Chapitre 6

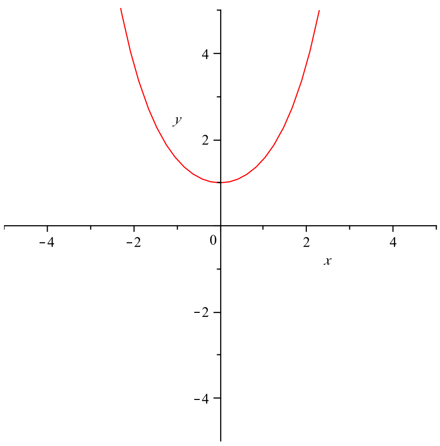
Fonctions usuelles

Fonctions hyperboliques

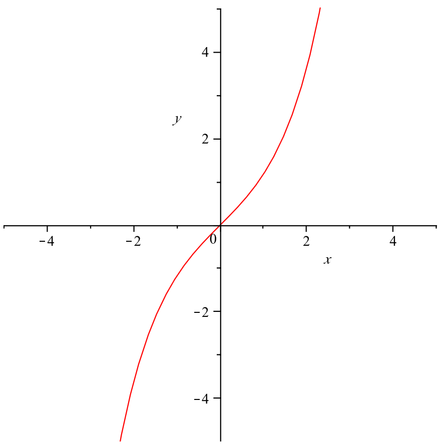
Définition : $\forall x \in \mathbb{R}, \qquad \mathrm{ch}(x) = \dfrac{e^x + e^{-x}}{2} \qquad \mathrm{sh}(x) = \dfrac{e^x - e^{-x}}{2} \quad$ et $\quad \mathrm{th}(x) = \dfrac{\mathrm{sh}(x)}{\mathrm{ch}(x)} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Propriétés : $\mathrm{ch}^2(x) - \mathrm{sh}^2(x) = 1 \quad$ et $\quad \dfrac{1}{\mathrm{ch}^2(x)} = 1 - \mathrm{th}^2(x)$

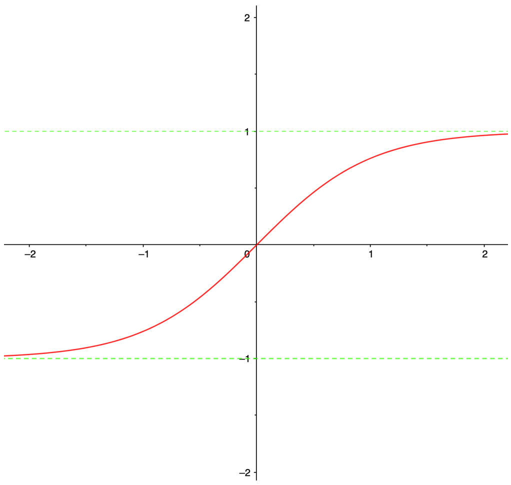
Cosinus hyperbolique : $x \mapsto \mathrm{ch}(x)$



Sinus hyperbolique : $x \mapsto \mathrm{sh}(x)$



Tangente hyperbolique : $x \mapsto \mathrm{th}(x)$



Fonctions hyperboliques réciproques

Propriétés : domaines de définition, expressions explicites (à savoir retrouver) et dérivées.

$$\mathrm{argch} : \begin{array}{ccc} [1, +\infty[& \to & \mathbb{R}_+ \\ x & \mapsto & \ln(x + \sqrt{x^2 - 1}) \end{array}$$

