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EFIN 301

Predicting Football Scores with Quadrinomial Trees

Introduction

Sports betting is legal in 30 U.S. states, Puerto Rico, and the District of Columbia. However, sports are unpredictable, which makes betting on sports a risky proposition, especially because sports books take a large cut of each bet. This paper aims to use a quadrinomial tree and forecasting to predict the likely outcome of future NFL games to find and exploit mispriced bets. These findings could potentially be used to make profitable bets on NFL games and to better understand team statistics.

Data

There were two main sets of data used. The first was drive-level data for all 32 teams in the NFL since 2002 (n=702). These data included items such as touchdowns, field goals, punts, safeties, and turnovers. These drive-level statistics show significant time trends, which can be seen in the graphs in Appendix A. The second set of data included the outcomes and betting odds for every NFL game since 2007 (n=9,722). The odds for each game included the away money line, home money line, home spread, and over-under.

Methods

As mentioned before, the model consisted of two parts—a quadrinomial tree to simulate an NFL game and a forecasting model to predict a team's performance. The quadrinomial was built in the Rust programming language, and the forecasting was done in R with RStudio.

The quadrinomial tree was responsible for simulating the games. It not only calculated the expected score but also the probability distribution of scores, allowing the change of any bet hitting to be calculated. Each quadrinomial tree represented one half of a game, and the results were multiplied by two to get the values for a whole game. Certain occurrences, such as punts returned for a touchdown, pick sixes (an interception returned for a touchdown), safeties, and overtime ended up being omitted from the tree. The punts and interceptions being returned for a touchdown were omitted because of the added complexity and computation time required and because of the infrequency of these events. The early versions of the tree included safeties and overtime, but these were later removed because they added a lot of computation time without contributing to the precision of the model.

The implementation of the quadrinomial tree is as follows. Each node represented a drive and had a score associated with it. Each node has four children—one representing a touchdown being scored, one representing a field goal, one representing no score, and one representing the end of the period. Each node was connected to its parent with a weighted edge, with the weight corresponding to the probability of that node following from the previous. The probability distribution at each step was calculated as the average between that of the offense and the opposing defense. The home team started with one point to account for home-field advantage. A typical tree had about 270,000 nodes.

The forecasting model aimed to predict a team's future statistics based on their past performance. To do this, a regression was performed relating the team's performance in one to its performance in the previous year. The model is shown in equation 1.

$$\sqrt{tdperdrive_t} = \hat{\beta}_0 + \hat{\beta}_1\sqrt{tdperdrive_{t-1}} + \hat{\beta}_2 playoff_{t-1} + \hat{\beta}_3 season \quad (1)$$

Tdperdrive is the number of touchdowns scored per drive, *playoff* is a dummy variable representing whether the team made the playoffs, and *season* is the season the model is trying to predict (to remove time trends). The square root of *tdperdrive* was taken to make the data more closely resemble a normal distribution. Only the *tdperdrive* and *fgperdrive* values were forecasted—the chance of the period ending was assumed to be unchanged and the chance of a drive ending without points was set to whatever value led to a valid probability distribution.

Not every model used all the terms in equation 1 as not all variables were significant in each model. *Tdperdrive* for the offense used all the terms, *fgperdrive* for the offense only used the time trend, *tdperdrive* for the defense used the lagged touchdown and time trends, and *fgperdrive* for the defense used the playoff and time trend terms. These prediction models were not very powerful, each having an R^2 of about 0.2-0.3.

To evaluate the performance of the model, the distributions of predicted games were used to find and exploit mispriced bets. For each portfolio, a minimum spread between the odds implied by the betting line and the odds calculated from the model was enforced. Additionally, two different models were used for each bet type: one where the bet was the same for all bets made, and one where the bet made was based on the spread between the odds.

Results

The average error between the actual final scores of NFL games and the simulated scores was -0.31 (S_x=9.2, n=9,722) when using the actual data from the relevant season. When using the forecasted models, the average error was -0.49 (S_x=10.04, n=9,466). As expected, when using the predicted values, the simulation was less accurate. Histograms of the errors are in Appendix B.

