Comp1081: Algorithms and Data Structures resit exam

Denim Adewumi

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Abstract

Here are all of the typed out solutions (in LaTeX) for the take home examination in Algorithms and Data Structures due 02/08/2024

Question 2: Hash Tables and Collision Handling

(a) **Separate Chaining**

Given:

- Keys: 6, 27, 13, 36, 25, 5, 17, 41, 23
- Hash Function: $h(k) = (4k + 13)11$

Hash Table:

$Index$	$Chain$
0	
1	
2	
3	36
4	$13 \to 41$
5	$6 \to 17 \to 23$
6	27
7	
8	5
9	
10	25

(b) **Linear Probing**

Hash Table:

Index	*Key*
0	
1	
2	
3	36
4	13
5	6
6	27
7	41
8	5
9	17
10	25
11	23

(c) **Quadratic Probing**

Hash Table:

Index	*Key*
0	
1	
2	
3	36
4	13
5	6
6	27
7	41
8	5
9	17
10	25
11	23

(d) **Double Hashing**

Given secondary hash function: $h'(k) = 5 - (k5)$

Hash Table:

Index	*Key*
0	23
1	36
2	13
3	
4	25
5	5
6	6
7	41
8	17
9	27
10	

Question 5: Asymptotic Notations

(a) $4x^2 - 2x$ is $O(x^2)$
True. The term $4x^2$ dominates $-2x$ as $x \to \infty$.

(b) $2x\sqrt{x}$ is $o(2^x)$
True. Exponential growth 2^x outgrows polynomial growth $2x\sqrt{x}$ as $x \to \infty$.

(c) $x + x\log_2 x$ is $\Theta(x)$
False. $x\log_2 x$ dominates x, so $x + x\log_2 x$ is $\Theta(x\log_2 x)$.

(d) $100x^4$ is $\Omega(16\log_2 x)$
True. Polynomial growth x^4 outgrows logarithmic growth $16\log_2 x$ as $x \to \infty$.

(e) $x + x^2 + 100x^3 + 10x^2\log_2 x$ is $\omega(x^3)$
False. $100x^3$ and $10x^2\log_2 x$ terms are bounded above by x^3.

Question 6: Master Theorem Applications

(a) $T(n) = 3T(n/9) + \sqrt{n}$
Case 1: $f(n) = \sqrt{n} = O(n^{\log_9 3 - \epsilon})$ where $\epsilon = 1/2$
Answer: $T(n) = \Theta(n^{\log_9 3}) = \Theta(n^{1/2})$

(b) $T(n) = 4T(n/4) + 5n\log_2 n$
Case 2: $f(n) = \Theta(n\log_2 n)$
Answer: $T(n) = \Theta(n\log^2 n)$

(c) $T(n) = 2T(n) + n$

Not solvable by Master Theorem: $a = 2$, $b = 1$ leads to an invalid form for Master Theorem.

(d) $T(n) = 4T(n/2) + n^3$
Case 3: $f(n) = \Omega(n^3) = \Omega(n^{\log_2 4+\epsilon})$
Answer: $T(n) = \Theta(n^3)$

(e) $T(n) = 4T(n/8) + 7n^{0.6}$
Case 1: $f(n) = O(n^{\log_8 4-\epsilon})$
Answer: $T(n) = \Theta(n^{\log_8 4}) = \Theta(n^{2/3})$

Question 7(b): Analysis of AverageQuickSort

(i) **Worst-Case Time Complexity**
In the worst case, the partition is extremely unbalanced each time (e.g., if all elements are the same), leading to $O(n^2)$.

(ii) **Constructing Worst-Case Inputs**
To construct the worst-case input, you can use a list where one element is much larger (or smaller) than the rest, repeatedly causing the partition to be extremely unbalanced. For instance, for decreasing order, use a list in ascending order.