Stochastic Optimal Stopping in a Multi-Bloc Global Order: A Global Games Refinement

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Abstract

In this paper, we develop a global games refinement of a multi-bloc stochastic stopping model of the global order. Sovereign blocs interact through a spectral exposure network while facing incomplete information about systemic fragility and political integration costs. Introducing idiosyncratic noisy signals eliminates equilibrium multiplicity and selects a unique Bayesian threshold equilibrium. Spectral centrality determines the sequencing of consolidation, while signal precision governs synchronization of regime transitions.

The paper ends with "The End"

1 Spectral Structure of the Global System

Let W be a nonnegative irreducible exposure matrix with Perron eigenvalue $\rho(W)$ and normalized eigenvector v satisfying

$$Wv = \rho(W)v, \qquad v_i > 0, \qquad \sum_i v_i = 1. \tag{1}$$

Systemic stability requires

$$\eta\rho(W) < 1, \tag{2}$$

where η measures contagion intensity.

Higher v_i implies greater systemic amplification weight in equilibrium spillovers.

2 Fundamental and Information Structure

Let the fundamental state θ measure global systemic fragility. Assume

$$\theta \sim \mathcal{N}(\bar{\theta}, \sigma_\theta^2). \tag{3}$$

Each bloc i observes a private signal

$$x_i = \theta + \varepsilon_i, \tag{4}$$

where $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ independently.

Signal precision is defined as

$$\alpha := \frac{1}{\sigma_\varepsilon^2}. \tag{5}$$

3 Bloc Payoffs

Bloc i chooses action $a_i \in 0, 1$, where $a_i = 1$ denotes consolidation.

Payoff from consolidation is

$$U_i^C = B - K_i + \gamma v_i \theta - c \sum_{j \neq i} a_j, \tag{6}$$

where K_i is private political cost and $c > 0$ captures strategic substitutability. The payoff from remaining fragmented is normalized to zero.

4 Posterior Beliefs

Under Gaussian priors and signals, the posterior expectation satisfies

$$\mathbb{E}[\theta \mid x_i] = \lambda x_i + (1 - \lambda)\bar{\theta}, \tag{7}$$

where

$$\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}. \tag{8}$$

Expected payoff conditional on x_i is

$$\mathbb{E}[U_i^C \mid x_i] = B - K_i + \gamma v_i \mathbb{E}[\theta \mid x_i] - c\mathbb{E}\left[\sum_{j \neq i} a_j \mid x_i\right]. \tag{9}$$

5 Global Games Equilibrium

Theorem 1 (Unique Bayesian Threshold Equilibrium)**.** *There exists a unique Bayesian Nash equilibrium in cutoff strategies. For each bloc i, there exists a threshold x_i^* such that consolidation occurs if and only if*

$$x_i \geq x_i^*. \tag{10}$$

Proof. The expected payoff difference is strictly increasing in x_i because $\mathbb{E}[\theta \mid x_i]$ is increasing in x_i. The single-crossing property ensures optimality of threshold strategies. The addition of arbitrarily small private noise eliminates multiple equilibria by the standard global games argument (Carlsson–van Damme; Morris–Shin). □

6 Symmetric Approximation

Under symmetry ($v_i = 1/n$, $K_i = K$), define the indifference condition

$$B - K + \frac{\gamma}{n}\mathbb{E}[\theta \mid x^*] - c(n-1)p(x^*) = 0, \tag{11}$$

where $p(x^*)$ is the equilibrium consolidation probability.

A first-order approximation yields a cutoff in fundamentals

$$\theta^* \approx \frac{n(B - K)}{\gamma}. \tag{12}$$

Higher spectral fragmentation (larger n) raises the coordination threshold.

7 Comparative Statics

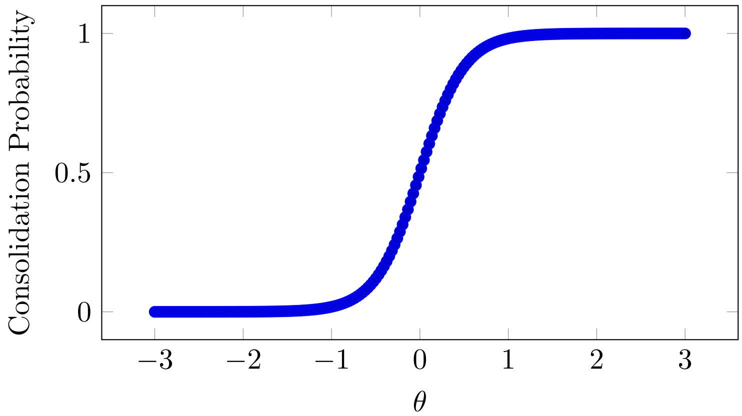
Proposition 1. *As signal precision α increases, consolidation becomes more synchronized.*

Proof. Higher precision increases λ, reducing dispersion of posterior beliefs. Cutoffs cluster, generating sharper aggregate transitions. □

Proposition 2. *Higher centrality v_i lowers the individual cutoff x_i^*.*

Proof. Expected marginal benefit of consolidation is increasing in v_i. Hence indifference occurs at a lower signal realization. □

8 Illustrative Transition Curve



Higher signal precision steepens the transition from fragmentation to consolidation.

9 Spectral Feedback

Let consolidation share be $p(\theta)$. The effective exposure matrix becomes $W(p)$ with spectral radius

$$\rho(p) = \rho(W(p)). \tag{13}$$

Corollary 1. *If $\partial\rho/\partial p < 0$, consolidation gradually reduces systemic amplification and yields a unique transition path.*

Private noise therefore selects a determinate regime path even near spectral instability.

10 Conclusion

The global games refinement eliminates multiplicity in the multi-bloc stopping environment. Informational noise selects a unique consolidation threshold, while spectral centrality governs sequencing. Signal precision determines synchronization intensity. The resulting framework integrates network amplification, stochastic fragility, and informational refinement into a coherent theory of regime transitions in the global order.

References

[1] H. Carlsson and E. van Damme (1993), Global games and equilibrium selection.

[2] S. Morris and H. Shin (2003), Global games: Theory and applications.

[3] E. Seneta, *Non-negative Matrices and Markov Chains*, Springer.

Glossary

Global Games Coordination games with noisy private signals that select a unique equilibrium.

Spectral Radius Dominant eigenvalue of the exposure matrix.

Perron Eigenvector Positive eigenvector associated with $\rho(W)$.

Signal Precision Inverse variance of private noise.

The End