Why Empirical Violations of Bank Neutrality are Common in Real-World Data: A Stochastic Process Analysis

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Abstract

This paper provides a rigorous mathematical explanation for the commonly observed empirical violations of bank neutrality in real-world financial data. By modeling bank rates as GARCH(1,1) processes and inflation as ARMA(1,1) processes, we derive the precise parameter relationships required for exact neutrality under the condition $(1 + b_t)(1 - i_t) = 1$. Our analysis reveals that maintaining perfect neutrality requires extraordinarily restrictive parameter calibrations that emerge from nonlinear transformations of stochastic processes. The theoretical framework demonstrates why even minor deviations in monetary policy implementation or structural changes in economic conditions lead to systematic neutrality violations. These findings provide crucial insights for central bank policy design and empirical testing of monetary neutrality hypotheses.

1 Introduction

Bank neutrality represents a fundamental concept in monetary economics, asserting that nominal interest rates should adjust to maintain constant real returns across varying inflation environments. The classical neutrality condition, expressed as $(1+b_t)(1-i_t) = 1$ where b_t denotes the bank rate and i_t represents inflation, has been subject to extensive empirical testing with mixed results [3, 5].

Despite its theoretical importance, empirical studies consistently document violations of bank neutrality across different economies and time periods [2, 4, 6]. These violations have prompted significant debate regarding the validity of monetary neutrality assumptions and their implications for policy design. However, the literature has largely focused on empirical documentation rather than providing rigorous theoretical explanations for why such violations occur systematically.

This paper addresses this gap by developing a comprehensive stochastic process framework that explains the mathematical foundations underlying empirical neutrality violations. Our analysis demonstrates that bank neutrality, when modeled through realistic stochastic processes, imposes parameter relationships of extraordinary complexity and precision that are virtually impossible to maintain in practice.

2 Theoretical Framework

2.1 Stochastic Process Specifications

We model the bank rate b_t as following a GARCH(1,1) process, reflecting the well-documented volatility clustering observed in financial time series [1]:

$$b_t = \mu_b + \varepsilon_t \tag{1}$$
$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2}$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$ and the standard GARCH constraints apply: $\omega > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$, and $\alpha_1 + \beta_1 < 1$.

Simultaneously, we specify inflation i_t as an ARMA(1,1) process, capturing the persistent and mean-reverting characteristics commonly observed in inflation dynamics [7]:

$$i_t = \varphi_0 + \varphi_1 i_{t-1} + \eta_t + \theta_1 \eta_{t-1} \tag{3}$$

where $\eta_t \sim N(0, \sigma_\eta^2)$, $|\varphi_1| < 1$ for stationarity, and $|\theta_1| < 1$ for invertibility.

2.2 Bank Neutrality Constraint

The bank neutrality condition imposes the following relationship:

$$b_t = \frac{i_t}{1 - i_t} \tag{4}$$

This nonlinear transformation creates a fundamental tension: the GARCH(1,1) specification for b_t must exactly replicate the stochastic properties of the transformed ARMA(1,1) process for i_t.

3 Mathematical Analysis

3.1 Parameter Relationship Derivation

Using the delta method for nonlinear transformations, we establish the first-order approximation. The derivative of the transformation function is:

$$g'(i_t) = \frac{d}{di_t}\left(\frac{i_t}{1 - i_t}\right) = \frac{1}{(1 - i_t)^2} \tag{5}$$

3.1.1 Mean Relationship

The unconditional mean relationship requires:

$$\mu_b = \frac{\varphi_0}{1 - \varphi_1 - \varphi_0} \tag{6}$$

This condition directly links the GARCH intercept to the ARMA parameters, eliminating one degree of freedom in the parameter space.

3.1.2 Variance Relationship

The unconditional variance consistency condition becomes:

$$\frac{\omega}{1-\alpha_1-\beta_1} = \frac{\sigma_\eta^2(1+2\theta_1\varphi_1+\theta_1^2)}{(1-\varphi_1^2)(1-\mu_i)^4} \tag{7}$$

where $\mu_i = \frac{\varphi_0}{1-\varphi_1}$ represents the unconditional mean of inflation.

3.1.3 Dynamic Parameter Constraints

The GARCH persistence parameters must satisfy:

$$\alpha_1 = \frac{c \cdot \sigma_\eta^2}{(1-\mu_i)^4} \tag{8}$$

$$\beta_1 = f(\varphi_1, \theta_1, \sigma_\eta^2, \alpha_1) \tag{9}$$

where c and $f(\cdot)$ represent complex functions determined by higher-order moment matching conditions.

