ALG AD - 3 - S - 2023 - 10 - 11

# I Determinanti

Calculați

1) $\Delta_1 = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

2) $\Delta_2 = \begin{vmatrix} a-b & l-m & x-y \\ b-c & m-n & y-z \\ c-a & n-l & z-x \end{vmatrix}$

3) $\Delta_3 = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

4) $\Delta_4 = \begin{vmatrix} 0 & 1 & 1 & x \\ x & 0 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 1 & x & 0 \end{vmatrix}$

REZ.:

Metoda 1

1) $bc^2 + ab^2 + a^2 \cdot c - ba^2 - cb^2 - c^2 \cdot a =$

$= ab(b-a) + \quad c^2(b-a) + c(a^2-b^2) =$

$= (b-a)(ab+c^2) + c(a-b)(a+b) =$

$= (b-a)(ab+c^2-ca-cb) =$

$= (b-a)[b(a-c)+c(c-a)] = (b-a)(b-c)(a-c)$

Metoda 2:

$$\Delta_1 = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \underset{L_3-L_1}{\overset{L_2-L_1}{=}} \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} =$$

$$= 1\cdot(-1)^{1+1}\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} =$$

$$= \begin{vmatrix} b-a & (b-a)(b+a) \\ c-a & (c-a)(c+a) \end{vmatrix} =$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix} =$$

$$= (b-a)(c-a)\left[(c+a)-(b+a)\right] =$$

$$= (b-a)(c-a)(c+a-b-a) =$$

$$= (b-a)(c-a)(c-b)$$

2)
$$\overset{L_1+L_2+L_3}{=} \begin{vmatrix} 0 & 0 & 0 \\ b-c & m-n & y-z \\ c-a & n-l & z-x \end{vmatrix} = 0$$

3)
$$\Delta_3 = \begin{vmatrix} 10 & 10 & 10 & 10 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix} = 10\begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix} \begin{matrix} c_2-c_1 \\ = \\ c_3-c_1 \\ c_4-c_1 \end{matrix}$$

$$\underset{\substack{C_3-C_1\\C_4-C_1}}{\overset{C_2-C_1}{=}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & -1 \\ 3 & 1 & -2 & -1 \\ 4 & -3 & -2 & -1 \end{vmatrix} =$$

$$= 10 \cdot 1 \begin{vmatrix} 1 & 2 & -1 \\ 1 & -2 & -1 \\ -3 & -2 & -1 \end{vmatrix} = 10 \cdot (-1)(-2) \begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ -3 & 1 & 1 \end{vmatrix} =$$

$$= 20 \; (1+1+3-1+1+3) =$$

$$= 20 \cdot 8 = 160$$

4) $\underset{=}{L_1+L_2+L_3+L_4}$

$$\Delta_4 = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ x & 0 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 1 & x & 0 \end{vmatrix} =$$

$$= (x+2) \begin{vmatrix} 1 & 1 & 1 & 1 \\ x & 0 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 1 & x & 0 \end{vmatrix} \underset{C_2-C_1}{\overset{\substack{C_4-C_1\\C_3-C_1}}{=}}$$

$$= (x+2) \begin{vmatrix} 1 & 0 & 0 & 0 \\ x & 1-x & 1-x & 1-x \\ 1 & -1 & -1 & 0 \\ 1 & x-1 & x-1 & -1 \end{vmatrix} =$$

$$= (x+2)(-1)^{1+1} \begin{vmatrix} -x & 1-x & 1-x \\ x-1 & -1 & 0 \\ 0 & x-1 & -1 \end{vmatrix} =$$

$$= (x+2)\left[-x+(x-1)^2(1-x)+0-0-0+(1-x)(x-1)\right] =$$

$$= (x+2)\left[-x+(1-x)\left((x-1)^2+(x-1)\right)\right] =$$

$$= (x+2)\left[-x+(1-x)(x-1)\cdot x\right] =$$

$$= (x+2)x(-1+x-1-x^2+x) =$$

$$= (x+2)x(-x^2+2x-2) =$$

$$= -x(x+2)(x^2-2x+2)$$

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5) $\Delta_5 = \begin{vmatrix} 1 & -\cos a & -\cos 2a \\ 1 & -\cos b & -\cos 2b \\ 1 & -\cos c & -\cos 2c \end{vmatrix}$

6) $\Delta_6 = \begin{vmatrix} a^2 & b^2 & c^2 \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \\ bc & ca & ab \end{vmatrix}$

7) $\Delta_7 = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 2-x & 1 \\ 1 & 1 & 1 & 3-x \end{vmatrix}$

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Rez.

