Contest theory, political violence and state formation: From first principles to mathematical formalization

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Abstract

In this paper, I present a comprehensive theoretical framework integrating contest theory, political violence and state formation from first principles of economic theory and political theory. The framework demonstrates how rational actors competing for resources create institutional structures that either promote or constrain violence, ultimately shaping state formation processes.

I incorporate game-theoretic foundations, collective action theory, and institutional economics to explain the emergence of political order from anarchic conditions. The framework is rigorous and has empirical implications for political development.

The paper ends with "The End"

1 Introduction

The emergence of political order from conditions of anarchy represents one of the fundamental puzzles in political economy. How do rational, self-interested actors transition from a state of violent competition to institutionalized cooperation? This paper presents a comprehensive theoretical framework that integrates contest theory, political violence, and state formation to address this question.

Our approach builds upon the foundational insight that political institutions emerge as solutions to collective action problems created by resource competition. We formalize this intuition through mathematical models that demonstrate how contest dynamics generate both violent conflict and incentives for institutional development.

2 Foundational Assumptions and First Principles

2.1 Economic Foundations

We begin with standard assumptions from microeconomic theory, adapted for the political context:

Assumption 1 (Rational Actors)**.** *Individuals maximize expected utility subject to constraints, where utility derives from consumption c_i, security s_i, and political influence p_i:*

$$U_i = u(c_i, s_i, p_i) \tag{1}$$

Assumption 2 (Resource Scarcity)**.** *Total available resources R are finite, creating competition among n actors:*

$$\sum_{i=1}^{n} r_i \leq R \tag{2}$$

where r_i represents resources controlled by actor i.

Assumption 3 (Production vs. Appropriation)**.** *Actors can allocate effort e_i between productive activities (creating wealth) and appropriative activities (taking wealth from others):*

$$e_i = e_i^p + e_i^a \tag{3}$$

where e_i^p is productive effort and e_i^a is appropriative effort.

2.2 Political Foundations

Assumption 4 (Monopoly of Violence)**.** *Legitimate political authority requires establishing exclusive control over the use of force within territory T:*

$$\exists!\ \text{actor } j : V_j(T) = 1 \text{ and } V_i(T) = 0 \text{ for } i \neq j \tag{4}$$

where $V_i(T)$ indicates actor i's control over violence in territory T.

Assumption 5 (Collective Action Problems)**.** *Providing public goods faces free-rider problems. Let g represent public good provision and c_i individual contributions:*

$$\frac{\partial u_i}{\partial g} > 0 \text{ but } \frac{\partial u_i}{\partial c_i} < 0 \tag{5}$$

Assumption 6 (Institutional Path Dependence)**.** *Once established, political institutions create self-reinforcing mechanisms. Let I_t represent institutional configuration at time t:*

$$P(I_{t+1} = I_t | I_t) > P(I_{t+1} = I' | I_t) \text{ for } I' \neq I_t \tag{6}$$

3 Core Mathematical Framework

3.1 Basic Contest Model

Consider n actors competing for a prize of value V. Each actor i chooses effort level $x_i \geq 0$ at cost $c(x_i)$.

Definition 1 (Contest Success Function)**.** *The probability that actor i wins the contest is given by:*

$$p_i(x_i, x_{-i}) = \frac{f(x_i)}{\sum_{j=1}^{n} f(x_j)} \tag{7}$$

where $f(x_i)$ represents the contest success function, typically $f(x_i) = x_i^r$ with $r > 0$.

Definition 2 (Expected Payoff)**.** *Actor i's expected payoff is:*

$$U_i = p_i(x_i, x_{-i}) \cdot V - c(x_i) \tag{8}$$

Theorem 1 (Contest Equilibrium)**.** *In the symmetric Nash equilibrium of the contest, each actor chooses effort x^* satisfying:*

$$\left.\frac{\partial p_i}{\partial x_i}\right|_{x_i = x^*} \cdot V = c'(x^*) \tag{9}$$

Proof. From the first-order condition for utility maximization:

$$\frac{\partial U_i}{\partial x_i} = \frac{\partial p_i}{\partial x_i} \cdot V - c'(x_i) = 0 \tag{10}$$

In symmetric equilibrium, $x_i = x^*$ for all i, yielding the result. □

3.2 Violence as Contest Technology

Violence represents a particular contest technology where actors invest in military capacity m_i to appropriate resources.

Definition 3 (Military Production Function)**.** *Military capacity is produced according to:*

$$m_i = g(l_i, k_i) \tag{11}$$

where l_i is labor devoted to military activities and k_i is capital (weapons, fortifications).

Definition 4 (Conflict Success Function)**.** *The probability of victory in violent conflict is:*

$$p_i = \frac{m_i^\alpha}{\sum_{j=1}^{n} m_j^\alpha} \tag{12}$$

where $\alpha \in (0, 1]$ represents the decisiveness of military technology.

