Practice problems 3: FRI sampling

EE 771: Recent Topics in Analytical Signal Processing, Spring 2018

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Instructions: The homeworks are to be submitted in Homework Box in EE Office on **06/04/18** by 5:00pm. Please do not copy. Copied homework (in full or part) will attract -5 marks for the entire submission. The negative marking is without a warning.

1. Consider a one dimensional periodic signal $x(t)$ with period $T = 1$ given by

$$x(t) = a_1 u(t - t_1) + a_2 u(t - t_2) - (a_1 + a_2)u(t - t_3)$$

   with $0 < t_1 < t_2 < t_3 < 1$.

   (a) How many samples of $x(t) * \text{sinc}(Bt)$ are sufficient for the reconstruction of $x(t)$? What value of B would you choose to ensure perfect reconstruction?

   (b) Repeat the exercise in part (a) if the signal to be sampled is $x_1(t)$ where $x_1(t) = x(t) + b_1\delta(t - t_1) - b_1\delta(t - t_3)$.

2. Why is the Yule-Walker system in Vetterli-Marziliano-Blu's algorithm invertible? Explain carefully. If it helps, you can assume $K = 3$ or $K = 4$ to justify your answer.

3. Let u_1, u_2 be roots of unity. Construct the annihilation filter for the Fourier series coefficients

$$X[m] = \sum_{r=0}^{3} c_r m^r u_1^m + \sum_{r=0}^{1} d_r m^r u_2^m.$$

   Your filter should have the shortest length impulse response in time-domain. Explain your answer.

4. Let $x(t) = \sum_{k=1}^{K} b_k \delta(t - t_k)$, where all the t_k are in $(0, 1)$. Two RC filters with resistance and capacitance values (R_1, C_1) and (R_2, C_2) are used in parallel to filter out $x(t)$. The impulse response of RC filters is given by

$$h_i(t) = e^{-R_i C_i t} u(t), \quad i = 1, 2$$

   where $u(t)$ is the unit-step signal. In terms of $t_1, \ldots, t_K$ and $b_1, \ldots, b_K$ derive conditions on the sampling interval T in terms of R_1, R_2, C_1, C_2 so that sampled signals of $x(t) * h_i(t)|_{t=nT}$ are sufficient to reconstruct the parameters of $x(t)$.