On GDP per Capita (PPP), the Bank Interest Rate and Population Density

Soumadeep Ghosh

Kolkata, India

Abstract

This paper presents a multiple linear regression analysis examining the relationship between GDP per capita adjusted for purchasing power parity (PPP) and two key explanatory variables: a composite measure incorporating prior-year GDP per capita and central bank interest rates, and population density. Using cross-sectional data from the G20 nations, we estimate the model $Y = a + b \cdot X_1 + c \cdot X_2$ where X_1 represents the interest-rate-adjusted prior GDP per capita and X_2 denotes population density. Our findings indicate a strong predictive relationship with $R^2 = 0.9545$, suggesting that 95.45% of the variation in current GDP per capita (PPP) is explained by the model.

The paper ends with "The End"

Contents

1 Introduction

Understanding the determinants of economic prosperity across nations remains a central question in macroeconomics. Gross Domestic Product per capita, adjusted for purchasing power parity (PPP), serves as a widely-used metric for comparing living standards across countries with different price levels [1].

This study investigates whether current GDP per capita (PPP) can be predicted using:

1. The previous year's GDP per capita (PPP), adjusted by a factor incorporating central bank interest rates;

2. Population density as a measure of economic agglomeration effects.

The G20 nations provide an ideal sample as they represent approximately 85% of global GDP and encompass diverse economic structures [2].

2 Data Description

2.1 Data Sources

Data were compiled from two primary sources:

- **World Bank World Development Indicators**: GDP per capita (PPP, current international $), population density (people per sq. km of land area)

- **TradingEconomics**: Central bank benchmark interest rates

2.2 G20 Dataset

Table 1 presents the complete dataset for all G20 member nations.

Table 1: G20 Nations Economic Indicators

Country	GDP/cap (PPP)	GDP/cap (Prev)	Interest (%)	Density
Argentina	26,505	25,822	32.00	16.7
Australia	64,674	62,725	4.35	3.5
Brazil	20,809	19,618	12.25	25.4
Canada	60,177	58,400	3.25	4.4
China	23,309	21,476	3.10	151.1
France	58,765	56,036	3.15	119.2
Germany	66,132	64,110	3.15	239.6
India	9,515	8,379	6.50	473.4
Indonesia	15,855	14,653	5.75	145.7
Italy	55,938	53,390	3.15	201.8
Japan	52,120	49,044	0.50	345.5
Mexico	22,745	21,531	10.00	66.4
Russia	38,410	36,485	21.00	9.0
Saudi Arabia	56,916	55,368	5.00	18.3
South Africa	16,091	15,464	7.50	49.5
South Korea	56,709	53,051	3.00	527.3
Türkiye	41,412	38,758	47.50	110.5
United Kingdom	58,880	56,471	4.75	277.9
United States	85,373	80,412	4.50	37.1
European Union	56,043	53,960	3.15	109.1

3 Methodology

3.1 Model Specification

We estimate the following multiple linear regression model:

$$Y_i = a + b \cdot X_{1i} + c \cdot X_{2i} + \varepsilon_i \tag{1}$$

where:

$$Y_i = \text{GDP per capita (PPP) for country } i$$
$$X_{1i} = \text{GDP per capita (PPP)}_{t-1} \times \left(1 + \frac{r_i}{100}\right)$$
$$X_{2i} = \text{Population density (per sq. km)}$$
$$\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$$

The transformation X_1 captures the interaction between prior economic output and monetary policy stance, hypothesizing that interest rates reflect expected economic conditions.

3.2 Estimation Procedure

Using Ordinary Least Squares (OLS), the coefficient vector $\boldsymbol{\beta} = (b, c)'$ is obtained by:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} \tag{2}$$

The normal equations yield:

$$\begin{pmatrix} \hat{b} \\ \hat{c} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}^{-1} \begin{pmatrix} S_{1y} \\ S_{2y} \end{pmatrix} \tag{3}$$

where S_{jk} denotes the centered sum of cross-products.

4 Results

4.1 Summary Statistics

Table 2 presents the summary statistics for the variables used in the regression.

Table 2: Summary Statistics

Statistic	Y	X_1	X_2
n	20	20	20
Mean	44,318.90	45,497.83	146.57
Sum	886,378	909,956.52	2,931.40
Sum of Squares	–	4.97×10^{10}	888,871.48

4.2 Regression Coefficients

The estimated regression equation is:

$$\boxed{\hat{Y} = -2{,}273.43 + 0.9943 \cdot X_1 + 9.33 \cdot X_2} \tag{4}$$

Table 3: Regression Coefficient Estimates

Parameter	Estimate	Interpretation
a (Intercept)	$-2{,}273.43$	Baseline adjustment
b (Adjusted GDP)	0.9943	Near unit elasticity with X_1
c (Density)	9.33	\$9.33 per person/km^2

4.3 Model Fit

The model demonstrates excellent explanatory power:

$$R^2 = \frac{SSR}{SST} = \frac{8{,}144{,}302{,}861}{8{,}532{,}495{,}656} = \mathbf{0.9545} \tag{5}$$

