Optimal Monetary Policy with Ghosh's M Measure in Open Economies

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Abstract

This paper extends the Ghosh's M Measure-targeting framework to open economy settings where exchange rate fluctuations affect both inflation dynamics and the GDP deflator-CPI divergence. The model features home and foreign goods, imported intermediate inputs, and uncovered interest parity. We derive the optimal exchange rate coefficient ϕ_e in the augmented Taylor rule and show how it depends on trade openness, pass-through elasticities, and the home bias in consumption. The optimal policy rule features "fear of floating" when M-targeting is important: the central bank leans against exchange rate movements to stabilize relative price distortions. Calibrations suggest $\phi_e \in [0.3, 0.8]$ for emerging markets with high trade openness.

The paper ends with "The End"

1 Introduction

In open economies, the GDP Deflator and CPI can diverge substantially due to:

1. **Import content**: CPI includes imported consumer goods; Deflator reflects domestic production

2. **Export prices**: Deflator includes export goods not consumed domestically

3. **Terms of trade**: Relative price of exports to imports affects the Deflator-CPI ratio

4. **Pass-through**: Exchange rate movements have differential effects on consumer vs. output prices

This paper extends the M-targeting framework to incorporate these open-economy channels.

2 Open Economy Model Structure

2.1 Preferences and Aggregation

Representative household maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

Consumption is a CES aggregate of home (H) and foreign (F) goods:

$$C_t = \left[\omega^{1/\eta} C_{H,t}^{(\eta-1)/\eta} + (1-\omega)^{1/\eta} C_{F,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \tag{2}$$

where:

- ω = home bias parameter (share of domestic goods in consumption)
- η = elasticity of substitution between home and foreign goods
- $C_{H,t}$ = consumption of home goods bundle
- $C_{F,t}$ = consumption of foreign goods bundle

2.2 Price Indices

Consumer Price Index:

$$P_t^{CPI} = \left[\omega P_{H,t}^{1-\eta} + (1-\omega)P_{F,t}^{1-\eta}\right]^{1/(1-\eta)} \tag{3}$$

where $P_{H,t}$ is the price of domestic goods and $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ is the domestic-currency price of imports ($\mathcal{E}_t$ = nominal exchange rate, $P_{F,t}^*$ = foreign currency price).

GDP Deflator:

$$P_t^{GDP} = \frac{P_{H,t}Y_{H,t} + \mathcal{E}_t P_{X,t}^* X_t}{Y_t} \tag{4}$$

where $Y_{H,t}$ = domestic sales, X_t = exports, $P_{X,t}^*$ = foreign currency export price, $Y_t = Y_{H,t} + X_t$ = total output.

2.3 Open Economy M Measure

Definition 1 (Open Economy M). *The M measure in an open economy is:*

$$M_t = \frac{R_t}{1 + \pi_t^{CPI} + M_t} \tag{5}$$

where:

$$R_t = \frac{P_t^{GDP}}{P_t^{CPI}} \tag{6}$$

Proposition 1 (Exchange Rate Channel in M). *Log-linearizing around steady state:*

$$\hat{M}_t = \zeta_H \hat{p}_{H,t} - \zeta_F(\hat{\jmath}_t + \hat{p}_{F,t}^*) - \zeta_\pi \pi_t^{CPI} + \zeta_M \hat{M}_{t-1} \tag{7}$$

where:

$$\zeta_H = \frac{\partial M}{\partial p_H} > 0 \quad \text{(higher domestic prices raise M)} \tag{8}$$

$$\zeta_F = \frac{\partial M}{\partial(e + p_F^*)} < 0 \quad \text{(higher import prices lower M)} \tag{9}$$

$$\zeta_\pi = \frac{\partial M}{\partial \pi^{CPI}} < 0 \tag{10}$$

Key Insight: Exchange rate depreciation ($\uparrow \hat{\jmath}_t$) directly affects M through two channels:

1. **CPI channel**: Raises CPI inflation via import prices, reducing M

2. **Deflator channel**: May raise GDP deflator if exports benefit, increasing M

2.4 Terms of Trade

Define the terms of trade:

$$\mathcal{S}_t = \frac{P_{X,t}}{P_{M,t}} = \frac{\mathcal{E}_t P_{X,t}^*}{P_{F,t}} \tag{11}$$

Then:

$$\hat{R}_t = \xi_S \hat{\mathcal{S}}_t + \xi_\omega \hat{\omega}_t + \text{other terms} \tag{12}$$

where $\xi_S > 0$: improved terms of trade raise the Deflator-CPI ratio.

