Classifying Economic Risk by Industries and Firms in Each Industry

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Abstract

This paper develops a comprehensive theoretical framework for classifying and measuring economic risk through a hierarchical decomposition across industries and firms. We propose a multi-level risk aggregation model that captures both idiosyncratic firm-level risks and systematic industry risks, incorporating correlation structures and diversification effects. Our framework provides closed-form expressions for industry-level and economy-wide risk measures under various correlation assumptions, and establishes conditions under which diversification reduces aggregate risk. The model integrates insights from portfolio theory, industrial organization, and macroeconomic risk analysis to provide a unified approach to economic risk classification.

The paper ends with "The End"

1 Introduction

The measurement and classification of economic risk represents a fundamental challenge in economics and finance. While traditional approaches focus either on aggregate macroeconomic risk or firm-specific microeconomic risk, there exists a critical intermediate layer: industry-level risk that reflects both the systematic exposures of sectors and the aggregation of heterogeneous firm risks within those sectors.

This paper develops a theoretical framework that explicitly models the hierarchical structure of economic risk, decomposing economy-wide risk into industry components and further into firm-level elements. Our approach builds on classical portfolio theory [12], systemic risk literature [1], and industrial organization theory [17] to provide a comprehensive risk classification system.

The key contributions of this paper are threefold. First, we provide rigorous mathematical formulations for aggregating firm-level risks into industry measures and industry risks into economy-wide metrics. Second, we characterize the role of correlation structures in determining diversification benefits across different hierarchical levels. Third, we derive conditions under which industry classification provides meaningful risk reduction compared to firm-level or economy-wide analysis.

2 The Basic Framework

2.1 Economic Structure

Consider an economy consisting of n distinct industries, indexed by $i \in \{1, 2, \ldots, n\}$. Each industry I_i contains $F_i \geq 1$ firms, indexed by $j \in \{1, 2, \ldots, F_i\}$.

Definition 1 (Firm-Level Risk-Return Profile)**.** *For firm j in industry i, define:*

- r_{ij}*: expected return of firm* (i, j)
- R_{ij}*: risk (standard deviation) of firm* (i, j)
- $\sigma_{ij}^2 = R_{ij}^2$*: variance of firm* (i, j)

The total number of firms in the economy is $N = \sum_{i=1}^{n} F_i$.

2.2 Vector Representation

For computational convenience, we can represent the risk structure in vector notation. For industry i, define:

$$\mathbf{r}_i = (r_{i1}, r_{i2}, \ldots, r_{iF_i})^\top \in \mathbb{R}^{F_i} \tag{1}$$

$$\mathbf{R}_i = (R_{i1}, R_{i2}, \ldots, R_{iF_i})^\top \in \mathbb{R}^{F_i} \tag{2}$$

For the entire economy:

$$\mathbf{r} = (\mathbf{r}_1^\top, \mathbf{r}_2^\top, \ldots, \mathbf{r}_n^\top)^\top \in \mathbb{R}^N \tag{3}$$

3 Industry-Level Risk Aggregation

3.1 General Formulation

Let w_{ij} denote the weight of firm j in industry i, satisfying $\sum_{j=1}^{F_i} w_{ij} = 1$ and $w_{ij} \geq 0$ for all j.

Definition 2 (Industry Return and Risk). *The industry-level return and risk are:*

$$r_i = \sum_{j=1}^{F_i} w_{ij} r_{ij} \tag{4}$$

$$\sigma_i^2 = \sum_{j=1}^{F_i} \sum_{k=1}^{F_i} w_{ij} w_{ik} Cov(R_{ij}, R_{ik}) \tag{5}$$

where $R_i = \sqrt{\sigma_i^2}$ is the industry risk.

3.2 Correlation Structure

Define the correlation coefficient between firms j and k in industry i as:

$$\rho_{ijk} = \frac{\mathrm{Cov}(R_{ij}, R_{ik})}{R_{ij} R_{ik}} \tag{6}$$

Then:

$$\sigma_i^2 = \sum_{j=1}^{F_i} \sum_{k=1}^{F_i} w_{ij} w_{ik} \rho_{ijk} R_{ij} R_{ik} \tag{7}$$

3.3 Special Cases

Proposition 1 (Independent Firms). *If firms within industry i are independent ($\rho_{ijk} = 0$ for $j \neq k$), then:*

$$R_i = \sqrt{\sum_{j=1}^{F_i} w_{ij}^2 R_{ij}^2} \tag{8}$$

Proposition 2 (Perfectly Correlated Firms). *If all firms in industry i are perfectly correlated ($\rho_{ijk} = 1$ for all j, k), then:*

$$R_i = \sum_{j=1}^{F_i} w_{ij} R_{ij} \tag{9}$$

Proposition 3 (Uniform Correlation). *If $\rho_{ijk} = \rho_i$ for all $j \neq k$ and $w_{ij} = 1/F_i$ (equal weights), then:*

$$R_i = \frac{1}{F_i} \sqrt{\sum_{j=1}^{F_i} R_{ij}^2 + \rho_i \sum_{j=1}^{F_i} \sum_{k \neq j} R_{ij} R_{ik}} \tag{10}$$

