Cornu's method for the determination of elastic constants of a Perspex beam

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Objective: Using Cornu's Method determine the elastic constants of a given transparent beam.

The Newton rings experiment is the basis of Cornu's method. As you may recall, the Newton rings experiment involves placing a plano-convex lens on a plane glass reflector and illuminating it with a monochromatic light, which results in formation of alternating bright and dark circular fringes around the point of contact. In the Cornu's method, the plane glass reflector is replaced by a bendable Perspex beam whose elastic constants (Young's modulus and Poisson's ratio) is to be determined.

The experimental set-up is shown in Fig. 1. It consists of (i) a light source (typically a sodium lamp); (ii) a plano-convex lens; (iii) a Perspex beam about 30 cm long with suitable arrangement for hanging weights at its two ends; (iv) a plane glass slide as a beam splitter, (v) identical pairs of weights, and (v) a stand with an attached traveling microscope for viewing the fringes.

When equal weights are hanged from the ends of the beam it bends along its length, this is called the longitudinal bending. The longitudinal bending will also result in a small amount of lateral bending, i.e., a small and not so easily discernible upward bending perpendicular to the length of the beam. Due to this, the shape of the Newton rings will change from circular to elliptical. To understand this, we recall that on any given ring the air-gap between the convex surface of the lens and the base reflector remains constant. If the lower reflecting surface is flat, the air-gap is constant along a circular path, which explains the circular shape of the rings. However, when the reflecting base is bent, the constant air-gap is traced along an elliptical path. Thus, the shape of the rings changes from circular to elliptical.

Let 'd_n' be the diameter of the nth circular ring in the absence of bending; and, d_n' and d_n'', the minor and the major axis, respectively, of the same ring upon bending. Due to large longitudinal bending along the length of the beam: $d_n' < d_n$; and a small upward bending in the lateral direction: $d_n'' \gtrsim d_n$. Stretching a body produces an internal force called stress which prevents the body from tearing apart. The ratio of stress to strain for a given material is called the Young's modulus. From the values of d_n, d_n' and d_n'' for various n under different values of attached weight W, one can determine the elastic constants: Poisson's ratio and Young's Modulus of the beam as explained below.

Figure 1: Experimental set-up for the Cornu's method: (from left) Sodium lamp source for producing Newton's rings, side view and front view of the apparatus with weights. Thin air-film between a plano-convex lens and the Perspex beam produces circular rings due to interference. Upon attaching weights at the ends of the beam and its consequent bending, the circular ring shape changes to an elliptical one.

