A Global-Pricing-Kernel State–Space Model of Sovereign Solvency Regimes

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Abstract

We develop an estimable multi-country state–space model in which a global pricing kernel jointly prices sovereign risk while country-specific solvency margins are driven by balance-sheet structure and fiscal capacity. Market prices load on both the global kernel and the national solvency margin. Discrete regimes (solvent, crisis, insolvent) arise as a data-driven discretisation of the latent solvency margin via cross-sectional clustering. The framework is designed for open economies, where arbitrage and external balance-sheet exposure dominate policy transmission.

The paper ends with "The End."

1 Economic objects

For countries $i = 1, \ldots, N$ and dates $t = 1, \ldots, T$, define a latent market-consistent solvency margin

$$z_{i,t} \equiv \mathbb{E}^{\mathbb{Q}_t}\left[\sum_{j\geq 0} m^{\text{global}}_{t,t+j} s_{i,t+j}\right] - B_{i,t}, \tag{1}$$

where $m^{\text{global}}_{t,t+j}$ is the global stochastic discount factor and $s_{i,t+j}$ denotes future primary surpluses.

A single global pricing state f_t summarises time variation in the international pricing kernel.

2 State equations

2.1 Global pricing kernel

$$f_t = \rho f_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \sigma_u^2). \tag{2}$$

2.2 Country solvency margins

Let $b_{i,t}$ denote a vector of slow-moving balance-sheet and fiscal exposure variables (external liabilities, foreign-currency share, maturity, primary balance, and growth–funding-rate differentials). The latent solvency margin obeys

$$z_{i,t} = \alpha_i + \lambda_i f_t + \theta' b_{i,t} + \varepsilon_{i,t}, \qquad \varepsilon_{i,t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon,i}). \tag{3}$$

3 Measurement equations

Let $y_{i,t} \in \mathbb{R}^m$ be a vector of observed market prices (USD sovereign CDS, bond spreads, CDS term slope, and FX-implied depreciation risk). Prices load on both latent states:

$$y_{i,t} = a_i z_{i,t} + c_i f_t + \xi_{i,t}, \qquad \xi_{i,t} \sim \mathcal{N}(0, \Sigma_{\xi,i}). \tag{4}$$

4 Stacked state–space representation

Define the stacked state vector

$$x_t = (f_t, z_{1,t}, \ldots, z_{N,t})'. \tag{5}$$

The transition system is

$$x_t = F x_{t-1} + d_t + G u_t + \eta_t, \tag{6}$$

where $d_t = (0, \alpha_1 + \theta' b_{1,t}, \ldots, \alpha_N + \theta' b_{N,t})'$ and the matrices F and G follow directly from the structural equations. The stacked measurement system is

$$Y_t = H x_t + \xi_t, \tag{7}$$

with $Y_t = (y'_{1,t}, \ldots, y'_{N,t})'$ and a sparse loading matrix H.

5 Identification

To remove scale and rotation indeterminacy, we impose

- $\mathrm{Var}(u_t) = 1$;
- one element of c_i (or equivalently one element of a_i) is normalised to unity.

All remaining parameters are estimated by maximum likelihood using a Kalman filter and smoother.

6 Solvency regimes

Economic regimes are defined by thresholds on the latent margin:

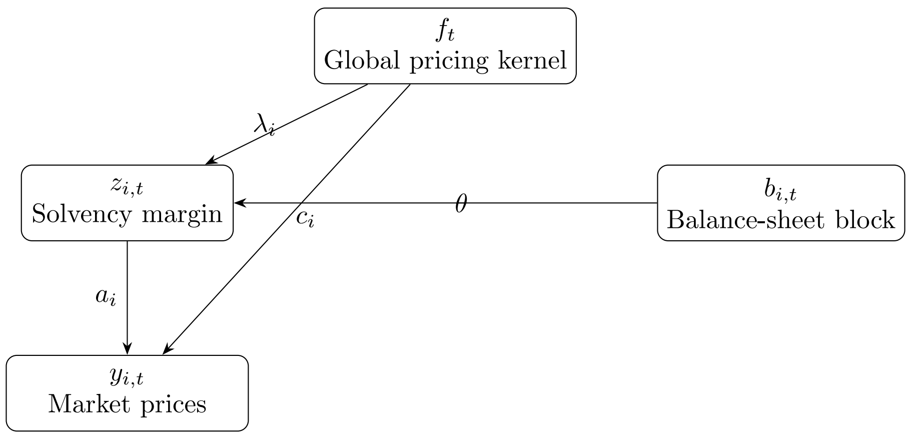
$$r_{i,t} = \begin{cases} 1, & z_{i,t} > \tau_1, \\ 2, & \tau_2 < z_{i,t} \leq \tau_1, \\ 3, & z_{i,t} \leq \tau_2. \end{cases} \tag{8}$$

Because $z_{i,t}$ is latent, (τ_1, τ_2) are approximated by applying K-means with $K = 3$ to the cross-section of smoothed estimates $\{\hat{z}_{i,t}\}_{i=1}^N$ at each date.

