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Probabilități Condiționate și Evenimente Independente

1. Se consideră evenimentele A, B, C - mutual indep.

A și $B \cup C$ - indep?

M, N - indep. $\iff \mathbb{P}(M \cap N) = \mathbb{P}(M) \cdot \mathbb{P}(N)$

$$\underbrace{\mathbb{P}(A \cap (B \cup C))}_{(**)} \stackrel{?}{=} \underbrace{\mathbb{P}(A) \cdot \mathbb{P}(B \cup C)}_{(*)}$$

$(*)$ $\mathbb{P}(A) \cdot [\mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B \cap C)] =$

$= \mathbb{P}(A) \cdot [\mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B) \cdot \mathbb{P}(C)] =$

$= \mathbb{P}(A) \cdot \mathbb{P}(B) + \mathbb{P}(A) \cdot \mathbb{P}(C) - \mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C)$

$(**)$ $\mathbb{P}(A \cap (B \cup C)) = \mathbb{P}[(A \cap B) \cup (A \cap C)] =$

$$= \mathbb{P}(A) \cdot \mathbb{P}(B) + \mathbb{P}(A) \cdot \mathbb{P}(C) - \mathbb{P}\underbrace{(A \cap B \cap C)}_{\mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C)}$$

2. $\underset{=r}{\mathbb{P}(A \mid \bar{B})}, \underset{=q}{\mathbb{P}(A \mid B)}, \underset{=r}{\mathbb{P}(\bar{B} \mid \bar{A})}$; $\mathbb{P}(A), \mathbb{P}(B)$

$r, q, r \in (0, 1)$

$$\boxed{\mathbb{P}(N \mid M) \stackrel{Def.}{=} \frac{\mathbb{P}(M \cap N)}{\underbrace{\mathbb{P}(M)}_{\neq 0}}}$$

2. $\overset{=p}{\mathbb{P}(A|\overline{B})}, \overset{=q}{\mathbb{P}(A|B)}, \overset{=r}{\mathbb{P}(\overline{B}|\overline{A})}; \mathbb{P}(A), \mathbb{P}(B)$

$p, q, r \in (0,1)$

$$\boxed{\text{Def.: } \mathbb{P}(N|M) = \frac{\mathbb{P}(M \cap N)}{\underbrace{\mathbb{P}(M)}_{\neq 0}}}$$

$$p = \mathbb{P}(A|\overline{B}) = \frac{\mathbb{P}(A \cap \overline{B})}{\mathbb{P}(\overline{B})} = \frac{\mathbb{P}(A \setminus B)}{1 - \mathbb{P}(B)} = \frac{\mathbb{P}(A) - \mathbb{P}(A \cap B)}{1 - \mathbb{P}(B)} = \boxed{p}$$

$$q = \mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} \Rightarrow \mathbb{P}(A \cap B) = q \cdot \mathbb{P}(B) = \boxed{q}$$

$$r = \mathbb{P}(\overline{B}|\overline{A}) = \frac{\mathbb{P}(\overline{A} \cap \overline{B})}{\mathbb{P}(\overline{A})} = \frac{\mathbb{P}(\overline{A \cup B})}{1 - \mathbb{P}(A)} = \frac{1 - \mathbb{P}(A \cup B)}{1 - \mathbb{P}(A)} =$$

$$= \frac{1 - \left(\mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)\right)}{1 - \mathbb{P}(A)} = \boxed{r}$$

3. Intr-o cutie se află 16 bile numerotate de la 1-16. Primele 3 bile sunt albe, urm. 10 sunt negre, iar ultimele 3 sunt roşii. Din cutie se extrage aleator o singură bilă.

