The Complete Treatise on Option Chain Analysis

Soumadeep Ghosh

Kolkata, India

Abstract

This treatise develops a comprehensive framework for *option chain analysis*: the extraction of actionable information from the cross-section of listed options across strikes and expiries. Drawing from probability, stochastic calculus, numerical analysis, market microstructure, and risk management, we formalize the option chain as a discretized slice of the risk-neutral valuation operator. We show how to (i) clean and align the chain, (ii) infer forwards and discount factors, (iii) compute and regularize Greeks, (iv) build and arbitrate the implied volatility (IV) smile and surface, (v) detect supply-demand imbalances via volume/open interest (OI) features, (vi) map chain states to strategy payoffs, and (vii) perform stress tests. Vector figures illustrate key constructions; algorithms provide reproducible workflows. The exposition emphasizes no-arbitrage structure, robust estimation, and live desk practicality.

The treatise ends with "The End"

Contents

1 Introduction

An *option chain* is the tabular collection of prices (quotes) for call and put options on a given underlying across a grid of strikes $\{K_j\}$ and maturities $\{T_i\}$. Each row typically corresponds to a strike, with columns for bid/ask, last, volume, OI, implied volatility, and Greeks for calls and puts. The chain is a discretization of the option price surface $C(K,T)$ and $P(K,T)$ under a risk-neutral measure $\mathbb{Q}$.

This treatise synthesizes the mathematics of no-arbitrage with practical data engineering to turn raw chains into analytics. Our guiding principle is: *good chain analysis is disciplined interpolation plus principled extrapolation under robust constraints.*

2 Preliminaries: Payoffs, Pricing, and Greeks

Let S_t be the underlying price process on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$ with short rate r_t. A European call and put with strike K and maturity T have payoffs $(S_T - K)^+$ and $(K - S_T)^+$.

2.1 Risk-neutral pricing and put-call parity

Under standard conditions,

$$C_t(K,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_u \, \mathrm{d}u} (S_T - K)^+\right], \tag{1}$$

$$P_t(K,T) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\int_t^T r_u \, \mathrm{d}u} (K - S_T)^+\right]. \tag{2}$$

Put-call parity for non-dividend-paying assets:

$$C_t(K,T) - P_t(K,T) = F_t(T)\, D_t(T) - K\, D_t(T), \tag{3}$$

where $D_t(T) = \exp(-\int_t^T r_u \, \mathrm{d}u)$ and $F_t(T) = \mathbb{E}_t^{\mathbb{Q}}[S_T]/D_t(T)$ is the forward. With discrete dividends or carry, suitable adjustments apply.

2.2 Implied volatility and Greeks

Given a price, the *implied volatility* $\sigma_{\mathrm{imp}}(K,T)$ is the σ such that the Black-Scholes price matches the market mid:

$$C^{\mathrm{BS}}(S_t, K, T, \sigma, q, r) = C^{\mathrm{mkt}}.$$

Greeks (Delta, Gamma, Vega, Theta, Rho) are partial derivatives of the pricing map; they summarize local sensitivities essential for hedging and risk control.

3 The Option Chain as a Data Object

We view a chain snapshot at time t as

$$\mathcal{C}_t = \left\{(K_j, T_i, q_{\{C,P\}}^{\mathrm{bid/ask}}, V, \mathrm{OI}, \sigma_{\mathrm{imp}}, \mathrm{Greeks})\right\}_{i=1..m,\, j=1..n}.$$

Key tasks:

1. Quote cleaning: stale quote detection, crossed markets, locked spreads, outlier filters.
2. Mid construction: microstructure-aware mid (e.g., *efficient price* using spread/queue signals).
3. Calendar/strike alignment: mapping raw strikes and expiries to numerical grids.

4. Static no-arbitrage checks: monotonicity/convexity in K; calendar monotonicity in T.

Definition 1 (Static no-arbitrage in strike)**.** *For fixed T, the call price $C(K,T)$ must be decreasing and convex in K. Discrete violations in a chain imply either noise or opportunities, typically removed via convex projection.*

Proposition 1 (Discretized convex projection)**.** *Given noisy quotes $\tilde{C}_j$ over strikes $\{K_j\}$, the projection onto the closed convex cone of decreasing-convex sequences yields the closest no-arbitrage vector (in a chosen norm), computable via quadratic programming or isotonic regression with convexity constraints.*

4 Forward, Discount, and Dividend Extraction from the Chain

From (3), using cleaned mid prices $\hat{C}, \hat{P}$:

$$\widehat{FD} = \hat{C}(K,T) - \hat{P}(K,T) + K\,\widehat{D}, \tag{4}$$

with $\widehat{D}$ estimated either from the yield curve, or free-form by minimizing parity errors across strikes. A practical estimator solves

$$\min_{F,D>0} \sum_j w_j \left[\hat{C}(K_j,T) - \hat{P}(K_j,T) - (F - K_j)D\right]^2, \tag{5}$$

with robust weights w_j favoring near-the-money quotes (highest signal-to-noise). With discrete dividends Q, use $F = S_0 e^{(r-q)T} - PV(Q)$ or treat q as an unknown carry and fit jointly.

