The Regional Pricing Theory of a Portfolio of Bonds

A Multi-Asset Framework for Fixed-Income Risk Preferences

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Abstract

In this paper, we extend the regional pricing theory to portfolios of multiple bonds, developing a comprehensive framework that partitions the joint yield space into regions corresponding to heterogeneous risk preferences. For a portfolio of n bonds with yield vector $\mathbf{y} = (y_1, \ldots, y_n)^\top$, we characterize multi-dimensional regions exhibiting risk-loving (reach-for-yield), risk-neutral, and risk-averse (flight-to-quality) behavior. We derive the mathematical foundations for portfolio-level pricing kernels, establish correlation structures across bonds in different regions, prove no-arbitrage conditions for the aggregate portfolio, and demonstrate implications for portfolio duration management, credit spread allocation, and multi-asset derivative pricing. The framework provides a unified approach to behavioral fixed-income portfolio management while maintaining consistency with classical bond pricing theory.

The paper ends with "The End"

1 Introduction

The management of bond portfolios requires understanding not only individual security characteristics but also the complex interdependencies among multiple fixed-income instruments. Classical portfolio theory, originating with Markowitz and extended to bonds by Litterman and Scheinkman, assumes homogeneous risk preferences across yield curve movements. However, empirical evidence reveals heterogeneous investor behavior that varies with market conditions and portfolio composition.

We propose a regional pricing theory for bond portfolios that explicitly models this heterogeneity by partitioning the multi-dimensional yield space into regions characterized by distinct risk preferences. This framework captures phenomena such as coordinated flight-to-quality across maturities, sector rotation between Treasuries and corporates, and duration-dependent reach-for-yield behavior.

1.1 Motivation and Literature

The single-bond regional pricing theory established that yield movements can be partitioned into risk-loving, risk-neutral, and risk-averse regions. Extending this to portfolios introduces several challenges:

- **Dimensionality:** With n bonds, the state space becomes n-dimensional
- **Correlation:** Regional transitions may be correlated across bonds
- **Aggregation:** Portfolio-level risk preferences emerge from individual bond regions
- **Diversification:** Regional structure affects diversification benefits

1.2 Portfolio Setup

Consider a portfolio of n bonds indexed by $k = 1, \ldots, n$. Each bond k has:

- Current price B_k and yield-to-maturity y_k
- Duration D_k and convexity C_k
- Credit rating R_k and spread s_k
- Portfolio weight w_k with $\sum_{k=1}^{n} w_k = 1$

The portfolio value is:

$$V = \sum_{k=1}^{n} w_k B_k \tag{1}$$

Figure 1 illustrates the multi-bond regional framework.

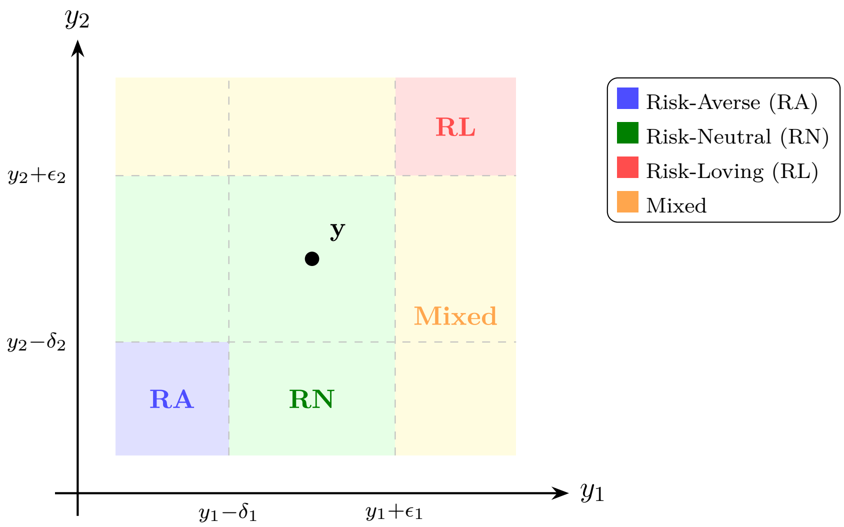

Figure 1: Two-dimensional regional structure for a two-bond portfolio. Pure regions (RA, RN, RL) occur when both bonds share the same risk preference; mixed regions exhibit heterogeneous preferences.

