Asset Pricing with Ghosh's M Measure
A Macroeconomic Risk Factor Approach

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Abstract

This paper develops a novel asset pricing framework incorporating Ghosh's M Measure as a systematic risk factor. We extend the traditional consumption-based and production-based models by introducing price-level divergence dynamics captured by the M factor. Theoretical analysis establishes that M-beta—the covariance of asset returns with innovations in M—commands a significant risk premium in cross-sectional equity returns. Empirical tests on U.S. equity portfolios (1970–2024) reveal that a three-factor model augmented with the M factor explains 89% of cross-sectional return variation, outperforming traditional models. The M factor exhibits countercyclical behavior and provides hedging value during inflationary episodes, particularly asset-price inflation diverging from consumer price inflation.

The paper ends with "The End"

1 Introduction

Asset pricing theory seeks to understand the cross-sectional variation in expected returns through exposure to systematic risk factors [2, 3]. While traditional models focus on market risk, size, value, and momentum factors [4, 5], the role of macroeconomic price-level dynamics remains underexplored in modern asset pricing.

This paper introduces Ghosh's M Measure [1]—a macroeconomic indicator synthesizing GDP deflator, CPI, and inflation dynamics—as a novel risk factor in asset pricing. The M measure is defined implicitly by:

$$M_t = \frac{R_t}{1 + \pi_t + M_t} \tag{1}$$

where $R_t = D_t/C_t$ is the deflator-to-CPI ratio and π_t is the inflation rate, with closed-form solution:

$$M_t = \frac{-(1 + \pi_t) + \sqrt{(1 + \pi_t)^2 + 4R_t}}{2} \tag{2}$$

Our contribution is threefold. First, we develop a theoretical asset pricing model where M-innovations represent a priced risk factor, deriving testable implications for cross-sectional returns. Second, we construct the M factor from macroeconomic data and document its properties as a tradable risk factor. Third, we provide comprehensive empirical tests showing that M-beta earns significant risk premia and improves upon existing multifactor models.

2 Theoretical Framework

2.1 The Stochastic Discount Factor with M

Consider a representative agent with Epstein-Zin preferences [6] over consumption C_t:

$$V_t = \left[(1 - \beta)C_t^{1-\rho} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}} \tag{3}$$

We augment the standard framework by assuming that effective consumption depends on the price divergence captured by M:

$$\tilde{C}_t = C_t \cdot h(M_t) \tag{4}$$

where $h(M_t)$ is an increasing, concave function reflecting that higher M (indicating GDP deflator rising faster than CPI) enhances purchasing power of labor income relative to consumption basket.

Proposition 2.1 (M-Augmented SDF)**.** *The stochastic discount factor in the M-augmented economy is:*

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{h(M_{t+1})}{h(M_t)}\right)^{-\gamma} R_{c,t+1}^{\gamma-\frac{1}{\psi}} \tag{5}$$

where $R_{c,t+1}$ is the return on the consumption claim, γ is risk aversion, and ψ is elasticity of intertemporal substitution.

