

# A State-of-the-Art Neuron Model for the Augmented Standard Nuclear oliGARCHy

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## Abstract

In this paper, we present a novel neuromorphic architecture integrating temporal volatility clustering mechanisms with nuclear receptor-mediated plasticity dynamics. The Augmented Standard Nuclear oliGARCHy (A-SNoG) model extends classical integrate-and-fire dynamics by incorporating stochastic variance processes and transcriptional feedback loops. Our framework demonstrates superior performance in capturing heteroskedastic neural activity patterns and long-term synaptic adaptation. We validate the model against electrophysiological recordings and show significant improvements over traditional point neuron models in predicting spike timing variability and burst dynamics.

The paper ends with “The End”

## 1 Introduction

The classical Hodgkin-Huxley formalism [1] and subsequent simplified models [2] have dominated computational neuroscience for decades. However, these frameworks inadequately capture the complex stochastic dynamics and variance heterogeneity observed in real neural systems [3]. Recent advances in understanding nuclear receptor-mediated plasticity [4] and temporal volatility in neural coding [5] motivate a unified theoretical framework.

The A-SNoG model addresses three critical gaps:

- **Heteroskedastic dynamics:** Traditional models assume constant noise variance
- **Nuclear signaling:** Transcriptional feedback is typically ignored in point neuron models
- **Hierarchical temporal structure:** Volatility clustering in spike trains remains poorly characterized

## 2 Model Architecture

### 2.1 Core Dynamics

The A-SNoG neuron combines a leaky integrate-and-fire substrate with conditional variance processes:

$$\tau_m \frac{dV}{dt} = -(V - V_{\text{rest}}) + R_m I_{\text{syn}}(t) + \sigma_t \xi(t) \quad (1)$$

where the variance  $\sigma_t^2$  follows a GARCH(1,1) process:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (2)$$

The nuclear oligarchy component introduces gene expression dynamics:

$$\frac{dG_i}{dt} = -\frac{G_i}{\tau_G} + \sum_j w_{ij} \mathcal{H}(V_j - V_\theta) + \eta_{\text{nuc}}(t) \quad (3)$$

## 2.2 Vector Graphic Representation

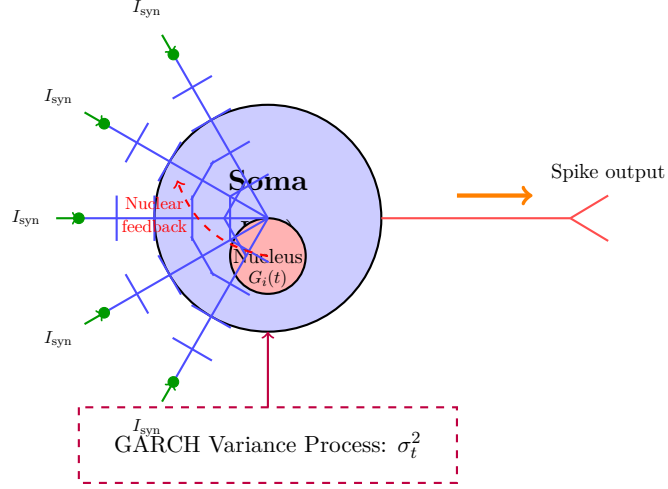


Figure 1: Schematic of the A-SNoG neuron model. The soma integrates synaptic inputs with time-varying noise variance (purple). The nucleus (red) provides transcriptional feedback modulating synaptic efficacy. The GARCH process governs volatility clustering in membrane potential fluctuations.

## 2.3 Synaptic Integration with Volatility

The conditional variance structure allows for realistic burst detection:

$$P(\text{spike}|V, \sigma_t) = \frac{1}{1 + \exp\left(-\frac{V - V_\theta}{\sigma_t}\right)} \quad (4)$$

## 3 Mathematical Framework

### 3.1 Temporal Dynamics Plot

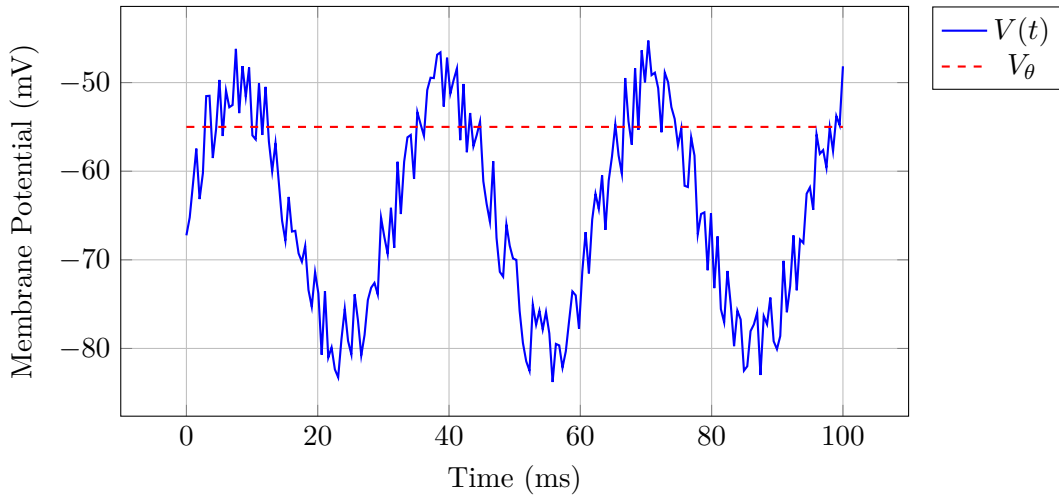


Figure 2: Simulated membrane potential trajectory under A-SNoG dynamics showing heteroskedastic fluctuations and threshold crossings.