Fie evenimente:

A — „bila extrasă e neagră"

B — „bila extrasă are un nr. mai mic sau egal decât 9"

Să se calc. $P(A|B)$, $\mathbb{P}(B|A)$, $\mathbb{P}(A|\bar{B})$

ALBE	1	2	3							
NEGRE	4	5	6	7	8	9	10	11	12	13
ROŞII	14	15	16							

A — bila neagră

B — ≤ 9

$\mathbb{P}(A|B) = \frac{6}{9}$

$\mathbb{P}(B|A) = \frac{6}{10}$

$\mathbb{P}(A|\bar{B}) = \frac{4}{7}$

$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B) - \mathbb{P}(B)}{\mathbb{P}(B)} = \frac{\frac{6}{\cancel{16}}}{\frac{9}{\cancel{16}}} = \frac{6}{9}$

4. 4 torpile

prob e p.

Pt. scufundarea navei sunt sufic. 2 torpile care ating ținta, iar dacă o singură torpilă atinge nava, ea se scuf cu prob. r.

a) Care e prob. ca ea să se scufunde

b) Cu ce prob. se știe că ea a fost scuf. cu 3 torpile.

Fie ev.:

A : „nava se scufundă"

A_k : „nava e lovită de exact 3 torpile"

a) $\mathbb{P}(A) = \sum_{i \in I} \mathbb{P}(A | A_i) \cdot \mathbb{P}(A_i) =$

$= \underbrace{\mathbb{P}(A | A_0)}_{0} \cdot \mathbb{P}(A_0) + \underbrace{\mathbb{P}(A | A_1)}_{r} \cdot \mathbb{P}(A | A_1) \cdot \mathbb{P}(A_1) +$

$+ \sum_{k=2}^{4} \underbrace{\mathbb{P}(A | A_k)}_{1} \cdot \mathbb{P}(A_k)$

$\mathbb{P}(A_k) = C_4^k \, p^k \cdot (1-p)^{4-k}, \quad k = 0 \dots 4$

$\mathbb{P}(A_1) = 4 \, p^1 \cdot (1-p)^3$

b) $\mathbb{P}(A_3 | A) = \dfrac{\mathbb{P}(A_3 \cap A)}{\mathbb{P}(A)} = \dfrac{\mathbb{P}(A_3) \cdot \mathbb{P}(A | A_3)}{\mathbb{P}(A)} =$

$$= \frac{\mathbb{P}(A_3)}{\mathbb{P}(A)}$$

$$\mathbb{P}(A) = 1 - \mathbb{P}(\bar{A}) = 1 - \Big(\mathbb{P}(A_0) + \overbrace{\mathbb{P}(A_1) \cdot \mathbb{P}(\bar{A} | A_1)}^{\mathbb{P}(A_1 \cap \bar{A})} \Big) =$$

$$= 1 - \mathbb{P}(A_0) - \mathbb{P}(A_1)(1 - r)$$

SCHEME DE PROBABILITATE

Se aruncă 3 zaruri clasice de 10 ori.

a) Care e prob. obț. sumei 6 de exact 4 ori.

b) Care e prob. obț. sumei 6 abia la a 10-a aruncare.

cazuri posibile $= 10^3 = 100$

$$\frac{10}{6^3} = p$$

$$1 - p = q = \frac{6^3 - 10}{6^3}$$

Schema lui Bernoulli

a) $p_{4/10} = C_{10}^{4} \cdot \left(\dfrac{10}{6^3}\right)^4 \cdot \left(\dfrac{6^3 - 10}{6^3}\right)^6$

$$\boxed{\mathbb{P}_{k/n} = p^k \cdot q^{n-k}, \quad p + q = 1 \qquad \text{Bernoulli}}$$

b) $\mathbb{P}_{10} = q^9 \cdot p = \left(\dfrac{6^3 - 10}{6^3}\right)^9 \cdot \dfrac{10}{6^3}$

$\mathbb{P}_{k/n} = C_n^k \cdot p^k \cdot q^{n-k}$, $p + q = 1$

$\mathbb{P}_n = q^{n-1} \cdot p$ $n \in \mathbb{N}^*$