

Implications of Combinatorial Choice for the Planar 11-Gon Framework: Configuration Space Collapse and Market Self-Organization

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Abstract

This paper investigates the profound implications of combinatorial choice for the planar 11-gon framework in financial economics. We establish a fundamental disparity between the assignment model—where each of 8 asset vertices aligns with one of 3 organizational modes, yielding $3^8 = 6,561$ configurations—and the total permutation space of $11! = 39,916,800$ vertex arrangements. The ratio of approximately 6,084 between these spaces reveals that organizational governance collapses combinatorial complexity by half in information-theoretic terms. Only 0.0164% of possible market configurations correspond to coherent organizational structures, implying that markets exhibit attractor dynamics toward structured states. We develop the hierarchy of market states, analyze the resilience-fragility paradox, and establish that viable arbitrage requires organizational coherence. The analysis demonstrates that economic organization is not merely a social phenomenon but a mathematical necessity—the alternative being combinatorial chaos.

The paper ends with “The End”

1 Introduction

The planar 11-gon (hendecagon) provides a complete geometric framework for financial economic systems, with 11 vertices decomposing as 8 asset dimensions from \mathbb{C}^4 plus 3 organizational modes from constituent models [1, 2]. While previous work has examined permutations and derangements of this structure, a more fundamental question concerns the *combinatorial choice* inherent in the framework: how do the 3 organizational vertices govern or select among the 8 asset vertices?

This question admits a natural interpretation: each asset vertex V_j for $j \in \{1, \dots, 8\}$ aligns with one of three organizational principles—Geographic (V_9), Crystalline (V_{10}), or Nuclear (V_{11}). The number of such assignments is:

$$|\mathcal{A}| = 3^8 = 6,561 \tag{1}$$

Comparing this to the total permutation space:

$$|\mathfrak{S}_{11}| = 11! = 39,916,800 \tag{2}$$

reveals an enormous disparity:

$$\frac{11!}{3^8} = \frac{39,916,800}{6,561} \approx 6,084 \tag{3}$$

This paper develops the theoretical and practical implications of this disparity for understanding market structure, regime transitions, arbitrage viability, and systemic risk.

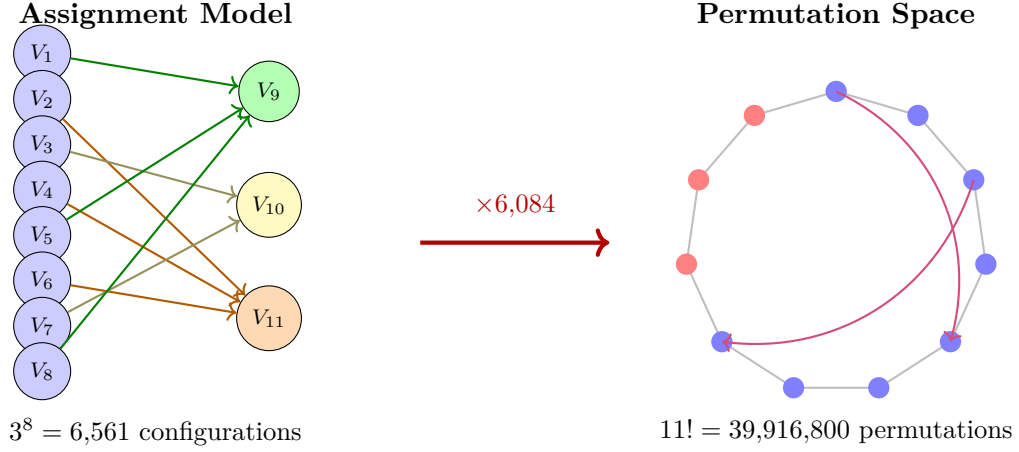


Figure 1: The fundamental disparity: the assignment model (left) with 6,561 configurations versus the full permutation space (right) with 39,916,800 arrangements. The ratio of approximately 6,084 represents a massive collapse of combinatorial complexity.