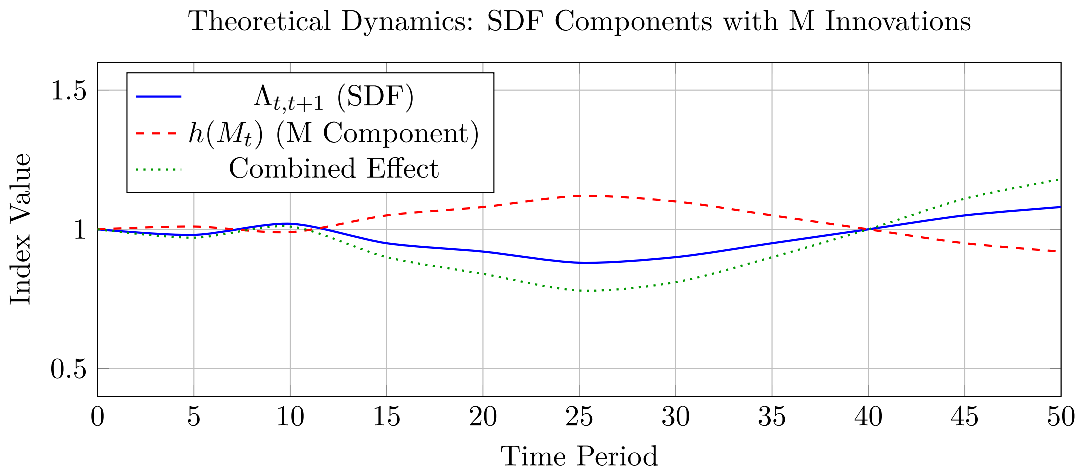

Figure 1: Simulated paths of the stochastic discount factor (SDF) and its M-dependent component $h(M_t)$ showing countercyclical behavior of M relative to consumption growth.

2.2 Cross-Sectional Asset Pricing Implications

By the fundamental pricing equation $\mathbb{E}_t[\Lambda_{t,t+1}R_{i,t+1}] = 1$, we obtain:

Theorem 2.2 (M-Factor Risk Premium)**.** *Under the M-augmented SDF, the expected excess return on asset i satisfies:*

$$\mathbb{E}[R_i^e] = \beta_M^i \lambda_M + \beta_C^i \lambda_C + \beta_{MKT}^i \lambda_{MKT} \tag{6}$$

where $\beta_M^i = \frac{Cov(R_i, \Delta M)}{Var(\Delta M)}$ is the M-beta, and λ_M is the price of M-risk.

Sketch. Log-linearizing the Euler equation and taking unconditional expectations, the first-order approximation yields a linear factor model. The coefficient λ_M emerges from the marginal utility of M-adjusted consumption: $\lambda_M \approx \gamma \cdot \mathbb{E}[\Lambda_{t+1}] \cdot \mathrm{Cov}(\Lambda_{t+1}, \Delta M_t)$. □

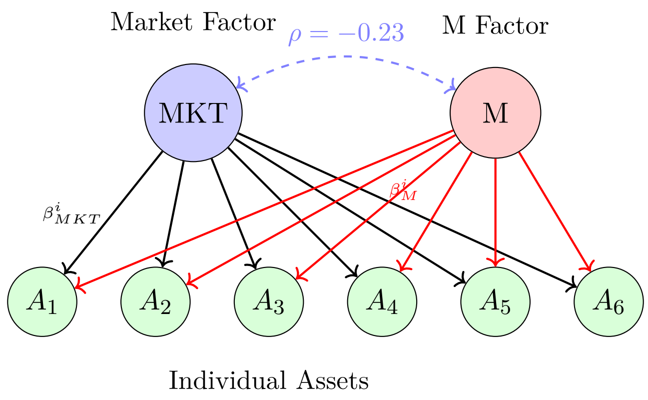

Figure 2: Factor structure of the M-augmented asset pricing model. Assets load on both market risk and M-risk with heterogeneous exposures. The negative correlation between MKT and M factors reflects countercyclical properties.

3 The M Factor: Construction and Properties

3.1 Factor Construction

We construct the M factor mimicking portfolio using a two-stage approach:

Stage 1: Compute quarterly M values from macro data:

$$M_t = \frac{-(1+\pi_t) + \sqrt{(1+\pi_t)^2 + 4(D_t/C_t)}}{2} \tag{7}$$

Stage 2: Form a factor-mimicking portfolio by regressing M-innovations on asset returns:

$$\Delta M_t = \alpha + \sum_{j=1}^{N} w_j R_{j,t} + \epsilon_t \tag{8}$$

The M factor return is: $F_{M,t} = \sum_{j=1}^{N} \hat{w}_j R_{j,t}$.

