Seven results on q-series

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Abstract

In this paper, I describe seven results on q-series.
The paper ends with "The End"

Introduction

In this paper, I describe seven results on q-series.

The QPolyGamma function

The QPolyGamma function $\psi_q(z)$ is defined by

$$\psi_q(z) = -\log(1-q) + \log(q) \sum_{n=0}^{\infty} \frac{q^{n+z}}{1-q^{n+z}}$$

and

$$\psi_q^{(n)}(z) = \frac{d^n \psi_q(z)}{dz^n}$$

The DoubleGamma function

The DoubleGamma function $G(n)$ is defined by

$$G(n) = \prod_{r=1}^{n-1} \Gamma(r)$$

Then we have the following seven results on q-series.

The first result on q-series

For $q > 1$ we have

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} q^{-ij} f(q) = \frac{\psi_{\frac{1}{q}}^{(0)}(1) + \log\left(\frac{q-1}{q}\right)}{\log\left(\frac{1}{q}\right)} f(q)$$

The second result on q-series

For $q > 1$ we have

$$\prod_{i=1}^{m}\prod_{j=1}^{n} q^{-ij} f(q) = \left(q^{-\frac{1}{2}n(n+1)}\right)^{\frac{1}{2}m(m+3)} \left(q^{\frac{1}{2}n(n+1)} f(q)^{n}\right)^{m}$$

The third result on q-series

For $q > 1$ we have

$$\prod_{i=1}^{\infty}\sum_{j=1}^{\infty} q^{-ij} + \prod_{k=0}^{\infty} \frac{1}{1-q^{k+1}} = 0$$

The fourth result on q-series

For $f(q) > 1$ we have

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} q f(q)^{-ij} = q^{\frac{\left(\psi^{(0)}_{\frac{1}{f(q)}}(1) + \log\left(1 - \frac{1}{f(q)}\right)\right)}{\log\left(\frac{1}{f(q)}\right)}}$$

The fifth result on q-series

For $f(q) > 1$ we have

$$\prod_{i=1}^{m}\prod_{j=1}^{n} q f(q)^{-ij} = \left(f(q)^{-\frac{1}{2}n(n+1)}\right)^{\frac{1}{2}m(m+3)} \left(q^{n} f(q)^{\frac{1}{2}n(n+1)}\right)^{m}$$

The sixth result on q-series

For $q > 1$ we have

$$\prod_{i=1}^{m}\prod_{j=1}^{n} \frac{q^{-ij}}{i!j!} f(q) = \left(\frac{1}{G(n+2)}\right)^{m} q^{\frac{1}{2}mn(n+3)} \left(q^{-\frac{1}{2}n(n+3)}\right)^{\frac{1}{2}m(m+3)} \left(\frac{q^{\frac{1}{2}m(m+1)} f(q)^{m}}{G(m+2)}\right)^{n}$$

The seventh result on q-series

For $f(q) > 1$ we have

$$\prod_{i=1}^{m}\prod_{j=1}^{n} q \frac{f(q)^{-ij}}{i!j!} = \left(\frac{1}{G(n+2)}\right)^{m} f(q)^{\frac{1}{2}mn(n+3)} \left(f(q)^{-\frac{1}{2}n(n+3)}\right)^{\frac{1}{2}m(m+3)} \left(\frac{q^{m} f(q)^{\frac{1}{2}m(m+1)}}{G(m+2)}\right)^{n}$$

The End