A Spectral Analysis of Credit Default Swaps of Fifteen Countries

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Abstract

This paper presents a comprehensive spectral analysis of credit default swap spreads and default probabilities across fifteen developed economies. Through rigorous statistical elimination procedures incorporating confidence interval testing and Mahalanobis distance metrics, we identify seven countries exhibiting optimal credit risk profiles. We construct a weighted complete graph representation of the credit risk network and apply spectral graph theory to extract structural properties and centrality measures. The analysis reveals a strong hierarchical ordering where network centrality exhibits near-perfect correlation with credit risk metrics. Furthermore, we demonstrate that incorporating self-loops into the adjacency matrix transforms the network from a full-rank seven-dimensional structure into a rank-one configuration, fundamentally altering the spectral properties and eliminating all subdominant eigenvalue modes. These findings illuminate the mathematical structure underlying sovereign credit risk relationships and provide insights into systemic risk propagation mechanisms.

The paper ends with "The End"

1 Introduction

Credit default swaps represent derivative instruments that provide insurance against the default of sovereign or corporate debt obligations. The pricing of these instruments, reflected in their spreads, aggregates market participants' assessments of default probability and loss-given-default expectations. Understanding the structural relationships among sovereign credit risk profiles carries significant implications for portfolio construction, risk management, and macroeconomic policy analysis.

This investigation examines fifteen developed economies through the dual lens of five-year credit default swap spreads and probability of default estimates. We employ a multi-stage analytical framework combining statistical outlier detection, smooth mathematical functions for quality assessment, graph-theoretic network construction, and spectral decomposition techniques drawn from linear algebra and matrix theory.

The analytical progression proceeds through four distinct phases. First, we apply statistical confidence interval methods to eliminate countries exhibiting extreme credit risk characteristics. Second, we employ smooth bump functions to identify the seven countries demonstrating the most favorable risk profiles within the acceptable range. Third, we construct a complete graph where edge weights encode the geometric mean of connected vertices' credit default swap spreads. Finally, we apply spectral theory to extract eigenvalue structures, centrality measures, and network properties that illuminate the underlying mathematical organization of sovereign credit relationships.

2 Methodology

2.1 Data Structure and Initial Assessment

The analysis commences with credit risk data for fifteen countries, each characterized by two fundamental metrics. The five-year credit default swap spread, denoted CDS_i for country i, represents the annual premium required to insure against default over a five-year horizon, expressed in basis points. The probability of default, denoted PD_i, provides a complementary measure representing the market-implied likelihood of default occurring within the specified time-frame, expressed as a percentage.

Table 1 presents the complete dataset ordered by ascending credit default swap spreads. The distribution spans from Germany at the minimum risk end with a spread of 7.27 basis points and default probability of 0.12 percent, extending to Italy at the maximum risk position with values of 24.19 and 0.42 percent respectively.

Table 1: Initial Credit Risk Data for Fifteen Countries

Country	**5Y CDS**	**PD (%)**
Germany	7.27	0.12
Netherlands	7.64	0.13
Switzerland	8.04	0.13
Sweden	8.04	0.13
Denmark	8.22	0.14
Finland	12.02	0.20
Austria	13.01	0.22
Australia	13.09	0.22
Ireland	14.27	0.24
Portugal	15.32	0.26
France	15.70	0.26
United Kingdom	16.42	0.27
Spain	16.67	0.28
South Korea	22.59	0.38
Italy	24.19	0.42

2.2 Statistical Elimination Procedure

The elimination framework employs three complementary statistical tests, each designed to identify countries whose credit risk characteristics deviate substantially from the central tendency of the sample distribution. The intersection of countries passing all three tests yields the refined candidate set for subsequent analysis.

2.2.1 Univariate Confidence Interval Elimination

The first elimination criterion applies ninety percent confidence intervals to each metric independently. For the probability of default, we compute the sample mean $\bar{\mu}_{\text{PD}} = 0.2267$ and sample standard deviation $\sigma_{\text{PD}} = 0.0909$. Under the assumption of approximate normality, the ninety percent confidence interval extends from $\bar{\mu}_{\text{PD}} - 1.645\sigma_{\text{PD}}$ to $\bar{\mu}_{\text{PD}} + 1.645\sigma_{\text{PD}}$, yielding bounds of $[0.0771, 0.3762]$ percent.