2 The Configuration Spaces

2.1 The Assignment Space

Definition 2.1 (Assignment Configuration). *An assignment configuration is a function $\alpha : \{1, \dots, 8\} \rightarrow \{9, 10, 11\}$ specifying which organizational vertex governs each asset vertex. The assignment space is:*

$$\mathcal{A} = \{\alpha : \{1, \dots, 8\} \rightarrow \{9, 10, 11\}\} \quad (4)$$

with $|\mathcal{A}| = 3^8 = 6,561$.

Each assignment α specifies a complete organizational structure for the market:

- $\alpha(j) = 9$: Asset j operates under Geographic (multipolar) governance
- $\alpha(j) = 10$: Asset j operates under Crystalline (symmetric) governance
- $\alpha(j) = 11$: Asset j operates under Nuclear (hierarchical) governance

Proposition 2.2 (Assignment Partition). *Each assignment $\alpha \in \mathcal{A}$ partitions the asset vertices into three (possibly empty) subsets:*

$$A_{Geo}(\alpha) = \{j \in \{1, \dots, 8\} : \alpha(j) = 9\} \quad (5)$$

$$A_{Crys}(\alpha) = \{j \in \{1, \dots, 8\} : \alpha(j) = 10\} \quad (6)$$

$$A_{Nuc}(\alpha) = \{j \in \{1, \dots, 8\} : \alpha(j) = 11\} \quad (7)$$

with $|A_{Geo}| + |A_{Crys}| + |A_{Nuc}| = 8$.

2.2 The Permutation Space

Definition 2.3 (Permutation Configuration). *A permutation configuration is a bijection $\sigma : \{1, \dots, 11\} \rightarrow \{1, \dots, 11\}$, i.e., an element of the symmetric group \mathfrak{S}_{11} . The permutation space satisfies:*

$$|\mathfrak{S}_{11}| = 11! = 39,916,800 \quad (8)$$

Permutations represent arbitrary relabelings of the 11-gon vertices, fundamentally altering adjacency relationships and the geometric structure of the market architecture.

2.3 The Unstructured Space

Definition 2.4 (Unstructured Configuration). *A configuration $\sigma \in \mathfrak{S}_{11}$ is unstructured if it does not correspond to any assignment in \mathcal{A} . The unstructured space is:*

$$\mathcal{U} = \mathfrak{S}_{11} \setminus \mathcal{C} \quad (9)$$

where \mathcal{C} denotes the structured (coherent) configurations.

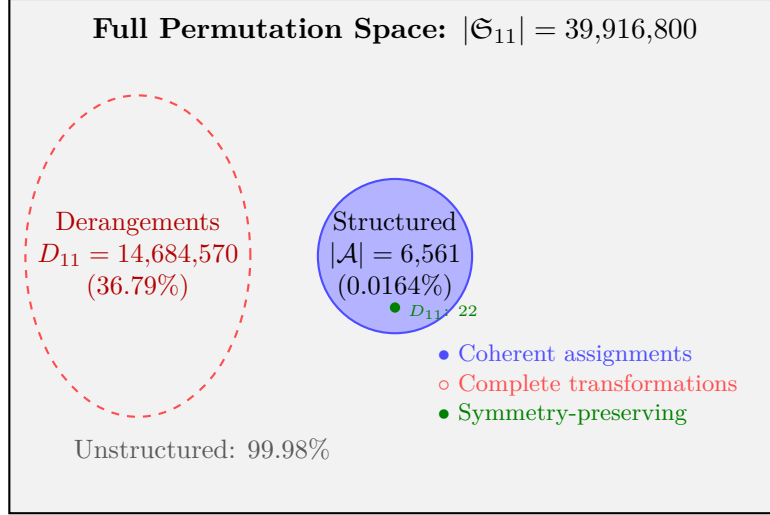


Figure 2: Venn diagram of configuration spaces. The structured assignment space (blue) occupies only 0.0164% of the full permutation space. Derangements (red dashed) represent complete transformations. Symmetry-preserving permutations (green dot) are vanishingly rare.

