Bayesian Causal Inference in a Small Closed Economy of Representative Agents of Nuclear Nations

Soumadeep Ghosh

Kolkata, India

Abstract

This paper develops a framework for Bayesian causal inference within a small closed economy populated by representative agents from nuclear-armed nations. We construct a model that incorporates strategic uncertainty, credible commitment problems, and the unique equilibrium properties that emerge when economic interdependence intersects with nuclear deterrence doctrine. The framework employs directed acyclic graphs to represent causal relationships, posterior updating mechanisms to capture belief revision under incomplete information, and game-theoretic foundations to analyze strategic interaction. Our analysis demonstrates that Bayesian updating of beliefs about adversarial intentions generates distinct equilibrium patterns compared to classical rational expectations models, particularly when agents face irreversible decisions under the shadow of catastrophic risk. The model provides theoretical foundations for understanding how economic sanctions, trade agreements, and financial integration affect strategic stability among nuclear powers.

The paper ends with "The End"

1 Introduction

The intersection of nuclear deterrence theory and international political economy presents unique challenges for causal inference. Traditional econometric approaches assume stable preference orderings and rely on large-sample asymptotic properties that become questionable when analyzing strategic interactions among a small number of nuclear-capable states [1, 2]. This paper develops a Bayesian framework specifically designed to address the epistemological constraints inherent in analyzing closed economic systems where representative agents possess both economic objectives and strategic nuclear capabilities.

The fundamental problem we address concerns the identification of causal effects when the number of strategic actors is small, their interactions are characterized by incomplete information, and the potential consequences of miscalculation include catastrophic outcomes. Classical frequentist inference requires repeated sampling from a well-defined population, an assumption that becomes untenable when studying unique strategic configurations among nuclear powers [3]. Bayesian methods offer a natural alternative by explicitly representing uncertainty through probability distributions and updating beliefs as new information becomes available.

We model a closed economy consisting of N representative agents, where each agent $i \in \{1, 2, \dots, N\}$ represents a nuclear-capable nation. The closure assumption reflects the reality that nuclear powers constitute a small, self-contained strategic system where external economic shocks are dominated by endogenous strategic dynamics. Each agent simultaneously pursues economic objectives while maintaining credible deterrence, creating a two-dimensional optimization problem that cannot be decomposed without loss of analytical tractability.

2 Theoretical Framework

2.1 The Economic Environment

Consider a closed economy with N representative agents indexed by $i \in \mathcal{N} = \{1, 2, \dots, N\}$. Agent i possesses an initial endowment $\omega_i \in \mathbb{R}^L_+$ of L commodities and preferences represented by a utility function $u_i : \mathbb{R}^L_+ \times \mathbb{R}_+ \times \Theta \to \mathbb{R}$, where the domain includes consumption bundles, security levels, and a parameter space Θ representing beliefs about adversarial intentions.

Definition 1 (Closed Economy Equilibrium)**.** A competitive equilibrium in the closed economy consists of allocations $(x_1^*, \dots, x_N^*)$ and prices p^* such that for each agent i:

$$x_i^* \in \arg\max_{x_i \in \mathbb{R}^L_+} u_i(x_i, s_i, \theta_i) \tag{1}$$

$$\text{subject to} \quad p^* \cdot x_i \leq p^* \cdot \omega_i - c_i(s_i) \tag{2}$$

$$\sum_{i=1}^{N} x_i^* = \sum_{i=1}^{N} \omega_i \tag{3}$$

where s_i represents agent i's security investment and $c_i(s_i)$ denotes the associated cost.

The security investment s_i reflects the resources allocated to maintaining nuclear deterrence capabilities, creating an immediate trade-off between economic consumption and strategic security. This trade-off becomes particularly acute in a closed economy where resources allocated to deterrence represent a direct opportunity cost in terms of foregone consumption.

2.2 Bayesian Belief Structure

Each agent maintains beliefs about the strategic types of other agents. Let $\theta_j \in \Theta$ represent agent j's type, which may be cooperative or adversarial. Agent i holds a prior distribution $\pi_i(\theta_j)$ over agent j's type and updates this distribution according to Bayes' rule upon observing economic actions and strategic signals.

