CS 310

Assignment 113

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Problem 1 — Algorithms often have the following properties:

- the steps are stated *unambiguously* so that there is no question how the algorithm proceeds
- the algorithm is *deterministic* so that repeating the algorithm on the same input produces the same output
- the algorithm is *finite* because it terminates after a finite number of steps have been performed
- the algorithm produces *correct* output for a given input

For the following algorithm, for each property listed above, determine whether the algorithm exhibits this property:

```
unsigned max3(unsigned a, unsigned b, unsigned c)
{
  unsigned result = a;
  if (b > result)
  {
    result = b;
  }
  if (c > result)
  {
    result = c;
  }
  return result;
}
```

Answer: This algorithm is unambiguous because the syntax for the operations is well-understood. It is deterministic because it always produces the same output for a given input. It is finite because the number of lines of code executed (including the header) is strictly between 3 and 7 inclusive. It is correct because for all possible valid input combinations it does in fact return a value equal to the maximum input value.

Problem 2 — Repeat problem 1 for the following algorithm. This algorithm empirically checks the correctness of Goldbach's conjecture, which states (in a modern interpretation) that every even number greater than 2 is the sum of two prime numbers. Assume `has_prime_addends` is a valid function that correctly determines whether its argument has two prime addends.

```
bool goldbach()
{
  unsigned value = 4;
  bool ok = true;
  while (ok)
  {
    if (!has_prime_addends(value))
```

```
        {
          ok = false;
        }
        else
        {
          value += 2;
        }
      }
      return ok;
    }
```

Answer: This algorithm is unambiguous because the syntax for the operation is clear and easy to understood. But it's non-deterministic since it does not produces the same output for a given input, in fact, if the given input is an even number and it's greater than 2, it would not have any output. This algorithm is also infinite because it would never reached the terminated condition if the given input is a even number and greater than 2. Lastly this algorithm does not correct since it does not produces correct output for a given input. At this example, the input is 4, which is an even number, so the while loop will goes on and on and never stop, thus, the while loop become a infinite loop and doesn't have any output. According to the analysis above, we can conclude that this algorithm is unambiguous, non-deterministic, infinite and not correct.

Problem 3 — What is the hexadecimal representation of 724_{10}?

Answer: The first few powers of 16 are:

$$\begin{aligned} 16^0 &= 1 \\ 16^1 &= 16 \\ 16^2 &= 256 \\ 16^3 &= 4096 \end{aligned}$$

Thus we have:

$$\begin{array}{r} 724 \\ \underline{-2 \times 256 = 512} \\ 212 \\ \underline{-13 \times 16 = 208} \\ 4 \\ \underline{-4 \times 1 = 2} \\ 0 \end{array}$$

And thus we have $724_{10} = 2d4_{16}$

.

Problem 4 — Based on the hexadecimal value found in the previous solution, what is the binary representation of 724_{10}?

Answer: According to the previous solution, we have the hex value 0x2d4. So we could convert the hex to binary one digit by one digit. So here is a binary to hex table:

$0000 = 0$	$1000 = 8$
$0001 = 1$	$1001 = 9$
$0010 = 2$	$1010 = a$
$0011 = 3$	$1011 = b$
$0100 = 4$	$1100 = c$
$0101 = 5$	$1101 = d$
$0110 = 6$	$1110 = e$
$0111 = 7$	$1111 = f$

Since $2_{16} = 0010$, $d_{16} = 1101$ and $4_{16} = 0100$. So the binary of $724_{10} = 0010\ 1101\ 0100$.

Problem 5 — What is the decimal representation of `0x2b3a`?

Answer: According to the table above, we could know that b in decimal is 11 and a in decimal is 10. So we have the following

$$\begin{aligned} 2b3a_{16} &= 2 \cdot 16^3 + 11 \cdot 16^2 + 3 \cdot 16^1 + 10 \cdot 16^0 \\ &= 2 \cdot 4096 + 11 \cdot 256 + 3 \cdot 16 + 10 \\ &= 8192 + 2816 + 48 + 10 \\ &= 11066 \end{aligned}$$

So, the decimal representation of `0x2b3a` is 11066.