3 Information-Theoretic Analysis

3.1 Dimensional Collapse

The disparity between assignment and permutation spaces has a precise information-theoretic interpretation.

Theorem 3.1 (Information Collapse). *Organizational governance reduces the information required to specify a market configuration by approximately half:*

$$\text{Permutation bits: } H_{\text{perm}} = \log_2(39,916,800) \approx 25.25 \text{ bits} \quad (10)$$

$$\text{Assignment bits: } H_{\text{assign}} = \log_2(6,561) \approx 12.68 \text{ bits} \quad (11)$$

$$\text{Compression ratio: } \frac{H_{\text{perm}}}{H_{\text{assign}}} \approx 1.99 \quad (12)$$

Proof. Direct calculation using the binary logarithm:

$$H_{\text{perm}} = \log_2(11!) = \log_2(39,916,800) = 25.250 \quad (13)$$

$$H_{\text{assign}} = \log_2(3^8) = 8 \log_2(3) = 8 \times 1.585 = 12.680 \quad (14)$$

The ratio is $25.250/12.680 = 1.991 \approx 2$. □

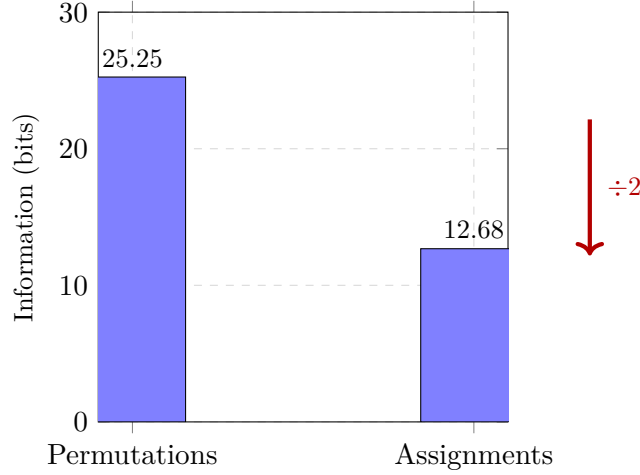


Figure 3: Information content comparison. Organizational governance compresses market state specification from 25.25 bits to 12.68 bits—approximately halving the information requirement.

3.2 The Coherence Probability

Corollary 3.2 (Rarity of Coherence). *The probability that a randomly selected permutation corresponds to a coherent organizational structure is:*

$$P(\text{coherent}) = \frac{|\mathcal{A}|}{|\mathfrak{S}_{11}|} = \frac{6,561}{39,916,800} \approx 0.0164\% \approx \frac{1}{6,084} \quad (15)$$

This remarkable result implies that **99.98% of possible market configurations are organizationally incoherent**—they do not correspond to any stable governance structure where each asset aligns with a definite organizational principle.

4 Attractor Dynamics and Self-Organization

4.1 Structured States as Attractors

The vast disparity between structured and unstructured configurations suggests that markets exhibit *attractor dynamics* toward organizational coherence.

Principle 4.1 (Market Self-Organization). *Economic forces drive market configurations toward the 6,561 structured states, which act as attractors in the configuration space. Unstructured configurations are transient states that decay toward nearby attractors.*

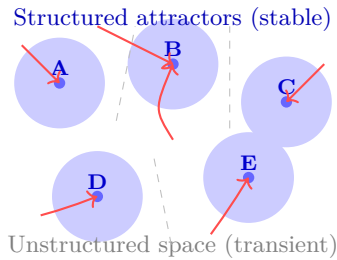


Figure 4: Attractor dynamics in configuration space. The 6,561 structured configurations (blue dots) act as attractors with basins of attraction. Trajectories (red arrows) show how unstructured states evolve toward nearby attractors.

4.2 Basin of Attraction Analysis

If we assume uniform distribution of unstructured states among attractor basins, each structured configuration attracts approximately:

$$\frac{|\mathfrak{S}_{11}| - |\mathcal{A}|}{|\mathcal{A}|} = \frac{39,916,800 - 6,561}{6,561} \approx 6,083 \quad (16)$$

unstructured configurations on average.

