

# The Four-Nation Complete Information Paradox: Why Asymmetric Information is Essential for Sovereign Debt Market Viability

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## Abstract

We establish a fundamental paradox in sovereign debt markets: under complete information symmetry across nations, at least one sovereign must default with positive probability due to binding planetary resource constraints. Using a four-nation model with stochastic discount factor theory, we prove that information asymmetry is not merely a market friction but a necessary condition for sovereign debt market existence. The model decomposes each nation's information into public ( $P_j$ ), private ( $p_j$ ), and hidden ( $H_j$ ) sets, demonstrating that the opacity created by private and hidden information prevents immediate market collapse. Our results have profound implications for transparency initiatives, credit rating agency function, and the stability of international debt markets.

The paper ends with "The End"

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## 1 Introduction

Asymmetric information has long been studied as a source of inefficiency in financial markets [1, 5]. In sovereign debt markets, governments possess superior information about their fiscal positions, policy intentions, and willingness to repay compared to international investors [2]. The conventional wisdom treats this information asymmetry as a problem to be minimized through enhanced disclosure, independent fiscal councils, and credit rating agencies.

This paper challenges that conventional wisdom. We demonstrate that complete information symmetry in sovereign debt markets leads to a logical contradiction: if all nations possess complete information about global resource constraints and debt obligations, then rational pricing implies certain default by at least one nation, potentially collapsing market liquidity entirely.

### 1.1 Main Contributions

Our analysis makes three primary contributions:

- 1. Formal characterization of information sets:** We decompose national information into public, private, and hidden components, providing a rigorous framework for analyzing information asymmetry effects.
- 2. Resource constraint theorem:** We prove that under complete information and bounded planetary resources, aggregate sovereign debt obligations exceed debt-serviceable resources with positive probability.
- 3. Inevitability of default:** Using stochastic discount factor theory, we establish that complete information symmetry makes default inevitable and immediately priced, fundamentally undermining market function.

## 1.2 Intuition and Economic Mechanism

The paradox emerges from three interrelated features of sovereign debt markets:

**First**, sovereign debt is a claim on future national output, which is bounded by planetary resource constraints in a closed economic system. Unlike corporate debt, which can be secured by specific assets, sovereign debt represents a general claim on tax revenues derived from economic activity.

**Second**, the stochastic discount factor framework ensures that under complete markets with symmetric information, asset prices reflect all available information about future payoffs. When nations share complete information, they collectively know that aggregate debt claims exceed available resources in certain states of the world.

**Third**, rational expectations and no-arbitrage conditions force immediate price adjustment. If default in certain states is common knowledge, it must be priced from the initial issuance, potentially making debt issuance prohibitively expensive or impossible for some nations.

The key insight is that *information opacity creates the uncertainty necessary for sovereign debt markets to function*. Hidden contingent liabilities, uncertain policy responses, and unpredictable political shocks generate a probability distribution over outcomes that allows for continued borrowing and lending even when the aggregate system faces resource constraints.

## 2 Model Setup and Information Structure

### 2.1 The Planetary Economy

Consider a planetary economy consisting of four nations, indexed by  $j \in \{1, 2, 3, 4\}$ . Time is continuous over  $[0, T]$  with uncertainty represented by a filtered probability space  $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in [0, T]}, \mathbb{P})$ .

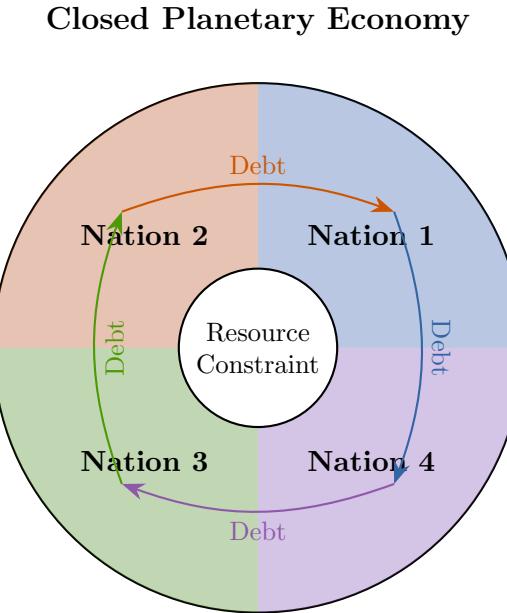


Figure 1: Four-nation planetary economy with debt flows and central resource constraint.

Each nation is both creditor and debtor to others, creating a closed financial system bounded by total planetary resources.

