Filter Design Assignment

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Filter Number 23

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Contents

1 First Filter Specifications

1.1 Un-normalized Discrete Time Filter Specifications

- Sampling Frequency (f_s) = 140kHz
- $B_L = 9.4kHz$
- $B_H = 14.4kHz$
- Monotonic Passband (Butterworth)

1.2 Normalized Digital Filter Specifications

- $\omega_{p1} = \frac{9.4}{140}2\pi = 0.4219$
- $\omega_{p2} = \frac{14.4}{140}2\pi = 0.6463$
- $\omega_{s1} = \frac{8.4}{140}2\pi = 0.3770$
- $\omega_{s2} = \frac{15.4}{140}2\pi = 0.6912$

1.3 Analog Filter Specifications

- $\Omega_{p1} = tan(\frac{\omega_{p1}}{2}) = 0.2141$
- $\Omega_{p2} = tan(\frac{\omega_{p2}}{2}) = 0.3349$
- $\Omega_{s1} = tan(\frac{\omega_{s1}}{2}) = 0.1908$
- $\Omega_{s2} = tan(\frac{\omega_{s2}}{2}) = 0.3600$

1.4 Frequency transformation to be employed

$$s_L = \frac{s^2 + \Omega_0^2}{Bs} \tag{1}$$

with $B = \Omega_{p2} - \Omega_{p1} = 0.1208$ and $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.2678$
This translates to

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{2}$$

1.5 Frequency transformed lowpass analog filter specifications

From 2

- $\Omega_{Lp} = 1$
- $\Omega_{Ls} = min(\Omega_{Ls1}, \Omega_{Ls2}) = 1.3321$

1.6 The analog Low pass Transfer Function

```
H_s_L =

                                                     1
  -------------------------------------------------------------------------------------------------------
  0.4533 s^11 + 3.423 s^10 + 12.92 s^9 + 32.07 s^8 + 57.98 s^7 + 80.04 s^6 + 86.01 s^5 + 71.95 s^4 + 45.95 s^3 + 21.38 s^2 + 6.539 s + 1

Continuous-time transfer function.
```

Figure 1: Analog Low Pass Transfer Function

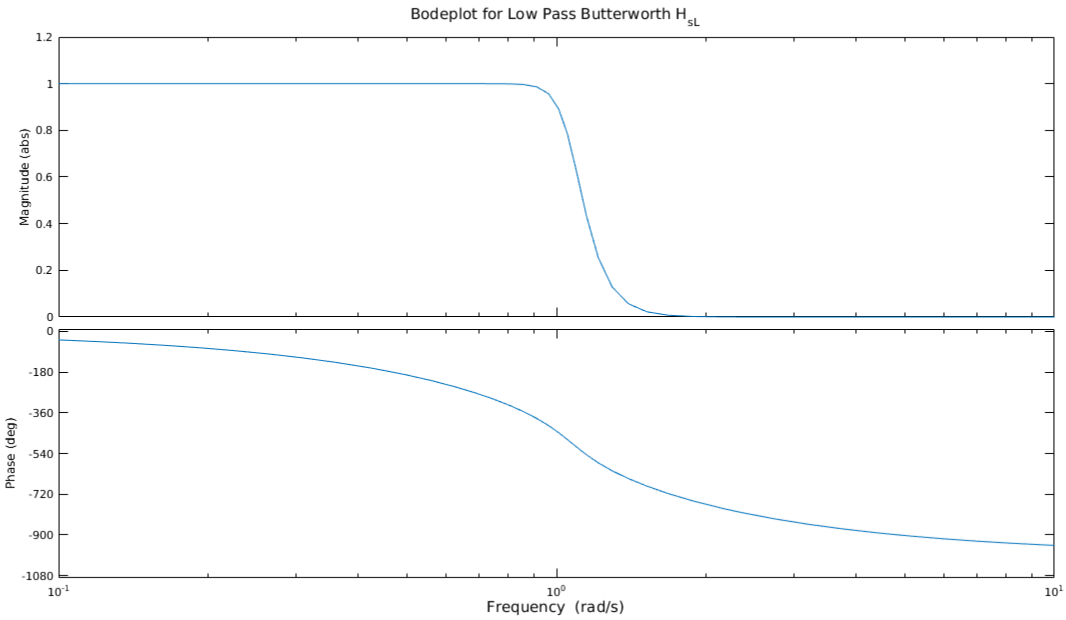

Figure 2: Analog Low Pass Bode plot

1.7 The Analog BandPass Transfer Function

```
H_s_bp =

                                              7.959e-11 s^11

  ----------------------------------------------------------------------------------------------------------

  0.4533 s^22 + 0.4133 s^21 + 0.5459 s^20 + 0.3528 s^19 + 0.2621 s^18 + 0.1301 s^17 + 0.0689 s^16 + 0.02732 s^15 + 0.01122 s^14

       + 0.003626 s^13 + 0.001198 s^12 + 0.0003179 s^11 + 8.587e-05 s^10 + 1.864e-05 s^9 + 4.135e-06 s^8 + 7.222e-07 s^7 + 1.306e-07 s^6

                                 + 1.768e-08 s^5 + 2.554e-09 s^4 + 2.465e-10 s^3 + 2.735e-11 s^2 + 1.485e-12 s + 1.168e-13

Continuous-time transfer function.
```

