The Theory and Statistics of Land and Water on the Planets Earth and Mars

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Abstract

This paper presents a theoretical framework connecting planetary surface area to fundamental physical properties through gravitational mechanics. We derive a relationship between total surface area, planetary mass, and surface gravity, then apply this framework to analyze the land and water distribution on Earth and Mars using contemporary NASA observational data.

The paper ends with "The End"

1 Introduction

The surface characteristics of terrestrial planets represent a complex interplay between gravitational physics, planetary formation, and geological evolution. Understanding the relationship between a planet's fundamental properties and its surface area provides insight into planetary structure and comparative planetology [2].

2 Theoretical Framework

2.1 Surface Area of a Spherical Planet

For an idealized spherical planet with radius R, the total surface area A is given by the classical geometric formula:

$$A = 4\pi R^2 \tag{1}$$

This represents the fundamental relationship between a planet's linear dimension and its two-dimensional surface extent.

2.2 Gravitational Acceleration at the Surface

The acceleration due to gravity g at the surface of a spherical planet with mass M and radius R follows from Newton's law of universal gravitation [1]:

$$g = \frac{GM}{R^2} \tag{2}$$

where $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the gravitational constant [3].

2.3 Elimination of Radius: The Mass-Gravity-Area Relation

From equation (1), we can express the radius squared as:

$$R^2 = \frac{A}{4\pi} \tag{3}$$

Substituting this into equation (2) yields:

$$g = \frac{GM}{A/(4\pi)} = \frac{4\pi GM}{A} \tag{4}$$

Rearranging to solve for surface area:

$$A = \frac{4\pi GM}{g} \tag{5}$$

This remarkable relation demonstrates that a planet's total surface area is uniquely determined by its mass and surface gravity, independent of explicit knowledge of its radius.

2.4 Partitioning into Land and Water

For planets with distinct surface types, we can partition the total surface area into land area L and water area W:

$$A = L + W \tag{6}$$

Combining equations (5) and (6):

$$L + W = \frac{4\pi GM}{g} \tag{7}$$

The land fraction f_L and water fraction f_W are then:

$$f_L = \frac{L}{L+W}, \quad f_W = \frac{W}{L+W}, \quad f_L + f_W = 1 \tag{8}$$

3 Application to Earth

3.1 Earth's Physical Parameters

Using data from the NASA planetary fact sheet [4], Earth's fundamental properties are:

$$M_\oplus = 5.9722 \times 10^{24} \text{ kg} \tag{9}$$

$$g_\oplus = 9.820 \text{ m/s}^2 \tag{10}$$

3.2 Calculation of Earth's Surface Area

Applying equation (7):

$$\begin{aligned} L_\oplus + W_\oplus &= \frac{4\pi \times 6.67430 \times 10^{-11} \times 5.9722 \times 10^{24}}{9.820} && (11)\\ &= \frac{5.0066 \times 10^{15}}{9.820} && (12)\\ &= 5.10 \times 10^{14} \text{ m}^2 && (13)\\ &= 510 \text{ million km}^2 && (14) \end{aligned}$$

3.3 Earth's Land-Water Distribution

Observational data yields [4]:

$$W_\oplus \approx 361 \text{ million km}^2 \quad (70.8\%) \tag{15}$$

$$L_\oplus \approx 149 \text{ million km}^2 \quad (29.2\%) \tag{16}$$

This distribution reflects Earth's hydrosphere, with liquid water covering approximately 71 percent of the planetary surface.

4 Application to Mars

4.1 Mars's Physical Parameters

From NASA's Mars fact sheet [5]:

$$M_{\text{Mars}} = 6.4174 \times 10^{23} \text{ kg} \tag{17}$$

$$g_{\text{Mars}} = 3.71 \text{ m/s}^2 \tag{18}$$

Note that Mars has approximately 10.7 percent of Earth's mass and 37.8 percent of Earth's surface gravity.

4.2 Calculation of Mars's Surface Area

Applying equation (7):

$$L_{\text{Mars}} + W_{\text{Mars}} = \frac{4\pi \times 6.67430 \times 10^{-11} \times 6.4174 \times 10^{23}}{3.71} \tag{19}$$

$$= \frac{5.3829 \times 10^{14}}{3.71} \tag{20}$$

$$= 1.45 \times 10^{14} \text{ m}^2 \tag{21}$$

$$= 145 \text{ million km}^2 \tag{22}$$

4.3 Mars's Land-Water Distribution

Mars currently possesses no liquid water on its surface under ambient conditions [6]. Therefore:

$$W_{\text{Mars}} \approx 0 \text{ million km}^2 \quad (0.0\%) \tag{23}$$

$$L_{\text{Mars}} \approx 145 \text{ million km}^2 \quad (100.0\%) \tag{24}$$

While water ice exists at the polar caps and subsurface [7], the Martian surface is entirely terrestrial under current atmospheric conditions.

5 Comparative Analysis

5.1 Surface Area Ratio

The ratio of Mars's surface area to Earth's:

$$\frac{A_{\text{Mars}}}{A_{\oplus}} = \frac{145}{510} \approx 0.284 \tag{25}$$

Mars possesses approximately 28.4 percent of Earth's surface area, which is remarkably close to Earth's land area alone.

