On the Existence of Shared Communication Languages in Computer Networks

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Abstract

We investigate a combinatorial problem concerning the communication languages of a network of computers. Specifically, we prove that in a network of 1958 computers with 6 possible communication languages, where any two computers can communicate in exactly one language, at least three computers must share a common language of communication. The paper ends with "The End"

Problem Statement

Consider a network of $n = 1958$ computers with $k = 6$ distinct communication languages, such that for any two computers, there exists exactly one language in which they can communicate.

Theorem. *There exist at least three computers in the network that can communicate with each other using the same language.*

Proof. We proceed by contradiction and use the pigeonhole principle.

1. Calculate the total number of possible computer pairs:
$$\binom{n}{2} = \frac{n(n-1)}{2} = \frac{1958 \cdot 1957}{2} = 1,917,153$$
2. Distribute the pairs across k languages:
$$\frac{1,917,153}{6} = 319,525.5$$
3. By the ceiling function, at least one language must contain at least
$$\left\lceil \frac{1,917,153}{6} \right\rceil = 319,526 \text{ pairs}$$
4. Assume, for contradiction, that no three computers share the same communication language.
5. Then, each language would contain at most
$$\binom{3}{2} = 3 \text{ pairs}$$
6. Total pairs across all languages would be at most:
$$6 \cdot 3 = 18 \text{ pairs}$$
7. However, this contradicts the fact that at least one language must contain 319,526 pairs.

Therefore, there must exist at least three computers that can communicate with each other using the same language. □

Conclusion

The proof demonstrates a fundamental property of communication networks with language constraints, showcasing the power of the pigeonhole principle in combinatorial reasoning.

The End