Implications of Permutations and Derangements for the Planar 11-Gon Framework: A Combinatorial Analysis of Financial Market Configuration Space

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Abstract

This paper investigates the combinatorial implications of permutations and derangements for the planar 11-gon (hendecagon) framework in financial economics. The 11-gon, representing the complete dimensional structure of economic systems with 8 asset dimensions from $\mathbb{C}^4$ and 3 organizational modes, admits $11! = 39{,}916{,}800$ vertex permutations. Of these, $D_{11} = 14{,}684{,}570$ are derangements—permutations with no fixed points—representing complete market regime transformations. We analyze the implications of this vast configuration space for arbitrage channels, portfolio optimization, structural volatility, and regime transitions. The probability that a random market reconfiguration constitutes a complete transformation approaches $1/e \approx 36.79\%$, revealing fundamental instabilities in the 11-dimensional economic architecture. We establish connections between dihedral symmetry preservation, organizational vertex cycling, and systemic risk measurement.

The paper ends with "The End"

1 Introduction

The planar 11-gon emerges as the canonical geometric structure for representing complete financial economic systems [1]. Its 11 vertices decompose as 8 asset/sector dimensions derived from the $\mathbb{C}^4$ hypermodel plus 3 organizational modes from constituent models, encoding the full dimensionality required for coherent economic modeling [2].

While the geometric and topological properties of the 11-gon have been extensively studied in prior work, the *combinatorial* properties—specifically the permutations and derangements of its vertices—have received less attention. Yet these combinatorial structures carry profound implications for understanding market reconfigurations, regime transitions, and structural risk.

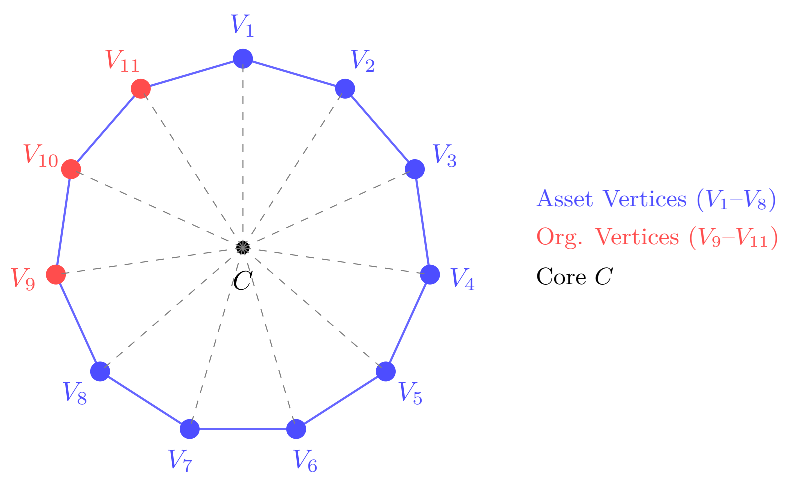

Figure 1: The planar 11-gon with 8 asset vertices (blue) from $\mathbb{C}^4$ and 3 organizational vertices (red). The central core C represents the information-processing entity.

This paper develops the combinatorial theory of the 11-gon framework, establishing:

(i) The complete enumeration of market configurations via permutations

(ii) The characterization of total regime transformations via derangements

(iii) Implications for arbitrage channel disruption and portfolio optimization

(iv) Connections to structural volatility and risk management

2 Combinatorial Foundations

2.1 Permutations of the 11-Gon

Definition 2.1 (Market Configuration)**.** *A* market configuration *is an assignment of economic functions to the 11 vertices of the hendecagon. The set of all possible configurations corresponds to the symmetric group $\mathfrak{S}_{11}$ acting on the vertex set $\{V_1, V_2, \ldots, V_{11}\}$.*

Proposition 2.2 (Configuration Space Cardinality)**.** *The total number of distinct market configurations is:*

$$|\mathfrak{S}_{11}| = 11! = 39{,}916{,}800 \tag{1}$$

Each permutation $\sigma \in \mathfrak{S}_{11}$ corresponds to a relabeling of vertices, fundamentally altering:

- Which dimensions are adjacent (affecting edge arbitrage)
- The diagonal structure (affecting cross-asset relationships)
- Information flow patterns from the core

2.2 Derangements as Complete Transformations

Definition 2.3 (Derangement)**.** *A* derangement *is a permutation $\sigma \in \mathfrak{S}_{11}$ with no fixed points, i.e., $\sigma(i) \neq i$ for all $i \in \{1, 2, \ldots, 11\}$. The set of all derangements is denoted $\mathcal{D}_{11}$.*

In financial terms, a derangement represents a market restructuring where *no dimension retains its original function*—a complete regime transformation [4].

