Tensor Network Diagrams

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Abstract

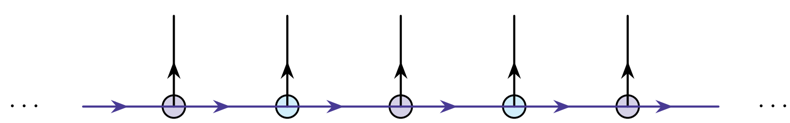
In this document, we compile tensor network diagrams which we frequently use, and will probably save a lot of time down the road...

Conventions

- *Physical* legs run vertically, while *virtual* legs run horizontally.
- MPS tensors of a give state (e.g G-cluster state) are represented as (blue) circles. Both the physical- and the virtual legs will, in general, have arrows.
- MPO tensors are represented as (grey) boxes, with label specifying the operator. The legs of the MPO tensor will have arrows when the legs are physical. When the legs are virtual, the legs will have no arrows since these are summed over in practice.

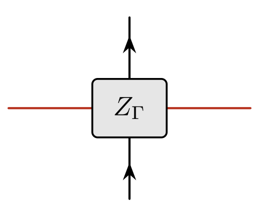
1 MPOs AND MPS

The diagrammtic presentation of the G-cluster state reads:



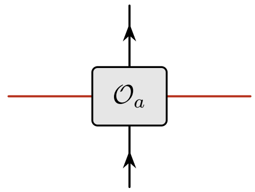
(1)

In the context of the G-cluster state, we regularly use the Z_Γ-MPOs:



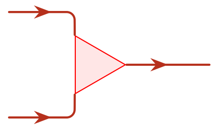
(2)

A general MPO labeled by $\mathcal{O}_a$:

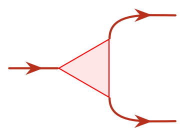


(3)

Multiplication of two MPOs defines fusion- and splitting tensors:

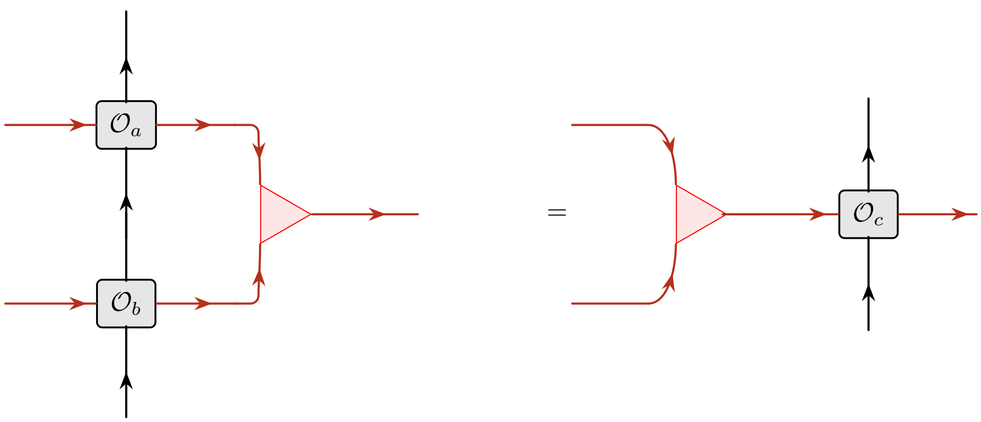


(4)



(5)

Multiplication of two MPOs satisfies the so-called *zipper equation*:



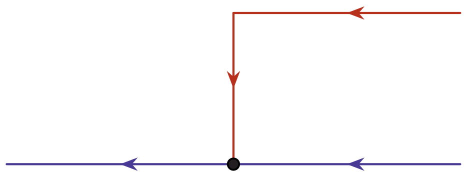
(6)

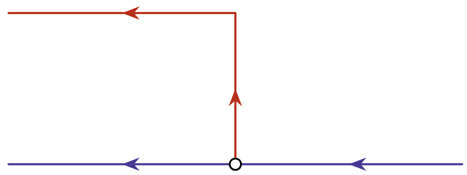
The action tensors are defined through by pulling an MPO acting on the physical leg of an MPS thorugh to the virtual legs of the MPS:

(7)

