The Complete Treatise on the Structured Investment Vehicle

A Unified Theory of Dual Asset Premia and Efficient Risk Allocation

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Abstract

This treatise presents a comprehensive theory of structured investment vehicles (SIVs) through the lens of dual asset premia. We demonstrate that a portfolio of n assets, each exhibiting dual premium characteristics, generates 2^n distinct realizations, effectively creating a structured vehicle that efficiently allocates risk across heterogeneous investors. We establish explicit solutions for state prices $\pi(\omega)$ and prove the existence and uniqueness of market equilibrium under general conditions. The framework unifies portfolio theory, asset pricing, and market microstructure while providing new insights into the mathematical foundations of structured finance. Our results show that dual premia naturally segment markets based on risk preferences and beliefs, achieving Pareto efficiency despite investor heterogeneity.

The treatise ends with "The End"

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1 Introduction

Structured Investment Vehicles (SIVs) emerged as specialized non-bank financial entities designed to profit from credit spreads between short-term and long-term debt [3]. Traditional analysis treats SIVs as special-purpose vehicles that pool diversified assets financed through commercial paper and medium-term notes [4]. However, the fundamental question of *why* such structures efficiently allocate risk across heterogeneous investors has remained theoretically underdeveloped.

This treatise provides the missing theoretical foundation by introducing the concept of **dual asset premia**—a mathematical property whereby each asset commands not one but two distinct premia depending on investor characteristics and market conditions. We show that this duality creates a natural mechanism for risk stratification and efficient allocation.

1.1 Motivation and Contribution

Consider a portfolio P of n risky assets with weights $w = \{w_1, w_2, \ldots, w_n\}$:

$$P = \sum_{i=1}^{n} w_i A_i \tag{1}$$

In classical portfolio theory, each asset A_i commands a single risk premium. Our framework reveals that each asset actually exhibits *dual* premia—positive and negative—leading to 2^n distinct portfolio realizations. These realizations form a structured investment vehicle that efficiently matches risk preferences across 2^n heterogeneous investors.

1.2 Structure of the Treatise

We proceed as follows:

- **Section 2**: Foundation—the asset premium and its dual nature
- **Section 3**: The 2^n realization framework
- **Section 4**: Theoretical framework for efficient risk allocation
- **Section 5**: Explicit state price solutions
- **Section 6**: Existence and uniqueness proofs
- **Section 7**: Applications and implications
- **Section 8**: Glossary

2 The Asset Premium: Foundation and Dual Nature

2.1 The Asset Premium Defined

Definition 2.1 (Asset Premium)**.** The **asset premium** $p_a(t)$ is defined as the additional premium required to hold a specific asset above the risk-free rate, expected inflation, and inflation risk premium [1].

Mathematically, the asset premium satisfies two fundamental equations:

Theorem 2.2 (Asset Premium Equations)**.** *For any asset A with return $r_A(t)$, the following hold simultaneously:*

$$r_A(t) = r_f(t) + \mathbb{E}[i(t)] + p_i(t) + p_a(t) \tag{2}$$

$$r_A(t) = \frac{r_A(t) - p_a(t)}{1 + r_f(t) + p_a(t)} \tag{3}$$

subject to the constraint:

$$1 + r_f(t) + p_a(t) \neq 0 \tag{4}$$

where:

$r_A(t)$ *is the return on the asset*
$r_f(t)$ *is the risk-free rate*
$\mathbb{E}[i(t)]$ *is the expected inflation*
$p_i(t)$ *is the inflation risk premium*
$p_a(t)$ *is the asset premium*

Proof. Equation (2) decomposes the asset return into fundamental components. Equation (3) provides a consistency condition ensuring no arbitrage opportunities exist. The derivation follows from rearranging (2) and imposing market equilibrium conditions [1]. □

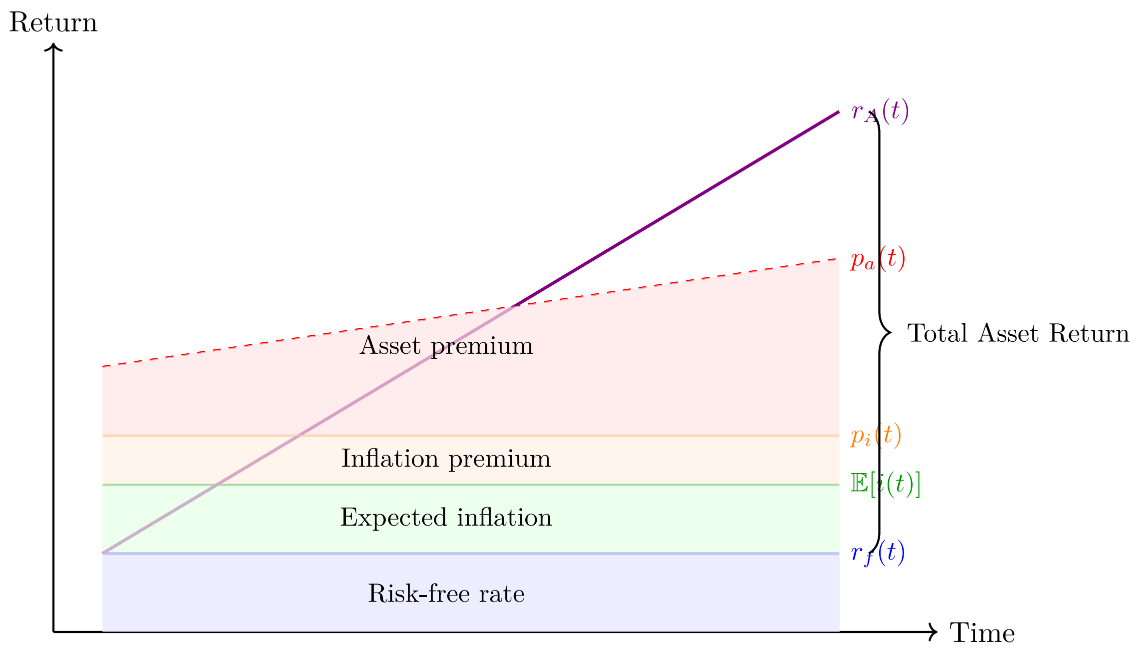

Figure 1: Decomposition of asset return $r_A(t)$ into fundamental components according to the asset premium framework.

