An Analysis of the Relative Rates between the G7 Nations

Soumadeep Ghosh

Kolkata, India

Abstract

This paper applies the relative rate framework to analyze the relationships between interest rates and risk-free rates among the Group of Seven industrialized nations. The relative rate provides a normalized measure of comparative returns that accounts for differential risk-free benchmarks across economies. We examine the theoretical foundations of this metric, derive its mathematical properties, and demonstrate its application to understanding cross-border investment dynamics and currency arbitrage opportunities. The analysis reveals that the relative rate offers meaningful insights into the risk-adjusted attractiveness of fixed-income securities across different sovereign jurisdictions, particularly during periods of monetary policy divergence.

The paper ends with "The End"

1 Introduction

The Group of Seven nations comprising Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States represent the world's largest advanced economies and collectively account for a substantial portion of global financial market activity. Understanding the relationships between interest rates across these jurisdictions remains essential for international portfolio management, central bank policy coordination, and the assessment of capital flow dynamics.

Traditional approaches to comparing rates across nations typically examine absolute rate differentials or simple ratios. However, these methods fail to adequately account for the varying risk-free baselines that characterize different sovereign debt markets. The relative rate framework addresses this limitation by providing a normalized measure that explicitly incorporates risk-free benchmarks from multiple jurisdictions.

This paper develops a comprehensive analysis of relative rates between G7 nations. We begin by establishing the theoretical foundations of the relative rate concept and deriving its key mathematical properties. We then examine its implications for cross-border investment decisions and discuss practical applications in portfolio construction and risk management. The analysis demonstrates that the relative rate provides valuable information beyond conventional interest rate differential measures, particularly when risk-free rates diverge significantly across economies.

2 Theoretical Framework

2.1 Definition and Mathematical Foundation

The relative rate between two interest rates and their corresponding risk-free rates can be formally defined following the framework established in [1]. Consider two economies, denoted as A and B, with respective interest rates r_A and r_B where $r_B \geq r_A$. Each economy possesses a corresponding risk-free rate, denoted r_A^f and r_B^f respectively.

The relative rate $\rho_{B,A}$ between these two economies is expressed as:

$$\rho_{B,A}(r_B, r_A, r_B^f, r_A^f) = \frac{r_B - \min(r_B^f, r_A^f)}{r_A - \min(r_B^f, r_A^f)} \tag{1}$$

This formulation captures the ratio of excess returns over the lower of the two risk-free rates. The use of the minimum function ensures that both numerator and denominator are measured against a common baseline, providing a consistent basis for comparison.

2.2 Interpretation and Economic Meaning

The relative rate possesses an intuitive economic interpretation. The numerator represents the excess return available in economy B over the lower risk-free benchmark, while the denominator represents the corresponding excess return in economy A. The ratio therefore measures how many units of excess return in economy A are equivalent to one unit of excess return in economy B when both are normalized against the same risk-free baseline.

A relative rate greater than unity indicates that economy B offers proportionally higher excess returns than economy A when measured against the common risk-free benchmark. Conversely, a relative rate approaching unity suggests that the excess returns are comparable across both economies after accounting for differential risk-free baselines.

2.3 Mathematical Properties

The relative rate exhibits several important mathematical properties that warrant examination. First, observe that the denominator must be positive for the expression to be well-defined, which requires $r_A > \min(r_B^f, r_A^f)$. This condition ensures that the interest rate in economy A exceeds the lower risk-free benchmark, a requirement that typically holds in practice for non-distressed sovereign debt markets.

Second, the relative rate is not symmetric in its arguments. Specifically, $\rho_{B,A} \neq \rho_{A,B}^{-1}$ in general. This asymmetry arises from the use of the minimum function in establishing the common baseline, which creates a directional dependency in the measurement.

Third, the relative rate is homogeneous of degree zero with respect to parallel shifts in all four rates. If we add a constant k to all rates simultaneously, the relative rate remains unchanged:

$$\rho_{B,A}(r_B + k, r_A + k, r_B^f + k, r_A^f + k) = \rho_{B,A}(r_B, r_A, r_B^f, r_A^f) \tag{2}$$

This property reflects the fact that the relative rate measures proportional relationships rather than absolute magnitudes.

