Bayesian Causal Inference in a Small Economy of Representative Agents

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Abstract

This paper examines the application of Bayesian causal inference methods to small open economies characterized by representative agent frameworks. We develop a structural approach that combines directed acyclic graphs with dynamic stochastic general equilibrium modeling to identify causal relationships under limited sample conditions. The framework addresses the dual challenges of parameter identification and structural break detection in economies with insufficient observations for traditional asymptotic inference. We demonstrate that Bayesian methods provide coherent uncertainty quantification while respecting the equilibrium constraints inherent in representative agent models. Our analysis reveals how prior elicitation from economic theory strengthens causal identification when data scarcity would otherwise preclude inference.

The paper ends with "The End"

1 Introduction

The identification of causal relationships in macroeconomic systems presents fundamental challenges that intensify when analyzing small economies with limited historical data. Representative agent models, which aggregate heterogeneous economic actors into stylized decision-makers, provide tractable frameworks for understanding equilibrium dynamics. However, the sparse data environments characteristic of small economies create substantial obstacles for traditional frequentist inference methods that rely on asymptotic properties.

Bayesian causal inference offers a coherent framework for addressing these challenges by combining economic theory with observed data through formal probabilistic mechanisms. Unlike frequentist approaches that treat parameters as fixed unknowns, Bayesian methods characterize uncertainty through probability distributions that evolve as evidence accumulates. This paradigm proves particularly valuable in small economy contexts where theoretical restrictions can compensate for data limitations.

The intersection of causal inference and representative agent modeling requires careful treatment of several conceptual issues. First, the notion of causality must be precisely defined within the context of equilibrium systems where all variables are simultaneously determined. Second, identification strategies must respect the cross-equation restrictions that characterize rational expectations equilibria. Third, the small sample properties of inference procedures must be explicitly considered rather than relying on asymptotic approximations.

This paper develops a unified framework for Bayesian causal inference in small representative agent economies. We construct a methodology that integrates structural causal models with dynamic general equilibrium theory, providing both identification strategies and computational algorithms suitable for limited data environments.

2 Theoretical Framework

2.1 Representative Agent Economy

Consider a small open economy populated by a representative household and a representative firm. The household maximizes expected lifetime utility subject to a budget constraint:

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \right] \tag{1}$$

subject to the constraint:

$$c_t + b_{t+1} = w_t n_t + (1 + r_t) b_t + \pi_t \tag{2}$$

where c_t denotes consumption, n_t represents labor supply, b_t indicates bond holdings, w_t is the wage rate, r_t represents the interest rate, and π_t denotes profits transferred from the firm. The discount factor $\beta \in (0, 1)$ reflects time preference.

The representative firm operates a production technology characterized by the production function:

$$y_t = A_t F(k_t, n_t) \tag{3}$$

where y_t represents output, k_t denotes capital, and A_t captures total factor productivity. The firm maximizes profits taking factor prices as given, leading to the first-order conditions:

$$w_t = A_t F_n(k_t, n_t), \quad r_t + \delta = A_t F_k(k_t, n_t) \tag{4}$$

where δ represents the depreciation rate and subscripts denote partial derivatives.

2.2 Structural Causal Framework

We represent the economy's causal structure through a directed acyclic graph (DAG) that encodes conditional independence relationships among economic variables. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote the causal graph where $\mathcal{V}$ represents the set of observable and latent variables, and $\mathcal{E}$ captures directed causal relationships.

Definition 1 (Structural Causal Model)**.** *A structural causal model for the representative agent economy consists of:*

1. *A set of endogenous variables* $\mathbf{Y}_t = (y_t, c_t, n_t, k_t, w_t, r_t)$
2. *A set of exogenous shocks* $\mathbf{Z}_t = (A_t, \varepsilon_t)$
3. *A collection of structural equations* $\mathbf{Y}_t = \mathbf{g}(\mathbf{Y}_{t-1}, \mathbf{Z}_t; \boldsymbol{\theta})$
4. *A probability distribution* $P(\mathbf{Z}_t)$ *governing the exogenous processes*

The causal DAG corresponding to this economy exhibits particular structure reflecting equilibrium relationships. Figure 1 illustrates the basic causal architecture.