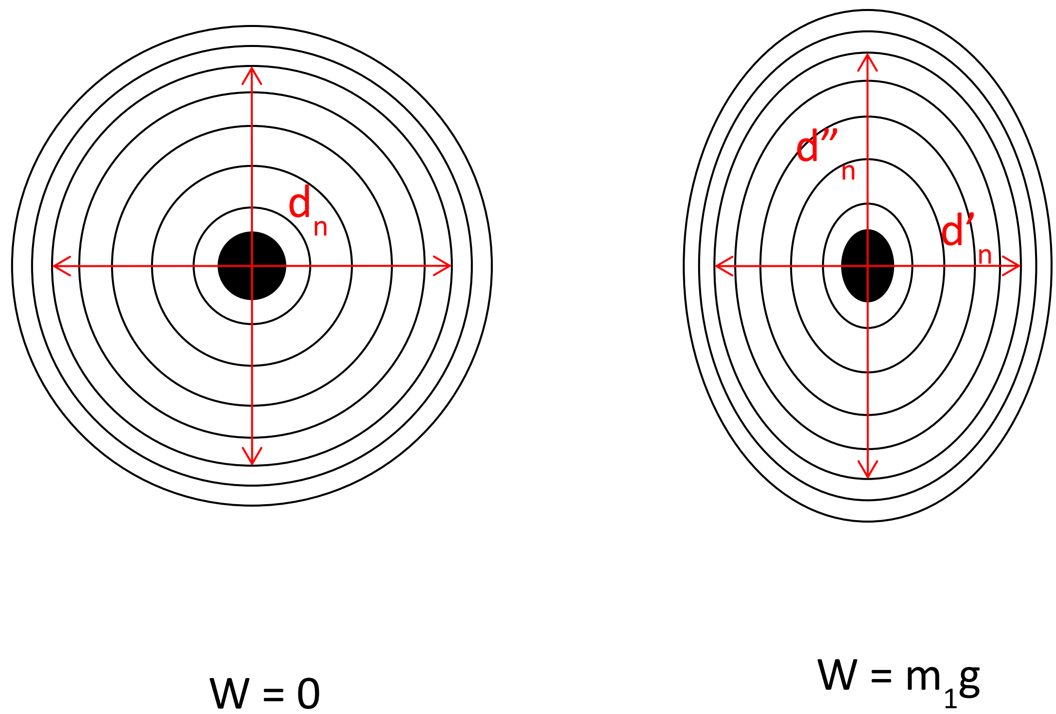

Figure2: Newton rings with and without bending shown, respectively, in the left and right panels. Note that bending is accompanied by change in shape of the rings from circular to elliptical. The diameter of the nth dark ring *decreases* greatly along the beam length and *increases* slightly perpendicular to the beam length in accordance with change in thickness of the air film sandwiched between the beam and the convex surface of the plano-convex lens (i.e., $d_n' < d_n < d_n''$).

A brief outline of the analysis used:

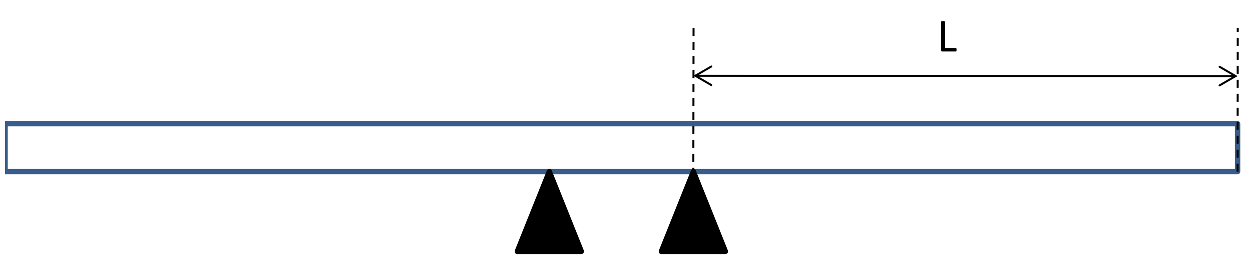

Figure 3: Side views: The unloaded condition of the beam whose elastic constants are to be determined. The beam is supported on two triangular pegs (black triangular shape objects in the figure). The rectangle in the right (w x h) shows the cross-section of the beam.

