Assignment 1

Mathias Porsgaard, Mikkel Reich, Noah Carelse

Faculty of Social Sciences
University of Copenhagen

1 Cobb-Douglas production with CRS

In this assignment we want to test whether the Cobb-Douglas production function (CD) exhibits constant returns to scale (CRS) at a firm level using real data and panel data methods. The Cobb-Douglas production function is given by,

$$F(K, L) = AK^{\beta_K} L^{\beta_L}$$

where K is capital, L is labor, A is total factor productivity (TFP) and β_K, β_L are the output elasticities of capital and labor, respectively. It follows that for CD to have CRS, it must be homogeneous of degree one, i.e. $\beta_K + \beta_L = 1$, as seen from,

$$F(\lambda K, \lambda L) = A(\lambda K)^{\beta_K} (\lambda L)^{\beta_L} = A\lambda^{\beta_K+\beta_L} K^{\beta_K} L^{\beta_L} = \lambda F(K, L) \iff \beta_K + \beta_L = 1.$$

which forms the linear hypothesis we want to test i.e. $H_0 : \beta_K + \beta_L = 1$ vs. $H_A : \beta_K + \beta_L \neq 1$. While K and L are certainly important production inputs, we cannot hope to observe **all** input factors that make up production. To investigate the functional form of the production function, we estimate a simple (two-way) fixed effects (FE) model, to remove the time-invarying and time-varying unobserved heterogeneity. Note, that these factors with control for much more than just TFP, but also other unobserved factors that may affect production.

We start by taking logs of the production function to linearize it and add an idiosyncratic error term, ε_{it}, to get,

$$\begin{aligned} \ln Y_{it} &= \ln A_{it} + \beta_K \ln K_{it} + \beta_L \ln L_{it} + \varepsilon_{it} \\ y_{it} &= \beta_K k_{it} + \beta_L l_{it} + u_{it}, \quad u_{it} = \ln A_{it} \end{aligned}$$

Suppose that $\ln A_{it}$ can be decomposed into a time-invariant firm-specific effect, α_i, a time-varying effect, γ_t, and add an idiosyncratic error, ε_{it}, i.e. $u_{it} = \alpha_i + \gamma_t + \varepsilon_{it}$. The FE model is then given by,

$$\begin{aligned} u_{it} &= \alpha_i + \gamma_t + \varepsilon_{it}, \\ y_{it} &= \alpha_i + \gamma_t + \beta_K k_{it} + \beta_L l_{it} + \varepsilon_{it}, \end{aligned}$$

where $y_{it} = \ln Y_{it}$, $k_{it} = \ln K_{it}$ and $l_{it} = \ln L_{it}$.

Gameplan:

Write stuff about assumptions, and then transformations, prob go from inconsistent pooled OLS to consistent FE. Right the estimator, list assumptions for identification and consistency (strict/cont exog, non-generacy) -¿ show consistency by LLN convergence in plim. Next, talk about asymptotic normality, CLT, variance estimation (efficiency - robust, cluster). Finally, hypothesis testing (Wald). Then proceed to empirical application (include wald-test code, since it is not given or done in any exercises).

2 Empirical results

The data consists of $N = 441$ French manufacturing firms over 12 years ($T = 12$) from 1968 to 1979, and is thus a panel data set with $NT = 5292$ observations. The data is balanced.

Maybe variable def.

We perform the two-way or double-demean transformation of our data matrices as described in the methodology section (Ref). The estimates along with standard errors and p-values are reported in Table **??**.

Next we proceed to test the null hypothesis of constant returns to scale. We reject the null at the 1 % significance level with a p-value of ≈ 0. Thus, we conclude that the (Cobb-Douglas) production function does not exhibit constant returns to scale, for French manufacturing firms in the period 1968-1979.

In addition one might want to test whether the production function exhibits increasing or decreasing returns to scale. This can be done by testing the null hypothesis of $\beta_K + \beta_L \geq 1$ against the alternative of $\beta_K + \beta_L < 1$ (decreasing returns to scale) or vice versa for increasing returns to scale. One might suspect that this particular sector exhibits increasing returns to scale, as firms in manufacturing often benefit from economies of scale.