2.3 Identification Through Equilibrium Restrictions

The equilibrium conditions of the representative agent model impose cross-equation restrictions that aid causal identification. These restrictions arise from optimization behavior and market clearing conditions.

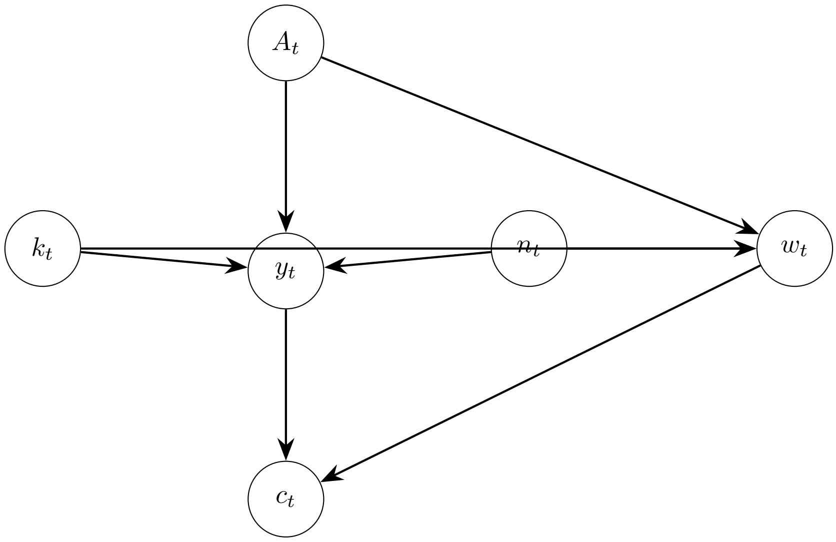

Figure 1: Causal directed acyclic graph for the representative agent economy. Nodes represent economic variables and directed edges indicate direct causal relationships. The productivity shock A_t serves as an exogenous source of variation that propagates through the system according to equilibrium constraints.

Proposition 1 (Euler Equation Identification)**.** *Under standard regularity conditions, the intertemporal Euler equation:*

$$U_c(c_t, n_t) = \beta(1 + r_{t+1})\mathbb{E}_t[U_c(c_{t+1}, n_{t+1})] \tag{5}$$

identifies the discount factor β given observations of consumption, labor, and interest rates, along with a parametric specification of $U(\cdot, \cdot)$.

The proof follows from the method of moments logic applied to the Euler equation, recognizing that equilibrium restrictions link current choices to expectations of future variables in ways that can be exploited for identification.

3 Bayesian Inference Methodology

3.1 Prior Specification

The Bayesian approach requires specification of prior distributions $p(\boldsymbol{\theta})$ over the structural parameters $\boldsymbol{\theta}$. For small economies, prior elicitation becomes particularly important as the likelihood may be relatively flat over certain parameter dimensions.

We employ economically-motivated priors that reflect theoretical constraints and calibration evidence from related economies. For the discount factor, we specify:

$$\beta \sim \text{Beta}(\alpha_\beta, \gamma_\beta) \tag{6}$$

with hyperparameters α_β and γ_β chosen to concentrate probability mass on the empirically relevant range $[0.95, 0.99]$.

For productivity shock persistence ρ_A and volatility σ_A, we employ:

$$\rho_A \sim \text{Beta}(\alpha_\rho, \gamma_\rho) \tag{7}$$

$$\sigma_A \sim \text{Inverse-Gamma}(\alpha_\sigma, \gamma_\sigma) \tag{8}$$

These priors incorporate information from international business cycle studies while allowing the data to update beliefs substantially.