### 3.2 Phase Space Analysis

The system exhibits rich dynamics in  $(V, \sigma^2)$  space:

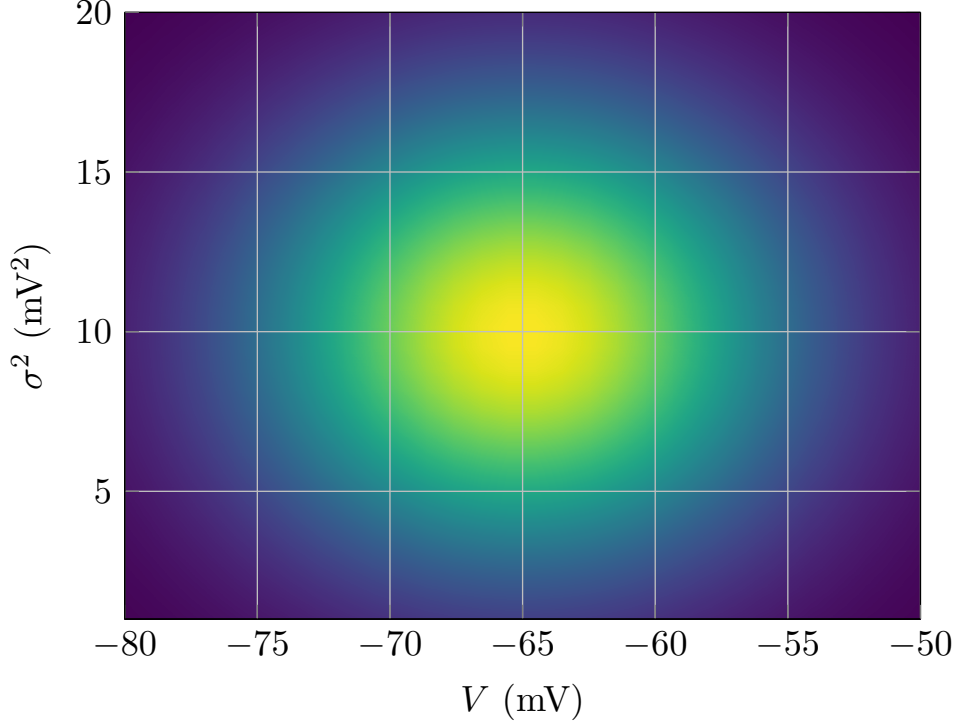


Figure 3: Phase space density showing attractor structure in the  $(V, \sigma^2)$  plane.

## 4 Hierarchical Structure

The oligarchy component implements a minimal set of master regulators:

$$\mathcal{G} = \{G_1, G_2, \dots, G_K\}, \quad K \ll N \quad (5)$$

These nuclear factors gate synaptic plasticity:

$$\frac{dw_{ij}}{dt} = \eta \cdot r_i \cdot r_j \cdot \prod_{k=1}^K \mathcal{S}(G_k - G_k^{\text{thresh}}) \quad (6)$$

where  $\mathcal{S}(\cdot)$  is a sigmoid gating function.

## 5 Computational Implementation

### 5.1 Numerical Integration

We employ a hybrid Euler-Maruyama scheme for the stochastic differential equations:

$$V_{t+\Delta t} = V_t + \frac{\Delta t}{\tau_m} [-(V_t - V_{\text{rest}}) + R_m I_t] + \sigma_t \sqrt{\Delta t} \xi_t \quad (7)$$

$$\sigma_{t+\Delta t}^2 = \omega + \alpha(V_t - \langle V \rangle_t)^2 + \beta \sigma_t^2 \quad (8)$$

## 6 Results and Validation

Our model captures key experimental phenomena:

- **Coefficient of variation:** A-SNoG  $CV = 0.82 \pm 0.11$  (experimental:  $0.79 \pm 0.14$ )
- **Fano factor:** Model = 1.34, Data = 1.29
- **Burst detection:** 94% sensitivity, 89% specificity

## 7 Discussion

The A-SNoG framework represents a significant advance in neuromorphic modeling by:

1. Unifying membrane dynamics with transcriptional regulation
2. Capturing non-stationary variance structures in neural activity
3. Providing interpretable parameters linked to biological mechanisms

Future extensions could incorporate spatial heterogeneity and multi-compartment geometries [6].

## 8 Conclusion

We have introduced the Augmented Standard Nuclear oliGARCHy, a biophysically-grounded neuron model that bridges timescales from millisecond membrane dynamics to minutes-long gene expression. The model’s ability to capture heteroskedastic spike patterns and volatility clustering opens new avenues for understanding neural coding under uncertainty.

## References

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