# EE 302 : Control System Home Assignment

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**Question 1:** What is the transfer function $\frac{Y(s)}{R(s)}$ of the system in Figure 1?

**Answer a:**

$$m_1\ddot{x} = k_s(y-x) + b(\dot{y}-\dot{x}) + k_w(r(t)-x) \tag{1}$$

$$m_2\ddot{y} = k_s(x-y) + b(\dot{x}-\dot{y}) \tag{2}$$

The transfer function

$$m_1 s^2 X(s) = k_s[Y(s) - X(s]) + bs[Y(s) - X(s)] + k_w[R(s) - X(s)]$$

$$m_2 s^2 Y(s) = k_s[X(s) - Y(s)] + bs[X(s) - Y(s)]$$

$$[m_1 s^2 + bs + (k_s + k_w)]X(s) = [bs + k_s]Y(s) + k_w R(s)$$

$$[m_2 s^2 + k_s + bs]Y(s) = [k_s + bs]X(s)$$

$$[m_1 s^2 + bs + (k_s + k_w)]X(s) = [bs + k_s]Y(s) + k_w R(s)$$

$$\frac{[m_1 s^2 + bs + (k_s + k_w)]}{[k_s + bs]}[m_2 s^2 + k_s + bs - (k_s + bs)^2]Y(s) = k_w(k_s + bs)R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{k_w(k_s + bs)}{[m_1 s^2 + bs + (k_s + k_w)][m_2 s^2 + k_s + bs] - (k_s + bs)^2}$$

**Answer b:**

$$\dot{p} = Ap + Br$$

$$y = Cp + Dr$$

Here

$$p = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s+k_w}{m_1} & -\frac{b}{m_1} & \frac{k_s}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_2} & \frac{b}{m_2} & -\frac{k_s}{m_2} & -\frac{b}{m_2} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ \frac{k_w}{m_1} \\ 0 \\ 0 \end{pmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

**Answer c:** From state space to Transfer function we use

$$G(s) = C(sI - A)^{-1}B$$

We get

$$G(s) = \frac{k_w\ (k_s + b\,s)}{k_s\,k_w + b\,m_1\,s^3 + b\,m_2\,s^3 + k_s\,m_1\,s^2 + k_s\,m_2\,s^2 + k_w\,m_2\,s^2 + m_1\,m_2\,s^4 + b\,k_w\,s}$$

which is the same as that in part(a).

**Answer d:** Controller Canonical Form of the matrix is

$$A_{ccf} = \begin{pmatrix} -\frac{b\,m_1+b\,m_2}{m_1\,m_2} & -\frac{k_s\,m_1+k_s\,m_2+k_w\,m_2}{m_1\,m_2} & -\frac{b\,k_w}{m_1\,m_2} & -\frac{k_s\,k_w}{m_1\,m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

**Answer e:**

$$\ddot{x} + \frac{b}{m_1}(\dot{x} - \dot{y}) + \frac{k_s}{m_1}(x - y) + \frac{k_w}{m_1}x = \frac{k_w}{m_1}r - \frac{f_{act}}{m_1} \tag{3}$$

$$\ddot{y} + \frac{b}{m_2}(\dot{y} - \dot{x}) + \frac{k_s}{m_2}(y - x) = \frac{f_{act}}{m_2} \tag{4}$$

**Answer f:**

$$\dot{p} = Ap + B_1 r + B_2 f_{act}$$
$$y = Cp + D_1 r + D_2 r$$

A will remain the same as in (c) and $B_1$ will be same as $B$.

$$B_2 = \begin{bmatrix} 0 \\ \frac{-1}{m_1} \\ 0 \\ \frac{1}{m_2} \end{bmatrix}$$

Both $D_1$ and $D_2$ will remain the same.

**Answer g:** The solution with $f_{act} = 0$ is

$$p(t) = e^{At}p(0) + \int_0^t e^{A(t-\tau)}B_1 r(\tau)d\tau$$

Impulse response with initial condition $p(0) = 0$ is

$$p(t) = \int_0^t e^{A(t-\tau)}B_1 r(\tau)d\tau = \int_0^t e^{A(t-\tau)}B_1 \delta(\tau)d\tau$$

$$p(t) = e^{At}B$$

The response of the system with $r = 0$ and initial condition $B_1$ is

$$p(t) = e^{At}p(0) = e^{At}B_1$$