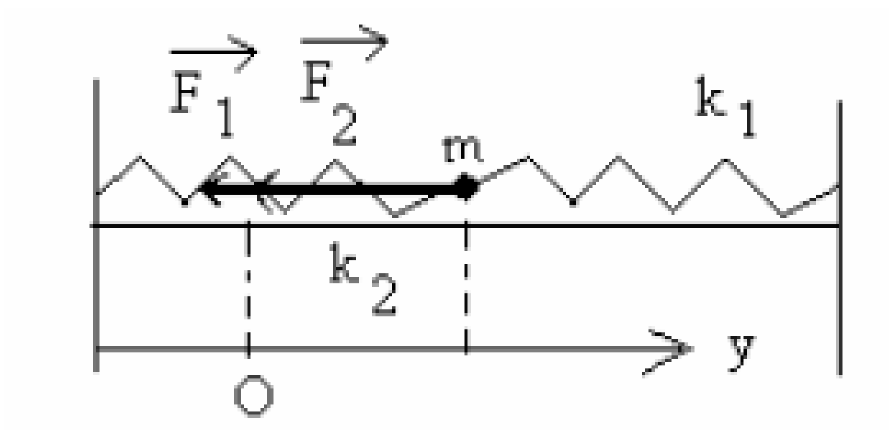
Fizica Generala

Curs 4

Compunerea oscilatiilor

Compunerea mişcărilor oscilatorii armonice

- Compunerea oscilaţiilor armonice paralele de aceeaşi pulsaţie

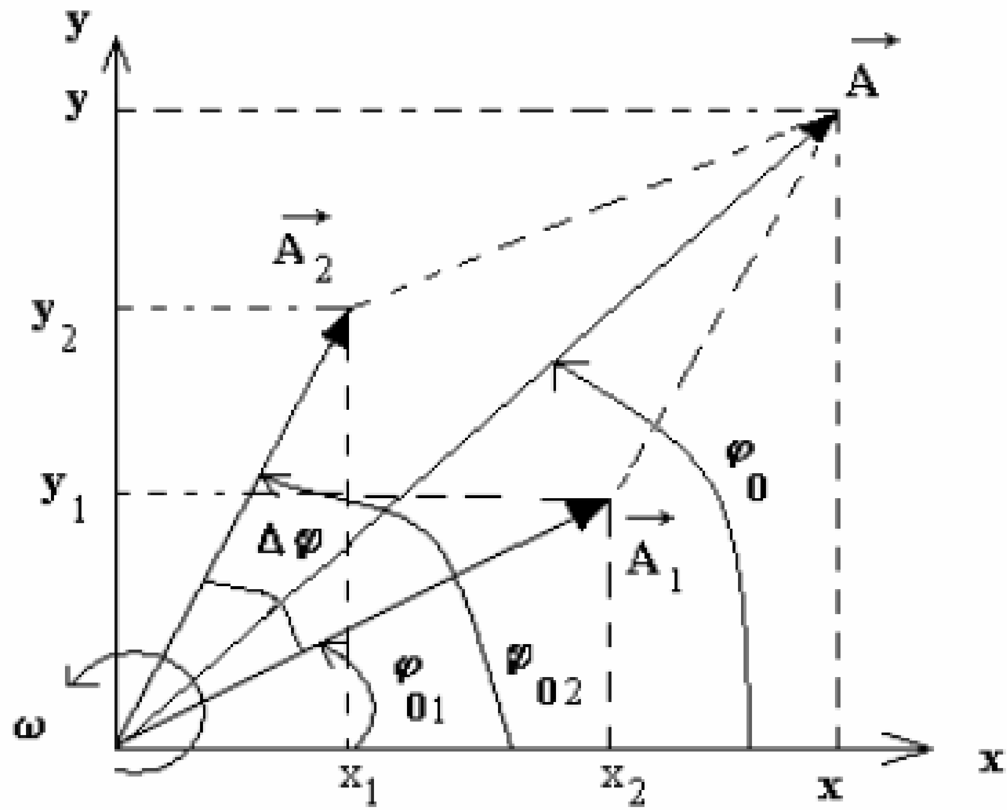


$$y_1(t) = A_1 \sin(\omega t + \varphi_{01})$$

$$y_2(t) = A_2 \sin(\omega t + \varphi_{02})$$

$$y(t) = y_1(t) + y_2(t)$$

$$y(t) = A \sin(\omega t + \varphi_0)$$



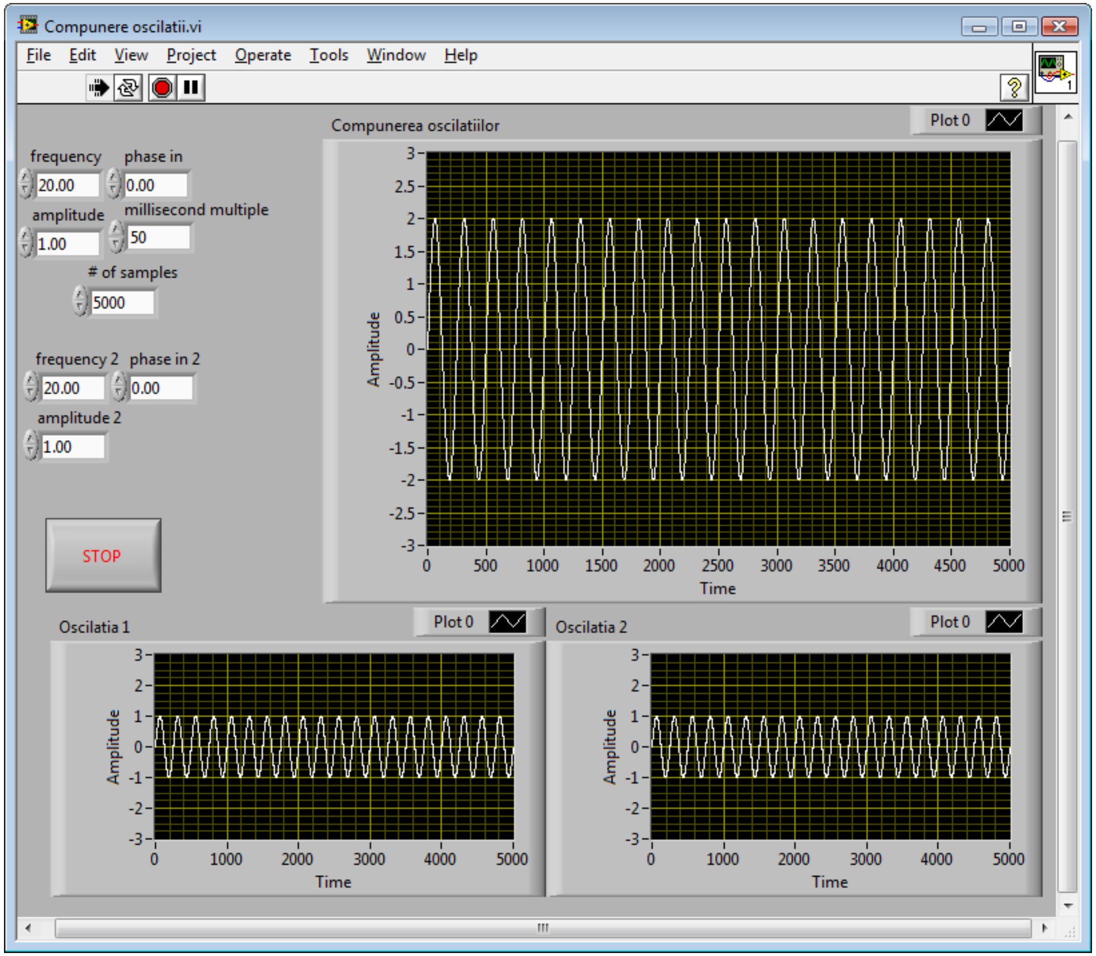
$$\Delta\varphi = \varphi_2(t) - \varphi_1(t) = \omega t + \varphi_{02} - \omega t - \varphi_{01} = \varphi_{02} - \varphi_{01}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\left(\varphi_{02} - \varphi_{01}\right)}$$

$$\mathrm{tg}\varphi_0 = \frac{y}{x} = \frac{y_1 + y_2}{x_1 + x_2} = \frac{A_1\sin\varphi_{01} + A_2\sin\varphi_{02}}{A_1\cos\varphi_{01} + A_2\cos\varphi_{02}}$$

Cazuri particulare

- I
  - Daca $\Delta\varphi = 0 \Rightarrow A = A_1 + A_2 \Rightarrow$ oscilatorii sunt *în fază*.
- II
  - Daca $\Delta\varphi = \pi \Rightarrow A = |A_1 - A_2| \Rightarrow$ oscilatorii sunt *în opoziţie de fază* (daca $A_1 = A_2 \Rightarrow A=0$ oscilatia se stinge)
- III
  - Daca $\Delta\varphi = \pi/2 \Rightarrow A = \sqrt{A_1 + A_2} \Rightarrow$ oscilatorii sunt în *cuadratură de fază*



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Compunerea oscilatiilor