Figure 3: Analog Band Pass Transfer Function

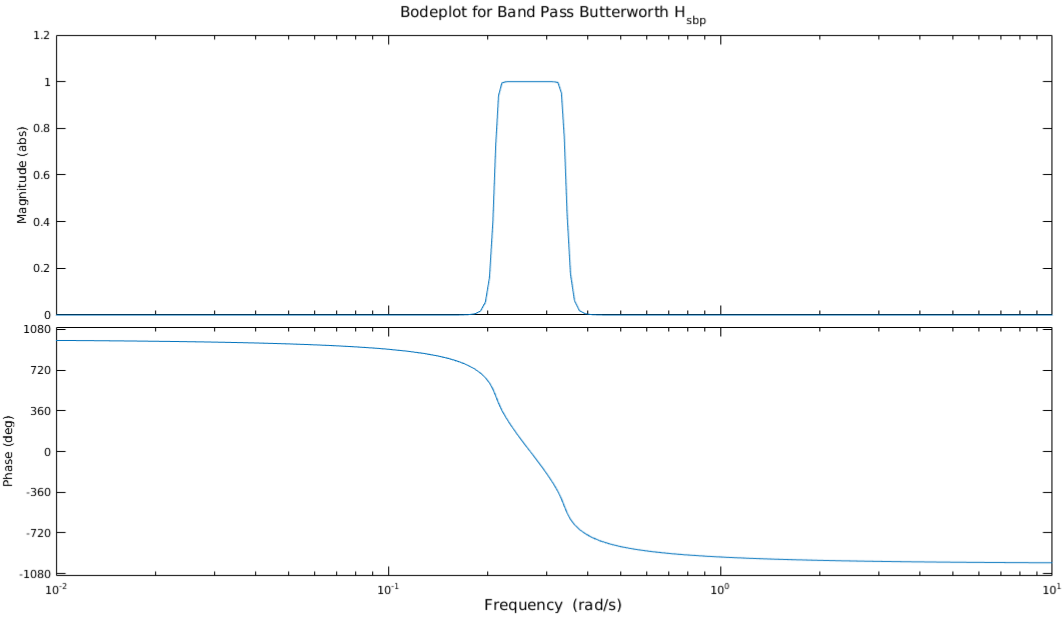

Figure 4: Analog Band Pass Bode Plot

1.8 The Discrete Time Filter Transfer Function

```
H_z_bp_actual =


  3.506e-11 + 2.132e-14 z^-1 - 3.86e-10 z^-2 + 2.501e-12 z^-3 + 1.916e-09 z^-4 + 4.729e-11 z^-5 - 5.919e-09 z^-6 + 2.983e-10 z^

         -7 + 1.099e-08 z^-8 + 9.459e-10 z^-9 - 1.746e-08 z^-10 + 1.368e-09 z^-11 + 1.487e-08 z^-12 + 1.062e-09 z^-13 - 1.23e-08 z^-14

         + 4.184e-10 z^-15 + 5.588e-09 z^-16 + 7.458e-11 z^-17 - 1.951e-09 z^-18 + 5.372e-12 z^-19 + 3.847e-10 z^-20 + 9.726e-14 z^-21

                                                                                                                  - 3.506e-11 z^-22

  ---------------------------------------------------------------------------------------------------------------------------------

  1 - 17.59 z^-1 + 150 z^-2 - 824.9 z^-3 + 3279 z^-4 - 1.002e04 z^-5 + 2.444e04 z^-6 - 4.874e04 z^-7 + 8.081e04 z^-8 - 1.126e05 z^-9

         + 1.329e05 z^-10 - 1.333e05 z^-11 + 1.139e05 z^-12 - 8.269e04 z^-13 + 5.083e04 z^-14 - 2.627e04 z^-15 + 1.129e04 z^-16

                                                     - 3966 z^-17 + 1112 z^-18 - 239.7 z^-19 + 37.36 z^-20 - 3.754 z^-21 + 0.1829 z^-22


Sample time: 7.1429e-06 seconds
Discrete-time transfer function.
```

