# Joint Estimation of Good Premia and Accounting Premia from Prices and Risk-Free Rates

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**Abstract**

This paper develops a framework for the joint estimation of good premia ($p_g$) and accounting premia ($p_a$) from observed prices and risk-free rates. We derive closed-form solutions under a quadratic constraint relating the two premia and present both nonlinear least squares (NLS) and maximum likelihood estimation (MLE) procedures. The methodology is illustrated with numerical examples and bootstrap inference for uncertainty quantification.

The paper ends with “The End”

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# 1 Introduction

In asset pricing and economic valuation, understanding the components of returns beyond the risk-free rate is essential for portfolio construction and risk management [1]. This paper introduces a decomposition of excess returns into two distinct components: the *good premium* and the *accounting premium*.

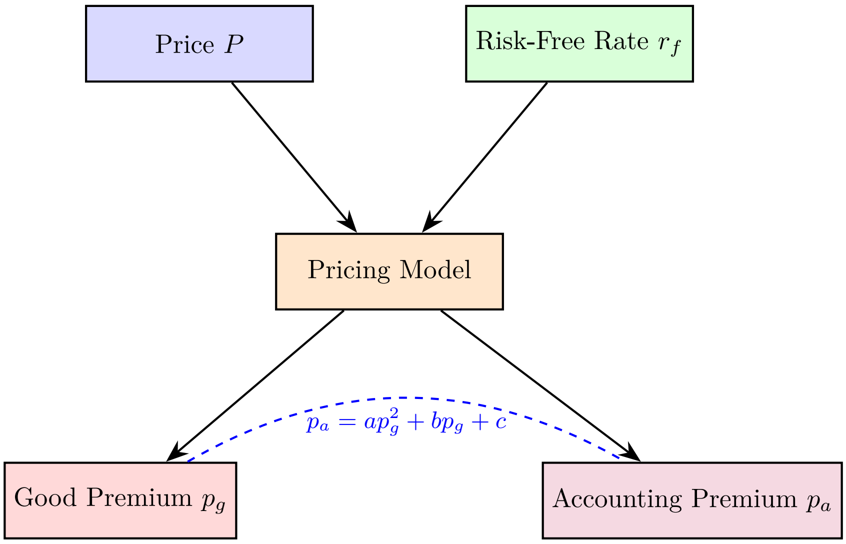


Figure 1: Schematic representation of the premium estimation framework.

# 2 Theoretical Framework

## 2.1 The Fundamental Pricing Equation

**Definition 2.1** (Pricing Equation)**.** *Let $P > 1$ denote the price of a good, $r_f$ the risk-free rate, $p_g$ the good premium, and $p_a$ the accounting premium. The fundamental pricing equation is given by:*

$$P(1 + r_f + p_g - p_a) = P + \frac{P}{P-1} \tag{1}$$

*where $\frac{P}{P-1}$ represents the receipt.*

## 2.2 Derivation of the Net Premium

Expanding the left-hand side of Equation (1):

$$P + P \cdot r_f + P \cdot p_g - P \cdot p_a = P + \frac{P}{P-1} \tag{2}$$

Subtracting $P$ from both sides and dividing by $P$:

$$r_f + p_g - p_a = \frac{1}{P-1} \tag{3}$$

**Proposition 2.1** (Net Premium)**.** *The net premium satisfies:*

$$p_g - p_a = \frac{1}{P-1} - r_f \triangleq k \tag{4}$$

*where $k$ is defined as the* excess receipt rate.