### 2.2 Information Set Decomposition

**Definition 2.1** (Information Structure). *For each nation  $j \in \{1, 2, 3, 4\}$ , we decompose its information into three disjoint sets:*

- $P_j$ : **Public information set** — observable to all nations (published GDP, debt-to-GDP ratios, budget deficits, demographic data)
- $p_j$ : **Private information set** — known only to nation  $j$  (true contingent liabilities, off-balance-sheet obligations, actual policy intentions, undisclosed fiscal risks)
- $H_j$ : **Hidden information set** — unknown even to nation  $j$  itself (future political regime changes, natural disasters, technological disruptions, unforeseen crisis responses)

The complete information set for nation  $j$  is:

$$C_j = P_j \cup p_j \cup H_j$$

The complete planetary information set is:

$$\mathcal{C} = C_1 \cup C_2 \cup C_3 \cup C_4 = \bigcup_{j=1}^4 (P_j \cup p_j \cup H_j)$$

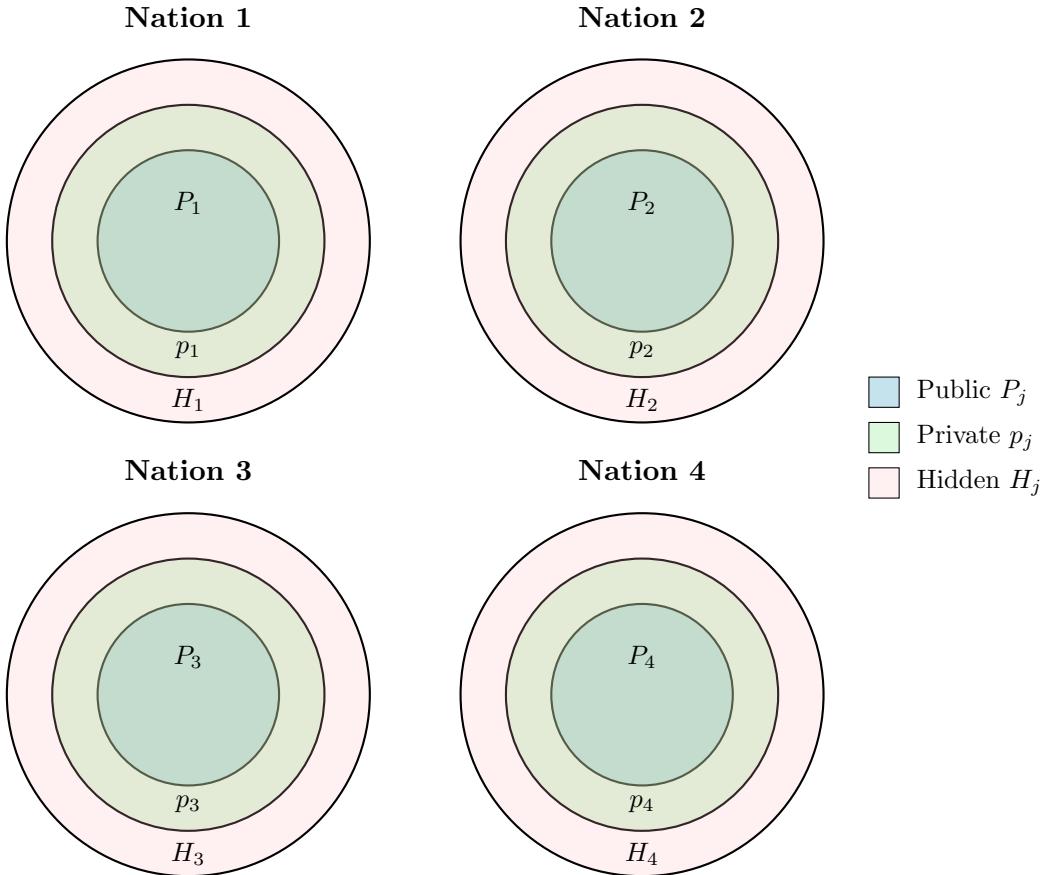


Figure 2: Decomposition of information sets for each nation.

Public information  $P_j$  is observable by all; private information  $p_j$  is known only to nation  $j$ ; hidden information  $H_j$  is unknown even to the nation itself.

**Assumption 1** (Disjointness of Information Sets). *Within each nation's complete information:*

$$P_j \cap p_j = \emptyset, \quad P_j \cap H_j = \emptyset, \quad p_j \cap H_j = \emptyset$$

This assumption ensures clean decomposition: information cannot simultaneously be public and private, or known and hidden.

### 2.3 Observable vs. Complete Information

In the standard sovereign debt market, each nation  $j$  observes:

$$\mathcal{I}_j = P_j \cup p_j \cup \bigcup_{k \neq j} P_k$$

That is, nation  $j$  knows its own public and private information, plus all other nations' public information, but does *not* observe other nations' private information  $\{p_k\}_{k \neq j}$  or any hidden information.

The **complete information assumption** we examine is:

$$\mathcal{I}_j = \mathcal{C} \quad \forall j \in \{1, 2, 3, 4\}$$

This means all nations observe the complete planetary information set, including:

- All other nations' private information (contingent liabilities, true policy intentions)
- All hidden information (future shocks become known at  $t = 0$ )

## 3 Resource Constraints and Output Process

### 3.1 National Output Process

**Definition 3.1** (Output Process). *For each nation  $j$ , define the output process:*

$$Y_{j,t} : \Omega \times [0, T] \rightarrow \mathbb{R}_+$$

which is  $\mathcal{F}_t$ -adapted and satisfies:

$$\mathbb{E} \left[ \int_0^T Y_{j,t} dt \right] < \infty$$

Total planetary output at time  $t$  in state  $\omega$ :

$$W_t(\omega) = \sum_{j=1}^4 Y_{j,t}(\omega)$$

Under complete information  $\mathcal{C}$ , the entire path  $\{W_t\}_{t \in [0, T]}$  is known at  $t = 0$  (i.e.,  $W_t$  is  $\mathcal{F}_0$ -measurable).

