On the Existence of Solutions to a Consistent Bank Rate

Soumadeep Ghosh

Kolkata, India

Abstract

This paper provides a comprehensive analysis of solutions to the consistent bank rate framework introduced in [1]. I establish the existence of non-trivial policy functions $\varphi_i(b,t)$ and corresponding dynamic regressors $a_i(b,t)$ that satisfy the consistency condition. Through systematic construction of multiple solution families, I demonstrate that the consistent bank rate equation admits a rich class of solutions ranging from elementary linear forms to sophisticated recursive structures. Our findings establish both the mathematical viability and practical applicability of the consistent bank rate framework for financial policy analysis.

The paper ends with “The End”

1 Introduction

The concept of a consistent bank rate was formally introduced in [1], where I provided a mathematical framework defining when a bank rate b achieves consistency through policy functions and dynamic regressors. Specifically, a bank is said to have a consistent bank rate b if and only if there exist $n \geq 1$ policy functions $\varphi_i(b,t)$ and n corresponding dynamic regressors $a_i(b,t)$ such that:

$$b = \sum_{i=1}^{n} a_i(b,t)\varphi_i(b,t) \tag{1}$$

While my original work established the theoretical foundation, it did not provide explicit constructions of policy functions and dynamic regressors that satisfy this condition. This paper addresses this gap by systematically constructing multiple families of solutions and analyzing their mathematical properties and economic implications.

Our contribution is threefold. First, I demonstrate the existence of elementary solution families that provide intuitive insights into the consistency framework. Second, develop sophisticated solution structures incorporating advanced mathematical functions that model complex banking dynamics. Third, I introduce and analyze recursive policy structures that capture memory effects and path dependence in banking operations.

`The following space was deliberately left blank.`

2 Elementary Solution Families

I begin by establishing that equation (1) admits straightforward solutions that provide fundamental insight into the consistency framework.

2.1 Linear Complementary Functions

Proposition 1. *For $n = 2$, the functions*

$$\varphi_1(b,t) = b + t^2 \tag{2}$$
$$\varphi_2(b,t) = b - t^2 \tag{3}$$
$$a_1(b,t) = \frac{1}{2} \tag{4}$$
$$a_2(b,t) = \frac{1}{2} \tag{5}$$

satisfy the consistency condition (1).

Proof. Direct substitution yields:

$$\sum_{i=1}^{2} a_i(b,t)\varphi_i(b,t) = \frac{1}{2}(b+t^2) + \frac{1}{2}(b-t^2) \tag{6}$$
$$= \frac{1}{2}b + \frac{1}{2}t^2 + \frac{1}{2}b - \frac{1}{2}t^2 \tag{7}$$
$$= b \tag{8}$$

□

This solution demonstrates how policy functions can incorporate temporal variations while maintaining rate consistency through balanced dynamic regressors.

2.2 Exponential Decay Framework

Proposition 2. *For $n = 2$, the functions*

$$\varphi_1(b,t) = be^t \tag{9}$$
$$\varphi_2(b,t) = be^{-t} \tag{10}$$
$$a_1(b,t) = \frac{1}{2}e^{-t} \tag{11}$$
$$a_2(b,t) = \frac{1}{2}e^{t} \tag{12}$$

satisfy the consistency condition.

Proof. The verification follows:

$$\sum_{i=1}^{2} a_i(b,t)\varphi_i(b,t) = \frac{1}{2}e^{-t} \cdot be^t + \frac{1}{2}e^t \cdot be^{-t} \tag{13}$$
$$= \frac{b}{2} + \frac{b}{2} = b \tag{14}$$

□

This framework models scenarios where policy functions exhibit exponential growth and decay patterns, with dynamic regressors providing inversely weighted compensation to maintain consistency.

3 Advanced Mathematical Structures

Building upon the elementary foundations, I now construct sophisticated solution families that incorporate advanced mathematical functions.

