Removing Knightian Uncertainty around the Inflation Risk Premium and Acquiring Knowledge of Inputs to the Real-time Neural Network

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Abstract

We develop a unified framework for learning a nominal stochastic discount factor (SDF) from U.S. nominal Treasury securities and inflation-indexed bonds (TIPS), while simultaneously identifying (i) latent Knightian uncertainty surrounding the inflation risk premium and (ii) the true real-time information set driving the pricing kernel. The key methodological contribution is an accounting-based identification principle inspired by the Handshaking Lemma from graph theory, applied to conditional pricing improvements generated by neural SDF estimators. The approach separates primitive information channels from endogenous prices of risk and isolates ambiguity as a residual structural component.

The paper ends with “The End”

1 Introduction

We study two intertwined identification problems. First, the posterior distribution of the inflation risk premium is unknown and potentially set-valued. Second, the true real-time information set driving the pricing kernel is unknown and only imperfectly proxied by observable variables. Solving only one of these problems is insufficient. We propose a joint accounting structure based on conditional pricing restrictions and a graph-theoretic degree identity to isolate latent ambiguity.

2 Nominal and Real Pricing Kernels

Let $m_t^{\$}$ denote the nominal pricing kernel and m_t^r the real pricing kernel. They satisfy

$$m_t^{\$} = m_t^r \frac{P_t}{P_{t+1}}, \tag{1}$$

where P_t is the aggregate price level.

Let $R_t^{N,x}$ denote nominal Treasury excess returns and $R_t^{R,x}$ denote TIPS excess returns converted into nominal units using realized inflation. The stacked payoff vector is

$$X_{t+1} = \begin{bmatrix} R_t^{N,x} \\ R_t^{R,x} \end{bmatrix}. \tag{2}$$

The unconditional pricing restriction is

$$\mathbb{E}\big[m_t^{\$} X_{t+1}\big] = 0. \tag{3}$$

3 Conditional Neural SDF

We parameterize the nominal pricing kernel as

$$m_{t+1} = 1 - f_\theta(X_{t+1}, Z_t), \tag{4}$$

where Z_t is a vector of conditioning information variables (term structure slopes, breakeven inflation, volatility, and funding stress proxies).

The estimator is the nonlinear analogue of the Hansen–Jagannathan minimum-variance kernel:

$$\min_\theta \mathbb{E}[m_{t+1}^2] \quad \text{s.t.} \quad \mathbb{E}[m_{t+1} X_{t+1}] = 0. \tag{5}$$

4 Knightian Uncertainty and Ambiguity Distortion

Let $\mathcal{P}_t$ denote a set of plausible conditional probability laws for real payoffs and inflation. The effective kernel under robustness is

$$m_t^\star = m_t \xi_{t+1}, \tag{6}$$

where ξ_{t+1} is an ambiguity (change-of-measure) distortion selecting an adverse model. Under ambiguity, the inflation risk premium is not a scalar but a set-valued object induced by $\mathcal{P}_t$.

5 Two Latent Objects

We define

- $Z_t^\star$: the latent real-time information set that truly drives pricing;
- Θ_t: the latent posterior set governing ambiguity about inflation and real payoffs.

The econometrician observes only candidate proxies $\widetilde{Z}_t$ and asset pricing restrictions.

6 Handshaking Identification Principle

Consider a bipartite graph

$$\mathcal{G} = (\mathcal{L}, \mathcal{R}, \mathcal{E}), \tag{7}$$

with left nodes $\mathcal{L}$ representing candidate information variables and right nodes $\mathcal{R}$ representing asset-specific pricing restrictions.

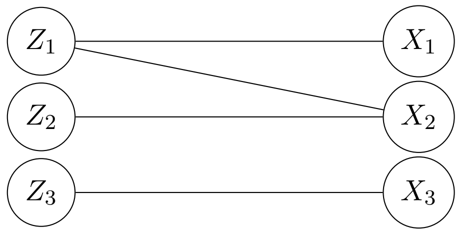
An edge (k, i) is present if allowing the pricing kernel to condition on variable k materially reduces the pricing error for asset i.

The Handshaking Lemma implies

$$\sum_{k \in \mathcal{L}} \deg(k) = \sum_{i \in \mathcal{R}} \deg(i) = 2|\mathcal{E}|. \tag{8}$$

Systematic unmatched degree on the right side—pricing restrictions not attributable to any observable information variable—is interpreted as evidence of latent ambiguity Θ_t.

