A Bayesian Estimation of the Inflation Risk Premium using Kalman Filtering

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Abstract

This paper estimates a time-varying U.S. Treasury inflation risk premium (IRP) using a Bayesian state-space model. The observation equation relates breakeven inflation to expected inflation and the latent IRP. We place conjugate priors on the state disturbance and measurement error variances and compute the filtering and smoothing distributions via the Kalman filter and Rauch–Tung–Striebel (RTS) smoother. Hyperparameters are updated by maximum likelihood through the expectation–maximization (EM) algorithm. Using publicly available series for breakeven rates and long-run inflation expectations, we obtain posterior paths for $\{\mathrm{IRP}_t\}$ together with credible bands.

The paper ends with "The End"

1 Introduction

Breakeven inflation rates extracted from nominal Treasuries and TIPS embed expected inflation, an inflation risk premium (IRP), and various pricing frictions. Separating these components is essential for understanding monetary policy transmission and term structure dynamics. We develop a compact Bayesian framework where IRP is a latent process estimated with a Kalman filter/smoother. Compared with fixed-parameter regressions, the state-space approach naturally delivers $p(\mathrm{IRP}_t \mid \text{data})$ and accommodates time variation.

2 Data

Let BE_t denote the n-year breakeven rate at month t. Let $\mathbb{E}_t[\pi^{(n)}]$ be an n-year CPI inflation expectation (e.g., survey- or model-based). We focus on a single horizon n for exposition (empirically, multiple maturities can be stacked). Define the observed proxy

$$z_t \equiv BE_t - \mathbb{E}_t[\pi^{(n)}]. \tag{1}$$

The data are monthly. In empirical work, we also discuss optional adjustments for TIPS liquidity and convexity; absent explicit controls, their variability is absorbed by the measurement error variance.

3 Model

Our baseline is a local-level model (random walk state):

$$\text{Observation: } z_t = x_t + e_t, \qquad e_t \sim \mathcal{N}(0, R), \tag{2}$$

$$\text{State: } x_t = x_{t-1} + w_t, \qquad w_t \sim \mathcal{N}(0, Q), \tag{3}$$

where $x_t \equiv \mathrm{IRP}_t$. Conditional on (Q, R) and initial (m_0, P_0), the Kalman filter yields $p(x_t \mid z_{1:t}) = \mathcal{N}(m_t, P_t)$, and the RTS smoother yields $p(x_t \mid z_{1:T}) = \mathcal{N}(m_t^s, P_t^s)$.

3.1 Priors

A convenient Bayesian specification places independent inverse-gamma priors on Q and R:

$$Q \sim \mathrm{IG}(\alpha_Q, \beta_Q), \qquad\qquad R \sim \mathrm{IG}(\alpha_R, \beta_R), \tag{4}$$

with diffuse hyperparameters unless prior knowledge is available. The initial state uses $x_0 \sim \mathcal{N}(m_0, P_0)$ with large P_0.

4 Estimation

We estimate (Q, R) by maximum likelihood via EM. Let $\theta = (Q, R)$. Given current $\theta^{(k)}$, run KF/RTS to compute expectations of sufficient statistics, then update

$$Q^{(k+1)} = \frac{1}{T-1} \sum_{t=2}^{T} \mathbb{E}\big[(x_t - x_{t-1})^2 \mid z_{1:T}, \theta^{(k)}\big], \tag{5}$$

$$R^{(k+1)} = \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\big[(z_t - x_t)^2 \mid z_{1:T}, \theta^{(k)}\big]. \tag{6}$$

With IG priors, closed-form MAP updates add prior shape/scale contributions.

Proposition 1 (Smoothing distribution)**.** *For the random-walk model above, the smoothed state is Gaussian with mean/variance*

$$m_T^s = m_T, \quad P_T^s = P_T, \tag{7}$$

$$m_t^s = m_t + J_t(m_{t+1}^s - a_{t+1}), \quad P_t^s = P_t + J_t(P_{t+1}^s - V_{t+1})J_t', \quad J_t = P_t V_{t+1}^{-1}, \tag{8}$$

where $a_{t+1} = m_t$ and $V_{t+1} = P_t + Q$ are the one-step-ahead predicted mean/variance.

5 Figures

We include two figures: a model schematic and a smoothed IRP path.