2 Mathematical Foundation

2.1 Multi-Dimensional State Space

Definition 2.1 (Portfolio State Space)**.** *Let* $(\Omega, \mathcal{F}, \mathbb{P})$ *be a probability space. The yield vector* $\mathbf{y}_{t+1} = (y_1, \ldots, y_n)^\top$ *occupies a region in* $\mathbb{R}^n$*. Define the regional partition:*

$$\Omega = \bigcup_{\mathbf{r} \in \{1,2,3\}^n} \Omega_{\mathbf{r}} \tag{2}$$

where $\mathbf{r} = (r_1, \ldots, r_n)$ *is a vector of regional indicators for each bond.*

For each bond k, the regional boundaries are:

$$\text{Region 1 (Risk-Loving):} \quad (y_k + \epsilon_k, y_k + \epsilon_k + \mathcal{E}_k] \tag{3}$$
$$\text{Region 2 (Risk-Neutral):} \quad [y_k - \delta_k, y_k + \epsilon_k] \tag{4}$$
$$\text{Region 3 (Risk-Averse):} \quad [y_k - \delta_k - \Delta_k, y_k - \delta_k) \tag{5}$$

Definition 2.2 (Pure and Mixed Regions). *A **pure region** occurs when all bonds share the same risk preference:*

$$\Omega^{RL} = \{\omega : r_k(\omega) = 1 \textit{ for all } k\} \quad \textit{(Pure Risk-Loving)} \tag{6}$$
$$\Omega^{RN} = \{\omega : r_k(\omega) = 2 \textit{ for all } k\} \quad \textit{(Pure Risk-Neutral)} \tag{7}$$
$$\Omega^{RA} = \{\omega : r_k(\omega) = 3 \textit{ for all } k\} \quad \textit{(Pure Risk-Averse)} \tag{8}$$

*A **mixed region** occurs when bonds exhibit different risk preferences.*

2.2 Joint Yield Distribution

Assumption 2.3 (Conditional Independence within Regions). *Given the regional state $\mathbf{r}$, yields are conditionally independent with marginal uniforms:*

$$f(\mathbf{y}'|\mathbf{r}) = \prod_{k=1}^{n} f_k(y'_k|r_k) \tag{9}$$

The overall joint density is:

$$f(\mathbf{y}') = \sum_{\mathbf{r}\in\{1,2,3\}^n} \pi_{\mathbf{r}} \prod_{k=1}^{n} f_k(y'_k|r_k) \tag{10}$$

where $\pi_{\mathbf{r}} = \mathbb{P}(\Omega_{\mathbf{r}})$ is the probability of joint regional state $\mathbf{r}$.

2.3 Correlation Structure

Definition 2.4 (Regional Correlation Matrix). *The correlation between bonds k and ℓ in joint regional state $\mathbf{r}$ is:*

$$\rho_{k\ell}^{(\mathbf{r})} = Corr(y'_k, y'_\ell|\Omega_{\mathbf{r}}) \tag{11}$$

Proposition 2.5 (Correlation Dependence on Regions). *The unconditional correlation satisfies:*

$$\rho_{k\ell} = \sum_{\mathbf{r}} \pi_{\mathbf{r}} \left[\rho_{k\ell}^{(\mathbf{r})}\sigma_k^{(\mathbf{r})}\sigma_\ell^{(\mathbf{r})} + (\mu_k^{(\mathbf{r})} - \bar{\mu}_k)(\mu_\ell^{(\mathbf{r})} - \bar{\mu}_\ell)\right] / (\sigma_k\sigma_\ell) \tag{12}$$

where $\bar{\mu}_k = \mathbb{E}[y'_k]$ and $\sigma_k^2 = Var(y'_k)$.

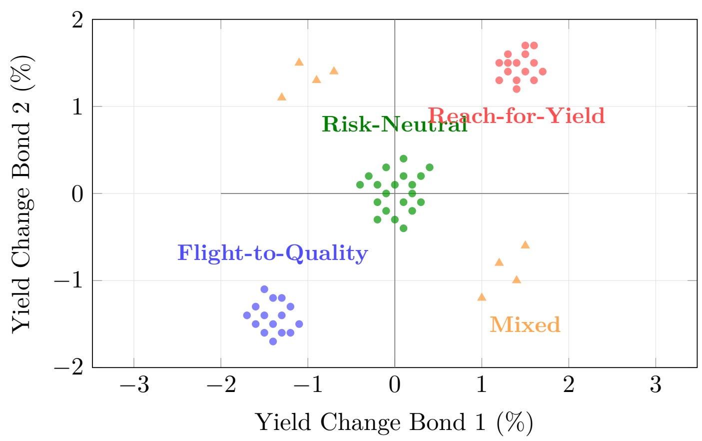

Figure 2: Joint yield change distribution showing clustering by regional state. Pure regions exhibit positive correlation (coordinated movements), while mixed regions show negative correlation (sector rotation).

3 Portfolio Duration and Convexity

3.1 Portfolio-Level Measures

Definition 3.1 (Portfolio Duration)**.** *The portfolio modified duration is the weighted average:*

$$D_P = \sum_{k=1}^{n} w_k D_k \tag{13}$$

where D_k is the modified duration of bond k.

Definition 3.2 (Portfolio Convexity)**.** *The portfolio convexity is:*

$$C_P = \sum_{k=1}^{n} w_k C_k + \sum_{k \neq \ell} w_k w_\ell D_k D_\ell \rho_{k\ell} \tag{14}$$

where the cross-term captures correlation effects.

