CS 753 | Assignment 2 | Problem 1

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Notation: $B(i,o) = \Pr(o|i)$ is the probability of outputting a primary observation symbol o at state i. $L(i,o,l) = \Pr(l|i,o)$ is the probability of choosing a length $l \geq 0$ for the auxiliary sequence emitted at state i conditioned on emitting primary observation symbol o. $A(i,o,x_1,...,x_l) = \Pr(x_1,..,x_l|i,o,l)$ is the probability of emitting a specific l-long auxiliary sequence $x_1,...,x_l$ on reaching state i, conditioned on the primary observation symbol emitted being o and the auxiliary sequence being of length . a_{ij} denote the probability of transition from state i to j. Also, let there be N states in HMM.

We note that

$$B(i,o).L(i,o,l).A(i,o,x_1,...,x_l). = \Pr(o,l,x_1,...,x_l|i)$$

Define a new Viterbi recursion where we calculate $v_{t,k}(i)$ for each time step t and number of auxilary outputs finished k. We calculate following for $0 \leq k \leq T'$ and $1 \leq t \leq T$.

$$v_{1,k}(i) = a_{0i}.B(1,o_1).L(1,o_1,k).A(1,o_1,x_1,...,x_k)$$

$$v_{t,k}(j) = \max_{1\leq i\leq N} \max_{0\leq l\leq k} v_{t-1,l}(i).a_{ij}.B(j,o_t).L(j,o_t,k-l).A(j,o_t,x_{l+1},...,x_k)$$

Finally we calculate best score as

$$P^* = \max_{1\leq i\leq N} a_{iF}.v_{T,T'}(i)$$

For Viterbi backtrace, we define $b_{t,k}(i)$ in a very similar way as

$$b_{1,k}(i) = 0$$

$$b_{t,k}(j) = \arg\max_{1\leq i\leq N} \max_{0\leq l\leq k} v_{t-1,l}(i).a_{ij}.B(j,o_t).L(j,o_t,k-l).A(j,o_t,x_{l+1},...,x_k)$$

$$b^*_T = \arg\max_{1\leq i\leq N} a_{iF}.v_{T,T'}(i)$$