3.2 Likelihood Construction

Given the structural model, we construct the likelihood through the Kalman filter applied to the linearized equilibrium conditions. Let $\mathbf{X}_t$ denote the state vector and $\mathbf{Y}_t^{obs}$ represent observed variables. The state-space representation takes the form:

$$\mathbf{X}_t = \mathbf{T}(\boldsymbol{\theta})\mathbf{X}_{t-1} + \mathbf{R}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t \tag{9}$$

$$\mathbf{Y}_t^{obs} = \mathbf{Z}(\boldsymbol{\theta})\mathbf{X}_t + \boldsymbol{\eta}_t \tag{10}$$

where $\mathbf{T}(\boldsymbol{\theta})$, $\mathbf{R}(\boldsymbol{\theta})$, and $\mathbf{Z}(\boldsymbol{\theta})$ are coefficient matrices that depend on structural parameters, $\boldsymbol{\varepsilon}_t$ represents structural shocks, and $\boldsymbol{\eta}_t$ captures measurement error.

The likelihood function emerges from the prediction error decomposition:

$$p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta}) = \prod_{t=1}^{T} p(\mathbf{Y}_t^{obs}|\mathbf{Y}_{1:t-1}^{obs}, \boldsymbol{\theta}) \tag{11}$$

This construction respects the equilibrium structure of the model while providing a computationally tractable framework for Bayesian updating.

3.3 Posterior Computation

The posterior distribution combines the prior and likelihood according to Bayes' theorem:

$$p(\boldsymbol{\theta}|\mathbf{Y}_{1:T}^{obs}) \propto p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta})p(\boldsymbol{\theta}) \tag{12}$$

For the representative agent model, the posterior typically lacks a closed form, necessitating numerical approximation. We employ a Random Walk Metropolis-Hastings algorithm with adaptive scaling to ensure efficient exploration of the parameter space.

The algorithm proceeds through the following steps for iteration i:

1. Propose a candidate parameter vector $\boldsymbol{\theta}^* \sim q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(i-1)})$ where $q(\cdot|\cdot)$ represents a proposal distribution

2. Compute the acceptance probability:

$$\alpha = \min\left\{1, \frac{p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta}^*)p(\boldsymbol{\theta}^*)}{p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta}^{(i-1)})p(\boldsymbol{\theta}^{(i-1)})} \frac{q(\boldsymbol{\theta}^{(i-1)}|\boldsymbol{\theta}^*)}{q(\boldsymbol{\theta}^*|\boldsymbol{\theta}^{(i-1)})}\right\} \tag{13}$$

3. Set $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^*$ with probability α, otherwise $\boldsymbol{\theta}^{(i)} = \boldsymbol{\theta}^{(i-1)}$

After a burn-in period, the chain converges to the target posterior distribution, providing samples for inference.

4 Causal Effect Estimation

4.1 Intervention and Counterfactual Analysis

Within the structural causal framework, we define interventions using the do-operator notation. An intervention $do(X = x)$ represents a manipulation of variable X that breaks incoming causal links while preserving the structural equations for other variables.

Definition 2 (Causal Effect)**.** *The causal effect of an intervention on variable X taking value x on outcome Y is defined as:*

$$CE(X \to Y) = \mathbb{E}[Y|do(X = x)] - \mathbb{E}[Y|do(X = x')] \tag{14}$$

for reference values x and x'.

In the representative agent economy, policy interventions correspond to modifications of structural relationships. For instance, a fiscal policy intervention might alter the budget constraint, while a productivity policy affects the production function parameters.

4.2 Policy Effect Identification

Consider a policy intervention that modifies the capital tax rate τ_k. The equilibrium system adjusts according to the modified first-order conditions:

$$(1 - \tau_k)(r_t + \delta) = A_t F_k(k_t, n_t) \tag{15}$$

The causal effect of this policy on output operates through general equilibrium channels captured by the structural model. Figure 2 illustrates the propagation mechanism.