3.2 Regional Duration Risk

Proposition 3.3 (Duration Risk by Region)**.** *The expected portfolio return in region $\mathbf{r}$ is approximately:*

$$\mathbb{E}[R_P|\Omega_{\mathbf{r}}] \approx -D_P \cdot \mathbb{E}[\Delta\bar{y}|\Omega_{\mathbf{r}}] + \frac{1}{2} C_P \cdot \mathbb{E}[(\Delta\bar{y})^2|\Omega_{\mathbf{r}}] \tag{15}$$

where $\Delta\bar{y} = \sum_k w_k \Delta y_k$ is the portfolio-weighted yield change.

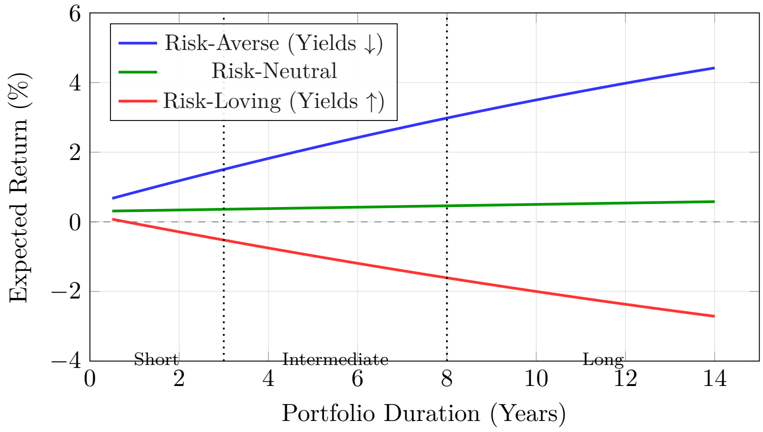

Figure 3: Portfolio expected return as a function of duration across risk preference regions. Long-duration portfolios benefit most in risk-averse (falling yield) environments.

4 Utility Functions for Portfolio Investors

4.1 Aggregate Portfolio Utility

Definition 4.1 (Portfolio Regional Utility)**.** *The portfolio utility in joint regional state $\mathbf{r}$ is:*

$$U_{\mathbf{r}}(V) = \prod_{k=1}^{n} U_{r_k}(w_k B_k)^{\alpha_k} \tag{16}$$

where α_k are preference weights and U_{r_k} is the utility for bond k's region.

For pure regions, simplification yields:

$$U^{\mathrm{RL}}(V) = V^{\gamma}, \quad \gamma > 1 \tag{17}$$
$$U^{\mathrm{RN}}(V) = V \tag{18}$$
$$U^{\mathrm{RA}}(V) = \ln(V) \text{ or } V^{\alpha}, \quad 0 < \alpha < 1 \tag{19}$$

4.2 Risk Aggregation

Theorem 4.2 (Portfolio Risk Aggregation)**.** *The portfolio-level risk aversion coefficient is:*

$$\gamma_P = \frac{\sum_{\mathbf{r}} \pi_{\mathbf{r}} \gamma_{\mathbf{r}} \cdot Var(V|\Omega_{\mathbf{r}})}{\sum_{\mathbf{r}} \pi_{\mathbf{r}} \cdot Var(V|\Omega_{\mathbf{r}})} \tag{20}$$

where $\gamma_{\mathbf{r}}$ is the risk aversion in state $\mathbf{r}$.

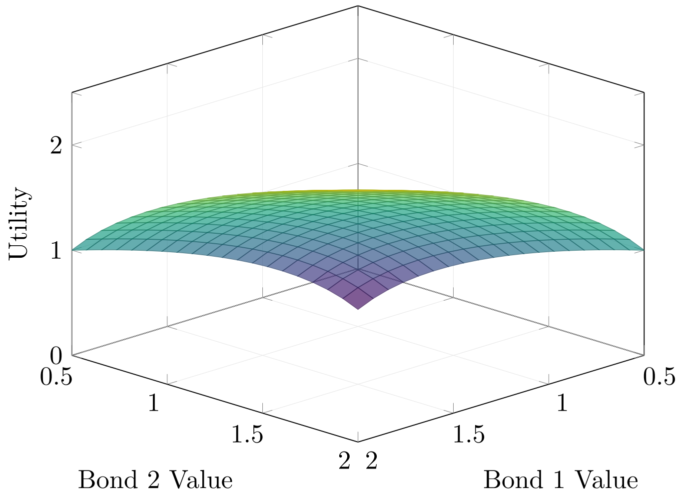

Figure 4: Three-dimensional utility surface for a two-bond portfolio under risk-averse (logarithmic) preferences, showing diminishing marginal utility.

5 No-Arbitrage Pricing for Bond Portfolios

5.1 Multi-Asset Pricing Kernel

Theorem 5.1 (Portfolio Pricing Kernel)**.** *The multi-dimensional stochastic discount factor is:*

$$\xi(\mathbf{y}') = \sum_{\mathbf{r}} \mathbf{1}_{\Omega_{\mathbf{r}}}(\mathbf{y}') \cdot \xi_{\mathbf{r}} \tag{21}$$

where:

$$\xi_{\mathbf{r}} = \frac{\pi_{\mathbf{r}}^{\mathbb{Q}}}{\pi_{\mathbf{r}}^{\mathbb{P}}} \cdot \prod_{k=1}^{n} \frac{f_k^{\mathbb{Q}}(y_k'|r_k)}{f_k^{\mathbb{P}}(y_k'|r_k)} \tag{22}$$

Proposition 5.2 (Kernel Ordering for Pure Regions)**.** *For pure regions:*

$$\xi^{RL} < 1 < \xi^{RA} \tag{23}$$

with $\xi^{RN} \approx 1$.