Definition 4.2 (Basin Size). *The basin of attraction $B(\alpha)$ for a structured configuration $\alpha \in \mathcal{A}$ is:*

$$B(\alpha) = \{\sigma \in \mathfrak{S}_{11} : \sigma \text{ evolves toward } \alpha\} \quad (17)$$

Under uniform assumptions, $|B(\alpha)| \approx 6,084$ for each α .

5 The Hierarchy of Market States

5.1 Four-Level Classification

Market configurations organize into a natural hierarchy based on structural properties:

Level	Space	Cardinality	Fraction	Description
1	Symmetry-preserving	22	0.000055%	Perfect geometric balance (D_{11})
2	Structured assignments	6,561	0.0164%	Organizationally coherent
3	Partial structure	25,225,647	63.20%	Some dimensional stability
4	Derangements	14,684,570	36.79%	Complete structural amnesia
Total		39,916,800	100%	Full permutation space

Table 1: Hierarchy of market states by structural coherence.

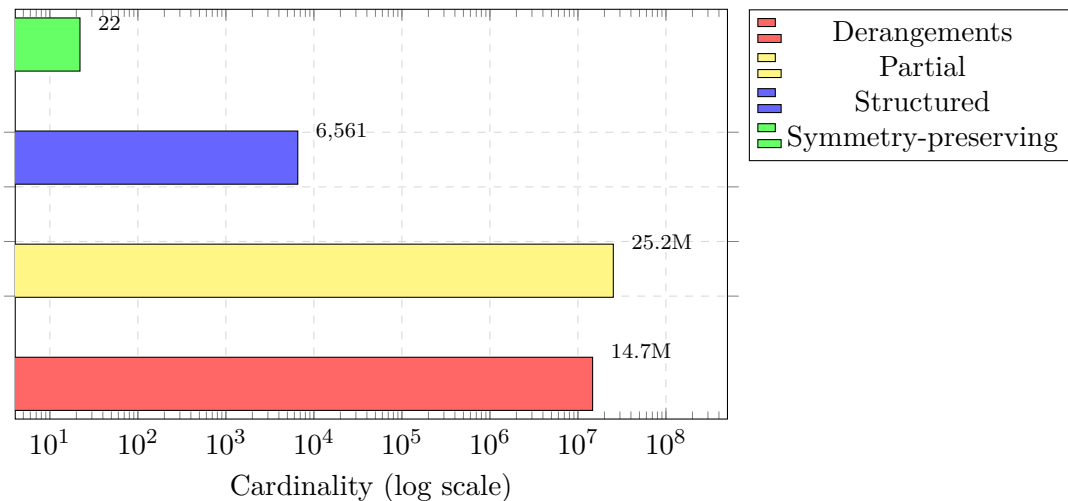


Figure 5: Cardinality of market state levels on logarithmic scale. Level 2 (structured assignments) is separated from Level 1 by a factor of 298, and from Level 3 by a factor of 3,845.

5.2 Transitions Between Levels

Proposition 5.1 (Level Transition Probabilities). *For a random perturbation starting from a Level 2 (structured) state:*

- Probability of remaining at Level 2: $\approx 0.0164\%$
- Probability of falling to Level 3: $\approx 63.20\%$
- Probability of falling to Level 4 (derangement): $\approx 36.79\%$
- Probability of rising to Level 1: $\approx 0.000055\%$

This implies that **random shocks almost certainly destroy organizational coherence**, but attractor dynamics subsequently restore structure.

