# The Ghosh quintic of three integers $\alpha$, $a \neq 0$ and $b$

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**Abstract**

In this paper, I describe the Ghosh quintic of three integers $\alpha$, $a$ and $b$.
The paper ends with "The End"

## Introduction

Unknown to most mathematicians, given three integers $\alpha$, $a \neq 0$ and $b$, there always exists **the Ghosh quintic of $\alpha$, $a$ and $b$** such that one of its roots is $\alpha$ and the remaining four roots are radical expressions of only $a$ and $b$.

In this paper, I describe the Ghosh quintic of three integers $\alpha$, $a \neq 0$, and $b$.

## The Ghosh quintic of three integers $\alpha$, $a \neq 0$, and $b$

The Ghosh quintic of three integers $\alpha$, $a \neq 0$, and $b$ is

$$f(x) = ax^5 + (a + 2b - a\alpha)x^4 + (a - 2b\alpha - a\alpha)x^3 + (a + 2b - a\alpha)x^2 + (a - 2b\alpha - a\alpha)x - a\alpha$$

## The roots of the Ghosh quintic of three integers $\alpha$, $a \neq 0$, $b$

One of the roots of $f(x) = 0$ is $x_1 = \alpha$.

The remaining four roots of $f(x) = 0$ are radical expressions of only $a$ and $b$:

$$x_2 = -\frac{1}{2}\sqrt{\frac{(a+2b)^2}{4a^2} + 1} - \frac{1}{2}\sqrt{\frac{(a+2b)^2}{2a^2} - \frac{-\frac{(a+2b)^3}{a^3} - \frac{4(a+2b)}{a}}{4\sqrt{\frac{(a+2b)^2}{4a^2} + 1}} - 3} - \frac{a+2b}{4a}$$

$$x_3 = -\frac{1}{2}\sqrt{\frac{(a+2b)^2}{4a^2} + 1} + \frac{1}{2}\sqrt{\frac{(a+2b)^2}{2a^2} - \frac{-\frac{(a+2b)^3}{a^3} - \frac{4(a+2b)}{a}}{4\sqrt{\frac{(a+2b)^2}{4a^2} + 1}} - 3} - \frac{a+2b}{4a}$$

$$x_4 = \frac{1}{2}\sqrt{\frac{(a+2b)^2}{4a^2} + 1} - \frac{1}{2}\sqrt{\frac{(a+2b)^2}{2a^2} + \frac{-\frac{(a+2b)^3}{a^3} - \frac{4(a+2b)}{a}}{4\sqrt{\frac{(a+2b)^2}{4a^2} + 1}} - 3} - \frac{a+2b}{4a}$$

$$x_5 = \frac{1}{2}\sqrt{\frac{(a+2b)^2}{4a^2} + 1} + \frac{1}{2}\sqrt{\frac{(a+2b)^2}{2a^2} + \frac{-\frac{(a+2b)^3}{a^3} - \frac{4(a+2b)}{a}}{4\sqrt{\frac{(a+2b)^2}{4a^2} + 1}} - 3} - \frac{a+2b}{4a}$$

## The End