Figure 1: State-Space Schematic

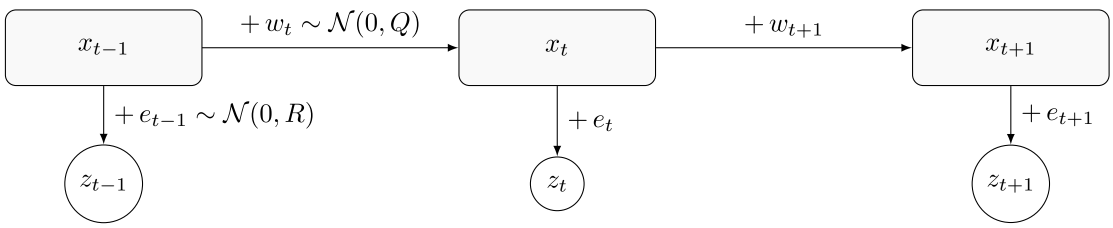

Figure 1: Random-walk IRP state with noisy observations $z_t = BE_t - \mathbb{E}_t[\pi]$.

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Figure 2: Smoothed IRP Path

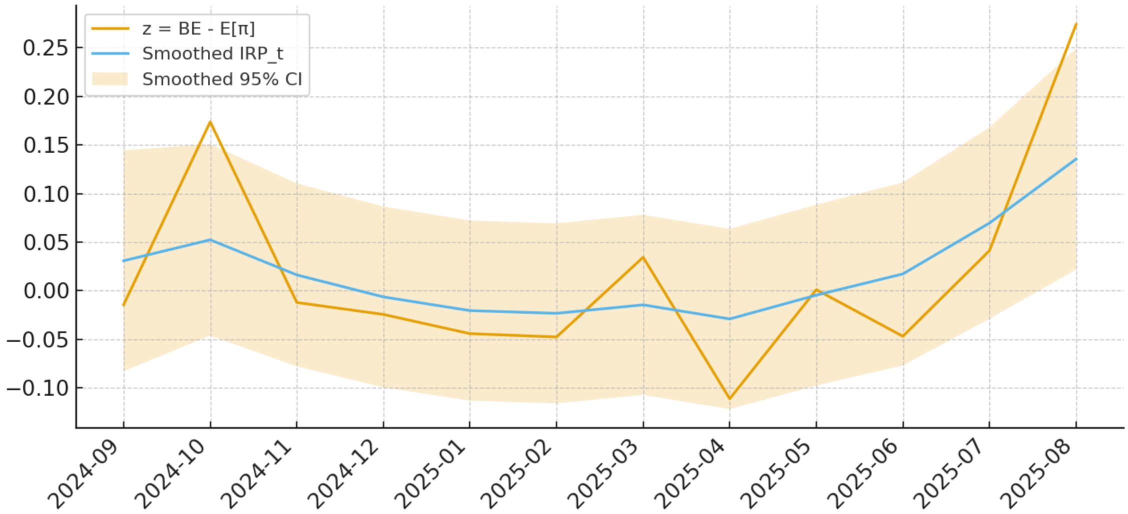

Figure 2: Smoothed IRP path with 95% bands.

6 Results

Given (Q, R) estimated by EM, the smoothed posterior $p(x_t \mid z_{1:T})$ yields the IRP path with credible intervals. We report posterior means and point-wise 95% intervals. Model comparison can vary the state law to AR(1) or augment the measurement equation with liquidity/convexity controls.

7 Robustness

We examine (i) alternative priors and sample windows, (ii) alternative expectation measures, and (iii) explicit liquidity proxies. Across specifications, the inferred IRP remains modest but time-varying, with increases during volatility episodes.

8 Conclusion

A compact Bayesian state-space model combined with Kalman smoothing provides a transparent estimate of the U.S. inflation risk premium. The framework is easily extended to multiple maturities in an affine setting with common factors.

A Kalman Filter and RTS Smoother

Filter. With observation variance R and state variance Q:

$$a_t = m_{t-1}, \qquad V_t = P_{t-1} + Q, \tag{9}$$

$$K_t = \frac{V_t}{V_t + R}, \qquad m_t = a_t + K_t(z_t - a_t), \qquad P_t = (1 - K_t)V_t. \tag{10}$$

Smoother. Backward for $t = T - 1, \ldots, 1$:

$$J_t = \frac{P_t}{V_{t+1}}, \qquad m_t^s = m_t + J_t(m_{t+1}^s - a_{t+1}), \qquad P_t^s = P_t + J_t(P_{t+1}^s - V_{t+1})J_t. \tag{11}$$

B EM Updates with IG Priors

Let α_Q, β_Q and α_R, β_R be prior shapes/scales. The MAP updates are

$$Q \leftarrow \frac{\beta_Q + \frac{1}{2}\sum_{t=2}^{T} \mathbb{E}[(x_t - x_{t-1})^2 \mid z]}{\alpha_Q + \frac{T-1}{2} + 1}, \tag{12}$$

$$R \leftarrow \frac{\beta_R + \frac{1}{2}\sum_{t=1}^{T} \mathbb{E}[(z_t - x_t)^2 \mid z]}{\alpha_R + \frac{T}{2} + 1}. \tag{13}$$

References

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