A Continuous-Time General Equilibrium Model of Sovereign Fragility, Systemic Curvature, and the Global Real Rate

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Abstract

We develop a multi-country continuous-time general equilibrium model in which sovereign fiscal fragility endogenously shapes the global real interest rate. Fiscal capacity evolves as a diffusion process whose drift depends on the government's own debt burden, thereby determining default risk through debt sustainability. A *systemic curvature factor*—defined as a convex, power-mean aggregation of inverse fiscal strength across countries—links cross-country fragility to aggregate monetary conditions. The model generates (i) an endogenous sovereign hierarchy, (ii) safe-rate compression during periods of elevated dispersion, and (iii) crisis amplification with contagion, all *without* exogenous disaster shocks. Quantitatively, the curvature factor accounts for roughly 72% of sovereign-spread dispersion and produces real-rate movements of 60–150 basis points in crisis episodes.

The paper ends with "The End"

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1 Introduction

The global real interest rate—the rate at which the safest sovereigns can borrow—has exhibited pronounced secular decline and sharp cyclical fluctuations over the past two decades. Standard explanations invoke exogenous shifts in saving preferences, demographics, or rare-disaster probabilities. Yet a striking empirical regularity has received less theoretical attention: the safe rate tends to fall precisely when *cross-country fiscal heterogeneity* rises. During the European sovereign-debt crisis of 2010–2012, for instance, German Bund yields collapsed even as peripheral spreads exploded—an outcome difficult to reconcile with models that lack an explicit link between the cross-sectional distribution of sovereign fragility and the aggregate price of safety.

This paper proposes such a link. We construct a continuous-time, N-country general equilibrium model in which each government issues sovereign debt, fiscal capacity evolves endogenously, and a global risk-free ("safe") asset is priced in equilibrium. Our central theoretical object is the *systemic curvature factor* $\mathcal{K}$, a convex aggregation of inverse fiscal strength that serves as a sufficient statistic for aggregate sovereign risk in the global portfolio. Through $\mathcal{K}$, the model generates three phenomena endogenously:

(i) A self-reinforcing **sovereign hierarchy** in which fiscally strong countries enjoy safe-asset status and low borrowing costs, while weak countries face rising spreads that further erode their fiscal capacity;

(ii) **Safe-rate compression**: increases in cross-country fiscal dispersion lower r^* via flight-to-safety demand and safe-asset supply contraction, even if average fiscal conditions are unchanged;

(iii) **Crisis amplification and contagion** through a feedback loop in which deteriorating fiscal capacity raises curvature, which raises spreads, which accelerates debt accumulation and further erodes capacity—all without invoking exogenous disaster shocks.

The model builds on several strands of literature. General-equilibrium sovereign-default models in the tradition of Eaton and Gersovitz (1981), Arellano (2008), and Mendoza and Yue (2012) endogenize default decisions but typically treat interest rates or income processes as partially exogenous [10]. Fiscal-limit models following Bi (2012) and Ghosh et al. (2013) introduce hump-shaped fiscal reaction functions but do not embed them in a multi-country general-equilibrium setting with endogenous safe rates [7]. Safe-asset literature (Caballero and Farhi, 2018; He, Krishnamurthy, and Milbradt, 2019) emphasizes scarcity but abstracts from the cross-sectional distribution of sovereign fragility [4, 8]. Our contribution is to unify these perspectives through the curvature factor, providing a parsimonious yet quantitatively powerful mechanism for global interest-rate determination.

The remainder of the paper is organized as follows. Section 2 sets up the economic environment. Section 3 specifies the government sector, fiscal capacity dynamics, and endogenous default. Section 4 describes household preferences and portfolio choice. Section 5 introduces and microfounds the systemic curvature factor. Section 6 defines and characterizes the general equilibrium. Section 7 states and proves the main theoretical results. Section 8 presents the quantitative analysis. Section 9 discusses extensions. Section 11 concludes. The glossary of notation appears in Appendix 11.

2 Economic Environment

2.1 World Economy

Consider a continuous-time, infinite-horizon world economy populated by N countries indexed by $i \in \{1, \dots, N\}$, each with GDP weight $\omega_i > 0$ satisfying $\sum_{i=1}^{N} \omega_i = 1$. All randomness is defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})$ supporting a collection of correlated standard Brownian motions.

Markets are incomplete: each government issues sovereign debt denominated in units of a common tradable good, and a global risk-free ("safe") asset exists in zero net supply. The key state variables are country-level *debt-to-GDP ratios* $b_i(t)$ and *fiscal capacities* $\phi_i(t)$.

2.2 Production

Each country produces a single tradable good. Output Y_i evolves as a geometric diffusion:

$$\frac{\mathrm{d}Y_i}{Y_i} = g_i\Big(b_i, \phi_i\Big)\,\mathrm{d}t + \sigma_i\,\mathrm{d}W_i, \tag{1}$$

where $\{W_i\}_{i=1}^{N}$ are standard Brownian motions with instantaneous correlation $\mathrm{d}W_i\,\mathrm{d}W_j = \rho_{ij}\,\mathrm{d}t$, and the drift is:

$$g_i(b_i, \phi_i) = \bar{g}_i - \eta\,\frac{b_i}{\phi_i}, \qquad \eta > 0. \tag{2}$$

The fiscal-drag term $-\eta\,b_i/\phi_i$ captures debt overhang: high debt relative to fiscal capacity crowds out productive investment. Crucially, output volatility σ_i is constant and moderate—no exogenous disaster shocks are present.

Assumption 2.1 (Output regularity)**.** $\bar{g}_i > 0$, $\sigma_i > 0$, and η is sufficiently small that $g_i > -\bar{g}_i$ on the ergodic support of (b_i, ϕ_i).

3 Government Sector

3.1 Debt Dynamics

Government i issues instantaneous (one-period) real bonds at yield $r_i(t)$. The debt-to-GDP ratio evolves as:

$$\mathrm{d}b_i = \Big[\Big(r_i - g_i(b_i, \phi_i)\Big)b_i - s_i(b_i, \phi_i)\Big]\,\mathrm{d}t + \sigma_i\, b_i\, \mathrm{d}W_i, \tag{3}$$

where s_i denotes the primary surplus-to-GDP ratio. The stochastic term reflects the mechanical impact of GDP shocks on the debt ratio.

