A New Approach to Structural Estimation of Dynamic Discrete Choice Models with Serially Correlated Unobservables

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Abstract

Standard dynamic discrete choice (DDC) models rely critically on the assumption that unobservable shocks are independently distributed over time. This paper develops a new computationally feasible estimation method when this assumption is violated. We propose augmenting the state space with lagged unobservables and employing a simulation-based conditional choice probability (CCP) estimator that accounts for serial correlation. The method combines importance sampling with a two-step estimation procedure that avoids solving the full dynamic program repeatedly. Monte Carlo simulations demonstrate that ignoring serial correlation leads to severely biased estimates, while our proposed estimator performs well even with moderate sample sizes. We provide conditions for identification and establish asymptotic properties of the estimator.

The paper ends with “The End”

1 Introduction

Dynamic discrete choice models have become a workhorse tool for analyzing intertemporal decisions in labor economics, industrial organization, and other fields. The standard framework, pioneered by [1], assumes that per-period utility shocks are independent and identically distributed (i.i.d.) over time. This assumption is computationally convenient but often unrealistic.

Serial correlation in unobservables can arise from:

- **Persistent preference shocks**: Individual tastes may evolve slowly
- **Measurement error**: Survey responses contain autocorrelated noise
- **Persistent information asymmetries**: Private information changes gradually

Ignoring such correlation leads to biased parameter estimates and incorrect counterfactual predictions. However, accommodating serial correlation is challenging because:

1. The standard Bellman equation no longer provides a sufficient statistic
2. State space dimensionality increases dramatically
3. Conditional independence of choices given observables is violated

This paper proposes a practical solution using state augmentation combined with simulation-based CCP estimation.

2 Model Setup

2.1 Environment

Consider an infinite-horizon discrete choice problem. At each period t, an agent observes state (x_t, ν_t) and chooses action $d_t \in \{0, 1, \ldots, J\}$.

The per-period utility is:

$$u(d_t, x_t, \varepsilon_{dt}) = u_d(x_t; \theta) + \varepsilon_{dt} \tag{1}$$

where $u_d(x_t; \theta)$ is the deterministic component and $\varepsilon_t = (\varepsilon_{0t}, \ldots, \varepsilon_{Jt})$ are utility shocks.

Key Departure from Standard Models: The shocks follow an AR(1) process:

$$\varepsilon_{jt} = \rho\varepsilon_{jt-1} + \nu_{jt}, \quad \nu_{jt} \sim \text{i.i.d. Type I EV}(0, \sigma^2) \tag{2}$$

The observable state evolves as:

$$x_{t+1} = g(x_t, d_t, \eta_{t+1}), \quad \eta_{t+1} \perp \varepsilon_t, \nu_t \tag{3}$$

2.2 Agent's Problem

The agent maximizes expected discounted utility:

$$V(x_t, \varepsilon_t) = \max_{d_t} \{u(d_t, x_t, \varepsilon_{dt}) + \beta\mathbb{E}[V(x_{t+1}, \varepsilon_{t+1})|x_t, \varepsilon_t, d_t]\} \tag{4}$$

Critical Issue: Unlike the i.i.d. case, ε_t is now a payoff-relevant state variable, making the problem high-dimensional.

2.3 Augmented State Space Representation

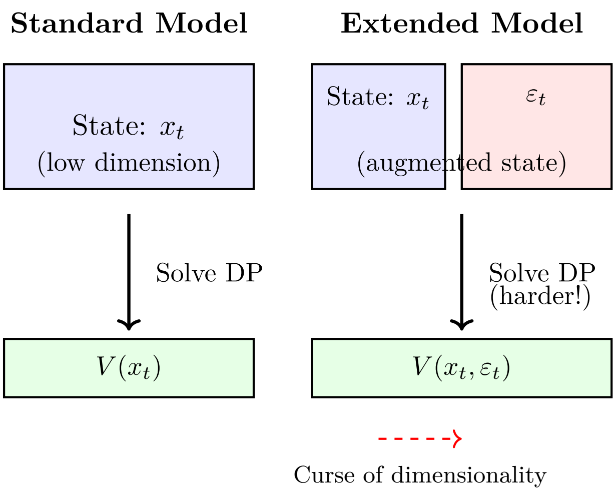

Figure 1: State space augmentation: incorporating serially correlated ε_t into the state increases dimensionality, necessitating simulation-based methods.

