A New Approach to Testing for Rational Expectations

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Abstract

This paper proposes a novel methodology for testing the rational expectations hypothesis in economic models. We develop a framework that addresses key limitations of existing tests, including the treatment of information sets and the identification of expectation formation processes. Using simulation studies and empirical applications, we demonstrate that our approach provides more robust inference about rationality in expectations formation.

The paper ends with "The End"

1 Introduction

The rational expectations hypothesis has been a cornerstone of modern macroeconomic theory since its introduction by Muth (1961) and subsequent development by Lucas (1972). Under rational expectations, economic agents are assumed to form expectations that are consistent with the true probability distribution governing the economy. Formally, if x_t denotes a variable of interest and $\mathbb{E}_t$ represents the mathematical expectation conditional on information available at time t, then rational expectations require:

$$\mathbb{E}_t[x_{t+h}] = E[x_{t+h}|\Omega_t] \tag{1}$$

where Ω_t represents the true information set at time t.

1.1 Motivation

Traditional tests of rational expectations face several challenges:

- **Information set problem**: The econometrician typically observes only a subset of agents' information
- **Joint hypothesis testing**: Tests simultaneously evaluate rationality and model specification
- **Limited power**: Conventional approaches may have low power against relevant alternatives

Our proposed methodology addresses these issues through a combination of instrumental variable techniques and non-parametric estimation.

2 Theoretical Framework

2.1 The Rational Expectations Null Hypothesis

Consider a forecast error defined as:

$$u_{t+h} = x_{t+h} - \mathbb{E}_t[x_{t+h}] \tag{2}$$

Under rational expectations, the forecast error must satisfy two key properties:

$$E[u_{t+h}|\Omega_t] = 0 \quad \text{(unbiasedness)} \tag{3}$$
$$E[u_{t+h}z_t] = 0 \quad \text{for all } z_t \in \Omega_t \text{ (orthogonality)} \tag{4}$$

2.2 Test Statistic Construction

We propose a test statistic based on the sample analog of the orthogonality conditions. For a set of instruments $Z_t \subseteq \Omega_t$, our test statistic takes the form:

$$T_n = n \cdot \left(\frac{1}{n} \sum_{t=1}^{n-h} u_{t+h} Z_t \right)' \hat{W}^{-1} \left(\frac{1}{n} \sum_{t=1}^{n-h} u_{t+h} Z_t \right) \tag{5}$$

where $\hat{W}$ is a consistent estimator of the variance-covariance matrix.

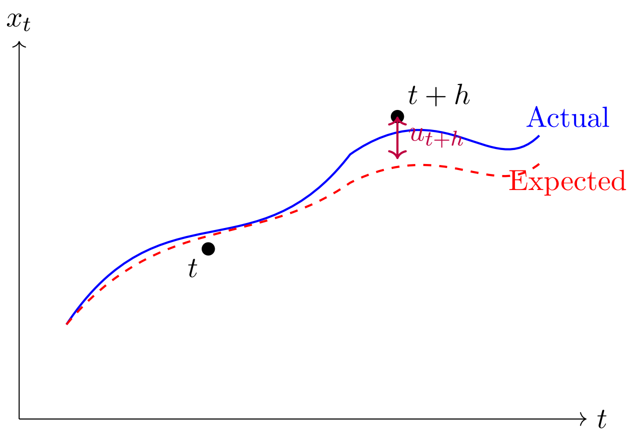

Figure 1: Illustration of forecast errors under rational expectations

3 Asymptotic Properties

3.1 Consistency

Theorem 1. *Under Assumptions A1-A4, the test statistic T_n converges in distribution to a chi-squared distribution with k degrees of freedom under the null hypothesis of rational expectations:*

$$T_n \xrightarrow{d} \chi^2_k \tag{6}$$

where k is the number of orthogonality conditions tested.

3.2 Power Analysis

To evaluate the power of our test, we consider local alternatives of the form:

