

Dynamics of High-IQ Survival under Differential Fitness

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1 Introduction

We develop deterministic and stochastic models in which a subpopulation with higher IQ survives and dominates almost surely, conditioned on non-extinction. The analysis uses replicator dynamics and continuous-time multitype branching processes.

2 Deterministic Model: Replicator Dynamics

Let $p_H(t)$ and $p_L(t)$ denote proportions of high-IQ and low-IQ individuals, respectively, with $p_H + p_L = 1$. Assign constant fitnesses $w_H > w_L > 0$. The replicator equation reads

$$\dot{p}_H(t) = p_H(t)(w_H - \bar{w}(t)), \quad \bar{w}(t) = p_H(t)w_H + (1 - p_H(t))w_L. \quad (1)$$

This simplifies to

$$\dot{p}_H(t) = p_H(t)(1 - p_H(t))(w_H - w_L), \quad (2)$$

which is logistic. The solution is

$$p_H(t) = \frac{1}{1 + \left(\frac{1 - p_H(0)}{p_H(0)} \right) e^{-(w_H - w_L)t}}, \quad (3)$$

so $p_H(t) \rightarrow 1$ exponentially fast as $t \rightarrow \infty$.

3 Stochastic Model: Continuous-Time Multitype Branching Process

Definition 1. A two-type continuous-time branching process $\mathbf{Z}(t) = (Z_H(t), Z_L(t))$ evolves as follows: each individual of type $i \in \{H, L\}$ reproduces at rate β_i and dies at rate δ_i . Births produce offspring of the same type (mutation-free case). Net growth rates are $r_i = \beta_i - \delta_i$.

Assume $r_H > r_L$ and $r_H > 0$.

Theorem 1 (Almost Sure Dominance). Suppose the initial population contains at least one high-IQ individual. Let $P_H(t) = Z_H(t)/(Z_H(t) + Z_L(t))$ when the denominator is positive. Then, conditioned on non-extinction of the total population,

$$\lim_{t \rightarrow \infty} P_H(t) = 1, \quad \text{almost surely.} \quad (4)$$

Sketch. The mean matrix of the process has dominant eigenvalue $\lambda^* = r_H$ and Perron eigenvector v concentrated on the H coordinate. By the Kesten–Stigum theorem, $e^{-\lambda^* t} \mathbf{Z}(t) \rightarrow Wv$ almost surely, where $W > 0$ on the survival event. Thus $Z_L(t)/Z_H(t) \rightarrow 0$ a.s., implying $P_H(t) \rightarrow 1$. \square

4 Vector Graphic Illustration

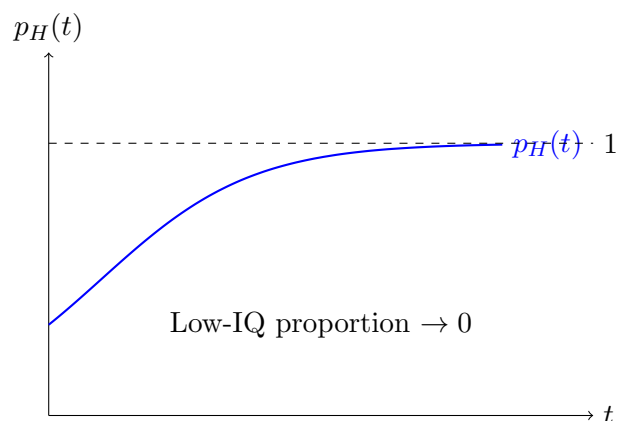


Figure 1: Replicator dynamics: $p_H(t)$ converges to 1.

5 Remarks

The result extends to models with small mutations $L \rightarrow H$ and to continuous IQ traits with fitness $r(\text{IQ})$. Spatial structure and density dependence may alter the conclusion.

References

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