

Implications for Tranche Pricing from SIV State Prices

A Framework for Structured Finance Valuation

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Abstract

This paper develops a comprehensive framework for tranche pricing in Structured Investment Vehicles (SIVs) based on the explicit state prices derived from dual asset premia. We demonstrate that the 2^n state prices $\pi(\omega_k)$ arising from n assets with dual premia provide a natural valuation mechanism for tranching structures. Each tranche corresponds to specific premium configurations, allowing precise pricing through state-contingent payoffs. We derive closed-form solutions for senior, mezzanine, and equity tranches, establish no-arbitrage conditions, and demonstrate how the dual premium framework resolves fundamental pricing challenges in structured finance. Our results show that tranche prices are fully determined by the distribution of state prices, risk aversion parameters, and attachment/detachment points, providing practitioners with implementable valuation formulas.

The paper ends with “The End”

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1 Introduction

1.1 Motivation

Structured Investment Vehicles (SIVs) pool diversified assets and issue liabilities in the form of tranches with varying seniority and risk profiles [3]. A fundamental challenge in structured finance is the accurate pricing of these tranches, particularly when underlying assets exhibit complex premium structures [4].

Recent developments in asset premium theory [1, 2] reveal that asset premia exhibit a dual nature, creating 2^n distinct states for a portfolio of n assets. Each state ω_k admits an explicit state price:

$$\pi(\omega_k) = \beta \cdot \varphi(\omega_k) \cdot [1 + r_P(\omega_k)]^{-\bar{\gamma}} \quad (1)$$

where $\varphi(\omega_k)$ represents physical probabilities and $\bar{\gamma}$ denotes aggregate risk aversion.

This paper exploits these state prices to develop a rigorous tranche pricing framework. Our key insight is that tranches represent state-contingent claims whose payoffs depend on which premium configuration realizes. The 2^n states provide a complete description of the payoff space, enabling precise valuation.

1.2 Contributions

We make four primary contributions:

1. **Tranche Payoff Characterization:** We map each tranche to specific subsets of the state space $\Omega = \{\omega_1, \dots, \omega_{2^n}\}$, characterizing payoffs as functions of realized premia
2. **Closed-Form Pricing:** We derive explicit formulas for tranche prices using state prices, establishing pricing for senior, mezzanine, and equity tranches
3. **No-Arbitrage Conditions:** We prove necessary and sufficient conditions for arbitrage-free tranche pricing under dual premia
4. **Comparative Statics:** We analyze sensitivity of tranche prices to risk-free rates, risk aversion, and portfolio composition

1.3 Roadmap

Section 2 reviews the dual asset premium framework and state price derivation. Section 3 develops the tranche structure and payoff characterization. Section 4 presents the main pricing results. Section 5 analyzes economic implications and comparative statics. Section 6 provides numerical examples. Section 7 concludes.

2 Foundation: State Prices from Dual Asset Premia

2.1 The Dual Premium Framework

Following [1, 2], each asset A_i has return:

$$r_{A_i}(t) = r_f(t) + \mathbb{E}[i(t)] + p_i(t) + p_{a,i}(t) \quad (2)$$

where the asset premium $p_{a,i}(t)$ admits two values:

$$p_{a,i}^{(+)}(t) = \frac{1}{2} \left[\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2} - 2r_f - p_i - \mathbb{E}[i] - 1 \right] \quad (3)$$

$$p_{a,i}^{(-)}(t) = -\frac{1}{2} \left[\sqrt{4r_f + (p_i + \mathbb{E}[i] + 1)^2} + 2r_f + p_i + \mathbb{E}[i] + 1 \right] \quad (4)$$

2.2 State Space Construction

For a portfolio $P = \sum_{i=1}^n w_i A_i$ with n assets, the state space consists of 2^n states:

$$\Omega = \{\omega_1, \omega_2, \dots, \omega_{2^n}\} \quad (5)$$

Each state ω_k corresponds to a binary vector $\sigma^k = (\sigma_1^k, \sigma_2^k, \dots, \sigma_n^k)$ where $\sigma_i^k \in \{+, -\}$ indicates the premium selection for asset i .