Figure 5: Discrete Time Band Pass Transfer Function

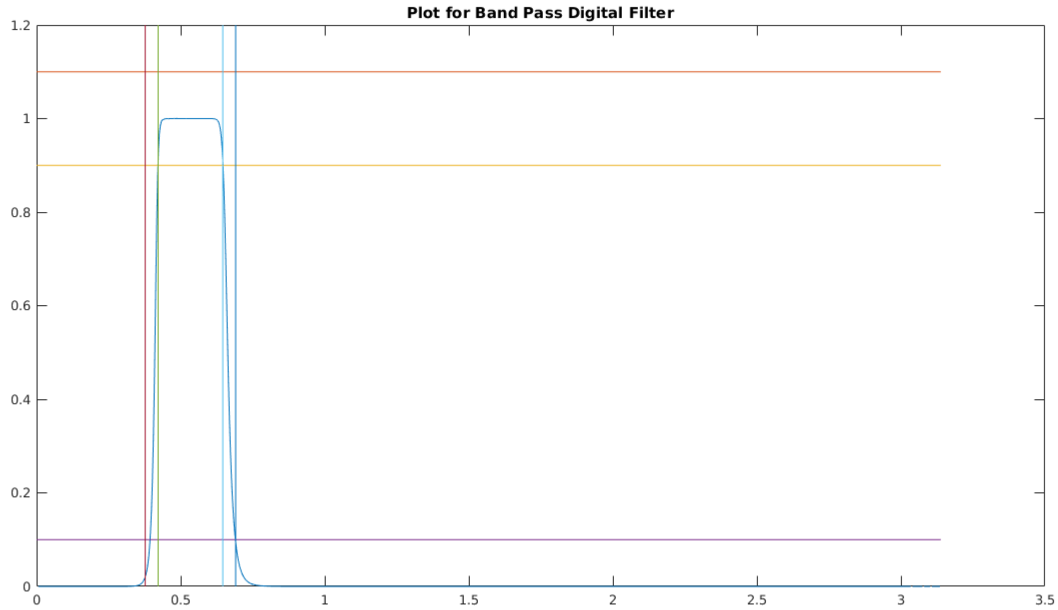

Figure 6: Frequency Magnitude Response of Discrete Time Band Pass Filter

1.9 Direct Form 2 Realization of the Discrete Filter

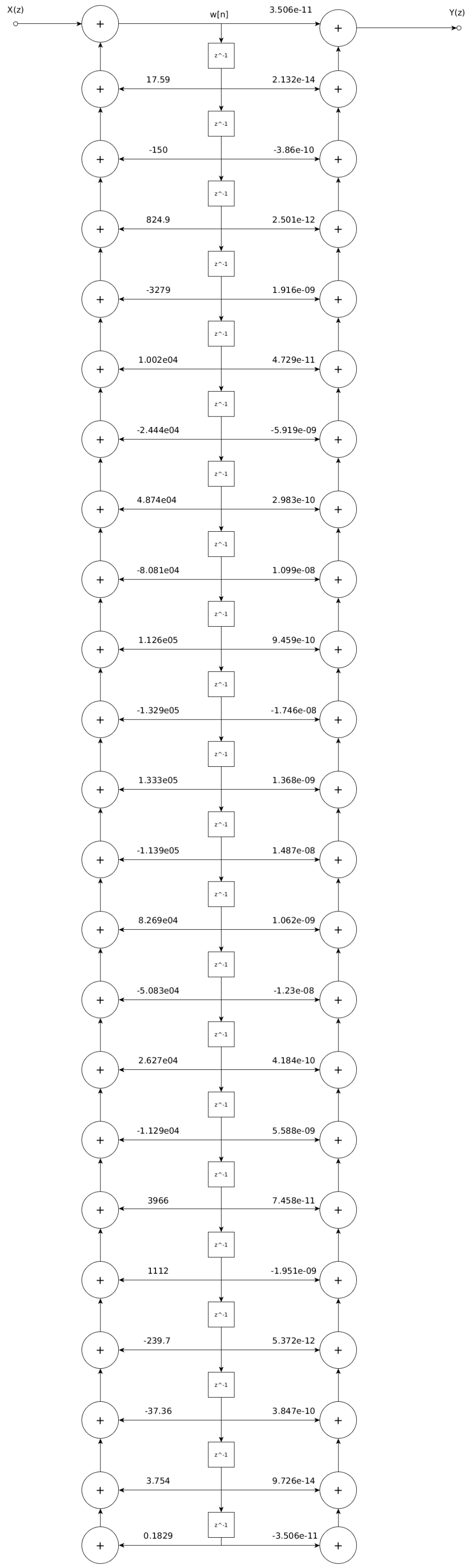

Figure 7: Direct Form 2 Signal Flow Graph for Band Pass IIR filter (Zoom In To see the Diagram)

1.10 FIR Filter Transfer Function to get the same specifications

Given that $\delta = 0.1$, we get

$$A = -20log_{10}(\delta) = -20log_{10}(0.1) = 20$$

For $A < 21$ we get $\alpha = 0$ and hence $\beta = 0$. Also we have $\Delta\omega = w_s - w_p$ We then compute

$$N \geq \frac{A - 8}{2 * 2.285 * \Delta\omega} = 58.507$$

Therefore we choose

$$N = 59$$

.

But with $N = 59$ the resulting FIR doesn't meet all the specifications. Here is the plot for the same

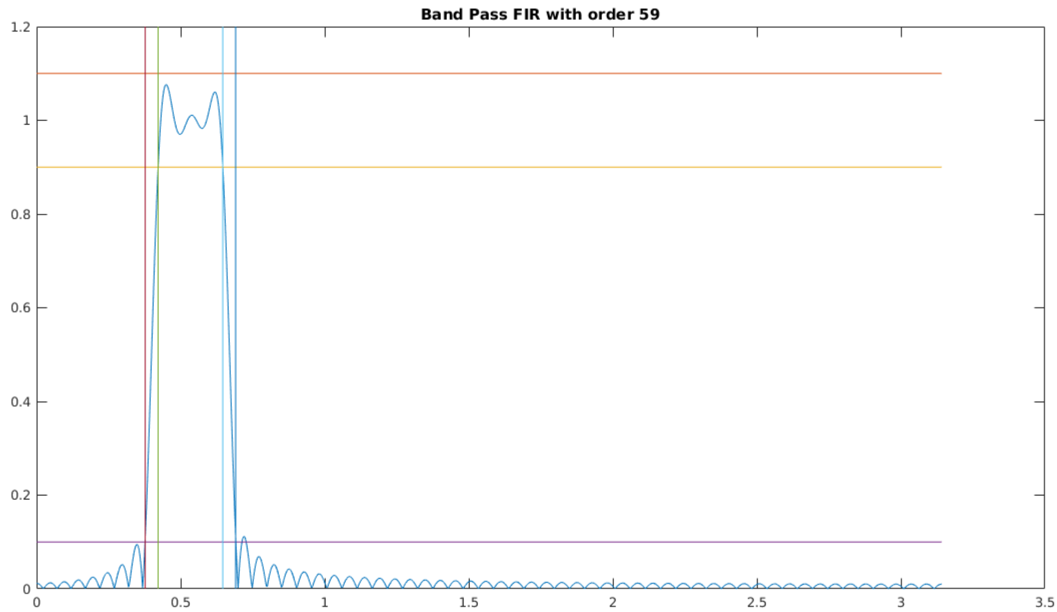

Figure 8: Order 59 FIR Band Pass

On increasing N to $N = 63$, we get an FIR that meets the specifications.

