

# Assignment 1

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# 1 Cobb-Douglas production with CRS

In this assignment we want to test whether the Cobb-Douglas production function (CD) exhibits constant returns to scale (CRS) at a firm level using real data and panel data methods. The Cobb-Douglas production function is given by,

$$F(K, L) = AK^{\beta_K} L^{\beta_L}$$

where  $K$  is capital,  $L$  is labor,  $A$  is total factor productivity (TFP) and  $\beta_K, \beta_L$  are the output elasticities of capital and labor, respectively. It follows that for CD to have CRS, it must be homogeneous of degree one, i.e.  $\beta_K + \beta_L = 1$ , as seen from,

$$F(\lambda K, \lambda L) = A(\lambda K)^{\beta_K} (\lambda L)^{\beta_L} = A\lambda^{\beta_K + \beta_L} K^{\beta_K} L^{\beta_L} = \lambda F(K, L) \iff \beta_K + \beta_L = 1.$$

which forms the linear hypothesis we want to test i.e.  $H_0 : \beta_K + \beta_L = 1$  vs.  $H_A : \beta_K + \beta_L \neq 1$ . While  $K$  and  $L$  are certainly important production inputs, we cannot hope to observe **all** input factors that make up production. To investigate the functional form of the production function, we estimate a simple (two-way) fixed effects (FE) model, to remove the time-invariant and time-varying unobserved heterogeneity. Note, that these factors with control for much more than just TFP, but also other unobserved factors that may affect production.

We start by taking logs of the production function to linearize it and add an idiosyncratic error term,  $\varepsilon_{it}$ , to get,

$$\begin{aligned} \ln Y_{it} &= \ln A_{it} + \beta_K \ln K_{it} + \beta_L \ln L_{it} + \varepsilon_{it} \\ y_{it} &= \beta_K k_{it} + \beta_L l_{it} + u_{it}, \quad u_{it} = \ln A_{it} \end{aligned}$$

Suppose that  $\ln A_{it}$  can be decomposed into a time-invariant firm-specific effect,  $\alpha_i$ , a time-varying effect,  $\gamma_t$ , and add an idiosyncratic error,  $\varepsilon_{it}$ , i.e.  $u_{it} = \alpha_i + \gamma_t + \varepsilon_{it}$ . The FE model is then given by,

$$\begin{aligned} u_{it} &= \alpha_i + \gamma_t + \varepsilon_{it}, \\ y_{it} &= \alpha_i + \gamma_t + \beta_K k_{it} + \beta_L l_{it} + \varepsilon_{it}, \end{aligned}$$

where  $y_{it} = \ln Y_{it}$ ,  $k_{it} = \ln K_{it}$  and  $l_{it} = \ln L_{it}$ .

## Gameplan:

Write stuff about assumptions, and then transformations, prob go from inconsistent pooled OLS to consistent FE. Right the estimator, list assumptions for identification and consistency (strict/cont exog, non-generacy) - show consistency by LLN convergence in plim. Next, talk about asymptotic normality, CLT, variance estimation (efficiency - robust, cluster). Finally, hypothesis testing (Wald). Then proceed to empirical application (include wald-test code, since it is not given or done in any exercises).