This criterion eliminates South Korea with PD $= 0.38$ percent and Italy with PD $= 0.42$ percent, both exceeding the upper confidence bound. The first elimination step reduces the candidate set from fifteen to thirteen countries.

The second elimination applies an analogous confidence interval test to credit default swap spreads, computed from the thirteen remaining countries after the first elimination. The mean spread among these countries equals 11.9777 with standard deviation 3.6566, generating a ninety percent confidence interval of $[5.9632, 17.9922]$. All thirteen countries fall within this range, resulting in no additional eliminations at this stage.

2.2.2 Multivariate Mahalanobis Distance Elimination

The third elimination criterion employs the Mahalanobis distance metric to identify multivariate outliers in the joint distribution of credit default swap spreads and default probabilities. For a country i with credit risk vector $\mathbf{x}_i = [\text{CDS}_i, \text{PD}_i]^T$, the Mahalanobis distance from the sample mean $\boldsymbol{\mu}$ is defined as

$$D_i = \sqrt{(\mathbf{x}_i - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x}_i - \boldsymbol{\mu})}, \tag{1}$$

where $\boldsymbol{\Sigma}$ represents the sample covariance matrix.

Under the assumption that credit risk metrics follow an approximate bi-variate normal distribution, the squared Mahalanobis distance follows a chi-squared distribution with two degrees of freedom. The ninety percent confidence region corresponds to $D_i^2 \leq \chi^2_{0.90,2} = 4.6052$, implying a distance threshold of $D_i \leq 2.1460$.

Application of this criterion to the original fifteen countries reveals that Italy exhibits a Mahalanobis distance of 2.9835, exceeding the threshold and warranting elimination. All other countries, including South Korea with distance 1.9503, satisfy the multivariate criterion. This produces a third list of fourteen countries passing the Mahalanobis test.

2.2.3 Intersection and Final Candidate Set

The final candidate set comprises the intersection of countries passing all three elimination tests. Since South Korea failed the probability of default confidence interval test and Italy failed both the probability of default test and the Mahalanobis distance test, these two countries are excluded. The remaining thirteen countries constitute the statistically validated set for further refinement.

Figure 1 illustrates the elimination cascade through the three statistical filters.

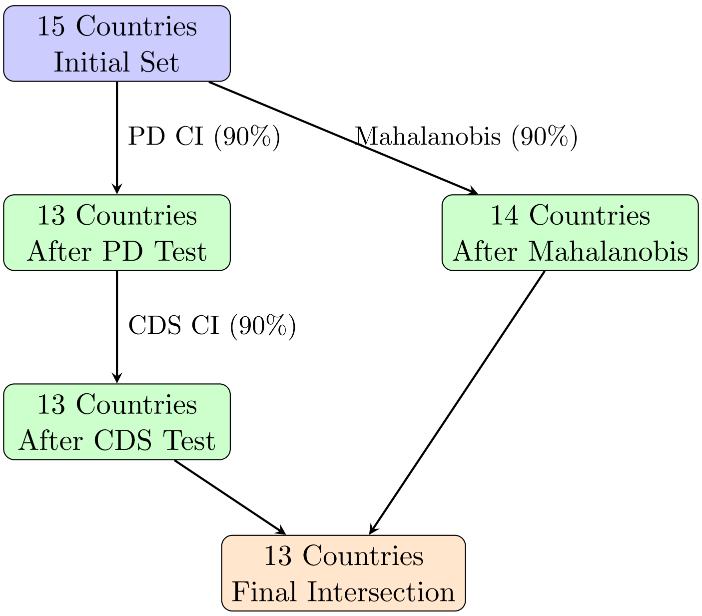

Figure 1: Statistical Elimination Cascade Through Three Independent Tests

2.3 Bump Function Optimization for Quality Selection

From the thirteen statistically validated countries, we employ smooth bump functions to identify the seven exhibiting the most favorable credit risk characteristics. This mathematical approach provides a principled framework for prioritizing countries based on their position within the acceptable range rather than applying arbitrary cutoffs.