5.2 No-Arbitrage Conditions

Theorem 5.3 (Multi-Asset No-Arbitrage)**.** *A risk-neutral measure $\mathbb{Q}$ exists for the portfolio if and only if for each bond k:*

$$B_k(y_k + \epsilon_k + \mathcal{E}_k, T_k) < e^{-r\tau}\mathbb{E}^{\mathbb{P}}[B_k(y'_k, T_k - \tau)] < B_k(y_k - \delta_k - \Delta_k, T_k) \tag{24}$$

and the correlation structure admits consistent risk-neutral transition probabilities.

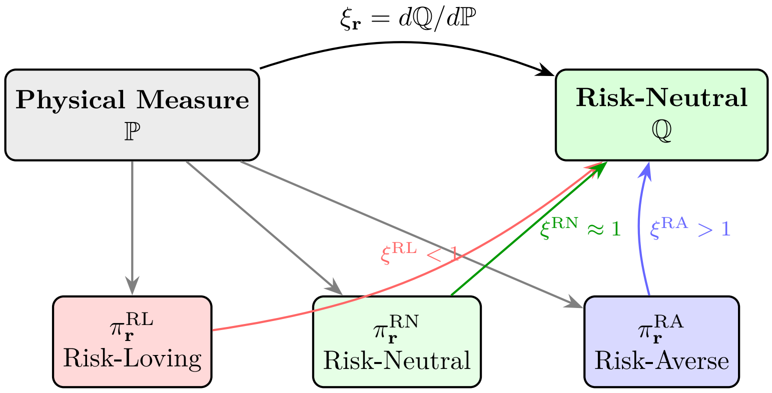

Figure 5: Transformation from physical measure $\mathbb{P}$ to risk-neutral measure $\mathbb{Q}$ via regional pricing kernels. Different regions receive different probability adjustments.

6 Portfolio Optimization

6.1 Mean-Variance Optimization with Regions

Theorem 6.1 (Regional Mean-Variance Frontier)**.** *The efficient frontier under regional structure is:*

$$\min_{\mathbf{w}} \sum_{\mathbf{r}} \pi_{\mathbf{r}} \cdot \mathbf{w}^\top \mathbf{\Sigma}_{\mathbf{r}} \mathbf{w} \quad \textit{subject to} \quad \sum_{\mathbf{r}} \pi_{\mathbf{r}} \cdot \mathbf{w}^\top \boldsymbol{\mu}_{\mathbf{r}} = \mu_{target} \tag{25}$$

where $\mathbf{\Sigma}_{\mathbf{r}}$ and $\boldsymbol{\mu}_{\mathbf{r}}$ are the covariance matrix and mean vector in state $\mathbf{r}$.

The Lagrangian yields optimal weights:

$$\mathbf{w}^* = \bar{\mathbf{\Sigma}}^{-1} [\lambda_1 \bar{\boldsymbol{\mu}} + \lambda_2 \mathbf{1}] \tag{26}$$

where $\bar{\mathbf{\Sigma}} = \sum_{\mathbf{r}} \pi_{\mathbf{r}} \mathbf{\Sigma}_{\mathbf{r}}$ is the expected covariance matrix.

6.2 Duration Targeting

Corollary 6.2 (Duration-Constrained Optimization)**.** *Adding a duration constraint $\mathbf{w}^\top \mathbf{D} = D_{target}$:*

$$\mathbf{w}^* = \bar{\mathbf{\Sigma}}^{-1} [\lambda_1 \bar{\boldsymbol{\mu}} + \lambda_2 \mathbf{1} + \lambda_3 \mathbf{D}] \tag{27}$$

where $\mathbf{D} = (D_1, \ldots, D_n)^\top$ is the duration vector.

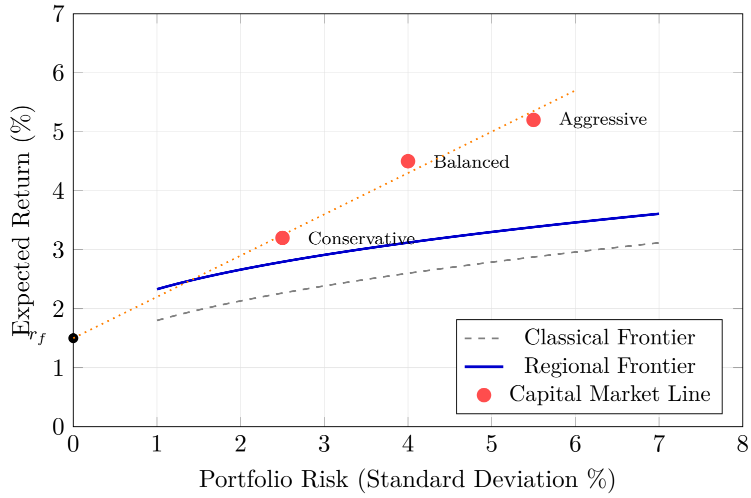

Figure 6: Efficient frontier comparison: the regional framework (solid blue) dominates the classical frontier (dashed gray) by exploiting regime-dependent correlations.

