{1}{5}$       (D)  $\frac{1}{4}$

NESA 2016 Mathematics HSC Examination

**15 M 4** The probability that Mel's soccer team wins this weekend is  $\frac{5}{7}$ . The probability that **M**

**1**

Mel's rugby league team wins this weekend is  $\frac{2}{3}$ . What is the probability that neither team wins this weekend?

- (A)  $\frac{2}{21}$       (B)  $\frac{10}{21}$       (C)  $\frac{13}{21}$       (D)  $\frac{19}{21}$

NESA 2015 Mathematics HSC Examination

**15** **14** Weather records for a town suggest that:

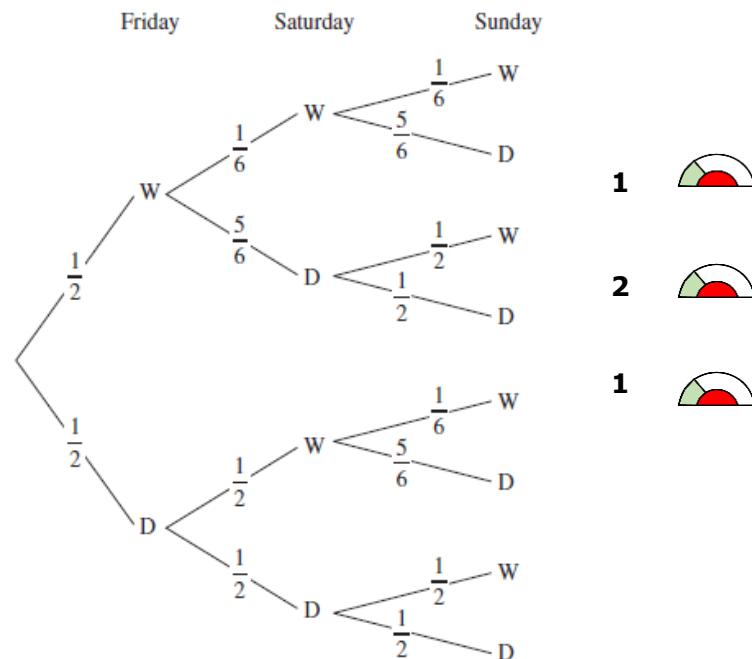
### **Solution**

- M b**

  - if a particular day is wet ( $W$ ), the probability of the next day being dry is  $\frac{5}{6}$ .
  - if a particular day is dry ( $D$ ), the probability of the next day being dry is  $\frac{1}{2}$ .

In a specific week Thursday is dry.  
The tree diagram shows the possible  
outcomes for the next three days:  
Friday, Saturday and Sunday.

- (i) Show that the probability of Saturday being dry is  $\frac{2}{3}$ .
  - (ii) What is the probability of both Saturday and Sunday being wet?
  - (iii) What is the probability of at least one of Saturday and Sunday being dry?



NESA 2015 Mathematics HSC Examination

**14** **10** Three runners compete in a race. The probabilities that the three runners finish the race in under 10 seconds are  $\frac{1}{4}$ ,  $\frac{1}{6}$  and  $\frac{2}{5}$  respectively.

## **Solution**

What is the probability that at least one of the three runners will finish the race in under 10 seconds?

- (A)  $\frac{1}{60}$       (B)  $\frac{37}{60}$       (C)  $\frac{3}{8}$       (D)  $\frac{5}{8}$

NESA 2014 Mathematics HSC Examination

**14** **12** A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.

- (i) Draw a tree diagram to show the possible outcomes. Include the probability on each branch.  
(ii) What is the probability that the two lollies are of different colours?

NESA 2014 Mathematics HSC Examination

**14 16** In Mathsville, there are on average eight rainy days in October. Which expression could be used to find a value for the probability that it will rain on two consecutive days in October in Mathsville?

- (A)  $\frac{8}{31} \times \frac{7}{30}$       (B)  $\frac{8}{31} \times \frac{7}{31}$       (C)  $\frac{8}{31} \times \frac{8}{30}$       (D)  $\frac{8}{31} \times \frac{8}{31}$

NESA 2014 Mathematics General 2 HSC Examination

**13**   **5**   A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random without replacement.

- What is the probability that at least one of the marbles selected is red?

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{2}$       (C)  $\frac{5}{6}$       (D)  $\frac{29}{30}$

NESA 2013 Mathematics HSC Examination

- 13 18** Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6 are rolled.  
**GM** What is the probability of obtaining a sum of 6?

(A)  $\frac{1}{6}$       (B)  $\frac{1}{12}$       (C)  $\frac{5}{12}$       (D)  $\frac{5}{36}$

**1** [Solution](#)

NESA 2013 General Mathematics HSC Examination

- 12 13** Two buckets each contain red marbles and white marbles. Bucket A contains 3 red  
**M c** and 2 white marbles. Bucket B contains 3 red and 4 white marbles. Chris randomly  
chooses one marble from each bucket.
- What is the probability that both marbles are red?
  - What is the probability that at least one of the marbles is white?
  - What is the probability that both marbles are the same colour?

**1** [Solution](#)

NESA 2012 Mathematics HSC Examination

- 12 12** Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled. What  
**GM** is the probability of a 6 appearing on at least one of the dice?

(A)  $\frac{1}{6}$       (B)  $\frac{11}{36}$       (C)  $\frac{25}{36}$       (D)  $\frac{5}{6}$

**1** [Solution](#)

NESA 2012 General Mathematics HSC Examination

- 11 1g** A batch of 800 items is examined. The probability that an item from this batch is  
**M** defective is 0.02. How many items from this batch are defective?

**1** [Solution](#)

NESA 2011 Mathematics HSC Examination

- 11 5b** Kim has three red shirts and two yellow shirts. On each of the three days, Monday,  
**M** Tuesday and Wednesday, she selects one shirt at random to wear. Kim wears each  
shirt that she selects only once.
- What is the probability that Kim wears a red shirt on Monday?
  - What is the probability that Kim wears a shirt of the same colour on all  
three days?
  - What is the probability that Kim does not wear a shirt of the same colour on  
consecutive days?

**1** [Solution](#)

NESA 2011 Mathematics HSC Examination

- 10 4c** There are twelve chocolates in a box. Four of the chocolates have mint centres,  
**M** four have caramel centres and four have strawberry centres. Ali randomly selects  
two chocolates and eats them.
- What is the probability that the two chocolates have mint centres?
  - What is the probability that the two chocolates have same centres?
  - What is the probability that the two chocolates have different centres?

**1** [Solution](#)

NESA 2010 Mathematics HSC Examination

- 10 8b** Two identical biased coins are tossed together, and the outcome is recorded. After  
**M** a large number of trials it is observed that the probability that both coins land  
showing heads is 0.36. What is the probability that both coins land showing tails?

**2** [Solution](#)

NESA 2010 Mathematics HSC Examination

- 09 5b** On each working day James parks his car in a parking station which has three  
**M** levels. He parks his car on a randomly chosen level. He always forgets where he  
has parked so when he leaves work he chooses a level at random and searches for  
his car. If his car is not on that level, he chooses a different level and continues in  
this way until he finds his car.
- What is the probability that his car is on the first level he searches?
  - What is the probability that he must search all three levels before he finds  
his car?
  - What is the probability that on every one of the five working days in a week,  
his car is not on the first level he searches?

**1** [Solution](#)

NESA 2009 Mathematics HSC Examination

- 09 9a** Each week Van and Marie take part in a raffle at their respective workplaces. The probability that Van wins a prize in his raffle is  $\frac{1}{9}$ . The probability that Marie wins a prize in her raffle is  $\frac{1}{16}$ . What is the probability that, during the next three weeks, at least one of them wins a prize? 2 [Solution](#)



NESA 2009 Mathematics HSC Examination

- 09 27** In each of three raffles, 100 tickets are sold and one prize is awarded. 4 [Solution](#)
- GM c** Mary buys two tickets in one raffle. Jane buys one ticket in each of the other two raffles. Determine who has the better chance of winning at least one prize. Justify your response using probability calculations.



NESA 2009 General Mathematics HSC Examination

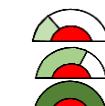
- 09 28** In an experiment, two unbiased dice, with faces numbered 1, 2, 3, 4, 5, 6, are rolled 18 times. The difference between the numbers on their uppermost faces is recorded each time. Juan performs this experiment twice and his results are shown in the tables. 4 [Solution](#)
- Juan states that Experiment 2 has given results that are closer to what he expected than the results given by Experiment 1. Is he correct? Explain your answer by finding the sample space for the dice differences and using theoretical probability.
- | Experiment 1 |           | Experiment 2 |           |
|--------------|-----------|--------------|-----------|
| Difference   | Frequency | Difference   | Frequency |
| 0            | 3         | 0            | 4         |
| 1            | 3         | 1            | 4         |
| 2            | 2         | 2            | 3         |
| 3            | 4         | 3            | 3         |
| 4            | 3         | 4            | 2         |
| 5            | 3         | 5            | 2         |



- 08 7c** Xena and Gabrielle compete in a series of games. The series finishes when one player has won two games. In any game, the probability that Xena wins is  $\frac{2}{3}$  and the probability that Gabrielle wins is  $\frac{1}{3}$ . [Solution](#)
- First game                      Second game                      Third game
- 

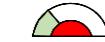


- (i) Copy and complete the tree diagram.  
(ii) What is the probability that Gabrielle wins the series?  
(iii) What is the probability that three games are played in the series? 1  
2  
2



NESA 2008 Mathematics HSC Examination

- 08 9a** It is estimated that 85% of students in Australia own a mobile phone. [Solution](#)
- (i) Two students are selected at random. What is the probability that neither of them owns a mobile phone? 2
- (ii) Based on a recent survey, 20% of the students who own a mobile phone have used their mobile phone during class time. A student is selected at random. What is the probability that the student owns a mobile phone and has used it during classtime? 1



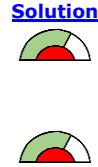
NESA 2008 Mathematics HSC Examination

- 
- 07 4b** Two ordinary dice are rolled. The score is the sum of the numbers on the top faces.  
**M**
- (i) What is the probability that the score is 10?  
(ii) What is the probability that the score is not 10?

**2**  
**1**

NESA 2007 Mathematics HSC Examination

- 
- 07 9b** A pack of 52 cards consists of four suits with 13 cards in each suit.  
**M**
- (i) One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table. What is the probability that the second card is from a different suit to the first?  
(ii) The two cards are replaced and the pack shuffled. Four cards are chosen from the pack and placed side by side on the table. What is the probability that these four cards are all from different suits?

**1**  
**2**

NESA 2007 Mathematics HSC Examination

# Year 11: Statistical Analysis

## S1.2 Discrete Probability Distributions



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

- define and categorise random variables
  - know that a random variable describes some aspect in a population from which samples can be drawn
  - know the difference between a discrete random variable and a continuous random variable
- use discrete random variables and associated probabilities to solve practical problems  
(ACMMM142) **AAM**
  - use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)
  - recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)
  - examine simple examples of non-uniform discrete random variables, and recognise that for any random variable,  $X$ , the sum of the probabilities is 1 (ACMMM139)
  - recognise the mean or expected value,  $E(X) = \mu$ , of a discrete random variable  $X$  as a measure of centre, and evaluate it in simple cases (ACMMM140)
  - recognise the variance,  $\text{Var}(X)$ , and standard deviation ( $\sigma$ ) of a discrete random variable as measures of spread, and evaluate them in simple cases (ACMMM141)
  - use  $\text{Var}(X) = E((X - \mu)^2) = E(X^2) - \mu^2$  for a random variable and  $\text{Var}(x) = \sigma^2$  for a dataset
- understand that a sample mean,  $\bar{x}$ , is an estimate of the associated population mean  $\mu$ , and that the sample standard deviation,  $s$ , is an estimate of the associated population standard deviation,  $\sigma$ , and that these estimates get better as the sample size increases and when we have independent observations

[Reference Sheet](#)

**23 12** The table shows the probability distribution of a  
**MA** discrete random variable.

- Show that the expected value  $E(X) = 2$ .
- Calculate the standard deviation, correct to one decimal place.

$x$	0	1	2	3	4
$P(X=x)$	0	0.3	0.5	0.1	0.1

[Solution](#)

**1**

**2**

[Solution](#)

**21 2** The probability distribution table for a  
**MA** random variable  $X$  is shown.  
What is the expected value for  $X$ ?

- 0.6
- 1.0
- 1.5
- 2.0

$x$	$P(X=x)$
1	0.6
2	0.3
3	0.1

**1**

[Solution](#)

NESA 2023 Mathematics Advanced HSC Examination

- MA** **19** A discrete random variable  $X$  has the probability distribution table shown.  
**SP** **Band 3-6** By finding the value of  $m$ , calculate the expected value and the variance of  $X$ .

$X = x$	11	12	13	14
$P(x)$	0.2	0.3	$m$	0.4

**3** [Solution](#)

- TG** **1** The total number of cars to be sold next week is described by the following probability distribution.

Determine the expected value and standard deviation of  $X$ , the number of cars sold.

$X$	0	1	2	3	4
$p(X)$	0.05	0.15	0.35	0.25	0.20

[Solution](#)

NESA Mathematics Advanced Year 11 Topic Guide: Statistical analysis

# Year 12: Functions

## F2 Graphing Techniques



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- apply transformations to sketch functions of the form  $y = kf(a(x + b)) + c$ , where  $f(x)$  is a polynomial, reciprocal, absolute value, exponential or logarithmic function and  $a, b, c$  and  $k$  are constants
  - examine translations and the graphs of  $y = f(x) + c$  and  $y = f(x + b)$  using technology
  - examine dilations and the graphs of  $y = kf(x)$  and  $y = f(ax)$  using technology
  - recognise that the order in which transformations are applied is important in the construction of the resulting function or graph
- use graphical methods with supporting algebraic working to solve a variety of practical problems involving any of the functions within the scope of this syllabus, in both real-life and abstract contexts **AAM**
  - select and use an appropriate method to graph a given function, including finding intercepts, considering the sign of  $f(x)$  and using symmetry
  - determine asymptotes and discontinuities where appropriate (vertical and horizontal asymptotes only)
  - determine the number of solutions of an equation by considering appropriate graphs
  - solve linear and quadratic inequalities by sketching appropriate graphs

**S  
T  
A**

[Reference Sheet](#)

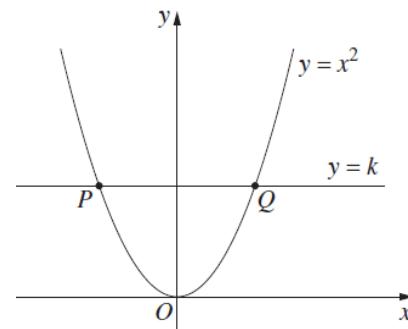
- 23 10 MA** The graph  $y = x^2$  meets the line  $y = k$  (where  $k > 0$ ) at points  $P$  and  $Q$  as shown in the diagram.

The length of the interval  $PQ$  is  $L$ .

Let  $a$  be a positive number. The graph  $y = \frac{x^2}{a^2}$  meets the line  $y = k$  at points  $S$  and  $T$ .

What is the length of  $ST$ ?

- A.  $\frac{L}{a}$       B.  $\frac{L}{a^2}$   
 C.  $aL$       D.  $a^2L$



**1** [Solution](#)



NESA 2023 Mathematics Advanced HSC Examination

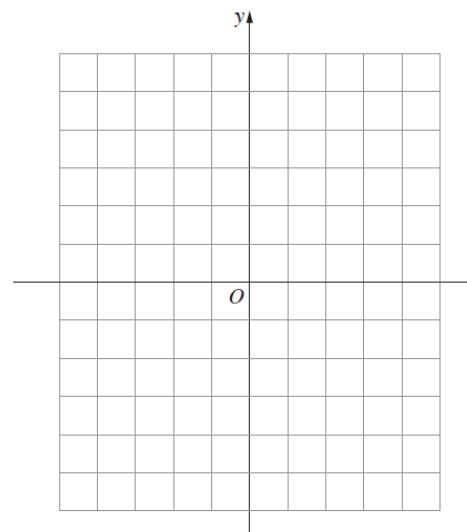
**23 19** (a) Sketch the graphs of the functions

**MA**

$f(x) = x - 1$  and  $g(x) = (1 - x)(3 + x)$   
showing the  $x$ -intercepts.

(b) Hence, or otherwise, solve the inequality

$$x - 1 < (1 - x)(3 + x).$$



**2**

**Solution**



**2**



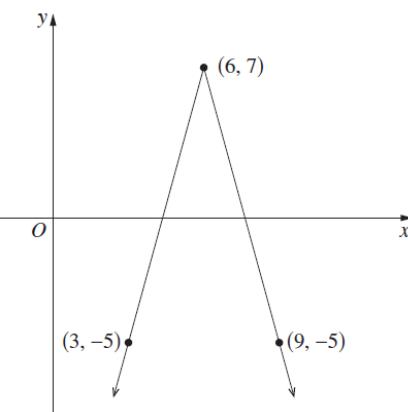
NESA 2023 Mathematics Advanced HSC Examination

**23 27** The graph of  $y = f(x)$ , where

**MA**  $f(x) = a|x - b| + c$ , passes through the points  $(3, -5)$ ,  $(6, 7)$  and  $(9, -5)$  as shown in the diagram.

(a) Find the values of  $a$ ,  $b$  and  $c$ .

(b) The line  $y = mx$  cuts the graph of  $y = f(x)$  in two distinct places.  
Find all possible values of  $m$ .



**Solution**



**3**



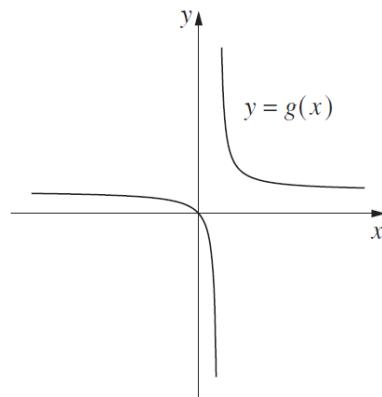
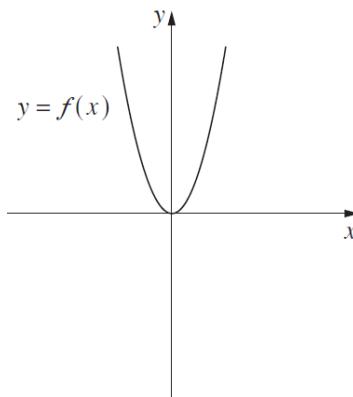
**2**



NESA 2023 Mathematics Advanced HSC Examination

**22 10** The graphs of  $y = f(x)$  and  $y = g(x)$  are shown.

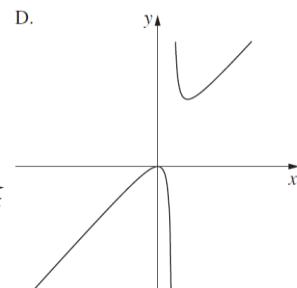
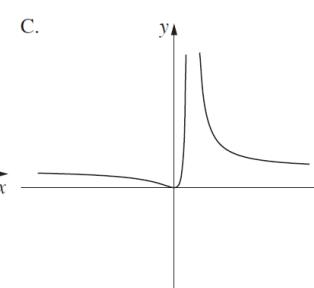
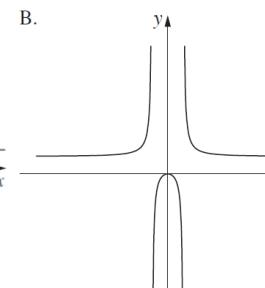
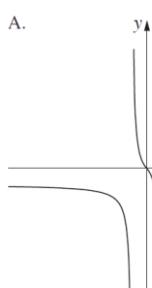
**MA**



**1** [Solution](#)



Which graph best represents  $y = g(f(x))$ ?



NESA 2022 Mathematics Advanced HSC Examination

**22 19** The graph of the function  $f(x) = x^2$  is translated  $m$  units to the right, dilated vertically by a scale factor of  $k$  and then translated 5 units down.

The equation of the transformed function is  $g(x) = 3x^2 - 12x + 7$ .  
Find the values of  $m$  and  $k$ .

**3** [Solution](#)



NESA 2022 Mathematics Advanced HSC Examination

**21 16** For what values of  $x$  is  $f(x) = x^2 - 2x^3$  increasing?

**MA**

**3** [Solution](#)



NESA 2021 Mathematics Advanced HSC Examination

**21 19** Without using calculus, sketch the graph of  $y = 2 + \frac{1}{x+4}$ , showing the asymptotes and the  $x$  and  $y$  intercepts.

**MA**

**3** [Solution](#)



NESA 2021 Mathematics Advanced HSC Examination

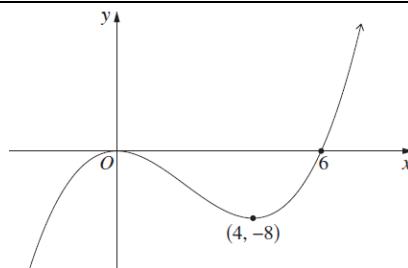
**21 21** Consider the graph of  $y = f(x)$  as shown.

**MA**

**2** [Solution](#)



Sketch the graph of  $y = 4f(2x)$  showing the  $x$ -intercepts and the coordinates of the turning points.



NESA 2021 Mathematics Advanced HSC Examination

- 20 2** The function  $f(x) = x^3$  is transformed to  $g(x) = (x - 2)^2 + 5$  by a horizontal translation of 2 units followed by a vertical translation of 5 units.

Which row of the table shows the directions of the translations?

	Horizontal translation of 2 units	Vertical translation of 5 units
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

**1** [Solution](#)



- 20 24** The circle  $x^2 - 6x + y^2 + 4y - 3 = 0$  is reflected in the  $x$ -axis.

**MA**

Sketch the reflected circle, showing the coordinates of the centre and the radius.

NESA 2020 Mathematics Advanced HSC Examination

**3** [Solution](#)

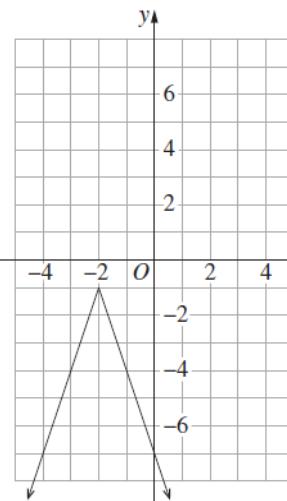


- MA 24** The function  $f(x) = |x|$  is transformed and the equation of the new function is of the form  $y = kf(x + b) + c$ , where  $k$ ,  $b$  and  $c$  are constants.

SP Band 3-5

The graph of the new function is shown.

What are the values of  $k$ ,  $b$  and  $c$ ?



**2** [Solution](#)

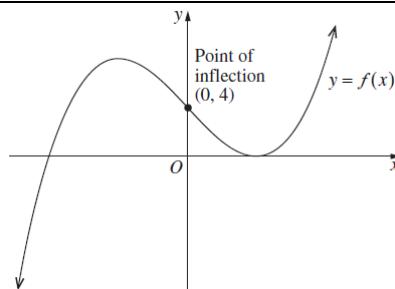


NESA Mathematics Advanced Sample HSC Examination Paper (2020)

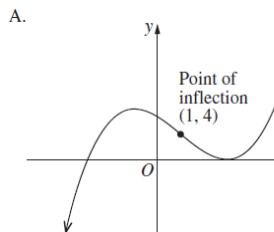
- MA 4** The diagram shows the graph of  $y = f(x)$ .

**SQ 2019**

**1** [Solution](#)



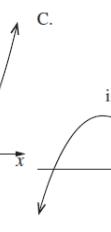
Which of these graphs represents  $y = f(x - 1)$ ?



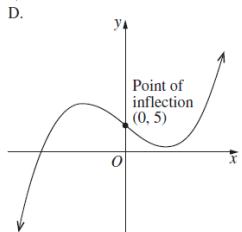
B.



C.



D.



NESA Mathematics Advanced Sample examination materials (2019)

- TG 1** Sketch on the same axes the graphs of  $y = f(x)$ ,  $y = g(x)$  and  $y = h(x)$ , where  $f(x) = 3^x$ ,  $g(x) = 3^{2x}$  and  $h(x) = 2(3^x)$ .

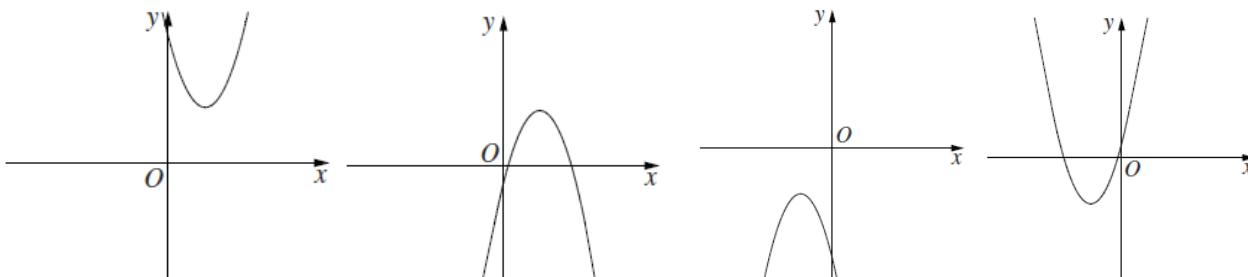
[Solution](#)



NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques

<b>TG 2</b>	Sketch the graph of the function $g(x) = x^2 - 9$ , state the range of $g(x)$ , and from the symmetry of the graph identify if $g(x)$ is an odd function, an even function or neither odd nor even.	<a href="#">Solution</a>
	Hence sketch the graph of the function $f(x) = (x - 1)^2 + 9$ , and from the symmetry of the graph, identify if $f(x)$ is an odd function, an even function or neither odd nor even.	
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 3</b>	Sketch the graph of $y = f(x)$ where $f(x) = \frac{1}{x+3}$ , identifying any asymptotes and any intercepts on the axes.	<a href="#">Solution</a>
	Over what intervals is the function (a) increasing? (b) decreasing?	
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 4</b>	Give a set of simple transformations which, when applied in succession, change the graph of $y = x^2$ to the graph with equation $y = 4 - 2x + x^2$ .	<a href="#">Solution</a>
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 5</b>	Show that any function $f$ of the form $f(x) = a^x$ , where $a > 0$ , has the property that for all values of $b$ , $f(x + b) = f(x)f(b)$ . Interpret this property in terms of transformations of graphs.	<a href="#">Solution</a>
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 6</b>	The curve $y = kx^2 + c$ is transformed by a translation, a dilation and a reflection respectively. The translation is 2 units in the negative $x$ -direction, the dilation is in the $y$ -direction with factor of 4 and the reflection is in the $y$ -axis. The equation of the final curve is $y = 12x^2 - 48x + 20$ .	<a href="#">Solution</a>
	(a) Find the equation of the graph:  (i) before the reflection  (ii) before the dilation.	
	(b) Find the values of $k$ and $c$ .	
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 7</b>	Sketch the graphs of $y = f(x)$ and $y = g(x)$ where $f(x) = \frac{4}{2x+1}$ and $g(x) = 1 + f(x)$ .	<a href="#">Solution</a>
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 8</b>	Sketch the graph of $y = f(x)$ where $f(x) = \frac{x-3}{x-1}$ , and state the domain and range.	<a href="#">Solution</a>
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	
<b>TG 9</b>	The graph of $y = -3(x + 1)^4$ has been produced from the graph of $y = x^4$ by three successive transformations: a translation, a dilation and then a reflection.	<a href="#">Solution</a>
	Define each of the transformations clearly, and state the equation of the graph after each of the first two transformations. Hence sketch the graph of $y = -3(x + 1)^4$ .	
	NESMA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques	

- TG 10** Which diagram best shows the graph of the parabola  $y = 3 - (x - 2)^2$ ? 1 [Solution](#)
- 16 3 M** (A) (B) (C) (D)



NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques

NESA 2016 Mathematics HSC Examination

- TG 11** On the same set of axes sketch the graphs of  $f(x) = x^2 - 3$  and  $g(x) = 2x$ . [Solution](#)
- Hence or otherwise solve  $x^2 - 3 > 2x$ .

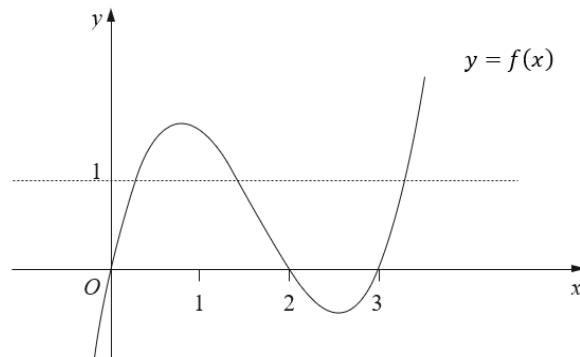
NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques

- TG 12** (a) Sketch the graph  $y = |2x - 3|$ . 1 [Solution](#)
- 13 15 M c** (b) Using the graph from part (a), or otherwise, find all values of  $m$  for which the equation  $|2x - 3| = mx + 1$  has exactly one solution. 2 [Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques  
NESA 2013 Mathematics HSC Examination

- TG 13** The diagram shows the graph of a function  $y = f(x)$ .

On the same set of axes,  
sketch the graph of  
 $y = 1 - f(x)$ .



NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques

- TG 14** A particle is released and moves in a straight line so that its displacement  $x$  m from a fixed point  $O$  at time  $t$  seconds is given by  $x = 2 - \frac{4}{t+1}$ .

- Find its initial displacement.
- At what time(s) is the particle at  $O$ ?
- Sketch the  $x$ - $t$  graph.
- A second particle is released at the same time.

It moves on the same straight line with displacement  $x$  m from the point  $O$  at time  $t$  seconds given by  $x = 4 - t$ .

When and where will the particles will collide?

NESA Mathematics Advanced Year 12 Topic Guide: Graphing Techniques

<b>17</b>	<b>12</b>	(i) Carefully sketch the graphs of $y =  x + 1 $ and $y = 3 -  x - 2 $ on the same axes, showing all intercepts.	<b>3</b>	<a href="#">Solution</a>
<b>MX</b>	<b>b</b>	(ii) Using the graphs from part (i), or otherwise, find the range of values of $x$ for which $ x + 1  +  x - 2  = 3$ .	<b>1</b>	
NESA 2017 Mathematics Extension 1 HSC Examination				
<b>10</b>	<b>2b</b>	Solve the inequality $x^2 - x - 12 < 0$ .	<b>2</b>	<a href="#">Solution</a>
NESA 2010 Mathematics HSC Examination				
<b>08</b>	<b>1f</b>	Let $f(x) = \log_e[(x - 3)(5 - x)]$ . What is the domain of $f(x)$ ?	<b>2</b>	<a href="#">Solution</a>
<b>MX</b>	<b>1</b>			
NESA 2008 Mathematics Extension 1 HSC Examination				
<b>08</b>	<b>3a</b>	(i) Sketch the graph of $y =  2x - 1 $ . (ii) Hence, or otherwise, solve $ 2x - 1  \leq  x - 3 $ .	<b>1</b> <b>3</b>	<a href="#">Solution</a>
<b>MX</b>	<b>1</b>			
NESA 2008 Mathematics Extension 1 HSC Examination				

# Year 12: Trigonometric Functions

## T3 Trigonometric Functions and Graphs



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- examine and apply transformations to sketch functions of the form  $y = kf(a(x + b)) + c$ , where  $a, b, c$  and  $k$  are constants, in a variety of contexts, where  $f(x)$  is one of  $\sin x$ ,  $\cos x$  or  $\tan x$ , stating the domain and range when appropriate
  - use technology or otherwise to examine the effect on the graphs of changing the amplitude (where appropriate),  $y = kf(x)$ , the period,  $y = f(ax)$ , the phase,  $y = f(x + b)$ , and the vertical shift,  $y = f(x) + c$
  - use  $k, a, b, c$  to describe transformational shifts and sketch graphs
- solve trigonometric equations involving functions of the form  $kf(a(x + b)) + c$ , using technology or otherwise, within a specified domain **AAM**
- use trigonometric functions of the form  $kf(a(x + b)) + c$  to model and/or solve practical problems involving periodic phenomena **AAM**

Reference Sheet

**23 20**

**MA** Find all the values of  $\theta$ , where  $0^\circ \leq \theta \leq 360^\circ$ , such that  $\sin(\theta - 60^\circ) = -\frac{\sqrt{3}}{2}$ .

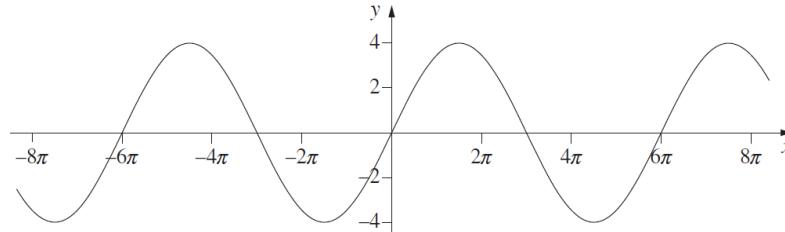
**3** Solution



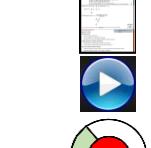
NESA 2023 Mathematics Advanced HSC Examination

**22 14**

**MA** The graph of  $y = k\sin(ax)$  is shown. What are the values of  $k$  and  $a$ ?



**2**



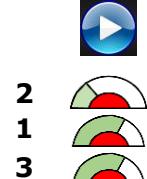
NESA 2022 Mathematics Advanced HSC Examination

**22 23** The depth of water in a bay rises and falls with the tide.

**MA** On a particular day the depth of the water,  $d$  metres, can be modelled by the equation

$$d = 1.3 - 0.6 \cos\left(\frac{4\pi}{25}t\right) \text{ where } t \text{ is the time in hours since low tide.}$$

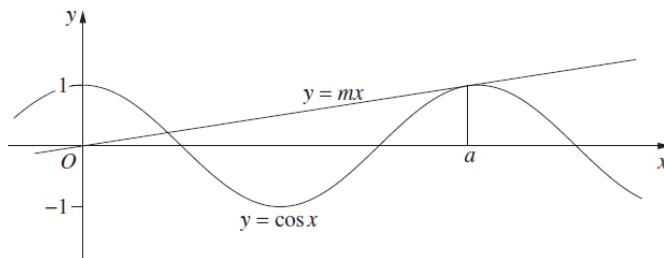
- Find the depth of water at low tide and at high tide.
- What is the time interval, in hours, between two successive low tides?
- For how long between successive low tides will the depth of water be at least 1 metre?



NESA 2022 Mathematics Advanced HSC Examination

- 21 10** The line  $y = mx$  is a tangent to the curve  $y = \cos x$  at the point where  $x = a$ , as shown in the diagram.  
Which of the following statements is true?

- A.  $m < \frac{1}{a} < \frac{1}{2\pi}$       B.  $\frac{1}{2\pi} < m < \frac{1}{a}$       C.  $\frac{1}{2\pi} < \frac{1}{a} < m$       D.  $m < \frac{1}{2\pi} < \frac{1}{a}$

**1** [Solution](#)

- 21 20** For what values of  $x$ , in the interval  $0 \leq x \leq \frac{\pi}{4}$ , does the line  $y = 1$  intersect the graph of  $y = 2 \sin 4x$ ?

NESAA 2021 Mathematics Advanced HSC Examination

**2** [Solution](#)

- 20 6** Which interval gives the range of the function  $y = 5 + 2 \cos 3x$ ?

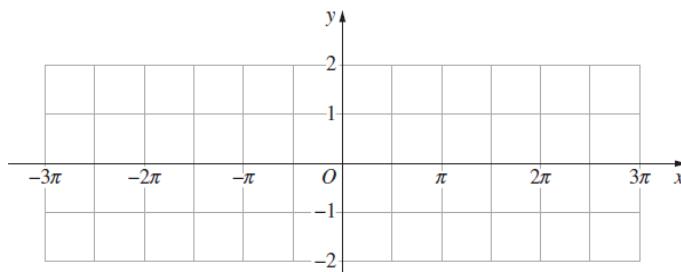
**1** [Solution](#)

- MA** A. [2, 8]      B. [3, 7]      C. [4, 6]      D. [5, 9]



NESAA 2020 Mathematics Advanced HSC Examination

- MA SP 26 Band 3-6** By drawing graphs on the number plane, determine how many solutions there are to the equation  $\sin x = \frac{|x|}{5}$  in the domain  $(-\infty, \infty)$ .

**3** [Solution](#)

NESAA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA SP 27 Band 2-3** The function  $f(x) = \cos x$  is transformed to  $g(x) = 3 \cos 2x$ .  
Describe in words how both the amplitude and period change in this transformation.

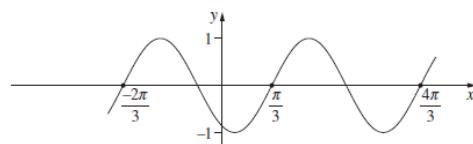
**2** [Solution](#)

NESAA Mathematics Advanced Sample Examination Paper (2020)

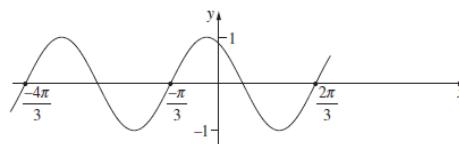
- TG 1** Which diagram shows the graph  $y = \sin(2x + \frac{\pi}{3})$ ?

**1** [Solution](#)

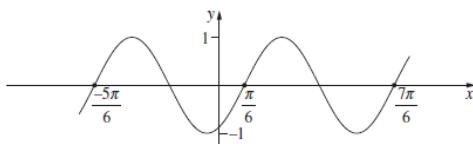
- 13 6** (A)



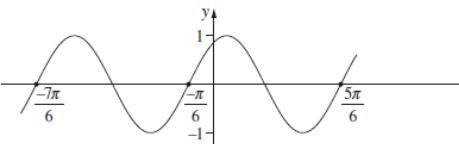
(B)



(C)



(D)

NESAA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESAA 2013 Mathematics HSC Examination

**TG 2** The diagram shows part of the graph of

$y = a \sin(bx) + 4$ .

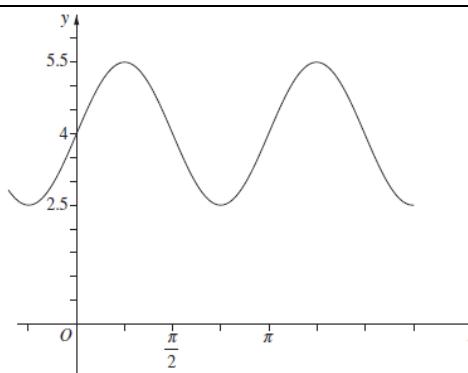
**19 7** What are the values of  $a$  and  $b$ ?

**M** A.  $a = 3$   $b = \frac{1}{2}$

B.  $a = 3$   $b = 2$

C.  $a = 1.5$   $b = \frac{1}{2}$

D.  $a = 1.5$   $b = 2$

**1** [Solution](#)NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 2019 Mathematics HSC Examination**TG 3** What is the period of the function  $f(x) = \tan(3x)$ ?**1** [Solution](#)

**16 6** M (A)  $\frac{\pi}{3}$  (B)  $\frac{2\pi}{3}$  (C)  $3\pi$  (D)  $6\pi$

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 2016 Mathematics HSC Examination**TG 4** Sketch the curve  $y = 1 - \sin 2x$  for  $0 \leq x \leq \pi$ .**3** [Solution](#)**00 6a****M**NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 2000 Mathematics HSC Examination**TG 5** (a) Sketch the graph of  $y = 2 \cos x$  for  $0 \leq x \leq 2\pi$ .**5** [Solution](#)**96 7a** (b) On the same set of axes, sketch the graph of  $y = 2 \cos x - 1$  for  $0 \leq x \leq 2\pi$ .**M** (c) Find the exact values of the  $x$  coordinates of the points where the graph of  $y = 2 \cos x - 1$  crosses the  $x$  axis in the domain  $0 \leq x \leq 2\pi$ .NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 1996 Mathematics HSC Examination**TG 6** Solve  $\sin\left(\frac{x}{2}\right) = \frac{1}{2}$  for  $0 \leq x \leq 2\pi$ ?**2** [Solution](#)**16 11** M gNESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 2016 Mathematics HSC Examination**TG 7** Solve  $2\sin^2 x - 3\sin x - 2 = 0$  for  $0 \leq x \leq 2\pi$ .[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric functions

**TG 8** (a) Draw the graphs of  $y = 4 \cos x$  and  $y = 2 - x$  on the same set of axes for[Solution](#)**95 10** M a (b)  $-2\pi \leq x \leq 2\pi$ .[Solution](#)Explain why all the solutions of the equation  $4 \cos x = 2 - x$  must lie between  $x = -2$  and  $x = 6$ .NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric functions  
NESA 1995 Mathematics HSC Examination**TG 9** (a) Show that  $x = \frac{\pi}{3}$  is a solution of  $\sin x = \frac{1}{2} \tan x$ .**1** [Solution](#)**99 10** M a (b) On the same set of axes, sketch the graphs of the functions  $y = \sin x$  and  $y = \frac{1}{2} \tan x$  for  $-\pi \leq x \leq \pi$ .**2** [Solution](#)(c) Hence find all solutions of  $\sin x = \frac{1}{2} \tan x$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .**1** [Solution](#)(c) Use your graphs to solve  $\sin x \leq \frac{1}{2} \tan x$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .**2** [Solution](#)NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions  
NESA 1999 Mathematics HSC Examination

**TG 10** The graph of  $y = 3 \cos(2x + \alpha)$  can be obtained from the graph of  $y = \cos x$  by a translation followed by two dilations. [Solution](#)

- (a) Describe each of these three transformations, and give the number of roots of the equation  $3\cos(2x + \alpha) = k$ , in the interval  $0 \leq x \leq 2\pi$ , where  $-3 < k < 3$ . 

- (b) Generalise your answer to give the number of roots of the equation 

$a \cos(nx + \alpha) = k$  in the interval  $0 \leq x \leq 2\pi$ , where  $a < 0$ ,  $n$  is a positive integer and  $-a < k < a$ . 

- (c) How does your answer to part (b) change if  $n$  is a negative integer? 

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions

**TG 11** A particle moves in a straight line. At time  $t$  seconds its distance  $x$  metres from a fixed point  $O$  in the line is given by  $x = 2 - 2 \cos 2t$ . [Solution](#)

- (a) Sketch the graph of  $x$  as a function of  $t$ . 

- (b) Find the times when the particle is at rest and the position of the particle at those times. 

- (c) Describe the motion. 

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions

**TG 12** The length of daylight,  $L(t)$ , is defined as the number of hours from sunrise to sunset, and can be modelled by the equation  $L(t) = 12 + 2 \cos\left(\frac{2\pi t}{366}\right)$ , where  $t$  is [Solution](#)

**18 15** the number of days after 21 December 2015, for  $0 \leq t \leq 366$ . 

**M a** the number of days after 21 December 2015, for  $0 \leq t \leq 366$ . 

- (a) Find the length of daylight on 21 December 2015. 

- (b) What is the shortest length of daylight? 

- (c) What are the two values of  $t$  for which the length of daylight is 11? 

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions

NESA 2018 Mathematics HSC Examination

**TG 13** When humans breathe, they do not inflate their lungs to full capacity. When resting, each inhalation adds approximately 0.5 L of air and this same volume of air is removed upon exhalation. When exhalation is completed, the volume of air that remains in the lungs, called the functional residual capacity, is approximately 2.2 L. On average the time taken to complete an inhale-exhale cycle is approximately 5 seconds.

The volume of air in the lungs can be modelled by the function  $V = k \sin(at) + c$  where  $V$  is the volume of air in litres and  $t$  is time in seconds.

- (a) Use the time for an inhale-exhale cycle to show that the period of this function is  $\frac{2\pi}{5}$ . 

- (b) Explain why  $k = 0.25$ . 

- (c) Find the value of  $c$ . 

- (d) Sketch the graph of  $V = k \sin(at) + c$  for  $0 \leq t \leq 15$  using these values of  $k$ ,  $a$  and  $c$ . 

- (e) When exercising, the volume of air inhaled and exhaled rises and breathing occurs more rapidly. Explain the effect this would have on the values of  $k$ ,  $a$  and  $c$ . 

- (f) Humans have a full lung capacity of approximately 6 L. An athlete who is exercising vigorously inhales approximately 4.6 L of air. Calculate the athlete's residual lung capacity. 

NESA Mathematics Advanced Year 12 Topic Guide: Trigonometric Functions

- 17 14** Sketch the curve  $y = 4 + 3 \sin 2x$  for  $0 \leq x \leq 2\pi$ .  
**M a**

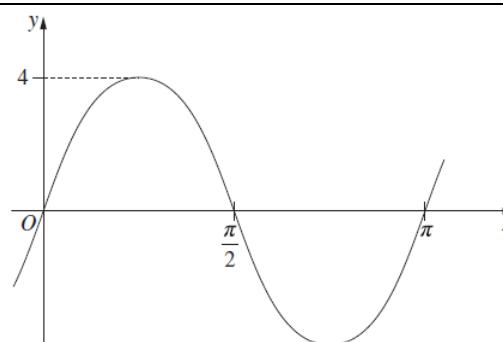
**3** [Solution](#)

NESA 2017 Mathematics HSC Examination

- 13 13** The population of a herd of wild horses is given by  $P(t) = 400 + 50 \cos\left(\frac{\pi}{6}t\right)$ , where  $t$  is time in months.
- Find all times during the first 12 months when the population equals 375 horses.
  - Sketch the graph of  $P(t)$  for  $0 \leq t \leq 12$ .

[Solution](#)**2** **2**

- 10 8c** The graph shown is  $y = A \sin bx$ .
- M**
- Write down the value of  $A$ .
  - Find the value of  $b$ .
  - Copy or trace the graph into your writing booklet. On the same set of axes, draw the graph  $y = 3 \sin x + 1$ , for  $0 \leq x \leq \pi$ .

[Solution](#)**1** **1** **2** 

NESA 2010 Mathematics HSC Examination

- 09 7b** Between 5 am and 5 pm on 3 March 2009, the height,  $h$ , of the tide in a harbour was given by  $h = 1 + 0.7 \sin \frac{\pi}{6}t$  for  $0 \leq t \leq 12$ , where  $h$  is in minutes and  $t$  is in hours, with  $t = 0$  at 5 am.
- What is the period of the function  $h$ ?
  - What was the value of  $h$  at low tide, and at what time did low tide occur?
  - A ship is able to enter the harbour only if the height of the tide is at least 1.35 m. Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour.

[Solution](#)**1** **2** **3**

NESA 2009 Mathematics HSC Examination

- 08 6a** Solve  $2 \sin^2 \frac{x}{3} = 1$  for  $-\pi \leq x \leq \pi$ .  
**M**

**3** [Solution](#)

NESA 2008 Mathematics HSC Examination

## Year 12: Calculus

### C2.1 Differentiation of Trigonometric, Exponential and Logarithmic Functions

**Syllabus:** updated November 2019. Latest version @<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- establish the formulae  $\frac{d}{dx}(\sin x) = \cos x$  and  $\frac{d}{dx}(\cos x) = -\sin x$  by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102)
- calculate derivatives of trigonometric functions
- establish and use the formula  $\frac{d}{dx}(a^x) = (\ln a)a^x$ 
  - using graphing software or otherwise, sketch and explore the gradient function for a given exponential function, recognise it as another exponential function and hence determine the relationship between exponential functions and their derivatives
- calculate the derivative of the natural logarithm function  $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- establish and use the formula  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

[Reference Sheet](#)
**21 13** Find the exact gradient of the tangent to the curve  $y = x \tan x$  at the point where**3** [Solution](#)

**MA**  $x = \frac{\pi}{3}$ .



NESA 2021 Mathematics Advanced HSC Examination

**21 23** A population,  $P$ , which is initially 5000, varies according to the formula**4** [Solution](#)

**MA**  $P = 5000b^{\frac{-t}{10}}$ , where  $b$  is a positive constant and  $t$  is time in years,  $t \geq 0$ .



The population is 1250 after 20 years.

Find the value of  $t$ , correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

NESA 2021 Mathematics Advanced HSC Examination

**MA** **4** What is the derivative of  $5^{2x+3}$ ?**1** [Solution](#)

- SP** **Band 3-4**
- A.  $2 \times 5^{2x+3}$
  - B.  $(2x + 3) \times 5^{2x+2}$
  - C.  $\ln 5 \times 5^{2x+3}$

- D.  $\ln 5 \times 2 \times 5^{2x+3}$



NESA Mathematics Advanced Sample Examination Paper (2020)

**MA** **16** Differentiate  $e^{\sin(\pi x)}$ .**2** [Solution](#)

NESA Mathematics Advanced Sample Examination Paper (2020)

**TG** **1** An object is attached to the end of a vertical spring.[Solution](#)The object is released at time  $t = 0$  and its position at time  $t$  is given by  $f(t) = 5 \cos t$ .  
Find the velocity and acceleration of the object and describe its motion.

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

**TG** **2** Donald is using the function  $f(x) = 2.55 + 14.6 \ln x$  to model the percentage of women in the workforce of the USA, for the years between 1960 and 2002, with  $x$  being the number of years after 1950.[Solution](#)In this context, find and interpret the values of  $f(12)$ ,  $f'(12)$ ,  $f(22)$ ,  $f'(22)$ .

Use your results to describe how the number of women in the workforce has changed over the time period. Suggest reasons for your answer.

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

**Year 12: Calculus****C2.2 Rules of Differentiation****Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>**Students:**

- apply the product, quotient and chain rules to differentiate functions of the form  $f(x)g(x)$ ,  $\frac{f(x)}{g(x)}$  and  $f(g(x))$  where  $f(x)$  and  $g(x)$  are any of the functions covered in the scope of this syllabus, for example  $xe^x$ ,  $\tan x$ ,  $\frac{1}{x^n}$ ,  $x \sin x$ ,  $e^{-x} \sin x$  and  $f(ax + b)$  (ACMMM106)
- use the composite function rule (chain rule) to establish that  $\frac{d}{dx}\{e^{f(x)}\} = f'(x)e^{f(x)}$
- use the composite function rule (chain rule) to establish that  $\frac{d}{dx}\{\ln f(x)\} = \frac{f'(x)}{f(x)}$
- use the logarithmic laws to simplify an expression before differentiating
- use the composite function rule (chain rule) to establish and use the derivatives of  $\sin(f(x))$ ,  $\cos(f(x))$  and  $\tan(f(x))$

[Reference Sheet](#)

<b>MA SP</b>	<b>10 Band 5-6</b>	Given the function $y = \log_7(x^x)$ , which expression is equal to $\frac{dy}{dx}$ ?	<b>1</b>	<a href="#">Solution</a>
		A. $\frac{1}{x \ln 7}$ B. $\frac{1}{\ln 7} \times \log_7(x^{x-1})$ C. $\frac{1}{x^x \ln 7}$ D. $\log_7 x + \frac{1}{\ln 7}$		

NESA Mathematics Advanced Sample Examination Paper (2020)

<b>MA SP</b>	<b>23 Band 3-5</b>	The function $f(x) = \tan^3 x$ is given. If $f'(x) = 3\tan^m x + 3\tan^2 x$ , find the value of $m$ .	<b>2</b>	<a href="#">Solution</a>
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NESA Mathematics Advanced Sample Examination Paper (2020)

<b>TG</b>	<b>1</b>	Find the derivative of: (a) $f(x) = \sin(3x + 5)$ (b) $f(t) = \cos^2 \pi t$ (c) $y = 5 \tan\left(\frac{x}{7}\right)$ (d) $f(t) = e^{-0.5t} \sin 2t$ (e) $y = 5 \log(5 - x)$ (f) $f(x) = \sin x^\circ$	<a href="#">Solution</a>
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NESA Mathematics Advanced Year 12 Topic Guide: Calculus

<b>19</b>	<b>11</b>	Differentiate $x^2 \sin x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>	<b>b</b>			

NESA 2019 Mathematics HSC Examination

<b>18</b>	<b>5</b>	What is the derivative of $\sin(\ln x)$ ?	<b>1</b>	<a href="#">Solution</a>
<b>M</b>		A. $\cos\left(\frac{1}{x}\right)$ B. $\cos(\ln x)$ C. $\cos\left(\frac{\ln x}{x}\right)$ D. $\frac{\cos(\ln x)}{x}$		

NESA 2018 Mathematics HSC Examination

<b>18</b>	<b>11</b>	Differentiate $x^2 \tan x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>	<b>f</b>			

NESA 2018 Mathematics HSC Examination

<b>18</b>	<b>11</b>	<b>M</b>	<b>g</b>	Differentiate $\frac{e^x}{x+1}$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2018 Mathematics HSC Examination						
<b>17</b>	<b>3</b>	<b>M</b>		What is the derivative of $e^{x^2}$ ?	<b>1</b>	<a href="#">Solution</a>
NESAA 2017 Mathematics HSC Examination						
<b>17</b>	<b>11</b>	<b>M</b>	<b>c</b>	Differentiate $\frac{\sin x}{x}$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2017 Mathematics HSC Examination						
<b>17</b>	<b>11</b>	<b>M</b>	<b>d</b>	Differentiate $x^3 \ln x$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2017 Mathematics HSC Examination						
<b>16</b>	<b>5</b>	<b>M</b>		What is the derivative of $\ln(\cos x)$ ?	<b>1</b>	<a href="#">Solution</a>
NESAA 2016 Mathematics HSC Examination						
<b>15</b>	<b>6</b>	<b>M</b>		What is the value of the derivative of $y = 2\sin 3x - 3\tan x$ at $x = 0$ ?	<b>1</b>	<a href="#">Solution</a>
NESAA 2015 Mathematics HSC Examination						
<b>15</b>	<b>11</b>	<b>M</b>	<b>e</b>	Differentiate $(e^x + x)^5$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2015 Mathematics HSC Examination						
<b>15</b>	<b>11</b>	<b>M</b>	<b>f</b>	Differentiate $y = (x + 4)\ln x$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2015 Mathematics HSC Examination						
<b>14</b>	<b>11</b>	<b>MX</b>	<b>f</b>	Differentiate $\frac{e^x \ln x}{x}$ .	<b>2</b>	<a href="#">Solution</a>
<b>1</b>				NESAA 2014 Mathematics Extension 1 HSC Examination		
<b>13</b>	<b>4</b>	<b>M</b>		What is the derivative of $\frac{x}{\cos x}$ ?	<b>1</b>	<a href="#">Solution</a>
NESAA 2013 Mathematics HSC Examination						
<b>13</b>	<b>11</b>	<b>M</b>	<b>c</b>	Differentiate $(\sin x - 1)^8$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2013 Mathematics HSC Examination						
<b>13</b>	<b>11</b>	<b>M</b>	<b>d</b>	Differentiate $x^2 e^x$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2013 Mathematics HSC Examination						
<b>12</b>	<b>11</b>	<b>M</b>	<b>d</b>	Differentiate $(3 + e^{2x})^5$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2012 Mathematics HSC Examination						
<b>12</b>	<b>12</b>	<b>M</b>		Differentiate with respect to $x$ :	<b>2</b>	<a href="#">Solution</a>
NESAA 2012 Mathematics HSC Examination						
<b>12</b>	<b>12</b>	<b>M</b>	<b>a</b>	(i) $(x - 1)\log_e x$		
NESAA 2012 Mathematics HSC Examination						
<b>12</b>	<b>12</b>	<b>M</b>	<b>a</b>	(ii) $\frac{\cos x}{x^2}$	<b>2</b>	<a href="#">Solution</a>
NESAA 2012 Mathematics HSC Examination						
<b>12</b>	<b>11</b>	<b>MX</b>	<b>b</b>	Differentiate $x^2 \tan x$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
NESAA 2012 Mathematics Extension 1 HSC Examination						

<b>11</b>	<b>1d</b>	Differentiate $\ln(5x + 2)$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2011 Mathematics HSC Examination
<b>11</b>	<b>4a</b>	Differentiate $\frac{x}{\sin x}$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2011 Mathematics HSC Examination
<b>11</b>	<b>1b</b>	Differentiate $\frac{\sin^2 x}{x}$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
<b>MX</b>				
				NESA 2011 Mathematics Extension 1 HSC Examination
<b>10</b>	<b>1e</b>	Differentiate $x^2 \tan x$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2010 Mathematics HSC Examination
<b>10</b>	<b>2a</b>	Differentiate $\frac{\cos x}{x}$ with respect to $x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2010 Mathematics HSC Examination
<b>09</b>	<b>2a</b>	Differentiate with respect to $x$ : (i) $x \sin x$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2009 Mathematics HSC Examination
<b>09</b>	<b>2a</b>	Differentiate with respect to $x$ : (ii) $(e^x + 1)^2$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2009 Mathematics HSC Examination
<b>09</b>	<b>1e</b>	Differentiate $x \cos^2 x$ .	<b>2</b>	<a href="#">Solution</a>
<b>MX</b>				
				NESA 2009 Mathematics Extension 1 HSC Examination
<b>08</b>	<b>2a</b>	(ii) Differentiate with respect to $x$ : $x^2 \log_e x$	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2008 Mathematics HSC Examination
<b>08</b>	<b>2a</b>	(iii) Differentiate with respect to $x$ : $\frac{\sin x}{x + 4}$	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2008 Mathematics HSC Examination
<b>07</b>	<b>2a</b>	(i) Differentiate with respect to $x$ : $\frac{2x}{e^x + 1}$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2007 Mathematics HSC Examination
<b>07</b>	<b>2a</b>	(ii) Differentiate with respect to $x$ : $(1 + \tan x)^{10}$ .	<b>2</b>	<a href="#">Solution</a>
<b>M</b>				
				NESA 2007 Mathematics HSC Examination

## Year 12: Calculus

### C3.1 The First and Second Derivatives

**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- use the first derivative to investigate the shape of the graph of a function
  - deduce from the sign of the first derivative whether a function is increasing, decreasing or stationary at a given point or in a given interval
  - use the first derivative to find intervals over which a function is increasing or decreasing, and where its stationary points are located
  - use the first derivative to investigate a stationary point of a function over a given domain, classifying it as a local maximum, local minimum or neither
  - determine the greatest or least value of a function over a given domain (if the domain is not given, the natural domain of the function is assumed) and distinguish between local and global minima and maxima
- define and interpret the concept of the second derivative as the rate of change of the first derivative function in a variety of contexts, for example recognise acceleration as the second derivative of displacement with respect to time (ACMMM108, ACMMM109) **AAM**
- understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)
- use the second derivative to determine concavity and the nature of stationary points
- understand that when the second derivative is equal to 0 this does not necessarily represent a point of inflection

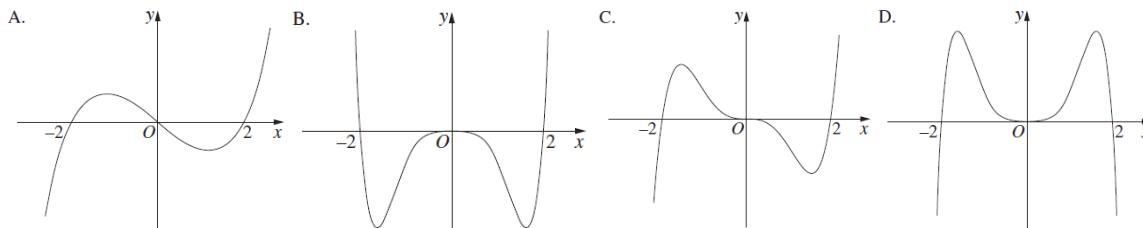
[Reference Sheet](#)

**23 MA 6** The following table gives the signs of the first and second derivatives of a function

$$y = f(x) \text{ for different values of } x.$$

Which of the following is a possible sketch of  $y = f(x)$ ?