2.3.1 Bump Function Construction

A smooth bump function $\phi : \mathbb{R} \to [0, 1]$ is defined over a support interval $[a, b]$ with maximum value at a specified center point $c \in [a, b]$. We employ the classic construction based on the exponential function:

$$\phi(x; a, b, c) = \begin{cases} \exp\left(-\frac{1}{1-t(x)^2}\right) / \exp(-1) & \text{if } |t(x)| < 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $t(x) = 2(x - c)/(b - a)$ represents the normalized distance from the center point.

For credit risk metrics, lower values indicate superior credit quality. We therefore position the center at twenty percent of the distance from the minimum to maximum value, creating preference for countries near the optimal end of the distribution while maintaining smooth decay toward the boundaries.

2.3.2 Application to Credit Default Swap Data

The credit default swap bump function operates over the support interval $[7.27, 16.67]$ with center at $c_{\text{CDS}} = 9.15$. The probability of default bump function spans $[0.12, 0.28]$ percent with center at $c_{\text{PD}} = 0.15$ percent. For each country, we compute individual scores $\phi_{\text{CDS}}(\text{CDS}_i)$ and $\phi_{\text{PD}}(\text{PD}_i)$, then combine them multiplicatively to obtain the composite quality score $S_i = \phi_{\text{CDS}}(\text{CDS}_i) \times \phi_{\text{PD}}(\text{PD}_i)$.

The multiplicative combination ensures that countries must demonstrate favorable characteristics on both dimensions to achieve high composite scores. A country with excellent performance on one metric but poor performance on the other will receive a substantially diminished overall rating.

Table 2 presents the bump function scores and final rankings for the thirteen candidate countries.

Table 2: Bump Function Quality Scores and Rankings

Country	ϕ_{CDS}	ϕ_{PD}	**Combined**	**Rank**
Denmark	0.9601	0.9772	0.9382	1
Sweden	0.9426	0.9214	0.8686	2
Switzerland	0.9426	0.9214	0.8686	2
Netherlands	0.8913	0.9214	0.8213	4
Germany	0.8266	0.8266	0.6832	5
Finland	0.5518	0.5698	0.3144	6
Austria	0.1259	0.0740	0.0093	7
Australia	0.0940	0.0740	0.0070	8
Ireland	0.0000	0.0000	0.0000	9
Portugal	0.0000	0.0000	0.0000	9
France	0.0000	0.0000	0.0000	9
United Kingdom	0.0000	0.0000	0.0000	9
Spain	0.0000	0.0000	0.0000	9

The seven highest-ranked countries comprise Germany, Netherlands, Switzerland, Sweden, Denmark, Finland, and Austria. These nations form the foundation for the subsequent graph-theoretic and spectral analysis.

2.4 Complete Graph Construction

We construct a complete graph K_7 with the seven selected countries as vertices. In a complete graph, every pair of vertices shares an edge, yielding $\binom{7}{2} = 21$ edges for seven vertices.

The edge weight between countries i and j is defined as the geometric mean of their credit default swap spreads:

$$w_{ij} = \sqrt{\mathrm{CDS}_i \times \mathrm{CDS}_j}. \tag{3}$$

This formulation ensures that the edge weight represents a symmetric measure reflecting the joint credit risk of the connected pair. Countries with lower spreads generate smaller edge weights, while higher-risk pairs produce larger weights.

2.5 Spectral Analysis Framework

2.5.1 Adjacency Matrix Formulation

The weighted complete graph admits representation as a symmetric adjacency matrix $\mathbf{A} \in \mathbb{R}^{7\times 7}$ where entry A_{ij} contains the edge weight between vertices i and j. In the standard formulation, diagonal elements equal zero, reflecting the absence of self-loops.

