4_C_2023_10_13

13.10.2023

AN MATE

$$\lim_{n\to\infty} \frac{[n\pi]}{n}$$

$[x]$ = cel mai mare nr. întreg mai mic sau egal cu $x$

$[5,7]$

$\ldots, -2, -1, 0, 1, 2, 3, 4, \boxed{5}$

$[-5,7]$

$\ldots, -8, -7, -6$

$[x] \in \mathbb{Z}$

$$[x] \le x < [x]+1$$

$$\Updownarrow$$

$$x-1 < [x] \le x$$

$$n\pi < [n\pi] \le n\pi$$

$$\pi - \frac{1}{n} < \frac{[n\pi]}{n} \le \pi$$

$$\downarrow$$

$$\pi$$

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

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$S$   $r\%$ pe an   $S + S \cdot r = $
$= S(1+r)$

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pe un semestru

$S + S \cdot \frac{r}{2} =$
$= S\left(1 + \frac{r}{2}\right)$

→ la finele anului   $S\left(1 + \frac{r}{2}\right) + S\left(1 + \frac{r}{2}\right) \cdot \frac{r}{2} =$
$= S\left(1 + \frac{r}{2}\right)\left(1 + \frac{r}{2}\right) = S\left(1 + \frac{r}{2}\right)^2$

1/an   $S\left(1 + r\right)$

2/an   $S\left(1 + \frac{r}{2}\right)^2$

4/an   $S\left(1 + \frac{r}{4}\right)^4$

12/an   $S\left(1 + \frac{r}{12}\right)^{12}$

52/an   $S\left(1 + \frac{r}{52}\right)^{52}$

365/an   $S\left(1 + \frac{r}{365}\right)^{365}$

n/an   $S\left(1 + \frac{r}{n}\right)^n$

$$\lim_{n\to\infty} S\left(1+\frac{r}{n}\right)^n \qquad \begin{array}{l} S = 1 \\ r = 1\% \end{array}$$

$$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n \overset{not}{=} e$$

$$\boxed{\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{\frac{1}{\frac{1}{n}}} = e}$$

$(x_n)_n \subseteq \mathbb{R}$

$\lim_{n\to\infty} x_n = 0$

$$\Rightarrow \lim_{n\to\infty} (1+x_n)^{\frac{1}{x_n}} = e$$

$$\Downarrow \ln$$

$$\boxed{\lim_{n\to\infty} \frac{\ln(1+x_n)}{x_n} = 1}$$

$$\boxed{\lim_{n\to\infty} \frac{e^{x_n}-1}{x_n} \overset{a>0}{=} \ln e}$$

$$\boxed{\lim_{n\to\infty} \frac{(1+x_n)^r-1}{x_n} = r}$$

„limite fundamentale"

Ex.: $\lim\limits_{n\to\infty} \left(1+\sqrt{n+1}-\sqrt{n}\right)^{-\sqrt{n}} = ?$

$$\lim_{n\to\infty} \sqrt{n+1}-\sqrt{n} \overset{\infty-\infty}{=} \lim_{n\to\infty} \frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}} =$$

$$= \lim_{n\to\infty} \frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \lim_{n\to\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}} \overset{\frac{1}{\infty}}{=} 0$$

$$\lim_{n\to\infty} \Big(1+\underbrace{(\sqrt{n+1}-\sqrt{n})}_{x_n\to 0}\Big)^{-\sqrt{n}} =$$

$$= \lim_{n\to\infty} \left[\left(1+\sqrt{n+1}-\sqrt{n}\right)^{\frac{1}{\sqrt{n+1}-\sqrt{n}}}\right]^{(\sqrt{n+1}-\sqrt{n})(-\sqrt{n})} =$$

$$= e^{-\lim\limits_{n\to\infty} \underbrace{(\sqrt{n+1}-\sqrt{n})}_{\infty-\infty}\underbrace{\sqrt{n}}_{\infty}} = e^{-\lim\limits_{n\to\infty} \frac{\overbrace{\sqrt{n}}^{\to\infty}}{\underbrace{\sqrt{n+1}+\sqrt{n}}_{\to\infty}}} =$$

$$= e^{-\lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n(1+\frac{1}{n})}+\sqrt{n}}} =$$

$$= e^{-\lim\limits_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}\left(\sqrt{1+\frac{1}{n}}+1\right)}} = e^{-\lim\limits_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1}} =$$

$$= e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right)^n$$

$$\lim_{n\to\infty}\sqrt[n]{2} = \lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1$$

$$\lim_{n\to\infty}\sqrt[n]{3} = 1$$

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}\right) \overset{1^\infty}{=} \lim_{n\to\infty}\left(1+\underbrace{\frac{\sqrt[n]{2}+\sqrt[n]{3}}{2}-1}_{x_n\to 0}\right)^n =$$

$$= \lim_{n\to\infty}\left[(1+x_n)^{\frac{1}{x_n}}\right]^{x_n\cdot n} = e^{\lim_{n\to\infty} n : x_n} =$$

$$= e^{\lim_{n\to\infty} n\cdot\frac{\sqrt[n]{2}+\sqrt[n]{3}-2}{2}} = e^{\frac{1}{2}\lim_{n\to\infty} n(\sqrt[n]{2}-1+\sqrt[n]{3}-1)} =$$

$$= e^{\frac{1}{2}\lim_{n\to\infty}\frac{\sqrt[n]{2}-1+\sqrt[n]{3}-1}{\frac{1}{n}}} =$$

$$= e^{\frac{1}{2}\lim_{n\to\infty}\frac{2^{\frac{1}{n}}-1}{\frac{1}{n}}+\frac{3^{\frac{1}{n}}-1}{\frac{1}{n}}} =$$

$$= e^{\frac{1}{2}(\ln 2+\ln 3)} = e^{\frac{1}{2}\ln 6} =$$

$$= (e^{\ln 6})^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6}$$