Empirical Construction of Five Double-Weighted Portfolios

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Abstract

This paper presents the empirical construction of five double-weighted portfolios: the Sector-Region Equity Portfolio, the Asset Class-Economic Regime Portfolio, the Hierarchical Asset Allocation Portfolio, the Multi-Period Portfolio, and the Enhanced Diversification Portfolio. We formalize the double-weighted portfolio structure, provide illustrative vector graphics and tables for each portfolio type, and discuss the empirical implications. The double-weighted approach allows for nuanced allocation across two dimensions, enhancing flexibility and diversification in portfolio construction.

The paper ends with "The End"

1 Introduction

The double-weighted portfolio is a flexible framework for allocating capital across two hierarchical or interacting dimensions, such as sectors and regions, asset classes and economic regimes, or time periods and risk factors. The mathematical formulation, as introduced in [1], is:

$$1 = \sum_{i=1}^{m} \sum_{j=1}^{n_i} w(i,j) \tag{1}$$

$$P = \sum_{i=1}^{m} \sum_{j=1}^{n_i} w(i,j) p(i,j) \tag{2}$$

where
m is the number of primary categories
n_i is the number of subcategories within the i-th category
$w(i,j)$ is the weight assigned to the j-th subcategory within the i-th primary category
$p(i,j)$ is the price or return of that component
P is the total portfolio price or return

Table 1 summarizes the notation:

Table 1: Summary of Variables in Double-Weighted Portfolio

Symbol	Description
m	Number of first-level categories (e.g., sectors, asset classes)
n_i	Number of second-level categories in i (e.g., regions, regimes)
$w(i,j)$	Weight for the j-th subcategory of the i-th category
$p(i,j)$	Price or return for the j-th subcategory of the i-th category
P	Total portfolio price or return

2 Weight Matrix Representation

Let m be the number of first-level categories (e.g., sectors), and n_i the number of second-level categories (e.g., regions) within the i-th first-level category. The weight matrix W is then:

$$W = \begin{bmatrix} w(1,1) & w(1,2) & \cdots & w(1,n_1) \\ w(2,1) & w(2,2) & \cdots & w(2,n_2) \\ \vdots & \vdots & \ddots & \vdots \\ w(m,1) & w(m,2) & \cdots & w(m,n_m) \end{bmatrix}$$

where $w(i,j)$ is the weight for the j-th subcategory of the i-th category, and the sum of all weights is 1:

$$1 = \sum_{i=1}^{m} \sum_{j=1}^{n_i} w(i,j)$$

The portfolio value is:

$$P = \sum_{i=1}^{m} \sum_{j=1}^{n_i} w(i,j)p(i,j)$$

where $p(i,j)$ is the price or return of the (i,j)-th component

3 Portfolio Types and Empirical Construction

3.1 Sector-Region Equity Portfolio

This portfolio allocates capital first by sector (e.g., Technology, Healthcare, Finance, Other), then by region (e.g., North America, Europe, Asia) within each sector.

Weight Matrix Representation

$$\mathbf{W}_{\text{Sector,Region}} = \begin{pmatrix} w(1,1) & w(1,2) & w(1,3) \\ w(2,1) & w(2,2) & w(2,3) \\ w(3,1) & w(3,2) & w(3,3) \\ w(4,1) & w(4,2) & w(4,3) \end{pmatrix}$$

where rows represent sectors and columns represent regions.

Example Table

Table 2: Sector-Region Equity Portfolio Weights and Returns

Sector	N. America	Europe	Asia	Sector Total
Technology	0.10 (8%)	0.08 (7%)	0.07 (9%)	0.25
Healthcare	0.12 (6%)	0.08 (5%)	0.05 (8%)	0.25
Finance	0.10 (7%)	0.15 (6%)	0.05 (7%)	0.30
Other	0.10 (5%)	0.05 (4%)	0.05 (6%)	0.20
Region Total	0.42	0.36	0.22	1.00

Heatmap Visualization

Figure 1: Heatmap of Sector-Region Portfolio Weights

3.2 Asset Class-Economic Regime Portfolio

This portfolio first allocates by asset class (e.g., Equities, Bonds, Commodities, Alternatives), then by economic regime (e.g., Expansion, Recession, Stagflation).

Weight Matrix Representation

$$\mathbf{W}_{\text{Asset,Regime}} = \begin{pmatrix} w(1,1) & w(1,2) & w(1,3) \\ w(2,1) & w(2,2) & w(2,3) \\ w(3,1) & w(3,2) & w(3,3) \\ w(4,1) & w(4,2) & w(4,3) \end{pmatrix}$$

where rows are asset classes and columns are regimes.

