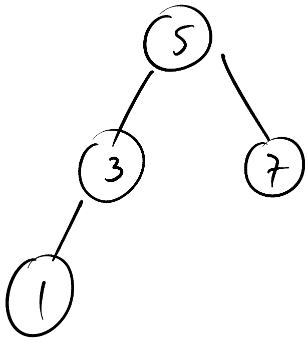
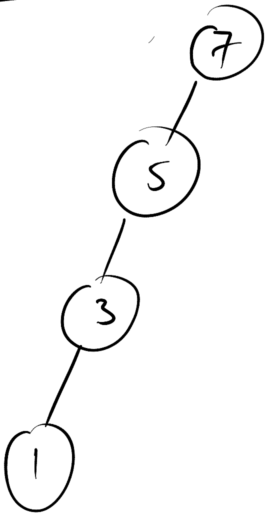
AVL Trees

Thursday, 16 December 2021 17:10

Why not simple BST?





Solution:

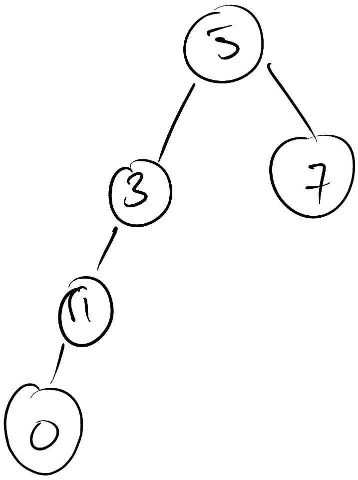
AVL!

other solutions:

2-3 Trees

3-4 Trees

Red-black trees

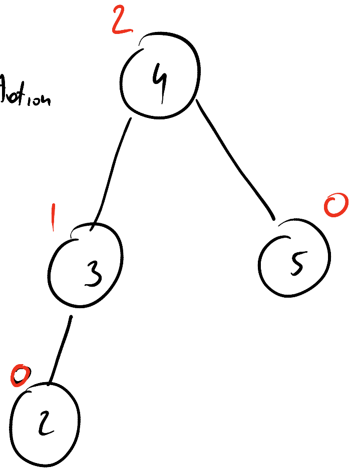


Height

$H(\text{None}) = -1$ ← for implementation

$H(\text{leaf}) = 0$

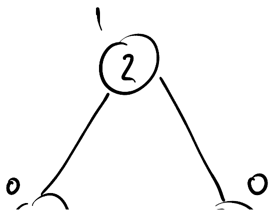
$H(N) = \text{Max}(H(N_L), H(N_R)) + 1$

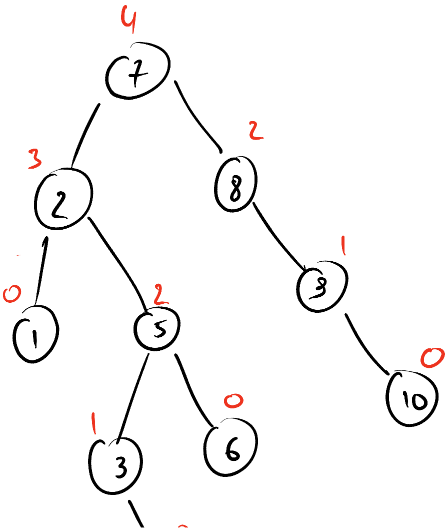


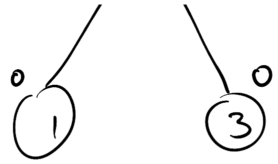
Balance

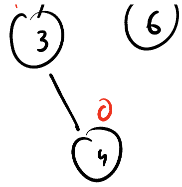
Balance factor

$B(T) = H(T_L) - H(T_R)$



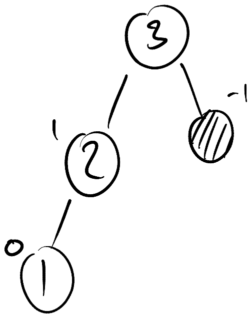






$B(2) = H(1) - H(3) =$

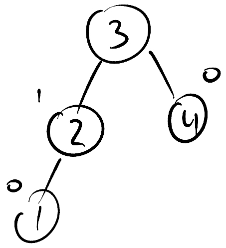
$= 0 - 0 = 0$



$B(3) = ?$

$B(3) = H(2) - H(None) =$

$= 1 - (-1) = 2$

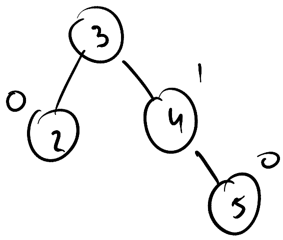


$B(3) = H(2) - H(4) =$

$= 1 - 0 =$

$= 1$

Tree is balanced $\iff |B_f| \leq 1$ AVL



$B(3) = H(2) - H(4) =$

$= 0 - 1 = -1$

$B_f > 0 \rightarrow$ left-heavy tree

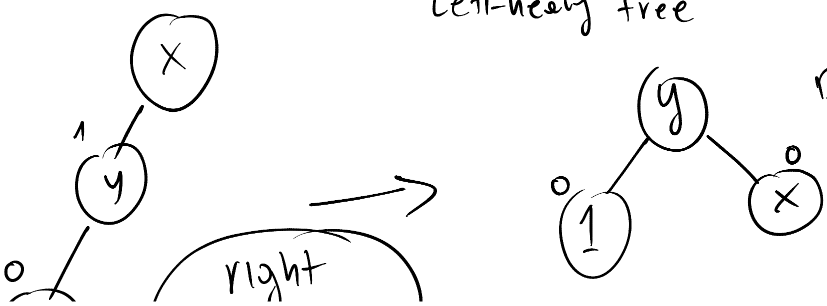
$B_f < 0 \rightarrow$ right-heavy tree

AVL Trees

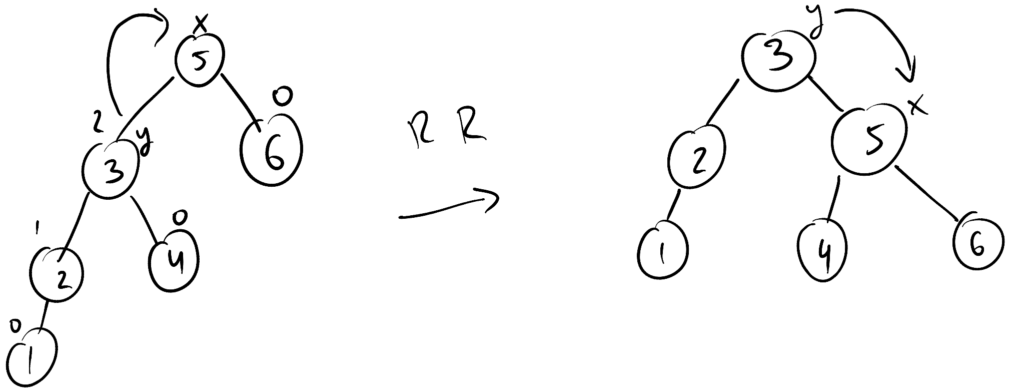
RR - Right Right Rotation

$B_f(x) = 1 - (-1) = 2$

Left-heavy tree

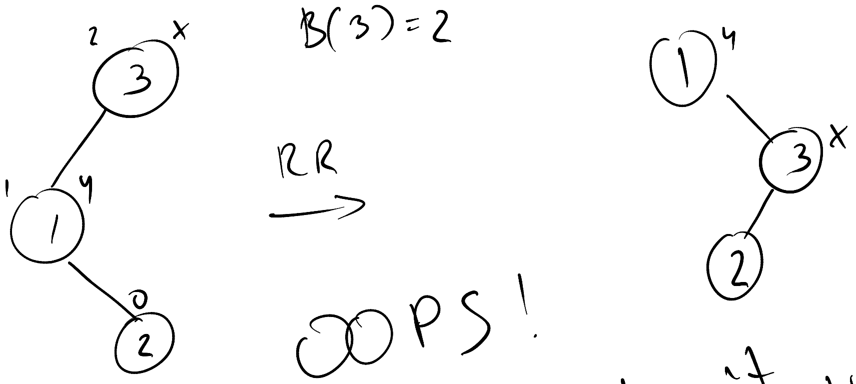


$B(y) = 0 - 0 = 0$



Why not RR in this case?