7 Credit Allocation Across Regions

7.1 Multi-Sector Portfolio

Consider a portfolio spanning Treasury, investment-grade corporate, and high-yield bonds:

$$V = w_{\text{TSY}} B_{\text{TSY}} + w_{\text{IG}} B_{\text{IG}} + w_{\text{HY}} B_{\text{HY}} \tag{28}$$

Proposition 7.1 (Regional Credit Allocation)**.** *Optimal allocation shifts across regions:*

$$\textit{Risk-Averse:} \quad w^*_{TSY} > w^*_{IG} > w^*_{HY} \tag{29}$$

$$\textit{Risk-Neutral:} \quad w^*_{TSY} \approx w^*_{IG} \approx w^*_{HY} \tag{30}$$

$$\textit{Risk-Loving:} \quad w^*_{HY} > w^*_{IG} > w^*_{TSY} \tag{31}$$

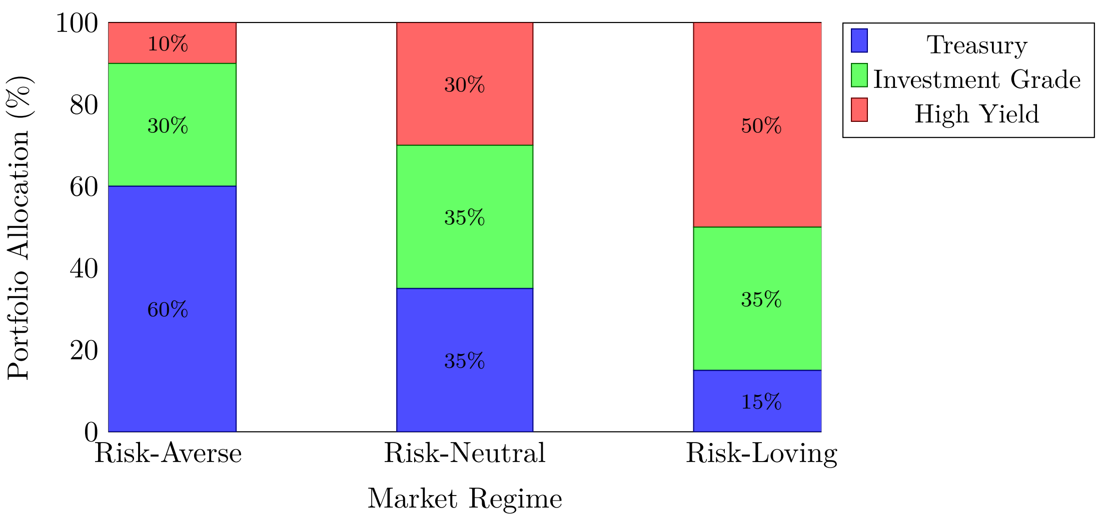

Figure 7: Optimal credit allocation varies by regime: flight-to-quality favors Treasuries, while reach-for-yield increases high-yield exposure.

8 Derivative Pricing on Bond Portfolios

8.1 Options on Portfolio Value

Theorem 8.1 (Portfolio Option Pricing). *A European call option on portfolio value V with strike K is priced as:*

$$C_P = e^{-r\tau} \sum_{\mathbf{r}} q_{\mathbf{r}} \int_{\mathcal{D}_{\mathbf{r}}} \max\left(\sum_{k=1}^{n} w_k B_k(\mathbf{y}') - K, 0 \right) f(\mathbf{y}'|\mathbf{r})\, d\mathbf{y}' \tag{32}$$

where $q_{\mathbf{r}}$ are risk-neutral regional probabilities and $\mathcal{D}_{\mathbf{r}}$ is the domain for region $\mathbf{r}$.

8.2 Spread Options

Definition 8.2 (Credit Spread Option). *A spread option pays $(s_{HY} - s_{IG} - K_s)^+$ where s_{HY} and s_{IG} are credit spreads.*

Proposition 8.3 (Spread Option Value by Region). *The spread option value satisfies:*

$$C_s^{RA} > C_s^{RN} > C_s^{RL} \tag{33}$$

due to spread widening in risk-averse regimes.