5.2 Visual Comparison

5.3 Tabular Summary

6 Physical Interpretation

The derived relationship in equation (7) reveals that planetary surface area scales linearly with mass but inversely with surface gravity. For planets of similar density, lower surface gravity (smaller radius relative to mass) results in larger surface area, while higher surface gravity (more compact structure) yields smaller surface area.

The stark contrast in water distribution between Earth and Mars reflects differences in atmospheric pressure, temperature, and geological history. Earth's thick atmosphere maintains liquid water stability, while Mars's thin atmosphere (approximately 0.6 percent of Earth's) causes water to sublimate directly from ice to vapor under most surface conditions [8].

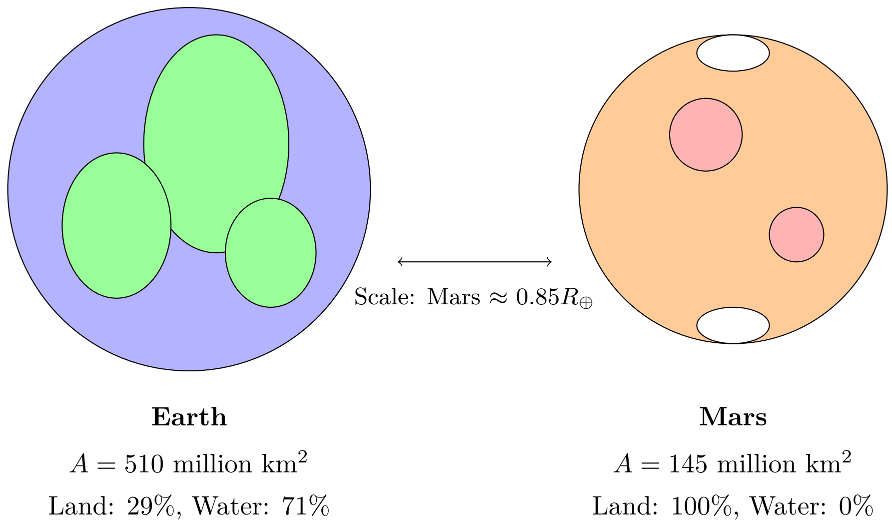

Figure 1: Comparative visualization of Earth and Mars showing relative sizes and surface composition. Earth exhibits significant water coverage (blue) with continental landmasses (green), while Mars is entirely terrestrial (orange/red) with polar ice caps (white).

Property	**Earth**	**Mars**
Mass (kg)	5.97×10^{24}	6.42×10^{23}
Surface gravity (m/s^2)	9.82	3.71
Total surface area (million km^2)	510	145
Land area (million km^2)	149	145
Water area (million km^2)	361	0
Land fraction (%)	29.2	100.0
Water fraction (%)	70.8	0.0

Table 1: Comparison of physical and surface properties for Earth and Mars.

7 Conclusion

We have established a theoretical framework connecting planetary surface area to mass and gravity, demonstrating that $A = 4\pi GM/g$. Application to Earth and Mars using NASA observational data confirms the validity of this relation and highlights the dramatic differences in surface composition between these neighboring terrestrial planets. Earth's surface is dominated by liquid water oceans, while Mars presents an entirely terrestrial landscape under current conditions. This analysis provides a foundation for comparative planetology and understanding the role of atmospheric and hydrological processes in shaping planetary surfaces.

References

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[3] CODATA (2018). Recommended Values of the Fundamental Physical Constants: 2018. *Reviews of Modern Physics*, 93(2), 025010.

[4] NASA (2024). Earth Fact Sheet. NASA Goddard Space Flight Center. `https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html`

[5] NASA (2024). Mars Fact Sheet. NASA Goddard Space Flight Center. `https://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html`

[6] Carr, M. H., & Head, J. W. (2010). Geologic history of Mars. *Earth and Planetary Science Letters*, 294(3-4), 185-203.

[7] Smith, P. H., et al. (2009). H_2O at the Phoenix Landing Site. *Science*, 325(5936), 58-61.

[8] Haberle, R. M., et al. (2017). *The Atmosphere and Climate of Mars.* Cambridge University Press.

Glossary

Surface Area (A) The total two-dimensional extent of a planet's outer boundary, measured in square meters or square kilometers.

Radius (R) The distance from a planet's center to its surface, assuming a spherical geometry.

Mass (M) The total quantity of matter contained within a planetary body, measured in kilograms.

Surface Gravity (g) The gravitational acceleration experienced at a planet's surface, measured in meters per second squared (m/s^2).

Gravitational Constant (G) A fundamental physical constant appearing in Newton's law of universal gravitation, with value $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

Land Area (L) The portion of a planet's surface area consisting of solid terrain above any liquid surfaces.

Water Area (W) The portion of a planet's surface area covered by liquid water.

Land Fraction (f_L) The ratio of land area to total surface area, expressed as $f_L = L/(L + W)$.

Water Fraction (f_W) The ratio of water area to total surface area, expressed as $f_W = W/(L + W)$.

Terrestrial Planet A planet composed primarily of silicate rocks and metals, with a solid surface. Earth and Mars are terrestrial planets.

Hydrosphere The total amount of water on a planet's surface, subsurface, and atmosphere.

Sublimation The phase transition of a substance directly from solid to gas without passing through the liquid phase, occurring for water on Mars due to low atmospheric pressure.

The End