I.6 CAPACITANCE MEASURING CIRCUIT

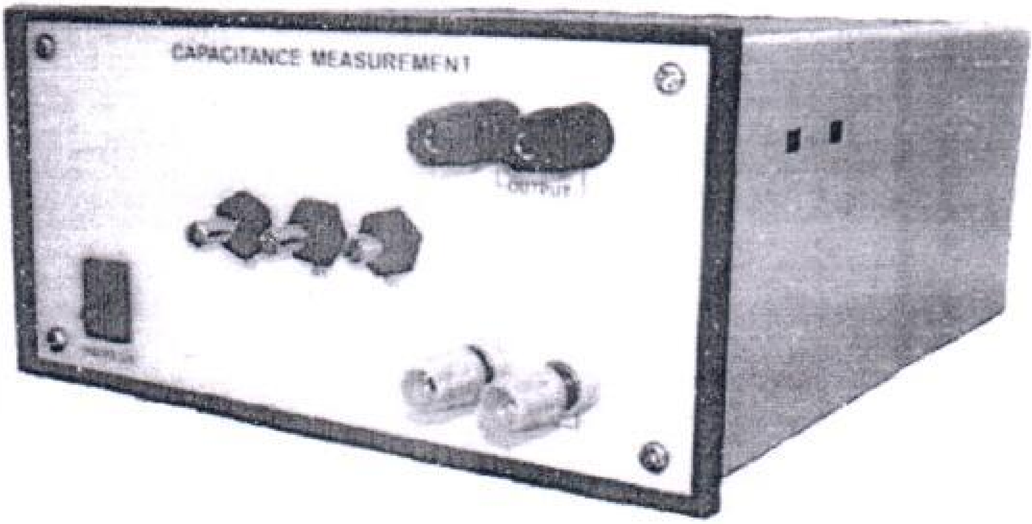

Figure I.6.1: Capacitance circuit

1. INTRODUCTION

The capacitance measuring circuit can be used to measure capacitances up to 250 pfd. It gives a DC output in volts proportional to the value of the capacitance. It is useful for measuring dielectric constant of non-polar liquids with a cylindrical capacitor and dipole moment of a polar molecule like acetone. These experiments are described in Section II.6.1 to 3.

2. DESCRIPTION OF THE INSTRUMENT

At the lower left corner of the front panel is the mains switch. When it is pressed down the instrument is activated. On the lower right corner are two banana terminals marked C. The unknown capacitance is connected to these two terminals. In the middle of the front panel are three preset pots marked R1, R1' and R2. Their values are preset in the factory. But one may change the values if one wishes by turning the screws. For a given value of the unknown capacitance, the DC output will vary as these screws are turned. At the top right are two banana terminals. A DMM in the appropriate DC range is connected to these terminals to measure the output.

3. PRINCIPLE OF CAPACITANCE MEASUREMENT:

Inside the instrument there is a capacitance of 2 nfd (i.e. 2000 pfd). This capacitance is connected in series with a resistance R1 that is in the 100 k to 500 k range. The unknown capacitor C is connected to another high resistance R2 in the same range. When a DC voltage 12 V is applied, both capacitors get charged. But the time constant of the two circuits will depend on the product $C_{std}R1$ and $C_{unknown}R2$. Since $C_{unknown}$ is more than ten times less than C_{std} and R1 and R2 are nearly of the same order, the time constant $\tau_{unknown}$ is less than the time constant τ_{std}. So the unknown capacitance charges faster than the standard capacitance.