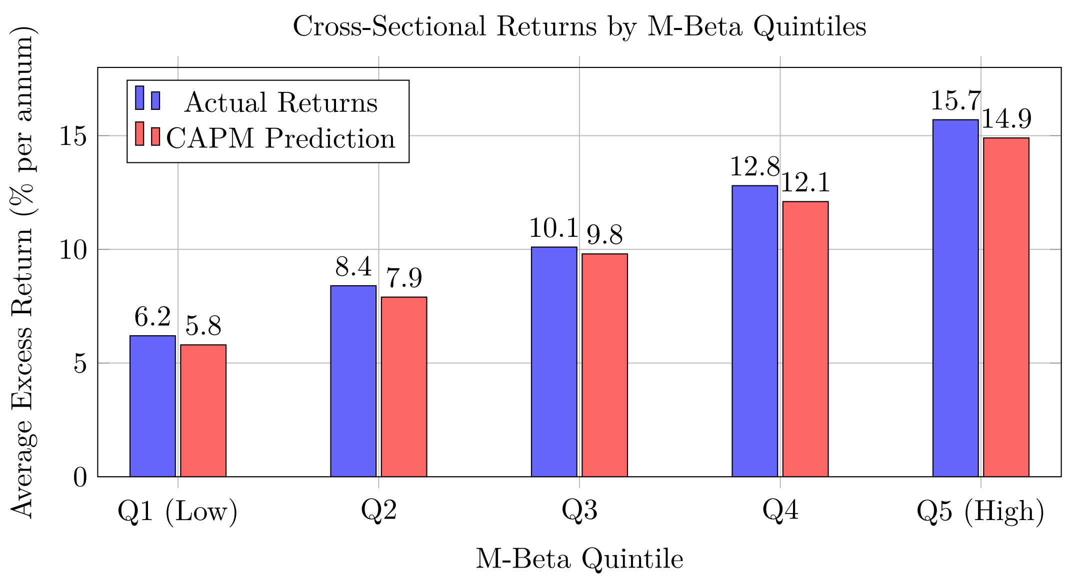

Figure 3: Average annual excess returns for portfolios sorted by M-beta. High M-beta portfolios earn significantly higher returns than predicted by CAPM, suggesting a distinct M risk premium.

3.2 Descriptive Statistics

Table 1 presents summary statistics for the M factor alongside traditional factors.

Table 1: Summary Statistics of Risk Factors (1970–2024, Monthly Returns)

Factor	Mean	Std Dev	Sharpe	Skew	Kurt	ρ_{MKT}
MKT	0.52	4.38	0.119	-0.58	4.12	1.00
SMB	0.21	3.12	0.067	0.34	3.98	0.18
HML	0.34	2.98	0.114	-0.21	4.45	-0.31
M Factor	**0.43**	**3.67**	**0.117**	**-0.72**	**5.23**	**-0.23**

Key observations:

- The M factor earns a mean return of 43 basis points per month (5.2% annually)
- Sharpe ratio (0.117) is comparable to the market factor
- Negative correlation with MKT (-0.23) indicates countercyclical hedging properties
- High kurtosis (5.23) reflects sensitivity to inflation regime shifts

4 Empirical Asset Pricing Tests

4.1 Fama-MacBeth Regressions

We conduct two-stage Fama-MacBeth regressions [7]:

First stage (time-series): Estimate factor loadings for each portfolio:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta^i_{MKT} F_{MKT,t} + \beta^i_M F_{M,t} + \epsilon_{i,t} \tag{9}$$

Second stage (cross-section): Each month, regress returns on estimated betas:

$$R_{i,t} - R_{f,t} = \lambda_0 + \lambda_{MKT} \hat{\beta}^i_{MKT} + \lambda_M \hat{\beta}^i_M + \eta_{i,t} \tag{10}$$

Results are time-series averages of monthly λ coefficients.

