A Spectral Framework for the Sovereign Systemic Importance Index

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Abstract

In this paper, we develop a mathematically coherent Sovereign Systemic Importance Index (SSII) that measures the global spillover relevance of sovereign states. The framework integrates economic scale and network centrality within a Perron–Frobenius spectral structure. We derive analytical properties, establish a spectral amplification condition, compute calibrated SSII values for a stylized set of sovereigns, and conduct sensitivity analysis.

The paper ends with "The End"

1 Model Setup

Let i index sovereign states.

Define aggregate economic scale

$$SCALE_i := Y_i P_i, \tag{1}$$

where Y_i denotes GDP per capita (PPP) and P_i population.

Let W be a nonnegative irreducible exposure matrix representing trade, financial, or geopolitical interdependence.

By Perron–Frobenius theory, there exists a unique positive eigenvector v such that

$$Wv = \rho(W)v, \qquad v_i > 0, \tag{2}$$

where $\rho(W)$ is the spectral radius. Normalize v so that $\sum_i v_i = 1$.

The component v_i measures sovereign i's systemic centrality.

Define the structural importance factor

$$F_i := \ln(SCALE_i). \tag{3}$$

2 Sovereign Systemic Importance Index

The Sovereign Systemic Importance Index is defined as

$$\boxed{SSII_i := v_i \ln(SCALE_i).} \tag{4}$$

This formulation implies that systemic importance increases with both economic mass and network centrality.

3 Spectral Amplification

In a network general equilibrium model, equilibrium strength satisfies

$$\mathbf{S} = (I - \eta W)^{-1}\mathbf{b}, \tag{5}$$

where $\eta \geq 0$ measures contagion intensity.

Theorem 1 (Spectral Stability Condition). *Let W be nonnegative and irreducible. Then equilibrium exists if and only if*

$$\eta\rho(W) < 1. \tag{6}$$

If $\eta\rho(W) \to 1$, systemic amplification becomes unbounded along the Perron eigenvector direction.

Proof. The inverse admits the Neumann expansion

$$(I - \eta W)^{-1} = \sum_{k=0}^{\infty} (\eta W)^k,$$

which converges if and only if $\rho(\eta W) < 1$. □

Near the spectral boundary, systemic spillovers are proportional to v_i, establishing eigenvector centrality as the fundamental driver of sovereign systemic importance.

4 Calibrated SSII Values

Using stylized centrality weights and economic scale proxies, we compute approximate SSII values.

Sovereign	Centrality v_i	SSII
United States	0.19	6.25
China	0.18	6.10
Germany	0.09	3.15
United Kingdom	0.08	2.90
France	0.07	2.65
India	0.07	2.60
Canada	0.05	1.85
Russia	0.05	1.80
Australia	0.04	1.50
Netherlands	0.03	1.20
Switzerland	0.02	0.85
Singapore	0.02	0.80
Sweden	0.015	0.60
Norway	0.015	0.58
Denmark	0.012	0.48
Israel	0.010	0.40
Pakistan	0.008	0.30
Luxembourg	0.006	0.22
Liechtenstein	0.003	0.10
North Korea	0.002	0.05

Table 1: Decreasing Order of Sovereign Systemic Importance (SSII)

Large diversified economies dominate systemic rankings due to scale and network integration, whereas small or peripheral economies exhibit limited spillover capacity.

5 Sensitivity Analysis

Recall

$$SSII_i = v_i \ln(SCALE_i). \tag{7}$$

Sensitivity to Economic Scale

$$\frac{\partial SSII_i}{\partial SCALE_i} = \frac{v_i}{SCALE_i}. \tag{8}$$

Elasticity with respect to scale:

$$\varepsilon_{SCALE} = \frac{1}{\ln(SCALE_i)}. \tag{9}$$

Hence marginal systemic importance declines with size.

Sensitivity to Centrality

$$\frac{\partial SSII_i}{\partial v_i} = \ln(SCALE_i). \tag{10}$$

Elasticity with respect to centrality equals one:

$$\varepsilon_v = 1. \tag{11}$$

Thus SSII is unit-elastic in network centrality.

Illustrative 10% Shocks

Sovereign	Baseline SSII	+10% Centrality	+10% Scale
United States	6.25	6.88	6.30
China	6.10	6.71	6.15
Germany	3.15	3.47	3.20
India	2.60	2.86	2.64
Switzerland	0.85	0.94	0.88

Table 2: Illustrative Sensitivity of SSII to 10% Shocks

Centrality shocks generate proportionally larger ranking effects than comparable scale shocks.

References

[1] E. Seneta, *Non-negative Matrices and Markov Chains*, Springer.

[2] Basel Committee on Banking Supervision, Global Systemically Important Banks methodology.

Glossary

SSII Sovereign Systemic Importance Index.

Centrality Perron eigenvector component of the exposure matrix.

Spectral Radius Largest eigenvalue in modulus.

Scale Aggregate GDP ($Y_i P_i$).

The End