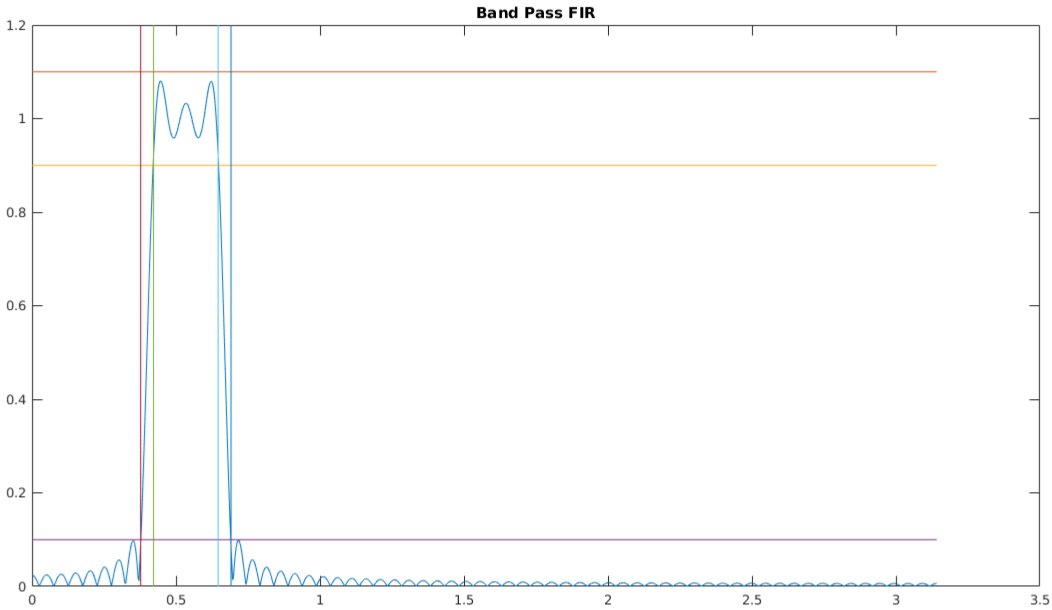

Figure 9: Order 63 FIR Band Pass

The complete Frequency Response is

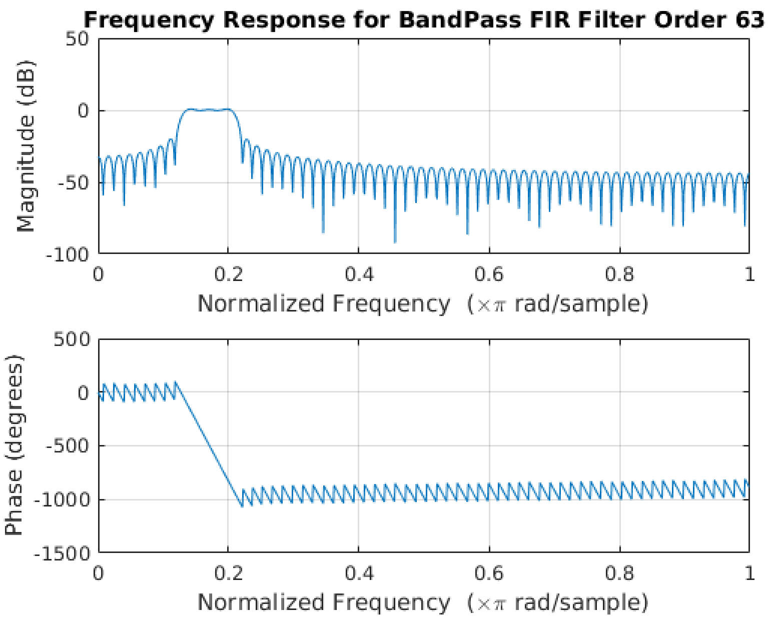

Figure 10: Frequency Response

$H_{fir\_bp} = 0.010699z^{-59} + 0.0099216z^{-58} + 0.0061769z^{-57} + 0.00067888z^{-56} + -0.0047345z^{-55} + -0.0083073z^{-54} + -0.0090412z^{-53} + -0.0070586z^{-52} + -0.0035005z^{-51} + 1.5549e - 17z^{-50} + 0.0020437z^{-49} + 0.0020753z^{-48} + 0.0006074z^{-47} + -0.0010474z^{-46} + -0.0014292z^{-45} + 0.00031922z^{-44} + 0.0039434z^{-43} + 0.0080615z^{-42} + 0.010683z^{-41} + 0.010067z^{-40} + 0.0055652z^{-39} + -0.0019308z^{-38} + -0.010166z^{-37} + -0.016293z^{-36} + -0.017965z^{-35} + -0.014297z^{-34} + -0.0062993z^{-33} + 0.0034278z^{-32} + 0.011658z^{-31} + 0.015779z^{-30} + 0.014805z^{-29} + 0.009742z^{-28} + 0.0031172z^{-27} + -0.0021396z^{-26} + -0.0040068z^{-25} + -0.0023031z^{-24} + 0.0011925z^{-23} + 0.003537z^{-22} + 0.0020435z^{-21} + -0.0042678z^{-20} + -0.013846z^{-19} + -0.022858z^{-18} + -0.026534z^{-17} + -0.021167z^{-16} + -0.0059818z^{-15} + 0.01592z^{-14} +$

