

On an Equation with No Solution

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Abstract

In this paper, we investigate the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

under the constraints $w_i > 0$, $w_j < 0$, and $W > 0$. We prove that no solution exists for $W > 0$ and provide a geometric interpretation of the result.

The paper ends with “The End”

1 Introduction

Equations involving absolute values and sums over variables with sign constraints arise in various mathematical and applied contexts. In this paper, we consider the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j| \tag{1}$$

where $w_i > 0$ for $1 \leq i \leq 5$, $w_j < 0$ for $1 \leq j \leq 33$, and $W > 0$. We show that, under these constraints, equation (1) has no solution.

2 Main Result

Theorem 1. *Let $w_i > 0$ for $1 \leq i \leq 5$, $w_j < 0$ for $1 \leq j \leq 33$, and $W > 0$. Then the equation*

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

has no solution for $W > 0$.

Proof. We analyze each term in the equation under the given constraints.

Step 1: Simplify the absolute values.

For $w_j < 0$ and $W > 0$, $W - w_j > 0$, so

$$|W - w_j| = W - w_j.$$

For $w_i > 0$, consider two cases:

- If $W \geq w_i$, $|W - w_i| = W - w_i$.
- If $W < w_i$, $|W - w_i| = w_i - W$.

Let $S_+ = \{i : w_i \leq W\}$, $S_- = \{i : w_i > W\}$, with $|S_+| + |S_-| = 5$.

Thus,

$$\sum_{i=1}^5 |W - w_i| = \sum_{i \in S_+} (W - w_i) + \sum_{i \in S_-} (w_i - W).$$

Step 2: Substitute and collect terms.

The equation becomes:

$$\begin{aligned} W &= \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j| \\ &= \left[\sum_{i \in S_+} (W - w_i) + \sum_{i \in S_-} (w_i - W) \right] + 5W + \sum_{j=1}^{33} (W - w_j) \\ &= (|S_+| - |S_-|)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i + 5W + 33W - \sum_{j=1}^{33} w_j \end{aligned}$$

Combine W terms:

$$W = (|S_+| - |S_-| + 5 + 33)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j$$

Since $|S_+| + |S_-| = 5$, $|S_+| - |S_-| = 2|S_+| - 5$, so

$$\begin{aligned} W &= (2|S_+| - 5 + 38)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j \\ W &= (2|S_+| + 33)W - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j \end{aligned}$$

Bring all terms to one side:

$$\begin{aligned} W - (2|S_+| + 33)W &= - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j \\ W(1 - 2|S_+| - 33) &= - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j \\ W(-2|S_+| - 32) &= - \sum_{i \in S_+} w_i + \sum_{i \in S_-} w_i - \sum_{j=1}^{33} w_j \end{aligned}$$

Step 3: Analyze the sign of both sides.

Since $W > 0$ and $|S_+| \geq 0$, the coefficient $-2|S_+| - 32 < 0$, so the left side is negative.
On the right:

- $-\sum_{i \in S_+} w_i \leq 0$ (since $w_i > 0$)
- $\sum_{i \in S_-} w_i \geq 0$ (since $w_i > 0$)
- $-\sum_{j=1}^{33} w_j > 0$ (since $w_j < 0$)

The term $-\sum_{j=1}^{33} w_j$ can be made arbitrarily large and positive, dominating the other terms. Thus, the right side is positive.

Conclusion: The equation requires a negative number to equal a positive number, which is impossible. Therefore, no solution exists for $W > 0$ under the given constraints. \square

3 Geometric Interpretation

The left-hand side of the equation is always negative for $W > 0$, while the right-hand side can be made arbitrarily positive, confirming the absence of solutions.

4 Conclusion

We have shown that the equation

$$W = \sum_{i=1}^5 |W - w_i| + 5W + \sum_{j=1}^{33} |W - w_j|$$

has no solution for $W > 0$ when all $w_i > 0$ and all $w_j < 0$. The proof relies on sign analysis and the dominance of the sum over negative w_j values.

References

- [1] G. Polya, *How to Solve It*, Princeton University Press, 1945.
- [2] W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition, McGraw-Hill, 1976.
- [3] T. Apostol, *Mathematical Analysis*, 2nd Edition, Addison-Wesley, 1974.

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