Theoretical fuel for a two-stage rocket

Soumadeep Ghosh

Kolkata, India

Abstract

In this paper, I describe the theoretical fuel for a two-stage rocket.
The paper ends with "The End"

Introduction

In a previous paper, I described the theoretical fuel for a simple rocket.

A **two-stage rocket** has two stages, with dry masses $d_1 > 0$ and $d_2 > 0$, fuel $f_1 > 0$ and $f_2 > 0$, payload $p > 0$, exhaust velocity $u_1 > 0$ and $u_2 > 0$,
target velocities $v_1 > u$ and $v_2 > v_1$ and zero initial velocity.

In this paper, I describe the theoretical fuel for a two-stage rocket.

Delta velocities

The **delta velocities** of a two-stage rocket are $\Delta v_1 = v_1 - 0 = v_1$ and $\Delta v_2 = v_2 - v_1$

The theoretical rocket equations

The theoretical rocket equations are

$$\Delta v = \Delta v_1 + \Delta v_2$$

$$\Delta v_1 = u_1 \ln\left(\frac{d_1 + d_2 + f_1 + f_2 + p}{d_1 + d_2 + f_2 + p}\right)$$

$$\Delta v_2 = u_2 \ln\left(\frac{d_2 + f_2 + p}{d_2 + p}\right)$$

Theoretical fuel for a two-stage rocket

Solving the theoretical rocket equations for f_1 and f_2 gives us the theoretical fuel for a two-stage rocket

$$f_1 = e^{-\frac{\Delta v_1}{u_2}}(e^{\frac{\Delta v_1}{u_1}} - 1)(d_1 e^{\frac{\Delta v_1}{u_2}} + e^{\frac{\Delta v_1 + \Delta v_2}{u_2}}(d_2 + p))$$

$$f_2 = (e^{\frac{\Delta v_2}{u_2}} - 1)(d_2 + p)$$

The End