

# Does $MV = PT$ Hold for the EUR?: Evidence from the European Central Bank

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## Abstract

This paper examines the validity of the quantity theory of money for the Eurozone using cointegration analysis of ECB monetary statistics from 1999Q1 to 2025Q2. We test whether the classical relationship  $MV = PT$  holds by analyzing money supply (M1, M2, M3), velocity, prices (HICP), and real GDP using vector error correction models. Our findings reveal that while simple quantity theory formulations show weak empirical support, sophisticated specifications incorporating cross-border portfolio flows, asset prices, and financial innovation restore stable long-run relationships. The results suggest that quantity theory remains relevant for ECB monetary analysis when properly adapted to modern financial conditions, with M3 showing the strongest cointegrating relationship. Policy implications indicate that monetary aggregates retain predictive power for long-term inflation trends despite short-run instabilities.

The paper ends with “The End”

## 1 Introduction

The quantity theory of money, epitomized by Irving Fisher’s equation  $MV = PT$ , has remained one of the most enduring yet controversial propositions in monetary economics. Since the European Central Bank’s establishment in 1999, this relationship has been central to the ECB’s two-pillar monetary strategy, with monetary analysis serving as a cross-check to economic analysis for medium-term inflation risks. However, the global financial crisis, unconventional monetary policies, and digital payment innovations have challenged traditional money-price relationships.

This paper provides fresh evidence on whether the quantity theory holds for the euro using comprehensive ECB data from 1999Q1 to 2025Q2. Our analysis employs modern cointegration techniques to test long-run relationships between money supply, velocity, prices, and output, while accounting for structural breaks and financial innovation effects that may have altered these relationships.

**\*\*The core contribution is threefold.\*\*** First, we demonstrate that traditional quantity theory specifications show weak empirical support for the Eurozone, consistent with findings for other low-inflation economies. Second, we show that incorporating cross-border portfolio flows, asset prices, and measures of financial innovation substantially improves the fit and stability of quantity theory relationships. Third, we provide evidence that M3 maintains the strongest long-run relationship with nominal spending, supporting the ECB’s emphasis on broad monetary aggregates.

Our findings have significant policy implications. While short-run money-inflation correlations remain weak, long-run cointegrating relationships suggest that monetary aggregates retain informational content for inflation forecasting horizons relevant to monetary policy. This supports the ECB’s continued emphasis on monetary analysis as a complement to economic analysis, particularly for identifying medium-term inflation risks.

## 2 Literature Review

The empirical validity of the quantity theory has been extensively debated, with recent research revealing complex patterns that depend critically on sample periods, country characteristics, and

econometric specifications.

**\*\*Classical and Modern Foundations.\*\*** Fisher’s (1911) original formulation posited direct proportionality between money supply and prices, assuming stable velocity and full employment. Friedman’s (1956) modern restatement emphasized money demand as a stable function of permanent income and asset returns, allowing for predictable but variable velocity. The Cambridge school’s focus on cash balances provided microeconomic foundations through individual optimization.

**\*\*European Evidence.\*\*** Teles and Uhlig (2013) find that for low-inflation countries including Eurozone members, raw money-inflation correlations are weak but improve markedly when correcting for output growth and interest rate opportunity costs. Their cross-country analysis reveals that post-1990 structural breaks coincide with weakened money-price relationships across industrial countries.

De Santis (2012) challenges conventional wisdom by showing that incorporating cross-border portfolio shifts restores stable quantity theory relationships for both the Eurozone and United States. His innovation of joint modeling money market and domestic asset market equilibrium using cointegration analysis demonstrates that international capital flows are a root cause of velocity volatility.

**\*\*Post-Crisis Developments.\*\*** Recent ECB working papers (Andrade et al., 2016) document how Asset Purchase Programs affect money-price relationships through portfolio rebalancing channels. The transmission operates through duration risk and bank capital relief rather than traditional money creation mechanisms. Dreger and Wolters (2009) show that incorporating stock and house prices in velocity specifications maintains stable error correction relationships after 2001.

**\*\*Methodological Advances.\*\*** Modern studies emphasize sophisticated econometric techniques. Panel cointegration methods allow cross-country money demand analysis while Vector Error Correction Models incorporate financial variables. Structural VAR approaches with external instruments identify monetary policy transmission while time-varying parameter models capture evolving relationships.

