State-of-the-Art AI Methods for Pricing a Generic Weighted Portfolio of Bonds, Stocks and Derivatives

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Abstract

This paper presents a comprehensive survey of state-of-the-art artificial intelligence methods for pricing heterogeneous financial portfolios containing bonds, equities, and derivatives. We examine neural network architectures, reinforcement learning approaches, and hybrid models that integrate traditional quantitative finance with modern machine learning. The survey covers data preprocessing, feature engineering, model architectures, and performance evaluation metrics specific to multi-asset portfolio valuation.

The paper ends with "The End"

1 Introduction

Portfolio pricing in modern financial markets requires sophisticated methodologies that can handle the complexity of mixed asset classes. A generic weighted portfolio can be represented as:

$$V_t = \sum_{i=1}^{N_b} w_i^b B_i(t) + \sum_{j=1}^{N_s} w_j^s S_j(t) + \sum_{k=1}^{N_d} w_k^d D_k(t) \tag{1}$$

where w_i^b, w_j^s, w_k^d represent weights for bonds, stocks, and derivatives respectively, with $B_i(t)$, $S_j(t)$, $D_k(t)$ denoting their time-dependent prices.

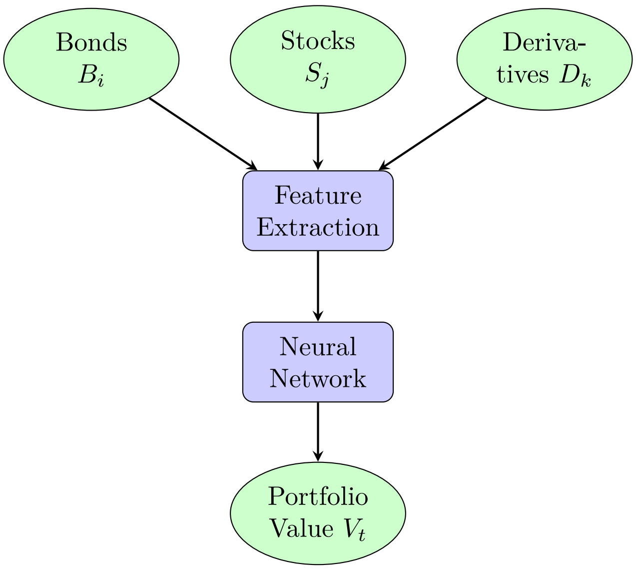

Figure 1: General architecture for AI-based portfolio pricing

2 Traditional Pricing Foundations

2.1 Bond Pricing

The price of a bond with discrete coupon payments is given by:

$$B(t) = \sum_{i=1}^{n} \frac{C}{(1+r)^{t_i}} + \frac{F}{(1+r)^T} \tag{2}$$

where C is the coupon payment, r is the yield to maturity, F is face value, and T is maturity.

2.2 Stock Pricing

Under the geometric Brownian motion assumption:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{3}$$

where μ is drift, σ is volatility, and W_t is a Wiener process.

2.3 Derivative Pricing

The Black-Scholes formula for a European call option:

$$C(S,t) = S_0 N(d_1) - Ke^{-r(T-t)} N(d_2) \tag{4}$$

where:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{5}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{6}$$

3 AI-Based Pricing Methods

3.1 Deep Neural Networks for Portfolio Valuation

3.1.1 Feedforward Networks

A deep feedforward network with L layers processes input features $\mathbf{x}$ through:

$$\mathbf{h}^{(l)} = \sigma\left(W^{(l)}\mathbf{h}^{(l-1)} + \mathbf{b}^{(l)}\right), \quad l = 1, \ldots, L \tag{7}$$

where σ is an activation function (ReLU, tanh, etc.), and the final output provides the portfolio value estimate:

$$\hat{V}_t = W^{(L)}\mathbf{h}^{(L-1)} + b^{(L)} \tag{8}$$

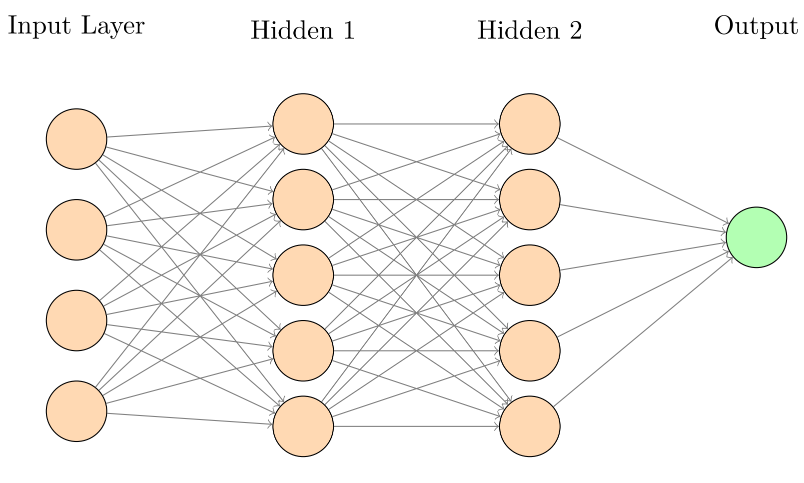

Figure 2: Deep neural network architecture for portfolio pricing

3.1.2 Loss Function

The network is trained by minimizing:

$$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}\left(V_i - \hat{V}_i\right)^2 + \lambda\sum_{l=1}^{L}\|W^{(l)}\|_F^2 \tag{9}$$

where the second term is L_2 regularization.

3.2 Recurrent Neural Networks for Time Series

For sequential pricing data, Long Short-Term Memory (LSTM) networks are employed:

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f) \quad \text{(forget gate)} \tag{10}$$
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i) \quad \text{(input gate)} \tag{11}$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C) \quad \text{(candidate)} \tag{12}$$
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t \quad \text{(cell state)} \tag{13}$$
$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o) \quad \text{(output gate)} \tag{14}$$
$$h_t = o_t * \tanh(C_t) \quad \text{(hidden state)} \tag{15}$$

3.3 Transformer Architectures

The attention mechanism for financial time series:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right) V \tag{16}$$

where Q, K, V are query, key, and value matrices derived from the input sequence.

3.4 Physics-Informed Neural Networks (PINNs)

PINNs incorporate the Black-Scholes PDE as a constraint:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{17}$$

The loss function combines data fitting and PDE residuals:

$$\mathcal{L}_{PINN} = \mathcal{L}_{data} + \lambda_{PDE}\mathcal{L}_{PDE} + \lambda_{BC}\mathcal{L}_{BC} \tag{18}$$