$0.038073z^{-13} + 0.05258z^{-12} + 0.052901z^{-11} + 0.036481z^{-10} + 0.0062323z^{-9} + -0.029837z^{-8} + -0.060854z^{-7} + -0.07654z^{-6} + -0.070733z^{-5} + -0.04374z^{-4} + -0.0026197z^{-3} + 0.040796z^{-2} + 0.073555z^{-1} + 0.085714z^{0} + 0.073555z^{1} + 0.040796z^{2} + -0.0026197z^{3} + -0.04374z^{4} + -0.070733z^{5} + -0.07654z^{6} + -0.060854z^{7} + -0.029837z^{8} + 0.0062323z^{9} + 0.036481z^{10} + 0.052901z^{11} + 0.05258z^{12} + 0.038073z^{13} + 0.01592z^{14} + -0.0059818z^{15} + -0.021167z^{16} + -0.026534z^{17} + -0.022858z^{18} + -0.013846z^{19} + -0.0042678z^{20} + 0.0020435z^{21} + 0.003537z^{22} + 0.0011925z^{23} + -0.0023031z^{24} + -0.0040068z^{25} + -0.0021396z^{26} + 0.0031172z^{27} + 0.009742z^{28} + 0.014805z^{29} + 0.015779z^{30} + 0.011658z^{31} + 0.0034278z^{32} + -0.0062993z^{33} + -0.014297z^{34} + -0.017965z^{35} + -0.016293z^{36} + -0.010166z^{37} + -0.0019308z^{38} + 0.0055652z^{39} + 0.010067z^{40} + 0.010683z^{41} + 0.0080615z^{42} + 0.0039434z^{43} + 0.00031922z^{44} + -0.0014292z^{45} + -0.0010474z^{46} + 0.0006074z^{47} + 0.0020753z^{48} + 0.0020437z^{49} + 1.5549e-17z^{50} + -0.0035005z^{51} + -0.0070586z^{52} + -0.0090412z^{53} + -0.0083073z^{54} + -0.0047345z^{55} + 0.00067888z^{56} + 0.0061769z^{57} + 0.0099216z^{58} + 0.010699z^{59}$

2 Second Filter Specifications

2.1 Un-normalized Discrete Time Filter Specifications

- Sampling Frequency (f_s) = 90kHz
- $B_L = 7.7kHz$
- $B_H = 10.7kHz$
- Equiripple Passband (Chebyshev)

2.2 Normalized Digital Filter Specifications

- $\omega_{p1} = \frac{6.7}{90}2\pi = 0.4677$
- $\omega_{p2} = \frac{11.7}{90}2\pi = 0.8168$
- $\omega_{s1} = \frac{7.7}{90}2\pi = 0.5376$
- $\omega_{s2} = \frac{10.7}{90}2\pi = 0.7470$

2.3 Analog Filter Specifications

- $\Omega_{p1} = tan(\frac{\omega_{p1}}{2}) = 0.2382$
- $\Omega_{p2} = tan(\frac{\omega_{p2}}{2}) = 0.4327$
- $\Omega_{s1} = tan(\frac{\omega_{s1}}{2}) = 0.2754$
- $\Omega_{s2} = tan(\frac{\omega_{s2}}{2}) = 0.3919$

2.4 Frequency transformation to be employed

$$s_L = \frac{Bs}{s^2 + \Omega_0^2} \tag{3}$$

with $B = \Omega_{p2} - \Omega_{p1} = 0.1945$ and $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.3211$
This translates to

$$\Omega_L = \frac{B\Omega}{\Omega^2 - \Omega_0^2} \tag{4}$$

2.5 Frequency transformed lowpass analog filter specifications

From 4

- $\Omega_{Lp} = 1$
- $\Omega_{Ls} = min(\Omega_{Ls1}, \Omega_{Ls2}) = 1.5097$

2.6 The analog Low pass Transfer Function

```
G_sl =

                        1
  ---------------------------------------------------
  3.875 s^4 + 3.811 s^3 + 5.749 s^2 + 3.004 s + 1.111

Continuous-time transfer function.
```

Figure 11: Analog Low Pass Transfer Function For Band Stop

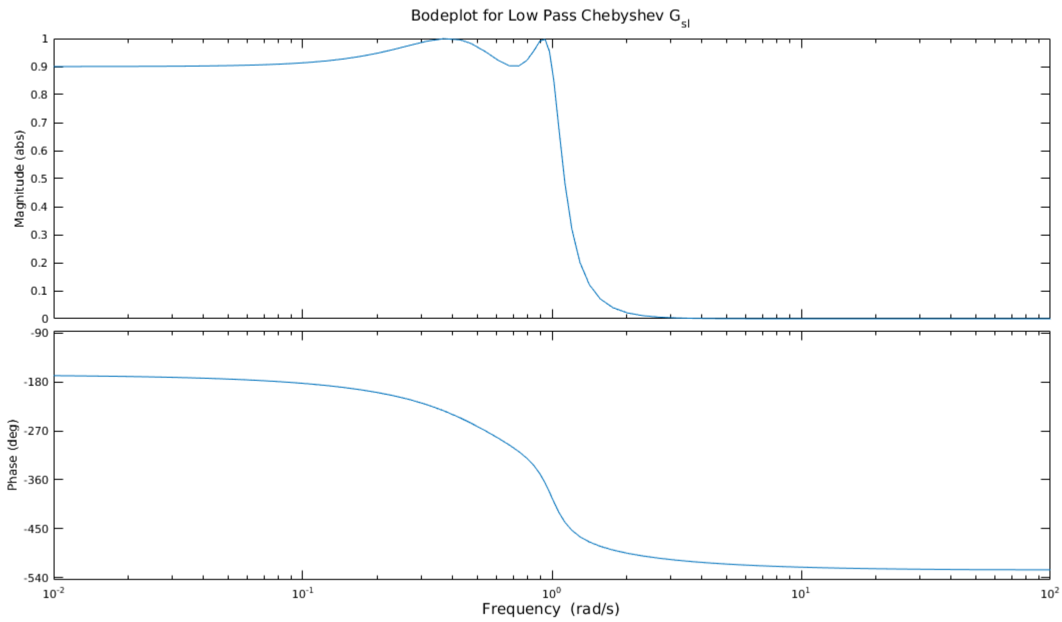

Figure 12: Analog Low Pass Bode plot For Band Stop

2.7 The Analog BandStop Transfer Function

```
G_bs =

                       s^8 + 0.4124 s^6 + 0.06377 s^4 + 0.004383 s^2 + 0.000113
  -----------------------------------------------------------------------------------------------------
  1.111 s^8 + 0.5842 s^7 + 0.6757 s^6 + 0.2087 s^5 + 0.1212 s^4 + 0.02152 s^3 + 0.007181 s^2 + 0.0006401 s + 0.0001255

Continuous-time transfer function.
```