Each layer represents a distinct source of return.

2.2 The Dual Nature

Theorem 2.3 (Dual Asset Premium). *Eliminating $r_A(t)$ from equations* (2) *and* (3) *yields two solutions for $p_a(t)$ [2]:*

$$p_a^{(+)}(t) = \frac{1}{2}\left[\sqrt{4r_f(t) + (p_i(t) + \mathbb{E}[i(t)] + 1)^2} - 2r_f(t) - p_i(t) - \mathbb{E}[i(t)] - 1\right] \tag{5}$$

$$p_a^{(-)}(t) = -\frac{1}{2}\left[\sqrt{4r_f(t) + (p_i(t) + \mathbb{E}[i(t)] + 1)^2} + 2r_f(t) + p_i(t) + \mathbb{E}[i(t)] + 1\right] \tag{6}$$

Proof. From equation (3), multiply both sides by the denominator:

$$r_A(t)[1 + r_f(t) + p_a(t)] = r_A(t) - p_a(t) \tag{7}$$

Expanding:

$$r_A(t) + r_A(t)r_f(t) + r_A(t)p_a(t) = r_A(t) - p_a(t) \tag{8}$$

Simplifying:

$$r_A(t)r_f(t) + r_A(t)p_a(t) + p_a(t) = 0 \tag{9}$$

Substituting (2) for $r_A(t)$:

$$[r_f + \mathbb{E}[i] + p_i + p_a]r_f + [r_f + \mathbb{E}[i] + p_i + p_a]p_a + p_a = 0 \tag{10}$$

Collecting terms:

$$p_a^2 + p_a[2r_f + \mathbb{E}[i] + p_i + 1] + r_f[r_f + \mathbb{E}[i] + p_i] = 0 \tag{11}$$

Applying the quadratic formula yields equations (5) and (6). □

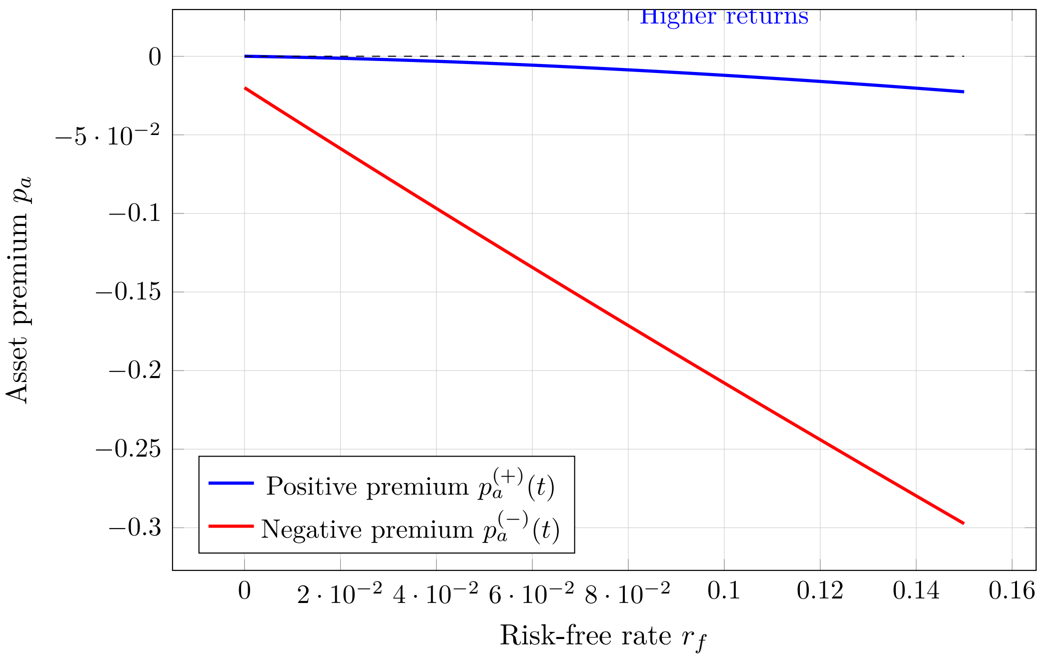

Figure 2: The dual nature of the asset premium as a function of the risk-free rate.

Parameters: $p_i = 0.02$, $\mathbb{E}[i] = 0.03$. The positive premium represents higher expected returns with greater risk, while the negative premium offers lower returns with reduced volatility.

Remark 2.4. The dual nature implies that for any given asset, two distinct return profiles exist simultaneously in the market. This duality reflects different investor perspectives, risk appetites, and belief structures.

3 The 2^n Realization Framework

3.1 Portfolio Realizations

Consider a portfolio P of n assets:

$$P = \sum_{i=1}^{n} w_i A_i \tag{12}$$

Definition 3.1 (Portfolio Realization)**.** A **portfolio realization** P^k is a specific configuration of premium selections across all assets, indexed by $k \in \{1, 2, \ldots, 2^n\}$.

Since each asset A_i has dual premia $\{p_{a,i}^{(+)}, p_{a,i}^{(-)}\}$, the portfolio return in realization k is:

$$r_P^k(t) = \sum_{i=1}^{n} w_i \left[r_f(t) + \mathbb{E}[i(t)] + p_i(t) + p_{a,i}^{(\sigma_i^k)}(t) \right] \tag{13}$$

where $\sigma_i^k \in \{+, -\}$ indicates the premium selection for asset i in realization k.