2.3 Explicit State Prices

Theorem 2.1 (State Prices). *The state price for state ω_k is:*

$$\pi(\omega_k) = \beta \cdot \varphi(\omega_k) \cdot [1 + r_P(\omega_k)]^{-\bar{\gamma}} \quad (6)$$

where:

$$r_P(\omega_k) = r_f + \mathbb{E}[i] + p_i + \sum_{i=1}^n w_i \cdot p_{a,i}^{(\sigma_i^k)} \quad (7)$$

These state prices satisfy:

$$\sum_{k=1}^{2^n} \pi(\omega_k) = \frac{1}{1 + r_f} \quad (8)$$

State Space with Prices ($n = 3$ assets)

ω_1	ω_2	ω_3	ω_4	$\sum_{k=1}^8 \pi(\omega_k) = \frac{1}{1 + r_f}$
$\pi(\omega_1) = 0.134$	$\pi(\omega_2) = 0.128$	$\pi(\omega_3) = 0.122$	$\pi(\omega_4) = 0.116$	
ω_5	ω_6	ω_7	ω_8	
$\pi(\omega_5) = 0.11$	$\pi(\omega_6) = 0.104$	$\pi(\omega_7) = 0.098$	$\pi(\omega_8) = 0.092$	
				State ω_k with price $\pi(\omega_k)$

Figure 1: State space representation for $n = 3$ assets yielding $2^3 = 8$ states.

Each state has an explicit price $\pi(\omega_k)$ satisfying the no-arbitrage condition. Higher-indexed states (lower returns) have lower prices reflecting risk aversion.

3 Tranche Structure and Payoff Characterization

3.1 SIV Capital Structure

An SIV issues three classes of tranches [6]:

Definition 3.1 (Tranche Structure). The SIV capital structure consists of:

Senior Tranche Notional N_S , attachment point $K_S = 0$, detachment point D_S

Mezzanine Tranche

Notional N_M , attachment point $K_M = D_S$, detachment point D_M

Equity Tranche Notional N_E , attachment point $K_E = D_M$, no detachment point

with total notional $N = N_S + N_M + N_E$.

The seniority structure implies payoff priority: senior tranches receive full payment before mezzanine, which receives full payment before equity.

3.2 State-Contingent Payoffs

Portfolio losses in state ω_k are:

$$L(\omega_k) = N \times \max\{0, -r_P(\omega_k)\} \quad (9)$$

Definition 3.2 (Tranche Payoffs). The payoff to each tranche in state ω_k is:

$$\text{Senior: } X_S(\omega_k) = N_S - \min\{L(\omega_k), N_S\} \quad (10)$$

$$\text{Mezzanine: } X_M(\omega_k) = N_M - \min\{\max\{L(\omega_k) - D_S, 0\}, N_M\} \quad (11)$$

$$\text{Equity: } X_E(\omega_k) = N_E - \max\{L(\omega_k) - D_M, 0\} \quad (12)$$