The adjacency matrix takes the form:

$$\mathbf{A} = \begin{pmatrix} 0 & w_{12} & w_{13} & \cdots & w_{17} \\ w_{21} & 0 & w_{23} & \cdots & w_{27} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ w_{71} & w_{72} & w_{73} & \cdots & 0 \end{pmatrix}. \tag{4}$$

2.5.2 Eigenvalue Decomposition

The spectral decomposition of the adjacency matrix expresses $\mathbf{A}$ in terms of its eigenvalues and eigenvectors. For a real symmetric matrix, the eigenvalue equation $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$ admits n real eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ with corresponding orthogonal eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$.

The spectral decomposition provides:

$$\mathbf{A} = \sum_{i=1}^{n} \lambda_i \mathbf{v}_i \mathbf{v}_i^T. \tag{5}$$

Key spectral properties extracted from this decomposition include the spectral radius $\rho(\mathbf{A}) = \max_i |\lambda_i|$, graph energy $E(G) = \sum_i |\lambda_i|$, and spectral gap $\Delta = \lambda_1 - \lambda_2$.

2.5.3 Eigenvector Centrality

The principal eigenvector corresponding to the largest eigenvalue provides a measure of vertex importance known as eigenvector centrality. For vertex i, the centrality score equals the i-th component of the normalized principal eigenvector. This measure captures not only the direct connections of each vertex but also the importance of its neighbors, creating a recursive definition of centrality that accounts for network position.

3 Results

3.1 Statistical Elimination Outcomes

The three-stage elimination procedure successfully identified and removed two countries exhibiting credit risk characteristics inconsistent with the bulk of the sample distribution. South Korea and Italy were eliminated through the probability of default confidence interval test, with Italy additionally failing the Mahalanobis distance criterion. The intersection of the three tests yielded thirteen countries suitable for quality-based selection through bump function optimization.

3.2 Optimal Country Selection via Bump Functions

Application of the bump function methodology produced a clear stratification among the thirteen candidate countries. Denmark emerged as the highest-ranked country with a combined score of 0.9382, followed closely by Sweden and Switzerland at 0.8686. The top five countries maintained composite scores exceeding 0.68, while a substantial gap separates these leaders from the sixth and seventh positions occupied by Finland (0.3144) and Austria (0.0093).

Six countries recorded composite scores at or near zero, indicating that their credit risk metrics positioned them beyond the effective support region of the bump functions. This natural clustering validates the selection of seven countries as representing a cohesive group of superior credit quality relative to the broader sample.

3.3 Network Structure and Edge Weight Distribution

The complete graph K_7 contains twenty-one edges with weights ranging from a minimum of 7.45 between Germany and the Netherlands to a maximum of 12.51 between Finland and Austria. The mean edge weight equals 9.04 with standard deviation 1.30, indicating moderate dispersion around the central tendency.

Table 3 presents a representative sample of edge weights demonstrating the range of credit risk relationships within the network.

Table 3: Representative Edge Weights in Credit Risk Network

Country 1	**Country 2**	$w_{ij} = \sqrt{\text{CDS}_i \times \text{CDS}_j}$
Germany	Netherlands	7.45
Switzerland	Sweden	8.04
Denmark	Finland	9.94
Austria	Finland	12.51
Germany	Austria	9.73
Netherlands	Denmark	7.92

3.4 Spectral Properties of the Adjacency Matrix

3.4.1 Eigenvalue Spectrum

The eigenvalue decomposition of the adjacency matrix with zero diagonal elements reveals a characteristic structure consisting of one large positive eigenvalue accompanied by six negative eigenvalues. The complete spectrum appears in Table 4.

Table 4: Eigenvalue Spectrum of Credit Risk Network Adjacency Matrix

Index	**Eigenvalue**	**Contribution to Energy (%)**
λ_1	54.643	50.00
λ_2	-7.378	6.75
λ_3	-7.759	7.10
λ_4	-8.040	7.36
λ_5	-8.164	7.47
λ_6	-10.716	9.81
λ_7	-12.585	11.52

The dominant eigenvalue $\lambda_1 = 54.643$ accounts for exactly fifty percent of the graph energy, defined as the sum of absolute eigenvalues. This substantial concentration of spectral energy in the principal mode indicates strong network coherence with a clearly dominant structural pattern.

