

Challenge problem: Alternating Harmonic and sloooooow convergence

0. This presentation revolves around two series. First is the Harmonic Series, $\sum_{n=1}^{\infty} \frac{1}{n}$ and the second is the Alternating Harmonic Series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. We know about the first at present and Dr T will explain why the second converges next week.
1. Identify your platform (Matlab or what you are using). Remind students that the harmonic series diverges ($p=1$) and the alternating harmonic converges (by Alternating Series Test). Good to put these 2 series on a side board with their names (so they can follow your presentation on the doc cam/computer while keeping the series in mind).
2. Now use Matlab series option to show how slowly the harmonic series sums to infinity. A loop with input the partial sum you are passing (like, 12), and the output is the number of terms n to pass that number.
3. Do again with alternating harmonic series, which sums to $\ln 2$. So you can see how soon to pass .69, .693, .6931, etc. If you are going PAST the number, then automatically your last term is one of the odd terms, $1/(2n-1)$, since you ADD to go larger than the sum.
4. Now mention that you can rearrange the alternating harmonic to converge to ANY number (the first TARGET number). So the way this happens is you add odd terms to pass the TARGET number ($1 + 1/3 + 1/5 + 1/7$, etc to pass the number). At that point, you print the partial sum, and k , the number of terms of $\sum_{n=1}^k \frac{1}{2n-1}$ you used. In the code, you rename TARGET to be the partial sum minus the old Target number. In other words, in successive runs, the TARGET is a measurement of the error between the partial sum and the total sum, the original target. Then you repeat with the new TARGET, the sum of even terms: $\sum_{n=1}^j \frac{1}{2n}$ and these have to pass the new TARGET amount (this takes only one term, usually). This will be ending up with a different number of terms. The next step is create a new TARGET, same way. Then you start at the counter $k+1$ to go past the new target. I think 5 passes (add odds, subtract evens, add odds, subtract evens, add odds) should be enough for folks to understand what is going on.