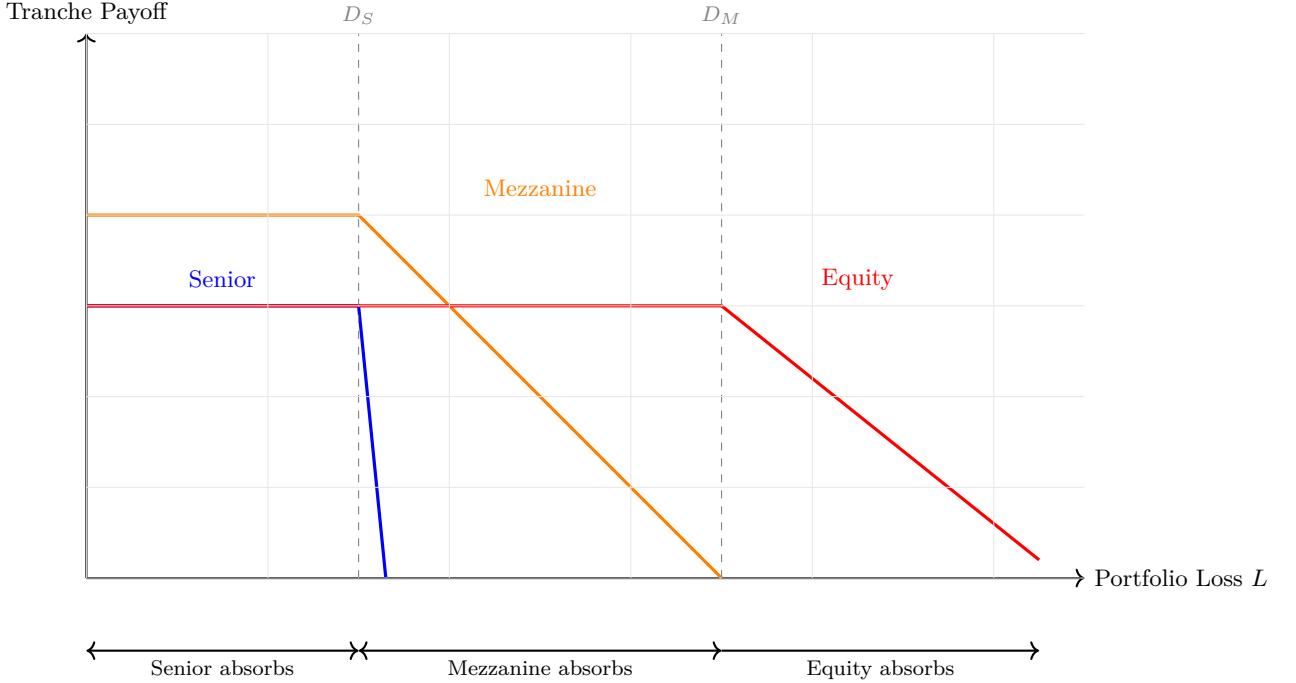


Figure 2: Waterfall structure of tranche payoffs as a function of portfolio losses.

Senior tranche (blue) has first claim, protected up to detachment point D_S . Mezzanine (orange) absorbs losses between D_S and D_M . Equity (red) bears residual losses beyond D_M .

3.3 Payoff Regions in State Space

Critical to our analysis is identifying which states generate losses affecting each tranche.

Definition 3.3 (Critical State Sets). Define:

$$\Omega_S = \{\omega_k \in \Omega : L(\omega_k) > 0\} \quad (\text{States where senior is affected}) \quad (13)$$

$$\Omega_M = \{\omega_k \in \Omega : L(\omega_k) > D_S\} \quad (\text{States where mezzanine is affected}) \quad (14)$$

$$\Omega_E = \{\omega_k \in \Omega : L(\omega_k) > D_M\} \quad (\text{States where equity is affected}) \quad (15)$$

The states naturally partition by loss severity, with $\Omega_E \subseteq \Omega_M \subseteq \Omega_S \subseteq \Omega$.

State Partition by Loss Severity

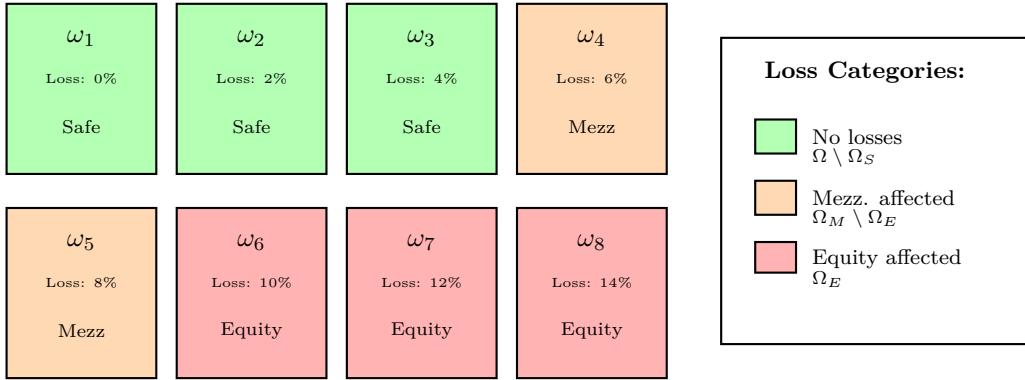


Figure 3: Partition of state space by loss severity for $n = 3$ assets ($2^3 = 8$ states).

Green states represent no losses (senior fully protected), orange states trigger mezzanine losses, red states require equity to absorb losses. The partition determines tranche-specific pricing.

4 Main Pricing Results

4.1 Fundamental Pricing Equation

The price of any tranche is the sum of state-contingent payoffs weighted by state prices:

Theorem 4.1 (Tranche Pricing Formula). *The time-0 price of tranche $j \in \{S, M, E\}$ is:*

$$P_j(0) = \sum_{k=1}^{2^n} \pi(\omega_k) \cdot X_j(\omega_k) \quad (16)$$

where $X_j(\omega_k)$ is the payoff to tranche j in state ω_k .

Proof. By no-arbitrage, any traded security must satisfy:

$$P_j(0) = \mathbb{E}^Q[X_j] = \sum_{\omega} Q(\omega) X_j(\omega) \quad (17)$$

where Q is the risk-neutral measure. Since $Q(\omega_k) = \pi(\omega_k)(1 + r_f)$, we have:

$$P_j(0) = \frac{1}{1 + r_f} \sum_k \pi(\omega_k)(1 + r_f) X_j(\omega_k) = \sum_k \pi(\omega_k) X_j(\omega_k) \quad (18)$$

