Eliminating the BEST Countries

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Abstract

We construct a two–stage trimming rule on a cross–section of sovereign credit risk. First, we retain only those countries whose five–year cumulative probability of default (PD) lies within a mean–based 95% confidence interval. Second, on this restricted set, we retain only those countries whose five–year CDS spread lies within the corresponding conditional 95% confidence interval. The procedure eliminates extreme tail sovereigns in a transparent and model–light manner.

The paper ends with "The End"

1 Data

Let p_i denote the five–year cumulative probability of default and s_i the five–year CDS spread (in basis points) for country i. The cross–section contains $n = 30$ sovereigns.

2 Stage I: PD filter

We compute the sample mean and standard deviation

$$\bar{p} = 0.8707\%, \qquad s_p = 1.1602\%. \tag{1}$$

The mean–based 95% confidence interval is

$$CI_p = \bar{p} \pm 1.96 s_p = [-1.4033\%; 3.1447\%]. \tag{2}$$

Because probabilities are non–negative, the effective interval is $[0, 3.1447\%]$.

Only two sovereigns lie outside this interval and are eliminated in Stage I:

- Turkey
- Egypt

The remaining sample size is $n_1 = 28$.

3 Stage II: CDS filter conditional on Stage I

On the restricted sample we compute

$$\bar{s} = 36.553 \text{ bps}, \qquad s_s = 37.286 \text{ bps}. \tag{3}$$

The conditional 95% interval is

$$CI_{s|p} = \bar{s} \pm 1.96 s_s = [-36.53; 109.63] \text{ bps}. \tag{4}$$

Imposing non–negativity of spreads yields the effective interval $[0, 109.63]$ bps.

Two additional countries are eliminated at this stage:

- Brazil
- South Africa

4 Resulting elimination set

The full set of eliminated sovereigns is therefore

$$\mathcal{E} = \{\text{Brazil}, \text{Egypt}, \text{South Africa}, \text{Turkey}\} \tag{5}$$

All remaining countries constitute the trimmed cross–section used for subsequent modelling.

5 Geometric interpretation

The procedure corresponds to trimming the right tail of the joint distribution of (s_i, p_i) in two steps: first along the PD dimension, then along the CDS dimension conditional on the first cut.

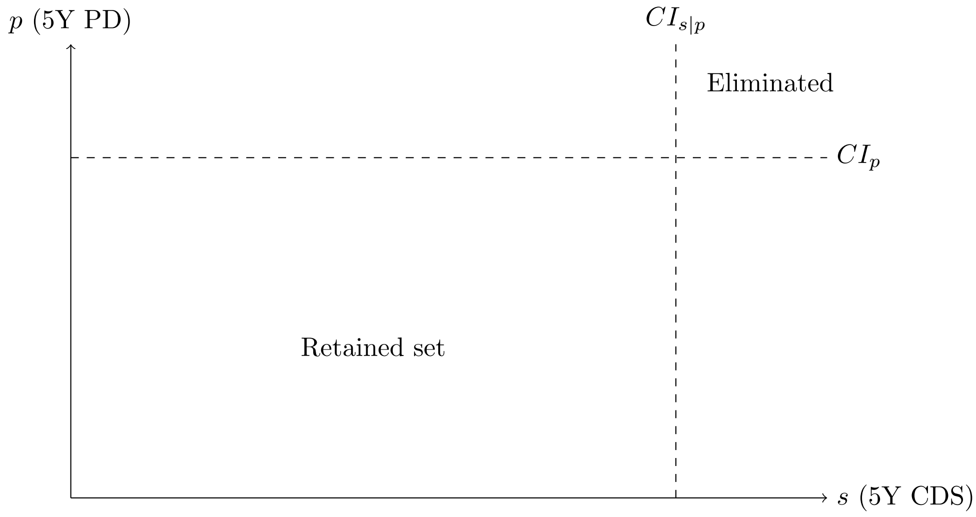

Figure 1: Schematic two–stage trimming in the (s, p) plane.

6 Interpretation

The PD filter removes only the extreme default–risk tail. The conditional CDS filter then removes additional spread outliers even when their PDs are not extreme. This sequential rule isolates the sovereigns that dominate tail risk in both reduced–form default probabilities and market–priced spread risk.

