# Closed-form solution to the Ghosh equations

Soumadeep Ghosh

Kolkata, India

**Abstract**

In this paper, I describe the closed-form solution of the Ghosh equations.
The paper ends with "The End"

## Introduction

**The Ghosh combat model** generalizes **the Lanchester combat model** by taking into account both growth and attrition of two militaries $A$ and $B$.
The Ghosh equations are differential equations describing the time dependence of two militaries' strengths A and B as a function of time, with the function depending on both A and B.
In this paper, I describe the closed-form solution to the Ghosh equations.

## The Ghosh equations

The Ghosh equations are

$$\frac{\partial A(t)}{\partial t} = aA(t) - \beta B(t)$$

$$\frac{\partial B(t)}{\partial t} = bB(t) - \alpha A(t)$$

## The closed-form solution to the Ghosh equations

The closed-form solution to the Ghosh equations is

$$A(t) = e^{\frac{1}{2}t(a+b)} \left( c_1 \cosh\left(\frac{1}{2}t\sqrt{(a-b)^2 + 4\alpha\beta}\right) + \frac{(c_1(a-b) - 2\beta c_2)\sinh\left(\frac{1}{2}t\sqrt{(a-b)^2 + 4\alpha\beta}\right)}{\sqrt{(a-b)^2 + 4\alpha\beta}} \right)$$

$$B(t) = e^{\frac{1}{2}t(a+b)} \left( c_2 \cosh\left(\frac{1}{2}t\sqrt{(a-b)^2 + 4\alpha\beta}\right) + \frac{(c_2(b-a) - 2\alpha c_1)\sinh\left(\frac{1}{2}t\sqrt{(a-b)^2 + 4\alpha\beta}\right)}{\sqrt{(a-b)^2 + 4\alpha\beta}} \right)$$

where
$c_1$ and $c_2$ are coefficients to be determined from battle data.

## The End