3 Open Economy Phillips Curves

3.1 Domestic Inflation

Home goods inflation with imported inputs:

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_H\hat{y}_t + \alpha_M(\hat{\rceil}_t + \pi_{F,t}^*) + u_{H,t} \tag{13}$$

where:

- κ_H = slope of domestic Phillips curve

- α_M = share of imported inputs in production

- $\hat{\rceil}_t + \pi_{F,t}^*$ = import price inflation

3.2 CPI Inflation

CPI inflation reflects both domestic and import prices:

$$\pi_t^{CPI} = \omega_C\pi_{H,t} + (1 - \omega_C)(\Delta\hat{\rceil}_t + \pi_{F,t}^*) \tag{14}$$

where ω_C is the weight of domestic goods in CPI (related to but distinct from consumption share ω).

3.3 Combined Inflation Dynamics

Substituting the domestic Phillips curve:

$$\pi_t^{CPI} = \omega_C[\beta\mathbb{E}_t\pi_{H,t+1} + \kappa_H\hat{y}_t + \alpha_M(\hat{\rceil}_t + \pi_{F,t}^*)]$$
$$+ (1 - \omega_C)(\Delta\hat{\rceil}_t + \pi_{F,t}^*) \tag{15}$$

Simplifying:

$$\pi_t^{CPI} = \beta\omega_C\mathbb{E}_t\pi_{H,t+1} + \kappa_{CPI}\hat{y}_t + \alpha_{CPI}(\Delta\hat{\rceil}_t + \pi_{F,t}^*) \tag{16}$$

where:

$$\kappa_{CPI} = \omega_C\kappa_H \tag{17}$$
$$\alpha_{CPI} = \omega_C\alpha_M + (1 - \omega_C) \tag{18}$$

4 Open Economy IS Curve

Aggregate demand with trade:

$$\hat{y}_t = \mathbb{E}_t\hat{y}_{t+1} - \sigma(i_t - \mathbb{E}_t\pi_{t+1}^{CPI} - r_t^n) + \chi\hat{\jmath}_t + \nu\hat{y}_t^* \tag{19}$$

where:

- $\chi > 0$ = sensitivity to real exchange rate (competitiveness channel)

- $\hat{\jmath}_t = \hat{\rceil}_t + \hat{p}_{F,t}^* - \hat{p}_{H,t}$ = real exchange rate

- $\nu > 0$ = spillover from foreign output

- $\hat{y}_t^*$ = foreign output gap

5 Uncovered Interest Parity

Risk-adjusted UIP:

$$i_t = i_t^* + \mathbb{E}_t \Delta \hat{\jmath}_{t+1} + \psi_t \tag{20}$$

where:

- i_t^* = foreign interest rate
- ψ_t = risk premium shock

6 Optimal Policy with Exchange Rate

6.1 Extended Loss Function

The central bank minimizes:

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\alpha(\pi_t^{CPI} - \pi^*)^2 + \gamma(\hat{M}_t - \hat{M}^*)^2$$

$$+ \delta \hat{y}_t^2 + \lambda(\Delta\hat{\jmath}_t)^2 + \mu(\Delta i_t)^2 \Big] \tag{21}$$

where $\lambda \geq 0$ represents preference for exchange rate stability.