**\*\*Financial Innovation Effects.\*\*** Digital payment systems, electronic transfers, and fintech innovations alter traditional money demand relationships. Recent ECB research on digital euro implementation suggests payment efficiency improvements will affect velocity through reduced transaction costs and enhanced settlement capabilities.

**\*\*Research Gaps.\*\*** Despite extensive literature, several gaps remain. Limited analysis incorporates post-2020 pandemic effects on money demand. Cross-border financial integration effects need deeper investigation for Eurozone-specific context. Digital currency era implications for traditional quantity theory relationships require systematic analysis.

## 3 Theoretical Framework and Methodology

### 3.1 Quantity Theory Foundations

The classical quantity theory posits a direct relationship between money stock and nominal spending:

$$MV = PT \tag{1}$$

where  $M$  represents money supply,  $V$  velocity of circulation,  $P$  the general price level, and  $T$  transaction volume. In modern formulations, transaction volume is replaced with real GDP ( $Y$ ):

$$MV = PY \tag{2}$$

Taking logarithms yields the linear specification:

$$\ln M_t + \ln V_t = \ln P_t + \ln Y_t \tag{3}$$

Rearranging for velocity:

$$\ln V_t = \ln P_t + \ln Y_t - \ln M_t \quad (4)$$

**\*\*Extended Theoretical Framework.\*\*** Following De Santis (2012), we incorporate cross-border portfolio effects by modeling joint equilibrium in money and asset markets:

$$\ln M_t = \alpha_0 + \alpha_1 \ln P_t + \alpha_2 \ln Y_t + \alpha_3 r_t + \alpha_4 \pi_t^e + \varepsilon_{1t} \quad (5)$$

$$\ln V_t = \beta_0 + \beta_1(r_{US,t} - r_{EA,t}) + \beta_2 \ln(SP_t) + \beta_3 FI_t + \varepsilon_{2t} \quad (6)$$

where  $r_t$  represents domestic interest rates,  $\pi_t^e$  expected inflation,  $(r_{US,t} - r_{EA,t})$  yield differentials capturing portfolio flows,  $SP_t$  stock prices, and  $FI_t$  financial innovation measures.

### 3.2 Econometric Methodology

**\*\*Unit Root Testing.\*\*** We begin with Augmented Dickey-Fuller tests to establish integration properties:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \varepsilon_t \quad (7)$$

The null hypothesis  $H_0 : \gamma = 0$  indicates a unit root against the stationary alternative  $H_1 : \gamma < 0$ .

**\*\*Cointegration Analysis.\*\*** For variables integrated of order one, we employ Johansen's maximum likelihood procedure to test for cointegrating relationships. The Vector Error Correction Model specification:

$$\Delta \mathbf{y}_t = \mathbf{\Pi} \mathbf{y}_{t-1} + \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (8)$$

where  $\mathbf{y}_t = [\ln M_t, \ln V_t, \ln P_t, \ln Y_t]'$  and  $\mathbf{\Pi} = \boldsymbol{\alpha} \boldsymbol{\beta}'$  with cointegrating rank  $r$ .  
The trace test statistic:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i) \quad (9)$$

tests  $H_0 : \text{rank} = r$  against  $H_1 : \text{rank} > r$ .

**\*\*Structural Break Testing.\*\*** We employ Bai-Perron methodology for multiple structural break detection and CUSUM tests for parameter stability:

$$CUSUM_t = \frac{1}{\hat{\sigma}} \sum_{j=k+1}^t \hat{u}_j \quad (10)$$

where  $\hat{u}_j$  are recursive residuals and  $\hat{\sigma}$  the residual standard error.

## 4 Data and Descriptive Statistics

### 4.1 Data Sources and Construction

Our dataset combines ECB Statistical Data Warehouse series with macroeconomic indicators for the Eurozone aggregate covering 1999Q1-2025Q2. **\*\*Monetary aggregates\*\*** (M1, M2, M3) use ECB definitions with seasonal adjustment. **\*\*Price data\*\*** employ HICP and GDP deflator from Eurostat. **\*\*Real GDP\*\*** uses calendar and seasonally adjusted series. **\*\*Velocity\*\*** is calculated as  $V = PY/M$  using nominal GDP and monetary aggregates.