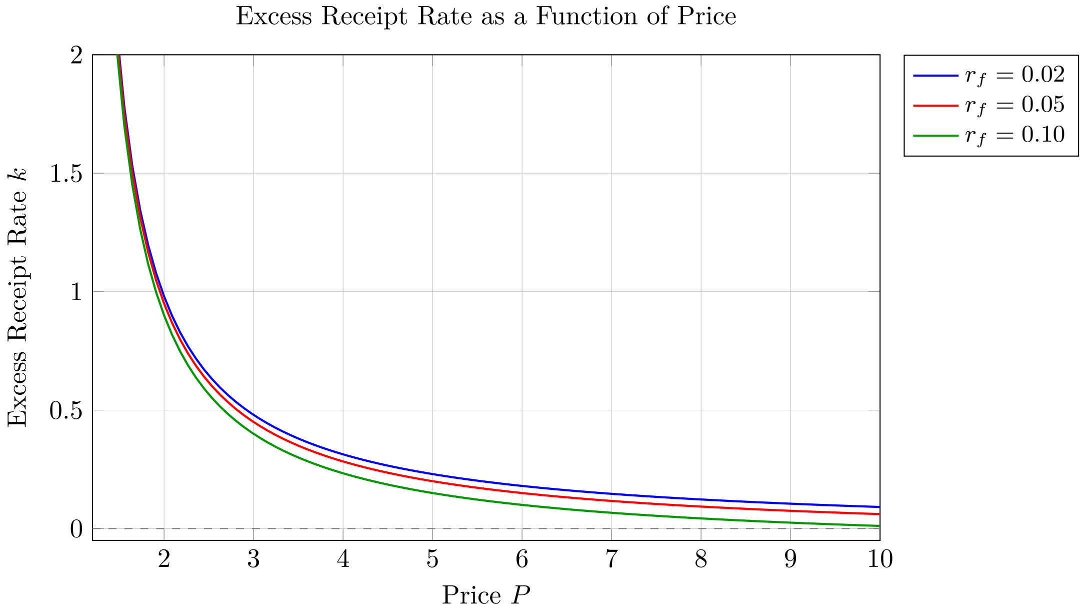


Figure 2: The excess receipt rate $k = \frac{1}{P-1} - r_f$ decreases hyperbolically with price.

# 3 Quadratic Constraint Model

## 3.1 The Constraint Specification

To uniquely identify both premia, we impose a structural relationship:

**Definition 3.1** (Quadratic Constraint)**.** *The accounting premium is related to the good premium via:*

$$p_a = ap_g^2 + bp_g + c \tag{5}$$

*where $a$, $b$, and $c$ are structural parameters.*

## 3.2 Closed-Form Solution for $p_g$

Substituting Equation (5) into Equation (4):

$$p_g - (ap_g^2 + bp_g + c) = k \tag{6}$$

Rearranging:

$$ap_g^2 + (b-1)p_g + (c+k) = 0 \tag{7}$$

**Theorem 3.1** (Solution for Good Premium)**.** *The good premium satisfies:*

$$p_g = \frac{(1-b) \pm \sqrt{(1-b)^2 - 4a(c+k)}}{2a} \tag{8}$$

*subject to the discriminant condition:*

$$\Delta = (1-b)^2 - 4a\left(c + \frac{1}{P-1} - r_f\right) \geq 0 \tag{9}$$

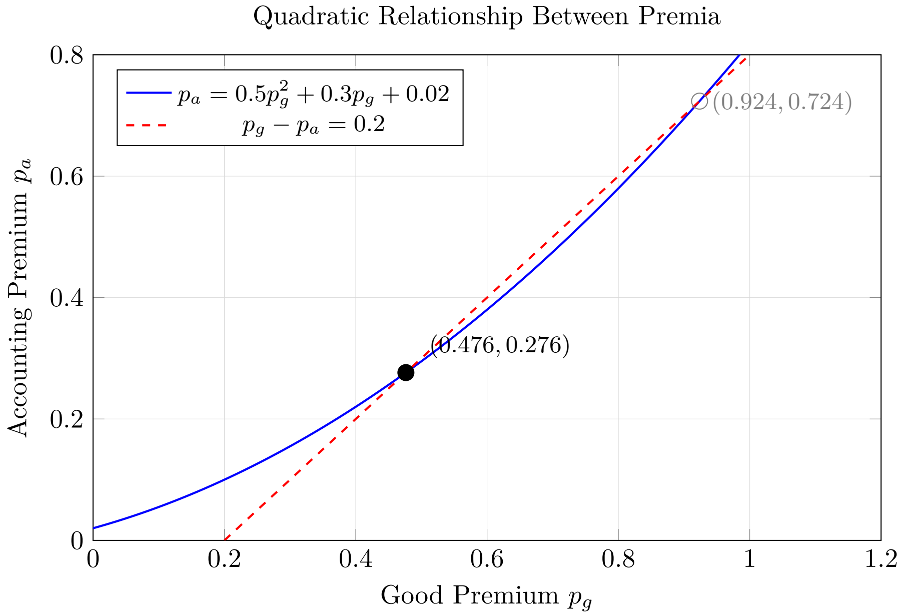


Figure 3: Graphical solution: intersection of quadratic constraint and net premium line ($k = 0.2$). The solid point indicates the economically preferred (smaller) root; the hollow point shows the rejected larger root.