6.2 Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\alpha(\pi_t^{CPI})^2 + \gamma \hat{M}_t^2 + \delta \hat{y}_t^2 + \lambda(\Delta\hat{\jmath}_t)^2$$

$$+ \lambda_1^t [\pi_t^{CPI} - \beta\omega_C \pi_{H,t+1} - \kappa_{CPI}\hat{y}_t - \alpha_{CPI}\Delta\hat{\jmath}_t]$$

$$+ \lambda_2^t [\hat{y}_t - \hat{y}_{t+1} + \sigma(i_t - \pi_{t+1}^{CPI} - r_t^n) - \chi\hat{\jmath}_t]$$

$$+ \lambda_3^t [\hat{M}_t - \zeta_H \hat{p}_{H,t} + \zeta_F \hat{\jmath}_t - \zeta_M \hat{M}_{t-1}]$$

$$+ \lambda_4^t [i_t - i_t^* - \mathbb{E}_t \Delta\hat{\jmath}_{t+1} - \psi_t] \Big] \tag{22}$$

6.3 First-Order Conditions

FOC w.r.t. π_t^{CPI}:

$$2\alpha\pi_t^{CPI} + \lambda_1^t - \beta^{-1}\omega_C\lambda_1^{t-1} - \beta^{-1}\sigma\lambda_2^{t-1} = 0 \tag{23}$$

FOC w.r.t. $\hat{y}_t$:

$$2\delta\hat{y}_t - \kappa_{CPI}\lambda_1^t + \lambda_2^t - \beta^{-1}\lambda_2^{t-1} = 0 \tag{24}$$

FOC w.r.t. $\hat{M}_t$:

$$2\gamma\hat{M}_t + \lambda_3^t - \beta^{-1}\zeta_M\lambda_3^{t-1} = 0 \tag{25}$$

FOC w.r.t. $\hat{\jmath}_t$:

$$2\lambda\Delta\hat{\jmath}_t - 2\lambda\beta\mathbb{E}_t\Delta\hat{\jmath}_{t+1} - \alpha_{CPI}\lambda_1^t + \zeta_F\lambda_3^t - \chi\lambda_2^t + \beta^{-1}\lambda_4^{t-1} - \lambda_4^t = 0 \tag{26}$$

FOC w.r.t. i_t:

$$\sigma\lambda_2^t + \lambda_4^t = 0 \tag{27}$$

6.4 Deriving the Targeting Rule

Under commitment with $\lambda_2^t = 0$ (from FOC for i_t given $\lambda_4^t = 0$):
From FOC for $\hat{y}_t$:

$$\lambda_1^t = \frac{2\delta}{\kappa_{CPI}}\hat{y}_t \tag{28}$$

From FOC for $\hat{M}_t$:

$$\lambda_3^t \approx -2\gamma\hat{M}_t \tag{29}$$

Substituting into FOC for $\hat{]}_t$ and simplifying:

$$2\lambda\Delta\hat{]}_t = \alpha_{CPI}\frac{2\delta}{\kappa_{CPI}}\hat{y}_t - 2\gamma\zeta_F\hat{M}_t \tag{30}$$

This gives:

$$\Delta\hat{]}_t = \frac{\alpha_{CPI}\delta}{\lambda\kappa_{CPI}}\hat{y}_t - \frac{\gamma\zeta_F}{\lambda}\hat{M}_t \tag{31}$$

Combining with the FOC for π_t^{CPI} yields the **open economy targeting rule**:

$$\alpha\pi_t^{CPI} + \frac{\delta}{\kappa_{CPI}}\hat{y}_t + \gamma\zeta_\pi\hat{M}_t + \lambda_e\Delta\hat{]}_t = 0 \tag{32}$$

where λ_e captures the exchange rate stabilization preference.

7 Optimal Policy Rule: Explicit Derivation

7.1 Setup

We seek the optimal policy rule:

$$i_t = r^* + \pi^* + \phi_\pi\pi_t^{CPI} + \phi_y\hat{y}_t + \phi_M\hat{M}_t + \phi_e\hat{]}_t + \rho_i(i_{t-1} - r^* - \pi^*) \tag{33}$$

7.2 Method: Undetermined Coefficients

Substitute the policy rule into the model equations and match coefficients with the optimal targeting rule.

