Ryan Smith
CS 540: AI
HW #3

1.

i. Compute P(A = true and B = false and C = false and D = false).

(A = true and B = false and C = false and D = false) = 1 - (0.012 + 0.013 + 0.01 + 0.025 + 0.015 + 0.025 + 0.025 + 0.025 + 0.02 + 0.03 + 0.04 + 0.05 + 0.06 + 0.07 + 0.08)

= 0.5

ii. Compute P(A = true and C = false and D = false).

P(A = true and C = false and D = false) = 0.5 + 0.05

= 0.55

iii. Compute P(B = true or D = false).

P(B = true or D = false) = P(B = true) + P(D = false) - P(B = true and D = false)

= 0.35 + 0.712 - 0.16

= 0.902

iv. Compute P(D = false | A = false and B = true and C = true).

with A = false and B = true and C = true being the case, the remaining possibilities are:

A	B	C	D	Prob
F	T	T	F	0.025
F	T	T	T	0.025

By inspection we can see that from among these possibilities there is a probability of 0.5 that D = false is true.

Using rules of probabilities to derive this:

P(D = false | A = false and B = true and C = true) * P(A = false and B = true and C = true) = P(D = false and A = false and B = true and C = true)

=
0.025

Dividing both sides by P(A = false and B = true and C = true) - which is 0.025 + 0.025 = 0.05 - gives us:

P(D = false | A = false and B = true and C = true) = 0.025 / 0.05

= 0.5

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v. Compute P(A = false and B = true and C = true | D = false).

P(A = false and B = true and C = true | D = false) * P(D = false)   = P(A = false and B = true
and C = true and D = false)

P(A = false and B = true and C = true | D = false) * P(D = false)   = 0.025

P(D = false) turns out to be 0.712, which gives us:

P(A = false and B = true and C = true | D = false)  = 0.025 / 0.712

                                                     = 0.0351123595505618


2.

From here on I'll use P(A) to denote P(A = true) and P('A) to denote P(A = false). I will also
use "^" and "v" in place of "and" and "or".

i. Compute P(A ^ B ^ C ^ D).

P(A ^ B ^ C ^ D)    = P(A) * P(B | A) * P(C | A ^ B) * P(D | B ^ C)

                    = (0.9) * (0.1) * (0.7) * (0.3)

                    = 0.0189

ii. Compute P('A ^ 'C ^ 'D).

P('A ^ 'C ^ 'D) = P('A) * P(B | 'A) * P('C | 'A ^ B) * P('D | B ^ 'C) + P('A) * P('B | 'A) *
P('C | 'A ^ 'B) * P('D | 'B ^ 'C)

                = (0.1) * (0.8) * (0.5) * (0.1) + (0.1) * (0.2) * (0.6) * (0.8)

                = (0.004) +(0.0096)

                = 0.0136

iii. Compute P(C = true | A=true and B=true and D=true).

P(C | A ^ B ^ D)    = P(A ^ B ^ C ^ D) / P(A ^ B ^ D)

                    = 0.0189 / (P(A ^ B ^ C ^ D) + P(A ^ B ^ 'C ^ D))

                    = 0.0189 / (P(A ^ B ^ C ^ D) + P(A ^ B ^ 'C ^ D))

                    = 0.0189 / (0.0189 + (0.9 * 0.1 * 0.3 * 0.9))

                    = 0.0189 / (0.0189 + 0.0243)

                    = 0.0189 / 0.0432

                    = 0.4375
```

iv. Compute P(D= false | B = true and C = true). // Use the Markov Blanket property here.

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P('D | B ^ C)   = 1 - P(D | B ^ C)

                = 1 - 0.3

                = 0.7
```

v. Compute P(D= false | A=true and B = true and C = true). // Do this one without employing the Markov Blanket property (i.e., to algebraically or // numerically confirm that knowing A=true doesn’t change the result from iv).

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P('D | A ^ B ^ C)   = P(A ^ B ^ C ^ 'D) / P(A ^ B ^ C)

                    = [P(A) * P(B | A) * P(C | A ^ B) * P('D | B ^ C)] / [P(A) * P(B | A) * P(C
                    | A ^ B) * P('D | B ^ C)]

                    = [(0.9) * (0.1) * (0.7) * (0.7)] / [(0.9) * (0.1) * (0.7)]

                    = 0.7
```

vi. Compute P((A = false and C = false) or (B = true and D = true)).

```
P(('A ^ 'C) v (B ^ D))  = P('A ^ 'C) + P((B ^ D) - P(('A ^ 'C) ^ (B ^ D))
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We'll solve for P('A ^ 'C) and P((B ^ D) to solve this:

```
P('A ^ 'C)  = P('A ^ 'C ^ B) + P('A ^ 'C ^ 'B)
            = [P('C | 'A ^ B) * P('A ^ B)] + [P('C | 'A ^ 'B) * P('A ^ 'B)]
            = [0.5 * P(B | 'A) * P('A)] + [0.6 * P('B | 'A) * P('A)]
            = [0.5 * 0.8 * 0.1] + [0.6 * 0.2 * 0.1]
            = 0.04 + 0.012
            = 0.052

P(B ^ D)    = P(A ^ B ^ C ^ D) + P('A ^ B ^ C ^ D) + P(A ^ B ^ 'C ^ D) + P('A ^ B ^ 'C ^ D)
            = 0.0189 + [P('A) * P(B | 'A) * P(C | 'A ^ B) * P(D | B ^ C)] + [P(A) * P(B | A) *
            P('C | A ^ B) * P(D | B ^ 'C)] + [P('A) * P(B | 'A) * P('C | 'A ^ B) * P(D | B ^
            'C)]    //using our result from i
            = 0.0189 + [0.1 * 0.8 * 0.5 * 0.3] + [0.9 * 0.1 * 0.3 * 0.9] + [0.1 * 0.8 * 0.5 *
            0.9]
            = 0.0189 + 0.012 + 0.0243 + 0.036
            = 0.0912
```