$x$	-2	0	2
$f'(x)$	+	0	+
$f''(x)$	-	0	+

**1****Solution**

NESA 2023 Mathematics Advanced HSC Examination

**22 MA 25** Let  $f(x) = \sin(2x)$ .

**3****Solution**

Find the value of  $x$ , for  $0 < x < \pi$ , for which  $f'(x) = -\sqrt{3}$  AND  $f''(x) = 2$ .

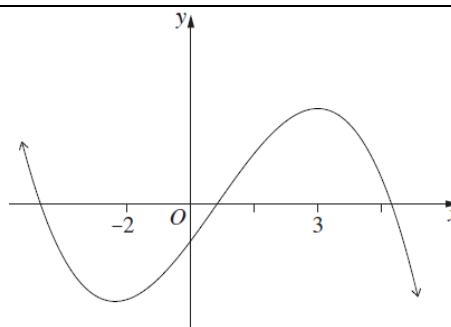


NESA 2022 Mathematics Advanced HSC Examination

- 21 MA 7** The diagram shows part of  $y = f(x)$  which has a local minimum at  $x = -2$  and a local maximum at  $x = 3$ .

Which of the following shows the correct relationship between  $f''(-2)$ ,  $f(0)$  and  $f'(-3)$ ?

- A.  $f(0) < f'(-3) < f''(-2)$
- B.  $f(0) < f''(-2) < f'(-3)$
- C.  $f''(-2) < f'(-3) < f(0)$
- D.  $f''(-2) < f(0) < f'(-3)$



**1** [Solution](#)



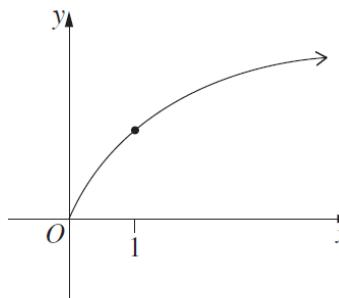
NESA 2021 Mathematics Advanced HSC Examination

- 20 MA 8** The graph of  $y = f(x)$  is shown.

MA

Which of the following inequalities is correct?

- A.  $f''(1) < 0 < f'(1) < f(1)$
- B.  $f''(1) < 0 < f(1) < f'(1)$
- C.  $0 < f''(1) < f'(1) < f(1)$
- D.  $0 < f''(1) < f(1) < f'(1)$



**1** [Solution](#)



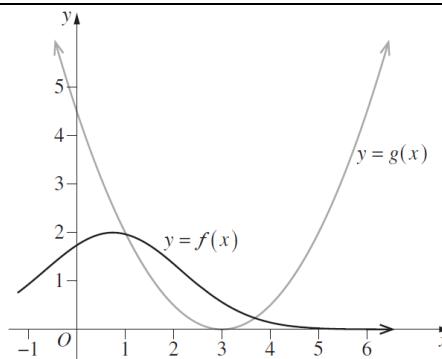
NESA 2020 Mathematics Advanced HSC Examination

- 20 MA 10** The graph shows two functions  $y = f(x)$  and  $y = g(x)$ .

Define  $h(x) = f(g(x))$ .

How many stationary points does  $y = h(x)$  have for  $1 \leq x \leq 5$ ?

- A. 0
- B. 1
- C. 2
- D. 3



**1** [Solution](#)



NESA 2020 Mathematics Advanced HSC Examination

- 20 MA 16** Sketch the graph of the curve  $y = -x^3 + 3x^2 - 1$ , labelling the stationary points and point of inflection. Do NOT determine the  $x$  intercepts of the curve.

**4** [Solution](#)



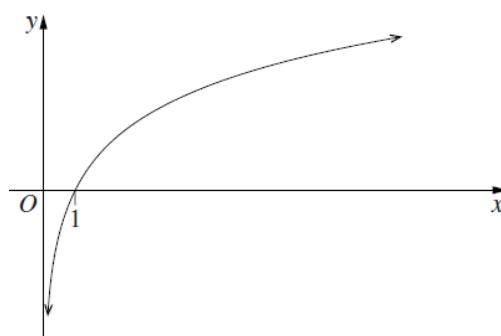
NESA 2020 Mathematics Advanced HSC Examination

- 20 MA 29** The diagram shows the graph of  $y = c \ln x$ ,  $c > 0$ .

- (a) Show that the equation of the tangent to  $y = c \ln x$ , at  $x = p$ , where  $p > 0$  is

$$y = \frac{c}{p}x - c + c \ln p.$$

- (b) Find the value of  $c$  such that the tangent from part (a) has a gradient of 1 and passes through the origin.



**2** [Solution](#)



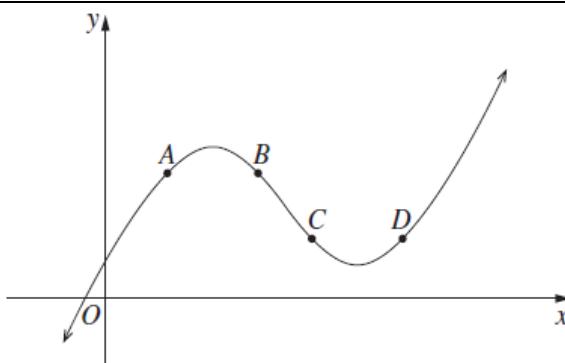
**2** [Solution](#)



NESA 2020 Mathematics Advanced HSC Examination

- MA 2** At which point on this curve are the first and second derivatives BOTH negative?
- SP Band 2-3**

- A. A  
B. B  
C. C  
D. D



**1** [Solution](#)



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- TG 1** Sketch the graph of the function  $f(x) = x^3 + 3x^2 - 9x - 9$  by identifying stationary points and determining their nature.

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

[Solution](#)



- TG 2** By considering the sign of the first derivative, show that the function  $f(x) = \frac{1}{3x-2}$  is decreasing throughout its domain.

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

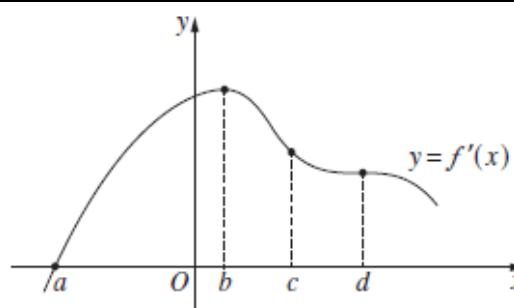
**1** [Solution](#)



- 18 9** The diagram shows the graph of  $y = f'(x)$ , the derivative of a function.

For what value of  $x$  does the graph of the function  $f(x)$  have a point of inflection?

- A.  $x = a$   
B.  $x = b$   
C.  $x = c$   
D.  $x = d$



NESA 2018 Mathematics HSC Examination

- 17 4** The function  $f(x)$  is defined for  $a \leq x \leq b$ . On this interval,  $f'(x) > 0$  and  $f''(x) < 0$ . Which graph best represents  $y = f(x)$ ?

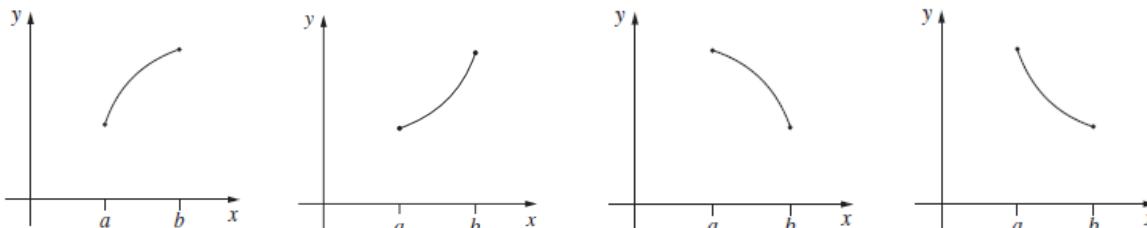
**1** [Solution](#)



- (A) (B)

- (C)

- (D)



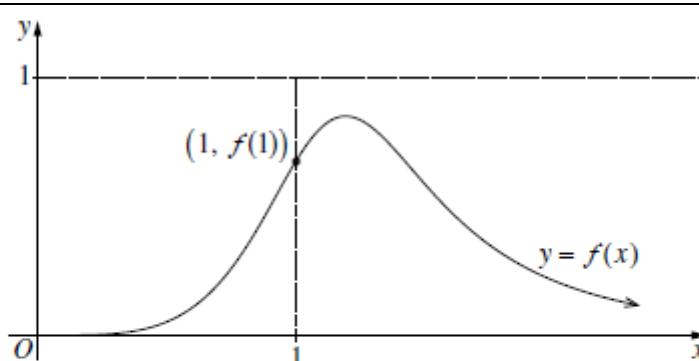
NESA 2017 Mathematics HSC Examination

- 16 9** The diagram shows the graph of  $y = f(x)$ . Which of the following is a correct statement?

**1** [Solution](#)



- (A)  $f''(1) < f(1) < 1 < f'(1)$   
(B)  $f''(1) < f'(1) < f(1) < 1$   
(C)  $f(1) < 1 < f'(1) < f''(1)$   
(D)  $f'(1) < f(1) < 1 < f''(1)$



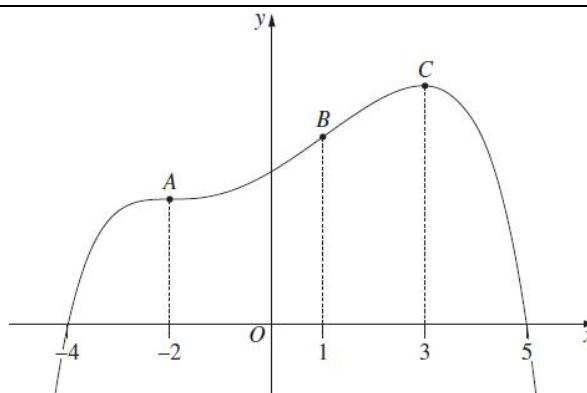
Not to scale

NESA 2016 Mathematics Extension 1 HSC Examination

- 14 14 M e** The diagram shows the graph of a function  $f(x)$ .

The graph has a horizontal point of inflection at  $A$ , a point of inflection at  $B$  and a maximum turning point at  $C$ .

Sketch the graph of the derivative  $f'(x)$ .



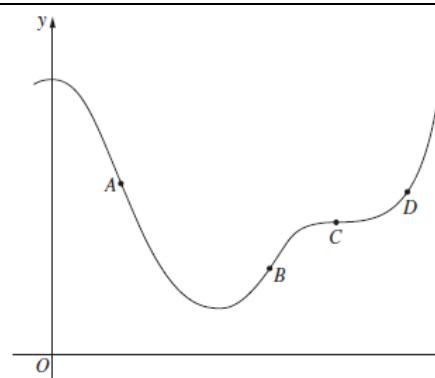
NESA 2014 Mathematics HSC Examination

**3** [Solution](#)

- 13 8 M** The diagram shows the points  $A$ ,  $B$ ,  $C$  and  $D$  on the graph  $y = f(x)$ .

At which point is  $f'(x) > 0$  and  $f''(x) = 0$ .

- (A)  $A$
- (B)  $B$
- (C)  $C$
- (D)  $D$



NESA 2013 Mathematics HSC Examination

**1** [Solution](#)

- 13 12 M** The cubic  $y = ax^3 + bx^2 + cx + d$  has a point of inflection at  $x = p$ .

Show that  $p = -\frac{b}{3a}$ .

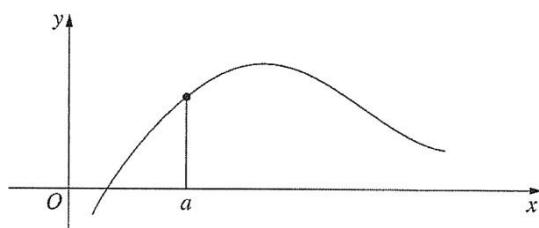
**2** [Solution](#)

NESA 2013 Mathematics HSC Examination

- 12 4 M** The diagram shows the graph of  $y = f(x)$ .

Which of the following statements is true?  
(A)  $f'(a) > 0$  and  $f''(a) < 0$

- (B)  $f'(a) > 0$  and  $f''(a) > 0$
- (C)  $f'(a) < 0$  and  $f''(a) < 0$
- (D)  $f'(a) < 0$  and  $f''(a) > 0$

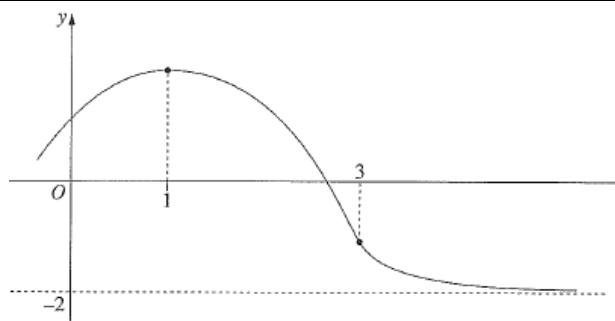


NESA 2012 Mathematics HSC Examination

**1** [Solution](#)

- 11 9c M** The graph  $y = f(x)$  in the diagram has a stationary point when  $x = 1$ , a point of inflection when  $x = 3$ , and a horizontal asymptote  $y = -2$ .

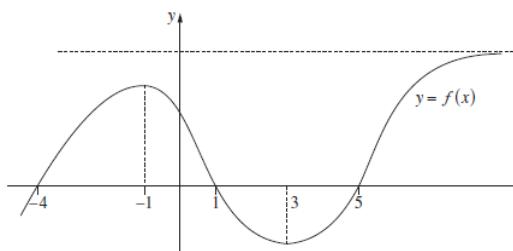
Sketch the graph  $y = f'(x)$ , clearly indicating its features at  $x = 1$  and at  $x = 3$ , and the shape of the graph as  $x \rightarrow \infty$ .



NESA 2011 Mathematics HSC Examination

**3** [Solution](#)

- 09 M 8a** The diagram shows the graph of a function  $y = f(x)$ .
- (i) For which values of  $x$  is the derivative,  $f'(x)$ , negative?
  - (ii) What happens to  $f'(x)$  for large values of  $x$ ?
  - (iii) Sketch the graph of  $y = f'(x)$ .

[Solution](#)**1****1****2**

NESA 2009 Mathematics HSC Examination

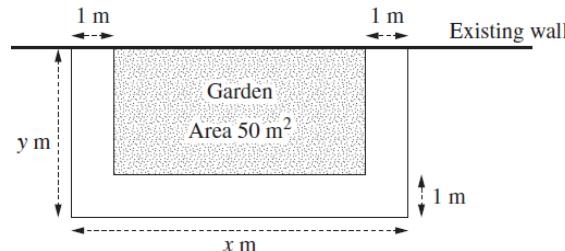
**Year 12: Calculus****C3.2 Applications of the Derivative** Back**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>**Students:**

- use any of the functions covered in the scope of this syllabus and their derivatives to solve practical and abstract problems AAM
- use calculus to determine and verify the nature of stationary points, find local and global maxima and minima and points of inflection (horizontal or otherwise), examine behaviour of a function as  $x \rightarrow \infty$  and  $x \rightarrow -\infty$  and hence sketch the graph of the function (ACMMM095)   
- solve optimisation problems for any of the functions covered in the scope of this syllabus, in a wide variety of contexts including displacement, velocity, acceleration, area, volume, business, finance and growth and decay AAM   
- define variables and construct functions to represent the relationships between variables related to contexts involving optimisation, sketching diagrams or completing diagrams if necessary
- use calculus to establish the location of local and global maxima and minima, including checking endpoints of an interval if required
- evaluate solutions and their reasonableness given the constraints of the domain and formulate appropriate conclusions to optimisation problems

 Reference Sheet

- 23 24 MA** A gardener wants to build a rectangular garden of area  $50 \text{ m}^2$  against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.

Let  $x$  and  $y$  be the dimensions, in metres, of the outer rectangle as shown.

 Solution

(a) Show that  $y = \frac{50}{x-2} + 1$ .

 1

- (b) Find the value of  $x$  such that the area of the concrete path is a minimum.

 4

Show that your answer gives a minimum area.

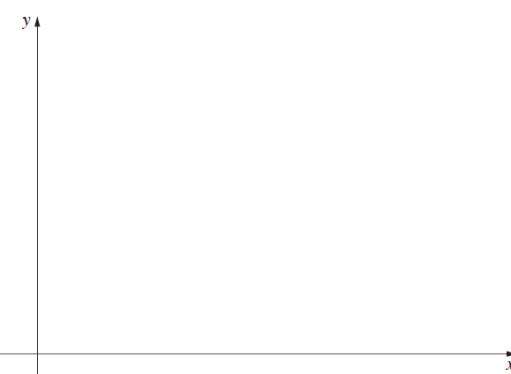
NESA 2023 Mathematics Advanced HSC Examination

- 23 30 MA** Let  $f(x) = e^{-x \sin x}$ .

- (a) Find the coordinates of the stationary points of  $f(x)$  for  $0 \leq x \leq 2\pi$ .

You do NOT need to check the nature of the stationary points.

- (b) Without using any further calculus, sketch the graph of  $f(x)$  for  $0 \leq x \leq 2\pi$ , showing stationary points and intercepts.

 Solution

 2

- 22 20** A scientist is studying the growth of bacteria.  
**MA** The scientist models the number of bacteria,  $N$ , by the equation  $N(t) = 200e^{0.013t}$ , where  $t$  is the number of hours after starting the experiment.
- What is the initial number of bacteria in the experiment? **1**
  - What is the number of bacteria 24 hours after starting the experiment? **1**
  - What is the rate of increase in the number of bacteria 24 hours after starting the experiment? **2**

NESA 2022 Mathematics Advanced HSC Examination

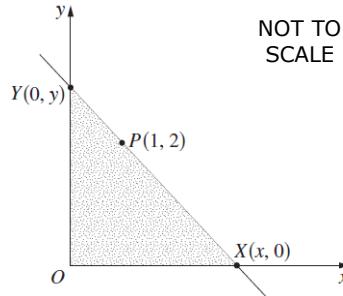
- 22 22** Find the global maximum and minimum values of  $y = x^3 - 6x^2 + 8$ , where  $-1 \leq x \leq 7$ . **4**

NESA 2022 Mathematics Advanced HSC Examination

- 22 27** Let  $f(x) = xe^{-2x}$ .  
**MA** It is given that  $f'(x) = e^{-2x} - 2xe^{-2x}$ .
- Show that  $f''(x) = 4(x - 1)e^{-2x}$ . **2**
  - Find any stationary points of  $f(x)$  and determine their nature. **2**
  - Sketch the curve  $y = xe^{-2x}$ , showing any stationary points, points of inflection and intercepts with the axes. **3**

NESA 2022 Mathematics Advanced HSC Examination

- 22 31** A line passes through the point  $P(1, 2)$  and meets the axes at  $X(x, 0)$  and  $Y(0, y)$ , where  $x > 1$ .
- Show that  $y = \frac{2x}{x-1}$ . **2**
  - Find the minimum value of the area of triangle  $XOY$ . **4**



NESA 2022 Mathematics Advanced HSC Examination

- 21 26** A particle is shot vertically upwards from a point 100 metres above ground level. **Solution**
- MA** The position of the particle,  $y$  metres above the ground after  $t$  seconds, is given by  $y(t) = -5t^2 + 70t + 100$ .
- Find the maximum height above ground level reached by the particle. **2**
  - Find the velocity of the particle, in metres per second, immediately before it hits the ground, leaving your answer in the form  $a\sqrt{b}$ , where  $a$  and  $b$  are integers. **3**

NESA 2021 Mathematics Advanced HSC Examination

**20 21** Hot tea is poured into a cup.[Solution](#)

**MA** The temperature of tea can be modelled by  $T = 25 + 70(1.5)^{-0.4t}$ , where  $T$  is the temperature of the tea, in degrees Celsius,  $t$  minutes after it is poured.

(a) What is the temperature of the tea 4 minutes after it has been poured?

1



(b) At what rate is the tea cooling 4 minutes after it has been poured?

2



(c) How long after the tea is poured will it take for its temperature to reach 55

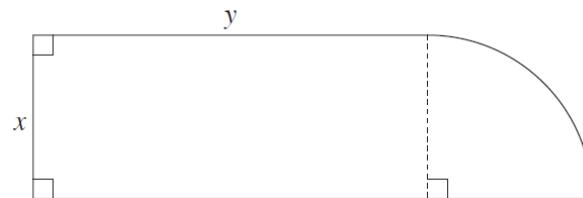
3



NESA 2020 Mathematics Advanced HSC Examination

**20 25** A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle.[Solution](#)

Let  $x$  and  $y$  be the dimensions of the rectangle in metres, as shown in the diagram.



The garden bed is required to have an area of  $36 \text{ m}^2$  and to have a perimeter which is as small as possible. Let  $P$  metres be the perimeter of the garden bed.

(a) Show that  $P = 2x + \frac{72}{x}$ .

3



(b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum perimeter.

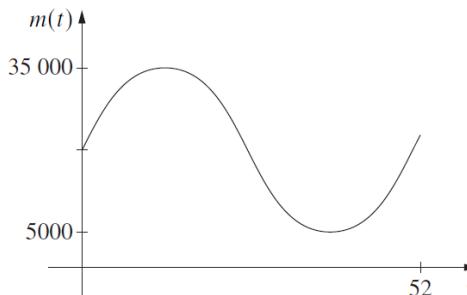
4



NESA 2020 Mathematics Advanced HSC Examination

**20 31** The population of mice on an isolated island can be modelled by the function[Solution](#)
 $m = a \sin\left(\frac{\pi}{26}t\right) + b$ , where  $t$  is the time in weeks and  $0 \leq t \leq 52$ . The population of mice reaches a maximum of 35 000 when  $t = 13$  and a minimum of 5000 when  $t = 39$ .

The graph of  $m(t)$  is shown.

(a) What are the values of  $a$  and  $b$ ?

2



(b) On the same island, the population of cats can be modelled by the function

3



$$c(t) = -80 \cos\left(\frac{\pi}{26}(t-10)\right) + 120.$$

Consider the graph of  $m(t)$  and the graph of  $c(t)$ .

Find the values of  $t$ ,  $0 \leq t \leq 52$ , for which both populations are increasing.

(c) Find the rate of change of the mice population when the cat population reaches a maximum.

2



NESA 2020 Mathematics Advanced HSC Examination

**MA 14** A function is given by  $f(x) = 18x^2 - x^4$ .[Solution](#)**SP Band 2-5** (a) Find the stationary points and determine their nature.

4



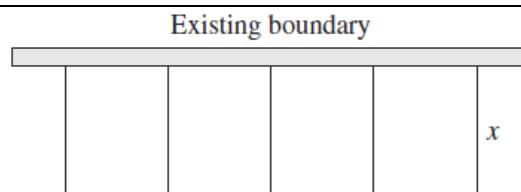
(b) Sketch the curve, labelling the stationary points and axis intercepts.

2



NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 32** A farmer wishes to make a rectangular enclosure of area  $720 \text{ m}^2$ . She uses an existing straight boundary as one side of the enclosure. She uses wire fencing for the remaining three sides and also to divide the enclosure into four equal rectangular areas of width  $x \text{ m}$  as shown.
- SP Band 2-5**
- 16 M 14 c**



**3** [Solution](#)

The total length,  $\ell \text{ m}$ , of the wire fencing is given by  $\ell = 5x + \frac{720}{x}$ .

(Do NOT prove this.)

Find the minimum length of wire fencing required, showing why this is the minimum length.

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

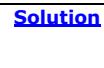
NESA 2016 Mathematics HSC Examination

- TG 1** By finding stationary points and their nature and intercepts on axes, sketch the curve  $y = e^{-x} \sin 2x$  over the domain  $0 \leq x \leq \pi$ .

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

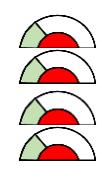


- TG 2** Consider the curve  $y = \frac{1}{4}x^4 - x^3$ .



- Find any stationary points and determine their nature.
- Find any points of inflection.
- Sketch the curve for  $-1.5 \leq x \leq 4.5$ , indicating where the curve crosses the  $x$ -axis.
- For what values of  $x$  is the curve concave down?

NESA Mathematics Advanced Year 12 Topic Guide: Calculus



- TG 3** A box without a lid is made by cutting out four equal squares from the corners of a sheet of heavy card, then folding up the sides.



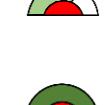
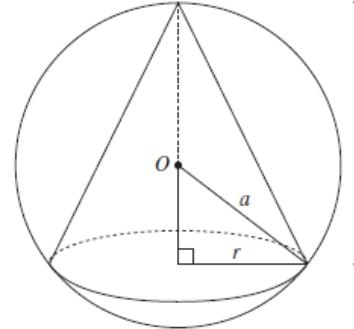
If the card has dimensions 20 cm by 10 cm, what are the dimensions of the box with largest volume that can be constructed in this way?

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

- TG 4** A right circular cone is inscribed in a sphere of radius  $a$ , centred at  $O$ . The distance from the base of the cone to the top of the sphere is  $x$  and the radius of the base is  $r$ , as shown in the diagram.



- 06 M 9c**
- (a) Show that the volume,  $V$ , of the cone is given by  $V = \frac{1}{3}\pi(2ax^2 - x^3)$ .



- (b) Find the value of  $x$  for which the volume of the cone is a maximum. Give reasons why this value of  $x$  gives the maximum volume.

NESA Mathematics Advanced Year 12 Topic Guide: Calculus  
NESA 2006 Mathematics HSC Examination

- 19 10** A particle is moving along a straight line with displacement  $x$  at time  $t$ .

**1** [Solution](#)



The particle is stationary when  $t = 11$  and when  $t = 13$ .

Which of the following MUST be true in this case?

- The particle changes direction at some time between  $t = 11$  and  $t = 13$ .
- The displacement function of the particle has a stationary point at some time between  $t = 11$  and  $t = 13$ .
- The acceleration of the particle is 0 at some time between  $t = 11$  and  $t = 13$ .
- The acceleration function of the particle has a stationary point at some time between  $t = 11$  and  $t = 13$ .

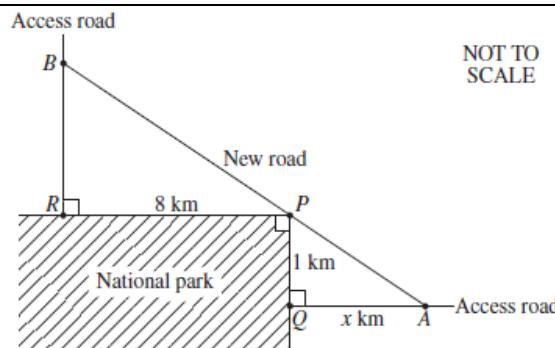
NESA 2019 Mathematics HSC Examination

- 19 12** The number of leaves,  $L(t)$ , on a tree  $t$  days after the start of autumn can be modelled by  $L(t) = 200\ 000e^{-0.14t}$ . [Solution](#)
- (i) What is the number of leaves on the tree when  $t = 31$ ? 1
- (ii) What is the rate of change of the number of leaves on the tree when  $t = 31$ ? 2
- (iii) For what value of  $t$  are there 100 leaves on the tree? 2

NESA 2019 Mathematics HSC Examination

- 19 14** The derivative of a function  $y = f(x)$  is given by  $f'(x) = 3x^2 + 2x - 1$ . [Solution](#)
- M b** (i) Find the  $x$ -values of the two stationary points of  $y = f(x)$ , and determine the nature of the stationary points. 2
- (ii) The curve passes through the point  $(0, 4)$ . Find an expression for  $f(x)$ . 2
- (iii) Hence sketch the curve, clearly indicating the stationary points. 2
- (iv) For what values of  $x$  is the curve concave down? 1

NESA 2019 Mathematics HSC Examination

- 19 15** The entry points,  $R$  and  $Q$ , to a national park can be reached via two straight access roads. The access roads meet the national park boundaries at right angles. The corner,  $P$ , of the national park is 8 km from  $R$  and 1 km from  $Q$ . The boundaries of the national park form a right angle at  $P$ . A new straight road is to be built joining these roads and passing through  $P$ . Points  $A$  and  $B$  on the access roads are to be chosen to minimise the distance,  $D$  km, from  $A$  to  $B$  along the new road. Let the distance  $QA$  be  $x$  km.
- 
- NOT TO SCALE
- (i) Show that  $D^2 = (x + 8)^2 + \left(\frac{8}{x} + 1\right)^2$  3
- (ii) Show that  $x = 2$  gives the minimum value of  $D^2$ . 3

NESA 2019 Mathematics HSC Examination

- 18 12** Find the equation of the tangent to the curve  $y = \cos 2x$  at  $x = \frac{\pi}{6}$ . 3 [Solution](#)

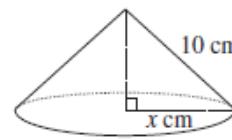
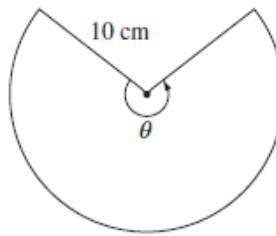
NESA 2018 Mathematics HSC Examination

- 18 13** Consider the curve  $y = 6x^2 - x^3$ . [Solution](#)
- M a** (i) Find the stationary points and determine their nature. 3
- (ii) Given that the point  $(2, 16)$  lies on the curve, show that it is a point of inflection. 2
- (iii) Sketch the curve, showing the stationary points, the point of inflection and the  $x$  and  $y$  intercepts. 2

NESA 2018 Mathematics HSC Examination

**18 16** A sector with radius

- M a** 10 cm and angle  $\theta$  is used to form the curved surface of a cone with base radius  $x$  cm, as shown in the diagram.



NOT TO SCALE

[Solution](#)

The volume of a cone of radius  $r$  and height  $h$  is given by  $V = \frac{1}{3} \pi r^2 h$ .

- (i) Show that the volume  $V$  cm<sup>3</sup>, of the cone described above is given by

$$V = \frac{1}{3} \pi x^2 \sqrt{100 - x^2}.$$

**1**

- (ii) Show that  $\frac{dV}{dx} = \frac{\pi x(200 - 3x^2)}{3\sqrt{100 - x^2}}$ .

**2**

- (iii) Find the exact value of  $\theta$  for which  $V$  is a maximum.

**3**

NESA 2018 Mathematics HSC Examination

**17 13** Consider the curve  $y = 2x^3 + 3x^2 - 12x + 7$ .

- M b** (i) Find the stationary points and determine their nature.  
(ii) Sketch the curve, labelling the stationary points.  
(iii) Hence, or otherwise, find the values of  $x$  for which  $\frac{dy}{dx}$  is positive.

**4****2****1**

NESA 2017 Mathematics HSC Examination

**17 16** John's home is at point  $A$  and his school is at point  $B$ . A straight river runs nearby. The point on the river closest to  $A$  is point  $C$ , which is 5 km from  $A$ . The point on the river closest to  $B$  is point  $D$ , which is 7 km from  $B$ . The distance from  $C$  to  $D$  is 9 km. To get some exercise, John cycles from home directly to point  $E$  on the river,  $x$  km from  $C$ , before cycling directly to school at  $B$ , as shown in the diagram.

The total distance John cycles from home to school is  $L$  km.

- (i) Show that  $L = \sqrt{x^2 + 25} + \sqrt{49 + (9 - x)^2}$ .

**1**

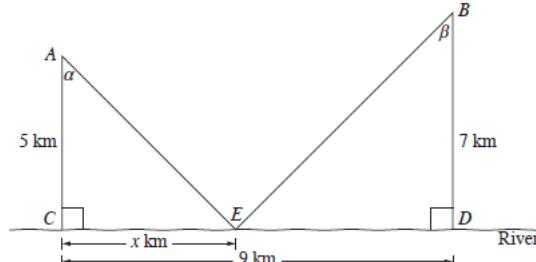
- (ii) Show that if  $\frac{dL}{dx} = 0$ , then  $\sin \alpha = \sin \beta$ .

**3**

- (iii) Find the value of  $x$  such that  $\sin \alpha = \sin \beta$ .

**2**

- (iv) Explain why this value of  $x$  gives a minimum for  $L$ .

**1**[Solution](#)

**16 11** Find the gradient of the tangent to the curve  $y = \tan x$  at the point where  $x = \frac{\pi}{8}$ .

**2**

Give your answer correct to 3 significant figures.

NESA 2016 Mathematics HSC Examination

**16 13** Consider the function  $y = 4x^3 - x^4$ .

- M a**
- (i) Find the two stationary points and determine their nature.
  - (ii) Sketch the graph of the function, clearly showing the stationary points and the  $x$  and  $y$  intercepts.

[Solution](#)4  
2

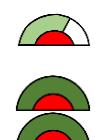
NESA 2016 Mathematics HSC Examination

**16 16** Some yabbies are introduced into a small dam. The size of the population,  $y$ , of

- M b** yabbies can be modelled by the function  $y = \frac{200}{1 + 19e^{-0.5t}}$ , where  $t$  is the time in months after the yabbies are introduced into the dam.

[Solution](#)

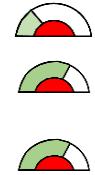
- (i) Show that the rate of growth of the size of the population is  $\frac{1900e^{-0.5t}}{(1 + 19e^{-0.5t})^2}$ .
- (ii) Find the range of the function  $y$ , justifying your answer.
- (iii) Show that the rate of growth of the size of the population can be rewritten as  $\frac{y}{400}(200 - y)$ .
- (iv) Hence, find the size of the population when it is growing at its fastest rate.

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NESA 2016 Mathematics HSC Examination

**15 13** Consider the curve  $y = x^3 - x^2 - x + 3$ .

- M c**
- (i) Find the stationary points and determine their nature.
  - (ii) Given that the point  $P(\frac{1}{3}, \frac{70}{27})$  lies on the curve, prove that there is a point of inflection at  $P$ .
  - (iii) Sketch the curve, labelling the stationary points, point of inflection and  $y$ -intercept.

[Solution](#)4  
2  
2

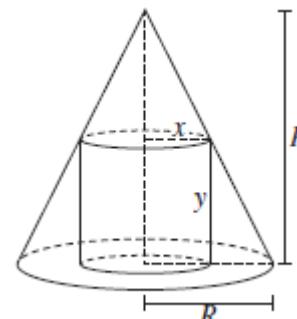
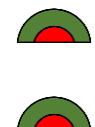
NESA 2015 Mathematics HSC Examination

**15 16** The diagram shows a cylinder of radius  $x$  and height  $y$  inscribed in a cone of radius  $R$  and height  $H$ , where  $R$  and  $H$  are constants.

The volume of a cone of radius  $r$  and height  $h$  is  $\frac{1}{3}\pi r^2 h$ .

The volume of a cylinder of radius  $r$  and height  $h$  is  $\pi r^2 h$ .

- (i) Show that the volume,  $V$ , of the cylinder can be written as  $V = \frac{H}{R} \pi x^2(R - x)$ .
- (ii) By considering the inscribed cylinder of maximum volume, show that the volume of any inscribed cylinder does not exceed  $\frac{4}{9}$  of the volume of the cone.

[Solution](#)3  
4

NESA 2015 Mathematics HSC Examination

**14 14** Find the coordinates of the stationary point on the graph  $y = e^x - ex$  and determine its nature.

3

[Solution](#)

NESA 2014 Mathematics HSC Examination

**14 16** The diagram shows a window consisting of two sections.

[Solution](#)

**M c** The top section is a semicircle of diameter  $x$  m. The bottom section is a rectangle of width  $x$  m and height  $y$  m.

The entire frame of the window, including the piece that separates the two sections, is made using 10 m of thin metal.

The semicircular section is made of coloured glass and the rectangular section is made of clear glass.

Under test conditions the amount of light coming through one square metre of the coloured glass is 1 unit and the amount of light coming through one square metre of the clear glass is 3 units. The total amount of light coming through the window under test conditions is  $L$  units.

(i) Show that  $y = 5 - x\left(1 + \frac{\pi}{4}\right)$ .

2



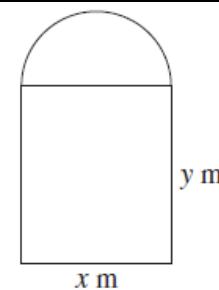
(ii) Show that  $L = 15x - x^2\left(3 + \frac{5\pi}{8}\right)$ .

2



(iii) Find the values of  $x$  and  $y$  that maximise the amount of light coming through the window under test conditions.

3



NESA 2014 Mathematics HSC Examination

**13 10** A particle is moving along the  $x$ -axis. The displacement of the particle at time

1

[Solution](#)

**M**  $t$  seconds is  $x$  metres. At a certain time,  $\dot{x} = -3 \text{ m s}^{-1}$  and  $\ddot{x} = 2 \text{ m s}^{-2}$ .

Which statement describes the motion of the particle at that time?

- (A) The particle is moving to the right with increasing speed.
- (B) The particle is moving to the left with increasing speed.
- (C) The particle is moving to the right with decreasing speed.
- (D) The particle is moving to the left with decreasing speed.

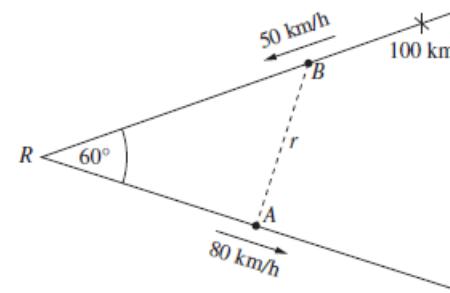
NESA 2013 Mathematics HSC Examination

**13 14** Two straight roads meet at  $R$  at an angle of  $60^\circ$ .

[Solution](#)

**M b** At time  $t = 0$  car A leaves  $R$  on one road, and car B is 100 km from  $R$  on the other road. Car A travels away from  $R$  at a speed of 80 km/h, and car B travels towards  $R$  at a speed of 50 km/h. The distance between the cars at time  $t$  hours is  $r$  km.

- (i) Show that  $r^2 = 12900t^2 - 18000t + 10000$ .
- (ii) Find the minimum distance between the cars.



2



3



NESA 2013 Mathematics HSC Examination

**12 14** A function is given by  $f(x) = 3x^4 + 4x^3 - 12x^2$ .

[Solution](#)

- M a** (i) Find the nature of the stationary points of  $f(x)$  and determine their nature.
- (ii) Hence, sketch the graph of  $y = f(x)$  showing the stationary points.
- (iii) For what values of  $x$  is the function increasing?
- (iv) For what values of  $k$  will  $3x^4 + 4x^3 - 12x^2 + k = 0$  have no solution?

3



2



1



1

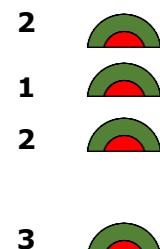
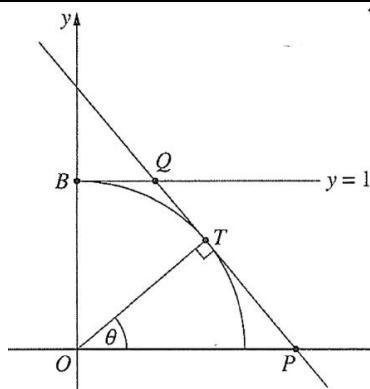


NESA 2012 Mathematics HSC Examination

- 12 16** The diagram shows a point  $T$  on the unit circle  $x^2 + y^2 = 1$  at angle  $\theta$  from the positive  $x$ -axis, where  $0 < \theta < \frac{\pi}{2}$ .

The tangent to the circle at  $T$  is perpendicular to  $OT$ , and intersects the  $x$ -axis at  $P$ , and the line  $y = 1$  at  $Q$ . The line  $y = 1$  intersects the  $y$ -axis at  $B$ .

- Show that the equation of the line  $PT$  is  $x \cos \theta + y \sin \theta = 1$ .
- Find the length of  $BQ$  in terms of  $\theta$ .
- Show that the area,  $A$ , of the trapezium  $OPQB$  is given by  $A = \frac{2 - \sin \theta}{2 \cos \theta}$ .
- Find the angle  $\theta$  that gives the minimum area of the trapezium.



NESA 2012 Mathematics HSC Examination

- 11 7a** Let  $f(x) = x^3 - 3x + 2$ .

- Find the coordinates of the stationary points of  $y = f(x)$ , and determine their nature.
- Hence, sketch the graph  $y = f(x)$  showing all stationary points and the  $y$ -intercept.

[Solution](#)

NESA 2011 Mathematics HSC Examination

- 11 4a** Consider the function  $f(x) = e^{-x} - 2e^{-2x}$ .

- Find  $f'(x)$
- The graph  $y = f(x)$  has one maximum turning point. Find the coordinates of the maximum turning point.
- Evaluate  $f(\ln 2)$ .
- Describe the behaviour of  $f(x)$  as  $x \rightarrow \infty$ .
- Find the  $y$ -intercept of the graph  $y = f(x)$ .
- Sketch the graph  $y = f(x)$  showing the features from parts (ii) – (v). You are not required to find any points of inflection.

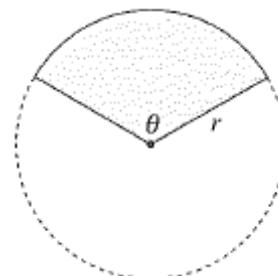
[Solution](#)

NESA 2011 Mathematics Extension 1 HSC Examination

- 11 10** A farmer is fencing a paddock using  $P$  metres of fencing.

- M b** The paddock is to be in the shape of a sector of a circle with radius  $r$  and sector  $\theta$  in radians, as shown in the diagram.

- Show that the length of fencing required to fence the perimeter of the paddock is  $P = r(\theta + 2)$ .
- Show that the area of the sector is  $A = \frac{1}{2}Pr - r^2$ .
- Find the radius of the sector, in terms of  $P$ , that will maximize the area of the paddock.
- Find the angle  $\theta$  that gives the maximum area of the paddock.
- Explain why it is only possible to construct a paddock in the shape of a sector if  $\frac{P}{2(\pi + 1)} < r < \frac{P}{2}$ .



NESA 2011 Mathematics HSC Examination

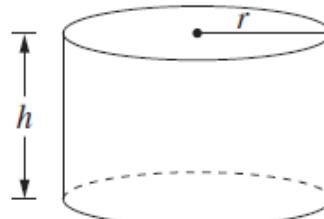
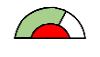
- 10 2c** Find the gradient of the tangent to the curve  $y = \ln(3x)$  at the point where  $x = 2$ .

**2**

NESA 2010 Mathematics HSC Examination

- 10 5a** A rainwater tank is to be designed in the shape of a cylinder with radius  $r$  metres and height  $h$  metres. The volume of the tank is to be 10 cubic metres. Let  $A$  be the surface area of the tank, including its top and base, in square metres.

- Given that  $A = 2\pi r^2 + 2\pi rh$ , show that  $A = 2\pi r^2 + \frac{20}{r}$ .
- Show that  $A$  has a minimum value and find the value of  $r$  for which the minimum occurs.

**2****3**

NESA 2010 Mathematics HSC Examination

- 10 6a** Let  $f(x) = (x + 2)(x^2 + 4)$ .

- Show that the graph of  $y = f(x)$  has no stationary points.
- Find the values of  $x$  for which the graph  $y = f(x)$  is concave down, and the values for which it is concave up.
- Sketch the graph of  $y = f(x)$ , indicating the values of the  $x$  and  $y$  intercepts.

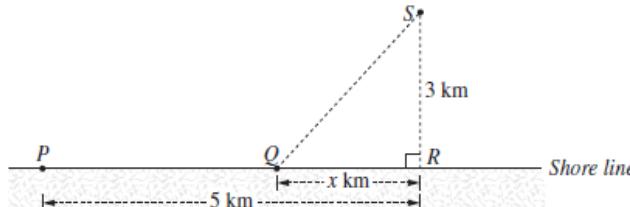
**2****2****2**

NESA 2010 Mathematics HSC Examination

- 09 9b** An oil rig,  $S$ , is 3 km offshore. A power station,  $P$ , is on the shore. A cable is to be laid from  $P$  to  $S$ . It costs \$1000 per kilometre to lay the cable along the shore and \$2600 per kilometre to lay the cable underwater from the shore to  $S$ . The point  $R$  is the point on the shore closest to  $S$ , and the distance  $PR$  is 5 km. The point  $Q$  is on the shore, at a distance of  $x$  km from  $R$ , as shown in the diagram.

**Solution**

- Find the total cost of laying the cable in a straight line from  $P$  to  $R$  and then in a straight line from  $R$  to  $S$ .

**1**

- Find the cost of laying the cable in a straight line from  $P$  to  $S$ .

**1**

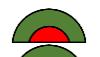
- Let  $\$C$  be the total cost of laying the cable in a straight line from  $P$  to  $Q$ , and then in a straight line from  $Q$  to  $S$ .

**2**

Show that  $C = 1000(5 - x + 2.6\sqrt{x^2 + 9})$ .

**3**

- Find the minimum cost of laying the cable.

**1**

- New technology means that the cost of laying the cable underwater can be reduced to \$1100 per kilometre. Determine the path for laying the cable in order to minimise the cost in this case.

NESA 2009 Mathematics HSC Examination

- 08 5a** The gradient of a curve is given by  $\frac{dy}{dx} = 1 - 6\sin 3x$ .

**3**

The curve passes through the point  $(0, 7)$ .

What is the equation of the curve?

NESA 2008 Mathematics HSC Examination

- 08 5c** Light intensity is measured in lux. The light intensity at the surface of a lake is 6000 lux. The light intensity,  $I$  lux, a distance  $s$  metres below the surface of the lake is given by  $I = Ae^{-ks}$  where  $A$  and  $k$  are constants.
- Write down the value of  $A$ .
  - The light intensity 6 metres below the surface of the lake is 1000 lux. Find the value of  $k$ .
  - At what rate, in lux per metre, is the light intensity decreasing 6 metres below the surface of the lake?

[Solution](#)

1



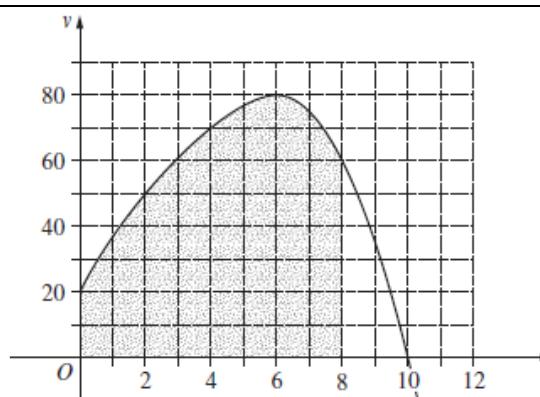
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2



- 08 6b** The graph shows the velocity of a particle,  $v$  metres per second, as a function of time,  $t$  seconds.
- What is the initial velocity of the particle?
  - When is the velocity of the particle equal to zero?
  - When is the acceleration of the particle equal to zero?
  - deleted in Maths Adv

[Solution](#)

NESA 2008 Mathematics HSC Examination

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- 08 8a** Let  $f(x) = x^4 - 8x^2$ .
- Find the coordinates of the points where the graph of  $y = f(x)$  crosses the axes.
  - Show that  $f(x)$  is an even function.
  - Find the coordinates of the stationary points of  $y = f(x)$  and determine their nature.
  - Sketch the graph of  $y = f(x)$ .

[Solution](#)

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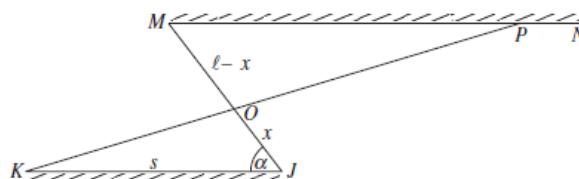


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NESA 2008 Mathematics HSC Examination

- 08 10 b** The diagram shows two parallel brick walls  $KJ$  and  $MN$  joined by a fence from  $J$  to  $M$ . The wall  $KJ$  is  $s$  metres long and  $\angle KJM = \alpha$ . The fence  $JM$  is  $l$  metres long.

[Solution](#)

A new fence is to be built from  $K$  to a point  $P$  somewhere on  $MN$ . The new fence  $KP$  will cross the original fence  $JM$  at  $O$ .

Let  $OJ = x$  metres, where  $0 < x < l$ .

- Show that the total area,  $A$  square metres, enclosed by  $\Delta OKJ$  and  $\Delta OMP$  is given by  $A = s(x - l + \frac{l^2}{2x})\sin \alpha$ .
- Find the value of  $x$  that makes  $A$  as small as possible. Justify the fact that this value of  $x$  gives the minimum value for  $A$ .
- Hence, find the length of  $MP$  when  $A$  is as small as possible.

3



3



1



NESA 2008 Mathematics HSC Examination

- 07 2c** The point  $P(\pi, 0)$  lies on the curve  $y = x \sin x$ .  
Find the equation of the tangent to the curve at  $P$ .

3

[Solution](#)

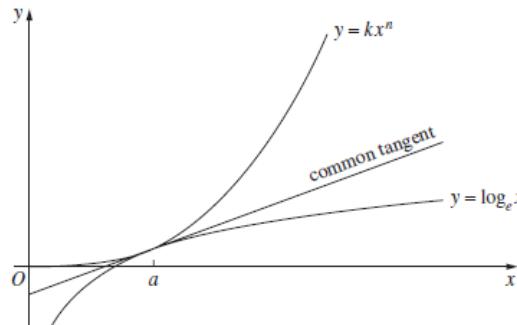
NESA 2007 Mathematics HSC Examination

- 07 6b** Let  $f(x) = x^4 - 4x^3$ .
- M**
- Find the coordinates of the points where the curve crosses the axes.
  - Find the coordinates of the stationary points and determine their nature.
  - Find the coordinates of the points of inflection.
  - Sketch the graph of  $y = f(x)$ , indicating clearly the intercepts, stationary points and points of inflection.

[Solution](#)**2****4****1****3**

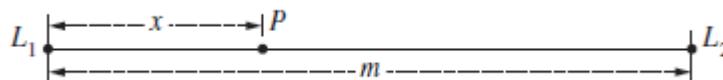
NESA 2007 Mathematics HSC Examination

- 07 7a** The graphs of the functions  $y = kx^n$  and  $y = \log_e x$  have  $x = a$  as shown in the diagram.
- MX 1**
- By considering gradients, show that  $a^n = \frac{1}{nk}$ .
  - Express  $k$  as a function of  $n$  by eliminating  $a$ .