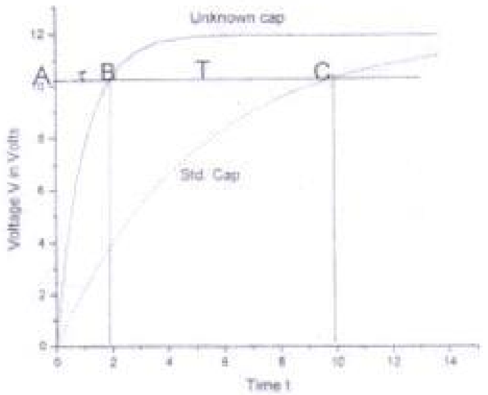

Figure 1.6.2: Unknown capacitor charges faster than standard capacitor. Unknown capacitor reaches 9 V in time τ sec while standard capacitor reaches 9 V time T sec. T>τ.

While both the capacitors are charging, a fixed current passes through a transistor and produces a voltage of 10 Volts across a 1 k resistor. When the unknown capacitor reaches a voltage of 9 V a Schmitt trigger circuit switches off the current through the transistor. So current through the transistor flows for a time τ which is proportional to $\tau_{unknown}$ i.e. to $C_{unknown}R_2$. The standard capacitor C_{std} reaches 9 V at a later time T which is longer than τ since τ_{std} is larger than $\tau_{unknown}$. The Schmitt triggers discharge both the capacitors when the standard condenser reaches 9V after a time T. The charging process begins again. The voltage across 1k appears in pulses of width τ as shown in Figure 1.6.3.

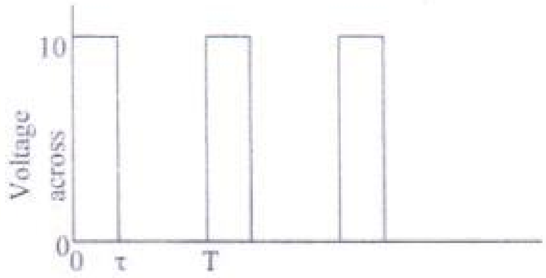

Figure 1.6.3: Voltage pulse across 1 k resistor appears for a time τ in every T seconds.

Thus in 1 second the charge-discharge process takes place at a frequency 1/T that is in the range of a few kilo Hertz. The DMM reads the voltage across the 1 k resistor averaged over many charge-discharge periods. This average voltage will be $10\tau/T$. τ is proportional to $\tau_{unknown}$ i.e. to $C_{unknown}$ since R_2 is fixed. So the output DC voltage is proportional to the unknown capacitance. This is the principle on which the instrument works.

A box is provided with the instrument. This contains three capacitors 100 pfd, 47 pfd and 22 pfd connected at one end to a common terminal. These are nominal values. At the factory the resistances R1, R1' and R2 are adjusted so that when 100 pfd capacitor is connected to the terminals marked C, the DC voltage at the output terminals O/P is about 2 to 2.5 V. The linearity of the readings can be checked by connecting the 47 pfd and 22 pfd capacitors.

It is recommended that the settings of the resistors R1, R1' and R2 are NOT altered. They have been set to give a convenient output which is linear in the capacitance and which is suitable for measuring the dielectric constant of benzene and dipole moment of acetone with the cylindrical capacitor provided with the instrument.

CAUTION:

DO NOT USE THE INSTRUMENT FOR MEASURING DIELECTRIC CONSTANT OF POLAR LIQUIDS LIKE ACETONE OR WATER DIRECTLY. THE CIRCUIT WILL GET DAMAGED.

II.6.1 COMPARISON OF CAPACITANCES AND VERIFICATION OF THE LAW OF ADDITION OF CAPACITANCES

1. INTRODUCTION

In a small box three capacitances with nominal values of 100 pfd, 47 pfd and 22 pfd are soldered respectively between a Red, Yellow, and Black terminal and a common (Green) terminal. Thus between green and black terminals one has 22 pfd, between green and yellow 47 pfd and between green and red 100 pfd. Between black and yellow terminals the 22 pfd and 47 pfd are in series. Between black and red, the 22 pfd and 100 pfd are in series and between yellow and red the 47 pfd and 100 pfd are in series. If black and yellow terminals are externally connected, and we measure between green and black (or yellow) terminals, we will have 22 pfd in parallel with 47 pfd. If red and black are externally connected, and we measure between green and black (or red) terminals, we will have 22 pfd and 100 pfd in parallel. When yellow and red terminals are externally connected, and we measure between green and yellow (or red) terminals we have 47 pfd in parallel with 100 pfd.

