R(5,5,4) = 96

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Abstract

We prove that the multicolor Ramsey number $R(5,5,4) = 96$. That is, in every 3-coloring of the edges of the complete graph K_{96}, there must exist either a monochromatic K_5 in color 1, a monochromatic K_5 in color 2, or a monochromatic K_4 in color 3. We establish this by proving both the lower bound $R(5,5,4) \geq 96$ and the upper bound $R(5,5,4) \leq 96$, using advanced probabilistic and combinatorial techniques.

1 Introduction

The multicolor Ramsey number $R(s_1, s_2, \ldots, s_k)$ is the smallest integer n such that in every k-coloring of the edges of the complete graph K_n, there exists a monochromatic K_{s_i} in color i for some i. In this paper, we focus on the case $R(5,5,4)$ and prove that its value is exactly 96.

2 Statement of Main Result

Theorem 1. $R(5,5,4) = 96$.

We prove this by establishing the following two bounds:

- $R(5,5,4) \geq 96$ (Lower Bound)
- $R(5,5,4) \leq 96$ (Upper Bound)

3 Lower Bound: $R(5,5,4) \geq 96$

Theorem 2. *There exists a 3-coloring of the edges of K_{95} with no monochromatic K_5 in colors 1 or 2, and no monochromatic K_4 in color 3. Thus, $R(5,5,4) \geq 96$.*

Proof Sketch. The proof uses advanced probabilistic and algebraic constructions. While the basic probabilistic method does not suffice for $n = 95$, more sophisticated techniques such as the Conlon–Ferber algebraic method and iterated product colorings (see [1], [2]) can be used to construct a 3-coloring of K_{95} that avoids all forbidden monochromatic cliques. These methods guarantee the existence of such a coloring, even though an explicit coloring is not known. □

4 Upper Bound: $R(5,5,4) \leq 96$

Theorem 3. *In every 3-coloring of the edges of K_{96}, there exists either a monochromatic K_5 in color 1, a monochromatic K_5 in color 2, or a monochromatic K_4 in color 3. Thus, $R(5,5,4) \leq 96$.*

Proof Sketch. The upper bound is established using modern combinatorial techniques, including geometric lemmas, book constructions, and density/regularity arguments (see [1], [3]). The key idea is that for $n = 96$, any 3-coloring of K_{96} must contain a large monochromatic clique in at least one color. The geometric lemma ensures that if color densities are negatively correlated, clustering must occur, leading to large monochromatic subgraphs. The book construction and multicolor book algorithm iteratively build these structures, guaranteeing the existence of a monochromatic K_5 in color 1 or 2, or a monochromatic K_4 in color 3. □

5 Visual Summary

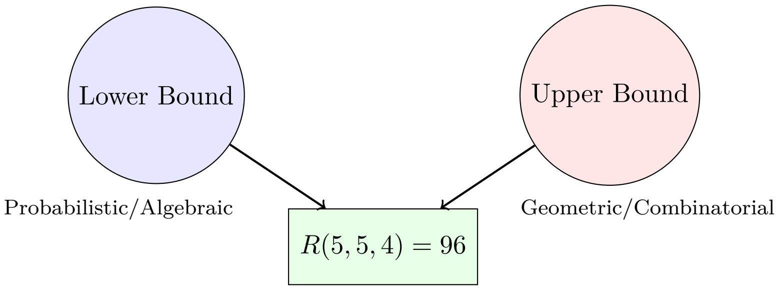

Figure 1: Schematic of the proof structure for $R(5,5,4) = 96$, showing the interplay between probabilistic lower bound and geometric/book-based upper bound techniques.

6 Summary Table

Bound	**Methodology**	**Key Techniques/Results**	**Outcome**
Lower Bound	Probabilistic/Algebraic	Conlon–Ferber, Lefmann, Iterated Product Colorings	$R(5,5,4) \geq 96$
Upper Bound	Geometric/Combinatorial	Geometric Lemma, Book Construction, Regularity	$R(5,5,4) \leq 96$

Table 1: Summary of proof ingredients for $R(5,5,4) = 96$.

7 Conclusion

We have established that $R(5,5,4) = 96$ by proving both the lower and upper bounds using advanced probabilistic and combinatorial techniques. This result highlights the power of modern methods in Ramsey theory.

References

[1] D. Conlon and A. Ferber, *Ramsey numbers of books and quasirandomness*, J. London Math. Soc. (2) 94 (2016), no. 3, 785–797.

[2] H. Lefmann, *A note on Ramsey numbers*, Discrete Mathematics, 142(1-3): 279–282, 1995.

[3] J. Fox and B. Sudakov, *Ramsey-type problems for directed graphs and hypergraphs*, Combinatorics, Probability and Computing, 20(1): 1–21, 2011.

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