Figure 13: Analog Band Stop Transfer Function

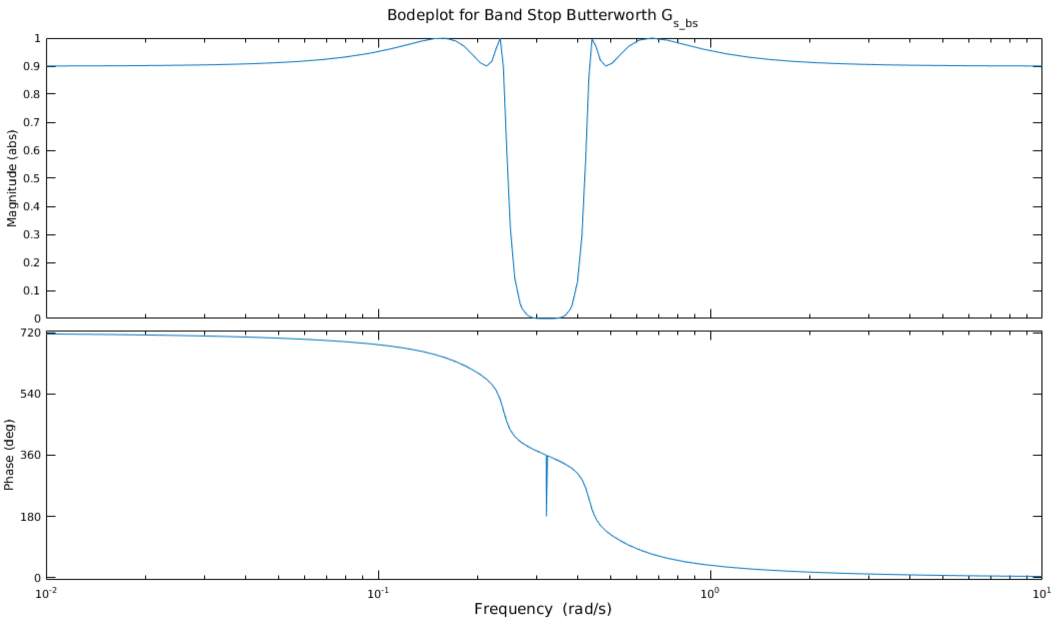

Figure 14: Analog Band Stop Bode Plot

2.8 The Discrete Time Filter Transfer Function

```
G_z_bs_actual =

  0.5423 - 3.527 z^-1 + 10.77 z^-2 - 19.91 z^-3 + 24.25 z^-4 - 19.91 z^-5 + 10.77 z^-6 - 3.527 z^-7 + 0.5423 z^-8
  ----------------------------------------------------------------------------------------------------------------
    1 - 5.654 z^-1 + 15.05 z^-2 - 24.39 z^-3 + 26.25 z^-4 - 19.23 z^-5 + 9.388 z^-6 - 2.815 z^-7 + 0.4029 z^-8

Sample time: 1.1111e-05 seconds
Discrete-time transfer function.
```

