Two solutions to the Emden–Chandrasekhar equation

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Abstract

In this paper, I describe two solutions to the Emden–Chandrasekhar equation.
The paper ends with "The End"

Introduction

The Emden–Chandrasekhar equation[1] is

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2\frac{\partial\psi(\xi)}{\partial\xi}\right) = e^{-\psi(\xi)}$$

In this paper, I describe two solutions to the Emden–Chandrasekhar equation with $\psi(0) = 0$ and $\psi'(0) = 0$.

A real solution to the Emden–Chandrasekhar equation

A real solution to the Emden–Chandrasekhar equation is

$$\psi(\xi) = \{ \begin{array}{cc} 0 & \xi = 0 \\ W\left(\frac{\xi^2}{a^2\xi^2+6a\xi+6}\right) & \xi \neq 0 \end{array}$$

where
$W(z)$ is the ProductLog function
$a = -0.03026108998\ldots$
$\xi = \frac{5}{32}$

A complex solution to the Emden–Chandrasekhar equation

A complex solution to the Emden–Chandrasekhar equation is

$$\psi(\xi) = \{ \begin{array}{cc} 0 & \xi = 0 \\ W\left(\frac{\xi^2}{a^2\xi^2+6a\xi+6}\right) & \xi \neq 0 \end{array}$$

where
$W(z)$ is the ProductLog function
$a = -\frac{38415128}{18684955} + i\frac{14818544}{13914029}$
$\xi = 1.63054458\ldots - i\,0.37887616\ldots$

References

[1] https://en.wikipedia.org/wiki/Emden-Chandrasekhar_equation

The End