5 Implied Volatility Smile and Surface

Define moneyness $\kappa = \ln(K/F)$. A *smile* is $\sigma_{\text{imp}}(\kappa,T)$; a *surface* is $\sigma_{\text{imp}}(\kappa,T)$ across T. Construction choices:

- Model-free smoothing: spline or local polynomial in $(\kappa, \sqrt{T})$ under arbitrage constraints.
- Parametric: SVI/GSVI parameterizations ensuring convexity and calendar no-arbitrage.
- Dynamic regularization: temporal smoothness for intraday stability.

5.1 Breeden-Litzenberger density

When $C(K,T)$ is twice differentiable in K, the risk-neutral density satisfies

$$f_{S_T}(K) = \frac{\partial^2 C(K,T)}{\partial K^2} \Big/ D_t(T). \tag{6}$$

On a discrete chain, finite differences with convex projection can recover a stable density proxy.

6 Microstructure Features: Volume, Open Interest, and Flow

Volume and OI encode supply-demand persistence. Features such as the *Call-Put OI imbalance*, *Near-ATM volume ratio*, and *Skew-Change conditional on flow* can signal dealer positioning and potential gamma/vega regimes. Caution: interpret with microstructure filters (roll-over effects near expiry, opening/closing trades, spread trades).

7 Risk and Hedging from the Chain

Given a portfolio Π of options, first-order P&L over a small horizon Δt:

$$\Delta\Pi \approx \Delta\,\Delta S + \tfrac{1}{2}\Gamma(\Delta S)^2 + \mathcal{V}\,\Delta\sigma + \Theta\,\Delta t + \text{higher order},$$

aggregating Greeks across legs. Stress testing sweeps ΔS and $\Delta\sigma$, possibly with correlation (e.g., sticky-delta vs sticky-strike rules for IV moves).

8 Arbitrage Diagnostics

- Vertical (butterfly) arbitrage: convexity violations in strike.
- Calendar arbitrage: $C(K, T_2) \geq C(K, T_1)$ for $T_2 > T_1$.
- Box spread bounds: $C(K_1) - C(K_2) - (P(K_1) - P(K_2))$ bounded by discounted strike differences.

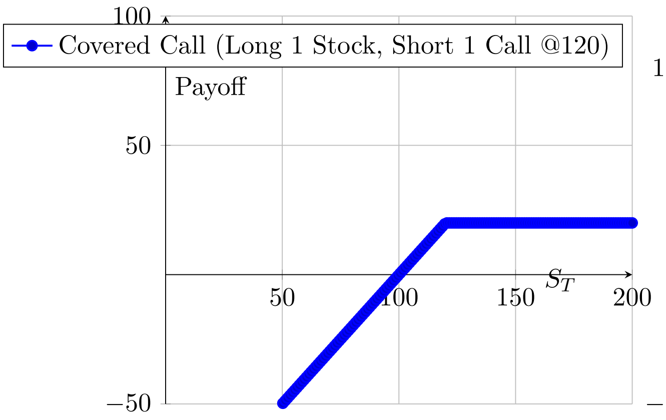
Violations can be detected and corrected by solving a constrained least-squares fit to a *no-arbitrage feasible set.*

9 Option Chain Schematic

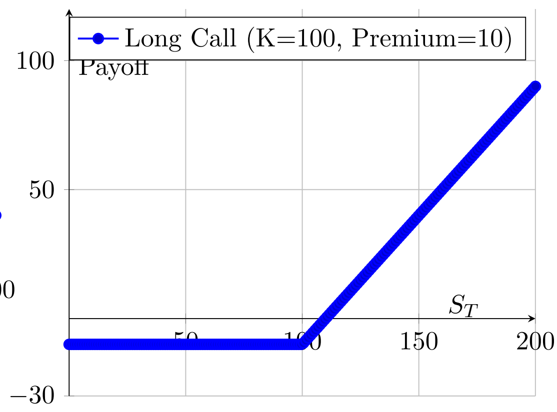
Table 1: Option Chain Snapshot

Strike	Call Bid	Call Ask	Call IV	Vol	OI	Put IV	Put Bid	Put Ask
95	1.95	2.10	0.24	1,200	15,000	0.25	0.95	1.05
100 (ATM)	1.45	1.60	0.22	3,200	22,000	0.23	1.40	1.55
105	1.05	1.20	0.21	5,400	28,000	0.22	1.95	2.10
110	0.75	0.90	0.21	4,800	26,000	0.21	2.80	2.95
115	0.55	0.68	0.22	3,000	21,000	0.22	3.80	3.95

10 Vector Graphics: Strategy Payoff Diagrams



(a) Covered call payoff.