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phase in 2
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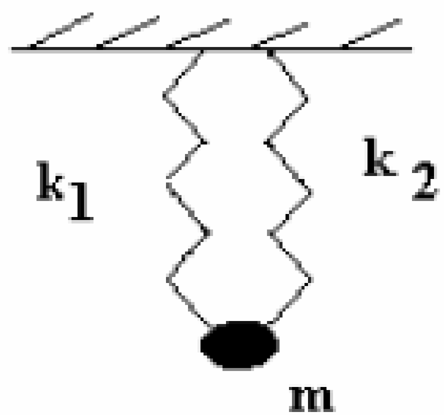
amplitude 2
1.00
STOP
Amplitude
Time
Oscilatia 1
Oscilatia 2

Compunerea oscilaţiilor armonice paralele de frecvenţă diferită

- Considerăm două oscilaţii armonice individuale ale punctului material de masă m de forma:

$$\begin{cases} y_1(t) = A_1 \sin(\omega_1 t + \varphi_1) \\ y_2(t) = A_2 \sin(\omega_2 t + \varphi_2) \end{cases}$$

- cu pulsatiile proprii ω_1 si ω_2 putin diferite



$$\omega_1 = \omega + \Delta\omega$$

$$\begin{cases} y_1(t) = A_1 \sin[(\omega + \Delta\omega)t + \varphi_1] \\ y_2(t) = A_2 \sin[(\omega - \Delta\omega)t + \varphi_2] \end{cases}$$

$$\omega_2 = \omega - \Delta\omega$$

$$y = y_1 + y_2 = A_1 \sin[\omega t + (\Delta\omega t + \varphi_1)] + A_2 \sin[\omega t + (-\Delta\omega t + \varphi_2)]$$

$$\begin{aligned} y = {} & A_1[\sin\omega t \cos(\Delta\omega t + \varphi_1) + \cos\omega t \sin(\Delta\omega t + \varphi_1)] + \\ & + A_2[\sin\omega t \cos(-\Delta\omega t + \varphi_2) + \cos\omega t \sin(-\Delta\omega t + \varphi_2)] \end{aligned}$$

$$\begin{aligned} y = {} & [\,A_1 \cos(\Delta\omega t + \varphi_1) + A_2 \ \cos(\Delta\omega t - \varphi_2)]\sin\omega t + \\ & + [\,A_1 \sin(\Delta\omega t + \varphi_1) - A_2 \ \sin(\,\Delta\omega t - \varphi_2)]\cos\omega t \end{aligned}$$