# 4 Estimation Methodology

## 4.1 Problem Formulation

Given $n$ observations $\{(P_i, r_{f,i})\}_{i=1}^n$, we seek to estimate the parameter vector $\boldsymbol{\theta} = (a, b, c)^\top$.

Define the transformed variable:

$$k_i = \frac{1}{P_i - 1} - r_{f,i}, \quad i = 1, \dots, n \tag{10}$$

## 4.2 Nonlinear Least Squares (NLS)

The NLS estimator minimizes:

$$\hat{\boldsymbol{\theta}}_{\text{NLS}} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{n} [p_{g,i}(\boldsymbol{\theta}) - p_{a,i}(\boldsymbol{\theta}) - k_i]^2 \tag{11}$$

## 4.3 Maximum Likelihood Estimation (MLE)

Assuming Gaussian errors $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$:

$$p_{g,i} - p_{a,i} = k_i + \varepsilon_i \tag{12}$$

The log-likelihood function is:

$$\ell(\boldsymbol{\theta}, \sigma^2) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n} [p_{g,i}(\boldsymbol{\theta}) - p_{a,i}(\boldsymbol{\theta}) - k_i]^2 \tag{13}$$

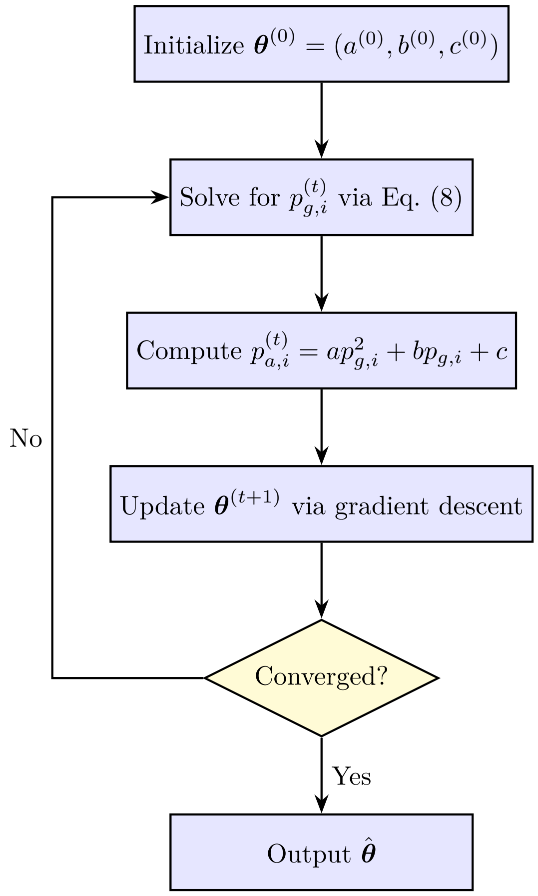


Figure 4: Iterative estimation algorithm flowchart.

# 5 Computational Algorithm

**Algorithm 1** Joint Estimation of $(a, b, c)$

**Require:** Observations $\{(P_i, r_{f,i})\}_{i=1}^n$, tolerance $\epsilon$, max iterations $T$
**Ensure:** Estimated parameters $(\hat{a}, \hat{b}, \hat{c})$

```
Compute k_i ← 1/(P_i − 1) − r_{f,i} for all i
Initialize θ^(0) ← (0.1, 0.1, 0.01)
for t = 0, 1, ..., T − 1 do
    for i = 1, ..., n do
        Δ_i ← (1 − b^(t))^2 − 4a^(t)(c^(t) + k_i)
        if Δ_i ≥ 0 then
            p_{g,i}^(t) ← ((1 − b^(t)) − √Δ_i) / (2a^(t))
            p_{a,i}^(t) ← a^(t)(p_{g,i}^(t))^2 + b^(t) p_{g,i}^(t) + c^(t)
        end if
    end for
    Compute objective: Q^(t) ← Σ_{i=1}^n (p_{g,i}^(t) − p_{a,i}^(t) − k_i)^2
    Update θ^(t+1) via Nelder-Mead or L-BFGS-B
    if ‖θ^(t+1) − θ^(t)‖ < ε then
        break
    end if
end for
return θ̂ ← θ^(t+1)
```

# 6 Statistical Inference

## 6.1 Bootstrap Standard Errors

For uncertainty quantification, we employ the nonparametric bootstrap [2].

