Ghosh theta functions

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Abstract

In this paper, I describe my theta functions.
The paper ends with "The End"

Introduction

My theta functions are easy to define but have many interesting properties.
In this paper, I describe my theta functions.

Ghosh theta functions

Ghosh theta functions are defined as follows:

For $\theta\sin(\theta) \neq 1, f(\theta) = \frac{1}{1-\theta\sin\theta}$

For $\theta\cos(\theta) \neq 1, g(\theta) = \frac{1}{1-\theta\cos\theta}$

For $\frac{\theta}{\pi} \notin \mathbb{Z} \wedge \theta\csc\theta \neq 1, p(\theta) = \frac{1}{1-\theta\csc\theta}$

For $\frac{1}{2} + \frac{\theta}{\pi} \notin \mathbb{Z} \wedge \theta\sec\theta \neq 1, q(\theta) = \frac{1}{1-\theta\sec\theta}$

For $\frac{1}{2} + \frac{\theta}{\pi} \notin \mathbb{Z} \wedge \theta\tan\theta \neq 1, u(\theta) = \frac{1}{1-\theta\tan\theta}$

For $\frac{\theta}{\pi} \notin \mathbb{Z} \wedge \theta\cot\theta \neq 1, v(\theta) = \frac{1}{1-\theta\cot\theta}$

Identities of Ghosh theta functions

Whenever all terms are well-defined, we have

$$\left(\frac{1-f(\theta)}{f(\theta)}\right)^2 + \left(\frac{1-g(\theta)}{g(\theta)}\right)^2 = \theta^2$$

$$\frac{1}{\left(\frac{p(\theta)}{1-p(\theta)}\right)^2 + \left(\frac{q(\theta)}{1-q(\theta)}\right)^2} = \theta^2$$

$$\left(\frac{1-u(\theta)}{u(\theta)}\right)\left(\frac{1-v(\theta)}{v(\theta)}\right) = \theta^2$$

Ghosh theta functions as infinite series

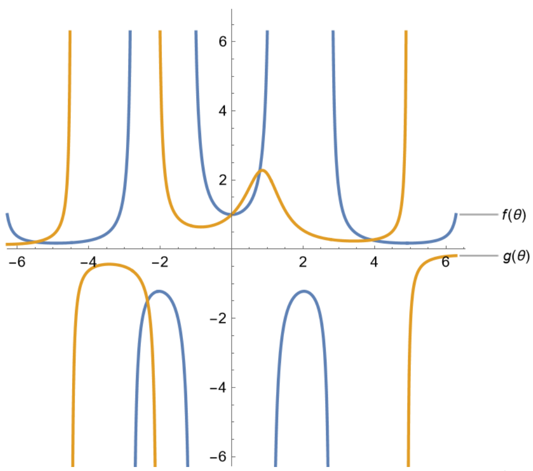
Whenever all terms are well-defined, we have

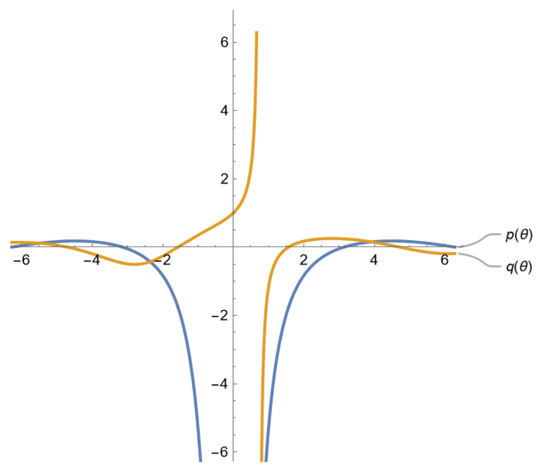
$$f(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\csc^i \theta} \qquad g(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\sec^i \theta}$$

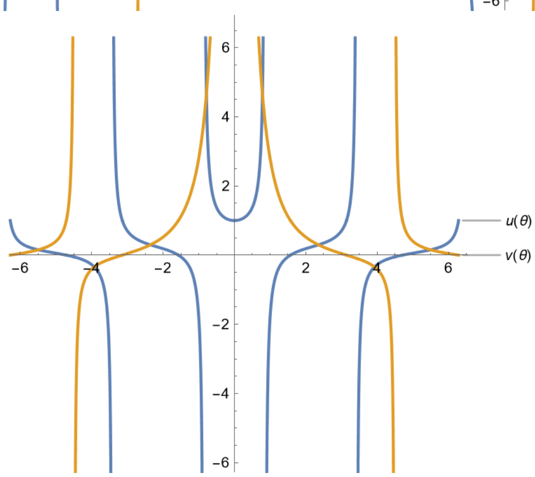
$$p(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\sin^i \theta} \qquad q(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\cos^i \theta}$$

$$u(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\cot^i \theta} \qquad v(\theta) = \sum_{i=0}^{\infty} \frac{\theta^i}{\tan^i \theta}$$

Graphs of Ghosh theta functions







The End