$$y=\left[A_1\cos(\Delta\omega t+\varphi_1)+A_2\cos(\Delta\omega t-\varphi_2)\right]\sin\omega t+ \\ +\left[A_1\sin(\Delta\omega t+\varphi_1)-A_2\sin(\Delta\omega t-\varphi_2)\right]\cos\omega t$$

$$y(t)=A\sin(\omega t+\varphi)$$

$$y(t)=A\sin(\omega t+\varphi)=A\sin\omega t\cos\varphi+A\cos\omega t\sin\varphi$$

$$\begin{cases} A\cos\varphi=A_1\cos(\Delta\omega t+\varphi_1)+A_2\cos(\Delta\omega t-\varphi_2) \\ A\sin\varphi=A_1\sin(\Delta\omega t+\varphi_1)-A_2\sin(\Delta\omega t-\varphi_2) \end{cases}$$

$$\mathrm{tg}\varphi=\frac{A_1\sin(\Delta\omega t+\varphi_1)-A_2\sin(\Delta\omega t-\varphi_2)}{A_1\cos(\Delta\omega t+\varphi_1)+A_2\cos(\Delta\omega t-\varphi_2)}$$

$$A^2=A_1^2+A_2^2+2A_1A_2\left[\cos(\Delta\omega t+\varphi_1)\cos(\Delta\omega t-\varphi_2)-\sin(\Delta\omega t+\varphi_1)\sin(\Delta\omega t-\varphi_2)\right]$$

$$A^2=A_1^2+A_2^2+2A_1A_2\cos\left[(\Delta\omega t+\varphi_1)+(\Delta\omega t-\varphi_2)\right]$$

$$A^2=A_1^2+A_2^2+2A_1A_2\cos\left[2\Delta\omega t+\varphi_1-\varphi_2\right]$$

- Consideram ca $A_1 = A_2 = A_0$, şi $\varphi_1 = \varphi_2 = 0$ =>

$$A = A_0\sqrt{1+1+2\cos(2\Delta\omega t)} = 2A_0\cos(\Delta\omega t)$$

$$y = 2A_0\cos(\Delta\omega t)\sin(\omega t)$$

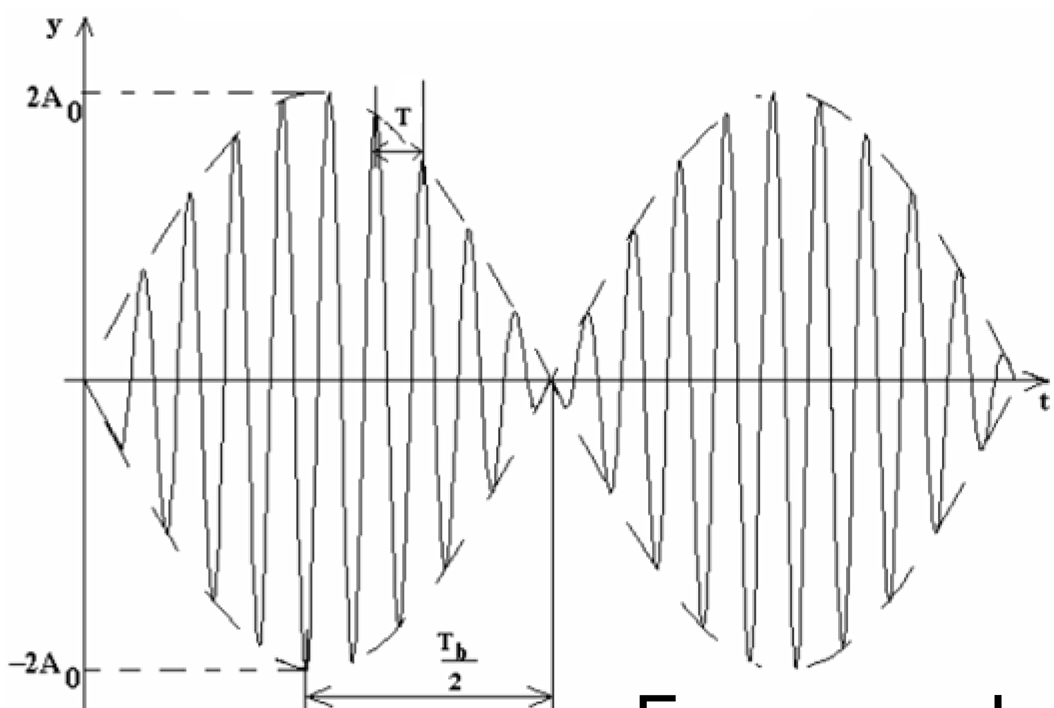
$$\Delta\omega = \frac{\omega_1 - \omega_2}{2}, \text{ iar } \omega = \frac{\omega_1 + \omega_2}{2}$$

