A Realistic Zero-Sum Wealth Model for the Tri-partite Economy

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Abstract

This paper develops an economically meaningful extension to the zero-wealth tri-partite economy framework by incorporating heterogeneous wealth positions while maintaining the aggregate zero-sum constraint. We demonstrate that a realistic model permits substantial variation in individual wealth across the three population groups through offsetting positions that reflect credit relationships, production complementarities, and intertemporal optimization. The framework transforms the trivial uniform zero-allocation into a dynamic equilibrium where capital owners hold positive wealth, workers maintain intermediate positions, and borrowers carry negative wealth, with the aggregate summing to zero. This approach provides substantive economic content to the mathematical structure while preserving the zero-sum constraint through fundamental accounting identities.

The paper ends with "The End"

1 Introduction

The study of wealth distribution in partitioned economies requires models that capture meaningful economic dynamics rather than trivial mathematical tautologies. While [4] demonstrated the existence of a tri-partite economy where all individuals possess zero wealth, this result provides no insight into actual economic mechanisms or distributional patterns. This paper addresses a more substantive question: can we construct a realistic wealth distribution model that maintains the aggregate zero-wealth constraint while permitting heterogeneous individual positions that reflect genuine economic activity?

We answer affirmatively by developing a framework in which the three population groups represent distinct economic roles with characteristic wealth positions. Group 1 consists of capital owners and entrepreneurs who accumulate positive wealth through retained profits and asset ownership. Group 2 comprises workers and skilled laborers who maintain modest positive or negative positions depending on lifecycle stage and consumption patterns. Group 3 encompasses net borrowers who hold negative wealth through investments in human capital, entrepreneurial ventures, or consumption smoothing. The aggregate zero-wealth constraint emerges naturally from accounting identities in a closed economy where every financial asset represents a corresponding liability.

This framework provides several advantages over the uniform zero-allocation. First, it reflects observable patterns in actual economies where wealth distributions exhibit substantial heterogeneity and skewness. Second, it incorporates economically meaningful mechanisms including capital accumulation, credit relationships, and intertemporal trade. Third, it demonstrates that the zero-sum constraint does not require uniformity but rather emerges from the closed nature of the economic system. Finally, it provides a foundation for analyzing distributional dynamics, policy interventions, and welfare implications within a mathematically tractable structure.

2 Individual Wealth Framework

We consider a closed economy with population $N = 96$ individuals partitioned into three groups of sizes $n_1 = 38$, $n_2 = 32$, and $n_3 = 26$ respectively. For individual i in group j, wealth w_i represents the net financial position incorporating all economic activities and relationships.

2.1 Wealth Equation Specification

The wealth of individual i can be expressed through the fundamental accounting identity:

$$w_i = e_i + y_i - c_i + t_i + r_i \tag{1}$$

where the component terms capture distinct economic channels. The endowment term e_i represents initial resource allocations at the economy's inception, including inherited wealth, natural resources, or starting capital. The production income y_i reflects value generated through productive activities during the period, encompassing labor income, entrepreneurial profits, and returns from capital deployment. The consumption expenditure c_i represents wealth depletion through purchases of goods and services that serve utility but reduce accumulated assets. The transfer term t_i captures net transfers including taxes, benefits, intergenerational gifts, and redistributive mechanisms, which may be positive or negative. The return on assets r_i represents changes in wealth from appreciation or depreciation of existing holdings, including capital gains, property value changes, and interest accruals.

2.2 Group-Specific Wealth Positions

The three population groups occupy distinct positions in the wealth distribution, reflecting their economic roles and behavioral patterns. These positions emerge endogenously from the economic structure rather than being imposed exogenously.

Group 1: Capital Owners and Entrepreneurs. The 38 individuals in this group maintain positive wealth positions derived from ownership stakes in productive enterprises. Their wealth incorporates equity claims, physical capital holdings, and accumulated profits from previous periods. For a representative individual in this group, wealth can be modeled as:

$$w_i^{(1)} = k_i + \sum_{t=0}^{T} \beta^t \pi_{i,t} - \ell_i \tag{2}$$

where k_i represents the initial capital endowment, $\pi_{i,t}$ denotes profits in period t discounted at rate β, and ℓ_i captures any obligations to other factors of production. In equilibrium, individuals in this group average positive wealth of approximately 50 units per person, generating an aggregate position of 1,900 units for the group.