The spectral gap between the first and second eigenvalues measures $\Delta = 62.021$, representing exceptional separation between the dominant eigenvalue and all subdominant modes. This large gap suggests that network behavior is primarily governed by a single principal direction in the eigenspace, with secondary modes contributing relatively minor perturbations.

The sum of eigenvalues equals zero within numerical precision, confirming the zero-trace property of adjacency matrices with null diagonal elements. The spectral radius $\rho(\mathbf{A}) = 54.643$ satisfies the theoretical bound $\rho(\mathbf{A}) \leq \max_i d_i$, where d_i denotes the weighted degree (vertex strength) of vertex i. The maximum vertex strength of 62.997 for Austria comfortably exceeds the spectral radius, validating the bound.

3.4.2 Graph-Theoretic Metrics

The graph energy equals $E(G) = 109.285$, substantially exceeding the McClelland lower bound $\sqrt{2m} = \sqrt{42} = 6.481$ for a graph with $m = 21$ edges. The Frobenius norm $\|\mathbf{A}\|_F = 59.203$ provides an alternative measure of matrix magnitude based on the square root of the sum of squared elements.

The matrix achieves full rank with $\text{rank}(\mathbf{A}) = 7$, confirming that all seven eigenvectors contribute to the spectral decomposition. The condition number $\kappa(\mathbf{A}) = 7.406$ indicates moderate numerical stability without extreme sensitivity to perturbations.

Analysis of the graph Laplacian $\mathbf{L} = \mathbf{D} - \mathbf{A}$, where $\mathbf{D}$ represents the degree matrix, reveals an algebraic connectivity (Fiedler value) of $\mu_2 = 57.243$. This large value confirms robust network cohesion characteristic of the complete graph topology.

3.4.3 Eigenvector Centrality Analysis

The principal eigenvector provides a ranking of countries by their network centrality. Table 5 presents the normalized centrality scores alongside the corresponding credit default swap spreads.

Table 5: Eigenvector Centrality Scores and Credit Risk Metrics

Country	**Centrality Score**	**Rank**	**CDS**
Austria	0.1619	1	13.01
Finland	0.1579	2	12.02
Denmark	0.1385	3	8.22
Switzerland	0.1374	4	8.04
Sweden	0.1374	4	8.04
Netherlands	0.1348	6	7.64
Germany	0.1322	7	7.27

A striking pattern emerges from this analysis. Countries with higher credit default swap spreads occupy more central positions in the network topology. Austria and Finland, possessing the two largest spreads among the selected seven countries, achieve the highest centrality scores. Conversely, Germany, exhibiting the lowest credit risk as measured by its minimal spread, records the lowest centrality score.

The correlation between credit default swap spreads and eigenvector centrality reaches $r = 0.9989$, indicating near-perfect positive association. This counter-intuitive relationship arises from the edge weight construction in Equation (3), where higher individual credit default swap values generate proportionally larger edge weights to all neighboring vertices through the multiplicative structure of the geometric mean.

Figure 2 illustrates the eigenvalue spectrum and its energy distribution.

