pq identities

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Abstract

In this paper, I describe pq identities.
The paper ends with "The End"

Introduction

pq identities are useful in mathematics especially in the fields of algebra, geometry, co-ordinate geometry and analytical geometry.
In this paper, I describe pq identities.

pq identities

pq identities are of the form

$$\frac{a^{p+q}+b^{p+q}+c^{p+q}}{p+q}-\frac{a^p+b^p+c^p}{p}\frac{a^q+b^q+c^q}{q}=k(a+b+c)^r f(a,b,c)$$

where
p, q are natural numbers
r is a whole number
k is a constant
$f(a,b,c)$ is an expression in a, b and c

A list of known pq identities

1.
$$\frac{a^2+b^2+c^2}{2}-\frac{a+b+c}{1}\,\frac{a+b+c}{1}=-\frac{1}{2}\left(a^2+4ab+4ac+b^2+4bc+c^2\right)$$

2.
$$\frac{a^3+b^3+c^3}{3}-\frac{a^2+b^2+c^2}{2}\,\frac{a+b+c}{1}=-\frac{1}{6}\left(a^3+3a^2b+3a^2c+3ab^2+3ac^2+b^3+3b^2c+3bc^2+c^3\right)$$

3.
$$\frac{a^4+b^4+c^4}{4}-\frac{a^2+b^2+c^2}{2}\,\frac{a^2+b^2+c^2}{2}=-\frac{1}{2}\left(a^2b^2+b^2c^2+c^2a^2\right)$$

4.
$$\frac{a^4+b^4+c^4}{4}-\frac{a^3+b^3+c^3}{3}\,\frac{a+b+c}{1}=-\frac{1}{12}\left(a^4+4a^3b+4a^3c+4ab^3+4ac^3+b^4+4b^3c+4bc^3+c^4\right)$$

5.
$$\frac{a^5+b^5+c^5}{5}-\frac{a^3+b^3+c^3}{3}\,\frac{a^2+b^2+c^2}{2}=\frac{1}{30}(a+b+c)^2\left(a^3-2a^2b-2a^2c-2ab^2+6abc-2ac^2+b^3-2b^2c-2bc^2+c^3\right)$$

6.
$$\frac{a^6+b^6+c^6}{6}-\frac{a^3+b^3+c^3}{3}\,\frac{a^3+b^3+c^3}{3}=\frac{1}{18}\left(a^6-4a^3b^3-4a^3c^3+b^6-4b^3c^3+c^6\right)$$

7.
$$\frac{a^5+b^5+c^5}{5}-\frac{a^4+b^4+c^4}{4}\,\frac{a+b+c}{1}=-\frac{1}{20}\left(a^5+5a^4b+5a^4c+5ab^4+5ac^4+b^5+5b^4c+5bc^4+c^5\right)$$

8.
$$\frac{a^6+b^6+c^6}{6}-\frac{a^4+b^4+c^4}{4}\,\frac{a^2+b^2+c^2}{2}=\frac{1}{24}\left(a^6-3a^4b^2-3a^4c^2-3a^2b^4-3a^2c^4+b^6-3b^4c^2-3b^2c^4+c^6\right)$$

9.
$$\frac{a^7+b^7+c^7}{7}-\frac{a^4+b^4+c^4}{4}\,\frac{a^3+b^3+c^3}{3}=\frac{1}{84}\left(5a^7-7a^4b^3-7a^4c^3-7a^3b^4-7a^3c^4+5b^7-7b^4c^3-7b^3c^4+5c^7\right)$$

10.
$$\frac{a^8+b^8+c^8}{8}-\frac{a^4+b^4+c^4}{4}\,\frac{a^4+b^4+c^4}{4}=\frac{1}{16}\left(a^2-b^2-c^2\right)\left(a^2+b^2-c^2\right)\left(a^2-b^2+c^2\right)\left(a^2+b^2+c^2\right)$$

11.
$$\frac{a^6+b^6+c^6}{6}-\frac{a^5+b^5+c^5}{5}\,\frac{a+b+c}{1}=-\frac{1}{30}\left(a^6+6a^5b+6ab^5+b^6+6a^5c+6b^5c+6ac^5+6bc^5+c^6\right)$$

12.

$$\frac{a^7+b^7+c^7}{7} - \frac{a^5+b^5+c^5}{5}\,\frac{a^2+b^2+c^2}{2} = \frac{1}{70}(a+b+c)(3a^6-3a^5b-3a^5c-4a^4b^2+6a^4bc-4a^4c^2+4a^3b^3-2a^3b^2c-2a^3bc^2+4a^3c^3-4a^2b^4-2a^2b^3c$$
$$+4a^2b^2c^2-2a^2bc^3-4a^2c^4-3ab^5+6ab^4c-2ab^3c^2-2ab^2c^3+6abc^4-3ac^5+3b^6-3b^5c-4b^4c^2+4b^3c^3-4b^2c^4-3bc^5+3c^6)$$

13.

$$\frac{a^8+b^8+c^8}{8} - \frac{a^5+b^5+c^5}{5}\,\frac{a^3+b^3+c^3}{3} = \frac{1}{120}\left(7a^8-8a^5b^3-8a^5c^3-8a^3b^5-8a^3c^5+7b^8-8b^5c^3-8b^3c^5+7c^8\right)$$

14.

$$\frac{a^9+b^9+c^9}{9} - \frac{a^5+b^5+c^5}{5}\,\frac{a^4+b^4+c^4}{4} = \frac{1}{180}\left(11a^9-9a^5b^4-9a^5c^4-9a^4b^5-9a^4c^5+11b^9-9b^5c^4-9b^4c^5+11c^9\right)$$

15.

$$\frac{a^{10}+b^{10}+c^{10}}{10} - \frac{a^5+b^5+c^5}{5}\,\frac{a^5+b^5+c^5}{5} = \frac{1}{50}\left(3a^{10}-4a^5b^5-4a^5c^5+3b^{10}-4b^5c^5+3c^{10}\right)$$

Corollary

For pq identities with the factor $(a+b+c)$ in the right hand side,
we have the left hand side equals zero whenever $a+b+c=0$

The End