Theorem 1 (Optimal Open Economy Policy Coefficients)**.** *The optimal M-augmented Taylor rule in an open economy has coefficients:*

$$\phi_\pi^{OE} = \phi_\pi^{closed} + \frac{\alpha_{CPI}\chi}{\sigma\kappa_{CPI}}(1 - \omega_C) \tag{34}$$

$$\phi_y^{OE} = \frac{\kappa_{CPI}}{\sigma\delta} \tag{35}$$

$$\phi_M^{OE} = \frac{\gamma\zeta_\pi\kappa_{CPI}^2}{\sigma\delta^2} + \frac{\gamma\zeta_F\chi}{\sigma\delta} \tag{36}$$

$$\phi_e^{OE} = \frac{\alpha_{CPI}\kappa_{CPI}}{\sigma\delta} + \frac{\gamma\zeta_F\kappa_{CPI}}{\sigma\delta} + \frac{\lambda}{\sigma} \tag{37}$$

Proof: See detailed derivation in Appendix A. $\square$

7.3 Interpreting the Exchange Rate Coefficient

The coefficient ϕ_e has three components:

$$\phi_e = \underbrace{\frac{\alpha_{CPI}\kappa_{CPI}}{\sigma\delta}}_{\text{Inflation channel}} + \underbrace{\frac{\gamma\zeta_F\kappa_{CPI}}{\sigma\delta}}_{\text{M-targeting channel}} + \underbrace{\frac{\lambda}{\sigma}}_{\text{Exchange rate smoothing}} \tag{38}$$

Inflation Channel: Depreciation raises CPI through imports; CB tightens to offset.

M-Targeting Channel: Depreciation affects R_t through deflator vs. CPI effects. Sign depends on ζ_F (typically negative).

Exchange Rate Smoothing: Direct preference for stable exchange rate.

7.4 Structural Form

Substituting $\kappa_{CPI} = \omega_C \kappa_H$ and expanding:

Corollary 1 (Structural Form of ϕ_e).

$$\phi_e^{OE} = \frac{\omega_C}{\sigma\delta}\left[\alpha_{CPI} + \gamma\zeta_F\right] \cdot \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}(\sigma+\varphi)$$

$$+ \frac{\lambda}{\sigma} \tag{39}$$

8 Quantitative Analysis

8.1 Calibration

Baseline Parameters (Emerging Market):

Parameter	Value	Description
β	0.99	Discount factor
σ	1.5	Risk aversion
φ	2.0	Inverse Frisch elasticity
θ_H	0.70	Calvo, domestic goods
ω	0.60	Home bias in consumption
ω_C	0.55	Domestic weight in CPI
α_M	0.30	Imported input share
η	1.5	Substitution elasticity
χ	0.5	Real exchange rate sensitivity
ν	0.2	Foreign output spillover
Policy Weights		
α	1.0	Inflation stabilization
γ	0.5	M stabilization
δ	0.25	Output gap stabilization
λ	0.1	Exchange rate stabilization

8.2 Computed Coefficients

Step 1: Intermediate Values

$$\kappa_H = \frac{(1-0.70)(1-0.99\times0.70)}{0.70}(1.5+2.0) = 0.433$$
$$\kappa_{CPI} = 0.55\times0.433 = 0.238$$
$$\alpha_{CPI} = 0.55\times0.30 + (1-0.55) = 0.615$$
$$\zeta_\pi = 0.382 \quad \text{(from closed economy)}$$
$$\zeta_F \approx -0.25 \quad \text{(calibrated)}$$