The generated portfolios saw mixed results. For reference, when making random bets, one would expect to lose 3-4.5%. With over/under bets, the expected return was about -3.5%; however, when using a minimum spread of 30%, the losses increased to -26%. No explanation could be found for this extreme drop. With money line bets, some positive returns were possible. The best portfolio of money line bets consisted of a minimum threshold of 30% and used dynamic bets. This portfolio had a return of 3.8%. The performing portfolio came from the spread bets. With a minimum spread of 30% and dynamic bets, a return of 13.2% was achieved. This was the best result by a significant margin. Full tables of results are available in Appendices C-E.

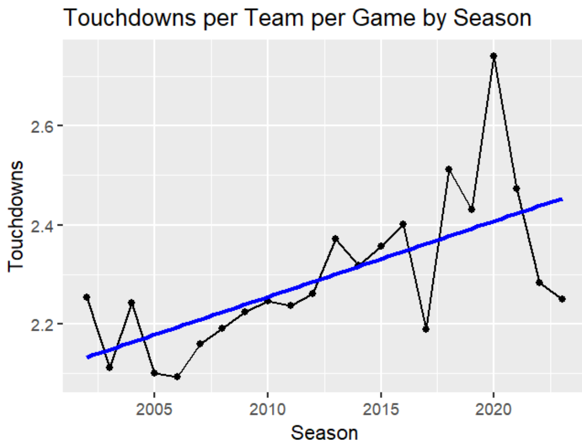
There was no clear indication of whether constant or dynamic bets are the better strategy. Depending on the exact parameters, one or the other could perform better, often by significant margins.

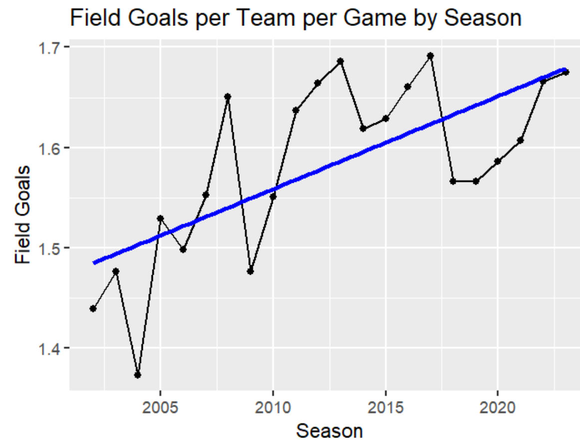
Conclusion

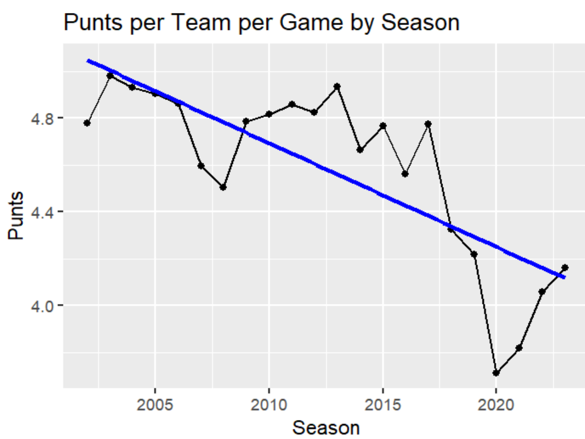
Some portfolios of bets generated by the model did yield positive returns. However, most portfolios yielded losses, with some performing even worse than making random bets. Furthermore, there are some cautions with using this model. It is highly unstable, and even slight changes to its parameters lead to very different results. There is also the possibility that the model overfits the historical data, limiting its predictive abilities. The model still has unexplained behavior, like how the returns on over/under bets suddenly drop unexpectedly. Further investigations into different dynamic betting models could lead to better returns than the one found here; the dynamic betting model could itself be tuned for better performance.