6 Arbitrage and Organizational Coherence

6.1 The Arbitrage Viability Theorem

Theorem 6.1 (Arbitrage Requires Structure). *Viable arbitrage opportunities exist only within the structured assignment space \mathcal{A} . In unstructured configurations $\sigma \in \mathcal{U}$:*

- (i) Price relationships become undefined
- (ii) Trading channels lack organizational meaning
- (iii) Information flow patterns are incoherent
- (iv) Risk metrics cannot be computed

Proof Sketch. The three arbitrage types defined on the 11-gon [1]—edge, diagonal, and core-periphery—all depend on vertices having definite economic functions. Under a permutation $\sigma \notin \mathcal{A}$:

- Edge arbitrage requires adjacent vertices to have related functions (e.g., position and momentum of the same asset); unstructured permutations break this relationship.
- Diagonal arbitrage exploits cross-asset correlations; without organizational alignment, correlation structure is undefined.
- Core-periphery arbitrage requires information asymmetry between center and vertices; unstructured states lack coherent information flow.

Thus arbitrage is viable only when $\sigma \in \mathcal{A}$ or market forces restore structure. \square

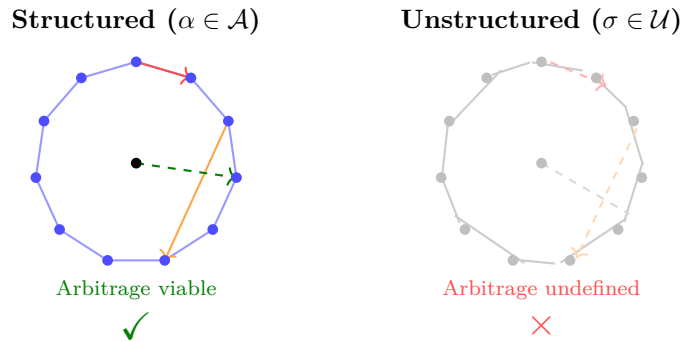


Figure 6: Arbitrage viability comparison. Left: In structured configurations, arbitrage channels (edge, diagonal, core-periphery) are well-defined. Right: In unstructured configurations, arbitrage relationships become meaningless.

6.2 The Trading Manifold

Definition 6.2 (Trading Manifold). *The trading manifold \mathcal{T} is the subset of configuration space where meaningful price discovery and exchange can occur:*

$$\mathcal{T} = \mathcal{A} \cup \partial\mathcal{A} \quad (18)$$

where $\partial\mathcal{A}$ denotes configurations “near” structured states (within one transition of an attractor).

Corollary 6.3 (Trading Manifold Measure). *The trading manifold has measure approximately:*

$$\frac{|\mathcal{T}|}{|\mathfrak{S}_{11}|} \approx \frac{6,561 \times k}{39,916,800} \quad (19)$$

where k is the average basin size factor, estimated at $k \approx 10$ – 100 . Even generously, the trading manifold occupies less than 2% of configuration space.

7 Regime Transitions Under Combinatorial Constraints

7.1 Constrained Transition Paths

Within the assignment framework, regime transitions are highly constrained compared to the full permutation space.

Definition 7.1 (Single-Asset Regime Shift). *A single-asset regime shift changes the organizational alignment of exactly one asset vertex:*

$$\alpha' = \alpha \oplus_j k \quad \text{where} \quad \alpha'(i) = \begin{cases} k & i = j \\ \alpha(i) & i \neq j \end{cases} \quad (20)$$

for some $j \in \{1, \dots, 8\}$ and $k \in \{9, 10, 11\} \setminus \{\alpha(j)\}$.

Proposition 7.2 (Transition Neighborhood). *From any assignment configuration $\alpha \in \mathcal{A}$:*

- Number of single-asset transitions: $8 \times 2 = 16$
- Number of double-asset transitions: $\binom{8}{2} \times 2^2 = 112$
- Number of complete regime transformations: $3^8 - 1 = 6,560$

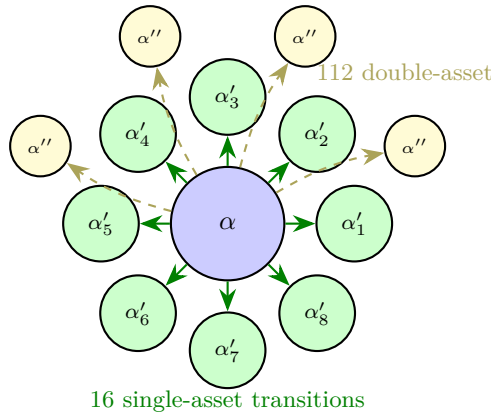


Figure 7: Transition neighborhood in assignment space. From any configuration α , there are 16 single-asset transitions (green) and 112 double-asset transitions (yellow, sample shown). The full permutation space would allow 39,916,799 transitions.