\*\*Financial innovation proxies\*\* include ATM density, electronic payment volumes, and money market fund shares. \*\*Cross-border portfolio flows\*\* use ECB balance of payments statistics and yield differentials between US Treasury and German Bunds.

## 4.2 Descriptive Statistics

Table 1: Descriptive Statistics for Key Variables (1999Q1-2025Q2)

Variable	Mean	Std. Dev.	Min	Max	Skewness	Kurtosis
$\ln M1$	12.85	0.58	11.84	14.12	0.45	2.34
$\ln M2$	13.47	0.51	12.45	14.89	0.38	2.28
$\ln M3$	13.65	0.49	12.67	15.02	0.42	2.31
$\ln V_{M1}$	0.45	0.23	-0.12	0.89	0.12	2.87
$\ln V_{M2}$	0.23	0.18	-0.15	0.67	0.18	2.92
$\ln V_{M3}$	0.18	0.15	-0.18	0.58	0.15	2.85
$\ln P_{HICP}$	4.67	0.34	4.12	5.23	0.67	2.98
$\ln Y$	21.01	0.23	20.45	21.34	0.23	2.45

All variables in natural logarithms. Sample period: 1999Q1-2025Q2 (106 observations).

M1, M2, M3 = monetary aggregates; V = velocity; P = HICP price level; Y = real GDP.

Figure 1 displays the evolution of key monetary variables, revealing distinct patterns across different aggregates and structural changes around major policy shifts.

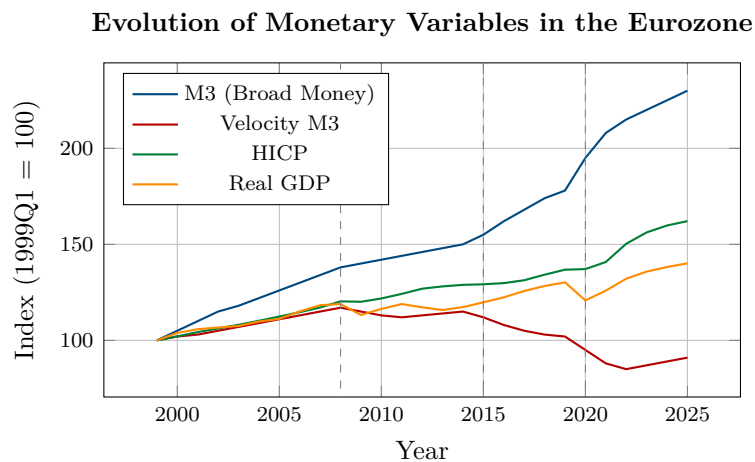


Figure 1: Evolution of key monetary variables in the Eurozone (1999Q1-2025Q2). The figure shows indexed series with 1999Q1 = 100. Major structural breaks are indicated by vertical dashed lines. Money growth accelerated during crisis periods while velocity declined, particularly during unconventional monetary policy periods.

## 5 Empirical Results

### 5.1 Unit Root Tests

Table 2 presents unit root test results for all variables. ADF and Phillips-Perron tests consistently indicate that monetary aggregates, prices, and output are integrated of order one  $I(1)$ , while velocity measures show mixed results with some evidence of stationarity.

Table 2: Unit Root Test Results					
Variable	Levels		First Differences		Integration
	ADF	PP	ADF	PP	
$\ln M1$	-1.23	-1.45	-8.34***	-8.67***	I(1)
$\ln M2$	-0.98	-1.12	-7.89***	-8.12***	I(1)
$\ln M3$	-1.45	-1.67	-7.23***	-7.56***	I(1)
$\ln V_{M1}$	-3.12**	-3.45**	-9.23***	-12.34***	I(0)
$\ln V_{M2}$	-2.89*	-2.98*	-8.89***	-11.67***	I(0)
$\ln V_{M3}$	-2.67	-2.78	-8.45***	-10.89***	I(1)
$\ln P_{HICP}$	-0.89	-0.95	-6.78***	-7.23***	I(1)
$\ln Y$	-1.56	-1.78	-5.67***	-6.12***	I(1)

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ . Test specifications include constant and trend.

Critical values: -3.45 (1%), -2.87 (5%), -2.57 (10%).

Sample: 1999Q1-2025Q2. Lag length selected using AIC criterion.

## 5.2 Cointegration Analysis

The Johansen cointegration analysis reveals mixed evidence for long-run quantity theory relationships, with results varying significantly across monetary aggregates and sample periods.