Assumption 1 (Common Prior)**.** All agents share a common prior distribution $\pi_0(\theta)$ over the type space Θ, but receive private signals that lead to heterogeneous posteriors.

The posterior belief of agent i about agent j after observing signal y is given by:

$$\pi_i(\theta_j | y) = \frac{f(y|\theta_j)\pi_0(\theta_j)}{\int_\Theta f(y|\theta)\pi_0(\theta) d\theta} \tag{4}$$

where $f(y|\theta_j)$ represents the likelihood function mapping types to observable signals.

This Bayesian updating mechanism captures the fundamental uncertainty that characterizes strategic interactions among nuclear powers. Unlike complete information models, agents must form beliefs about adversarial intentions based on noisy signals, including economic policy choices, diplomatic communications, and observable military deployments.

2.3 Causal Structure

We represent the causal relationships among economic variables, strategic choices, and belief updating through a directed acyclic graph (DAG). Figure 1 illustrates the fundamental causal structure.

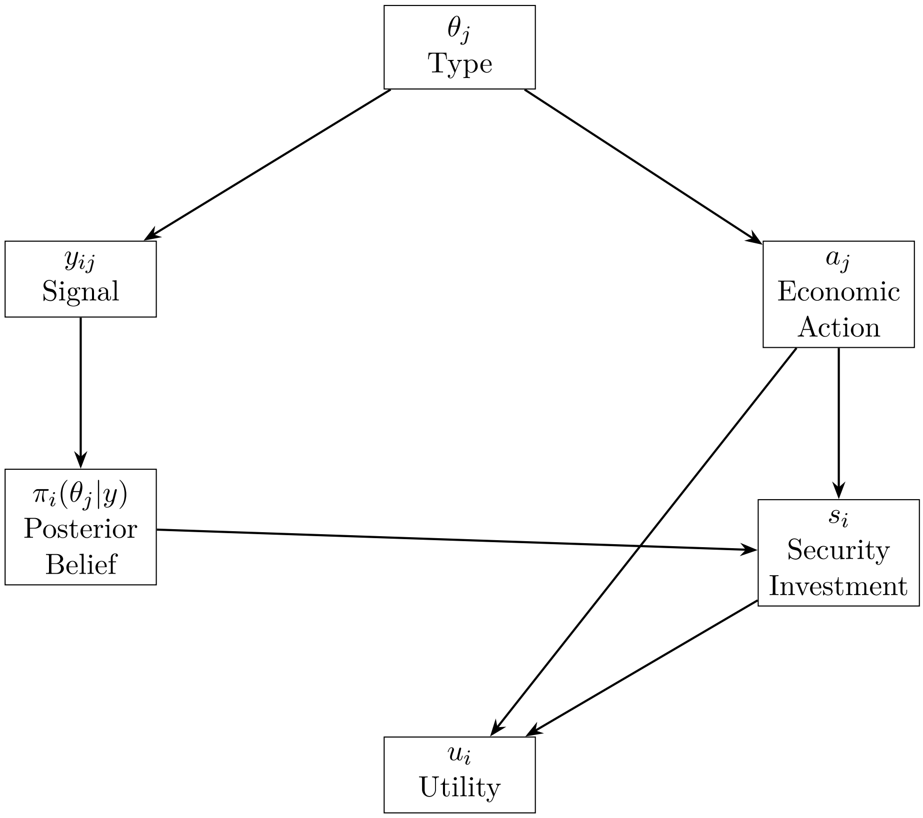

Figure 1: Causal DAG representing the relationship between strategic types, observable signals, belief updating, and economic outcomes in a nuclear-capable representative agent framework.

The causal structure reveals several critical features of the model. First, the strategic type θ_j functions as a common cause of both the signal y_{ij} and the economic action a_j, creating a confounding relationship that must be addressed through appropriate conditioning. Second, the posterior belief $\pi_i(\theta_j|y)$ serves as a mediator between observed signals and security investment decisions, highlighting the central role of belief updating in determining equilibrium outcomes.

