The mathematics and the paradox of the duel

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Abstract

In this paper, I describe the mathematics and the paradox of the duel.
The paper ends with "The End"

Introduction

There is demand of knowledge of the mathematics and the paradox of **the duel**.

In this paper, I describe the mathematics and the paradox of the duel.

To do so, I first describe two types of duel: the **ideal duel** and the **practical duel** and then describe **the paradox of the duel**.

The ideal duel

Preliminaries

1. The duel is **fair**.
2. Each of the two participants can **fire** a **weapon** with a probability to **kill** the other participant.
3. Both weapons **don't backfire** on the participant firing the weapon.

The mathematics of the ideal duel

The mathematics of the ideal duel is

$$f_0 + f_A + f_B + f_A f_B = 1$$

$$k_0 + k_A + k_B + k_A k_B = 1$$

$$f_0 k_0 + f_A k_B + f_B k_A + f_A f_B k_B k_A = 1$$

where
f_0 is the probability that neither participant fires their weapons
f_A is the probability that participant A fires his/her weapon
f_B is the probability that participant B fires his/her weapon
k_0 is the probability that neither participant is killed
k_A is the probability that participant A is killed
k_B is the probability that participant B is killed

The solutions to the mathematics of the ideal duel

The solutions to mathematics of the ideal duel are

1. Neither fires, neither are killed:

$$f_0 = 1, f_A = 0, f_B = 0, k_0 = 1, k_A = 0, k_B = 0$$

2. A fires, B is killed:

$$f_0 = 0, f_A = 1, f_B = 0, k_0 = 0, k_A = 0, k_B = 1$$

3. B fires, A is killed:

$$f_0 = 0, f_A = 0, f_B = 1, k_0 = 0, k_A = 1, k_B = 0$$

The practical duel

Preliminaries

1. The duel is **fair**.
2. Each of the two participants can **fire** a **weapon** with a probability to **kill** the other participant.
3. Both weapons **may backfire** on the participant firing the weapon.

The mathematics of the practical duel

The mathematics of the practical duel is

$$f_0 + f_A + f_B + f_A f_B = 1$$

$$k_0 + k_A + k_B + k_A k_B = 1$$

$$f_0 k_0 + f_A(k_B + k_A) + f_B(k_A + k_B) + f_A f_B k_B k_A = 1$$

where
f_0 is the probability that neither participant fires their weapons
f_A is the probability that participant A fires his/her weapon
f_B is the probability that participant B fires his/her weapon
k_0 is the probability that neither participant is killed
k_A is the probability that participant A is killed
k_B is the probability that participant B is killed

The solutions to the mathematics of the practical duel

The solutions to mathematics of the practical duel are

1. Neither fires, neither are killed:
$$f_0 = 1, f_A = 0, f_B = 0, k_0 = 1, k_A = 0, k_B = 0$$
2. A backfires, A is killed:
$$f_0 = 0, f_A = 1, f_B = 0, k_0 = 0, k_A = 1, k_B = 0$$
3. A fires, B is killed:
$$f_0 = 0, f_A = 1, f_B = 0, k_0 = 0, k_A = 0, k_B = 1$$
4. B backfires, B is killed:
$$f_0 = 0, f_A = 0, f_B = 1, k_0 = 0, k_A = 0, k_B = 1$$
5. B fires, A is killed:
$$f_0 = 0, f_A = 0, f_B = 1, k_0 = 0, k_A = 1, k_B = 0$$

The paradox of the duel

Note that the **frequentist probability by counting solutions** that A (or B) is killed in the ideal duel is $\frac{1}{3}$. Note that the **frequentist probability by counting solutions** that A (or B) is killed in the practical duel is $\frac{2}{5}$. Both these probabilities are not found in the solutions of neither the ideal duel nor the practical duel.

Hence, the **paradox of the duel**.

The End