[Solution](#)**1****2**

NESA 2007 Mathematics Extension 1 HSC Examination

- 07 10 b** The noise level,  $N$ , at a distance  $d$  metres from a single sound source of loudness  $L$  is given by the formula  $N = \frac{L}{d^2}$ .

[Solution](#)

Two sound sources, of loudness  $L_1$  and  $L_2$ , are placed  $m$  metres apart.

The point  $P$  lies on the line between the sound sources and is  $x$  metres from the sound source with loudness  $L_1$ .

- Write down a formula for the sum of the noise levels at  $P$  in terms of  $x$ .
- There is a point on the line between the sound sources where the sum of the noise levels is a minimum.

**1****4**

Find an expression for  $x$  in terms of  $m$ ,  $L_1$  and  $L_2$  if  $P$  is chosen to be this point.

NESA 2007 Mathematics HSC Examination

# Year 12: Calculus

## C4.1 The Anti-Dervative



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- define anti-differentiation as the reverse of differentiation and use the notation  $\int f(x) dx$  for anti-derivatives or indefinite integrals (ACMMM114, ACMMM115)
- recognise that any two anti-derivatives of  $f(x)$  differ by a constant
- establish and use the formula  $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$ , for  $n \neq -1$  (ACMMM116) \*
- establish and use the formula  $\int f'(x)[f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$  where  $n \neq -1$  (the reverse chain rule) \*
- establish and use the formulae for the anti-derivatives of  $\sin(ax+b)$ ,  $\cos(ax+b)$  and  $\sec^2(ax+b)$
- establish and use the formulae  $\int e^x dx = e^x + c$  and  $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$
- establish and use the formulae  $\int \frac{1}{x} dx = \ln|x| + c$  and  $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$  for  $x \neq 0, f(x) \neq 0$ , respectively
- establish and use the formulae  $\int a^x dx = \frac{a^x}{\ln a} + c$
- recognise and use linearity of anti-differentiation (ACMMM119)
  - examine families of anti-derivatives of a given function graphically
- determine indefinite integrals of the form  $\int f(ax+b) dx$  (ACMMM120)
- determine  $f(x)$ , given  $f'(x)$  and an initial condition  $f(a) = b$  in a range of practical and abstract applications including coordinate geometry, business and science

[Reference Sheet](#)

**23 13**

**MA** Let  $P(t)$  be a function such that  $\frac{dP}{dt} = 3000e^{2t}$ .

**2** [Solution](#)



When  $t = 0$ ,  $P = 4000$ .

Find an expression for  $P(t)$ .

NESA 2023 Mathematics Advanced HSC Examination

**23 17**

**MA** Find  $\int x\sqrt{x^2 + 1} dx$ .

**2** [Solution](#)



NESA 2023 Mathematics Advanced HSC Examination

**22 6**

**MA** What is  $\int \frac{1}{(2x+1)^2} dx$ ?

**1** [Solution](#)



- A.  $\frac{-2}{2x+1} + C$       B.  $\frac{-1}{2(2x+1)} + C$       C.  $2 \ln(2x+1) + C$       D.  $\frac{1}{2} \ln(2x+1) + C$

NESA 2022 Mathematics Advanced HSC Examination

**22 18** (a) Differentiate  $y = (x^2 + 1)^4$ .**2****MA**(b) Hence, or otherwise, find  $\int x(x^2 + 1)^3 dx$ .**1**

NESA 2022 Mathematics Advanced HSC Examination

**21 15** Evaluate  $\int_{-2}^0 \sqrt{2x + 4} dx$ .**2****MA**

NESA 2021 Mathematics Advanced HSC Examination

**20 4** What is  $\int (e + e^{3x}) dx$ ?**1****MA**

- A.  $ex + 3e^{3x} + c$       B.  $ex + \frac{1}{3}e^{3x} + c$       C.  $e + 3e^{3x} + c$       D.  $e + \frac{1}{3}e^{3x} + c$



NESA 2020 Mathematics Advanced HSC Examination

**20 17** Find  $\int \frac{x}{4+x^2} dx$ .**2****MA**

NESA 2020 Mathematics Advanced HSC Examination

**20 18** (a) Differentiate  $e^{2x}(2x + 1)$ **2****MA**(b) Hence, find  $\int (x+1)e^{2x} dx$ .**1**

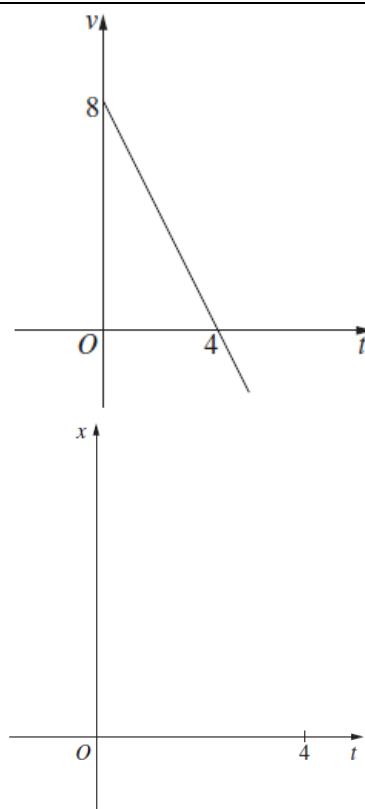
NESA 2020 Mathematics Advanced HSC Examination

**MA 30** The population,  $P$ , of rabbits on an island is given by  $P(t)$ , where  $t$  is the time in years after the rabbits were introduced.**3** [Solution](#)**SP Band 3-5**The rabbit population changes at a rate modelled by the function  $\frac{dP}{dt} = 30e^{1.25t}$ .Calculate the increase in the number of rabbits at the end of the first 10 years.  
Give your answer correct to two significant figures.

NESA Mathematics Advanced Sample Examination Paper (2020)

**MA 33** A particle is moving along the  $x$ -axis.**SP Band 4-6** The graph shows its velocity  $v$  metres per second at time  $t$  seconds.

When  $t = 0$  the displacement  $x$  is equal to 2 metres.

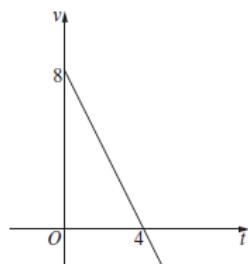
**2**[Solution](#)

On the axes draw a graph that shows the particle's displacement,  $x$  metres from the origin, at a time  $t$  seconds between  $t = 0$  and  $t = 4$ . Label the coordinates of the endpoints of your graph.

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA 6** A particle is moving along the  $x$ -axis.**SQ 2019 Band 5-6** The graph shows its velocity  $v$  metres per second at time  $t$  seconds. When  $t = 0$  the displacement  $x$  is equal to 2 metres. What is the maximum value of the displacement  $x$ ?**15 M 9**

- (A) 8 m      (B) 14 m  
 (C) 16 m      (D) 1

**1**[Solution](#)

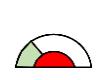
NESA Mathematics Advanced Sample examination materials (2019)

**TG 1** A function  $y(x)$  has the derivative  $\frac{dy}{dx} = 4x^3 + x^2 - 5x + 1$ , and  $y(1) = 6$ .

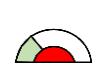
Find the function.

**TG 2** Given that the function  $f(x)$  has derivative  $f'(x) = 4x + 3$  and that  $f(2) = 9$ , find the function.**TG 3** Determine the anti-derivatives of each function below:  
 (a)  $f(x) = (3x - 2)^5$ 

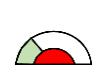
(b)  $f(p) = \frac{1}{\sqrt{p}}$



(c)  $f(t) = \frac{1}{\sqrt{t+1}}$



(d)  $g(r) = \frac{r^5 - 2r^3}{r^2}$



NESA Mathematics Advanced Year 12 Topic Guide: Calculus

<b>TG</b>	<b>4</b>	The rate of change of the number of internet users in a large country is estimated as $\frac{dy}{dt} = -2.54t + 31.2$ , where $y$ is the number of millions of internet users and $t$ is the number of years since 1995.	<b>Solution</b>
	(a)	If there were 57 million users of the internet in this country in 1997, find the number of internet users in 2003.	
	(b)	Discuss any limitations of this model, including the implications for large values of $t$ .	
<b>19</b>	<b>8</b>	A particle is moving along a straight line. The graph shows the acceleration of the particle. For what value of $t$ is the velocity $v$ a maximum?	NESA Mathematics Advanced Year 12 Topic Guide: Calculus
<b>M</b>		A. 1      B. 2 C. 3      D. 5	
<b>19</b>	<b>9</b>	Which expression is equal to $\int \tan^2 x \, dx$ ?	<b>1</b> <b>Solution</b>
<b>M</b>		A. $\tan x - x + C$ B. $\tan x - 1 + C$ C. $\frac{\tan^3 x^2}{6} + C$ D. $\frac{\tan^3 x}{3} + C$	
<b>19</b>	<b>13</b>	(i) Differentiate $(\ln x)^2$ . (ii) Hence, or otherwise, find $\int \frac{\ln x}{x} \, dx$ .	NESA 2019 Mathematics HSC Examination
<b>M</b>	<b>c</b>		<b>2</b> <b>Solution</b> <b>1</b>
<b>19</b>	<b>14</b>	A particle is moving along a straight line. The particle is initially at rest.	NESA 2019 Mathematics HSC Examination
<b>M</b>	<b>a</b>	The acceleration of the particle at time $t$ seconds is given by $a = e^{2t} - 4$ , where $t \geq 0$ . Find an expression, in terms of $t$ , for the velocity of the particle.	<b>2</b> <b>Solution</b>
<b>17</b>	<b>9</b>	The graph of $y = f'(x)$ is shown. The curve $y = f(x)$ has maximum value of 12. What is the equation of the curve $y = f(x)$ ? (A) $y = x^2 - 4x + 12$ (B) $y = 4 + 4x - x^2$ (C) $y = 8 + 4x - x^2$ (D) $y = x^2 - 4x + 16$	NESA 2019 Mathematics HSC Examination
<b>M</b>			
<b>17</b>	<b>11</b>	Find $\int (2x + 1)^4 \, dx$ .	<b>1</b> <b>Solution</b>
<b>M</b>	<b>b</b>		
<b>17</b>	<b>13</b>	The rate at which water flows into a tank is given by $\frac{dV}{dt} = \frac{2t}{1+t^2}$ , where $V$ is the volume of water in the tank in litres and $t$ is the time in seconds. Initially the tank is empty. Find the exact amount of water in the tank after 10 seconds.	NESA 2017 Mathematics HSC Examination
<b>M</b>	<b>d</b>		<b>3</b> <b>Solution</b>

**17 15** Two particles move along the  $x$ -axis. [Solution](#)

- M c** When  $t = 0$ , particle  $P_1$  is at the origin and moving with velocity 3. For  $t \geq 0$ , particle  $P_1$  has acceleration given by  $a_1 = 6t + e^{-t}$ .

(i) Show that the velocity of particle  $P_1$  is given by  $v_1 = 3t^2 + 4 - e^{-t}$ . 2 When  $t = 0$ , particle  $P_2$  is also at the origin.For  $t \geq 0$ , particle  $P_2$  has velocity given by  $v_2 = 6t + 1 - e^{-t}$ .(ii) When do the two particles have the same velocity? 2 (iii) Show that the two particles do not meet for  $t > 0$ . 3 

NESA 2017 Mathematics HSC Examination

**16 16** A particle moves in a straight line. [Solution](#)

- M a** Its velocity  $v$  ms<sup>-1</sup> at time  $t$  seconds is given by  $v = 2 - \frac{4}{t+1}$ .

- (i) Find the initial velocity. 1 
- (ii) Find the acceleration of the particle when the particle is stationary. 2 
- (iii) By considering the behavior of  $v$  for large  $t$ , sketch a graph of  $v$  against  $t$  for  $t \geq 0$ , showing any intercepts. 2 
- (iv) Find the exact distance travelled by the particle in the first 7 seconds. ~~delete~~ 3 

NESA 2016 Mathematics HSC Examination

**15 11** **M h** Find  $\int \frac{x}{x^2 - 3} dx$ . 2 

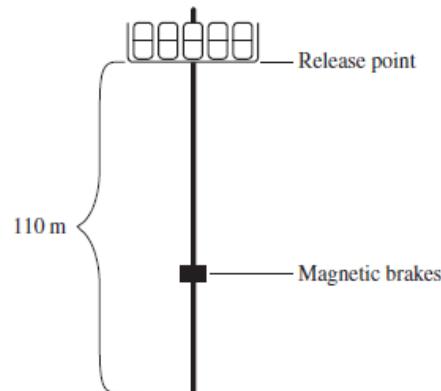
NESA 2015 Mathematics HSC Examination

**15 14** In a theme park ride, a chair is released from a height of 110 metres and falls vertically. [Solution](#)

- M a** Magnetic brakes are applied when the velocity of the chair reaches -37 metres per second. The height of the chair at time  $t$  seconds is  $x$  metres. The acceleration of the chair is given

by  $\ddot{x} = -10$ . At the release point,  $t = 0$ , $x = 110$  and  $\dot{x} = 0$ .

- (i) Using calculus, show that  $x = -5t^2 + 110$ . 2 
- (ii) How far has the chair fallen when the magnetic brakes are applied? 2 



Not to scale

NESA 2015 Mathematics HSC Examination

**15 15** Water is flowing in and out of a rock pool. The volume of water in the pool at time  $t$  hours is  $V$  litres. The rate of change of the volume is given by  $\frac{dV}{dt} = 80 \sin(0.5t)$ . [Solution](#)

- M c** At time  $t = 0$ , the volume of water in the pool is 1200 litres and is increasing.

- (i) After what time does the volume of water first start to decrease? 2 
- (ii) Find the volume of water in the pool when  $t = 3$ . 2 
- (iii) What is the greatest volume of water in the pool? 1 

NESA 2015 Mathematics HSC Examination

**14 4** Which expression is equal to  $\int e^{2x} dx$ ? 1 

(A)  $e^{2x} + c$

(B)  $2e^{2x} + c$

(C)  $\frac{e^{2x}}{2} + c$

(D)  $\frac{e^{2x+1}}{2x+1} + c$

NESA 2014 Mathematics HSC Examination

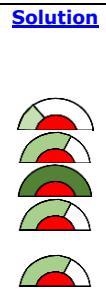
<b>14</b>	<b>11</b>	<b>d</b>	Find $\int \frac{1}{(x+3)^2} dx$ .	<b>2</b>	<a href="#">Solution</a>
NESA 2014 Mathematics HSC Examination					
<b>14</b>	<b>11f</b>	<b>M</b>	The gradient function of a curve $y = f(x)$ is given by $f'(x) = 4x - 5$ . The curve passes through the point $(2, 3)$ . Find the equation of the curve.	<b>2</b>	<a href="#">Solution</a>
NESA 2014 Mathematics HSC Examination					
<b>14</b>	<b>13</b>	<b>a</b>	(i) Differentiate $3 + \sin 2x$ . (ii) Hence, or otherwise, find $\int \frac{\cos 2x}{3 + \sin 2x} dx$ .	<b>1</b> <b>2</b>	<a href="#">Solution</a>
NESA 2014 Mathematics HSC Examination					
<b>13</b>	<b>11</b>	<b>e</b>	Find $\int e^{4x+1} dx$ .	<b>2</b>	<a href="#">Solution</a>
NESA 2013 Mathematics HSC Examination					
<b>13</b>	<b>14</b>	<b>a</b>	The velocity of a particle moving along the $x$ -axis is given by $\dot{x} = 10 - 2t$ , where $x$ is the displacement from the origin in metres and $t$ is the time in seconds. Initially the particle is 5 metres to the right of the origin. (i) Show that the acceleration of the particle is constant. (ii) Find the time when the particle is at rest. (iii) Show that the position of the particle after 7 seconds is 26 metres to the right of the origin. (iv) Find the distance travelled by the particle during the first 7 seconds. <del>delete</del>	<b>1</b> <b>1</b> <b>2</b> <b>2</b>	<a href="#">Solution</a>
NESA 2013 Mathematics HSC Examination					
<b>13</b>	<b>16</b>	<b>a</b>	The derivative of a function $f(x)$ is $f'(x) = 4x - 3$ . The line $y = 5x - 7$ is tangent to the graph of $f(x)$ . Find the function $f(x)$ .	<b>3</b>	<a href="#">Solution</a>
NESA 2013 Mathematics HSC Examination					
<b>12</b>	<b>12</b>	<b>b</b>	Find $\int \frac{4x}{x^2 + 6} dx$ .	<b>2</b>	<a href="#">Solution</a>
NESA 2012 Mathematics HSC Examination					
<b>12</b>	<b>15</b>	<b>b</b>	The velocity of a particle is given by $\dot{x} = 1 - 2\cos t$ , where $x$ is the displacement in metres and $t$ is the time in seconds. Initially the particle is 3 m to the right of the origin. (i) Find the initial velocity of the particle. (ii) Find the maximum velocity of the particle. (iii) Find the displacement, $x$ , of the particle in terms of $t$ . (iv) Find the position of the particle when it is at rest for the first time.	<b>1</b> <b>1</b> <b>2</b> <b>2</b>	<a href="#">Solution</a>
NESA 2012 Mathematics HSC Examination					
<b>11</b>	<b>2e</b>	<b>M</b>	Find $\int \frac{1}{3x^2} dx$ .	<b>2</b>	<a href="#">Solution</a>
NESA 2011 Mathematics HSC Examination					
<b>11</b>	<b>4c</b>	<b>M</b>	The gradient of a curve is given by $\frac{dy}{dx} = 6x - 2$ . The curve passes through the point $(-1, 4)$ . What is the equation of the curve?	<b>2</b>	<a href="#">Solution</a>
NESA 2011 Mathematics HSC Examination					
<b>11</b>	<b>4d</b>	<b>M</b>	(i) Differentiate $y = \sqrt{9 - x^2}$ with respect to $x$ . (ii) Hence, or otherwise, find $\int \frac{6x}{\sqrt{9 - x^2}} dx$ .	<b>2</b> <b>2</b>	<a href="#">Solution</a>
NESA 2011 Mathematics HSC Examination					

**11 M 7b**

The velocity of a particle moving along the  $x$ -axis is given by  $\dot{x} = 8 - 8e^{-2t}$ , where  $t$  is the time in seconds and  $x$  is the displacement in metres.

- (i) Show that the particle is initially at rest. **1**
- (ii) Show that the acceleration of the particle is always positive. **1**
- (iii) Explain why the particle is moving in the positive direction for all  $t > 0$ . **2**
- (iv) As  $t \rightarrow \infty$ , the velocity of the particle approaches a constant. **1**  
Find the value of this constant.
- (v) Sketch the graph of the particle's velocity as a function of time. **2**

NESA 2011 Mathematics HSC Examination

**11 M 9b**

A tap releases liquid A into a tank at the rate of  $\left(2 + \frac{t^2}{t+1}\right)$  litres per minute, where

$t$  is time in minutes. A second tap releases liquid B into the same tank at the rate of  $\left(1 + \frac{1}{t+1}\right)$  litres per minute. The taps are opened at the same time and release the liquids into an empty tank.

- (i) Show that the rate of liquid A is greater than the rate of flow of liquid B by  $t$  litres per minute. **1**
- (ii) The taps are closed after 4 minutes. By how many litres is the volume of liquid A greater than the volume of liquid B in the tank when the taps are closed? **2**

NESA 2011 Mathematics HSC Examination

**10 M 2d**

- (i) Find  $\int \sqrt{5x+1} dx$ . **2**



NESA 2010 Mathematics HSC Examination

**10 M 2d**

- (ii) Find  $\int \frac{x}{4+x^2} dx$ . **2**



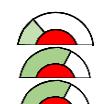
NESA 2010 Mathematics HSC Examination

**10 M 7a**

The acceleration of a particle is given by  $\ddot{x} = 4 \cos 2t$ , where  $x$  is displacement in metres and  $t$  is time in seconds. Initially the particle is at the origin with a velocity of  $1 \text{ m s}^{-1}$ .

- (i) Show that the velocity of the particle is given by  $\dot{x} = 2 \sin 2t + 1$ . **2**
- (ii) Find the time when the particle first comes to rest. **2**
- (iii) Find the displacement,  $x$ , of the particle in terms of  $t$ . **2**

NESA 2010 Mathematics HSC Examination

**09 M 2b**

- (i) Find  $\int 5 dx$ . **1**



NESA 2009 Mathematics HSC Examination

**09 M 2b**

- (ii) Find  $\int \frac{3}{(x-6)^2} dx$ . **2**



NESA 2009 Mathematics HSC Examination

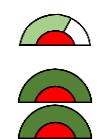
**09 M 7a**

The acceleration of a particle is given by  $\ddot{x} = 8e^{-2t} + 3e^{-t}$ , where  $x$  is displacement in metres and  $t$  is time in seconds.

Initially its velocity is  $-6 \text{ ms}^{-1}$  and its displacement is 5 m.

- (i) Show that the displacement of the particle is given by  $x = 2e^{-2t} + 3e^{-t} + t$ . **2**
- (ii) Find the time when the particle comes to rest. **3**
- (iii) Find the displacement when the particle comes to rest. **1**

NESA 2009 Mathematics HSC Examination



<b>08</b>	<b>2c</b>	(i) Find $\int \frac{dx}{x+5}$	<b>1</b>	<a href="#">Solution</a>
	<b>M</b>			
				NESA 2008 Mathematics HSC Examination
<b>07</b>	<b>2b</b>	(i) Find $\int (1 + \cos 3x) dx$ .	<b>2</b>	<a href="#">Solution</a>
	<b>M</b>			
				NESA 2007 Mathematics HSC Examination
<b>07</b>	<b>5b</b>	A particle is moving on the $x$ -axis and is initially at the origin. Its velocity, $v$ metres per second, at time $t$ seconds is given by $v = \frac{2t}{16+t^2}$ .		<a href="#">Solution</a>
		(i) What is the initial velocity of the particle?	<b>1</b>	
		(ii) Find an expression for the acceleration of the particle.	<b>2</b>	
		(iii) Find the time when the acceleration of the particle is zero.	<b>1</b>	
		(iv) Find the position of the particle when $t = 4$ .	<b>3</b>	
				NESA 2007 Mathematics HSC Examination

**Year 12: Calculus****C4.2 Areas and the Definite Integral****Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>**Students:**

- know that 'the area under a curve' refers to the area between a function and the  $x$ -axis, bounded by two values of the independent variable and interpret the area under a curve in a variety of contexts AAM
- determine the approximate area under a curve using a variety of shapes including squares, rectangles (inner and outer rectangles), triangles or trapezia
  - consider functions which cannot be integrated in the scope of this syllabus, for example  $f(x) = \ln x$ , and explore the effect of increasing the number of shapes used
- use the notation of the definite integral  $\int_a^b f(x) dx$  for the area under the curve  $y = f(x)$  from  $x = a$  to  $x = b$  if  $f(x) \geq 0$
- use the Trapezoidal rule to estimate areas under curves AAM
  - use geometric arguments (rather than substitution into a given formula) to approximate a definite integral of the form  $\int_a^b f(x) dx$ , where  $f(x) \geq 0$ , on the interval  $a \leq x \leq b$ , by dividing the area into a given number of trapezia with equal widths
  - demonstrate understanding of the formula:  

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(a) + f(b) + 2\{f(x_1) + \dots + f(x_{n-1})\}]$$
where  $a = x_0$  and  $b = x_n$ , and the values of  $x_0, x_1, x_2, \dots, x_n$  are found by dividing the interval  $a \leq x \leq b$  into  $n$  equal sub-intervals
- use geometric ideas to find the definite integral  $\int_a^b f(x) dx$  where  $f(x)$  is positive throughout an interval  $a \leq x \leq b$  and the shape of  $f(x)$  allows such calculations, for example when  $f(x)$  is a straight line in the interval or  $f(x)$  is a semicircle in the interval AAM
- understand the relationship of position to signed areas, namely that the signed area above the horizontal axis is positive and the signed area below the horizontal axis is negative
- using technology or otherwise, investigate the link between the anti-derivative and the area under a curve
  - interpret  $\int_a^b f(x) dx$  as a sum of signed areas (ACMMM127)
  - understand the concept of the signed area function  $F(x) = \int_a^x f(t) dt$  (ACMMM129)
- use the formula  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F(x)$  is the anti-derivative of  $f(x)$ , to calculate definite integrals (ACMMM131) AAM
  - understand and use the Fundamental Theorem of Calculus,  $F'(x) = \frac{d}{dx} [\int_a^x f(t) dt] = f(x)$  and illustrate its proof geometrically (ACMMM130)
  - use symmetry properties of even and odd functions to simplify calculations of area
  - recognise and use the additivity and linearity of definite integrals (ACMMM128)
  - calculate total change by integrating instantaneous rate of change
- calculate the area under a curve (ACMMM132)
- calculate areas between curves determined by any functions within the scope of this syllabus (ACMMM134) AAM
- integrate functions and find indefinite or definite integrals and apply this technique to solving practical problems AAM

[Reference Sheet](#)

- 23 5** The diagram shows the graph  $y = f(x)$ , where  $f(x)$  is an odd function.

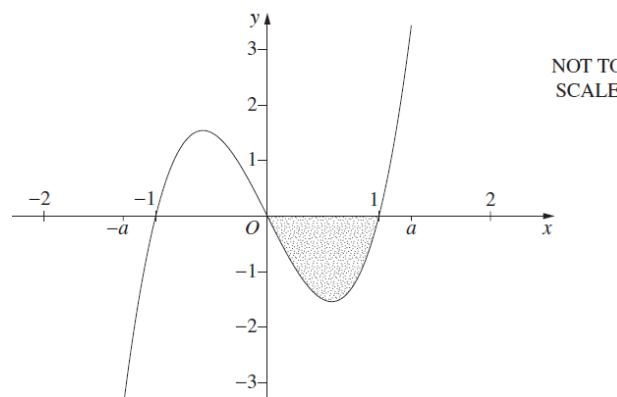
The shaded area is 1 square unit.

The number  $a$ , where  $a > 1$ , is chosen

$$\text{so that } \int_0^a f(x) dx = 0.$$

$$\text{What is the value of } \int_{-a}^1 f(x) dx?$$

- A. -1      B. 0  
C. 1      D. 3



**1** Solution

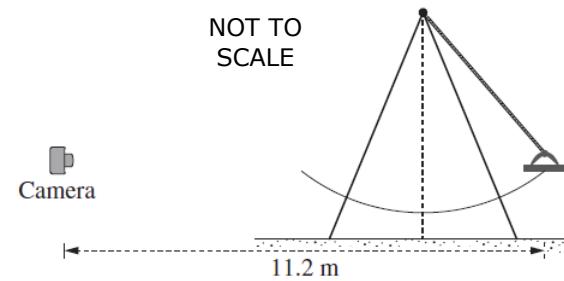


- 23 26** A camera films the motion of a swing in a park.

Let  $x(t)$  be the horizontal distance, in metres, from the camera to the seat of the swing at  $t$  seconds.

The seat is released from rest at a horizontal distance of 11.2 m from the camera.

- (a) The rate of change of  $x$  can be modelled by the equation  $\frac{dx}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right)$ .  
Find an expression for  $x(t)$ .
- (b) How many times does the swing reach the closest point to the camera during the first 10 seconds?



**Solution**



- 23 28** The curve  $y = f(x)$  is shown on the diagram.

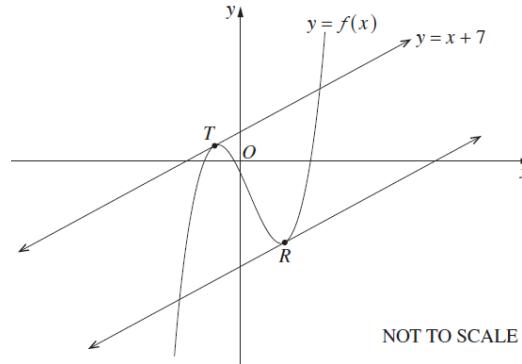
The equation of the tangent to the curve at point  $T(-1, 6)$  is  $y = x + 7$ . At a point  $R$ , another tangent parallel to the tangent at  $T$  is drawn.

The gradient function of the curve is given by

$$\frac{dy}{dx} = 3x^2 - 6x - 8.$$

Find the coordinates of  $R$ .

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**4** Solution



**23 32**

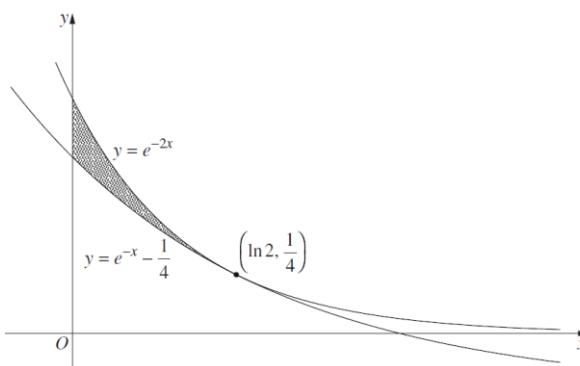
**MA** The curves  $y = e^{-2x}$  and  $y = e^{-x} - \frac{1}{4}$

intersect at exactly one point as shown in the diagram. The point of intersection has coordinates  $(\ln 2, \frac{1}{4})$ .

(Do NOT prove this.)

- (a) Show that the area bounded by the two curves and the  $y$ -axis, as shaded in the diagram, is  $\frac{1}{4} \ln 2 - \frac{1}{8}$ .

- (b) Find the values of  $k$  such that the curves  $y = e^{-2x}$  and  $y = e^{-x} + k$  intersect at two points.

**Solution****3****3**

NESA 2023 Mathematics Advanced HSC Examination

**22 8**

**MA** The graph of the even function  $y = f(x)$  is shown.

The area of the shaded region  $A$  is  $\frac{1}{2}$  and the area of the shaded region  $B$  is  $\frac{3}{2}$ .

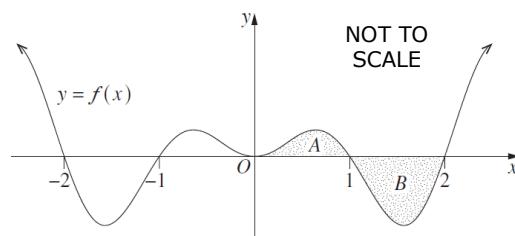
What is the value of  $\int_{-2}^2 f(x) dx$ ?

A. 4

B. 2

C. -2

D. -4

**1**

NESA 2022 Mathematics Advanced HSC Examination

**22 13**

**MA** Use two applications of the trapezoidal rule to find an approximate value of

$$\int_0^2 \sqrt{1+x^2} dx.$$

Give your answer correct to 2 decimal places.

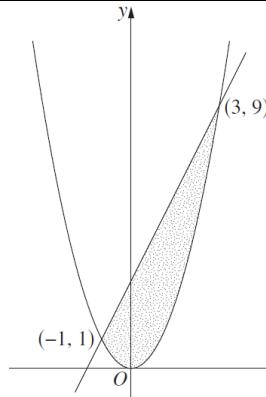
**2**

NESA 2022 Mathematics Advanced HSC Examination

**22 16**

**MA** The parabola  $y = x^2$  meets the line  $y = 2x + 3$  at the points  $(-1, 1)$  and  $(3, 9)$  as shown in the diagram.

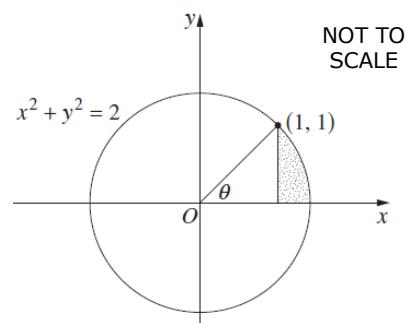
Find the area enclosed by the parabola and the line.

**3**

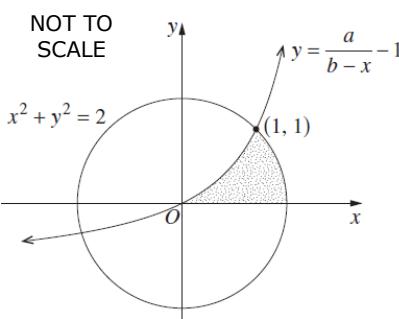
NESA 2022 Mathematics Advanced HSC Examination

- 22 28 MA** The graph of the circle  $x^2 + y^2 = 2$  is shown. The interval connecting the origin,  $O$ , and the point  $(1, 1)$  makes an angle  $\theta$  with the positive  $x$ -axis.  
 (a) By considering the value of  $\theta$ , find the exact area of the shaded region, as shown on the diagram.

Part of the hyperbola  $y = \frac{a}{b-x} - 1$  which passes through the points  $(0, 0)$  and  $(1, 1)$  is drawn with the circle  $x^2 + y^2 = 2$  as shown.  
 (b) Show that  $a = b = 2$ .  
 (c) Using parts (a) and (b), find the exact area of the region bounded by the hyperbola, the positive  $x$ -axis and the circle as shown on the diagram.

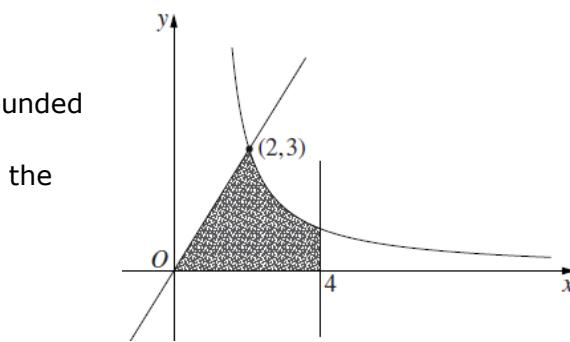


2

2  
3

NESA 2022 Mathematics Advanced HSC Examination

- 21 24 M A** The curve  $y = \frac{3}{x-1}$  intersects the line  $y = \frac{3}{2}x$  at the point  $(2, 3)$ . The region bounded by the curve  $y = \frac{3}{x-1}$ , the line  $y = \frac{3}{2}x$ , the  $x$ -axis and the line  $x = 4$  is shaded in the diagram.  
 Find the exact area of the shaded region.

3 [Solution](#)

NESA 2021 Mathematics Advanced HSC Examination

- 21 27** Kenzo has a solar powered phone charger. Its power,  $P$ , can be modelled by the function  $P(t) = 400 \sin\left(\frac{\pi}{12}t\right)$ ,  $0 \leq t \leq 12$ , where  $t$  is the number of hours after sunrise.

(a) Sketch the graph of  $P$  for  $0 \leq t \leq 12$ .

2



Power is the rate of change of energy. Hence the amount of energy,  $E$  units, generated by the solar powered phone charger from  $t = a$  to  $t = b$ , where

$0 \leq a \leq b \leq 12$  is given by  $E = \int_a^b P(t) dt$ .

2



(b) Show that  $E = \frac{4800}{\pi} \left( \cos \frac{a\pi}{12} - \cos \frac{b\pi}{12} \right)$ .

3



(c) To make a phone call, a phone battery needs at least 300 units of energy. Kenzo

woke up 3 hours after sunrise and found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery.

Find the least amount of time he needed to wait before he could make a phone call. Give your answer correct to the nearest minute.

(d) The next day, Kenzo woke up 6 hours after sunrise and again found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery.

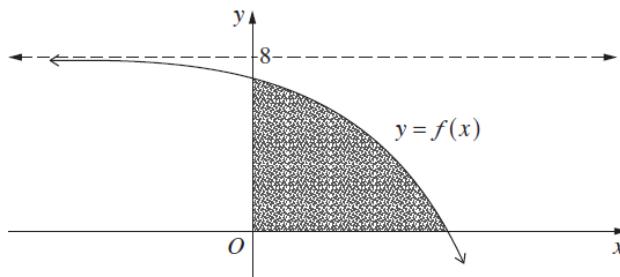
Would it take more time or less time or the same amount of time, compared to the answer in part (c), to charge his phone battery in order to make a phone call? Explain your answer by referring to the graph drawn in part (a).

NESA 2021 Mathematics Advanced HSC Examination

1



- 21 28** The region bounded by the graph of the function  $f(x) = 8 - 2^x$  and the coordinate axes is shown.

[Solution](#)

(a) Show that the exact area of the shaded region is given by  $24 - \frac{7}{\ln 2}$ .

3



(b) A new function  $g(x)$  is found by taking the graph of  $y = -f(-x)$  and translating it by 5 units to the right.

2



Sketch the graph of  $y = g(x)$  showing the  $x$ -intercept and the asymptote.

(c) Hence, find the exact value of  $\int_2^5 g(x) dx$ .

1

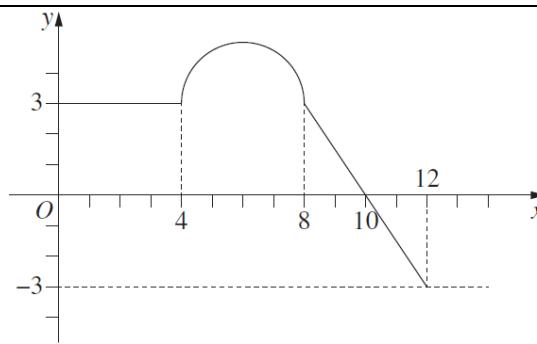


NESA 2021 Mathematics Advanced HSC Examination

- 20 7** The diagram shows the graph  $y = f(x)$ , which is made up of line segments and a semicircle.

What is the value of  $\int_0^{12} f(x) dx$ ?

- A.  $24 + 2\pi$   
B.  $24 + 4\pi$   
C.  $30 + 2\pi$   
D.  $30 + 4\pi$



NESA 2020 Mathematics Advanced HSC Examination

**1** [Solution](#)

- 20 13** **MA** Evaluate  $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ .

NESA 2020 Mathematics Advanced HSC Examination

**2** [Solution](#)

- 20 20** Kenzo is driving his car along a road while his friend records the velocity of the car,  $v(t)$ , in km/h every minute over a 5-minute period. The table gives the velocity  $v(t)$ , at time  $t$  hours.

$t$	0	$\frac{1}{60}$	$\frac{2}{60}$	$\frac{3}{60}$	$\frac{4}{60}$	$\frac{5}{60}$
$v(t)$	60	55	65	68	70	67

The distance covered by the car over the  $\frac{5}{60}$  5-minute period is given by  $\int_0^{\frac{5}{60}} v(t) dt$ .

Use the trapezoidal rule and the velocity at each of the six time values to find the approximate distance in kilometres the car has travelled in the 5-minute period. Give your answer correct to one decimal place,

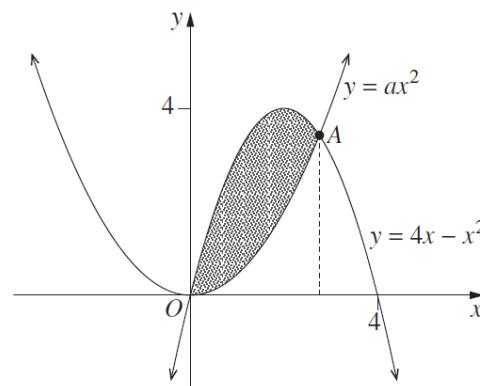
NESA 2020 Mathematics Advanced HSC Examination

**2** [Solution](#)

- 20 30** The diagram shows two parabolas  $y = 4x - x^2$  and  $y = ax^2$ , where  $a > 0$ .

The two parabolas intersect at the origin,  $O$ , and at  $A$ .

- (a) Show that the  $x$ -coordinate of  $A$  is  $\frac{4}{a+1}$ .  
(b) Find the value of  $a$  such that the shaded area is  $\frac{16}{3}$ .

[Solution](#)**2** [Solution](#)**4** [Solution](#)

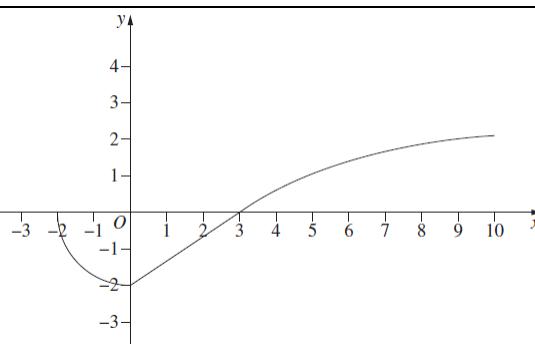
NESA 2020 Mathematics Advanced HSC Examination

- MA 18** The diagram shows a continuous function  $y = f(x)$  defined in the domain  $[-2, 10]$ .  
**SP Band 2-5**

The function consists of a quarter of a circle centred at  $(0, 0)$  with radius 2, a straight line segment and a logarithmic function  $f(x) = \ln(x - 2)$  in the domain  $[3, 10]$ .

- (a) Find the exact area bounded by the function  $y = f(x)$  and the  $x$ -axis in the domain  $[-2, 3]$ .

- (b) Hence, find the exact value of  $\int_3^{10} \ln(x - 2) dx$ , given that  $\int_{-2}^{10} f(x) dx = 8 \ln 8 - 10 - \pi$ .



2



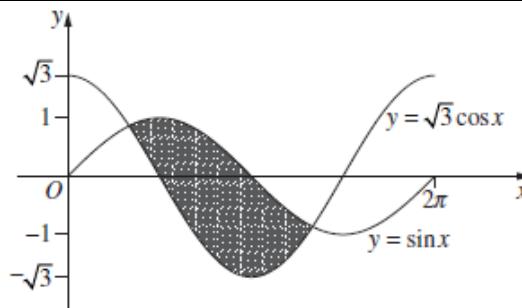
2



NESAA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 35** The diagram shows the curves  $y = \sin x$  and  $y = \sqrt{3} \cos x$ .  
**SP Band 3-6**

Find the area of the shaded region.



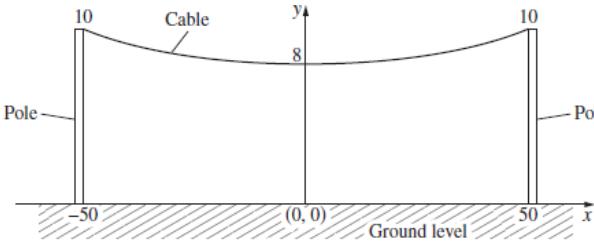
4



NESAA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 38** A cable is freely suspended between two 10 m poles, as shown. The poles are 100 m apart and the minimum height of the cable is 8 metres.  
**SP Band 3-6**  
The height of the cable is given as  $y = c(e^{kx} + e^{-kx})$ , where  $c$  and  $k$  are positive constants.

- (a) Show that the value of  $c$  is 4.

[Solution](#)

1



- (b) Use the result in part (a) to show that one value of  $k$  is  $\frac{\ln 2}{50}$ .

4



- (c) Hence find the area between the poles, the cable and the ground.

3



NESAA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 11** (a) Sketch the graph of  $y = \ln x$  in the space provided.  
**SQ Band 2-5** (b) Use the trapezoidal rule with three function values to find an approximation

$$\text{to } \int_1^3 \ln x dx.$$

1



- (c) State whether the approximation found in part (b) is greater than or less

1

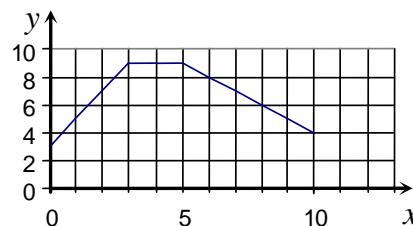


$$\text{than the exact value of } \int_1^3 \ln x dx. \text{ Justify your answer.}$$

NESAA Mathematics Advanced Sample examination materials (2019)

**TG 1** The graph represents the function  $y = f(x)$ .

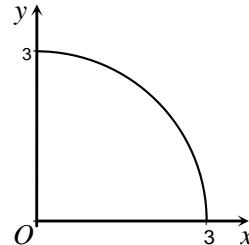
Use the graph to evaluate the integral  $\int_0^{10} f(x) dx$ .



## Solution

**TG 2** The graph represents the function  $y = g(x)$ .

Use the formula for the area of a circle to find  $\int_0^3 g(x) dx$ .



## Solution

**TG 3** The following table shows the velocity (in metres per second) of a moving object evaluated at 10-second intervals. Use the trapezoidal rule to obtain an estimate of the distance travelled by the object over the time interval  $30 \leq t \leq 70$ .

Time	30	40	50	60	70
Velocity	0	4.6	5.7	8	9.9

## Solution

**TG 4** An object is moving on the  $x$ -axis.

**07** **10** The graph shows the velocity,  $\frac{dx}{dt}$ , of the object as a function of time  $t$ .  
**M** **a** The coordinates of the point where the graph crosses the  $t$ -axis are  $(t_1, v_1)$ .

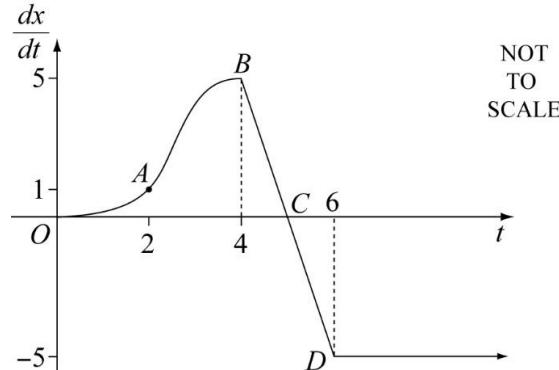
The coordinates of the points shown on the graph are  $A(2, 1)$ ,  $B(4, 5)$ ,  $C(5, 0)$  and  $D(6, -5)$ .  
The velocity is constant for  $t \geq 6$ .

(a) Use the trapezoidal rule to estimate the distance travelled between  $t = 0$  and  $t = 4$ , using 2 strips\*. \*: projectmaths

(b) The object is initially at the origin.

When is the displacement of the object decreasing?

(c) Estimate the time at which the object returns to the origin. Justify your answer.  
 (d) Sketch the displacement  $x$  as a function of time.



### **Solution**

1

**TG 5** Find the area bounded by the graph of  $y = 3x^2 + 6$ , the  $x$ -axis, and the lines  $x = -2$  and  $x = 2$ .

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

TG 6

(a) Show that  $\int_{-2}^2 x^3 dx = 0$ .

(b) Explain why this is not representative of the area bounded by the graph of  $y = x^3$ , the  $x$ -axis, and the lines  $x = -2$  and  $x = 2$ .

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

**TG 7** Find the area bounded by the line  $y = 5$  and the curve  $y = x^2 - 4$ .

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

**TG 8**Given  $Q(x) = \int 8x^3 dx$ , and  $Q(0) = 5$ , determine  $Q(x)$ .[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Calculus

**TG 9** Sketch the region bounded by the curve  $y = x^2$  and the lines  $y = 4$ ,  $y = 9$ .[Solution](#)

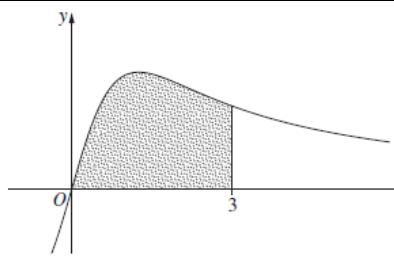
Evaluate the area of this region.



NESA Mathematics Advanced Year 12 Topic Guide: Calculus

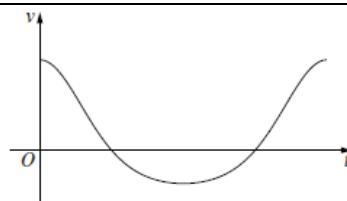
**19 11 M e** Evaluate  $\int_0^1 \frac{1}{(3x+2)^2} dx$ .**3** [Solution](#)

NESA 2019 Mathematics HSC Examination

**19 12 M d** The diagram shows the graph of  $y = \frac{3x}{x^2 + 1}$ .**2** [Solution](#)The region enclosed by the graph, the  $x$ -axis and the line  $x = 3$  is shaded.

NESA 2019 Mathematics HSC Examination

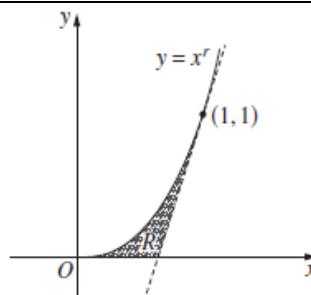
Calculate the exact value of the area of the shaded region.

**19 16 M b** A particle moves in a straight line, starting at the origin.**3** [Solution](#)Its velocity,  $v$  ms $^{-1}$ , is given by  $v = e^{\cos t} - 1$ , where  $t$  is in seconds.

The diagram shows the graph of the velocity against time.

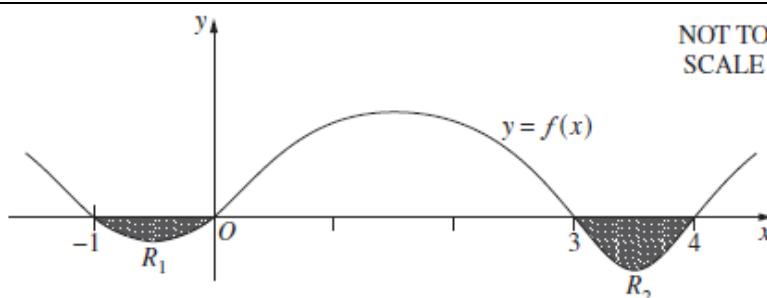
Using one application of *Trapezoidal rule\**, estimate theposition of the particle when it first comes to rest. Give your answer correct to two decimal places. \**Changed from Simpson's rule by projectmaths*.

NESA 2019 Mathematics HSC Examination

**19 16 M c** The diagram shows the region  $R$ , bounded by the curve  $y = x^r$ , where  $r \geq 1$ , the  $x$ -axis and the tangent to the curve at the point  $(1, 1)$ .[Solution](#)(i) Show that the tangent to the curve at  $(1, 1)$  meets the  $x$ -axis at  $(\frac{r-1}{r}, 0)$ .**2**(ii) Using the result of part (i), or otherwise, show that the area of the region  $R$  is  $\frac{r-1}{2r(r+1)}$ .**2**(iii) Find the exact value of  $r$  for which the area of  $R$  is a maximum.**3**

NESA 2019 Mathematics HSC Examination

- 18 7 M** The diagram shows the graph of  $y = f(x)$  with intercepts at  $x = -1, 0, 3$  and  $4$ . The area of shaded region  $R_1$  is  $2$ . The area of shaded region  $R_2$  is  $3$ .

**1** [Solution](#)

It is given that  $\int_0^4 f(x) dx = 10$ . What is the value of  $\int_{-1}^3 f(x) dx$ ?

- A. 5      B. 9      C. 11      D. 15

NESA 2018 Mathematics HSC Examination

- 18 10 M** A trigonometric function  $f(x)$  satisfies the condition  $\int_0^\pi f(x) dx \neq \int_\pi^{2\pi} f(x) dx$ .

**1** [Solution](#)

Which function could  $f(x)$  be?

- A.  $f(x) = \sin(2x)$       B.  $f(x) = \cos(2x)$       C.  $f(x) = \sin\left(\frac{x}{2}\right)$       D.  $f(x) = \cos\left(\frac{x}{2}\right)$

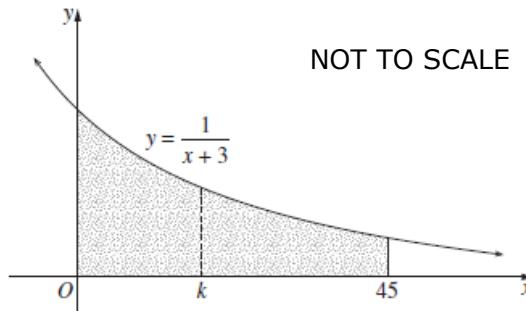
NESA 2018 Mathematics HSC Examination

- 18 11 M e** Evaluate  $\int_0^3 e^{5x} dx$ .

**2** [Solution](#)

NESA 2018 Mathematics HSC Examination

- 18 15 M b** The diagram shows the region bounded by the curve  $y = \frac{1}{x+3}$  and the lines  $x = 0$ ,  $x = 45$  and  $y = 0$ . The region is divided into two parts of equal area by the line  $x = k$ , where  $k$  is a positive integer. What is the value of the integer  $k$ , given that the two parts have equal area?

**3** [Solution](#)

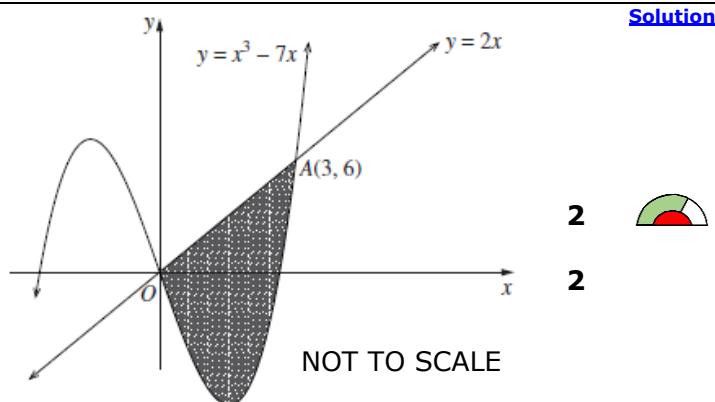
NESA 2018 Mathematics HSC Examination

- 18 15 M c** The shaded region is enclosed by the curve  $y = x^3 - 7x$  and the line  $y = 2x$  as shown in the diagram. The line  $y = 2x$  meets the curve  $y = x^3 - 7x$  at  $O(0, 0)$  and  $A(3, 6)$ . DO NOT prove this.

- Use integration to find the area of the shaded region.
- Verify that one application of Simpson's rule gives the exact area of the shaded region. *Not in Maths A*

The point  $P$  is chosen on the curve  $y = x^3 - 7x$  so that the tangent at  $P$  is parallel to the line  $y = 2x$  and the  $x$ -coordinate of  $P$  is positive.

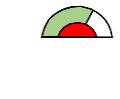
- Show that the coordinates of  $P$  are  $(\sqrt{3}, -4\sqrt{3})$ .
- Find the area of  $\triangle OAP$ . *Not in Maths A*



2



2



NESA 2018 Mathematics HSC Examination

**17 14 M b**

- Find the exact value of  $\int_0^{\frac{\pi}{3}} \cos x \, dx$ .

1



- Using *Trapezoidal rule\** with three function values, find an approximation to the

2



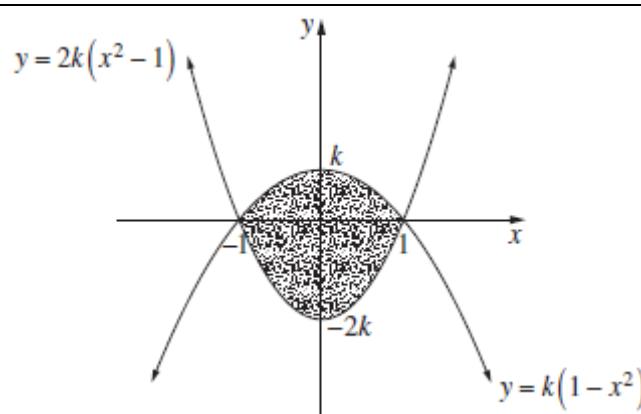
integral  $\int_0^{\frac{\pi}{3}} \cos x \, dx$ , leaving your answer in terms of  $\pi$  and  $\sqrt{3}$ .

