$$I_1 = \int_{-\frac{1}{2}}^{\left(\frac{-\pi-1}{2}\right)} \frac{1}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}} \cdot \operatorname{arctg} \frac{t}{\frac{\sqrt{3}}{2}} \Bigg|_{-\frac{1}{2}}^{-\frac{(\pi-1)}{2}}$$

$$\Big|_{0}^{-\pi/2} \quad x - \frac{1}{2} = t$$

$$dx = dt$$

Șiruri Cauchy

$(x_n)_n \subseteq$ convergent dacă există $l \in \mathbb{R}$ a.î.

$\lim\limits_{n\to\infty} x_n = l$ i.e. $(\forall)\, \varepsilon > 0\, (\exists)\, n_\varepsilon \in \mathbb{N}\ (\forall)\, n \in \mathbb{N},\ n \geq n_\varepsilon$

$|x_n - l| < \varepsilon$

$\Downarrow$

$(\forall)\, m, n \in \mathbb{N}$

$m, n \geq n_\varepsilon$

$|x_m - x_n| = |x_m - l + l - x_n| \leq$

$\leq |x_m - l| + |l - x_n| < \varepsilon + \varepsilon = 2\varepsilon$

Def.: $(x_n)_n \subseteq \mathbb{R}$ șir Cauchy dacă

$(\forall)\, \varepsilon > 0 \quad (\exists)\, n_\varepsilon \in \mathbb{N} \quad (\forall)\, m, n \geq n_\varepsilon$

$|x_m - x_n| < \varepsilon$

R $(x_n)_n \subseteq \mathbb{R}$ convergent $\iff$ $(x_n)_n$ șir Cauchy

Dacă $(S_n)_n$ are limita $S \in \overline{\mathbb{R}}$, atunci

$S = x_1 + x_2 + \ldots + x_n + \ldots =$

$= \text{suma seriei } \sum\limits_{n=1}^{\infty} x_n$

$\sum\limits_{n} x_n \qquad \sum\limits_{n\in\mathbb{N}} x_n$

$$\sum_{n\in\mathbb{N}} \xrightarrow{\text{cantitativ}} \sum_{n\in\mathbb{N}} x_n = ?$$

$$\sum_{n\in\mathbb{N}} \xrightarrow{\text{calitativă}} \sum x_n \text{ convergentă? } \lim_{n\to\infty} S_n \in \mathbb{R}$$

(R) $\sum x_n$ - convergentă $\Rightarrow \lim_{n\to\infty} x_n = 0$

$$x_n \downarrow S_n - S_{n-1}$$

(R) $\sum |x_n|$ - convergentă $\Rightarrow \sum x_n$ - convergentă!

(C_1) x_n, y_n

$0 \leq x_n \leq y_n$

$(\forall)\, n \in \mathbb{N}$

α) $\sum y_n$ - convergentă $\Rightarrow \sum x_n$ - convergentă

β) $\sum x_n$ - divergentă $\rightarrow \sum y_n$ divergentă

(C_2) $0 < x_n, y_n \ (\forall)\, n \in \mathbb{N}$

$$\lim_{n\to\infty} \frac{x_n}{y_n} = l \in (0, \infty) \Rightarrow \sum x_n \sim \sum y_n$$

$$\sum_{n\in\mathbb{N}} \alpha^n \text{ convergentă} \iff |\alpha| < 1$$

$$\sum_{n\in\mathbb{N}} \frac{1}{n^\alpha} \text{ convergentă} \iff \alpha > 1$$

$\sum x_n$, $x_n > 0$

$\lim\limits_{n\to\infty} \frac{x_n+1}{x_n} = l$

$\left[\lim\limits_{n\to\infty} \sqrt[n]{x_n} = l\right]$

$l < 1 \Rightarrow \sum x_n$ convergentă

$l > 1 \Rightarrow \sum x_n$ divergentă

$l = 1 \Rightarrow$???

$\lim\limits_{n\to\infty} n\left(\frac{x_n}{x_{n+1}} - 1\right) = l$

$l < 1 \Rightarrow \sum x_n$ divergentă

$l > 1 \Rightarrow \sum x_n$ convergentă

$l = 1 \Rightarrow$???

Criteriul lui ABEL

$$\left.\begin{array}{l} (x_n)_n \subseteq \mathbb{R} \\ \lim\limits_{n\to\infty} x_n = 0 \\ \quad x_{n+1} \le x_n \ (\forall)\, n \in \mathbb{N} \\ (y_n)_n \subseteq \mathbb{R} \\ (\exists)\, M \in \mathbb{R} \quad (\forall)\, n \in \mathbb{N} \quad |y_1 + y_2 + \dots + y_n| \le M \end{array}\right\} \Rightarrow$$

$\Rightarrow \sum x_n \cdot y_n$ convergentă

$$\sum_{n\in\mathbb{N}} \alpha^n \quad \text{convergentă} \iff |\alpha|<1$$

$$\sum_{n\in\mathbb{N}} \frac{1}{n^\alpha} \quad \text{convergentă} \iff \alpha>1$$

$\sum x_n$, $x_n > 0$

$\lim\limits_{n\to\infty} \frac{x_{n+1}}{x_n} = l$

$l<1 \Rightarrow \sum x_n$ convergentă

$l>1 \Rightarrow \sum x_n$ divergentă

$l=1 \Rightarrow$???

$\left[\lim\limits_{n\to\infty} \sqrt[n]{x_n} = l\right]$

$\lim\limits_{n\to\infty} n\left(\frac{x_n}{x_{n+1}} - 1\right) = l$

$l<1 \Rightarrow \sum x_n$ divergentă

$l>1 \Rightarrow \sum x_n$ convergentă

$l=1 \Rightarrow$???

Criteriul lui ABEL

$$\left.\begin{array}{l} (x_n)_n \subseteq \mathbb{R} \\ \lim\limits_{n\to\infty} x_n = 0 \\ \quad x_{n+1} \le x_n \ (\forall)\, n\in\mathbb{N} \\ (y_n)_n \subseteq \mathbb{R} \\ (\exists)\, M\in\mathbb{R} \quad (\forall)\, n\in\mathbb{N} \quad |y_1+y_2+\dots+y_n| \le M \end{array}\right\} \Rightarrow$$

$\Rightarrow \sum x_n \cdot y_n$ convergentă