Contemporary approaches in asset pricing: A survey of leading methodologies

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Abstract

In this paper, I survey the primary approaches developed by the most cited authors in the field of asset pricing, organized by their theoretical foundations and practical applications.
The paper ends with "The End"

1 Introduction

Asset pricing theory has evolved through several paradigmatic shifts, each addressing limitations of previous models while incorporating new empirical insights.

In this paper, I survey the primary approaches developed by the most cited authors in the field of asset pricing, organized by their theoretical foundations and practical applications.

2 Foundational Equilibrium Models

2.1 Capital Asset Pricing Model Framework

The foundational work by Sharpe (1964) established the single-factor CAPM, which relates expected returns to systematic risk through the security market line:

$$E[R_i] = R_f + \beta_i(E[R_m] - R_f) \tag{1}$$

where $\beta_i = \frac{\text{Cov}(R_i,R_m)}{\text{Var}(R_m)}$ represents the systematic risk of asset i relative to the market portfolio. The model assumes homogeneous expectations, mean-variance optimization, and frictionless markets. Despite its theoretical elegance, empirical tests by Fama and French (1992) revealed significant limitations in explaining cross-sectional return variations.

2.2 Arbitrage Pricing Theory

Ross (1976) developed the APT as a more flexible alternative, assuming that asset returns follow a K-factor structure:

$$R_i = E[R_i] + \sum_{k=1}^{K} \beta_{ik} F_k + \epsilon_i \tag{2}$$

where F_k are systematic factors and ϵ_i is idiosyncratic risk. Under the no-arbitrage condition, expected returns satisfy:

$$E[R_i] = R_f + \sum_{k=1}^{K} \beta_{ik} \lambda_k \tag{3}$$

where λ_k represents the risk premium for factor k. The APT's theoretical generality comes at the cost of limited guidance on factor identification.

3 Consumption-Based Asset Pricing

3.1 Consumption CAPM and Stochastic Discount Factor Models

The consumption-based approach by Breeden (1979) grounds asset pricing in fundamental economic theory. The stochastic discount factor M_{t+1} satisfies:

$$E_t[M_{t+1}R_{i,t+1}] = 1 \tag{4}$$

Under power utility with relative risk aversion γ, the discount factor becomes:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{5}$$

where C_t represents consumption at time t. This framework addresses CAPM's theoretical shortcomings but faces the equity premium puzzle documented by Mehra and Prescott (1985).

3.2 Long-Run Risk Models

Bansal and Yaron (2004) extended consumption-based models by incorporating time-varying expected consumption growth x_t and volatility σ_t:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1} \tag{6}$$

$$x_{t+1} = \rho x_t + \phi_e \sigma_t e_{t+1} \tag{7}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \tag{8}$$

where η_{t+1}, e_{t+1}, and w_{t+1} are independent shocks. This framework successfully explains asset pricing puzzles through investors' concerns about long-term economic uncertainty.

4 Behavioral Asset Pricing

4.1 Prospect Theory Applications

Behavioral models incorporate Kahneman and Tversky (1979) prospect theory preferences. The value function exhibits loss aversion:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases} \tag{9}$$

where $\lambda > 1$ captures loss aversion and $\alpha, \beta < 1$ reflect diminishing sensitivity. Barberis et al. (2001) demonstrate how these preferences generate predictable return patterns.

4.2 Sentiment and Limits to Arbitrage

Shleifer and Vishny (1997) show that arbitrage limitations prevent complete elimination of mispricing. When noise traders create demand shocks ρ_t, fundamental arbitrageurs with capital W_t face:

$$p_t = f_t + \frac{\rho_t}{1 + \eta W_t} \tag{10}$$

where η measures arbitrage effectiveness. This framework explains persistent mispricing in financial markets.

5 Empirical Factor Models

5.1 Fama-French Multi-Factor Models

Fama and French (1993) revolutionized empirical asset pricing with their three-factor model:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i(R_{m,t} - R_{f,t}) + s_i \text{SMB}_t + h_i \text{HML}_t + \epsilon_{i,t} \tag{11}$$

where SMB captures size effects and HML captures value effects. Fama and French (2015) extended this to a five-factor model incorporating profitability (RMW) and investment (CMA) factors:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i \text{MKT}_t + s_i \text{SMB}_t + h_i \text{HML}_t + r_i \text{RMW}_t + c_i \text{CMA}_t + \epsilon_{i,t} \tag{12}$$

5.2 Momentum and Reversal Strategies

Jegadeesh and Titman (1993) documented momentum effects through portfolio strategies that buy past winners and sell past losers. The momentum factor can be expressed as:

$$\text{MOM}_t = \frac{1}{n} \sum_{i \in \text{Winners}} R_{i,t} - \frac{1}{m} \sum_{j \in \text{Losers}} R_{j,t} \tag{13}$$

De Bondt and Thaler (1985) established long-term reversal patterns, suggesting that behavioral biases create systematic return predictability.