### 3.2 Feasibility Constraints

**Definition 3.2** (Planetary Feasibility). *Let  $C_{j,t}(\omega)$  denote consumption by nation  $j$ . The planetary feasibility constraint requires:*

$$\sum_{j=1}^4 C_{j,t}(\omega) + I_t(\omega) + G_t(\omega) \leq W_t(\omega) \quad \forall (\omega, t)$$

where:

- $I_t(\omega)$ : total planetary investment
- $G_t(\omega)$ : total public goods provision

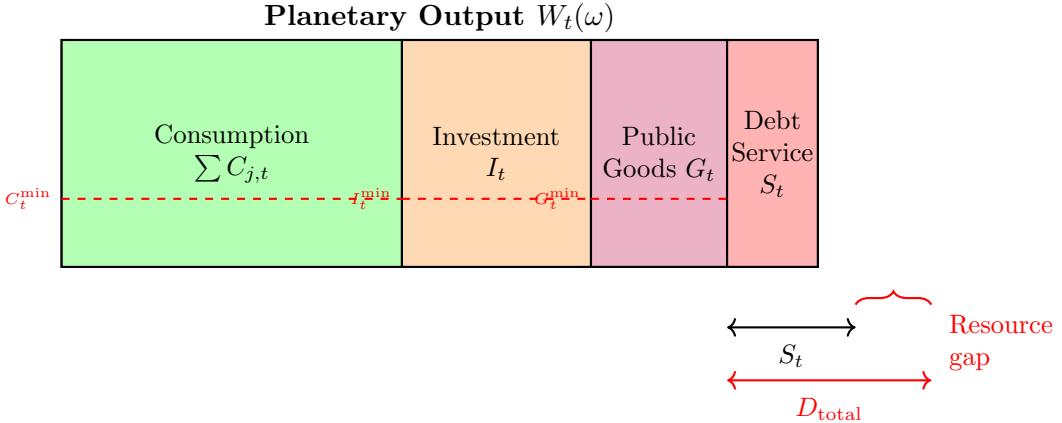


Figure 3: Allocation of planetary output  $W_t(\omega)$  among competing uses.

Debt service capacity  $S_t$  is limited by minimum required consumption, investment, and public goods. When total debt obligations  $D_{\text{total}}$  exceed  $S_t$ , default becomes inevitable.

**Definition 3.3** (Debt-Serviceable Resources). *Define the maximum resources available for debt service at time  $T$ :*

$$S_T(\omega) := W_T(\omega) - C_T^{\min}(\omega) - I_T^{\min}(\omega) - G_T^{\min}(\omega)$$

where minimum allocations satisfy political economy and subsistence constraints.

**Assumption 2** (Binding Minimum Constraints). *The minimum allocations are strictly positive fractions of output:*

$$C_T^{\min}(\omega) \geq \kappa_C \cdot W_T(\omega) \quad (1)$$

$$I_T^{\min}(\omega) \geq \kappa_I \cdot W_T(\omega) \quad (2)$$

$$G_T^{\min}(\omega) \geq \kappa_G \cdot W_T(\omega) \quad (3)$$

where  $\kappa_C, \kappa_I, \kappa_G > 0$  and  $\kappa_C + \kappa_I + \kappa_G =: \kappa < 1$ .

This implies:

$$S_T(\omega) \leq (1 - \kappa) \cdot W_T(\omega) =: \alpha \cdot W_T(\omega)$$

where  $\alpha = 1 - \kappa < 1$  is the maximum fraction of output available for debt service.

### 3.3 Debt Structure

**Definition 3.4** (Sovereign Debt Obligations). *Each nation  $j$  issues zero-coupon bonds at  $t = 0$  maturing at  $T$  with promised payment:*

$$D_j(\omega) = B_j \cdot (1 + r_j)^T$$

where  $B_j$  is face value and  $r_j$  is the promised yield.

Total promised planetary debt payments:

$$D_{\text{total}}(\omega) := \sum_{j=1}^4 D_j(\omega)$$

In a closed planetary system, debt relationships form a network. Some nations are net creditors, others net debtors, but all debt is held internally within the four-nation system.

## 4 The Binding Resource Constraint Theorem

We now establish that under reasonable conditions, aggregate debt obligations exceed debt-serviceable resources with positive probability.