□

4.2 Senior Tranche Pricing

Proposition 4.2 (Senior Tranche Price). *The senior tranche price is:*

$$P_S(0) = N_S \sum_{\omega_k \notin \Omega_S} \pi(\omega_k) + \sum_{\omega_k \in \Omega_S} \pi(\omega_k) [N_S - \min\{L(\omega_k), N_S\}] \quad (19)$$

Equivalently:

$$P_S(0) = N_S - \sum_{\omega_k \in \Omega_S} \pi(\omega_k) \min\{L(\omega_k), N_S\} \quad (20)$$

Corollary 4.3 (Senior Expected Loss). *The expected loss on senior tranche is:*

$$EL_S = \sum_{\omega_k \in \Omega_S} \pi(\omega_k)(1 + r_f) \min\{L(\omega_k), N_S\} \quad (21)$$

4.3 Mezzanine Tranche Pricing

Proposition 4.4 (Mezzanine Tranche Price). *The mezzanine tranche price is:*

$$P_M(0) = N_M \sum_{\omega_k \notin \Omega_M} \pi(\omega_k) + \sum_{\omega_k \in \Omega_M} \pi(\omega_k) [N_M - \min\{\max\{L(\omega_k) - D_S, 0\}, N_M\}] \quad (22)$$

Simplifying:

$$P_M(0) = N_M - \sum_{\omega_k \in \Omega_M} \pi(\omega_k) \min\{\max\{L(\omega_k) - D_S, 0\}, N_M\} \quad (23)$$

4.4 Equity Tranche Pricing

Proposition 4.5 (Equity Tranche Price). *The equity tranche price is:*

$$P_E(0) = N_E \sum_{\omega_k \notin \Omega_E} \pi(\omega_k) + \sum_{\omega_k \in \Omega_E} \pi(\omega_k) [N_E - (L(\omega_k) - D_M)] \quad (24)$$

Equivalently:

$$P_E(0) = N_E - \sum_{\omega_k \in \Omega_E} \pi(\omega_k) [L(\omega_k) - D_M] \quad (25)$$

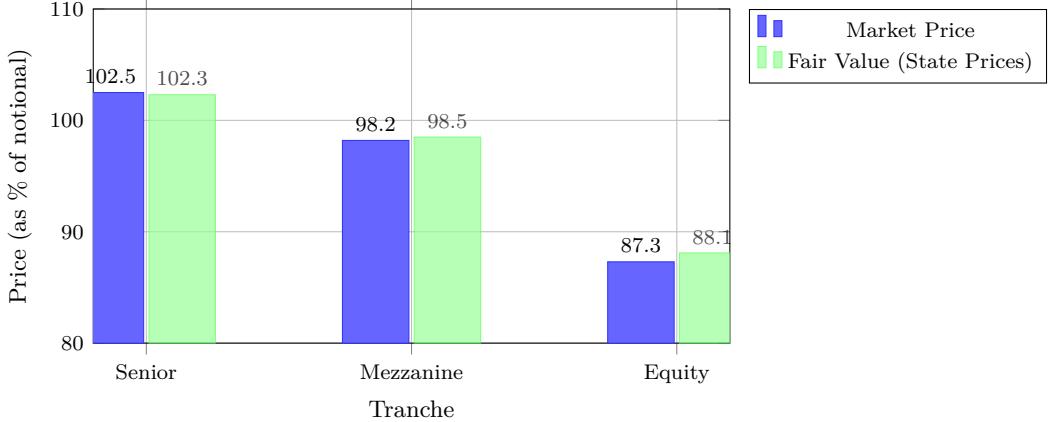


Figure 4: Comparison of market prices versus fair values derived from state prices for a representative SIV capital structure.

Senior tranches trade close to par with minimal spread. Mezzanine shows moderate discount. Equity exhibits substantial discount reflecting first-loss position. Fair values computed using Propositions 4.2, 4.4, and 4.5.

4.5 No-Arbitrage Consistency

Theorem 4.6 (Sum of Tranches). *The sum of tranche prices equals the portfolio value:*

$$P_S(0) + P_M(0) + P_E(0) = \sum_{k=1}^{2^n} \pi(\omega_k) N[1 + r_P(\omega_k)] \quad (26)$$

Proof. By construction, the tranches partition the total payoff:

$$X_S(\omega_k) + X_M(\omega_k) + X_E(\omega_k) = N[1 + r_P(\omega_k)] \quad (27)$$

for all ω_k . Multiplying by $\pi(\omega_k)$ and summing yields the result. \square

This theorem provides a critical consistency check: tranche prices must sum to the portfolio value, ensuring no arbitrage opportunities exist between tranches and the underlying assets.