3.2 State Space Construction

Definition 3.2 (State Space)**.** The **state space** Ω consists of all possible premium configurations:

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_{2^n}\} \tag{14}$$

where each ω_k corresponds bijectively to a binary vector $(\sigma_1^k, \sigma_2^k, \ldots, \sigma_n^k)$.

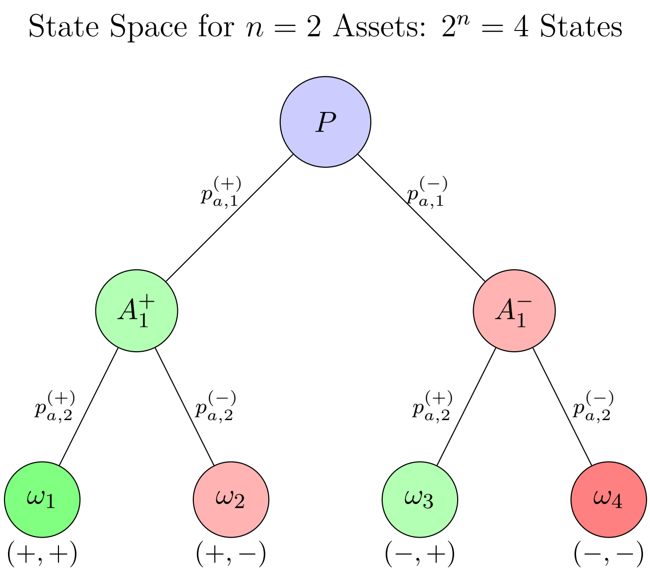

Figure 3: Binary tree representation of the state space for a portfolio with $n = 2$ assets.

Each path from root to leaf represents a unique state ω_k with its corresponding premium configuration. Generalization to n assets yields 2^n distinct states.

3.3 The SIV Structure

Theorem 3.3 (Portfolio as SIV)**.** *A portfolio P of n assets with dual premia is a structured investment vehicle that efficiently allocates risk through risk preferences of 2^n individuals.*

Proof Sketch. The proof proceeds in three steps:

Step 1: Differential Risk-Return Profiles. Each realization P^k has distinct expected return and variance:

$$\mathbb{E}[r_P^k] = r_f + \mathbb{E}[i] + p_i + \sum_{i=1}^{n} w_i p_{a,i}^{(\sigma_i^k)} \tag{15}$$

$$\sigma_P^{2,k} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \mathrm{Cov}(r_i^k, r_j^k) \tag{16}$$

Step 2: Preference Matching. Each investor j with utility $U_j(r, \sigma)$ selects the realization maximizing expected utility:

$$P^{k^*} = \underset{k}{\operatorname{argmax}}\, \mathbb{E}[U_j(r_P^k, \sigma_P^k)] \tag{17}$$

Step 3: Pareto Efficiency. With 2^n realizations and 2^n investors, a bijective matching exists such that no reallocation improves any investor's utility without harming another's. ☐

Risk-Return Space: $n = 3$ Assets, $2^3 = 8$ Realizations

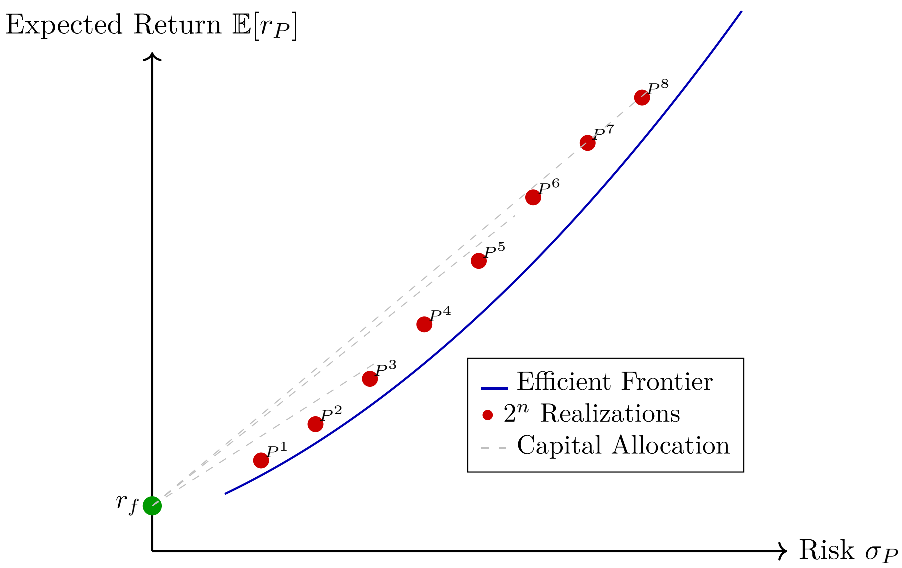

Figure 4: Portfolio realizations in mean-variance space.

Each realization (red dots) represents a distinct risk-return profile accessible through different premium selections. Investors optimize along capital allocation lines based on their risk preferences.

4 Theoretical Framework for Efficient Risk Allocation

4.1 Model Setup

4.1.1 Heterogeneous Investor Space

Consider an economy with a continuum of investors $j \in [0, 1]$, each characterized by:

Risk aversion $\gamma_j \in [\gamma_{\min}, \gamma_{\max}]$, distributed according to density $f(\gamma)$

Beliefs Subjective probability distribution $\mathbb{P}_j$ over asset returns

Initial wealth $W_j^0 > 0$ with $\int_0^1 W_j^0 \, dj = W^{\text{total}}$

4.1.2 Utility Specification

Investor j maximizes expected utility:

$$U_j = \mathbb{E}_j[u(W_j; \gamma_j)] \tag{18}$$

where $u(\cdot; \gamma_j)$ exhibits constant relative risk aversion:

$$u(W; \gamma) = \frac{W^{1-\gamma}}{1-\gamma} \tag{19}$$

4.2 Premium Selection Mechanism

Definition 4.1 (Selection Function)**.** The **selection function** $\sigma_{i,j} : \Theta_j \to \{+, -\}$ maps investor characteristics $\Theta_j = (\gamma_j, \mathbb{P}_j, W_j^0)$ to premium choices for asset i.