**Theorem 4.1** (Binding Resource Constraint). *Assume:*

1. All nations know  $\mathcal{C}$  at  $t = 0$  (i.e.,  $\mathcal{F}_0 = \mathcal{F}_T$ )
2. Each nation  $j$  has run fiscal deficits:  $\int_0^T [G_{j,s} - \text{Tax}_{j,s}] ds > 0$
3. Planetary growth is bounded:  $\mathbb{E}[W_T] < \infty$
4. Output volatility:  $\text{Var}(W_T) > 0$

*Then:*

$$\mathbb{P}(D_{\text{total}}(\omega) > S_T(\omega)) > 0$$

*Proof.* By assumption 2, each nation has accumulated debt obligations over  $[0, T]$ . In a closed planetary system, aggregate debt reflects cumulative deficits:

$$D_{\text{total}} = \sum_{j=1}^4 \int_0^T [G_{j,s} - \text{Tax}_{j,s}] ds \cdot e^{r \cdot (T-s)} > 0$$

Consider the set of low-output states:

$$\Omega^{\text{crisis}} := \{\omega \in \Omega : W_T(\omega) < \mathbb{E}[W_T] - \sigma_W\}$$

where  $\sigma_W = \sqrt{\text{Var}(W_T)} > 0$  by assumption 4.

By Chebyshev's inequality:

$$\mathbb{P}(\Omega^{\text{crisis}}) \geq \mathbb{P}(|W_T - \mathbb{E}[W_T]| \geq \sigma_W) \geq \frac{1}{4} > 0$$

In these crisis states:

$$S_T(\omega) = \alpha \cdot W_T(\omega) < \alpha \cdot (\mathbb{E}[W_T] - \sigma_W)$$

Meanwhile, debt obligations  $D_{\text{total}}$  were set at  $t = 0$  based on expected ability to pay. Under optimistic or median expectations:

$$D_{\text{total}} \approx \beta \cdot \mathbb{E}[S_T] = \beta \cdot \alpha \cdot \mathbb{E}[W_T]$$

where  $\beta > 1$  reflects risk premium and intertemporal borrowing preferences.

For  $\omega \in \Omega^{\text{crisis}}$ :

$$D_{\text{total}} - S_T(\omega) \geq \beta \cdot \alpha \cdot \mathbb{E}[W_T] - \alpha \cdot (\mathbb{E}[W_T] - \sigma_W) \quad (4)$$

$$= \alpha [(\beta - 1)\mathbb{E}[W_T] + \sigma_W] > 0 \quad (5)$$

Therefore:

$$\mathbb{P}(D_{\text{total}} > S_T) \geq \mathbb{P}(\Omega^{\text{crisis}}) > 0$$

□

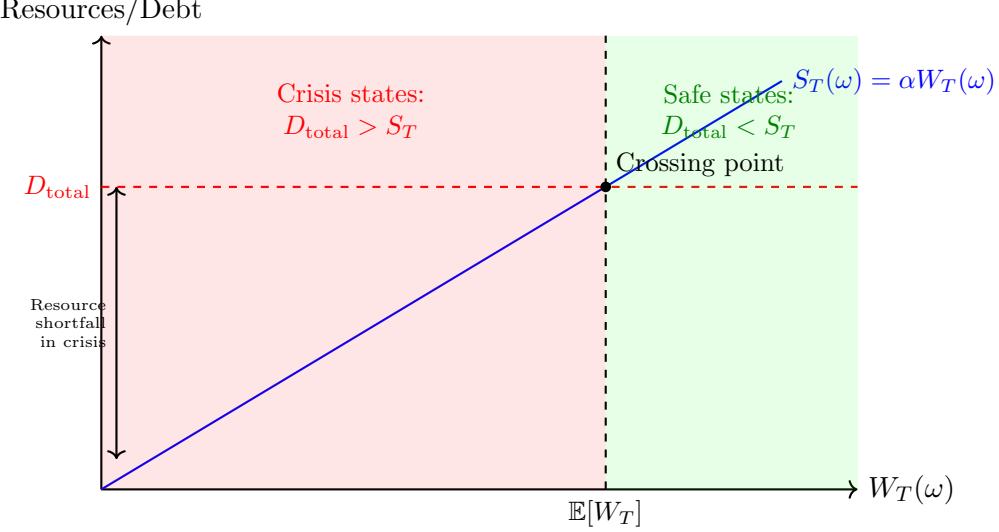


Figure 4: Resource constraint binding in low-output states.

When output  $W_T(\omega)$  falls below expected levels, debt-serviceable resources  $S_T(\omega) = \alpha W_T(\omega)$  fall below total debt obligations  $D_{\text{total}}$ , making default inevitable in crisis states.

## 5 Stochastic Discount Factor and Asset Pricing

### 5.1 The SDF Framework

In complete markets, all asset prices can be represented using a stochastic discount factor (SDF), also called the pricing kernel [3, 4].

**Definition 5.1** (Stochastic Discount Factor). *The SDF  $M_T$  is a strictly positive random variable satisfying:*

$$q = \mathbb{E}[M_T \cdot X]$$

for any asset with current price  $q$  and future payoff  $X$ .

For a zero-coupon sovereign bond issued by nation  $j$  at  $t = 0$  with maturity  $T$ :

$$q_j = \mathbb{E}[M_T \cdot R_j]$$

where  $R_j \in [0, 1]$  is the recovery rate:

$$R_j = \begin{cases} 1 & \text{if nation } j \text{ repays in full} \\ \delta_j \in [0, 1) & \text{if nation } j \text{ defaults} \end{cases}$$

Under standard utility maximization, the SDF can be derived from the representative agent's marginal utility:

$$M_T = \beta \cdot \frac{u'(C_T)}{u'(C_0)}$$

where  $\beta$  is the time preference parameter and  $u(\cdot)$  is the utility function.