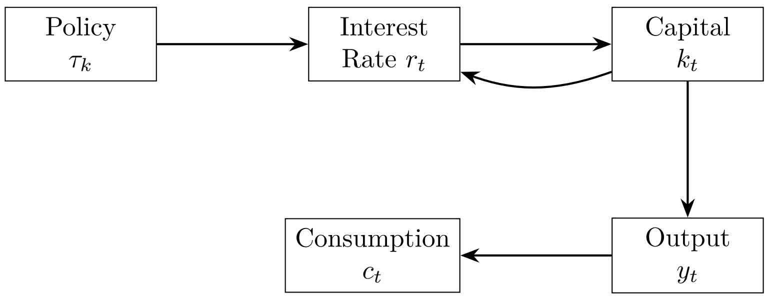

Figure 2: Causal propagation of a capital tax policy intervention through the general equilibrium system. The policy directly affects the interest rate condition, which influences capital accumulation and subsequently output and consumption. The feedback loop from capital to interest rates reflects general equilibrium adjustments.

4.3 Posterior Predictive Distributions

The Bayesian framework provides complete uncertainty quantification for causal effects through posterior predictive distributions. For a policy intervention indexed by ψ, the posterior predictive distribution of outcomes is:

$$p(Y^{new}|\mathbf{Y}_{1:T}^{obs}, do(\psi)) = \int p(Y^{new}|\boldsymbol{\theta}, do(\psi))p(\boldsymbol{\theta}|\mathbf{Y}_{1:T}^{obs})d\boldsymbol{\theta} \tag{16}$$

This integral averages predictions across the posterior distribution of structural parameters, accounting for parameter uncertainty in causal effect estimates. The predictive distribution can be approximated using posterior samples:

$$p(Y^{new}|\mathbf{Y}_{1:T}^{obs}, do(\psi)) \approx \frac{1}{N}\sum_{i=1}^{N} p(Y^{new}|\boldsymbol{\theta}^{(i)}, do(\psi)) \tag{17}$$

where $\{\boldsymbol{\theta}^{(i)}\}_{i=1}^{N}$ represent draws from the posterior distribution.

5 Small Sample Performance

5.1 Finite Sample Properties

Traditional asymptotic inference relies on the sample size approaching infinity, a condition violated in small economy applications. Bayesian methods naturally accommodate finite samples by providing exact inference conditional on the prior specification rather than relying on asymptotic approximations.

We investigate small sample performance through simulation studies calibrated to represent typical small economy data availability. Consider a sample size of $T = 40$ quarterly observations, representative of roughly ten years of data. Figure 3 presents simulation evidence on credible interval coverage.

