

Siruri Cauchy

$(x_n)_n \subseteq \mathbb{R}$  convergent dacă există  $l \in \mathbb{R}$  s.t.

$$\lim_{n \rightarrow \infty} x_n = l \quad \text{i.e. } (\forall) \varepsilon > 0 \quad (\exists) n_\varepsilon \in \mathbb{N} \quad (\forall) m, n \in \mathbb{N} \quad n > n_\varepsilon$$

$$|x_n - l| < \varepsilon$$



$$(\forall) m, n \in \mathbb{N}$$

$$m, n > n_\varepsilon$$

$$|x_m - x_n| = |x_m - l + l - x_n| \leq$$

$$\leq |x_m - l| + |l - x_n| < \varepsilon + \varepsilon = 2\varepsilon$$

Def.:  $(x_n)_n \subseteq \mathbb{R}$  și Cauchy dacă

$$(\forall) \varepsilon > 0 \quad (\exists) n_\varepsilon \in \mathbb{N} \quad (\forall) m, n \geq n_\varepsilon$$

$$|x_m - x_n| < \varepsilon$$

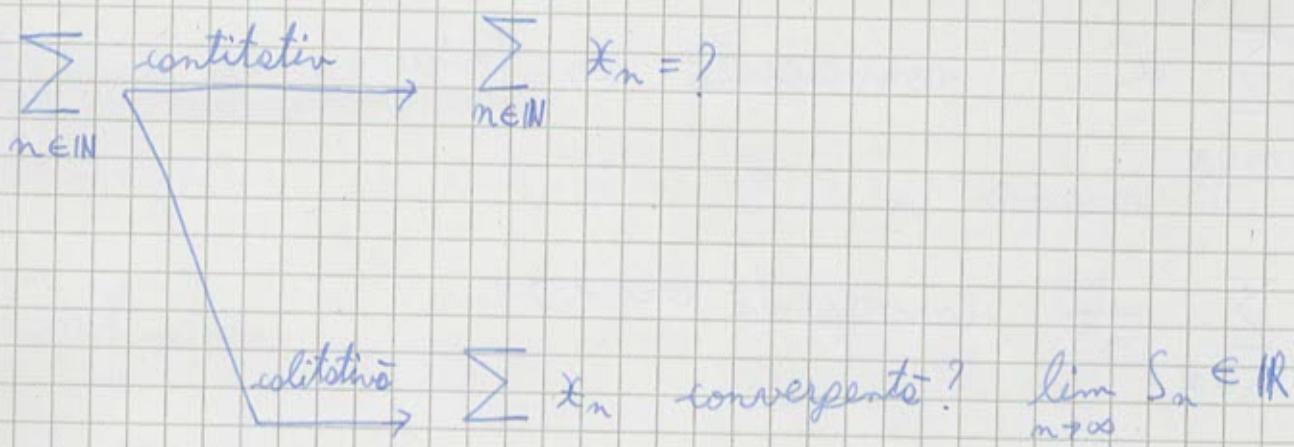
R  $(x_n)_n \subseteq \mathbb{R}$  convergent  $\Leftrightarrow (x_n)_n$  și Cauchy

Dacă  $(s_n)_n$  are limite  $s \in \bar{\mathbb{R}}$ , sănătă

$$\begin{aligned} S &= x_1 + x_2 + \dots + x_n + \dots = \\ &= \text{suma seriei } \sum_{n=1}^{\infty} x_n \end{aligned}$$

$$\sum_n x_n$$

$$\sum_{n \in N} x_n$$



R  $\sum x_n \text{ -konvergent} \Rightarrow \lim_{n \rightarrow \infty} x_n = 0$

$\downarrow$

$$S_n - S_{n-1}$$

R  $\sum |x_n| \text{ -konvergent} \Rightarrow \sum x_n \text{ -konvergent!}$

C<sub>1</sub>  $x_n, y_n$   
 $0 \leq x_n \leq y_n$   
 $(\forall) n \in \mathbb{N}$

$\alpha) \sum y_n \text{ -konvergent} \Rightarrow \sum x_n \text{ -konvergent}$

$\beta) \sum x_n \text{ -divergent} \rightarrow \sum y_n \text{ divergent}$

C<sub>2</sub>  $0 < x_n, y_n (\forall) n \in \mathbb{N}$   
 $\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = l \in (0, \infty) \Rightarrow \sum x_n \sim \sum y_n$

$$\sum_{n \in \mathbb{N}} \alpha^n \text{-convergent} \Leftrightarrow |\alpha| < 1$$

$$\sum_{n \in \mathbb{N}} \frac{1}{n^\alpha} \text{-convergent} \Leftrightarrow \alpha > 1$$

$$\sum x_n, x_n > 0$$

$$\lim_{n \rightarrow \infty} \frac{x_n + 1}{x_n} = l$$

$l < 1 \Rightarrow \sum x_n \text{ convergent}$

$l > 1 \Rightarrow \sum x_n \text{ divergent}$

$l = 1 \Rightarrow ???$

$$\left[ \lim_{n \rightarrow \infty} \sqrt[n]{x_n} = l \right]$$

$$\lim_{n \rightarrow \infty} n \left( \frac{x_n}{x_{n+1}} - 1 \right) = l$$

$l < 1 \Rightarrow \sum x_n \text{ divergent}$

$l > 1 \Rightarrow \sum x_n \text{ convergent}$

$l = 1 \Rightarrow ???$

### Criteriul lui ABEL

$$(x_n)_n \subseteq \mathbb{R}$$

$$\lim_{n \rightarrow \infty} x_n = 0$$

$$x_{n+1} \leq x_n \quad (\forall) n \in \mathbb{N}$$

$$(y_n)_n \subseteq \mathbb{R}$$

$$(\exists) n \in \mathbb{R} \quad (\forall) n \in \mathbb{N} \quad |y_1 + y_2 + \dots + y_n| \leq M$$

$$\Rightarrow \sum x_n \cdot y_n \text{-convergent}$$

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## Criteriul lui LEIBNIZ

$y_n = (-1)^n$   $(x_n)_n \searrow 0 \Rightarrow \sum (-1)^n x_n$  -convergentă

Continuare

