A rigorous model of the political economy

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Abstract

In this paper, I present a rigorous model of the political economy that integrates electoral competition with economic policy formation. We develop a framework where politicians compete for votes by proposing economic policies, while voters make decisions based on their economic preferences and expectations. The model incorporates key elements including voter utility maximization, political party optimization, equilibrium conditions, and dynamic feedback mechanisms between political and economic systems. Our analysis reveals conditions under which political equilibria exist and examines the efficiency properties of politically determined economic outcomes.

The paper ends with "The End"

Introduction

Political economy studies the interaction between political institutions and economic outcomes. Traditional economic models often assume policy decisions are made by benevolent planners seeking to maximize social welfare. However, in democratic societies, economic policies emerge from political competition where politicians seek to maximize their probability of election or re-election.

This paper develops a comprehensive mathematical framework that captures the essential features of this political-economic interaction. We model a system where voters have heterogeneous preferences over economic outcomes, politicians propose policy platforms to maximize their electoral success, and the resulting political equilibrium determines economic policy implementation.

The model builds upon the seminal work of Downs (1957) on spatial voting theory, incorporates elements from Hotelling's (1929) location model, and extends recent advances in the theory of the political economy. Our framework provides insights into questions such as: Under what conditions do political equilibria exist? How do distributional concerns affect policy outcomes? What is the relationship between political competition and economic efficiency?

Model Setup

Economic Environment

Consider an economy with a continuum of agents indexed by $i \in [0, 1]$. Each agent is characterized by their economic type θ_i, which determines their preferences over economic policies. We assume θ_i is distributed according to a continuous distribution function $F(\theta)$ with density $f(\theta)$ on support $[\underline{\theta}, \overline{\theta}]$.

The economic policy space is represented by $\mathbf{p} \in \mathbb{R}^k$, where k is the dimension of policy choices. For simplicity, we initially consider the one-dimensional case where $p \in \mathbb{R}$ represents a single policy parameter (e.g., tax rate, level of government spending, or degree of redistribution).

Voter Preferences

Agent i with type θ_i derives utility from economic policy p according to:

$$U_i(p, \theta_i) = u(p, \theta_i) - \epsilon_i \tag{1}$$

where $u(p, \theta_i)$ represents the deterministic component of utility and ϵ_i is an idiosyncratic preference shock. We assume:

$$u(p, \theta_i) = -\alpha(\theta_i)(p - \theta_i)^2 + \beta(\theta_i) \cdot g(p) \tag{2}$$
$$\alpha(\theta_i) > 0 \quad \forall \theta_i \tag{3}$$
$$g'(p) \geq 0, \quad g''(p) \leq 0 \tag{4}$$

The first term captures the agent's preference for policies close to their ideal point θ_i, with $\alpha(\theta_i)$ determining the intensity of this preference. The second term $g(p)$ represents the public good or aggregate benefit component of policy, weighted by individual-specific parameter $\beta(\theta_i)$.

Political Competition

We model political competition as a two-party system where parties A and B simultaneously choose policy platforms p_A and p_B respectively. Each party seeks to maximize its probability of winning the election.

The probability that party A wins is given by:

$$\Pi_A(p_A, p_B) = \Pr\left[\int_0^1 \mathbf{1}[U_i(p_A, \theta_i) > U_i(p_B, \theta_i)]di > \frac{1}{2}\right] \tag{5}$$

where $\mathbf{1}[\cdot]$ is the indicator function.

Equilibrium Analysis

Voter Decision Making

Given policy platforms (p_A, p_B), voter i supports party A if and only if:

$$U_i(p_A, \theta_i) > U_i(p_B, \theta_i) \tag{6}$$

Substituting our utility specification:

$$-\alpha(\theta_i)(p_A - \theta_i)^2 + \beta(\theta_i) \cdot g(p_A) \tag{7}$$
$$> -\alpha(\theta_i)(p_B - \theta_i)^2 + \beta(\theta_i) \cdot g(p_B) \tag{8}$$

Rearranging:

$$\alpha(\theta_i)[(p_B - \theta_i)^2 - (p_A - \theta_i)^2] > \beta(\theta_i)[g(p_B) - g(p_A)] \tag{9}$$

This condition defines the set of voter types who support each party.

Political Equilibrium

A Nash equilibrium in the political competition game consists of policy platforms (p_A^*, p_B^*) such that:

$$p_A^* = \arg\max_{p_A} \Pi_A(p_A, p_B^*) \tag{10}$$

$$p_B^* = \arg\max_{p_B} \Pi_B(p_A^*, p_B) \tag{11}$$

where $\Pi_B = 1 - \Pi_A$.