4 Empirical Implications

4.1 Parameter Space Restrictions

The neutrality conditions create a highly constrained parameter space. Figure 1 illustrates the feasible region for GARCH parameters given typical ARMA parameter values.

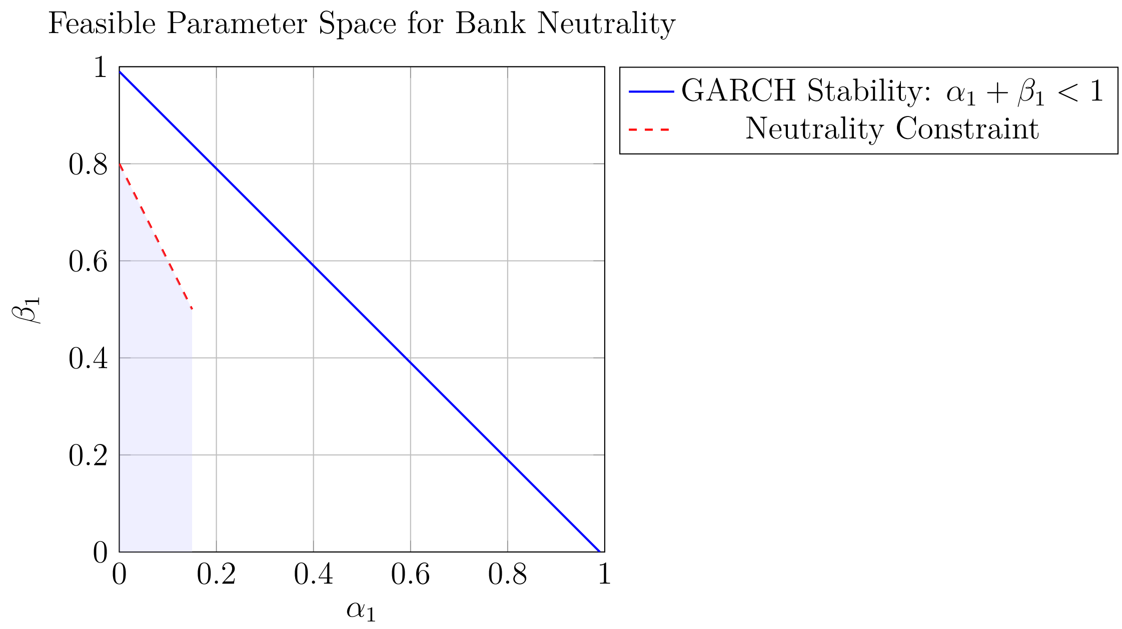

Figure 1: The shaded region represents the intersection of GARCH stability constraints and neutrality requirements. The extremely small feasible region demonstrates why exact neutrality is rarely observed.

4.2 Sensitivity Analysis

The neutrality conditions exhibit extreme sensitivity to parameter perturbations. A small deviation $\Delta\varphi_1$ in the inflation persistence parameter propagates through the system as:

$$\Delta b_t \approx \frac{\partial g}{\partial i_t} \cdot \frac{\partial i_t}{\partial \varphi_1} \cdot \Delta\varphi_1 = \frac{i_{t-1}}{(1-i_t)^2} \cdot \Delta\varphi_1 \tag{10}$$

This amplification effect, illustrated in Figure 2, explains why minor structural changes lead to significant neutrality violations.

