The Complete Treatise on Cubic Voting

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Abstract

Cubic Voting (CV) is an innovative voting system designed to allow voters to express the intensity of their preferences by purchasing votes, with the cost of each additional vote increasing cubically. This treatise provides a comprehensive overview of cubic voting, including its conceptual foundations, mathematical formulation, equilibrium analysis, welfare properties, historical context, applications, criticisms, and alternatives. Vector graphics are used to illustrate key concepts.

The treatise ends with "The End"

1 Introduction

Democratic decision-making often confronts a trade-off between equal voice and efficient aggregation of preference intensity. Traditional one-person-one-vote (1p1v) systems treat all preferences equally but fail to capture intensity, leading to potential tyranny of the majority and disenfranchisement of minorities with strongly held views. Quadratic Voting (QV) addresses this by allowing voters to purchase v votes at cost v^2, aligning marginal vote prices with marginal utilities in large populations. However, QV can still permit large vote disparities when wealth or valuation variance is high.

Cubic Voting (CV) extends this paradigm by imposing a cubic cost on vote purchases: the cost of v votes is proportional to $|v|^3$. This steeper penalty further dampens extreme vote accumulation, potentially enhancing equity and welfare in settings with high preference heterogeneity or wealth disparities. This treatise formalizes CV, derives its equilibrium properties, assesses its welfare characteristics relative to QV and 1p1v, and discusses practical implementations and extensions.

2 Concept and Mathematical Foundations

2.1 Definition

In Cubic Voting, each voter is allocated a fixed budget of *voice credits* (or, in some variants, can spend wealth directly). The cost to cast n votes for a single option is n^3 credits. Voters may distribute their votes across multiple options, subject to their total budget.

Definition 1 (Cubic Voting Cost Function)**.** *The cost of casting v votes is $c(v) = \alpha|v|^3$, with $\alpha > 0$ a scaling parameter.*

2.2 Mathematical Formulation

Let B be the total budget of credits for a voter, and n_i the number of votes cast for option i (with k total options). The constraint is:

$$\sum_{i=1}^{k} |n_i|^3 \leq B$$

The cubic cost function is steeper than quadratic or linear, making it increasingly expensive to concentrate votes on a single option.

2.3 Vector Graphic: Cubic Cost Function

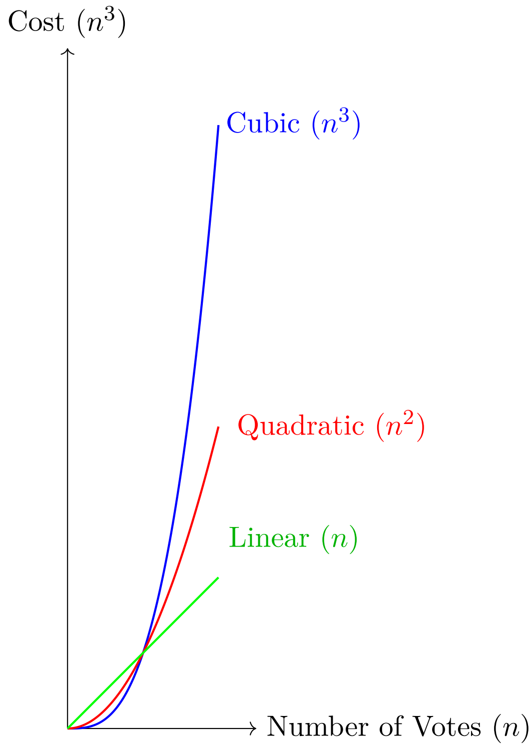

Figure 1: Cost functions for casting n votes: cubic (blue), quadratic (red), and linear (green).

3 Historical Context and Related Work

Cubic Voting is a recent extension of Quadratic Voting, which was introduced by Glen Weyl and collaborators in the 2010s. The cubic cost function was proposed to further discourage vote concentration and address strategic manipulation. The mathematical study of how votes translate to outcomes dates back to the 'law of the cubic proportion' in electoral systems, as described by Kendall and Stuart in 1950. Most practical experience comes from QV, which has been piloted in corporate governance and civic technology initiatives, while CV remains largely theoretical with some academic and blockchain-based experiments.

4 Equilibrium and Welfare Analysis

4.1 Price-Taking Equilibrium

In the price-taking approximation, voters take as given the "vote-price of influence" p, the votes needed for unit impact. For CV, the first-order condition for interior $v_i > 0$ is:

$$u_i \cdot \frac{\partial \Pr(\text{win})}{\partial v_i} = 3\alpha v_i^2$$

Under the large-population approximation where $\partial \Pr / \partial v_i \approx p$, we obtain:

$$v_i^* = \sqrt{\frac{u_i p}{3\alpha}}$$

Market clearing $\sum_i v_i^* = S$ determines p, yielding a unique price-taking equilibrium analogous to QV but with a square-root dependence on u_i.

4.2 Nash Equilibrium

In finite populations, type-symmetric Bayes-Nash equilibria exist under independent private values. The cubic cost ensures strict convexity and concavity of payoff differences, guaranteeing existence of pure-strategy equilibria. Uniqueness follows from incremental passivity arguments similar to QV but requires stronger curvature assumptions due to the cubic penalty.