(b) Long call payoff.

Figure 1: Canonical option strategy payoffs at expiry.

11 Implied Volatility Smile and Surface: Vector Plots

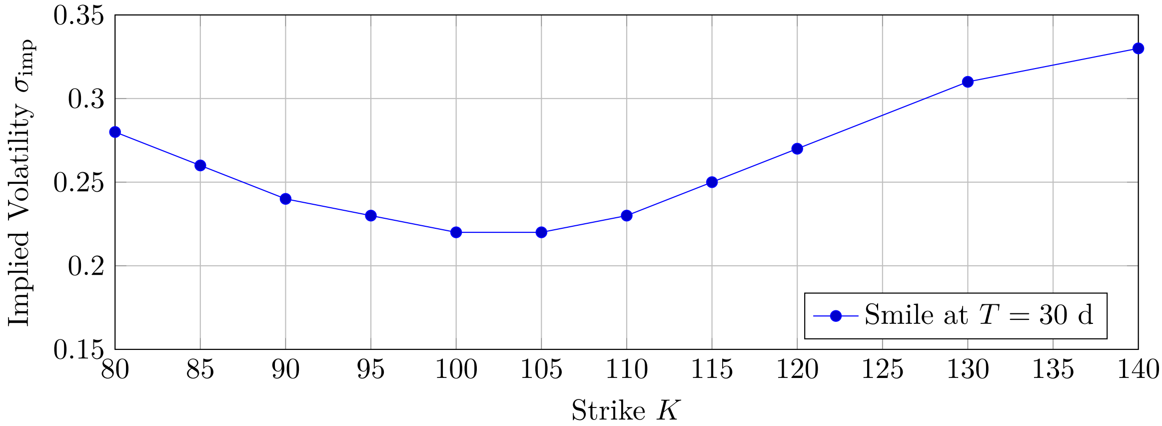


Figure 2: Example IV smile: skewed with higher IV for OTM puts (left wing).

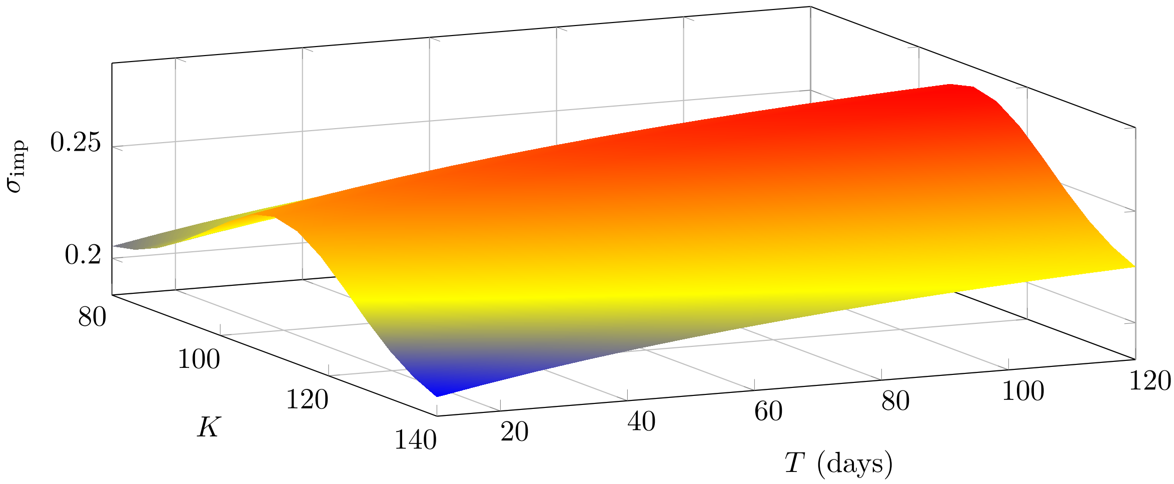


Figure 3: Illustrative IV surface with term structure and skew.