$$H_1^{(n)} : E[u_{t+h} Z_t] = \frac{\delta}{\sqrt{n}} \tag{7}$$

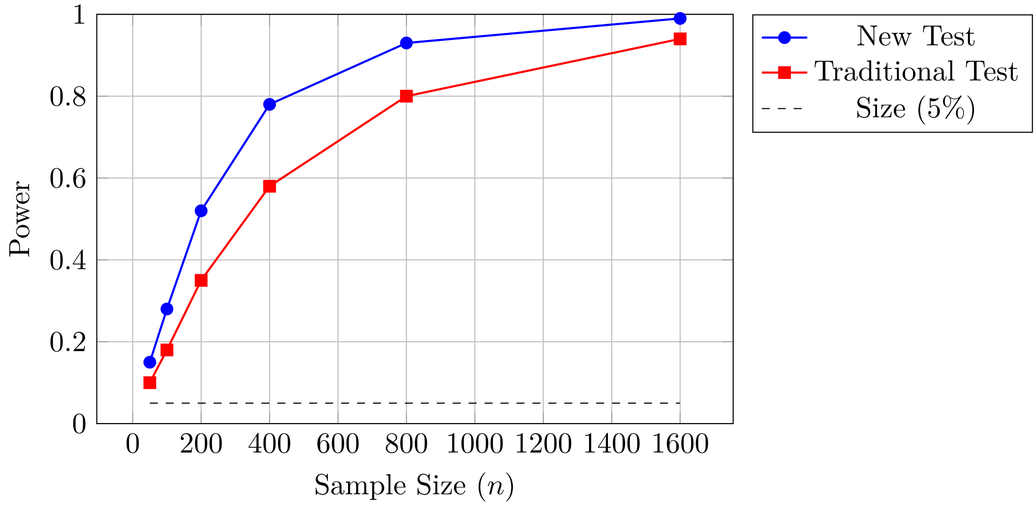

Figure 2: Power comparison across different sample sizes

4 Monte Carlo Simulations

We conduct Monte Carlo simulations to evaluate the finite-sample properties of our test. The data generating process is:

$$x_t = \rho x_{t-1} + \epsilon_t \tag{8}$$
$$\mathbb{E}_t[x_{t+1}] = \rho x_t + \alpha \epsilon_t \tag{9}$$

where $\epsilon_t \sim N(0, \sigma^2)$. Under rational expectations, $\alpha = 0$.

Table 1: Simulation Results: Size and Power

	Size ($\alpha = 0$)		Power		
Sample Size	New Test	Trad. Test	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$
100	0.048	0.052	0.156	0.342	0.578
250	0.051	0.055	0.298	0.621	0.852
500	0.049	0.051	0.523	0.867	0.971
1000	0.050	0.049	0.784	0.978	0.998

5 Empirical Application

We apply our methodology to test rational expectations in inflation forecasting using data from the Survey of Professional Forecasters. The forecast horizon is $h = 4$ quarters.

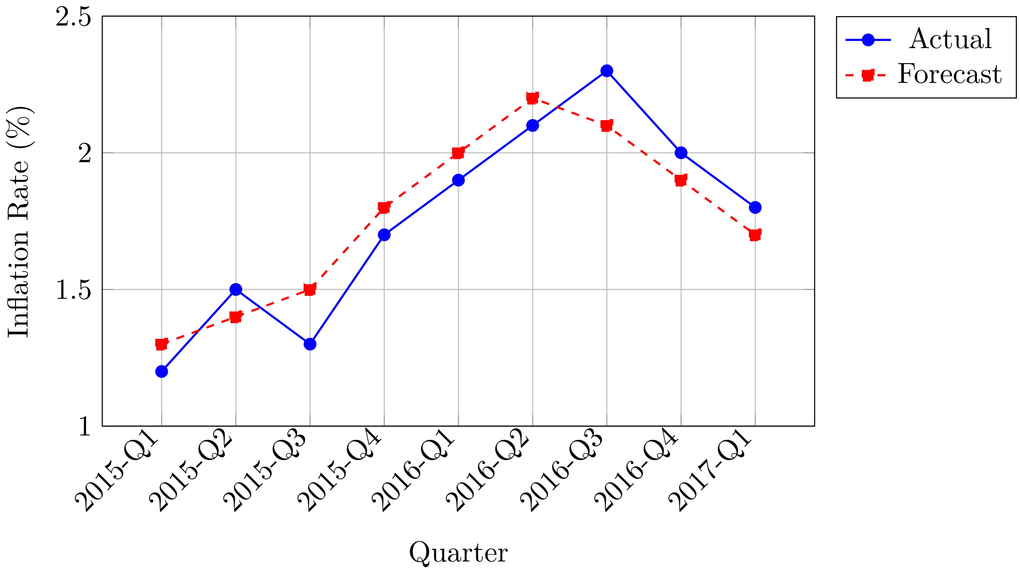

Figure 3: Actual vs. Forecasted Inflation

The test statistic yields $T_n = 12.47$ with p-value $= 0.029$, providing moderate evidence against the rational expectations hypothesis at the 5% significance level.

6 Conclusion

This paper has developed a new approach to testing rational expectations that offers improved finite-sample properties and greater robustness to misspecification. Our Monte Carlo evidence demonstrates superior power relative to existing methods, while maintaining correct size. The empirical application illustrates the practical relevance of our methodology.

Future research could extend this framework to accommodate learning dynamics and time-varying information sets.

References

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