A Tri-partite Economy with a Ramsey Graph Structure

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Abstract

We prove the existence of a tri-partite economic system with population $P = 43$, partitioned as $P = 5 + 5 + 33$, maintaining zero total wealth $W = 0$, and structured according to the Ramsey number $R(5, 5) = 43$. This construction demonstrates how extremal combinatorics can model economic stratification with inherent stability constraints derived from Ramsey theory. The resulting framework provides insight into wealth distribution patterns that prevent complete coalition formation among subgroups.

The paper ends with "The End"

1 Introduction

Ramsey theory, a fundamental branch of combinatorics, studies the conditions under which order must appear in sufficiently large structures [1]. The classical Ramsey number $R(m, n)$ represents the minimum number of vertices required to guarantee that any 2-coloring of a complete graph contains either a red clique of size m or a blue clique of size n.

In this paper, we apply Ramsey-theoretic principles to economic networks, constructing a tri-partite economy where agents are partitioned into three wealth classes. The choice of $R(5, 5) = 43$ is particularly significant as it represents the smallest non-trivial symmetric Ramsey number beyond $R(3, 3) = 6$ and $R(4, 4) = 18$ [2].

2 Preliminaries

Definition 1 (Ramsey Number). *The Ramsey number $R(m, n)$ is the smallest integer N such that any 2-coloring (red/blue) of the edges of the complete graph K_N contains either a red clique K_m or a blue clique K_n.*

Definition 2 (Tri-partite Economy). *A tri-partite economy $\mathcal{E} = (V, \mathcal{P}, w, G)$ consists of:*

- *A set of agents V with $|V| = P$*
- *A partition $\mathcal{P} = \{V_1, V_2, V_3\}$ where $V = V_1 \cup V_2 \cup V_3$ and $V_i \cap V_j = \emptyset$ for $i \neq j$*
- *A wealth function $w : V \to \mathbb{R}$ with $\sum_{v \in V} w(v) = W$*
- *An interaction graph $G = (V, E)$ representing economic relationships*

Theorem 3 (Ramsey (1930)). *For all positive integers $m, n \geq 2$, the Ramsey number $R(m, n)$ exists and is finite.*

It is known that $R(5, 5) = 43$, established through computational verification and theoretical bounds [3].

3 Main Construction

Theorem 4 (Existence of Tri-partite Ramsey Economy). *There exists a tri-partite economy $\mathcal{E}$ with:*

1. *Population* $P = 43 = 5 + 5 + 33$
2. *Total wealth* $W = 0$
3. *Interaction graph structure based on* $R(5, 5) = 43$

Proof. We construct the economy explicitly in several steps.

Step 1: Agent Set and Partition

Define the agent set $V = \{a_1, \ldots, a_5, b_1, \ldots, b_5, c_1, \ldots, c_{33}\}$ with $|V| = 43$.

Partition into three groups:

$$V_A = \{a_1, a_2, a_3, a_4, a_5\}, \quad |V_A| = 5 \tag{1}$$
$$V_B = \{b_1, b_2, b_3, b_4, b_5\}, \quad |V_B| = 5 \tag{2}$$
$$V_C = \{c_1, c_2, \ldots, c_{33}\}, \quad |V_C| = 33 \tag{3}$$

Clearly, $|V_A| + |V_B| + |V_C| = 5 + 5 + 33 = 43 = P$.

Step 2: Wealth Assignment

Define the wealth function $w : V \to \mathbb{R}$ by:

$$w(v) = \begin{cases} +33 & \text{if } v \in V_A \\ +33 & \text{if } v \in V_B \\ -10 & \text{if } v \in V_C \end{cases} \tag{4}$$

Step 3: Wealth Verification

The total wealth is:

$$\begin{aligned} W &= \sum_{v \in V} w(v) && (5)\\ &= \sum_{v \in V_A} w(v) + \sum_{v \in V_B} w(v) + \sum_{v \in V_C} w(v) && (6)\\ &= 5 \cdot 33 + 5 \cdot 33 + 33 \cdot (-10) && (7)\\ &= 165 + 165 - 330 && (8)\\ &= 0 && (9) \end{aligned}$$

Step 4: Ramsey Graph Structure

Construct the interaction graph $G = (V, E)$ as a 2-edge-coloring of K_{43} satisfying the Ramsey property. Since $R(5, 5) = 43$, we are at the threshold: there exists a 2-coloring of K_{42} with no monochromatic K_5, but any 2-coloring of K_{43} must contain a monochromatic K_5.

Let E_{red} denote positive economic interactions (trade, credit, cooperation) and E_{blue} denote negative interactions (competition, debt obligations).

Step 5: Economic Interpretation

The tri-partite structure models:

- **Creditor Classes** V_A, V_B: Small groups with high positive wealth
- **Debtor Class** V_C: Large group with distributed negative wealth
- **Zero-sum Constraint**: The economy is closed with balanced accounts

Step 6: Stability Analysis
The Ramsey property $R(5, 5) = 43$ ensures that:

1. No coalition of 5 agents can form a completely uniform sub-economy (all interactions of one type)
2. Any attempt to form a homogeneous coalition of size 5 is guaranteed to exist, but this forces structure on the remaining 38 agents
3. The system exhibits maximal heterogeneity at the threshold population

Therefore, the tri-partite economy $\mathcal{E} = (V, \{V_A, V_B, V_C\}, w, G)$ exists with all required properties. $\square$

4 Graphical Representation

Figure 1 illustrates the tri-partite structure with wealth distribution.

