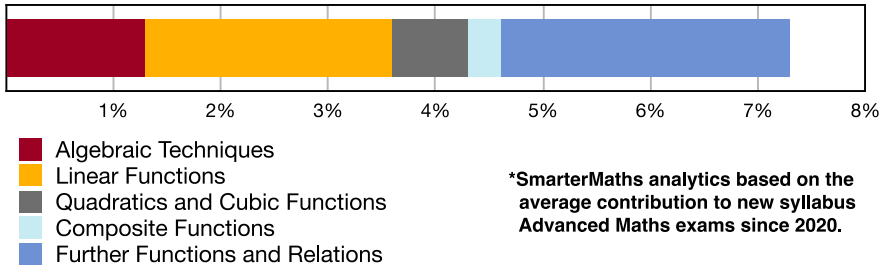
ADVANCED MATHEMATICS
Functions (Adv), F1 Working with Functions (Adv)

Algebraic Techniques (Y11)

Teacher: Cathyanne Horvat
Exam Equivalent Time: 30 minutes *(based on allocation of 1.5 minutes per mark)*

F1 Working With Functions



***SmarterMaths analytics based on the average contribution to new syllabus Advanced Maths exams since 2020.**

HISTORICAL CONTRIBUTION

- *F1 Working with Functions* is a Year 11 topic whose content represents the lowest of low hanging fruit in the new Advanced course.
- *F1 Working with Functions* has contributed a healthy 7.3% per exam since the new syllabus was introduced in 2020.
- We have split the topic into 5 categories for analysis purposes: *1-Algebraic Techniques, 2-Linear Functions, 3-Quadratics and Cubic Functions, 4-Composite Functions and 5-Further Functions and Relations.*
- This analysis looks at *Algebraic Techniques (1.3%).*

HSC ANALYSIS - What to expect and common pitfalls

- *Algebraic Techniques* were regularly tested in the old *Mathematics* course, with surd calculations representing the dominant question type.
- Surds have historically been examined in 2 out of every 3 years (not examined in 2021 or 2020). Note that virtually all past questions have involved compound surds in the denominator which are no longer examinable. This fact is reflected in the database along with supplementary questions that look at examinable surd denominators (i.e. non-compound surds).
- Algebraic Fractions were last examined in the 2021 Advanced paper and deserve attention, particularly as they represent an overlap between the new Advanced and Standard 2 content.
- Rounding questions can require students to know standard decimal place rounding, significant figures (2019) and scientific notation (2015).

Questions

1. Functions, 2ADV F1 2014 HSC 1 MC

   What is the value of $\frac{\pi^2}{6}$, correct to 3 significant figures?

   (A) 1.64
   (B) 1.65
   (C) 1.644
   (D) 1.645

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2. Functions, 2ADV F1 SM-Bank 53

   i. If $\frac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^x$, find x. *(1 mark)*

   ii. Calculate the value of $\frac{1}{\sqrt[3]{7+\pi}}$ to 3 significant figures. *(1 mark)*

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3. Functions, 2ADV F1 2005 HSC 1d

   Express $\frac{(2x-3)}{2} - \frac{(x-1)}{5}$ as a single fraction in its simplest form. *(2 marks)*

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4. Functions, 2ADV F1 2008 HSC 1e

   Expand and simplify $\left(\sqrt{3}-1\right)\left(2\sqrt{3}+5\right)$. *(2 marks)*

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5. Functions, 2ADV F1 2021 HSC 11

   Solve $x + \frac{x-1}{2} = 9$ *(2 marks)*

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6. Functions, 2ADV F1 SM-Bank 46

   Find a and b such that a, b are real numbers and

   $$\frac{8-\sqrt{27}}{2\sqrt{3}} = a + b\sqrt{3}.$$ *(2 marks)*

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7. Functions, 2ADV F1 SM-Bank 55

   Simplify $\frac{4p-12p^2}{3} \times \frac{6p}{3p^2-p}$. *(2 marks)*

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8. Functions, 2ADV F1 EQ-Bank 22

Worker A picks a bucket of blueberries in a hours. Worker B picks a bucket of blueberries in b hours.