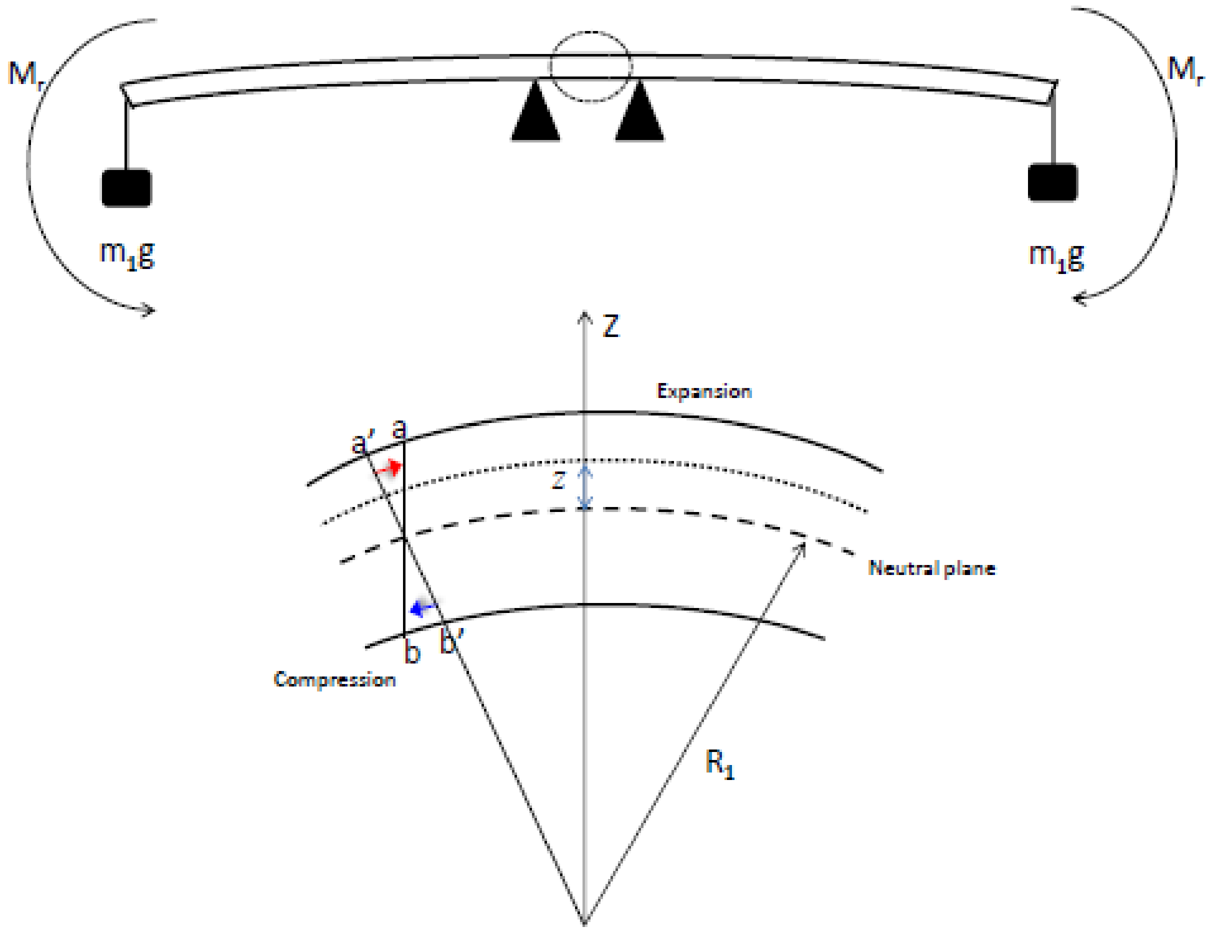

Figure 4: Bending of a beam under an applied weight W = m_1g at each end. M_r is the bending moment (= $m_1g.L$). Lower panel shows an expanded view of the region over which the Newton rings form (region enclosed within the dotted circle in the upper panel). Due to bending, the upper half (i.e., half above the neutral plane) of the beam will undergo "expansion" while the lower half (below the neutral plane) will undergo compression, resulting in stress (section ab of the beam became a'b' upon bending). The *stress* generated is indicated by arrows. Being very small, **we approximate a small section (region over which the Newton rings form) by an arc of a circle of radius R_1.**

Calculation of Strain ε(z):

Elongation (contraction) of a plane at a distance "z" above (below) the neutral plane (see fig. 4) is given by:

$$\varepsilon(z) = \frac{(R_1 + z)\theta - R_1\theta}{R_1\theta} = \frac{z}{R_1} \quad (1)$$

Calculation of Stress σ(z):

We know that Young's modulus (Y) = Stress/Strain = σ/ε therefore, $\sigma(z) = Y.z/R_1$ (2)

The bending moment is therefore given by:

$$M_r = 2\int_0^{h/2} \sigma(z).z\,(w.dz) \quad (3)$$

$$= \frac{Y\,w\,h^3}{12\,R_1} \quad (4)$$

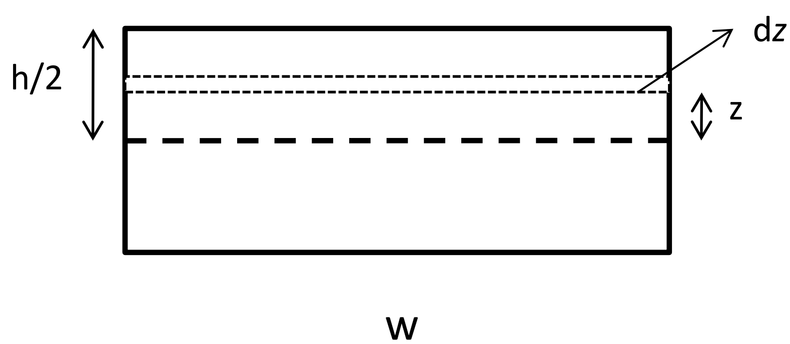

Figure 4: Cross-section of a bent beam. Broken line indicates neutral plane. See figure 6 for details.