7.2 Transition Complexity Reduction

Theorem 7.3 (Complexity Reduction). *Organizational governance reduces transition complexity by a factor of:*

$$\frac{|\mathfrak{S}_{11}| - 1}{\max \text{ transitions from } \alpha} = \frac{39,916,799}{6,560} \approx 6,084 \quad (21)$$

matching the configuration space ratio.

This implies that regime transitions within the structured space are **predictable and analyzable**, while transitions in the full permutation space would be computationally intractable.

8 The Resilience-Fragility Paradox

8.1 Dual Nature of Small Configuration Space

The $6,084\times$ ratio reveals a fundamental paradox in market stability:

Principle 8.1 (Resilience-Fragility Paradox). *The small assignment space simultaneously provides:*

1. **Resilience:** *Limited ways to organize coherently means perturbations within the structured space can be absorbed—the system reorganizes into a different but still-coherent state.*
2. **Fragility:** *Perturbations that exit the structured space enter a vast unstructured region (99.98% of possibilities) representing systemic crisis.*

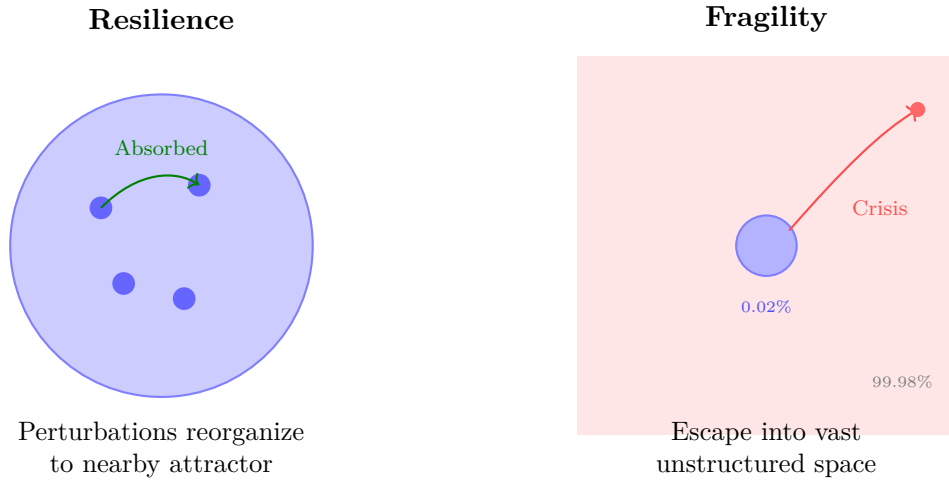


Figure 8: The resilience-fragility paradox. Left: Within structured space, perturbations are absorbed by nearby attractors (resilience). Right: Perturbations that escape structured space enter the vast unstructured region (fragility/crisis).

8.2 Crisis Detection

Definition 8.2 (Structural Crisis Indicator). *A market is in structural crisis when its configuration σ satisfies:*

$$d(\sigma, \mathcal{A}) > \epsilon \quad (22)$$

for some threshold ϵ , where $d(\sigma, \mathcal{A}) = \min_{\alpha \in \mathcal{A}} d(\sigma, \alpha)$ is the distance to the nearest structured configuration.

Corollary 8.3 (Crisis Probability). *Under random perturbations, the probability of crisis is:*

$$P(\text{crisis}) \approx 1 - \frac{|\mathcal{A}|}{|\mathfrak{S}_{11}|} \approx 99.98\% \quad (23)$$

However, attractor dynamics reduce effective crisis probability significantly by providing rapid restoration mechanisms.