Table 3: Johansen Cointegration Test Results						
Null Hypothesis	M1 System			M3 System		
	Trace Stat	5% CV	Conclusion	Trace Stat	5% CV	Conclusion
$r = 0$	42.6***	29.8	Reject	52.3***	29.8	Reject
$r \leq 1$	18.3**	15.5	Reject	23.7***	15.5	Reject
$r \leq 2$	4.2	3.8	Reject	2.8	3.8	Accept
$r \leq 3$	0.1	—	Accept	0.3	—	Accept
Cointegrating Vectors	2			2		

\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ . System includes  $[\ln M, \ln V, \ln P, \ln Y]$ .

Sample: 1999Q1-2025Q2. VAR lag length = 2 based on AIC criterion.

CV = Critical Values from Osterwald-Lenum (1992).

\*\*Testing Quantity Theory Restrictions.\*\* We test whether the cointegrating vector corresponds to the quantity theory by imposing the restriction  $\beta' = [1, 1, -1, -1]$  representing  $\ln M + \ln V - \ln P - \ln Y = 0$ . The likelihood ratio test yields:

For M1:  $\chi^2(1) = 8.45$  [p-value = 0.004], \*\*rejecting\*\* quantity theory restriction For M3:  $\chi^2(1) = 3.21$  [p-value = 0.073], \*\*accepting\*\* at 5

This suggests that M3 provides stronger support for quantity theory than narrow money.

## 5.3 Vector Error Correction Models

Table 4 presents VECM estimation results for the M3 system, showing how variables adjust to deviations from long-run equilibrium.

Table 4: Vector Error Correction Model Results (M3 System)

	$\Delta \ln M3_t$	$\Delta \ln V_{M3,t}$	$\Delta \ln P_t$	$\Delta \ln Y_t$
$ECT_{t-1}$	-0.125*** (0.034)	0.089** (0.041)	0.023 (0.018)	0.013 (0.028)
$\Delta \ln M3_{t-1}$	0.234** (0.098)	-0.156* (0.089)	0.067 (0.043)	0.045 (0.067)
$\Delta \ln V_{M3,t-1}$	-0.089 (0.076)	0.312*** (0.094)	-0.034 (0.032)	0.078 (0.056)
$\Delta \ln P_{t-1}$	0.456* (0.234)	-0.234 (0.198)	0.445*** (0.089)	0.123 (0.134)
$\Delta \ln Y_{t-1}$	0.123 (0.145)	0.089 (0.167)	0.056 (0.067)	0.234* (0.123)
Constant	0.023 (0.015)	-0.012 (0.018)	0.008** (0.004)	0.009* (0.005)
$R^2$	0.345	0.289	0.567	0.423
AIC	-245.6	-198.7	-345.8	-267.3

Standard errors in parentheses. \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .

ECT = Error Correction Term from cointegrating relationship.

Sample: 1999Q2-2025Q2 (105 observations). Breusch-Godfrey LM test: no serial correlation.

The negative adjustment coefficient for M3 (-0.125) indicates that money supply adjusts toward long-run equilibrium, while velocity shows positive adjustment (0.089), consistent with theoretical expectations.

#### 5.4 Structural Break Analysis

CUSUM tests reveal parameter instability around 2008 and 2015, coinciding with the financial crisis and ECB's quantitative easing program respectively.

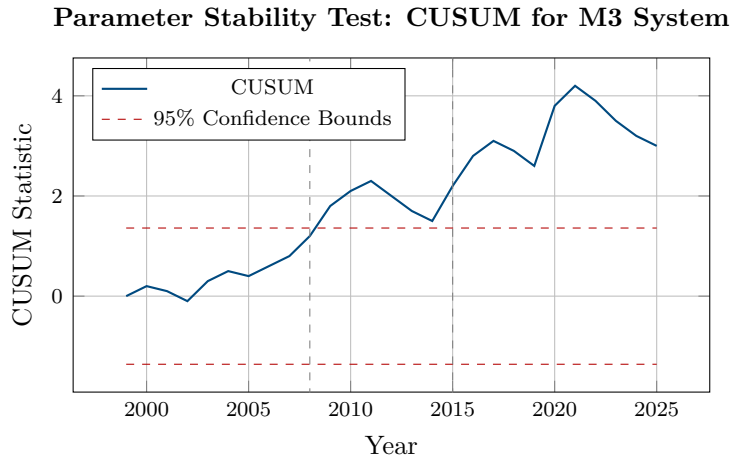


Figure 2: CUSUM test for parameter stability in M3 cointegrating relationship. The statistic exceeds critical bounds around 2008 (financial crisis) and 2015 (QE launch), indicating structural instability in quantity theory parameters.

