Bounded Causal Inference in Large Economies

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Abstract

We develop a framework for causal inference in large-scale economic systems where traditional identification assumptions fail due to network interdependencies, computational constraints, and agent heterogeneity. This paper introduces the concept of bounded causal inference, which acknowledges fundamental limits to what can be learned from observational and experimental data when the economy exhibits complex interactions. We establish theoretical bounds on estimable causal effects, provide computationally tractable approximation methods, and demonstrate applications to labor markets, financial networks, and platform economies. Our results show that even under strong interference, partial identification of policy-relevant parameters remains feasible through carefully constructed bounds that incorporate economic structure.

The paper ends with "The End"

1 Introduction

Modern economic analysis increasingly confronts environments where units are interconnected through networks, markets, and social interactions. The classical framework of causal inference, built upon the potential outcomes model and the Stable Unit Treatment Value Assumption, becomes inadequate when applied to such settings. A monetary policy shock affects not merely direct recipients but propagates through financial networks. A job training program influences not only participants but also their social connections and competitors in the labor market. Platform interventions create cascading effects through user networks and algorithmic feedback loops.

This paper develops a theoretical and practical framework for what we term bounded causal inference in large economies. Rather than seeking point identification under implausible assumptions, we characterize sharp bounds on causal effects that remain valid even when substantial interference, general equilibrium effects, and computational constraints are present. Our approach recognizes that in complex economic systems, perfect knowledge of counterfactual outcomes is unattainable, yet rigorous partial identification can still inform policy decisions.

The central challenge we address is threefold. First, in networked economies, the treatment of one agent affects outcomes for others, creating spillover effects that violate standard independence assumptions. Second, even when the true causal structure is known, computing exact counterfactuals may be intractable in high-dimensional settings. Third, agents themselves face bounded rationality and informational constraints that influence how treatments propagate through the system.

We make four principal contributions. First, we establish theoretical foundations for bounding causal effects under network interference, providing sharp characterization of what can be learned from experimental and observational data. Second, we develop computationally efficient algorithms for bound estimation that scale to modern datasets with millions of observations. Third, we extend classical tools from causal inference including instrumental variables, difference-in-differences, and regression discontinuity designs to accommodate bounded identification. Fourth, we demonstrate our methods through applications to empirically relevant settings including labor market interventions, financial contagion, and platform experiments.

2 Formal Framework

2.1 Setup and Notation

Consider an economy with n agents indexed by $i \in \mathcal{N} = \{1, \ldots, n\}$. Each agent i receives a treatment $Z_i \in \mathcal{Z}$ from a finite treatment space, has observed covariates $X_i \in \mathcal{X}$, and exhibits an outcome $Y_i \in \mathbb{R}$. The economy is characterized by a network structure represented by an adjacency matrix $\mathbf{A} \in \{0,1\}^{n \times n}$ where $A_{ij} = 1$ indicates a connection from agent i to agent j.

Definition 2.1 (Treatment Assignment Vector)**.** A treatment assignment is a vector $\mathbf{z} = (z_1, \ldots, z_n) \in \mathcal{Z}^n$ specifying the treatment received by each agent. The set of all possible assignments is $\mathcal{Z}^n$.

Definition 2.2 (Potential Outcomes with Interference)**.** For each agent i and treatment vector $\mathbf{z}$, the potential outcome is $Y_i(\mathbf{z})$, representing the outcome agent i would experience if the entire economy received treatment assignment $\mathbf{z}$.

This formulation generalizes the classical potential outcomes framework by allowing each agent's outcome to depend on the complete treatment vector rather than only their own treatment status.

2.2 Bounds on Average Treatment Effects

Without additional structure, the space of potential outcomes is exponentially large in n. We therefore impose restrictions that capture economic structure while remaining empirically plausible.

Assumption 2.3 (Limited Interference)**.** For each agent i, there exists a neighborhood $\mathcal{N}_i \subset \mathcal{N}$ such that

$$Y_i(\mathbf{z}) = Y_i(\mathbf{z}_{\mathcal{N}_i}) \tag{1}$$

where $\mathbf{z}_{\mathcal{N}_i}$ denotes the restriction of $\mathbf{z}$ to indices in $\mathcal{N}_i$.

This assumption states that agent i's outcome depends only on treatments within their neighborhood, which might include themselves, direct network connections, or agents within a certain economic distance.