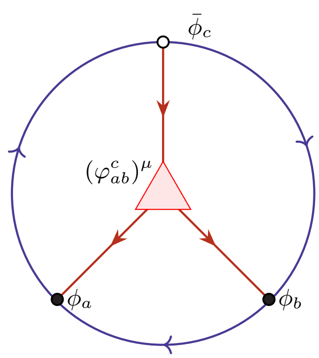
2 MPO ACTIONS ON PHYSICAL- AND VIRTUAL LEVELS

Action tensors:

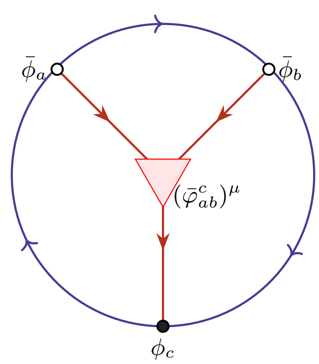
$(\phi) =$  (8)

$(\bar{\phi})_i =$  (9)

L-symbol wheels:

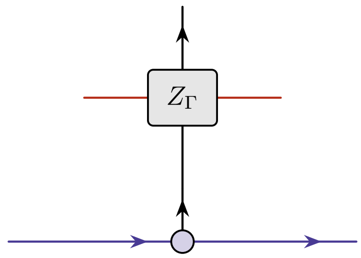


(10)



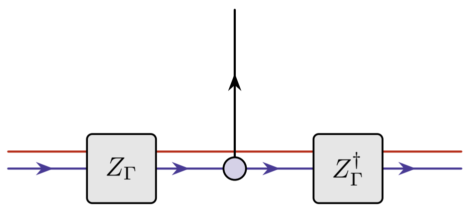
(11)

Action of the Z_Γ-MPO on the physical leg of the G-cluster state MPS (even sites):



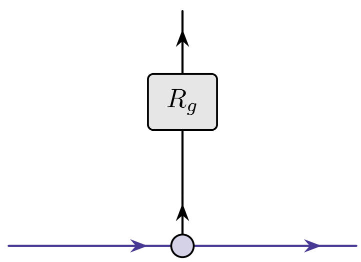
(12)

Pulling this action through to the virual legs of the G-cluster state MPS:



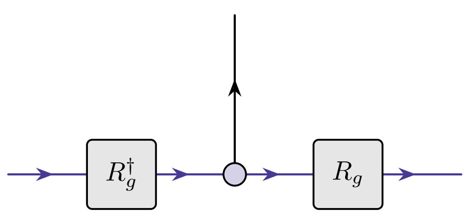
(13)

Action of $\overleftarrow{X}_g \equiv R_g$ on the physical leg of the G-cluster state MPS (odd sites):



(14)

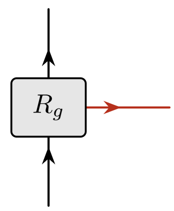
Pulling this through to the virtual legs of the G-cluster state MPS:



(15)

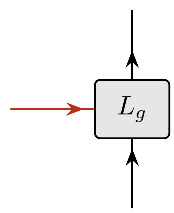
3 GAUGING PROJECTORS AND -DISENTANGLERS

For the gauging procedure of some group G, we need to construct a projector $\mathcal{P}^{(i)}$ which, after expanding the Hilbert space with the gauge Hilbert spaces, projects onto the locally gauge invariant subspace. This projector will contain, for every incoming leg of the matter site, a right-group-multiplication factor:



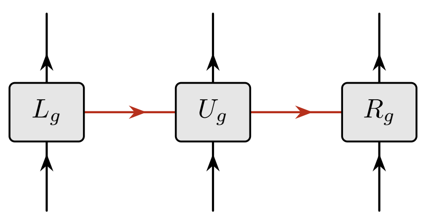
(16)

Similarly, for every outgoing leg of the matter site, we have a left-group-multiplication factor:



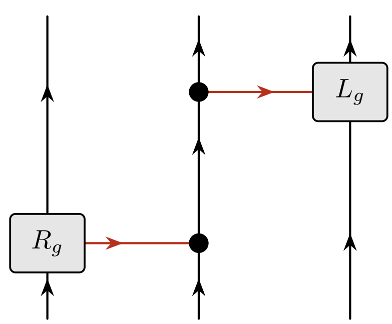
(17)

An example of a projector acting on a single matter site i reads:



(18)

We also use the disentangler, which trivializes the matter d.o.f:



(19)