AnMATE_S_13_2023_11_14 14.11.2023

Calculati

a) $\lim\limits_{x\to 0} \dfrac{1-\cos^3 x}{x \sin 2x} \overset{\frac{0}{0}}{=} \dfrac{-[(\cos x)^3]'}{x\cdot(\sin 2x)' + x'\cdot \sin 2x} =$

$= \dfrac{3\cos^2 x \cdot \sin x}{2x\cdot \cos 2x + \sin 2x} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 0} \dfrac{(3\cos^2 x)'\sin x + 3\cos^2 x(\sin x)'}{(2x)'\cos 2x + 2x(\cos 2x)' + \sin 2x} =$

$= \lim\limits_{x\to 0} \dfrac{6\cos x \sin x \sin x + 3\cos^2 x \cos x}{2\cos 2x - 2x\sin 2x + 2\sin 2x} =$

$= \lim\limits_{x\to 0} \dfrac{-6\cos x \sin^2 x + 3\cos^3 x}{2\cos 2x - 2x\sin 2x + 2\sin 2x} = \dfrac{3}{2}$

b) $\lim\limits_{x\to\infty} x^2\left(e^{\frac{1}{x}} - e^{\frac{1}{x+1}}\right) \overset{0\cdot\infty}{=}$

$= \lim\limits_{x\to\infty} \dfrac{e^{\frac{1}{x}} - e^{\frac{1}{x+1}}}{\frac{1}{x^2}} =$

$= \lim\limits_{x\to 0} \dfrac{e^{\frac{1}{x}} - 1}{\frac{1}{x^2}\cdot\frac{1}{x}}\cdot\dfrac{1}{x} - \dfrac{e^{\frac{1}{x+1}} - 1}{\frac{1}{x^2}\cdot\frac{1}{x+1}}\cdot\dfrac{1}{x+1} =$

$\to 1$ $\to 1$

$= \lim\limits_{x\to\infty} \dfrac{x+1}{x} - \dfrac{x^2}{x+1} = \lim\limits_{x\to\infty} \dfrac{x^2+x-x^2}{x+1} =$

$= \lim\limits_{x\to\infty} \dfrac{x}{x+1} = 1$

d) $\lim_{x \to 0} \left(\frac{x}{x+1}\right)^{\frac{1}{\ln(x^2+1)}} \overset{0^\infty}{=} \lim_{x \to 0} e^{\frac{1}{\ln(x+1)} \cdot \ln\left(\frac{x}{x+1}\right)} =$

$= \lim_{x \to 0} e^{\frac{1}{\ln(x+1)}(\ln x - \ln(x+1))}$

$\lim_{x \to 0} \frac{\ln x}{e^{\ln(x+1)}} - 1 = \lim_{x \to 0} e^{-\infty} = 0$

Polinomul lui Taylor

$f : [a, b] \to \mathbb{R}$, f - derivabilă de $k+1$ ori pe $[a, b]$. Atunci $(\exists)$ $x_0 \in (a, b)$

a.î.

$$f(x) = \underbrace{f(x_0) + \frac{f'(x_0)}{1!} \cdot (x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k}_{T_k(f, k)} + \underbrace{\frac{f^{(k+1)}(c)}{(k+1)!} \cdot (x - x_0)^{k+1}}_{\mathbb{R}_{k+1}(f, x)},$$

$T_k(f, x)$ - polinomul lui Taylor de grad k în x_0.

$\mathbb{R}_{k+1}(f, x)$ - restul Lagrange.

$\ln\left(\frac{1}{2}\right)$

$f : (-\infty, 1) \to \mathbb{R}$, $f(x) = \ln(1 - x)$

$1 - x > 0$

$-x > -1$

$x < 1$

$\lim_{k \to \infty} R_{k+1} = 0$

$$T_2(f, x) = \frac{-1}{1!} \cdot x - \frac{1}{2!} \cdot x^2 = -x - \frac{1}{2}x^2$$

$$\ln(1-x) \simeq -x - \frac{1}{2}x^2$$

$$x = \frac{1}{2} \Rightarrow \ln\left(1 - \frac{1}{2}\right) \simeq -\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{4}$$

$$\ln\left(\frac{1}{2}\right) \simeq -\frac{1}{2} - \frac{1}{8}$$

$$\simeq -\frac{5}{8}$$

$$\lim_{x \to 0} \frac{\ln(1+2x) - \sin 2x + 2x^2}{x^3} = \underbrace{\lim_{x \to 0} \frac{(1+2x) + x^2}{x^3}}_{\ell_1} - \underbrace{\lim_{x \to 0} \frac{\sin 2x - x^2}{x^2}}_{\ell_2}$$

Ex. $f : D \to \mathbb{R}$, $f(x) = \ln(1+2x)$ – calculăm

$T_3(f, x)$ în $x_0 = 0$

$$T_3(f, x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} \cdot x^2 + \frac{f'''(0)}{3!} \cdot x^3$$

$$f'(x) = (\ln(1+2x))' = \frac{1}{(1+2x)} \cdot ((1+2x))' = \frac{1}{1+2x} \cdot 2$$

$$f''(x) = \left(\frac{2}{1+2x}\right)' = \frac{2' \cdot (1+2x) - 2(1+2x)'}{(1+2x)^2} = \frac{-2 \cdot 2}{(1+2x)^2} =$$

$$= \frac{-4}{(1+2x)^2}$$

$$f'''(x) = \frac{(-4)'[(1+2x)^2]' - (-4)\cdot[(1+2x)^2]'}{(1+2x)^4} = \frac{-16}{2(1+2x)^3} \Rightarrow$$

$$\Rightarrow f'(0) = \frac{2}{2} = 2 \qquad \Rightarrow f'''(0) = 16$$

Temă: De rezolvat din nou:

Calculați cu polinomul lui Taylor de 3 ori

$$\left(\lim_{x \to 0} \frac{\ln(1+2x) - \sin 2x + 2x^2}{x^3} \right)$$

$T_3(f, x)$ în $x_0 = 0$ pt $f(x) = \ln(1+2x) - \sin 2x$

* Vezi +T_AnMate_S_13_2023_11_14