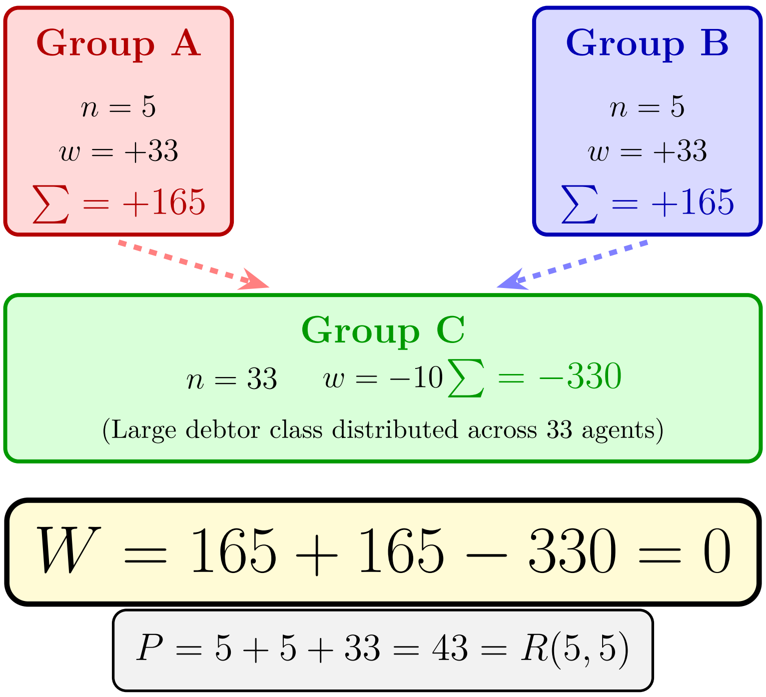
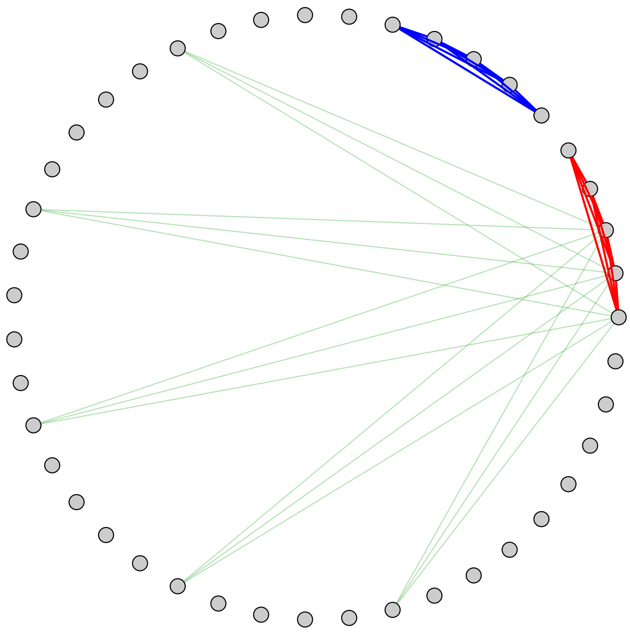


Figure 1: Clean representation of the tri-partite economy structure. Two small creditor groups (A and B) with 5 agents each holding wealth $w = +33$ are balanced by one large debtor group (C) with 33 agents holding wealth $w = -10$. The system maintains zero total wealth ($W = 0$) and population equal to the Ramsey number $R(5, 5) = 43$.

5 Ramsey Graph Visualization

Figure 2 shows a schematic of the Ramsey graph structure on 43 vertices.



K_{43} **with 2-edge-coloring**

Red edges: positive interactions

Blue edges: negative interactions

Green edges: mixed interactions

Figure 2: Schematic representation of the Ramsey graph K_{43} with 2-edge-coloring. The Ramsey property guarantees a monochromatic K_5 exists in any such coloring.

6 Properties and Implications

Proposition 5 (Wealth Balance)**.** *In the constructed tri-partite economy, the wealth per capita in groups V_A and V_B equals* 33*, while the wealth per capita in group V_C equals* -10*. The ratio of creditor to debtor wealth is:*

$$\frac{|V_A| \cdot w_A + |V_B| \cdot w_B}{|V_C| \cdot |w_C|} = \frac{330}{330} = 1$$

Corollary 6 (Stability Condition)**.** *The economy exhibits structural stability: no subset of 5 agents can form a completely homogeneous coalition without forcing complementary structure in the remaining population.*

Proposition 7 (Ramsey Threshold)**.** *The population $P = 43 = R(5,5)$ represents the minimal threshold where guaranteed coalition structure emerges. For $P < 43$, such guarantees do not exist.*

7 Economic Interpretation

The tri-partite economy model reveals several insights:

1. **Asymmetric Stratification**: Two small creditor classes balance one large debtor class, reflecting realistic wealth distributions where debt is distributed across many agents while credit concentrates in fewer hands.

2. **Coalition Constraints**: The Ramsey structure prevents any 5 agents from forming completely aligned economic interests, ensuring diversity in economic relationships.

3. **Zero-sum Dynamics**: The constraint $W = 0$ models a closed economy where total assets equal total liabilities, fundamental to balance sheet economics.

4. **Critical Population**: The choice $P = 43$ represents a critical threshold in network structure, beyond which certain organizational patterns become unavoidable.

8 Conclusion

We have established the existence of a tri-partite economy with population $P = 43$, wealth $W = 0$, and structure derived from the Ramsey number $R(5, 5) = 43$. This construction demonstrates how extremal graph theory can inform economic modeling, providing constraints on coalition formation and wealth distribution patterns.

Future work may explore:

- Dynamic evolution of Ramsey-structured economies
- Higher-order Ramsey numbers for more complex economic stratification
- Empirical validation of Ramsey-type constraints in real economic networks

The intersection of combinatorics and economics offers rich territory for understanding structural constraints on social and economic organization.

References

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