Theorem 1 (Causal Identification)**.** *Under the Markov condition and faithfulness assumption, the causal effect of economic action a_j on agent i's utility is identified by* *conditioning on the posterior belief $\pi_i(\theta_j|y)$:*

$$\mathbb{E}[u_i|do(a_j)] = \int \mathbb{E}[u_i|a_j, \pi_i]p(\pi_i)d\pi_i \tag{5}$$

where $do(a_j)$ represents Pearl's intervention operator.

The identification strategy relies on the observation that conditioning on posterior beliefs blocks all backdoor paths from actions to utilities through the type variable, enabling causal interpretation of the estimated effects despite the presence of unobserved strategic types.

3 Strategic Equilibrium Under Bayesian Updating

The strategic interaction among nuclear-capable representative agents generates a sequential game with incomplete information. Each period, agents simultaneously choose economic actions a_i and security investments s_i while updating beliefs about adversarial types based on observed behavior.

3.1 Sequential Equilibrium

Definition 2 (Perfect Bayesian Equilibrium)**.** A Perfect Bayesian Equilibrium consists of strategy profiles $(a_1^*, \ldots, a_N^*, s_1^*, \ldots, s_N^*)$ and belief systems $(\pi_1, \ldots, \pi_N)$ such that:

1. Each agent's strategy maximizes expected utility given beliefs and others' strategies
2. Beliefs are derived from Bayes' rule whenever possible
3. Off-equilibrium beliefs satisfy reasonable refinement criteria

The Perfect Bayesian Equilibrium concept ensures that strategies are sequentially rational given beliefs, and beliefs are consistent with observed behavior through Bayesian updating. This equilibrium concept proves particularly suitable for analyzing nuclear deterrence scenarios where credibility of threats depends critically on belief consistency.

3.2 Comparative Statics

We analyze how changes in prior beliefs affect equilibrium security investments and economic welfare. Let $\lambda \in [0, 1]$ parameterize the prior probability that an adversary is cooperative rather than hostile.

Proposition 1 (Belief Monotonicity)**.** *If the utility function exhibits decreasing absolute risk aversion and the likelihood ratio $f(y|\theta_{hostile})/f(y|\theta_{coop})$ is monotone increasing in y, then equilibrium security investment $s_i^*(\lambda)$ is decreasing in the prior probability of cooperation λ.*

This proposition formalizes the intuitive result that as agents become more optimistic about adversarial intentions, they reduce security investments and reallocate resources toward economic consumption. However, the relationship between beliefs and welfare proves more complex due to the strategic complementarities in security investment.

Figure 2 illustrates the relationship between prior beliefs and equilibrium outcomes for a two-agent economy.