7 Mathematical appendix: Trimming as a projection operator

Let $(s_i, p_i)_{i=1}^n$ denote the observed cross–section of 5Y CDS spreads and 5Y cumulative probabilities of default. Assume the sample is drawn from an unknown joint law $F_{S,P}$ on $\mathbb{R}_+ \times \mathbb{R}_+$.

Stage I: PD truncation

Define the empirical moments

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i, \qquad \hat{\sigma}_p^2 = \frac{1}{n-1}\sum_{i=1}^{n}(p_i - \bar{p})^2. \tag{6}$$

The PD cut–off is

$$c_p = \bar{p} + 1.96\,\hat{\sigma}_p. \tag{7}$$

The first–stage retained index set is

$$I_p = \{\, i : p_i \le c_p \,\}. \tag{8}$$

Stage II: conditional CDS truncation

On the restricted set I_p, define

$$\bar{s}_p = \frac{1}{|I_p|}\sum_{i \in I_p} s_i, \qquad \hat{\sigma}_{s,p}^2 = \frac{1}{|I_p| - 1}\sum_{i \in I_p}(s_i - \bar{s}_p)^2. \tag{9}$$

The conditional CDS cut–off is

$$c_s = \bar{s}_p + 1.96\,\hat{\sigma}_{s,p}. \tag{10}$$

The final retained sample is

$$\mathcal{T} = \{(s_i, p_i) : i \in I_p,\ s_i \le c_s\}. \tag{11}$$

The trimming operator

Let $\mathcal{D} = \{(s_i, p_i)\}_{i=1}^n$ denote the original dataset. Define the data–dependent trimming operator

$$\Pi(\mathcal{D}) = \{(s_i, p_i) \in \mathcal{D} : \; p_i \le c_p(\mathcal{D}), \; s_i \le c_s(\mathcal{D})\}, \tag{12}$$

where the cut–offs $c_p(\mathcal{D})$ and $c_s(\mathcal{D})$ are computed from $\mathcal{D}$ using the two stages above.

Because the CDS threshold is computed *after* truncating by PD, the operator is generally not idempotent:

$$\Pi\big(\Pi(\mathcal{D})\big) \ne \Pi(\mathcal{D}). \tag{13}$$

Relation to tail–robust estimation

Let $\theta(F_{S,P})$ be any functional of the joint law of (S, P) (for example, a covariance element or a tail–dependence coefficient). The trimmed estimator is defined as

$$\hat{\theta}_\Pi = \theta\Big(\hat{F}^{\Pi}_{S,P}\Big), \tag{14}$$

where $\hat{F}^{\Pi}_{S,P}$ is the empirical distribution generated by the retained sample $\mathcal{T}$.

The procedure is therefore a form of data–dependent truncation. Unlike fixed trimming, the CDS boundary is conditional on the PD filter.

Implication for oliGARCHy–type models

Let

$$\Sigma = \mathrm{Var}(S, P) \tag{15}$$

denote the cross–sectional dispersion operator entering an oliGARCHy–type system.

The trimmed analogue is

$$\Sigma_\Pi = \mathrm{Var}_{\hat{F}^{\Pi}_{S,P}}(S, P). \tag{16}$$

Since the right tail in the PD direction and the conditional right tail in the CDS direction are removed, extreme sovereign observations no longer dominate the second moments. For heavy–tailed sovereign cross–sections, this implies generically

$$\lambda_{\max}(\Sigma_\Pi) < \lambda_{\max}(\Sigma), \tag{17}$$

providing a theoretical justification for using the trimmed cross–section as a stability–preserving input for oliGARCHy calibration and systemic risk propagation.

References

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[3] P. Embrechts, C. Kl"uppelberg, and T. Mikosch. *Modelling Extremal Events for Insurance and Finance.* Springer, 1997.

Glossary

5Y CDS Five–year sovereign credit default swap spread, quoted in basis points.

PD Five–year cumulative probability of sovereign default implied from a reduced–form credit model.

95% confidence interval Mean–based interval given by $\bar{x} \pm 1.96s$.

Conditional CI A confidence interval computed after restricting the sample using a previous filter.

Elimination set $\mathcal{E}$ The set of sovereigns removed by the two–stage trimming rule.

The End