An exact solution to the van der Pol oscillator when the parameter is a function of time

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Abstract

In this paper, I describe an exact solution to the van der Pol oscillator
when the parameter μ is a function of time t.
The paper ends with "The End"

Introduction

The van der Pol oscillator[1] is given by the second-order differential equation

$$\frac{\partial^2 x(t)}{\partial t^2} - \mu \left(1 - x(t)^2\right) \frac{\partial x(t)}{\partial t} + x(t) = 0$$

where μ is a parameter

According to [2], "there are no exact solutions of the van der Pol oscillator".

However, when μ is a function of time, the van der Pol oscillator **does** have an exact solution.

In this paper, I describe an exact solution to the van der Pol oscillator when μ is a function of time t.

An exact solution to the van der Pol oscillator when the parameter μ is a function of time t

When $t(t-2)\left(t^4 - e^{2t}\right) \neq 0$ and

$$\mu(t) = \frac{2e^{2t}(t-1)^2}{t(t-2)\left(t^4 - e^{2t}\right)}$$

is defined, then $x(t) = e^{-t}t^2$ is an exact solution to the van der Pol oscillator.

References

1. https://en.wikipedia.org/wiki/Van_der_Pol_oscillator

2. https://onlinelibrary.wiley.com/doi/10.1002/zamm.200310040

The End