Proposition 1: Under the assumption that ϵ_i are independently and identically distributed with a continuous distribution, the vote share for party A is:

$$V_A(p_A, p_B) = \int_0^1 \Pr[\epsilon_i^B - \epsilon_i^A > \Delta u_i(p_A, p_B)] f(\theta_i) d\theta_i \tag{12}$$

where $\Delta u_i(p_A, p_B) = u(p_A, \theta_i) - u(p_B, \theta_i)$.

Median Voter Theorem Extension

In the classical median voter model, political equilibrium converges to the median voter's preferred policy. Our framework extends this result.

Theorem 1 (Generalized Median Voter): Suppose $\alpha(\theta) = \alpha > 0$ is constant and $\beta(\theta) = \beta > 0$ is constant for all θ. If $g(p)$ is sufficiently small relative to the quadratic preference component, then the unique political equilibrium satisfies:

$$p_A^* = p_B^* = \theta_m + \frac{\beta g'(\theta_m)}{2\alpha} \tag{13}$$

where θ_m is the median of the distribution $F(\theta)$.

Proof sketch: The first-order conditions for vote maximization yield:

$$\frac{\partial V_A}{\partial p_A} = \int_0^1 [2\alpha(p_A - \theta_i) + \beta g'(p_A)]\, \phi(\Delta u_i) f(\theta_i) d\theta_i = 0 \tag{14}$$

where ϕ is the density of the difference in idiosyncratic shocks. By symmetry and the properties of the median, this leads to the stated result.

Welfare Analysis

Social Welfare Function

We define social welfare as the utilitarian sum of individual utilities:

$$W(p) = \int_0^1 u(p, \theta_i) f(\theta_i) d\theta_i \tag{15}$$

The socially optimal policy p^{SO} maximizes this welfare function:

$$p^{SO} = \arg\max_{p} W(p) \tag{16}$$

Efficiency of Political Equilibrium

Proposition 2: The political equilibrium policy p^* generally differs from the socially optimal policy p^{SO}. The welfare loss from political competition is:

$$\mathcal{L} = W(p^{SO}) - W(p^*) = \int_0^1 [u(p^{SO}, \theta_i) - u(p^*, \theta_i)] f(\theta_i) d\theta_i \tag{17}$$

For our quadratic utility specification:

$$\mathcal{L} = \int_0^1 \left[-\alpha(\theta_i)(p^{SO} - \theta_i)^2 + \alpha(\theta_i)(p^* - \theta_i)^2\right] f(\theta_i) d\theta_i \tag{18}$$

$$+ \int_0^1 \beta(\theta_i)[g(p^{SO}) - g(p^*)] f(\theta_i) d\theta_i \tag{19}$$

Dynamic Extensions

Repeated Elections

Consider a dynamic setting where elections occur at times $t = 1, 2, 3, \ldots$ and policies implemented in period t affect economic outcomes observed in period $t + 1$.

Let $y_t = h(p_t, \xi_t)$ represent the economic outcome in period t, where p_t is the implemented policy and ξ_t is a random shock. Voters update their beliefs about the relationship between policies and outcomes based on observed data.

The dynamic voter utility becomes:

$$U_i^t(p_t, \theta_i) = E_t[u(h(p_t, \xi_{t+1}), \theta_i)] - \epsilon_i^t \tag{20}$$

where $E_t[\cdot]$ denotes expectations conditional on information available at time t.

Learning and Adaptation

Voters learn about the policy-outcome relationship through a Bayesian updating process. Let Θ_t represent the posterior distribution over possible parameter values governing $h(\cdot, \cdot)$ at time t.

The updating rule follows:

$$\Theta_{t+1} = \frac{L(y_{t+1}|p_t, \Theta_t) \cdot \Theta_t}{\int L(y_{t+1}|p_t, \theta) d\Theta_t(\theta)} \tag{21}$$

where $L(\cdot)$ is the likelihood function.

Extensions and Applications

Multi-Dimensional Policy Space

Extending to k-dimensional policy space $\mathbf{p} \in \mathbb{R}^k$, voter utility becomes:

$$u(\mathbf{p}, \boldsymbol{\theta}_i) = -\frac{1}{2}(\mathbf{p} - \boldsymbol{\theta}_i)^T \mathbf{A}_i (\mathbf{p} - \boldsymbol{\theta}_i) + \boldsymbol{\beta}_i^T \mathbf{g}(\mathbf{p}) \tag{22}$$

where $\mathbf{A}_i$ is a positive definite matrix representing preference intensities and cross-policy interactions.

