The Pigeon-Hole Principle Applied to Economies and Risk-Free Rates

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Abstract

The pigeon-hole principle is a simple combinatorial truth: placing more objects than containers forces at least one container to hold multiple objects. By treating each economy as a "pigeon" and each risk-free-rate category as a "pigeon-hole", we obtain immediate, rigorous statements about the inevitable overlap of rate environments. In this paper, I illustrate the argument for three generic buckets and, more concretely, for the sign-based buckets (negative, zero, positive). The resulting insights touch on diversification, policy benchmarking, and relative-value opportunities.

The paper ends with "The End"

1 The Pigeon-Hole Principle

Let n objects be placed into k containers. If $n > k$, then at least one container contains at least

$$\left\lceil \frac{n}{k} \right\rceil$$

objects. The proof is elementary: assume every container held at most $\lceil n/k \rceil - 1$ objects; then the total number of objects would be at most $k\big(\lceil n/k \rceil - 1\big) < n$, a contradiction.

2 Economies as Pigeons, Rate Buckets as Holes

Consider five distinct economies, each characterized by its own risk-free rate. Suppose we pre-define three mutually exclusive "rate buckets":

1. Low (e.g. $0\%-2\%$),

2. Medium (e.g. $2\%-5\%$),

3. High (e.g. $> 5\%$).

Here the economies are the *pigeons* and the buckets are the *pigeon-holes*. Applying the principle with $n = 5$ and $k = 3$ gives

$$\left\lceil \frac{5}{3} \right\rceil = 2,$$

so at least one bucket must contain *two or more* economies. Consequently, perfect diversification across three disjoint rate levels is impossible; some overlap is unavoidable.

3 Sign-Based Buckets: Negative, Zero, Positive

Now let the three holes correspond to the *sign* of the risk-free rate:

- **Negative** ($r < 0$),

- **Zero** ($r = 0$),

- **Positive** ($r > 0$).

Again we have $n = 5$ economies and $k = 3$ signs, so at least two economies must share the same sign. Several useful corollaries follow.

3.1 Corollaries

1. If all three signs appear among the five economies, the remaining two economies must duplicate one of the existing signs. Hence at least one sign bucket contains *two or more* economies.

2. If only two signs appear (e.g., no zero-rate economy), the pigeon-hole principle forces one of those signs to host at least $\lceil 5/2 \rceil = 3$ economies.

3. If a single sign appears, all five economies lie in the same monetary-policy regime (e.g., all at the zero lower bound).

3.2 Economic implications

Clustering of monetary environments The forced overlap means that at least two economies experience the same qualitative stance (e.g., both dealing with negative rates), leading to similar policy challenges and potential coordination.

Diversification limits A portfolio seeking distinct "rate-sign'' exposures cannot obtain five independent sign positions; overlapping sign exposure reduces the diversification benefit derived solely from the sign of the risk-free rate.

Benchmarking Economies sharing a sign become natural peers for policy comparison, facilitating the exchange of best practices.

Relative-value opportunities Once the sign is common, the *magnitude* of the rates differentiates economies, opening carry-trade or yield-curve arbitrage possibilities.

4 Conclusion

By treating economies as pigeons and risk-free-rate categories as pigeon-holes, the pigeon-hole principle delivers immediate, rigorous statements about the inevitability of rate-environment overlap. Whether the buckets are defined by magnitude (low/medium/high) or by sign (negative/zero/positive), the principle guarantees that at least two economies share the same bucket, shaping diversification, benchmarking, and arbitrage considerations in international finance.

References

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