\*Changed from Simpson's rule by projectmaths.

(iii) deleted

NESA 2017 Mathematics HSC Examination

- 17 14 M d** The shaded region shown is enclosed by two parabolas, each with  $x$ -intercepts at  $x = -1$  and  $x = 1$ . The parabolas have equations  $y = 2k(x^2 - 1)$  and  $y = k(1 - x^2)$ , where  $k > 0$ . Given that the area of the shaded region is 8, find the value of  $k$ .



3



- 16 9 M** What is the value of  $\int_{-3}^2 |x + 1| \, dx$ ?

1



- (A)  $\frac{5}{2}$       (B)  $\frac{11}{2}$       (C)  $\frac{13}{2}$       (D)  $\frac{17}{2}$

NESA 2016 Mathematics HSC Examination

- 16 11 M d** Evaluate  $\int_0^1 (2x + 1)^3 \, dx$ .

2



NESA 2016 Mathematics HSC Examination

**16 M 12** (i) Differentiate  $y = xe^{3x}$ .

**1** [Solution](#)

(ii) Hence find the exact value of  $\int_0^2 e^{3x}(3+9x) dx$ .

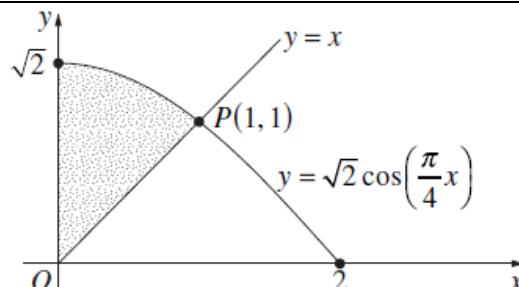
**2** 

NESA 2016 Mathematics HSC Examination

**16 M 13** The curve  $y = \sqrt{2} \cos\left(\frac{\pi}{4}x\right)$  meets the line  $y = x$  at  $P(1, 1)$ , as shown in the diagram.

**3** [Solution](#)

Find the exact value of the shaded area.

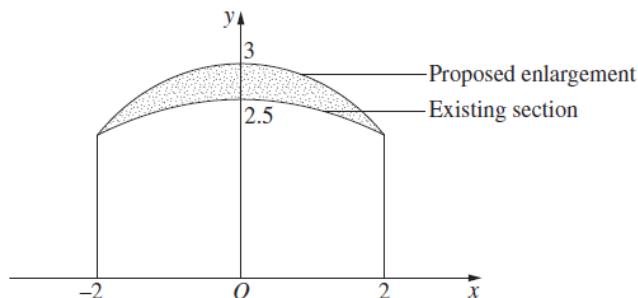


NESA 2016 Mathematics HSC Examination

**16 M 14** The diagram shows the cross-section of a tunnel and a proposed enlargement.

**3** [Solution](#)

a



The heights, in metres, of the existing section at 1 metre intervals are shown in Table A.

Table A: Existing heights

$x$	-2	-1	0	1	2
$y$	2	2.38	2.5	2.38	2

The heights, in metres, of the proposed enlargement are shown in Table B.

Table B: Proposed heights

$x$	-2	-1	0	1	2
$y$	2	2.78	3	2.78	2

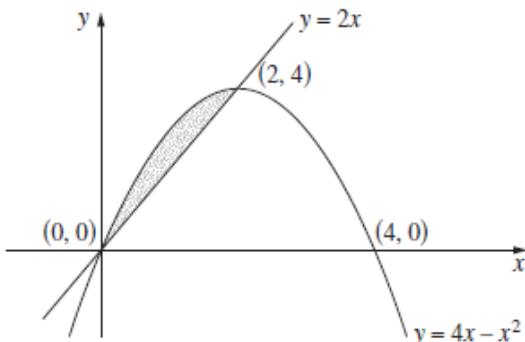
Use the *Trapezoidal rule\** with the measurements given to calculate the approximate increase in area. \**Changed from Simpson's rule by projectmaths.*

NESA 2016 Mathematics HSC Examination

**15 M 7** The diagram shows the parabola  $y = 4x - x^2$  meeting the line  $y = 2x$  at  $(0, 0)$  and  $(2, 4)$ . Which expression gives the area of the shaded region bounded by the parabola and the line?

**1** 

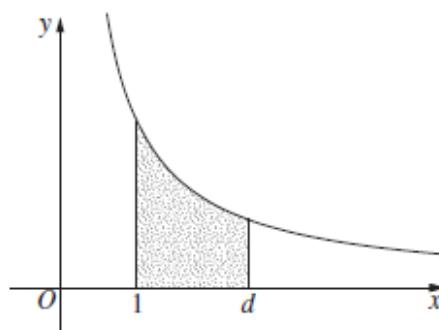
- (A)  $\int_0^2 x^2 - 2x \, dx$       (B)  $\int_0^2 2x - x^2 \, dx$   
 (C)  $\int_0^4 x^2 - 2x \, dx$       (D)  $\int_0^4 2x - x^2 \, dx$



NESA 2015 Mathematics HSC Examination

- 15 10** The diagram shows the area under the curve  $y = \frac{2}{x}$  from  $x = 1$  to  $x = d$ . What value of  $d$  makes the shaded area equal to 2?

- (A)  $e$       (B)  $e + 1$   
 (C)  $2e$       (D)  $e^3$



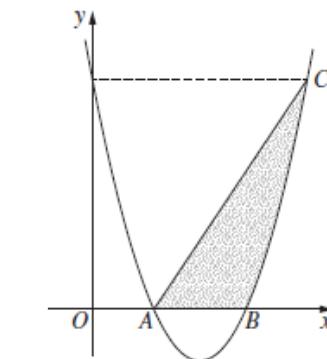
NESAA 2015 Mathematics HSC Examination

**1** [Solution](#)

- 15 11** **M g** Evaluate  $\int_0^{\frac{\pi}{4}} \cos 2x \, dx$ .

**2** [Solution](#)

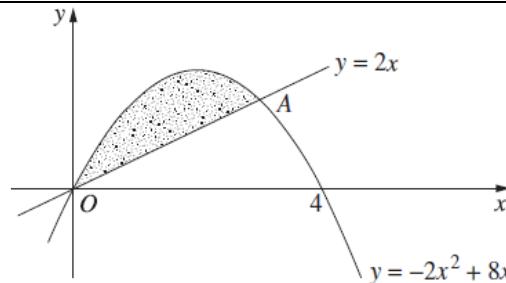
- 15 16** **M a** The diagram shows the curve with equation  $y = x^2 - 7x + 10$ . The curve intersects the  $x$ -axis at points  $A$  and  $B$ . The point  $C$  on the curve has the  $y$ -coordinate as the  $y$ -intercept of the curve.
- Find the  $x$ -coordinates of points  $A$  and  $B$ .
  - Write down the coordinates of  $C$ .
  - Evaluate  $\int_0^2 (x^2 - 7x + 10) \, dx$ .
  - Hence, or otherwise, find the area of the shaded region.

[Solution](#)**1****1****2**

- 14 11** **M e** Evaluate  $\int_0^{\frac{\pi}{2}} \sin \frac{x}{2} \, dx$ .

**3** [Solution](#)

- 14 12** **M d** The parabola  $y = -2x^2 + 8x$  and the line  $y = 2x$  intersect at the origin and at the point  $A$ .
- Find the  $x$ -coordinate of the point  $A$ .
  - Calculate the area enclosed by the parabola and the line.

[Solution](#)**1****3**

- 14 16** **M a** Use the *Trapezoidal rule\** with five function values to show that

**3** [Solution](#)

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec dx \approx \frac{\pi}{6} [3 + \frac{4}{\sqrt{3}}]. \quad *Changed from Simpson's rule by projectmaths.$$

NESAA 2014 Mathematics HSC Examination

- 13 11** **M f** Evaluate  $\int_0^1 \frac{x^2}{x^3 + 1} \, dx$ .

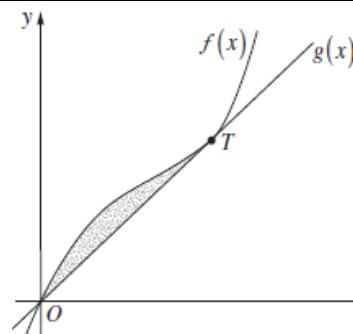
**3** [Solution](#)

NESAA 2013 Mathematics HSC Examination

- 13 13** The diagram shows the graphs of the functions  
**M b**  $f(x) = 4x^3 - 4x^2 + 3x$  and  $g(x) = 2x$ .

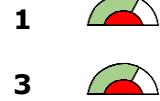
The graphs meet at  $O$  and at  $T$ .

- Find the  $x$ -coordinate of  $T$ .
- Find the area of the shaded region between the graphs of the functions  $f(x)$  and  $g(x)$ .



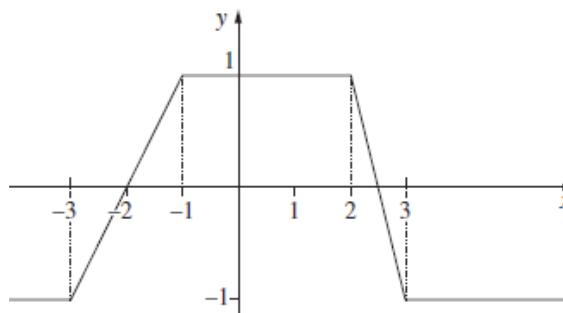
NESA 2013 Mathematics HSC Examination

[Solution](#)



- 13 14** The diagram shows the graph  $y = f(x)$ .  
**M d**

What is the value of  $a$ , where  $a > 0$ ,  
so that  $\int_{-a}^a f(x) dx = 0$ .



NESA 2013 Mathematics HSC Examination

[Solution](#)



- 12 9** What is the value of  $\int_1^4 \frac{1}{3x} dx$ ?

- (A)  $\frac{1}{3} \ln 3$       (B)  $\frac{1}{3} \ln 4$       (C)  $\ln 9$       (D)  $\ln 12$

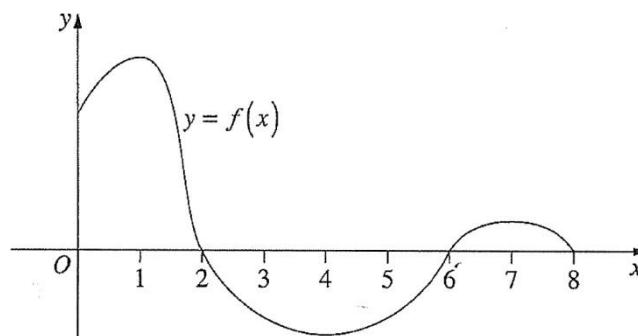
NESA 2012 Mathematics HSC Examination

[Solution](#)



- 12 10** The graph of  $y = f(x)$  has been drawn to scale for  $0 \leq x \leq 8$ .  
**M** Which of the following integrals has the greatest value?

- (A)  $\int_0^1 f(x) dx$       (B)  $\int_0^2 f(x) dx$   
(C)  $\int_0^7 f(x) dx$       (D)  $\int_0^8 f(x) dx$



NESA 2012 Mathematics HSC Examination

[Solution](#)



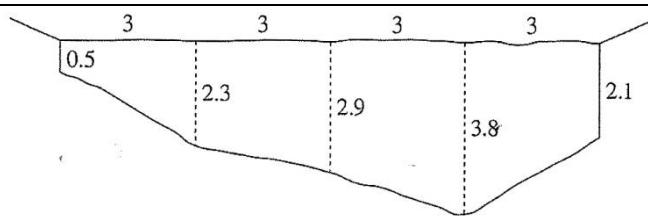
- 12 11** Find  $\int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2} dx$ .

NESA 2012 Mathematics HSC Examination

[Solution](#)



- 12 12 M d** At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3 metre intervals. The cross-section is shown.

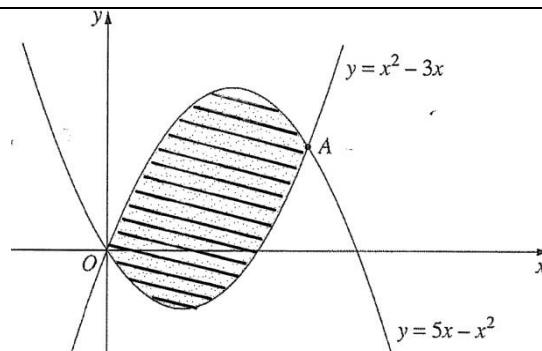
[Solution](#)**3**

- (i) Use *Trapezoidal Rule\** with the five depth measurements to calculate the approximate area of the cross-section.  
*\*Changed from Simpson's rule by projectmaths.*

- (ii) The river flows at 0.4 metres per second. Calculate the approximate volume of water flowing through the cross-section in 10 seconds.

NESA 2012 Mathematics HSC Examination

- 12 13 M b** The diagram shows the parabolas  $y = 5x - x^2$  and  $y = x^2 - 3x$ . The parabolas intersect at the origin  $O$  and the point  $A$ . The region between the two parabolas is shaded.

[Solution](#)**1**

- (i) Find the  $x$ -coordinate of the point  $A$ .
- (ii) Find the area of the shaded region.

NESA 2012 Mathematics HSC Examination

- 11 4b M** Evaluate  $\int_{e}^{e^3} \frac{5}{x} dx$ .

**2**

NESA 2011 Mathematics HSC Examination

- 11 5c M** The table gives the speed  $v$  of a jogger at time  $t$  in minutes over a 20-minute period. The speed  $v$

$t$	0	5	10	15	20
$v$	173	81	127	195	168

**3**

is measured in metres per minute, in intervals of 5 minutes. The distance covered by the jogger over the 20-minute period is given by  $\int_0^{20} v dt$ .

Use *Trapezoidal Rule\** and the speed at each of the five time values to find the approximate distance the jogger covers in the 20-minute period.

*\*Changed from Simpson's rule by projectmaths.*

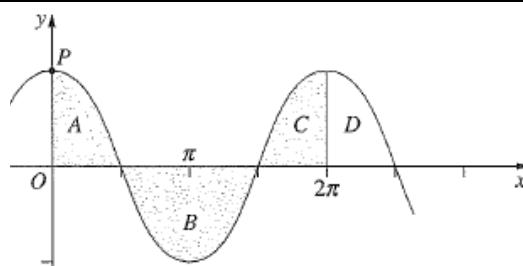
NESA 2011 Mathematics HSC Examination

- 11 6c** The diagram shows the graph  
M  $y = 2 \cos x$ .

[Solution](#)

- (i) State the coordinates of  $P$ .  
(ii) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} 2 \cos x \, dx.$$

1  
2

- (iii) Indicate which area in the diagram,  $A$ ,  $B$ ,  $C$  or  $D$ , is represented by the

1



$$\text{integral } \int_{\frac{3\pi}{2}}^{2\pi} 2 \cos x \, dx.$$

- (iii) Using parts (ii) and (iii), or otherwise, find the area of the region bounded by the curve  $y = 2 \cos x$  and the  $x$ -axis, between  $x = 0$  and  $x = 2\pi$ .

1



- (v) Using the parts above, write down the value of  $\int_{\frac{\pi}{2}}^{2\pi} 2 \cos x \, dx$ .

1



$$\int_{\frac{\pi}{2}}^{2\pi} 2 \cos x \, dx.$$

NESA 2011 Mathematics HSC Examination

- 10 2e**  
M

Given that  $\int_0^6 (x + k) \, dx = 30$ , and  $k$  is a constant, find the value of  $k$ .

2



NESA 2010 Mathematics HSC Examination

- 10 3b**  
M

- (i) Sketch the curve  $y = \ln x$ .  
(ii) Use the trapezoidal rule with three function values to find an approximation to  $\int_1^3 \ln x \, dx$ .  
(iii) State whether the approximation found in (ii) is greater than or less than the exact value of  $\int_1^3 \ln x \, dx$ . Justify your answer.

1  
2

1



NESA 2010 Mathematics HSC Examination

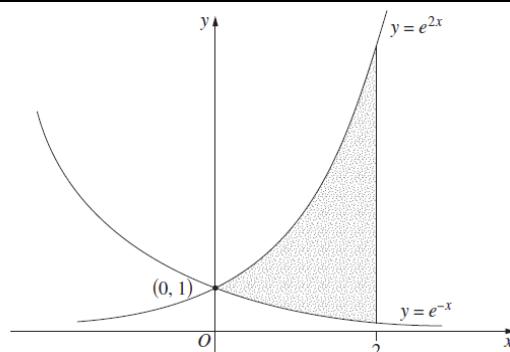
- 10 4b**  
M

The curves  $y = e^{2x}$  and  $y = e^{-x}$  intersect at the point  $(0, 1)$  as shown in the diagram.

3



Find the exact area enclosed by the curves and the line  $x = 2$ .



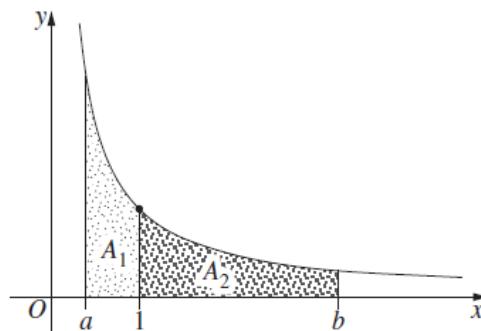
NESA 2010 Mathematics HSC Examination

**10 5c****M**

The diagram shows the curve  $y = \frac{1}{x}$ ,

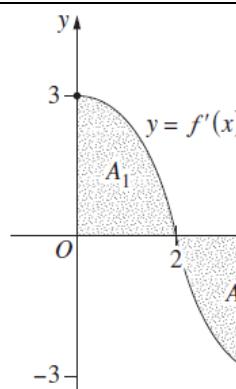
for  $x > 0$ .

The area under the curve between  $x = a$  and  $x = 1$  is  $A_1$ . The area under the curve between  $x = 1$  and  $x = b$  is  $A_2$ .  
The area  $A_1$  and  $A_2$  are each equal to 1 square unit.  
Find the values of  $a$  and  $b$ .

**3**[Solution](#)**10 9b****M**

Let  $y = f(x)$  be a function defined for  $0 \leq x \leq 6$ , with  $f(0) = 0$ . The diagram shows the graph of the derivative of  $f$ ,  $y = f'(x)$ . The shaded region  $A_1$  has area 4 square units. The shaded region  $A_2$  has area 4 square units.

- For which values of  $x$  is  $f(x)$  increasing?
- What is the maximum value of  $f(x)$ ?
- Find the value of  $f(6)$ .
- Draw a graph of  $y = f(x)$  for  $0 \leq x \leq 6$ .

[Solution](#)**09 2b****M**

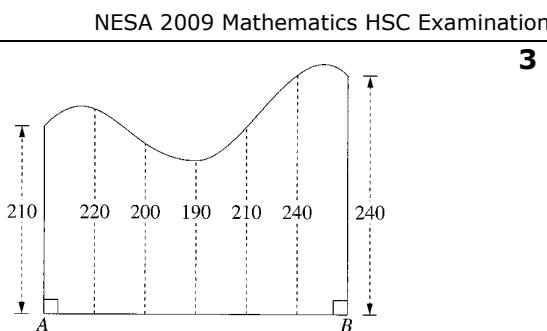
(iii) Find  $\int_1^4 x^2 + \sqrt{x} \, dx$ .

**3**[Solution](#)**09 3d****M**

The diagram shows a block of land and its dimensions, in metres. The block of land is bounded on one side by a river. Measurements are taken perpendicular to the line  $AB$ , from  $AB$  to the river, at equal intervals of 50 m.

Use Trapezoidal Rule\* with six subintervals to find an approximation to the area of the block of land.

\*Changed from Simpson's rule by projectmaths.

**3**[Solution](#)**08 2c****M**

(ii) Evaluate  $\int_0^{\frac{\pi}{12}} \sec^2 3x \, dx$ .

**3**[Solution](#)**08 3b****M**

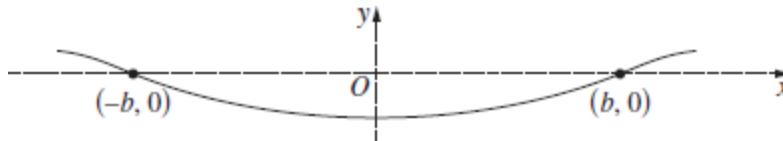
(i) Differentiate  $\log_e(\cos x)$  with respect to  $x$ .

(ii) Hence, or otherwise, evaluate  $\int_0^{\frac{\pi}{4}} \tan x \, dx$ .

**2**[Solution](#)

**08 4c** Consider the parabola  $x^2 = 8(y - 3)$ .**M** (iv) Calculate the area bounded by the parabola and the line  $y = 5$ .**3**

NESA 2008 Mathematics HSC Examination

**08 9c** A beam is supported at  $(-b, 0)$  and  $(b, 0)$  as shown in the diagram.**M**[Solution](#)

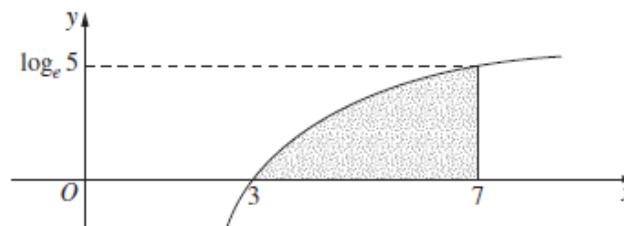
It is known that the shape formed by the beam has equation  $y = f(x)$ , where  $f(x)$  satisfies  $f''(x) = k(b^2 - x^2)$  ( $k$  is a positive constant) and  $f'(-b) = -f'(b)$ .

(i) Show that  $f'(x) = k(b^2x - \frac{x^3}{3})$ **2**(ii) How far is the beam below the  $x$ -axis at  $x = 0$ ?**2**

NESA 2008 Mathematics HSC Examination

**08 10** In the diagram, the shaded region is**M a** bounded by  $y = \log_e(x - 2)$ ,  
the  $x$ -axis and the line  $x = 7$ .**5**

Find the exact value of the area of the shaded region.



NESA 2008 Mathematics HSC Examination

**07 2b** ii. Evaluate  $\int_1^4 \frac{8}{x^2} dx$ .**3**

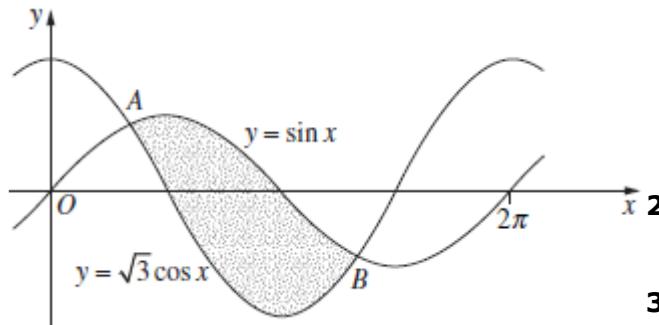
NESA 2007 Mathematics HSC Examination

**07 7b** The diagram shows the graphs of**M**  $y = \sqrt{3} \cos x$  and  $y = \sin x$ .The first two points of intersection to the right of the  $y$ -axis are labelled  $A$  and  $B$ .[Solution](#)

(i) Solve the equation

 $\sqrt{3} \cos x = \sin x$  to find the  $x$ -coordinates of  $A$  and  $B$ .

(ii) Find the area of the shaded region in the diagram.

**3**

NESA 2007 Mathematics HSC Examination

# Year 12: Financial Mathematics

## M1.1 Modelling Investments and Loans



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- solve compound interest problems involving financial decisions, including a home loan, a savings account, a car loan or superannuation **AAM**
- identify an annuity (present or future value) as an investment account with regular, equal contributions and interest compounding at the end of each period, or a single-sum investment from which regular, equal withdrawals are made
- use technology to model an annuity as a recurrence relation and investigate (numerically or graphically) the effect of varying the interest rate or the amount and frequency of each contribution or a withdrawal on the duration and/or future or present value of the annuity
- use a table of interest factors to perform annuity calculations, eg calculating the present or future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity

STANDARD  
2

[Reference Sheet](#)

**23 15 MA** A table of future value interest factors for an annuity of \$1 is shown.

Period \ Rate	1.5%	3%	4.5%	6%
5	5.152	5.309	5.471	5.637
10	10.703	11.464	12.288	13.181
20	23.124	26.870	31.371	36.786
40	54.268	75.401	107.030	154.762

- (a) Micky wants to save \$450 000 over the next 10 years.

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

- (b) Instead, Micky decides to contribute \$8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly.

How much will Micky have at the end of 10 years?

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

[Solution](#)

**2**



[Solution](#)

**3**



**23 29 MS 2** The table shows monthly repayments for each \$1000 borrowed.

- (a) A couple borrows \$520 000 to buy a house at 8% per annum over 25 years.

How much does the couple repay in total for this loan?

- (b) Chris borrows some money at 7% per annum. Chris will repay

the loan over 15 years, paying \$3596 per month.

How much money does Chris borrow?

**Monthly repayment table**

Principal and Interest per \$1000 borrowed

Interest rate (per annum)	Term of loan (years)					
	5	10	15	20	25	30
6.5%	19.57	11.35	8.71	7.46	6.75	6.32
7.0%	19.80	11.61	8.99	7.75	7.07	6.65
7.5%	20.04	11.87	9.27	8.06	7.39	6.99
8.0%	20.28	12.13	9.56	8.36	7.72	7.34

**3**



**1**



NESA 2023 Mathematics Standard 2 HSC Examination

**22 21** Eli is choosing between two investment options.

**MA**

Option 1: Depositing a single amount of \$40 000 today, earning interest of 1.2% per annum, compounded monthly.

Option 2: Depositing \$1000 at the end of each quarter, earning interest of 2.4% per annum, compounded quarterly.

A table of future value interest factors for an annuity of \$1 is shown.

- (a) What is the value of Eli's investment after 10 years using Option 1?  
 (b) What is the difference between the future

values after 10 years using Option 1 and Option 2?

N	r	Interest rate per period as a decimal					
		0.002	0.006	0.020	0.024	0.060	0.240
10		10.09048	10.27437	10.94972	11.15211	13.18079	31.64344
20		20.38460	21.18211	24.29737	25.28909	36.78559	303.60062
30		30.88646	32.76227	40.56808	43.20983	79.05819	2640.91639
40		41.60026	45.05630	60.40198	65.92708	154.76197	22 728.80260

**Solution****2****2****3****Solution**

**21 25** A table of future value interest factors for an annuity of \$1 is shown.

Simone deposits \$1000 into a savings account at the end of each year for 8 years.

The interest rate for these 8 years is 0.75% per annum, compounded annually.

After the 8th deposit, Simone stops making deposits but leaves the money in the savings account. The money in her savings account then earns interest at 1.25% per annum, compounded annually, for a further two years.

Find the amount of money in Simone's savings account at the end of ten years.

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**20 37** Wilma deposited a lump sum into a new bank account which earns 2% per annum compound interest.

Present value interest factors for an annuity of \$1 for various interest rates ( $r$ ) and number of periods ( $N$ ) are given in the table.

Wilma was able to make the following withdrawals from this account.

- \$1000 at the end of each year for twenty years (starting one year after the account is opened)
- \$3000 each year for ten years starting 21 years after the account is opened.

Calculate the minimum sum Wilma must have deposited when she opened the new account.

Table of future value interest factors

Number of periods	Interest rate per period				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

**3****Solution**

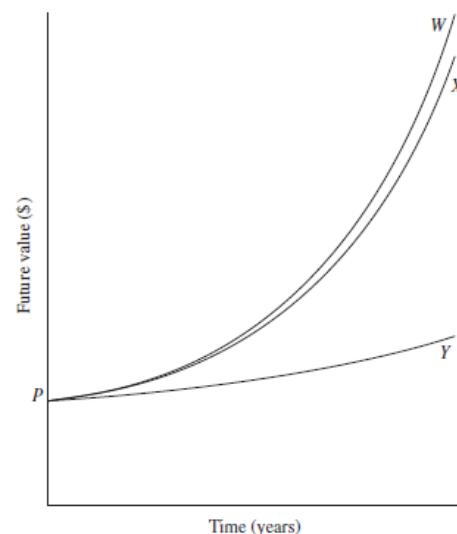
Table of present value interest factors

N	r	Interest rate per period as a decimal			
		0.01	0.015	0.02	0.025
10		9.471	9.222	8.983	8.752
20		18.046	17.169	16.351	15.589
30		25.808	24.016	22.396	20.930

**Solution**

- MA 8** The graphs show the future values over time of  $P$ , invested at three different rates of compound interest.  
Which of the following correctly identifies each graph?

- 19 MS 2 13**
- $W$ : 5% pa, compounding annually  
 $X$ : 10% pa, compounding annually  
 $Y$ : 10% pa, compounding quarterly
  - $W$ : 5% pa, compounding annually  
 $X$ : 10% pa, compounding quarterly  
 $Y$ : 10% pa, compounding annually
  - $W$ : 10% pa, compounding quarterly  
 $X$ : 10% pa, compounding annually  
 $Y$ : 5% pa, compounding annually
  - $W$ : 10% pa, compounding annually  
 $X$ : 10% pa, compounding quarterly  
 $Y$ : 5% pa, compounding annually

**1** [Solution](#)NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

- MA SP Band 2-5 34** The table shows the future values of an annuity of \$1 for different interest rates for 4, 5 and 6 years. The contributions are made at the end of each year.
- 19 MS 2 42** An annuity account is opened and contributions of \$2000 are made at the end of each year for 7 years.

For the first 6 years, the interest rate is 4% per annum, compounded annually.  
For the 7<sup>th</sup> year, the interest rate increases to 5% per annum, compounded annually.

Calculate the amount in the account immediately after the 7<sup>th</sup> contribution is made.

NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

Years	Future value of an annuity of \$1			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

**3** [Solution](#)

- TG 1** A spreadsheet is used to calculate values in a table of loan repayments.  
Calculate the row for  $n = 5$  in the spreadsheet.

Loan table				
N	Principal ( $P$ )	Interest ( $I$ )	This table assumes the same number of days in each month,	
			1%	2%
			4.060	4.122
1	50 000	\$416.67	50 416.67	49 816.67
2	49 816.67	\$415.14	50 231.81	49 631.81
3	49 631.81	\$413.60	50 045.40	49 445.40
4	49 445.40	\$412.05	49 857.45	49 257.45
5				

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial Mathematics

- TG 2** An amount of \$3000 is invested and compounded annually at 5%. Use the table of compounded values of \$1 to find the value of the investment after three years.

Periods	Compounded values of \$1				
	Interest rate per period	1%	5%	10%	15%
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.464	1.749	2.074

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial Mathematics

<b>TG 3</b>	Grandparents wish to save \$10 000 for their grandchild's university expenses, and to have this amount available in eight years' time. Calculate the single sum they need to invest at 5% pa compounded annually.	<a href="#">Solution</a>																																																								
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESAA Mathematics Standard 2 Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 4</b>	Determine the single sum to be deposited if \$10 000 is required in five years' time and a rate of 3% pa, compounded quarterly, is available.	<a href="#">Solution</a>																																																								
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESAA Mathematics Standard 2 Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 5</b>	A principal of \$1000 is invested for three years at an interest rate of 5.6% pa compounded half-yearly. Determine how much needs to be invested to achieve the same interest if the interest rate was 3.5% pa compounded monthly.	<a href="#">Solution</a>																																																								
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESAA Mathematics Standard 2 Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 6</b>	It is predicted that a particular painting will appreciate at a rate of 5% per annum. Calculate its predicted value in 2020 if it was purchased in 2010 for \$48 000.	<a href="#">Solution</a>																																																								
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESAA Mathematics Standard 2 Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 7</b>	An amount of \$2000 is invested at an interest rate of 7% per annum. Find the value of the investment after three years if the interest is compounded	<a href="#">Solution</a>																																																								
	(a) annually (b) quarterly (c) monthly.																																																									
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 8</b>	The value of an asset, currently priced at \$150 000, is expected to increase by 12% per annum. After how many years will it be expected to be worth at least \$1.2 million?	<a href="#">Solution</a>																																																								
	NESAA Mathematics Advanced Year 12 Topic Guide: Financial mathematics																																																									
<b>TG 9</b>	Use the table of future value interest factors to find the future value of an annuity of \$1200 per year for three years at 5% per annum.	<p><i>Table of future value interest factors</i></p> <table border="1"> <thead> <tr> <th rowspan="2"><i>Period</i></th> <th colspan="5"><i>Interest rate per period</i></th> </tr> <tr> <th>1%</th> <th>2%</th> <th>3%</th> <th>4%</th> <th>5%</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1.0000</td> <td>1.0000</td> <td>1.0000</td> <td>1.0000</td> <td>1.0000</td> </tr> <tr> <td>2</td> <td>2.0100</td> <td>2.0200</td> <td>2.0300</td> <td>2.0400</td> <td>2.0500</td> </tr> <tr> <td>3</td> <td>3.0301</td> <td>3.0604</td> <td>3.0909</td> <td>3.1216</td> <td>3.1525</td> </tr> </tbody> </table> <p>NESAA Mathematics Advanced Year 12 Topic Guide: Financial Mathematics</p>	<i>Period</i>	<i>Interest rate per period</i>					1%	2%	3%	4%	5%	1	1.0000	1.0000	1.0000	1.0000	1.0000	2	2.0100	2.0200	2.0300	2.0400	2.0500	3	3.0301	3.0604	3.0909	3.1216	3.1525																											
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<b>TG 10</b>	Use the table to calculate the monthly repayment for a car loan of \$8000 at 9% per annum for four years.	<p><i>Table of present value interest factors</i></p> <table border="1"> <thead> <tr> <th><i>r</i></th> <th>0.0060</th> <th>0.0065</th> <th>0.0070</th> <th><b>0.0075</b></th> <th>0.0080</th> <th>0.0085</th> </tr> <tr> <th><i>n</i></th> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </thead> <tbody> <tr> <td>45</td> <td>39.33406</td> <td>38.90738</td> <td>38.48712</td> <td>38.07318</td> <td>37.66545</td> <td>37.26383</td> </tr> <tr> <td>46</td> <td>40.09350</td> <td>39.64965</td> <td>39.21263</td> <td>38.78231</td> <td>38.35859</td> <td>37.94133</td> </tr> <tr> <td>47</td> <td>40.84841</td> <td>40.38714</td> <td>39.93310</td> <td>39.48617</td> <td>39.04622</td> <td>38.61311</td> </tr> <tr> <td><b>48</b></td> <td>41.59882</td> <td>41.11986</td> <td>40.64856</td> <td><b>40.18478</b></td> <td>39.72839</td> <td>39.27924</td> </tr> <tr> <td>49</td> <td>42.34475</td> <td>41.84785</td> <td>41.35905</td> <td>40.87820</td> <td>40.40515</td> <td>39.93975</td> </tr> <tr> <td>50</td> <td>43.08623</td> <td>42.57113</td> <td>42.06459</td> <td>41.56645</td> <td>41.07653</td> <td>40.59470</td> </tr> </tbody> </table> <p>NESAA Mathematics Advanced Year 12 Topic Guide: Financial Mathematics</p>	<i>r</i>	0.0060	0.0065	0.0070	<b>0.0075</b>	0.0080	0.0085	<i>n</i>							45	39.33406	38.90738	38.48712	38.07318	37.66545	37.26383	46	40.09350	39.64965	39.21263	38.78231	38.35859	37.94133	47	40.84841	40.38714	39.93310	39.48617	39.04622	38.61311	<b>48</b>	41.59882	41.11986	40.64856	<b>40.18478</b>	39.72839	39.27924	49	42.34475	41.84785	41.35905	40.87820	40.40515	39.93975	50	43.08623	42.57113	42.06459	41.56645	41.07653	40.59470
<i>r</i>	0.0060	0.0065	0.0070	<b>0.0075</b>	0.0080	0.0085																																																				
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- MS 4** The table gives the present value interest factors for an annuity of \$1 per period, for various interest rates ( $r$ ) and number of periods ( $N$ ). A man plans to invest \$200 each month for 6 years. His investment will earn interest per month at the rate of 9.6% per annum.

Based on the information in the table, what is the present value of this annuity?

- A. \$10 395.24      B. \$10 914.19      C. \$14 400.00      D. \$15 782.40

		Table of present value interest factors				
<i>N</i>	<i>r</i>	Interest rate per period (as a decimal)				
		0.0075	0.0080	0.0085	0.0090	0.0095
70		54.30462	53.43960	52.59397	51.76724	50.95891
71		54.89293	54.00754	53.14226	52.29657	51.46995
72		55.47685	54.57097	53.68593	52.82118	51.97618
73		56.05643	55.12993	54.22502	53.34111	52.47764
74		56.63169	55.68446	54.75957	53.85641	52.97438

**1** [Solution](#)



- MS 16** The table shows the future value of an annuity of \$1 for varying interest rates and time periods. The contribution is made at the beginning of each period.

- (a) Ken invests \$200 at the start of each year for 8 years at an interest rate of 5% per annum. Calculate the future value of Ken's investment.
- (b) Holly is planning to take a holiday in 3 years. She needs \$5000 for this holiday. She is going to make regular quarterly payments into an account that earns interest at the rate of 4% pa. compounded quarterly. What is the minimum investment, to the nearest \$100, Holly needs to make each quarter in order to take this holiday? Support your answer with an explanation.

n	1%	2%	3%	4%	5%
1	1.0100	1.0200	1.0300	1.0400	1.0500
2	2.0301	2.0604	2.0909	2.1216	2.1525
3	3.0604	3.1216	3.1836	3.2465	3.3101
4	4.1010	4.2040	4.3091	4.4163	4.5256
5	5.1520	5.3081	5.4684	5.6330	5.8019
6	6.2135	6.4343	6.6625	6.8983	7.1420
7	7.2857	7.5830	7.8923	8.2142	8.5491
8	8.3685	8.7546	9.1591	9.5828	10.0265
9	9.4622	9.9497	10.4639	11.0061	11.5779
10	10.5668	11.1687	11.8078	12.4864	13.2068
11	11.6825	12.4121	13.1920	14.0258	14.9171
12	12.8093	13.6803	14.6178	15.6268	16.7130

[Solution](#)



- MS 17** The table shows the future value of an annuity of \$1 at different interest rates over different time periods. The contribution is made at the end of each period.

- (a) What is the value of an annual annuity that would provide a future value of \$250 000 after 6 years at 3% per annum compound interest?
- (b) An annuity of \$2000 is invested each half year at 4% per annum, compounded half-yearly for 4 years. What amount of interest will be earned?

Future values of a \$1 annuity

Time period	Interest rate			
	1%	2%	3%	4%
1	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400
3	3.0301	3.0604	3.0909	3.1216
4	4.0604	4.1216	4.1836	4.2465
5	5.1010	5.2040	5.3091	5.4163
6	6.1520	6.3081	6.4684	6.6330
7	7.2135	7.4343	7.6625	7.8983
8	8.2857	8.5830	8.8923	9.2142

[Solution](#)



NESA Mathematics Standard 2 Sample examination materials

- 19 MS 2** **42** The table shows the future values of an annuity of \$1 for different interest rates for 4, 5 and 6 years. The contributions are made at the end of each year.

An annuity account is opened and contributions of \$2000 are made at the end of each year for 7 years.

For the first 6 years, the interest rate is 4% per annum, compounded annually. For the 7<sup>th</sup> year, the interest rate increases to 5% per annum, compounded annually.

Calculate the amount in the account immediately after the 7<sup>th</sup> contribution is made.

Years	Future value of an annuity of \$1			
	Interest rate per annum	1%	2%	3%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

**3** [Solution](#)



- 17 MG** **27c** A table of future value interest factors for an annuity of \$1 is shown. An annuity involves contributions of \$12 000 per annum for 5 years. The interest rate of 4% per annum, compounded annually.

(i) Calculate the future value of this annuity.

(ii) Calculate the interest earned on this annuity.

Period	Table of future value interest factors				
	Interest rate per period				
	1%	2%	3%	4%	5%
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019

[Solution](#)



- 16 MG** **28d** The table gives the contribution per period for an annuity with a future value of \$1 at different interest rates and different period of time. Margaret needs to save \$75 000 over 6 years for a deposit on a new apartment. She makes regular quarterly contributions

into an investment account which pays interest at 3% pa. How much will Margaret need to contribute each quarter to reach her savings goal?

Contribution per period for an annuity with a future value of \$1

Number of periods	Interest rate (% per period)					
	0.25%	0.5%	0.75%	1%	1.25%	1.5%
6	0.1656	0.1646	0.1636	0.1625	0.1615	0.1605
12	0.0822	0.0811	0.0800	0.0788	0.0778	0.0767
18	0.0544	0.0532	0.0521	0.0510	0.0499	0.0488
24	0.0405	0.0393	0.0382	0.0371	0.0360	0.0349
30	0.0321	0.0310	0.0298	0.0287	0.0277	0.0266
36	0.0266	0.0254	0.0243	0.0232	0.0222	0.0212

**1** [Solution](#)



**2** [Solution](#)



- 15 30c MG** The table gives the present value interest factors for an annuity of \$1 per period. For various interest rates ( $R$ ) and number of periods ( $N$ ).

- (i) Oscar plans to invest \$200 each month for 74 months. His investment will earn interest at the rate of 0.0080 (as a decimal) per month. Use this information in the table to calculate the present value of this annuity.
- (ii) Lucy is using the same table to calculate the loan repayment for her car loan. Her loan is \$21 500 and will be repaid in equal monthly repayments over 6 years. The interest rate on her loan is 10.8% per annum. Calculate the amount of each monthly repayment, correct to the nearest dollar.

Table of present value interest factors					
$N$	Interest rate per period (as a decimal)				
	0.0075	0.0080	0.0085	0.0090	0.0095
70	54.30462	53.43960	52.59397	51.76724	50.95891
71	54.89293	54.00754	53.14226	52.29657	51.46995
72	55.47685	54.57097	53.68593	52.82118	51.97618
73	56.05643	55.12993	54.22502	53.34111	52.47764
74	56.63169	55.68446	54.75957	53.85641	52.97438

**Solution****1****2**

- 14 21 MG** A table of future value interest factors is shown. A certain annuity involves making equal contributions of \$25 000 into an account every 6 months for 2 years at an interest rate of 4% per annum. Based on the information provided, what is the future value of this annuity?

- (A) \$50 500      (B) \$51 000      (C) \$103 040      (D) \$106 162

NESA 2014 Mathematics General HSC Examination

Period	Interest rate per period				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101

**1**

- 13 23 GM** Zina opened an account to save for a new car. Six months after opening the account, she made her first deposit of \$1200 and continued depositing \$1200 at the end of each six month period. Interest was paid at 3% per annum, compounded half-yearly. How much was in Zina's account two years after first opening it?

- (A) \$4909.08      (B) \$4982.72      (C) \$5018.16      (D) \$5094.55

NESA 2013 General Mathematics HSC Examination

- 11 23c GM** An amount of \$5000 is invested at 10% per annum, compounded six-monthly.

Use the table to find the value of this investment at the end of three years.

Period	Compounded values of \$1				
	1%	5%	10%	15%	20%
1	1.010	1.050	1.100	1.150	1.200
2	1.020	1.103	1.210	1.323	1.440
3	1.030	1.158	1.331	1.521	1.728
4	1.041	1.216	1.464	1.750	2.074
5	1.051	1.276	1.611	2.011	2.488
6	1.062	1.340	1.772	2.313	2.986

**2**

NESA 2011 General Mathematics HSC Examination

**09  
GM**

**27a** The table shows the future value of a \$1 annuity at different interest rates over different numbers of time periods.

- What would be the future value of a \$5000 per year annuity at 3% per annum for 6 years, with interest compounding yearly?
- What is the value of an annuity that would provide a future value of \$407 100 after 7 years at 5% per annum compound interest?
- An annuity of \$1000 per quarter is invested at 4% per annum, compounded quarterly for 2 years. What will be the amount of interest earned?

Future values of a \$1 annuity

Time Period	Interest rate				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

[Solution](#)**1****1****3**

NESA 2009 General Mathematics HSC Examination

# Year 12: Financial Mathematics

## M1.2 Arithmetic Sequences and Series



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

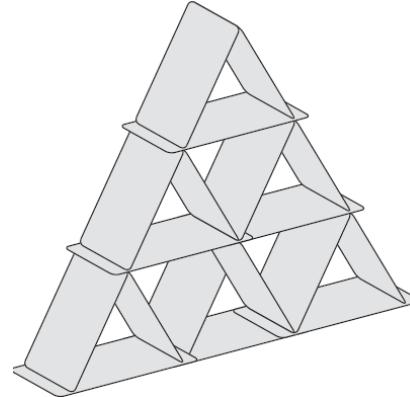
- know the difference between a sequence and a series
- recognise and use the recursive definition of an arithmetic sequence:  $T_n = T_{n-1} + d$ ,  $T_1 = a$  AAM
- establish and use the formula for the  $n^{\text{th}}$  term (where  $n$  is a positive integer) of an arithmetic sequence:  $T_n = a + (n - 1)d$ , where  $a$  is the first term and  $d$  is the common difference, and recognise its linear nature AAM
- establish and use the formulae for the sum of the first  $n$  terms of an arithmetic sequence:  
 $S_n = \frac{n}{2}(a + l)$  where  $l$  is the last term in the sequence and  $S_n = \frac{n}{2}\{2a + (n - 1)d\}$  AAM
- identify and use arithmetic sequences and arithmetic series in contexts involving discrete linear growth or decay such as simple interest (ACMMM070) AAM

[Reference Sheet](#)

**23 11 MA** The first three terms of an arithmetic sequence are 3, 7 and 11.  
Find the 15th term.

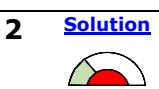


**22 17 MA** Cards are stacked to build a 'house of cards'.  
A house of cards with 3 rows is shown.  
A house of cards requires 3 cards in the top row,  
6 cards in the next row, and each successive row  
has 3 more cards than the previous row.  
(a) Show that a house of cards with 12 rows  
has a total of 234 cards.  
(b) Another house of cards has a total of 828  
cards.  
How many rows are in this house of cards?



NESA 2022 Mathematics Advanced HSC Examination

**21 14 MA** The first term of an arithmetic sequence is 5. The sum of the first 43 terms is 2021.  
What is the common difference of the sequence?



NESA 2021 Mathematics Advanced HSC Examination

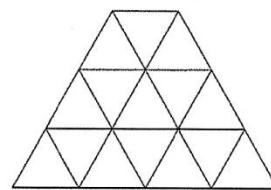
**20 12 MA** Calculate the sum of the arithmetic series  $4 + 10 + 16 + \dots + 1354$ .



NESA 2020 Mathematics Advanced HSC Examination

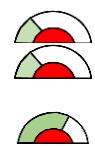
- MA 17** Jay is making a pattern using triangular tiles. The pattern has 3 tiles in the first row, 5 tiles in the second row, and each successive row has 2 more tiles than the previous row.

- SQ Band 2-5 2019**
- 12 12** tiles than the previous row.
- M c**
- How many tiles would Jay use in row 20?
  - How many tiles would Jay use altogether to make the first 20 rows?
  - Jay has only 200 tiles. How many complete rows of the pattern can Jay make?

[Solution](#)

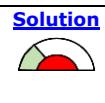
Row 1  
Row 2  
Row 3

**2**  
**1**



NESA Mathematics Advanced Sample examination materials (2019)  
NESA 2012 Mathematics HSC Examination

- TG 1** An office assistant is employed at an initial salary of \$40 200 per annum. At the beginning of each subsequent year he receives an increment of \$900. What is his salary in his ninth year, and what will be his total earnings for the first nine years?



NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 2** A young business owner's new business makes a profit of \$500 in its first month, with the profit then increasing by \$500 each month.



What is its total profit at the end of 20 months?

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 3** You are offered a choice of two salary packages. Package A offers \$40 000 in the first year and increases of \$2000 each year, while package B offers \$38 000 in the first year and increases of \$2200 each year. Which package would pay more in the tenth year?



NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 4** The first five terms of a sequence are 4, 6, 10, 18, 34, ... Which of the following recurrence relations could generate this sequence?



- A.  $T_1 = 4, T_n = T_{n-1} + 2, n \geq 2$   
B.  $T_1 = 4, T_n = 2T_{n-1} + 4, n \geq 2$   
C.  $T_1 = 4, T_n = 2T_{n-1} - 4, n \geq 2$   
D.  $T_1 = 4, T_n = 2T_{n-1} - 2, n \geq 2$

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 5** The first three terms of an arithmetic series are 3, 7 and 11. What is the 15<sup>th</sup> term of this series?



- A. 59      B. 63      C. 465      D. 495

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 6** In an arithmetic series, the fourth term is 7 and the sum of the first 16 terms is 160. Find the common difference.



NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- 19 12** In an arithmetic series, the fourth term is 6 and the sum of the first 16 terms is 120. **3**  
**M b** Find the common difference.



NESA 2019 Mathematics HSC Examination

- 18 11** In an arithmetic series, the third term is 8 and the twentieth term is 59.

[Solution](#)

- M d**
- Find the common difference.
  - Find the 50<sup>th</sup> term.

**1**  
**2**

NESA 2018 Mathematics HSC Examination

- 17 12** In an arithmetic series, the fifth term is 200 and the sum of the first four terms is 1200. Find the value of the tenth term.



NESA 2017 Mathematics HSC Examination

- 16 14** Write  $\log 2 + \log 4 + \log 8 + \dots + \log 512$  in the form of  $a \log b$  where  $a$  and  $b$  are integers greater than 1.



NESA 2016 Mathematics HSC Examination



# Year 12: Financial Mathematics

## M1.3 Geometric Sequences and Series



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- recognise and use the recursive definition of a geometric sequence:  $T_n = rT_{n-1}$ ,  $T_1 = a$  (ACMMM072) AAM
- establish and use the formula for the  $n^{\text{th}}$  term of a geometric sequence:  $T_n = ar^{n-1}$ , where  $a$  is the first term,  $r$  is the common ratio and  $n$  is a positive integer, and recognise its exponential nature (ACMMM073) AAM
- establish and use the formula for the sum of the first  $n$  terms of a geometric sequence:  

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$$
 (ACMMM075) AAM ⚡
- derive and use the formula for the limiting sum of a geometric series with  $|r| < 1$ :  $S = \frac{a}{1-r}$  AAM ⚡
  - understand the limiting behaviour as  $n \rightarrow \infty$  and its application to a geometric series as a limiting sum
  - use the notation  $\lim_{n \rightarrow \infty} r^n = 0$  for  $|r| < 1$

[Reference Sheet](#)

**23 MA 21** The fourth term of a geometric sequence is 48.

**3** [Solution](#)

The eighth term of the same sequence is  $\frac{3}{16}$ .

Find the possible value(s) of the common ratio and the corresponding first term(s).



NESA 2023 Mathematics Advanced HSC Examination

**22 MA 29** (a) The diagram shows the graph of  $y = 2^{-x}$ .  
Also shown on the diagram are the first 5 of an infinite number of rectangular strips of width 1 unit and height  $y = 2^{-x}$  for non-negative integer values of  $x$ .

**1**

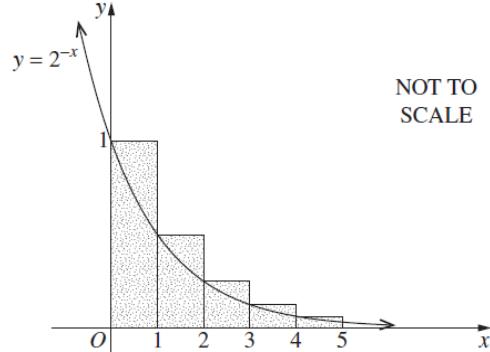
[Solution](#)

For example, the second rectangle shown

has width 1 and height  $\frac{1}{2}$ .

The sum of the areas of the rectangles forms a geometric series.

Show that the limiting sum of this series is 2.



(b) Show that  $\int_0^4 2^{-x} dx = \frac{15}{16 \ln 2}$ .

**2**



(c) Use parts (a) and (b) to show that  $e^{15} < 2^{32}$ .

**2**



NESA 2022 Mathematics Advanced HSC Examination

- 21 34** A discrete random variable has probability distribution as shown in the table where  
**MA**  $n$  is a finite positive integer.

**3** [Solution](#)

$x$	$r$	$r^2$	$r^3$	...	$r^k$	...	$r^n$
$P(X = x)$	$r^n$	$r^{n-1}$	$r^{n-2}$	...	$r^{n-k+1}$	...	$r$

Show that  $E(X) = n(2r - 1)$ 

NESA 2021 Mathematics Advanced HSC Examination

- TG 1** A sequence has an  $n^{th}$  term given by  $T_n = 21 + 3n^2$ . Find the first four terms of the sequence and state whether it is arithmetic, geometric, or neither.

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 2** Which expression is a term of the geometric series  $3x - 6x^2 + 12x^3 - \dots$ ? **1** [Solution](#)  
**14 8** (A)  $3072x^{10}$  (B)  $-3072x^{10}$  (C)  $3072x^{11}$  (D)  $-3072x^{11}$   
**M**

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

NESA 2014 Mathematics HSC Examination

- TG 3** A certain kind of washing process removes 60% of a stain on a fabric.

[Solution](#)

(a) What proportion of the original stain is removed by three successive washes?

(b) How many washes are needed to remove 95% of the original stain?

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 4** A student is constructing a family tree.

[Solution](#)

How many great-great-great-grandparents should be included?

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 5** As an employee earning \$50 000 per year, would you prefer being given a \$1200 pay increase each year for 10 years, or a 2% pay increase each year for 10 years?

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 6** How many terms of the geometric series  $2 + 8 + 32 + \dots$  are needed to obtain a sum of 699 050?

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 7** By summing the geometric series  $1 + x + x^2 + x^3 + x^4$ , or otherwise,  
**16 14** **d** find  $\lim_{x \rightarrow 1} \frac{x^5 - 1}{x - 1}$ .

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

NESA 2016 Mathematics HSC Examination

- TG 8** Jane had something posted on her Facebook page, asking her to share the post on five other friends' walls. She does this and then they each post on five other people's walls. If the progression is not interrupted, how many people would have the post on their wall after four progressions?

[Solution](#)

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

- TG 9** At the beginning of every 8-hour period, a patient is given 10 mL of a particular drug. During each of these 8-hour periods, the patient's body partially breaks down

[Solution](#)

- 14 14** **d** the drug. Only  $\frac{1}{3}$  of the total amount of the drug present in the patient's body at the beginning of each 8-hour period remains at the end of that period.

(i) How much of the drug is in the patient's body immediately after the second dose is given?

**1**

(ii) Show that the total amount of the drug in the patient's body never exceeds 15 mL.