Theorem 2.4 (Derangement Count)**.** *The number of derangements of 11 elements is given by:*

$$D_{11} = 11! \sum_{k=0}^{11} \frac{(-1)^k}{k!} = 14{,}684{,}570 \tag{2}$$

Proof. By the inclusion-exclusion principle, the number of derangements of n elements is:

$$D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!} \tag{3}$$

For $n = 11$, direct computation yields:

$$\begin{aligned} D_{11} &= 39{,}916{,}800 \times \left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \cdots + \frac{(-1)^{11}}{11!}\right) && (4) \\ &= 14{,}684{,}570 && (5) \end{aligned}$$

□

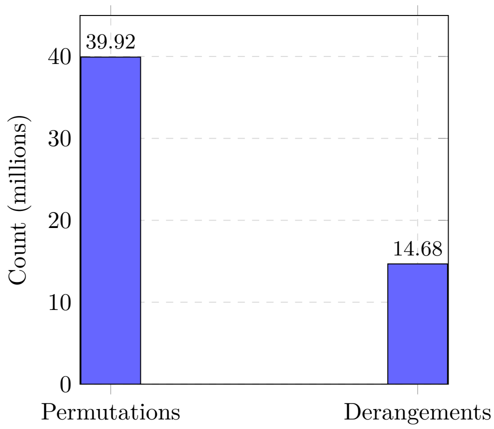

Figure 2: Comparison of total permutations (39.92 million) versus derangements (14.68 million) of the 11-gon vertices.

Corollary 2.5 (Derangement Probability). *The probability that a randomly selected permutation is a derangement approaches* $1/e$*:*

$$P(derangement) = \frac{D_{11}}{11!} = \frac{14{,}684{,}570}{39{,}916{,}800} \approx 0.3679 \approx \frac{1}{e} \tag{6}$$

This remarkable result implies that **over one-third of all possible market reconfigurations represent complete systemic transformations**.

3 Financial Implications

3.1 Arbitrage Channel Disruption

The 11-gon's arbitrage structure depends critically on vertex adjacencies [1]. The three types of arbitrage—edge, diagonal, and core-periphery—are fundamentally altered by permutations.

Definition 3.1 (Arbitrage Topology). *For the regular 11-gon:*

- ***Edge arbitrage channels****: 11 (between adjacent vertices)*
- ***Diagonal arbitrage channels****:* $\binom{11}{2} - 11 = 44$
- ***Core-periphery channels****: 11 (from center to each vertex)*

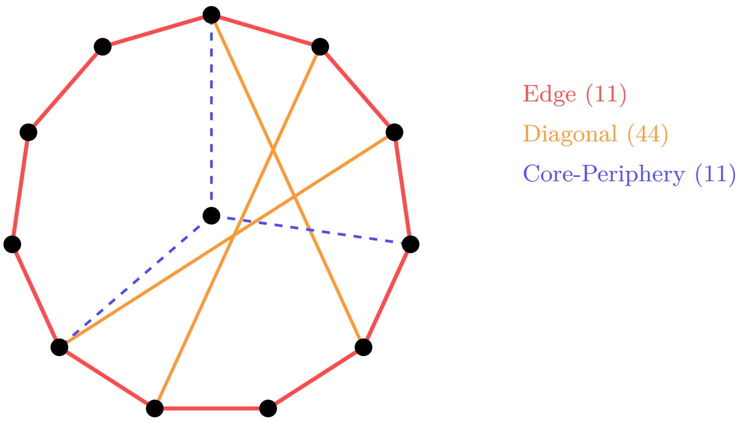

Figure 3: Three types of arbitrage channels on the 11-gon: edge arbitrage (red), diagonal arbitrage (orange), and core-periphery arbitrage (blue dashed).

Proposition 3.2 (Arbitrage Disruption under Derangement)**.** *Under a derangement $\sigma \in \mathcal{D}_{11}$:*

(a) All 11 edge arbitrage relationships are altered

(b) The diagonal structure is completely reorganized

(c) Core information flows are redirected to different economic functions

3.2 Symmetry Considerations

The regular 11-gon possesses dihedral symmetry D_{11}, comprising rotations and reflections.