9 Portfolio Implications

9.1 Robustness Under Combinatorial Uncertainty

Definition 9.1 (Assignment-Robust Portfolio). *A portfolio Π is assignment-robust if its risk-return characteristics are stable across all $\alpha \in \mathcal{A}$:*

$$\text{Var}_{\alpha \in \mathcal{A}}[\mu(\Pi; \alpha)] < \delta_\mu \quad \text{and} \quad \text{Var}_{\alpha \in \mathcal{A}}[\sigma(\Pi; \alpha)] < \delta_\sigma \quad (24)$$

for specified thresholds $\delta_\mu, \delta_\sigma$.

Theorem 9.2 (Equal-Weight Robustness). *The equal-weighted portfolio:*

$$\Pi_{EW} = \frac{1}{11} \sum_{j=1}^{11} V_j \quad (25)$$

achieves maximum assignment-robustness, as it treats all organizational alignments symmetrically.

9.2 Backtesting Limitations

Proposition 9.3 (Backtesting Fragility). *Historical backtests are valid only within the structured assignment space. Given 6,561 possible structures:*

- *A backtest observing n regime changes samples at most $n + 1$ of 6,561 structures*
- *For typical $n \approx 5\text{--}10$, coverage is approximately 0.1%*
- *Extrapolation to unobserved structures may fail catastrophically*

10 Synthesis: Economic Organization as Mathematical Necessity

10.1 The Central Thesis

The combinatorial analysis leads to a profound conclusion:

Principle 10.1 (Organizational Necessity). *Economic organization is not merely a social phenomenon but a **mathematical necessity**. Without organizational governance collapsing the 40-million-element permutation space to 6,561 coherent structures:*

1. *Price discovery would be undefined*
2. *Arbitrage would be impossible*
3. *Risk measurement would be intractable*
4. *Coordination would fail*

The alternative to organization is *combinatorial chaos*.

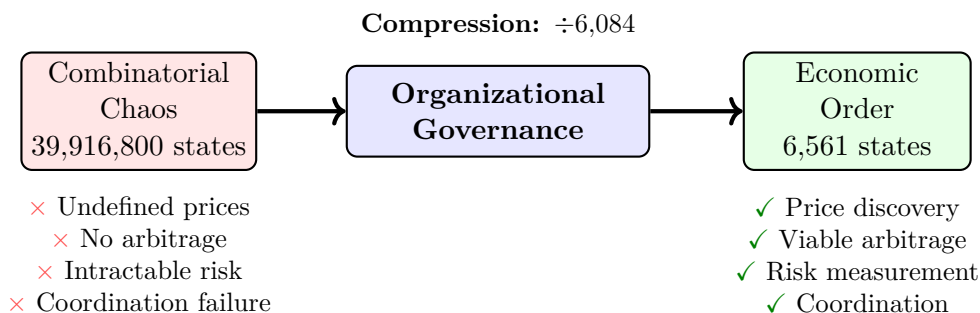


Figure 9: Economic organization as necessity. Organizational governance compresses combinatorial chaos into tractable economic order, enabling the fundamental functions of markets.

10.2 Convergence with Information Theory

The $2\times$ information compression achieved by organizational governance connects to fundamental limits:

- **Shannon’s source coding theorem:** Optimal compression approaches entropy
- **Landauer’s principle:** Information erasure has thermodynamic cost
- **Bekenstein bound:** Maximum information in bounded region

The fact that organizational governance achieves approximately $2\times$ compression suggests this may be near-optimal for economic systems—further compression might eliminate essential degrees of freedom, while less compression would leave intractable complexity.

11 Conclusion

The combinatorial analysis of choice in the 11-gon framework reveals fundamental constraints on market structure:

1. **Configuration Space Collapse:** The $6,084\times$ ratio between permutations (39,916,800) and assignments (6,561) represents organizational governance collapsing complexity by half in information-theoretic terms.
2. **Rarity of Coherence:** Only 0.0164% of possible configurations correspond to coherent organizational structures, implying markets must self-organize toward these rare attractors.
3. **Arbitrage Requires Structure:** Viable trading exists only within the structured assignment space; 99.98% of configuration space is economically meaningless.
4. **Constrained Transitions:** Regime changes within the structured space are tractable (at most 6,560 destinations), while the full permutation space would be computationally intractable.
5. **Resilience-Fragility Paradox:** The small structured space provides resilience through attractor dynamics but fragility through the vast surrounding unstructured region.