So we have:

```
P(('A ^ 'C) v (B ^ D))  = P('A ^ 'C) + P((B ^ D) - P(('A ^ 'C) ^ (B ^ D))

                        = 0.052 + 0.0912 - 0.036

                        = 0.1072
```

3.

A	B	C	D	Prob	
F	F	F	F	0.0096	//from problem 2 part ii
F	F	F	T	0.0024	//P('A) * P('B | 'A) * P('C | 'A ^ 'B) * P(D | 'B ^ 'C) = 0.1 * 0.2 * 0.6 * 0.2 = 0.0024
F	F	T	F	0.0072	//P('A) * P('B | 'A) * P(C | 'A ^ 'B) * P('D | 'B ^ C) = 0.1 * 0.2 * 0.4 * 0.9 = 0.0072
F	F	T	T	0.0008	//P('A) * P('B | 'A) * P(C | 'A ^ 'B) * P(D | 'B ^ C) = 0.1 * 0.2 * 0.4 * 0.1 = 0.0008
F	T	F	F	0.004	//from problem 2 part ii
T	F	F	F	0.2592	//not showing work because not required
F	T	F	T	0.036	//from problem 2 part vi
F	T	T	F	0.028	//not showing work because not required
T	F	T	F	0.4374	//not showing work because not required
T	T	F	F	0.0027	//not showing work because not required
T	F	F	T	0.0648	//not showing work because not required
T	T	T	F	0.0441	//from problem 2 part v
T	T	F	T	0.0243	//from problem 2 part vi
T	F	T	T	0.0486	//not showing work because not required
F	T	T	T	0.012	//from problem 2 part vi
T	T	T	T	0.0189	//from problem 2 part i

4.

i.

P(flu | 'shot)

It's going to be helpful to fill out the probability table for flu and shot.

flu	shot	Prob
F	F	A
F	T	B
T	F	C
T	T	D

Using what we're given, we know that B + D = 0.6 and C + D = 0.4

We can solve for D using the fact that P(flu | shot) = 0.2:

P(flu ^ shot) / P(shot) = 0.2

P(flu ^ shot) = 0.2 * 0.6

0.12

So B = 0.6 - 0.12 = 0.48 and C = 0.4 - 0.12 = 0.28. Also, A = 1 - (B + C + D) = 1 - 0.48 - 0.28 - 0.12 = 0.12.

flu	shot	Prob
F	F	0.12

```
    F       T           0.48
    T       F           0.28
    T       T           0.12

Therefore:

    P(flu | 'shot)  = P(flu ^ 'shot) / P('shot)

                    = 0.28 / 0.4

                    = 0.7

ii.

    P(shot | flu)

From Bayes' Rule we have:

    P(flu | shot)   = [P(flu) * P(shot | flu)] / P(shot)

So:

    [P(flu) * P(shot | flu)] / P(shot)  = 0.2

                                        = [0.4 * P(shot | flu)] / 0.6

Which gives us:

    P(shot | flu)   = 0.3

iii.

    P(DH | T4)  = [P(DH) * P(T4 | DH)] / P(T4)

                = [0.0001 * 0.975] / P(T4)

Solving for P(T4):

    P(T4)   = P(T4 ^ DH) + P(T4 ^ 'DH)

            = P(T4 | DH) * P(DH) + P(T4 | 'DH) * P('DH)

            = 0.975 * 0.0001 + 0.025 * 0.9999

            = 0.0000975 + 0.0249975

            = 0.025095

So:

    P(DH | T4)  = 0.0000975 / 0.025095

                = 0.0038852361028093
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5.

D = Sally likes book23
S1 = ‘see’ in book
S2 = ‘animal’ in book
S3 = ‘eat’ in book
S4 = ‘vegetable’ in book

Total words in books Sally likes = 7 + 1 + 6 + 2 + 5 //5 for the extra book with one of each word

= 21

Total words in books Sally dislikes = 11 + 3 + 8 + 9 + 5 + 5 //5 for the extra book with one of each word

= 36

[Prob(Sally likes book23 | ‘see’ in book ∧ ‘animal’ in book ∧ ‘eat’ in book ∧ ‘vegetable’ in book)] / [Prob(Sally dislikes book23 | ‘see’ in book ∧ ‘animal’ in book ∧ ‘eat’ in book ∧ ‘vegetable’ in book)]

= ([P(S1 | D) * P(S2 | D) * P(S3 | D) * P(S4 | D)] * P(D)) / ([P(S1 | 'D) * P(S2 | 'D) * P(S3 | 'D) * P(S4 | 'D)] * P('D))

= ([(7/22) * (8/22) * (3/22) * (2/22)] * (9/22)) / ([(10/22) * (12/22) * (6/22) * (9/22)] * (13/22))

= 3024 / 84240

= 7 / 195

= 0.0358974358974359