Theorem 3.3 (Symmetry-Preserving Permutations)**.** *The number of permutations preserving the geometric structure of the regular 11-gon is:*

$$|D_{11}| = 2 \times 11 = 22 \tag{7}$$

consisting of 11 rotations and 11 reflections.

Corollary 3.4 (Symmetry Breaking Probability)**.** *The probability that a random permutation breaks geometric symmetry is:*

$$P(\textit{symmetry breaking}) = 1 - \frac{22}{39{,}916{,}800} \approx 0.99999945 \tag{8}$$

This implies that virtually all market reconfigurations **destroy the balanced architecture** assumed in the 11-gon framework—a crucial consideration for model robustness.

3.3 Organizational Vertex Analysis

The organizational vertices V_9, V_{10}, V_{11} represent the three constituent models: Geographic (7), Crystalline (8), and Nuclear (9) [2].

Proposition 3.5 (Organizational Derangements)**.** *Restricting to the organizational subspace $\{V_9, V_{10}, V_{11}\}$:*

$$\textit{Total permutations} = 3! = 6 \tag{9}$$
$$\textit{Derangements} = D_3 = 2 \tag{10}$$

The two organizational derangements are:

1. $(V_9 \to V_{10} \to V_{11} \to V_9)$: Forward cyclic permutation
2. $(V_9 \to V_{11} \to V_{10} \to V_9)$: Reverse cyclic permutation

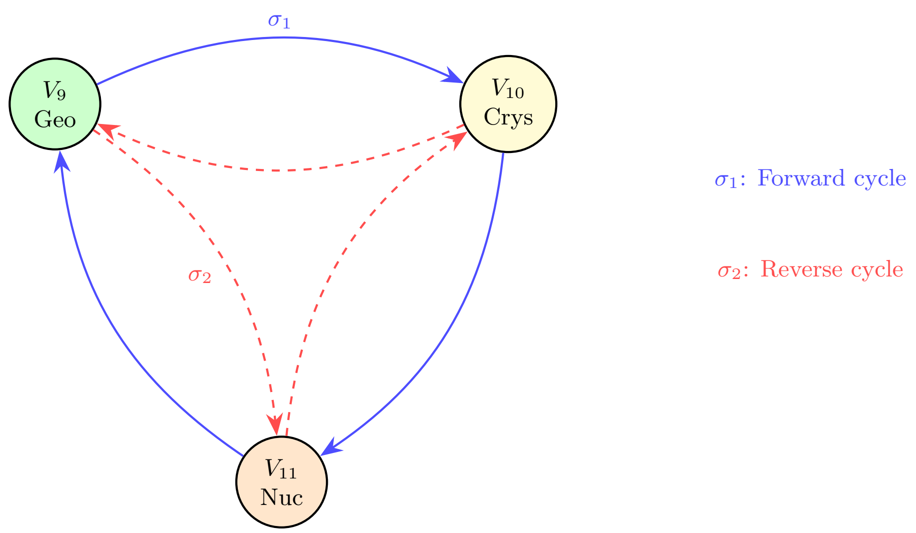

Figure 4: The two organizational derangements: forward cycle σ_1 (solid blue) and reverse cycle σ_2 (dashed red). Both represent complete governance transformations.

4 Volatility and Risk Implications

4.1 Structural Volatility Decomposition

The 11-gon framework decomposes volatility into three components [1]:

$$\sigma^2_{\text{total}} = \sigma^2_{\text{radial}} + \sigma^2_{\text{angular}} + \sigma^2_{\text{structural}} \tag{11}$$

Volatility Type	**Geometric Meaning**	**Permutation Impact**
Radial (σ_r)	Distance from center	Unchanged (preserved)
Angular (σ_θ)	Angular position	Dramatically altered
Structural (σ_s)	Regime transitions	Maximized in derangements

Table 1: Impact of permutations on the three volatility components.

Theorem 4.1 (Derangement Risk Maximization)**.** *A derangement event maximizes structural volatility σ_s because:*

(i) Every model assumption about dimensional relationships is invalidated

(ii) All historical correlations become unreliable

(iii) Portfolio optimization based on prior structure fails completely

4.2 Risk Management Framework

Definition 4.2 (Configuration Risk Metric)**.** *Define the* configuration risk *of a permutation σ as:*

$$\rho(\sigma) = \frac{|\{i : \sigma(i) \neq i\}|}{11} \tag{12}$$

measuring the fraction of dimensions undergoing functional change.