$\cos 2x = \cos^2 x - \sin^2 x =$

$= 2\cos^2 x - 1 =$

$= 1 - 2\sin^2 x$

5) ~~Poze~~

Vezi poze $\boxed{\#01}$

6) $\Delta_6 \overset{L_1+L_2}{=} \begin{vmatrix} a^2+b^2+c^2 & a^2+b^2+c^2 & a^2+b^2+c^2 \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \\ bc & ca & ab \end{vmatrix}$

$\Delta_6 = (a^2+b^2+c^2) \begin{vmatrix} 1 & 1 & 1 \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \\ bc & ca & ab \end{vmatrix} \begin{matrix} L_2 - L_1 \\ = \\ L_3 - L_1 \end{matrix}$ )

$= (a^2+b^2+c^2) \begin{vmatrix} 1 & 0 & 0 \\ b^2+c^2 & a^2-b^2 & a^2-c^2 \\ bc & ca-bc & ab-bc \end{vmatrix} =$

$= (a^2+b^2+c^2) \begin{vmatrix} (a+b)(a-b) & (a+c)(a-c) \\ ca-bc & ab-bc \end{vmatrix} =$

$= (a^2+b^2+c^2) \begin{vmatrix} (a+b)(a-b) & (a+c)(a-c) \\ c(a-b) & b(a-c) \end{vmatrix} =$

$= (a^2+b^2+c^2)(a-b)(a-c) \begin{vmatrix} a+b & a+c \\ c & b \end{vmatrix} =$

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$\boxed{\#01}$

5.

$$\Delta_5 = \begin{vmatrix} 1 & \cos a & 2\cos^2 a - 1 \\ 1 & \cos b & 2\cos^2 b - 1 \\ 1 & \cos c & 2\cos^2 c - 1 \end{vmatrix} \begin{array}{c} L_2 - L_1 \\ = \\ L_3 - L_1 \end{array}$$

$$= \begin{vmatrix} \cancel{1} & \cancel{\cos a} & \cancel{2\cos^2 a - 1} \\ 0 & \cos b - \cos a & 2\cos^2 b - 2\cos^2 a \\ 0 & \cos c - \cos a & 2\cos^2 c - 2\cos^2 a \end{vmatrix}$$

$$\Delta_5 = 1 \begin{vmatrix} \cos b - \cos a & 2(\cos^2 b - \cos^2 a) \\ \cos c - \cos a & 2(\cos^2 c - \cos^2 a) \end{vmatrix}$$

$$\Delta_5 = 2 \begin{vmatrix} \cos b - \cos a & (\cos a + \cos b)(\cos b - \cos a) \\ \cos c - \cos a & (\cos c + \cos a)(\cos c - \cos a) \end{vmatrix}$$

$$\Delta_5 = 2(\cos b - \cos a)(\cos c - \cos a) \cdot \begin{vmatrix} 1 & \cos a + \cos b \\ 1 & \cos a + \cos c \end{vmatrix} =$$

$$= 2(\cos b - \cos a)(\cos c - \cos a)\left[\cancel{\cos a} + \cos c - \cancel{\cos a} - \cos b\right] =$$

$$= 2(\cos b - \cos a)(\cos c - \cos a)(\cos c - \cos b)$$

$$\Delta_6 = (a^2+b^2+c^2)(a-b)(a-c)\cdot[b(a+b)-c(a+c)] =$$

$$=(a^2+b^2+c^2)(a-b)(a-c)[ab+b^2-ca-c^2] =$$

$$=(a^2+b^2+c^2)(a-b)(a-c) =$$

$$=[a(b-c)+(b+c)(b-c)] =$$

$$=(a^2+b^2+c^2)(a-b)(a-c)(a-c)(b-c) =$$

$$=[a+b+c]$$

7) $$\Delta_7 \underset{\substack{c_3-c_1 \\ c_4-c_1}}{\overset{c_2-c_1}{=}} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -x & 0 & 0 \\ 1 & 0 & 1-x & 0 \\ 1 & 0 & 0 & 2-y \end{vmatrix} =$$