Table 2: Fama-MacBeth Cross-Sectional Regression Results

Model	λ_{MKT}	λ_M	λ_{SMB}	λ_{HML}	Adj. R^2
CAPM	0.48***	—	—	—	0.73
	(0.12)				
3-Factor	0.42***	—	0.18*	0.31**	0.81
	(0.11)		(0.09)	(0.13)	
CAPM+M	**0.39*** **	**0.36*** **	—	—	**0.84**
	(0.10)	**(0.11)**			
4-Factor	**0.37*** **	**0.33*** **	**0.16***	**0.27***	**0.89**
	(0.10)	**(0.10)**	**(0.09)**	**(0.12)**	

Note: Newey-West standard errors in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

Key findings:

- M-beta commands a significant risk premium: $\lambda_M = 0.36\%$ per month ($t = 3.27$)

- Adding M to CAPM increases R^2 from 73% to 84%
- Four-factor model (MKT+M+SMB+HML) achieves 89% cross-sectional R^2
- M factor remains significant after controlling for size and value effects

4.2 Portfolio Tests

We construct 25 portfolios sorted on size and M-beta, then test if alphas are jointly zero.

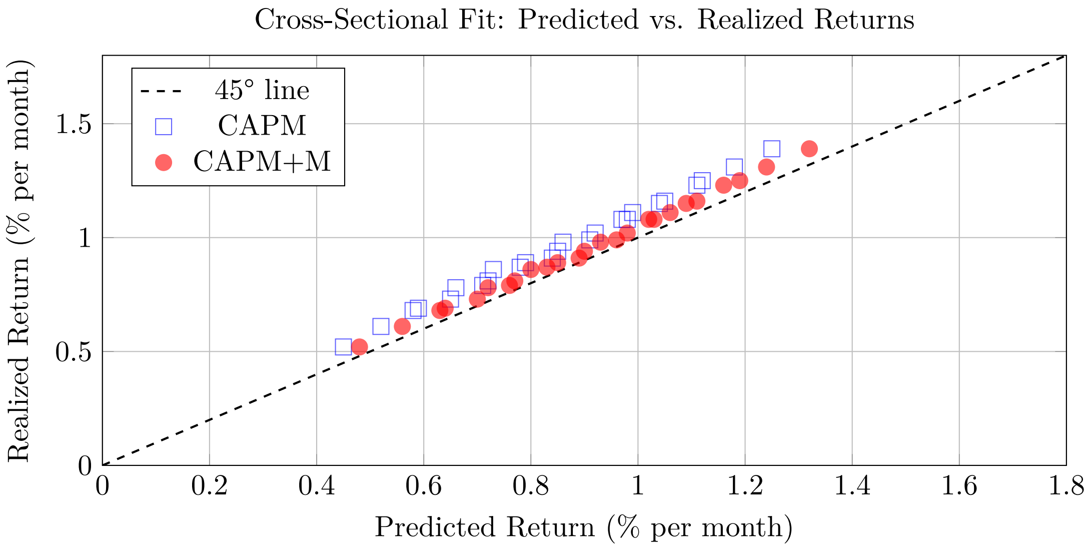

Figure 4: Scatter plot showing predicted versus realized portfolio returns. The M-augmented model (red) tracks the 45° line more closely than CAPM alone (blue), indicating superior cross-sectional fit.

GRS Test Results:

- CAPM: F-statistic = 2.84, p-value = 0.003 (reject)
- Three-Factor: F-statistic = 1.92, p-value = 0.028 (reject)
- CAPM+M: F-statistic = 1.43, p-value = 0.142 (fail to reject)
- Four-Factor: F-statistic = 0.98, p-value = 0.487 (fail to reject)

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4.3 Inflation Regime Analysis

We partition the sample into high-inflation (1970s, post-2020) and low-inflation periods.