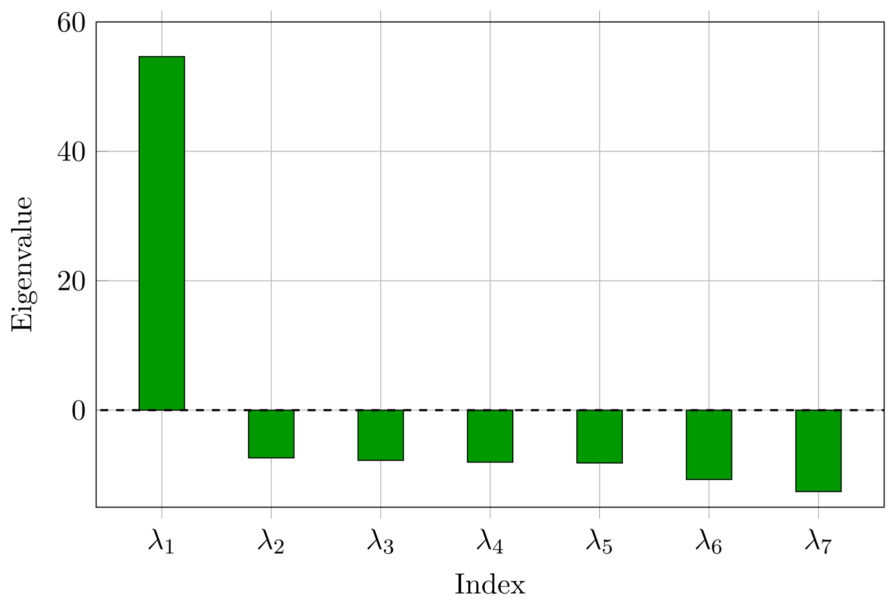

Figure 2: Eigenvalue Spectrum of Credit Risk Network ($\lambda_1 = 54.64$, remaining eigenvalues negative)

3.5 Impact of Diagonal Modification

We examine the effect of modifying the adjacency matrix by setting diagonal elements equal to the credit default swap values rather than zero. This transformation fundamentally alters the matrix structure and spectral properties.

3.5.1 Rank Reduction and Structural Transformation

The modified matrix with $A_{ii} = \text{CDS}_i$ exhibits a remarkable property. Every element satisfies $A_{ij} = \sqrt{\text{CDS}_i \times \text{CDS}_j}$, including diagonal entries where $A_{ii} = \sqrt{\text{CDS}_i \times \text{CDS}_i} = \text{CDS}_i$. This construction represents a rank-one outer product of the form $\mathbf{A} = \mathbf{v}\mathbf{v}^T$, where $\mathbf{v} = [\sqrt{\text{CDS}_1}, \sqrt{\text{CDS}_2}, \ldots, \sqrt{\text{CDS}_7}]^T$.

The rank reduction from seven to one produces dramatic changes in the eigenvalue spectrum. The modified matrix supports precisely one non-zero eigenvalue equal to $\lambda_1 = \mathbf{v}^T\mathbf{v} = \sum_{i=1}^{7} \text{CDS}_i = 64.240$, while all remaining eigenvalues collapse to zero within numerical precision.

Table 6 presents the comparative eigenvalue structures.

Table 6: Eigenvalue Comparison: Original versus Modified Adjacency Matrix

Index	**Original (diag=0)**	**Modified (diag=CDS)**	**Difference**
λ_1	54.643	64.240	+9.597
λ_2	−7.378	0.000	+7.378
λ_3	−7.759	0.000	+7.759
λ_4	−8.040	0.000	+8.040
λ_5	−8.164	0.000	+8.164
λ_6	−10.716	0.000	+10.716
λ_7	−12.585	0.000	+12.585

3.5.2 Spectral Metric Transformations

The structural transformation induces several notable changes in graph-theoretic quantities. The spectral radius increases by 17.56 percent from 54.643 to 64.240, reflecting the additional contribution from diagonal elements. The spectral gap expands to its theoretical maximum of $\Delta = 64.240$, equal to the sole non-zero eigenvalue, representing perfect separation between the dominant mode and the zero subspace.

Paradoxically, despite the increased spectral radius, the graph energy decreases by 41.22 percent from 109.285 to 64.240. This apparent contradiction resolves upon recognizing that graph energy sums absolute eigenvalues across all modes. The original matrix distributed energy among seven eigenvalues with varying magnitudes, while the modified matrix concentrates all energy in a single eigenvalue, eliminating contributions from six previously significant modes.

The Frobenius norm increases modestly from 59.203 to 64.240, while the trace transforms from zero to 64.240, exactly equaling the sum of credit default swap values as required by the diagonal modification.

3.5.3 Eigenvector Centrality Under Diagonal Modification

The centrality rankings remain unchanged under diagonal modification, with Austria retaining the highest position and Germany the lowest. However, the centrality scores undergo systematic shifts that amplify existing disparities. High-risk countries experience centrality gains, with Austria increasing by 0.0093 and Finland by 0.0066. Low-risk countries experience reductions, with Germany declining by 0.0043 and the Netherlands by 0.0036.