Figure 15: Discrete Time Band Stop Transfer Function

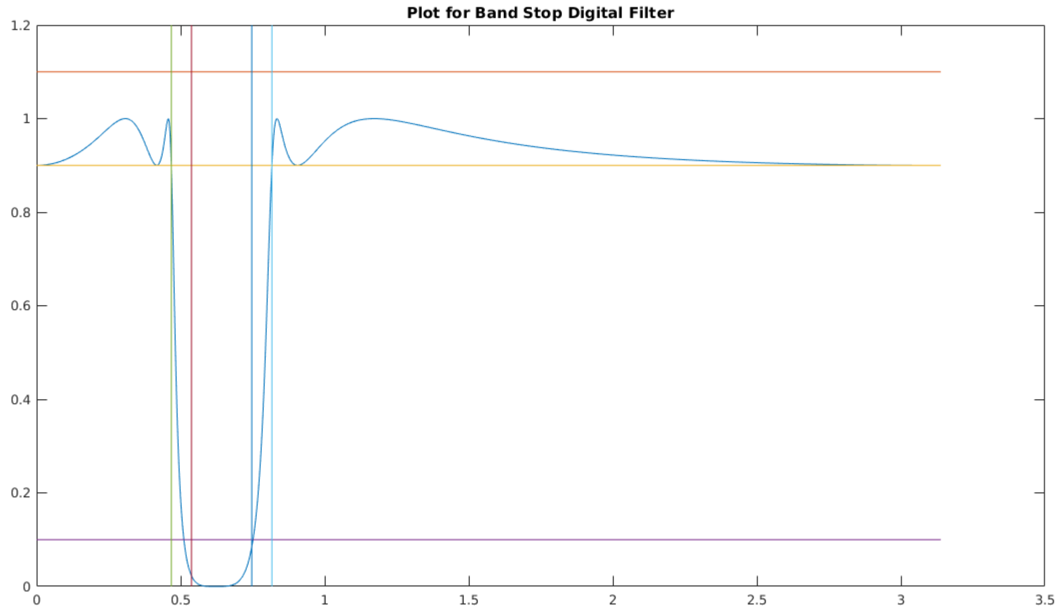

Figure 16: Frequency Magnitude Response of Discrete Time Band Stop Filter

2.9 Direct Form 2 Realization of the Discrete Filter

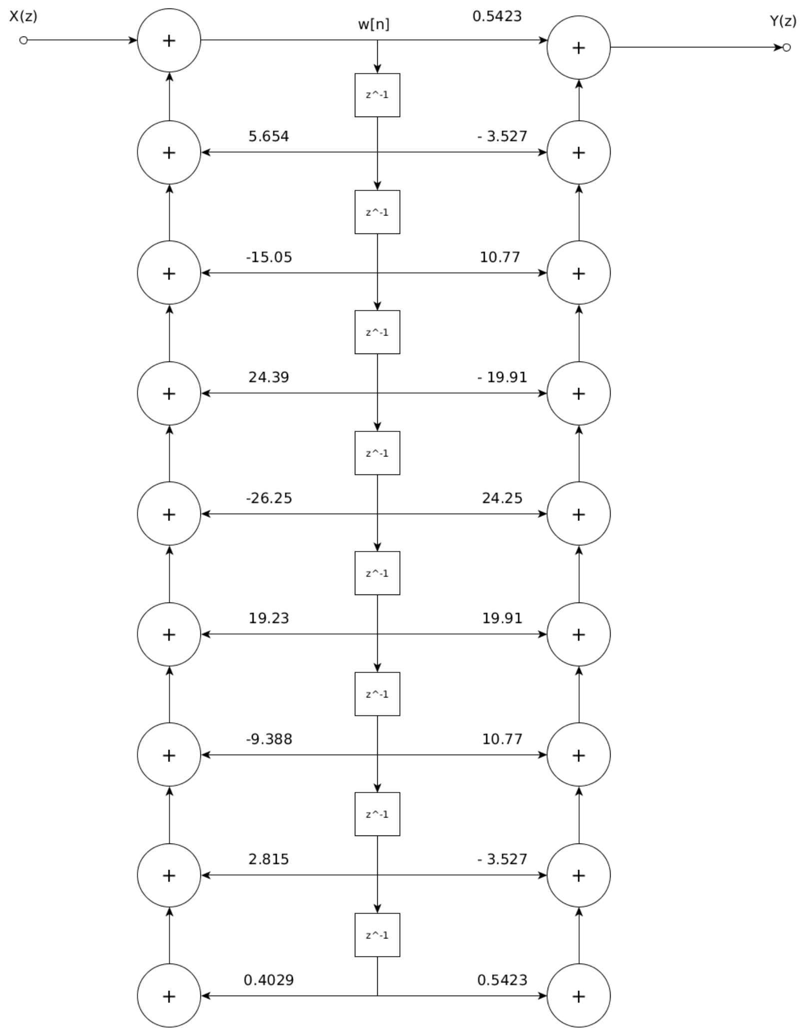

Figure 17: Direct Form 2 Signal Flow Graph for Band Stop IIR filter (Zoom In To see the Diagram)

2.10 FIR Filter Transfer Function to get the same specifications

Given that $\delta = 0.1$, we get

$$A = -20log_{10}(\delta) = -20log_{10}(0.1) = 20$$

For $A < 21$ we get $\alpha = 0$ and hence $\beta = 0$. Also we have $\Delta\omega = w_s - w_p$ We then compute

$$N \geq \frac{A - 8}{2 * 2.285 * \Delta\omega} = 37.6121$$

Therefore we choose

$$N = 38$$

.

But with $N = 38$ the resulting FIR doesn't meet all the specifications. Here is the plot for the same

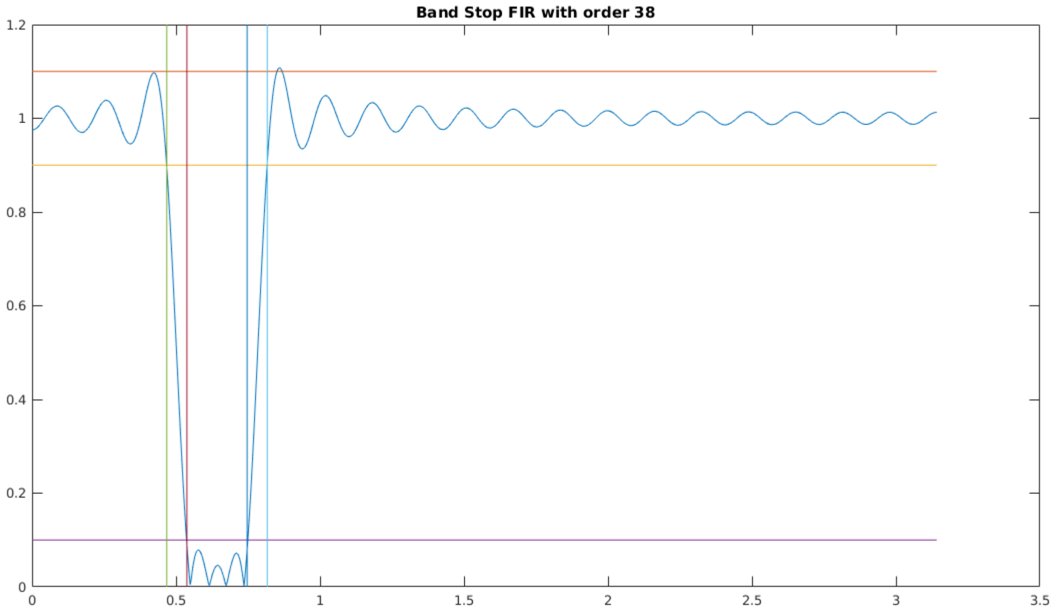

Figure 18: Order 38 FIR Band Stop

On increasing N to $N = 39$, we get an FIR that meets the specifications.