i. Write an algebraic expression for the fraction of a bucket of blueberries that could be picked in one hour if A and B worked together. ***(2 marks)***

ii. What does the reciprocal of this fraction represent? ***(1 mark)***

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9. Functions, 2ADV F1 2008 HSC 1c

Simplify $\dfrac{2}{n} - \dfrac{1}{n+1}$. ***(2 marks)***

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10. Functions, 2ADV F1 EQ-Bank 21

Simplify $\left(\dfrac{p}{q}\right)^3 \div \left(pq^{-2}\right)$. ***(2 marks)***

Worked Solutions

1. Functions, 2ADV F1 2014 HSC 1 MC

$$\begin{aligned}\frac{\pi^2}{6} &= 1.6449\ldots \\ &= 1.64 \text{ (3 sig. figures)} \\ \Rightarrow\ & A\end{aligned}$$

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2. Functions, 2ADV F1 SM-Bank 53

i. $\dfrac{1}{\sqrt[3]{7+\pi}} = (7+\pi)^{-\frac{1}{3}}$

ii. $\begin{aligned}\dfrac{1}{\sqrt[3]{7+\pi}} &= 0.4619\ldots \\ &= 0.462 \text{ (to 3 sig. fig.)}\end{aligned}$

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3. Functions, 2ADV F1 2005 HSC 1d

$$\begin{aligned}&\frac{(2x-3)}{2} - \frac{(x-1)}{5} \\ &= \frac{5(2x-3)-2(x-1)}{10} \\ &= \frac{10x-15-2x+2}{10} \\ &= \frac{8x-13}{10}\end{aligned}$$

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4. Functions, 2ADV F1 2008 HSC 1e

$$\begin{aligned}&\left(\sqrt{3}-1\right)\left(2\sqrt{3}+5\right) \\ &= 2\times 3 + 5\sqrt{3} - 2\sqrt{3} - 5 \\ &= 1 + 3\sqrt{3}\end{aligned}$$

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5. Functions, 2ADV F1 2021 HSC 11

$$\begin{aligned}x + \frac{x-1}{2} &= 9 \\ 2x + x - 1 &= 18 \\ 3x &= 19 \\ x &= \frac{19}{3}\end{aligned}$$

Worked Solutions

6. Functions, 2ADV F1 SM-Bank 46

$$\frac{8-\sqrt{27}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{3}\left(8-3\sqrt{3}\right)}{\left(2\sqrt{3}\right)^2}$$

$$= \frac{16\sqrt{3}-18}{12}$$

$$= -\frac{3}{2} + \frac{4}{3}\sqrt{3}$$

$$\therefore a = -\frac{3}{2},\ b = \frac{4}{3}$$

7. Functions, 2ADV F1 SM-Bank 55

$$\frac{4p-12p^2}{3} \times \frac{6p}{3p^2-p} = \frac{4p(1-3p)}{3} \times \frac{6p}{p(3p-1)}$$

$$= \frac{8p(1-3p)}{3p-1}$$

$$= -8p$$

8. Functions, 2ADV F1 EQ-Bank 22

i. In one hour:

Worker A picks $\frac{1}{a}$ **bucket.**

Worker B picks $\frac{1}{b}$ **bucket.**

COMMENT: Note that the question asks for "a fraction".

$\therefore$ **Fraction picked in 1 hour working together**

$$= \frac{1}{a} + \frac{1}{b}$$

$$= \frac{a+b}{ab}$$

ii. The reciprocal represents the number of hours it would take to fill one bucket, with A and B working together.

9. Functions, 2ADV F1 2008 HSC 1c

$$\frac{2}{n} - \frac{1}{n+1}$$

$$= \frac{2(n+1)-1(n)}{n(n+1)}$$

$$= \frac{2n+2-n}{n(n+1)}$$

$$= \frac{n+2}{n(n+1)}$$

10. Functions, 2ADV F1 EQ-Bank 21

$$\left(\frac{p}{q}\right)^3 \div \left(pq^{-2}\right) = \frac{p^3}{q^3} \div \frac{p}{q^2}$$

$$= \frac{p^3}{q^3} \times \frac{q^2}{p}$$

$$= \frac{p^2}{q}$$