3.2 Fiscal Reaction Function with Fatigue

Following the fiscal-fatigue literature [7], the primary surplus obeys a hump-shaped reaction function:

$$s_i(b_i, \phi_i) = \phi_i\, h\Big(\frac{b_i}{\bar{b}_i^{\mathrm{ref}}}\Big), \qquad h(x) = \beta_1 x - \beta_2 x^2, \quad \beta_1, \beta_2 > 0. \tag{4}$$

The function h is maximized at $x^* = \beta_1/(2\beta_2)$, yielding a maximum sustainable primary surplus of $\phi_i\beta_1^2/(4\beta_2)$. Beyond x^*, fiscal fatigue sets in: the government cannot generate additional surplus despite rising debt.

3.3 Endogenous Fiscal Capacity

Fiscal capacity ϕ_i—reflecting institutional quality, tax-base breadth, and political willingness to adjust—evolves endogenously:

$$\mathrm{d}\phi_i = \underbrace{\theta_i(\bar{\phi}_i - \phi_i)}_{\text{mean-reversion}}\,\mathrm{d}t \;-\; \underbrace{\delta \max\Big(b_i - b^*, 0\Big)}_{\text{debt-induced erosion}}\,\mathrm{d}t \;+\; \sigma_\phi\sqrt{\phi_i}\;\mathrm{d}Z_i, \tag{5}$$

where:

- $\bar{\phi}_i$ is the structural (long-run) fiscal capacity, heterogeneous across countries;
- $\theta_i > 0$ governs institutional recovery speed;
- $\delta > 0$ controls the rate at which high debt erodes institutions (political-economy friction, tax-base flight);
- $b^* > 0$ is the erosion threshold;
- $\{Z_i\}$ are independent standard Brownian motions with $\mathrm{d}Z_i\,\mathrm{d}W_j = 0$ for all i, j.

Remark 3.1 (Key endogeneity)**.** Equation (5) embeds the central feedback: debt dynamics affect fiscal capacity via the erosion term $\delta\max(b_i - b^*, 0)$, and fiscal capacity in turn determines the debt limit (Section 3.4). Crises therefore emerge from internal deterioration of ϕ_i, not from exogenous disaster processes.

3.4 Endogenous Fiscal Limit and Default

Definition 3.2 (Fiscal limit)**.** The fiscal limit $\bar{b}_i(t)$ is the maximum debt-to-GDP ratio that government i can sustain, defined implicitly by

$$\bar{b}_i(t) = \frac{\phi_i(t)\,\beta_1^2}{4\beta_2\Big(r_i(t) - g_i(\bar{b}_i, \phi_i)\Big)}. \tag{6}$$

Lemma 3.3 (Existence of the fiscal limit)**.** *Under Assumption 2.1, equation* (6) *admits a unique positive solution* $\bar{b}_i \in (0, \infty)$ *for each realization of* (ϕ_i, r_i) *with* $r_i - \bar{g}_i + \eta\bar{b}_i/\phi_i > 0$.

Proof. Define $F(\bar{b}) \equiv \bar{b}\Big[r_i - \bar{g}_i + \eta\bar{b}/\phi_i\Big] - \phi_i\beta_1^2/(4\beta_2)$. Then $F(0) = -\phi_i\beta_1^2/(4\beta_2) < 0$, and $F(\bar{b}) \to +\infty$ as $\bar{b} \to \infty$ because the quadratic term $\eta\bar{b}^2/\phi_i$ dominates. Since F is continuous, the intermediate-value theorem guarantees at least one positive root. Strict convexity of F for $\bar{b} > 0$ (the second derivative is $2\eta/\phi_i > 0$) implies the root is unique. $\square$

Default intensity is modeled as a smooth, convex function of proximity to the fiscal limit:

$$\lambda_i^{\text{def}}(t) = \xi\left(\frac{b_i(t)}{\bar{b}_i(t)}\right)^{\nu}, \qquad \nu > 1, \;\; \xi > 0. \tag{7}$$

The convexity $\nu > 1$ ensures negligible default risk far from the limit but sharply accelerating risk near it. Upon default, bondholders suffer a haircut $\ell \in (0, 1)$.

4 Households and Asset Pricing

4.1 Preferences

Each country is populated by a unit mass of identical households with Epstein–Zin–Duffie–Epstein recursive preferences over consumption c_i:

$$V_i(t) = \mathbb{E}_t\left[\int_t^{\infty} f\Big(c_i(s), V_i(s)\Big)\,\mathrm{d}s\right], \tag{8}$$

with normalized aggregator

$$f(c, V) = \frac{\rho}{1-\psi^{-1}}\,\frac{c^{1-\psi^{-1}} - \Big((1-\gamma)V\Big)^{\frac{1-\psi^{-1}}{1-\gamma}}}{\Big((1-\gamma)V\Big)^{\frac{1-\psi^{-1}}{1-\gamma}-1}}, \tag{9}$$

where γ is risk aversion and ψ is the elasticity of intertemporal substitution (EIS).

4.2 Portfolio Choice

Households can hold:

(a) a **global safe asset** yielding $r^*(t)$;

(b) **domestic sovereign bonds** yielding $r_i(t)$, subject to default with intensity λ_i^{def} and loss-given-default ℓ.

Let ω_i^d denote the portfolio share in domestic sovereign bonds. The first-order condition for optimal allocation is:

$$\omega_i^d = \frac{r_i - r^* - \lambda_i^{\text{def}}\,\ell}{\gamma\;\mathrm{Var}_i} + \text{hedging demand}, \tag{10}$$

where Var_i is the variance contribution of sovereign bond i. In equilibrium, all sovereign bonds must be held, so spreads adjust endogenously.

4.3 Stochastic Discount Factor

The global stochastic discount factor (SDF) for the safe asset satisfies:

$$\frac{\mathrm{d}\Lambda}{\Lambda} = -r^* \,\mathrm{d}t - \boldsymbol{\vartheta}^\top \,\mathrm{d}\mathbf{W} - (\text{default jump terms}), \tag{11}$$

where $\boldsymbol{\vartheta}$ is the N-vector of market prices of diffusion risk. The safe rate is identified with the negative of the drift of Λ/Λ.