7 Estimation algorithm

1. Extract a preliminary global pricing factor $\hat{f}_t$ from the international panel of market prices by principal components or a dynamic factor model.
2. Initialise the state–space system using $\hat{f}_t$.
3. Estimate all structural and measurement parameters by maximum likelihood.
4. Recover smoothed solvency margins $\hat{z}_{i,t}$.
5. Apply cross-sectional K-means to $\hat{z}_{i,t}$ to obtain regime labels.

8 Vector-graphics representation



9 Model implications

The latent margin $z_{i,t}$ is a forward-looking, pricing-kernel-consistent distance to the sovereign intertemporal budget constraint. Monetary policy primarily affects prices through the global factor f_t, whereas fiscal and balance-sheet policies operate through $b_{i,t}$ and hence directly shift the solvency margin and the location of regime boundaries.

10 Empirical implementation with CDS Gram matrices and spectral filtering

This section connects the state–space model to a CDS-based spectral pipeline.

10.1 CDS panel and preprocessing

Let $s_{i,t}$ denote the mid-quote of the USD 5y sovereign CDS spread of country i at date t. Define log-spread changes

$$r_{i,t} = \Delta \log s_{i,t}. \tag{9}$$

Let $R_t = (r_{1,t}, \ldots, r_{N,t})'$.

10.2 Rolling Gram matrix

For a rolling window of length W, define the empirical Gram matrix

$$G_t = \frac{1}{W} \sum_{\tau=t-W+1}^{t} R_\tau R_\tau', \tag{10}$$

which estimates the cross-country covariance of CDS innovations.

10.3 Spectral extraction of the global pricing kernel

Let the eigendecomposition be

$$G_t = V_t \Lambda_t V_t'. \tag{11}$$

The leading eigenvector $v_{1,t}$ captures the dominant common component in sovereign credit risk. We define the spectral proxy for the global pricing kernel as

$$\hat{f}_t = v_{1,t}' R_t. \tag{12}$$

This construction is consistent with a single global pricing kernel driving contemporaneous re-pricing across countries.

10.4 Kernel purification of CDS changes

Country-specific CDS innovations purged of the global kernel are obtained as

$$\tilde{r}_{i,t} = r_{i,t} - \beta_i \hat{f}_t, \tag{13}$$

where β_i is estimated by a rolling regression of $r_{i,t}$ on $\hat{f}_t$.

10.5 Market-based solvency signal

Let $\tilde{R}_t = (\tilde{r}_{1,t}, \ldots, \tilde{r}_{N,t})'$. A second Gram matrix is formed as

$$\tilde{G}_t = \frac{1}{W} \sum_{\tau=t-W+1}^{t} \tilde{R}_\tau \tilde{R}_\tau', \tag{14}$$

whose leading eigenvector isolates the dominant country-specific stress direction, net of the global kernel. The resulting scalar market signal is

$$\hat{z}_{i,t}^{(\text{price})} = w_{i,t}' \tilde{R}_t, \tag{15}$$

where $w_{i,t}$ is the i-th element of the leading eigenvector of $\tilde{G}_t$.

10.6 Link to the state–space model

The spectral signal $\hat{z}_{i,t}^{(\text{price})}$ is used as the empirical counterpart of the measurement-driven component of the latent solvency margin in the state–space system. In practice, $\hat{z}_{i,t}^{(\text{price})}$ is treated as a noisy observation of $z_{i,t}$ when initialising the Kalman filter, and as a validation target for the smoothed estimates $\hat{z}_{i,t}$.

10.7 Regime construction and diagnostics

At each date t, K-means with $K = 3$ is applied to the cross-section $\{\hat{z}_{i,t}\}_{i=1}^{N}$ to obtain the discrete regimes. Diagnostics are based on (i) regime transition matrices, (ii) conditional CDS volatility across regimes, and (iii) out-of-sample predictability of CDS changes using lagged $\hat{z}_{i,t}$ and $\hat{f}_t$.

References

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Glossary

Global pricing kernel The common stochastic discount factor that prices internationally traded risky assets.

Solvency margin The difference between the global-kernel present value of future primary surpluses and the outstanding public debt stock.

Measurement block The mapping from latent states to observed market prices.

Structural block The law of motion linking latent solvency to balance-sheet structure and the global pricing state.

Regime discretisation The partition of a continuous latent solvency margin into a finite number of economic states.

External balance-sheet exposure The sensitivity of national solvency to foreign-currency liabilities and international funding conditions.

The End