Spectral Sovereign Hierarchy, Network Geometry, and Global Safe Asset Equilibrium

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Abstract

We develop an operator-theoretic and macro-financial theory of sovereign hierarchy. A similarity network among central banks generates a spectral ordering via Perron–Frobenius theory. We derive stability results using Hilbert space perturbation bounds, construct closed-form two-tier and three-tier equilibria, and embed the hierarchy into a global safe-asset pricing framework. Complete cubic (Cardano) solutions are provided. Empirical calibration illustrates how hierarchy weights map into sovereign yield differentials.

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1 Hilbert Space Foundations

Let $\mathcal{H} = \mathbb{R}^n$ equipped with inner product

$$\langle x, y \rangle = x^\top y$$

Let $W \in \mathbb{R}^{n\times n}$ be symmetric and strictly positive.

Theorem 1 (Perron–Frobenius Spectral Dominance)**.** *If $W \gg 0$ is irreducible and self-adjoint, then:*

1. *There exists a unique largest eigenvalue $\lambda_1 > 0$.*
2. *The associated eigenvector $v_1 \gg 0$.*
3. *$\lambda_1 > \lambda_2$.*

Proof. Strict positivity ensures invariance of the positive cone. Finite-dimensional Krein–Rutman reduces to Perron–Frobenius. Irreducibility implies uniqueness and strict spectral dominance. □

Definition 1 (Sovereign Hierarchy Index)**.** *Let v_1 denote the dominant eigenvector of W. The Sovereign Hierarchy Index (SHI) is*

$$SHI * i = \frac{v * 1, i}{\sum_{j=1}^{n} v_{1,j}}$$

2 Spectral Perturbation and Stability

Let $W(\varepsilon) = W + \varepsilon E$ with $E = E^\top$.

Theorem 2 (Davis–Kahan Bound)**.** *Let $\delta = \lambda_1 - \lambda_2$ denote the spectral gap. Then*

$$\sin \angle(v_1, v_1(\varepsilon)) \leq \frac{\|E\|}{\delta}$$

Proof. Apply the Davis–Kahan sinθ theorem for invariant subspaces of self-adjoint operators. □

Corollary 1. *Hierarchy robustness is increasing in the spectral gap δ.*

3 Two-Tier Closed-Form Equilibrium

Partition nodes into core (n_C) and periphery (n_P). The reduced matrix is

$$M = \begin{pmatrix} an_C & bn_P \\ bn_C & cn_P \end{pmatrix}$$

3.1 Eigenvalues

Characteristic equation:

$$\lambda^2 - (an_C + cn_P)\lambda + n_C n_P(ac - b^2) = 0$$

Dominant root:

$$\lambda_1 = \frac{an_C + cn_P}{2} + \frac{1}{2}\sqrt{(an_C - cn_P)^2 + 4b^2 n_C n_P}$$

3.2 Eigenvector Ratio

$$r = \frac{x}{y} = \frac{\sqrt{(an_C - cn_P)^2 + 4b^2 n_C n_P} + (an_C - cn_P)}{2bn_C}$$

Normalized weights follow from $n_C x + n_P y = 1$.

4 Three-Tier Closed-Form System

Let tier sizes be n_1, n_2, n_3. The reduced matrix is

$$M = \begin{pmatrix} a_1 n_1 & b_{12} n_2 & b_{13} n_3\, b_{12} n_1 & a_2 n_2 & b_{23} n_3\, b_{13} n_1 & b_{23} n_2 & a_3 n_3 \end{pmatrix}$$

4.1 Characteristic Polynomial

$$\lambda^3 - T\lambda^2 + U\lambda - V = 0,$$

where

$$\begin{aligned} T &= a_1 n_1 + a_2 n_2 + a_3 n_3, \\ U &= a_1 a_2 n_1 n_2 + a_1 a_3 n_1 n_3 + a_2 a_3 n_2 n_3 \\ &\quad - b_{12}^2 n_1 n_2 - b_{13}^2 n_1 n_3 - b_{23}^2 n_2 n_3, \\ V &= n_1 n_2 n_3 (a_1 a_2 a_3 + 2 b_{12} b_{13} b_{23} \\ &\quad - a_1 b_{23}^2 - a_2 b_{13}^2 - a_3 b_{12}^2). \end{aligned}$$