### 5.2 Complete Information and the SDF

**Lemma 5.2** (SDF under Complete Information). *When all nations observe  $\mathcal{C}$  at  $t = 0$  (i.e.,  $\mathcal{F}_0 = \mathcal{F}_T$ ), the stochastic discount factor  $M_T$  is  $\mathcal{F}_0$ -measurable and therefore deterministic conditional on time-0 information.*

*Proof.* Under complete information, the entire consumption path  $\{C_t\}_{t \in [0, T]}$  is known at  $t = 0$ . Therefore:

$$M_T = \beta \cdot \frac{u'(C_T)}{u'(C_0)}$$

is a known constant (conditional on time-0 information), not a random variable.

Equivalently, since  $\mathcal{F}_0 = \mathcal{F}_T$ , any  $\mathcal{F}_T$ -measurable random variable is also  $\mathcal{F}_0$ -measurable.  $\square$

This lemma has profound implications: under complete information, all uncertainty is resolved at  $t = 0$ . Asset prices fully reflect all information about future payoffs, including which nations will default.

## 6 The Default Inevitability Theorem

We now prove the central result: under complete information, at least one nation must default with positive probability.

**Theorem 6.1** (Inevitable Default under Complete Information). *Assume:*

1. *Complete information:  $\mathcal{F}_0 = \mathcal{F}_T$  (all nations know  $\mathcal{C}$ )*
2. *No-arbitrage: there exists a unique SDF  $M_T > 0$  almost surely*
3. *Binding resource constraint:  $\mathbb{P}(D_{\text{total}} > S_T) > 0$  (by Theorem 4.1)*
4. *Rational bond pricing:  $q_j = \mathbb{E}[M_T \cdot R_j]$  where  $R_j \in [0, 1]$*

*Then:*

$$\exists j \in \{1, 2, 3, 4\} \text{ such that } \mathbb{P}(R_j < 1) > 0$$

*That is, at least one nation defaults with positive probability.*

*Proof.* Suppose, for contradiction, that  $R_j = 1$  almost surely for all  $j \in \{1, 2, 3, 4\}$  (no nation defaults).

Then actual debt payments equal promised payments:

$$\sum_{j=1}^4 R_j \cdot D_j = \sum_{j=1}^4 D_j = D_{\text{total}}$$

But debt service must come from available resources. The feasibility constraint requires:

$$\sum_{j=1}^4 R_j \cdot D_j \leq S_T(\omega) \quad \text{for all } \omega$$

Combining these:

$$D_{\text{total}}(\omega) \leq S_T(\omega) \quad \text{for all } \omega$$

This implies:

$$\mathbb{P}(D_{\text{total}} > S_T) = 0$$

But this contradicts assumption 3, which establishes:

$$\mathbb{P}(D_{\text{total}} > S_T) > 0$$

Therefore, our supposition must be false. There exists at least one nation  $j^*$  and a set  $\Omega_{j^*} \subseteq \Omega$  with  $\mathbb{P}(\Omega_{j^*}) > 0$  such that:

$$R_{j^*}(\omega) < 1 \quad \text{for } \omega \in \Omega_{j^*}$$

By the rational pricing condition (assumption 4):

$$q_{j^*} = \mathbb{E}[M_T \cdot R_{j^*}] < \mathbb{E}[M_T \cdot 1] = \mathbb{E}[M_T]$$

Therefore nation  $j^*$  faces a credit spread reflecting positive default probability, and this is known to all nations at time 0 under complete information.  $\square$

