Theoretical prices of European call and put options on a zero-dividend stock using time-series processes

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Abstract

In this paper, I describe theoretical prices of European call and put options
on a zero-dividend stock using time-series processes.
The paper ends with "The End"

Introduction

European call and put options on a zero-dividend stock are easy to analyze and price using time-series processes.

In this paper, I describe theoretical prices of European call and put options
on a zero-dividend stock using a time-series process.

Notation

We use the standard notation:
$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2} ds$ is **the error function**
$erfc(x) = 1 - erf(x)$ is **the complimentary error function**
t is time
T is time of maturity
r is the bond rate
$K > 0$ is the strike price
$S(t)$ is the stock price as a function of time
$C(t)$ is the call option price as a function of time
$P(t)$ is the put option price as a function of time

The put-call parity

The put-call parity **requires**

$$P(t) = C(t) + Ke^{-r(T-t)} - S(t)$$

Therefore, it suffices to know $C(t)$ to know $P(t)$

Theoretical prices of a European call option on a zero-dividend stock using a time-series process

Theoretical prices of a European call option on a zero-dividend stock using a time-series process are:

1. **Brownian Motion process (also known as Wiener process) with drift μ and volatility $\sigma > 0$**

$$C(t) = \frac{\sigma\sqrt{t}}{\sqrt{2\pi}}\, e^{-\frac{(K-\mu t)^2}{2\sigma^2 t}} - \frac{1}{2}(K-\mu t)\, erfc\left(\frac{K-\mu t}{\sqrt{2}\sigma\sqrt{t}}\right)$$

2. **Geometric Brownian Motion process with drift μ, volatility $\sigma > 0$ and initial value $S_0 > 0$**

$$C(t) = \frac{1}{2}\left(S_0 e^{\mu t}\left(1 + \, erf\left(\frac{2\log\frac{S_0}{K} + 2\mu t + \sigma^2 t}{2\sqrt{2}\sigma\sqrt{t}}\right)\right) - K\, erfc\left(\frac{2\log\frac{K}{S_0} - 2\mu t + \sigma^2 t}{2\sqrt{2}\sigma\sqrt{t}}\right)\right)$$

3. **Auto-Regressive Moving-Average $(\alpha,\ \beta)$ process with normal white noise variance $\nu > 0$**

$$C(t) = \sqrt{\frac{2\alpha\beta\nu + \beta^2\nu + \nu}{2\pi - 2\pi\alpha^2}}\, e^{\frac{\left(\alpha^2-1\right)K^2}{2\nu\left(1+2\alpha\beta+\beta^2\right)}} - \frac{1}{2}\, K\, erfc\left(\frac{K}{\sqrt{2}\sqrt{\frac{\nu(1+2\alpha\beta+\beta^2)}{1-\alpha^2}}}\right)$$

4. **Ornstein-Uhlenbeck process (also known as Vasicek process) with long-term mean μ, volatility $\sigma > 0$ and mean reversion speed $\theta > 0$**

$$C(t) = \frac{1}{2}\left((\mu - K)\, erfc\left(\frac{\sqrt{\theta}(K-\mu)}{\sigma}\right) + \frac{\sigma}{\sqrt{\pi}\sqrt{\theta}} e^{-\frac{\theta(K-\mu)^2}{\sigma^2}}\right)$$

5. **Ito $(\alpha,\ \beta)$ process**

$$C(t) = \frac{1}{2}\sqrt{\frac{1}{\beta^2}}\sqrt{\beta^2}\left((\alpha t - K)\, erfc\left(\frac{\sqrt{\frac{1}{\beta^2 t}}(K - \alpha t)}{\sqrt{2}}\right) + \frac{\sqrt{\frac{2}{\pi}}}{\sqrt{\frac{1}{\beta^2 t}}} e^{-\frac{(K-\alpha t)^2}{2\beta^2 t}}\right)$$

The End