Table of Contents

Basic Electricity

Definitions

Voltage is energy per unit charge. = $\frac{dw}{dq}\left(\frac{J}{C}=V\right)$

Current is rate of charge flow. = $\frac{dq}{dt}\left(\frac{C}{s}=A\right)$

Power is energy per unit time. = $\frac{dw}{dt}\ (W = VA = \frac{J}{s})$

Resistance is the capacity to impede flow of current. = $R\ (\Omega)$

Nodes are points where two or more elements join

Loops are any closed path in circuit

Meshes are loops not containing other loops

Relationships

$$p = vi = i^2R = \frac{v^2}{R}$$
$$v = iR$$

Passive Sign Convention

Passive Element(Load) = Power Dissipated > 0

Active Element = Power Dissipated < 0

Circuit Elements

Independent

Ideal Voltage Sources deliver a prescribed voltage independent of current flow.

Idea Current Sources deliver a prescribed current independent of voltage

Dependent

Voltage/**C**urrent **C**ontrol **V**oltage/**C**urrent **S**ources depend on Independent Elements.

VCVS. = $v_s = \alpha v_x$

CCVS. = $v_s = \beta i_x$

VCCS. = $i_s = \gamma v_x$

CCCS. = $i_s = \gamma i_x$

Resistors

Resistors impede the flow of charge. **Ohm's Law** states $v = iR$. **Shorts** are when voltage across is 0. Ex: $R = 0 \ \rightarrow V = i(0) = 0$. **Opens** are when current is 0. Ex: $R = \ \infty \rightarrow i = \frac{v}{\infty} = 0$.

Series:$R_{eq} = R_1 + R_2 + \cdots + R_n = \sum_n R_n\ , V_n = \frac{R_n}{R_{eq}}V_s \rightarrow V_1 = \frac{R_1}{R_1+R_2}V_s$

Parallel: $G_{eq} = G_1 + G_2 + \cdots + G_n = \sum_n G_n, i_n = \frac{\frac{1}{R_n}}{\frac{1}{R_{eq}}}i_s = \frac{G_n}{G_{eq}}i_s \rightarrow i_1 = \frac{R_2}{R_1+R_2}i_s, R_{eq} = \sum_n 1/R_n \rightarrow R_{eq\ TWO} = \frac{R_1R_2}{R_1+R_2}$

Capacitors

Capacitors store energy and are model as an open in DC steady state. Series add in parallel and parallel add in series.

$q = cv, i = c\frac{dv}{dt} \rightarrow i = c(0) = 0, v = \frac{1}{c}\int_{t_0}^{t} i\ dt + v(t_0), w_c = \frac{1}{2}cv^2$

Inductors

Inductors store energy and are model as closed in DC steady state. Series add in series and parallel add in parallel.

$v = L\frac{di}{dt} \to v = L(0) = 0,\ i = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0),\ w_L = \frac{1}{2}Li^2$

Op Amps

Op Amps can add, subtract, multiply or divide input voltage signals.

$v_p = v_n,\ R_{in} \to \infty,\ R_{out} = 0,\ A = \frac{V_o}{V_s} = \frac{V_o}{V_p - V_n} \to A \to \infty,\ -V_{cc} \le V_o \le V_{cc}$

Procedure to solve for v_o

1) Solve for v_p or v_n (whichever is not connected to v_o).
2) Use Node-Voltage Method for the one connected to v_o.
3) Substitute Equation from step (1) and (2) and solve.

Voltmeter

Voltmeters find voltages & are connected in parallel.

$R_{estiance\ into\ circuit} = \frac{v_{fullscale}}{y} \to Ideally\ \infty,\ R_v = \frac{v_r}{y},\ v_R = fullscale - x$

Ammeters

Ammeters find currents & are connected in series.