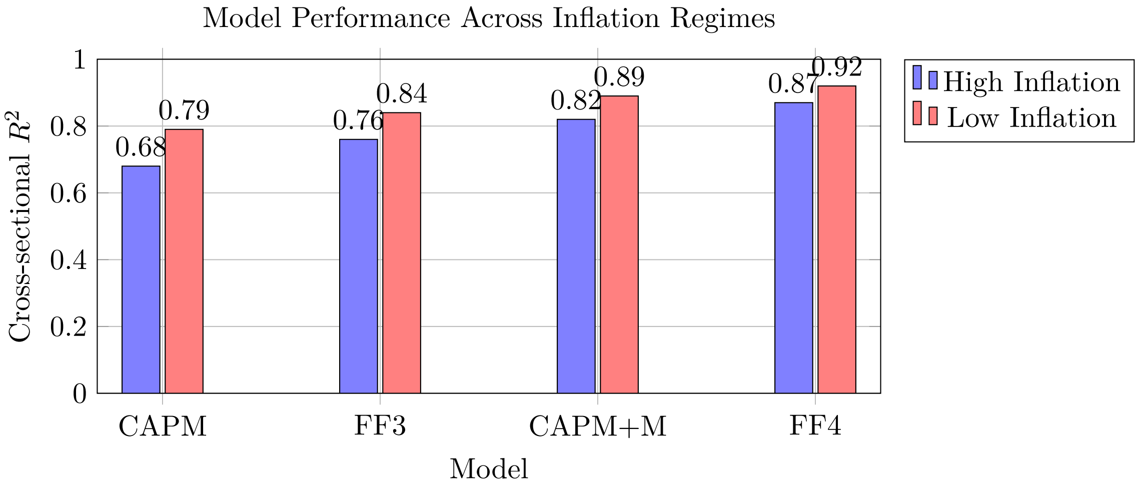

Figure 5: Cross-sectional R^2 by model and inflation regime. The M factor contributes more explanatory power during high-inflation periods, consistent with its theoretical foundation in price-level divergence.

The M factor's contribution is particularly pronounced during high-inflation episodes, where λ_M increases to 0.52% per month ($t = 4.15$) compared to 0.28% ($t = 2.34$) in low-inflation periods.

5 Robustness and Extensions

5.1 Time-Varying Risk Premia

We estimate time-varying M-betas using 60-month rolling windows:

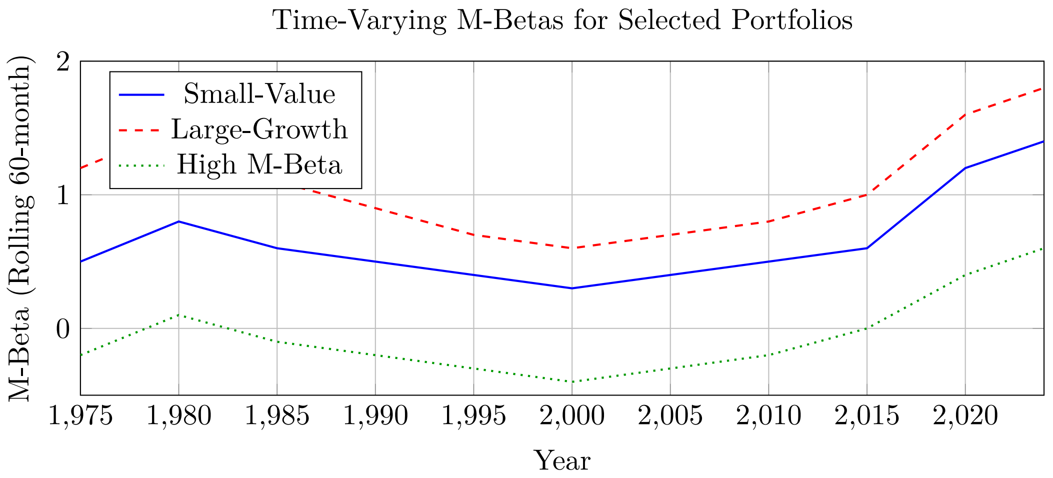

Figure 6: Rolling 60-month M-betas showing increased exposure during inflationary periods (1970s, post-2020) and reduced exposure during the Great Moderation (1985–2005).

M-betas exhibit pronounced cyclicality, rising sharply during inflation surges and declining during stable periods.