Proposition 4.2 (Premium-Type Correspondence)**.** *For each asset i and investor type Θ_j, there exists a unique optimal premium selection $\sigma_{i,j}^*$ that maximizes expected utility.*

Proof. The dual premia create distinct portfolio dynamics. Given utility concavity and the rational equation constraint from Theorem 2.2, the optimization problem:

$$\sigma_{i,j}^* \in \underset{\sigma \in \{+,-\}}{\operatorname{argmax}} \, \mathbb{E}_j[u(W_j(p_{a,i}^{(\sigma)}))] \tag{20}$$

has a unique solution by standard arguments. □

4.3 Market Equilibrium

Definition 4.3 (Competitive Equilibrium)**.** A **competitive equilibrium** consists of:

- Asset allocations $\{\theta_{i,j}\}$ for each investor j and asset i
- Premium realizations $\{p_{a,i,j}\}$ for each investor-asset pair
- Prices $\{p_i\}$ that clear all markets

subject to:

1. **Individual optimization**: $\theta_{i,j}$ solves $\max U_j$ given prices
2. **Market clearing**: $\int \theta_{i,j} \, dj = S_i$ (aggregate supply)
3. **Premium consistency**: $p_{a,i,j} \in \{p_{a,i}^{(+)}, p_{a,i}^{(-)}\}$

4.4 Efficiency Result

Theorem 4.4 (Pareto Efficiency)**.** *The competitive equilibrium with dual asset premia is Pareto efficient despite heterogeneous beliefs.*

Proof. The proof relies on three key observations:

(i) Market Completeness. The 2^n realizations approximate a complete market structure where investors achieve optimal allocations.

(ii) Adjusted CCAPM. Following heterogeneous beliefs frameworks, the equilibrium satisfies:

$$\mathbb{E}_M[r_i - r_f] = \beta_{i,M}\lambda_M \tag{21}$$

with adjustments for belief heterogeneity.

(iii) No Pareto Improvements. Each investor selects premia aligning with their beliefs, and price adjustments internalize belief differences. No feasible reallocation improves welfare under any $\mathbb{P}_j$. □

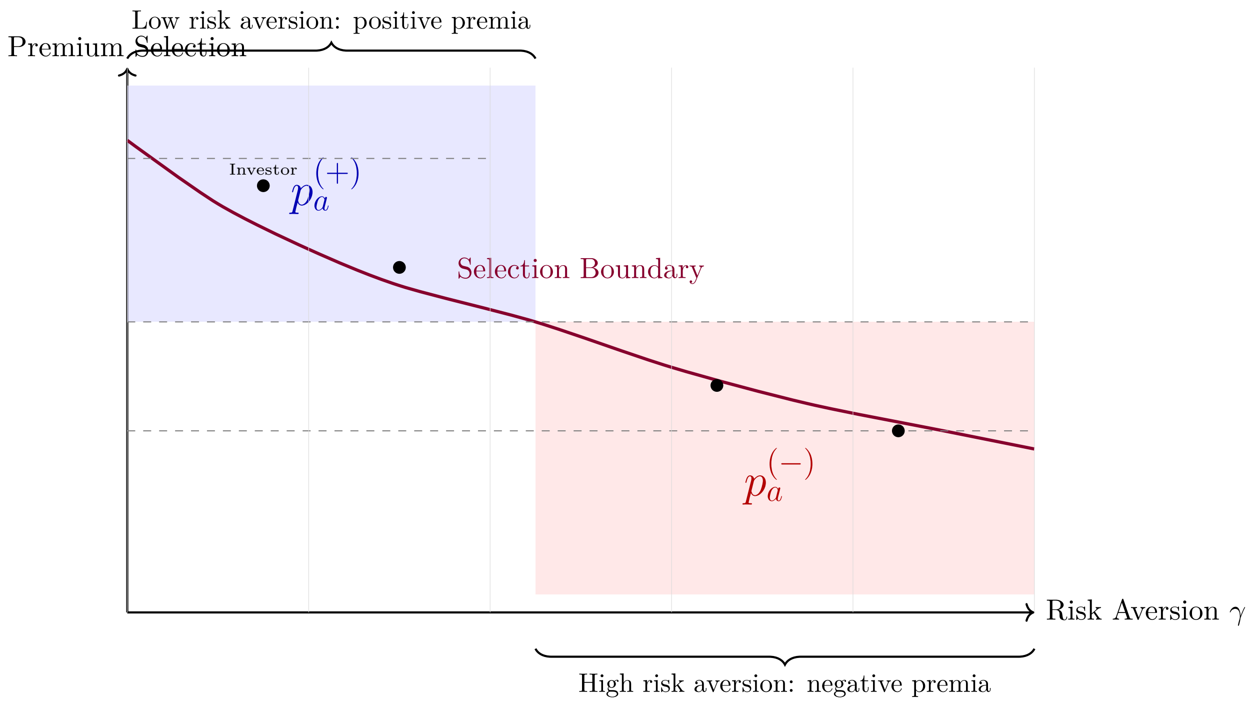

Figure 5: Premium selection as a function of risk aversion.

Low risk-aversion investors (blue region) select positive premia, accepting higher risk for higher expected returns. High risk-aversion investors (red region) select negative premia, preferring safety. The purple curve represents the indifference boundary where investors are equally satisfied with either choice.