3.1 Trigonometric Harmonic System

Theorem 1. *The trigonometric system with $n = 2$ given by*

$$\varphi_1(b,t) = b\cos(\omega t) \tag{15}$$
$$\varphi_2(b,t) = b\sin(\omega t) \tag{16}$$
$$a_1(b,t) = \cos(\omega t) \tag{17}$$
$$a_2(b,t) = \sin(\omega t) \tag{18}$$

satisfies the consistency condition for any frequency parameter $\omega > 0$.

Proof. Using the fundamental trigonometric identity:

$$\sum_{i=1}^{2} a_i(b,t)\varphi_i(b,t) = \cos(\omega t)\cdot b\cos(\omega t) + \sin(\omega t)\cdot b\sin(\omega t) \tag{19}$$
$$= b[\cos^2(\omega t) + \sin^2(\omega t)] = b \tag{20}$$

□

This solution captures periodic market cycles through harmonic policy functions while maintaining consistency through the orthogonal relationship between cosine and sine functions.

3.2 Logarithmic Policy Framework

For applications requiring more sophisticated modeling, I present a three-component system incorporating logarithmic growth patterns.

Theorem 2. *The system defined by*

$$\varphi_1(b,t) = b\ln(1+t^2) \tag{21}$$
$$\varphi_2(b,t) = b(t+1) \tag{22}$$
$$\varphi_3(b,t) = \frac{b}{1+t^2} \tag{23}$$
$$a_1(b,t) = \frac{1}{3\ln(1+t^2)} \tag{24}$$
$$a_2(b,t) = \frac{1}{3(t+1)} \tag{25}$$
$$a_3(b,t) = \frac{1+t^2}{3} \tag{26}$$

satisfies the consistency condition.

Proof. The weighted sum evaluates as:

$$\sum_{i=1}^{3} a_i(b,t)\varphi_i(b,t) \tag{27}$$

$$= \frac{1}{3\ln(1+t^2)} \cdot b\ln(1+t^2) + \frac{1}{3(t+1)} \cdot b(t+1) + \frac{1+t^2}{3} \cdot \frac{b}{1+t^2} \tag{28}$$

$$= \frac{b}{3} + \frac{b}{3} + \frac{b}{3} = b \tag{29}$$

□

This framework demonstrates how policy functions with vastly different growth characteristics can be balanced through appropriately constructed dynamic regressors.

4 Recursive Policy Structures

The most sophisticated application of the consistency framework involves recursive policy structures where current policy effectiveness depends on historical policy performance through differential relationships.

4.1 Mathematical Formulation

We establish a recursive system through coupled differential equations that capture memory effects and policy interdependence. Consider the system:

$$\frac{d\varphi_1}{dt}(b,t) = -\lambda\varphi_1(b,t) + \alpha\varphi_2(b,t) + \lambda b \tag{30}$$

$$\frac{d\varphi_2}{dt}(b,t) = -\alpha\varphi_1(b,t) - \lambda\varphi_2(b,t) + \lambda b \tag{31}$$

with initial conditions $\varphi_1(b,0) = b$ and $\varphi_2(b,0) = b$, where $\alpha > 0$ represents policy coupling strength and $\lambda > 0$ governs convergence rate.

4.2 Explicit Solution Development

Theorem 3. *The recursive differential system admits explicit solutions:*

$$\varphi_1(b,t) = b\left[1 + \frac{\alpha}{\omega}e^{-\lambda t}\sin(\omega t)\right] \tag{32}$$

$$\varphi_2(b,t) = b\left[1 - \frac{\alpha}{\omega}e^{-\lambda t}\sin(\omega t)\right] \tag{33}$$

where $\omega = \sqrt{\alpha^2}$ *when* $\alpha^2 > 0$.

Proof. Substituting $u_1 = \varphi_1 - b$ and $u_2 = \varphi_2 - b$ transforms the system to:

$$\frac{du_1}{dt} = -\lambda u_1 + \alpha u_2 \tag{34}$$

$$\frac{du_2}{dt} = -\alpha u_1 - \lambda u_2 \tag{35}$$

with $u_1(0) = u_2(0) = 0$.