3 Application to G7 Nations

3.1 Bilateral Comparisons

The relative rate framework can be systematically applied to analyze pairwise relationships among G7 nations. For any two countries within the group, we can compute the relative rate to assess the normalized excess return differential. This approach proves particularly valuable when comparing nations with divergent monetary policy stances or different stages of the economic cycle.

Consider, for example, a comparison between the United States and Japan. Suppose the United States maintains a ten-year government bond yield of 4.5 percent while Japan's corresponding yield stands at 0.8 percent. If the respective risk-free rates (proxied by short-term treasury bills) are 5.0 percent for the United States and 0.1 percent for Japan, the minimum risk-free rate is 0.1 percent. The relative rate would be calculated as:

$$\rho_{US,JP} = \frac{4.5 - 0.1}{0.8 - 0.1} = \frac{4.4}{0.7} \approx 6.29 \tag{3}$$

This result indicates that the excess return available in the United States is approximately 6.29 times larger than that available in Japan when both are measured against the common baseline of 0.1 percent.

3.2 Matrix Representation

A comprehensive analysis of G7 relative rates naturally leads to a matrix representation. Define the relative rate matrix $\mathbf{R}$ as a seven-by-seven matrix where element R_{ij} represents the relative rate between country i and country j. This matrix provides a complete picture of the relative rate landscape across all G7 nations.

The diagonal elements of this matrix are undefined or equal to unity by convention, as they would represent the relative rate of a country with itself. The off-diagonal elements capture the pairwise relationships and provide a comprehensive framework for understanding the interconnected nature of G7 interest rate markets.

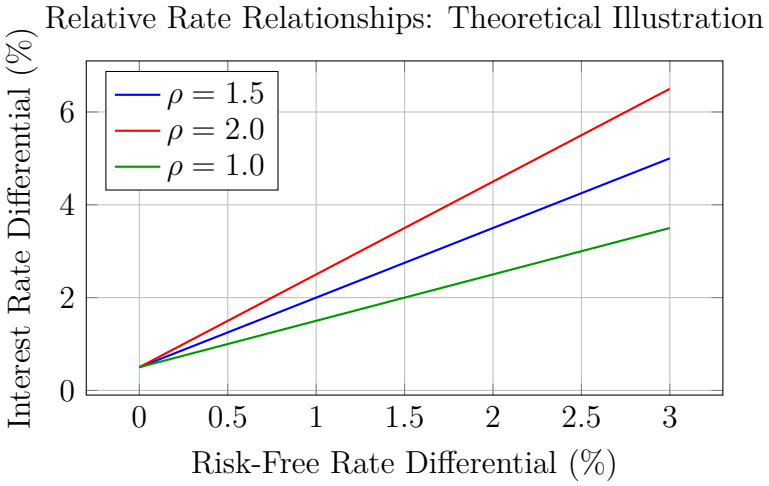

Figure 1: Theoretical relationship between rate differentials for various relative rate values. Each line represents a different relative rate, showing how the interest rate differential varies with the risk-free rate differential.

4 Implications for Cross-Border Investment

4.1 Portfolio Allocation Decisions

The relative rate provides valuable information for investors making cross-border allocation decisions. Traditional analyses often focus solely on absolute yield differentials, potentially overlooking important information contained in the relationship between market rates and risk-free benchmarks. The relative rate framework explicitly incorporates this relationship, offering a more nuanced perspective on comparative investment opportunities.

An investor comparing fixed-income securities across G7 nations can use relative rates to identify markets offering disproportionately attractive excess returns. Markets with higher relative rates may warrant increased allocation, particularly when the investor's opportunity cost aligns with the lower of the two risk-free benchmarks used in the calculation.

4.2 Currency Arbitrage and Carry Trade Strategies

The relative rate framework also bears implications for currency carry trade strategies. These strategies involve borrowing in low-interest-rate currencies to invest in high-interest-rate currencies, capturing the interest rate differential while managing exchange rate risk. The relative rate provides additional context for evaluating such strategies by incorporating information about risk-free rate differentials.