**2**

NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics

NESA 2014 Mathematics HSC Examination

**19 11** What is the limiting sum of the following geometric series?**M d**

$$2000 - 1200 + 720 - 432 \dots$$

**2** [Solution](#)

NESA 2019 Mathematics HSC Examination

**18 14** An artist posted a song online. Each day there were  $2^n + n$  downloads, where  $n$  is the number of days after the song was posted.**M d**

- (i) Find the number of downloads on each of the first 3 days after the song was posted.  
(ii) What is the total number of times the song was downloaded in the first 20 days after it was posted?

**1****2**

NESA 2018 Mathematics HSC Examination

**17 16** A geometric series has first term  $a$  and limiting sum 2. Find all possible values of  $a$ .**M b****3**

NESA 2017 Mathematics HSC Examination

**16 15** An eight-sided die is marked with numbers 1, 2, ..., 8. A game is played by rolling the die until an 8 appears on the uppermost face. At this point the game ends.**M b**

- (i) Using a tree diagram, or otherwise, explain why the probability of the game ending before the fourth roll is  $\frac{1}{8} + \frac{7}{8} \times \frac{1}{8} + \left(\frac{7}{8}\right)^2 \times \frac{1}{8}$ .  
(ii) What is the smallest value of  $n$  for which the probability of the game ending before the  $n$ th roll is more than  $\frac{3}{4}$ ?

**2****3**

NESA 2016 Mathematics HSC Examination

**15 11** Find the limiting sum of the geometric series  $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots$ **M d****2**

NESA 2015 Mathematics HSC Examination

**13 12c** Kim and Alex start jobs at the beginning of the same year. Kim's annual salary in the first year is \$30 000, and increases by 5% at the beginning of each subsequent year. Alex's annual salary in the first year is \$33 000, and increases by \$1500 at the beginning of each subsequent year.**M**[Solution](#)

- (i) Show that in the 10<sup>th</sup> year Kim's annual salary is higher than Alex's annual salary.  
(ii) In the first 10 years how much, in total, does Kim earn?  
(iii) Every year, Alex saves  $\frac{1}{3}$  of her annual salary. How many years does it take her to save \$87 500?

**2****2****3**

NESA 2013 Mathematics HSC Examination

**13 15** Pat and Chandra are playing a game. They take turns throwing two dice.**M d**[Solution](#)

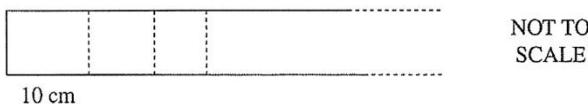
The game is won by the first player to throw a double six. Pat starts the game.

- (i) Find the probability that Pat wins the game on the first throw.  
(ii) What is the probability that Pat wins the game on the first or on the second throw?  
(iii) Find the probability that Pat eventually wins the game.

**1****2****2**

NESA 2013 Mathematics HSC Examination

- 12 15** Rectangles of the same height are cut from a strip and arranged in a row. The first **M a** rectangle has width 10 cm. The width of each subsequent rectangle is 96% of the width of the previous rectangle.

[Solution](#)

- (i) Find the length of the strip required to make the first ten rectangles.  
(ii) Explain why a strip of length 3 m is sufficient to make any number of rectangles.

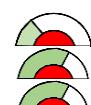
2  
1

NESA 2012 Mathematics HSC Examination

- 11 5a** The number of members of a new social networking site doubles every day. On Day 1 there were 27 members and on Day 2 there were 54 members.

[Solution](#)

- (i) How many members were there on Day 12?  
(ii) On which day was the number of members first greater than 10 million?  
(iii) The site earns 0.5 cents per member per day. How much money did the site earn in the first 12 days? Give your answer to the nearest dollar.

1  
2  
2

NESA 2011 Mathematics HSC Examination

- 10 1f** Find the limiting sum of the geometric series  $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

2



NESA 2010 Mathematics HSC Examination

- 09 4a** A tree grows from ground level to a height of 1.2 metres in one year.

2

[Solution](#)

- M** In each subsequent year, it grows  $\frac{9}{10}$  as much as it did in the previous year. Find the limiting height of the tree.



NESA 2009 Mathematics HSC Examination

- 08 4b** The zoom function in a software package multiplies the dimensions of an image by 1.2. In an image, the height of a building is 50 mm. After the zoom function is applied once, the height of the building in the image is 60 mm. After a second application, its height is 72 mm.

[Solution](#)

- (i) Calculate the height of the building in the image after the zoom function has been applied eight times. Give your answer to the nearest mm.  
(ii) The height of the building in the image is required to be more than 400 mm. Starting from the original image, what is the least number of times the zoom function must be applied?

2  
2

NESA 2008 Mathematics HSC Examination

- 08 5b** Consider the geometric series  $5 + 10x + 20x^2 + 40x^3 + \dots$

[Solution](#)

- M** (i) For what values of  $x$  does this series have a limiting sum?  
(ii) The limiting sum of this series is 100. Find the value of  $x$ .

2  
2

NESA 2008 Mathematics HSC Examination

- 07 1d** Find the limiting sum of the geometric series  $\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \dots$

2



NESA 2007 Mathematics HSC Examination

**Year 12: Financial Mathematics****M1.4 Financial Applications of Sequences and Series****Syllabus:** updated November 2019. Latest version @
<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- use geometric sequences to model and analyse practical problems involving exponential growth and decay (ACMMM076) **AAM**
- calculate the effective annual rate of interest and use results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
- solve problems involving compound interest loans or investments, eg determining the future value of an investment or loan, the number of compounding periods for an investment to exceed a given value and/or the interest rate needed for an investment to exceed a given value (ACMGM096)
- recognise a reducing balance loan as a compound interest loan with periodic repayments, and solve problems including the amount owing on a reducing balance loan after each payment is made
- solve problems involving financial decisions, including a home loan, a savings account, a car loan or superannuation **AAM**
- calculate the future value or present value of an annuity by developing an expression for the sum of the calculated compounded values of each contribution and using the formula for the sum of the first  $n$  terms of a geometric sequence
- verify entries in tables of future values or annuities by using geometric series

**STANDA**  
 STANDARDS OF ASSESSMENT

[Reference Sheet](#)

- 23 MA 25** On the first day of November, Jia deposits \$10 000 into a new account which earns 0.4% interest per month, compounded monthly. At the end of each month, after the interest is added to the account, Jia intends to withdraw \$ $M$  from the account. Let  $A_n$  be the amount (in dollars) in Jia's account at the end of  $n$  months.

**Solution**

(a) Show that  $A_2 = 10\ 000(1.004)^2 - M(1.004) - M$

**1**

(b) Show that  $A_n = (10\ 000 - 250M)(1.004)^n + 250M$ .

**3**

(c) Jia wants to be able to make at least 100 withdrawals.

**2**

What is the largest value of  $M$  that will enable Jia to do this?

NESA 2023 Mathematics Advanced HSC Examination

- 22 MA 32** In a reducing-balance loan, an amount  $P$  is borrowed for a period of  $n$  months at an interest rate of 0.25% per month, compounded monthly.

At the end of each month, a repayment of  $M$  is made.

After the  $n$ th repayment has been made, the amount owing,  $A$ , is given by

$$A_n = P(1.0025)^n - M(1 + (1.0025)^1 + (1.0025)^2 + \dots + (1.0025)^{n-1}).$$

(Do NOT prove this.)

- (a) Jane borrows \$200 000 in a reducing-balance loan as described.



2

The loan is to be repaid in 180 monthly repayments.

Show that  $M = 1381.16$ , when rounded to the nearest cent.

- (b) After 100 repayments of \$1381.16 have been made, the interest rate changes to 0.35% per month.



3

At this stage, the amount owing to the nearest dollar is \$100 032.

(Do NOT prove this.)

Jane continues to make the same monthly repayments.

For how many more months will Jane need to make full monthly payments of \$1381.16?

- (c) The final repayment will be less than \$1381.16.



2

How much will Jane need to pay in the final payment in order to pay off the loan?

NESA 2022 Mathematics Advanced HSC Examination

- 21 MA 29** (a) On the day that Megan was born, her grandfather deposited \$5000 into an account earning 3% per annum compounded annually. On each birthday after this, her grandfather deposited \$1000 into the same account, making his final deposit on Megan's 17th birthday. That is, a total of 18 deposits were made. Let  $A_n$  be the amount in the account on Megan's  $n$ th birthday, after the deposit is made.

2



Show that  $A_3 = \$8554.54$ .

- (b) On her 17th birthday, just after the final deposit is made, Megan has

3



\$30 025.83 in her account. You are NOT required to show this. Megan then decides to leave all the money in the same account continuing to earn interest at 3% per annum compounded annually. On her 18th birthday, and on each birthday after this, Megan withdraws \$2000 from the account.

How many withdrawals of \$2000 will Megan be able to make?

NESA 2021 Mathematics Advanced HSC Examination

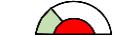
- 20 MA 26** Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

[Solution](#)

The amount in the account immediately after  $n$ th withdrawal can be determined using the recurrence relation  $A_n = A_{n-1}(1.005) - 800$ , where  $n = 1, 2, 3, \dots$  and  $A_0 = 60 000$ .

- (a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. \*
- (b) Calculate the amount of interest earned in the first three months. \*
- (c) Calculate the amount of money in the account immediately after the 94<sup>th</sup> withdrawal.

2



2



3



\* COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

<b>MA</b>	<b>36</b>	An island initially has 16 100 trees. The number of trees increases by 1% per annum. The people on the island cut down 1161 trees at the end of each year.	<a href="#">Solution</a>
<b>SP</b>	<b>Band 2-6</b>	(a) Show that after the first year there are 15 100 trees. (b) Show that at the end of 2 years the number of trees remaining is given by the expression $T_2 = 16\ 100 \times (1.01)^2 - 1161(1 + 1.01)$ . (c) Show that at the end of $n$ years the number of trees remaining is given by the expression $T_n = 16\ 100 - 100\ 000 \times (1.01)^n$ . (d) For how many years will the people on the island be able to cut down 1161 trees annually?	<b>1</b> <b>2</b> <b>2</b> <b>1</b>
		NESA Mathematics Advanced Sample Examination Paper (2020)	
<b>MA</b>	<b>2</b>	What amount must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to \$60 000? A. \$8919      B. \$11 156      C. \$49 173      D. \$49 316	<b>1</b>
<b>SQ</b>	<b>Band 3-6</b>		<a href="#">Solution</a>
		NESA Mathematics Advanced Sample examination materials (2019)	
<b>TG</b>	<b>1</b>	A gardener develops an eco-friendly spray that will kill harmful insects on fruit trees without contaminating the fruit. A trial is to be conducted with 100 000 insects. The gardener expects the spray to kill 35% of the insects each day and exactly 5000 new insects will be produced each day.	<a href="#">Solution</a>
<b>16</b>	<b>14</b>	The number of insects expected at the end of the $n$ th day of the trial is $A_n$ .	
<b>M</b>	<b>b</b>	(i) Show that $A_2 = 0.65(0.65 \times 100\ 000 + 5000) + 5000$ . (ii) Show that $A_n = 0.65^n \times 100\ 000 + 5000 \frac{(1 - 0.65^n)}{0.35}$ . (iii) Find the expected insect population at the end of the fourteenth day, correct to the nearest 100.	<b>2</b> <b>1</b> <b>1</b>
		NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESA 2016 Mathematics HSC Examination	
<b>TG</b>	<b>2</b>	At the start of a month, Joe opens a bank account and makes a deposit of \$500. At the start of each subsequent month, Joe makes a deposit which is 1% more than the previous deposit. At the end of each month, the bank pays interest of 0.3% (per month) on the balance of the account.	<a href="#">Solution</a>
<b>14</b>	<b>16b</b>	(i) Explain why the balance of the account at the end of the second month is \$500(1.003) <sup>2</sup> + \$500(1.01)(1.003). (ii) Find the balance of the account at the end of the 60 <sup>th</sup> month, correct to the nearest dollar.	<b>2</b> <b>3</b>
		NESA Mathematics Advanced Year 12 Topic Guide: Financial mathematics NESA 2014 Mathematics HSC Examination	
<b>MS</b>	<b>FM</b>	Mia wants to invest \$42 000 for a total of 5 years. She has three investment options.	<b>5</b>
<b>SQ</b>	<b>11</b>	Option A – simple interest is paid at the rate of 6% per annum Option B – compound interest is paid at a rate of 5.5% per annum, compounded annually Option C – compound interest is paid at a rate of 4.8% per annum, compounded quarterly Determine Mia's best investment option. Support your answer with calculations.	
		NESA Mathematics Standard 2 Sample examination materials	
<b>MS</b>	<b>FM</b>	A house was purchased at the start of 1986 for \$45 000.	<b>2</b>
<b>SQ</b>	<b>19</b>	Assume that the value of the house has increased by 8% per annum since then. What is the value of the house at the end of 2019, to the nearest \$1000?	
		NESA Mathematics Standard 2 Sample examination materials	

- 19 16** A person wins \$1 000 000 in a competition and decides to invest this money in an account that earns interest at 6% per annum compounded quarterly. The person decides to withdraw \$80 000 from this account at the end of every fourth quarter.

[Solution](#)

Let  $A$  be the amount remaining in the account after the  $n$ th withdrawal.

(i) Show that the amount remaining in the account after the withdrawal at the end of the eighth quarter is  $A_2 = 1 000 000 \times 1.015^8 - 80 000(1 + 1.015^4)$ .

2



(ii) For how many years can the full amount of \$80 000 be withdrawn?

3



NESA 2019 Mathematics HSC Examination

- 18 16c** Kara deposits an amount of \$300 000 into an account which pays compound interest of 4% per annum, added to the account at the end of each year.

[Solution](#)

Immediately after the interest is added, Kara makes a withdrawal for expenses for the coming year. The first withdrawal is  $\$P$ . Each subsequent withdrawal is 5% greater than the previous one. Let  $\$A_n$  be the amount in the account after the  $n$ th withdrawal.

(i) Show that  $A_2 = 300 000(1.04)^2 - P[(1.04) + (1.05)]$

1



(ii) Show that  $A_3 = 300 000(1.04)^3 - P[(1.04)^2 + (1.04)(1.05) + (1.05)^2]$ .

1



(iii) Show that there will be money in the account when  $\left(\frac{105}{104}\right)^n < 1 + \frac{3000}{P}$ .

3



NESA 2018 Mathematics HSC Examination

- 17 15b** Anita opens a savings account. At the start of each month she deposits  $\$X$  into the savings account. At the end of the month, after interest is added into the savings account, the bank withdraws \$2500 from the savings account as a loan repayment.

[Solution](#)

Let  $M_n$  be the amount in the savings account after the  $n$ th withdrawal.

The savings account pays interest at 4.2% per annum compounded monthly.

(i) Show that after the second withdrawal the amount in the savings account is given by  $M_2 = X(1.0035^2 + 1.0035) - 2500(1.0035 + 1)$ .

2



(ii) Find the value of  $X$  so that the amount in the savings account is \$80 000 after the last withdrawal of the fourth year.

3



NESA 2017 Mathematics HSC Examination

- 15 14c** Sam borrows \$100 000 to be repaid at a reducible interest rate of 0.6% per month. Let  $\$A_n$  be the amount owing at the end of  $n$  months and  $\$M$  be the monthly repayment.

[Solution](#)

(i) Show that  $A_2 = 100 000(1.006)^2 - M(1 + 1.006)$ .

1



(ii) Show that  $A_n = 100 000(1.006)^n - M\left(\frac{(1.006)^n - 1}{0.006}\right)$ .

2



(iii) Sam makes monthly repayments of \$780. Show that after making 120 monthly repayments the amount owing is \$68 500 to the nearest \$100.

1



(iv) Immediately after making the 120th repayment, Sam makes a one-off payment, reducing the amount owing to \$48 500. The interest rate and monthly repayment remain unchanged. After how many more months will the amount owing be completely repaid?

3



NESA 2015 Mathematics HSC Examination

- 13 13d M** A family borrows \$500 000 to buy a house. The loan is to be repaid in equal monthly instalments. The interest, which is charged at 6% per annum, is reducible and calculated monthly. The amount owing after  $n$  months,  $A_n$ , is given by  $A_n = Pr^n - M(1 + r + r^2 + \dots + r^{n-1})$ , (Do NOT prove this) where \$P\$ is the amount borrowed,  $r = 1.005$  and \$M\$ is the monthly repayment.

[Solution](#)

- The loan is to be repaid over 30 years. Show that the monthly repayment is \$2998 to the nearest dollar. **2**
- Show that the balance owing after 20 years is \$270 000 to the nearest thousand dollars. **1**
- After 20 years the family borrows an extra amount, so that the family then owes a total of \$370 000. The monthly repayment remains \$2998, and the interest rate remains the same. How long will it take to repay the \$370 000? **2**



NESA 2013 Mathematics HSC Examination

- 12 15c M** Ari takes out a loan of \$360 000. The loan is to be repaid in equal monthly repayments, \$M, at the end of each month, over 25 years (300 months). Reducible interest is charged at 6% per annum, calculated monthly. Let  $A_n$  be the amount owing after the  $n$ th repayment.
- Write down an expression for the amount owing after two months,  $A_2$ . **1**
  - Show that the monthly repayment is approximately \$2319.50. **2**
  - After how many months will the amount owing,  $A_n$ , become less than \$180 000? **3**

[Solution](#)

NESA 2012 Mathematics HSC Examination

- 11 8c M** When Jules started working she began paying \$100 at the beginning of each month into a superannuation fund. The contributions are compounded monthly at an interest rate of 6% per annum. She intends to retire after having worked for 35 years.
- Let \$P\$ be the final value of Jules's superannuation when she retires after 35 years (420 months). Show that  $P = \$143\,183$  to the nearest dollar.
  - Fifteen years after she started working Jules read a magazine article about retirement, and realized that she would need \$800 000 in her fund when she retires. At the time of reading the magazine article she had \$29 227 in her fund. For the remaining 20 years she intends to work, she decides to pay a total of \$M into her fund at the beginning of each month. The contributions continue to attract the same interest rate of 6% per annum, compounded monthly. At the end of  $n$  months after starting the new contributions, the amount in the fund is  $A_n$ .
    - Show that  $A_2 = 29\,227 \times 1.005^2 + M(1.005 + 1.005^2)$ . **1**
    - Find the value of  $M$  so that Jules will have \$800 000 in her fund after the remaining 20 years (240 months). **3**

[Solution](#)

NESA 2011 Mathematics HSC Examination

- 10 9a M** (i) When Chris started a new job, \$500 was deposited into his superannuation fund at the beginning of each month. The money was invested at 0.5% per month, compounded monthly. Let \$P\$ be the value of the investment after 240 months, when Chris retires. Show that  $P = 232\,175.55$
- After retirement, Chris withdraws \$2000 from the account at the end of each month, without making any further deposits. The account continues to earn interest at 0.5% per month. Let  $A_n$  be the amount left in the account  $n$  months after Chris's retirement.
    - Show that  $A_n = (P - 400\,000) \times 1.005^n + 400\,000$ . **3**
    - For how many months after retirement will there be money left in the account? **2**

[Solution](#)

NESA 2010 Mathematics HSC Examination

<b>09</b>	<b>8b</b>	One year ago Daniel borrowed \$350 000 to buy a house. The interest rate was 9% per annum, compounded monthly. He agreed to repay the loan in 25 years with equal monthly repayments of \$2937.	<a href="#">Solution</a>
<b>M</b>		(i) Calculate how much Daniel owed after his monthly repayment. (ii) Daniel has just made his 12 <sup>th</sup> monthly repayment. He now owes \$346 095. The interest rate now decreases to 6% per annum, compounded monthly. The amount $A_n$ , owing on the loan after the $n^{\text{th}}$ monthly repayment is now calculated using the formula $A_n = 346\ 095 \times 1.005^n - 1.005^{n-1} M - \dots - 1.005M - M$ where \$M is the monthly repayment and $n = 1, 2, \dots, 288$ . (Do NOT prove this formula.) Calculate the monthly repayment if the loan is to be repaid over the remaining 24 years (288 months).	<b>1</b>  <b>3</b> 
		(iii) Daniel chooses to keep his monthly repayments at \$2937. Use the formula in part (ii) to calculate how long it will take him to repay the \$346 095. (iv) How much will Daniel save over the term of the loan by keeping his monthly repayments at \$2937, rather than reducing his repayments to the amount calculated in part (ii)?	<b>3</b>  <b>1</b> 
		NESA 2009 Mathematics HSC Examination	
<b>08</b>	<b>9b</b>	Peter retires with a lump sum of \$100 000. The money is invested in a fund which pays interest each month at a rate of 6% per annum, and Peter receives a fixed monthly payment of \$M from the fund. Thus, the amount left in the fund after the first monthly payment is $\$(100\ 500 - M)$ .	<a href="#">Solution</a>
<b>M</b>		(i) Find a formula for the amount, $A_n$ , left in the fund after $n$ monthly payments. (ii) Peter chooses the value of $M$ so that there will be nothing left in the fund at the end of the 12 <sup>th</sup> year (after 144 payments). Find the value of $M$ .	<b>2</b>  <b>3</b> 
		NESA 2008 Mathematics HSC Examination	
<b>07</b>	<b>9c</b>	Mr and Mrs Caine each decide to invest some money each year to help pay for their son's university education. The parents choose different investment strategies.	<a href="#">Solution</a>
<b>M</b>		(i) Mr Caine makes 18 yearly contributions of \$1000 into an investment fund. He makes his first contribution on the day his son is born, and his final contribution on his son's seventeenth birthday. His investment earns 6% compound interest per annum. Find the total value of Mr Caine's investment on his son's eighteenth birthday. (ii) Mrs Caine makes her contributions into another fund. She contributes \$1000 on the day of her son's birth, and increases her annual contribution by 6% each year. Her investment also earns 6% compound interest per annum. Find the total value of Mrs Caine's investment on her son's third birthday (just before she makes her fourth contribution). (iii) Mrs Caine also makes her final contribution on her son's seventeenth birthday. Find the total value of Mrs Caine's investment on her son's eighteenth birthday.	<b>3</b>  <b>2</b>  <b>1</b> 
		NESA 2007 Mathematics HSC Examination	

# Year 12: Statistical Analysis

## S2.1 Data (grouped and ungrouped) and Summary Statistics

 Back**Syllabus: updated November 2019. Latest version @**<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- classify data relating to a single random variable 
- organise, interpret and display data into appropriate tabular and/or graphical representations including Pareto charts, cumulative frequency distribution tables or graphs, parallel box-plots and two-way tables AAM 
- compare the suitability of different methods of data presentation in real-world contexts (ACMEM048)
- summarise and interpret grouped and ungrouped data through appropriate graphs and summary statistics AAM 
- calculate measures of central tendency and spread and investigate their suitability in real-world contexts and use to compare large datasets 
- investigate real-world examples from the media illustrating appropriate and inappropriate uses or misuses of measures of central tendency and spread (ACMEM056) AAM
- identify outliers and investigate and describe the effect of outliers on summary statistics 
- use different approaches for identifying outliers, for example consideration of the distance from the mean or median, or the use of below  $Q_1 - 1.5 \times IQR$  and above  $Q_3 + 1.5 \times IQR$  as criteria, recognising and justifying when each approach is appropriate
- investigate and recognise the effect of outliers on the mean, median and standard deviation
- describe, compare and interpret the distributions of graphical displays and/or numerical datasets and report findings in a systematic and concise manner AAM 

 Reference SheetSTANDARD  
2**22 2** Consider the following dataset.**MA** 13    16    17    17    21    24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

	<i>Median</i>	<i>Mean</i>
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

**1** Solution

COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

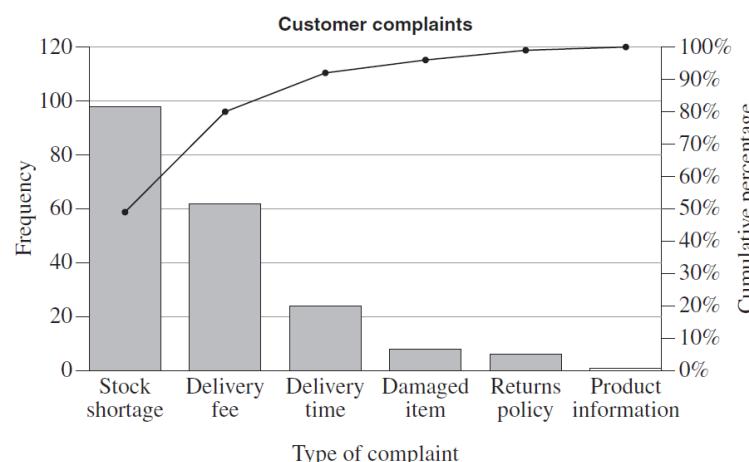
- 22 11 MA** The table shows the types of customer complaints received by an online business in a month.

(a) What are the values of **A** and **B**?

Type of complaint	Frequency	Cumulative frequency	Cumulative percentage
Stock shortage	98	98	49
Delivery fee	62	<b>A</b>	80
Delivery time	24	184	92
Damaged item	8	192	<b>B</b>
Returns policy	6	198	99
Product information	2	200	100
Total	200		

(b) The data from the table are shown in the following Pareto chart.

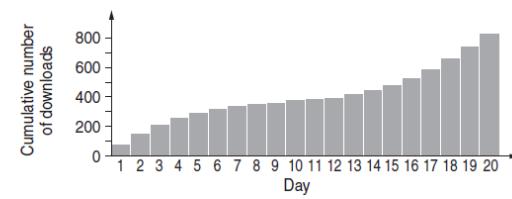
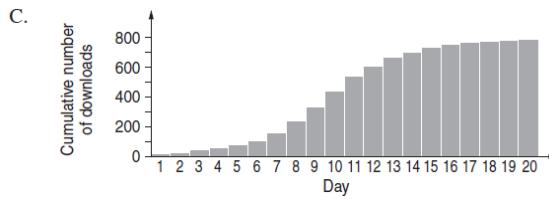
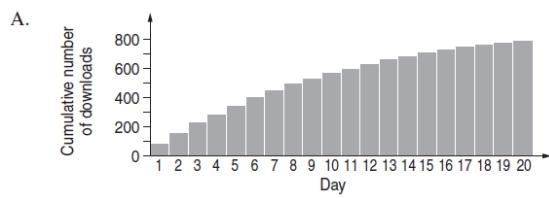
The manager will address 80% of the complaints.  
Which type of complaints will the manager address?



COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

- 21 4 MA** The number of downloads of a song on each of twenty consecutive days is shown in the following graph.

Which of the following graphs best shows the cumulative number of downloads up to and including each day?



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**Solution**



**2**



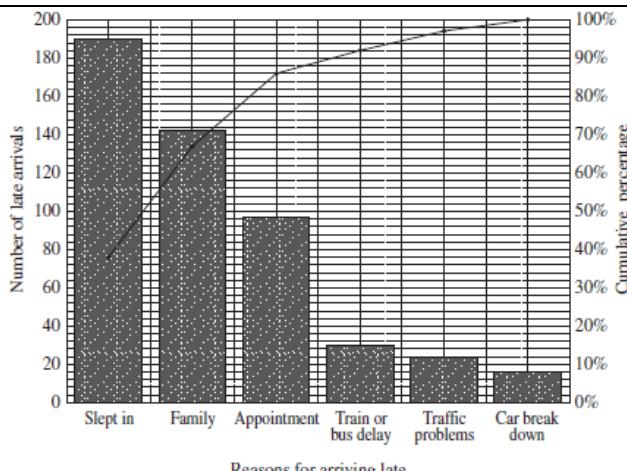
**1**



- MA 5** A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected

- 19 10** MS 2 What percentage of students gave the reason 'Train or bus delay'?

- A. 6%  
B. 15%  
C. 30%  
D. 92%

**1** [Solution](#)NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

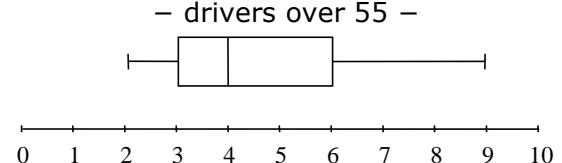
- TG 1** A dataset of nine scores has a median of 7. The scores 6, 6, 12 and 17 are added to this data set. What is the median of the data set now?  
**11 14** MG (A) 6 (B) 7 (C) 8 (D) 9

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis  
NESA 2011 Mathematics General HSC Examination

- TG 2**
- Using the box-plot,

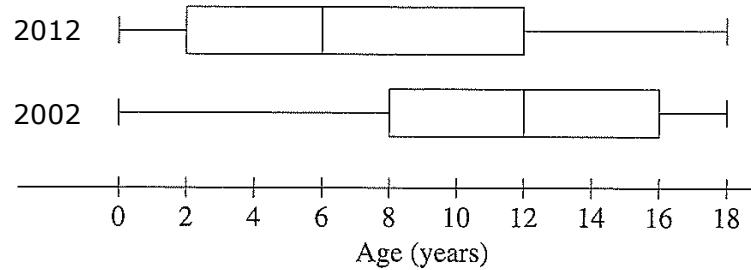
- (a) what percentage of drivers in this sample have reaction times of three or more seconds?  
(b) what percentage of drivers in this sample have reaction times between four and nine seconds?  
(c) What is the interquartile range for this dataset?

Reaction time in seconds prior to braking

[Solution](#)[Solution](#)

- TG 3** The box-plots show the distribution of the ages of children in Numbertown in 2002 and 2012. The number of children aged 12–18 years was the same in both 2002 and 2012. By considering the data, provide advice to town planners about recreational facilities that should be offered, giving statistical reasons.

Distribution of the ages of children in Numbertown

[Solution](#)

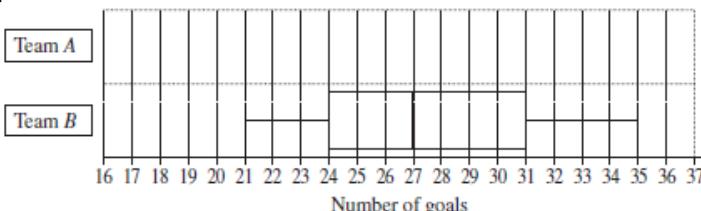
- 19 19** MS 2 The heights, in centimetres, of 10 players on a basketball team are shown.  
170, 180, 185, 188, 192, 193, 193, 194, 196, 202  
Is the height of the shortest player on the team considered an outlier?  
Justify your answer with calculations.

**3** [Solution](#)

NESA 2019 Mathematics Standard 2 HSC Examination

- 19 MS 2** **39** Two netball teams, Team A and Team B, each played 15 games in a tournament. For each team, the number of goals scored in each game was recorded. The frequency table shows the data for Team A.

The data for Team B was analysed to create the box plot below.



Number of goals	Frequency
19	1
20	0
21	1
22	1
23	1
24	3
25	0
26	4
27	3
28	1

**5** [Solution](#)

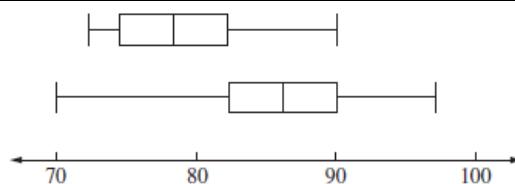


Compare the distributions of the number of goals scored by the two teams.

Support your answer with the construction of a box-plot for the data for Team A.

NESA 2019 Mathematics Standard 2 HSC Examination

- MS SA** The box-plots show the results of a test sat by two classes, A and B.  
**SQ 10** Which of the following statements is true about Class B's results when compared to those of Class A?



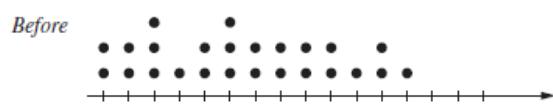
**1** [Solution](#)



- A. The IQR for Class B's results is larger than that for Class A's results.
- B. The results for Class B are more consistent than those of Class A.
- C. 25% of Class B's results are higher than all of Class A's results.
- D. 25% of Class B's results are lower than half of Class A's results.

NESA Mathematics Standard 2 Sample examination materials

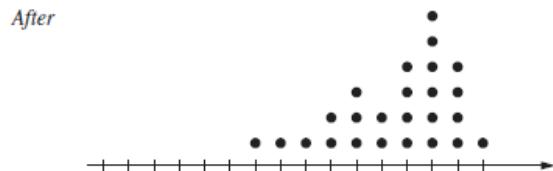
- MS SA** The dot plots show the class scores in tests taken before and after a unit of work was completed.



**1** [Solution](#)



They are drawn on the same scale.



- A. The mean increased and the standard deviation decreased.
- B. The mean increased and the standard deviation increased.
- C. The mean decreased and the standard deviation decreased.
- D. The mean decreased and the standard deviation increased.

NESA Mathematics Standard 2 Sample examination materials

- MS SA** In a computer game, a player scored 180, 183, 184, 186 and 192 in the first five rounds. After playing a sixth round, the player's average score increased by 1. What was the player's score in the sixth round?

**1** [Solution](#)



- A. 185
- B. 186
- C. 191
- D. 193

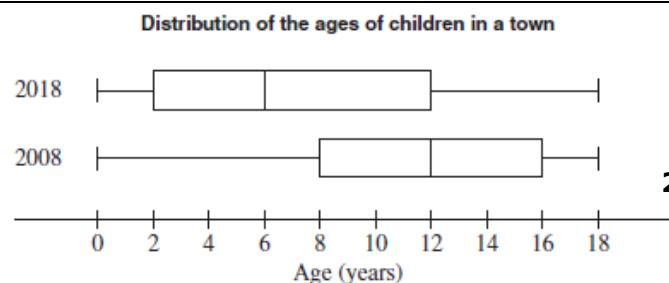
NESA Mathematics Standard 2 Sample examination materials

- MS SA** The diagram shows the distribution of the ages of children in a town in 2008 and 2018.  
**SQ 17**

In 2008 there were 1750 children aged 0 – 18 years.

(a) The number of children aged 12 – 18 years was the same in both 2008 and 2018. How many children aged 0 – 18 years were there in 2018?

(b) What would be ONE possible implication for government planning as a consequence of the change in the distribution of ages from 2008 to 2018? Justify your answer with reference to the data provided.

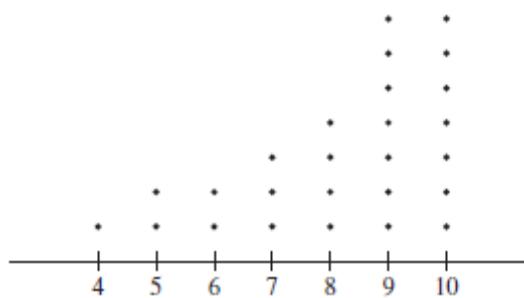
**Solution****2****2**

- MS SA** Write down the five-number summary for the dataset 2, 4, 6, 9, 12, 18.  
**SQ 24**

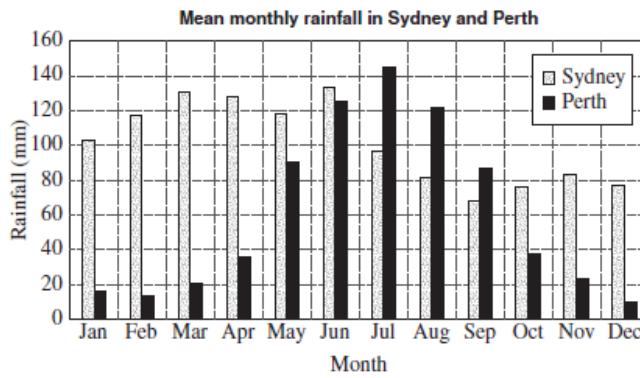
**2****Solution**

NESA Mathematics Standard 2 Sample examination materials

- 18 MG 6** A set of data is displayed in this dot plot. Which of the following best describes this set of data?  
 A. Symmetrical  
 B. Positively skewed  
 C. Negatively skewed  
 D. Normally distributed

**1****Solution**

- 18 MG 26 d** The graph displays the mean monthly rainfall in Sydney and Perth.  
 (i) For how many months is the mean monthly rainfall higher in Perth than Sydney?  
 (ii) For which of the two cities is the standard deviation of the mean monthly rainfall smaller? Justify your answer WITHOUT calculations.

**Solution****1****1**

- 18 MG 26 e** A cumulative frequency table for a data set is shown.

What is the interquartile range for this data set?

Score	Cumulative frequency
1	5
2	9
3	16
4	20
5	34
6	42

**2****Solution**

NESA 2018 Mathematics General HSC Examination

**17 30** A set of data has a lower quartile ( $Q_L$ ) of 10 and an upper quartile ( $Q_U$ ) of 16.

**MG a** What is the maximum possible range for this set of data if there are no outliers?

**2**[Solution](#)

NESA 2017 Mathematics General HSC Examination



**16 19** A soccer referee wrote down the number of goals scored in 9 different games during the season.

**1**[Solution](#)

2, 3, 3, 3, 5, 5, 8, 9, □

The last number has been omitted. The range of the data is 10.

What is the five-number summary for this data set?

- (A) 2, 3, 5, 8.5, 12 (B) 2, 3, 5, 8.5, 10 (C) 2, 3, 5, 8, 12 (D) 2, 3, 5, 8, 10

NESA 2016 Mathematics General HSC Examination

**1**[Solution](#)

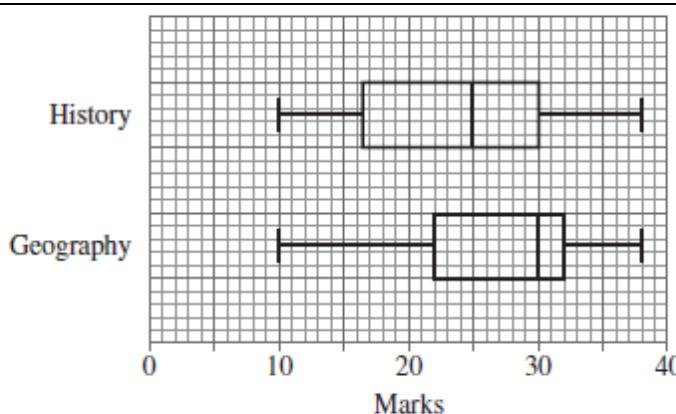
**16 22** The box-and-whisker plots show the results of a History test and a Geography test.

In History, 112 students completed the test.

The number of students who scored above 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

- (A) 8 (B) 50 (C) 56 (D) 112



NESA 2016 Mathematics General HSC Examination

**2**[Solution](#)

**16 27** A small population consists of three students of heights 153 cm, 168 cm and

**MG b** 174 cm. Samples of varying sizes can be taken from this population. What is the mean of the mean heights of all the possible samples? Justify your answer.

NESA 2016 Mathematics General HSC Examination

**16 29** The ages of members of a dance class are shown in the back-to-back stem-and-leaf plot. Pat claims that the women who attend the dance class are generally older than the men. Is Pat correct? Justify your answer by referring to the median and skewness of the two sets of data.

Women	Men
2	3 4 6
4 2	4 2 2 5 6 8
8 8 5 4 0 0	5 3
9 4 3 3	6 3

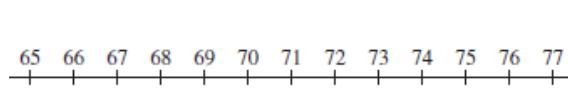
**3**[Solution](#)

NESA 2016 Mathematics General HSC Examination

**14 29** Terry and Kim each sat twenty class tests. Terry's results on the tests are displayed in the box-and-whisker plot.

[Solution](#)

- (i) Kim's 5-number summary for the tests is 67, 69, 71, 73, 75. Draw a box-and-whisker plot to display Kim's results below that of Terry's results.

**1**

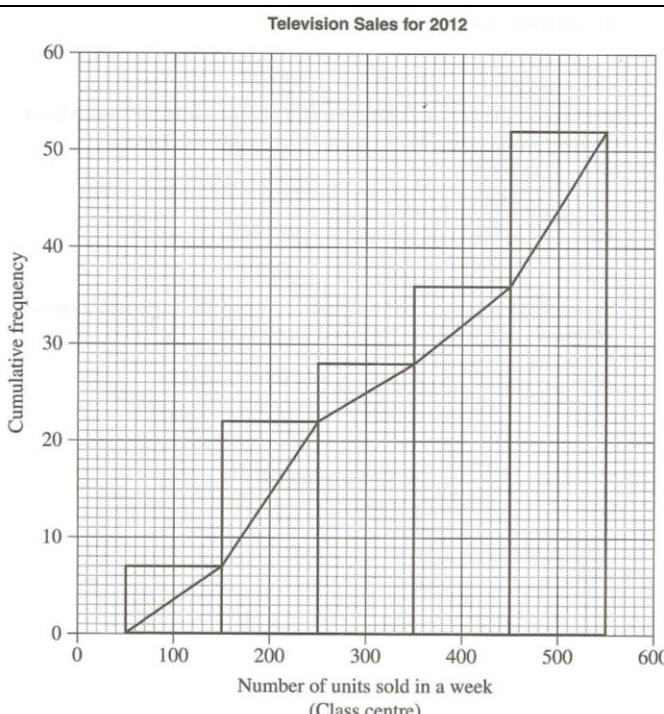
- (ii) What percentage of Terry's results were below 69?  
 (iii) Terry claims that his results were better than Kim's. Is he correct? Justify your answer by referring to the summary statistics and the skewness of the distributions.

**4**[Solution](#)

NESA 2014 Mathematics General HSC Examination

- 13 27 MG c** A retailer has collected data on the number of televisions that he sold each week in 2012. He grouped the data into classes and displayed the data using a cumulative frequency histogram and polygon (ogive).

- (i) Use the cumulative frequency polygon to determine the interquartile range.
- (ii) Oscar said that the retailer sold 300 televisions in 6 of the weeks in 2012. Is he correct? Give a reason for your answer.

[Solution](#)

2



1



- 11 17 MG** The heights of the players in a basketball team were recorded as 1.8 m, 1.83 m, 1.84 m, 1.86 m and 1.92 m. When a sixth player joined the team, the average height of the players increased by 1 centimetre. What was the height of the sixth player?

- (A) 1.85 m      (B) 1.86 m      (C) 1.91 m      (D) 1.93 m

NESA 2011 Mathematics General HSC Examination

1

[Solution](#)

- 09 21 MG** The mean of a set of ten scores is 14. Another two scores are included and the new mean is 16.

What is the mean of the two additional scores?

- (A) 4      (B) 16      (C) 18      (D) 26

NESA 2009 Mathematics General HSC Examination

1

[Solution](#)

- 08 13 MG** The height of each student in a class was measured and it was found that the mean height was 160 cm. Two students were absent. When their heights were included in the data for the class, the mean height did not change. Which of the following heights are possible for the two absent students?

- (A) 155 cm and 162 cm      (B) 152 cm and 167 cm  
 (C) 149 cm and 171 cm      (D) 143 cm and 178 cm

NESA 2008 Mathematics General HSC Examination

1

[Solution](#)

- 08 23 MG f** Christina has completed three Mathematics tests. Her mean mark is 72%. What mark (out of 100) does she have to get in her next test to increase her mean mark to 73%?

NESA 2008 Mathematics General HSC Examination

2

[Solution](#)

- 07 21 MG** This set of data is arranged in order from smallest to largest.

$$5, 6, 11, x, 13, 18, 25$$

The range is six less than twice the value of  $x$ . Which one of the following is true?

- (A) The median is 12 and the interquartile range is 7.  
 (B) The median is 12 and the interquartile range is 12.  
 (C) The median is 13 and the interquartile range is 7.  
 (D) The median is 13 and the interquartile range is 12.

NESA 2007 Mathematics General HSC Examination

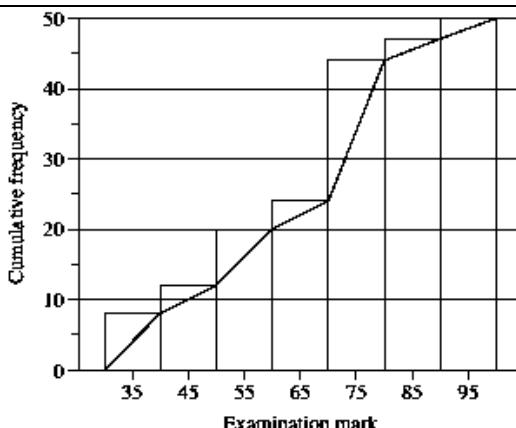
1

[Solution](#)

**07 22 MG** A set of examination results is displayed in a cumulative frequency histogram and polygon (ogive). Sanath knows that his examination mark is in the 4th decile. Which of the following could have been Sanath's examination mark?

- (A) 37                    (B) 57  
(C) 67                    (D) 77

**1** [Solution](#)



NESA 2007 Mathematics General HSC Examination



Syllabus: updated November 2019. Latest version @ [education.govt.nz](https://www.education.govt.nz/curriculum/national-curriculum/syllabuses)

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

### Students:

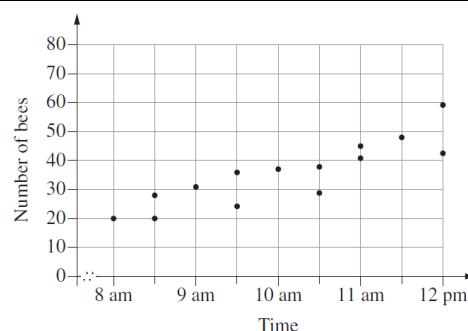
- construct a bivariate scatterplot to identify patterns in the data that suggest the presence of an association (ACMGM052) 
  - use bivariate scatterplots (constructing them where needed), to describe the patterns, features and associations of bivariate datasets, justifying any conclusions AAM 
    - describe bivariate datasets in terms of form (linear/non-linear) and in the case of linear, also the direction (positive/negative) and strength of association (strong/moderate/weak)
    - identify the dependent and independent variables within bivariate datasets where appropriate
    - describe and interpret a variety of bivariate datasets involving two numerical variables using real-world examples in the media or those freely available from government or business datasets  
  - calculate and interpret Pearson's correlation coefficient ( $r$ ) using technology to quantify the strength of a linear association of a sample (ACMGM054) 
  - model a linear relationship by fitting an appropriate line of best fit to a scatterplot and using it to describe and quantify associations AAM 
    - fit a line of best fit to the data by eye and using technology (ACMEM141, ACMEM142)
    - fit a least-squares regression line to the data using technology (ACMGM057)
    - interpret the intercept and gradient of the fitted line (ACMGM059)
  - use the appropriate line of best fit, both found by eye and by applying the equation of the fitted line, to make predictions by either interpolation or extrapolation AAM 
    - distinguish between interpolation and extrapolation, recognising the limitations of using the fitted line to make predictions, and interpolate from plotted data to make predictions where appropriate 
  - solve problems that involve identifying, analysing and describing associations between two numeric variables AAM 
  - construct, interpret and analyse scatterplots for bivariate numerical data in practical contexts AAM 
    - demonstrate an awareness of issues of privacy and bias, ethics, and responsiveness to diverse groups and cultures when collecting and using data    

## Reference Sheet

**23**    **1** The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.

Which Pearson's correlation coefficient best describes the observations?

- A. -0.8      B. -0.2  
C. 0.2      D. 0.8



---

1 Solution



COMMON QUESTION: NESQ 2023 Mathematics Standard 2 and Advanced HSC Examinations

**23 18**

- MA** A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius ( $^{\circ}\text{C}$ ).

**Solution**

Using  $x$  as the average daily outside temperature and  $y$  as the total daily gas usage, the equation of the least-squares regression line was found.

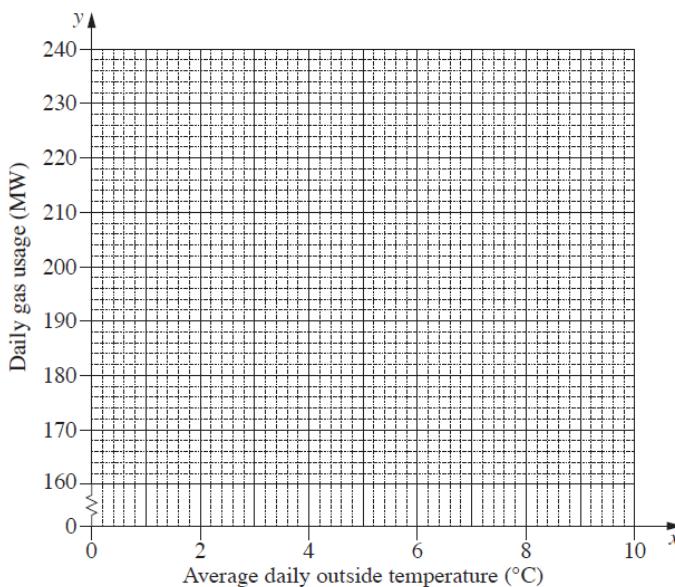
The equation of the regression line predicts that when the temperature is  $0^{\circ}\text{C}$ , the daily gas usage is 236 MW.

The ten temperatures measured were:  $0^{\circ}$ ,  $0^{\circ}$ ,  $0^{\circ}$ ,  $2^{\circ}$ ,  $5^{\circ}$ ,  $7^{\circ}$ ,  $8^{\circ}$ ,  $9^{\circ}$ ,  $9^{\circ}$ ,  $10^{\circ}$ .

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point  $(\bar{x}, \bar{y})$ , where  $\bar{x}$  is the sample mean of the  $x$ -values and  $\bar{y}$  is the sample mean of the  $y$ -values.

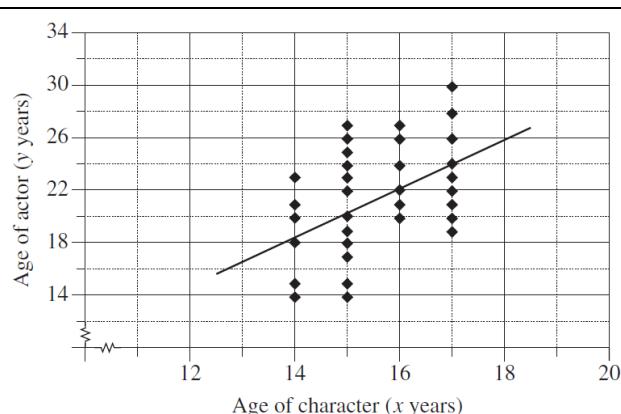
- Using the information provided, plot the point  $(\bar{x}, \bar{y})$  and the  $y$ -intercept of the least-squares regression line on the grid.
- What is the equation of the regression line?
- In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is  $23^{\circ}\text{C}$ .

**3****2****1**

**COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations**

**22 24**

- MA** Jo is researching the relationship between the ages of teenage characters in television series and the ages of actors playing these characters. After collecting the data, Jo finds that the correlation coefficient is 0.4564. A scatterplot showing the data is drawn. The line of best fit with equation  $y = -7.51 + 1.85x$ , is also drawn. Describe and interpret the data and other information provided, with reference to the context given.

**4**

**COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations**

- 21 17 MA** For a sample of 17 inland towns in Australia, the height above sea level,  $x$  (metres), and the average maximum daily temperature,  $y$  ( $^{\circ}\text{C}$ ), were recorded.

The graph shows the data as well as a regression line. The equation of the regression line is  $y = 29.2 - 0.011x$ .

The correlation coefficient is  $r = -0.494$ .

- (a) (i) By using the equation of the regression line, predict the average maximum daily temperature, in degrees Celsius, for a town that is 540 m above sea level. Give your answer correct to one decimal place.  
(ii) The gradient of the regression line is  $-0.011$ . Interpret the value of this gradient in the given context.

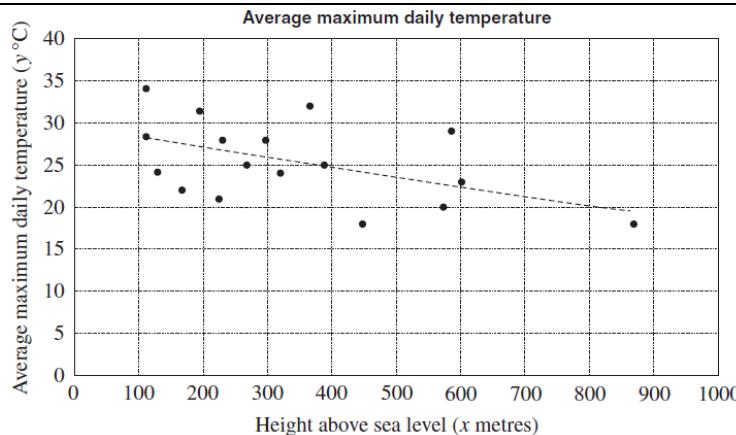
- (b) The graph shows the relationship between the latitude,  $x$  (degrees south), and the average maximum daily temperature,  $y$  ( $^{\circ}\text{C}$ ), for the same 17 towns, as well as a regression line.

The equation of the regression line is  $y = 45.6 - 0.683x$ .

The correlation coefficient is  $r = -0.897$ .

Another inland town in Australia is 540 m above sea level. Its latitude is 28 degrees south. Which measurement, height above sea level or latitude, would be better to use to predict this town's average maximum daily temperature? Give a reason for your answer.

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations



[Solution](#)

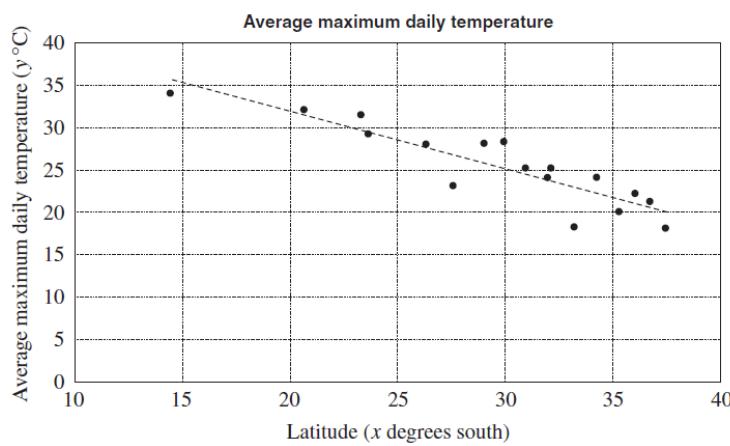
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**2**



**1**

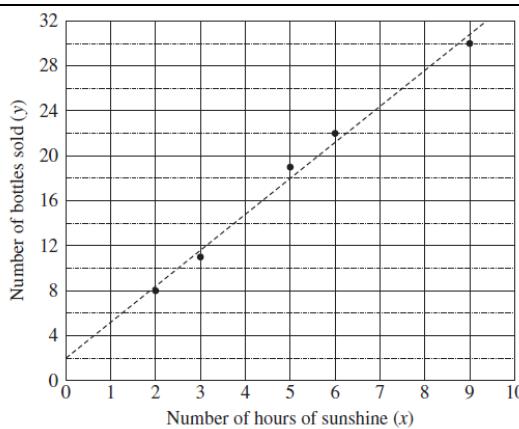


- 21 MS 2** **28** A salesperson is interested in the relationship between the number of bottles of lemonade sold per day and the number of hours of sunshine in the day.

The diagram shows the dataset used in the investigation and the least-squares regression line.