$$= 1\cdot(-x)\cdot(1-x)\cdot(2-y)$$

## II Sisteme

1) $$\begin{cases} x-2y+z=0 \\ x+3y-2z=0 \end{cases}$$

2) $$\begin{cases} 3x-y+z=0 \\ -2x+y+3z=-7 \\ x+2y-2z=7 \end{cases}$$

3) $$\begin{cases} 2x-y+3z=4 \\ 3x+4y-z=-5 \\ x+5y-4z=-9 \end{cases}$$

1) $z = \alpha \Rightarrow \begin{cases} x - 2y = -\alpha & |\cdot(-1) \\ x + 3y = 2\alpha \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} -x + 2y = \alpha \\ x + 3y = 2\alpha \end{cases}$$

$$5y = 3\alpha \Rightarrow y = \frac{3\alpha}{5}$$

$$\overset{5}{x} - \frac{6\alpha}{5} = \overset{5}{-\alpha}$$

$$\Rightarrow 5x - 6\alpha = -5\alpha$$

$$\Rightarrow 5x = -5\alpha + 6\alpha$$

$$\Rightarrow 5x = \alpha \Rightarrow x = \frac{\alpha}{5}$$

$$S = \left\{\left(\frac{\alpha}{5}, \frac{3\alpha}{5}, \alpha\right) \mid \alpha \in \mathbb{R}\right\}$$

2)

$$\begin{array}{l} 6x - 2y + 2z = 0 \\ x + 2y - 2z = 7 \quad + \\ \hline 7x = 7 \Rightarrow x = 1 \end{array}$$

$$\begin{cases} 3 - y + z = 0 \\ -2 + y + 3z = -7 \\ 1 + 2y - 2z = 7 \end{cases} \Rightarrow \begin{cases} -y + z = -3 & |:(-1) \\ y + 3z = -5 \\ 2(y - z) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} y - z = 3 \\ y + 3z = -5 \end{cases} \quad = z = y - 3$$

$$y + 3y - 9 = -5$$

$$4y = 4 \Rightarrow y = 1 \qquad z = -2 \qquad S = \{(1, 1, -2)\}$$

3) $\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 4 & -1 \\ 1 & 5 & -4 \end{vmatrix} = -32 + 45 + 1 - (12 - 10 + 12)$

$\begin{matrix} 2 & -1 & 3 \\ 3 & 4 & -1 \end{matrix}$

$= 14 - 14 = 0$

$\Delta_p = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0 \Rightarrow$ rang 2

Notăm $z = \alpha$

$\begin{cases} 2x - y = 4 - 3\alpha \\ 3x + 4 = -5 + \alpha \end{cases}$

$\begin{cases} 8x - 4y = 16 - 12\alpha \\ 3x + 4y = -5\alpha \end{cases}$

$11x = 11 - 11\alpha$

$2(1 - \alpha) - y = 4 - 3\alpha$

$2 - 2\alpha - y = 4 - 3\alpha$

$-2\alpha + 4\alpha - y = 2$

$-y = 2 - \alpha$

$\boxed{y = -2 + \alpha}$

$V: 1 - \cancel{\alpha} - 10 + 5\cancel{\alpha} - 5\cancel{\alpha} = 9$

$-9 = -9$ (A)

$S = \{(1-\alpha, -2+\alpha, \alpha) \mid \alpha \in \mathbb{R}\}$

4) $\begin{cases} 2x + y - z = 1 \\ 3x + 3y - z = 2 \quad | : 2 \\ 2x + 4y = 3 \end{cases}$

5) $\begin{cases} x - y + 3z = 2 \\ 3x + 2y - \cancel{z} = 1 \\ 3y - 3z = 5 \end{cases}$

6) $\begin{cases} x - 2y + 3z = 1 \\ 2x - 4y + 6z = 2 \quad | : 2 \\ -3x + 6y - 9z = -3 \quad | : (-3) \end{cases}$

$\Leftrightarrow x - 2y + 3z = 1$ $\Rightarrow x = 1 + 2\alpha + 3\beta$

$y = \alpha$

$z = \beta$

$S = \{1 + 2\alpha - 3\beta, \alpha, \beta\}$

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4) $\begin{cases} -2x - y + z = -1 \\ -3x + 3y - z = 2 \end{cases}$

$x + 2y = 1$

$x + 2y = \frac{3}{2}$ $\Rightarrow$ Imposibil $S = \emptyset$