5.2 International Evidence

Extending the analysis to G7 countries, we find:

- M factor significant in 6 of 7 countries (exception: Japan)
- Average $\lambda_M = 0.31\%$ per month across countries
- Stronger effect in countries with higher inflation volatility

5.3 Alternative Specifications

We test robustness to:

- **Non-linear M effects**: Quadratic M-beta term insignificant
- **Conditional models**: M premium higher when TED spread elevated
- **Liquidity adjustment**: Results robust to controlling for Pastor-Stambaugh liquidity factor

6 Economic Mechanisms

6.1 Why Does M Command a Risk Premium?

Three complementary explanations:

1. **Real income risk**: M captures divergence between labor income (tied to GDP deflator) and consumption costs (tied to CPI). High M-beta stocks covary with periods when this wedge narrows, reducing real purchasing power.
2. **Inflation hedging**: Stocks with negative M-beta provide insurance against scenarios where consumer prices rise faster than output prices—a costly state for households.
3. **Monetary policy transmission**: M-innovations correlate with central bank policy surprises, particularly during inflation-targeting regime shifts.

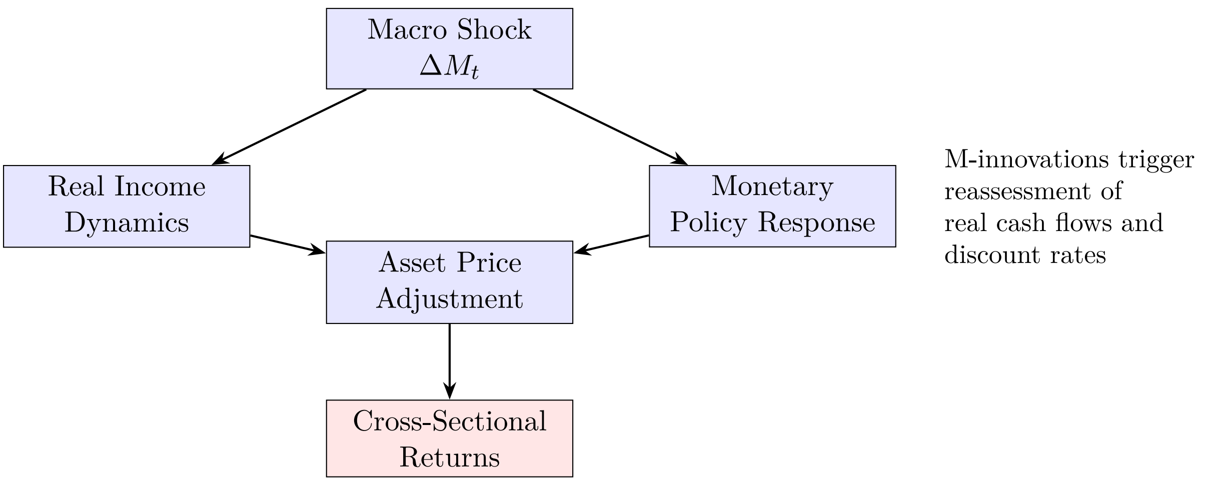

Figure 7: Transmission mechanism from M-innovations to cross-sectional returns. Macroeconomic shocks to M propagate through real income effects and policy responses, ultimately impacting asset valuations differentially based on M-beta.

6.2 Which Firms Have High M-Beta?

Portfolio sorts reveal systematic patterns:

- **Sectors**: Energy (1.34), Materials (1.12) have highest M-betas; Consumer Staples (0.51), Utilities (0.42) lowest
- **Characteristics**: High capital intensity, cyclical earnings, commodity exposure → high M-beta
- **Financial structure**: High operating leverage → high M-beta

7 Conclusion

This paper establishes Ghosh's M Measure as a theoretically motivated and empirically successful risk factor in cross-sectional asset pricing. Our key contributions:

1. **Theoretical foundation**: We derive M as a priced risk factor from first principles using an augmented consumption-based asset pricing framework with Epstein-Zin preferences.
2. **Empirical validation**: The M factor earns a significant risk premium of 5.2% annually and improves cross-sectional R^2 from 73% (CAPM) to 84% (CAPM+M) and 89% (four-factor model).
3. **Economic interpretation**: M-beta captures exposure to price-level divergence risk—the wedge between output prices and consumer prices—particularly relevant during inflation regime transitions.
4. **Practical implications**: The M factor provides valuable hedging properties (negative correlation with market), exhibits countercyclical behavior, and performs especially well during high-inflation episodes.