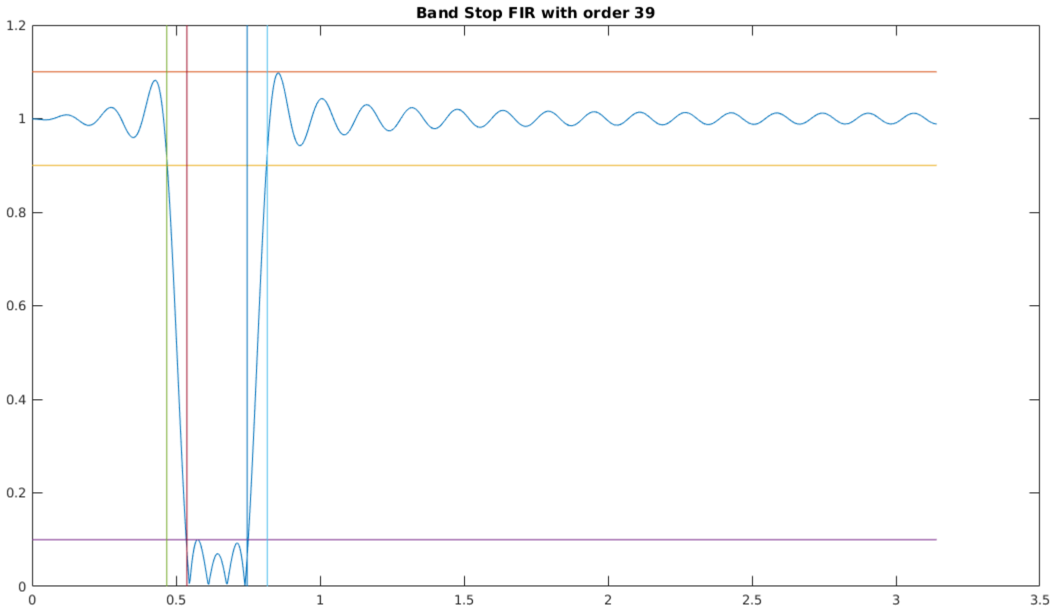

Figure 19: Order 39 FIR Band Stop

The complete Frequency Response is

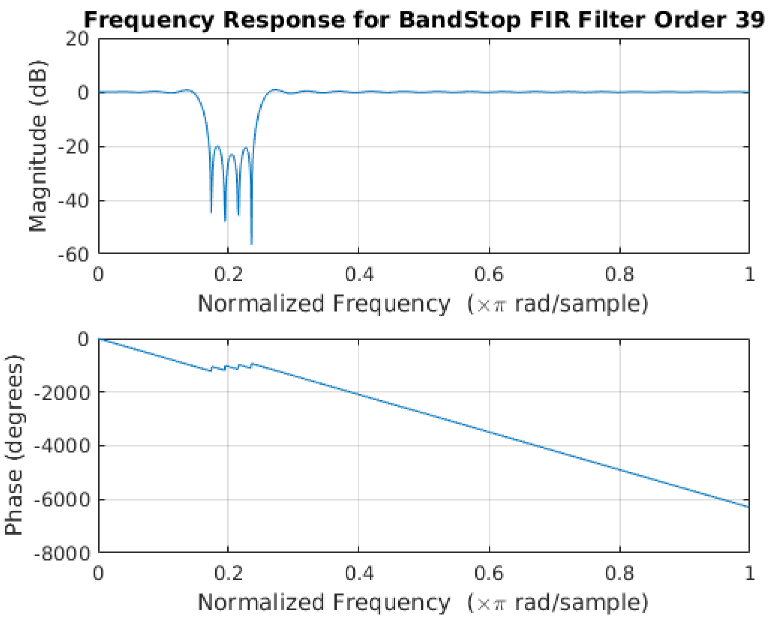

Figure 20: Frequency Response

$H_{fir\_bs} = 0.012088z^{-39} + 0.010386z^{-38} + 0.0031096z^{-37} + -0.0071609z^{-36} + -0.015816z^{-35} + -0.018492z^{-34} + -0.013424z^{-33} + -0.002553z^{-32} + 0.0092892z^{-31} + 0.016789z^{-30} + 0.016869z^{-29} + 0.010236z^{-28} + 0.00087022z^{-27} + -0.0062943z^{-26} + -0.0081842z^{-25} + -0.0052796z^{-24} + -0.0011458z^{-23} + -1.4092e-05z^{-22} + -0.0038107z^{-21} + -0.010465z^{-20} + -0.014705z^{-19} + -0.011139z^{-18} + 0.0019957z^{-17} + 0.020653z^{-16} + 0.035952z^{-15} + 0.038273z^{-14} + 0.0226z^{-13} + -0.0077075z^{-12} + -0.041041z^{-11} + -0.062085z^{-10} + -0.058952z^{-9} + -0.029547z^{-8} + 0.01619z^{-7} + 0.059689z^{-6} + 0.081643z^{-5} + 0.070893z^{-4} + 0.030086z^{-3} + -0.024772z^{-2} + -0.070945z^{-1} + 0.91111z^{0} + -0.070945z^{1} + -0.024772z^{2} + 0.030086z^{3} + 0.070893z^{4} + 0.081643z^{5} + 0.059689z^{6} + 0.01619z^{7} + -0.029547z^{8} +$

$-0.058952z^9 + -0.062085z^{10} + -0.041041z^{11} + -0.0077075z^{12} + 0.0226z^{13} + 0.038273z^{14} + 0.035952z^{15} + 0.020653z^{16} + 0.0019957z^{17} + -0.011139z^{18} + -0.014705z^{19} + -0.010465z^{20} + -0.0038107z^{21} + -1.4092e{-}05z^{22} + -0.0011458z^{23} + -0.0052796z^{24} + -0.0081842z^{25} + -0.0062943z^{26} + 0.00087022z^{27} + 0.010236z^{28} + 0.016869z^{29} + 0.016789z^{30} + 0.0092892z^{31} + -0.002553z^{32} + -0.013424z^{33} + -0.018492z^{34} + -0.015816z^{35} + -0.0071609z^{36} + 0.0031096z^{37} + 0.010386z^{38} + 0.012088z^{39}$