Example Table

Table 3: Asset Class-Economic Regime Portfolio Weights

Asset Class	Expansion	Recession	Stagflation	Class Total
Equities	0.18	0.07	0.05	0.30
Bonds	0.10	0.20	0.05	0.35
Commodities	0.07	0.05	0.08	0.20
Alternatives	0.05	0.05	0.05	0.15
Regime Total	0.40	0.37	0.23	1.00

Heatmap Visualization

Figure 2: Heatmap of Asset Class-Economic Regime Portfolio Weights

3.3 Hierarchical Asset Allocation Portfolio

This portfolio uses a hierarchical structure, such as asset class → sector → region.

Three-Level Weight Tensor

$$w(i, j, k)$$

where i indexes asset class, j indexes sector, and k indexes region.

Example Table (Flattened)

Table 4: Hierarchical Asset Allocation Portfolio

Asset Class	Sector	Region	Weight $w(i, j, k)$
Equities	Technology	N. America	0.16
Equities	Technology	Europe	0.09
Equities	Healthcare	Asia	0.06
Bonds	Government	N. America	0.31
Bonds	Corporate	Europe	0.25
Commodities	Energy	Asia	0.13

Hierarchical Allocation Diagram

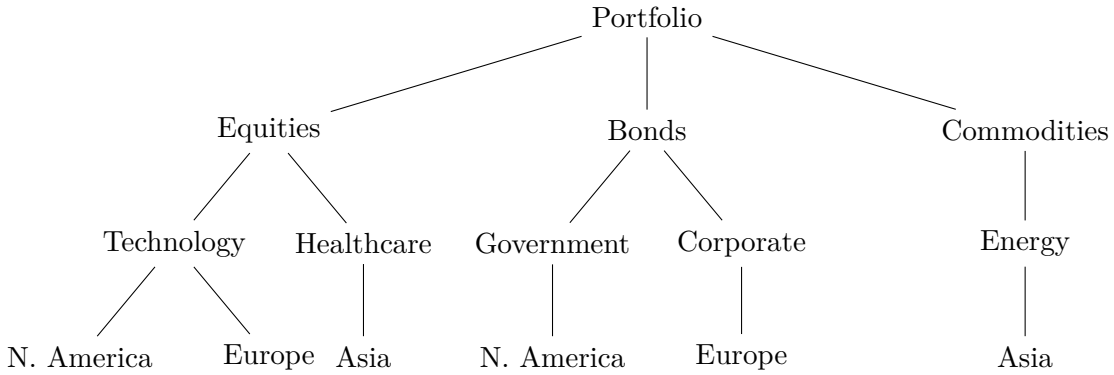

Figure 3: Hierarchical Asset Allocation Structure

3.4 Multi-Period Portfolio

This portfolio allocates across time, such as years and quarters, or investment horizons and sub-periods.

Weight Matrix Representation

$$\mathbf{W}_{\text{Year,Quarter}} = \begin{pmatrix} w(1,1) & w(1,2) & w(1,3) & w(1,4) \\ w(2,1) & w(2,2) & w(2,3) & w(2,4) \\ w(3,1) & w(3,2) & w(3,3) & w(3,4) \end{pmatrix}$$

where rows are years and columns are quarters.

Example Table

Table 5: Multi-Period Portfolio Weights

Year	Q1	Q2	Q3	Q4	Year Total
2024	0.05	0.07	0.08	0.10	0.30
2025	0.10	0.12	0.13	0.15	0.50
2026	0.05	0.07	0.04	0.04	0.20
Quarter Total	0.20	0.26	0.25	0.29	1.00

Heatmap Visualization

Figure 4: Heatmap of Multi-Period Portfolio Weights

3.5 Enhanced Diversification Portfolio

This portfolio is constructed to maximize diversification across two risk factors, such as volatility and correlation buckets.

Weight Matrix Representation

$$\mathbf{W}_{\text{Vol,Cor}} = \begin{pmatrix} w(1,1) & w(1,2) & w(1,3) \\ w(2,1) & w(2,2) & w(2,3) \\ w(3,1) & w(3,2) & w(3,3) \end{pmatrix}$$

where rows are volatility buckets (Low, Medium, High) and columns are correlation buckets (Low, Medium, High).

Example Table

Table 6: Enhanced Diversification Portfolio Weights

Volatility \ Correlation	Low	Medium	High	Vol Total
Low	0.10	0.08	0.07	0.25
Medium	0.12	0.10	0.08	0.30
High	0.10	0.15	0.20	0.45
Corr Total	0.32	0.33	0.35	1.00

Heatmap Visualization

Figure 5: Heatmap of Enhanced Diversification Portfolio Weights

4 Conclusion

The double-weighted portfolio framework provides a powerful and flexible approach to portfolio construction, enabling nuanced allocation across two dimensions. The empirical examples above demonstrate its application to sector-region, asset class-regime, hierarchical, multi-period, and risk-factor-diversified portfolios. The use of vector graphics and heatmaps enhances interpretability and transparency in portfolio design.

References

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The End