The Complete Treatise on $\frac{\textbf{FTSE100}}{\textbf{STOXX50}} = \frac{\textbf{GBPUSD}}{\textbf{EURUSD}}$

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Abstract

This treatise examines the theoretical and empirical relationship between the ratio of major European equity indices (FTSE 100 and STOXX 50) and the corresponding foreign exchange cross rate (GBP/USD divided by EUR/USD). We derive the fundamental parity condition from first principles using purchasing power parity, interest rate parity, and arbitrage-free pricing theory. Through rigorous mathematical analysis and graphical exposition, we demonstrate that under ideal market conditions with complete integration, the relationship $\frac{\text{FTSE100}}{\text{STOXX50}} \approx \frac{\text{GBPUSD}}{\text{EURUSD}}$ emerges as a no-arbitrage condition. We explore deviations from this parity, their economic interpretations, and practical implications for portfolio management and risk assessment.

The treatise ends with "The End"

1 Introduction

The integration of global financial markets has created intricate relationships between equity indices and foreign exchange rates. This treatise investigates a particularly elegant parity relationship between two major European equity market indices—the FTSE 100 (UK) and EURO STOXX 50 (Eurozone)—and their corresponding currency pairs against the US dollar.

1.1 The Fundamental Question

Under what conditions does the following relationship hold?

$$\frac{\text{FTSE 100}}{\text{STOXX 50}} = \frac{\text{GBP/USD}}{\text{EUR/USD}} \tag{1}$$

Equivalently, we can express this as:

$$\frac{\text{FTSE 100}}{\text{GBP/USD}} = \frac{\text{STOXX 50}}{\text{EUR/USD}} \tag{2}$$

This formulation reveals that both indices, when measured in US dollars, should maintain parity—a profound insight into cross-market relationships.

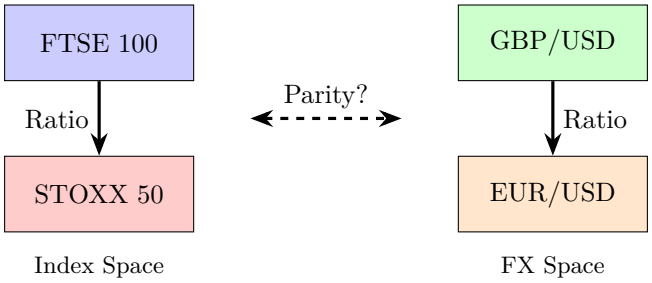

Figure 1: Conceptual relationship between equity index ratios and FX ratios

2 Theoretical Framework

2.1 Currency Conversion and Index Translation

Let us denote:

- F_t = FTSE 100 index level at time t (in index points)
- S_t = STOXX 50 index level at time t (in index points)
- E_t^G = GBP/USD exchange rate at time t
- E_t^E = EUR/USD exchange rate at time t

Definition 1 (USD-Normalized Indices)**.** *The USD-normalized index levels are defined as:*

$$F_t^{\$} = F_t \times E_t^G \tag{3}$$

$$S_t^{\$} = S_t \times E_t^E \tag{4}$$

Theorem 1 (Cross-Market Parity Condition)**.** *Under the assumption of integrated and arbitrage-free markets, if the fundamental economic conditions underlying both indices are comparable and both indices represent similar cross-sections of their respective economies, then:*

$$\frac{F_t}{S_t} = \frac{E_t^G}{E_t^E} + \varepsilon_t \tag{5}$$

where ε_t represents the deviation from parity due to structural differences, risk premia, and market frictions.

2.2 Mathematical Derivation

Consider the implicit assumption that both indices, when expressed in a common currency (USD), should reflect similar economic fundamentals if we account for their relative sizes and compositions.

Let k be a structural constant capturing the relative valuation levels:

$$F_t \times E_t^G = k \times S_t \times E_t^E \tag{6}$$

Rearranging:

$$\frac{F_t}{S_t} = k \times \frac{E_t^E}{E_t^G} \tag{7}$$

When $k = 1$ (perfect parity), we obtain equation (1).