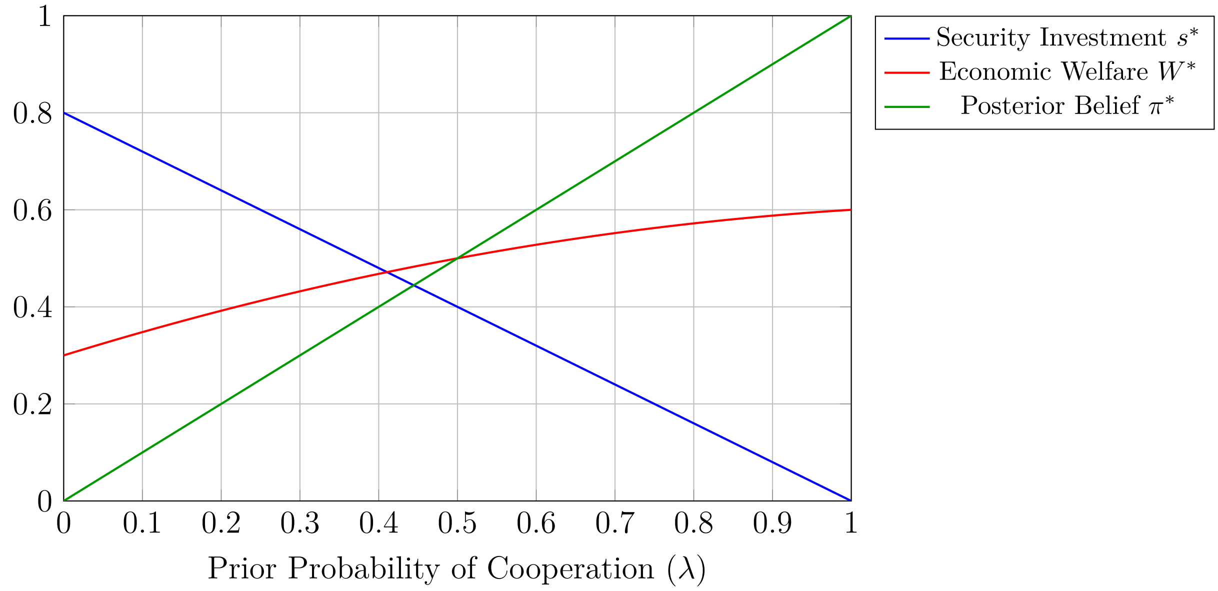

Figure 2: Equilibrium comparative statics showing the relationship between prior beliefs about cooperation, security investment, economic welfare, and posterior beliefs in a symmetric two-agent nuclear economy.

The comparative statics reveal a nonmonotonic relationship between prior optimism and welfare. At low levels of cooperation probability, increased optimism generates welfare gains through reduced security expenditures. However, at high cooperation probabilities, further increases in optimism may reduce welfare by creating vulnerability to exploitation if the adversary proves hostile.

4 Empirical Implementation

4.1 Identification Strategy

The empirical implementation of Bayesian causal inference in this context faces significant challenges due to the small sample size and strategic nature of interactions. We employ a hierarchical Bayesian model that pools information across time periods while respecting the sequential structure of belief updating.

The hierarchical model specifies:

$$u_{it} \sim \mathcal{N}(x_{it}'\beta + s_{it}'\gamma, \sigma^2) \tag{6}$$

$$\beta \sim \mathcal{N}(\mu_\beta, \Sigma_\beta) \tag{7}$$

$$\gamma \sim \mathcal{N}(\mu_\gamma, \Sigma_\gamma) \tag{8}$$

where subscript t indexes time periods, x_{it} represents economic variables, and s_{it} denotes security investments.

The hierarchical structure allows us to estimate causal effects even with limited data by borrowing strength across time periods while maintaining flexibility to capture temporal variation in strategic relationships.

4.2 Prior Specification

The specification of prior distributions requires careful consideration of both theoretical constraints and available information. For the utility parameters, we employ weakly informative priors centered on economically plausible values:

$$\mu_\beta \sim \mathcal{N}(0, 10^2 I), \quad \Sigma_\beta \sim \text{Inv-Wishart}(\nu_0, S_0) \tag{9}$$

For the security investment parameters, we incorporate the theoretical constraint that increased security should provide positive utility gains, implemented through a truncated normal prior:

$$\mu_\gamma \sim \mathcal{N}^+(0, 5^2) \tag{10}$$

where $\mathcal{N}^+$ denotes the positive-truncated normal distribution.

4.3 Posterior Computation

We employ Markov Chain Monte Carlo (MCMC) methods to compute posterior distributions of model parameters. The Gibbs sampler alternates between sampling from the conditional posteriors:

$$p(\beta | u, x, s, \gamma, \mu_\beta, \Sigma_\beta, \sigma^2) \tag{11}$$
$$p(\gamma | u, x, s, \beta, \mu_\gamma, \Sigma_\gamma, \sigma^2) \tag{12}$$
$$p(\sigma^2 | u, x, s, \beta, \gamma) \tag{13}$$

Convergence diagnostics include trace plots, Gelman-Rubin statistics, and effective sample size calculations to ensure reliable inference.