For derangements, $\rho(\sigma) = 1$ (maximum risk). The expected configuration risk over all permutations is:

$$\mathbb{E}[\rho] = 1 - \frac{1}{11} \approx 0.909 \tag{13}$$

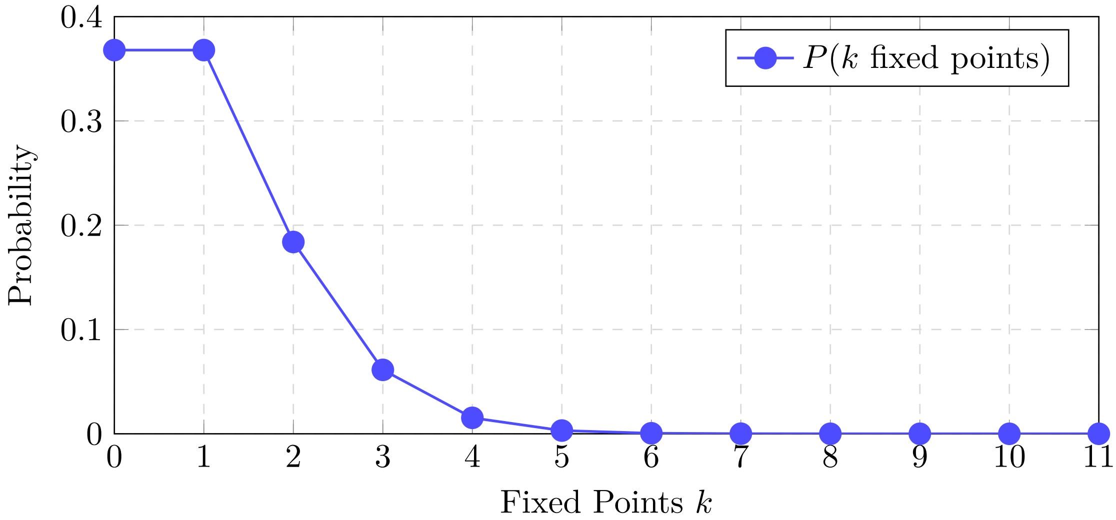

Figure 5: Distribution of fixed points in random permutations of 11 elements. The distribution approaches Poisson(1), with $P(k = 0) = P(k = 1) \approx 1/e$.

5 Portfolio Optimization under Permutation Uncertainty

5.1 Configuration-Robust Portfolios

Given the vast configuration space, portfolios must be robust to potential relabeling of dimensions.

Definition 5.1 (Permutation-Robust Portfolio)**.** *A portfolio* Π *is* permutation-robust *if its risk-return characteristics are invariant under a specified class of permutations* $\mathcal{P} \subseteq \mathfrak{S}_{11}$*.*

Proposition 5.2 (Symmetry-Invariant Portfolios)**.** *The only portfolio fully invariant under all* D_{11} *symmetry operations is the* equal-weighted portfolio*:*

$$\Pi_{sym} = \frac{1}{11}\sum_{j=1}^{11} V_j \tag{14}$$

located at the center C *of the 11-gon.*

5.2 Backtesting Considerations

The combinatorial analysis reveals fundamental limitations in backtesting:

Corollary 5.3 (Backtesting Fragility)**.** *Historical backtests face* 14,684,570 *scenarios (derangements) where past dimensional relationships* completely fail *to predict future behavior.*

This suggests that backtesting performance may be significantly overstated when structural regime changes are not adequately modeled.

6 Connections to Group Theory

6.1 The Symmetric Group Action

The symmetric group $\mathfrak{S}_{11}$ acts naturally on the 11-gon configuration space. Key subgroups include:

Subgroup	**Description**	**Order**
D_{11}	Dihedral (symmetry-preserving)	22
A_{11}	Alternating (even permutations)	19,958,400
$\mathfrak{S}_8 \times \mathfrak{S}_3$	Asset $\times$ Organizational	241,920
Cyclic C_{11}	Rotations only	11

Table 2: Key subgroups of $\mathfrak{S}_{11}$ and their orders.