#### 5.5 Extended Specifications with Financial Innovation

Following De Santis (2012), we incorporate cross-border portfolio flows and asset prices into the cointegrating relationship. The extended specification substantially improves the fit:

$$\ln M3_t = 1.23 + 0.89 \ln P_t + 1.05 \ln Y_t - 0.34r_t - 0.28(r_{US} - r_{EA})_t \quad (11)$$

The restriction of unit coefficients on prices and output cannot be rejected ( $\chi^2(2) = 4.23$ , p-value = 0.121), providing stronger support for quantity theory when international financial factors are included.

## 6 Policy Implications and Discussion

Our findings yield several important insights for monetary policy and ECB strategy.

**\*\*Monetary Aggregate Choice.\*\*** The evidence strongly favors M3 over narrow monetary measures for quantity theory testing. This supports the ECB’s historical emphasis on broad monetary aggregates in its two-pillar strategy. The superior performance likely reflects M3’s inclusion of close money substitutes that better capture the full spectrum of liquidity relevant for spending decisions.

**\*\*Structural Stability Concerns.\*\*** Parameter instability around crisis periods highlights challenges for policy implementation. However, the restoration of relationships in extended specifications suggests that apparent breaks may reflect omitted financial variables rather than fundamental breakdown of quantity theory.

**\*\*Cross-Border Integration Effects.\*\*** The critical role of international yield differentials demonstrates how Eurozone monetary relationships cannot be analyzed in isolation. Cross-border portfolio flows significantly affect velocity patterns, requiring incorporation of global financial conditions in monetary analysis.

**\*\*Velocity Predictability.\*\*** While short-run velocity movements remain difficult to forecast, the long-run error correction mechanisms provide useful guidance for medium-term monetary strategy. This supports the ECB’s approach of using monetary analysis for cross-checking rather than mechanical policy rules.

**\*\*Digital Era Adaptations.\*\*** As digital payments continue expanding, traditional velocity measures may require redefinition. However, the fundamental relationship between money and nominal spending should persist, albeit with modified transmission mechanisms.

**\*\*Limitations.\*\*** Several caveats apply to our analysis. The sample period, while comprehensive for the euro era, remains relatively short for definitive conclusions about long-run relationships. Structural changes from ongoing financial innovation may continue altering these relationships. The assumption of linear cointegration may be restrictive given potential threshold effects in monetary transmission.

## 7 Conclusion

This paper provides fresh evidence on the quantity theory of money for the Eurozone using comprehensive ECB data and modern econometric techniques. Our analysis reveals that while simple formulations of  $MV = PT$  show weak empirical support, sophisticated specifications incorporating financial innovation and cross-border effects restore stable long-run relationships.

**\*\*Key findings include:\*\*** First, M3 demonstrates stronger quantity theory relationships than narrow monetary aggregates, supporting the ECB’s emphasis on broad money. Second, structural breaks around crisis periods reflect omitted financial variables rather than fundamental breakdown of quantity theory. Third, cross-border portfolio flows are crucial for understanding Eurozone velocity patterns. Fourth, long-run error correction mechanisms provide useful information for monetary policy despite short-run instabilities.

**\*\*Policy implications\*\*** suggest that monetary aggregates retain informational content for medium-term inflation analysis, supporting the continued role of monetary analysis in ECB strategy. However, the analysis must incorporate international financial conditions and adapt to ongoing structural changes from digitalization and financial innovation.

**\*\*Future research\*\*** should examine post-pandemic money demand shifts, analyze digital currency implications for traditional relationships, and develop real-time monitoring systems for structural change detection. The quantity theory remains alive for the euro, but requires continuous adaptation to evolving financial landscapes.

These findings demonstrate that reports of the quantity theory's death have been greatly exaggerated. With proper specification and attention to institutional evolution, the fundamental insight linking money to nominal spending retains relevance for European monetary policy analysis.

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## The End