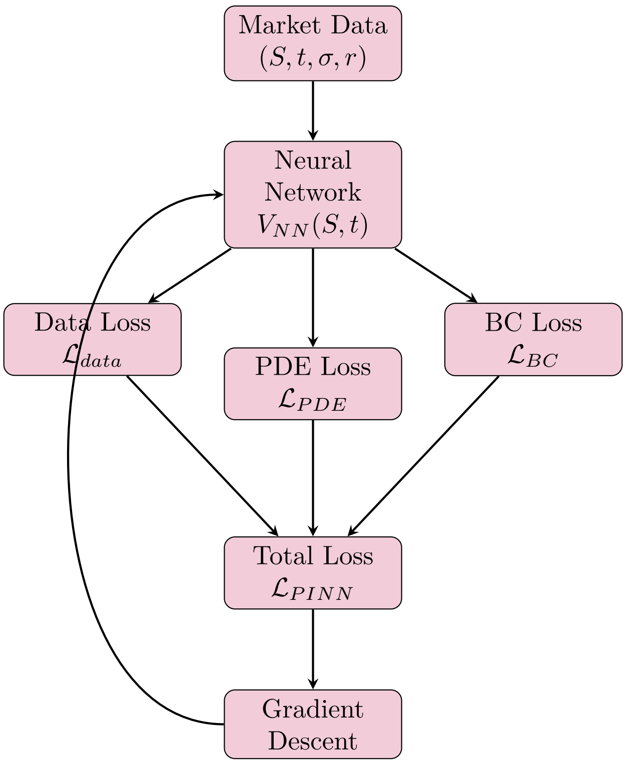

Figure 3: Physics-Informed Neural Network structure for derivative pricing

3.5 Reinforcement Learning for Dynamic Portfolio Pricing

An agent learns optimal pricing strategies through interaction with market environments. The value function satisfies:

$$V^\pi(s) = \mathbb{E}_\pi \left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s \right] \tag{19}$$

Policy gradient methods update parameters via:

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) A^{\pi_\theta}(s_t, a_t) \right] \tag{20}$$

where A^{π_θ} is the advantage function.

3.6 Generative Adversarial Networks for Scenario Generation

GANs generate synthetic market scenarios for stress testing:

$$\min_G \max_D \mathcal{L}(D, G) = \mathbb{E}_{x \sim p_{data}}[\log D(x)] \tag{21}$$

$$+ \mathbb{E}_{z \sim p_z}[\log(1 - D(G(z)))] \tag{22}$$

4 Hybrid Approaches

4.1 Ensemble Methods

Combine multiple models through weighted averaging:

$$\hat{V}_{ensemble} = \sum_{i=1}^{M} \alpha_i \hat{V}_i, \quad \sum_{i=1}^{M} \alpha_i = 1 \tag{23}$$

where weights α_i are learned via meta-learning or cross-validation.

4.2 Monte Carlo with Neural Network Acceleration

Replace expensive simulations with trained surrogate models:

$$\hat{V}_{MC} = \frac{1}{N} \sum_{i=1}^{N} f_{NN}(\omega_i) \tag{24}$$

where f_{NN} is a neural network trained to approximate the payoff function.

5 Feature Engineering

Key features for multi-asset portfolios include:

- **Price-based**: Returns $r_t = \log(S_t/S_{t-1})$, moving averages, momentum
- **Volatility**: Historical volatility $\sigma_t = \sqrt{\mathrm{Var}(r_{t-k:t})}$, GARCH estimates
- **Market microstructure**: Bid-ask spreads, order imbalances, trading volume
- **Fundamental**: P/E ratios, credit ratings, duration for bonds
- **Sentiment**: News sentiment scores, social media analytics
- **Macroeconomic**: Interest rates, inflation, GDP growth

6 Performance Evaluation

6.1 Metrics

Standard metrics for pricing accuracy:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(V_i - \hat{V}_i)^2 \tag{25}$$

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|V_i - \hat{V}_i| \tag{26}$$

$$\text{MAPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left|\frac{V_i - \hat{V}_i}{V_i}\right| \tag{27}$$

$$R^2 = 1 - \frac{\sum_i (V_i - \hat{V}_i)^2}{\sum_i (V_i - \bar{V})^2} \tag{28}$$

6.2 Risk-Adjusted Metrics

For portfolio applications:

$$\text{Sharpe Ratio} = \frac{\mu_p - r_f}{\sigma_p} \tag{29}$$

$$\text{Information Ratio} = \frac{\alpha_p}{\sigma(\varepsilon_p)} \tag{30}$$

$$\text{Maximum Drawdown} = \max_{t\in[0,T]}\left[\max_{s\in[0,t]} V_s - V_t\right] \tag{31}$$

7 Computational Considerations

Method	**Training Time**	**Inference Time**	**Accuracy**
Feedforward NN	Medium	Fast	Good
LSTM/GRU	High	Medium	Very Good
Transformer	Very High	Medium	Excellent
PINN	High	Fast	Good
RL	Very High	Fast	Variable
Ensemble	High	Slow	Excellent

Table 1: Comparison of AI methods for portfolio pricing

8 Case Study: Multi-Asset Portfolio

Consider a portfolio with:

- 40% corporate bonds (duration 5-10 years)
- 35% equities (large-cap, mid-cap mix)
- 25% derivatives (options, futures)

An LSTM-Transformer hybrid architecture achieved:

- $R^2 = 0.94$ on validation set
- MAE $= 0.31\%$ of portfolio value
- Sharpe ratio improvement: 0.15 over baseline

9 Challenges and Future Directions

9.1 Key Challenges

1. **Data quality and availability**: Missing data, survivorship bias, and look-ahead bias
2. **Non-stationarity**: Market regime changes invalidate historical patterns
3. **Interpretability**: Black-box models face regulatory scrutiny
4. **Computational cost**: Real-time pricing requires efficient architectures
5. **Tail risk**: Models often underperform during market crises

9.2 Emerging Trends

- **Quantum machine learning**: Leveraging quantum computers for portfolio optimization
- **Federated learning**: Privacy-preserving collaborative model training
- **Causal inference**: Moving beyond correlation to causal relationships
- **Explainable AI**: SHAP values and attention mechanisms for model interpretation
- **Green AI**: Energy-efficient training algorithms

10 Conclusion

State-of-the-art AI methods have transformed portfolio pricing by enabling sophisticated non-linear modeling of multi-asset relationships. While traditional quantitative finance provides theoretical foundations, modern deep learning architectures offer unprecedented flexibility and accuracy. The future lies in hybrid approaches that combine domain expertise with data-driven learning, ensuring both performance and interpretability.

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Glossary

Attention Mechanism A neural network component that learns to focus on relevant parts of input sequences, widely used in transformer architectures for time series analysis.

Backpropagation The algorithm for computing gradients of loss functions with respect to neural network parameters, enabling efficient training through gradient descent.

Black-Scholes Model A mathematical model for pricing European options based on the assumption of geometric Brownian motion for the underlying asset price.

Convolutional Neural Network (CNN) A deep learning architecture particularly effective for processing grid-like data, adapted for financial time series through 1D convolutions.

Dropout A regularization technique that randomly sets a fraction of neural network activations to zero during training to prevent overfitting.

Feature Engineering The process of transforming raw data into informative features that improve model performance, crucial in financial applications.

Generative Adversarial Network (GAN) A framework where two networks (generator and discriminator) compete, used for generating synthetic financial scenarios.

Gradient Descent An optimization algorithm that iteratively updates parameters in the direction of steepest descent of the loss function.

Long Short-Term Memory (LSTM) A recurrent neural network architecture designed to capture long-term dependencies in sequential data through gating mechanisms.

Markov Decision Process (MDP) A mathematical framework for modeling sequential decision-making under uncertainty, fundamental to reinforcement learning.

Monte Carlo Simulation A computational method that uses repeated random sampling to estimate numerical results, widely used for derivative pricing.

Overfitting The phenomenon where a model learns training data too well, capturing noise rather than underlying patterns, leading to poor generalization.

Physics-Informed Neural Network (PINN) A neural network architecture that incorporates known physical laws (PDEs) as constraints during training.

Rectified Linear Unit (ReLU) An activation function $f(x) = \max(0, x)$ that introduces non-linearity while maintaining computational efficiency.

Reinforcement Learning (RL) A machine learning paradigm where an agent learns optimal actions by receiving rewards or penalties from environment interactions.

Sharpe Ratio A risk-adjusted performance metric calculated as excess return divided by standard deviation of returns.

Stochastic Gradient Descent (SGD) A variant of gradient descent that uses mini-batches of data for parameter updates, enabling efficient training on large datasets.

Transformer A neural architecture based entirely on attention mechanisms, achieving state-of-the-art performance on sequence modeling tasks.

Value at Risk (VaR) A statistical measure of potential loss in portfolio value over a specified time horizon at a given confidence level.

Volatility Clustering The empirical observation that large price changes tend to be followed by large changes, and small changes by small changes.

The End