EE 302: Home assignment on designing an electromagnetic car suspension system.

This home assignment is about designing an electromagnetic car suspension system using state-feedback controllers. This, in principle, is same as the Bose suspension system, whose video was shown in the class.

Question 1 Suppose you have a car with a suspension system modelled as a two-mass system (Figure 1). Assume that the car has a mass of 1580 kg, including the four wheels, which have a mass of 20 kg each. By placing a known weight directly over a wheel and measuring the car's deflection, you found the spring constant k_s of the car suspension to be 130,000 N/m. Measuring the wheel's deflection for the same applied weight, the tire compressibility k_w is found to be 1,000,000 N/m. The shock absorber b has a friction constant 9800 N-sec/m.

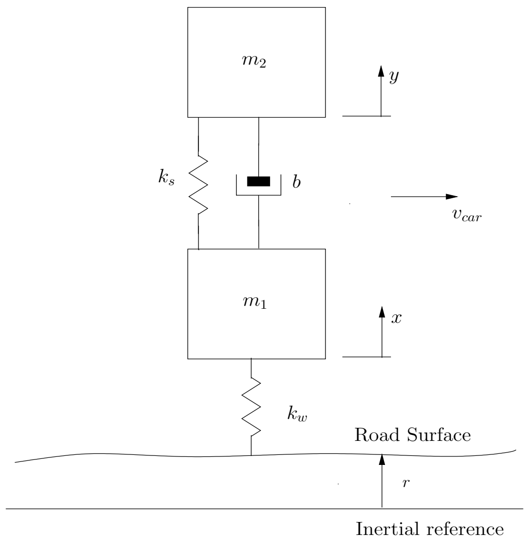

Figure 1: Two mass-model of car suspension system.

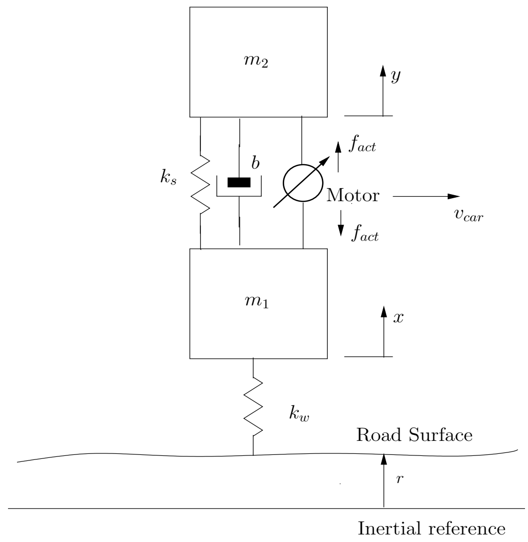

Figure 2: Two mass-model of car suspension system with a motor.

(a) What is the transfer function $\frac{Y(s)}{R(s)}$ of the system in Figure 1?

(b) Let $p := \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$ be the state vector of the system. Find the state space representation of the system with p as state-vector and r, y as input and output, respectively.

(c) Show that the state-space representation in (b) results in the transfer function obtained in (a).

(d) Find the state space representation of the system in controller canonical form.

A motor is connected between the body frame of the car and the wheels. This acts as a shock absorber for the car suspension system. Let the actuator force be f_{act}.

(e) Write down the equations of motion for the system in Figure 2.

(f) Find the state space representation of the system shown in Figure 2 in the form

$$\frac{d}{dt}p = Ap + B_1 r + B_2 f_{act} \quad \text{and} \quad y = Cp + D_1 r + D_2 f_{act}.$$

(g) Show that the impulse response of the system in Figure 2 with $f_{act} = 0$ and initial condition $p(0) = 0$ is the same as the response of the system with $f_{act} = 0$ and $r = 0$ and initial condition B_2.

(h) Design a state-feedback controller $f_{act} = -Kp$ such that the second order approximation of the system achieves a

- peak overshoot of 2.5%,
- settling time of $0.39\,sec$,

in the position of the car's body from equilibrium when the car encounters a step input in the road surface. Place the other two poles of the system at -60 and -100.

(i) Obtain the observer canonical form of the system shown in Figure 1.

(j) Design a full-state observer to estimate the state variables of the system such that the poles of the observer are placed at $\{-60, -100, -300, -400\}$. Assume r, y, f_{act} are inputs to the observer.

(k) Consider that the states estimated by the observer designed in (j) are used as the input to the controller designed in (h).

  (i) What will be the closed loop state-space representation of the entire system.

  (ii) Draw the block diagram of the entire closed-loop system.

  (iii) Plot the step response of the system (with r as input and y as output).

(l) For $r = 0$, design a controller of the form $f_{act} = -Fp$ such that it minimizes the performance index

$$J = \int_0^\infty 2y^2(t) + f_{act}^2(t).$$

This controller is known as the *linear quadratic regulator* (LQR).

(m) Plot the step and impulse responses (like before, take r as input and y as output) of the closed-loop system with the controller being the one designed in (l).