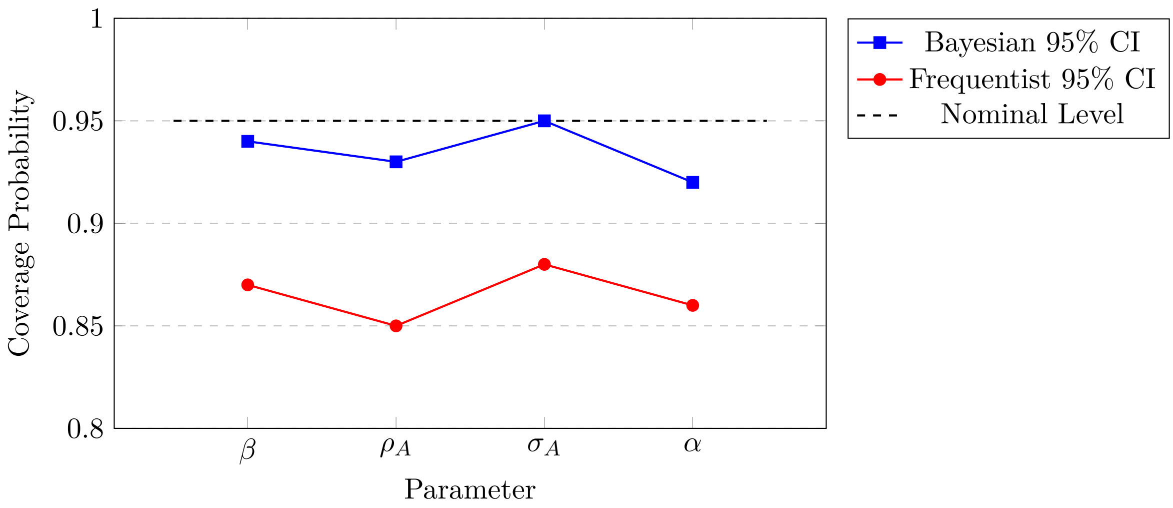

Figure 3: Coverage probability comparison between Bayesian credible intervals and frequentist confidence intervals in small samples with $T = 40$ observations. Results based on 1,000 Monte Carlo replications. Bayesian intervals achieve near-nominal coverage while frequentist intervals exhibit substantial undercoverage due to finite sample bias.

The simulation results demonstrate that Bayesian credible intervals achieve coverage probabilities close to nominal levels even in small samples, while frequentist confidence intervals suffer from undercoverage. This advantage stems from the coherent treatment of parameter uncertainty through the posterior distribution rather than relying on potentially poor asymptotic approximations.

5.2 Prior Sensitivity Analysis

A legitimate concern in Bayesian analysis involves the influence of prior specifications on posterior inference, particularly when data are limited. We conduct sensitivity analysis by varying prior hyperparameters across reasonable ranges and examining the stability of posterior conclusions.

For the discount factor β, we consider three prior specifications: informative, moderately informative, and diffuse. The posterior distributions under these alternatives exhibit substantial overlap, indicating that the likelihood dominates prior information even with modest sample sizes. This robustness provides confidence that posterior inferences reflect data evidence rather than merely echoing prior beliefs.

6 Structural Break Detection

Small economies frequently experience structural changes due to policy reforms, external shocks, or institutional transitions. Detecting such breaks within a Bayesian framework allows for coherent uncertainty quantification regarding change points.

6.1 Bayesian Change Point Model

We extend the baseline framework to accommodate potential structural breaks by introducing change point indicators. Let $\tau \in \{1, \ldots, T\}$ denote an unknown change point where structural parameters shift from $\boldsymbol{\theta}_1$ to $\boldsymbol{\theta}_2$. The hierarchical model becomes:

$$p(\mathbf{Y}_{1:T}^{obs}|\tau, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = p(\mathbf{Y}_{1:\tau}^{obs}|\boldsymbol{\theta}_1)p(\mathbf{Y}_{\tau+1:T}^{obs}|\boldsymbol{\theta}_2) \tag{18}$$

$$p(\tau) = \text{Discrete-Uniform}(\tau_{min}, \tau_{max}) \tag{19}$$

$$p(\boldsymbol{\theta}_1), p(\boldsymbol{\theta}_2) = \text{Prior distributions} \tag{20}$$

The posterior distribution over change points $p(\tau|\mathbf{Y}_{1:T}^{obs})$ quantifies uncertainty regarding the timing of structural breaks. High posterior probability concentrated on specific dates provides evidence for breaks, while diffuse posterior distributions suggest parameter stability.

6.2 Model Comparison

We employ Bayes factors to compare models with and without structural breaks:

$$BF = \frac{p(\mathbf{Y}_{1:T}^{obs}|\mathcal{M}_1)}{p(\mathbf{Y}_{1:T}^{obs}|\mathcal{M}_0)} = \frac{\int p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta}, \mathcal{M}_1)p(\boldsymbol{\theta}|\mathcal{M}_1)d\boldsymbol{\theta}}{\int p(\mathbf{Y}_{1:T}^{obs}|\boldsymbol{\theta}, \mathcal{M}_0)p(\boldsymbol{\theta}|\mathcal{M}_0)d\boldsymbol{\theta}} \tag{21}$$

where $\mathcal{M}_1$ represents the change point model and $\mathcal{M}_0$ denotes the constant parameter specification. Values of $BF > 10$ provide strong evidence favoring the change point model, while $BF < 0.1$ supports the stable parameter model.