$$y = 2A_0\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)$$

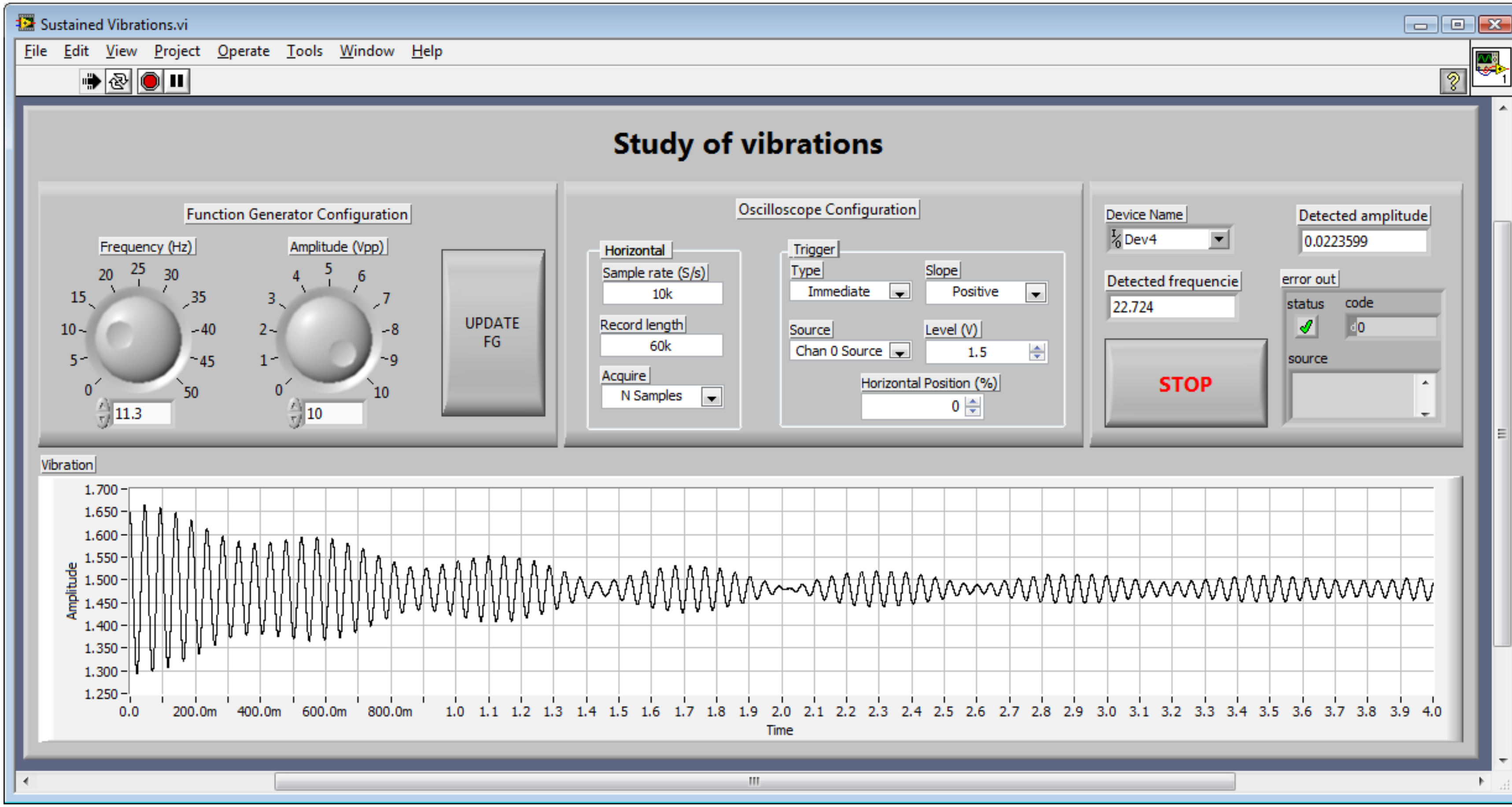
- => apare fenomenul de *batai* ce constă în modularea amplitudinii oscilaţiei.

Perioada bătăilor este intervalul de timp între două treceri succesive ale amplitudinii rezultante prin valoarea minimă sau maximă si este data de relatia:

$$T_b = \frac{2\pi}{\Delta\omega} = \frac{2\pi}{\frac{\omega_1 - \omega_2}{2}}$$



Fenomenul de bătăi


Sustained Vibrations.vi
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Study of vibrations
Function Generator Configuration
Frequency (Hz)
11.3
Amplitude (Vpp)
10
UPDATE FG
Oscilloscope Configuration
Horizontal
Sample rate (S/s)
10k
Record length
60k
Acquire
N Samples
Trigger
Type
Immediate
Slope
Positive
Source
Chan 0 Source
Level (V)
1.5
Horizontal Position (%)
0
Device Name
Dev4
Detected amplitude
0.0223599
Detected frequencie
22.724
STOP
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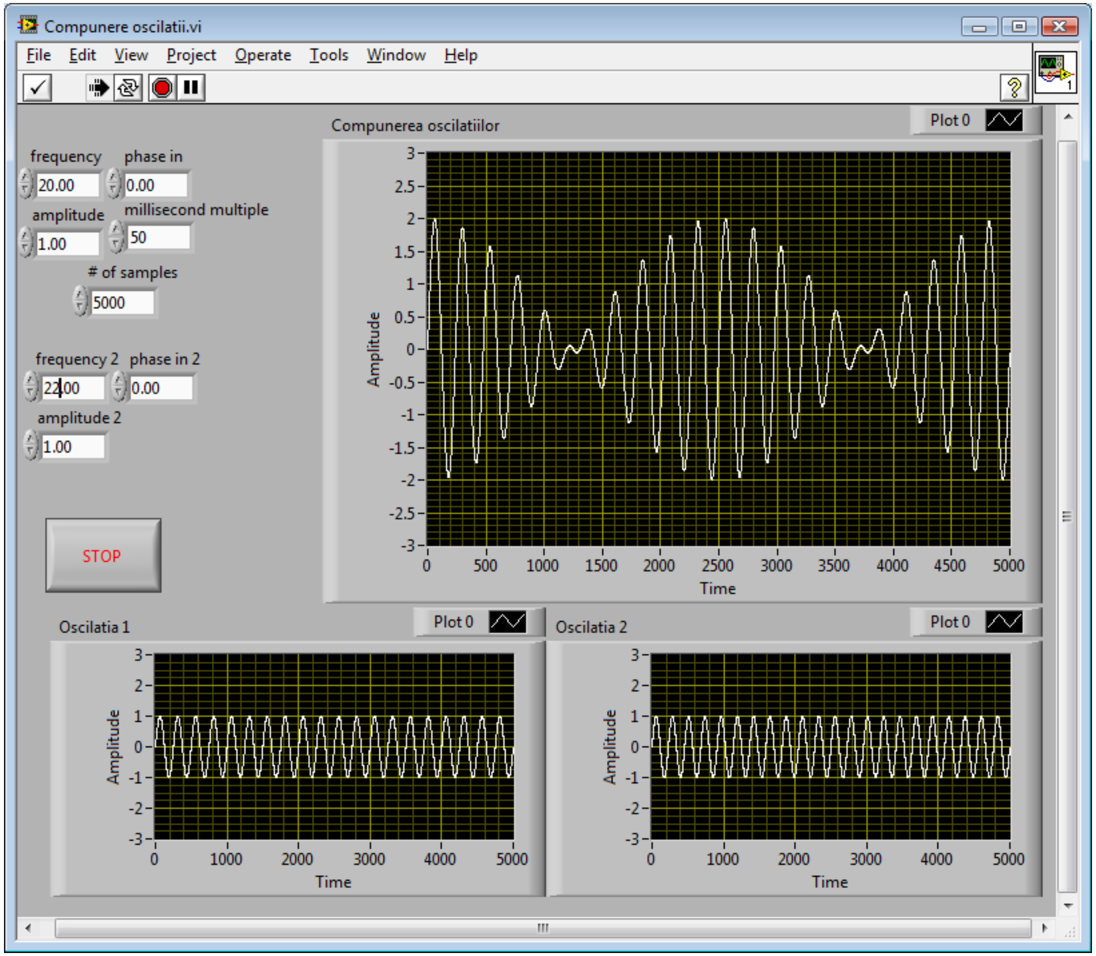
Vibration
Amplitude
Time