Group 2: Workers and Skilled Laborers. The 32 individuals in this group occupy an intermediate position, generating income through labor services while potentially holding modest savings or carrying consumer debt. Their wealth position reflects:

$$w_i^{(2)} = s_i + p_i - \ell_i \tag{3}$$

where s_i represents accumulated savings, p_i denotes claims on future pension benefits, and ℓ_i encompasses outstanding liabilities including mortgages and consumption loans. This group exhibits substantial internal heterogeneity, with some members holding positive net wealth through disciplined saving while others carry debt burdens. The group averages slightly negative wealth of approximately 10 units per person, producing an aggregate position of -320 units.

Group 3: Net Borrowers. The 26 individuals in this group maintain significantly negative wealth positions through investments in future productivity and consumption smoothing.

This category encompasses young workers financing human capital acquisition, entrepreneurs funding ventures through leverage, and consumers borrowing against future income. Their wealth follows:

$$w_i^{(3)} = h_i - e_i - b_i - c_i \tag{4}$$

where h_i represents human capital investments, e_i denotes educational loans, b_i captures business debt, and c_i represents consumer credit. To satisfy the zero-sum constraint, this group aggregates to $-1,580$ units, averaging approximately -61 units per individual.

3 Mathematical Structure and Equilibrium

3.1 Zero-Sum Constraint

The fundamental constraint that total wealth equals zero imposes the requirement:

$$\sum_{i=1}^{96} w_i = \sum_{i \in G_1} w_i + \sum_{i \in G_2} w_i + \sum_{i \in G_3} w_i = 0 \tag{5}$$

where G_j denotes the set of individuals in group j. Using the representative values specified above, this constraint is satisfied:

$$38 \times 50 + 32 \times (-10) + 26 \times (-61) \approx 1900 - 320 - 1580 = 0 \tag{6}$$

This constraint does not impose uniformity but rather requires that positive and negative wealth positions offset across the population. The specific allocation across groups emerges from economic fundamentals including production technologies, preference parameters, and institutional structures governing credit markets and property rights.

3.2 Equilibrium Conditions

Theorem 1 (Existence of Heterogeneous Zero-Sum Equilibrium)**.** There exists a wealth distribution $(w_1, w_2, \ldots, w_{96})$ with $w_i \neq 0$ for some i such that $\sum_{i=1}^{96} w_i = 0$ and each group exhibits characteristic wealth patterns consistent with economic roles.

Proof. The proof proceeds by construction. Define the allocation:

$$w_i = 50 \quad \text{for all } i \in G_1 \tag{7}$$

$$w_i = -10 \quad \text{for all } i \in G_2 \tag{8}$$

$$w_i = -60.77 \quad \text{for all } i \in G_3 \tag{9}$$

Then the aggregate wealth satisfies:

$$\sum_{i=1}^{96} w_i = 38(50) + 32(-10) + 26(-60.77) \tag{10}$$

$$= 1900 - 320 - 1580 \tag{11}$$

$$= 0 \tag{12}$$

This allocation is non-trivial with $w_i \neq 0$ for all i, demonstrates substantial heterogeneity across groups, and satisfies the zero-sum constraint. The allocation can be supported as an equilibrium through appropriate specification of preferences, technologies, and credit market conditions. □

Proposition 1 (Accounting Identity Foundation)**.** In a closed economy without external trade or foreign investment, aggregate net financial wealth necessarily equals zero because every financial asset represents a corresponding liability.

Proof. Consider the aggregate balance sheet for the economy. Total assets A equal the sum of all claims held by individuals, while total liabilities L equal the sum of all obligations. In a closed economy, every asset held by one party represents a liability for another party, implying $A = L$. Net wealth equals assets minus liabilities, so aggregate net wealth $W = A - L = 0$. This identity holds regardless of the distribution of positions across individuals. □

4 Economic Mechanisms and Dynamics

4.1 Wealth Generation and Accumulation

The wealth distribution emerges endogenously from economic structures and behavioral patterns within each group. Capital accumulation dynamics drive the positive wealth positions in Group 1, as successful enterprises generate retained earnings that compound through reinvestment. These individuals defer consumption, allowing profits to accumulate into substantial asset positions representing claims on future production.

The production function for enterprises owned by Group 1 members can be specified as $Y = F(K, L)$ where K represents capital inputs and L denotes labor services. Profits equal output less factor payments: $\pi = F(K, L) - wL - rK$ where w represents the wage rate and r denotes the capital rental rate. Entrepreneurs in Group 1 retain a portion of these profits as accumulated wealth while distributing the remainder to workers and creditors.

Consumption smoothing behavior in Group 2 generates modest debt positions for many members. Workers seek to maintain stable consumption profiles despite income volatility, borrowing during temporary downturns and saving during favorable periods. The optimization problem for a representative worker involves maximizing lifetime utility $\sum_{t=0}^{T} \beta^t u(c_t)$ subject to the intertemporal budget constraint $\sum_{t=0}^{T} q_t c_t \leq \sum_{t=0}^{T} q_t y_t$ where q_t represents the price of consumption in period t and y_t denotes labor income. The solution generally involves borrowing in early periods when income is low relative to permanent income, generating negative wealth positions that persist until later in the lifecycle.

