$$x_{n+1} = \ln\left(e^{x_n} - x_n\right)$$

Suppose the sequence converges to a limit L. That is,

$$\lim_{n\to\infty} x_n = L.$$

Taking limits on both sides,

$$L = \ln\left(e^L - L\right)$$

$$e^L = e^L - L$$

$$\Rightarrow L = 0.$$

Hence, the sequence converges to 0.

$$e^{x_{n+1}} = e^{x_n} - x_n$$

$$x_n = e^{x_n} - e^{x_{n+1}}$$

The infinite series $x_0 + x_1 + x_2 + ...$ can be written as

$$\sum_{k=0}^{\infty} x_k .$$

Applying the method of differences,

$$\begin{aligned}
\sum_{k=0}^{\infty} x_k &= \sum_{k=0}^{\infty}\left(e^{x_k} - e^{x_{k+1}}\right) \\
&= \lim_{n\to\infty}\left(e^{x_0} - e^{x_{n+1}}\right) \\
&= e^{x_0} - \lim_{n\to\infty} e^{x_{n+1}} \\
&= e - \lim_{n\to\infty} e^{x_{n+1}} \\
&= e - 1. \quad \because \lim_{n\to\infty} e^{x_{n+1}} = \exp\left(\lim_{n\to\infty} x_{n+1}\right) = 0
\end{aligned}$$