EE 771 : Recent Topics in Analytical Signal Processing Assignment 2

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Q1

We are given the bandlimited field $g(x,y)$ as

$$g(x,y) = \sum_{l=-10}^{10}\sum_{k=-5}^{5} a[k,l]exp(j2\pi kx + j2\pi ly)$$

We are also given that we move along the path $y = \sqrt{2}x$ and this path is denoted by L. g is parametrized by time as

$$h(t) = g(t, \sqrt(2)t) \qquad 0 \le t \le 1/\sqrt{2}$$

Let the samples taken along the path L be separated by distances of Δ. Slope of the line is $\sqrt{2}$ and thus its projections of x-axis and y-axis are $\Delta_x = \Delta/\sqrt{3}$ and $\Delta_y = \Delta\sqrt{2}/\sqrt{3}$

1a

We need to find the degrees of freedom along the two axes of the 2d field g. First we consider along the x-axis i.e. y is a constant (here taken to be y_0) and x is variable.

$$g(x,y_0) = \sum_{l=-10}^{10}\sum_{k=-5}^{5} a[k,l]exp(j2\pi kx + j2\pi ly_0)$$

$$g(x,y_0) = \sum_{k=-5}^{5} exp(j2\pi kx)\sum_{l=-10}^{10} a[k,l]exp(j2\pi ly_0)$$

Let $\alpha[k] = \sum_{l=-10}^{10} a[k,l]exp(j2\pi ly_0)$

$$g(x,y_0) = \sum_{k=-5}^{5} exp(j2\pi kx)\alpha[k] \tag{1}$$

Therefore we can reconstruct $g(x,y_0)$ from the values of $\alpha[k]$ and 11 such values are required. Therefore degrees of freedom along x-axis is 11.

For y-axis we have constant x (say x_0) and variable y.

$$\beta[l] = \sum_{k=-5}^{5} a[k,l]exp(j2\pi kx_0)$$

$$g(x_0,y) = \sum_{l=-10}^{10} exp(j2\pi ly)\beta[l] \tag{2}$$

Again, we can reconstruct $g(x_0,y)$ from the values of $\beta[l]$ and 21 such values are required. Therefore degrees of freedom along y-axis is 21.

1b

We note that the representations in 1 and 2 readily suggest the bandlimitness of the 1d representation. For the case of $g(x, 0.5)$ we note that 1 directly implies that the maximum frequency requried would be when $k = +5$ or $k = -5$ both of which correspond to 5Hz. Also we note that this value is independent of the value of y_0. Similarly for the case of $g(0.25, y)$ we have maximum frequency of 10Hz. Therefore both 1d representations are bandlimited.

1c

We first note that $g(x, y)$ is bandlimited to $[-\rho_x, \rho_x]x[-\rho_y, \rho_y]$ where $\rho_x = 10\pi$ and $\rho_y = 20\pi$. For $h(t)$ to cover all degrees of freedom of $g(x, y)$ we need that there should be no aliasing in either dimension. That is, we need $\Delta_x \leq \pi/\rho_x$ and $\Delta_y \leq \pi/\rho_y$. That is:

$$\Delta/\sqrt{3} \leq 1/10$$

$$\Delta\sqrt{2}/\sqrt{3} \leq 1/20$$

The above reduces to $\Delta \leq \sqrt{3}/10$ and $\Delta \leq \sqrt{3}/(20\sqrt{2})$. Clearly the second inequality is stronger and therefore we need $\Delta \leq \sqrt{3}/(20 * \sqrt{2})$ so that $h(t)$ can capture all degrees of freedom of $g(x, y)$.

1d

As shown in above part, if $\Delta \leq \sqrt{3}/(20/\sqrt{2})$ then we can reconstruct $g(x, y)$ from $h(t)$.

Q2

We are given that $W(x)$ is a white noise process. We are also given that $S_W(\omega) \propto \frac{1}{\omega}$ for $|\omega| \geq \frac{\pi}{X}$ where X is the sampling distance.

From power spectral density theory we know that:

$$S_{W_s} = \frac{1}{X} \sum_{k=-\infty}^{\infty} S_X(\omega - \frac{2\pi k}{T}) \qquad |\omega| \leq \tfrac{\pi}{T}$$

Q3

Q4