EE 771 : Recent Topics in Analytical Signal Processing
Paper Review 2

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Q1

The paper is different from the previous works in the following ways:

- Other works in this topic have focused on the additive noise and its effect on signal reconstruction whereas reconstruction error for stream of diracs with respect to quantization noise has been less studied.

- This work analyzes maximum possible reconstruction error due to quantization.

Q2

In general it is not possible $\max_k |t_k - \hat{t_k}|$ to decrease faster than $O(2^{-L})$. From eq(44) we have a lower bound proportional to $||\hat{V} - V||$. Since ∞ norm is less than 2-norm. The lower bound in proportional to $||\hat{V} - V||_\infty$.

We further note:

$$||\hat{V} - V||_\infty = \sum_{k=1}^{K} |sin(l\pi(t_k - \hat{t_k})|$$

We know $sin(2x) \geq x$ when $x \in [0, \pi/4]$. Clearly if $l \geq 2$ then the inequality is always followed.

$$||\hat{V} - V||_\infty \geq \frac{1}{2} \sum_{k=1}^{K} |(l\pi(t_k - \hat{t_k})| \geq \frac{1}{2} \max_k l\pi |t_k - \hat{t_k}|$$

Therefore the lower bound is also of the order $O(2^{-L})$. Hence $\max_k |t_k - \hat{t_k}|$ cannot decrease faster than $O(2^{-L})$ unless $\hat{V}$ and V are extremely close to each other which occurs when the approximations t_k and $\hat{t_k}$ are very close. So in general cases, the lower bound is also of the order $O(2^{-L})$.

Q3

Part C is the most difficult to analyze because we are trying to give bounds on the inverse of $\hat{\chi}$.
To solve this, the authors do the following:

- First get the singular value decomposition of $\chi = VDV^T$. Here V is a vandermonde system with $V_{ij} = exp(-j2\pi(i-1)t_j)$. And $D = diag(c_1, c_2, \ldots, c_K)$.

- Then they use the fact $\sigma_K(\chi) \geq [\sigma_K(V)]^2 \min_k |c_k|$.

- Next, they find the ∞ norm of V^{-1} and use $||V^{-1}|| = [\sigma_K(V)]^{-1} \leq \sqrt{K}||V^{-1}||_\infty$. Combining this with the previous step gives them a bound on $\sigma_K(\chi)$ as

$$\sigma_K(\chi) \geq [(\prod_{v=1}^{\lfloor \frac{K}{2} \rfloor} 4sin^2(\pi v \delta_1)) \frac{1}{2^{K-1}\sqrt{K}}]^2 c_{min}$$

- The authors further use the theorem that if A, B are two nxn invertible matrices with $\sigma_1(B) < \sigma_n(A)$ then $A + B$ is always invertible. The authors had previously assumed $||\hat{\chi} - \chi|| \leq \sigma_K(\chi)/2$ and therefore this confirms the invertibility of $\hat{\chi}$.

Q4

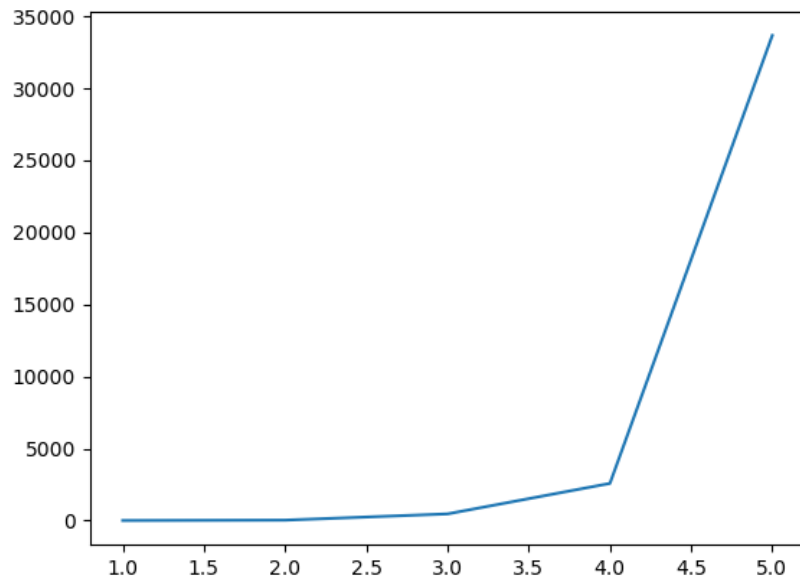Code is attached in the zip file. A plot of stability constant ϵ vs K is shown here.

Figure 1: Plot of ϵ vs K