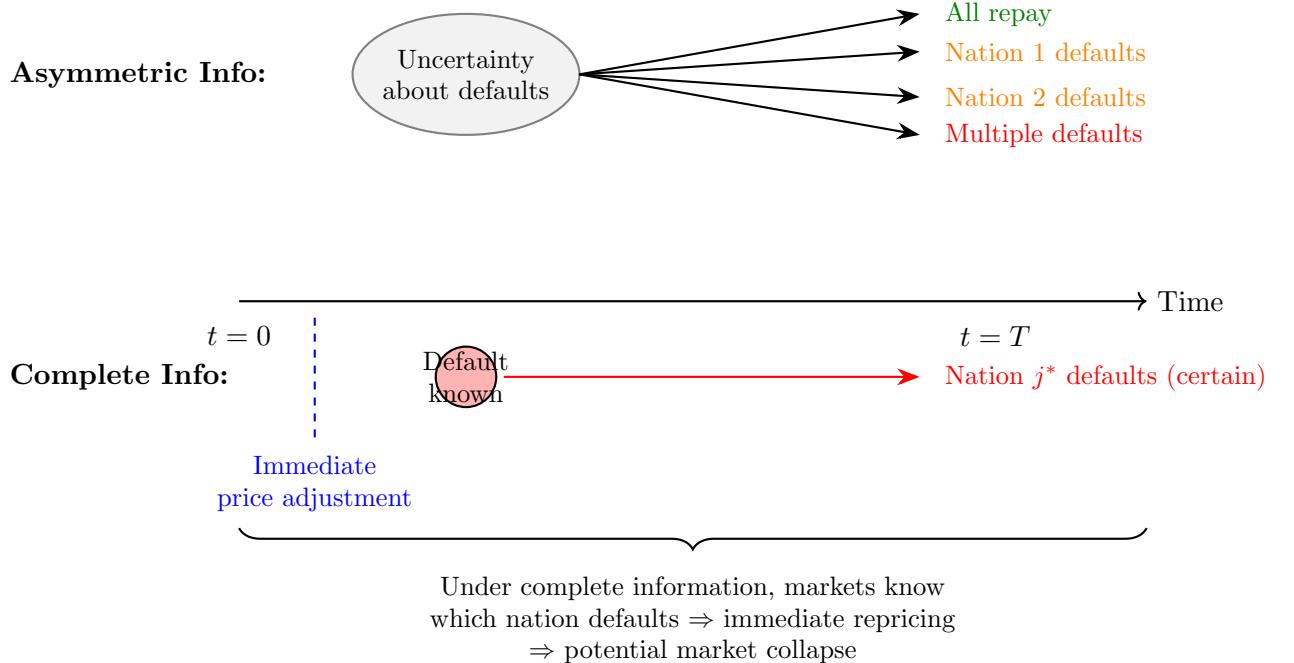


Figure 5: Comparison of market dynamics under asymmetric vs. complete information.

With asymmetric information, uncertainty about defaults allows markets to function. Under complete information, defaults are known at  $t = 0$ , forcing immediate price adjustments that may prevent market formation.

## 6.1 Recovery Rates and Aggregate Constraints

**Corollary 6.2** (Aggregate Recovery Rate). *Define the aggregate recovery rate:*

$$\rho(\omega) := \frac{\sum_{j=1}^4 R_j(\omega) \cdot D_j(\omega)}{D_{total}(\omega)}$$

Then under the conditions of Theorem 6.1:

$$\mathbb{E}[\rho] \leq \frac{\mathbb{E}[S_T]}{\mathbb{E}[D_{total}]}$$

If debt is set optimistically (i.e.,  $\mathbb{E}[D_{total}] > \mathbb{E}[S_T]$ ), then:

$$\mathbb{E}[\rho] < 1$$

*Proof.* By the feasibility constraint:

$$\sum_{j=1}^4 R_j(\omega) \cdot D_j(\omega) \leq S_T(\omega) \quad \text{for all } \omega$$

Therefore:

$$\rho(\omega) \leq \frac{S_T(\omega)}{D_{\text{total}}(\omega)}$$

Taking expectations:

$$\mathbb{E}[\rho] \leq \mathbb{E}\left[\frac{S_T}{D_{\text{total}}}\right]$$

By Jensen's inequality (since  $1/D_{\text{total}}$  is convex):

$$\mathbb{E}\left[\frac{S_T}{D_{\text{total}}}\right] \geq \frac{\mathbb{E}[S_T]}{\mathbb{E}[D_{\text{total}}]}$$

If  $\mathbb{E}[D_{\text{total}}] > \mathbb{E}[S_T]$ , then:

$$\mathbb{E}[\rho] < 1$$

□

This corollary shows that under optimistic debt issuance (which is typical given political economy incentives), the expected aggregate recovery rate is strictly less than 100%, confirming that losses to creditors are inevitable.

## 7 The Complete Information Paradox

### 7.1 Market Viability under Information Asymmetry

The key insight from Theorems 4.1 and 6.1 is that *information asymmetry is not a bug but a feature* of sovereign debt markets.

**Under asymmetric information** (the real-world case):

- Each nation observes  $\mathcal{I}_j = P_j \cup p_j \cup \bigcup_{k \neq j} P_k$
- Hidden information sets  $\{H_j\}$  create genuine uncertainty about future states
- Private information sets  $\{p_j\}$  prevent other nations from knowing true fiscal positions
- This uncertainty allows for:
  - Probability distributions over default outcomes
  - Risk pricing based on incomplete information
  - Continued market liquidity as investors hold heterogeneous beliefs
  - Nations to borrow even when aggregate constraints bind in some states

**Under complete information** (the hypothetical case examined):

- All nations observe  $\mathcal{C}$  at  $t = 0$
- All uncertainty is resolved immediately ( $\mathcal{F}_0 = \mathcal{F}_T$ )
- Nations know exactly which sovereign(s) will default
- Bond prices immediately reflect default probabilities
- The nation(s) certain to default may find borrowing impossible
- Market liquidity potentially collapses as lenders refuse to purchase bonds from nations known to default

