Dynamics of High-IQ Survival under Differential Fitness

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1 Introduction

In this paper, we develop deterministic and stochastic models in which a subpopulation with higher IQ survives and dominates almost surely, conditioned on non-extinction. The analysis uses replicator dynamics and continuous-time multitype branching processes. The paper ends with "The End"

2 Deterministic Model: Replicator Dynamics

Let $p_H(t)$ and $p_L(t)$ denote proportions of high-IQ and low-IQ individuals, respectively, with $p_H + p_L = 1$. Assign constant fitnesses $w_H > w_L > 0$. The replicator equation reads

$$\dot{p}_H(t) = p_H(t)\big(w_H - \bar{w}(t)\big), \quad \bar{w}(t) = p_H(t)w_H + (1 - p_H(t))w_L. \tag{1}$$

This simplifies to

$$\dot{p}_H(t) = p_H(t)(1 - p_H(t))(w_H - w_L), \tag{2}$$

which is logistic. The solution is

$$p_H(t) = \frac{1}{1 + \left(\frac{1-p_H(0)}{p_H(0)}\right) e^{-(w_H - w_L)t}}, \tag{3}$$

so $p_H(t) \to 1$ exponentially fast as $t \to \infty$.

3 Stochastic Model: Continuous-Time Multitype Branching Process

Definition 1. *A two-type continuous-time branching process $\boldsymbol{Z}(t) = (Z_H(t), Z_L(t))$ evolves as follows: each individual of type $i \in \{H, L\}$ reproduces at rate β_i and dies at rate δ_i. Births produce offspring of the same type (mutation-free case). Net growth rates are $r_i = \beta_i - \delta_i$.*

Assume $r_H > r_L$ and $r_H > 0$.

Theorem 1 (Almost Sure Dominance)**.** *Suppose the initial population contains at least one high-IQ individual. Let $P_H(t) = Z_H(t)/(Z_H(t) + Z_L(t))$ when the denominator is positive. Then, conditioned on non-extinction of the total population,*

$$\lim_{t\to\infty} P_H(t) = 1, \quad \textit{almost surely.} \tag{4}$$

Sketch. The mean matrix of the process has dominant eigenvalue $\lambda^* = r_H$ and Perron eigenvector v concentrated on the H coordinate. By the Kesten–Stigum theorem, $e^{-\lambda^* t}\boldsymbol{Z}(t) \to Wv$ almost surely, where $W > 0$ on the survival event. Thus $Z_L(t)/Z_H(t) \to 0$ a.s., implying $P_H(t) \to 1$. □

4 Vector Graphic Illustration

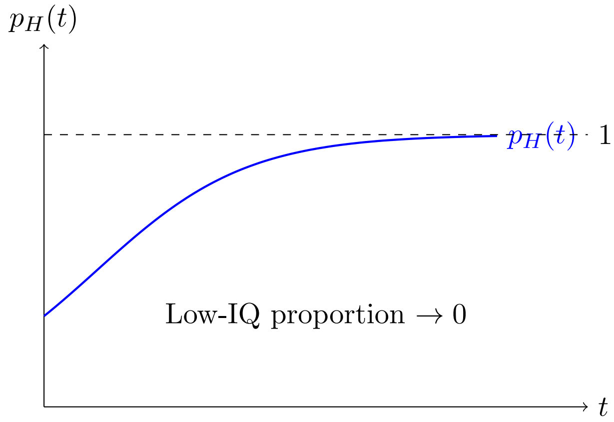

Figure 1: Replicator dynamics: $p_H(t)$ converges to 1.

5 Remarks

The result extends to models with small mutations $L \to H$ and to continuous IQ traits with fitness $r(\text{IQ})$. Spatial structure and density dependence may alter the conclusion.

References

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[3] Nowak, M. A. *Evolutionary Dynamics: Exploring the Equations of Life*. 2006.

The End