

數學常用公式 2

求 n 取 m 的排列

- $$P_k^n = \frac{n!}{(n - k)!}$$

本末倒置(數值反轉後的差必為 9 的倍數)

- 證明數值反轉後的差必為 9 的倍數：
 - 反轉
 - 4321 ⇒ 1234
 - 96321 ⇒ 12369
 - 二位數的本末倒置

10 - 1 = 9	20 - 2 = 18	30 - 3 = 27	40 - 4 = 36	50 - 5 = 45	60 - 6 = 54	70 - 7 = 63	80 - 8 = 72	90 - 9 = 81
	21 - 12 = 9	31 - 13 = 18	41 - 14 = 27	51 - 15 = 36	61 - 16 = 45	71 - 17 = 54	81 - 18 = 63	91 - 19 = 72
12 - 21 = 9		32 - 23 = 9	42 - 24 = 18	52 - 25 = 27	62 - 26 = 36	72 - 27 = 45	82 - 28 = 54	92 - 29 = 63
13 - 31 = 18	23 - 32 = 9		43 - 34 = 9	53 - 35 = 18	63 - 36 = 27	73 - 37 = 36	83 - 38 = 45	93 - 39 = 54
14 - 41 = 27	24 - 42 = 18	34 - 43 = 9		54 - 45 = 9	64 - 46 = 18	74 - 47 = 27	84 - 48 = 36	94 - 49 = 45
15 - 51 = 36	25 - 52 = 27	35 - 53 = 18	45 - 54 = 9		65 - 56 = 9	75 - 57 = 18	85 - 58 = 27	95 - 59 = 36
16 - 61 = 45	26 - 62 = 36	36 - 63 = 27	46 - 64 = 18	56 - 65 = 9		76 - 67 = 9	86 - 68 = 18	96 - 69 = 27
17 - 71 = 54	27 - 72 = 45	37 - 73 = 36	47 - 74 = 27	57 - 75 = 18	67 - 76 = 9		87 - 78 = 9	97 - 79 = 18
18 - 81 = 63	28 - 82 = 54	38 - 83 = 45	48 - 84 = 36	58 - 85 = 27	68 - 86 = 18	78 - 87 = 9		98 - 89 = 9
19 - 91 = 72	29 - 92 = 63	39 - 93 = 54	49 - 94 = 45	59 - 95 = 36	69 - 96 = 27	79 - 97 = 18	89 - 98 = 9	

- 公式推導
 - 令 $a = 10x + y$
 - $0 < x \leq 9$
 - $0 \leq y \leq 9$
 - $x \neq y$
 - ⇒ $|(10x + y) - (10y + x)| = |9x - 9y| = 9|x - y|$
 - ⇒ 反轉後的差必為 9 的倍數

- 三位數的本末倒置

100 - 1 = 99	200 - 2 = 198	300 - 3 = 297	...	700 - 7 = 693	800 - 8 = 792	900 - 9 = 891
	201 - 102 = 99	301 - 103 = 198	...	701 - 107 = 594	801 - 108 = 693	901 - 109 = 792
102 - 201 = 99		302 - 203 = 99	...	702 - 207 = 495	802 - 208 = 594	902 - 209 = 693
⋮	⋮	⋮	⋮	⋮	⋮	⋮
197 - 791 = 594	297 - 792 = 495	397 - 793 = 396	...		897 - 798 = 99	997 - 799 = 198
198 - 891 = 693	298 - 892 = 594	398 - 893 = 495	...	798 - 897 = 99		998 - 899 = 99
199 - 991 = 792	299 - 992 = 693	399 - 993 = 594	...	799 - 997 = 198	899 - 998 = 99	

- 公式推導

- 令 $a = 100x + 10y + z$

- $0 < x \leq 9$

- $0 \leq y \leq 9$

- $0 \leq z \leq 9$

- $x \neq z$

- $\Rightarrow |(100x + 10y + z) - (100z + 10y + x)| = |99x - 99z| = 99|x - z| = 9 * 11|x - z|$

- \Rightarrow 反轉後的差必為 99 的倍數，也就是 9 的倍數

多項式

- $$(a + b)^k = x_1 a^k + x_2 a^{k-1} b + x_3 a^{k-2} b^2 + \dots + x_{k+1} b^k$$

$$x_i = C_k^i$$

平方和公式 + 等差級數公式

- $$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} = \frac{n(n+1)(2n+1)}{6}$$

$$S_n = \frac{n}{2} (a + a_n)$$

- $$= \frac{n}{2} [2a + (n-1)d]$$

$$= an + d \cdot \frac{n(n-1)}{2}$$