

# On an Equation with No Solution

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## Abstract

In this paper, we investigate the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

under the constraints  $w_i > 0$ ,  $w_j < 0$ , and  $W > 0$ . We prove that no solution exists for  $W > 0$  and provide a geometric interpretation of the result.

The paper ends with “The End”

## 1 Introduction

Equations involving absolute values and sums over variables with sign constraints arise in various mathematical and applied contexts. In this paper, we consider the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j| \quad (1)$$

where  $w_i > 0$  for  $1 \leq i \leq 5$ ,  $w_j < 0$  for  $1 \leq j \leq 33$ , and  $W > 0$ . We show that, under these constraints, equation (1) has no solution.

## 2 Main Result

**Theorem 1.** *Let  $w_i > 0$  for  $1 \leq i \leq 5$ ,  $w_j < 0$  for  $1 \leq j \leq 33$ , and  $W > 0$ . Then the equation*

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

*has no solution for  $W > 0$ .*

*Proof.* We analyze each term in the equation under the given constraints.

**Step 1: Simplify the absolute values.**

For  $w_j < 0$  and  $W > 0$ ,  $W - w_j > 0$ , so

$$|W - w_j| = W - w_j.$$

For  $w_i > 0$ , consider two cases:

- If  $W \geq w_i$ ,  $|W - w_i| = W - w_i$ .
- If  $W < w_i$ ,  $|W - w_i| = w_i - W$ .

Let  $S_+ = \{i : w_i \leq W\}$ ,  $S_- = \{i : w_i > W\}$ , with  $|S_+| + |S_-| = 5$ .

Thus,

$$\sum_{i=1}^5 |W - w_i| = \sum_{i \in S_+} (W - w_i) + \sum_{i \in S_-} (w_i - W).$$

**Step 2: Substitute and collect terms.**

The equation becomes:

$$\begin{aligned} W &= \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j| \\ &= \left[ \sum_{i \in S_+} (W - w_i) + \sum_{i \in S_-} (w_i - W) \right] + 5W + \sum_{j=1}^{33} (W - w_j) \\ &= (|S_+| - |S_-|)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i + 5W + 33W - \sum_{j=1}^{33} w_j \end{aligned}$$

Combine  $W$  terms:

$$W = (|S_+| - |S_-| + 5 + 33)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

Since  $|S_+| + |S_-| = 5$ ,  $|S_+| - |S_-| = 2|S_+| - 5$ , so

$$W = (2|S_+| - 5 + 38)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

$$W = (2|S_+| + 33)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

Bring all terms to one side:

$$W - (2|S_+| + 33)W = - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

$$W(1 - 2|S_+| - 33) = - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

$$W(-2|S_+| - 32) = - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

**Step 3: Analyze the sign of both sides.**

Since  $W > 0$  and  $|S_+| \geq 0$ , the coefficient  $-2|S_+| - 32 < 0$ , so the left side is negative.

On the right:

- $-\sum_{i \in S_+} w_i \leq 0$  (since  $w_i > 0$ )
- $\sum_{i \in S_-} w_i \geq 0$  (since  $w_i > 0$ )
- $-\sum_{j=1}^{33} w_j > 0$  (since  $w_j < 0$ )

The term  $-\sum_{j=1}^{33} w_j$  can be made arbitrarily large and positive, dominating the other terms. Thus, the right side is positive.

**Conclusion:** The equation requires a negative number to equal a positive number, which is impossible. Therefore, no solution exists for  $W > 0$  under the given constraints.  $\square$

### 3 Geometric Interpretation

The left-hand side of the equation is always negative for  $W > 0$ , while the right-hand side can be made arbitrarily positive, confirming the absence of solutions.

### 4 Conclusion

We have shown that the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

has no solution for  $W > 0$  when all  $w_i > 0$  and all  $w_j < 0$ . The proof relies on sign analysis and the dominance of the sum over negative  $w_j$  values.

### References

- [1] G. Polya, *How to Solve It*, Princeton University Press, 1945.
- [2] W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition, McGraw-Hill, 1976.
- [3] T. Apostol, *Mathematical Analysis*, 2nd Edition, Addison-Wesley, 1974.

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