EE 302 : Control System Home Assignment

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Answer a: The freebody diagrams are as follows:

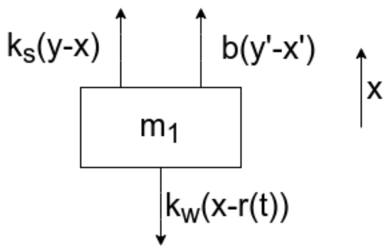

Figure 1: Free body diagram of m1

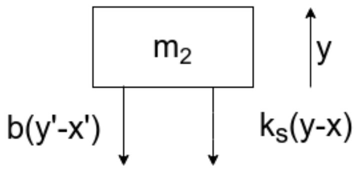

Figure 2: Free body diagram of m2

$$m_1\ddot{x} = k_s(y-x) + b(\dot{y}-\dot{x}) + k_w(r(t)-x) \tag{1}$$

$$m_2\ddot{y} = k_s(x-y) + b(\dot{x}-\dot{y}) \tag{2}$$

Converting (1) to the Laplace Domain and taking initial conditions as 0. We get

$$m_1 s^2 X(s) = k_s[Y(s)-X(s)] + bs[Y(s)-X(s)] + k_w[R(s)-X(s)] \tag{3}$$

Rearranging the terms we get

$$[m_1 s^2 + bs + (k_s+k_w)]X(s) = [bs+k_s]Y(s) + k_w R(s) \tag{4}$$

Converting (2) to the Laplace Domain and taking initial conditions as 0. We get

$$m_2 s^2 Y(s) = k_s[X(s)-Y(s)] + bs[X(s)-Y(s)] \tag{5}$$

Rearranging the terms we get

$$[m_2 s^2 + k_s + bs]Y(s) = [k_s + bs]X(s) \tag{6}$$

Put X(s) from (6) into (4) we get

$$\frac{[m_1 s^2 + bs + (k_s+k_w)]}{[k_s+bs]}[m_2 s^2 + k_s + bs - (k_s+bs)^2]Y(s) = k_w(k_s+bs)R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{k_w(k_s + bs)}{[m_1 s^2 + bs + (k_s + k_w)][m_2 s^2 + k_s + bs] - (k_s + bs)^2]}$$

Answer b:

$$\dot{p} = Ap + Br$$

$$y = Cp + Dr$$

Here

$$p = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

Making use of (1) and (2) to get the second and fourth row of A. We get

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s + k_w}{m_1} & -\frac{b}{m_1} & \frac{k_s}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_2} & \frac{b}{m_2} & -\frac{k_s}{m_2} & -\frac{b}{m_2} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ \frac{k_w}{m_1} \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

Answer c: From state space to Transfer function we use

$$G(s) = C(sI - A)^{-1}B$$

We get

$$G(s) = \frac{k_w\ (k_s + b\,s)}{m_1\,m_2\,s^4 + (b\,m_1 + b\,m_2)\ s^3 + (k_s\,m_1 + k_s\,m_2 + k_w\,m_2)\ s^2 + b\,k_w\,s + k_s\,k_w}$$

which is the same as that in part(a).

Answer d: The controller canonical form of a state space model is given by

$$A_{ccf} = \begin{bmatrix} -a_3 & -a_2 & -a_1 & -a_0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

where the characteristic equation is

$$s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Controller Canonical Form of the matrix is

$$A_{ccf} = \begin{pmatrix} -\frac{b\,m_1 + b\,m_2}{m_1\,m_2} & -\frac{k_s\,m_1 + k_s\,m_2 + k_w\,m_2}{m_1\,m_2} & -\frac{b\,k_w}{m_1\,m_2} & -\frac{k_s\,k_w}{m_1\,m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Answer e: From 1 we have

$$m_1\ddot{x} = k_s(y - x) + b(\dot{y} - \dot{x}) + k_w(r(t) - x) - f_{act}$$

Rearranging the terms we get

$$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}x = \frac{k_w}{m_1}r - \frac{f_{act}}{m_1} \tag{7}$$

From 2 we have

$$m_2\ddot{y} = k_s(x-y) + b(\dot{x}-\dot{y})$$

Rearranging the terms we get

$$\ddot{y} + \frac{b}{m_2}(\dot{y}-\dot{x}) + \frac{k_s}{m_2}(y-x) = \frac{f_{act}}{m_2} \tag{8}$$

Answer f:

$$\dot{p} = Ap + B_1 r + B_2 f_{act}$$
$$y = Cp + D_1 r + D_2 f_{act}$$

A will remain the same as in (c) and B_1 will be same as B. Using 7 and 8 we get

$$B_2 = \begin{bmatrix} 0 \\ \frac{-1}{m_1} \\ 0 \\ \frac{1}{m_2} \end{bmatrix}$$

Both D_1 and D_2 turn out to be 0.