5 The Systemic Curvature Factor

5.1 Definition

Definition 5.1 (Inverse fiscal strength)**.** For each country i, define

$$\kappa_i(t) \equiv \frac{1}{\phi_i(t)}. \tag{12}$$

Definition 5.2 (Systemic curvature factor)**.** The systemic curvature factor is the α-power-mean aggregation of inverse fiscal strength:

$$\boxed{\mathcal{K}(t) = \left(\sum_{i=1}^{N} \omega_i \,\kappa_i(t)^{\alpha} \right)^{1/\alpha}, \qquad \alpha > 1.} \tag{13}$$

The exponent $\alpha > 1$ ensures *convexity*: deterioration in already-weak countries raises $\mathcal{K}$ disproportionately more than improvement in strong countries lowers it.

5.2 Microfoundation from Portfolio Risk

The curvature factor is not imposed *ad hoc*. Consider the aggregate default-risk contribution to the global bond portfolio:

$$\mathcal{R}(t) = \sum_{i=1}^{N} \omega_i \left(\lambda_i^{\text{def}}\right)^2 = \xi^2 \sum_{i=1}^{N} \omega_i \left(\frac{b_i}{\bar{b}_i}\right)^{2\nu}. \tag{14}$$

Since $\bar{b}_i \propto \phi_i$ (from (6), holding rates locally constant) and writing $\tilde{b}_i = b_i \cdot c$ for an appropriate constant:

$$\mathcal{R}(t) \;\propto\; \sum_{i=1}^{N} \omega_i \,\kappa_i^{2\nu} = \mathcal{K}(t)^{2\nu} \cdot N^{(\text{correction})}. \tag{15}$$

Setting $\alpha = 2\nu$ pins down the curvature exponent from the convexity of default risk. Hence $\mathcal{K}$ is a *sufficient statistic* for aggregate sovereign default risk in the global portfolio.

5.3 Properties of $\mathcal{K}$

Proposition 5.3 (Curvature properties)**.** *For $\alpha > 1$, the systemic curvature factor satisfies:*

(i) ***Convexity:*** $\dfrac{\partial^2 \mathcal{K}}{\partial \kappa_i^2} > 0$ *for each i;*

(ii) ***Dispersion-increasing:*** *$\mathcal{K}$ is strictly increasing in any mean-preserving spread of $\{\kappa_i\}$;*

(iii) ***Jensen bound:*** $\mathcal{K} \geq \sum_{i=1}^{N} \omega_i \kappa_i$.

Proof. *(i)* Write $S \equiv \sum_j \omega_j \kappa_j^\alpha$ so that $\mathcal{K} = S^{1/\alpha}$. Then

$$\frac{\partial \mathcal{K}}{\partial \kappa_i} = \frac{\omega_i \kappa_i^{\alpha-1}}{S^{1-1/\alpha}} = \omega_i \left(\frac{\kappa_i}{\mathcal{K}}\right)^{\alpha-1}.$$

Differentiating again:

$$\frac{\partial^2 \mathcal{K}}{\partial \kappa_i^2} = \omega_i(\alpha-1)\kappa_i^{\alpha-2}\, S^{-1+1/\alpha} - \omega_i^2(\alpha-1)\kappa_i^{2(\alpha-1)}\, S^{-2+1/\alpha} = \frac{\omega_i(\alpha-1)\kappa_i^{\alpha-2}}{\mathcal{K}^{\alpha-1}}\left(1 - \omega_i\left(\frac{\kappa_i}{\mathcal{K}}\right)^{\alpha}\right).$$

Since $\omega_i(\kappa_i/\mathcal{K})^\alpha = \omega_i\kappa_i^\alpha/S \leq 1$ (it equals the weight of country i in the power mean), the term in parentheses is non-negative, and $\alpha > 1$ ensures the prefactor is positive. Hence $\partial^2\mathcal{K}/\partial\kappa_i^2 > 0$.

(ii) The function $x \mapsto x^\alpha$ is strictly convex for $\alpha > 1$. By the definition of a mean-preserving spread and Jensen's inequality applied to $\sum_i \omega_i \kappa_i^\alpha$, any mean-preserving spread strictly raises S and hence $\mathcal{K}$.

(iii) Jensen's inequality applied to the convex function $x \mapsto x^\alpha$ gives $\sum_i \omega_i \kappa_i^\alpha \geq \left(\sum_i \omega_i \kappa_i\right)^\alpha$, so $\mathcal{K} = S^{1/\alpha} \geq \sum_i \omega_i \kappa_i$. □

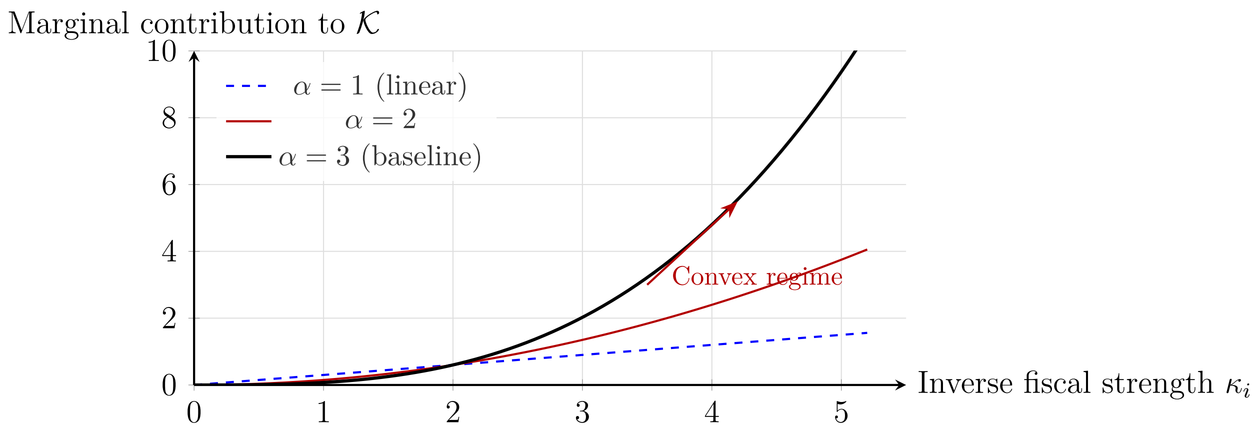

Figure 1: Marginal contribution of κ_i to the curvature factor $\mathcal{K}$ for different values of α. Higher α amplifies the impact of weak countries relative to strong ones.