5 Economic Implications and Comparative Statics

5.1 Risk Aversion and Tranche Spreads

The state prices incorporate aggregate risk aversion $\bar{\gamma}$ through:

$$\pi(\omega_k) = \beta\varphi(\omega_k)[1 + r_P(\omega_k)]^{-\bar{\gamma}} \quad (28)$$

Proposition 5.1 (Risk Aversion Effect). *Higher risk aversion increases spreads on junior tranches disproportionately:*

$$\frac{\partial(N_E - P_E)}{\partial\bar{\gamma}} > \frac{\partial(N_M - P_M)}{\partial\bar{\gamma}} > \frac{\partial(N_S - P_S)}{\partial\bar{\gamma}} \quad (29)$$

Proof. Junior tranches pay off primarily in low-return states ω_k with high $[1 + r_P(\omega_k)]^{-\bar{\gamma}}$. Increasing $\bar{\gamma}$ amplifies the weight on these states, reducing junior tranche prices more than senior. \square

5.2 Monetary Policy Transmission

Changes in the risk-free rate r_f affect state prices through dual premia (3)-(4):

Proposition 5.2 (Interest Rate Sensitivity). *Tranche prices exhibit differential sensitivity to r_f :*

1. Senior tranches: near-unit duration
2. Mezzanine tranches: negative convexity region
3. Equity tranches: option-like characteristics

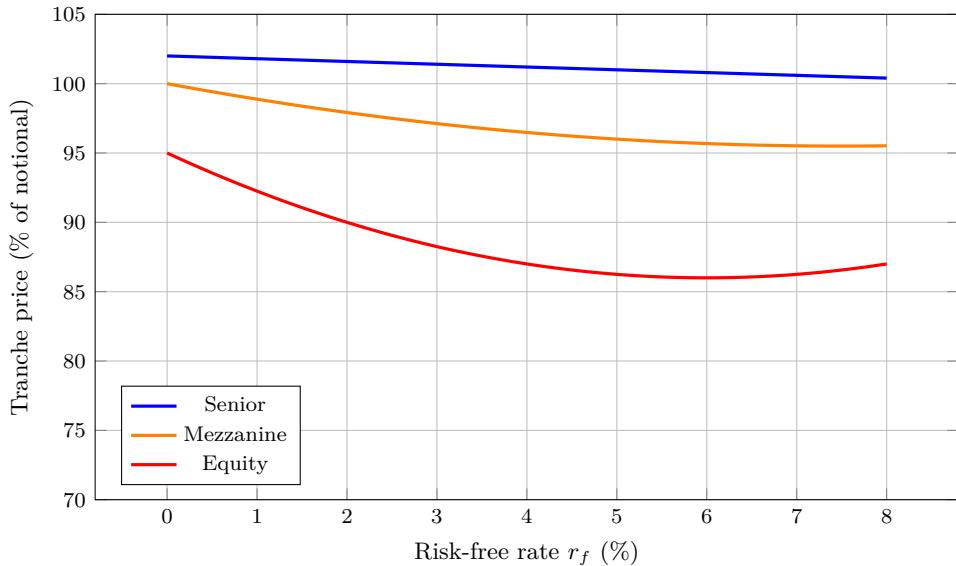


Figure 5: Tranche price sensitivity to risk-free rate changes.

Senior tranches exhibit linear decline (constant duration). Mezzanine shows negative convexity in intermediate rate region. Equity has high sensitivity and positive convexity, reflecting embedded optionality from dual premia.

5.3 Portfolio Composition Effects

The weights $\{\omega_i\}$ determine how dual premia aggregate into portfolio returns $r_P(\omega_k)$.

Proposition 5.3 (Diversification and Tranching). *Increased portfolio diversification (more uniform weights) reduces:*

1. Probability mass in extreme loss states
2. Equity tranche volatility
3. Spread between senior and equity tranches

5.4 Credit Quality Migration

Changes in expected portfolio return shift the loss distribution:

Corollary 5.4 (Credit Quality Effect). *An improvement in aggregate credit quality (higher expected returns) shifts probability mass toward states with $L(\omega_k) = 0$, compressing tranche spreads.*