4.4 Graphical Analysis

Figure 1 illustrates the relationship between the primary explanatory variable X_1 and GDP per capita (PPP).
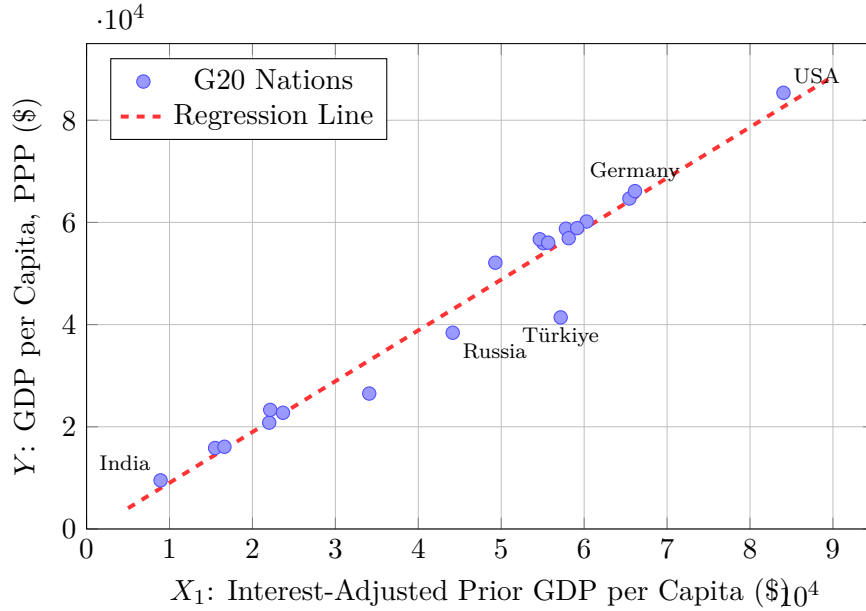

Figure 1: Scatter plot of GDP per Capita (PPP) against Interest-Adjusted Prior GDP

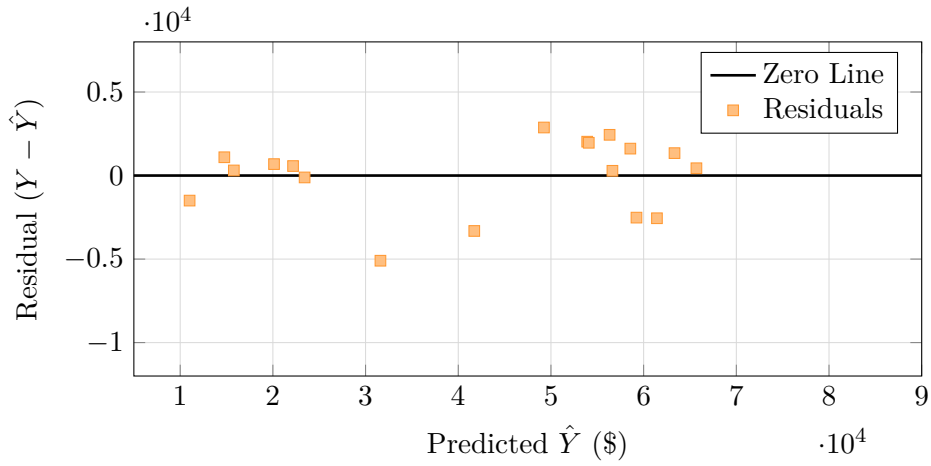Figure 2 displays the residual analysis for model diagnostics.

Figure 2: Residual plot for regression diagnostics

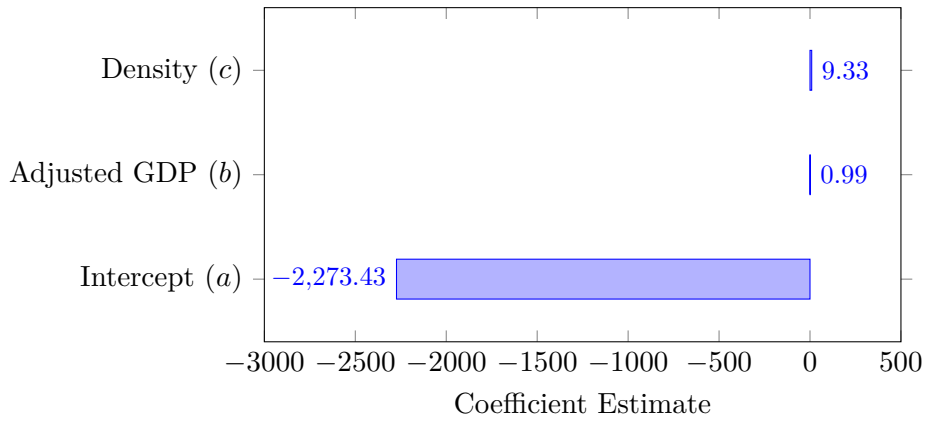Figure 3 provides a visual representation of the coefficient magnitudes.