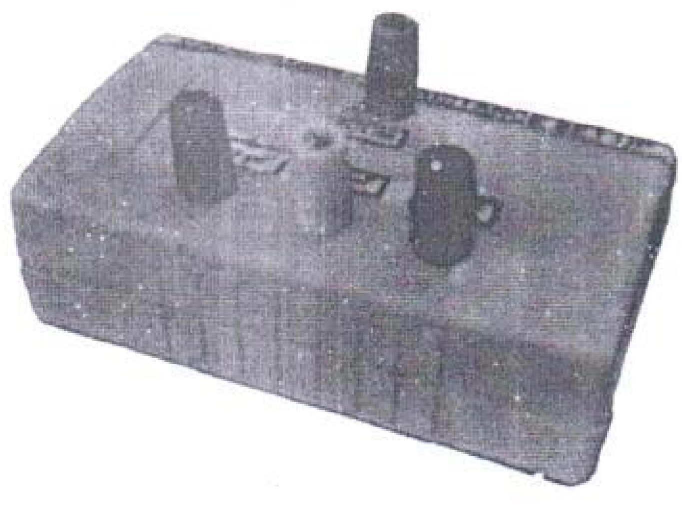

Figure II.6.1.1 Connections in the capacitance box

2. APPARATUS REQUIRED:

Capacitance measuring circuit, capacitance box and a DMM measuring DC 2 V to three decimal places.

3. PROCEDURE

Connect the terminals marked C on the capacitance circuit described in Section I to the black and red terminals on the box. Switch on the capacitance circuit and note the reading on a DMM in the DC 20 V range connected to the output terminals of the capacitance meter. Then change the connection from the red terminal to the yellow and black in succession and note the readings. These three readings will correspond to the nominal values of the capacitances 100, 47 and 22 pfd respectively.

To measure the capacitances in series connect the terminals red and yellow on the box to the capacitance circuit and measure the DC multimeter reading. This corresponds to 100 pfd in series with 47 pfd. Then change the connections on the box to the pair red and black and then yellow and black. This measures the capacitance of the series combination of 100 pfd and 22 pfd and the series combination of 47 pfd and 22 pfd respectively.

Short the yellow and red terminals on the box with a wire. Connect the green and red terminals on the box to the terminals marked C on the capacitance circuit. The DMM reading now corresponds to the parallel combination of 100 and 47 pfd capacitances. Short red and black terminals on the box and connect the red and green terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 100 and 22 pfd capacitances. Then short the yellow and black terminals on the box and connect the green and black terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 47 and 22 pfd capacitances.

Taking the 100 pfd capacitance as standard and assuming the output voltage is proportional to the capacitance, calculate the values of the capacitances for the various combinations. This is shown in Table II.6.1.1 below.

Table II.6.1.1: Comparison of capacitances

Assume capacitance between red and green is 100 pfd

Connection	DC Volts	Calc Cap µf	Nominal Value µf
Green and black	0.45	20.5	22
Green & Yellow	1.10	50.0	47
Green & red	2.20	100	100
Black & Yellow	0.31	14.5	15.0
Black & Red	0.35	17.0	18.0
Yellow & red	0.70	33.3	32.0
Black & Yellow short			
Between black & Green	1.69	76.8	69
Red and black short			
Between black & green	2.83	128.6	122
Red and Yellow short			
Between yellow & green	3.45	156.8	147

We see that the results are in reasonable agreement with the nominal values. This shows that the output voltage of our capacitance circuit varies linearly with capacitance. We have also verified the law of addition of capacitances.