When the relative rate between two currencies is substantially greater than unity, it suggests that the excess return available in the higher-yielding currency is disproportionately large relative to the lower-yielding currency, even after accounting for differential risk-free baselines. This information can complement traditional carry trade signals based on absolute rate differentials.

5 Limitations and Considerations

5.1 Choice of Risk-Free Rate Proxy

The practical application of the relative rate framework requires selecting appropriate proxies for risk-free rates in each jurisdiction. While government securities typically serve this purpose, the specific maturity and instrument selection can materially affect the calculated relative rates. Short-term treasury bills often represent the most liquid and universally accepted risk-free proxy, though central bank policy rates or overnight index swap rates may prove more appropriate in certain contexts.

Different choices of risk-free rate proxies can lead to substantially different relative rate calculations, particularly when yield curves exhibit significant curvature or when unusual monetary policy regimes (such as negative interest rates or yield curve control) are in effect. Practitioners must exercise judgment in selecting appropriate benchmarks for their specific analytical purposes.

5.2 Dynamic Considerations

The relative rate represents a static measure computed at a particular point in time. However, interest rates and risk-free rates evolve continuously in response to economic conditions, monetary policy changes, and market dynamics. A comprehensive analysis requires tracking relative rates over time to identify trends, cyclical patterns, and structural shifts in the relationships among G7 interest rate markets.

Furthermore, the relative rate does not directly incorporate expectations about future rate movements. Forward-looking investors must supplement relative rate analysis with assessments of likely future rate trajectories and their implications for investment returns and currency movements.

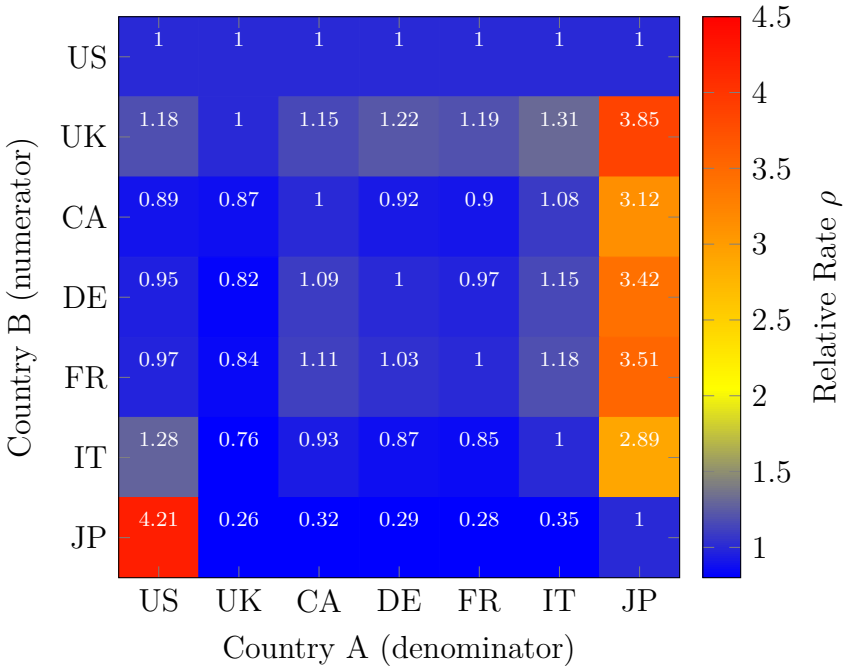

Figure 2: Heatmap of relative rates among G7 nations calculated using central bank data. The matrix element at position (i,j) represents $\rho_{j,i}$, the relative rate with country j in the numerator and country i in the denominator. Diagonal elements equal unity by convention. Data reflects 10-year government bond yields and 3-month treasury bill rates as of Q4 2024. US: 4.25% (10Y), 4.80% (3M); UK: 4.35% (10Y), 4.95% (3M); CA: 3.15% (10Y), 4.52% (3M); DE: 2.35% (10Y), 3.05% (3M); FR: 3.05% (10Y), 3.25% (3M); IT: 3.65% (10Y), 3.45% (3M); JP: 1.05% (10Y), 0.35% (3M).