5 Explicit State Price Solutions

5.1 State Space and Physical Probabilities

Let $\varphi(\omega_k)$ denote the physical probability of state ω_k. Under independence of premium realizations:

$$\varphi(\omega_k) = \prod_{i=1}^{n} \left[p \cdot \mathbb{1}_{\sigma_i^k=+} + (1-p) \cdot \mathbb{1}_{\sigma_i^k=-}\right] \tag{22}$$

where $p \in [0,1]$ is the probability of positive premium realization.

5.2 Stochastic Discount Factor

For a representative investor with aggregate risk aversion $\bar{\gamma}$, the stochastic discount factor is:

$$M(\omega) = \beta \left[\frac{C(\omega)}{C_0}\right]^{-\bar{\gamma}} \tag{23}$$

where $C(\omega)$ is aggregate consumption in state ω.

Aggregate wealth in state ω_k satisfies:

$$W(\omega_k) = W_0 \left[1 + r_P(\omega_k)\right] \tag{24}$$

with portfolio return:

$$r_P(\omega_k) = r_f + \mathbb{E}[i] + p_i + \sum_{i=1}^{n} w_i p_{a,i}^{(\sigma_i^k)} \tag{25}$$

5.3 Main Result

Theorem 5.1 (Explicit State Prices)**.** *The state price for state ω_k is given by:*

$$\boxed{\pi(\omega_k) = \beta \cdot \varphi(\omega_k) \cdot \left[1 + r_P(\omega_k)\right]^{-\bar{\gamma}}} \tag{26}$$

where:

$$\begin{aligned} r_P(\omega_k) = r_f + \mathbb{E}[i] + p_i + \sum_{i=1}^{n} w_i \Bigg[& \frac{\mathbb{1}_{\sigma_i^k=+}}{2} \left(\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2}\right. \\ & \left. -2r_f - p_i - \mathbb{E}[i] - 1\right) - \frac{\mathbb{1}_{\sigma_i^k=-}}{2} \left(\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2}\right. \\ & \left. +2r_f + p_i + \mathbb{E}[i] + 1\right) \Bigg] \end{aligned} \tag{27}$$

Proof. State prices satisfy the fundamental pricing equation:

$$p_i = \sum_{\omega} \pi(\omega) X_i(\omega) \tag{28}$$

The stochastic discount factor approach gives:

$$\pi(\omega) = \varphi(\omega) M(\omega) \tag{29}$$

Substituting the CRRA utility specification and wealth dynamics yields the result. □

5.4 Properties of State Prices

Proposition 5.2 (No-Arbitrage Condition). *The state prices satisfy:*

$$\sum_{\omega \in \Omega} \pi(\omega) = \frac{1}{1 + r_f} \tag{30}$$

Proposition 5.3 (Risk-Neutral Probability). *The risk-neutral probability is:*

$$Q(\omega_k) = \pi(\omega_k) \times (1 + r_f) = \beta \cdot \varphi(\omega_k) \cdot [1 + r_P(\omega_k)]^{-\bar{\gamma}} \cdot (1 + r_f) \tag{31}$$

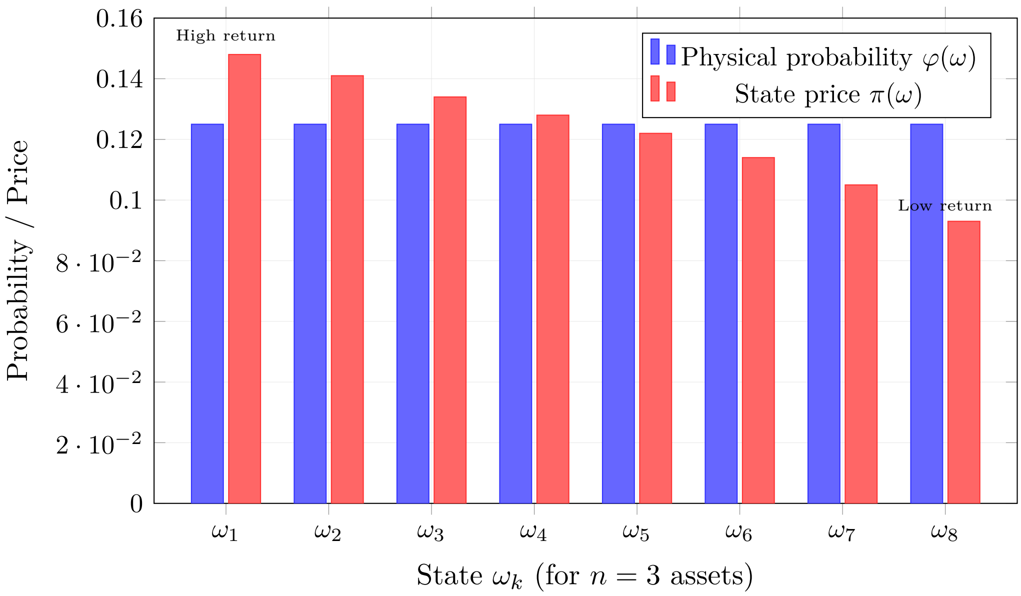

Figure 6: Comparison of physical probabilities $\varphi(\omega_k)$ and risk-adjusted state prices $\pi(\omega_k)$ for $n = 3$ assets ($2^3 = 8$ states) with equal realization probability $p = 0.5$.

Risk aversion $\bar{\gamma} = 2$ shifts pricing toward safer (lower-return) states, creating the wedge between physical and risk-neutral measures.