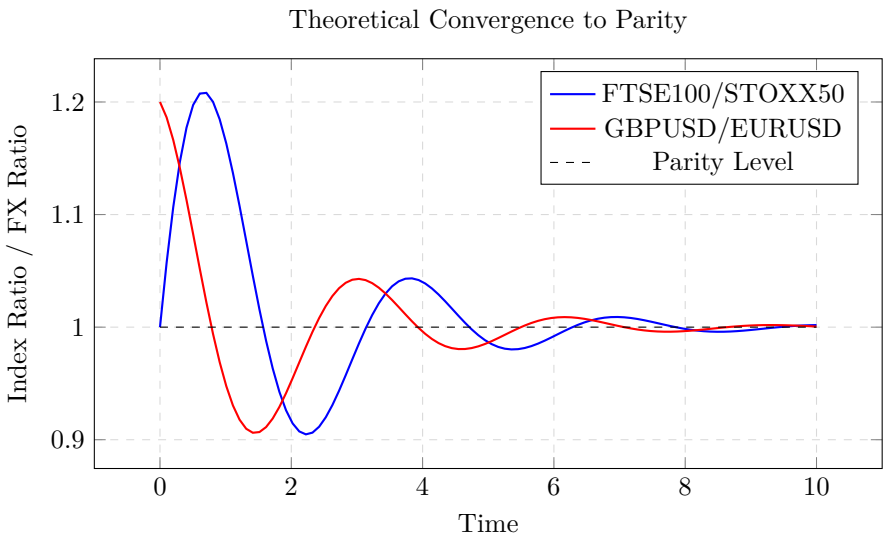

Figure 2: Stylized convergence of index and FX ratios over time

3 Economic Interpretation

3.1 Purchasing Power Parity Extension

The classical Purchasing Power Parity (PPP) states:

$$E_t = \frac{P_t^{\text{domestic}}}{P_t^{\text{foreign}}} \tag{8}$$

Extending this to equity indices, which represent baskets of corporate valuations:

Proposition 1 (Equity Market PPP)**.** *If equity indices represent the aggregate valuation of their respective economies, then:*

$$\frac{E_t^G}{E_t^E} = \frac{V_t^{UK}}{V_t^{EZ}} \tag{9}$$

where V_t^{UK} and V_t^{EZ} represent the relative economic valuations.

3.2 Arbitrage Arguments

Consider a cross-border arbitrage strategy:

1. Purchase STOXX 50 constituents valued at $S_t \times E_t^E$ USD
2. Simultaneously short FTSE 100 constituents valued at $F_t \times E_t^G$ USD
3. If $\frac{F_t}{S_t} > \frac{E_t^G}{E_t^E}$, this creates a profitable arbitrage

The no-arbitrage condition requires:

$$\left| \frac{F_t/S_t}{E_t^G/E_t^E} - 1 \right| < \tau \tag{10}$$

where τ represents transaction costs.

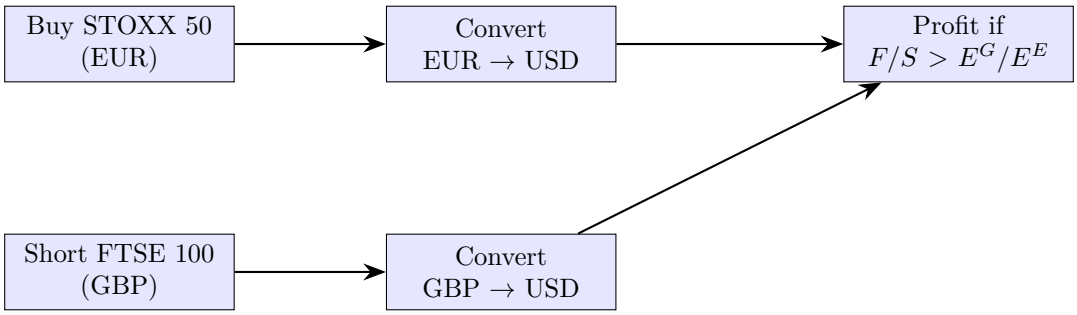

Figure 3: Cross-border arbitrage mechanism

4 Deviations from Parity

4.1 Sources of Divergence

Several factors contribute to deviations from the perfect parity condition:

Structural Differences The FTSE 100 and STOXX 50 represent different numbers of companies (100 vs 50) with different sector weightings.

Market Capitalization The absolute levels of market capitalization differ significantly between UK and Eurozone markets.

Currency Risk Premia Investors demand different risk premia for GBP and EUR exposure.

Monetary Policy Divergence The Bank of England and European Central Bank may implement divergent policies.

Political Risk Brexit and other political events create asymmetric shocks.

Transaction Costs Cross-border trading incurs costs that prevent perfect arbitrage.