12 Algorithmic Workflows

12.1 Chain cleaning and forward extraction

Algorithm 1 Microstructure-aware chain cleaning and forward/discount estimation

Require: Raw quotes $\{(K_j, T_i, \text{bid}, \text{ask}, V, \text{OI})\}$

1: Remove stale quotes (age $> \tau$), crossed/locked markets, extreme spreads.
2: Compute mids $M = (\text{bid} + \text{ask})/2$ and quality weights $w \propto V/\text{spread}$.
3: **for** each expiry T_i **do**
4: Select near-ATM set $\mathcal{J}_i$ by $|\ln(K_j/F_0)| < \epsilon$ (seed with $F_0 \approx S_0$).
5: Estimate $(F_i, D_i) = \arg\min_{F,D>0} \sum_{j\in\mathcal{J}_i} w_{ij} \left[M^C_{ij} - M^P_{ij} - (F - K_j)D\right]^2$.
6: Recompute ATM set with updated F_i; iterate until convergence.
7: **end for**
8: Enforce calendar smoothness of $\{(F_i, D_i)\}$ (e.g., Tikhonov regularization).

12.2 No-arbitrage convex projection in strike

Algorithm 2 Convex projection of call prices over strike (fixed T)

Require: $\{\tilde{C}_j\}$ at strikes $\{K_j\}$, weights $\{w_j\}$

1: Solve $\min_C \sum_j w_j(\tilde{C}_j - C_j)^2$ subject to $C_{j+1} \le C_j$ and $C_{j+1} - C_j \le C_{j+2} - C_{j+1}$.

2: Return $C^\star$ and finite-difference density proxy $\Delta_K^2 C^\star$.

13 Descriptive and Predictive Statistics from Chains

13.1 Descriptive

- **Smile metrics**: ATM IV, $25\Delta/75\Delta$ skew, kurtosis of smile.
- **Term metrics**: slope $T^{-1/2}\partial_T \sigma_{\text{imp}}$, calendar spreads.
- **Flow metrics**: OI/volume imbalances, roll pressure near expiry.

13.2 Predictive lenses (caveat: regime-dependent)

- Sticky-delta vs sticky-strike responses to price shocks.
- Dealer gamma regime inference from chain convexity and OI clustering.
- Variance risk premium proxies via difference between model-free implied variance and realized variance forecasts.

14 Feature Summary Table

Table 2: Core features derived from a cleaned option chain.

Category	**Feature**	**Definition / Use**
Smile	ATM IV	$\sigma_{\text{imp}}(\kappa{=}0, T)$; baseline uncertainty level.
Smile	Skew	$\sigma_{\text{imp}}(-\kappa_0, T) - \sigma_{\text{imp}}(+\kappa_0, T)$; downside premium.
Surface	Term slope	$\partial_T \sigma_{\text{imp}}$ at ATM; decay/persistence of uncertainty.
Flow	OI imbalance	$\frac{\text{OI}_{\text{call}} - \text{OI}_{\text{put}}}{\text{OI}_{\text{call}} + \text{OI}_{\text{put}}}$; positioning tilt.
Microstructure	Quality score	Function of volume, spreads, quote age; weights analytics.
Arb checks	Butterfly value	$C(K-\Delta) - 2C(K) + C(K+\Delta) \ge 0$; convexity.
Risk	Vega concentration	Share of portfolio Vega within $\pm$ one strike/tenor bucket.

15 End-to-End Synthetic Example

Using synthetic quotes (as in Table 1), we fit (F, D) for T=30d, build the smile (Fig. 2), and assemble the surface (Fig. 3). We then compute Greeks from Black-Scholes at the fitted F and recover a density proxy via convex projection. Finally, we simulate a $\pm 5\%$ spot shock under sticky-delta and reprice to obtain a simple stress report.

A Black-Scholes, Merton, and Local/Stochastic Volatility

Under Black-Scholes with dividend yield q and constant σ,

$$C = S_0 e^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2), \tag{7}$$

$$d_{1,2} = \frac{\ln(S_0/K) + (r - q \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \tag{8}$$

Greeks follow by differentiation. In stochastic volatility (e.g., Heston), σ_{imp} emerges from the model parameters; in practice we *invert* the market to get σ_{imp} and then interpolate with no-arb constraints (SVI), see [5].

B No-Arbitrage Constraints for IV Surfaces

In terms of total implied variance $w(\kappa, T) = \sigma_{\text{imp}}^2 T$, sufficient conditions for absence of butterfly and calendar arbitrage are known for SVI parameterizations; practitioners fit SVI slices and then enforce calendar monotonicity by vertical convexity and horizontal slope constraints.

C Model-Free Implied Variance

Model-free variance for maturity T:

$$\mathrm{IVar}(T) \approx \frac{2e^{rT}}{T}\left(\int_0^F \frac{P(K,T)}{K^2}\,\mathrm{d}K + \int_F^\infty \frac{C(K,T)}{K^2}\,\mathrm{d}K\right),$$

discretized over chain strikes with trapezoidal rules and careful tail extrapolation.

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The End