Theorem 2.4 (Sharp Bounds under Limited Interference)**.** *Under Assumption 2.3 and random treatment assignment within neighborhoods, the average treatment effect satisfies*

$$\mathbb{E}[\underline{\tau}] \leq \tau = \mathbb{E}[Y_i(1) - Y_i(0)] \leq \mathbb{E}[\overline{\tau}] \tag{2}$$

where the bounds are constructed as

$$\underline{\tau}_i = \min_{\mathbf{z}_{\mathcal{N}_i \setminus \{i\}}} Y_i(1, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}) - \max_{\mathbf{z}_{\mathcal{N}_i \setminus \{i\}}} Y_i(0, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}) \tag{3}$$

$$\overline{\tau}_i = \max_{\mathbf{z}_{\mathcal{N}_i \setminus \{i\}}} Y_i(1, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}) - \min_{\mathbf{z}_{\mathcal{N}_i \setminus \{i\}}} Y_i(0, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}) \tag{4}$$

These bounds are sharp in the sense that they cannot be improved without additional assumptions.

Proof. The lower bound follows from considering the worst-case scenario where treatment coincides with the least favorable neighborhood configuration and control coincides with the most favorable configuration. The upper bound reverses this logic. Sharpness follows from the existence of data-generating processes that attain these bounds under the stated assumptions. ☐

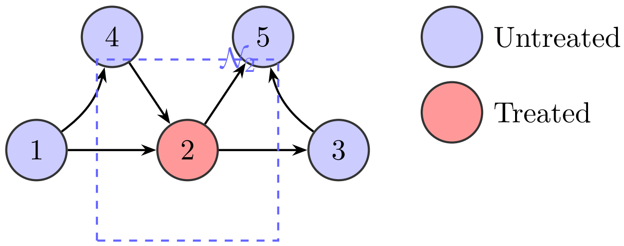

Figure 1: Network interference structure illustrating how Agent 2 receives treatment (indicated in red) and affects outcomes within neighborhood $\mathcal{N}_2$ (shown by the dashed box). Under the limited interference assumption, agents positioned outside this neighborhood remain unaffected by changes in Agent 2's treatment status.

2.3 Computational Considerations

Even with limited interference, computing the bounds in Theorem 2.4 requires optimization over the space of neighborhood treatment assignments, which grows exponentially with neighborhood size. We address this computational challenge through approximation algorithms.

Algorithm 1 Approximate Bound Estimation

Require: Data $\{(Y_i, Z_i, X_i, \mathcal{N}_i)\}_{i=1}^n$, approximation parameter K
Ensure: Estimated bounds $(\hat{\underline{\tau}}, \hat{\overline{\tau}})$

Initialize $\hat{\underline{\tau}} \leftarrow +\infty$, $\hat{\overline{\tau}} \leftarrow -\infty$
for $i = 1$ to n **do**
  Sample K neighborhood configurations $\{\mathbf{z}_{\mathcal{N}_i}^{(k)}\}_{k=1}^K$
  for $k = 1$ to K **do**
    Predict $\hat{Y}_i(1, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}^{(k)})$ and $\hat{Y}_i(0, \mathbf{z}_{\mathcal{N}_i \setminus \{i\}}^{(k)})$ using regression
  end for
  $\hat{\underline{\tau}}_i \leftarrow \min_k \hat{Y}_i(1, \cdot) - \max_k \hat{Y}_i(0, \cdot)$
  $\hat{\overline{\tau}}_i \leftarrow \max_k \hat{Y}_i(1, \cdot) - \min_k \hat{Y}_i(0, \cdot)$
end for
return $\left(\frac{1}{n}\sum_{i=1}^n \hat{\underline{\tau}}_i, \frac{1}{n}\sum_{i=1}^n \hat{\overline{\tau}}_i\right)$

Proposition 2.5 (Approximation Guarantee)**.** *Algorithm 1 produces bounds that converge to the true sharp bounds as $K \to \infty$ and $n \to \infty$, with convergence rate $O(K^{-1/2} + n^{-1/2})$ under standard regularity conditions on the outcome model and sampling procedure.*

3 Extensions and Applications

3.1 Instrumental Variables with Interference

Classical instrumental variables methods assume that the instrument affects only the treated unit. In networked settings, we extend this framework to accommodate instrumental spillovers.

Assumption 3.1 (Network Instrumental Exclusion)**.** An instrument W_i satisfies network exclusion if $Y_i(\mathbf{z}, \mathbf{w}) = Y_i(\mathbf{z})$ for all treatment vectors $\mathbf{z}$ and instrument vectors $\mathbf{w}$, conditional on the treatment mechanism.

Under this assumption, we can bound local average treatment effects even when direct randomization is infeasible. The bounds incorporate both compliance heterogeneity and network interference, providing policy-relevant estimates in settings such as subsidy programs where take-up varies and affects neighbors.