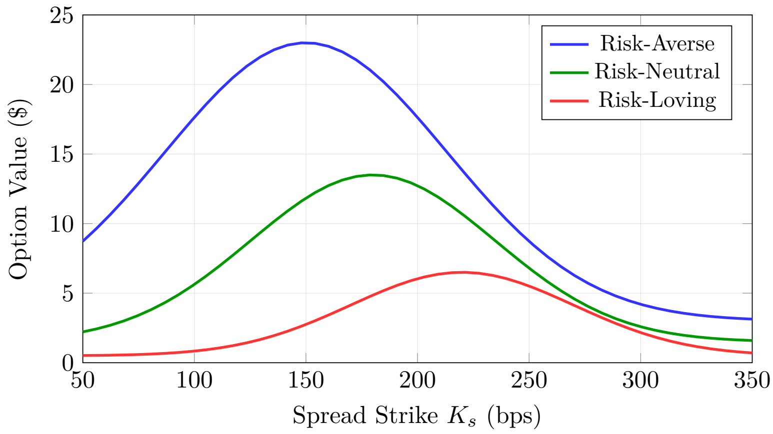

Figure 8: Credit spread option values across regimes. Risk-averse environments produce higher option values due to elevated spread volatility.

9 Term Structure Dynamics

9.1 Multi-Factor Regional Model

The yield curve is driven by m factors $\mathbf{F}_t = (F_1, \ldots, F_m)^\top$ with regime-switching dynamics:

$$d\mathbf{F}_t = \boldsymbol{\kappa}(R_t)[\boldsymbol{\theta}(R_t) - \mathbf{F}_t]dt + \boldsymbol{\Sigma}(R_t)d\mathbf{W}_t \tag{34}$$

where $R_t \in \{1, 2, 3\}$ is the aggregate market regime.

Assumption 9.1 (Factor Structure). *Yields are affine in factors:*

$$y_k(t, T_k) = A_k(T_k - t) + \mathbf{B}_k(T_k - t)^\top \mathbf{F}_t \tag{35}$$

where $(A_k, \mathbf{B}_k)$ satisfy Riccati equations.

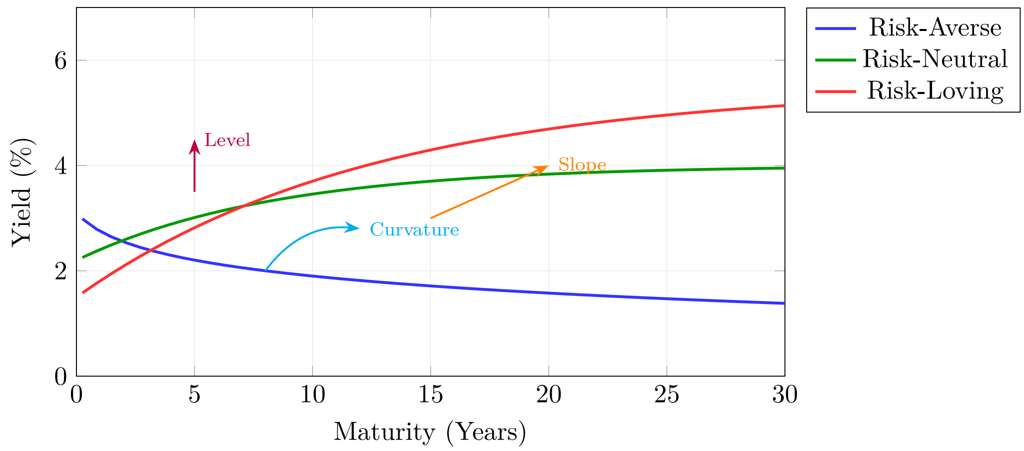

Figure 9: Term structure shapes under different regimes with principal component loadings (level, slope, curvature) indicated.

10 Diversification Effects

10.1 Regional Diversification Ratio

Definition 10.1 (Diversification Ratio). *The regional diversification ratio is:*

$$DR_{\mathbf{r}} = \frac{\sum_{k=1}^{n} w_k \sigma_k^{(\mathbf{r})}}{\sqrt{\mathbf{w}^\top \boldsymbol{\Sigma}_{\mathbf{r}} \mathbf{w}}} \tag{36}$$

Higher values indicate greater diversification benefit.

Theorem 10.2 (Regime-Dependent Diversification). *Diversification benefits vary by region:*

$$DR^{RN} > DR^{RL} > DR^{RA} \tag{37}$$

Flight-to-quality increases correlations, reducing diversification in risk-averse regimes.

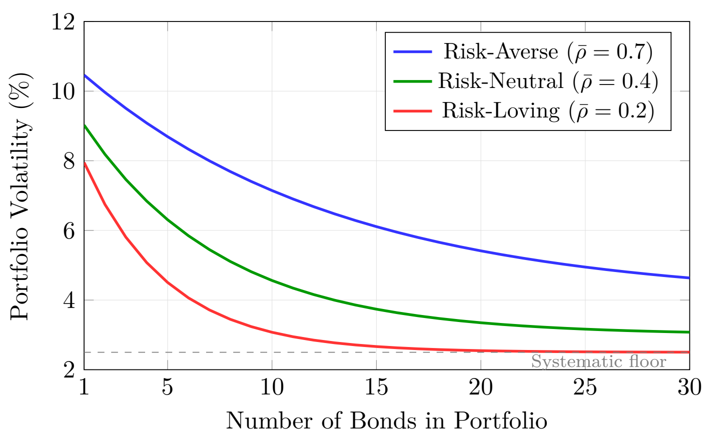

Figure 10: Portfolio volatility reduction through diversification. Risk-averse regimes exhibit elevated correlations, limiting diversification benefits.