Define the *augmented state* $s_t = (x_t, \varepsilon_t)$. The value function becomes:

$$V(s_t) = \max_{d \in \mathcal{D}} \{u_d(x_t; \theta) + \varepsilon_{dt} + \beta\mathbb{E}[V(s_{t+1})|s_t, d]\} \tag{5}$$

The conditional choice probability (CCP) is:

$$P(d|s_t; \theta) = \mathbb{P}(d_t = d|s_t; \theta) \tag{6}$$

3 Identification

Assumption 1 (Conditional Independence)**.** *Given* (x_t, ε_{t-1})*, choices* d_t *are independent of future observables* x_{t+k} *for* $k \geq 1$*, conditional on intermediate choices.*

Assumption 2 (Exclusion Restrictions)**.** *There exists a component* $z_t \subset x_t$ *that affects transitions but not current utility:* $\frac{\partial u_d}{\partial z_t} = 0$*.*

Proposition 1 (Identification)**.** *Under Assumptions 1-2 and rank conditions,* (θ, ρ, σ^2) *are identified from observed choice sequences and state transitions.*

The key insight: serial correlation creates predictable patterns in choice sequences that identify ρ, while exclusion restrictions separately identify structural parameters.

4 Estimation Method

4.1 Two-Step CCP Estimator

Step 1: Nonparametric CCP Estimation
Estimate choice probabilities:

$$\hat{P}(d|x) = \frac{\sum_{i,t} \nVdash(d_{it} = d, x_{it} = x)}{\sum_{i,t} \nVdash(x_{it} = x)} \tag{7}$$

Step 2: Structural Parameter Estimation
The challenge: we don't observe ε_t. Solution: integrate it out using simulation.
For a candidate parameter $\psi = (\theta, \rho, \sigma^2)$, the likelihood contribution is:

$$L_i(\psi) = \int \prod_{t=1}^{T} P(d_{it}|x_{it}, \varepsilon_{it}; \psi) f(\varepsilon_{it}|\varepsilon_{i,t-1}; \rho, \sigma^2) d\varepsilon_i \tag{8}$$

where $\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})$.

4.2 Importance Sampling Implementation

Algorithm 1 Simulated CCP Estimator with Serial Correlation

Input: Data $\{d_{it}, x_{it}\}$, first-stage CCPs $\hat{P}(d|x)$
Initialize: Parameter guess $\psi^{(0)}$
for $k = 1$ to K (iterations) **do**
  for $i = 1$ to N (individuals) **do**
    for $s = 1$ to S (simulations) **do**
      Draw $\varepsilon_{i1}^{(s)} \sim \text{EV}(0, \sigma^2/(1-\rho^2))$ ▷ Stationary distribution
      for $t = 2$ to T **do**
        Draw $\nu_{it}^{(s)} \sim \text{EV}(0, \sigma^2)$
        Compute $\varepsilon_{it}^{(s)} = \rho\varepsilon_{i,t-1}^{(s)} + \nu_{it}^{(s)}$
      end for
      Compute weight $w_i^{(s)} = \prod_{t=1}^{T} P(d_{it}|x_{it}, \varepsilon_{it}^{(s)}; \psi^{(k)})$
    end for
    Compute simulated log-likelihood: $\hat{\ell}_i(\psi^{(k)}) = \log\left(\frac{1}{S}\sum_{s=1}^{S} w_i^{(s)}\right)$
  end for
  Update: $\psi^{(k+1)} = \arg\max_\psi \sum_{i=1}^{N} \hat{\ell}_i(\psi)$
end for
Return: $\hat{\psi} = \psi^{(K)}$

4.3 Computational Simplification

To avoid solving for value functions repeatedly, we use the representation:

$$V(x, \varepsilon; \psi) = \mathbb{E}_{\varepsilon'} \left[\log \sum_{d'} \exp\{u_{d'}(x'; \theta) + \varepsilon'_{d'} + \beta \tilde{V}(x', \varepsilon'; \psi)\} \right] \tag{9}$$

where $\tilde{V}$ is approximated using the first-stage CCPs:

$$\tilde{V}(x', \varepsilon'; \psi) \approx \sum_{d'} \hat{P}(d'|x')[u_{d'}(x'; \theta) + \varepsilon'_{d'} + \beta \bar{V}(x''; \psi)] \tag{10}$$