3.4 Diversification Benefits Within Industries

Theorem 4 (Industry Diversification Effect)**.** *Under equal weights and uniform correlation $\rho_i < 1$, as $F_i \to \infty$:*

$$\lim_{F_i \to \infty} R_i = \sqrt{\rho_i} \cdot \bar{R}_i \tag{11}$$

where $\bar{R}_i = \lim_{F_i \to \infty} \frac{1}{F_i} \sum_{j=1}^{F_i} R_{ij}$ is the average firm risk.

This theorem demonstrates that diversification within an industry eliminates all idiosyncratic risk, leaving only the systematic component $\sqrt{\rho_i} \cdot \bar{R}_i$.

4 Economy-Wide Risk Aggregation

4.1 General Framework

Let ω_i denote the weight of industry i in the economy, where $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \geq 0$.

Definition 3 (Economy-Wide Return and Risk)**.** *The aggregate economic return and risk are:*

$$r_E = \sum_{i=1}^{n} \omega_i r_i \tag{12}$$

$$\sigma_E^2 = \sum_{i=1}^{n} \sum_{k=1}^{n} \omega_i \omega_k Cov(r_i, r_k) \tag{13}$$

where $R_E = \sqrt{\sigma_E^2}$ is the economy-wide risk.

4.2 Inter-Industry Correlation

Define the correlation between industries i and k as:

$$\rho_{ik}^{IND} = \frac{\text{Cov}(r_i, r_k)}{R_i R_k} \tag{14}$$

Then:

$$R_E = \sqrt{\sum_{i=1}^{n} \sum_{k=1}^{n} \omega_i \omega_k \rho_{ik}^{IND} R_i R_k} \tag{15}$$

4.3 Two-Level Decomposition

The economy-wide risk can be decomposed into within-industry and between-industry components.

Theorem 5 (Risk Decomposition)**.** *The total economic variance can be written as:*

$$\sigma_E^2 = \underbrace{\sum_{i=1}^{n} \omega_i^2 \sigma_i^2}_{\text{Within-industry}} + \underbrace{\sum_{i=1}^{n} \sum_{k \neq i} \omega_i \omega_k \rho_{ik}^{IND} R_i R_k}_{\text{Between-industry}} \tag{16}$$

4.4 Multi-Level Diversification

Theorem 6 (Hierarchical Diversification)**.** *Under uniform correlation structures at both levels:*

- *Within industries: $\rho_{ijk} = \rho^{FIRM}$ for all $j \neq k$ in industry i*
- *Between industries: $\rho_{ik}^{IND} = \rho^{IND}$ for all $i \neq k$*

As $n \to \infty$ and $F_i \to \infty$ for all i, the economy-wide risk converges to:

$$R_E \to \sqrt{\rho^{IND} \cdot \rho^{FIRM}} \cdot \bar{R} \tag{17}$$

where $\bar{R}$ is the average firm risk across the entire economy.

This result shows that hierarchical diversification compounds: both within-industry and between-industry diversification contribute to aggregate risk reduction.

5 Risk Classification Metrics

5.1 Industry Risk Contribution

The marginal contribution of industry i to total economic risk is:

$$\mathrm{MRC}_i = \frac{\partial R_E}{\partial \omega_i} = \frac{1}{R_E} \sum_{k=1}^{n} \omega_k \rho_{ik}^{IND} R_i R_k \tag{18}$$

5.2 Systematic vs. Idiosyncratic Risk

For industry i, we can decompose firm risk into systematic and idiosyncratic components:

Definition 4 (Risk Decomposition at Firm Level)**.** *For firm j in industry i:*

$$R_{ij}^2 = \underbrace{\beta_{ij}^2 R_i^2}_{\textit{Systematic}} + \underbrace{\epsilon_{ij}^2}_{\textit{Idiosyncratic}} \tag{19}$$

where β_{ij} measures the exposure of firm (i, j) to industry i risk.

5.3 Risk Classification Index

We propose a classification index that measures the importance of industry structure:

Definition 5 (Industry Structure Importance Index)**.**

$$ISI = 1 - \frac{R_E^{industry}}{R_E^{flat}} \tag{20}$$

where $R_E^{industry}$ uses the hierarchical structure and R_E^{flat} treats all firms equally without industry grouping.