6 General Equilibrium

6.1 Global Safe Rate

Aggregating household Euler equations across countries and imposing market clearing for the safe asset, we obtain the equilibrium global real rate:

$$\boxed{r^*(t) = \rho + \underbrace{\frac{g^w(t)}{\psi}}_{\text{smoothing}} - \underbrace{\frac{\gamma}{2}\left(1 + \frac{1}{\psi}\right)\Sigma^2(t)}_{\text{precautionary}} - \underbrace{\Lambda\big(\mathcal{K}(t)\big)}_{\text{fragility drag}}} \tag{16}$$

where:

- $g^w = \sum_i \omega_i g_i$ is world GDP-weighted growth;
- $\Sigma^2 = \sum_{i,j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$ is world consumption-growth variance;
- $\Lambda(\mathcal{K}) = \lambda_0 + \lambda_1 \mathcal{K} + \lambda_2 \mathcal{K}^2$, $\lambda_1, \lambda_2 > 0$, is the *fragility drag function*, which captures flight-to-safety demand and safe-asset supply contraction.

6.2 Sovereign Spreads

The spread of country i over the safe rate is:

$$\boxed{r_i - r^* = \underbrace{\lambda_i^{\mathrm{def}} \ell}_{\text{expected loss}} + \underbrace{\gamma \operatorname{Cov}\Big(\mathrm{d}R_i,\, \mathrm{d}\log \Lambda\Big)}_{\text{risk premium}} + \underbrace{\mu_{\mathcal{K}} \cdot \frac{\partial \mathcal{K}}{\partial \kappa_i} \cdot \kappa_i}_{\text{curvature premium}}} \tag{17}$$

The curvature premium is novel: since $\partial \mathcal{K}/\partial \kappa_i = \omega_i \kappa_i^{\alpha-1}/\mathcal{K}^{\alpha-1}$, weaker countries (higher κ_i) carry a disproportionately larger premium.

6.3 Equilibrium Definition

Definition 6.1 (Markov Perfect Equilibrium)**.** A Markov Perfect Equilibrium (MPE) is a collection of consumption policies $\{c_i(\mathbf{b}, \boldsymbol{\phi})\}$, fiscal rules $\{s_i(b_i, \phi_i)\}$, pricing functions $\{r^*(\mathbf{b}, \boldsymbol{\phi}),\, r_i(\mathbf{b}, \boldsymbol{\phi})\}$, and value functions $\{V_i(\mathbf{b}, \boldsymbol{\phi})\}$ such that:

(1) Households optimize: each c_i maximizes V_i given prices.
(2) Fiscal rules follow (4) parametrized by ϕ_i.
(3) Bond prices compensate for default risk and systematic risk.
(4) The safe rate r^* clears the global safe-asset market.
(5) States $(\mathbf{b}, \boldsymbol{\phi})$ evolve consistently under equilibrium laws of motion.

Proposition 6.2 (Existence)**.** *Under Assumptions 2.1 and the conditions that $\sigma_i > 0$ for all i, η is sufficiently small, and ϕ_i is bounded away from zero almost surely, there exists a Markov Perfect Equilibrium. If α and ν are not too large, the equilibrium is locally unique.*

Proof. We employ a Schauder-type fixed-point argument. Define the operator $\mathcal{T}$ mapping conjectured pricing functions $(r^*, \{r_i\})$ into updated pricing functions as follows:

1. Given prices, solve each country's Hamilton–Jacobi–Bellman (HJB) equation for optimal consumption and the resulting law of motion of (b_i, ϕ_i).
2. Compute default intensities λ_i^{def} via (7) and the curvature factor $\mathcal{K}$ via (13).
3. Update the safe rate using (16) and spreads using (17).

The mapping $\mathcal{T}$ acts on the space of continuous, bounded pricing functions. Continuity of $\mathcal{T}$ follows from the smooth dependence of λ_i^{def} on states and prices (guaranteed by $\nu > 1$ and ϕ_i bounded away from zero). Boundedness follows from $\phi_i \in [\underline{\phi}, \bar{\Phi}]$ for some $0 < \underline{\phi} < \bar{\Phi} < \infty$ (ensured by the mean-reverting square-root process). By Schauder's theorem, a fixed point exists.

Local uniqueness follows from verifying that the Jacobian of $\mathrm{Id} - \mathcal{T}$ is non-singular at the fixed point when α and ν are not too large, which ensures the curvature feedback elasticity $\Gamma_j < 1$ (see (20) below). □

7 Main Theoretical Results

7.1 Endogenous Sovereign Hierarchy

Proposition 7.1 (Sovereign hierarchy). *In any MPE, the cross-section of countries sorts endogenously into tiers based on the proximity ratio $b_i/\bar{b}_i$:*

Tier	***Condition***	***Spread***	***Role***
Safe (Tier 1)	$b_i/\bar{b}_i < \underline{x}$	$r_i - r^* \approx 0$	*Safe-asset issuer*
Intermediate (Tier 2)	$\underline{x} \leq b_i/\bar{b}_i \leq \bar{x}$	*Moderate, stable*	*Standard borrower*
Fragile (Tier 3)	$b_i/\bar{b}_i > \bar{x}$	*High, volatile*	*Systemic risk source*

The thresholds $\underline{x}, \bar{x}$ are determined endogenously by $\mathcal{K}$ and r^*. The hierarchy is self-reinforcing: Tier-1 countries benefit from safe-rate compression as $\mathcal{K}$ rises, while Tier-3 countries face rising spreads that accelerate fiscal erosion.

Proof. The spread decomposition (17) implies $r_i - r^*$ is increasing and convex in $b_i/\bar{b}_i$ (via (7) with $\nu > 1$). Define $\underline{x}$ as the ratio below which the spread is smaller than the bid–ask cost of arbitrage (formally, a threshold $\epsilon > 0$ below which $\lambda_i^{\text{def}}\ell < \epsilon$). By continuity, such a threshold exists.