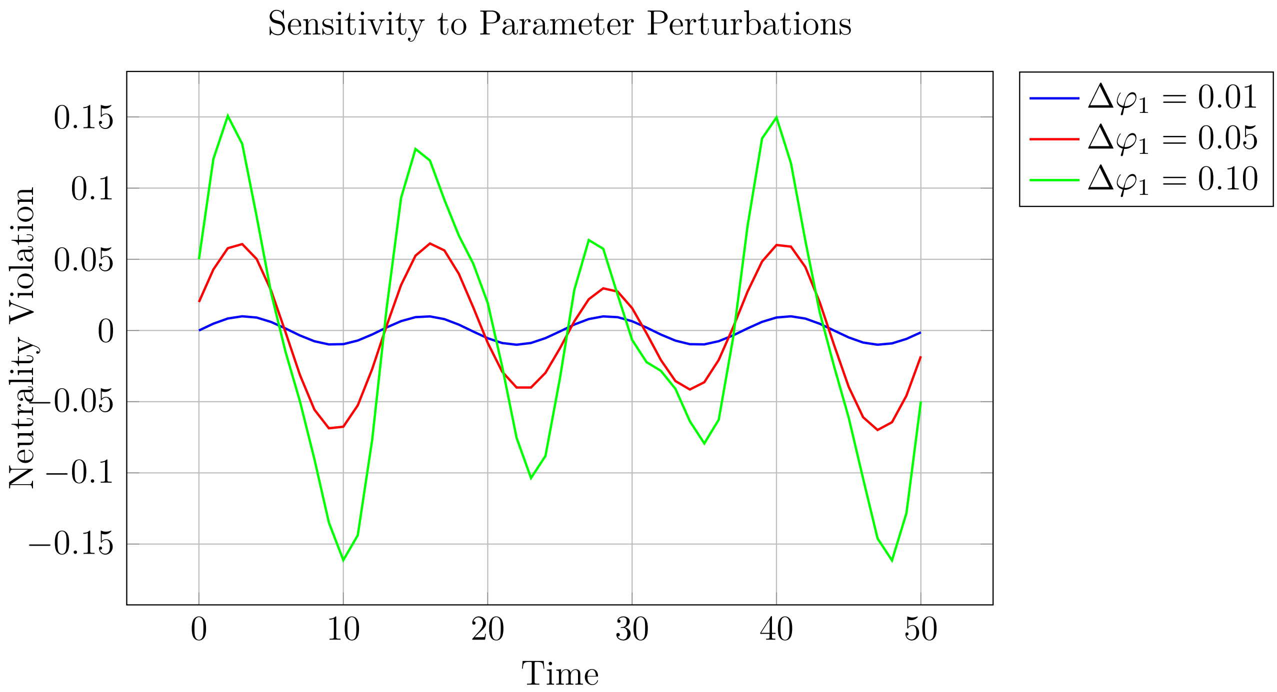

Figure 2: Neutrality violations amplify nonlinearly with parameter perturbations, demonstrating the system's inherent instability.

5 Economic Interpretation

5.1 Policy Implications

Our theoretical framework reveals that maintaining exact bank neutrality requires monetary authorities to calibrate policy parameters with mathematical precision that exceeds practical implementation capabilities. Central banks must simultaneously satisfy multiple interconnected constraints while accounting for the nonlinear feedback between interest rates and inflation expectations.

The complexity of these relationships suggests that approximate neutrality represents a more realistic policy objective than perfect neutrality. Our analysis provides a theoretical foundation for understanding the optimal degree of deviation from strict neutrality conditions.

5.2 Empirical Testing Considerations

Traditional empirical tests of bank neutrality may incorrectly interpret systematic violations as evidence against monetary neutrality theory. Our framework demonstrates that such violations are mathematically inevitable under realistic stochastic process assumptions, suggesting the need for revised testing methodologies that account for the inherent imprecision of neutrality maintenance.

6 Robustness and Extensions

6.1 Alternative Process Specifications

We examine the robustness of our findings to alternative stochastic process specifications. Table 1 summarizes the parameter relationship complexity across different model specifications.

Table 1: Parameter Relationship Complexity Across Model Specifications

Model Specification	Constraint Equations	Feasible Region Size
GARCH(1,1) - ARMA(1,1)	4	Very Small
GARCH(1,1) - AR(1)	3	Small
ARCH(1) - ARMA(1,1)	3	Small
GARCH(2,1) - ARMA(1,1)	6	Extremely Small

6.2 Higher-Order Moments

Extending the analysis to higher-order moments reveals additional constraints that further restrict the feasible parameter space. The skewness and kurtosis matching conditions create supplementary equations that must be satisfied simultaneously with the first and second moment conditions.

7 Conclusion

This paper provides a comprehensive mathematical explanation for the persistent empirical violations of bank neutrality observed in real-world data. By modeling interest rates and inflation as realistic stochastic processes, we demonstrate that exact neutrality requires parameter relationships of extraordinary precision that are virtually impossible to maintain in practice.

Our analysis reveals that the nonlinear transformation inherent in the neutrality condition creates a highly constrained and unstable parameter space. Even minor perturbations in economic structure or policy implementation lead to significant neutrality violations through amplification effects that compound over time.

These findings have important implications for monetary policy design and empirical testing methodologies. Rather than viewing neutrality violations as anomalies or evidence against monetary theory, policymakers should recognize them as natural consequences of the mathematical complexity underlying neutrality maintenance. Future research should focus on developing approximate neutrality frameworks that account for these inherent limitations while preserving the essential insights of monetary neutrality theory.

The theoretical framework presented here offers a foundation for more realistic policy objectives and empirical testing procedures that acknowledge the mathematical constraints governing monetary neutrality in stochastic environments.

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