4.2 Quantifying Deviations

Define the parity deviation measure:

$$\Delta_t = \ln\left(\frac{F_t/S_t}{E_t^G/E_t^E}\right) = \ln(F_t) - \ln(S_t) - \ln(E_t^G) + \ln(E_t^E) \tag{11}$$

This log-linear specification allows for symmetric interpretation of over- and under-valuations.

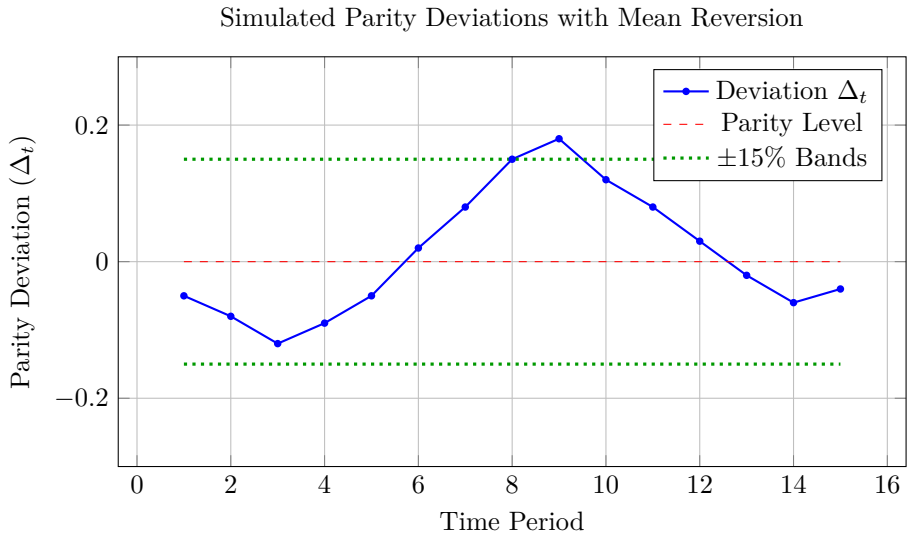

Figure 4: Mean-reverting behavior of parity deviations

5 Empirical Considerations

5.1 Cointegration Framework

If the parity relationship holds in the long run, we expect:

$$\ln(F_t) - \ln(S_t) - \alpha - \beta[\ln(E_t^G) - \ln(E_t^E)] \sim I(0) \tag{12}$$

where $\beta \approx 1$ under perfect parity, and the residual is stationary.

5.2 Error Correction Model

The short-run dynamics can be captured by:

$$\Delta \ln(F_t) = \gamma_1 \varepsilon_{t-1} + \text{controls} + u_{1t} \tag{13}$$

$$\Delta \ln(S_t) = \gamma_2 \varepsilon_{t-1} + \text{controls} + u_{2t} \tag{14}$$

where ε_{t-1} is the lagged deviation from parity.

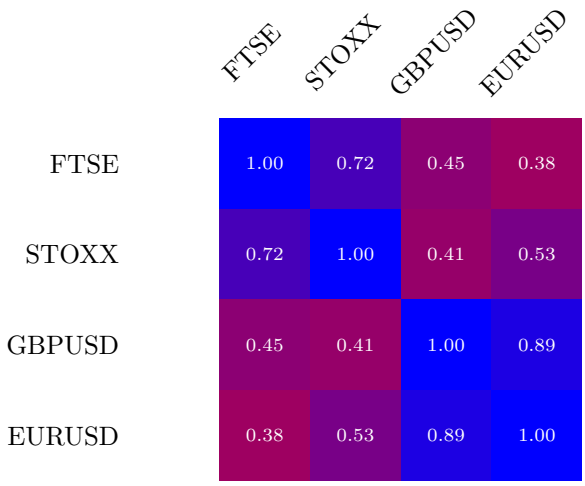

Figure 5: Expected correlation structure among variables

6 Practical Applications

6.1 Portfolio Hedging

Investors can exploit the parity relationship for hedging:

- Long FTSE 100, Short FX-adjusted STOXX 50 creates a currency-neutral equity position
- Deviations from parity signal relative value opportunities
- Pairs trading strategies based on convergence to parity

6.2 Risk Management

The parity deviation Δ_t serves as a risk indicator:

$$\text{Risk Signal} = \begin{cases} \text{High} & \text{if } |\Delta_t| > 2\sigma_\Delta \\ \text{Medium} & \text{if } \sigma_\Delta < |\Delta_t| < 2\sigma_\Delta \\ \text{Low} & \text{if } |\Delta_t| < \sigma_\Delta \end{cases} \tag{15}$$