- (a) Find the equation of the least-squares regression line relating to the dataset.

- (b) Suppose a sixth data point was collected on a day which had 10 hours of sunshine. On that day 45 bottles of lemonade were sold. What would happen to the gradient found in part (a)?



[Solution](#)

**2**

**1**

NESA 2021 Mathematics Standard 2 HSC Examination

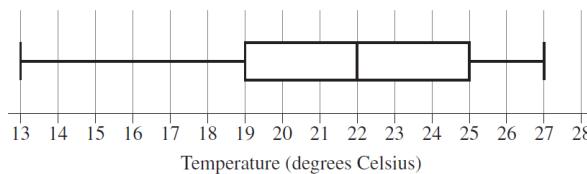
- 20 MA 27** A cricket is an insect. The male cricket produces a chirping sound. A scientist wants to explore the relationship between the temperature in degrees Celsius and the number of cricket chirps heard in a 15-second time interval. Once a day for 20 days, the scientist collects data.

**5** [Solution](#)



Based on the 20 data points, the scientist provides the information below.

- A box-plot of the temperature data is shown.
- The mean temperature in the dataset is  $0.525^{\circ}\text{C}$  below the median temperature in the dataset.
- A total of 684 chirps was counted when collecting the 20 data points.



The scientist fits a least-squares regression line using the data  $(x, y)$ , where  $x$  is the temperature in degrees Celsius and  $y$  is the number of chirps heard in a 15-second time interval. The equation of this line is  $y = -10.6063 + bx$ , where  $b$  is the slope of the regression line.

The least-squares regression line passes through the point  $(\bar{x}, \bar{y})$  where  $\bar{x}$  is the sample mean of the temperature data and  $\bar{y}$  is the sample mean of the chirp data. Calculate the number of chirps expected in a 15-second interval when the temperature is  $19^{\circ}\text{ Celsius}$ . Give your answer to the nearest whole number.

**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

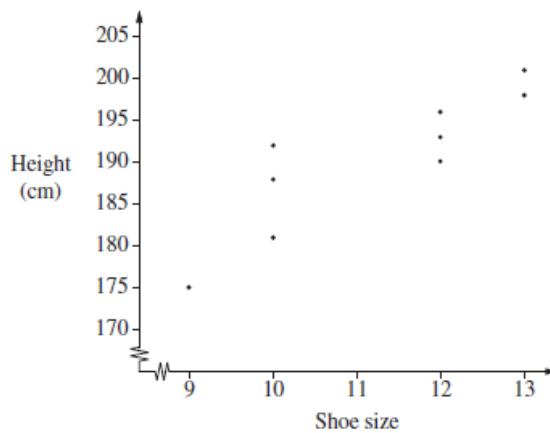
- MA** **37**  
**SP** **Band  
2-5** A set of bivariate data is collected by measuring the height and recording the shoe size of nine basketball players.

The collected data is shown in the table and graphed in the scatterplot shown.

- Determine the equation of the least-squares regression line for this data.
- A player with a shoe size of 11 was absent when the data was collected.

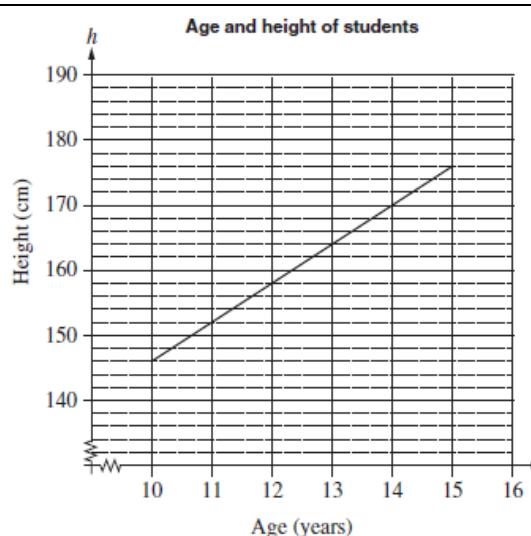
Calculate the predicted height for the missing basketball player using your answer to part (a).

<i>Shoe size</i>	10	12	12	12	10	9	13	10	13
<i>Height (cm)</i>	181	190	193	196	188	175	198	192	201

**Solution****1****1**

- MS** **SA**  
**SQ** **0** Data collected on the age ( $a$ ) and the height ( $h$ ) of 10- to 15-year-olds were used to create a scatterplot. A line of best fit to model the relationship between the age and height of students was then constructed as shown.

- Determine the equation of the line of best fit shown.
- Based on the line of best fit, what is the height of a typical 15-year-old?
- Why would this model NOT be useful for predicting the height of a typical 35-year-old?

**Solution****3****1****1**

NESA Mathematics Standard 2 Sample examination materials

- TG** **1** A table of data is given as follows:

- Construct a scatterplot.
- Compute the correlation coefficient,  $r$ .
- Compute the coefficients of the linear regression line,  $y = mx + c$ .
- What is the estimated value for  $y$  when  $x = 7$ .

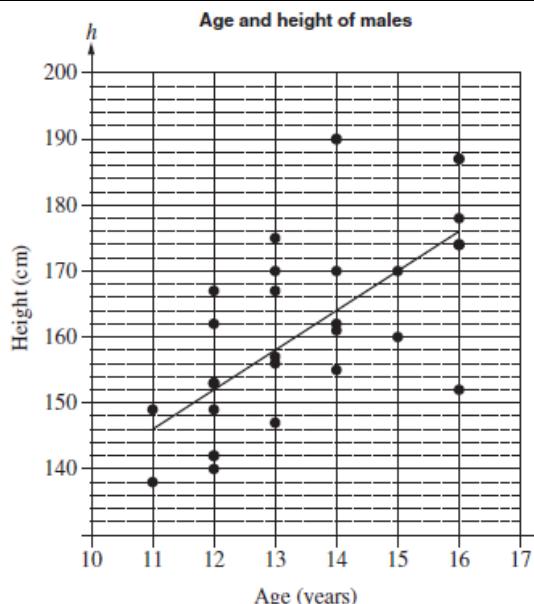
$x$	1	2	3	2	4	5	2	8	9
$y$	16	23	35	28	44	40	22	61	82

**Solution**

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

- TG 2** Ahmed collected data on the age ( $a$ ) and height ( $h$ ) of males aged 11 to 16 years. He created a scatterplot of the data and constructed a line of best fit to model the relationship between the age and height of males.

- Determine the gradient of the line of best fit shown on the graph.
- Explain the meaning of the gradient in the context of the data.
- Determine the equation of the line of best fit shown on the graph.
- Use the line of best fit to predict the height of a typical 17-year-old male.
- Why would this model not be useful for predicting the height of a typical 45-year-old male?

[Solution](#)

- 1  
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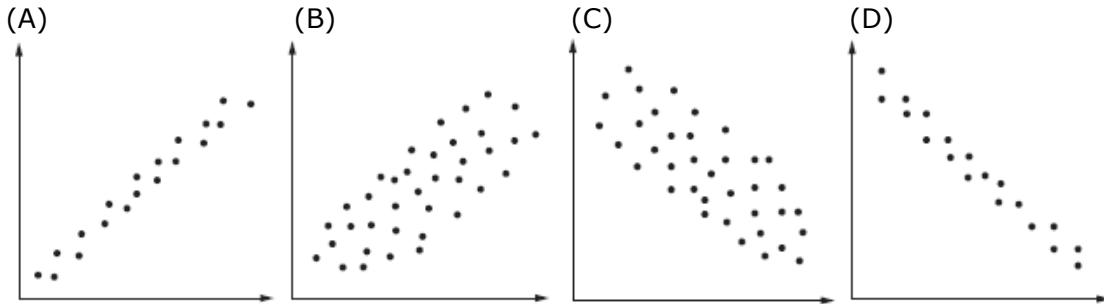
- TG 3** The height of 10 high school students and the length of their right foot were measured. The results were tabulated as follows:

- Using technology, calculate the Pearson correlation coefficient for the data.
- Describe the strength of the association between height and length of the right foot for this dataset.

Height (cm)	165	153	146	138	149	172	170	158	163	154
Right foot (cm)	26	21	20	19	22	24	25	23	22	25

[Solution](#)

- MS SA 7** Which of the following scatter graphs is most likely to have a correlation coefficient of  $-0.2$ ?



1

[Solution](#)

- MS SA 18** A student claimed that as study time increases, test scores increase. After collecting and analyzing some data, the student found the correlation coefficient,  $r$ , to be  $0.83$ .

What does this correlation indicate about the relationship between time spent on study and test scores?

1

[Solution](#)

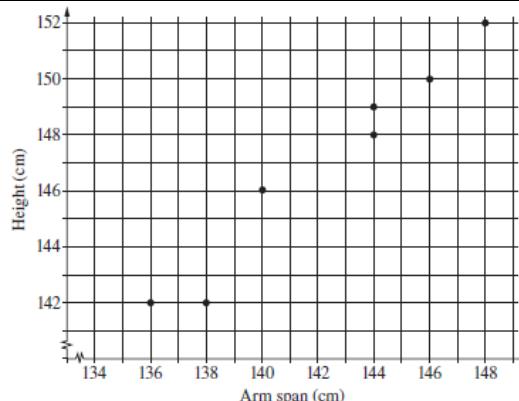
NESA Mathematics Standard 2 Sample examination materials

- 19 MS 2** **23** A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatterplot of these measurements.

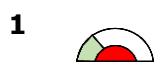
- Calculate Pearson's correlation for the data, correct to two decimal places.
- Identify the direction and the strength of the linear association between the height and arm span.
- The equation of the least-squares regression line is shown.  
 $\text{Height} = 0.866 \times (\text{arm span}) + 23.7$

A child has an arm span of 143 cm.

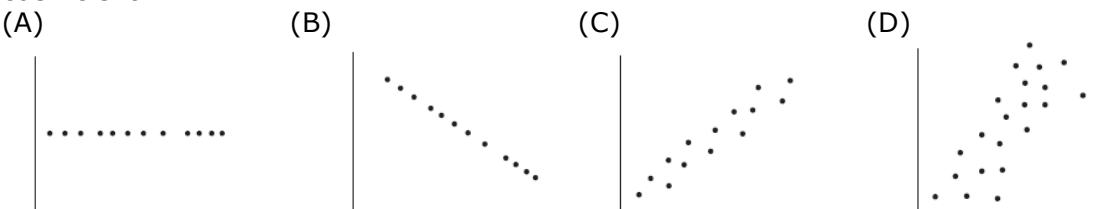
Calculate the predicted height for this child using the equation of the least-squares regression line.



[Solution](#)



- 17 MG** **12** Which of the data sets graphed below has the largest positive correlation coefficient?

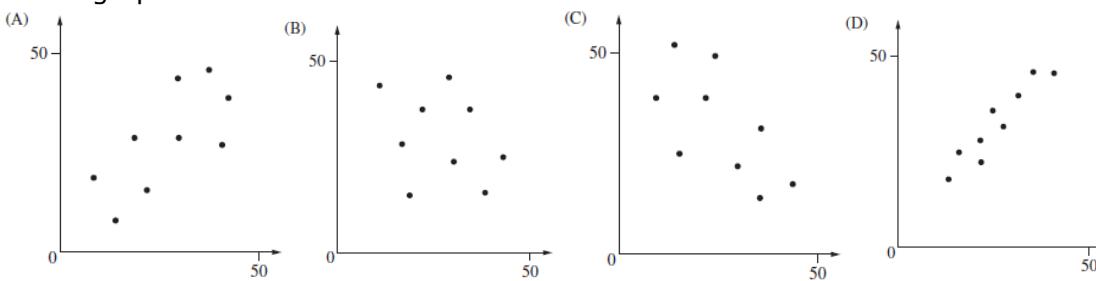


NESA 2019 Mathematics Standard 2 HSC Examination

[Solution](#)



- 13 MG** **2** Which graph best shows data with a correlation closest to 0.3?



NESA 2013 Mathematics General HSC Examination

[Solution](#)



- 12 MG** **11** Which of the following relationships would most likely show a negative correlation?

- The population of a town and the number of hospitals in that town.
- The hours spent training for a race and the time taken to complete the race.
- The price per litre of petrol and the number of people riding bicycles to work.
- The number of pets per household and the number of computers per household

[Solution](#)



NESA 2012 Mathematics General HSC Examination

# Year 12: Statistical Analysis

## S3.1 Continuous Random Variables



**Syllabus:** updated November 2019. Latest version @

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand and use the concepts of a probability density function of a continuous random variable **AAM**
  - know the two properties of a probability density function:  $f(x) \geq 0$  for all real  $x$  and  $\int_{-\infty}^{\infty} f(x)dx = 1$
  - define the probability as the area under the graph of the probability density function using the notation  $P(X \leq r) = \int_a^r f(x)dx$ , where  $f(x)$  is the probability density function defined on  $[a, b]$
  - examine simple types of continuous random variables and use them in appropriate contexts
  - explore properties of a continuous random variable that is uniformly distributed
  - find the mode from a given probability density function
- obtain and analyse a cumulative distribution function with respect to a given probability density function
  - understand the meaning of a cumulative distribution function with respect to a given probability density function
  - use a cumulative distribution function to calculate the median and other percentiles

[Reference Sheet](#)

**23 29** A continuous random variable  $X$  has probability density function  $f(x)$  given by

**MA**

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of } x \end{cases}$$

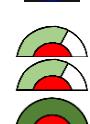
- Find the mode of  $X$ .
- Find the cumulative distribution function for the given probability density function.
- Without calculating the median, show that the mode is greater than the median.

**2**

**2**

**2**

[Solution](#)

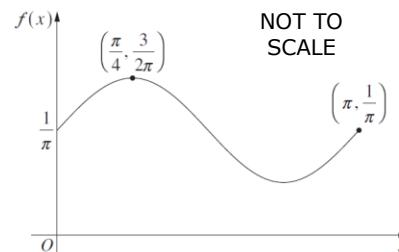


NESA 2023 Mathematics Advanced HSC Examination

**22 7** Consider the following graph of a probability density function  $f(x)$ .

What is the value of the mode?

- A.  $\frac{1}{\pi}$       B.  $\frac{3}{2\pi}$   
     C.  $\frac{\pi}{4}$       D.  $\pi$



**1**

[Solution](#)



NESA 2022 Mathematics Advanced HSC Examination

- 22 30** A continuous random variable  $X$  has cumulative distribution function given by:

- (a) Show that  $k = 3$ .  
 (b) Given that  $P(X < c) = 2P(X > c)$ , find the exact value of  $c$ .

$$F(x) = \begin{cases} 1 & x > e^3 \\ \frac{1}{k} \ln x & 1 \leq x \leq e^3 \\ 0 & x < 1 \end{cases}$$



1  
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NESA 2022 Mathematics Advanced HSC Examination

- 21 30** The number of hours for which light bulbs will work before failing can be modelled by the random variable  $X$  with cumulative distribution function.

**2** [Solution](#)

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

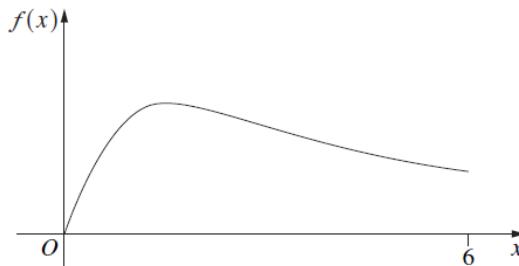
Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs. Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal places

NESA 2021 Mathematics Advanced HSC Examination

- 21 33** People are given a maximum of six hours to complete a puzzle. The time spent on the puzzle, in hours, can be modelled using the continuous random variable  $X$  which has probability density function

$$f(x) = \begin{cases} \frac{Ax}{x^2 + 4}, & \text{for } 0 \leq x \leq 6, (\text{where } A > 0) \\ 0, & \text{for all other values of } x \end{cases}$$

The graph of the probability density function is shown below.  
 The graph has a local maximum.



- (a) Show that  $A = \frac{2}{\ln 10}$ .  
 (b) Show that the mode of  $X$  is two hours.  
 (c) Show that  $P(X < 2) = \log_{10} 2$ .  
 (d) The Intelligence Quotient (IQ) scores of people are normally distributed with a mean of 100 and standard deviation of 15. It has been observed that the puzzle is generally completed more quickly by people with a high IQ. It is known that 80% of people with an IQ greater than 130 can complete the puzzle in less than two hours. A person chosen at random can complete the puzzle in less than two hours.

2  
2  
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2

What is the probability that this person has an IQ greater than 130? Give your answer correct to three decimal places.

NESA 2020 Mathematics Advanced HSC Examination

**20 23** A continuous random variable,  $X$ , has the following probability density functions. [Solution](#)

**MA**

$$f(x) = \begin{cases} \sin x & \text{for } 0 \leq x \leq k \\ 0 & \text{for all other values of } k \end{cases}$$

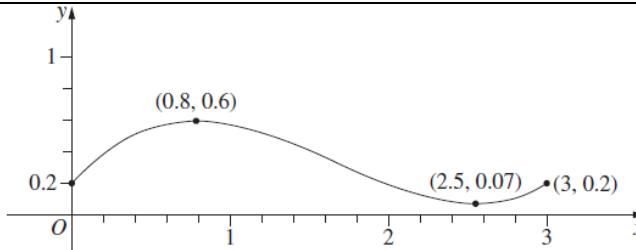
(a) Find the value of  $k$ . 2 (b) Find  $P(X \leq 1)$ . Give your answer correct to four decimal places. 2 

NESA 2020 Mathematics Advanced HSC Examination

**MA** **7** The diagram shows the graph of a continuous probability density function.

Which of the following is the mode?

- A. 0.07      B. 0.6  
C. 0.8      D. 3



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA** **31** A bid made at an auction for a real estate property, in millions of dollars, can be modelled by the random variable  $X$  with the probability density function

$$f(x) = \begin{cases} k(16 - x^2) & 1 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that the value of  $k$  is  $\frac{1}{27}$ . 2 (b) Find the cumulative distribution function. 2 (c) Find the probability that a bid of more than 3 million dollars will be made. 1 

NESA Mathematics Advanced Sample Examination Paper (2020)

**MA** **13** A continuous random variable  $X$  has a probability density function  $f$  given by

**SQ**  
**2019**

$$f(x) = \begin{cases} Ax + B & 1 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

where  $A$  and  $B$  are constants.The median of  $X$  is 2.Find the values of  $A$  and  $B$ .

NESA Mathematics Advanced Sample examination materials (2019)

**TG** **1** Let  $X$  be a continuous random variable whose probability density function is  $f(x) = 3x^2$  for  $0 < x < 1$ . [Solution](#)

(a) What is the probability that  $X$  falls between  $\frac{1}{2}$  and 1? (b) Show that  $P(X = \frac{1}{2}) = 0$ . 

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

**TG** **2** Let  $X$  be a continuous random variable whose probability density function is  $f(x) = \frac{x^3}{4}$  for an interval  $0 < x < c$ .

[Solution](#)What is the value of the constant  $c$  that makes  $f(x)$  a valid probability density function?

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

**TG** **3** Let  $X$  be a continuous random variable with the following probability density function:

$$f(x) = \frac{1}{2} \text{ for } 0 < x < 2.$$

[Solution](#)What are the lower quartile, median and upper quartile of  $X$ ?

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

# Year 12: Statistical Analysis

## S3.2 The normal Distribution



**Syllabus: updated November 2019. Latest version @**

<https://educationstandards.nsw.edu.au/wps/portal/nesa/11-12/stage-6-learning-areas/stage-6-mathematics/mathematics-advanced-2017>

Students:

- identify the numerical and graphical properties of data that is normally distributed ⓘ
- calculate probabilities and quantiles associated with a given normal distribution using technology and otherwise, and use these to solve practical problems (ACMMM170) AAM ⚙️💻
  - identify contexts that are suitable for modelling by normal random variables, eg the height of a group of students (ACMMM168)
  - recognise features of the graph of the probability density function of the normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , and the use of the standard normal distribution (ACMMM169)
  - visually represent probabilities by shading areas under the normal curve, eg identifying the value above which the top 10% of data lies
- understand and calculate the  $z$ -score (standardised score) corresponding to a particular value in a dataset AAM ⓘ
  - use the formula  $z = \frac{x-\mu}{\sigma}$ , where  $\mu$  is the mean and  $\sigma$  is the standard deviation 💻
  - describe the  $z$ -score as the number of standard deviations a value lies above or below the mean
- use  $z$ -scores to compare scores from different datasets, for example comparing students' subject examination scores AAM ⓘ
- use collected data to illustrate the empirical rules for normally distributed random variables ⓘ
  - apply the empirical rule to a variety of problems
  - sketch the graphs of  $f(x) = e^{-x^2}$  and the probability density function for the normal distribution  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$  using technology 💻
  - verify, using the Trapezoidal rule, the results concerning the areas under the normal curve
- use  $z$ -scores to identify probabilities of events less or more extreme than a given event AAM ⓘ
  - use statistical tables to determine probabilities 📈
  - use technology to determine probabilities 💻
- use  $z$ -scores to make judgements related to outcomes of a given event or sets of data AAM ⓘ

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[Reference Sheet](#)

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**23 23**

- MA** A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below  $z$  for some positive values of  $z$ .

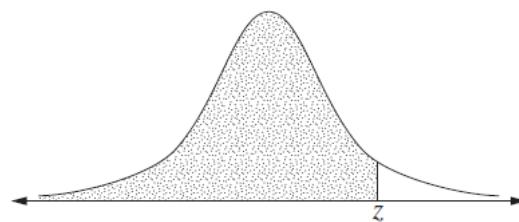
**4****Solution**

$z$	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.

The weights of adult male koalas form a normal distribution with mean  $\mu = 10.40$  kg, and standard deviation  $\sigma = 1.15$  kg.

In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?



**COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations**

**22 26**

- MA** The life span of batteries from a particular factory is normally distributed with a mean of 840 hours and a standard deviation of 80 hours. It is known from statistical tables that for this distribution approximately 60% of the batteries have a life span of less than 860 hours. What is the approximate percentage of batteries with a life span between 820 and 920 hours?

**3****Solution**

**COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations**

**22 13****MS 2**

- A random variable is normally distributed with mean 0 and standard deviation 1.

The table gives the probability that this random variable lies below  $z$  for some positive values of  $z$ .

The probability values given in the table are represented by the shaded area in the following diagram.

What is the probability that a normally distributed random variable with mean 0 and standard deviation 1 lies between 0 and 1.94?

A. 0.0262

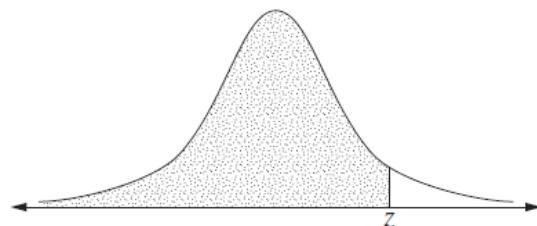
B. 0.4738

C. 0.5262

D. 0.9738

**1****Solution**

$z$	1.90	1.91	1.92	1.93	1.94	1.95	1.96	1.97	1.98	1.99
Probability	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767



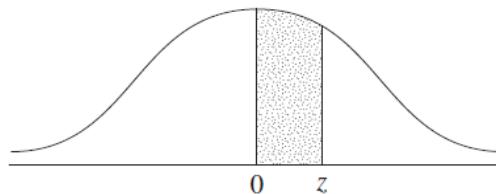
NESA 2022 Mathematics Standard 2 HSC Examination

**21 22** A random variable is normally distributed with mean 0 and standard deviation 1. [Solution](#)

**MA** The table gives the probability that this random variable lies between 0 and  $z$  for different values of  $z$ .

$z$	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of  $z$  are represented by the shaded area in the following diagram.

**1**

- (a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1 lies between 0.1 and 0.5.

- (b) Birth weights are normally distributed with a mean of 3300 grams and a standard deviation of 570 grams. By first calculating a  $z$ -score, find how many babies, out of 1000 born, are expected to have a birth weight greater than 3528 grams.

**3**

**COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations**

**21 32** In a particular city, the heights of adult females and the heights of adult males are each normally distributed. [Solution](#)

Information relating to two females from that city is given in Table 1.

Table 1

Height	Gender	Percentage of females in this city shorter than this person
175 cm	Female	97.5%
160.6 cm	Female	16%

The means and standard deviations of adult females and males, in centimetres, are given in Table 2.

Table 2

	Mean	Standard deviation
Females	$\mu$	$\sigma$
Males	$1.05\mu$	$1.1\sigma$

A selected male is taller than 84% of the population of adult males in this city. By first labelling the normal distribution curve with the heights of the two females given in Table 1, calculate the height of the selected male, in centimetres, correct to two decimal places.

**COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations**

**20 3** John recently did a class test in each of three subjects. The class scores on each test are normally distributed. [Solution](#)

The table shows the subjects and John's scores as well as the mean and standard deviation of the class scores on each test.

Relative to the rest of the class, which row of the table below shows John's strongest subject and his weakest subject?

Subject	John's score	Mean	Standard deviation
French	82	70	8
Commerce	80	65	5
Music	74	50	12

**1**

	Strongest subject	Weakest subject
A.	Commerce	French
B.	French	Music
C.	Music	French
D.	Commerce	Music

**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

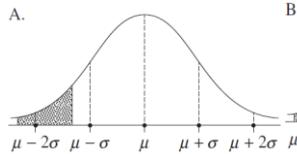
- 20 9 MA** Suppose the weight of melons is normally distributed with a mean of  $\mu$  and a standard deviation of  $\sigma$ .

**1** [Solution](#)

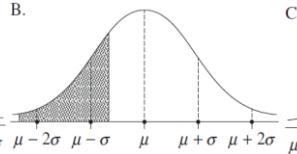
A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?

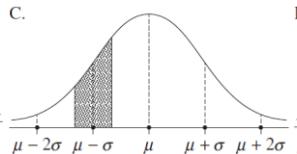
A.



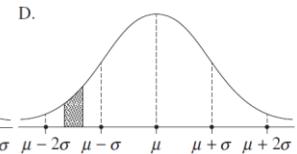
B.



C.



D.



NESA 2020 Mathematics Advanced HSC Examination

- 20 28 MA** In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of \$25 and a standard deviation of \$5.

[Solution](#)**3**

- (a) Two adults who both work are chosen at random.

Find the probability that at least one of them earns between \$15 and \$30 per hour.

- (b) The number of adults who work is equal to three times the number of adults who do not work.

**2**

One adult is chosen at random.

Find the probability that the chosen adult works and earns more than \$25 per hour.

NESA 2020 Mathematics Advanced HSC Examination

- 20 MS 35 2** The Intelligence Quotient (IQ) scores for adults in City A are normally distributed with a mean of 108 and a standard deviation of 10.

[Solution](#)

The IQ scores for adults in City B are normally distributed with a mean of 112 and a standard deviation of 16.

- (a) Yin is an adult who lives in City A and has an IQ score of 128.

**2**

What percentage of the adults in this city have an IQ score higher than Yin's?

- (b) There are 1 000 000 adults living in City B. Calculate the number of adults in City B that would be expected to have an IQ score lower than Yin's?

**2**

- (c) Simon, an adult who lives in City A, moves to City B. The z-score corresponding to his IQ score in City A is the same as the z-score corresponding to his IQ score in City B.

**3**

By first forming an equation, calculate Simon's IQ score. Give your answer correct to one decimal place.

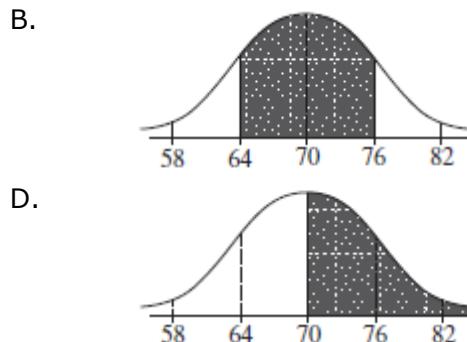
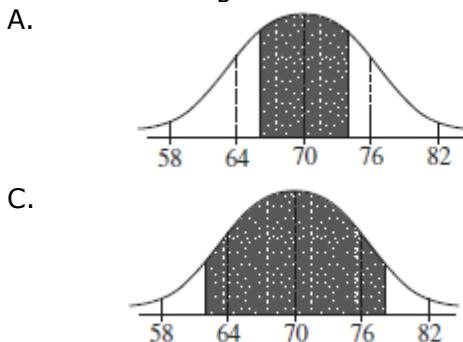
NESA 2020 Mathematics Standard 2 HSC Examination

- MA 9 SP Band 5-6** The scores on an examination are normally distributed with a mean of 70 and a standard deviation of 6. Michael received a score on the examination between the lower quartile and the upper quartile of his scores.

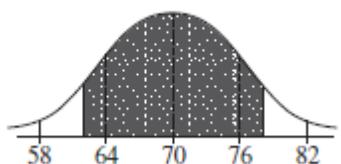
Which shaded region most accurately represents where Michael's score lies?

**1**

**19 MS 2** **15**



D.



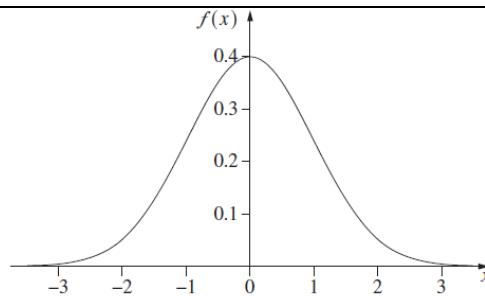
NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

- MA 29 SP Band 3-5** Let  $X$  denote a normal random variable with mean 0 and standard deviation 1. The random variable  $X$  has a probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \text{ where}$$

$$-\infty < x < \infty.$$

The diagram shows the graph of  $y = f(x)$ .



- (a) Complete the table of values for the function given.

Give your answer correct to four significant figures.

$X = x$	0	1	2	3
$f(x)$	0.3989	0.242		0.004432

- (b) Using the trapezoidal rule and the 4 function values in the table in part (a),

$$\text{show that } P(-3 \leq X \leq 3) = \int_{-3}^3 f(x) dx \approx 0.9953.$$

**1**

- (c) The IQ (Intelligence Quotient) scores for a large population are normally distributed with a mean of 100 and a standard deviation of 15.

Using the result obtained in part (b), calculate the probability of randomly selecting a person with an IQ score above 145 from this large population.

**2**

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

<b>MA SQ 2019</b>	<b>15</b> <b>Band 2-5</b>	<p>The diastolic measurement for blood pressure in 50-year-old people is normally distributed, with a mean of 85 and a standard deviation of 20.</p> <p>(a) A person is considered to have low blood pressure if the diastolic measurement is 65 or less. What percentage of 50-year-old people have low blood pressure?</p> <p>(b) Calculate the z-score for a diastolic measurement of 60.</p> <p>(c) The probability that a 50-year-old person has a diastolic measurement for blood pressure between 60 and 65 can be found by evaluating <math>\int_a^b f(x) dx</math> where <math>a</math> and <math>b</math> are constants and where <math>f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}</math> is the normal probability density function with mean 0 and standard deviation 1.</p> <p>By first finding the values of <math>a</math> and <math>b</math>, calculate an approximate value for this probability by using the trapezoidal rule with 3 function values.</p> <p>(d) Using the results above, find the approximate probability that a 50-year-old person chosen at random has a diastolic measurement for blood pressure of 60 or less.</p>	<a href="#">Solution</a>
			
			

NESA Mathematics Advanced Sample examination materials (2019)

<b>TG 1</b>	Packets of rice are each labelled as having a mass of 1 kg. The mass of these packets is normally distributed with a mean of 1.06 kg and a standard deviation of 0.03 kg.	<table border="1" style="margin-left: auto; margin-right: 0;"> <tr> <td>Mass in kg</td><td>1.00</td><td>1.03</td><td>1.06</td><td>1.09</td><td>1.12</td></tr> <tr> <td>z-score</td><td></td><td></td><td>0</td><td>1</td><td></td></tr> </table>	Mass in kg	1.00	1.03	1.06	1.09	1.12	z-score			0	1		
Mass in kg	1.00	1.03	1.06	1.09	1.12										
z-score			0	1											

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

<b>TG 2</b>	A machine is set for the production of cylinders. The diameters of the cylinders produced are normally distributed with a mean of 5.00 cm and a standard deviation 0.020 cm.	<a href="#">Solution</a>
	<p>(a) Within what interval will 95% of the diameters lie?</p> <p>(b) If a cylinder, randomly selected from this production, has a diameter of 5.070 cm, what conclusion could be drawn?</p>	
		

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

<b>TG 3</b>	The number of runs scored by Amira in each of her innings is listed below: 34, 33, 31, 33, 32, 32, 33, 31, 33, 33.	<a href="#">Solution</a>
	<p>(a) What was her batting average correct to two decimal places?</p>	
	<p>(b) What was her population standard deviation correct to two decimal places?</p>	
	<p>(c) What was her z-score in her final innings, correct to two decimal places?</p>	
	<p>(d) What was the z-score of her highest score, correct to two decimal places?</p>	

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

**TG 4** Most IQ tests have means of 100 and standard deviations of 15.

[Solution](#)

If you take an IQ test and get the score of 125 are you super-smart or just mediocre?

Explain your answer with careful mathematical reasoning and justification.

*Projectmaths has provided this Cumulative Normal Distribution Table below:*

<b><i>z</i></b>	<b>.0</b>	<b>.1</b>	<b>.2</b>	<b>.3</b>	<b>.4</b>	<b>.5</b>	<b>.6</b>	<b>.7</b>	<b>.8</b>	<b>.9</b>
<b>0.</b>	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
<b>1.</b>	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
<b>2.</b>	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
<b>3.</b>	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

NESA Mathematics Advanced Year 12 Topic Guide: Statistical Analysis

**TG 5** Find the probability that a person selected at random from a pool of people that took a test on which the mean was 100 and the standard deviation was 15 will have a score:

[Solution](#)

- (a) between 100 and 120.
- (b) of at least 120.
- (c) of greater than 120.

*Projectmaths has provided this Cumulative Normal Distribution Table below:*

<b><i>z</i></b>	<b>.0</b>	<b>.1</b>	<b>.2</b>	<b>.3</b>	<b>.4</b>	<b>.5</b>	<b>.6</b>	<b>.7</b>	<b>.8</b>	<b>.9</b>
<b>0.</b>	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
<b>1.</b>	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
<b>2.</b>	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
<b>3.</b>	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

NESA Mathematics Advanced Year 12 Topic Guide: Statistical analysis

**TG 6** The lifetime of a particular make of lightbulb is normally distributed with mean

[Solution](#)

1020 hours and standard deviation 85 hours.

Find the probability that a lightbulb of the same make chosen at random has a lifetime between 1003 and 1088 hours.

*Projectmaths has provided this Cumulative Normal Distribution Table below:*

<b><i>z</i></b>	<b>.0</b>	<b>.1</b>	<b>.2</b>	<b>.3</b>	<b>.4</b>	<b>.5</b>	<b>.6</b>	<b>.7</b>	<b>.8</b>	<b>.9</b>
<b>0.</b>	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
<b>1.</b>	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
<b>2.</b>	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
<b>3.</b>	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

NESA Mathematics Standard 2 Year 12 Topic Guide: Statistical analysis

- TG 7** A section of highway is undergoing roadworks and a speed limit of 80 km/h has been imposed. Police will use a mobile speed camera to detect the speed of vehicles passing through this section of highway. An infringement notice will be issued to the driver if the speed limit is detected to be over 80 km/h.

[Solution](#)

From data collected from similar roadworks it is known that:

- the speeds of vehicles are normally distributed with mean,  $\mu$ , and standard deviation,  $\sigma$
- 12.3% of vehicles have speeds of more than 82.64 km/h
- 24.2% of vehicles have speeds of less than 75.2 km/h.

(a) Find the values of  $\mu$  and  $\sigma$ .

(b) A vehicle passes the mobile speed camera. What is the probability that an infringement notice will be issued to the driver?



*Projectmaths has provided this Cumulative Normal Distribution Table extract below:*

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
...	...	...	...	...	...	...	...	...	...	...
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015

NESA Mathematics Standard 2 Year 12 Topic Guide: Statistical analysis

- MA SQ 3 Band 3-4** The number of hours working during a week by casual staff in the hospitality industry is normally distributed with a mean of 18 hours and a standard deviation of 2.5 hours. What is the percentage of casual staff working fewer than 23 hours in a week? **1** [Solution](#)

- A. 47.5%      B. 84%      C. 95%      D. 97.5%

NESA Mathematics Advanced Sample examination materials (2019)



- MS SA 9** In a class, the shoe sizes of the students are normally distributed. The mean shoe size of these students is 6.5 and the standard deviation is 2. Approximately what percentage of the class has a shoe size larger than 8.5? **1** [Solution](#)

- A. 16%      B. 34%      C. 68%      D. 84%

NESA Mathematics Standard 2 Sample examination materials



- MS SA 13** There will be 90 000 students sitting an examination. If their examination results form a normal distribution, how many students would be expected to score a result between 1 and 2 standard deviations above the mean? **1** [Solution](#)

- A. 12 150      B. 24 300      C. 30 600      D. 42 750

NESA Mathematics Standard 2 Sample examination materials



- MS SA 19** In Broken Hill, the maximum temperature for each day is recorded. The mean of these maximum temperatures during spring is  $25.8^{\circ}\text{C}$ , and the standard deviation is  $4.2^{\circ}\text{C}$ . You may assume that these maximum temperatures are normally distributed.
- (a) What temperature has a z-score of -1?  
 (b) What percentage of spring days in Broken Hill would have maximum temperatures between  $21.6^{\circ}\text{C}$  and  $38.4^{\circ}\text{C}$ ? **2** [Solution](#)

NESA Mathematics Standard 2 Sample examination materials





NESA 2016 Mathematics General HSC Examination



- 16 MG** **30d** The formula to calculate z-scores can be rearranged to give  $\bar{x} = x - sz$  where  $\bar{x}$  is the mean,  $x$  is the score,  $s$  is the standard deviation and  $z$  is the z-score.

## **Solution**

- (i) In an examination, Aaron achieved a score of 88, which corresponds to a z-score of 2.4.

Substitute these values into the rearranged formula above to form an equation.

- (ii) In the same examination, Brock achieved a score of 52, which corresponds to a  $z$ -score of -1.2.

Using this information, form another equation and solve it simultaneously

with the equation from part (i) to find the values of  $\bar{x}$  and  $s$ .

NESA 2016 Mathematics General HSC Examination



NESA 2015 Mathematics General HSC Examination



- 15 MG 28b** The results of two tests are normally distributed. The mean and standard deviation for each test are displayed in the table. Kristoff scored 74 in Mathematics and 80 in English.

	Mathematics	English
$\bar{x}$	70	75
$s$	6.5	8

He claims that he has performed better in English. Is Kristoff correct? Justify your answer using appropriate calculations.

NESA 2015 Mathematics General HSC Examination

- 12 MG 29b** A machine produces nails. When the machine is set correctly, the length of the nails are normally distributed with a mean of 6.000 cm and a standard deviation of 0.040 cm/ To confirm the setting of the machine, three nails are randomly selected. In one sample the lengths are 5.950, 5.983 and 6.140. The setting of the machine needs to be checked when the lengths of two or more nails in a sample lie more than 1 standard deviation from the mean. Does the setting on the machine need to be checked? Justify your answer with suitable calculations.

NESA 2012 Mathematics General HSC Examination



- 11 MG 27c** Two brands of light bulbs are being compared. For each brand, the life of the light bulbs is normally distributed.

(i) One of the Brand B light bulbs has a life of 400 hours. What is the z-score of the life of this light bulb?

(ii) A light bulb is considered defective if it lasts less than 400 hours. The following claim is made:

'Brand A light bulbs are more likely to be defective than Brand B light bulbs.'

Is this claim correct? Justify your answer, with reference to z-scores or standard deviations or the normal distribution.

Life of light bulbs (in hours)		
	Mean	Standard deviation
Brand A	450	25
Brand B	500	50

[Solution](#)

1



2



- 07 MG 25d** The results of two class tests are normally distributed. The means and standard deviations of the tests are displayed in the table.

(i) Stuart scored 63 in Test 1 and 62 in Test 2. He thinks that he has performed better in Test 1. Do you agree? Justify your answer using appropriate calculations.

(ii) If 150 students sat for Test 2, how many students would you expect to have scored less than 64?

NESA 2011 Mathematics General HSC Examination

	Test 1	Test 2
Mean	60	58
Standard deviation	6.2	6.0

[Solution](#)

2



2



NESA 2007 Mathematics General HSC Examination



NSW Education Standards Authority

2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

## Mathematics Advanced

## Mathematics Extension 1

## Mathematics Extension 2

### REFERENCE SHEET

#### Measurement

##### Length

$$l = \frac{\theta}{360} \times 2\pi r$$

##### Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

##### Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

##### Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

#### Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For  $ax^3 + bx^2 + cx + d = 0$ :

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$a\beta + a\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

#### Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

#### Financial Mathematics

$$A = P(1 + r)^n$$

##### Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

#### Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

**Trigonometric Functions**

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

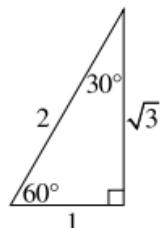
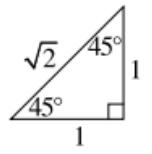
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

**Trigonometric identities**

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

**Compound angles**

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

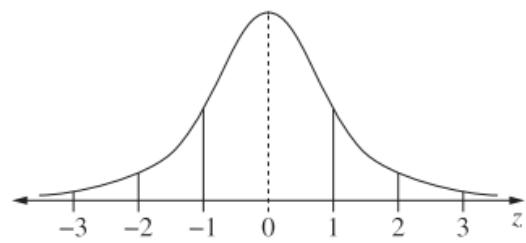
$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

**Statistical Analysis**

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score less than  $Q_1 - 1.5 \times IQR$  or more than  $Q_3 + 1.5 \times IQR$

**Normal distribution**

- approximately 68% of scores have  $z$ -scores between  $-1$  and  $1$
- approximately 95% of scores have  $z$ -scores between  $-2$  and  $2$
- approximately 99.7% of scores have  $z$ -scores between  $-3$  and  $3$

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

**Probability**

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

**Continuous random variables**

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

**Binomial distribution**

$$P(X = r) = {}^n C_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= {}^n C_x p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

**Differential Calculus****Function**

$$y = f(x)^n$$

**Derivative**

$$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

**Integral Calculus**

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where  $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2 \left[ f(x_1) + \dots + f(x_{n-1}) \right] \right\}$$

where  $a = x_0$  and  $b = x_n$

**Combinatorics**

$${}^n P_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1} x^{n-1} a + \cdots + \binom{n}{r} x^{n-r} a^r + \cdots + a^n$$

**Vectors**

$$|\underline{u}| = \left| x_1 \underline{i} + y_1 \underline{j} \right| = \sqrt{x_1^2 + y_1^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1 x_2 + y_1 y_2,$$

where  $\underline{u} = x_1 \underline{i} + y_1 \underline{j}$

and  $\underline{v} = x_2 \underline{i} + y_2 \underline{j}$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

**Complex Numbers**

$$z = a + ib = r(\cos \theta + i \sin \theta)$$

$$= r e^{i\theta}$$

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) \\ = r^n e^{in\theta}$$

**Mechanics**

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

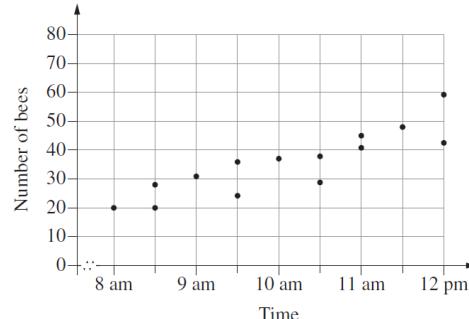
$$\ddot{x} = -n^2(x - c)$$

## 2023 HSC Paper

- 23 MA 1** The number of bees leaving a hive was observed and recorded over 14 days at different times of the day.

Which Pearson's correlation coefficient best describes the observations?

- A.  $-0.8$       B.  $-0.2$   
C.  $0.2$       D.  $0.8$

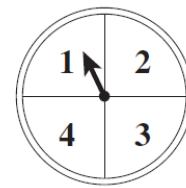
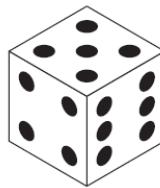
**1 Solution**

- 23 MA 2** A game involves throwing a die and spinning a spinner.

The sum of the two numbers obtained is the score. The table of scores below is partially completed.

What is the probability of getting a score of 7 or more?

- A.  $\frac{1}{6}$       B.  $\frac{1}{4}$   
C.  $\frac{5}{18}$       D.  $\frac{5}{12}$



		SPINNER			
		1	2	3	4
DIE	1	2	3	4	
	2	3	4	5	
	3		5	6	
	4			7	
	5				
	6				

COMMON QUESTION: NES 2023 Mathematics Standard 2 and Advanced HSC Examinations

- 23 MA 3** What is the domain of  $f(x) = \frac{1}{\sqrt{1-x}}$ ?

- A.  $x < 1$       B.  $x \leq 1$       C.  $x > 1$       D.  $x \geq 1$

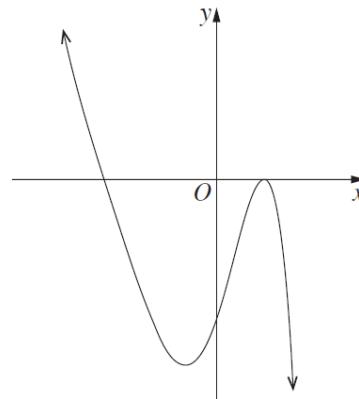
NES 2023 Mathematics Advanced HSC Examination

**1 Solution**

- 23 MA 4** The graph of a polynomial is shown.

Which row of the table is correct for this polynomial?

	Equation	Value of $b$	Value of $c$
A.	$y = -(x - b)(x - c)^2$	$b > 0$	$c < 0$
B.	$y = -(x - b)(x - c)^2$	$b < 0$	$c > 0$
C.	$y = -x(x - b)(x - c)$	$b > 0$	$c < 0$
D.	$y = -x(x - b)(x - c)$	$b < 0$	$c > 0$



NES 2023 Mathematics Advanced HSC Examination

**1 Solution**

- 23 MA 5** The diagram shows the graph  $y = f(x)$ , where  $f(x)$  is an odd function.

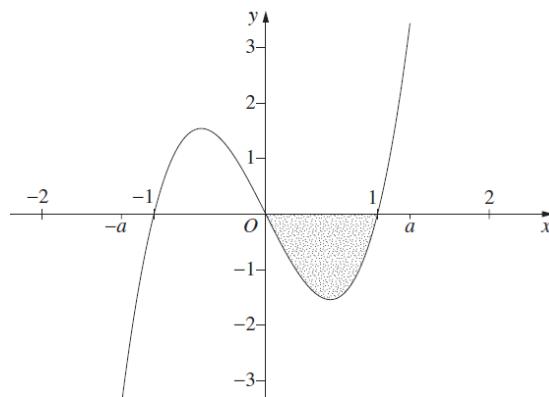
The shaded area is 1 square unit.

The number  $a$ , where  $a > 1$ , is chosen

so that  $\int_0^a f(x) dx = 0$ .

What is the value of  $\int_{-a}^1 f(x) dx$ ?

- A. -1      B. 0  
C. 1      D. 3



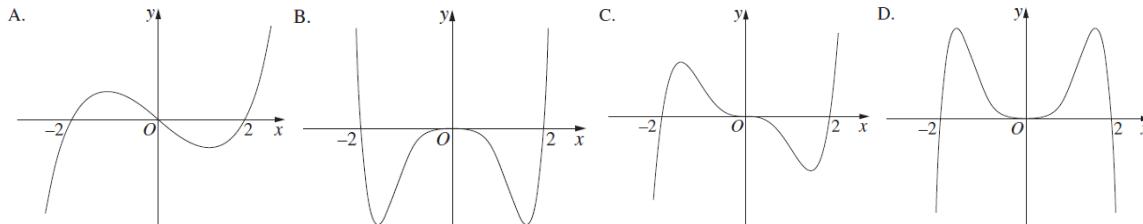
NOT TO SCALE

**1** **Solution**



- 23 MA 6** The following table gives the signs of the first and second derivatives of a function  $y = f(x)$  for different values of  $x$ .

Which of the following is a possible sketch of  $y = f(x)$ ?



NESA 2023 Mathematics Advanced HSC Examination

$x$	-2	0	2
$f'(x)$	+	0	+
$f''(x)$	-	0	+

**1** **Solution**



- 23 MA 7** It is given that  $y = f(g(x))$ , where  $f(1) = 3$ ,  $f'(1) = -4$ ,  $g(5) = 1$  and  $g'(5) = 2$ .

What is the value of  $y'$  at  $x = 5$ ?

- A. -8      B. -4      C. 3      D. 6

NESA 2023 Mathematics Advanced HSC Examination

**1** **Solution**



- 23 MA 8** What is the solution of the equation  $\log_a x^3 = b$ , where  $a$  and  $b$  are positive constants?

- A.  $x = b^{\frac{a}{3}}$       B.  $x = a^{\frac{b}{3}}$       C.  $x = \frac{b^a}{3}$       D.  $x = \frac{a^b}{3}$

NESA 2023 Mathematics Advanced HSC Examination

**1** **Solution**



- 23 MA 9** Let  $f(x)$  be any function with domain all real numbers.

Which of the following is an even function, regardless of the choice of  $f(x)$ ?

- A.  $2f(x)$       B.  $f(f(x))$       C.  $f(-x)^2$       D.  $f(x)f(-x)$

NESA 2023 Mathematics Advanced HSC Examination

**1** **Solution**



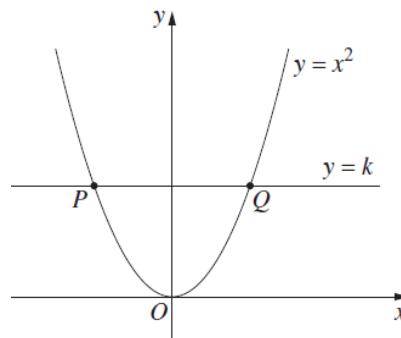
- 23 10** The graph  $y = x^2$  meets the line  $y = k$  (where  $k > 0$ ) at points  $P$  and  $Q$  as shown in the diagram.

The length of the interval  $PQ$  is  $L$ .

Let  $a$  be a positive number. The graph  $y = \frac{x^2}{a^2}$  meets the line  $y = k$  at points  $S$  and  $T$ .

What is the length of  $ST$ ?

- A.  $\frac{L}{a}$   
 B.  $\frac{L}{a^2}$   
 C.  $aL$   
 D.  $a^2L$



**1** [Solution](#)



- 23 11** The first three terms of an arithmetic sequence are 3, 7 and 11.  
**MA** Find the 15th term.

NESA 2023 Mathematics Advanced HSC Examination

**2** [Solution](#)



- 23 12** The table shows the probability distribution of a discrete random variable.  
**MA**
- Show that the expected value  $E(X) = 2$ .
  - Calculate the standard deviation, correct to one decimal place.

$x$	0	1	2	3	4
$P(X=x)$	0	0.3	0.5	0.1	0.1

[Solution](#)



NESA 2023 Mathematics Advanced HSC Examination

**1**



- 23 13** Let  $P(t)$  be a function such that  $\frac{dP}{dt} = 3000e^{2t}$ .

When  $t = 0$ ,  $P = 4000$ .

Find an expression for  $P(t)$ .

NESA 2023 Mathematics Advanced HSC Examination

**2** [Solution](#)



- 23 14** Find the equation of the tangent to the curve  $y = (2x + 1)^3$  at the point  $(0, 1)$ .  
**MA**

NESA 2023 Mathematics Advanced HSC Examination

**3** [Solution](#)



- 23 15** A table of future value interest factors for an annuity of \$1 is shown.

- (a) Micky wants to save \$450 000 over the next 10 years.

If the interest rate is 6% per annum compounding annually, how much should Micky contribute each year? Give your answer to the nearest dollar.

- (b) Instead, Micky decides to contribute \$8535 every three months for 10 years to an annuity paying 6% per annum, compounding quarterly.

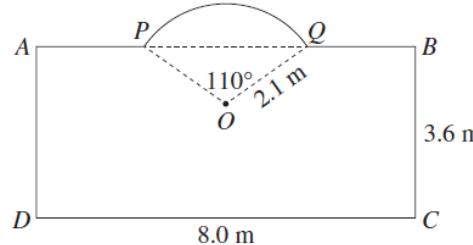
How much will Micky have at the end of 10 years?

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

- 23 16** The diagram shows a shape  $APQBCD$ . The shape consists of a rectangle  $ABCD$  with an arc  $PQ$  on side  $AB$  and with side lengths  $BC = 3.6$  m and  $CD = 8.0$  m.

The arc  $PQ$  is an arc of a circle with centre  $O$  and radius 2.1 m and  $\angle POQ = 110^\circ$ .

What is the perimeter of the shape  $APQBCD$ ?  
Give your answer correct to one decimal place.



NOT TO SCALE

**4** Solution

**2**



**3**



- 23 17**

**MA** Find  $\int x\sqrt{x^2 + 1} dx$ .

**2**



NESA 2023 Mathematics Advanced HSC Examination

**23 18**

- MA** A university uses gas to heat its buildings. Over a period of 10 weekdays during winter, the gas used each day was measured in megawatts (MW) and the average outside temperature each day was recorded in degrees Celsius ( $^{\circ}\text{C}$ ).



Using  $x$  as the average daily outside temperature and  $y$  as the total daily gas usage, the equation of the least-squares regression line was found.

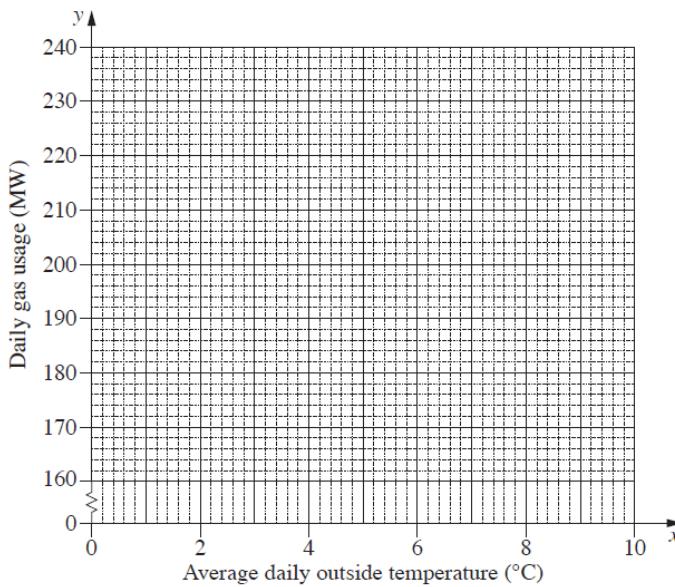
The equation of the regression line predicts that when the temperature is  $0^{\circ}\text{C}$ , the daily gas usage is 236 MW.