5 Credible Commitment and Time Inconsistency

A central feature of nuclear deterrence theory concerns the credibility of threats and promises. In our framework, this translates into a time inconsistency problem where agents face incentives to deviate from announced policies after other agents have taken irreversible actions.

Definition 3 (Credible Commitment Equilibrium)**.** A strategy profile constitutes a Credible Commitment Equilibrium if it satisfies Perfect Bayesian Equilibrium conditions and additionally requires that no agent can profitably deviate after observing others' economic commitments, even when such deviation would not trigger immediate retaliation.

The credible commitment constraint tightens the equilibrium conditions by requiring that strategies remain optimal even off the equilibrium path. This proves particularly important when analyzing economic sanctions or trade agreements that involve irreversible investments by one or more parties.

Figure 3 illustrates the commitment problem through a simplified extensive form representation.

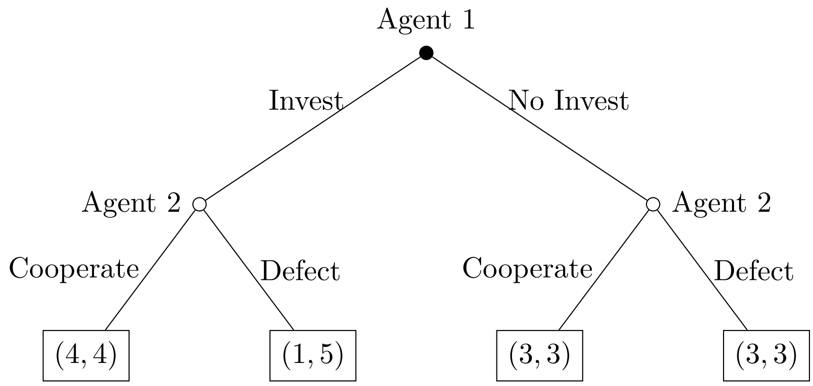

Figure 3: Extensive form representation of the commitment problem in economic cooperation among nuclear powers. Agent 1 first decides whether to make an irreversible economic investment, then Agent 2 chooses whether to cooperate or defect. Payoffs (u_1, u_2) reflect both economic gains and strategic considerations.

The game tree reveals the fundamental commitment problem. Agent 1's investment decision creates vulnerability that Agent 2 could exploit through defection. The credible commitment equilibrium requires that Agent 2's reputation costs or future retaliation threats make cooperation optimal even after Agent 1 has already invested.

6 Welfare Analysis

We evaluate social welfare using both ex-ante and ex-post criteria. The ex-ante welfare measure evaluates allocations before types are revealed, using prior beliefs to compute expected utilities. The ex-post welfare criterion conditions on realized types and evaluates actual utility realizations.

Definition 4 (Expected Social Welfare)**.** The expected social welfare function aggregates individual utilities weighted by prior probabilities:

$$W = \sum_{i=1}^{N} \int_{\Theta^N} u_i(x_i^*, s_i^*, \theta)\pi_0(\theta)d\theta \tag{14}$$

where $\theta = (\theta_1, \ldots, \theta_N)$ represents the type profile.

An alternative welfare criterion focuses on ex-post efficiency, evaluating whether the allocation is Pareto optimal conditional on realized types. However, this criterion proves difficult to implement empirically since true types remain unobserved.

6.1 Constrained Efficiency

The presence of strategic uncertainty and nuclear capabilities introduces constraints on achievable efficiency. Even if agents could coordinate on a social planner's solution, information asymmetries prevent implementation of first-best allocations.

Theorem 2 (Constrained Efficiency Bound)**.** *In a closed economy with N nuclear-capable representative agents and asymmetric information about types, no mechanism can achieve first-best efficiency while satisfying individual rationality and incentive compatibility constraints simultaneously.*

This impossibility result follows from standard mechanism design theory but carries particular force in the nuclear deterrence context where the potential losses from strategic miscalculation vastly exceed typical economic inefficiencies. The theorem implies that any feasible allocation must accept some welfare loss relative to the complete-information benchmark.