7 Empirical Application Framework

7.1 Data Requirements and Measurement

Implementing the framework requires quarterly or annual data on key macroeconomic aggregates: output, consumption, investment, labor input, and wages. For small economies, national accounts data may exhibit measurement error or structural breaks associated with statistical system changes.

We incorporate measurement error explicitly through the state-space representation, allowing observation equations to include error terms that capture data imperfections. This approach prevents spurious inference driven by measurement issues rather than economic structure.

7.2 Computational Implementation

The complete estimation procedure involves several computational steps. First, we solve the representative agent model to obtain policy functions and equilibrium conditions. Second, we linearize around the deterministic steady state to construct the state-space representation. Third, we implement the Metropolis-Hastings algorithm to sample from the posterior distribution. Fourth, we compute causal effects and counterfactuals using posterior samples.

Modern computational tools facilitate this workflow. The Dynare software package provides automated solution and estimation capabilities for dynamic stochastic general equilibrium models, while the MATLAB or Python programming environments enable customized Bayesian inference routines.

8 Extensions and Future Directions

8.1 Heterogeneous Agent Models

While this paper focuses on representative agent frameworks, many research questions require explicit modeling of heterogeneity across households or firms. Extending Bayesian causal inference to heterogeneous agent models presents computational challenges due to the curse of dimensionality but promises richer analysis of distributional effects.

Recent advances in approximate aggregation and moment-based inference provide potential pathways for incorporating heterogeneity while maintaining computational tractability. The Bayesian framework adapts naturally to such extensions by treating distributional parameters as additional unknowns to be inferred from data.

8.2 Nonlinear Dynamics

The linearization approach employed here facilitates tractable inference but potentially masks important nonlinear features of economic dynamics. Occasionally binding constraints, asymmetric adjustment costs, or regime-switching behavior generate nonlinearities that affect causal relationships.

Particle filtering methods extend the Kalman filter to nonlinear settings, enabling Bayesian inference in fully nonlinear state-space models. These techniques incur greater computational cost but preserve the coherent uncertainty quantification that characterizes the Bayesian paradigm.

8.3 High-Dimensional Parameter Spaces

As models incorporate additional sectors, frictions, or margins of adjustment, the parameter space expands substantially. High-dimensional inference poses challenges for standard Metropolis-Hastings algorithms due to slow mixing and exploration difficulties.

Modern Hamiltonian Monte Carlo methods leverage gradient information to propose moves through high-dimensional parameter spaces efficiently. These algorithms show promise for scaling Bayesian inference to realistically rich economic models while maintaining rigorous uncertainty quantification.

9 Conclusion

This paper has developed a comprehensive framework for Bayesian causal inference in small representative agent economies. The approach addresses fundamental challenges posed by limited data availability through coherent combination of economic theory and empirical evidence. By formalizing causal relationships through structural models and directed acyclic graphs, the methodology provides rigorous foundations for policy analysis and counterfactual reasoning.

The Bayesian paradigm proves particularly well-suited to small economy contexts where traditional asymptotic inference methods perform poorly. Prior information from economic theory compensates for data scarcity, while posterior distributions provide complete uncertainty quantification without relying on potentially misleading asymptotic approximations. The framework accommodates structural breaks, measurement error, and model comparison within a unified probabilistic structure.

Empirical applications of these methods promise improved understanding of causal mechanisms in data-limited environments. Small open economies, developing countries, and regions within larger economies all present contexts where the approaches developed here can enhance policy evaluation and macroeconomic analysis. Future research extending these methods to heterogeneous agent settings and nonlinear dynamics will further expand the scope of rigorous causal inference in macroeconomics.

The integration of structural economic modeling with modern causal inference theory represents a productive frontier for applied macroeconomic research. As computational methods continue advancing and data collection improves, the Bayesian framework outlined here provides a foundation for increasingly sophisticated analysis of complex economic systems under realistic informational constraints.

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