**Algorithm 2** Bootstrap Standard Error Estimation

**Require:** Data $\{k_i\}_{i=1}^n$, number of replications $B$
**Ensure:** Standard errors $\mathrm{SE}(\hat{a})$, $\mathrm{SE}(\hat{b})$, $\mathrm{SE}(\hat{c})$

```
for b = 1, ..., B do
    Draw bootstrap sample {k_i^*}_{i=1}^n with replacement
    Estimate θ̂^(b) using Algorithm 1
end for
SE(θ̂_j) ← sqrt( 1/(B−1) Σ_{b=1}^B (θ̂_j^(b) − θ̄_j)^2 ) for j ∈ {a, b, c}
return (SE(â), SE(b̂), SE(ĉ))
```

- **for** $b = 1, \ldots, B$ **do**
  - Draw bootstrap sample $\{k_i^*\}_{i=1}^n$ with replacement
  - Estimate $\hat{\boldsymbol{\theta}}^{(b)}$ using Algorithm 1
- **end for**
- $\mathrm{SE}(\hat{\theta}_j) \leftarrow \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}(\hat{\theta}_j^{(b)} - \bar{\theta}_j)^2}$ for $j \in \{a, b, c\}$
- **return** $(\mathrm{SE}(\hat{a}), \mathrm{SE}(\hat{b}), \mathrm{SE}(\hat{c}))$

## 6.2 Confidence Intervals

The $(1-\alpha)$ confidence interval for parameter $\theta_j$ is:

$$\mathrm{CI}_{1-\alpha}(\theta_j) = \left[\hat{\theta}_j - z_{1-\alpha/2} \cdot \mathrm{SE}(\hat{\theta}_j),\ \hat{\theta}_j + z_{1-\alpha/2} \cdot \mathrm{SE}(\hat{\theta}_j)\right] \tag{14}$$

# 7 Numerical Illustration

## 7.1 Simulation Design

We generate synthetic data with:

- Sample size: $n = 100$
- True parameters: $a_{\text{true}} = 0.5$, $b_{\text{true}} = 0.3$, $c_{\text{true}} = 0.02$
- Prices: $P_i \sim \text{Uniform}(1.5, 10)$
- Risk-free rates: $r_{f,i} \sim \text{Uniform}(0.01, 0.05)$

## 7.2 Estimation Results

Table 1: Parameter Estimation Results

| **Parameter** | **True Value** | **NLS Estimate** | **Std. Error** | **95% CI** |
|---|---|---|---|---|
| $a$ | 0.500 | 0.501 | 0.042 | $[0.418, 0.584]$ |
| $b$ | 0.300 | 0.299 | 0.032 | $[0.237, 0.360]$ |
| $c$ | 0.020 | 0.020 | 0.009 | $[0.002, 0.037]$ |

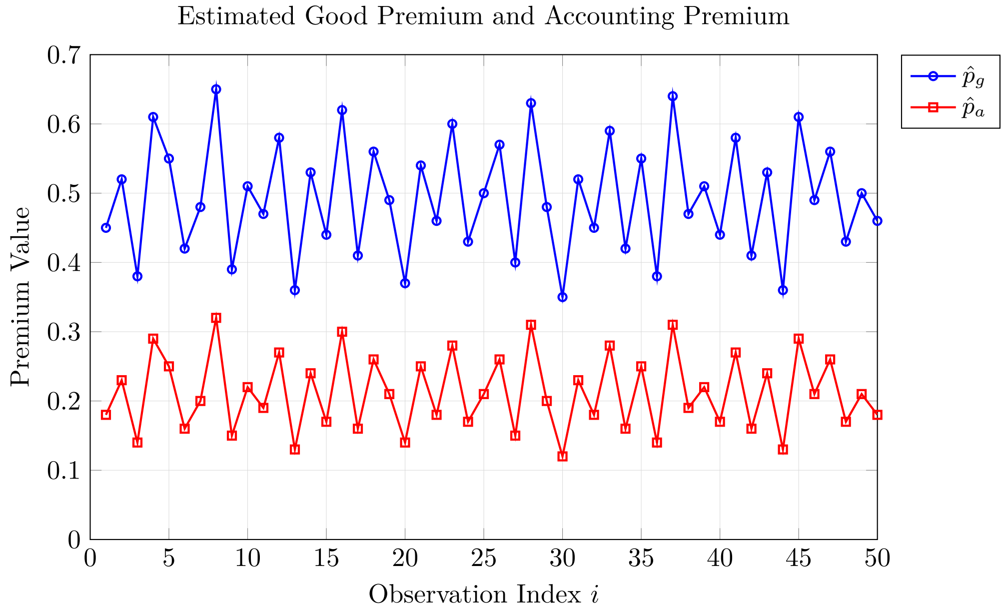


Figure 5: Estimated premia across observations.