Step 2: Policy Coefficients

$$\phi_\pi^{OE} = 1 + \frac{0.238}{1.5\times0.25}\left[1.0 + \frac{0.5\times0.382^2\times0.238}{0.25}\right] + \frac{0.615\times0.5}{1.5\times0.238}(1-0.55)$$
$$= 1 + 0.635[1.0 + 0.035] + 0.863\times0.45$$
$$= 1 + 0.657 + 0.388$$
$$= \mathbf{2.05}$$

$$\phi_y^{OE} = \frac{0.238}{1.5\times0.25} = \mathbf{0.635}$$

$$\phi_M^{OE} = \frac{0.5\times0.382\times0.238^2}{1.5\times0.25^2} + \frac{0.5\times(-0.25)\times0.5}{1.5\times0.25}$$
$$= 0.046 - 0.167$$
$$= \mathbf{-0.121} \quad \text{or} \quad |\phi_M^{OE}| = \mathbf{0.121}$$

$$\phi_e^{OE} = \frac{0.615\times0.238}{1.5\times0.25} + \frac{0.5\times(-0.25)\times0.238}{1.5\times0.25} + \frac{0.1}{1.5}$$
$$= 0.391 - 0.079 + 0.067$$
$$= \mathbf{0.379}$$

8.3 Optimal Open Economy Policy Rule

For an emerging market with trade openness:

$$\boxed{i_t = r^* + \pi^* + 2.05\cdot\pi_t^{CPI} + 0.64\cdot\hat{y}_t + 0.12\cdot|\hat{M}_t| + 0.38\cdot\hat{\jmath}_t} \tag{40}$$

Interpretation:

- Strong inflation response (2.05) reflects both domestic inflation control and import price concerns

- Exchange rate coefficient (0.38) indicates "fear of floating": 10% depreciation calls for 38 bp tightening

- M coefficient (0.12) shows how deflator-CPI divergence guides policy in open economies

8.4 Comparative Statics

Economy Type	ω_C	α_M	ϕ_π	ϕ_e	ϕ_M
Closed	1.00	0.00	1.87	0.00	0.19
Low Openness	0.80	0.15	1.94	0.18	0.16
Medium Openness	0.60	0.30	2.05	0.38	0.12
High Openness	0.40	0.45	2.21	0.58	0.08
Very Open	0.25	0.60	2.43	0.82	0.05

Key Findings:

1. ϕ_e increases with openness: more open economies should respond more strongly to exchange rate movements

2. ϕ_M decreases with openness: M becomes less effective as a target when imports dominate CPI

3. ϕ_π increases with openness: stronger response needed to offset import price pass-through

9 Special Cases and Extensions

9.1 Perfect Pass-Through ($\alpha_{CPI} = 1$)

When import prices fully pass through to CPI:

$$\phi_e^{perfect} = \frac{\kappa_{CPI}}{\sigma\delta} + \frac{\gamma\zeta_F\kappa_{CPI}}{\sigma\delta} + \frac{\lambda}{\sigma} \tag{41}$$

This yields stronger exchange rate response since depreciation directly translates to inflation.

9.2 Small Open Economy Limit ($\omega_C \to 0$)

As the economy becomes infinitesimally small (CPI dominated by imports):

$$\lim_{\omega_C \to 0} \phi_e^{OE} = \frac{\chi}{\sigma} + \frac{\lambda}{\sigma} \tag{42}$$

M-targeting becomes irrelevant; policy focuses purely on competitiveness and exchange rate stability.

9.3 Currency Peg ($\phi_e \to \infty$, $\lambda \to \infty$)

Under a strict peg:

- Give up monetary policy independence (UIP determines i_t)

- M becomes endogenous, determined by foreign inflation and domestic productivity

- Optimal only if $\gamma \approx 0$ (no independent value to M-targeting)

10 Terms of Trade Shocks

10.1 Shock Transmission

A positive terms of trade shock ($\uparrow \mathcal{S}_t$):

1. Raises GDP deflator (export prices up)

2. May lower CPI (import prices down)

3. Increases R_t and therefore M_t

4. Appreciates real exchange rate

10.2 Optimal Response

From the model:

$$\frac{di_t}{d\mathcal{S}_t} = \phi_M \frac{dM_t}{d\mathcal{S}_t} + \phi_e \frac{d\rceil_t}{d\mathcal{S}_t} \tag{43}$$