Proposition 7. *$ISI \in [0, 1]$, with higher values indicating greater importance of industry classification.*

6 Graphical Representation

Figure 1 illustrates the hierarchical structure of economic risk from firms to industries to the aggregate economy.

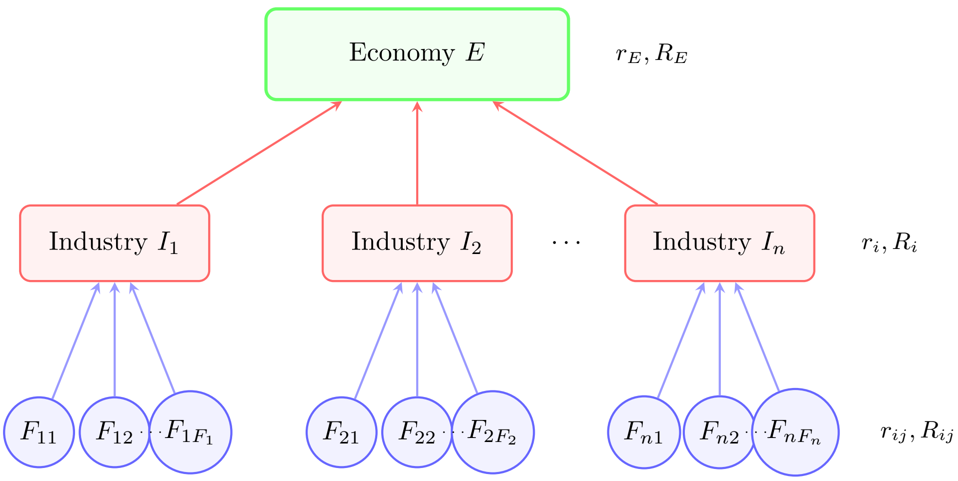

Figure 1: Hierarchical structure of economic risk aggregation from individual firms through industries to the economy-wide level.

Figure 2 shows the relationship between the number of assets and portfolio risk under different correlation assumptions.

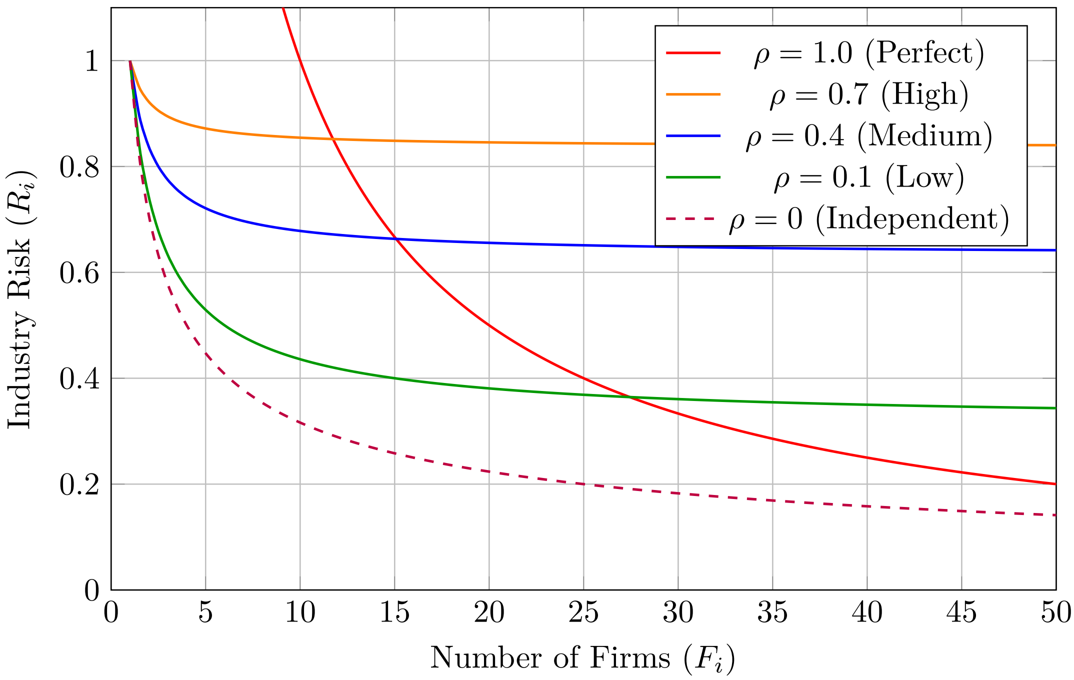

Figure 2: Diversification effect: Industry risk as a function of the number of firms under different correlation assumptions (normalized with average firm risk = 1).