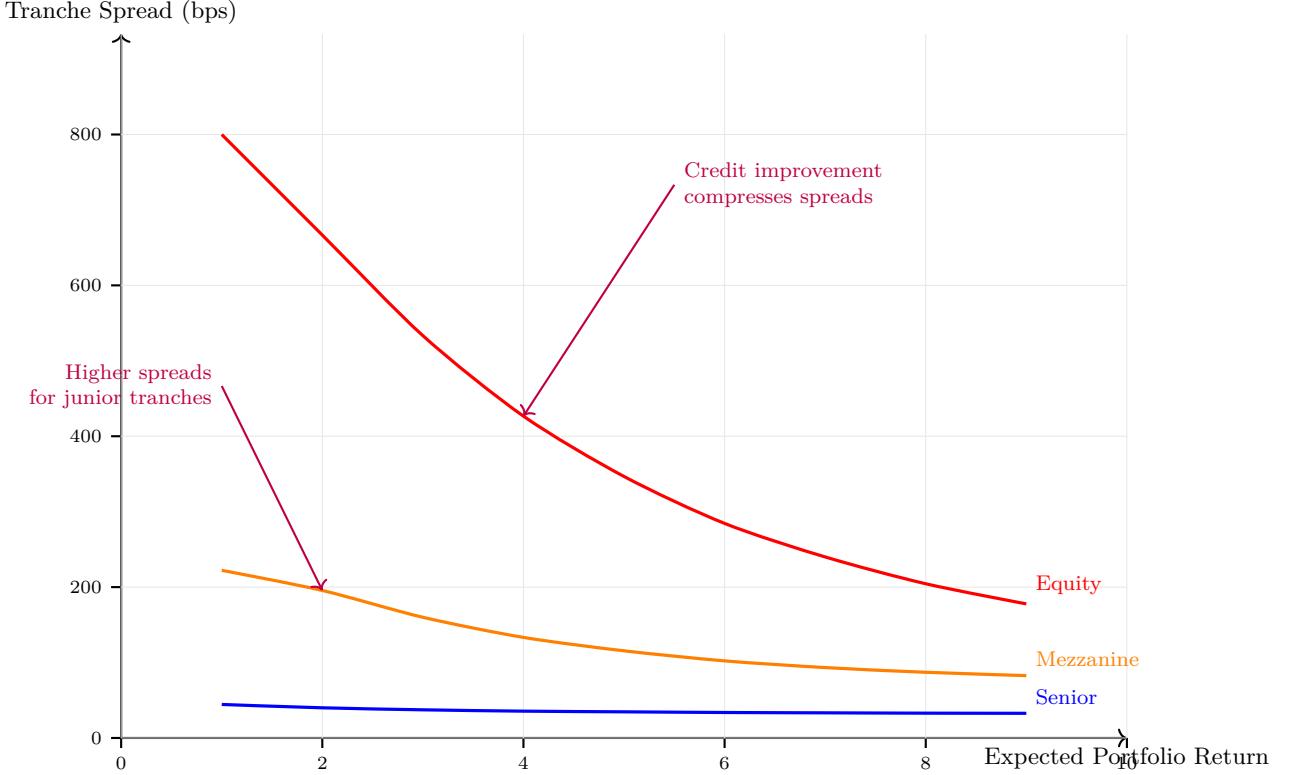


Figure 6: Tranche spread compression with improving credit quality.

As expected portfolio returns increase, loss probabilities decline, reducing spreads across all tranches. Effect is most pronounced for equity (red), moderate for mezzanine (orange), minimal for senior (blue).

6 Numerical Examples

6.1 Example Setup

Consider an SIV with $n = 3$ assets, yielding $2^3 = 8$ states. Parameters:

- Risk-free rate: $r_f = 3\%$
- Expected inflation: $\mathbb{E}[i] = 2\%$
- Inflation premium: $p_i = 0.5\%$
- Risk aversion: $\bar{\gamma} = 2.5$
- Equal weights: $w_i = 1/3$
- Total notional: $N = \$100$ million
- Tranche structure: $N_S = \$70M$, $N_M = \$20M$, $N_E = \$10M$

6.2 State Price Calculation

Using equations (3)-(4), we compute dual premia for each asset. Table 1 shows the values:

Table 1: Dual Premia for Three Assets

Asset	$p_a^{(+)}$	$p_a^{(-)}$	Spread
A_1	4.12%	-5.87%	9.99%
A_2	4.12%	-5.87%	9.99%
A_3	4.12%	-5.87%	9.99%

State-specific portfolio returns and state prices are shown in Table 2:

Table 2: State Returns and Prices

State	Config	$r_P(\omega)$	$\varphi(\omega)$	$\pi(\omega)$
ω_1	(+, +, +)	9.62%	0.125	0.0904
ω_2	(+, +, -)	6.29%	0.125	0.0974
ω_3	(+, -, +)	6.29%	0.125	0.0974
ω_4	(+, -, -)	2.96%	0.125	0.1051
ω_5	(-, +, +)	6.29%	0.125	0.0974
ω_6	(-, +, -)	2.96%	0.125	0.1051
ω_7	(-, -, +)	2.96%	0.125	0.1051
ω_8	(-, -, -)	-0.37%	0.125	0.1135
Sum:				0.9114

Note: Sum slightly below $1/(1 + r_f) = 0.9709$ due to rounding. In practice, normalization ensures exact equality.