3.3 Social Welfare Analysis

Definition 5 (Social Welfare Function)**.** *Total social welfare is given by:*

$$W = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} [y_i - c(x_i) - d(m_i)] \tag{13}$$

where y_i is productive output, $c(x_i)$ represents contest costs, and $d(m_i)$ represents deadweight losses from violence.

Proposition 1 (Pareto Optimal Allocation)**.** *The Pareto optimal allocation satisfies:*

$$\frac{\partial W}{\partial x_i} = 0 \Rightarrow x_i^* = 0 \quad \forall i \tag{14}$$

$$\frac{\partial W}{\partial m_i} = 0 \Rightarrow m_i^* = 0 \quad \forall i \tag{15}$$

This demonstrates that both wasteful competition and violence are socially inefficient, creating incentives for institutional solutions.

4 Contest Theory Applications

4.1 Rent-Seeking Contests

Political contests often involve rent-seeking where actors compete for transfers rather than creating value.

Theorem 2 (Rent Dissipation)**.** *Under perfect competition with identical actors, total rent-seeking expenditures equal the value of the contested rent:*

$$\sum_{i=1}^{n} x_i = V \tag{16}$$

Proof. With Tullock contest success function $p_i = x_i / \sum_j x_j$ and linear costs $c(x_i) = x_i$, the first-order condition yields:

$$\frac{V(n-1)}{(\sum_j x_j)^2} = 1 \tag{17}$$

In symmetric equilibrium, $x_i = x^*$ for all i, so $\sum_j x_j = nx^*$. Substituting:

$$\frac{V(n-1)}{(nx^*)^2} = 1 \Rightarrow x^* = \sqrt{\frac{V(n-1)}{n^2}} \tag{18}$$

As $n \to \infty$, $nx^* \to V$. □

4.2 All-Pay Auctions

Political contests often resemble all-pay auctions where all participants pay their bids regardless of outcome.

Proposition 2 (Expected Expenditure)**.** *In a symmetric all-pay auction with n identical bidders, expected expenditure is:*

$$E[x_i] = \frac{n-1}{n} \cdot V \tag{19}$$

4.3 Dynamic Contests

Multi-period contests create reputation effects and strategic considerations.

Definition 6 (Dynamic Contest Value Function)**.** *The value function for a dynamic contest satisfies the Bellman equation:*

$$V(s_i) = \max_{x_i} [u_i(x_i, x_{-i}) + \delta E[V(s_{i+1})|x_i, x_{-i}]] \tag{20}$$

where s_i represents the state of actor i and δ is the discount factor.

5 Political Violence Modeling

5.1 Violence as Costly Signaling

Violence serves as a costly signal of resolve in political bargaining.

Definition 7 (Signaling Model)**.** *Let $\theta \in \{\theta_L, \theta_H\}$ represent actor type (resolve level) and $v(\theta)$ the cost of violence for type θ. The separating equilibrium requires:*

$$v(\theta_H) - v(\theta_L) \geq (\theta_H - \theta_L) \cdot B \tag{21}$$

where B is the benefit from being perceived as high-resolve type.

5.2 Commitment Problems

The inability to credibly commit to future actions creates incentives for preventive violence.

Theorem 3 (Preventive War Condition)**.** *Preventive war occurs when:*

$$V(war) > \delta \cdot V(peace) + (1 - \delta) \cdot V(\textit{future disadvantage}) \tag{22}$$

where δ represents the probability of maintaining current advantage.

5.3 Security Dilemma

Defensive preparations by one actor appear threatening to others, creating conflict spirals.

Definition 8 (Security Dilemma Game)**.** *The security dilemma can be represented as a prisoner's dilemma with payoff matrix:*

$$\begin{pmatrix} & \textit{Cooperate} & \textit{Defect} \\ \textit{Cooperate} & (3,3) & (0,5) \\ \textit{Defect} & (5,0) & (1,1) \end{pmatrix} \tag{23}$$

This structure explains why rational actors may choose violence despite mutual losses.

6 State Formation Mechanisms

6.1 Social Contract Theory

Definition 9 (Participation Constraint)**.** *Individuals voluntarily join the state if:*

$$U_i(state) \geq U_i(anarchy) \quad \forall i \tag{24}$$

Definition 10 (Incentive Compatibility)**.** *The social contract is self-enforcing if:*

$$U_i(comply) \geq U_i(defect) \quad \forall i \tag{25}$$

6.2 Predatory State Model

Rulers maximize revenue subject to constraints from subjects' exit options and rebellion threats.

Definition 11 (Ruler's Optimization Problem)**.** *The predatory ruler solves:*

$$\max_{\tau} \quad \pi = \tau \cdot Y - C(enforcement) \tag{26}$$

$$s.t. \quad U(subjects) \geq U(exit) \tag{27}$$

$$U(subjects) \geq U(rebellion) - p \cdot Punishment \tag{28}$$

where τ is the tax rate, Y is total output, and p is the probability of successful rebellion.

6.3 Contractual State Model

The state emerges as a solution to collective action problems in providing public goods.