Compunere oscilatii.vi
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Compunerea oscilatiilor
Plot 0
frequency
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frequency 2
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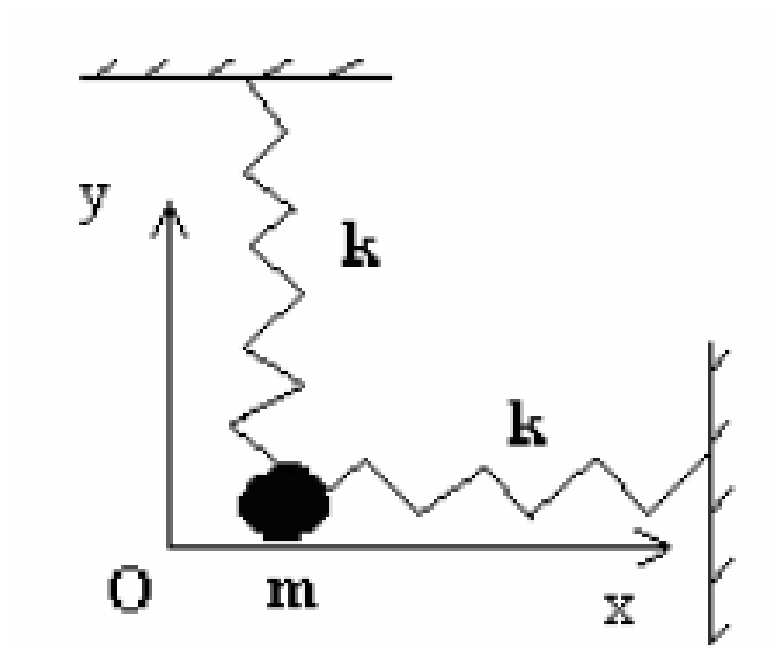
amplitude 2
1.00
STOP
Amplitude
Time
Oscilatia 1
Oscilatia 2

Compunerea oscilaţiilor perpendiculare

- Considerăm un punct material de masă m, care care este solicitat simultan să oscileze armonic sub acţiunea a două resorturi elastice identice legate pe două direcţii perpendiculare, ca în fig.

$$\begin{cases} x(t) = A_1 \sin(\omega t + \varphi_1) \\ y(t) = A_2 \sin(\omega t + \varphi_2) \end{cases}$$



$$\begin{cases} x(t) = A_1 \sin(\omega t + \varphi_1) \\ y(t) = A_2 \sin(\omega t + \varphi_2) \end{cases}$$

$$\begin{cases} \dfrac{x(t)}{A_1} = \sin(\omega t + \varphi_1) = \sin\omega t \cos\varphi_1 + \cos\omega t \sin\varphi_1 \quad \Big| \cos\varphi_2 \quad \Big| \sin\varphi_2 \\ \dfrac{y(t)}{A_2} = \sin(\omega t + \varphi_2) = \sin\omega t \cos\varphi_2 + \cos\omega t \sin\varphi_2 \quad \Big| \cos\varphi_1 \quad \Big| \sin\varphi_1 \end{cases}$$

$$\begin{cases} \dfrac{x}{A_1}\cos\varphi_2 - \dfrac{y}{A_2}\cos\varphi_1 = \cos\omega t\left(\sin\varphi_1 \cos\varphi_2 - \sin\varphi_2 \cos\varphi_1\right) \\ \sin\varphi_1 \cos\varphi_2 - \sin\varphi_2 \cos\varphi_1 = -\sin(\varphi_2 - \varphi_1) \\ \dfrac{x}{A_1}\sin\varphi_2 - \dfrac{y}{A_2}\sin\varphi_1 = \sin\omega t\left(\cos\varphi_1 \sin\varphi_2 - \cos\varphi_2 \sin\varphi_1\right) \end{cases}$$

$$\cos\varphi_1 \sin\varphi_2 - \cos\varphi_2 \sin\varphi_1 = \sin(\varphi_2 - \varphi_1)$$