These changes reflect the transformation of the principal eigenvector into exact proportionality with the square root of credit default swap values. The near-perfect correlation of $r = 0.9989$ in the original formulation becomes mathematically exact in the modified structure, eliminating all residual variation and reducing centrality to a deterministic function of individual credit risk metrics.

4 Discussion

4.1 Interpretation of Network Centrality

The strong positive correlation between credit default swap spreads and eigenvector centrality represents a counter-intuitive finding that warrants careful interpretation. In many network contexts, centrality connotes importance, influence, or desirability. Within this credit risk framework, however, centrality emerges not as a measure of quality but rather as a mathematical consequence of the edge weight construction.

The geometric mean weighting scheme in Equation (3) creates a multiplicative relationship where countries with elevated credit risk generate proportionally larger edge weights to all neighbors. Since eigenvector centrality depends on both the number of connections and their weights, countries maintaining higher spreads accumulate greater total weighted degree and consequently achieve higher centrality scores.

From a financial risk perspective, this property suggests that higher-risk sovereigns exert disproportionate influence on aggregate network measures when edge weights encode joint credit exposure through multiplicative combinations. This mathematical structure may carry implications for understanding systemic risk propagation, where the most vulnerable entities potentially contribute most significantly to network-level vulnerability through their connection weights.

4.2 Implications of Rank-One Structure

The transformation of the adjacency matrix from full rank to rank one through diagonal modification represents more than a technical curiosity. The rank-one structure implies that all network relationships reduce to perfect alignment with a single dimension, eliminating any subtlety or complexity in how countries relate to one another beyond their individual credit risk levels.

The original full-rank structure preserved the possibility that certain country pairs might exhibit relationships not fully explained by individual spreads alone, potentially reflecting correlated exposure to regional economic shocks, currency area effects, or financial integration patterns. The modified rank-one structure assumes complete information compression, where all such effects collapse into the scalar credit risk hierarchy.

This distinction matters for empirical applications. If actual credit market co-movements exhibit significant idiosyncratic correlation structures beyond what individual spreads capture, the full-rank representation provides a more flexible framework. Conversely, if credit relationships reduce predominantly to functions of individual risk metrics, the rank-one structure accurately reflects the underlying simplicity.

4.3 Spectral Gap and Network Dynamics

The substantial spectral gap in both matrix formulations carries implications for understanding network dynamics under iterative processes. In contexts where the adjacency matrix governs state evolution through repeated multiplication, the spectral gap determines the rate of convergence to the stationary distribution associated with the principal eigenvector.

The original matrix's gap of $\Delta = 62.021$ indicates rapid convergence, while the modified matrix's maximal gap of $\Delta = 64.240$ implies instantaneous equilibration with no transient dynamics. These properties suggest that credit risk networks characterized by geometric mean edge weights exhibit inherently stable configurations with limited scope for complex temporal evolution under diffusion-like dynamics.

4.4 Limitations and Extensions

Several methodological choices merit acknowledgment. The ninety percent confidence level for statistical elimination represents a moderately conservative threshold; alternative specifications could yield different candidate sets. The bump function center placement at twenty percent from the minimum embodies a subjective preference for low-risk countries that could be adjusted based on specific analytical objectives.

The geometric mean edge weighting in Equation (3) constitutes one among multiple reasonable choices for encoding pairwise credit relationships. Alternative formulations including arithmetic means, harmonic means, or correlation-based weights might produce different network structures and spectral properties worthy of comparative investigation.

Future research could extend this framework through temporal analysis examining how eigenvalue spectra evolve in response to macroeconomic shocks or policy interventions. Cross-sectional comparisons across different sets of countries or asset classes could illuminate whether the observed spectral properties generalize or represent characteristics specific to developed economy sovereigns during the particular measurement period.

5 Conclusion

This investigation applied rigorous statistical methods and spectral graph theory to analyze credit default swap relationships among fifteen developed economies. Through confidence interval testing and Mahalanobis distance metrics, we identified thirteen countries meeting statistical quality criteria, then employed smooth bump functions to select seven exhibiting optimal credit risk profiles.