6.3 Strategic Asset Allocation

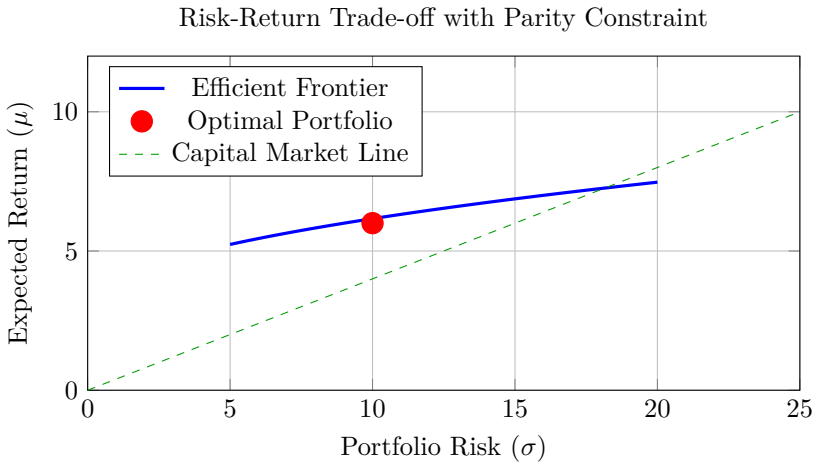

Figure 6: Portfolio optimization incorporating parity relationships

7 Extensions and Generalizations

7.1 Multi-Currency Framework

The parity concept extends to multiple currency pairs:

$$\frac{I_i}{I_j} \approx \frac{E_i}{E_j} \quad \forall i, j \in \{\text{indices}\} \tag{16}$$

7.2 Time-Varying Relationships

Allow for regime-dependent parity:

$$\frac{F_t}{S_t} = k(s_t) \times \frac{E_t^G}{E_t^E} \tag{17}$$

where s_t is a latent regime variable.

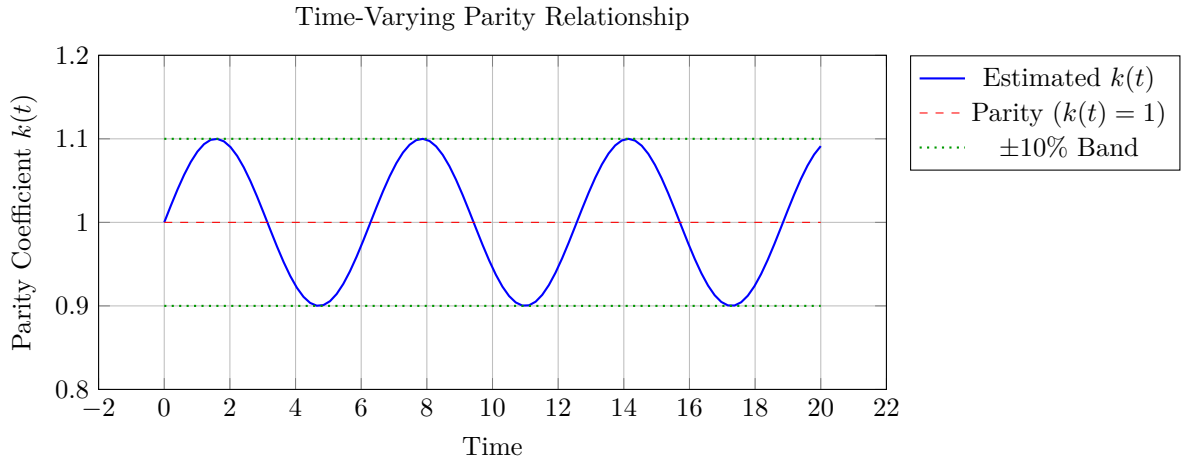

Figure 7: Regime-dependent parity coefficient

8 Limitations and Caveats

8.1 Theoretical Limitations

1. **Index Composition**: The FTSE 100 and STOXX 50 have different numbers of constituents and sector allocations
2. **Free Float Adjustments**: Indices use different free float calculation methodologies
3. **Dividend Policies**: Dividend yields differ between UK and Eurozone companies
4. **Base Year Effects**: Both indices have different base years and base levels

