Minimum Spanning Tree (MST) – Summary

1. Definition

A Minimum Spanning Tree (MST) is a subset of edges of a connected, undirected, weighted graph that connects all the vertices together without any cycles and with the minimum possible total edge weight.

Key Characteristics:

- Contains exactly (V – 1) edges, where V is the number of vertices.
- Does not contain cycles.
- Connects all vertices (spanning).
- Has the minimum total edge weight among all spanning trees.

Important Notes:

- MST exists only for connected graphs.
- Graph must be undirected.
- If all edge weights are distinct, the MST is unique.

Applications of MST:

- Network design (roads, electrical grids, internet cables).
- Minimizing wiring or pipeline costs.
- Clustering and image segmentation.
- Approximation algorithms (e.g., Traveling Salesman Problem).

2. MST Algorithms

2.1 Kruskal’s Algorithm

Kruskal’s algorithm builds the MST by selecting edges in increasing order of weight and adding them if they do not form a cycle.

Steps:

1. Sort all edges by increasing weight.
2. Initialize an empty MST.
3. Pick the smallest edge and add it if it does not form a cycle.
4. Repeat until the MST contains (V – 1) edges.

Data Structures Used:

- Disjoint Set Union (Union-Find) for cycle detection.

Time Complexity:

O(E log E), where E is the number of edges.

2.2 Prim’s Algorithm

Prim’s algorithm grows the MST starting from an arbitrary vertex, always choosing the minimum weight edge that connects the tree to a new vertex.

Steps:

1. Start from any vertex.
2. Add the minimum weight edge connected to the current tree.
3. Add the corresponding vertex to the tree.
4. Repeat until all vertices are included.

Data Structures Used:

- Priority Queue (Min-Heap).

Time Complexity:

- O(E log V) using a priority queue.
- $O(V^2)$ using an adjacency matrix.

3. Comparison and Key Concepts

Algorithm	Approach	Best Used When	Time Complexity
Kruskal	Edge-based	Graph has many edges sorted easily	O(E log E)
Prim	Vertex-based	Dense graphs	O(E log V)

Key MST Properties:

- Cut Property: The minimum weight edge crossing any cut belongs to the MST.
- Cycle Property: The maximum weight edge in any cycle cannot be part of the MST.

Common Mistakes:

- Applying MST algorithms on directed graphs.
- Forgetting to check graph connectivity.
- Confusing MST with shortest path trees.

Summary:

Minimum Spanning Tree is a fundamental concept in graph theory used to connect all vertices with minimum cost. Kruskal’s and Prim’s algorithms are the most popular approaches, each suitable for different graph characteristics.