# Gauging

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**Abstract**

## 1 INTRODUCTION

The process of promoting a global symmetry to a local symmetry, formally referred to as 'gauging', is introduced in the tensor-network framework. Succintly, this is done by introducing so-called gauge degrees of freedom, enlarging the Hilbert space, after which we then project onto the locally gauge-invariant subspace. This process is encapsulated in what we call the 'gauging map', and when supplemented with a 'disentangler', we retrieve Kramers-Wannier-like dualities.

## 2 THE GAUGING PROCEDURE

We gauge at the level of states, which should be interpreted as the map $\mathcal{G}$ : $\mathcal{H}_m \to \mathcal{H}_{m+g}$ such that the global symmetry $U(g)\,|\psi\rangle = \bigotimes_i u_i(g)\,|\psi\rangle = |\psi\rangle$ becomes a local symmetry $\hat{u}_i(g)\,|\hat{\psi}\rangle = |\hat{\psi}\rangle \;\; \forall i$. Herein, we have introduced gauge degrees of freedom (d.o.f) on the edges between the sites- denoted by $i+1/2$- such that the local Hilbert spaces become: $\hat{\mathcal{H}}_i \coloneqq \mathcal{H}_{i-1/2} \otimes \mathcal{H}_i \otimes \mathcal{H}_{i+1/2}$. For all practical purposes we shall also initiate the gauge d.o.f in the trivial state $|e\rangle_{i+\frac{1}{2}}$. For the case we will be considering, we use the group algebra $\mathbb{C}[G]$ to represent the gauge d.o.f labelled by the group elements: $\{|g\rangle \mid \; g \in G\}$. We will also be using the left- and right regular representations:

$$L_g^i \equiv L(g)^i \equiv \overrightarrow{X}_i(g) \coloneqq \sum_{h\in G} |gh\rangle \langle h| \tag{1}$$

$$R_g^i \equiv R^i(g) \equiv \overleftarrow{X}_i(g) \coloneqq \sum_{h\in G} |h\bar{g}\rangle \langle h| \tag{2}$$

The local Gauss law reads $\hat{u}_g^i = R_g^{i-\frac{1}{2}} \otimes u_g^i \otimes L_g^{i+\frac{1}{2}}$. Upon confirming that the left- and right regular representations acting on the same site commute:

$[L_g, R_h] = 0$, we can construct a projector onto the locally gauge-invariant subspace using the group-averaged type projector:

$$\mathcal{P}_i \coloneqq \frac{1}{|G|} \sum_{g \in G} u_g^i \tag{3}$$

Naturally, the global projector is taken over all sites: $\mathcal{P} = \Pi_i \mathcal{P}_i$. Summarizing: the enlarging of the local Hilbert spaces and subsequent projection onto the states wich are locally gauge-invariant is encapsulated in the gauging map $\mathcal{G} |\psi\rangle = \mathcal{P}(|\psi\rangle \bigotimes_i |e\rangle_{i+\frac{1}{2}})$.
Gauging of operators $O$ is defined by $\Gamma : \mathcal{L}(\mathcal{H}_m) \to \mathcal{L}(\mathcal{H}_{m+g})$, and satisfy:

$$\text{(i)} \;\; u_g^i \mathcal{G} = \mathcal{G} \tag{4}$$

$$\text{(ii)} \;\; [u_g^i, \Gamma[O]] = 0 \tag{5}$$

$$\text{(iii)} \;\; \mathcal{G} \cdot U_g = \mathcal{G} \tag{6}$$

$$\text{(iv)} \;\; \Gamma[O]\mathcal{G} |\psi\rangle = \mathcal{G} O |\psi\rangle \tag{7}$$

*Note: add gauging of a subgroup here?*
*Note: remember to add the fundamental theorem of MPS somewhere! What about gauging of so-called anomalous symmetries, and twisted gauging?*

Identifying the possible symmetries of the tensors which constitute the gauging map greatly aids in the diagrammatic derivations which will be used. For example [list some examples here with diagrams]. Similarly, we do this for the left- and right legs of the projectors: [insert symmetries of R and L here, wtih diagrams]

The gauging map, complemented with a so-called 'disentangler' $\mathcal{U}$ is what constitutes a 'duality'[explain what is meant by the term duality]. By pulling the disentangler through the gauging map, we possibly get a simpler action which reduces the complexity of identifying operators on each side of the duality.

# REFERENCES


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