Two useful results involving the Jacobian matrix of two general conics

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Abstract

In this paper, I describe two useful results involving the Jacobian matrix of two general conics.
The paper ends with "The End"

Introduction

Two general conics are

$$F(x, y) = ax^2 + 2hxy + by^2 + 2fx + 2gy + c$$

and

$$G(x, y) = \alpha x^2 + 2\eta xy + \beta y^2 + 2\phi x + 2\gamma y + \chi$$

In this paper, I describe two useful results involving the Jacobian matrix of two general conics.

The Jacobian matrix of two general conics

The Jacobian matrix $\mathbf{J}_{F,G}$ of $F(x, y)$ and $G(x, y)$ is

$$\mathbf{J}_{F,G} = \begin{pmatrix} \frac{\partial F(x,y)}{\partial x} & \frac{\partial F(x,y)}{\partial y} \\ \frac{\partial G(x,y)}{\partial x} & \frac{\partial G(x,y)}{\partial y} \end{pmatrix} = \begin{pmatrix} 2(ax + hy + f) & 2(hx + by + g) \\ 2(\alpha x + \eta y + \phi) & 2(\eta x + \beta y + \gamma) \end{pmatrix}$$

The transpose $\mathbf{J}_{F,G}{}^T$ of the Jacobian matrix $\mathbf{J}_{F,G}$ is

$$\mathbf{J}_{F,G}{}^T = \begin{pmatrix} 2(ax + hy + f) & 2(\alpha x + \eta y + \phi) \\ 2(hx + by + g) & 2(\eta x + \beta y + \gamma) \end{pmatrix}$$

The first useful result involving the Jacobian matrix of two general conics

The Jacobian matrix of two general conics and its transpose are both always divisible by 4.

The second useful result involving the Jacobian matrix of two general conics

$$\mathbf{J}_{F,G} = \mathbf{J}_{F,G}{}^T$$

$$\Longleftrightarrow$$

$$\alpha x + \eta y + \phi = by + hx + g$$

The End