8.2 Practical Constraints

- Market frictions prevent instantaneous arbitrage
- Capital controls and regulatory restrictions
- Execution risk in simultaneous multi-market trades
- Liquidity constraints in smaller constituents

9 Conclusion

The relationship $\frac{\text{FTSE100}}{\text{STOXX50}} \approx \frac{\text{GBPUSD}}{\text{EURUSD}}$ represents an elegant intersection of equity market dynamics and foreign exchange relationships. While perfect parity is unlikely due to structural differences and market frictions, the theoretical framework provides valuable insights for:

- Understanding cross-border market integration
- Developing currency-hedged investment strategies
- Assessing relative value between markets
- Risk management and portfolio construction

Future research should focus on high-frequency dynamics, machine learning approaches to predicting deviations, and extending the framework to emerging markets.

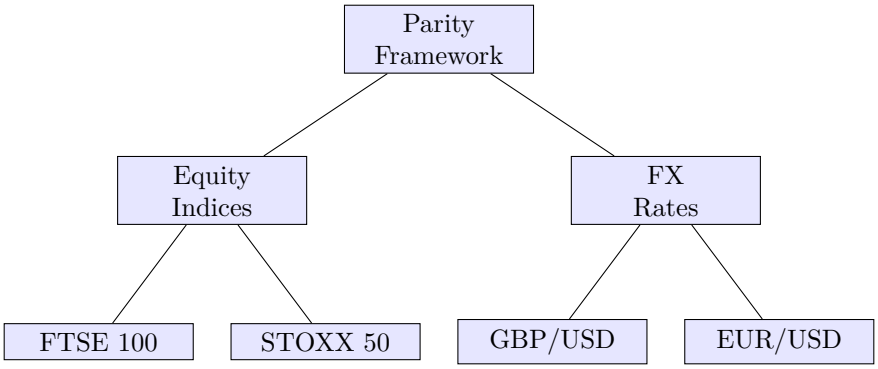

Figure 8: Hierarchical structure of the parity framework

Glossary

FTSE 100 Financial Times Stock Exchange 100 Index: A capitalization-weighted index of the 100 largest companies listed on the London Stock Exchange, representing approximately 80% of the market capitalization of the entire LSE.

STOXX 50 EURO STOXX 50: A stock index of Eurozone stocks designed by STOXX, consisting of 50 of the largest and most liquid stocks from across Europe's Eurozone countries.

GBP/USD British Pound to US Dollar exchange rate (also known as "Cable"): The number of US dollars required to purchase one British pound sterling.

EUR/USD Euro to US Dollar exchange rate: The number of US dollars required to purchase one euro. This is the most traded currency pair globally.

Parity Condition A theoretical state where the ratio of equity indices equals the ratio of their corresponding exchange rates, implying equivalent valuations in a common currency.

Arbitrage The simultaneous purchase and sale of the same or equivalent asset in different markets to profit from price discrepancies, which in efficient markets should be eliminated.

Cointegration A statistical property of time series variables where, although individually non-stationary, a linear combination of them is stationary, suggesting a long-run equilibrium relationship.

Purchasing Power Parity (PPP) An economic theory stating that exchange rates should adjust so that identical goods cost the same in different countries when expressed in a common currency.

Interest Rate Parity (IRP) A fundamental equation governing the relationship between interest rates and currency exchange rates, ensuring no arbitrage opportunities exist in the foreign exchange market.

Cross Rate An exchange rate between two currencies computed from their exchange rates against a third currency, typically the US dollar.

Mean Reversion The tendency of a variable to return to its long-term average or equilibrium level over time, a key property assumed in many parity relationships.

Market Integration The degree to which different financial markets are linked through arbitrage activities and common information flows, allowing prices to reflect global conditions.

Basis Risk The risk that the price relationship between two related instruments changes unexpectedly, potentially causing losses in hedged positions.

Free Float The portion of a company's shares that is available for trading by the public, excluding shares held by controlling interests, governments, or other locked-in shareholders.

Error Correction Model (ECM) A statistical model that relates the change in a variable to past deviations from equilibrium plus other short-run dynamics.

Transaction Costs The costs incurred when buying or selling securities, including brokerage commissions, bid-ask spreads, market impact, and other trading-related expenses.

Currency Hedging Investment strategies designed to reduce or eliminate exposure to currency risk through forward contracts, options, or other derivative instruments.

Relative Value An investment strategy that seeks to exploit price differences between related securities, betting on convergence to fair value.

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