Figure 3: Regression coefficient estimates showing intercept $a = -2273.43$, slope on adjusted prior GDP $b = 0.99$, and slope on population density $c = 9.33$

4.5 Variance Decomposition

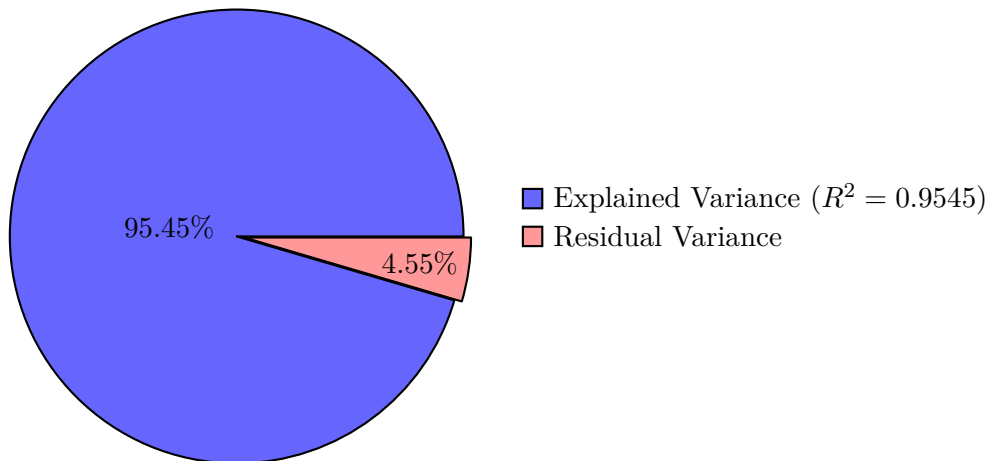Figure 4 illustrates the decomposition of total variance.

Figure 4: Variance decomposition: proportion of variance explained by the model

5 Discussion

5.1 Key Findings

1. **High Explanatory Power**: The R^2 of 0.9545 indicates that the model explains over 95% of the cross-sectional variation in GDP per capita (PPP) among G20 nations.

2. **Near-Unity Coefficient on X_1**: The coefficient $\hat{b} = 0.9943$ suggests that, holding population density constant, current GDP per capita tracks almost perfectly with the interest-rate-adjusted prior GDP.

3. **Positive Density Effect**: Contrary to simple correlations, the partial effect of population density ($\hat{c} = 9.33$) is positive, suggesting agglomeration economies after controlling for the composite income-interest variable.

4. **Outliers**: Nations with exceptionally high interest rates (Türkiye at 47.5%, Argentina at 32%) exhibit larger residuals, reflecting the complex relationship between monetary policy and economic output during periods of high inflation.

5.2 Limitations

- Cross-sectional analysis cannot establish causality

- Sample limited to G20 nations ($n = 20$)

- Interest rates may be endogenous to economic conditions

- PPP adjustments subject to measurement error

6 Conclusion

This analysis demonstrates that GDP per capita (PPP) can be effectively modeled as a function of interest-rate-adjusted prior output and population density. The strong model fit ($R^2 = 0.9545$) supports the specification, while the near-unity coefficient on the composite variable suggests economic persistence across years. Future research should explore panel data methods to account for country-specific effects and temporal dynamics.

Glossary

GDP (Gross Domestic Product) The total monetary value of all goods and services produced within a country's borders in a specific time period.

PPP (Purchasing Power Parity) An economic theory and method of adjustment that allows comparison of the purchasing power of different currencies by accounting for differences in price levels between countries.

GDP per Capita Gross Domestic Product divided by the total population, providing a per-person measure of economic output and a proxy for average living standards.

Bank Interest Rate The benchmark interest rate set by a country's central bank, which influences borrowing costs throughout the economy and serves as a monetary policy tool.

Population Density The number of people per unit area (typically per square kilometer), measuring how concentrated a population is within a geographic region.

OLS (Ordinary Least Squares) A statistical method for estimating regression coefficients by minimizing the sum of squared differences between observed and predicted values.

R-squared (R^2) The coefficient of determination; a statistical measure indicating the proportion of variance in the dependent variable explained by the independent variables.

Residual The difference between an observed value and the value predicted by a statistical model; $e_i = Y_i - \hat{Y}_i$.

G20 (Group of Twenty) An intergovernmental forum comprising 19 sovereign countries, the European Union, and the African Union, representing the world's major economies.

Agglomeration Effects Economic benefits arising from the spatial concentration of economic activity, including knowledge spillovers, labor pooling, and infrastructure sharing.

Cross-sectional Data Data collected by observing multiple subjects (e.g., countries) at a single point in time, as opposed to time-series or panel data.

References

[1] World Bank. (2024). *World Development Indicators*. Washington, DC: The World Bank Group. Available at: `https://databank.worldbank.org/source/world-development-indicators`

[2] Trading Economics. (2024). *Interest Rate by Country*. Available at: `https://tradingeconomics.com/country-list/interest-rate`

[3] International Monetary Fund. (2024). *World Economic Outlook Database*. Washington, DC: IMF.

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