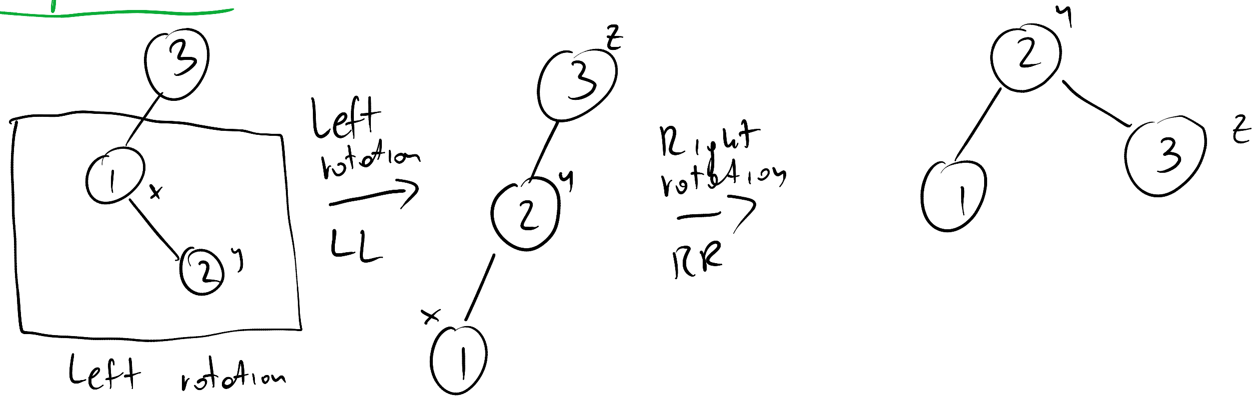
$B(3)=2$

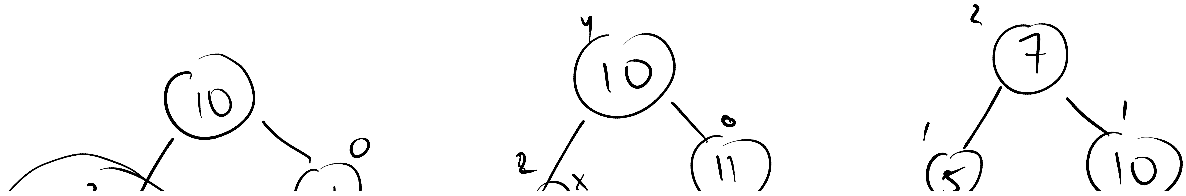


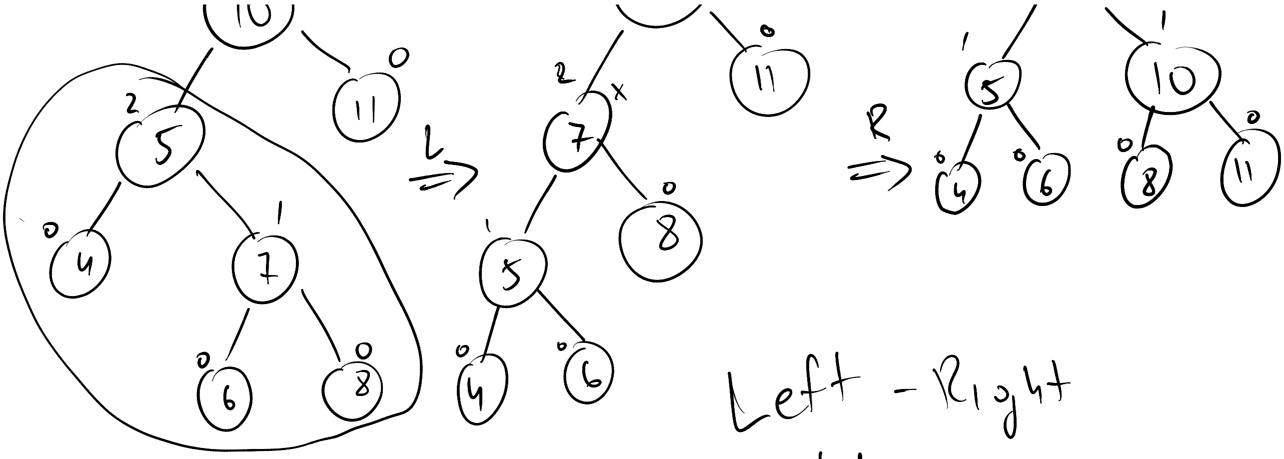
OOPS!