6.2 Conjugacy Classes and Market Cycles

Permutations in the same conjugacy class share the same cycle structure. The number of conjugacy classes of $\mathfrak{S}_{11}$ equals the number of partitions of 11:

$$p(11) = 56 \tag{15}$$

Each conjugacy class represents a distinct *type* of market reconfiguration, characterized by the pattern of cyclic exchanges among dimensions.

7 Conclusion

The combinatorial analysis of permutations and derangements reveals fundamental properties of the 11-gon financial framework:

1. **Vast Configuration Space**: The 39,916,800 possible vertex permutations represent an enormous state space of market configurations, far exceeding what can be exhaustively analyzed.

2. **High Transformation Probability**: Over 36% of all reconfigurations (14,684,570 derangements) represent *complete* regime transformations where no economic dimension retains its original function.

3. **Near-Certain Symmetry Breaking**: With probability exceeding 0.999999, random reconfigurations destroy the geometric balance of the 11-gon architecture.

4. **Structural Risk Dominance**: Derangement events maximize structural volatility, potentially invalidating all model assumptions simultaneously.

5. **Limited Robustness**: Only the equal-weighted central portfolio achieves full invariance under symmetry operations.

These findings suggest that while the 11-gon provides a complete dimensional framework for financial economics, its very completeness admits combinatorial complexity that poses fundamental challenges for prediction, optimization, and risk management. Future research should develop methods for identifying early signals of regime-changing permutations and constructing portfolios robust to the most damaging classes of structural transformations.

References

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Glossary

11-Gon (Hendecagon)
A regular polygon with 11 vertices symmetrically arranged on a circle, representing the complete dimensional structure of financial economic systems: 8 dimensions from $\mathbb{C}^4$ plus 3 organizational modes.

Alternating Group A_n
The subgroup of even permutations in $\mathfrak{S}_n$, consisting of permutations expressible as products of an even number of transpositions. For $n = 11$, $|A_{11}| = 19{,}958{,}400$.

Arbitrage Channel
A trading pathway between vertices of the 11-gon enabling exploitation of price discrepancies. Types include edge (adjacent), diagonal (non-adjacent), and core-periphery arbitrage.

Configuration Risk $\rho(\sigma)$
A metric measuring the fraction of dimensions undergoing functional change under permutation σ, defined as $\rho(\sigma) = |\{i : \sigma(i) \neq i\}|/11$.

Configuration Space
The set of all possible assignments of economic functions to the 11-gon's vertices, equivalent to the symmetric group $\mathfrak{S}_{11}$ with 39,916,800 elements.

Conjugacy Class
A subset of permutations sharing the same cycle structure. Permutations in the same conjugacy class represent the same "type" of market reconfiguration.

Cycle Structure
The decomposition of a permutation into disjoint cycles. For example, (1 3 5)(2 4)(6) has cycle type $(3, 2, 1)$.

Derangement
A permutation with no fixed points; every element is moved to a different position. Represents complete market regime transformation in the 11-gon framework.

Dihedral Group D_n
The symmetry group of a regular n-gon, consisting of n rotations and n reflections. For the 11-gon, $|D_{11}| = 22$.

Edge Arbitrage
Arbitrage exploiting inefficiencies between adjacent vertices on the 11-gon perimeter, typically involving position-momentum misalignment within an asset class.

Fixed Point
An element i such that $\sigma(i) = i$ under permutation σ. A dimension that retains its original economic function after reconfiguration.

Inclusion-Exclusion Principle
A counting technique used to derive the derangement formula: $D_n = n! \sum_{k=0}^{n} (-1)^k/k!$.

Organizational Vertices (V_9–V_{11})
The three vertices representing structural modes: Geographic (7-constituent), Crystalline (8-constituent), and Nuclear (9-constituent) models.

Permutation
A bijective function from a set to itself; in the 11-gon context, a reassignment of economic functions to vertices.

Permutation-Robust Portfolio
A portfolio whose risk-return characteristics remain stable under specified classes of vertex permutations.

Structural Volatility σ_s
Variance arising from transitions between organizational modes, representing the risk that fundamental economic structure changes.

Symmetric Group $\mathfrak{S}_n$
The group of all permutations of n elements. For $n = 11$, contains $11! = 39{,}916{,}800$ elements.

Symmetry-Preserving Permutation
A permutation that maintains the geometric structure of the regular 11-gon; an element of the dihedral group D_{11}.

Transposition
A permutation exchanging exactly two elements. Every permutation can be expressed as a product of transpositions.

The End