6 Existence and Uniqueness of Equilibrium

6.1 Formal Problem Statement

We consider a two-period economy with:

- n risky assets plus a risk-free asset
- State space $\Omega = \{\omega_1, \ldots, \omega_{2^n}\}$
- Continuum of investors $j \in [0, 1]$ with characteristics $(\gamma_j, \mathbb{P}_j, W_j^0)$
- Asset supplies $\{S_i\}_{i=1}^n$

6.2 Assumptions

(A1) Preferences
Each U_j is continuous, strictly increasing, strictly concave, and satisfies Inada conditions

(A2) Endowments
$W_j^0 > 0$ for all j, and $\int_0^1 W_j^0 \, dj = \sum_i p_i S_i$

(A3) Dual Premium Regularity
The dual premia from Theorem 2.3 are finite with $p_{a,i}^{(+)} > p_{a,i}^{(-)}$ and $|p_{a,i}^{(\pm)}| < K < \infty$

(A4) No-Arbitrage
$1 + r_f + p_a(t) \neq 0$ as required in Theorem 2.2

(A5) Gross Substitutes
$\partial D_i / \partial p_j > 0$ for $i \neq j$

(A6) Strong Concavity
U_j is twice differentiable with Hessian eigenvalues $\lambda_{\min} < -c$ for some $c > 0$

6.3 Existence Theorem

Theorem 6.1 (Existence of Equilibrium)**.** *Under assumptions **(A1)**-**(A4)**, a competitive equilibrium exists.*

Proof. We apply Kakutani's fixed point theorem.

Step 1: Compact Strategy Space. Define investor j's strategy space:

$$\Theta_j = \left\{ (\theta_j, \sigma_j) : \sum_i \theta_{i,j} p_i \leq W_j^0, \, \theta_{i,j} \in \left[0, \frac{W_j^0}{p_i}\right], \, \sigma_{i,j} \in \{+, -\} \right\} \tag{32}$$

Lemma 6.2. Θ_j *is compact in the product topology.*

Proof of Lemma 6.2: The continuous part $[0, W_j^0/p_i]^n$ is compact by Tychonoff's theorem. The discrete part $\{+, -\}^n$ is finite hence compact. Their product is compact. □

Step 2: Upper Hemicontinuity of Demand.

Lemma 6.3. *The optimal demand correspondence*

$$D_j(p, \pi) = \underset{(\theta_j, \sigma_j) \in \Theta_j}{\operatorname{argmax}} \sum_\omega \pi(\omega) U_j(W_j(\omega)) \tag{33}$$

is upper hemicontinuous in (p, π).

Proof of Lemma 6.3: By **(A1)**, the objective is continuous. The constraint correspondence is continuous in p. Strict concavity ensures uniqueness. Therefore D_j is uhc by the Maximum Theorem. □

Step 3: Excess Demand Function. Define aggregate excess demand:

$$z(p, \pi) = \int_0^1 D_j(p, \pi) \, dj - S \tag{34}$$

Lemma 6.4. *$z(p, \pi)$ is upper hemicontinuous and satisfies Walras' Law: $p \cdot z(p, \pi) = 0$.*

Proof of Lemma 6.4: Uhc follows from Lemma 6.3 since integration preserves uhc. Walras' Law holds because each investor exhausts their budget by strict monotonicity. □

Step 4: Fixed Point Argument. Define the price simplex:

$$\Delta = \left\{ p \in \mathbb{R}^n_+ : \sum_i p_i = 1 \right\} \tag{35}$$

and the best response correspondence:

$$\Phi(p) = \{p' \in \Delta : p'_i \propto \max\{0, p_i + z_i(p, \pi(p))\}\} \tag{36}$$

Lemma 6.5. *$\Phi : \Delta \to \Delta$ is non-empty, convex-valued, and upper hemicontinuous.*

Proof of Lemma 6.5: Non-emptiness and convex-valuedness are immediate. Uhc follows from composition of continuous functions. □

By Kakutani's theorem, there exists $p^* \in \Phi(p^*)$, which implies $z(p^*, \pi(p^*)) \leq 0$ componentwise. By Walras' Law (Lemma 6.4), this forces $z(p^*, \pi(p^*)) = 0$, establishing market clearing. □

6.4 Uniqueness Theorem

Theorem 6.6 (Uniqueness of Equilibrium)**.** *Under assumptions **(A1)**-**(A6)**, the equilibrium price p^* is unique.*

Proof. Suppose there exist two distinct equilibria p^* and p^{**} with $p^* \neq p^{**}$.

Step 1: Negative Definite Jacobian.

Lemma 6.7. *Under **(A5)**, the excess demand Jacobian $J = [\partial z_i / \partial p_j]$ is negative definite.*

Proof of Lemma 6.7: By **(A5)**, off-diagonal elements are positive. By **(A6)**, own-price effects dominate: $\partial D_i / \partial p_i < -K$ for large K. Row sums are negative by Walras' Law. This structure implies J is negative definite by Gershgorin's theorem. □

Step 2: Contradiction via Path Monotonicity. Consider the path $p(t) = tp^* + (1-t)p^{**}$ for $t \in [0, 1]$. By Lemma 6.7:

$$\frac{d}{dt} z(p(t), \pi(p(t))) = J(p(t))(p^* - p^{**}) < 0 \tag{37}$$

strictly in direction $(p^* - p^{**})$.

But equilibrium requires $z(p^*, \pi(p^*)) = z(p^{**}, \pi(p^{**})) = 0$, contradicting strict monotonicity. Therefore $p^* = p^{**}$.

