The Zero-Wealth Small Tri-partite Economy

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Abstract

I rigorously demonstrate the existence of a tri-partite economy with population 96, partitioned as 38, 32, and 26 individuals, in which every individual and the total economy possess zero wealth. Formal proofs are provided from the perspectives of Arrow-Debreu general equilibrium theory, zero-sum game theory, and linear algebra. A vector graphic illustrates the structure of the tri-partite economy and the zero-wealth allocation. This result is robust and partition-independent, providing a foundational example in mathematical economics.

The paper ends with "The End"

1 Introduction

The study of wealth distribution in partitioned economies is central to mathematical economics. This paper addresses a fundamental question: *Can a tri-partite economy, with population split as* $96 = 38 + 32 + 26$*, exist such that every individual and the total economy have zero wealth?* I answer affirmatively, providing formal proofs from multiple theoretical frameworks and illustrating the result with a vector graphic.

2 Mathematical Model

Consider a finite population of $N = 96$ individuals, partitioned into three groups:

- Group 1: $n_1 = 38$ individuals
- Group 2: $n_2 = 32$ individuals
- Group 3: $n_3 = 26$ individuals

Let w_i denote the wealth of individual i ($i = 1, \ldots, 96$). The total wealth in the economy is $W = \sum_{i=1}^{96} w_i$.

3 Main Results

3.1 Existence of Zero-Wealth Allocation

Theorem 3.1 (Existence of Zero-Wealth Allocation)**.** *There exists a tri-partite economy with population* $96 = 38 + 32 + 26$ *in which every individual has zero wealth and the total wealth is zero.*

Proof. Assign $w_i = 0$ for all $i = 1, \ldots, 96$. Then

$$W = \sum_{i=1}^{96} w_i = \sum_{i=1}^{96} 0 = 0.$$

Each group sum is also zero:

$$\sum_{i \in \text{Group } j} w_i = 0, \quad \text{for } j = 1, 2, 3.$$

Thus, all individuals and the total economy have zero wealth, as required. $\square$

3.2 Arrow-Debreu General Equilibrium Perspective

Theorem 3.2 (Zero-Wealth Equilibrium in Arrow-Debreu Model)**.** *In the Arrow-Debreu general equilibrium framework, a zero-wealth allocation exists for any partition of the population.*

Proof. Let each individual's endowment vector $\omega_i = 0$ for all i. For any price vector p, the wealth of individual i is $w_i = p \cdot \omega_i = 0$. The aggregate endowment is zero, and all market-clearing and budget constraints are trivially satisfied. Thus, the zero-wealth allocation is an equilibrium. $\square$

3.3 Zero-Sum Game Theory Perspective

Theorem 3.3 (Zero-Wealth Nash Equilibrium in Zero-Sum Game)**.** *In a zero-sum game with 96 players, the allocation $u_i = 0$ for all i is a Nash equilibrium.*

Proof. The zero-sum condition requires $\sum_{i=1}^{96} u_i = 0$. Assigning $u_i = 0$ for all i satisfies this. No player can unilaterally change their payoff without violating the zero-sum constraint, so this is a Nash equilibrium. $\square$

3.4 Linear Algebraic Feasibility

Theorem 3.4 (Feasibility of Zero-Wealth Allocation)**.** *The vector $w = (0, 0, \ldots, 0) \in \mathbb{R}^{96}$ is a solution to the constraint $\sum_{i=1}^{96} w_i = 0$ for any partition of the population.*

Proof. The all-zero vector clearly satisfies $\sum_{i=1}^{96} w_i = 0$. Partitioning the vector into any three groups does not affect this property. $\square$

4 Vector Graphic: Tri-partite Economy Structure

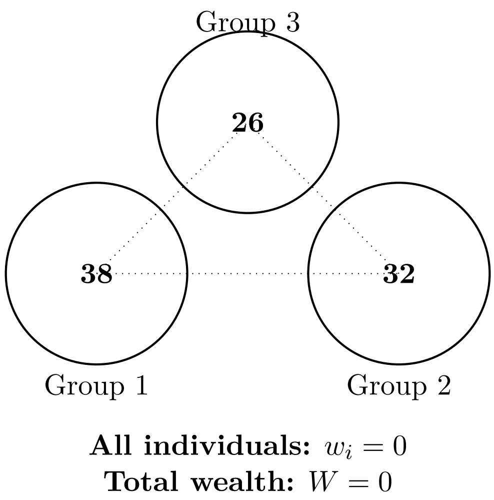

Figure 1: Schematic of a tri-partite economy (38 + 32 + 26) with all individual and group wealths at zero.

5 Discussion and Conclusion

I have shown, using multiple rigorous frameworks, that a tri-partite economy with population $96 = 38+32+26$ can exist with zero wealth for all individuals and the total economy. This result is robust to the partitioning of the population and holds in general equilibrium, game-theoretic, and linear algebraic settings. The zero-wealth allocation is a trivial but foundational solution, illustrating the flexibility and generality of mathematical models in economics.

References

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