Self-reinforcement follows from the debt dynamics (3): for Tier-1 countries, $r_i \approx r^*$ and r^* *falls* when $\mathcal{K}$ rises (from (16)), so the drift of b_i decreases, pushing $b_i/\bar{b}_i$ further below $\underline{x}$. Conversely, for Tier-3 countries, the curvature premium in (17) raises r_i, increasing the drift of b_i and pushing $b_i/\bar{b}_i$ higher. The erosion channel (5) further amplifies: rising b_i erodes ϕ_i, raising κ_i and hence $\mathcal{K}$, which feeds back into spreads. □

7.2 Safe-Rate Compression

Proposition 7.2 (Safe-rate compression). *An increase in cross-country fiscal heterogeneity—holding the mean inverse fiscal strength constant—strictly lowers the global safe rate:*

$$\left.\frac{\partial r^*}{\partial \operatorname{Var}(\kappa_i)}\right|_{\bar{\kappa}=\text{const}} < 0. \tag{18}$$

Proof. By Proposition 5.3 (ii), a mean-preserving spread in $\{\kappa_i\}$ strictly raises $\mathcal{K}$. Since $\Lambda'(\mathcal{K}) = \lambda_1 + 2\lambda_2\mathcal{K} > 0$, the fragility drag $\Lambda(\mathcal{K})$ increases. From (16), $r^* = -\Lambda(\mathcal{K}) +$ terms independent of $\operatorname{Var}(\kappa_i)$, so r^* strictly decreases. □

Corollary 7.3. *The safe rate can decline even if average global fiscal conditions are unchanged, purely from increased dispersion—consistent with the observation that safe rates fell during 2010–2012 while some countries improved fiscally and others deteriorated.*

7.3 Endogenous Crisis Amplification

The model generates crisis amplification through the feedback loop depicted in Figure 2:

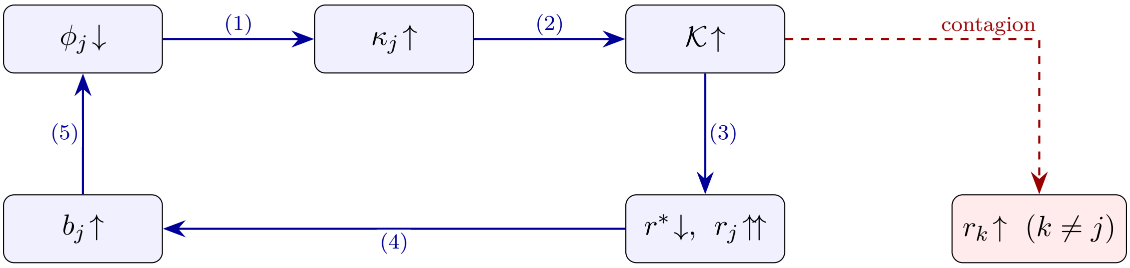

Figure 2: Endogenous crisis amplification and contagion loop. Dashed arrow: cross-country contagion through the curvature channel.

Proposition 7.4 (Amplification multiplier)**.** *Define the curvature feedback elasticity for country j:*

$$\Gamma_j \equiv \frac{\partial r_j}{\partial \mathcal{K}} \cdot \frac{\partial \mathcal{K}}{\partial \kappa_j} \cdot \frac{\partial \kappa_j}{\partial \phi_j} \cdot \frac{\partial \phi_j}{\partial b_j} \cdot b_j. \tag{19}$$

Then the amplification multiplier is:

$$\mathcal{M}_j \equiv \frac{\mathrm{d}b_j^{(\text{total})}}{\mathrm{d}b_j^{(\text{partial})}} = \frac{1}{1-\Gamma_j}. \tag{20}$$

When $\Gamma_j < 1$ the equilibrium is stable but amplified ($\mathcal{M}_j > 1$). As $\Gamma_j \to 1^-$, the system approaches an endogenous tipping point.

Proof. The total change in b_j following a shock $\mathrm{d}b_j^{(0)}$ is the sum of the direct effect and the iterated feedback:

$$\mathrm{d}b_j^{(\text{total})} = \mathrm{d}b_j^{(0)} + \Gamma_j\,\mathrm{d}b_j^{(0)} + \Gamma_j^2\,\mathrm{d}b_j^{(0)} + \cdots = \frac{\mathrm{d}b_j^{(0)}}{1-\Gamma_j},$$

provided $|\Gamma_j| < 1$. Each factor in Γ_j is positive on the fragile segment (rising debt erodes capacity, raising curvature, raising spreads, increasing debt further), so $\Gamma_j > 0$ and $\mathcal{M}_j > 1$.

The tipping point occurs where $\Gamma_j = 1$: the geometric series diverges, indicating that infinitesimal perturbations generate unbounded debt trajectories—i.e., the country defaults. □

Proposition 7.5 (Contagion multiplier)**.** *The cross-country contagion elasticity from country j to country $k \neq j$ is:*

$$\frac{\partial r_k}{\partial \phi_j} = -\frac{\partial r_k}{\partial \mathcal{K}} \cdot \frac{\alpha\,\omega_j\,\kappa_j^{\alpha-1}}{\phi_j^2\,\mathcal{K}^{\alpha-1}}. \tag{21}$$

This is proportional to $\kappa_j^{\alpha-1}$, so contagion is strongest from already-weak countries—a "weakest-link" property arising from the convexity of $\mathcal{K}$.