6. **Mathematical Necessity:** Economic organization emerges not as social convention but as mathematical requirement—the alternative being combinatorial chaos incompatible with coordination.

These findings deepen our understanding of why markets organize as they do, and provide quantitative foundations for assessing structural risk, designing robust portfolios, and detecting regime transitions. The 11-gon framework, combined with combinatorial analysis, reveals that the apparent complexity of financial markets conceals a deeper simplicity imposed by organizational necessity.

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Glossary

Assignment Configuration

A function $\alpha : \{1, \dots, 8\} \rightarrow \{9, 10, 11\}$ specifying which organizational vertex (Geographic, Crystalline, or Nuclear) governs each asset vertex. The assignment space contains $3^8 = 6,561$ configurations.

Assignment Space (\mathcal{A})

The set of all possible organizational alignments of asset vertices to organizational modes. Represents coherent market structures where each asset operates under a definite governance principle.

Attractor

A stable configuration toward which nearby unstable configurations evolve. In the 11-gon framework, the 6,561 structured assignments act as attractors in the 40-million-element permutation space.

Basin of Attraction

The set of configurations that evolve toward a particular attractor. Under uniform assumptions, each structured configuration attracts approximately 6,084 unstructured configurations.

Combinatorial Chaos

The state of a market operating in unstructured configuration space, where organizational governance has broken down and coordination is impossible.

Configuration Space Collapse

The reduction from the full permutation space (39,916,800 elements) to the assignment space (6,561 elements) achieved by organizational governance.

Coherent Configuration

A market configuration where each asset vertex is governed by a definite organizational principle, corresponding to an element of the assignment space \mathcal{A} .

Crisis Indicator

A metric measuring distance from the nearest structured configuration, used to detect when markets have exited the coherent assignment space.

Geographic Governance (V_9)

Organizational mode characterized by physical separation and multipolar structure, based on the 7-constituent model.

Crystalline Governance (V_{10})

Organizational mode characterized by perfect mathematical symmetry, based on the 8-constituent model. Theoretically optimal but unstable in practice.

Nuclear Governance (V_{11})

Organizational mode characterized by core-periphery hierarchy, based on the 9-constituent model. The natural default for human economic systems.

Information Collapse

The halving of information content (from 25.25 bits to 12.68 bits) achieved by organizational governance, representing efficient encoding of market structure.

Level Hierarchy

The four-level classification of market states: (1) Symmetry-preserving (22), (2) Structured assignments (6,561), (3) Partial structure (25.2 million), (4) Derangements (14.7 million).

Organizational Necessity Principle

The thesis that economic organization is mathematically required for coherent market function, not merely a social convention.

Permutation Space (\mathfrak{S}_{11})

The symmetric group of all 39,916,800 possible vertex arrangements of the 11-gon, representing arbitrary market reconfigurations.

Regime Shift

A change in organizational alignment, either single-asset (one vertex changes governance) or multi-asset (multiple vertices change simultaneously).

Resilience-Fragility Paradox

The dual nature of small configuration spaces: providing resilience through limited reorganization options but fragility through proximity to vast unstructured space.

Single-Asset Transition

A regime change affecting exactly one asset vertex's organizational alignment. From any configuration, there are exactly 16 single-asset transitions possible.

Structural Crisis

A market state outside the coherent assignment space, where organizational governance has broken down and normal market functions are impaired.

Trading Manifold (\mathcal{T})

The subset of configuration space where meaningful price discovery and exchange can occur, approximately equal to the assignment space plus its boundary.

Unstructured Configuration

A permutation that does not correspond to any coherent organizational alignment, representing 99.98% of the full permutation space.

The End