BAD, doesn't work!

Left-Right LR







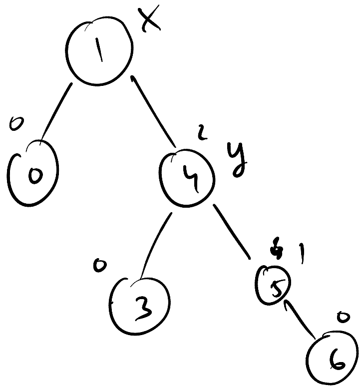
Left - Right

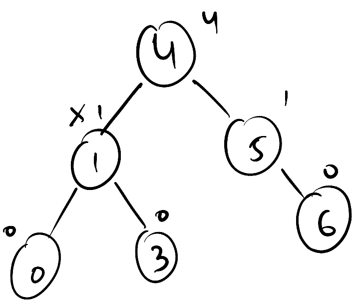
Rotation

RR	$B(T) > 1$	$\wedge$	$B(T_L) \geq 0$
LR	$B(T) > 1$	$\wedge$	$B(T_L) < 0$
LL	$B(T) < -1$	$\wedge$	$B(T_R) \leq 0$
RL	$B(T) < -1$	$\wedge$	$B(T_R) > 0$

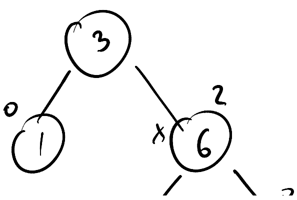
Important!

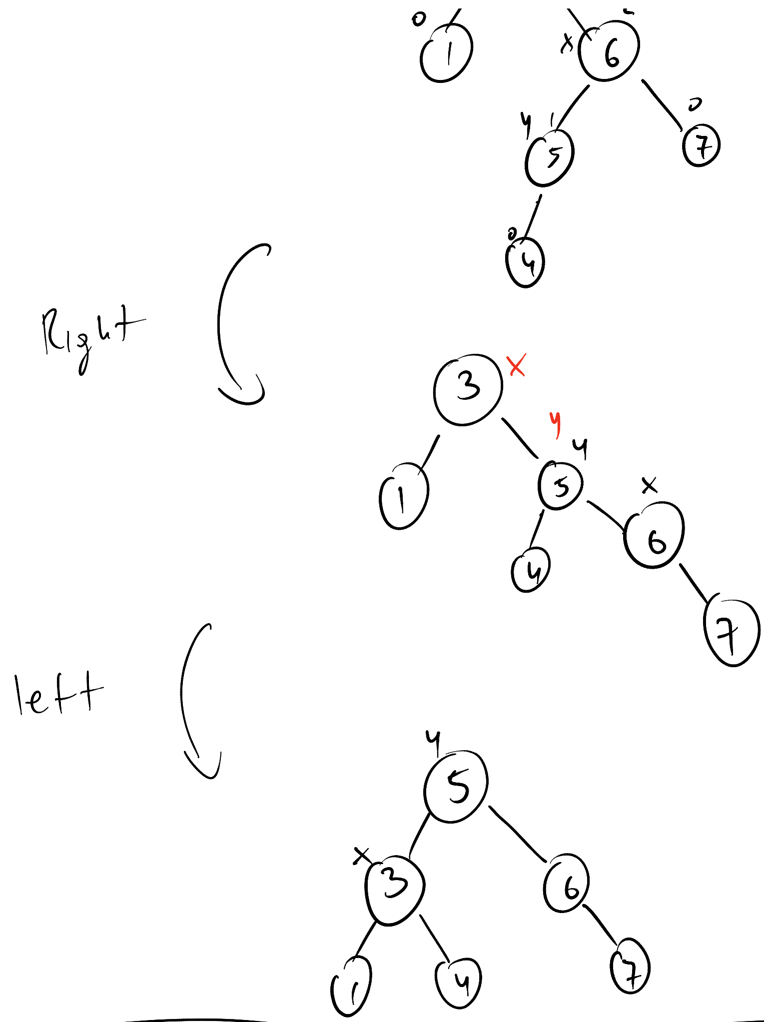
LL - left left





RL - Right-Left





<u>Search</u>

$O(\log n) \sim O(h)$ $h \sim \log(n)$