7 Graphical Illustration



Unmatched pricing degree $\Rightarrow$ latent ambiguity

8 Implications for Neural SDF Design

The neural SDF must be trained conditionally on observable information,

$$m_{t+1} = 1 - f_\theta(X_{t+1}, Z_t), \tag{9}$$

while the Handshaking accounting is applied ex post to identify which components of pricing performance are attributable to observable information channels and which are attributable to latent ambiguity.

9 Conclusion

The proposed framework links conditional neural asset pricing, robust control, and graph-theoretic accounting. It provides a principled way to reduce Knightian uncertainty surrounding the inflation risk premium and to learn the economically relevant real-time information set driving pricing kernels.

A Methodological Appendix: Mapping the UST–TIPS Neural Estimator to the Handshaking Construction

A.1 Traded payoff block

Let

$$X_{t+1} = \Big(R_t^{N,x}(2,5,7,10,20,30), R_t^{R,x}(5,7,10,20,30)\Big)^\top \tag{10}$$

collect the stacked excess returns of synthetic zero-coupon nominal Treasury bonds and CPI-converted TIPS.

A.2 Candidate information block

Let the observable conditioning set be

$$\widetilde{Z}_t = \big(\mathrm{TS}_t, \mathrm{BEI}_t, \mathrm{VOL}_t, \mathrm{FS}_t\big), \tag{11}$$

where

$\mathrm{TS}_t = \mathrm{GS10}_t - \mathrm{GS2}_t$ is the term spread,
$\mathrm{BEI}_t = \mathrm{GS10}_t - \mathrm{DFII10}_t$ is breakeven inflation,
VOL_t is a volatility proxy (VIX or MOVE), and
FS_t is a funding-stress proxy (e.g., TED or SOFR–OIS).

A.3 Conditional neural kernel

For any subset j of candidate information variables, the conditional kernel is

$$m_{t+1}(j) = 1 - f_{\theta_j}(X_{t+1}, Z_t^{(j)}), \tag{12}$$

with $Z_t^{(j)} \subseteq \widetilde{Z}_t$. For each j, the estimator solves

$$\min_{\theta_j} \mathbb{E}[m_{t+1}(j)^2] \quad \text{s.t.} \quad \mathbb{E}[m_{t+1}(j)X_{t+1}] = 0. \tag{13}$$

A.4 Edge construction

For asset i and conditioning variable k, define the pricing-error reduction

$$\Delta_{k,i} = \left|\widehat{\mathbb{E}}\left[m_{t+1}^{(-k)} X_{t+1}^i\right]\right| - \left|\widehat{\mathbb{E}}\left[m_{t+1}^{(+k)} X_{t+1}^i\right]\right| \tag{14}$$

where $m^{(-k)}$ is estimated without variable k and $m^{(+k)}$ is estimated with variable k included. An edge (k,i) is declared if $\Delta_{k,i}$ exceeds a prespecified tolerance.

A.5 Degrees and interpretation

The left degree is

$$d_L(k) = \sum_i \mathbf{1}(k,i) \in \mathcal{E}, \tag{15}$$

and the right degree is

$$d_R(i) = \sum_k \mathbf{1}(k,i) \in \mathcal{E}. \tag{16}$$

Persistent right-side degree not attributable to any $k \in \widetilde{Z}_t$ is attributed to the latent ambiguity component Θ_t.

A.6 Connection to inflation ambiguity

In the UST–TIPS system, unmatched degree typically concentrates in long-maturity nominal and real bonds. This residual structure is interpreted as Knightian uncertainty about the joint law of long-horizon inflation and real discounting, i.e., ambiguity surrounding the inflation risk premium.

A.7 Practical implementation

1. Fix the traded payoff panel X_{t+1}.
2. For each candidate variable k, re-estimate the conditional neural kernel with and without k.
3. Compute $\Delta_{k,i}$ for all assets.
4. Build the bipartite graph and its degree sequences (d_L, d_R).

References

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Glossary

Nominal SDF Pricing kernel that prices payoffs in nominal units.

Real SDF Pricing kernel that prices payoffs in real consumption units.

Inflation Risk Premium Compensation for exposure to inflation innovations and their interaction with marginal utility.

Knightian Uncertainty Uncertainty about the probability law itself, represented by a set of plausible models.

Ambiguity Distortion Change-of-measure term selecting adverse models under multiple priors.

Information set Z_t Conditioning variables that shift prices of risk but are not priced payoffs.

Handshaking Lemma Graph identity equating the sum of node degrees to twice the number of edges.

Pricing restriction Moment condition $\mathbb{E}[mX] = 0$ for a traded excess payoff.

Neural SDF Nonlinear parameterization of the pricing kernel via a neural network.

The End