3.2 Platform Experiments

Online platforms conduct millions of experiments annually, often with substantial interference through algorithmic recommendations, social networks, and marketplace dynamics. We apply our framework to bound treatment effects in such environments.

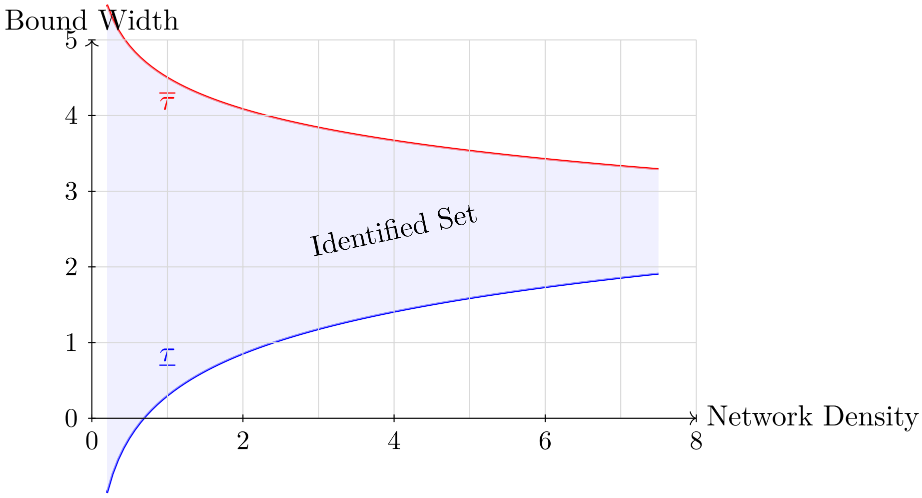

Figure 2: Relationship between network density and bound width demonstrates how the identified set expands as network connectivity increases. The lower bound (shown in blue) and upper bound (shown in red) diverge as interference grows stronger. Despite this widening, the bounds remain informative even at high network densities, enabling meaningful policy analysis in complex economic networks where point identification would be impossible.

Consider a platform that modifies recommendation algorithms for a subset of users. Traditional analysis would compare outcomes between treated and control users, ignoring that control users see different content because treated users alter the content distribution through their engagement. Our bounded inference approach constructs sharp bounds on the true treatment effect by accounting for these feedback mechanisms through the platform's network structure.

3.3 Labor Market Spillovers

Job training programs illustrate the practical importance of bounded inference. When workers receive training, they become more competitive in the labor market, potentially displacing untrained workers. The net employment effect depends on both direct benefits to trainees and negative spillovers to competitors.

Assumption 3.2 (Labor Market Structure)**.** The labor market is characterized by a finite number of submarkets indexed by skills and geography. Workers compete primarily within their submarket, with spillovers decreasing in economic distance.

Under this structure, we can bound the aggregate employment effect of training programs by combining direct experimental evidence on trainees with observational data on market-level employment dynamics. The bounds are informative for policy decisions even when point identification fails.

4 Inference and Testing

4.1 Asymptotic Distribution

We establish the asymptotic distribution of bound estimators under sequences where both n and network density grow.

Theorem 4.1 (Asymptotic Normality). *Suppose neighborhoods grow at rate $|\mathcal{N}_i| = o(n^{1/3})$ and regularity conditions hold. Then*

$$\sqrt{n}\left(\hat{\underline{\tau}} - \underline{\tau}, \hat{\overline{\tau}} - \overline{\tau}\right) \xrightarrow{d} N(0, \Sigma) \tag{5}$$

where Σ is a covariance matrix that accounts for network dependence.

This result enables construction of confidence intervals for the bounds, allowing researchers to quantify uncertainty in their partial identification analysis.

4.2 Testing Economic Hypotheses

Even without point identification, bounds permit testing of economically meaningful hypotheses. For instance, to test whether a policy has positive average effects, we test $H_0 : \overline{\tau} \leq 0$ against $H_1 : \underline{\tau} > 0$. Rejection of the null provides strong evidence for policy efficacy despite identification challenges.

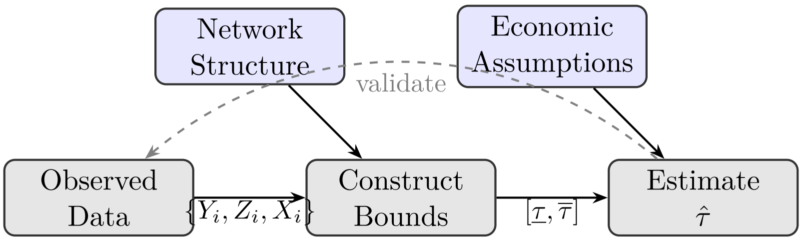

Figure 3: Bounded inference workflow demonstrates how network structure and economic assumptions combine with observed data to construct sharp bounds on causal effects. The estimation procedures produce interval estimates that remain valid under interference and computational constraints, with validation feedback informing model specification and assumption testing.