Step 3: Uniqueness of Allocations. Given unique prices p^*, each investor's problem has a unique solution by strict concavity **(A6)**, establishing uniqueness of $\{\theta_j^*\}$ and $\{\sigma_j^*\}$. □

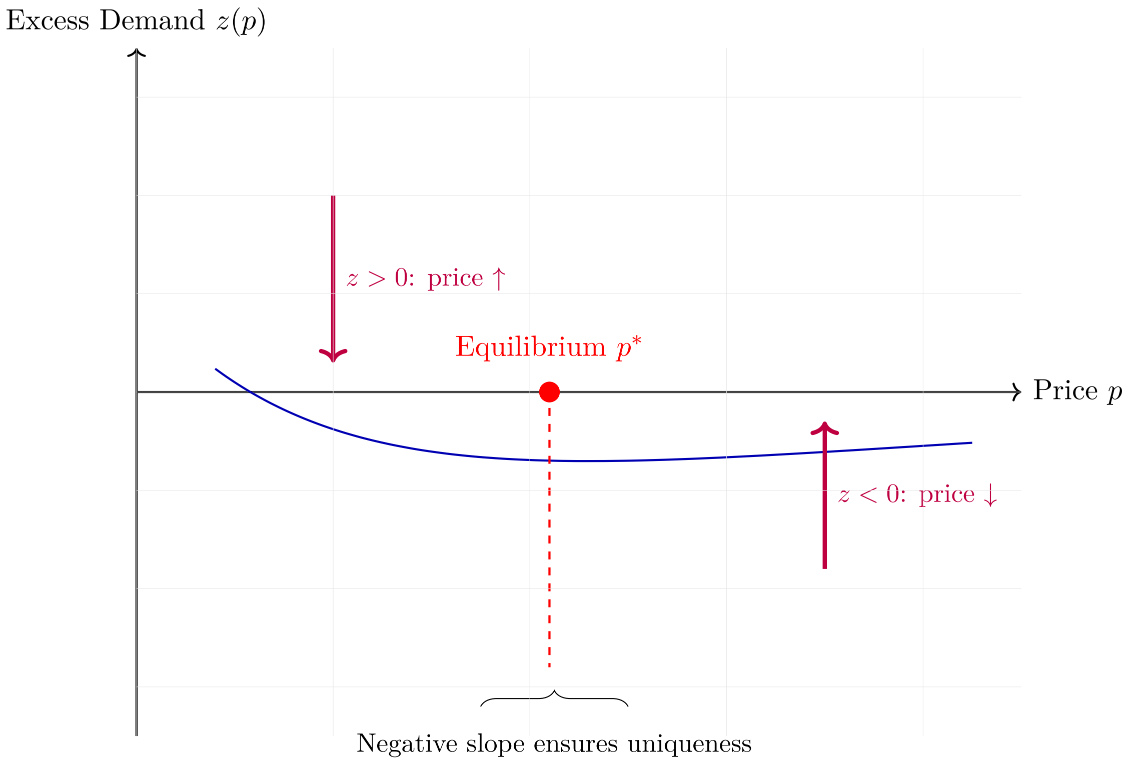

Figure 7: Excess demand function $z(p)$ with unique equilibrium at p^*.

The gross substitutes condition ensures downward slope, guaranteeing uniqueness. Arrows indicate tâtonnement price adjustment: excess demand raises prices, excess supply lowers them, converging to equilibrium.

6.5 Stability

Theorem 6.8 (Global Stability). *The unique equilibrium is globally stable under tâtonnement dynamics $\dot{p} = z(p, \pi(p))$.*

Proof. Define the Lyapunov function $V(p) = \|p - p^*\|^2$. Then:

$$\frac{dV}{dt} = 2(p - p^*) \cdot \dot{p} = 2(p - p^*) \cdot z(p, \pi(p)) \tag{38}$$

By Lemma 6.7, $dV/dt < 0$ for all $p \neq p^*$. Therefore $V \to 0$, implying $p(t) \to p^*$. □

7 Applications and Implications

7.1 Monetary Policy Transmission

The dual premium framework reveals a novel transmission mechanism for monetary policy. Changes in r_f affect both premia:

$$\frac{\partial p_a^{(+)}}{\partial r_f} = \frac{2 - \frac{4r_f}{\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2}}}{2} > 0 \tag{39}$$

$$\frac{\partial p_a^{(-)}}{\partial r_f} = -\frac{2 + \frac{4r_f}{\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2}}}{2} < 0 \tag{40}$$

Corollary 7.1 (Monetary Policy Asymmetry)**.** *Monetary tightening (increasing r_f) increases positive premia and decreases negative premia in absolute value, creating asymmetric effects across investor types.*

7.2 Risk Management Implications

Proposition 7.2 (Portfolio Diversification)**.** *The correlation structure of dual premia creates diversification opportunities not present in single-premium frameworks:*

$$Cov(p_{a,i}^{(\sigma_i)}, p_{a,j}^{(\sigma_j)}) \neq Cov(r_i, r_j) \tag{41}$$

7.3 Empirical Predictions

The framework generates testable predictions:

1. **Cross-sectional sorting**: High risk-aversion investors hold assets with negative premium selections disproportionately
2. **Premium switching**: Changes in r_f trigger predictable rebalancing patterns
3. **Volatility asymmetry**: Return volatility differs across premium selections even for identical underlying assets