The ten temperatures measured were:  $0^{\circ}$ ,  $0^{\circ}$ ,  $0^{\circ}$ ,  $2^{\circ}$ ,  $5^{\circ}$ ,  $7^{\circ}$ ,  $8^{\circ}$ ,  $9^{\circ}$ ,  $9^{\circ}$ ,  $10^{\circ}$ .

The total gas usage for the ten weekdays was 1840 MW.

In any bivariate dataset, the least-squares regression line passes through the point  $(\bar{x}, \bar{y})$ , where  $\bar{x}$  is the sample mean of the  $x$ -values and  $\bar{y}$  is the sample mean of the  $y$ -values.

- Using the information provided, plot the point  $(\bar{x}, \bar{y})$  and the  $y$ -intercept of the least-squares regression line on the grid.
- What is the equation of the regression line?
- In the context of the dataset, identify ONE problem with using the regression line to predict gas usage when the average outside temperature is  $23^{\circ}\text{C}$ .

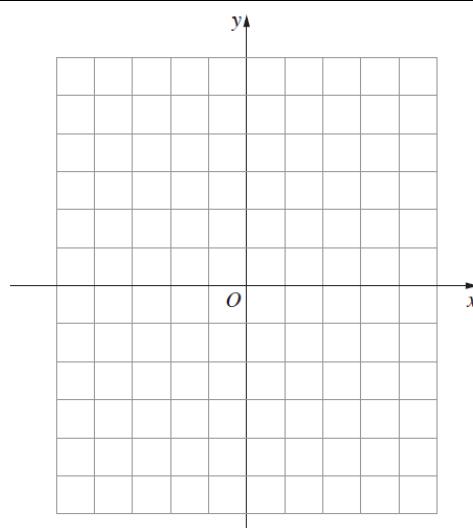
**3****2****1**

**COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations**

**23 19** (a) Sketch the graphs of the functions

**MA**  $f(x) = x - 1$  and  $g(x) = (1 - x)(3 + x)$  showing the  $x$ -intercepts.

- Hence, or otherwise, solve the inequality  $x - 1 < (1 - x)(3 + x)$ .

**2****2**

NESA 2023 Mathematics Advanced HSC Examination

**23 20**

- MA** Find all the values of  $\theta$ , where  $0^\circ \leq \theta \leq 360^\circ$ , such that  $\sin(\theta - 60^\circ) = -\frac{\sqrt{3}}{2}$ .

**3 Solution****23 21**

- MA** The fourth term of a geometric sequence is 48.

**3 Solution**

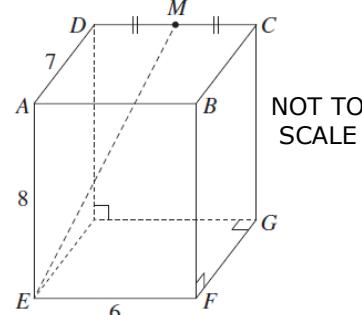
$$\text{The eighth term of the same sequence is } \frac{3}{16}.$$

Find the possible value(s) of the common ratio and the corresponding first term(s).

NESA 2023 Mathematics Advanced HSC Examination

**23 22** In the rectangular prism shown,  $AD = 7 \text{ cm}$ ,  $AE = 8 \text{ cm}$ ,**MA**  $EF = 6 \text{ cm}$ . Point  $M$  is the midpoint of  $CD$ .**3 Solution**

Find  $\angle AEM$ , to the nearest degree.



NESA 2023 Mathematics Advanced HSC Examination

**23 23**

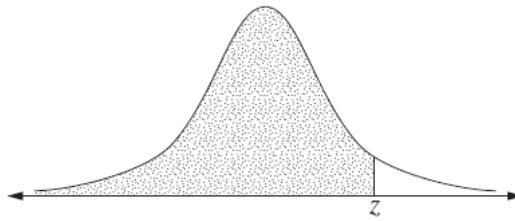
- MA** A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table gives the probability that this random variable lies below  $z$  for some positive values of  $z$ .

**4 Solution**

$z$	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

The probability values given in the table are represented by the shaded area in the following diagram.

The weights of adult male koalas form a normal distribution with mean  $\mu = 10.40 \text{ kg}$ , and standard deviation  $\sigma = 1.15 \text{ kg}$ .



In a group of 400 adult male koalas, how many would be expected to weigh more than 11.93 kg?

COMMON QUESTION: NESA 2023 Mathematics Standard 2 and Advanced HSC Examinations

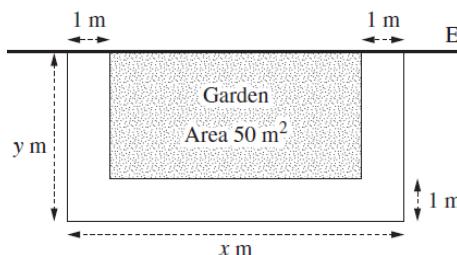
- 23 24 MA** A gardener wants to build a rectangular garden of area  $50 \text{ m}^2$  against an existing wall as shown in the diagram. A concrete path of width 1 metre is to be built around the other three sides of the garden.

Let  $x$  and  $y$  be the dimensions, in metres, of the outer rectangle as shown.

(a) Show that  $y = \frac{50}{x-2} + 1$ .

- (b) Find the value of  $x$  such that the area of the concrete path is a minimum.

Show that your answer gives a minimum area.

**Solution****1****4**

- 23 25 MA** On the first day of November, Jia deposits \$10 000 into a new account which earns 0.4% interest per month, compounded monthly. At the end of each month, after the interest is added to the account, Jia intends to withdraw \$ $M$  from the account. Let  $A_n$  be the amount (in dollars) in Jia's account at the end of  $n$  months.

(a) Show that  $A_2 = 10\ 000(1.004)^2 - M(1.004) - M$

**1**

(b) Show that  $A_n = (10\ 000 - 250M)(1.004)^n + 250M$ .

**3**

- (c) Jia wants to be able to make at least 100 withdrawals.

**2**

What is the largest value of  $M$  that will enable Jia to do this?

NESA 2023 Mathematics Advanced HSC Examination

- 23 26 MA** A camera films the motion of a swing in a park.

Let  $x(t)$  be the horizontal distance, in metres, from the camera to the seat of the swing at  $t$  seconds.

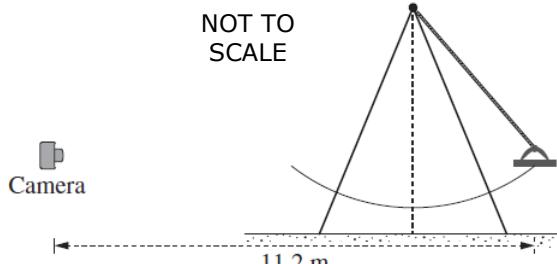
The seat is released from rest at a horizontal distance of 11.2 m from the camera.

(a) The rate of change of  $x$  can be modelled by the equation  $\frac{dx}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right)$ .

**2**

Find an expression for  $x(t)$ .

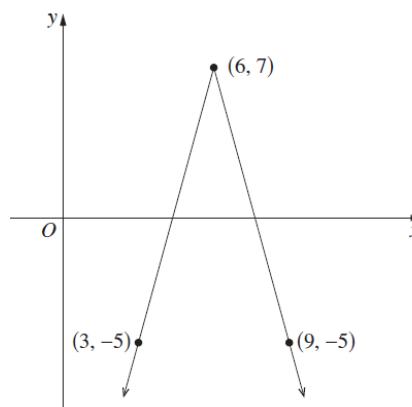
- (b) How many times does the swing reach the closest point to the camera during the first 10 seconds?

**2**

**23 27** The graph of  $y = f(x)$ , where

**MA**  $f(x) = a|x - b| + c$ , passes through the points  $(3, -5)$ ,  $(6, 7)$  and  $(9, -5)$  as shown in the diagram.

- Find the values of  $a$ ,  $b$  and  $c$ .
- The line  $y = mx$  cuts the graph of  $y = f(x)$  in two distinct places.  
Find all possible values of  $m$ .

**Solution****3****2**

NESA 2023 Mathematics Advanced HSC Examination

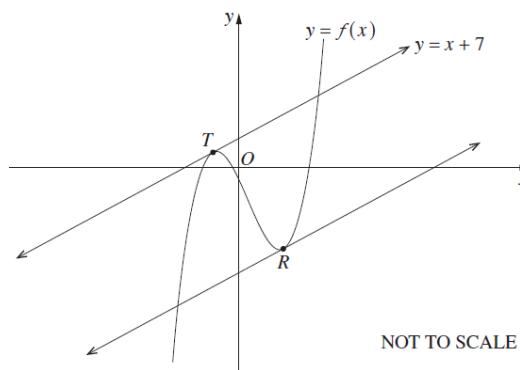
**23 28** The curve  $y = f(x)$  is shown on the diagram.

**MA** The equation of the tangent to the curve at point  $T(-1, 6)$  is  $y = x + 7$ . At a point  $R$ , another tangent parallel to the tangent at  $T$  is drawn.

The gradient function of the curve is given by  

$$\frac{dy}{dx} = 3x^2 - 6x - 8.$$

Find the coordinates of  $R$ .

**4**

NOT TO SCALE

NESA 2023 Mathematics Advanced HSC Examination

**23 29** A continuous random variable  $X$  has probability density function  $f(x)$  given by

**MA**

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Find the mode of  $X$ .

**2**

(b) Find the cumulative distribution function for the given probability density function.

**2**

(c) Without calculating the median, show that the mode is greater than the median.

**2**

NESA 2023 Mathematics Advanced HSC Examination

**23 30** Let  $f(x) = e^{-x \sin x}$ .

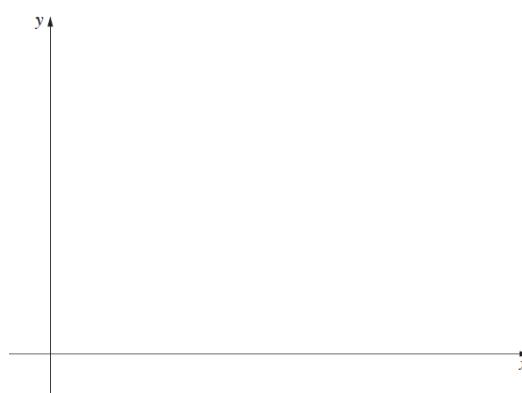
**MA** (a) Find the coordinates of the stationary

**3**

points of  $f(x)$  for  $0 \leq x \leq 2\pi$ .

You do NOT need to check the nature of the stationary points.

- Without using any further calculus, sketch the graph of  $f(x)$  for  $0 \leq x \leq 2\pi$ , showing stationary points and intercepts.

**2**

NESA 2023 Mathematics Advanced HSC Examination

- 23 31** Four Year 12 students want to organise a graduation party. All four students have the same probability,  $P(F)$ , of being available next Friday. All four students have the same probability,  $P(S)$ , of being available next Saturday.

It is given that  $P(F) = \frac{3}{10}$ ,  $P(S|F) = \frac{1}{3}$ , and  $P(F|S) = \frac{1}{8}$ .

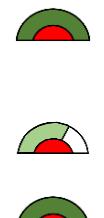
Kim is one of the four students.

- (a) Is Kim's availability next Friday independent from his availability next Saturday? **1**

Justify your answer.

- (b) Show that the probability that Kim is available next Saturday is  $\frac{4}{5}$ . **2**

- (c) What is the probability that at least one of the four students is NOT available next Saturday? **2**



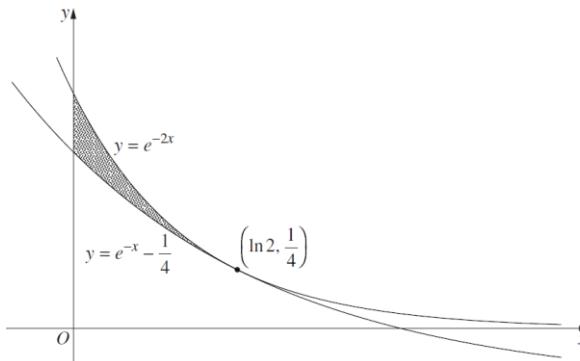
NESA 2023 Mathematics Advanced HSC Examination

- 23 32** The curves  $y = e^{-2x}$  and  $y = e^{-x} - \frac{1}{4}$  intersect at exactly one point as shown in the diagram. The point of intersection has coordinates  $(\ln 2, \frac{1}{4})$ .

(Do NOT prove this.)

- (a) Show that the area bounded by the two curves and the  $y$ -axis, as shaded in the diagram, is  $\frac{1}{4} \ln 2 - \frac{1}{8}$ .

- (b) Find the values of  $k$  such that the curves  $y = e^{-2x}$  and  $y = e^{-x} + k$  intersect at two points. **3**

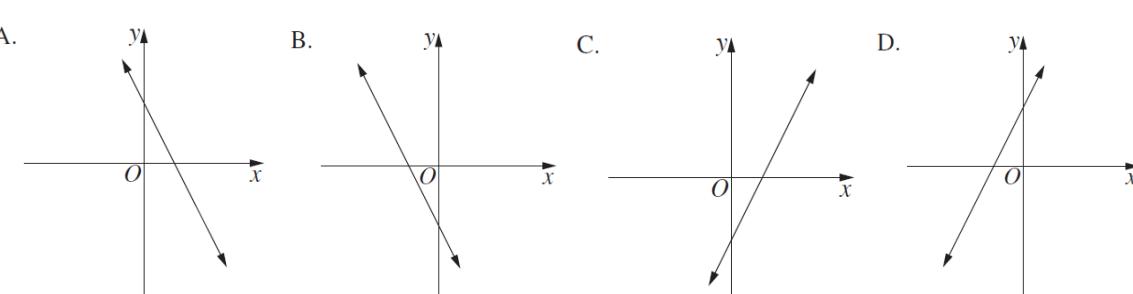


NESA 2023 Mathematics Advanced HSC Examination

# 2022 HSC Paper



**22 1** Which of the following could be the graph of  $y = -2x + 2$ ?

**MA**

**1** **Solution**



COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

**22 2** Consider the following dataset.

**MA**

13    16    17    17    21    24

Which row of the table shows how the median and mean are affected when a score of 5 is added to the dataset?

	Median	Mean
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

**1** **Solution**



COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

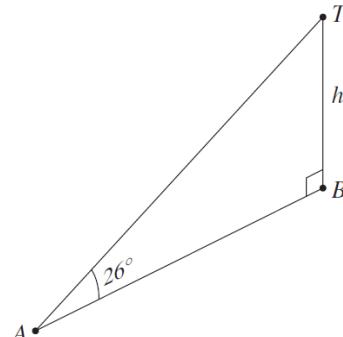
**22 3** A tower  $BT$  has height  $h$  metres.

**MA**

From point  $A$ , the angle of elevation to the top of the tower is  $26^\circ$  as shown.

Which of the following is the correct expression for the length of  $AB$ ?

- A.  $h \tan 26^\circ$
- B.  $h \cot 26^\circ$
- C.  $h \sin 26^\circ$
- D.  $h \cosec 26^\circ$



**1** **Solution**



NESA 2022 Mathematics Advanced HSC Examination

**22 4** Which of the following is the range of the function  $f(x) = x^2 - 1$ ?

**MA**

- A.  $[-1, \infty)$
- B.  $(-\infty, 1]$
- C.  $[-1, 1]$
- D.  $(-\infty, \infty)$

**1** **Solution**



NESA 2022 Mathematics Advanced HSC Examination

**22 5** Let  $h(x) = \frac{f(x)}{g(x)}$ , where

$$\begin{array}{ll} f(1) = 2 & f'(1) = 4 \\ g(1) = 8 & g'(1) = 12 \end{array}$$

What is the gradient of the tangent to the graph of  $y = h(x)$  at  $x = 1$ ?

- A.  $-8$
- B.  $8$
- C.  $-\frac{1}{8}$
- D.  $\frac{1}{8}$

**1** **Solution**



NESA 2022 Mathematics Advanced HSC Examination

**22 MA 6** What is  $\int \frac{1}{(2x+1)^2} dx$ ?

- A.  $\frac{-2}{2x+1} + C$       B.  $\frac{-1}{2(2x+1)} + C$       C.  $2 \ln(2x+1) + C$       D.  $\frac{1}{2} \ln(2x+1) + C$

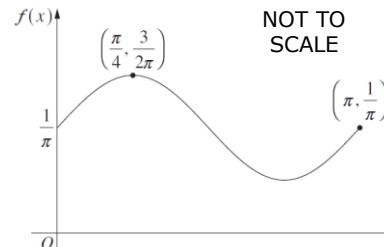


NESA 2022 Mathematics Advanced HSC Examination

**22 MA 7** Consider the following graph of a probability density function  $f(x)$ .

What is the value of the mode?

- A.  $\frac{1}{\pi}$       B.  $\frac{3}{2\pi}$   
C.  $\frac{\pi}{4}$       D.  $\pi$



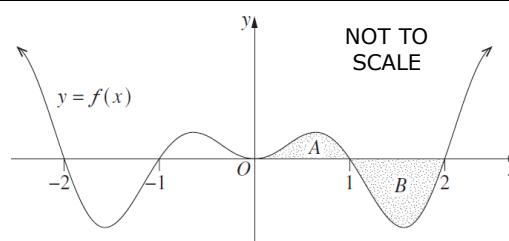
NESA 2022 Mathematics Advanced HSC Examination

**22 MA 8** The graph of the even function  $y = f(x)$  is shown.

The area of the shaded region A is  $\frac{1}{2}$  and the area of the shaded region B is  $\frac{3}{2}$ .

What is the value of  $\int_{-2}^2 f(x) dx$ ?

- A. 4      B. 2      C. -2      D. -4



NESA 2022 Mathematics Advanced HSC Examination

**22 MA 9** Liam is playing two games. He is equally likely to win each game.

The probability that Liam will win at least one of the games is 80%.

Which of the following is closest to the probability that Liam will win both games?

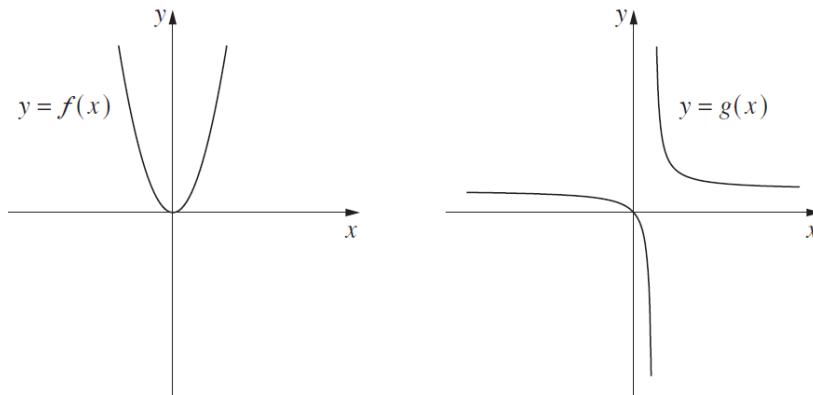
- A. 31%      B. 40%      C. 55%      D. 64%



NESA 2022 Mathematics Advanced HSC Examination

**22 10** The graphs of  $y = f(x)$  and  $y = g(x)$  are shown.

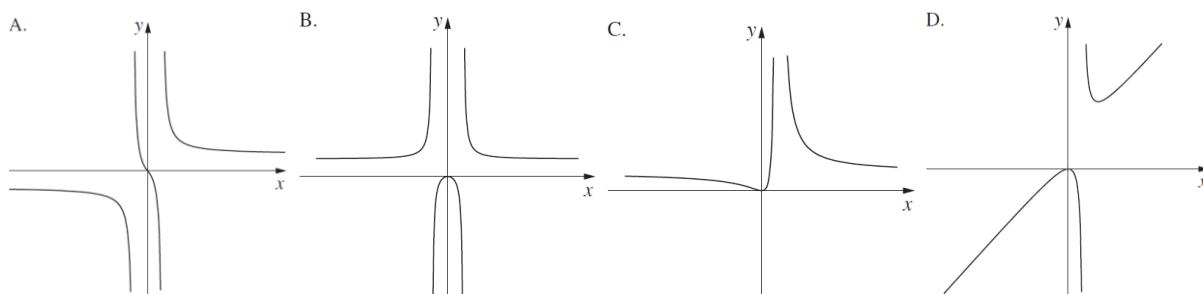
**MA**



**1** **Solution**



Which graph best represents  $y = g(f(x))$ ?



NESA 2022 Mathematics Advanced HSC Examination

**22 11** The table shows the types of customer complaints received by an online business in a month.

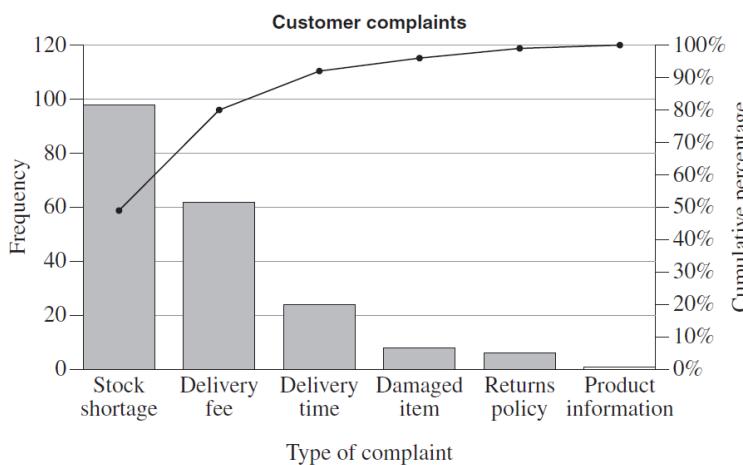
(a) What are the values of **A** and **B**?

Type of complaint	Frequency	Cumulative frequency	Cumulative percentage
Stock shortage	98	98	49
Delivery fee	62	<b>A</b>	80
Delivery time	24	184	92
Damaged item	8	192	<b>B</b>
Returns policy	6	198	99
Product information	2	200	100
Total	200		

(b) The data from the table are shown in the following Pareto chart.

The manager will address 80% of the complaints.

Which type of complaints will the manager address?



**Solution**



**2**



**1**



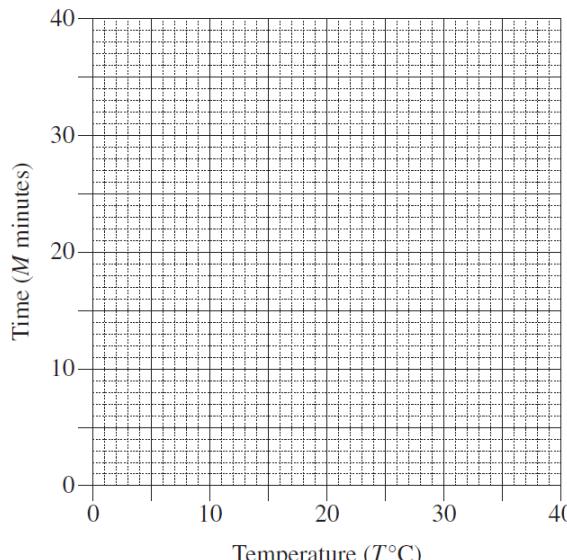
COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

- 22 12** A student believes that the time it takes for an ice cube to melt ( $M$  minutes) varies inversely with the room temperature ( $T^\circ\text{C}$ ).

The student observes that at a room temperature of  $15^\circ\text{C}$  it takes 12 minutes for an ice cube to melt.

- (a) Find the equation relating  $M$  and  $T$ .  
 (b) By first completing this table of values, graph the relationship between temperature and time from  $T = 5^\circ\text{C}$  to  $T = 30^\circ\text{C}$ .

$T$	5	15	30
$M$			



COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

- 22 13** Use two applications of the trapezoidal rule to find an approximate value of

$$\int_0^2 \sqrt{1+x^2} dx.$$

Give your answer correct to 2 decimal places.

**2** [Solution](#)

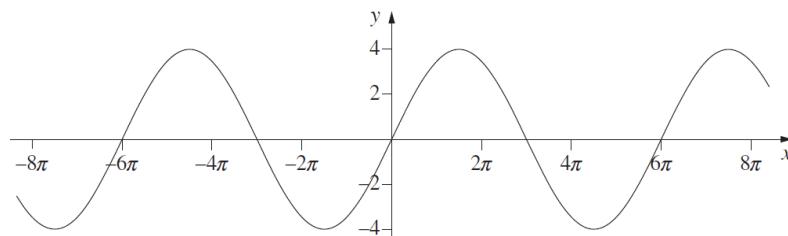


NESA 2022 Mathematics Advanced HSC Examination

- 22 14** The graph of  $y = k \sin(ax)$

**MA** is shown.

What are the values of  $k$  and  $a$ ?



NESA 2022 Mathematics Advanced HSC Examination

- 22 15** In a bag there are 3 six-sided dice.

**MA** Two of the dice have faces marked 1, 2, 3, 4, 5, 6.

The other is a special die with faces marked 1, 2, 3, 5, 5, 5.

One die is randomly selected and tossed.

(a) What is the probability that the die shows a 5?

(b) Given that the die shows a 5, what is the probability that it is the special die?

[Solution](#)

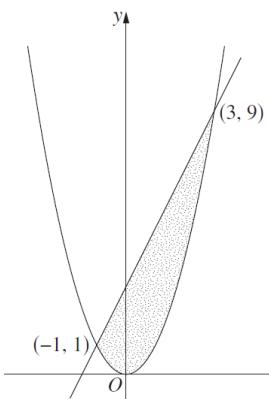


**1**  
**1**

NESA 2022 Mathematics Advanced HSC Examination

- 22 16** The parabola  $y = x^2$  meets the line  $y = 2x + 3$  at the points  $(-1, 1)$  and  $(3, 9)$  as shown in the diagram.

Find the area enclosed by the parabola and the line.



NESA 2022 Mathematics Advanced HSC Examination

**3** **Solution**



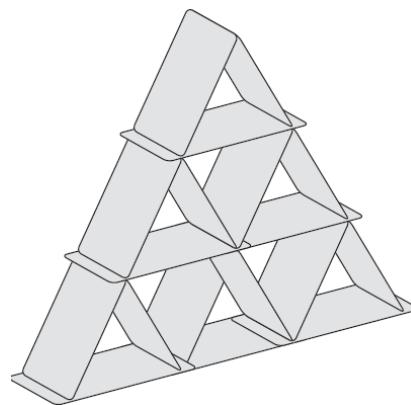
- 22 17** Cards are stacked to build a 'house of cards'.

**MA** A house of cards with 3 rows is shown.

A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.

- Show that a house of cards with 12 rows has a total of 234 cards.
- Another house of cards has a total of 828 cards.

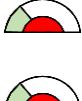
How many rows are in this house of cards?



**Solution**



**2**



**3**



NESA 2022 Mathematics Advanced HSC Examination

- 22 18** (a) Differentiate  $y = (x^2 + 1)^4$ .

**MA**

(b) Hence, or otherwise, find  $\int x(x^2 + 1)^3 dx$ .

**2** **Solution**



**1**



**3**



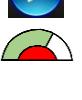
NESA 2022 Mathematics Advanced HSC Examination

- 22 19** The graph of the function  $f(x) = x^2$  is translated  $m$  units to the right, dilated vertically by a scale factor of  $k$  and then translated 5 units down.

The equation of the transformed function is  $g(x) = 3x^2 - 12x + 7$ .

Find the values of  $m$  and  $k$ .

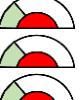
**3** **Solution**



**1**



**2**



NESA 2022 Mathematics Advanced HSC Examination

- 22 20** A scientist is studying the growth of bacteria.

**MA** The scientist models the number of bacteria,  $N$ , by the equation  $N(t) = 200e^{0.013t}$ , where  $t$  is the number of hours after starting the experiment.

- What is the initial number of bacteria in the experiment?

**Solution**



**1**



- What is the number of bacteria 24 hours after starting the experiment?

**1**



- What is the rate of increase in the number of bacteria 24 hours after starting the experiment?

**2**



NESA 2022 Mathematics Advanced HSC Examination

**22 21** Eli is choosing between two investment options.

**MA**

Option 1: Depositing a single amount of \$40 000 today, earning interest of 1.2% per annum, compounded monthly.

Option 2: Depositing \$1000 at the end of each quarter, earning interest of 2.4% per annum, compounded quarterly.

A table of future value interest factors for an annuity of \$1 is shown.

- What is the value of Eli's investment after 10 years using Option 1?
- What is the difference between the future values after 10 years using Option 1 and Option 2?

N	r	Interest rate per period as a decimal					
		0.002	0.006	0.020	0.024	0.060	0.240
10	10.09048	10.27437	10.94972	11.15211	13.18079	31.64344	
20	20.38460	21.18211	24.29737	25.28909	36.78559	303.60062	
30	30.88646	32.76227	40.56808	43.20983	79.05819	2640.91639	
40	41.60026	45.05630	60.40198	65.92708	154.76197	22728.80260	

**Solution**



**2**



**2**



**COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations**

**22 22** Find the global maximum and minimum values of  $y = x^3 - 6x^2 + 8$ , where  $-1 \leq x \leq 7$ .

**MA**

**4**

**Solution**



NESA 2022 Mathematics Advanced HSC Examination

**22 23** The depth of water in a bay rises and falls with the tide.

**MA** On a particular day the depth of the water,  $d$  metres, can be modelled by the equation

$$d = 1.3 - 0.6 \cos\left(\frac{4\pi}{25}t\right) \text{ where } t \text{ is the time in hours since low tide.}$$

- Find the depth of water at low tide and at high tide.
- What is the time interval, in hours, between two successive low tides?
- For how long between successive low tides will the depth of water be at least 1 metre?

**2**



**1**



**3**



NESA 2022 Mathematics Advanced HSC Examination

**22 24** Jo is researching the relationship between the ages of teenage characters in television series and the ages of actors playing these characters.

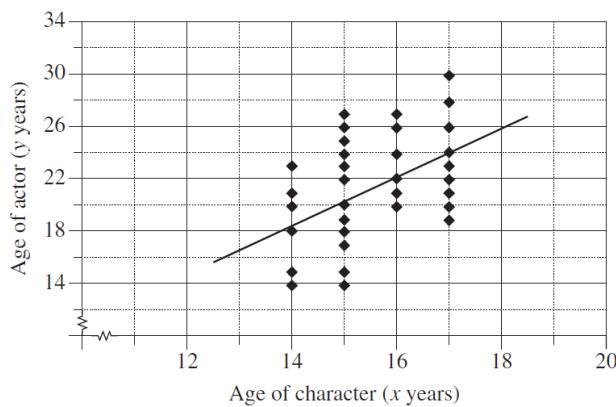
After collecting the data, Jo finds that the correlation coefficient is 0.4564 . A scatterplot showing the data is drawn.

The line of best fit with equation  $y = -7.51 + 1.85x$ , is also drawn.

Describe and interpret the data and other information provided, with reference to the context given.

**4**

**Solution**



**COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations**

**22 25** Let  $f(x) = \sin(2x)$ .

**MA** Find the value of  $x$ , for  $0 < x < \pi$ , for which  $f'(x) = -\sqrt{3}$  AND  $f''(x) = 2$ .

**3**

NESA 2022 Mathematics Advanced HSC Examination

**22 26** The life span of batteries from a particular factory is normally distributed with a mean of 840 hours and a standard deviation of 80 hours.

It is known from statistical tables that for this distribution approximately 60% of the batteries have a life span of less than 860 hours.

What is the approximate percentage of batteries with a life span between 820 and 920 hours?

**3**

COMMON QUESTION: NESA 2022 Mathematics Standard 2 and Advanced HSC Examinations

**22 27** Let  $f(x) = xe^{-2x}$ .

**MA** It is given that  $f'(x) = e^{-2x} - 2xe^{-2x}$ .

(a) Show that  $f''(x) = 4(x - 1)e^{-2x}$ .

**2**

(b) Find any stationary points of  $f(x)$  and determine their nature.

**2**

(c) Sketch the curve  $y = xe^{-2x}$ , showing any stationary points, points of inflection and intercepts with the axes.

**3**

NESA 2022 Mathematics Advanced HSC Examination

**22 28** The graph of the circle  $x^2 + y^2 = 2$  is shown. The interval connecting the origin,  $O$ , and the point  $(1, 1)$  makes an angle  $\theta$  with the positive  $x$ -axis.

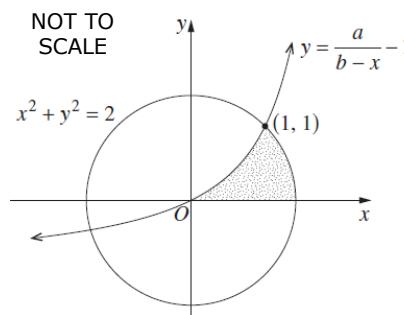
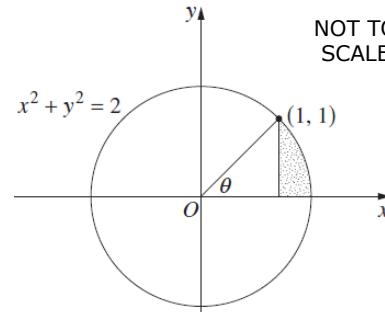
(a) By considering the value of  $\theta$ , find the exact area of the shaded region, as shown on the diagram.

Part of the hyperbola  $y = \frac{a}{b-x} - 1$  which passes through the points  $(0, 0)$  and  $(1, 1)$  is drawn with the circle  $x^2 + y^2 = 2$  as shown.

(b) Show that  $a = b = 2$ .

**2**

(c) Using parts (a) and (b), find the exact area of the region bounded by the hyperbola, the positive  $x$ -axis and the circle as shown on the diagram.

**3**

NESA 2022 Mathematics Advanced HSC Examination

- 22 29** (a) The diagram shows the graph of  $y = 2^{-x}$ .  
**MA** Also shown on the diagram are the first 5 of an infinite number of rectangular strips of width 1 unit and height  $y = 2^{-x}$  for non-negative integer values of  $x$ .

For example, the second rectangle shown

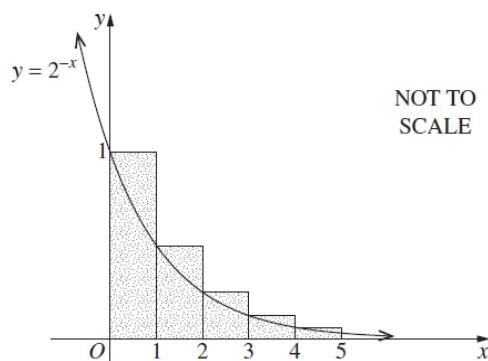
has width 1 and height  $\frac{1}{2}$ .

The sum of the areas of the rectangles forms a geometric series.

Show that the limiting sum of this series is 2.

(b) Show that  $\int_0^4 2^{-x} dx = \frac{15}{16\ln 2}$ .

(c) Use parts (a) and (b) to show that  $e^{15} < 2^{32}$ .



**1** Solution



**2**



**2**



NESA 2022 Mathematics Advanced HSC Examination

- 22 30** A continuous random variable  $X$  has cumulative distribution function given by:

(a) Show that  $k = 3$ .

(b) Given that  $P(X < c) = 2P(X > c)$ , find the exact value of  $c$ .

$$F(x) = \begin{cases} 1 & x > e^3 \\ \frac{1}{k} \ln x & 1 \leq x \leq e^3 \\ 0 & x < 1 \end{cases}$$

**Solution**



**1**



**2**

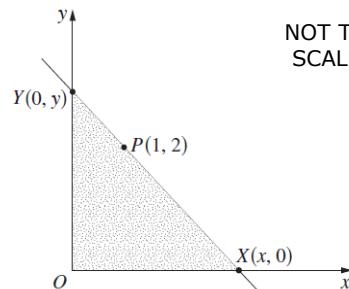


NESA 2022 Mathematics Advanced HSC Examination

- 22 31** A line passes through the point  $P(1, 2)$  and meets the axes at  $X(x, 0)$  and  $Y(0, y)$ , where  $x > 1$ .

(a) Show that  $y = \frac{2x}{x-1}$ .

(b) Find the minimum value of the area of triangle  $XOY$ .



**Solution**



**2**



**4**



NESA 2022 Mathematics Advanced HSC Examination

- 22 32 MA** In a reducing-balance loan, an amount  $\$P$  is borrowed for a period of  $n$  months at an interest rate of 0.25% per month, compounded monthly.

**Solution**

At the end of each month, a repayment of  $\$M$  is made.

After the  $n$ th repayment has been made, the amount owing,  $\$A$ , is given by

$$A_n = P(1.0025)^n - M(1 + (1.0025)^1 + (1.0025)^2 + \dots + (1.0025)^{n-1}).$$

(Do NOT prove this.)

- (a) Jane borrows \$200 000 in a reducing-balance loan as described.

**2**

The loan is to be repaid in 180 monthly repayments.

Show that  $M = 1381.16$ , when rounded to the nearest cent.

- (b) After 100 repayments of \$1381.16 have been made, the interest rate changes to 0.35% per month.

**3**

At this stage, the amount owing to the nearest dollar is \$100 032.

(Do NOT prove this.)

Jane continues to make the same monthly repayments.

For how many more months will Jane need to make full monthly payments of \$1381.16?

- (c) The final repayment will be less than \$1381.16.

**2**

How much will Jane need to pay in the final payment in order to pay off the loan?

NESA 2022 Mathematics Advanced HSC Examination

# 2021 HSC Paper



**21 1** Which of the following is equivalent to  $\sin^2 5x$ ?

**1** [Solution](#)

- MA** A.  $1 + \cos^2 5x$       B.  $1 - \cos^2 5x$       C.  $-1 + \cos^2 5x$       D.  $-1 - \cos^2 5x$



NESA 2021 Mathematics Advanced HSC Examination

**21 2** The probability distribution table for a random variable  $X$  is shown.

**1** [Solution](#)

What is the expected value for  $X$ ?  
 A. 0.6  
 B. 1.0  
 C. 1.5  
 D. 2.0

$x$	$P(X = x)$
1	0.6
2	0.3
3	0.1



NESA 2021 Mathematics Advanced HSC Examination

**21 3** Which of the following represents the domain of the function  $f(x) = \ln(1 - x)$ ?

**1** [Solution](#)

- MA** A.  $[1, \infty)$       B.  $(1, \infty)$       C.  $(-\infty, 1]$       D.  $(-\infty, 1)$

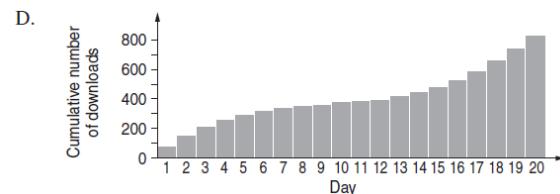
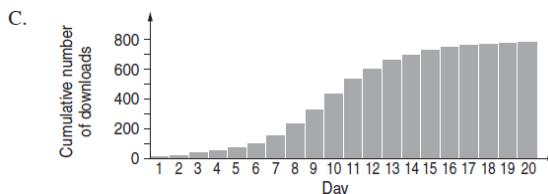
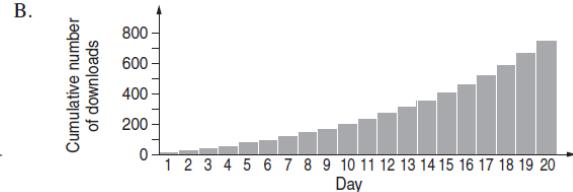
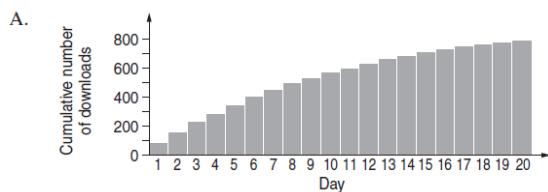
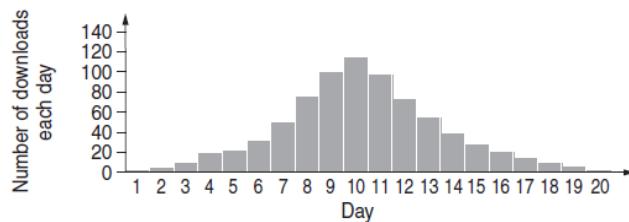


NESA 2021 Mathematics Advanced HSC Examination

**21 4** The number of downloads of a song on each of twenty consecutive days is shown in the following graph.

**1** [Solution](#)

Which of the following graphs best shows the cumulative number of downloads up to and including each day?

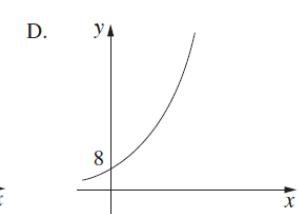
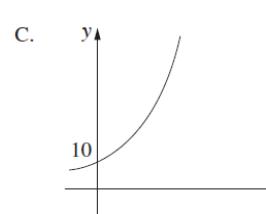
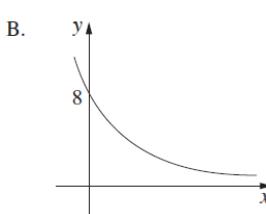
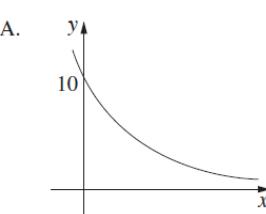


COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

**21 5** Which of the following best represents the graph of  $y = 10(0.8)^x$ ?

**1** [Solution](#)

**MA**

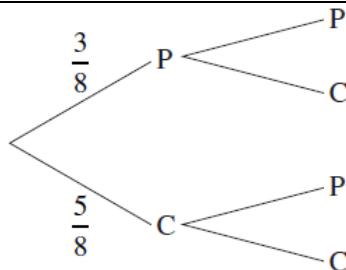


COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 6** There are 8 chocolates in a box.  
**MA** Three have peppermint centres (P) and five have caramel centres (C). Kim randomly chooses a chocolate from the box and eats it. Sam then randomly chooses and eats one of the remaining chocolates.  
A partially completed probability tree is shown.

What is the probability that Kim and Sam choose chocolates with different centres?

- A.  $\frac{15}{64}$       B.  $\frac{15}{56}$       C.  $\frac{15}{32}$       D.  $\frac{15}{28}$



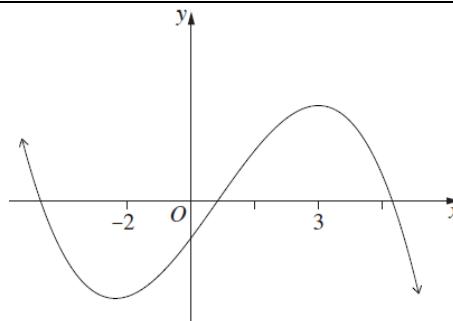
**1** [Solution](#)



- 21 7** The diagram shows part of  $y = f(x)$  which has a local minimum at  $x = -2$  and a local maximum at  $x = 3$ .

Which of the following shows the correct relationship between  $f''(-2)$ ,  $f(0)$  and  $f'(-3)$ ?

- A.  $f(0) < f'(-3) < f''(-2)$   
B.  $f(0) < f''(-2) < f'(-3)$   
C.  $f''(-2) < f'(-3) < f(0)$   
D.  $f''(-2) < f(0) < f'(-3)$



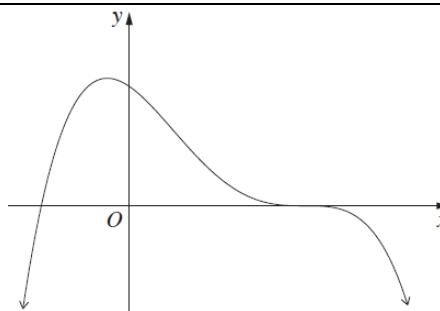
**1** [Solution](#)



- 21 8** The graph of  $y = f(x)$  is shown.

Which of the following could be the equation of this graph?

- A.  $y = (1 - x)(2 + x)^3$   
B.  $y = (x + 1)(x - 2)^3$   
C.  $y = (x + 1)(2 - x)^3$   
D.  $y = (x - 1)(2 + x)^3$



**1** [Solution](#)



- 21 9** Let  $h(x) = f(g(x))$  where the function  $f(x)$  is an odd function and the function  $g(x)$  is an even function.

The tangent to  $y = h(x)$  at  $x = k$ , where  $k > 0$ , has the equation  $y = mx + c$ .

What is the equation of the tangent to  $y = h(x)$  at  $x = -k$ ?

- A.  $y = mx + c$       B.  $y = -mx + c$       C.  $y = mx - c$       D.  $y = -mx - c$

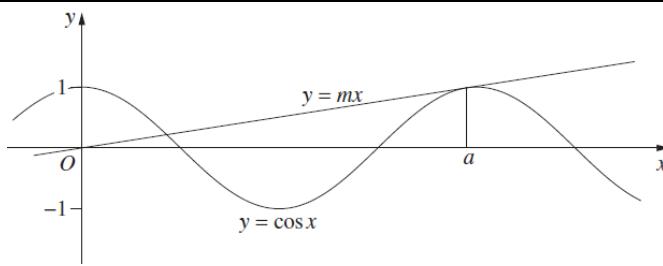
NESA 2021 Mathematics Advanced HSC Examination

**1** [Solution](#)



- 21 10** The line  $y = mx$  is a tangent to the curve  $y = \cos x$  at the point where  $x = a$ , as shown in the diagram.  
Which of the following statements is true?

A.  $m < \frac{1}{a} < \frac{1}{2\pi}$       B.  $\frac{1}{2\pi} < m < \frac{1}{a}$       C.  $\frac{1}{2\pi} < \frac{1}{a} < m$       D.  $m < \frac{1}{2\pi} < \frac{1}{a}$

**1** [Solution](#)

NESA 2021 Mathematics Advanced HSC Examination

- 21 11** Solve  $x + \frac{x-1}{2} = 9$ .

**2** [Solution](#)

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 12** A right-angled triangle XYZ is cut out from a semicircle with centre O. The length of the diameter XZ is 16 cm and  $\angle YXZ = 30^\circ$ , as shown on the diagram.

[Solution](#)

(a) Find the length of XY in cm, correct to two decimal places.

**2**

(b) Hence, find the area of the shaded region in square centimetres, correct to one decimal place.

**3**

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 13** Find the exact gradient of the tangent to the curve  $y = x \tan x$  at the point where

**3** [Solution](#)

$$x = \frac{\pi}{3}.$$



NESA 2021 Mathematics Advanced HSC Examination

- 21 14** The first term of an arithmetic sequence is 5. The sum of the first 43 terms is 2021. What is the common difference of the sequence?

**2** [Solution](#)

NESA 2021 Mathematics Advanced HSC Examination

- 21 15** Evaluate  $\int_{-2}^0 \sqrt{2x+4} dx$ .

**2**

NESA 2021 Mathematics Advanced HSC Examination

- 21 16** For what values of  $x$  is  $f(x) = x^2 - 2x^3$  increasing?

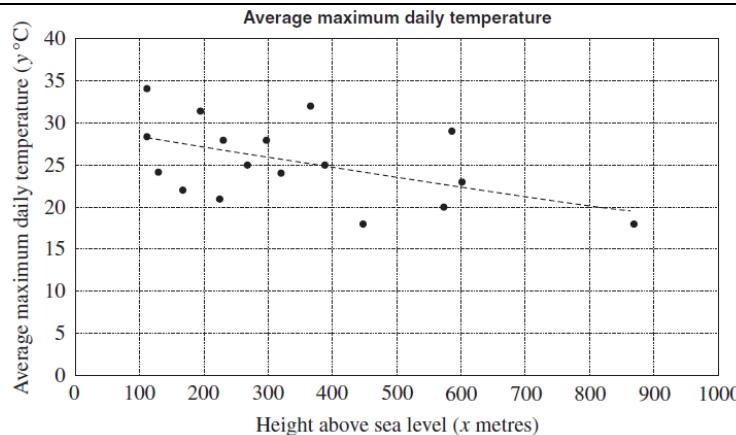
**3** [Solution](#)

NESA 2021 Mathematics Advanced HSC Examination

- 21 17** For a sample of 17 inland towns in Australia, the height above sea level,  $x$  (metres), and the average maximum daily temperature,  $y$  ( $^{\circ}\text{C}$ ), were recorded.

The graph shows the data as well as a regression line. The equation of the regression line is  $y = 29.2 - 0.011x$ .

The correlation coefficient is  $r = -0.494$ .



- (a) (i) By using the equation of the regression line, predict the average maximum daily temperature, in degrees Celsius, for a town that is 540 m above sea level. Give your answer correct to one decimal place.  
(ii) The gradient of the regression line is  $-0.011$ . Interpret the value of this gradient in the given context.

- (b) The graph shows the relationship between the latitude,  $x$  (degrees south), and the average maximum daily temperature,  $y$  ( $^{\circ}\text{C}$ ), for the same 17 towns, as well as a regression line. The equation of the regression line is  $y = 45.6 - 0.683x$ .

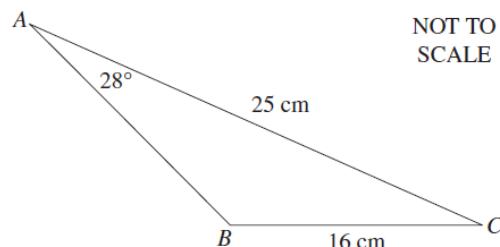
The correlation coefficient is  $r = -0.897$ .

Another inland town in Australia is 540 m above sea level. Its latitude is 28 degrees south. Which measurement, height above sea level or latitude, would be better to use to predict this town's average maximum daily temperature? Give a reason for your answer.

COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 18** The diagram shows a triangle  $ABC$  where  $AC = 25 \text{ cm}$ ,  $BC = 16 \text{ cm}$ ,  $\angle BAC = 28^\circ$  and angle  $ABC$  is obtuse.

Find the size of the obtuse angle  $ABC$  correct to the nearest degree.



**3** [Solution](#)

- 21 19** Without using calculus, sketch the graph of  $y = 2 + \frac{1}{x+4}$ , showing the asymptotes and the  $x$  and  $y$  intercepts.

**3** [Solution](#)

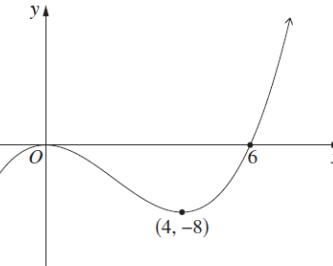
NESA 2021 Mathematics Advanced HSC Examination

- 21 20 MA** For what values of  $x$ , in the interval  $0 \leq x \leq \frac{\pi}{4}$ , does the line  $y = 1$  intersect the graph of  $y = 2 \sin 4x$ ? **2** [Solution](#)



NESA 2021 Mathematics Advanced HSC Examination

- 21 21 MA** Consider the graph of  $y = f(x)$  as shown.

**2** [Solution](#)

Sketch the graph of  $y = 4f(2x)$  showing the  $x$ -intercepts and the coordinates of the turning points.

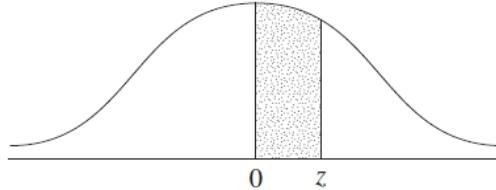
NESA 2021 Mathematics Advanced HSC Examination

- 21 22 MA** A random variable is normally distributed with mean 0 and standard deviation 1. **2** [Solution](#)

The table gives the probability that this random variable lies between 0 and  $z$  for different values of  $z$ .

$z$	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.0398	0.0793	0.1179	0.1554	0.1915	0.2257

The probability values given in the table for different values of  $z$  are represented by the shaded area in the following diagram.

**1**

- (a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1 lies between 0.1 and 0.5.

- (b) Birth weights are normally distributed with a mean of 3300 grams and a standard deviation of 570 grams. By first calculating a  $z$ -score, find how many babies, out of 1000 born, are expected to have a birth weight greater than 3528 grams. **3** [Solution](#)



COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

- 21 23 MA** A population,  $P$ , which is initially 5000, varies according to the formula **4** [Solution](#)

$$P = 5000b^{-t}, \text{ where } b \text{ is a positive constant and } t \text{ is time in years, } t \geq 0.$$



The population is 1250 after 20 years.

Find the value of  $t$ , correct to one decimal place, for which the instantaneous rate of decrease is 30 people per year.

NESA 2021 Mathematics Advanced HSC Examination

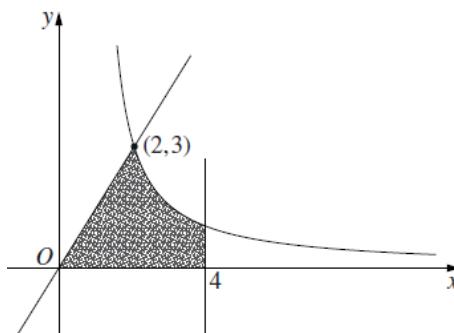
- 21 24** **MA** The curve  $y = \frac{3}{x-1}$  intersects the line

$$y = \frac{3}{2}x \text{ at the point } (2, 3).$$

The region bounded by the curve

$$y = \frac{3}{x-1}, \text{ the line } y = \frac{3}{2}x, \text{ the } x\text{-axis and}$$

the line  $x = 4$  is shaded in the diagram.



NESA 2021 Mathematics Advanced HSC Examination

**3** [Solution](#)

- 21 25** **MA** A table of future value interest factors for an annuity of \$1 is shown.

Simone deposits \$1000 into a savings account at the end of each year for 8 years.

The interest rate for these 8 years is 0.75% per annum, compounded annually.

After the 8th deposit, Simone stops making deposits but leaves the money in the savings account. The money in her savings account then earns interest at 1.25% per annum, compounded annually, for a further two years.

Find the amount of money in Simone's savings account at the end of ten years.

**COMMON QUESTION:** NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations

Table of future value interest factors

Number of periods	Interest rate per period				
	0.25%	0.5%	0.75%	1%	1.25%
2	2.0025	2.0050	2.0075	2.0100	2.0125
4	4.0150	4.0301	4.0452	4.0604	4.0756
6	6.0376	6.0755	6.1136	6.1520	6.1907
8	8.0704	8.1414	8.2132	8.2857	8.3589
10	10.1133	10.2280	10.3443	10.4622	10.5817

- 21 26** **MA** A particle is shot vertically upwards from a point 100 metres above ground level.

[Solution](#)

The position of the particle,  $y$  metres above the ground after  $t$  seconds, is given by

$$y(t) = -5t^2 + 70t + 100.$$

(a) Find the maximum height above ground level reached by the particle.

**2**

(b) Find the velocity of the particle, in metres per second, immediately before it hits

**3**

the ground, leaving your answer in the form  $a\sqrt{b}$ , where  $a$  and  $b$  are integers.