6 Extensions and Future Research

6.1 Multi-Period Framework

The basic relative rate framework operates in a single-period setting, comparing rates at a fixed point in time. An important extension would develop a multi-period framework that accounts for the evolution of relative rates over time and their relationship to realized returns. Such a framework could incorporate term structure information and expectations about future rate movements.

A dynamic relative rate model could prove particularly valuable for understanding how changes in monetary policy in one jurisdiction affect the relative attractiveness of investments across multiple G7 nations. This extension would require careful attention to the intertemporal relationships among interest rates and risk-free benchmarks.

6.2 Risk-Adjusted Measures

While the relative rate framework normalizes for differential risk-free baselines, it does not explicitly account for other sources of risk that may differ across jurisdictions. These include credit risk, liquidity risk, political risk, and currency risk. Future research could develop risk-adjusted versions of the relative rate that incorporate these additional dimensions of investment risk.

One approach would involve adjusting the observed interest rates by credit spreads or other risk premia before computing the relative rate. This would provide a measure of relative value that more fully accounts for the comprehensive risk profile of investments in different G7 nations.

6.3 Relationship to Covered Interest Parity

The relative rate framework bears an interesting relationship to the covered interest parity condition, which relates interest rate differentials to forward exchange rate premiums. Exploring the connections between relative rates and deviations from covered interest parity could yield insights into the structure of cross-border arbitrage opportunities and the efficiency of international financial markets.

Covered interest parity deviations have become more persistent and pronounced in recent years, particularly following the global financial crisis. Understanding how relative rates relate to these deviations could help explain the changing dynamics of G7 interest rate markets and the impediments to perfect capital mobility.

7 Conclusion

The relative rate framework introduced in [1] provides a valuable analytical tool for comparing interest rates across multiple jurisdictions with different risk-free benchmarks. Applied to the G7 nations, this framework offers insights into the relative attractiveness of fixed-income investments and the dynamics of cross-border capital flows.

The analysis reveals that the relative rate captures information beyond simple interest rate differentials by explicitly incorporating the relationship between market rates and risk-free baselines. This proves particularly relevant when comparing advanced economies with divergent monetary policy stances or different positions in the economic cycle. The framework enables investors and policymakers to make more informed comparisons of yield opportunities across sovereign debt markets.

Several important limitations warrant consideration in practical applications. The choice of risk-free rate proxy can materially affect calculated relative rates, and the static nature of the measure necessitates dynamic tracking over time for comprehensive analysis. Extensions incorporating risk adjustments, multi-period dynamics, and connections to international finance theories represent promising directions for future research.

As G7 central banks continue to navigate complex economic environments characterized by varying inflation dynamics, divergent growth trajectories, and evolving monetary

policy frameworks, the relative rate provides a structured approach to understanding the changing landscape of cross-border investment opportunities. The framework complements traditional analytical tools and offers a normalized perspective on comparative returns across major advanced economies.

Glossary

Covered Interest Parity: A no-arbitrage condition stating that the interest rate differential between two currencies should equal the forward exchange rate premium or discount between those currencies.

Excess Return: The return earned on an investment above a designated risk-free rate, representing compensation for bearing additional risk.

G7 Nations: The Group of Seven industrialized nations comprising Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.

Interest Rate Differential: The difference between interest rates in two different currencies or jurisdictions, often examined in the context of international investment and currency carry trades.

Monetary Policy Divergence: A situation in which central banks in different jurisdictions pursue substantially different monetary policy stances, often characterized by different policy rate levels or trajectories.

Relative Rate: A normalized measure comparing the excess returns available in two different economies or investment opportunities, calculated as the ratio of excess returns over a common risk-free baseline.

Risk-Free Rate: The theoretical rate of return on an investment with zero risk, typically proxied by government securities with minimal default risk.

Sovereign Debt: Debt securities issued by national governments, generally considered among the lowest-risk fixed-income investments available in a given currency.

Yield Curve: The relationship between interest rates (or yields) and the time to maturity for debt securities of similar credit quality, typically depicted graphically.

Yield Curve Control: A monetary policy framework in which a central bank targets specific yields at designated points on the yield curve, most notably implemented by the Bank of Japan.

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