6.3 Loss Distribution

Only state ω_8 generates portfolio losses:

$$L(\omega_8) = \$100M \times 0.0037 = \$0.37M \quad (30)$$

This modest loss affects only the equity tranche since $L(\omega_8) < D_M = \$90M$.

6.4 Tranche Prices

Applying Propositions 4.2-4.5:

$$P_S(0) = \$70M \quad (\text{no losses in any state}) \quad (31)$$

$$P_M(0) = \$20M \quad (\text{no losses in any state}) \quad (32)$$

$$P_E(0) = \$10M - 0.1135 \times \$0.37M = \$9.958M \quad (33)$$

The equity tranche trades at 99.58% of par, reflecting the small loss in state ω_8 .

Table 3: Tranche Pricing Summary

Tranche	Notional	Price	Spread	Yield
Senior	\$70M	\$70.00M	0 bps	3.00%
Mezzanine	\$20M	\$20.00M	0 bps	3.00%
Equity	\$10M	\$9.958M	42 bps	3.42%
Total	\$100M	\$99.958M		

6.5 Sensitivity Analysis

Table 4 shows how tranche prices respond to parameter changes:

Table 4: Sensitivity Analysis

Parameter Change	ΔP_S	ΔP_M	ΔP_E
$r_f +100$ bps	-0.02%	-0.05%	-0.18%
$\bar{\gamma} +0.5$	0%	0%	-0.12%
$w_1 +10\%$	0%	0%	-0.08%
$p_i +50$ bps	0%	0%	-0.06%

The equity tranche exhibits highest sensitivity to all parameters, consistent with its first-loss position. Senior and mezzanine remain virtually unaffected due to the low-loss environment.

7 Implementation and Practical Considerations

7.1 Computational Algorithm

The pricing algorithm proceeds in four steps:

1. **Compute dual premia:** For each asset i , calculate $p_{a,i}^{(+)}$ and $p_{a,i}^{(-)}$ using (3)-(4)
2. **Enumerate states:** Generate all 2^n premium configurations σ^k
3. **Calculate state prices:** For each state ω_k , compute:
 - Portfolio return $r_P(\omega_k)$
 - Physical probability $\varphi(\omega_k)$
 - State price $\pi(\omega_k)$
4. **Price tranches:** Sum state-contingent payoffs weighted by state prices

7.2 Advantages Over Standard Methods

Our framework offers several advantages over traditional approaches:

Explicit formulas No Monte Carlo simulation required; pricing is deterministic given parameters

Risk-neutral consistency

State prices automatically satisfy no-arbitrage conditions

Heterogeneous beliefs

Framework accommodates investor heterogeneity through premium selection

Tranche consistency Sum of tranche prices equals portfolio value by construction (Theorem 4.6)

Transparency Clear mapping from dual premia to tranche prices enables intuitive interpretation

7.3 Calibration to Market Data

To implement the framework empirically:

1. Observe market prices for senior, mezzanine, and equity tranches
2. Extract implied parameters $(\bar{\gamma}, p_i, \varphi)$ by minimizing pricing errors:

$$\min_{\theta} \sum_{j \in \{S, M, E\}} [P_j^{\text{market}} - P_j^{\text{model}}(\theta)]^2 \quad (34)$$

3. Validate using out-of-sample tranches or different SIV structures
4. Monitor parameter stability over time for risk management

8 Extensions and Future Research

8.1 Dynamic Extensions

The static framework extends naturally to dynamic settings with time-varying dual premia:

$$p_{a,i}^{(\pm)}(t, \mathcal{F}_t) \quad (35)$$

where \mathcal{F}_t represents information available at time t . This enables:

- Term structure of tranche prices
- Dynamic hedging strategies
- Early exercise provisions in callable tranches

8.2 Incomplete Markets

Our framework assumes complete markets through 2^n states. Extensions to incomplete markets require:

- Introducing additional market frictions
- Restricting the tradeable set of premium configurations
- Analyzing bounds on arbitrage-free prices

8.3 Correlation Structure

Current analysis assumes independent premium realizations. Future work should incorporate:

- Correlation between asset premia
- Copula-based dependence structures
- Regime-switching dynamics

8.4 Optimal Tranching

Given state prices, an optimal tranching problem emerges:

$$\max_{\{K_j, D_j\}} \sum_j P_j(0; K_j, D_j) \quad (36)$$

subject to constraints on tranche sizes and capital requirements.

