

# Sovereign Systemic Importance, Spectral Stability, and the Optimal Structure of the Global Order

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## Abstract

This paper develops a theoretical framework for a Sovereign Systemic Importance Index (SSII). The index integrates economic scale and eigenvector centrality within a Perron–Frobenius spectral network structure. We derive the spectral stability condition for systemic equilibrium, characterize optimal sovereign fragmentation under political and coordination costs, and incorporate security externalities. The resulting framework generates a core–periphery equilibrium structure and delivers calibrated optimal sovereign bloc sizes consistent with observed global macroeconomic organization.

The paper ends with “The End”

## 1 Introduction

The distribution of sovereign power in the global system is neither uniform nor accidental. A small number of large economies account for a disproportionate share of global output, trade, finance, and geopolitical influence. Traditional measures of national power focus on income, population, or military capacity in isolation. However, systemic importance is inherently relational: a sovereign’s relevance depends not only on its own scale, but also on its position within the global network of interdependence.

This paper proposes a unified metric – the Sovereign Systemic Importance Index (SSII) – that combines domestic economic mass with network centrality. The framework builds on spectral graph theory and general equilibrium reasoning. By embedding sovereign economies within an exposure matrix and applying Perron–Frobenius theory, we obtain a rigorous characterization of systemic amplification and stability.

The analysis proceeds in three steps. First, we define the SSII and derive the spectral stability condition. Second, we analyze optimal fragmentation of the global system under political and coordination costs. Third, we incorporate military and security externalities to determine how geopolitical risk alters optimal sovereign size.

## 2 Model Setup

Let  $i$  index sovereign states. Define aggregate economic scale as

$$S_i := Y_i P_i, \tag{1}$$

where  $Y_i$  denotes GDP per capita (PPP) and  $P_i$  population. Economic scale captures both productivity and demographic mass, reflecting a sovereign’s potential to influence global activity.

Let  $W$  be a nonnegative irreducible exposure matrix representing trade, financial, technological, or geopolitical interdependence among sovereigns. Entry  $W_{ij}$  measures the intensity of spillover from  $j$  to  $i$ .

By Perron–Frobenius theory, there exists a unique positive eigenvector  $v$  such that

$$Wv = \rho(W)v, \quad (2)$$

where  $\rho(W)$  is the spectral radius. Normalizing  $v$  so that  $\sum_i v_i = 1$  yields a measure of systemic centrality. The component  $v_i$  represents sovereign  $i$ 's relative importance in propagating network shocks.

### 3 Sovereign Systemic Importance Index

We define the Sovereign Systemic Importance Index as

$$SSII_i := v_i \ln(S_i). \quad (3)$$

This specification captures two complementary dimensions of systemic relevance. The logarithmic transformation reflects diminishing marginal influence of size, while eigenvector centrality accounts for indirect amplification through the network. A sovereign with moderate size but high centrality may therefore rank above a larger but peripheral economy.

The SSII can be interpreted as a reduced-form projection of a more general equilibrium model in which shocks propagate through interdependent production and financial linkages.

### 4 Spectral Stability Condition

In a network general equilibrium setting, equilibrium sovereign strength satisfies

$$\mathbf{S} = (I - \eta W)^{-1} \mathbf{b}, \quad (4)$$

where  $\eta \geq 0$  measures contagion intensity and  $\mathbf{b}$  collects domestic fundamentals.

**Theorem 1** (Spectral Stability). *Let  $W$  be nonnegative and irreducible. A finite equilibrium exists if and only if*

$$\eta \rho(W) < 1. \quad (5)$$

*If  $\eta \rho(W) \rightarrow 1$ , systemic amplification becomes unbounded along the Perron eigenvector direction.*

*Proof.* The inverse admits the Neumann expansion

$$(I - \eta W)^{-1} = \sum_{k=0}^{\infty} (\eta W)^k,$$

which converges if and only if  $\rho(\eta W) < 1$ .  $\square$

Near the spectral boundary, spillovers become concentrated along the dominant eigenvector, implying that central economies disproportionately transmit global shocks.

### 5 Optimal Fragmentation

To determine the optimal number of sovereign blocs, assume total world scale is fixed at  $\bar{S} = \sum_i S_i$  and that blocs are symmetric, so  $S_i = \bar{S}/n$ .

Define global loss as

$$\mathcal{L}(n) = \frac{1}{n} \ln^2! \left( \frac{\bar{S}}{n} \right) + \phi n + \psi n^2, \quad (6)$$

where  $\phi > 0$  represents political fragmentation cost and  $\psi > 0$  convex coordination cost.

The first-order condition is

$$\phi + 2\psi n = \frac{1}{n^2} \left[ \ln^2! \left( \frac{\bar{S}}{n} \right) + 2 \ln! \left( \frac{\bar{S}}{n} \right) \right]. \quad (7)$$

Calibrating  $\bar{S} = 100$ ,  $\phi = 0.15$ , and  $\psi = 0.02$  yields an interior optimum

$$n^* \approx 6, \quad (8)$$

corresponding to macro-sovereign GDP sizes of roughly 16–17 trillion (PPP).

## 6 Security Externalities

Geopolitical considerations alter the optimal fragmentation condition. Suppose scale confers defensive and deterrence benefits proportional to

$$\sigma\mu \ln! \left( \frac{\bar{S}}{n} \right), \quad (9)$$

where  $\sigma\mu > 0$  measures security intensity.

The modified first-order condition becomes

$$\phi + 2\psi n = \frac{1}{n^2} \left[ \ln^2! \left( \frac{\bar{S}}{n} \right) + 2 \ln! \left( \frac{\bar{S}}{n} \right) \right] - \frac{\sigma\mu}{n}. \quad (10)$$

Under moderate security intensity, the optimal number of sovereign blocs declines to approximately

$$n^* \approx 4\text{--}5. \quad (11)$$

Assuming bloc populations between 400 and 500 million, this implies optimal GDP per capita (PPP) in the range of 45,000–55,000.

## 7 Asymmetric Security Environments

When security intensity varies across regions, high-threat areas consolidate into larger blocs, while low-threat regions sustain smaller sovereign units. The equilibrium structure thus features a core–periphery configuration: a small number of highly central macro-blocs coexisting with peripheral states of limited systemic influence.

This asymmetric equilibrium aligns with observed geopolitical organization, in which a handful of major powers dominate global spillover channels.

## 8 Conclusion

The spectral SSII framework provides a unified lens through which to analyze sovereign systemic importance, optimal fragmentation, and bloc formation. The model highlights three central forces: economic scale, network centrality, and geopolitical externalities. Together, these forces determine both systemic stability and the optimal number of sovereign entities.

Under plausible calibrations, the world economy exhibits an interior optimal fragmentation with a limited number of macro-sovereign blocs. Security externalities further encourage consolidation, particularly in high-threat environments. The resulting equilibrium is neither fully fragmented nor globally unified, but instead characterized by structured hierarchy and spectral concentration.

## References

- [1] E. Seneta, *Non-negative Matrices and Markov Chains*, Springer.
- [2] Basel Committee on Banking Supervision, Global Systemically Important Banks methodology.

## Glossary

**SSII** Sovereign Systemic Importance Index.

**Spectral Radius** Largest eigenvalue in modulus.

**Perron Eigenvector** Positive eigenvector associated with the spectral radius.

**Fragmentation Cost** Political cost proportional to the number of sovereigns.

**Coordination Cost** Convex diplomatic and institutional complexity cost.

## The End