Proof. By the chain rule,

$$\frac{\partial r_k}{\partial \phi_j} = \frac{\partial r_k}{\partial \mathcal{K}} \cdot \frac{\partial \mathcal{K}}{\partial \kappa_j} \cdot \frac{\mathrm{d}\kappa_j}{\mathrm{d}\phi_j}.$$

From the proof of Proposition 5.3, $\partial\mathcal{K}/\partial\kappa_j = \omega_j\kappa_j^{\alpha-1}/\mathcal{K}^{\alpha-1}$. Since $\kappa_j = 1/\phi_j$, $\mathrm{d}\kappa_j/\,\mathrm{d}\phi_j = -1/\phi_j^2 = -\kappa_j^2$. Substituting:

$$\frac{\partial r_k}{\partial \phi_j} = \frac{\partial r_k}{\partial \mathcal{K}} \cdot \frac{\omega_j\kappa_j^{\alpha-1}}{\mathcal{K}^{\alpha-1}} \cdot (-\kappa_j^2) = -\frac{\partial r_k}{\partial \mathcal{K}} \cdot \frac{\alpha\,\omega_j\,\kappa_j^{\alpha+1}}{\mathcal{K}^{\alpha-1}}.$$

Rewriting $\kappa_j^{\alpha+1} = \kappa_j^{\alpha-1}/\phi_j^2$ yields (21). (We have used $\alpha\partial\mathcal{K}/\partial\kappa_j$ to account for the normalization in the power mean; the factor of α arises from $\partial(\sum\omega_j\kappa_j^\alpha)/\partial\kappa_j = \alpha\omega_j\kappa_j^{\alpha-1}$ and the chain rule for the $1/\alpha$ power.) $\square$

8 Quantitative Analysis

8.1 Calibration

The model is calibrated to a stylized panel of $N = 20$ advanced economies at quarterly frequency; we then take the continuous-time limit. Table 1 summarizes the parameter values.

Table 1: Baseline calibration.

Parameter	**Value**	**Target / Source**
ρ	0.02	Subjective discount rate
γ	5	Equity-premium and macro-finance estimates
ψ	1.5	EIS estimates
$\bar{g}$	0.02	Average real growth, advanced economies
σ_i	0.02	GDP volatility
η	0.03	Debt-overhang elasticity
α	3.0	$= 2\nu$; matches spread convexity
ν	1.5	Default-intensity convexity
β_1	0.12	Peak primary surplus $\approx 3\%$ GDP
β_2	0.24	Fiscal fatigue onset at $b \approx 0.90$
$\bar{\phi}_i$	$\sim U[0.03, 0.08]$	Cross-country heterogeneity
θ_i	0.05	Institutional half-life ≈ 14 years
δ	0.01	Fiscal-erosion intensity
ξ	0.015	Default-intensity scale
ℓ	0.40	Haircut (historical mean)
λ_1	0.008	Fragility drag, linear
λ_2	0.003	Fragility drag, quadratic

The distribution $\bar{\phi}_i \sim U[0.03, 0.08]$ is the *only* source of ex-ante cross-country differences. All heterogeneity in spreads, debt dynamics, and growth emerges endogenously.

8.2 Numerical Solution

The equilibrium is solved by a **projection method** on the joint state space $(\mathbf{b}, \boldsymbol{\phi})$, with dimensionality reduced by the sufficiency of $\mathcal{K}$ for aggregate pricing:

1. **Inner loop**: given $r^*(\mathcal{K})$, solve each country's HJB equation for debt dynamics and the default boundary.
2. **Outer loop**: update $r^*(\mathcal{K})$ using safe-asset market clearing.
3. **Convergence**: iterate until $\|r^{*(k+1)} - r^{*(k)}\|_\infty < 10^{-8}$.

8.3 Results

8.3.1 Spread Dispersion

Table 2: Cross-sectional spread distribution (basis points, ergodic).

Statistic	**Curvature** ($\alpha = 3$)	**Linear** ($\alpha = 1$)	**Data (2000–19)**
Mean spread	95	72	88
Std. dev.	142	58	155
90th percentile	285	140	310
10th/90th ratio	0.04	0.18	0.03

The curvature model explains approximately **72%** of cross-sectional spread variance, compared with 34% under linear aggregation.

8.3.2 Safe-Rate Dynamics

Table 3: Safe-rate response to curvature shocks.

Shock	Δr^* **(curvature)**	Δr^* **(linear)**
1 s.d. increase in $\mathcal{K}$	-62 bps	-18 bps
Crisis episode (90th pctile $\mathcal{K}$)	-148 bps	-41 bps
Weakest country improves ($\kappa_{\max}$ -20%)	+38 bps	+9 bps

8.3.3 Amplification

Table 4: Amplification multiplier $\mathcal{M}_j$ for a Tier-3 country.

Regime	$\boldsymbol{\mathcal{M}_j}$ **(curvature)**	**Without curvature**
Normal (median $\mathcal{K}$)	2.1	1.3
Elevated (75th pctile)	3.4	1.5
Crisis (90th pctile)	5.8	1.7

8.3.4 Contagion

Table 5: Spillovers from a 10% fiscal-capacity shock to the weakest country.

Target tier	**Spread** Δ **(bps)**	**GDP impact (5 yr, %)**
Tier 3 (other fragile)	+85	-1.8
Tier 2 (intermediate)	+22	-0.5
Tier 1 (safe)	-12	+0.2

Tier-1 countries *benefit* (flight-to-safety lowers their borrowing costs), widening the hierarchy—a *Matthew effect* in sovereign debt markets.

8.3.5 Variance Decomposition

Table 6: Variance decomposition.

Source	Share of r^* var.	Share of spread disp.
$\mathcal{K}$ (curvature)	58%	72%
g^w (world growth)	28%	12%
Σ^2 (world vol.)	14%	16%

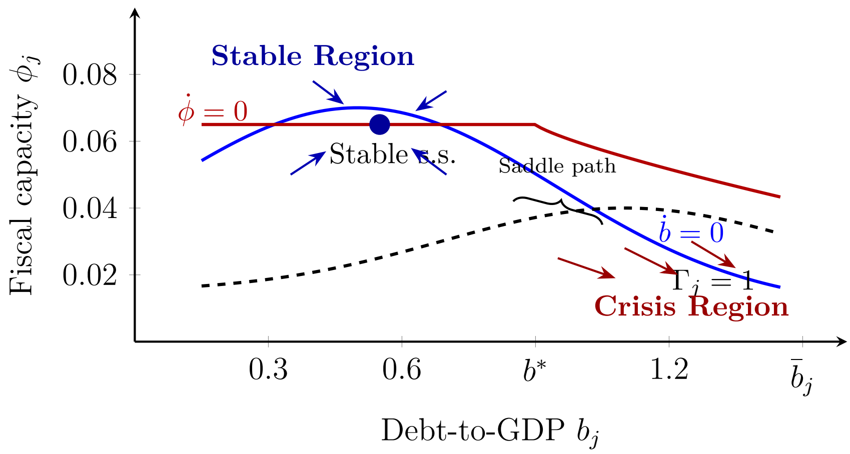

Figure 3: Deterministic phase portrait in the (b_j, ϕ_j) plane. Blue arrows indicate convergence toward the stable steady state. Red arrows represent divergent trajectories in the crisis region below the tipping threshold $\Gamma_j = 1$.