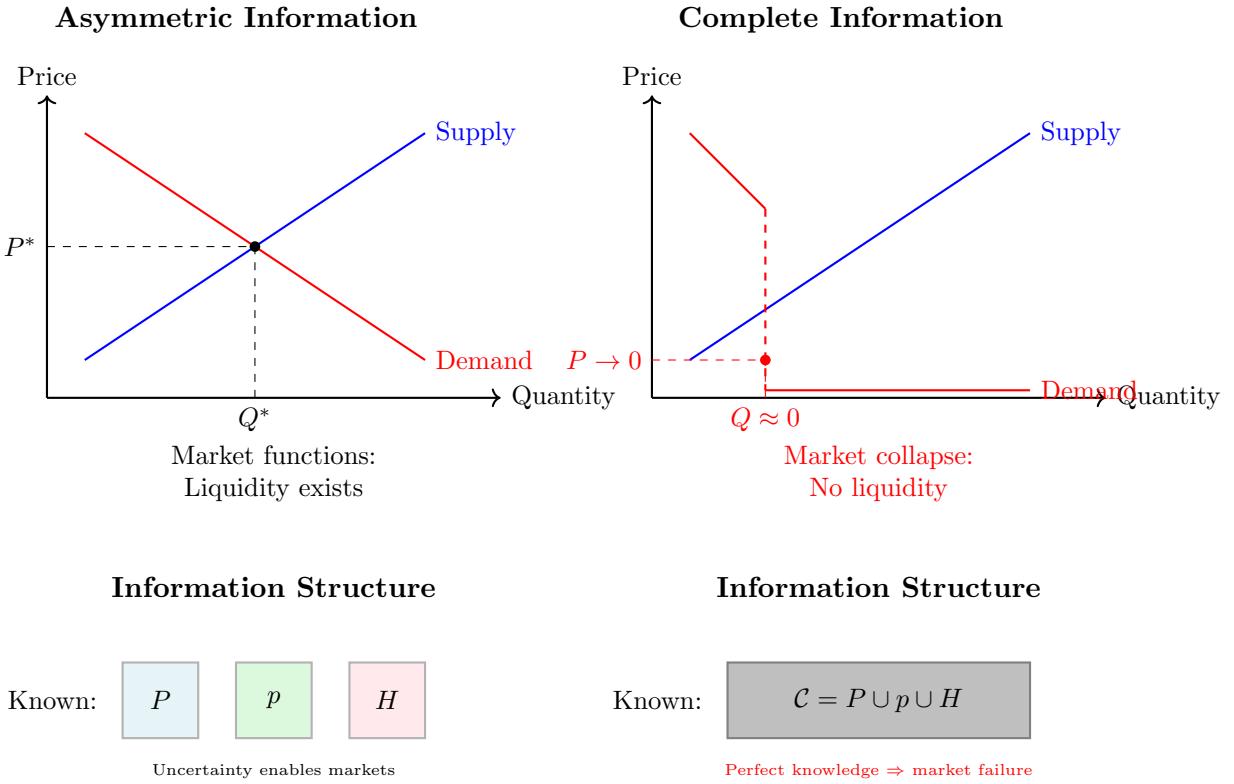


Figure 6: Comparison of market equilibrium under asymmetric vs. complete information.

Under asymmetric information (left), uncertainty created by private ( $p$ ) and hidden ( $H$ ) information sets allows normal market functioning with positive equilibrium quantity and price. Under complete information (right), perfect knowledge of which nations will default causes demand to collapse, eliminating market liquidity.

## 7.2 Policy Implications

Our results suggest several counterintuitive policy implications:

1. **Transparency has limits:** While enhanced disclosure of public information  $P_j$  is generally beneficial, forcing complete revelation of private information  $p_j$  could paradoxically destabilize markets by removing necessary uncertainty.
2. **Rating agencies provide valuable opacity:** Credit rating agencies aggregate information but also *smooth* it through ratings categories (AAA, AA, etc.), preventing continuous revelation of exact default probabilities that could trigger market collapse.
3. **Hidden shocks are market stabilizers:** The existence of truly unknown future shocks ( $H_j$ ) provides "common uncertainty" that keeps default from being deterministic, allowing markets to function.
4. **Information asymmetry as commitment device:** By maintaining private information about fiscal positions and policy intentions, nations preserve some strategic ambiguity that enables continued borrowing even in aggregate resource-constrained environments.

## 8 Extensions and Robustness

### 8.1 Continuous Time Formulation

The model can be extended to continuous time with stochastic output processes:

$$dY_{j,t} = \mu_j(t) dt + \sigma_j(t) dW_{j,t}$$

where  $W_{j,t}$  are correlated Brownian motions capturing idiosyncratic and systematic risks.

The resource constraint becomes:

$$\int_0^T \left( \sum_{j=1}^4 C_{j,t} + I_t + G_t \right) dt \leq \int_0^T W_t dt$$

The main results extend naturally: under complete information about the paths  $\{Y_{j,t}\}$ , the binding resource constraint still implies inevitable default.

### 8.2 Endogenous Information Revelation

Nations may strategically choose how much private information  $p_j$  to reveal. Let  $\gamma_j \in [0, 1]$  denote the fraction of  $p_j$  that nation  $j$  discloses.

A separating equilibrium may exist where:

- Strong fiscal nations choose high  $\gamma_j$  to signal creditworthiness
- Weak fiscal nations choose low  $\gamma_j$  to pool with stronger nations
- Credit spreads reflect both observed fundamentals and inference from disclosure choices

However, if all nations are forced to set  $\gamma_j = 1$  (complete transparency), the paradox re-emerges.

