1. Studiați monotonia șirului

$$x_n = \frac{3n+5}{4n+6}$$

$n \in \mathbb{N}$

$x_{n+1} - x_n =$

$$x_{n+1} = \frac{3\cdot(n+1)+5}{4(n+1)+6} = \frac{3n+8}{4n+1}$$

$$x_{n+1} - x_n = {}^{4n+7)}\frac{3n+8}{4n+1} - {}^{4n+1)}\frac{3n+5}{4n+7} = \frac{(4n+7)(4n+8)}{(4n+1)(4n+7)}$$

$$= \frac{(3n+8)(4n+7) - (3n+5)(4n+1)}{(4n+1)(4n+7)} =$$

$$= \frac{(12n+56) - (12n+5)}{16n+7} = \frac{51}{16n+7}$$

2. Calculați $\lim_{n\to\infty} n \cdot \ln\left(1-\frac{1}{n}\right) = 0\cdot\infty = l$

CN: $\frac{0}{0}$, $\frac{\infty}{\infty}$, 1^{∞}, $0\cdot\infty$, 0^{0}, $\infty-\infty$, 0^{∞}

Criteriul log.

$\lim_{n\to\infty} \frac{\ln(1+x_n)}{x_n} \overset{\frac{0}{0}}{=} 1$, pt. $\lim_{n\to\infty} x_n = 0$

$$l = \lim_{n\to\infty} \frac{n \cdot \ln(1-\underbrace{\tfrac{1}{n}}_{x_n})}{-\frac{1}{n}} \cdot \left(-\frac{1}{n}\right) = \lim_{n\to\infty} n\left(-\frac{1}{n}\right) = -1$$

M II. $l = \lim_{n\to\infty} \ln\left(1-\frac{1}{n}\right)^n = \ln\left(\lim_{n\to\infty} \underbrace{\left(1-\frac{1}{n}\right)^n}_{1^\infty}\right)$

$\lim_{n\to\infty} f(x_n) = f\left(\lim_{n\to\infty} x_n\right)$

Criteriul lui Euler

$\lim_{n\to\infty} (1+x_n)^{\frac{1}{x_n}} = e$; pt. $\lim_{n\to\infty} x_n = 0$

$= \ln\left[\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^{-n}\right]$

$$\lim_{n\to\infty}(x_n)^{y_n} = \left(\lim_{n\to\infty}, x_n\right)$$

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3) $$\lim_{x\to 0} \frac{\sqrt{1+x+x^2}-\sqrt{1+x^2}}{x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{1+x+x^2-(1+x^2)}{x(\sqrt{1+x+x^2}+\sqrt{1+x^2})} =$$

$$\ell = \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{1+x+x^2}+\sqrt{1+x^2})} = \frac{1}{2}$$

$$(\sqrt{a}-\sqrt{b})(a+\sqrt{b}) = a-b$$

$$(x-y)(x+y) = x^2-y^2$$

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4) Studiați continuitatea funcției

$$f(x) = \begin{cases} \frac{\sqrt{1+x}-x}{x}, & x<0 \\ \frac{\ln(1+2x)}{x}, & x \geq 0 \end{cases}$$

I $x<0 \Rightarrow f(x) = \frac{\sqrt{1+x}-1}{x}$ - elementară, deci continuă pe $x \in (-\infty, 0)$

II $x>0 \Rightarrow f(x) = \frac{\ln(1+2x)}{x}$ - elementară —"— pt $x \in (0,\infty)$

III $x=0$: f - continuă $\Leftrightarrow$ $ls(0) = ld(0) = f(0)$

$$ls(0) = \lim_{\substack{x \to 0 \\ x<0}} \frac{\overset{conj}{\sqrt{1+x}-1}}{x} = \lim_{\substack{x \to 0 \\ x<0}} \frac{1+x-1}{x(\sqrt{1+x}+1)} =$$

$$= \lim_{\substack{x \to 0 \\ x<0}} \frac{x}{x(\sqrt{1+x}+1)} = \frac{1}{2}$$

$$\lim_{\substack{x \to 0 \\ x>0}} \frac{\ln(1+2x)}{2x} \cdot 2 = 2$$

$x>0$

$ls(0) \neq ld(0) \Rightarrow f$ nu e cont.

5) a) $f(x) = \operatorname{arctg} \frac{1-x^2}{1+x^2}$, $f'(x) = ?$

$$f'(x) = \frac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2} \cdot \left(\frac{1-x^2}{1+x^2}\right)'$$

$$= \frac{1}{1+\left(\frac{1+x^2}{1+x^2}\right)^2} \cdot \frac{(-2x)(1+x^2)-(1-x^2)2x}{(1+x^2)^2} =$$

$$= \frac{1}{1\left(\frac{1+x^2}{1+x^2}\right)^2} \cdot \frac{-2x-2x^3-2x+\cancel{2x^3}}{(1+x^2)^2} =$$

$$= \frac{1}{1+\left(\frac{1-x^2}{1+x^2}\right)^2} \cdot -\left(\frac{4x}{(1+x^2)^2}\right)$$

$f(x) = x^x$

$a^b = e^{b \ln a}$

a^b

$f'(x) = (e^{x \cdot \ln x})' = (e^{u(x)})' = e^{u(x)} \cdot u'(x)$

$$\boxed{\begin{array}{l} (e^x)' = e^x \\ (e^{u(x)})' = e^{u(x)} \cdot u'(x) \end{array}}$$

$e^{x \cdot \ln x} \cdot (x' \cdot \ln x + x \cdot (\ln x)')$

$e^{x \cdot \ln x}(\ln x + x \frac{1}{x}) =$

$= e^{x \cdot \ln x} \ln x$

$f'(x_0) = \lim\limits_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim\limits_{x \to x_0} \frac{e^x - e^{x_0}}{x - x_0} \; \frac{0}{0} =$

<u>Criteriul exponential</u>

$\lim\limits_{x \to a} \frac{e^{f(x)} - 1}{f(x)} = 1$, pt. $\lim\limits_{x \to a} f(x) = 0$

$\lim\limits_{x \to x_0} \frac{e^{x_0}(e^{x - x_0} - 1)}{x - x_0} = e^{x_0}$

$\downarrow$

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6) $\int \frac{-\cos x - \sin x}{-\cos x + \sin x} dx = \int \frac{1}{t} dt = \ln t + C = \ln(-\sin x + \cos x) + C$

$\int f \cdot f' =$

$\int \frac{f'}{f} =$

$-\cos x + \sin x = t \mid d \Leftrightarrow (-\sin x + -\cos x) \cdot dx = 1 \cdot dt$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ (f. injective)

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (f. injective)

7) $\int \frac{x^3}{(1+x^2)^2} dx = \int \frac{x \cdot x^2}{(1+x^2)^2} dx = \int \frac{2x \cdot x^2}{(1+x^2)^2} \cdot \frac{1}{2} dx = \frac{1}{2} \int \frac{x^2}{1+x^2} 2x dx$

$(1+x^2)^2 = t \mid d \Leftrightarrow 2 \cdot (1+x^2)(1+x^2)' dx = dt$

$1+x^2 = t \mid d \Leftrightarrow 2x \, dx = dt$

$= \int \frac{1}{t} - \int t^2$

$= \ln(t) - \frac{t^{-2+1}}{-2+1} + C$