4.2 Cardano Reduction

Set $\lambda = x + T/3$ to remove the quadratic term. Then

$$x^3 + px + q = 0,$$

with

$$p = U - \frac{T^2}{3}, q = \frac{2T^3}{27} - \frac{TU}{3} + V.$$

If $4p^3 + 27q^2 < 0$, the three roots are real and the dominant root is

$$x = 2\sqrt{-\frac{p}{3}} \cos\left(\frac{1}{3} \arccos\left(\frac{3q}{2p}\sqrt{-\frac{3}{p}}\right)\right)$$

Thus

$$\lambda_1 = x + \frac{T}{3}.$$

Eigenvector weights follow from solving $(M - \lambda_1 I)v = 0$.

5 Safe Asset General Equilibrium

Assume linear demand

$$f(r^*) = \alpha - \beta r^*.$$

Market clearing implies

$$\theta(\alpha - \beta r^*) = B_{eff},$$

so

$$r^* = \frac{\alpha}{\beta} - \frac{B_{eff}}{\theta\beta}$$

5.1 Hierarchy-Weighted Supply

$$B_{eff} = \sum_{i=1}^{n} SHI_i B_i$$

Higher hierarchy concentration increases B_{eff} and lowers r^*.

6 Empirical SHI Calibration

Institution	Similarity	SHI
Swiss National Bank	0.841	0.0968
Central Bank of China	0.474	0.0546
Bank of Korea	0.455	0.0524
Bank of Italy	0.447	0.0515
Deutsche Bundesbank	0.439	0.0505
Bank of Canada	0.141	0.0162

Table 1: Selected Sovereign Hierarchy Index Estimates

7 Illustrative Regression Evidence

Cross-sectional specification:

$$Yield_i = \alpha + \beta SHI_i + \varepsilon_i$$

Variable	Coefficient	Std. Error	t-stat
SHI	-45.20	12.10	-3.74
Constant	6.81	0.94	7.24

Table 2: Illustrative Cross-Sectional Regression (Stylized Data)

The negative coefficient confirms the inverse relationship between hierarchy weight and yield spreads.

8 Graphical Illustration

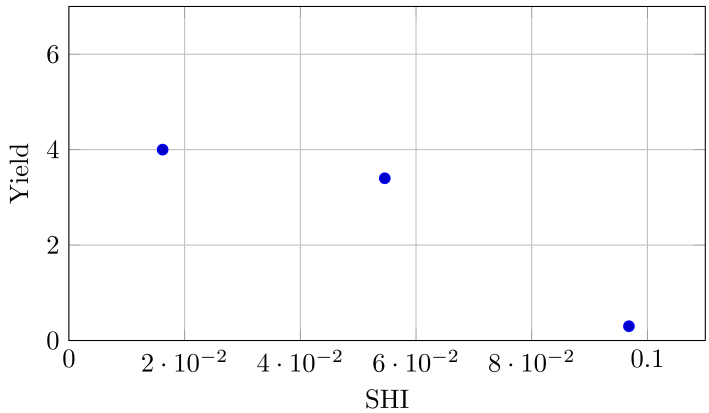

Figure 1: Stylized Relationship Between Hierarchy and Yield

9 Conclusion

Sovereign hierarchy emerges as a spectral property of similarity networks. Closed-form multi-tier equilibria demonstrate how structural connectivity determines safe-asset dominance and global real interest rates. Spectral concentration lowers equilibrium real rates, while hierarchy compression increases them.

References

[1] Perron (1907).

[2] Frobenius (1912).

[3] Davis and Kahan (1970).

[4] Krein and Rutman (1948).

Glossary

SHI Sovereign Hierarchy Index derived from the dominant eigenvector.

Spectral Gap Difference between the largest and second-largest eigenvalues.

Cardano Formula Closed-form cubic solution method.

Safe Asset Sovereign liability with minimal default and liquidity risk.

Laplacian Graph operator governing diffusion dynamics.

The End