Answer g: The solution with $f_{act} = 0$ is

$$p(t) = e^{At}p(0) + \int_0^t e^{A(t-\tau)}B_1 r(\tau)d\tau$$

Impulse response with initial condition $p(0) = 0$ is

$$p(t) = \int_0^t e^{A(t-\tau)}B_1 r(\tau)d\tau = \int_0^t e^{A(t-\tau)}B_1 \delta(\tau)d\tau$$

$$p(t) = e^{At}B_1$$

The response of the system with $r = 0$ and initial condition B_1 is

$$p(t) = e^{At}p(0) = e^{At}B_1$$

Since D_1 and D_2 are zero, we get

$$y = Cp$$

To prove that the impulse responses are the same, it suffices to show that the $p(t)$ is the same for both the cases.
Hence we have proved that the impulse response of the system with $f_{act} = 0$ and initial condition $p(0) = 0$ is the same as response of the system with $f_{act} = 0$ and $r = 0$ and initial condition B_1

Answer h: To get peak overshoot $M = 2.5\%$ and settling time $t_s = 0.39sec$, using second order approximation, we get the poles to be placed as

$$p_1 = -11.7949 - j10.0450$$

$$p_2 = -11.7949 - j10.0450$$

The other two poles to be placed are at

$$p_3 = -60$$

$$p_4 = -100$$

The required transfer function would have the characteristic polynomial as

$$G_{required}(s) = s^4 + 183.5898s^3 + 10014.389691s^2 + 179942.2705616s^1 + 1440130.14606$$

From this we get the required A matrix in canonical form as

$$A_{reqd\_canonical} = \begin{pmatrix} -183.5898 & -10014.38969101 & -179942.270561600 & -1440130.14606 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The current A matrix represented in canonical form is

$$A_{ccf} = \begin{pmatrix} -\frac{b\,m_1+b\,m_2}{m_1\,m_2} & -\frac{k_s\,m_1+k_s\,m_2+k_w\,m_2}{m_1\,m_2} & -\frac{b\,k_w}{m_1\,m_2} & -\frac{k_s\,k_w}{m_1\,m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

On replacing the corresponding values $k_w = 1000000$, $k_s = 130000$, $b = 9800$, $m_1 = 80$, $m_2 = 1500$,

$$A_{ccf} = \begin{pmatrix} -129.0333 & -14211.667 & -81666.6667 & -1083333.3333 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

So we find $K_{reqd\_canonical}$ as

$$K_{reqd\_canonical} = A_{ccf} - A_{reqd\_canonical} = \begin{pmatrix} 54.5564667 & -4197.27697 & 98275.6038 & 356796.81272 \end{pmatrix}$$

Now we convert this K to our current form, by multiplying T^{-1} where T is the transformation matrix required to transform the state space to control canonical form.

$$K = K_{reqd\_canonical}T^{-1} = \begin{pmatrix} 338\,065.6577 & -3735.5535 & 42\,815.6175 & 11\,793.0725 \end{pmatrix}$$

The new A is given as

$$A_{state\_feedback} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -9899.1793 & -169.1944 & 2160.1952 & 269.9134 \\ 0 & 0 & 0 & 1 \\ -138.7104 & 9.0237 & -115.2104 & -14.3954 \end{pmatrix}$$

Answer i: The observer canonical form is given by

Answer l: The performance index to be minimized is

$$J = \int_0^\infty 2y^2(t) + f_{act}^2(t)dt$$

This can be solved using the theory of Symmetric Root Locus, which provides solution of minimization the following performance index.

$$J = \int_0^\infty \rho y^2(t) + f_{act}^2(t)dt$$

where $f_{act} = -Kp$.
The optimal value of K is that which places the closed-loop poles at the stable roots (those in the LHP) of the symmetric root-locus (SRL) equation

$$1 + \rho G_0(-s)G_0(s) = 0$$

where

$$G_0(s) = \frac{Y(s)}{f_{act}(s)} = C(sI - A)^{-1}B_2$$