Under equilibrium, the internal bending moment (eq. 4) must be balanced by the moment due to weight m_1g attached to its ends (eq. 5),

$$M_r = m_1 g.L \quad (5)$$

Combining (4) and (5) gives:

$$Y = \frac{12\,m_1\,g\,L\,R_1}{w\,h^3} \quad (6)$$

Thus, if we can determine R_1 from our experiment, other quantities being known, the Young's modulus of the beam can be determined. However, before going for R_1, we should first estimate the radius of curvature (r_0) of the plano-convex lens used.

I. Determination of R_0 (radius of curvature of the plano-convex lens)

It is simple to show that:

$R_0 = \frac{d_n^2}{4n\lambda}$ (7)

Where d_n is the diameter of the n^{th} dark ring.

Plot 'd_n^2' as a function of 'n', which should be a straight line with slope $4\lambda R_0$ (since the absolute value of 'n' is not known with certainty, you should not try to calculate R_0 using the direct formula).

II. **Determination of R_1**

Using the same geometrical reasoning as used in deriving eq. (7), it can be easily shown that the radius of curvature of the beam (R_1) around the point of observation is given by:

$\frac{1}{R_1} = 4\,\lambda\,n\left(\frac{1}{d_n'^2} - \frac{1}{d_n^2}\right)$ (8)

By plotting (as done in part I) you can obtain R_1 from the slope of straight line, which leads to determination of Y.

We now consider a small but finite lateral bending arising due to the longitudinal bending (see, figure 6). This results in a lateral strain which is the reason why in figure 2 major axis (d_n'') of the elliptical rings is slightly greater than the diameter (d_n) of the corresponding circular ring. The measure of this tendency is called Poisson's ratio (ζ), given by:

ζ = lateral strain/longitudinal strain. With the help of eq. 1: $\zeta = R_1/R_2$. Typically R_2 is much larger than R_1. (9)