$R_{estiance\ into\ circuit} = \frac{x}{i_{fullscale}} \to Ideally\ 0, R_a = \frac{x}{i_r}; i_R = fullscale - y$

Analysis Methods

KCL

Kirchhoff's current law states charge is conserved. $\sum_{node} i = 0$

KVL

Kirchhoff's voltage law states energy is conserved. $\sum_{loop} v = 0$

Node-Voltage

1) Define a Reference Node to 0V
2) Label all independent nodes (voltage unknown)
3) Define current directions
4) Write current in terms of Node-Voltage ($i = \frac{V_A - v_B}{R}$) and substitute into KCL equations
5) Solve
6) Use Node-Voltage to find anything else in the circuit.

Super-Nodes

Super-Nodes are 2 non-reference nodes connected directly by a voltage source. To solve apply KVL in Loop with Super.

Current Mesh

1) Define current loop directions.
2) Express voltage in terms of Current Meshes ($v = i_1 R_1$) and substitute into KVL equations.
3) Solve
4) Use Mesh to find anything else in circuit

SuperMesh
To solve **SuperMeshes** find a closed path that does include the current source and apply KCL.

Thévenin Equivalent
A **Thévenin equivalent circuit** is a simple circuit with a voltage source and resistor in series.
1) Set all independent source to 0 (Short VS and open CS).
2) $R_{th} = R_{eq}$
3) $v_{th} = v_{OC}$ (Set load to open).

Norton Equivalent
A **Norton equivalent circuit** is a simple circuit with a current source and resistor in parallel.
1) $R_N = R_{th}$
2) $i_N = i_{sc}$ (Set load to closed).

Source Transform
A Thévenin equivalent circuit can be transformed into a Norton equivalent circuit and vice versa. $v_{th} = i_N R$ in Series is equal to $i_N = \frac{v_{th}}{R}$ in Parallel. The resistor remains constant and the sources change following Ohm's Law.

Inductors and Capacitors
1) Find time constant. = $\tau = Rc = \frac{L}{R}$
2) Capacitors: $v_{0-} = v_{0+}$, Inductors: $i_{0-} = i_{0+}$
3) Find X_f as $t \to \infty$
4) Standard form: $x(t) = x_f + (x_0 - x_f)e^{-\frac{t}{\tau}}$

Sinusoidal Steady State
1) Find ω
2) Convert from time domain to frequency domain
   a. Sinusoidal voltages and currents to phasors
   b. R, L, C to Impedance
3) Solve linear system $v = Zi$
4) Convert back to time domain

AC

Impedance and Reactance Equations

	Impedance	Reactance
Resistor	R	0
Inductor	$j\omega L$	ωL
Capacitor	$-\frac{j}{\omega c}$	$-\frac{1}{\omega c}$

Sinusoidal Functions
$v(t) = v_m \cos(\omega t + \phi) = v_m \angle \phi$, $T = \frac{2\pi}{\omega}$, $\omega = 2\pi f$

Power

Maximum Power: $Z_L = Z_{th} \to p_{L\,max} = \frac{v_{th}^2}{4R_{th}}(DC); Z_L = Z_{th}^* \to P_{L\,Max} = \frac{\left|v_{th\,(RMS)}\right|^2}{4R_L}(AC)$
Apparent (S), **Average**(P), and **Reactive**(Q) Power: $S = P + jQ = v_{RMS} i_{RMS}^*$
Power/**R**eactive **F**actor: $pf = \cos(\theta_v - \theta_i)\,, rf = \sin\,(\theta_v - \theta_i);$
Average Power: $P = \frac{1}{2} v_m i_m \cos(\theta_v - \theta_i) = v_{RMS} i_{RMS} \cos(\theta_v - \theta_i)$
Reactive Power: $Q = \frac{1}{2} v_m i_m \sin(\theta_v - \theta_i) = v_{RMS} i_{RMS} \sin(\theta_v - \theta_i)$
Units:$P \to watts, q \to VAR, S \to VA$