II.6.2 DIELECTRIC CONSTANT OF A NON-POLAR LIQUID

1. INTRODUCTION

A molecule in which the center of gravity (CG) of negative charge is displaced from the center of gravity of the positive charge is said to have a dipole moment. In a highly symmetric molecule, like Carbon tetrachloride CCl_4 and benzene C_6H_6, the CG of positive and negative charges will coincide (at the point marked x in Figure II.6.2.1 for benzene) and the molecule has no electrical dipole moment. In asymmetric molecules like $CHCl_3$ and acetone, there is a dipole moment per molecule.

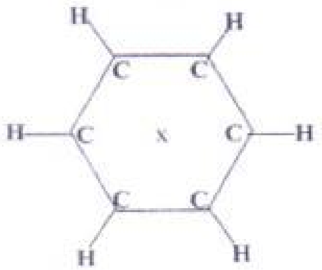

Figure II.6.2.1: Structure of the benzene molecule
C is carbon atom and H is hydrogen atom.
x marks the center of the molecule.

When an electric field is applied there can be a shift of the CG of negative charge relative to the CG of positive charge leading to a dipole moment induced in the direction of the field, even in molecules that do not have a permanent dipole moment, like a molecule of benzene. This leads to the relation

$$\mathbf{p}_{\text{induced}} = \varepsilon_0 \, \alpha \, \mathbf{E}_{\text{local}} \qquad \text{(II.6.2.1)}$$

Here ε_0 is the electric permittivity of free space and has the value $8.854\text{x}10^{-12}$ Farad/m and $\mathbf{E}_{\text{local}}$ is the local electric field at the site of the molecule. $\mathbf{p}_{\text{induced}}$ is the induced dipole moment and α is called the electronic polarizability of the molecule. The local electric field is the sum of the applied electric field **E** and the electric field arising from the induced dipoles. In a gas the density of the molecules is very low. So the field due to the induced dipoles can be neglected in comparison to the applied field **E**. If there are N molecules per unit volume the electric polarization **P**, which is the dipole moment per unit volume, is

$$\mathbf{P} = N\mathbf{p}_{\text{ind}} = \varepsilon_0 N\alpha \, \mathbf{E} \qquad \text{(II.6.2.2)}$$

The electrical displacement **D** is

$$\mathbf{D} = \varepsilon_0 \, \mathbf{E} + \mathbf{P} = \varepsilon_0 \, \varepsilon_r \, \mathbf{E} \qquad \text{(II.6.2.3)}$$

ε_r is the dielectric constant of the material.

So for a gas $$\varepsilon_r = 1 + N\alpha \qquad \text{(II.6.2.4)}$$

In a liquid the density is high and the local electric field is the sum of the applied electric field **E** and the electric field due to the polarization **P** in the medium.

$$\mathbf{E}_{local} = \mathbf{E} + \mathbf{P}/(3\,\varepsilon_o) \qquad \text{(II.6.2.5)}$$

The polarization **P** is itself given by

$$\mathbf{P} = \varepsilon_0 N\alpha\ \mathbf{E}_{local} = \varepsilon_0 N\alpha\,(\mathbf{E} + \mathbf{P}/3\,\varepsilon_o) \qquad \text{(II.6.2.6)}$$

Solving for **P** in terms of **E** we have

$$\mathbf{P} = \varepsilon_0 N\alpha\,\mathbf{E} / (1 - N\alpha/3) \qquad \text{(II.6.2.7)}$$

and

$$\mathbf{D} = \varepsilon_o\mathbf{E} + \mathbf{P} = \varepsilon_o\mathbf{E}\,[1 + (N\alpha / (1 - N\alpha/3))] = \varepsilon_o\,\varepsilon_r\,\mathbf{E} \qquad \text{(II.6.2.8)}$$

From this we get

$$(\varepsilon_r - 1)/(\varepsilon_r + 2) = N\alpha/3 \qquad \text{(II.6.2.9)}$$

This is called the Clausius-Mosotti relation.