Proposition 3: In the multi-dimensional case, pure strategy equilibria may fail to exist unless additional restrictions are imposed on the preference structure.

Redistribution Model

Consider a specific application where policy represents the degree of redistribution. Let $p \in [0, 1]$ be the tax rate on income, with proceeds distributed equally among all agents.

Agent i with income y_i has post-redistribution income:

$$c_i(p) = (1 - p)y_i + p\bar{y} \tag{23}$$

where $\bar{y} = \int_0^1 y_i f(\theta_i) d\theta_i$ is mean income.

Utility is:

$$u(p, y_i) = \log(c_i(p)) - \gamma\frac{p^2}{2} \tag{24}$$

The quadratic term captures administrative costs or distortions from taxation.

Proposition 4: The politically chosen redistribution level p^* exceeds the utilitarian optimum if and only if the median income is below the mean income, i.e., if the income distribution is right-skewed.

Empirical Implications

The model generates several testable predictions:

1. **Convergence Hypothesis:** In competitive two-party systems, policy platforms should converge toward the preferences of the median voter, adjusted for public good considerations.
2. **Skewness Effect:** The direction of bias in redistribution policy depends on the skewness of the income distribution.
3. **Preference Intensity:** Policies should be more responsive to groups with higher preference intensities (larger $\alpha(\theta_i)$).
4. **Information Effects:** Increased information about policy consequences should lead to outcomes closer to the social optimum.

Robustness and Extensions

Alternative Voting Rules

The framework can accommodate different voting systems:

- **Proportional Representation:** Policy outcome is a weighted average: $p = \lambda p_A + (1 - \lambda)p_B$ where λ is party A's vote share.
- **Runoff Systems:** Multi-stage competition with elimination of candidates.
- **Approval Voting:** Voters can support multiple candidates, changing the strategic considerations.

Interest Groups and Lobbying

Incorporate interest groups that provide campaign contributions $C_j(p_A, p_B)$ based on policy platforms:

$$\Pi_A^{total} = \Pi_A(p_A, p_B) + \mu \sum_j C_j^A(p_A, p_B) \tag{25}$$

where μ represents the relative importance of contributions versus votes.

Conclusion

This paper has developed a comprehensive mathematical framework for analyzing the political economy. The model demonstrates how electoral competition shapes economic policy, with outcomes generally differing from utilitarian social optima. Key insights include:

The median voter theorem provides a baseline for policy convergence, but must be modified to account for public good components and preference intensities. The efficiency of democratic policy-making depends critically on the information available to voters and the competitiveness of political markets.

The framework's flexibility allows for numerous extensions, including dynamic learning, multi-dimensional policy spaces, and alternative institutional arrangements. Future research could explore the quantitative calibration of the model parameters and empirical testing of its predictions using cross-national data on democratic institutions and policy outcomes.

The mathematical rigor of this approach provides a solid foundation for understanding one of the most important questions in social science: how democratic institutions translate individual preferences into collective policy choices, and whether these choices serve the broader public interest.

Mathematical Appendix

Proof of Existence of Political Equilibrium

Theorem 2: Under regularity conditions on the distribution of voter types and preference parameters, a pure strategy Nash equilibrium exists in the political competition game.

Proof: We apply Brouwer's fixed point theorem. Define the best response correspondence:

$$BR_A(p_B) = \arg\max_{p_A \in \mathcal{P}} V_A(p_A, p_B) \tag{26}$$

Where $\mathcal{P}$ is a compact convex subset of $\mathbb{R}$. Under our assumptions:

1. $\mathcal{P}$ is non-empty, compact, and convex
2. $V_A(p_A, p_B)$ is continuous in (p_A, p_B)
3. $V_A(p_A, p_B)$ is quasi-concave in p_A

These conditions ensure that BR_A is non-empty, compact-valued, and upper-semicontinuous. By symmetry, the same holds for BR_B. The Kakutani fixed point theorem then guarantees existence of a fixed point (p_A^*, p_B^*) such that $p_A^* \in BR_A(p_B^*)$ and $p_B^* \in BR_B(p_A^*)$.

Welfare Bounds

Proposition 5: The welfare loss from political competition is bounded by:

$$\mathcal{L} \leq \frac{\alpha}{2}\text{Var}(\theta) + \beta|g(p^{SO}) - g(p^*)| \tag{27}$$

where $\text{Var}(\theta)$ is the variance of the type distribution.

This bound shows that welfare losses are larger when voter preferences are more heterogeneous and when the public good component is more important.

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