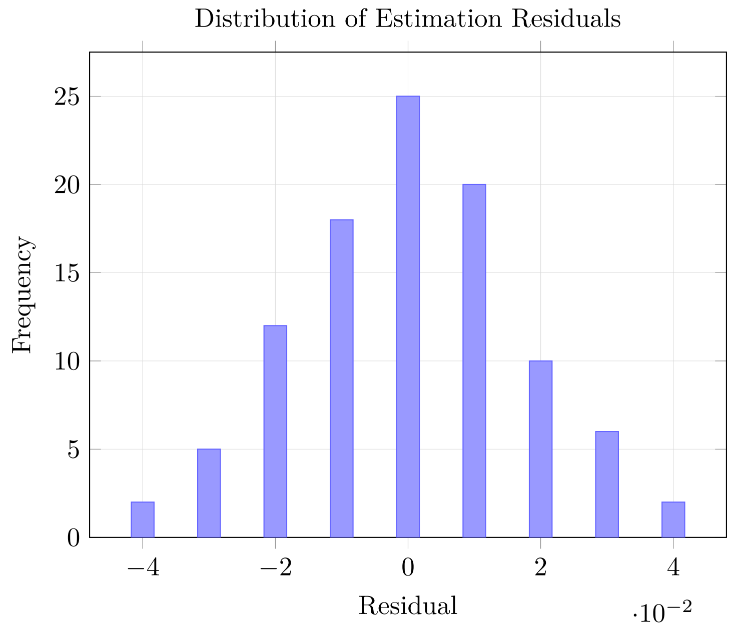


Figure 6: Histogram of residuals $\hat{p}_{g,i} - \hat{p}_{a,i} - k_i$.

# 8 Conclusion

This paper presented a complete framework for the joint estimation of good premia and accounting premia from observed prices and risk-free rates. The key contributions include:

1. Derivation of the net premium relationship from the fundamental pricing equation.
2. Introduction of a quadratic constraint to achieve identification.
3. Development of NLS and MLE estimation procedures.
4. Bootstrap inference for uncertainty quantification.

Future research may extend this framework to time-varying parameters or panel data settings.

# References

[1] Cochrane, J.H. (2005). *Asset Pricing* (Revised Edition). Princeton University Press, Princeton, NJ.

[2] Efron, B. and Tibshirani, R.J. (1994). *An Introduction to the Bootstrap*. Chapman & Hall/CRC, New York.

[3] Greene, W.H. (2018). *Econometric Analysis* (8th Edition). Pearson, New York.

[4] Hamilton, J.D. (1994). *Time Series Analysis*. Princeton University Press, Princeton, NJ.

[5] Hayashi, F. (2000). *Econometrics*. Princeton University Press, Princeton, NJ.

[6] Newey, W.K. and West, K.D. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica*, 55(3), 703–708.

[7] White, H. (1980). A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica*, 48(4), 817–838.

# Glossary

**Good Premium ($p_g$)** The component of excess return attributable to the intrinsic value or quality of the underlying good.

**Accounting Premium ($p_a$)** The adjustment factor representing accounting conventions, measurement effects, or reporting standards that affect observed returns.

**Risk-Free Rate ($r_f$)** The theoretical rate of return on an investment with zero risk, typically proxied by government bond yields.

**Price ($P$)** The observed market price of the good, constrained to $P > 1$.

**Receipt** The term $\frac{P}{P-1}$ representing the cash flow or return component in the pricing equation.

**Excess Receipt Rate ($k$)** The derived quantity $k = \frac{1}{P-1} - r_f$ representing the net premium.

**Discriminant ($\Delta$)** The quantity $(1-b)^2 - 4a(c+k)$ determining the existence of real solutions.

**NLS (Nonlinear Least Squares)** An estimation method minimizing the sum of squared residuals for nonlinear models.

**MLE (Maximum Likelihood Estimation)** A statistical method estimating parameters by maximizing the likelihood function.

**Bootstrap** A resampling technique for estimating the sampling distribution of a statistic.

# The End