6 Dynamic Asset Pricing Models

6.1 Habit Formation Models

Campbell and Cochrane (1999) developed habit formation models where utility depends on consumption relative to habit H_t:

$$U(C_t, H_t) = \frac{(C_t - H_t)^{1-\gamma}}{1-\gamma} \tag{14}$$

with habit evolution:

$$H_{t+1} = \delta H_t + (1-\delta) C_t \tag{15}$$

This creates time-varying risk aversion and explains the equity premium puzzle through countercyclical risk premiums.

6.2 Rare Disasters Framework

Barro (2006) addresses asset pricing puzzles by incorporating disaster probability p_t:

$$\log C_{t+1} = \log C_t + \mu + \sigma \epsilon_{t+1} - b_t D_{t+1} \tag{16}$$

where D_{t+1} is a disaster indicator and b_t represents disaster magnitude. This framework explains high equity premiums through investors' concern about catastrophic events.

7 Production-Based Asset Pricing

7.1 Investment-Based Models

Zhang (2005) developed production-based models linking asset returns to investment decisions. The firm's optimization problem:

$$\max_{I_t, K_{t+1}} E_t \sum_{s=0}^{\infty} M_{t,t+s} \Pi_{t+s}(K_{t+s}, I_{t+s}) \tag{17}$$

where Π_{t+s} represents profits and $M_{t,t+s}$ is the stochastic discount factor. The first-order condition implies:

$$1 = E_t[M_{t+1} \frac{\partial \Pi_{t+1}}{\partial K_{t+1}}] \tag{18}$$

This q-theory framework connects asset prices to investment through the marginal product of capital.

8 Machine Learning and Alternative Data Approaches

8.1 Factor Discovery Through Machine Learning

Kozak et al. (2020) apply machine learning techniques to factor model estimation using the penalized regression:

$$\hat{\beta} = \arg\min_{\beta} \frac{1}{N} \sum_{i=1}^{N} (R_i - X_i' \beta)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \tag{19}$$

where λ controls regularization strength. This approach identifies relevant factors from large characteristic sets while controlling for overfitting through cross-validation procedures.

9 Term Structure and Credit Risk Models

9.1 Affine Term Structure Models

Duffie and Kan (1996) developed affine term structure models where bond prices take the exponential-affine form:

$$P_t(\tau) = \exp(A(\tau) + B(\tau)' X_t) \tag{20}$$

where X_t follows an affine diffusion process:

$$dX_t = \kappa(\theta - X_t)dt + \Sigma \sqrt{S_t} dW_t \tag{21}$$

with $S_t = \alpha + \beta X_t$. This framework enables analytical solutions for many fixed-income derivatives.

9.2 Structural Credit Models

Merton (1974) developed the structural approach treating equity as a call option on firm assets. The equity value satisfies:

$$E_t = \max(V_t - D, 0) \tag{22}$$

where V_t represents firm value and D is debt. Under geometric Brownian motion for firm value, the Black-Scholes formula applies with default probability:

$$P(\text{default}) = N\left(\frac{\log(D/V_0) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \right) \tag{23}$$

10 Cross-Sectional Asset Pricing

10.1 Characteristics-Based Models

Daniel and Titman (1997) emphasize that firm characteristics, rather than factor loadings, drive cross-sectional returns. They argue that the success of factor models stems from correlation with fundamental characteristics rather than their role as risk factors. This approach suggests direct modeling of characteristic-return relationships:

$$E[R_i] = f(\text{characteristics}_i) \tag{24}$$

11 Conclusion

Contemporary asset pricing encompasses diverse approaches addressing different aspects of return generation and risk assessment. The field has evolved from simple single-factor models to sophisticated frameworks incorporating behavioral insights, alternative data sources, and advanced econometric techniques. The most robust approaches combine theoretical rigor with empirical validation, recognizing that no single model explains all observed return patterns. Integration of these various approaches continues advancing our understanding, with machine learning techniques and alternative data representing the current research frontier.

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