A Spectral Framework for Sovereign Stability and Collapse

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Abstract

In this paper, we develop a mathematically consistent framework for sovereign stability. A logistic Sovereign Collapse Probability Index (SCPI) is constructed from macroeconomic and institutional fundamentals. The model is extended to a continuous latent hierarchy, embedded in a network general equilibrium system, and analyzed using Perron–Frobenius spectral theory. The resulting structure yields an endogenous stability frontier and a centrality-weighted collapse ranking rule.

The paper ends with "The End"

1 Baseline Collapse Index

Let i index sovereign states. Define

$$Y_i := \text{GDP per capita (PPP, thousands)}, \tag{1}$$

$$A_i := 1_{\text{AAA rating}}, \tag{2}$$

$$N_i := 1_{\text{Nuclear weapons}}, \tag{3}$$

$$G_i := \text{Gender ratio (male/female)}. \tag{4}$$

Define the linear stability score

$$Z_i = \alpha - \beta \ln Y_i - \gamma A_i - \delta N_i + \theta |G_i - 1|, \tag{5}$$

where $\alpha, \beta, \gamma, \delta, \theta > 0$.

The Sovereign Collapse Probability Index is

$$\mathrm{SCPI}_i = \frac{1}{1 + e^{-Z_i}}. \tag{6}$$

Proposition 1. *If $\beta > 0$, then SCPI is strictly decreasing in income.*

Proof. Let $\Lambda(x) = 1/(1 + e^{-x})$. Since $\Lambda'(x) = \Lambda(x)(1 - \Lambda(x)) > 0$,

$$\frac{\partial \mathrm{SCPI}_i}{\partial \ln Y_i} = -\beta \Lambda'(Z_i) < 0.$$

□

2 Income Stability Frontier

Consider the reduced-form regression

$$\mathrm{logit}(\mathrm{SCPI}_i) = a + b \ln Y_i. \tag{7}$$

The stability frontier is defined by SCPI = 0.5. Since logit(0.5) = 0,

$$0 = a + b \ln Y^*. \tag{8}$$

Thus

$$\ln Y^* = -\frac{a}{b}. \tag{9}$$

Empirically, Y^* is approximately 24 (thousand PPP dollars), defining the income-based stability threshold.

3 Continuous Sovereign Hierarchy

Let latent sovereign strength be

$$S_i = \beta_1 \ln Y_i + \beta_2 C_i + \beta_3 D_i - \beta_4 R_i, \tag{10}$$

where

- $C_i \in [0, 1]$ is a continuous credibility index,
- $D_i \in [0, 1]$ is deterrence capacity,
- $R_i \geq 0$ captures structural risk.

Binary indicators are special cases: $C_i = A_i$, $D_i = N_i$.
Continuous-time collapse hazard:

$$\lambda_i = \lambda_0 e^{-S_i}, \qquad \lambda_0 > 0. \tag{11}$$

Collapse probability over horizon T:

$$1 - e^{-\lambda_0 e^{-S_i} T}. \tag{12}$$

The logistic SCPI arises as a reduced-form approximation:

$$\mathrm{SCPI}_i \approx \frac{1}{1 + e^{-\kappa S_i}}. \tag{13}$$

4 Network General Equilibrium

Let W be a nonnegative irreducible exposure matrix and $\eta \geq 0$ a contagion parameter.
Equilibrium strength satisfies

$$\mathbf{S} = (I - \eta W)^{-1}\mathbf{b}, \tag{14}$$

provided $\rho(\eta W) < 1$, where $\rho(\cdot)$ denotes spectral radius.

5 Spectral Contagion

Theorem 1 (Spectral Stability Condition). *Let W be nonnegative and irreducible. Then:*

1. *If $\eta\rho(W) < 1$, equilibrium exists and contagion decays geometrically.*
2. *If $\eta\rho(W) = 1$, the system is at a critical threshold.*
3. *If $\eta\rho(W) > 1$, contagion amplifies without bound.*

Proof. The Neumann series

$$(I - \eta W)^{-1} = \sum_{k=0}^{\infty} (\eta W)^k$$

converges if and only if $\rho(\eta W) < 1$. □

6 Centrality-Weighted Ranking

Let v be the Perron eigenvector satisfying

$$Wv = \rho(W)v, \qquad v_i > 0. \tag{15}$$

Normalize $\sum_i v_i = 1$.
Define the centrality-adjusted ranking score

$$R_i = \kappa(S_i - \chi v_i), \qquad \kappa, \chi > 0. \tag{16}$$

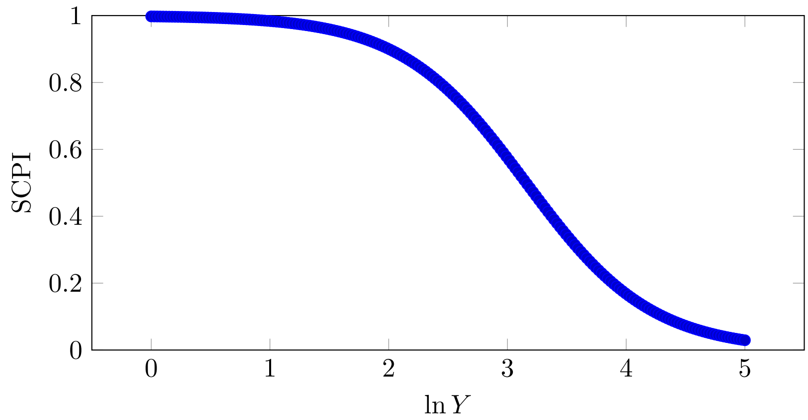
Ranking rule:

$$i \prec j \quad \text{if and only if} \quad R_i < R_j. \tag{17}$$

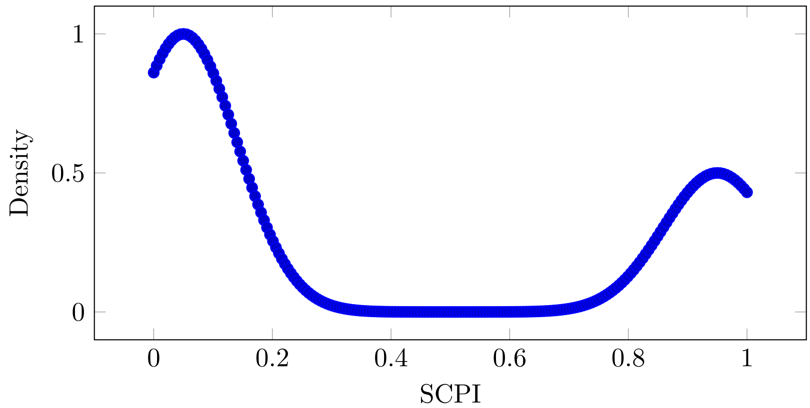
High systemic centrality reduces effective stability under contagion risk.

7 Illustrative Figures

Logistic Stability Curve



Stylized Bimodal Density



References

[1] E. Seneta, *Non-negative Matrices and Markov Chains*, Springer.

[2] C. Reinhart and K. Rogoff, *This Time Is Different*, Princeton University Press.

[3] D. Acemoglu and J. Robinson, *Why Nations Fail*, Crown.

Glossary

SCPI Sovereign Collapse Probability Index.

Spectral Radius Largest eigenvalue in modulus.

Perron Eigenvector Positive eigenvector associated with the spectral radius.

Stability Frontier Income level where SCPI equals 0.5.

Centrality Eigenvector-based systemic exposure measure.

The End