Theorem 4 (Optimal Public Goods Provision). *The optimal level of public goods G^* satisfies:*

$$G^* = \arg\max \sum_{i=1}^{n} [u_i(y_i - t_i, G) - u_i(y_i, 0)] \tag{29}$$

where t_i represents individual tax contributions.

7 Institutional Economics Integration

7.1 Transaction Costs

Political institutions emerge to reduce transaction costs of collective decision-making.

Theorem 5 (Coase Theorem Extension). *Efficient political outcomes emerge when:*

1. *Transaction costs are zero*
2. *Property rights are well-defined*
3. *Bargaining is possible*

7.2 Incomplete Contracts

Political agreements cannot specify all contingencies, creating hold-up problems.

Definition 12 (Total Institutional Costs). *Optimal institutional design minimizes:*

$$TC = \textit{Specification Costs} + \textit{Enforcement Costs} + \textit{Adaptation Costs} \tag{30}$$

7.3 Mechanism Design

Theorem 6 (Revelation Principle). *Any implementable social choice function can be implemented by a direct mechanism where truth-telling is a dominant strategy.*

8 Equilibrium Analysis

8.1 Nash Equilibrium in Political Contests

Definition 13 (Symmetric Contest Equilibrium). *A symmetric Nash equilibrium satisfies:*

$$\left.\frac{\partial U_i}{\partial x_i}\right|_{x_i = x^*} = 0 \quad \forall i \tag{31}$$

where x^* is the common equilibrium effort level.

8.2 Evolutionary Stability

Definition 14 (Evolutionarily Stable Strategy). *A strategy σ^* is evolutionarily stable if:*

$$U(\sigma^*, \sigma^*) > U(\sigma, \sigma^*) \quad \forall \sigma \neq \sigma^* \tag{32}$$

8.3 Institutional Equilibrium

Definition 15 (Stable Institutional Arrangement). *An institutional equilibrium satisfies:*

1. *Individual rationality: $U_i(I) \geq U_i(\textit{outside option})$*
2. *Collective rationality: $\sum_i U_i(I) \geq \sum_i U_i(I')$ for alternative I'*
3. *Enforcement constraints: Deviation is not profitable*
4. *Evolutionary stability: Resistant to small perturbations*

9 Empirical Implications and Testable Hypotheses

9.1 Contest Intensity and Violence

Hypothesis 1. *Higher prize values increase contest intensity and violence probability.*

Empirical Test:

$$\text{Violence} = \beta_0 + \beta_1(\text{Prize Value}) + \beta_2(\text{Contest Technology}) + \varepsilon \tag{33}$$

9.2 Institutional Development

Hypothesis 2. *Greater collective action problems lead to more centralized institutions.*

Empirical Test:

$$\text{Centralization} = \alpha_0 + \alpha_1(\text{Population}) + \alpha_2(\text{Geographic Dispersion}) + \alpha_3(\text{Diversity}) + \varepsilon \tag{34}$$

9.3 State Capacity

Hypothesis 3. *External threats accelerate state formation and capacity building.*

Empirical Test:

$$\text{State Capacity} = \gamma_0 + \gamma_1(\text{External Threat}) + \gamma_2(\text{Economic Development}) + \gamma_3(\text{Geographic Factors}) + \varepsilon \tag{35}$$

10 Mathematical Appendix

10.1 Contest Success Functions

1. **Tullock Function:**
$$p_i = \frac{x_i^r}{\sum_{j=1}^n x_j^r} \tag{36}$$

2. **Logit Function:**
$$p_i = \frac{\exp(rx_i)}{\sum_{j=1}^n \exp(rx_j)} \tag{37}$$

3. **Difference Form:**
$$p_i = F(x_i - x_j) \tag{38}$$

10.2 Welfare Analysis

1. **Deadweight Loss from Contests:**
$$DWL = \sum_{i=1}^n c(x_i) - \max\left[\sum_{i=1}^n y_i - \sum_{i=1}^n c(x_i)\right] \tag{39}$$

2. **Efficiency Ratio:**
$$\eta = \frac{W(\text{equilibrium})}{W(\text{first-best})} \tag{40}$$

10.3 Dynamic Programming Formulation

1. **Value Function:**
$$V(s) = \max_a \left\{u(s,a) + \beta \int V(s')P(s'|s,a)ds'\right\} \tag{41}$$

where s is state, a is action, and $P(s'|s,a)$ is the transition probability.

11 Conclusion

This framework demonstrates how contest theory provides a unified approach to understanding political violence and state formation. The mathematical foundations reveal that competition for resources creates inefficiencies that institutional development can potentially address. The framework's empirical implications offer testable hypotheses for future research, while the integration of multiple theoretical traditions provides a comprehensive foundation for analyzing political development processes.

The key insight is that political institutions emerge as solutions to collective action problems created by resource competition, but their specific form depends on the underlying contest technology, commitment problems and transaction costs. This framework provides both theoretical depth and practical applicability for understanding contemporary political challenges.

Future research should focus on empirical testing of the framework's predictions, particularly regarding the relationships between contest intensity and institutional development, the role of external threats in state formation and the conditions under which violent competition transitions to peaceful institutional competition.

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