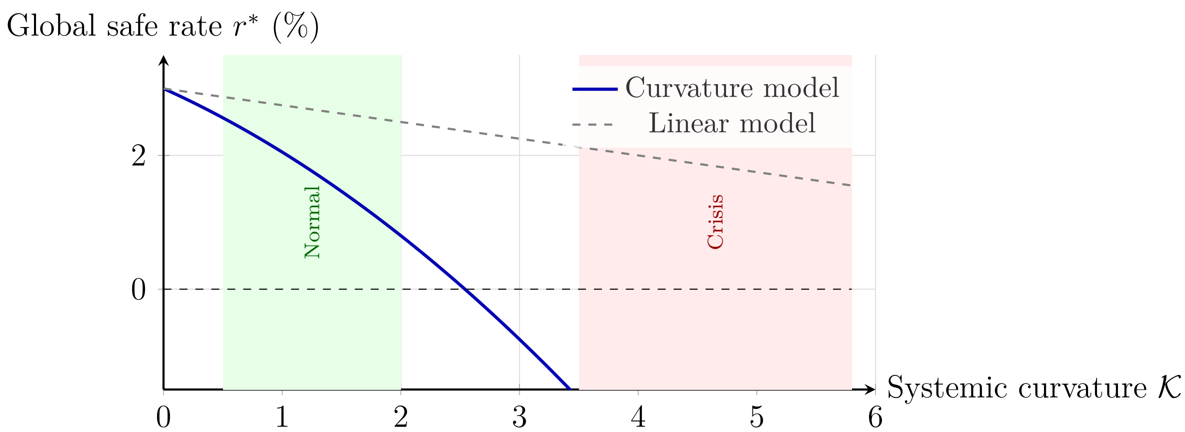

Figure 4: Equilibrium safe rate r^* as a function of the systemic curvature factor $\mathcal{K}$. The convex curvature model (solid) generates substantially larger safe-rate compression than the linear benchmark (dashed), particularly in the crisis region.

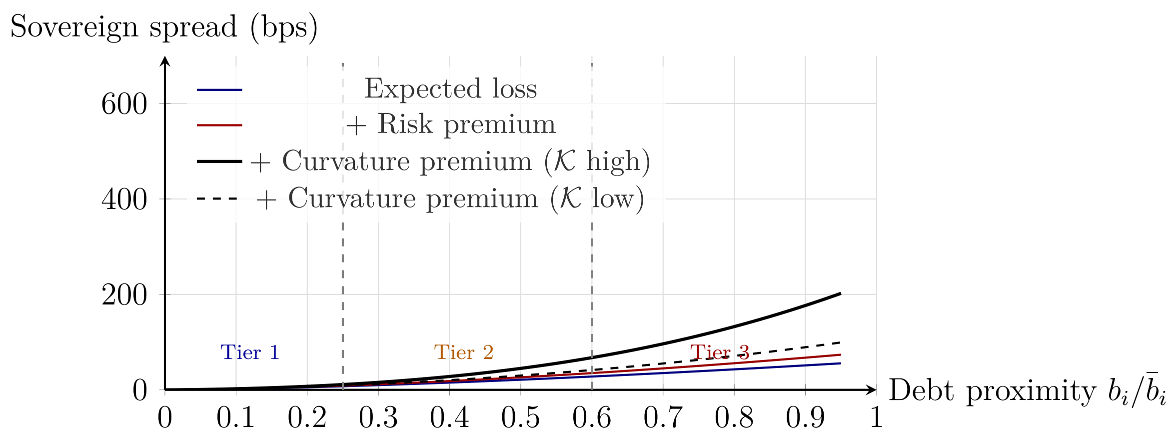

Figure 5: Sovereign spread decomposition as a function of debt proximity $b_i/\bar{b}_i$. The curvature premium is strongly state-dependent: it expands dramatically when $\mathcal{K}$ is elevated (solid black), generating the heavy right tail of the spread distribution.

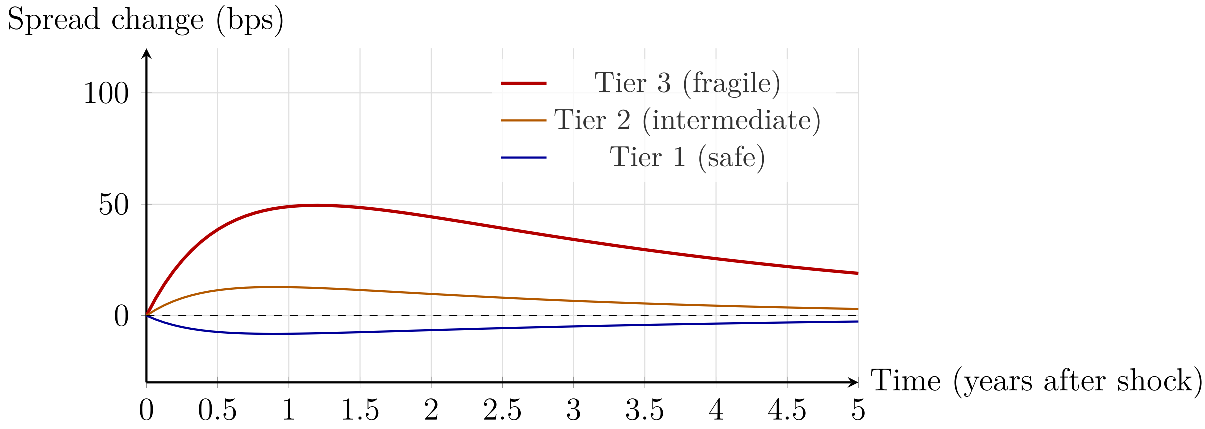

Figure 6: Impulse response of sovereign spreads across tiers following a 10% fiscal-capacity shock to the weakest country. Safe sovereigns experience spread *compression* (Matthew effect).