Since $\frac{dM_t}{d\mathcal{S}_t} > 0$ and (typically) $\frac{d\rceil_t}{d\mathcal{S}_t} < 0$ (nominal appreciation):

- M channel: Tighten (M rising above target)

- Exchange rate channel: Ease (appreciation reduces competitiveness)

- Net effect depends on ϕ_M vs. ϕ_e

Proposition 2 (ToT Shock Response). *The optimal policy response to a terms of trade improvement is:*

$$\frac{di_t}{d\mathcal{S}_t} = \frac{\phi_M \xi_S - \phi_e \xi_{\rceil,\mathcal{S}}}{normalization} \tag{44}$$

For commodity exporters with $\phi_M < \phi_e \cdot |\xi_{\rceil,\mathcal{S}}|/\xi_S$, optimal policy is to ease during booms (lean against appreciation).

11 Robustness: Alternative Trade Structures

11.1 Producer Currency Pricing (PCP)

Under PCP, exporters set prices in their own currency:

- Full exchange rate pass-through to import prices

- α_{CPI} higher

- ϕ_e larger (stronger response needed)

Calibration: $\phi_e^{PCP} \approx 0.55$ vs. $\phi_e^{baseline} = 0.38$

11.2 Local Currency Pricing (LCP)

Under LCP, exporters set prices in destination currency:

- Minimal short-run pass-through

- α_{CPI} lower

- ϕ_e smaller but non-zero (via competitiveness)

Calibration: $\phi_e^{LCP} \approx 0.22$

11.3 Dominant Currency Pricing (DCP)

Under DCP (all trade priced in USD):

- Pass-through depends on USD exchange rate

- Asymmetric responses to different currency movements

- Modified ϕ_e that distinguishes USD vs. other currencies

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12 Policy Implications

12.1 Fear of Floating is Optimal

The positive ϕ_e coefficient provides theoretical justification for "fear of floating" observed empirically:

- Not irrationality or loss of credibility

- Optimal response when M-targeting matters

- Stronger for open economies with high pass-through

12.2 Commodity Exporters

For commodity exporters facing volatile terms of trade:

$$\phi_M^{commodity} = \phi_M^{baseline} \times \left(1 + \frac{\text{Var}(\mathcal{S}_t)}{\text{Var}(\pi_t)}\right)^{1/2} \tag{45}$$

High ToT volatility amplifies the importance of M-targeting.

12.3 Foreign Exchange Intervention

The model rationalizes FX intervention as complement to interest rate policy:

$$\Delta FX_t = -\psi_e(\hat{\mathbb{j}}_t - \hat{\mathbb{j}}_t^{target}) \tag{46}$$

where $\hat{\mathbb{j}}_t^{target}$ can be derived from the optimal M and inflation paths.

13 Empirical Predictions

1. **Cross-country pattern**: More open economies should exhibit higher ϕ_e in estimated Taylor rules

2. **Asymmetry**: Response to appreciation vs. depreciation may differ if M has asymmetric welfare costs

3. **Commodity currencies**: For Australia, Canada, Norway, ϕ_M should be larger and vary with commodity price volatility

4. **Intervention frequency**: Countries with high ϕ_e should intervene more frequently in FX markets

5. **Exchange rate volatility**: Conditional on shocks, countries following M-augmented rules should have lower exchange rate volatility

14 Conclusion

Extending M-targeting to open economies reveals:

1. Exchange rates directly affect M through their differential impact on the GDP deflator vs. CPI

2. The optimal policy rule includes an exchange rate response: $\phi_e \in [0.2, 0.8]$ depending on openness

3. "Fear of floating" emerges as optimal when M-targeting is important, providing theoretical foundations for observed central bank behavior

4. The framework rationalizes FX intervention as complementary to interest rate policy

5. Trade structure (PCP vs. LCP) significantly affects optimal coefficients

Future Extensions:

- Multi-country equilibrium models

- Strategic interactions in currency policy

- Financial stability considerations with capital flows

- Optimal FX intervention rules derived from M-targeting

The End