7 Extensions and Robustness

7.1 Dynamic Learning

The baseline model assumes static beliefs updated only once per period. We extend the framework to incorporate dynamic learning where agents continuously update beliefs as new information arrives. This extension proves particularly relevant for understanding how economic integration affects strategic stability over time.

Let $\{\mathcal{F}_t\}$ denote a filtration representing the information available at time t. Agent i's beliefs evolve according to:

$$d\pi_{it}(\theta_j) = \pi_{it}(\theta_j) \left[\frac{f(dy_t|\theta_j)}{\int f(dy_t|\theta)\pi_{it}(\theta)d\theta} - 1 \right] \tag{15}$$

This stochastic differential equation describes how beliefs change continuously as agents observe economic flows, diplomatic signals, and strategic indicators.

7.2 Multiple Equilibria

The strategic complementarities in security investment can generate multiple equilibria with different welfare properties. In particular, the model admits both high-trust equilibria with low security investment and high economic integration, and low-trust equilibria with high security investment and limited economic cooperation.

Figure 4 illustrates the reaction function dynamics that generate multiplicity.

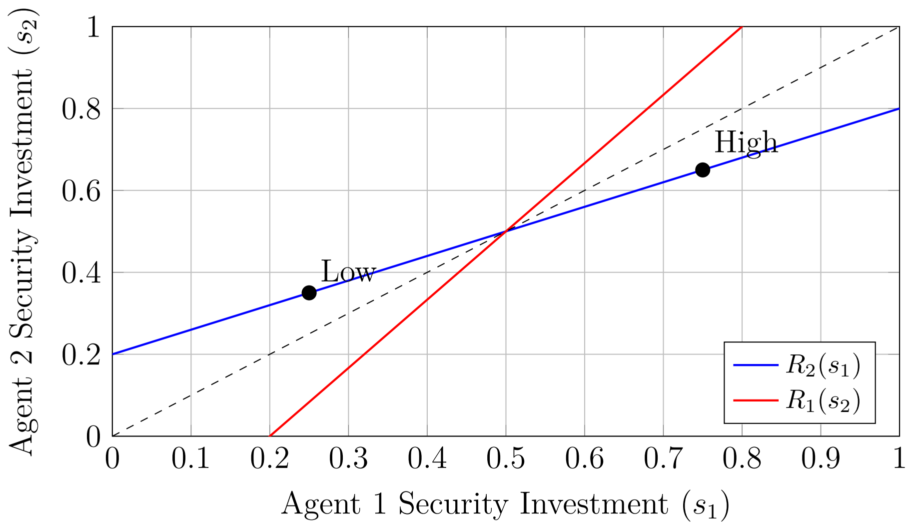

Figure 4: Reaction functions in security investment space showing multiple equilibria. The low-trust equilibrium exhibits high mutual security investment, while the high-trust equilibrium features low security investment and greater economic cooperation.

The existence of multiple equilibria raises important policy questions regarding equilibrium selection and the possibility of transitions between equilibrium states. Coordination on the efficient equilibrium may require external mechanisms such as international institutions or binding agreements.

7.3 Robustness to Prior Specification

A key concern in Bayesian analysis involves sensitivity to prior specification. We conduct robustness checks by varying the prior hyperparameters and examining how posterior inferences change. The analysis reveals that conclusions regarding the qualitative relationship between beliefs and security investment remain robust to reasonable prior specifications, though quantitative magnitudes vary.

We measure prior sensitivity through the prior predictive distance:

$$D(\pi_0, \pi_0') = \int |p(y|\pi_0) - p(y|\pi_0')| \, dy \tag{16}$$

where π_0 and π_0' represent alternative prior specifications.

8 Discussion and Policy Implications

The Bayesian framework developed here offers several insights for understanding strategic stability among nuclear powers in an economically integrated world. First, the model demonstrates that belief updating plays a central role in determining equilibrium security investments, suggesting that policies affecting information revelation carry strategic significance beyond their immediate economic effects.

