EE 771 : Recent Topics in Analytical Signal Processing
Assignment 2

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Q1

We are given a 1 dimensional periodic signal $x(t)$ with period $T = 1$ as:

$$x(t) = a_1 u(t - t_1) + a_2 u(t - t_2) - (a_1 + a_2) u(t - t_3)$$

with $0 < t_1 < t_2 < t_3$

1a

We want to find the number of samples of $x(t) * sinc(Bt)$ which are sufficient for the reconstrucion of $x(t)$ and a suitable value for B. We note that $x(t)$ is an example of a spline and therefore we can directly use Theorem 2 from the paper

1b

We are now given a new 1d periodic signal $x_1(t)$ with the same period $T = 1$ as:

$$x_1(t) = x(t) + b_1 \delta(t - t_1) - b_2 (t - t_3)$$

Clearly, this is an example of the stream of derivatives of dirac deltas and we can use Theorem 3 from the paper.

Q2

We want to prove that the Yule Walker system in the algorithm mentioned in the paper is invertible.

Denote the Yule Walker system matrix as A. We consider a $3x3$ matrix and note that the proof can be easily extended to any other nxn matrix.

$$A = \begin{bmatrix} X[0] & X[-1] & X[-2] \\ X[1] & X[0] & X[-1] \\ X[2] & X[1] & X[0] \end{bmatrix}$$

Here $X[m] = \frac{1}{\tau} \sum_{k=0}^{K-1} c_k exp(-i2\pi m t_k / \tau)$. Denote $u_k = exp(-i2\pi t_k / \tau)$. We can re-write $X[m] = \frac{1}{\tau} \sum_{k=0}^{K-1} c_k u_k^m$. In this case $K = 3$. Also it is clear that the value of τ wouldn't make a difference in the invertibility of the matrix A. Thus we can write the following:

$$X[0] = c_1 + c_2 + c_3$$

$$X[1] = c_1 u_1 + c_2 u_2 + c_3 u_3$$
$$X[2] = c_1 u_1^2 + c_2 u_2^2 + c_3 u_3^2$$
$$X[-1] = c_1 u_1^{-1} + c_2 u_2^{-1} + c_3 u_3^{-1}$$
$$X[-2] = c_1 u_1^{-2} + c_2 u_2^{-2} + c_3 u_3^{-2}$$

We further note that we can write A as $A = [A_1c|A_2c|A_3c]$. Here:

$$A_3 = \begin{bmatrix} u_1^{-2} & u_2^{-2} & u_3^{-2} \\ u_1^{-1} & u_1^{-1} & u_3^{-1} \\ 1 & 1 & 1 \end{bmatrix}$$

$$A_2 = UA_3$$
$$A_1 = U^2A_3$$

$$U = \begin{bmatrix} u1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{bmatrix}$$

Moreover, A_3 is a permutation of a vander monde matrix. Therefore A_3 is invertible and therefore is non-singular. Now denote the determinant of A by $det(A)$. We have

$$det(A) = det(A_3[U^2c|Uc|c]) = det(A_3)det([U^2c|Uc|c])$$

The first term on the rhs is non-zero since A_3 is non-singular. The second term on the rhs is

$$det([U^2c|Uc|c]) = c_1c_2c_3det(B)$$

$$B = \begin{bmatrix} u1^2 & u1 & 1 \\ u2^2 & u2 & 1 \\ u3^2 & u3 & 1 \end{bmatrix}$$

Clearly B is also a vander monde matrix and therefore, B is also non-singular. Also, c_1, c_2, c_3 are also non-zero (else there will be no diracs at those places and the dimension of the matrix will reduce). Therefore we have:

$$det(A) \neq 0$$

Consequently, we have proved that A is invertible.

Q3

We are given u_1, u_2 as the roots of unity. We want to construct the annhilation filter for the Fourier Series coefficients

$$X[m] = \sum_{r=0}^{3} c_r m^r u_1^m + \sum_{r=0}^{1} d_r m^r u_2^m$$

The annhilation filter $A(z)$ can be constructed in the following way (as noted in the paper).

Q4

We are given

$$x(t) = \sum_{k=1}^{K} b_k \delta(t - t_k)$$

Here all t_k are in $(0, 1)$. Two RC filters with values (R_1, C_1) and (R_2, C_2) are used in parallel to filter out $x(t)$. We are also given the impulse response of the RC filters as:

$$h_i = exp(-R_iC_i)u(t)$$

where $i = 1, 2$ and $u(t)$ is the unit time step signal. We need to find the conditions on sampling interval T so that the sampled signal $x(t) * h_i(t)|_{t=nT}$ are sufficient to reconstruct the parameters of $x(t)$.