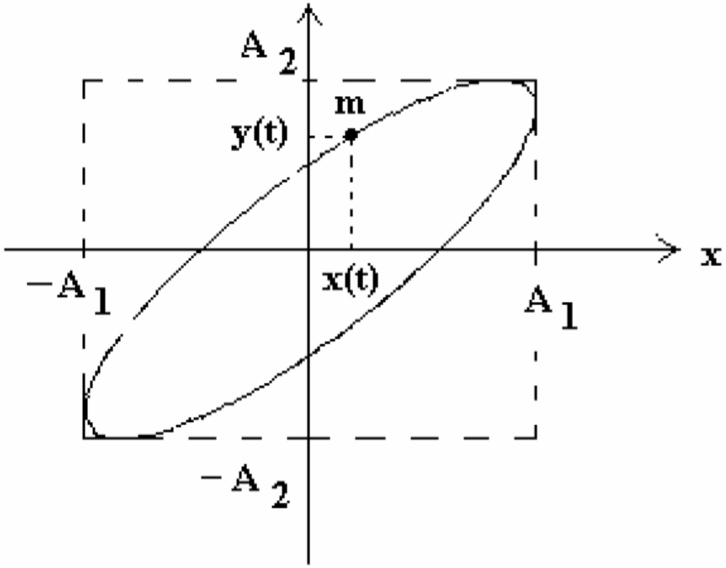
Prin ridicare la patrat se obtine:

$$\left(\frac{x}{A_1}\right)^2+\left(\frac{y}{A_2}\right)^2-2\frac{x}{A_1}\frac{y}{A_2}\left(\cos\varphi_1\cos\varphi_2+\sin\varphi_1\sin\varphi_2\right)=$$

$$=\sin^2(\varphi_2-\varphi_1)\left(\sin^2\omega t+\cos^2\omega t\right)$$

$$\boxed{\left(\frac{x}{A_1}\right)^2+\left(\frac{y}{A_2}\right)^2-2\frac{x}{A_1}\frac{y}{A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)} \qquad (*)$$

ecuaţia generalizată a elipsei, adică ecuaţia unei elipse rotite faţă de axele de coordonate



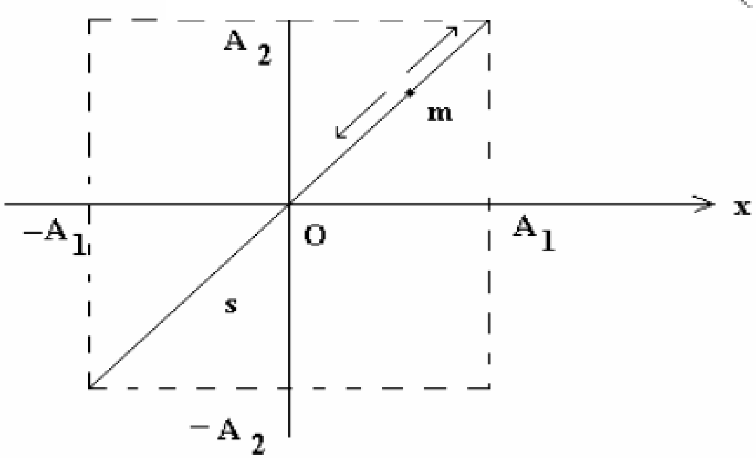
Cazuri particulare

- Dacă $\Delta\varphi = \varphi_2 - \varphi_1 = 2n\pi$ (oscilaţiile sunt în fază)=>

$$\left(\frac{x}{A_1}\right)^2 + \left(\frac{y}{A_2}\right)^2 - 2\frac{x}{A_1}\frac{y}{A_2} = 0$$

sau:

$$\left(\frac{x}{A_1} - \frac{y}{A_2}\right)^2 = 0$$



$$y = \frac{A_2}{A_1}x$$

- Dacă $\Delta\varphi = \varphi_2 - \varphi_1 = (2n+1)\pi$ (oscilaţiile sunt în opoziţie de fază =>

$$\left(\frac{x}{A_1}\right)^2 + \left(\frac{y}{A_2}\right)^2 + 2\frac{x}{A_1}\frac{y}{A_2} = 0$$

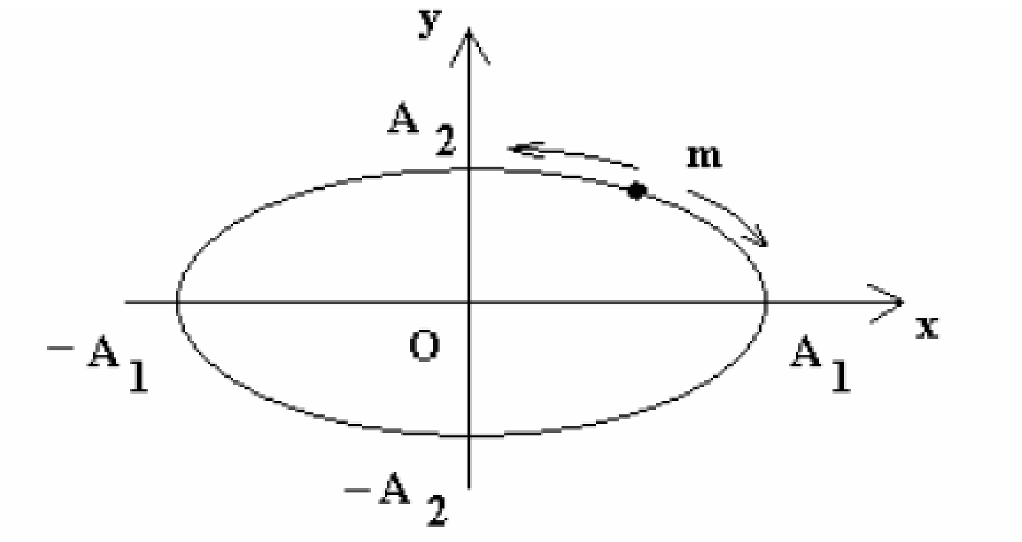
sau:

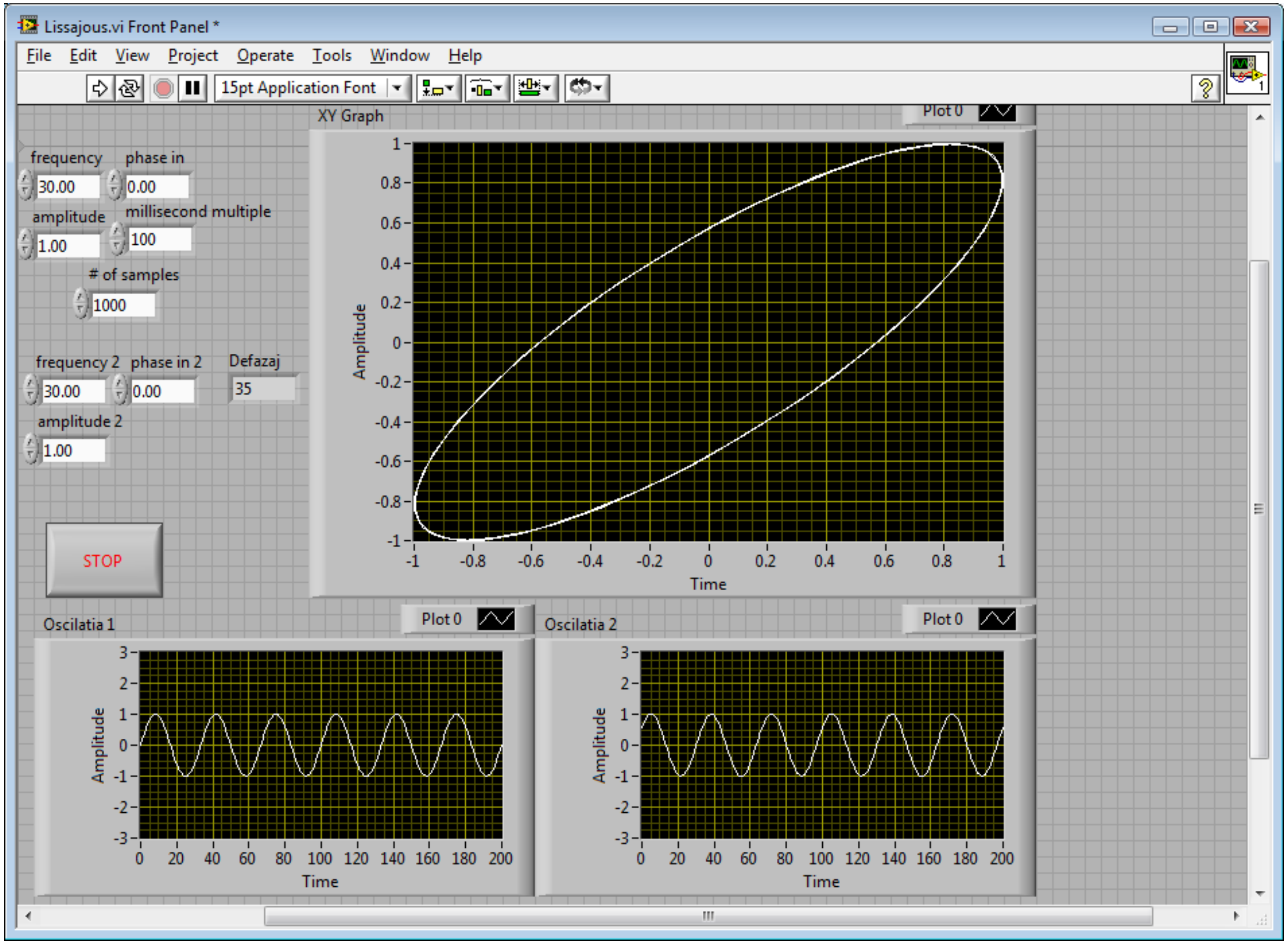
$$\left(\frac{x}{A_1} + \frac{y}{A_2}\right)^2 = 0$$

$$y = -\frac{A_2}{A_1}x$$

- Dacă $\Delta\varphi = \varphi_2 - \varphi_1 = (2n+1)\,\pi/2$ (oscilaţiile sunt în cuadratură) =>

$$\left(\frac{x}{A_1}\right)^2 + \left(\frac{y}{A_2}\right)^2 = 1$$





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STOP
XY Graph
Plot 0
Amplitude
Time
Oscilatia 1
Oscilatia 2

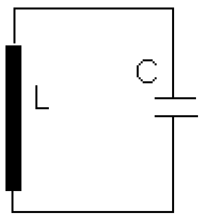
Oscilatii
Analogia mecano-electrica

Oscilatii electrice

- Studiem cazul circuitelor oscilante electrice
- Consideram cazul unui circuit oscilant simplu, format dintr-un condensator de capacitate C şi o bobină de inductanţă L

$$U_C - U_L = 0$$

$$U_C = \frac{q}{C}; \quad U_L = -L\frac{dI}{dt} = -L\ddot{q}$$ unde $$I = \frac{dq}{dt} = \dot{q}$$



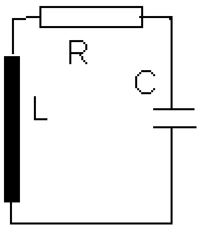
- =>ec. dif. este $\ddot{q} + \frac{1}{LC}q = 0 \quad \rightarrow \ddot{q} + \omega_0^2 q = 0$

- Solutia ec. dif. fiind $q = a\cos(\omega_0 t + \alpha)$ sarcina din circuit variază armonic

