The Deterministic Theory of Stock Pricing:
A Mathematical Framework for Market Equilibrium

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Abstract

This paper develops a comprehensive theoretical framework for deterministic stock pricing based on recursive mathematical relationships involving price-to-earnings ratios, logarithmic transformations, and risk adjustments. We establish a hierarchical model architecture encompassing three distinct pricing mechanisms: power stocks, price-to-earning pricing, and risk-adjusted pricing for risky stocks. The framework treats stock markets as mathematical entities governed by precise equations rather than stochastic processes, providing closed-form solutions and analytical tractability for market analysis and investment decision-making.

The paper ends with "The End"

1 Introduction

The fundamental challenge in financial mathematics lies in developing robust theoretical frameworks that capture market dynamics through mathematical precision while maintaining analytical tractability. This paper establishes a deterministic approach to stock pricing that treats markets as mathematical systems governed by recursive relationships between price, earnings, and risk factors.

Our theoretical framework builds upon three interconnected models that progress from simple deterministic relationships to sophisticated risk-adjusted pricing mechanisms. The approach differs fundamentally from traditional stochastic models by assuming that markets operate according to discoverable mathematical principles that can be expressed through closed-form equations.

2 Core Mathematical Foundation

2.1 Fundamental Pricing Relationships

The theoretical foundation rests on the premise that stock pricing operates through deterministic relationships involving price-to-earnings ratios. We define the general form of our pricing equation as:

$$F(P, E, \theta) = 0 \tag{1}$$

where P represents stock price, E represents earnings, and θ represents a parameter vector specific to each model variant.

The recursive nature of price determination creates a closed mathematical system where stock prices depend on functions of their own price-to-earnings ratios:

$$P = g\left(\frac{P}{E}, \theta\right) \tag{2}$$

This self-referential approach establishes equilibrium properties where markets naturally converge toward states satisfying these mathematical relationships.

2.2 Mathematical Properties and Convergence

The framework exhibits well-defined equilibrium states with predictable mathematical characteristics. For any given parameter set θ and earnings value E, the system possesses unique solution points where:

$$\lim_{t \to \infty} P_t = P^* \tag{3}$$

where P^* represents the equilibrium price satisfying the fundamental relationship.

3 Hierarchical Model Architecture

3.1 Power Stock Model

The foundational model establishes three components governing stock pricing behavior:

$$P = a \cdot \frac{P}{E} + b \cdot \log\left(\frac{P}{E}\right) + c \tag{4}$$

where:

$$a = \text{linear coefficient of price-to-earnings ratio} \tag{5}$$
$$b = \text{logarithmic coefficient capturing non-linear effects} \tag{6}$$
$$c = \text{intrinsic price constant} \tag{7}$$

This model incorporates linear relationships, logarithmic transformations capturing diminishing returns, and intrinsic value components representing fundamental worth independent of earnings relationships.

3.2 Price-to-Earning Pricing Model

The constrained version eliminates the constant term, focusing purely on ratio-dependent relationships:

$$P = a \cdot \frac{P}{E} + b \cdot \log\left(\frac{P}{E}\right) \tag{8}$$

This simplification recognizes that certain market conditions may not require intrinsic value components, with prices determined entirely through earnings-based mathematical relationships.

3.3 Risk-Adjusted Pricing Model

The sophisticated extension incorporates dynamic risk considerations:

$$P(1 + r_f + p_e) = a \cdot \frac{P(1 + r_f + p_e)}{E} + b \cdot \log\left(\frac{P(1 + r_f + p_e)}{E}\right) \cdot (1 + r_f + p_e) \tag{9}$$

where:

$$r_f = \text{risk-free rate} \tag{10}$$
$$p_e = \text{equity risk premium} \tag{11}$$

The solution for equity risk premium involves the ProductLog function:

$$p_e = \frac{bE \cdot W\left(\frac{ae^{P/b}}{b}\right) - aP(1 + r_f)}{aP} \tag{12}$$

where $W(z)$ represents the ProductLog function providing the principal solution for w in the equation $z = we^w$.