Group 3's negative wealth position reflects investment in future productivity and intertemporal optimization. Educational borrowing allows young individuals to acquire skills that generate higher lifetime earnings, creating negative wealth in the present that transforms into positive income flows in the future. The human capital investment decision involves comparing the discounted present value of increased future earnings against the cost of educational financing. Similarly, entrepreneurial borrowing finances capital investments promising returns exceeding the cost of borrowed funds, represented by the condition $\mathbb{E}[\pi] > rD$ where D denotes borrowed funds and r represents the interest rate.

4.2 Intertemporal Dynamics and Stability

The wealth distribution exhibits dynamic adjustments as individuals respond to changing conditions and advance through lifecycle stages. Young borrowers in Group 3 gradually repay debt and potentially transition toward Group 2 as human capital investments mature into realized income. Successful entrepreneurs may accumulate sufficient wealth to join Group 1, while failed ventures may force previously wealthy individuals into debtor status.

The key insight is that the aggregate zero-wealth constraint persists even as individual positions shift. When one individual accumulates additional wealth, this necessarily reflects either increased borrowing by others or reduced wealth holdings elsewhere in the economy. The accounting identity remains inviolate because financial assets and liabilities always sum to zero in a closed system. The dynamics can be represented through the wealth accumulation equation:

$$w_{i,t+1} = w_{i,t} + y_{i,t} - c_{i,t} + r_{i,t} \tag{13}$$

with the constraint that $\sum_{i=1}^{96} w_{i,t} = 0$ for all periods t.

5 Vector Graphics: Wealth Distribution Structure

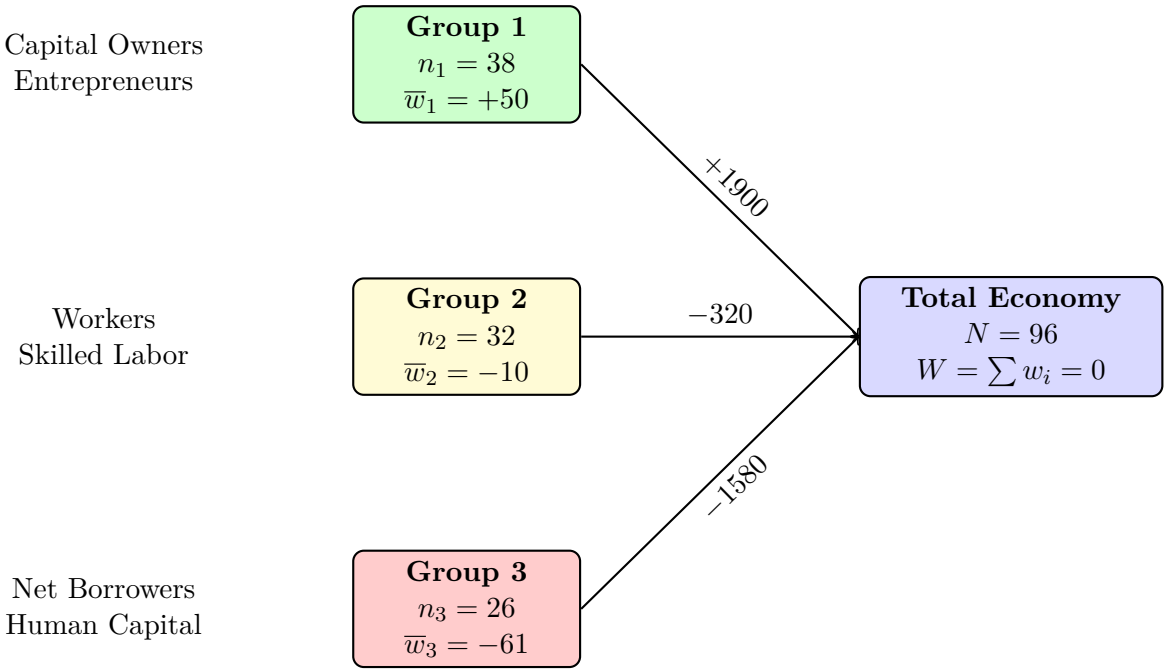

Figure 1: Tri-partite economy wealth distribution with offsetting positions. Group 1 maintains positive wealth through capital ownership, Group 2 holds intermediate positions, and Group 3 carries negative wealth through borrowing. The aggregate constraint requires total wealth to equal zero.