4.3 Welfare Properties

CV's steeper cost curve more heavily penalizes large vote purchases, reducing the influence of extremely high-valuation or wealthy voters relative to QV. This can improve utilitarian welfare when valuations are heavy-tailed or wealth inequality is pronounced, as it prevents a small coalition from dominating outcomes. However, if valuations are uniformly moderate, CV may under-aggregate intensity, leading to inefficiency compared to QV.

Proposition 1 (Welfare Comparison)**.** *Suppose voter valuations $\{u_i\}$ are i.i.d. from a distribution with finite fourth moment. Then, as $N \to \infty$, the social welfare under CV converges to*

$$W_{\mathrm{CV}} = C_1 \, \mathrm{E}[u_i^{3/2}],$$

whereas under QV, $W_{\mathrm{QV}} = C_2 \, \mathrm{E}[u_i]$. Thus, CV dominates QV in welfare when $\mathrm{E}[u_i^{3/2}]/\mathrm{E}[u_i]$ is sufficiently large.

5 Applications and Real-World Implementations

5.1 Corporate Governance

CV can be embedded in corporate charters to balance shareholder influence. By capping effective votes via cubic costs, CV protects minority shareholders from large stakeholders, potentially reducing managerial entrenchment and improving board accountability.

5.2 Blockchain Governance

In proof-of-stake systems, CV aligns with stake-weighted voting. The cubic cost curbs "whale" dominance, encouraging broader participation and decentralization. Pilot implementations could use smart contracts to enforce $c(v) = \alpha v^3$ and redistribute revenues.

5.3 Public Decision-Making

For public referenda, CV can be implemented via voice-credit budgets. Each voter receives B credits, allocating x_r credits to issue r, with votes $v_r = \mathrm{sgn}(x_r)\sqrt{|x_r|}$ and cost $\sum_r |x_r|^{3/2} \le B$. This ensures bounded spending while allowing intensity expression.

6 Extensions and Robustness

6.1 Multi-Issue Cubic Voting

For M binary issues, voter i allocates votes v_{ir} to issue r, paying $\alpha \sum_r |v_{ir}|^3$. The price-taking equilibrium generalizes component-wise, with separate vote-prices p_r per issue. Budget constraints across issues induce trade-offs, mirroring QV multi-issue analysis but with cubic penalties.

6.2 Budget-Limited Variant

To mitigate wealth effects, allocate each voter a fixed budget B of "voice credits." Votes cost $|v|^3$ credits, ensuring $\sum_r |v_{ir}|^3 \leq B$. This variant levels the playing field, focusing on preference intensity rather than wealth.

6.3 Collusion and Strategic Behavior

CV's steep cost curve increases the expense of vote-buying cartels. Splitting votes across identities (Sybil attacks) yields diminishing returns: n identities each casting v/n votes cost $n \cdot \alpha(v/n)^3 = \alpha v^3/n^2$, which vanishes as n grows. Thus, CV is robust to Sybil attacks without identity verification.

7 Criticisms and Limitations

Cubic Voting faces several criticisms:

- **Wealth-Based Influence:** The system can empower wealthier individuals, as those with more resources can purchase more votes, even with the cubic cost function.
- **Complexity:** The mathematical and conceptual complexity may reduce accessibility and public trust.
- **Strategic Voting:** Voters may attempt to game the system by allocating votes tactically.
- **Ethical Concerns:** The commodification of votes raises philosophical questions about democratic equality.
- **Implementation Challenges:** Robust mechanisms are needed to track and enforce vote purchases.

8 Alternatives to Cubic Voting

Several alternative voting systems address similar issues:

- **Quadratic Voting:** Uses a quadratic cost function (n^2), balancing intensity expression and equity.
- **Cumulative Voting:** Voters allocate multiple votes, but cost is linear.
- **Ranked-Choice Voting:** Voters rank options; votes are redistributed until a majority is achieved.
- **Approval Voting:** Voters approve as many options as they like; the most approved wins.
- **Proportional Representation:** Seats are allocated in proportion to votes received.

8.1 Vector Graphic: Comparison of Voting Systems

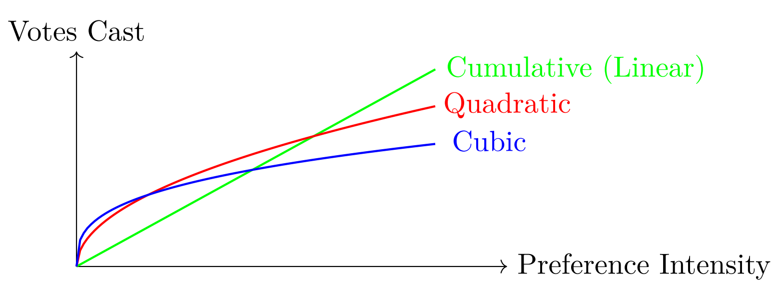

Figure 2: Relationship between preference intensity and votes cast under different systems (for a fixed budget).

9 Conclusion

Cubic Voting represents a significant theoretical advance in the design of voting systems, offering a mechanism for voters to express the intensity of their preferences while discouraging vote concentration. Its equilibrium properties align marginal costs with marginal benefits under a price-taking approximation, and welfare outcomes can dominate QV in high-variance environments. Applications span corporate charters, blockchain governance, and public referenda, with budget-limited variants enhancing equity. However, it faces practical, ethical, and philosophical challenges that must be addressed before widespread adoption. Ongoing research and experimentation will determine its future role in collective decision-making.

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