### 8.3 Multiple Equilibria

Under asymmetric information, multiple equilibria may exist:

- **Good equilibrium:** Investors have optimistic beliefs about hidden information  $H_j$ , allowing all nations to borrow at reasonable rates
- **Bad equilibrium:** Investors have pessimistic beliefs, leading to high spreads, self-fulfilling debt dynamics, and actual default

Complete information eliminates multiplicity by resolving all uncertainty, but at the cost of market collapse when fundamentals are weak.

### 8.4 Dynamic Information Accumulation

In reality, information evolves over time:  $\mathcal{I}_{j,t}$  grows as new data arrives. The key questions become:

1. At what rate does information accumulate?
2. Is there a critical threshold where "enough" information triggers market collapse?
3. Can nations strategically manage information release to maintain market stability?

These dynamic considerations are left for future research.

## 9 Conclusion

We have established a fundamental paradox in sovereign debt markets: complete information symmetry across nations leads to inevitable default and potential market collapse. Using a four-nation model with formal decomposition of information sets into public, private, and hidden components, we proved:

1. Under reasonable assumptions, aggregate debt obligations exceed debt-serviceable planetary resources with positive probability (Theorem 4.1)
2. When all nations possess complete information, at least one nation must default, and this is known at the time of bond issuance (Theorem 6.1)
3. Information asymmetry—far from being purely a source of inefficiency—is essential for sovereign debt market viability

These results challenge the conventional view that maximum transparency should be a policy goal in sovereign debt markets. Instead, we show that strategic opacity, maintained through private information and genuine uncertainty about future shocks, creates the conditions necessary for markets to function.

The policy implications are significant: regulators and international organizations should carefully consider whether transparency initiatives might inadvertently destabilize the very markets they seek to improve. A more nuanced approach would recognize that *optimal opacity*—not maximum transparency—may be the appropriate goal.

Future research should explore the optimal degree of information revelation, the role of time-varying information structures, and the strategic interactions between nations in managing their information disclosures. The complete information paradox established here provides a foundation for this investigation.

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## Glossary

### **Public Information ( $P_j$ )**

Information about nation  $j$  that is observable to all market participants, including published GDP figures, debt-to-GDP ratios, budget deficits, and demographic data.

### **Private Information ( $p_j$ )**

Information known only to nation  $j$  itself, including true contingent liabilities, off-balance-sheet obligations, actual (vs. stated) policy intentions, and undisclosed fiscal risks.

### **Hidden Information ( $H_j$ )**

Information unknown even to nation  $j$ , representing genuinely unpredictable future events such as political regime changes, natural disasters, technological disruptions, and crisis responses.

### **Complete Information Set ( $C_j$ )**

The union of public, private, and hidden information for nation  $j$ :  $C_j = P_j \cup p_j \cup H_j$ .

### **Planetary Information Set ( $\mathcal{C}$ )**

The union of all nations' complete information sets:  $\mathcal{C} = \bigcup_{j=1}^4 C_j$ .

### **Stochastic Discount Factor (SDF)**

Also called the pricing kernel,  $M_T$  is a strictly positive random variable that prices all assets:  $q = \mathbb{E}[M_T \cdot X]$  for an asset with current price  $q$  and future payoff  $X$ .

### **Recovery Rate ( $R_j$ )**

The fraction of promised debt payments that nation  $j$  actually makes:  $R_j = 1$  for full repayment,  $R_j \in [0, 1)$  for default with partial recovery.

### **Debt-Serviceable Resources ( $S_T$ )**

Maximum planetary resources available for debt service after accounting for minimum required consumption, investment, and public goods:  $S_T = W_T - C_T^{\min} - I_T^{\min} - G_T^{\min}$ .

### **Resource Constraint**

The fundamental limitation that total uses of planetary output cannot exceed total production:  $\sum_j C_j + I + G \leq W$ .

### No-Arbitrage Condition

The principle that there should be no riskless profit opportunities, which implies the existence of a strictly positive stochastic discount factor.

### Filtration ( $\mathcal{F}_t$ )

The increasing sequence of  $\sigma$ -algebras representing information available at time  $t$ . Complete information means  $\mathcal{F}_0 = \mathcal{F}_T$  (all future information known at time 0).

### Crisis States ( $\Omega^{\text{crisis}}$ )

States of the world where planetary output falls sufficiently low that debt obligations exceed debt-serviceable resources:  $\{\omega : D_{\text{total}}(\omega) > S_T(\omega)\}$ .

### Adverse Selection

Market failure arising when one party has private information, causing high-quality participants to exit the market (Akerlof's "lemons problem").

### Moral Hazard

The risk that a party with private information takes actions that are detrimental to the other party, which cannot be perfectly monitored.

### Separating Equilibrium

An equilibrium in which different types of agents choose different actions, allowing their types to be inferred from their choices.

### Pooling Equilibrium

An equilibrium in which different types of agents choose identical actions, preventing their types from being distinguished.

## The End