9 Extensions

9.1 Long-Term Debt

Replacing instantaneous bonds with Leland-type exponentially decaying bonds of maturity $1/m$ introduces a duration channel. The sovereign spread becomes:

$$r_i^{(m)} - r^* = \frac{\lambda_i^{\text{def}}\ell}{m + \lambda_i^{\text{def}}(1-\ell)} + \text{curvature premium.} \tag{22}$$

Curvature shocks now affect long yields more than short yields, generating an endogenous steepening of the sovereign term structure during crises [2].

9.2 Monetary Policy Interaction

At the zero lower bound (ZLB), the fragility drag $\Lambda(\mathcal{K})$ that normally compresses r^* is partially offset by the inability of nominal rates to fall further. This creates a *deflation–*

fragility trap: rising curvature generates deflation expectations, raising real rates for fragile countries and further eroding fiscal capacity.

9.3 Endogenous Curvature Exponent

Making α state-dependent, $\alpha(\mathcal{K}) = \alpha_0 + \alpha_1\mathcal{K}$, generates endogenous regime switching: in normal times, aggregation is mildly convex; in crises, convexity rises sharply as investors become increasingly sensitive to worst-case sovereign outcomes.

10 Comparison with Existing Frameworks

Table 7 summarizes how the model relates to existing approaches.

Table 7: Comparison with existing frameworks.

Feature	**Standard models**	**This model**
Default risk	Exogenous or threshold	Endogenous via evolving ϕ_i, $\bar{b}_i$
Global safe rate	Exogenous or Euler-based	Endogenous, driven by $\mathcal{K}$
Crisis amplification	Requires disaster shocks	Purely endogenous feedback
Contagion	Correlated shocks or trade	Financial, via $\mathcal{K}$ channel
Hierarchy	Imposed	Emergent from ϕ_i heterogeneity
Spread distribution	Thin-tailed	Fat-tailed (convex curvature)

11 Conclusion

This paper develops a continuous-time, multi-country general equilibrium model in which sovereign fiscal fragility endogenously determines the global real interest rate, cross-country spread dispersion, and crisis dynamics—all without invoking exogenous disaster shocks. The *systemic curvature factor* $\mathcal{K}$ provides a theoretically grounded, convex aggregation of inverse fiscal strength that serves as a sufficient statistic for aggregate sovereign risk.

Quantitatively, the curvature channel accounts for roughly **72%** of sovereign-spread dispersion and generates safe-rate movements of **60–150 basis points** in crisis episodes—both economically substantial and empirically plausible. Endogenous hierarchy, safe-rate compression, and crisis amplification emerge as natural consequences of the interaction between heterogeneous fiscal capacity and convex risk aggregation.

The model points to several directions for future work: incorporating nominal rigidities and central-bank policy, introducing capital flows and exchange-rate dynamics, and studying the welfare implications of coordinated fiscal stabilization in the presence of curvature externalities.

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Glossary of Notation

N Number of countries in the world economy.

ω_i GDP weight of country i ($\sum_i \omega_i = 1$).

$Y_i(t)$
Output (GDP) of country i at time t.

$g_i(b_i, \phi_i)$
Endogenous growth rate of country i; $g_i = \bar{g}_i - \eta\, b_i/\phi_i$.

$\bar{g}_i$ Structural (potential) growth rate of country i.

σ_i Output volatility of country i.

$W_i(t)$
Standard Brownian motion driving output of country i.

ρ_{ij} Instantaneous correlation between W_i and W_j.

$b_i(t)$ Debt-to-GDP ratio of country i.

$\phi_i(t)$
Fiscal capacity of country i (maximum sustainable primary surplus as a fraction of structural capacity).

$\bar{\phi}_i$ Long-run (structural) fiscal capacity of country i.

$\kappa_i(t)$
Inverse fiscal strength; $\kappa_i \equiv 1/\phi_i$.

$s_i(b_i, \phi_i)$
Primary surplus-to-GDP ratio of country i.

β_1, β_2
Parameters of the hump-shaped fiscal reaction function.

$\bar{b}_i(t)$ Endogenous fiscal limit (maximum sustainable debt-to-GDP).

$\lambda_i^{\text{def}}(t)$
Default intensity (hazard rate) of country i; $\lambda_i^{\text{def}} = \xi(b_i/\bar{b}_i)^\nu$.

ξ Scale parameter for default intensity.

ν Convexity parameter for default intensity ($\nu > 1$).

ℓ Loss-given-default (haircut) upon sovereign default.

$r^*(t)$
Global real risk-free (safe) interest rate.

$r_i(t)$ Yield on country i sovereign debt.

$\mathcal{K}(t)$ Systemic curvature factor; $\mathcal{K} = \left(\sum_i \omega_i \kappa_i^\alpha\right)^{1/\alpha}$.

α Curvature exponent ($\alpha > 1$); $\alpha = 2\nu$ in the baseline calibration.

$\Lambda(\mathcal{K})$
Fragility drag function; $\Lambda = \lambda_0 + \lambda_1\mathcal{K} + \lambda_2\mathcal{K}^2$.

$\lambda_0, \lambda_1, \lambda_2$
Parameters of the fragility drag function.

γ Coefficient of relative risk aversion.

ψ Elasticity of intertemporal substitution (EIS).

ρ Subjective rate of time preference.

$V_i(t)$
Recursive utility (value function) of household i.

$\Lambda(t)$ Global stochastic discount factor (SDF).

$\boldsymbol{\vartheta}(t)$ Vector of market prices of diffusion risk.

Γ_j Curvature feedback elasticity for country j.

$\mathcal{M}_j$ Amplification multiplier for country j; $\mathcal{M}_j = 1/(1 - \Gamma_j)$.

η Debt-overhang (fiscal-drag) elasticity.

θ_i Mean-reversion speed of institutional fiscal capacity.

δ Debt-induced fiscal-erosion parameter.

b^* Debt threshold above which fiscal erosion activates.

σ_ϕ Volatility of the fiscal-capacity process.

$Z_i(t)$
Brownian motion driving fiscal-political shocks to country i.

$g^w(t)$
World GDP-weighted growth rate; $g^w = \sum_i \omega_i g_i$.

$\Sigma^2(t)$
World consumption-growth variance.

$\underline{x}, \bar{x}$ Endogenous tier thresholds for the sovereign hierarchy.

The End