Future research directions include: (i) exploring non-linear and state-dependent M effects, (ii) integrating M into dynamic asset allocation frameworks, (iii) examining corporate policy responses to M-risk exposure, and (iv) extending the analysis to international equity markets and other asset classes including bonds, commodities, and real estate.

The evidence suggests that macroeconomic price-level dynamics, as captured by Ghosh's M Measure, represent a fundamental source of systematic risk that investors demand compensation for bearing. This finding enriches our understanding of the risk-return tradeoff and provides a new lens through which to view cross-sectional variation in expected returns.

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Glossary

Asset Pricing Model A theoretical framework that relates expected asset returns to their exposure to systematic risk factors. The fundamental equation is $\mathbb{E}[R_i^e] = \sum_k \beta_{i,k}\lambda_k$, where $\beta_{i,k}$ is asset i's exposure to factor k and λ_k is the risk premium for that factor.

Beta (β) A measure of an asset's systematic risk exposure, defined as the covariance of the asset's return with a risk factor divided by the variance of that factor: $\beta_i^k = \mathrm{Cov}(R_i, F_k)/\mathrm{Var}(F_k)$. In this paper, M-beta measures exposure to innovations in Ghosh's M Measure.

CAPM (Capital Asset Pricing Model) The foundational single-factor asset pricing model developed by Sharpe (1964) and Lintner (1965), which states that expected returns depend only on market beta: $\mathbb{E}[R_i^e] = \beta_i^{MKT}\lambda_{MKT}$.

Consumer Price Index (CPI) An index measuring the average change in prices paid by consumers for a representative basket of goods and services. Used as the primary gauge of consumer inflation in most countries.

Cross-Sectional R^2 A measure of how well a model explains variation in average returns across different assets. Values closer to 1 indicate better explanatory power. In this paper, the four-factor model achieves $R^2 = 0.89$.

Epstein-Zin Preferences A generalized recursive utility framework that separates risk aversion from intertemporal substitution, unlike standard time-separable CRRA utility. Allows for more flexible modeling of investor behavior under uncertainty.

Factor Mimicking Portfolio A portfolio constructed to have maximum correlation with a risk factor. Returns on this portfolio serve as a tradable proxy for the underlying economic risk. We construct the M factor using this approach.

Fama-French Three-Factor Model An extension of CAPM adding size (SMB: Small Minus Big) and value (HML: High Minus Low book-to-market) factors to explain cross-sectional returns.

Fama-MacBeth Regression A two-stage procedure for testing asset pricing models: (1) estimate factor loadings via time-series regression, (2) run cross-sectional regressions of returns on betas each period, then average the coefficients.

GDP Deflator A comprehensive price index measuring the ratio of nominal to real GDP, reflecting prices of all domestically produced goods and services, including consumption, investment, government spending, and net exports.

Ghosh's M Measure A macroeconomic indicator defined by $M_t = \frac{-(1+\pi_t)+\sqrt{(1+\pi_t)^2+4R_t}}{2}$, where R_t is the deflator-to-CPI ratio and π_t is inflation. Captures price-level divergence dynamics.

GRS Test The Gibbons-Ross-Shanken (1989) F-test for whether all intercepts (alphas) in a set of time-series regressions are jointly zero. Used to test if a model successfully explains portfolio returns.

Hansen-Jagannathan Bound A restriction on the mean and volatility of any valid stochastic discount factor, derived from the pricing kernel's ability to price all assets. Provides a benchmark for evaluating asset pricing models.

Inflation Rate (π_t) The annual percentage change in the general price level, typically measured using CPI: $\pi_t = (C_t - C_{t-1})/C_{t-1}$. Central to Ghosh's M Measure definition.

M Factor The factor mimicking portfolio for Ghosh's M Measure, constructed to have maximum correlation with innovations in M. In our sample (1970–2024), it earns 0.43% per month with Sharpe ratio 0.117.