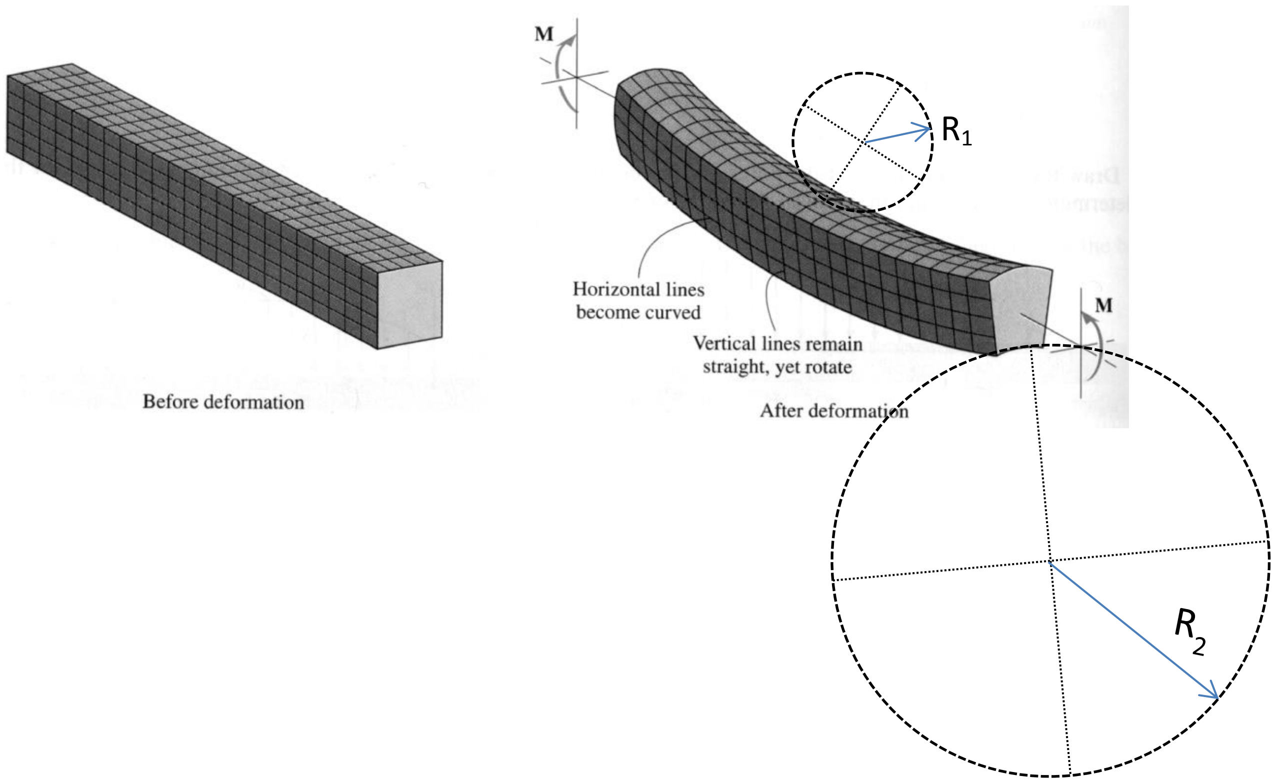

Figure. 6: Adapted from http://courses.washington.edu/me354a/chap3.pdf. Longitudinal bending (R_1) of a bean resulting in small lateral bending (R_2)

III. Determination of R_2

It can be shown that:

$$\frac{1}{R_2} = -\,4\,\lambda\,n\,\left(\frac{1}{d_n''^2} - \frac{1}{d_n^2}\right) \qquad (10)$$

By plotting (as done in part I and II) you can obtain R_2 from the slope of straight line, which leads to the experimental determination of Poisson's ratio z.

Q: Show that the fringe shape will change from circular to elliptical upon longitudinal bending?

Q: What would be the shape of interference fringes upon beam deformation if instead of a plano-convex lens a glass slide is used? Assume that prior to the deformation, the surface of the glass slide was fully in contact with beam - obviously, in this condition no fringes will be seen, the fringes will appear only after the beam is deformed?

Q = Derive equations 7, 8 and 9.

Hint: derivation of 7 is trivial and is furnished in most optics or general physics books (see, for example, Advanced Level Physics by M. Nelkon and P. Parker). For deriving eq. 8, the important point to keep in mind is that the thickness of the air-film corresponding to the n_{th} ring remains constant before and after bending –i.e., the ring moves inward in the longitudinal direction to preserve the thickness. In another words, because of the bending the air-gap thickness at a distance x from the centre increases from t to t', the ring therefore move inward to a distance x' (< x) where the air-gap is again t.

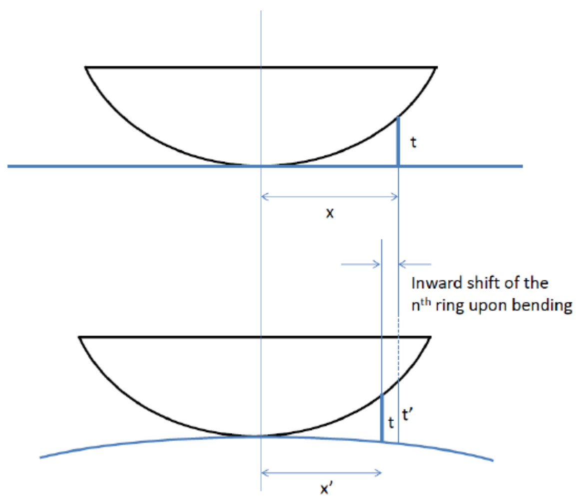

Fig. 7: Inward shift of a ring upon bending. The ring shifts its position in order to keep the air-gap thickness unchanged.