Second, the analysis reveals that economic integration among nuclear powers creates both opportunities for welfare gains and risks of strategic vulnerability. The optimal degree of integration depends critically on the precision of signals about adversarial intentions and the costs of maintaining deterrence credibility.

Third, the existence of multiple equilibria implies that historical accidents or coordinated interventions can have persistent effects on the level of trust and cooperation among nuclear-capable nations. This observation suggests a potential role for confidence-building measures and institutional design in facilitating coordination on efficient equilibria.

From a methodological perspective, the framework illustrates how Bayesian methods provide a natural approach to causal inference in strategic settings with small samples and incomplete information. The explicit representation of uncertainty through posterior distributions offers advantages over classical econometric techniques when dealing with unique strategic configurations.

9 Conclusion

This paper has developed a comprehensive framework for Bayesian causal inference in a closed economy of representative agents from nuclear-capable nations. The model integrates game-theoretic foundations with Bayesian updating mechanisms and causal identification strategies to analyze how economic interdependence affects strategic stability under incomplete information.

The theoretical analysis demonstrates that belief structures fundamentally shape equilibrium outcomes, with prior beliefs about adversarial intentions determining both security investments and economic welfare. The framework provides tools for rigorous causal inference despite the small-sample and strategic-interaction challenges inherent in analyzing nuclear deterrence relationships.

Future research directions include extending the model to incorporate asymmetric information about capabilities in addition to intentions, analyzing dynamic games with learning and reputation effects, and developing empirical applications using historical data on economic relations among nuclear powers. The framework also suggests promising avenues for incorporating recent developments in causal machine learning and algorithmic game theory.

The integration of Bayesian statistics, causal inference, and nuclear deterrence theory developed here demonstrates the value of interdisciplinary approaches to understanding strategic interactions in an economically integrated world. As nuclear proliferation continues and economic interdependence deepens, frameworks that explicitly account for strategic uncertainty and belief updating will become increasingly essential for both theoretical analysis and policy application.

References

[1] Morgan, P.M. (1983). *Deterrence: A Conceptual Analysis*. Second Edition. Beverly Hills, CA: Sage Publications.

[2] Schelling, T.C. (1980). *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

[3] Pearl, J. (2009). *Causality: Models, Reasoning, and Inference*. Second Edition. Cambridge: Cambridge University Press.

[4] Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A., & Rubin, D.B. (2013). *Bayesian Data Analysis*. Third Edition. Boca Raton, FL: CRC Press.

[5] Fudenberg, D., & Tirole, J. (1991). *Game Theory*. Cambridge, MA: MIT Press.

[6] Powell, R. (1990). *Nuclear Deterrence Theory: The Search for Credibility*. Cambridge: Cambridge University Press.

[7] Imbens, G.W., & Rubin, D.B. (2015). *Causal Inference for Statistics, Social, and Biomedical Sciences: An Introduction*. Cambridge: Cambridge University Press.

[8] Kreps, D.M., & Wilson, R. (1982). Sequential equilibria. *Econometrica*, 50(4), 863–894.

[9] Fearon, J.D. (1995). Rationalist explanations for war. *International Organization*, 49(3), 379–414.

[10] Mas-Colell, A., Whinston, M.D., & Green, J.R. (1995). *Microeconomic Theory*. Oxford: Oxford University Press.

[11] Robert, C.P., & Casella, G. (2004). *Monte Carlo Statistical Methods*. Second Edition. New York: Springer.

[12] Harsanyi, J.C. (1967). Games with incomplete information played by Bayesian players. *Management Science*, 14(3), 159–182.

[13] Jervis, R. (1976). *Perception and Misperception in International Politics*. Princeton, NJ: Princeton University Press.

[14] Keohane, R.O. (1984). *After Hegemony: Cooperation and Discord in the World Political Economy*. Princeton, NJ: Princeton University Press.

[15] Spirtes, P., Glymour, C., & Scheines, R. (2000). *Causation, Prediction, and Search*. Second Edition. Cambridge, MA: MIT Press.

The End