Construction of a weighted complete graph with edge weights encoding geometric means of credit default swap spreads enabled spectral analysis revealing a dominant eigenvalue accounting for fifty percent of graph energy, accompanied by six negative eigenvalues forming subdominant modes. Eigenvector centrality analysis uncovered a strong positive correlation between credit risk metrics and network position, reflecting the multiplicative structure of the edge weighting scheme.

Modification of the adjacency matrix through incorporation of self-loops weighted by credit default swap values transformed the network from full rank to rank one, concentrating all spectral energy in a single eigenvalue equal to the sum of spreads while eliminating subdominant modes entirely. This structural transformation illustrates how seemingly minor modifications to network representations can fundamentally alter mathematical properties and interpretive frameworks.

The methodological progression from statistical elimination through quality optimization to spectral decomposition demonstrates the value of integrating techniques from multiple quantitative disciplines. The findings illuminate structural properties of sovereign credit risk relationships while raising questions about systemic vulnerability propagation mechanisms that merit continued investigation.

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Glossary

Adjacency Matrix A square matrix representation of a graph where entry (i, j) contains the edge weight between vertices i and j, or zero if no edge exists.

Algebraic Connectivity The second-smallest eigenvalue of the graph Laplacian matrix, also known as the Fiedler value, which measures the robustness of network connectivity.

Bump Function A smooth function with compact support that achieves its maximum at a specified center point and decays continuously to zero at the boundary of its support interval.

Complete Graph A graph where every pair of distinct vertices is connected by an edge, denoted K_n for n vertices.

Condition Number The ratio of the largest to smallest singular value of a matrix, measuring sensitivity to perturbations and numerical stability.

Credit Default Swap (CDS) A derivative contract providing insurance against default on a debt obligation, with the spread representing the annual premium expressed in basis points.

Eigenvalue A scalar λ satisfying the equation $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$ for a matrix $\mathbf{A}$ and non-zero vector $\mathbf{v}$.

Eigenvector A non-zero vector $\mathbf{v}$ that satisfies $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$ for some eigenvalue λ.

Eigenvector Centrality A measure of vertex importance based on the principal eigenvector of the adjacency matrix, accounting for both the number and quality of connections.

Frobenius Norm The matrix norm defined as $\|\mathbf{A}\|_F = \sqrt{\sum_{i,j} A_{ij}^2}$, representing the square root of the sum of squared elements.

Geometric Mean For two positive numbers a and b, the geometric mean equals $\sqrt{ab}$, representing their multiplicative average.

Graph Energy The sum of absolute values of all eigenvalues of the adjacency matrix, denoted $E(G) = \sum_i |\lambda_i|$.

Graph Laplacian The matrix $\mathbf{L} = \mathbf{D} - \mathbf{A}$ where $\mathbf{D}$ is the diagonal degree matrix and $\mathbf{A}$ is the adjacency matrix.

Mahalanobis Distance A multivariate distance metric accounting for correlation structure, defined as $D = \sqrt{(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})}$.

Matrix Rank The number of linearly independent rows or columns in a matrix, equivalently the number of non-zero singular values.

Perron-Frobenius Theorem A theorem establishing that non-negative matrices possess a largest real eigenvalue with associated non-negative eigenvector, with uniqueness properties for irreducible matrices.

Probability of Default (PD) The market-implied likelihood that a debt obligation will default within a specified time-frame, typically expressed as a percentage.

Spectral Decomposition The representation of a matrix as a sum of rank-one matrices formed from its eigenvalues and eigenvectors.

Spectral Gap The difference between the largest and second-largest eigenvalue, denoted $\Delta = \lambda_1 - \lambda_2$, measuring separation of the dominant mode.

Spectral Radius The maximum absolute value among all eigenvalues, denoted $\rho(\mathbf{A}) = \max_i |\lambda_i|$.

Vertex Strength The sum of edge weights incident to a vertex, generalizing the degree concept to weighted graphs.

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