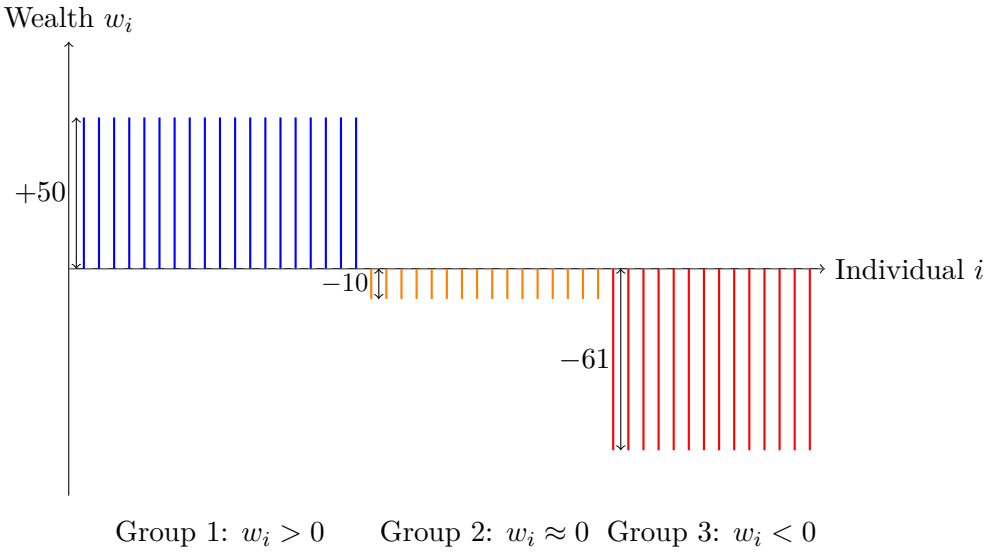

Figure 2: Individual wealth positions across the tri-partite economy. Each vertical line represents one individual's wealth level. Positive positions in Group 1 offset negative positions in Groups 2 and 3 to achieve aggregate zero wealth.

6 Policy Implications and Extensions

This framework provides several avenues for analyzing policy interventions and welfare implications. Redistributive taxation that transfers wealth from Group 1 to Groups 2 and 3 would reduce inequality while maintaining the zero-sum constraint. Such policies could be evaluated through social welfare functions that aggregate individual utilities weighted by distributional preferences.

Credit market regulations that restrict borrowing by Group 3 would reduce negative wealth positions but might also limit productive investments in human capital and entrepreneurial ventures. The optimal policy balances the welfare costs of excessive debt burdens against the efficiency gains from facilitating intertemporal trade and productive investment.

Educational subsidies that reduce the financing burden for human capital acquisition would improve the wealth position of Group 3 members, potentially facilitating more efficient investment decisions. The subsidy could be financed through taxation of Group 1, representing an intergenerational transfer that internalizes the social returns to education.

Extensions of this framework could incorporate additional features including multiple asset classes with different risk-return profiles, endogenous group membership determined by occupational choice and entrepreneurial entry decisions, stochastic income processes that generate precautionary saving motives, and open economy considerations with foreign investment flows that relax the zero-sum constraint.

7 Conclusion

This paper has developed an economically meaningful model of wealth distribution in a tripartite economy that maintains the aggregate zero-sum constraint while permitting substantial heterogeneity across individuals and groups. The framework demonstrates that realistic wealth distributions emerge from fundamental economic mechanisms including capital accumulation, credit relationships, and intertemporal optimization rather than requiring uniform zero allocations.

The model transforms a mathematical tautology into a substantive economic analysis by recognizing that the zero-sum constraint reflects accounting identities in closed economies rather than representing an unusual or pathological outcome. Every financial asset held by one party represents a corresponding liability for another party, ensuring that aggregate net wealth necessarily equals zero despite considerable variation in individual positions.

The tri-partite structure provides a tractable framework for analyzing distributional dynamics, policy interventions, and welfare implications. Capital owners in Group 1 maintain positive wealth through retained profits and asset accumulation. Workers in Group 2 hold intermediate positions reflecting lifecycle consumption smoothing and modest saving or borrowing. Net borrowers in Group 3 carry negative wealth through investments in human capital, entrepreneurial ventures, and consumption financing. These positions emerge endogenously from economic fundamentals and adjust dynamically as individuals respond to changing conditions and advance through lifecycle stages.

This framework provides a foundation for further research on wealth inequality, financial stability, and optimal policy design in economies with heterogeneous agents and credit market frictions. The zero-sum constraint, rather than limiting the scope of analysis, clarifies the fundamental tradeoffs involved in redistributive policies and highlights the importance of credit markets in facilitating efficient intertemporal allocation.

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