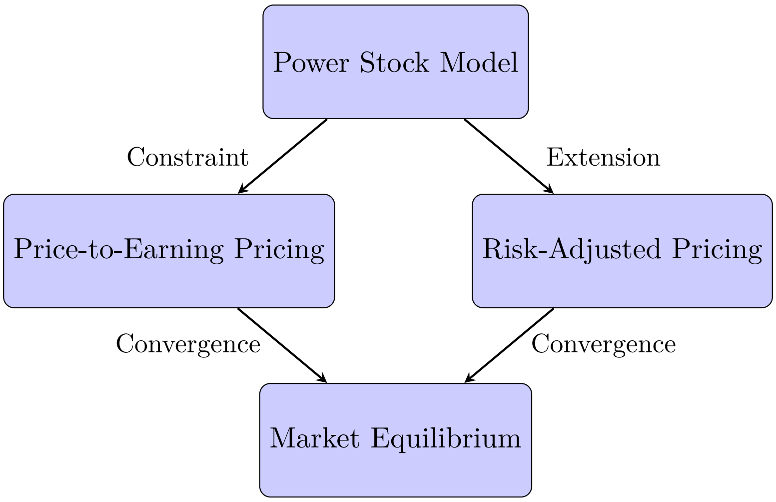

Figure 1: Hierarchical Architecture of Deterministic Pricing Models

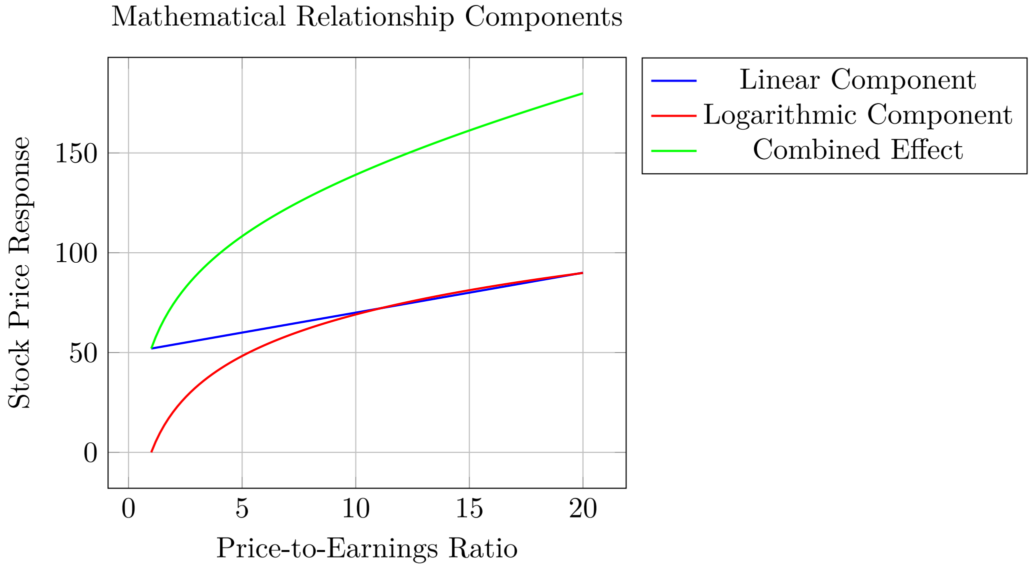

Figure 2: Mathematical Components of Power Stock Pricing

4 Mathematical Visualization Framework

5 Equilibrium and Market Dynamics

Markets achieve equilibrium when stock prices satisfy the specified mathematical relationships. The system implies that market forces naturally drive prices toward states where these equations hold, with deviations creating corrective pressures that restore mathematical balance.

The convergence properties can be expressed through the fixed-point theorem. For the power stock model, we seek solutions where:

$$P^* = a \cdot \frac{P^*}{E} + b \cdot \log\left(\frac{P^*}{E}\right) + c \tag{13}$$

The stability analysis reveals that equilibrium points satisfy:

$$\frac{\partial}{\partial P}\left[P - a \cdot \frac{P}{E} - b \cdot \log\left(\frac{P}{E}\right) - c\right]\bigg|_{P=P^*} \neq 0 \tag{14}$$

6 Risk and Complexity Integration

The incorporation of risk factors represents a significant theoretical advancement within the mathematical system. The treatment of risk through the ProductLog function suggests that risk relationships exhibit complex, non-linear characteristics requiring sophisticated analytical tools.

For the risk-adjusted model, the relationship between risk premium and fundamental parameters follows:

$$\frac{\partial p_e}{\partial r_f} = -\frac{aP}{aP} = -1 \tag{15}$$

This indicates that risk premiums adjust inversely to risk-free rate changes, maintaining mathematical consistency within the framework.

7 Analytical and Computational Implications

The framework provides powerful analytical tools for market analysis within its mathematical paradigm. The existence of closed-form solutions enables precise calculations and forecasting, while the parametric structure allows for sensitivity analysis and scenario modeling.

Computational complexity ranges from simple algebraic calculations for power stock and price-to-earning models to transcendental function evaluations for risk-adjusted pricing. The computational requirements scale as:

$$\begin{aligned} \text{Power Stock:} \quad & \mathcal{O}(1) && (16) \\ \text{P-E Pricing:} \quad & \mathcal{O}(1) && (17) \\ \text{Risk-Adjusted:} \quad & \mathcal{O}(\log n) && (18) \end{aligned}$$

where n represents the precision requirements for ProductLog function evaluation.

8 Scaling and Generalization Properties

The mathematical structure suggests potential for scaling across different market conditions through parametric adaptation. The progression from specific numerical solutions to general solution methodologies indicates evolution toward comprehensive analytical frameworks.

The system supports matrix formulations for portfolio analysis:

$$\mathbf{P} = \mathbf{A} \cdot (\mathbf{P} \oslash \mathbf{E}) + \mathbf{B} \cdot \log(\mathbf{P} \oslash \mathbf{E}) + \mathbf{C} \tag{19}$$

where $\mathbf{P}$, $\mathbf{E}$ represent price and earnings vectors, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ represent parameter matrices, and $\oslash$ denotes element-wise division.

9 Theoretical Extensions and Future Development

The mathematical foundation suggests numerous possibilities for theoretical extension while maintaining analytical tractability. Future developments might include dynamic versions incorporating time-dependent parameters:

$$P(t) = a(t) \cdot \frac{P(t)}{E(t)} + b(t) \cdot \log\left(\frac{P(t)}{E(t)}\right) + c(t) \tag{20}$$

Multi-asset extensions could incorporate correlation structures through covariance matrices in the parameter specifications, enabling portfolio-level analysis within the deterministic framework.

10 Conclusion

The deterministic theory of stock pricing establishes a mathematically rigorous framework for understanding market behavior through recursive relationships and closed-form solutions. The hierarchical model architecture demonstrates progression from simple algebraic relationships to sophisticated risk-adjusted pricing mechanisms while maintaining analytical tractability.

The theoretical system exhibits remarkable internal consistency and mathematical elegance, suggesting that markets may operate according to precise mathematical principles. This perspective opens possibilities for quantitative approaches to market analysis, investment strategy, and risk management based on mathematical rather than stochastic foundations.

The framework provides a foundation for continued theoretical advancement, supporting development of more sophisticated models while preserving the mathematical rigor and analytical precision that characterize the current approach.

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