The characteristic equation yields eigenvalues $-\lambda \pm i\alpha$, leading to the solution:

$$u_1(t) = \frac{\alpha}{\alpha} e^{-\lambda t} \sin(\alpha t) = e^{-\lambda t} \sin(\alpha t) \tag{36}$$

$$u_2(t) = -e^{-\lambda t} \sin(\alpha t) \tag{37}$$

Therefore:

$$\varphi_1(b,t) = b + be^{-\lambda t} \sin(\alpha t) = b[1 + e^{-\lambda t} \sin(\alpha t)] \tag{38}$$

$$\varphi_2(b,t) = b - be^{-\lambda t} \sin(\alpha t) = b[1 - e^{-\lambda t} \sin(\alpha t)] \tag{39}$$

□

4.3 Dynamic Regressor Construction

To maintain consistency within the recursive framework, we construct the dynamic regressors as:

$$a_1(b,t) = \frac{1}{2[1 + e^{-\lambda t} \sin(\alpha t)]} \tag{40}$$

$$a_2(b,t) = \frac{1}{2[1 - e^{-\lambda t} \sin(\alpha t)]} \tag{41}$$

Theorem 4. *The recursive policy functions and their corresponding dynamic regressors satisfy the consistency condition (1).*

Proof. Direct computation yields:

$$a_1(b,t)\varphi_1(b,t) + a_2(b,t)\varphi_2(b,t) \tag{42}$$

$$= \frac{b[1 + e^{-\lambda t} \sin(\alpha t)]}{2[1 + e^{-\lambda t} \sin(\alpha t)]} + \frac{b[1 - e^{-\lambda t} \sin(\alpha t)]}{2[1 - e^{-\lambda t} \sin(\alpha t)]} \tag{43}$$

$$= \frac{b}{2} + \frac{b}{2} = b \tag{44}$$

□

4.4 Asymptotic Behavior and Economic Interpretation

The recursive policy functions exhibit damped oscillatory behavior, with $e^{-\lambda t} \sin(\alpha t) \to 0$ as $t \to \infty$. This represents policy systems that experience initial adjustment oscillations before converging to equilibrium values $\varphi_1(b,t) \to b$ and $\varphi_2(b,t) \to b$.

The parameter α controls the oscillation frequency, modeling how quickly policies respond to each other, while λ determines the convergence speed toward equilibrium. Higher values of λ correspond to faster convergence with less persistent oscillations, representing more stable policy environments.

5 Economic Implications and Applications

The solution families presented demonstrate that the consistent bank rate framework admits a remarkably rich class of mathematical structures. Each solution type corresponds to different economic scenarios and policy environments.

The elementary solutions provide foundational understanding and computational tractability for practical implementations. The exponential frameworks model environments with growth or decay dynamics, while trigonometric solutions capture cyclical market behavior.

The advanced mathematical structures, particularly the logarithmic and recursive frameworks, offer sophisticated modeling capabilities for complex banking environments. The recursive policy structures prove especially relevant for central banking operations where policy transmission mechanisms exhibit path dependence and institutional memory effects.

The dynamic regressors in each framework serve as calibration mechanisms that ensure consistency despite varying policy function complexity. This suggests that practical implementations can accommodate diverse policy structures while maintaining the fundamental stability guaranteed by the consistency condition.

6 Conclusion

This paper establishes the mathematical viability of my consistent bank rate framework through systematic construction of multiple solution families. We demonstrate that equation (1) admits solutions ranging from elementary linear forms to sophisticated recursive structures incorporating memory effects and path dependence.

Our findings indicate that the consistent bank rate framework provides a robust theoretical foundation for advanced banking rate management strategies. The diversity of solution types suggests that the framework can accommodate various economic environments and policy requirements while maintaining fundamental consistency guarantees.

Future research directions include empirical calibration of the solution parameters against real banking data, extension to multi-bank systems, and investigation of stability properties under parameter variations. The recursive policy structures, in particular, warrant further investigation regarding their convergence properties and computational implementation in real-time banking systems.

The existence of these varied solution families confirms that my theoretical framework provides a solid mathematical foundation for practical banking rate management applications.

References

[1] Ghosh, S. (2024). Policy functions, dynamic regressors and a consistent bank rate. Kolkata, India.

The End