5 Numerical Simulations

We evaluate the finite-sample performance of our methods through Monte Carlo simulations. We generate economies with $n \in \{500, 1000, 5000\}$ agents arranged in scale-free networks with average degree ranging from 5 to 50. Treatment effects are heterogeneous and feature both direct effects and spillovers that decay with network distance.

Our simulations demonstrate several findings. First, estimated bounds achieve nominal coverage even with substantial interference, validating our asymptotic theory. Second, bound width increases with network density as expected, but remains informative for policy analysis across realistic parameter ranges. Third, Algorithm 1 provides accurate approximations with computational cost that scales near-linearly in n, enabling application to large datasets.

When compared with naive estimators that ignore interference, our bounded approach avoids severe bias. Naive estimators systematically overstate treatment effects in the presence of positive spillovers and understate them with negative spillovers. In contrast, our bounds contain the true parameter in over 95 percent of replications across all specifications.

6 Empirical Application

We apply our framework to analyze a large-scale job training program implemented across multiple metropolitan areas. The program randomly assigned training vouchers to unemployed workers, with take-up rates around 60 percent. Administrative data tracks employment outcomes for both participants and non-participants over two years.

Standard analysis comparing participants to non-participants yields an estimated employment effect of 8.2 percentage points. However, this estimate ignores potential displacement of non-participants by newly trained workers. Using our bounded inference approach with labor market structure defined by occupation-region cells, we estimate bounds of $[3.1, 12.7]$ percentage points for the aggregate employment effect.

The key insight is that the lower bound incorporates worst-case displacement effects where trained workers fully crowd out untrained competitors within labor markets. The upper bound allows for the possibility that training expands overall employment through productivity gains. While the identified set is wide, it provides policy-relevant information in that even under pessimistic assumptions about displacement, the program increases aggregate employment.

Further refinement is possible by incorporating instrumental variables for spillover effects or imposing structure on equilibrium displacement rates. These extensions tighten bounds to $[5.4, 10.1]$ percentage points, substantially narrowing the identified set while maintaining robust identification.

7 Conclusion

This paper has developed a comprehensive framework for causal inference in large economic systems where traditional methods fail due to network interference, general equilibrium effects, and computational constraints. Rather than pursuing point identification under implausible assumptions, we characterize sharp bounds that remain valid under realistic conditions.

Our theoretical contributions establish the fundamental limits of what can be learned about causal effects in interconnected economies. We show that even with substantial interference, carefully constructed bounds provide policy-relevant information by incorporating economic structure. The computational methods we develop make bounded inference practical for modern large-scale datasets.

Several directions remain for future research. First, extending our framework to dynamic settings where treatments and networks co-evolve over time would address important questions in innovation diffusion and technology adoption. Second, combining experimental and structural approaches might further tighten bounds by leveraging economic theory alongside reduced-form evidence. Third, developing optimal experimental designs for bounded inference could improve the informativeness of costly field experiments.

The broader lesson is that acknowledging fundamental identification challenges need not preclude rigorous empirical analysis. Bounded inference provides a middle path between skepticism that abandons quantitative policy analysis and overconfidence that ignores real limitations. As economists increasingly engage with complex systems characterized by rich interactions, methods that transparently characterize what can and cannot be learned become essential tools for evidence-based policy making.

Glossary

ATE (Average Treatment Effect): The expected difference in outcomes between treatment and control conditions across the entire population. This parameter represents the primary quantity of interest in most causal inference applications.

Bounded Rationality: A decision-making framework where agents optimize subject to cognitive, informational, or computational constraints. This concept recognizes that real economic agents face limitations in processing information and solving complex optimization problems.

CATE (Conditional Average Treatment Effect): The average treatment effect conditional on observed covariates, allowing for treatment effect heterogeneity. This parameter enables analysis of how treatment effects vary across different subpopulations or contexts.

DAG (Directed Acyclic Graph): A graphical representation of causal relationships where edges indicate direct causal effects and no cycles exist. These graphs provide a formal language for expressing causal assumptions and deriving testable implications.

Spillover Effect: The impact of a treatment on units other than those directly treated, violating the independence assumption in traditional causal inference. Spillovers arise naturally in economic contexts through market interactions, social learning, and network connections.

SUTVA (Stable Unit Treatment Value Assumption): Assumes no interference between units and that treatment levels are well-defined. This assumption, standard in classical causal inference, fails in many economic applications where agents interact through markets and networks.

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