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Oscilatii electrice

- In cazul real trebuie tinut cont de rezistenţa electrică în care sunt incluse contribuţiile rezistenţei conductorului bobinei reale şi rezistenţei dielectricului condensatorului real



$$U_R + U_C - U_L = 0$$

$$R\dot{q} + \frac{q}{C} + L\ddot{q} = 0$$

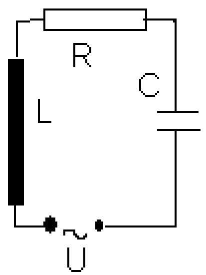
$$\ddot{q} + 2\beta\dot{q} + \omega_0^2 q = 0$$ unde $\omega_0^2 = \frac{1}{LC}, \quad 2\beta = \frac{R}{L}$

- pentru cazul unor rezistenţe mici are ca soluţie o oscilaţie amortizată a sarcinii în circuit

$$q = a\, e^{-\beta t} \cos(\omega t + \alpha)$$

Oscilatii electrice

- Pentru a întreţine oscilaţiile sarcinii din circuit, este necesar evident un aport energetic din exterior, ceea ce inseamnă aplicarea la bornele circuitului a unei tensiuni alternative de o anumită pulsaţie



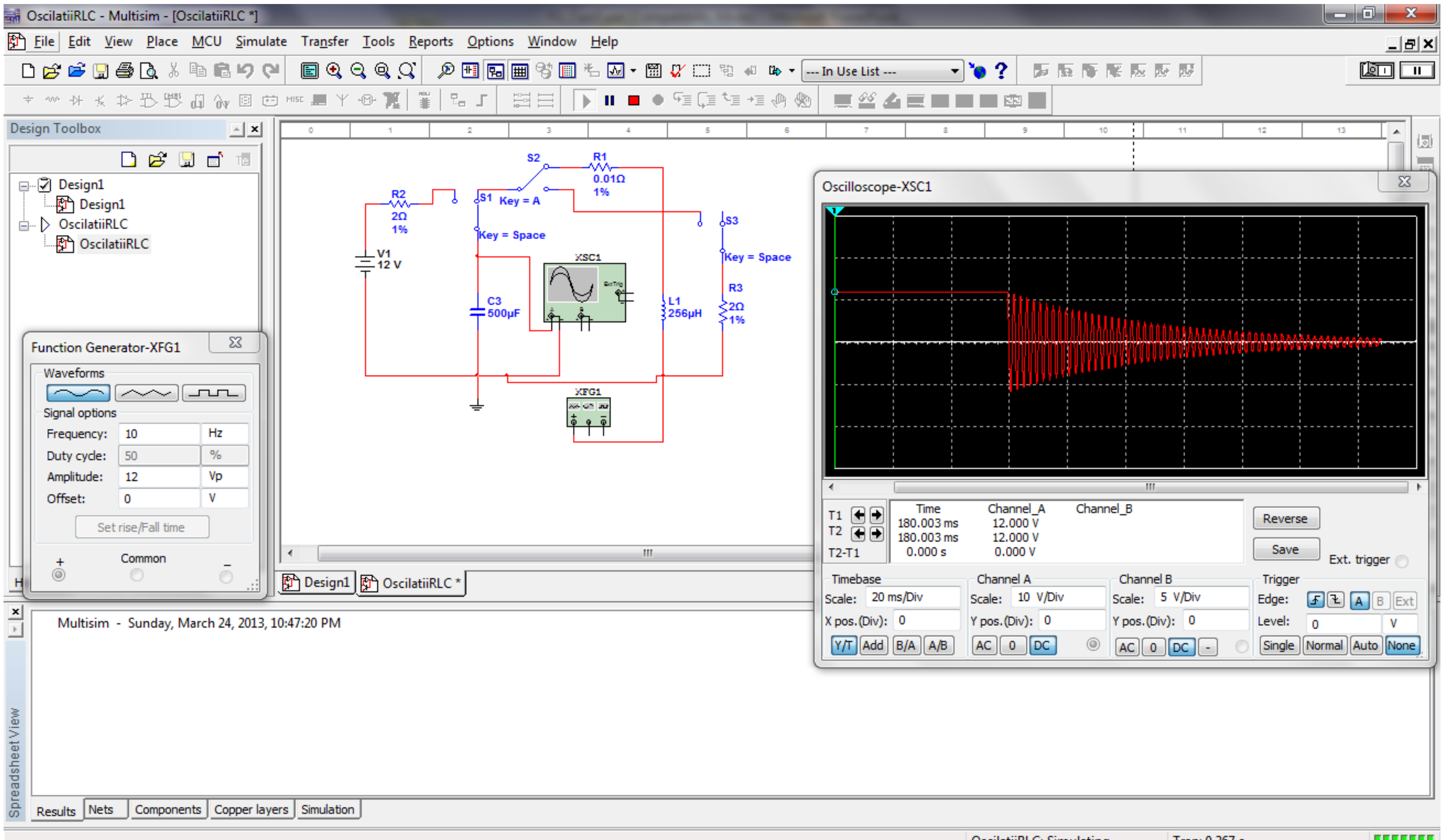
- Consideram $\tilde{U} = U_0 \cos(\omega_p t + \alpha)$

$$U_C + U_R - U_L = U_0 \cdot \cos(\omega_p t + \alpha)$$

- Avem în acest caz evident de-a face după trecerea timpului de relaxare cu oscilaţii forţate ale sarcinii în circuit, a căror ecuaţie matematică este:

$$q = \frac{\frac{U_0}{L}}{\sqrt{\left(\frac{1}{LC} - \omega_p^2\right)^2 + \frac{R^2}{L^2}\omega_p^2}} \cos(\omega_p t + \alpha - \varphi) \qquad A_p = \frac{f}{\sqrt{(\omega_0^2 - \omega_p^2)^2 + (2\beta\omega_p)^2}}$$

Oscilatii electrice



Exercitiu: Sa se calculeze frecventa de oscilatie si coeficientul de amortizare a circuitului