HW2 Theory

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Problem 1.13

Part a

Prove: $17 \in \mathrm{O}(1)$

Solution

$c >= 1$

$$\begin{aligned} 17 &<= c(1) \\ c &= 18 \\ 17 &<= 18 \\ 17 &\in \mathrm{O}(1) \end{aligned}$$

Part b

Prove: $n(n-1)/2 \in \mathrm{O}(n^2)$

Solution

$c >= n_0$

$n_0 > 0$

$$\begin{aligned} n(n-1)/2 &= \frac{n^2-n}{2} \\ \frac{n^2-n}{2} &<= cn^2 \\ c &= 10 \\ \frac{n^2-n}{2} &<= 10n^2 \\ n^2-n &<= 20n^2 \\ n-1 &<= 20n, \forall n >= 1 \\ n(n-1)/2 &\in \mathrm{O}(n^2) \end{aligned}$$

Part c

Prove: $\max(n^3, 10n^2) \in \mathrm{O}(n^3)$

Solution

First there must be proof that $10n^2 \in \mathrm{O}(n^3)$ to prove that the $\max(n^3, 10n^2)$ is n^3

$c >= n_0$

$n_0 > 0$

$$
\begin{aligned}
10n^2 <&= cn^3 \\
c &= 1 \\
10n^2 <&= n^3 \\
10 <= n, \forall n >&= 10 \\
\max(n^3, 10n^2) &\text{ is } n^3 \\
n^3 \in \mathrm{O}(n^3), &\forall n
\end{aligned}
$$

Part d

Prove: $\sum_{i=1}^{n} i^k \in \mathrm{O}(n^{k+1})$ and $\in \Omega(n^{k+1})$

Solution

BigOh

$$
\begin{aligned}
\sum_{i=1}^{n} i^k <&= cn^{k+1} \\
1 + \frac{\sum_{i=1}^{n-1} i^k}{n^k} <&= cn \\
\implies 0 < \frac{i^k}{n^k} < 1, \forall i &\in [1, n-1] \\
1 + \sum_{i=1}^{n-1} j <= cn, 0 < j < 1, c &>= 1 \\
\implies \sum_{i=1}^{n} i^k &\in \mathrm{O}(n^{k+1})
\end{aligned}
$$

BigOmega

$$
\begin{aligned}
c\sum_{i=1}^{n} i^k &>= n^{k+1} \\
c + \frac{c\sum_{i=1}^{n-1} i^k}{n^k} &>= n \\
1 + \frac{\sum_{i=1}^{n-1} i^k}{n^k} &>= \frac{n}{c} \\
\frac{n}{c} &<= n \\
\implies \sum_{i=1}^{n} i^k &\in \Omega(n^{k+1})
\end{aligned}
$$

Problem 1.16

In order or smallest growth rate to largest

1. $(\frac{1}{3})^n$ (h)

2. 17 (j)

3. $log(log(n))$ (d)

4.$log(n)$ (c)

5.$log^2(n)$ (e)

6. $\sqrt{n}$ (b)

7. $\sqrt{n}log^2(n)$ (g)

8. $\frac{n}{log(n)}$ (f)

9. n (a)

10. $(3/2)^n$ (i)

Problem 1.18

Part a

Part b

$T(n) = 2^{log(n)}$

Problem 2.9

When the if statement inside the while loop is $True$ the list will be lost. This is because the pointer at the current position is not preserved. After it is deleted it will have nothing to point to next.

Problem 2.11

First: $n^2 + 1$

End: $n^2 + n$

Next: $n^3 + n^2 + n$