7 Comparative Statics and Economic Implications

7.1 Effect of Correlation on Aggregate Risk

Proposition 8 (Monotonicity in Correlation)**.** *For fixed weights and firm risks:*

1. $\frac{\partial R_i}{\partial \rho_{ijk}} > 0$*: Industry risk increases in within-industry correlation*

2. $\frac{\partial R_E}{\partial \rho_{ik}^{IND}} > 0$*: Economy-wide risk increases in between-industry correlation*

7.2 Optimal Industry Composition

Given a target level of expected return $\bar{r}_E$, the optimal industry weights solve:

$$\begin{aligned} \min_{\{\omega_i\}} \quad & R_E = \sqrt{\sum_{i=1}^{n}\sum_{k=1}^{n} \omega_i \omega_k \rho_{ik}^{IND} R_i R_k} \\ \text{s.t.} \quad & \sum_{i=1}^{n} \omega_i r_i = \bar{r}_E \\ & \sum_{i=1}^{n} \omega_i = 1 \\ & \omega_i \geq 0 \quad \forall i \end{aligned} \tag{21}$$

This is analogous to the classical Markowitz portfolio optimization problem.

7.3 Industry Clustering and Contagion

Industries with high mutual correlation create contagion risk:

Definition 6 (Contagion Intensity)**.** *The contagion intensity between industries i and k is:*

$$C_{ik} = \omega_i \omega_k \rho_{ik}^{IND} R_i R_k \tag{22}$$

High values of C_{ik} indicate that shocks to one industry significantly affect the other.

8 Extensions and Generalizations

8.1 Time-Varying Risk

Allow risk parameters to evolve over time: $R_{ij}(t)$, $\rho_{ijk}(t)$, $\rho_{ik}^{IND}(t)$.

The dynamic economy-wide risk becomes:

$$R_E(t) = \sqrt{\sum_{i=1}^{n}\sum_{k=1}^{n}\omega_i(t)\omega_k(t)\rho_{ik}^{IND}(t)R_i(t)R_k(t)} \tag{23}$$

8.2 Multi-Factor Risk Models

Extend to include common risk factors $\mathbf{f} = (f_1, \ldots, f_K)$:

$$R_{ij} = \sum_{m=1}^{K}\beta_{ij,m}f_m + \epsilon_{ij} \tag{24}$$

This connects to APT and factor models in asset pricing [15].

8.3 Network Effects

Industries can be connected through input-output networks. Let $A = [a_{ik}]$ be the input-output matrix where a_{ik} represents the fraction of industry k's output used by industry i.

The network-adjusted risk becomes:

$$R_i^{NET} = R_i + \sum_{k=1}^{n} a_{ik}R_k \tag{25}$$

8.4 Tail Risk and Higher Moments

Beyond variance, we can consider:

- Skewness: $S_i = E[(R_i - \mu_i)^3]/\sigma_i^3$
- Kurtosis: $K_i = E[(R_i - \mu_i)^4]/\sigma_i^4$
- Value-at-Risk: $\mathrm{VaR}_\alpha(R_i)$
- Expected Shortfall: $\mathrm{ES}_\alpha(R_i)$

9 Empirical Considerations

9.1 Estimation Challenges

Estimating the full covariance structure requires:

$$\binom{N}{2} + N = \frac{N(N+1)}{2} \tag{26}$$

parameters for N firms, which grows quadratically.

The hierarchical structure reduces this to approximately:

$$\sum_{i=1}^{n}\binom{F_i}{2} + \binom{n}{2} \ll \binom{N}{2} \tag{27}$$

parameters under reasonable sparsity assumptions.

9.2 Industry Classification Schemes

Common classification systems include:

- SIC (Standard Industrial Classification)
- NAICS (North American Industry Classification System)
- GICS (Global Industry Classification Standard)
- ICB (Industry Classification Benchmark)

The choice of classification affects measured risk aggregation.

10 Conclusion

This paper has developed a comprehensive theoretical framework for classifying economic risk through a hierarchical decomposition across firms and industries. Our main contributions include:

1. Rigorous mathematical formulations for multi-level risk aggregation
2. Characterization of diversification benefits at industry and economy levels
3. Analytical results on the role of correlation structures
4. A unified approach integrating portfolio theory, industrial organization, and macroeconomic risk

The framework provides theoretical foundations for practical applications in risk management, regulatory policy, and macroeconomic analysis. Future research directions include empirical validation, dynamic extensions, and incorporation of network effects and systemic risk considerations.

The hierarchical approach demonstrates that proper industry classification is not merely a matter of taxonomic convenience but has substantive implications for understanding and managing economic risk. The degree to which diversification reduces risk depends critically on correlation structures both within and between industries, suggesting that careful attention to sectoral composition is essential for both private portfolio management and public policy design.

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