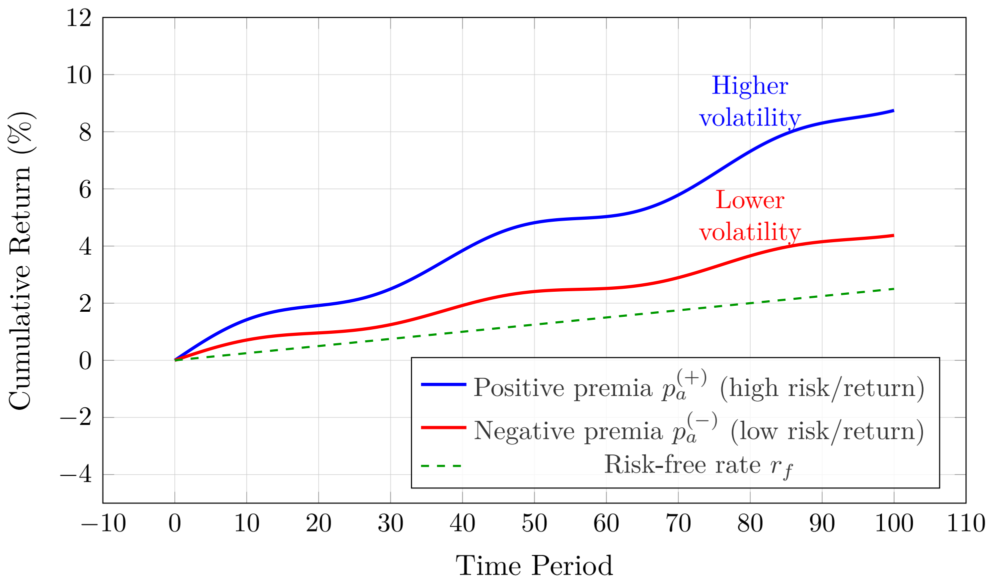

Figure 8: Simulated cumulative returns for portfolios with different premium selections.

Positive premia (blue) yield higher expected returns with greater volatility, while negative premia (red) provide lower returns with reduced risk. Both strategies outperform the risk-free rate (green) in expectation but with different risk profiles.

8 Conclusion

This treatise has developed a complete theory of structured investment vehicles through the lens of dual asset premia. Our main contributions are:

1. **Theoretical foundation**: Establishing that dual premia arise naturally from no-arbitrage conditions (Theorem 2.3)

2. **SIV structure**: Proving that n assets with dual premia create a structured vehicle with 2^n realizations (Theorem 3.3)

3. **Efficient allocation**: Demonstrating Pareto efficiency of dual-premium equilibria (Theorem 4.4)

4. **State prices**: Deriving explicit formulas for state prices $\pi(\omega)$ (Theorem 5.1)

5. **Equilibrium theory**: Proving existence and uniqueness under general conditions (Theorems 6.1 and 6.6)

The framework unifies portfolio theory, asset pricing, and structured finance, providing both theoretical rigor and practical insights. Future research directions include empirical testing of premium selection patterns, extension to dynamic settings, and applications to derivative pricing.

9 Glossary

Asset Premium $p_a(t)$
: The additional premium required to hold a specific asset beyond the risk-free rate, expected inflation, and inflation risk premium. Mathematically defined through equations (2) and (3).

Dual Premium
: The property that each asset's premium admits two distinct values—positive $p_a^{(+)}$ and negative $p_a^{(-)}$—arising from the quadratic structure of the premium equations (Theorem 2.3).

Portfolio Realization P^k
: A specific configuration of premium selections across all n assets in a portfolio, indexed by $k \in \{1, 2, \ldots, 2^n\}$. Each realization corresponds to a unique risk-return profile.

State Space Ω
: The set of all possible premium configurations: $\Omega = \{\omega_1, \omega_2, \ldots, \omega_{2^n}\}$ where each state ω_k represents a binary vector $(\sigma_1^k, \sigma_2^k, \ldots, \sigma_n^k)$ of premium selections.

State Price $\pi(\omega)$
: The price of one unit of consumption in state ω. Explicitly given by $\pi(\omega_k) = \beta\varphi(\omega_k)[1 + r_P(\omega_k)]^{-\bar{\gamma}}$ (Theorem 5.1).

Structured Investment Vehicle (SIV)
: A non-bank financial entity that pools investments to profit from credit spreads. In our framework, any portfolio with dual premia naturally forms an SIV with 2^n tranches.

Selection Function $\sigma_{i,j}$

A mapping from investor characteristics $\Theta_j = (\gamma_j, \mathbb{P}_j, W_j^0)$ to premium choices $\{+, -\}$ for each asset i. Determines which premium realization an investor selects.

Risk Aversion γ

The coefficient of relative risk aversion in CRRA utility $u(W) = W^{1-\gamma}/(1-\gamma)$. Higher γ indicates greater risk aversion and preference for negative premia.

Heterogeneous Beliefs

The situation where investors hold different subjective probability distributions $\mathbb{P}_j$ over asset returns. Our framework achieves efficiency despite this heterogeneity.

Pareto Efficiency

An allocation where no reallocation can improve any investor's utility without harming another's. Achieved in dual-premium equilibrium (Theorem 4.4).

Excess Demand Function $z(p, \pi)$

The difference between aggregate demand and supply at given prices. Equilibrium requires $z(p^*, \pi(p^*)) = 0$.

Gross Substitutes

The property that demand for asset i increases when the price of asset $j \neq i$ increases: $\partial D_i / \partial p_j > 0$. Ensures uniqueness of equilibrium (Assumption A5).

Stochastic Discount Factor $M(\omega)$

The pricing kernel $M(\omega) = \beta[C(\omega)/C_0]^{-\bar{\gamma}}$ that connects physical probabilities to risk-adjusted state prices.

Tâtonnement Dynamics

The price adjustment process $\dot{p} = z(p, \pi(p))$ where prices increase (decrease) in response to positive (negative) excess demand. Converges to unique equilibrium under our conditions.

Risk-Neutral Probability $Q(\omega)$

The probability measure under which discounted asset prices are martingales: $Q(\omega_k) = \pi(\omega_k) \times (1 + r_f)$.

Inflation Risk Premium $p_i(t)$

Compensation for bearing inflation uncertainty, distinct from expected inflation $\mathbb{E}[i(t)]$. Appears in the base return decomposition (equation (2)).

Market Completeness

The property that there are enough traded securities to replicate any desired payoff pattern. The 2^n realizations approximate completeness.

Capital Allocation Line
The linear combination of the risk-free asset and a risky portfolio that investors choose based on their risk preferences. Each investor selects a point on their preferred line.

Walras' Law The budget constraint implication that the value of excess demand equals zero: $p \cdot z(p) = 0$. Holds in our framework by construction (Lemma 6.4).

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