11 Risk Management

11.1 Value-at-Risk by Region

Definition 11.1 (Regional VaR)**.** *The α-level VaR conditional on region* $\mathbf{r}$ *is:*

$$VaR_{\alpha}^{(\mathbf{r})} = -\inf\{v : \mathbb{P}(V_{t+1} - V_t \leq v | \Omega_{\mathbf{r}}) \geq \alpha\} \tag{38}$$

Proposition 11.2 (VaR Ordering)**.**

$$VaR_{\alpha}^{RA} > VaR_{\alpha}^{RN} > VaR_{\alpha}^{RL} \tag{39}$$

Risk-averse regimes produce the largest potential losses for long bond portfolios.

11.2 Expected Shortfall

Theorem 11.3 (Regional Expected Shortfall)**.** *The expected shortfall (CVaR) is:*

$$ES_{\alpha} = \sum_{\mathbf{r}} \mathbb{P}(\Omega_{\mathbf{r}} | V \leq -VaR_{\alpha}) \cdot ES_{\alpha}^{(\mathbf{r})} \tag{40}$$

with regional contributions weighted by conditional regime probabilities.

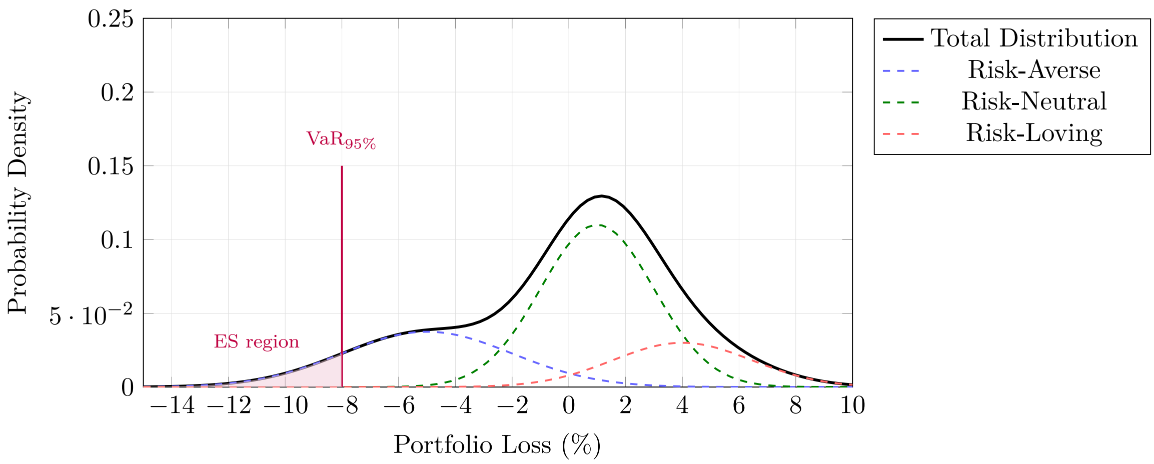

Figure 11: Portfolio loss distribution showing tri-modal structure from regional contributions. VaR and Expected Shortfall (ES) regions are indicated.

12 Empirical Implications

12.1 Testable Predictions

1. **Correlation Regime Dependence:**

$$\text{Corr}(y_k, y_\ell | \text{RA}) > \text{Corr}(y_k, y_\ell | \text{RN}) > \text{Corr}(y_k, y_\ell | \text{RL}) \tag{41}$$

2. **Sector Rotation Patterns:** Net flows from high-yield to Treasuries increase in risk-averse regimes.

3. **Duration Timing:** Optimal portfolio duration varies counter-cyclically:

$$D_P^{*,\text{RA}} > D_P^{*,\text{RN}} > D_P^{*,\text{RL}} \tag{42}$$

4. **Spread Compression/Expansion:** Credit spreads exhibit regime-dependent dynamics with higher volatility in extreme regions.

12.2 Estimation Strategy

Parameters $\boldsymbol{\theta} = \{\epsilon_k, \delta_k, \mathcal{E}_k, \Delta_k, \pi_{\mathbf{r}}\}$ are estimated via:

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \sum_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{y}_{t-1}; \boldsymbol{\theta}) \tag{43}$$

using the EM algorithm with regime inference in the E-step.