Note that from a measurement of the dielectric constant and knowledge of the density of the liquid one can obtain the value of the polarizability α of the molecule. The polarizability has the dimensions of volume and can be written in terms of (nano-meter)^3.

2. APPARATUS REQUIRED:

Capacitance measuring circuit, cylindrical capacitance in a jar, a DMM measuring to three decimal places in the DC 2 V range and Analar grade benzene.

3. EXPERIMENTAL DETAILS

For this measurement a cylindrical capacitor of 15 cm is provided. The cylindrical capacitor is suspended in a tall cylindrical graduated plastic jar (of volume 100 ml). The cylindrical capacitor consists of two coaxial metal tubes of different radii insulated from each other. Two leads taken out of the tubes are connected to banana plugs on the insulating mount on top of the jar. Such a cylindrical capacitor will have a capacitance of a few tens of pico-farads.

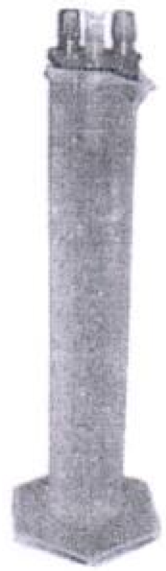

Figure II.6.2.2 Measuring jar with a cylindrical capacitor inside

The capacitance of the leads may be a sizeable fraction of the capacitance of the cylindrical condenser if the connecting leads are large in diameter and are close to each other. The capacitance of the cylindrical capacitor is of the order of 10 to 15 pfd. To reduce the lead capacitance use thin wires for connection and separate the wires by a large distance. Connect the black terminal on the cylindrical capacitance to the ground terminal of the pair C on the capacitance circuit. Bring the wire from the other terminal of the pair C close to the red terminal of the cylindrical capacitor. Without connecting it to the red terminal note the DC reading on the DMM (in the DC 2 V range) connected to the output of the capacitance meter. Let it be V_0. Then connect the lead to the red terminal and measure the voltage. Let it be V_{air}. Then $(V_{air}-V_0)$ is proportional to the capacitance of the cylindrical condenser with air as the medium.

Analar grade (this means high purity) benzene (or carbon tetrachloride) is added in approximately 10 ml quantities into the jar through a tube provided in the cap of the capacitor. If the liquid spills out it may damage the plastic terminals. The liquid is now added in steps of 10 ml and the DC voltage is noted on the DMM after each such addition. Please wait for a minute or two after adding the liquid for the reading on the DMM to settle down to a steady value. Note the level of the liquid in milli-liters from the graduation on the jar for each addition of the liquid.

The jars purchased in the market may be of different cross-sections. So in some the top of the cylindrical capacitor may be above the 100ml mark on the jar. In the others the top of the cylindrical capacitor may be below the 100ml mark. Take readings of the output voltage till you fill the jar up to 80 ml. Then note the height of the jar, h, from 0 to 100 ml. Find the height of the top of the cylindrical capacitor from the bottom of the jar. Let it be h'. So h' would correspond to $(h'/h)\times 100 = v$

milliliters of liquid. This is the volume of the liquid to fill the capacitor. In the case of the sample readings given below this corresponds to 90 ml.

A graph is plotted of the DC voltage against the volume of benzene. A straight line is fitted on the computer to the points and the equation of the straight line is noted as

$$V = V' + bm \qquad (II.6.2.10)$$

where V is the DC voltage with the liquid, V' is the intercept, b is the slope and m is the volume of the liquid in milliliters. The reading corresponding to the situation when the liquid is filled to reach the top of the cylindrical capacitor is obtained from equation (II.6.2.10) by putting m = v i.e.

$$V_{liq} = a + bv. \qquad (II.6.2.11)$$

V_{liq} includes the contribution V_0 of the leads. So the voltage due to cylindrical capacitance filled with liquid, omitting the capacitance of the leads, will be $(V_{liq}-V_0)$.