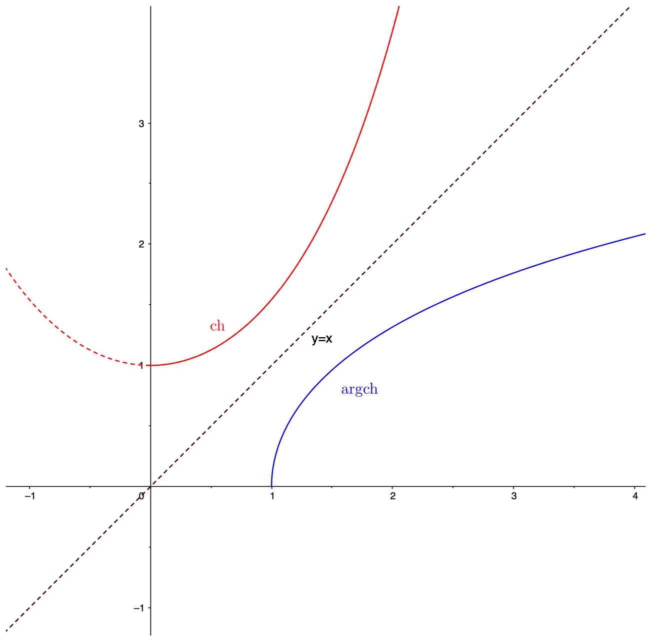
$$\forall x > 1, \quad \mathrm{argch}'(x) = \frac{1}{\sqrt{x^2 - 1}}$$

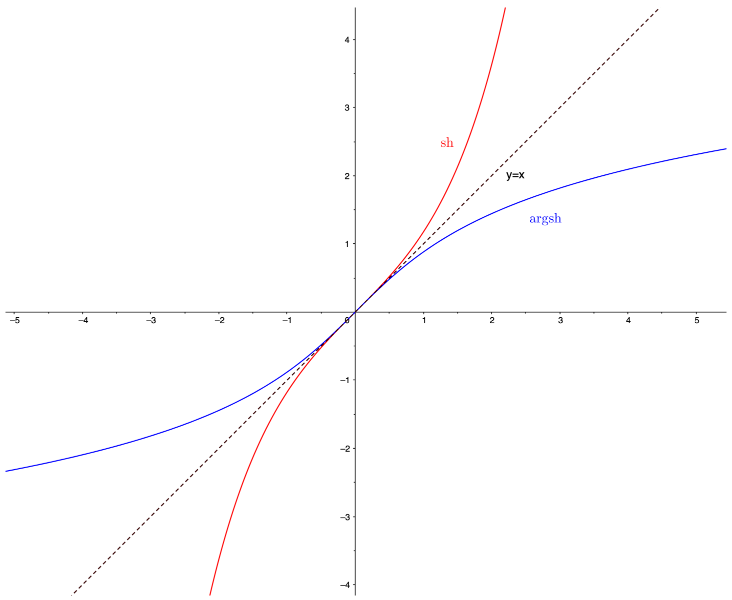
$$\mathrm{argsh} : \begin{array}{ccc} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & \ln(x + \sqrt{x^2 + 1}) \end{array}$$

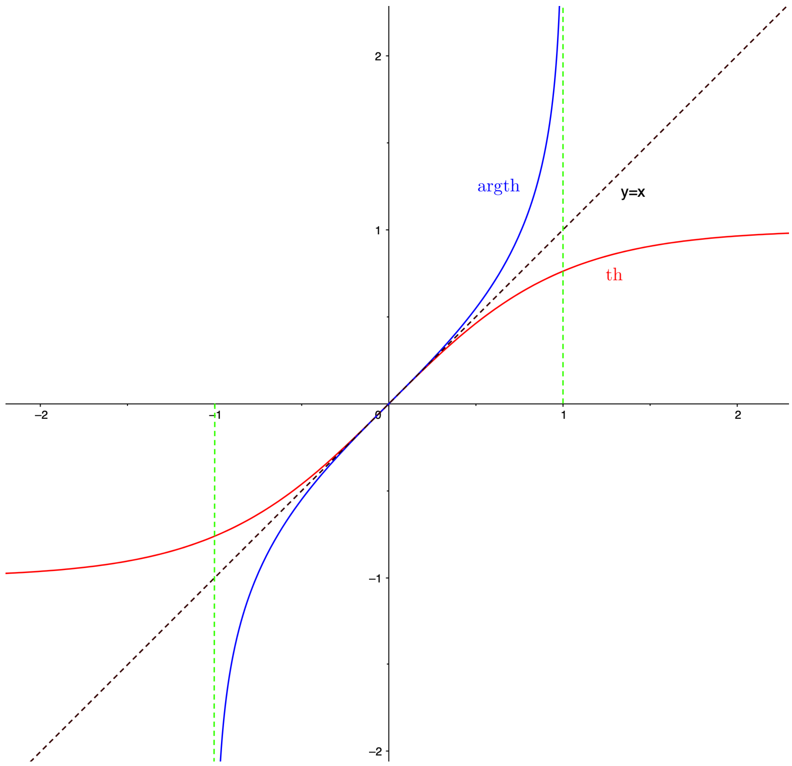
$$\forall x \in \mathbb{R}, \quad \mathrm{argsh}'(x) = \frac{1}{\sqrt{x^2 + 1}}$$