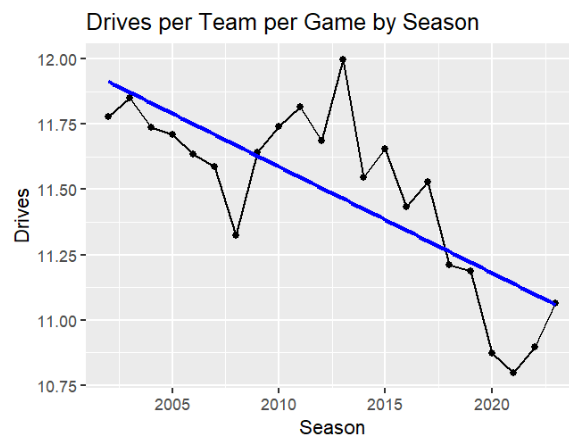
There are some limitations with this model. Firstly, it only considers the previous season when forecasting and does not consider the season as it has progressed so far. (For example, at the end of the season, a stronger prediction could be made from a team's performance earlier in the season than from the team's performance in the previous season.) Secondly, it dramatically simplifies the game of football to speed up computation. Lastly, it is possible that sportsbooks have improved their odds-making over time and that the optimal model for betting the oddsmakers changes over time as well.

Appendix A: Time Trend Plots

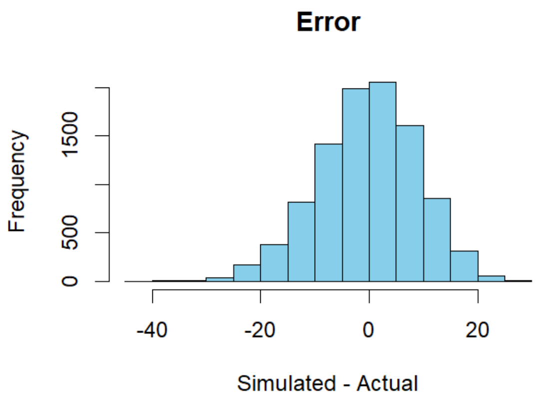


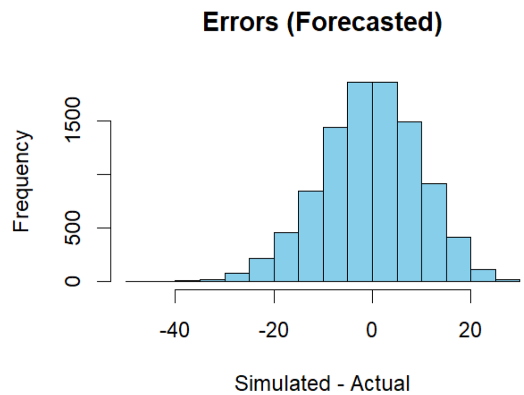






Appendix B: Error in Simulation





Appendix C: Over/Under Bet Returns

Difference in Odds	Percent of Games (in 1 Season)	Returns	Returns (Dynamic)
>0%	100% (272)	-3.9%	-4.4%
>5%	77.5% (211)	-3.5%	-4.1%
>10%	44.0% (120)	-5.3%	-5.0%
>20%	8.4% (23)	-4.0%	-4.4%
>25%	2.3% (6)	-1.1%	-2.3%
>30%	0.5% (1)	-26.5%	-26.6%

Appendix D: Money Line Bet Returns

Difference in Odds	Percent of Games (in 1 Season)	Returns	Returns (Dynamic)
>0%	100% (272)	-3.0%	-4.3%
>5%	76.3% (208)	-3.9%	-4.6%
>10%	53.2% (145)	-7.0%	-6.1%
>20%	18.3% (50)	-8.5%	-6.4%
>25%	8.2% (22)	1.9%	3.0%
>30%	3.4% (9)	1.8%	3.8%

Appendix E: Spread Bet Returns

Difference in Odds	Percent of Games (in 1 Season)	Returns	Returns (Dynamic)
>0%	100% (272)	-11.5%	-0.6%
>5%	97.1% (264)	-11.7%	-0.6%
>10%	91.5% (249)	-11.2%	-0.4%
>20%	77.3% (210)	-5.6%	2.8%
>25%	68.2% (186)	0.5%	7.1%
>30%	58.2% (158)	8.6%	13.2%