9 Conclusion

This paper develops a rigorous framework for tranche pricing in Structured Investment Vehicles based on state prices derived from dual asset premia. Our main contributions are:

1. **Theoretical foundation:** We establish that dual asset premia generate 2^n states with explicit prices $\pi(\omega_k)$, providing a complete description of the payoff space
2. **Pricing formulas:** We derive closed-form expressions for senior, mezzanine, and equity tranches as functions of state prices and attachment/detachment points
3. **No-arbitrage consistency:** We prove that tranche prices sum to portfolio value, ensuring internal consistency
4. **Economic insights:** We analyze comparative statics with respect to risk aversion, interest rates, and portfolio composition
5. **Practical implementation:** We provide numerical examples and computational algorithms

The framework resolves fundamental challenges in structured finance by exploiting the mathematical structure of dual premia. The explicit state prices enable transparent valuation without simulation, while accommodating heterogeneous investor beliefs through premium selection.

Future research should extend the framework to dynamic settings, incorporate richer correlation structures, and explore optimal tranching strategies. The dual premium approach opens new avenues for understanding structured finance beyond traditional credit models.

10 Glossary

Asset Premium $p_a(t)$

The additional return required to hold an asset beyond risk-free rate, expected inflation, and inflation risk premium. Exhibits dual nature with positive $p_a^{(+)}$ and negative $p_a^{(-)}$ values.

State Space Ω

The set of all possible premium configurations $\{\omega_1, \dots, \omega_{2^n}\}$ for a portfolio of n assets. Each state represents a unique combination of positive/negative premia.

State Price $\pi(\omega)$

The price of one unit of consumption in state ω , given by $\pi(\omega_k) = \beta\varphi(\omega_k)[1 + r_P(\omega_k)]^{-\bar{\gamma}}$. Used to value state-contingent claims.

Tranche

A slice of an SIV's capital structure with specific seniority. Senior tranches have priority in payment waterfall, followed by mezzanine, then equity.

Attachment Point K_j

The portfolio loss level at which tranche j begins to absorb losses. For senior tranche, $K_S = 0$.

Detachment Point D_j

The portfolio loss level at which tranche j is fully wiped out. Losses beyond D_j affect more junior tranches.

Waterfall Structure

The priority scheme for distributing portfolio returns to tranches. Senior tranches receive full payment before any funds flow to junior tranches.

Portfolio Loss $L(\omega)$

The realized loss in state ω , computed as $L(\omega) = N \times \max\{0, -r_P(\omega)\}$ where N is total notional.

Critical State Set Ω_j

The subset of states where tranche j experiences losses. For example, $\Omega_M = \{\omega : L(\omega) > D_S\}$ for mezzanine.

Expected Loss EL

The probability-weighted average loss on a tranche, computed as $\text{EL}_j = \sum_\omega \pi(\omega)(1+r_f) \times \text{Loss}_j(\omega)$.

Tranche Spread

The yield premium of a tranche over the risk-free rate, reflecting expected losses and risk premium. Equity tranches have highest spreads.

Risk Aversion $\bar{\gamma}$

Aggregate coefficient of relative risk aversion in the economy. Higher $\bar{\gamma}$ increases state prices for low-return states, widening junior tranche spreads.

Physical Probability $\varphi(\omega)$

The objective probability of state ω occurring. For independent premia, $\varphi(\omega_k) = \prod_i [p \mathbb{1}_{\sigma_i^k=+} + (1-p) \mathbb{1}_{\sigma_i^k=-}]$.

Risk-Neutral Probability $Q(\omega)$

The probability measure under which discounted asset prices are martingales. Related to state prices by $Q(\omega) = \pi(\omega)(1+r_f)$.

No-Arbitrage Condition

The requirement that $\sum_{\omega} \pi(\omega) = 1/(1 + r_f)$, ensuring no riskless profit opportunities exist.

Premium Configuration σ^k

A binary vector $(\sigma_1^k, \dots, \sigma_n^k)$ where $\sigma_i^k \in \{+, -\}$ specifies which premium each asset realizes in state ω_k .

Seniority

The ranking of tranches in the payment waterfall. Senior tranches have highest priority, equity tranches lowest.

First-Loss Position

The equity tranche's status as first to absorb portfolio losses. Provides downside protection to senior and mezzanine tranches.

Notional N_j

The face value of tranche j . Total notional $N = \sum_j N_j$ equals the portfolio value.

Dual Premium Selection

The investor-specific choice between positive $p_a^{(+)}$ and negative $p_a^{(-)}$ premia based on risk preferences and beliefs.

References

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