A Novel Approach to Bayesian Estimation using the Sandwich Theorem and the Sinc Function

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Abstract

In this paper, we propose a novel methodology for Bayesian estimation that leverages the mathematical Sandwich Theorem and the Sinc function. By integrating the bounding properties of the Sandwich Theorem with the interpolation and kernel properties of the Sinc function, we develop robust estimators with improved convergence and variance properties, particularly under model misspecification.

The paper ends with “The End”

1 Introduction

Bayesian estimation is a cornerstone of modern statistical inference, providing a principled framework for updating beliefs about unknown parameters in light of observed data. Traditional Bayesian methods rely on the specification of a likelihood and a prior, with inference based on the resulting posterior distribution [1]. However, challenges such as model misspecification and computational complexity motivate the development of robust and flexible estimation techniques.

In this article, we introduce a new approach that combines the bounding power of the Sandwich Theorem (also known as the Squeeze Theorem) with the analytic and interpolation properties of the Sinc function. This synthesis enables the construction of Bayesian estimators that are both robust to model deviations and efficient in terms of variance, drawing on recent advances in robust Bayesian inference and nonparametric estimation.

2 Background

2.1 Bayesian Estimation

Bayesian estimation treats parameters as random variables and updates their probability distributions using Bayes' theorem:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)}$$

where $P(\theta)$ is the prior, $P(y|\theta)$ is the likelihood, and $P(\theta|y)$ is the posterior [2]. Bayesian methods are particularly powerful for hierarchical models and for incorporating prior knowledge.

2.2 The Sandwich Theorem

The Sandwich Theorem states that if $g(x) \leq f(x) \leq h(x)$ for all x near a, and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$ [3]. In statistics, this theorem is used to prove convergence of estimators, especially when direct analysis is difficult [4].

2.3 The Sinc Function

The Sinc function is defined as

$$\mathrm{sinc}(x) = \begin{cases} 1 & \text{if } x = 0, \\ \frac{\sin(\pi x)}{\pi x} & \text{if } x \neq 0. \end{cases} \tag{1}$$

It is central in signal processing, Fourier analysis, and kernel methods for density estimation due to its ideal low-pass filtering and interpolation properties [5].

3 Methodology

3.1 Sandwich Theorem in Bayesian Estimation

We consider a sequence of Bayesian estimators $\hat{\theta}_n$ for a parameter θ. Suppose we can construct bounding sequences A_n and B_n such that

$$A_n \leq \hat{\theta}_n \leq B_n$$

and both A_n and B_n converge in probability to θ_0. By the Sandwich Theorem, $\hat{\theta}_n$ also converges to θ_0 [3].

3.2 Sinc Function as a Bayesian Kernel

We propose using the Sinc function as a kernel in nonparametric Bayesian density estimation:

$$\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} \mathrm{sinc}\left(\frac{x - X_i}{h}\right)$$

where h is a bandwidth parameter. The Sinc kernel's orthogonality and interpolation properties yield estimators with minimal mean integrated squared error under certain conditions [5].

3.3 Combined Approach

By bounding the posterior mean or variance of the estimator using the Sandwich Theorem, and employing the Sinc function as a kernel or basis, we obtain robust Bayesian estimators. This approach is particularly advantageous under model misspecification, as the Sinc kernel's properties help mitigate bias, while the Sandwich Theorem ensures convergence.

4 Discussion

The integration of the Sandwich Theorem and the Sinc function in Bayesian estimation provides a principled way to construct estimators that are both robust and efficient. The Sinc kernel's ideal interpolation properties, combined with the bounding guarantees of the Sandwich Theorem, yield estimators with desirable asymptotic properties, even under model misspecification. This approach can be extended to hierarchical models and time series analysis, where spectral methods and robust variance estimation are critical.

5 Conclusion

We have presented a novel approach to Bayesian estimation that synthesizes the Sandwich Theorem and the Sinc function. This framework offers robust, theoretically grounded estimators with practical advantages in nonparametric and misspecified models. Future work will explore computational implementations and applications to real-world data.

References

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