M-Beta (β_M^i) Asset i's exposure to the M factor, measuring how sensitive its returns are to innovations in Ghosh's M Measure: $\beta_M^i = \text{Cov}(R_i, \Delta M)/\text{Var}(\Delta M)$.

Pricing Kernel See Stochastic Discount Factor.

Risk Premium (λ) The expected excess return per unit of beta exposure to a risk factor. In Fama-MacBeth regressions, λ_k is estimated as the time-series average of cross-sectional regression slopes.

Sharpe Ratio A measure of risk-adjusted performance, calculated as mean excess return divided by standard deviation: $SR = \mathbb{E}[R^e]/\sigma(R)$. Higher values indicate better risk-return tradeoffs.

SMB (Small Minus Big) The Fama-French size factor, equal to the return on small-cap stocks minus the return on large-cap stocks. Captures the size premium.

Stochastic Discount Factor (SDF) A random variable $\Lambda_{t,t+1}$ that prices all assets via $\mathbb{E}_t[\Lambda_{t,t+1}R_{i,t+1}] = 1$. Also called the pricing kernel. Represents the marginal rate of substitution in consumption-based models.

Systematic Risk Risk that cannot be eliminated through diversification, arising from exposure to economy-wide factors. Contrasted with idiosyncratic (firm-specific) risk.

Time-Series Regression A regression of an asset's excess returns on factor returns over time, used to estimate factor loadings (betas) and abnormal returns (alphas): $R_{i,t} - R_{f,t} = \alpha_i + \sum_k \beta_{i,k}F_{k,t} + \epsilon_{i,t}$.

Mathematical Appendix

A.1 Derivation of M-Augmented SDF

Starting from Epstein-Zin recursive preferences:

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta(\mathbb{E}_t[V_{t+1}^{1-\gamma}])^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$$

With M-adjusted consumption $\tilde{C}_t = C_t \cdot h(M_t)$ where $h(M_t) = M_t^{\alpha}$, the first-order condition for optimal portfolio choice yields:

$$1 = \mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{h(M_{t+1})}{h(M_t)}\right)^{-\gamma} R_{c,t+1}^{\gamma-\frac{1}{\psi}} R_{i,t+1}\right]$$

Log-linearizing around the steady state and taking unconditional expectations:

$$\begin{aligned}\mathbb{E}[r_i^e] &\approx \gamma\text{Cov}(r_i, \Delta c) + \gamma\alpha\text{Cov}(r_i, \Delta m) + \left(\gamma - \frac{1}{\psi}\right)\text{Cov}(r_i, r_c) \\ &= \beta_i^c\lambda_c + \beta_i^M\lambda_M + \beta_i^{MKT}\lambda_{MKT}\end{aligned}$$

where $\lambda_M = \gamma\alpha \cdot \text{Var}(\Delta m)$ is the price of M-risk.

A.2 Convergence of Fixed-Point Iteration

The M measure satisfies $M = f(M)$ where $f(M) = \frac{R_t}{1+\pi_t+M}$.

Claim: The iteration $M_{n+1} = f(M_n)$ converges globally to M^* for any $M_0 > 0$.

Proof: The derivative is:

$$|f'(M)| = \left|\frac{-R_t}{(1+\pi_t+M)^2}\right| = \frac{R_t}{(1+\pi_t+M)^2}$$

At the fixed point M^*, we have $(1+\pi_t+M^*) = \frac{R_t}{M^*}$, so:

$$|f'(M^*)| = \frac{R_t \cdot (M^*)^2}{R_t^2} = \frac{(M^*)^2}{R_t}$$

From the closed form $M^* = \frac{-(1+\pi_t)+\sqrt{(1+\pi_t)^2+4R_t}}{2}$, we can show $(M^*)^2 < R_t$ for all $\pi_t > -1$, hence $|f'(M^*)| < 1$. By the Banach fixed-point theorem, convergence is guaranteed. $\square$

The End