$$\mathrm{argth} : \begin{array}{ccc}]-1, 1[& \to & \mathbb{R} \\ x & \mapsto & \frac{1}{2} \ln\left(\dfrac{1+x}{1-x}\right) \end{array}$$

$$\forall x \in]-1, 1[, \quad \mathrm{argth}'(x) = \frac{1}{1 - x^2}$$


ch
y=x
argch


sh
y=x
argsh


argth
y=x
th

Fonctions trigonométriques réciproques

Propriétés : domaines de définition et dérivées.

$$\arccos : [-1,1] \to [0,\pi]$$

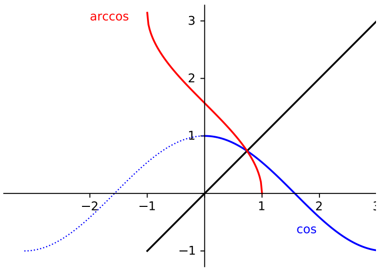
$$\forall x \in]-1,1[, \quad \arccos'(x) = \frac{-1}{\sqrt{1-x^2}}$$

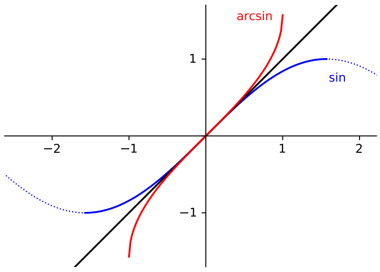
$$\arcsin : [-1,1] \to [-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$$

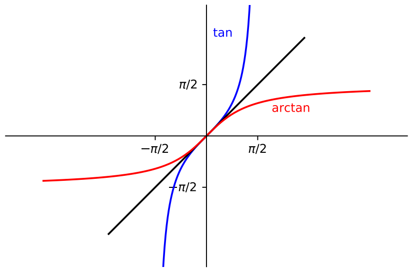
$$\forall x \in]-1,1[, \quad \arcsin'(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\arctan : \mathbb{R} \to]-\tfrac{\pi}{2}, \tfrac{\pi}{2}[$$

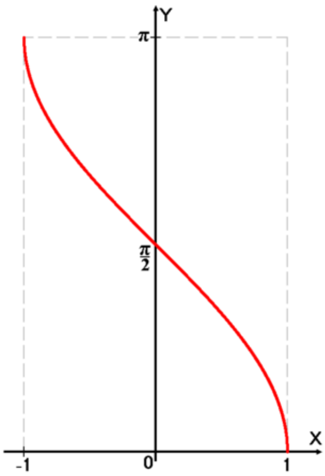
$$\forall x \in \mathbb{R}, \quad \arctan'(x) = \frac{1}{1+x^2}$$



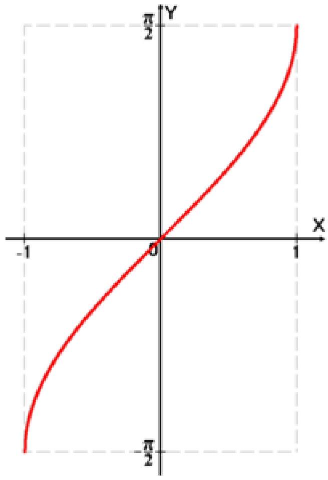




Arc cosinus : $x \mapsto \arccos(x)$



Arc sinus : $x \mapsto \arcsin(x)$



Arc tangente : $x \mapsto \arctan(x)$