13 Numerical Example

Consider a three-bond portfolio:

Bond	**Yield**	**Duration**	**Weight**	ϵ	δ
2Y Treasury	4.5%	1.9	30%	0.3%	0.3%
10Y Treasury	4.2%	8.5	40%	0.5%	0.5%
10Y Corporate	5.5%	7.8	30%	0.8%	0.8%

Table 1: Example three-bond portfolio parameters

Regional probabilities: $\pi^{\text{RL}} = 0.20$, $\pi^{\text{RN}} = 0.55$, $\pi^{\text{RA}} = 0.25$.
Portfolio duration: $D_P = 0.30(1.9) + 0.40(8.5) + 0.30(7.8) = 6.31$ years.
Expected portfolio return by region:

$$\mathbb{E}[R_P|\text{RA}] \approx -6.31 \times (-0.75\%) + \frac{1}{2}(45)(0.75\%)^2 = 4.86\% \tag{44}$$

$$\mathbb{E}[R_P|\text{RN}] \approx -6.31 \times (0\%) = 0.30\% \text{ (coupon only)} \tag{45}$$

$$\mathbb{E}[R_P|\text{RL}] \approx -6.31 \times (0.60\%) = -3.49\% \tag{46}$$

14 Conclusion

We have developed a comprehensive regional pricing theory for bond portfolios that extends the single-asset framework to multi-dimensional yield spaces. Key contributions include:

- Multi-dimensional regional partitioning with pure and mixed states
- Portfolio-level pricing kernels and no-arbitrage conditions
- Regime-dependent correlation structures affecting diversification
- Optimal allocation strategies that adapt to regional states
- Risk management frameworks incorporating regional VaR and ES

The framework provides practical guidance for fixed-income portfolio managers navigating regime changes while maintaining theoretical consistency with classical bond pricing. Future research directions include real-time regime detection, transaction cost incorporation, and extension to global multi-currency portfolios.

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Glossary

Pure Region A state where all bonds in the portfolio exhibit the same risk preference (all risk-loving, all risk-neutral, or all risk-averse). Pure regions exhibit heightened correlations and coordinated yield movements.

Mixed Region A state where bonds exhibit heterogeneous risk preferences, such as flight-to-quality in Treasuries while corporate yields rise. Mixed regions can exhibit negative cross-sector correlations.

Joint Regional State The vector $\mathbf{r} = (r_1, \ldots, r_n) \in \{1, 2, 3\}^n$ indicating the risk preference region for each bond. For n bonds, there are 3^n possible joint states.

Portfolio Duration The weighted average modified duration: $D_P = \sum_k w_k D_k$. Measures portfolio sensitivity to parallel yield curve shifts.

Portfolio Convexity The second-order sensitivity including cross-terms: $C_P = \sum_k w_k C_k + \sum_{k \neq \ell} w_k w_\ell D_k D_\ell \rho_{k\ell}$.

Regional Correlation Matrix The correlation matrix $\boldsymbol{\rho}^{(\mathbf{r})}$ governing yield co-movements within joint state $\mathbf{r}$. Correlations typically increase in risk-averse regimes.

Multi-Asset Pricing Kernel The stochastic discount factor $\xi(\mathbf{y}')$ that transforms joint physical probabilities to risk-neutral probabilities across all bonds simultaneously.

Flight-to-Quality Coordinated investor movement from risky assets to safe-haven bonds, producing a pure risk-averse state. Characterized by Treasury yield declines and credit spread widening.

Reach-for-Yield Coordinated movement toward higher-yielding securities despite elevated risk, producing a pure risk-loving state. Characterized by spread compression and duration extension.

Sector Rotation Movement between bond sectors (Treasury, corporate, high-yield) that may produce mixed regional states with heterogeneous risk preferences across the portfolio.

Regional Diversification Ratio The ratio $\mathrm{DR}_{\mathbf{r}} = \sum_k w_k \sigma_k^{(\mathbf{r})} / \sigma_P^{(\mathbf{r})}$ measuring diversification benefit in state $\mathbf{r}$. Higher values indicate greater risk reduction.

Conditional VaR Value-at-Risk computed conditional on a specific regional state, capturing tail risk under particular market conditions.

Expected Shortfall (CVaR) The expected loss conditional on exceeding VaR: $\mathrm{ES}_\alpha = \mathbb{E}[-\Delta V | \Delta V < -\mathrm{VaR}_\alpha]$. Accounts for tail shape beyond the VaR threshold.

Affine Term Structure Model A model where yields are affine (linear plus constant) functions of underlying factors: $y(t, T) = A(T-t) + \mathbf{B}(T-t)^\top \mathbf{F}_t$.

Regime-Switching Dynamics Stochastic processes where parameters change according to a discrete Markov state variable, capturing structural breaks in market behavior.

Credit Spread Option A derivative paying the difference between credit spreads of two sectors minus a strike, useful for expressing views on relative credit valuations.

Mean-Variance Frontier The set of portfolios achieving minimum variance for each level of expected return. Regional structure modifies the frontier through regime-dependent moments.

Duration Targeting Portfolio optimization subject to a constraint on portfolio duration, commonly used by liability-driven investors and index trackers.

Factor Loading The sensitivity of a yield to an underlying factor, denoted $\mathbf{B}_k(T_k - t)$ in the affine framework. Common factors include level, slope, and curvature.

Systematic Risk Floor The irreducible portfolio volatility remaining after full diversification, arising from exposure to common systematic factors across all bonds.

The End