NESA 2021 Mathematics Advanced HSC Examination

- 21 27** Kenzo has a solar powered phone charger. Its power,  $P$ , can be modelled by the function  $P(t) = 400 \sin\left(\frac{\pi}{12}t\right)$ ,  $0 \leq t \leq 12$ , where  $t$  is the number of hours after sunrise.
- (a) Sketch the graph of  $P$  for  $0 \leq t \leq 12$ .
- Power is the rate of change of energy. Hence the amount of energy,  $E$  units, generated by the solar powered phone charger from  $t = a$  to  $t = b$ , where  $0 \leq a \leq b \leq 12$  is given by  $E = \int_a^b P(t) dt$ .
- (b) Show that  $E = \frac{4800}{\pi} \left( \cos \frac{a\pi}{12} - \cos \frac{b\pi}{12} \right)$ .
- (c) To make a phone call, a phone battery needs at least 300 units of energy. Kenzo woke up 3 hours after sunrise and found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. Find the least amount of time he needed to wait before he could make a phone call. Give your answer correct to the nearest minute.
- (d) The next day, Kenzo woke up 6 hours after sunrise and again found that his phone battery had no units of energy. He immediately began to use his solar powered charger to charge his phone battery. Would it take more time or less time or the same amount of time, compared to the answer in part (c), to charge his phone battery in order to make a phone call? Explain your answer by referring to the graph drawn in part (a).

NESA 2021 Mathematics Advanced HSC Examination

[Solution](#)

2



2



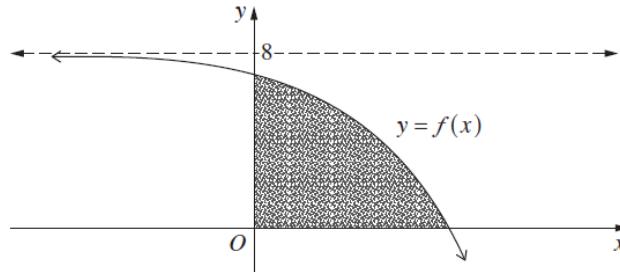
3



1



- 21 28** The region bounded by the graph of the function  $f(x) = 8 - 2^x$  and the coordinate axes is shown.



- (a) Show that the exact area of the shaded region is given by  $24 - \frac{7}{\ln 2}$ .

3



- (b) A new function  $g(x)$  is found by taking the graph of  $y = -f(-x)$  and translating it by 5 units to the right.

2



Sketch the graph of  $y = g(x)$  showing the  $x$ -intercept and the asymptote.

- (c) Hence, find the exact value of  $\int_2^5 g(x) dx$ .

1



NESA 2021 Mathematics Advanced HSC Examination

- 21 29** (a) On the day that Megan was born, her grandfather deposited \$5000 into an account earning 3% per annum compounded annually. On each birthday after this, her grandfather deposited \$1000 into the same account, making his final deposit on Megan's 17th birthday. That is, a total of 18 deposits were made. Let  $A_n$  be the amount in the account on Megan's  $n$ th birthday, after the deposit is made.
- Show that  $A_3 = \$8554.54$ .
- (b) On her 17th birthday, just after the final deposit is made, Megan has \$30 025.83 in her account. You are NOT required to show this. Megan then decides to leave all the money in the same account continuing to earn interest at 3% per annum compounded annually. On her 18th birthday, and on each birthday after this, Megan withdraws \$2000 from the account.
- How many withdrawals of \$2000 will Megan be able to make?

**2** [Solution](#)**3** [Solution](#)

- 21 30** The number of hours for which light bulbs will work before failing can be modelled by the random variable  $X$  with cumulative distribution function.

**2** [Solution](#)

$$F(x) = \begin{cases} 1 - e^{-0.01x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Jane sells light bulbs and promises that they will work for longer than exactly 99% of all light bulbs. Find how long, according to Jane's promise, a light bulb bought from her should work. Give your answer in hours, rounded to two decimal places.

NESA 2021 Mathematics Advanced HSC Examination

- 21 31** By considering the equation of the tangent to  $y = x^2 - 1$  at the point  $(a, a^2 - 1)$ , find the equations of the two tangents to  $y = x^2 - 1$  which pass through  $(3, -8)$ .

**4** [Solution](#)

NESA 2021 Mathematics Advanced HSC Examination

- 21 32** In a particular city, the heights of adult females and the heights of adult males are each normally distributed.

**4** [Solution](#)

Information relating to two females from that city is given in Table 1.

Table 1

Height	Gender	Percentage of females in this city shorter than this person
175 cm	Female	97.5%
160.6 cm	Female	16%

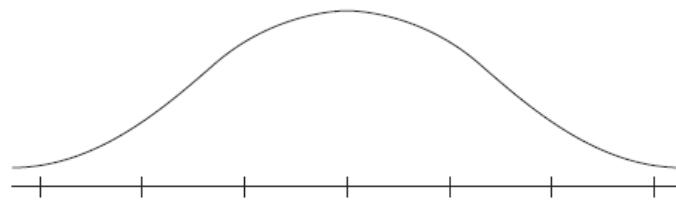
The means and standard deviations of adult females and males, in centimetres, are given in Table 2.

Table 2

	Mean	Standard deviation
Females	$\mu$	$\sigma$
Males	$1.05\mu$	$1.1\sigma$

A selected male is taller than 84% of the population of adult males in this city. By first labelling the normal distribution curve with the heights of the two females given in Table 1, calculate the height of the selected male, in centimetres, correct to two decimal places.

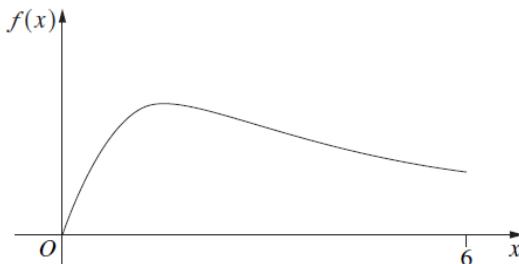
COMMON QUESTION: NESA 2021 Mathematics Standard 2 and Advanced HSC Examinations



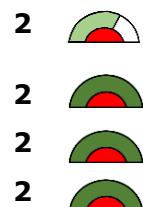
- 21 33** People are given a maximum of six hours to complete a puzzle. The time spent on the MA puzzle, in hours, can be modelled using the continuous random variable  $X$  which has probability density function

$$f(x) = \begin{cases} \frac{Ax}{x^2 + 4}, & \text{for } 0 \leq x \leq 6, (\text{where } A > 0) \\ 0, & \text{for all other values of } x \end{cases}$$

The graph of the probability density function is shown below. The graph has a local maximum.



- (a) Show that  $A = \frac{2}{\ln 10}$ .
- (b) Show that the mode of  $X$  is two hours.
- (c) Show that  $P(X < 2) = \log_{10} 2$ .
- (d) The Intelligence Quotient (IQ) scores of people are normally distributed with a mean of 100 and standard deviation of 15. It has been observed that the puzzle is generally completed more quickly by people with a high IQ. It is known that 80% of people with an IQ greater than 130 can complete the puzzle in less than two hours. A person chosen at random can complete the puzzle in less than two hours.



What is the probability that this person has an IQ greater than 130? Give your answer correct to three decimal places.

NESA 2021 Mathematics Advanced HSC Examination

- 21 34** A discrete random variable has probability distribution as shown in the table where MA  $n$  is a finite positive integer.

**3** [Solution](#)

$x$	$r$	$r^2$	$r^3$	...	$r^k$	...	$r^n$
$P(X = x)$	$r^n$	$r^{n-1}$	$r^{n-2}$	...	$r^{n-k+1}$	...	$r$

Show that  $E(X) = n(2r - 1)$

NESA 2021 Mathematics Advanced HSC Examination

# 2020 HSC Paper

[Back](#)

- 20 1** Which inequality gives the domain of  $\sqrt{2x - 3}$ ?

**MA**

- A.  $x < \frac{3}{2}$       B.  $x > \frac{3}{2}$       C.  $x \leq \frac{3}{2}$       D.  $x \geq \frac{3}{2}$

**1** [Solution](#)



- 20 2** The function  $f(x) = x^3$  is transformed to  $g(x) = (x - 2)^2 + 5$  by a horizontal translation of 2 units followed by a vertical translation of 5 units.

Which row of the table shows the directions of the translations?

	Horizontal translation of 2 units	Vertical translation of 5 units
A.	Left	Up
B.	Right	Up
C.	Left	Down
D.	Right	Down

**1** [Solution](#)



- 20 3** John recently did a class test in each of three subjects. The class scores on each test are normally distributed.

The table shows the subjects and John's scores as well as the mean and standard deviation of the class scores on each test.

Relative to the rest of the class, which row of the table below shows John's strongest subject and his weakest subject?

Subject	John's score	Mean	Standard deviation
French	82	70	8
Commerce	80	65	5
Music	74	50	12

**1** [Solution](#)



Strongest subject	Weakest subject
A. Commerce	French
B. French	Music
C. Music	French
D. Commerce	Music

**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

- 20 4** What is  $\int (e + e^{3x}) dx$ ?

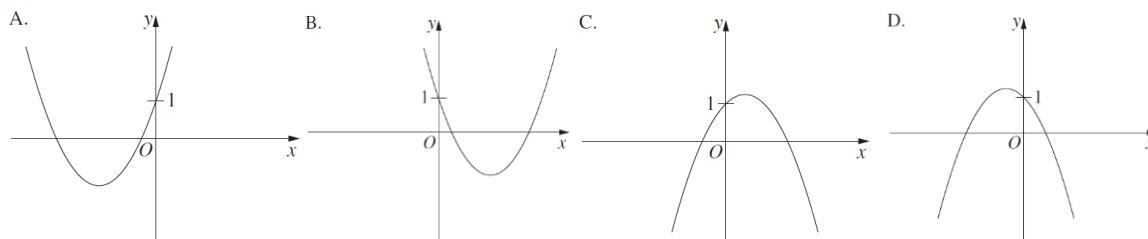
**MA**

- A.  $ex + 3e^{3x} + c$       B.  $ex + \frac{1}{3}e^{3x} + c$       C.  $e + 3e^{3x} + c$       D.  $e + \frac{1}{3}e^{3x} + c$

**1** [Solution](#)



- 20 5** Which of the following could represent the graph of  $y = -x^2 + bx + 1$ , where  $b > 0$ ?

**MA**

**1** [Solution](#)



- 20 6** Which interval gives the range of the function  $y = 5 + 2 \cos 3x$ ?

**MA**

- A.  $[2, 8]$       B.  $[3, 7]$       C.  $[4, 6]$       D.  $[5, 9]$

**1** [Solution](#)

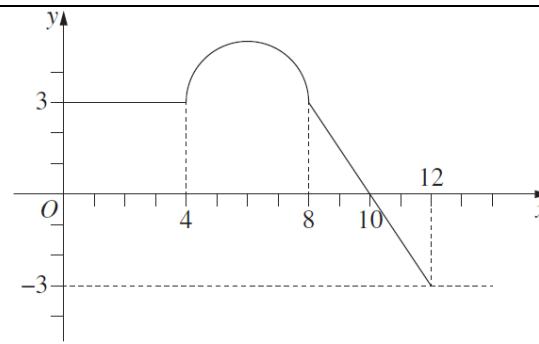


NESA 2020 Mathematics Advanced HSC Examination

- 20 7** The diagram shows the graph  $y = f(x)$ , which is made up of line segments and a semicircle.

What is the value of  $\int_0^{12} f(x) dx$ ?

- A.  $24 + 2\pi$       B.  $24 + 4\pi$   
C.  $30 + 2\pi$       D.  $30 + 4\pi$



NESA 2020 Mathematics Advanced HSC Examination

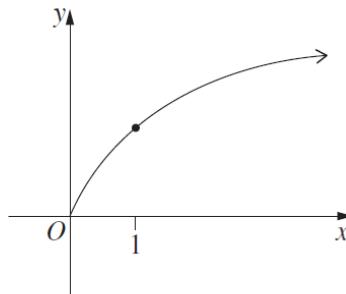
**1** [Solution](#)

- 20 8** The graph of  $y = f(x)$  is shown.

**MA**

Which of the following inequalities is correct?

- A.  $f''(1) < 0 < f'(1) < f(1)$   
B.  $f''(1) < 0 < f(1) < f'(1)$   
C.  $0 < f''(1) < f'(1) < f(1)$   
D.  $0 < f''(1) < f(1) < f'(1)$



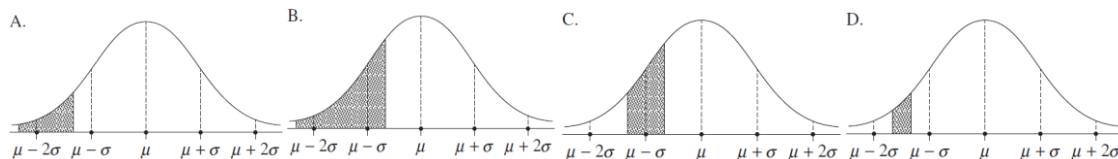
NESA 2020 Mathematics Advanced HSC Examination

**1** [Solution](#)

- 20 9** Suppose the weight of melons is normally distributed with a mean of  $\mu$  and a standard deviation of  $\sigma$ .

A melon has a weight below the lower quartile of the distribution but NOT in the bottom 10% of the distribution.

Which of the following most accurately represents the region in which the weight of this melon lies?



NESA 2020 Mathematics Advanced HSC Examination

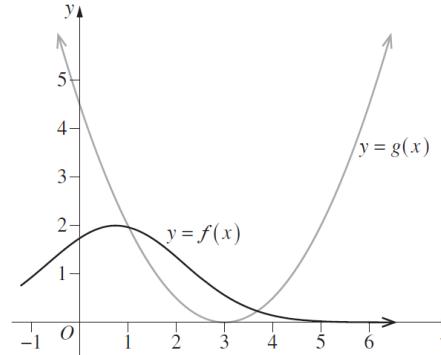
**1** [Solution](#)

- 20 10** The graph shows two functions  $y = f(x)$  and  $y = g(x)$ .

Define  $h(x) = f(g(x))$ .

How many stationary points does  $y = h(x)$  have for  $1 \leq x \leq 5$ ?

- A. 0      B. 1  
C. 2      D. 3



NESA 2020 Mathematics Advanced HSC Examination

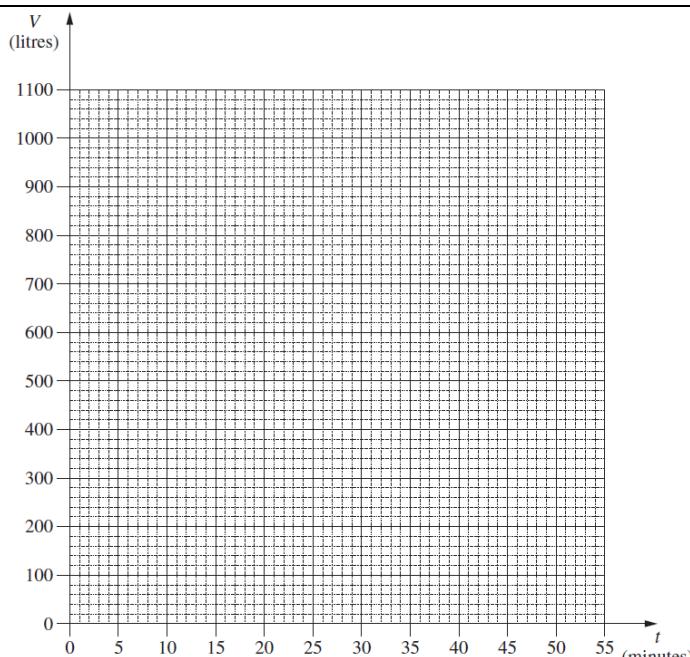
**1** [Solution](#)

- 20 11** There are two tanks on a property, Tank A and Tank B. Initially, Tank A holds 1000 litres of water and Tank B is empty.

- (a) Tank A begins to lose water at a constant rate of 20 litres per minute.

The volume of water in Tank A is modelled by  $V = 1000 - 20t$  where  $V$  is the volume in litres and  $t$  is the time in minutes from when the tank begins to lose water.

On the grid below, draw the graph of this model and label it as Tank A.

**1**

- (b) Tank B remains empty until  $t = 15$  when water is added to it at a constant rate of 30 litres per minute.

By drawing a line on the grid on the previous page, or otherwise, find the value of  $t$  when the two tanks contain the same volume of water.

- (c) Using the graphs drawn, or otherwise, find the value of  $t$  (where  $t > 0$ ) when the total volume of water in the two tanks is 1000 litres.

**2**

**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

- 20 12** Calculate the sum of the arithmetic series  $4 + 10 + 16 + \dots + 1354$ .

**3****MA**

NESA 2020 Mathematics Advanced HSC Examination

- 20 13**

**MA** Evaluate  $\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ .

**2**

NESA 2020 Mathematics Advanced HSC Examination

- 20 14** History and Geography are two subjects students may decide to study.

[Solution](#)

**MA** For a group of 40 students, the following is known.

- 7 students study neither History nor Geography
- 20 students study History
- 18 students study Geography

- (a) A student is chosen at random. By using a Venn diagram, or otherwise, find the probability that the student studies both History and Geography.
- (b) A student is chosen at random. Given that the student studies Geography, what is the probability that the student does NOT study History?
- (c) Two different students are chosen at random, one after the other. What is the probability that the first student studies History and the second student does NOT study History?

**2****1****2**

NESA 2020 Mathematics Advanced HSC Examination

- 20 15** Mr Ali, Ms Brown and a group of students were camping at the site located at  $P$ .

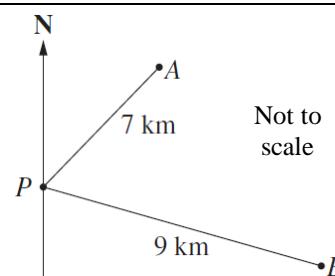
Mr Ali walked with some of the students on a bearing of  $035^\circ$  for 7 km to location  $A$ .

Ms Brown, with the rest of the students, walked on a bearing of  $100^\circ$  for 9 km to location  $B$ .

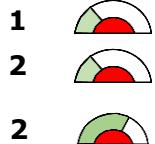
(a) Show that the angle  $APB$  is  $65^\circ$ .

(b) Find the distance  $AB$ .

(c) Find the bearing of Ms Brown's group from Mr Ali's group. Give your answer correct to the nearest degree.



[Solution](#)



**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

- 20 16** Sketch the graph of the curve  $y = -x^3 + 3x^2 - 1$ , labelling the stationary points and point of inflection. Do NOT determine the  $x$  intercepts of the curve.



NESA 2020 Mathematics Advanced HSC Examination

- 20 17** Find  $\int \frac{x}{4+x^2} dx$ .



NESA 2020 Mathematics Advanced HSC Examination

- 20 18** (a) Differentiate  $e^{2x}(2x + 1)$

[Solution](#)

- (b) Hence, find  $\int (x+1)e^{2x} dx$ .



NESA 2020 Mathematics Advanced HSC Examination

- 20 19** Prove that  $\sec \theta - \cos \theta = \sin \theta \tan \theta$

[Solution](#)

**MA**

NESA 2020 Mathematics Advanced HSC Examination

- 20 20** Kenzo is driving his car along a road while his friend records the velocity of the car,  $v(t)$ , in km/h every minute over a 5-minute period. The table gives the velocity  $v(t)$ , at time  $t$  hours.

$t$	0	$\frac{1}{60}$	$\frac{2}{60}$	$\frac{3}{60}$	$\frac{4}{60}$	$\frac{5}{60}$
$v(t)$	60	55	65	68	70	67

[Solution](#)



The distance covered by the car over the

5-minute period is given by  $\int_0^{\frac{5}{60}} v(t) dt$ .

Use the trapezoidal rule and the velocity at each of the six time values to find the approximate distance in kilometres the car has travelled in the 5-minute period. Give your answer correct to one decimal place,

NESA 2020 Mathematics Advanced HSC Examination

- 20 21** Hot tea is poured into a cup.

[Solution](#)

- MA** The temperature of tea can be modelled by  $T = 25 + 70(1.5)^{-0.4t}$ , where  $T$  is the temperature of the tea, in degrees Celsius,  $t$  minutes after it is poured.

(a) What is the temperature of the tea 4 minutes after it has been poured?



(b) At what rate is the tea cooling 4 minutes after it has been poured?



(c) How long after the tea is poured will it take for its temperature to reach 55



NESA 2020 Mathematics Advanced HSC Examination



- 20 22** The diagram shows a regular decagon (ten-sided shape with all sides equal and all interior angles equal).  
**MA**

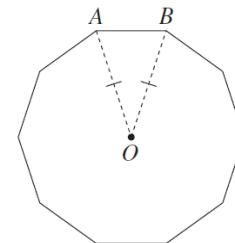
The decagon has centre  $O$ .

The perimeter of the shape is 80 cm.

By considering triangle  $OAB$ , calculate the area of the ten-sided shape.

Give your answer in square centimetres, correct to one decimal place.

**4** [Solution](#)



COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations

- 20 23** A continuous random variable,  $X$ , has the following probability density functions.

[Solution](#)

**MA**

$$f(x) = \begin{cases} \sin x & \text{for } 0 \leq x \leq k \\ 0 & \text{for all other values of } k \end{cases}$$

(a) Find the value of  $k$ .

**2**



(b) Find  $P(X \leq 1)$ . Give your answer correct to four decimal places.

**2**



NESA 2020 Mathematics Advanced HSC Examination

- 20 24** The circle  $x^2 - 6x + y^2 + 4y - 3 = 0$  is reflected in the  $x$ -axis.

**3** [Solution](#)

**MA**

Sketch the reflected circle, showing the coordinates of the centre and the radius.

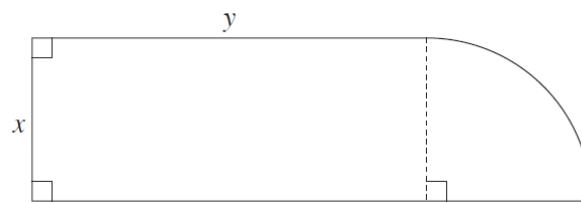


NESA 2020 Mathematics Advanced HSC Examination

- 20 25** A landscape gardener wants to build a garden bed in the shape of a rectangle attached to a quarter-circle.  
**MA**

[Solution](#)

Let  $x$  and  $y$  be the dimensions of the rectangle in metres, as shown in the diagram.



The garden bed is required to have an area of  $36 \text{ m}^2$  and to have a perimeter which is as small as possible. Let  $P$  metres be the perimeter of the garden bed.

(a) Show that  $P = 2x + \frac{72}{x}$ .

**3**



(b) Find the smallest possible perimeter of the garden bed, showing why this is the minimum perimeter.

**4**



NESA 2020 Mathematics Advanced HSC Examination

- 20 26** Tina inherits \$60 000 and invests it in an account earning interest at a rate of 0.5% per month. Each month, immediately after the interest has been paid, Tina withdraws \$800.

[Solution](#)

**MA**

The amount in the account immediately after  $n$ th withdrawal can be determined using the recurrence relation  $A_n = A_{n-1}(1.005) - 800$ , where  $n = 1, 2, 3, \dots$  and  $A_0 = 60\ 000$ .

(a) Use the recurrence relation to find the amount of money in the account immediately after the third withdrawal. \*

**2**



(b) Calculate the amount of interest earned in the first three months. \*

**2**



(c) Calculate the amount of money in the account immediately after the 94<sup>th</sup> withdrawal.

**3**



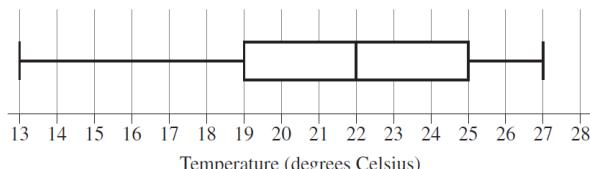
COMMON QUESTION: NESA 2020 Mathematics Standard 2\* and Advanced HSC Examinations

- 20 27** A cricket is an insect. The male cricket produces a chirping sound.  
**MA** A scientist wants to explore the relationship between the temperature in degrees Celsius and the number of cricket chirps heard in a 15-second time interval.

**5** [Solution](#)

Once a day for 20 days, the scientist collects data.  
 Based on the 20 data points, the scientist provides the information below.

- A box-plot of the temperature data is shown.



- The mean temperature in the dataset is  $0.525^{\circ}\text{C}$  below the median temperature in the dataset.

- A total of 684 chirps was counted when collecting the 20 data points.

The scientist fits a least-squares regression line using the data  $(x, y)$ , where  $x$  is the temperature in degrees Celsius and  $y$  is the number of chirps heard in a 15-second time interval. The equation of this line is  $y = -10.6063 + bx$ , where  $b$  is the slope of the regression line.

The least-squares regression line passes through the point  $(\bar{x}, \bar{y})$  where  $\bar{x}$  is the sample mean of the temperature data and  $\bar{y}$  is the sample mean of the chirp data. Calculate the number of chirps expected in a 15-second interval when the temperature is  $19^{\circ}$  Celsius. Give your answer to the nearest whole number.

**COMMON QUESTION: NESA 2020 Mathematics Standard 2 and Advanced HSC Examinations**

- 20 28** In a particular country, the hourly rate of pay for adults who work is normally distributed with a mean of \$25 and a standard deviation of \$5.

[Solution](#)

- (a) Two adults who both work are chosen at random.

Find the probability that at least one of them earns between \$15 and \$30 per hour.

- (b) The number of adults who work is equal to three times the number of adults who do not work.

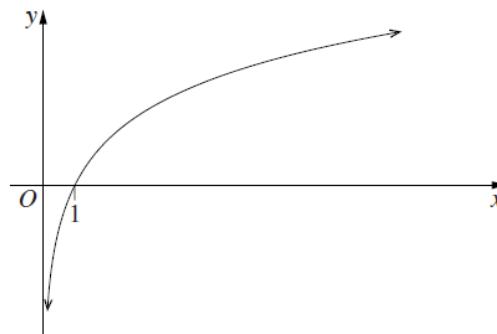
**2**

One adult is chosen at random.

Find the probability that the chosen adult works and earns more than \$25 per hour.

NESA 2020 Mathematics Advanced HSC Examination

- 20 29** The diagram shows the graph of  $y = c \ln x$ ,  $c > 0$ .

[Solution](#)

2



- (a) Show that the equation of the tangent to  $y = c \ln x$ , at  $x = p$ , where  $p > 0$  is

$$y = \frac{c}{p}x - c + c \ln p.$$

- (b) Find the value of  $c$  such that the tangent from part (a) has a gradient of 1 and passes through the origin.

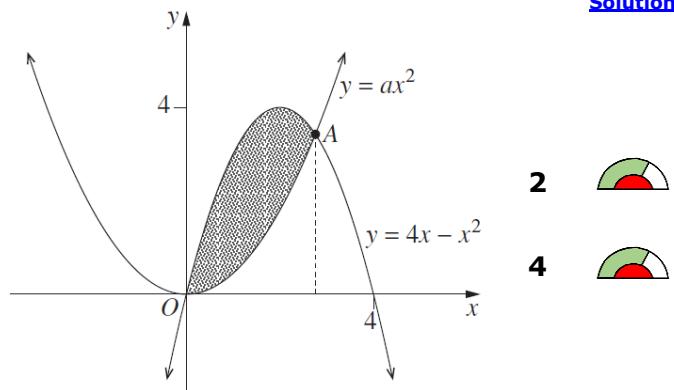
NESA 2020 Mathematics Advanced HSC Examination

- 20 30** The diagram shows two parabolas  $y = 4x - x^2$  and  $y = ax^2$ , where  $a > 0$ .

[Solution](#)

The two parabolas intersect at the origin,  $O$ , and at  $A$ .

- (a) Show that the  $x$ -coordinate of  $A$  is  $\frac{4}{a+1}$ .
- (b) Find the value of  $a$  such that the shaded area is  $\frac{16}{3}$ .



NESA 2020 Mathematics Advanced HSC Examination

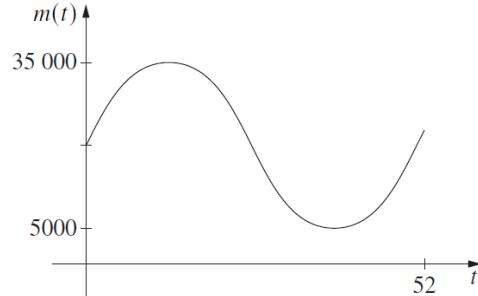
- 20 31** The population of mice on an isolated island can be modelled by the function

[Solution](#)

$$m = a \sin\left(\frac{\pi}{26}t\right) + b, \text{ where } t \text{ is the time in weeks}$$

and  $0 \leq t \leq 52$ . The population of mice reaches a maximum of 35 000 when  $t = 13$  and a minimum of 5000 when  $t = 39$ .

The graph of  $m(t)$  is shown.



2



- (a) What are the values of  $a$  and  $b$ ?

3



- (b) On the same island, the population of cats can be modelled by the function

$$c(t) = -80 \cos\left(\frac{\pi}{26}(t-10)\right) + 120.$$

Consider the graph of  $m(t)$  and the graph of  $c(t)$ .

Find the values of  $t$ ,  $0 \leq t \leq 52$ , for which both populations are increasing.

- (c) Find the rate of change of the mice population when the cat population reaches a maximum.

2

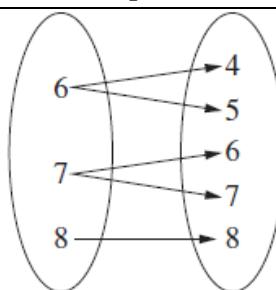


NESA 2020 Mathematics Advanced HSC Examination

# 2020 NESA Sample Paper

← Back
**MA**  
**SP**  
**1**  
 Band  
 2-3

What type of relation is shown?

**1** [Solution](#)


- A. Many-to-many    B. One-to-many    C. One-to-one    D. Many-to-one

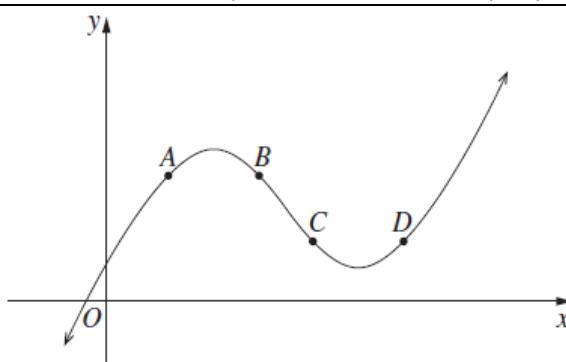
NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA**  
**SP**  
**2**  
 Band  
 2-3

At which point on this curve are the first and second derivatives BOTH negative?

**1** [Solution](#)

- A. A  
B. B  
C. C  
D. D



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA**  
**SP**  
**3**  
 Band  
 3-4
What is the gradient of any line perpendicular to  $3x + 2y = 5$ ?
**1** [Solution](#)

- A.  $\frac{2}{3}$     B.  $-\frac{2}{3}$     C.  $\frac{3}{2}$     D.  $-\frac{3}{2}$

NESA Mathematics Advanced Sample Examination Paper (2020)

**MA**  
**SP**  
**4**  
 Band  
 3-4
What is the derivative of  $5^{2x+3}$ ?
**1** [Solution](#)

- A.  $2 \times 5^{2x+3}$   
B.  $(2x + 3) \times 5^{2x+2}$   
C.  $\ln 5 \times 5^{2x+3}$   
D.  $\ln 5 \times 2 \times 5^{2x+3}$

NESA Mathematics Advanced Sample Examination Paper (2020)

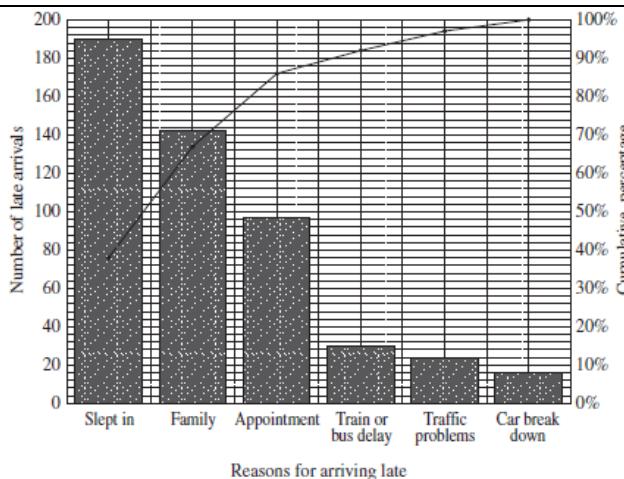
**MA**  
**SP**  
**5**  
 Band  
 3-4

A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.

**1** [Solution](#)
**19**  
**MS**  
**2**

What percentage of students gave the reason 'Train or bus delay'?

- A. 6%  
B. 15%  
C. 30%  
D. 92%

NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

- MA 6** What are the values of  $x$  for which  $|3 - 4x| = 4$ ? 1 [Solution](#)
- SP Band 4-5**
- A.  $x = -\frac{7}{4}$  and  $x = -\frac{1}{4}$
  - B.  $x = -\frac{7}{4}$  and  $x = \frac{1}{4}$
  - C.  $x = \frac{7}{4}$  and  $x = -\frac{1}{4}$
  - D.  $x = \frac{7}{4}$  and  $x = \frac{1}{4}$

NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 7** The diagram shows the graph of a continuous probability density function. Which of the following is the mode? 1 [Solution](#)
- SP Band 4-5**
- A. 0.07
  - B. 0.6
  - C. 0.8
  - D. 3
- 

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 8** The graphs show the future values over time of  $\$P$ , invested at three different rates of compound interest. Which of the following correctly identifies each graph? 1 [Solution](#)
- SP Band 5-6**
- 19 MS 2**
- A. W: 5% pa, compounding annually  
X: 10% pa, compounding annually  
Y: 10% pa, compounding quarterly
  - B. W: 5% pa, compounding annually  
X: 10% pa, compounding quarterly  
Y: 10% pa, compounding annually
  - C. W: 10% pa, compounding quarterly  
X: 10% pa, compounding annually  
Y: 5% pa, compounding annually
  - D. W: 10% pa, compounding annually  
X: 10% pa, compounding quarterly  
Y: 5% pa, compounding annually
- 

NESA Mathematics Advanced Sample Examination Paper (2020)

NESA 2019 Mathematics Standard 2 HSC Examination

- MA 9** The scores on an examination are normally distributed with a mean of 70 and a standard deviation of 6. Michael received a score on the examination between the lower quartile and the upper quartile of his scores. 1 [Solution](#)
- SP Band 5-6**
- 19 MS 2** Which shaded region most accurately represents where Michael's score lies?
- A.
  - B.
  - C.
  - D.

NESA Mathematics Advanced Sample Examination Paper (2020)

NESA 2019 Mathematics Standard 2 HSC Examination

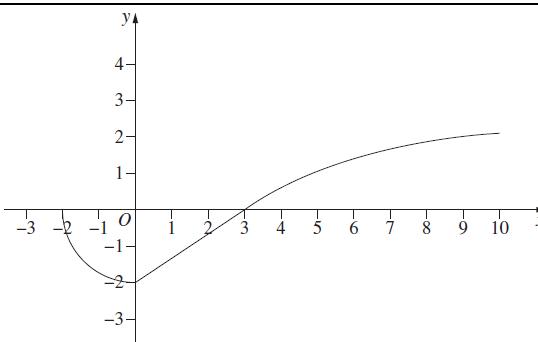
<b>MA</b>	<b>10</b>	Given the function $y = \log_7(x^x)$ , which expression is equal to $\frac{dy}{dx}$ ?	<b>1</b>	<a href="#">Solution</a>
<b>SP</b>	<b>Band 5-6</b>	A. $\frac{1}{x \ln 7}$ B. $\frac{1}{\ln 7} \times \log_7(x^{x-1})$ C. $\frac{1}{x^x \ln 7}$ D. $\log_7 x + \frac{1}{\ln 7}$		
NESA Mathematics Advanced Sample Examination Paper (2020)				
<b>MA</b>	<b>11</b>	Find the equation of the tangent to the curve $f(x) = x^3 + 1$ at the point $(1, 2)$ .	<b>2</b>	<a href="#">Solution</a>
<b>SP</b>	<b>Band 2-4</b>	NESA Mathematics Advanced Sample Examination Paper (2020)		
<b>MA</b>	<b>12</b>	The diagram shows a triangle with sides of length $x$ cm, 11 cm and 13 cm and an angle of $80^\circ$ .	<b>3</b>	<a href="#">Solution</a>
<b>SP</b>	<b>Band 2-4</b>	NOT TO SCALE		
<b>19</b>	<b>17</b>	Use the cosine rule to calculate the value of $x$ , correct to two significant figures.		
<b>MS</b>	<b>2</b>	NESA Mathematics Advanced Sample Examination Paper (2020) NESA 2019 Mathematics Standard 2 HSC Examination		
<b>MA</b>	<b>13</b>	A credit card requires a four-figure personal identification number (PIN) for purchases. The figures are chosen from the digits 0, 1, 2, 3, ..., 9. Repetition is allowed and the PIN can start with any of the 10 digits. The credit card is lost and the finder tries to guess the PIN by entering four digits.		<a href="#">Solution</a>
<b>SP</b>	<b>Band 2-4</b>	(a) What is the probability that the four digits entered are the correct PIN? (b) What is the probability that the finder will guess at least one digit in its correct order?	<b>1</b>	<b>1</b>
NESA Mathematics Advanced Sample Examination Paper (2020)				
<b>MA</b>	<b>14</b>	A function is given by $f(x) = 18x^2 - x^4$ .		<a href="#">Solution</a>
<b>SP</b>	<b>Band 2-5</b>	(a) Find the stationary points and determine their nature. (b) Sketch the curve, labelling the stationary points and axis intercepts.	<b>4</b>	<b>2</b>
NESA Mathematics Advanced Sample Examination Paper (2020)				
<b>MA</b>	<b>15</b>	Two teams play a game. There are only two possible ways to score points: hitting the red target or hitting the blue target.		<a href="#">Solution</a>
<b>SP</b>	<b>Band 3-5</b>	Hitting the red target scores $R$ points and hitting the blue target scores $B$ points. The results of a game are shown. By forming a pair of simultaneous equations, or otherwise, find the values of $R$ and $B$ .		
NESA Mathematics Advanced Sample Examination Paper (2020)				
<b>MA</b>	<b>16</b>	Differentiate $e^{\sin(\pi x)}$ .	<b>2</b>	<a href="#">Solution</a>
<b>SQ</b>	<b>Band 3-5</b>	NESA Mathematics Advanced Sample HSC Examination Paper (2020)		
<b>MA</b>	<b>17</b>	Given the function $f(x) = x^2 + 2$ and $g(x) = \sqrt{x-6}$ , sketch $y = f(g(x))$ over its natural domain.	<b>2</b>	<a href="#">Solution</a>
<b>SP</b>	<b>Band 4-5</b>	NESA Mathematics Advanced Sample HSC Examination Paper (2020)		

- MA 18** The diagram shows a continuous function  $y = f(x)$  defined in the domain  $[-2, 10]$ .

The function consists of a quarter of a circle centred at  $(0, 0)$  with radius 2, a straight line segment and a logarithmic function  $f(x) = \ln(x - 2)$  in the domain  $[3, 10]$ .

- (a) Find the exact area bounded by the function  $y = f(x)$  and the  $x$ -axis in the domain  $[-2, 3]$ .

- (b) Hence, find the exact value of  $\int_3^{10} \ln(x - 2) dx$ , given that  $\int_{-2}^{10} f(x) dx = 8 \ln 8 - 10 - \pi$ . **2**

**2****2**

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 19** A discrete random variable  $X$  has the probability distribution table shown. By finding the value of  $m$ , calculate the expected value and the variance of  $X$ .

$X = x$	11	12	13	14
$P(x)$	0.2	0.3	$m$	0.4

**3**[Solution](#)

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 20** A student was asked to differentiate  $f(x) = x^2 + 4x$  from first principles.

**SP Band 3-5** The student began the solution as shown below.

Complete the solution.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**2**[Solution](#)

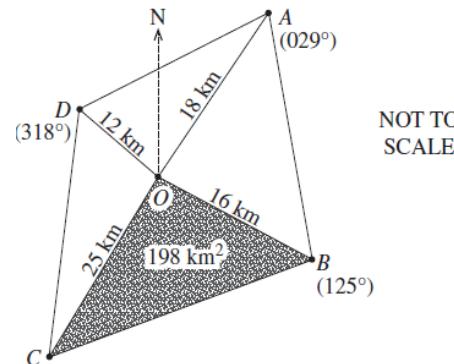
NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 21** The diagram shows the distances of four towns  $A, B, C$  and  $D$  from point  $O$ .

The true bearings of towns  $A, B$  and  $D$  from point  $O$  are also shown.

The area of the acute-angled triangle  $BOC$  is  $198 \text{ cm}^2$ .

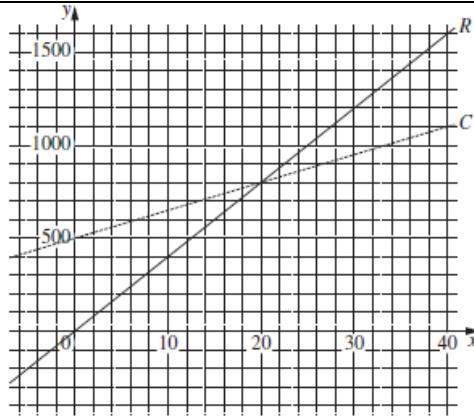
Calculate the true bearing of town  $C$  from point  $O$ , correct to the nearest degree.

**3**[Solution](#)

NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 22** A small business makes and sells bird houses. Technology was used to draw straight-line graphs to represent the cost of making bird houses ( $C$ ) and the revenue from selling bird house ( $R$ ). The  $x$ -axis displays the number of bird houses and the  $y$ -axis displays the cost/revenue in dollars.

- SP Band 2-5** 19 **36** (a) How many bird houses need to be sold to break even?  
 (b) By first forming equations for cost ( $C$ ) and revenue ( $R$ ), determine how many bird houses need to be sold to earn a profit of \$1900.

**1****3**

NESA Mathematics Advanced Sample Examination Paper (2020)

NESA 2019 Mathematics Standard 2 HSC Examination

**MA 23** The function  $f(x) = \tan^3 x$  is given.

**SP Band 3-5** If  $f'(x) = 3\tan^m x + 3\tan^2 x$ , find the value of  $m$ .

**2** [Solution](#)

NESA Mathematics Advanced Sample Examination Paper (2020)

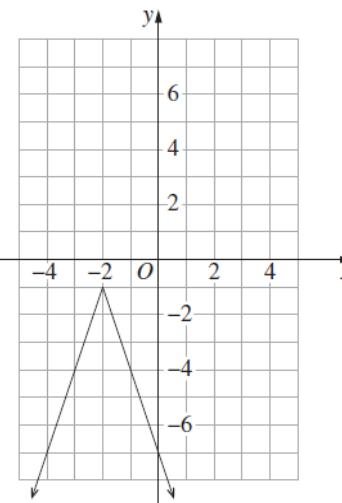
**MA 24** The function  $f(x) = |x|$  is transformed and the equation of the new function is of the form  $y = kf(x + b) + c$ , where  $k$ ,  $b$  and  $c$  are constants.

**SP Band 3-5**

The graph of the new function is shown.

What are the values of  $k$ ,  $b$  and  $c$ ?

**2** [Solution](#)



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA 25** A circle is given by the equation  $x^2 + y^2 + 4x - 10y = -16$ .

**SP Band 3-5**

Find the centre and radius of this circle.

**2** [Solution](#)

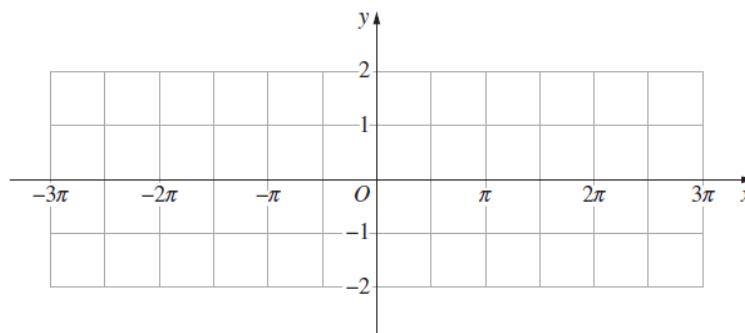
NESA Mathematics Advanced Sample Examination Paper (2020)

**MA 26** By drawing graphs on the number plane,

**SP Band 3-6** determine how many solutions there are to

the equation  $\sin x = \frac{|x|}{5}$   
in the domain  $(-\infty, \infty)$ .

**3** [Solution](#)



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

**MA 27** The function  $f(x) = \cos x$  is transformed to  $g(x) = 3 \cos 2x$ .

**SP Band 2-3**

Describe in words how both the amplitude and period change in this transformation.

**2** [Solution](#)

NESA Mathematics Advanced Sample Examination Paper (2020)

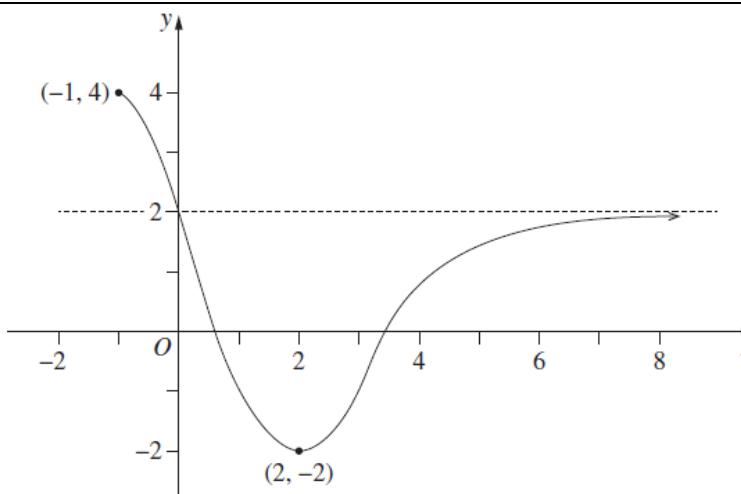
**MA 28** The graph of a function  $f(x)$  is shown.

**SP Band 3-4**

It has an asymptote at  $y = 2$ .

Using interval notation,  
state the domain and  
range of  $f(x)$ .

**2** [Solution](#)

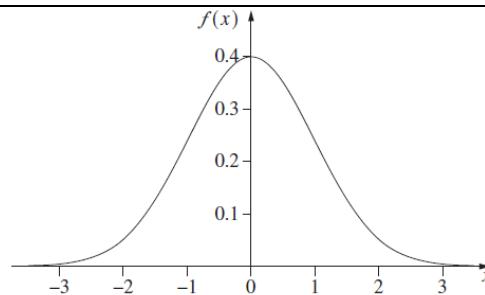


NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 29** Let  $X$  denote a normal random variable with mean 0 and standard deviation 1. The random variable  $X$  has a probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \text{ where } -\infty < x < \infty.$$

The diagram shows the graph of  $y = f(x)$ .



[Solution](#)

- (a) Complete the table of values for the function given.

Give your answer correct to four significant figures.

$X = x$	0	1	2	3
$f(x)$	0.3989	0.242		0.004432

1

- (b) Using the trapezoidal rule and the 4 function values in the table in part (a),

$$\text{show that } P(-3 \leq X \leq 3) = \int_{-3}^{3} f(x) dx \approx 0.9953.$$

2

- (c) The IQ (Intelligence Quotient) scores for a large population are normally distributed with a mean of 100 and a standard deviation of 15.

2

Using the result obtained in part (b), calculate the probability of randomly selecting a person with an IQ score above 145 from this large population.

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 30** The population,  $P$ , of rabbits on an island is given by  $P(t)$ , where  $t$  is the time in years after the rabbits were introduced.

[Solution](#)

The rabbit population changes at a rate modelled by the function  $\frac{dP}{dt} = 30e^{1.25t}$ .

Calculate the increase in the number of rabbits at the end of the first 10 years. Give your answer correct to two significant figures.

NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 31** A bid made at an auction for a real estate property, in millions of dollars, can be modelled by the random variable  $X$  with the probability density function

[Solution](#)

$$f(x) = \begin{cases} k(16 - x^2) & 1 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Show that the value of  $k$  is  $\frac{1}{27}$ .

2

- (b) Find the cumulative distribution function.

2

- (c) Find the probability that a bid of more than 3 million dollars will be made.

1

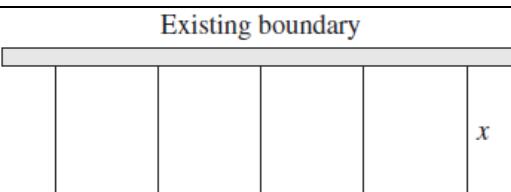
NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 32** A farmer wishes to make a rectangular enclosure of area  $720 \text{ m}^2$ . She uses an existing straight boundary as one side of the enclosure. She uses wire fencing for the remaining three sides and also to divide the enclosure into four equal rectangular areas of width  $x \text{ m}$  as shown.

The total length,  $\ell \text{ m}$ , of the wire fencing is given by  $\ell = 5x + \frac{720}{x}$ .

(Do NOT prove this.)

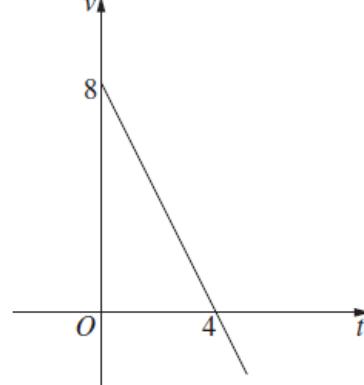
Find the minimum length of wire fencing required, showing why this is the minimum length.



**3** [Solution](#)

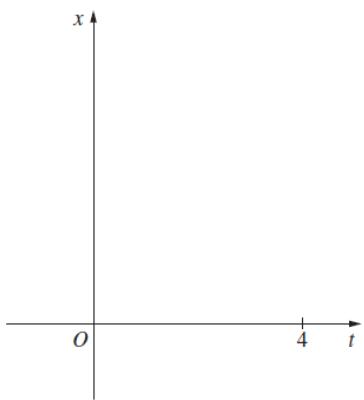
- MA 33** A particle is moving along the  $x$ -axis.  
**SP Band 4-6** The graph shows its velocity  $v$  metres per second at time  $t$  seconds.

When  $t = 0$  the displacement  $x$  is equal to 2 metres.



**2** [Solution](#)

On the axes draw a graph that shows the particle's displacement,  $x$  metres from the origin, at a time  $t$  seconds between  $t = 0$  and  $t = 4$ . Label the coordinates of the endpoints of your graph.



NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 34** The table shows the future values of an annuity of \$1 for different interest rates for 4, 5 and 6 years. The contributions are made at the end of each year.  
**SP Band 2-5** An annuity account is opened and contributions of \$2000 are made at the end of each year for 7 years.

For the first 6 years, the interest rate is 4% per annum, compounded annually.

For the 7<sup>th</sup> year, the interest rate increases to 5% per annum, compounded annually.

Calculate the amount in the account immediately after the 7<sup>th</sup> contribution is made.

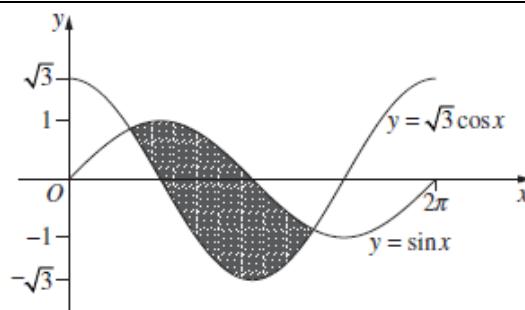
Years	Future value of an annuity of \$1			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

**3** [Solution](#)

NESA Mathematics Advanced Sample Examination Paper (2020)  
NESA 2019 Mathematics Standard 2 HSC Examination

- MA 35** The diagram shows the curves  $y = \sin x$   
**SP Band 3-6** and  $y = \sqrt{3} \cos x$ .

Find the area of the shaded region.



**4** [Solution](#)

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 36** An island initially has 16 100 trees. The number of trees increases by 1% per annum. The people on the island cut down 1161 trees at the end of each year.  
**SP Band 2-6**
- Show that after the first year there are 15 100 trees.
  - Show that at the end of 2 years the number of trees remaining is given by the expression  $T_2 = 16\ 100 \times (1.01)^2 - 1161(1 + 1.01)$ .
  - Show that at the end of  $n$  years the number of trees remaining is given by the expression  $T_n = 16\ 100 - 100\ 000 \times (1.01)^n$ .
  - For how many years will the people on the island be able to cut down 1161 trees annually?

[Solution](#)

**1**

**2**

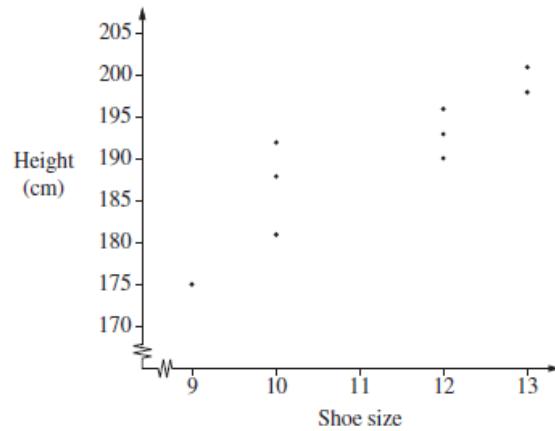
**2**

**1**

NESA Mathematics Advanced Sample Examination Paper (2020)

- MA 37** A set of bivariate data is collected by measuring the height and recording the shoe size of nine basketball players.  
**SP Band 2-5** The collected data is shown in the table and graphed in the scatterplot shown.

Shoe size	10	12	12	12	10	9	13	10	13
Height (cm)	181	190	193	196	188	175	198	192	201



[Solution](#)

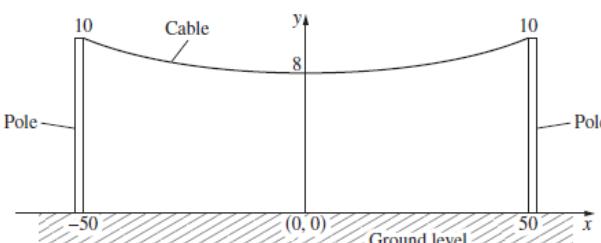
**1**

**1**

- Determine the equation of the least-squares regression line for this data.
  - A player with a shoe size of 11 was absent when the data was collected.
- Calculate the predicted height for the missing basketball player using your answer to part (a).

NESA Mathematics Advanced Sample HSC Examination Paper (2020)

- MA 38** A cable is freely suspended between two 10 m poles, as shown. The poles are 100 m apart and the minimum height of the cable is 8 metres.  
**SP Band 3-6** The height of the cable is given as  $y = c(e^{kx} + e^{-kx})$ , where  $c$  and  $k$  are positive constants.



[Solution](#)

**1**

**4**

**3**

- Show that the value of  $c$  is 4.
- Use the result in part (a) to show that one value of  $k$  is  $\frac{\ln 2}{50}$ .
- Hence find the area between the poles, the cable and the ground.

NESA Mathematics Advanced Sample HSC Examination Paper (2020)