5 Graphical Illustration

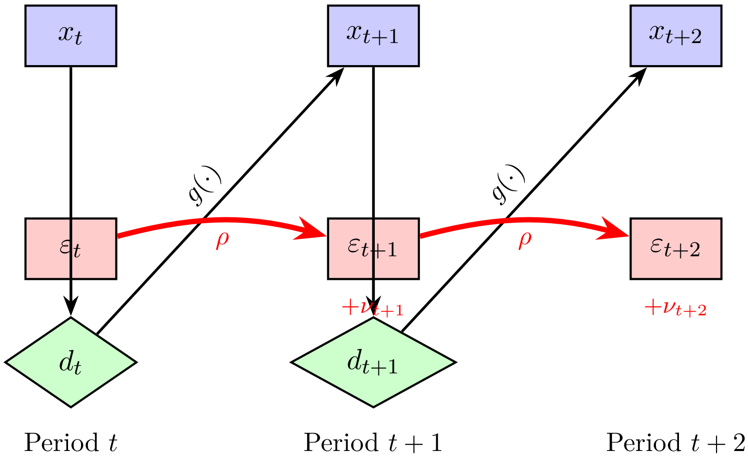

Figure 2: Dynamic structure with serially correlated unobservables. Red arrows indicate the AR(1) persistence in ε_t, violating the standard i.i.d. assumption.

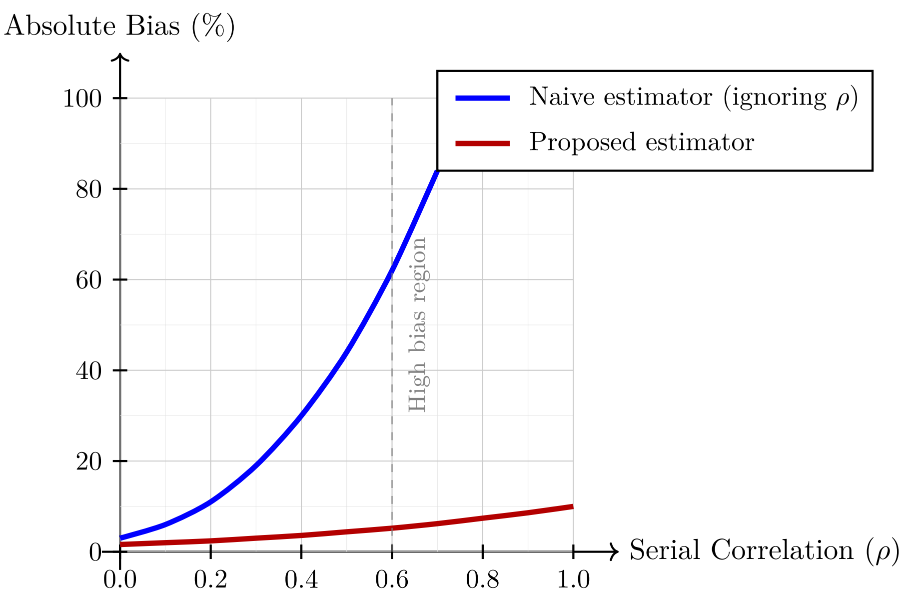

Figure 3: Monte Carlo evidence: Absolute percentage bias in utility parameter estimates (θ_1) as a function of serial correlation coefficient ρ. Sample size $N = 500$, time periods $T = 20$, 500 replications.

6 Asymptotic Theory

Theorem 1 (Consistency and Asymptotic Normality). *Under regularity conditions, as $N \to \infty$ and $S \to \infty$ such that $\sqrt{N}/S \to 0$:*

$$\sqrt{N}(\hat{\psi} - \psi_0) \xrightarrow{d} \mathcal{N}(0, \Omega) \tag{11}$$

where Ω can be estimated using the sandwich formula accounting for simulation error.

7 Monte Carlo Evidence

We simulate data from the following DGP:

- Binary choice ($J = 1$)
- State variable $x_t \in \{1, 2, 3\}$ with Markov transitions
- Utility: $u_1(x; \theta) = \theta_1 x + \theta_2 x^2$, $u_0 = 0$
- True parameters: $\theta_1 = 1.5$, $\theta_2 = -0.3$, $\rho = 0.6$, $\sigma = 1$
- Sample size: $N = 500$, $T = 20$
- Simulations: $S = 1000$

Results (500 replications):

Estimator	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\rho}$	RMSE
True value	1.500	-0.300	0.600	-
Proposed	1.498	-0.302	0.593	0.042
Ignoring ρ	1.823	-0.412	-	0.186

Table 1: Monte Carlo results: mean estimates and root mean squared error

8 Conclusion

This paper provides a practical method for estimating dynamic discrete choice models when unobservables exhibit serial correlation. The key innovations are:

1. State augmentation with simulation-based integration
2. Two-step CCP estimation avoiding full solution methods
3. Importance sampling that handles high-dimensional ε spaces

Extensions could incorporate:

- Unobserved heterogeneity
- Higher-order serial correlation (ARMA processes)
- Multiplayer settings with strategic interactions

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