The dielectric constant of the liquid is

$$\varepsilon_r = \text{capacitance with liquid/capacitance with air}$$
$$= (V_{liq} - V_0)/(V_{air}-V_0) \qquad (II.6.2.12)$$

A sample set of readings is given below:

Table II.6.2.1

The voltage due to leads V_0 = 0.020 V
The voltage of capacitance in air with leads V_{air} = 0.412 V
When benzene is filled in the jar the following readings were obtained

Milli-liter	V in mV
0	412
10	459
20	510
30	555
40	606
50	650
60	705
70	750
80	800

Figure II.6.2.2 shows a plot of the readings in Table II.6.2.1. The linear fit to the points is shown. The intercept is 411.2 mV and the slope is 4.85 mV/ml.

The voltage V_{liq} when the capacitor is filled with liquid is

$$V_{liq} = 411.2+4.85x90 = 847.7 \text{ mV} = 0.848 \text{ V}.$$

The dielectric constant of benzene is

$$\varepsilon_r = (V_{liq} - V_0)/(V_{air}-V_0) = (0.848-0.020)/(0.412-0.020) = 2.11 \qquad \text{(II.6.2. 13)}$$

For benzene the value of the dielectric constant from tables is 2.25 at 300 K.

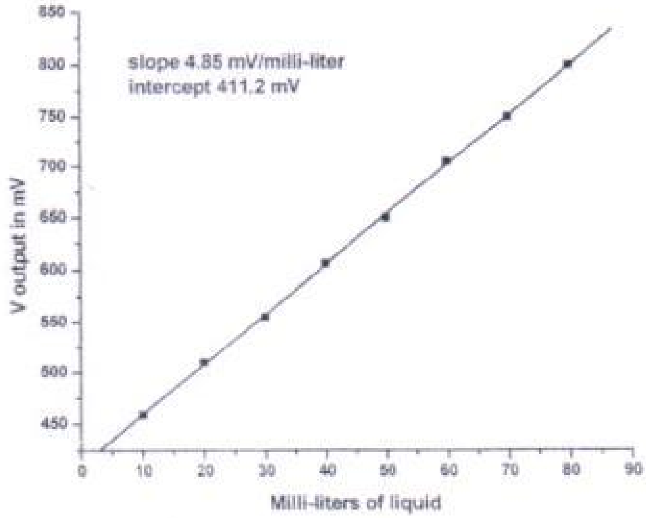

Figure II.6.2.3: Plot of DC Voltage against ml of benzene filled

Use the Clausius Mosotti relation

$$(\varepsilon-1)/(\varepsilon+2) = N_B\alpha/3$$

where N_B is the number of benzene molecules per unit volume and α is the electronic polarizability of benzene.

The molecular weight of benzene (C_6H_6) is 78 gm and its density, ρ_B, is 0.899 g/cc. So 78 grams of benzene contain the Avogadro number N_A (6.022×10^{23}) of molecules. 1cc of benzene has a mass 0.899 g. The number of molecules in 1 cc is

$$N_B = 6.022\times10^{23}\times0.899/78 = 6.94\times10^{21} \text{ molecules/cc.} \qquad \text{(II.6.2.14)}$$

In the SI units the unit of volume is 1 $m^3 = 10^6$cc. So the number N_B of molecules in 1 m^3 is

$$N_B = 6.94\times10^{21}\times10^6 = 6.94\times10^{27} /m^3 \qquad \text{(II.6.2.15)}$$

Substituting for□ ε and for N_B

α for benzene = $3\mathrm{x}((2.10-1)/(2.10+2))/6.94\mathrm{x}10^{27} = 0.116\mathrm{x}10^{-27}\ \mathrm{m}^3$

$= 0.116\ (\text{nano-meter})^3$

Note that the electronic polarizability of a molecule has the dimensions of volume and roughly gives the size of the molecule.