Filter Design Assignment

Arka Sadhu - 140070011
Filter Number 23

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Contents

1 First Filter Specifications

1.1 Un-normalized Discrete Time Filter Specifications

- Sampling Frequency (f_s) = 140kHz
- $B_L = 9.4kHz$
- $B_H = 14.4kHz$
- Monotonic Passband (Butterworth)

1.2 Normalized Digital Filter Specifications

- $\omega_{p1} = \frac{9.4}{140}2\pi = 0.4219$
- $\omega_{p2} = \frac{14.4}{140}2\pi = 0.6463$
- $\omega_{s1} = \frac{8.4}{140}2\pi = 0.3770$
- $\omega_{s2} = \frac{15.4}{140}2\pi = 0.6912$

1.3 Analog Filter Specifications

- $\Omega_{p1} = tan(\frac{\omega_{p1}}{2}) = 0.2141$
- $\Omega_{p2} = tan(\frac{\omega_{p2}}{2}) = 0.3349$
- $\Omega_{s1} = tan(\frac{\omega_{s1}}{2}) = 0.1908$
- $\Omega_{s2} = tan(\frac{\omega_{s2}}{2}) = 0.3600$

1.4 Frequency transformation to be employed

$$s_L = \frac{s^2 + \Omega_0^2}{Bs} \tag{1}$$

with $B = \Omega_{p2} - \Omega_{p1} = 0.1208$ and $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.2678$
This translates to

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{2}$$

1.5 Frequency transformed lowpass analog filter specifications

From 2

- $\Omega_{Lp} = 1$
- $\Omega_{Ls} = min(\Omega_{Ls1}, \Omega_{Ls2}) = 1.3321$

1.6 The analog Low pass Transfer Function

```
H_s_L =

                                                   1
  ----------------------------------------------------------------------------------------------------------
  0.4533 s^11 + 3.423 s^10 + 12.92 s^9 + 32.07 s^8 + 57.98 s^7 + 80.04 s^6 + 86.01 s^5 + 71.95 s^4 + 45.95 s^3 + 21.38 s^2 + 6.539 s + 1

Continuous-time transfer function.
```

Figure 1: Analog Low Pass Transfer Function

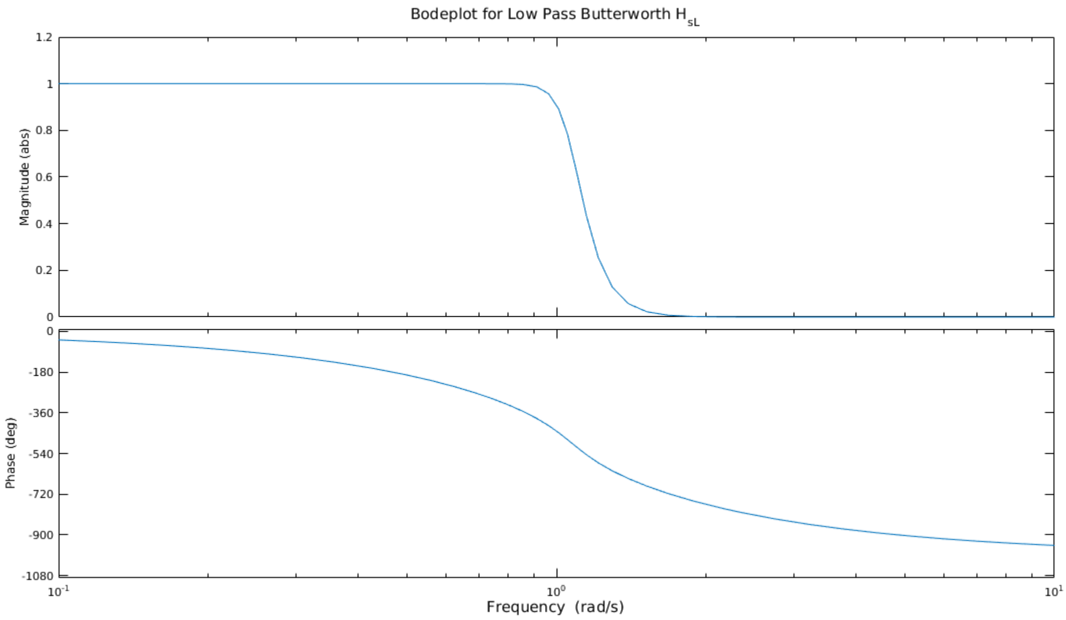

Figure 2: Analog Low Pass Bode plot

1.7 The Analog BandPass Transfer Function

```
H_s_bp =

                                              7.959e-11 s^11

  ----------------------------------------------------------------------------------------------------

  0.4533 s^22 + 0.4133 s^21 + 0.5459 s^20 + 0.3528 s^19 + 0.2621 s^18 + 0.1301 s^17 + 0.0689 s^16 + 0.02732 s^15 + 0.01122 s^14

       + 0.003626 s^13 + 0.001198 s^12 + 0.0003179 s^11 + 8.587e-05 s^10 + 1.864e-05 s^9 + 4.135e-06 s^8 + 7.222e-07 s^7 + 1.306e-07 s^6

                                  + 1.768e-08 s^5 + 2.554e-09 s^4 + 2.465e-10 s^3 + 2.735e-11 s^2 + 1.485e-12 s + 1.168e-13

Continuous-time transfer function.
```

Figure 3: Analog Band Pass Transfer Function

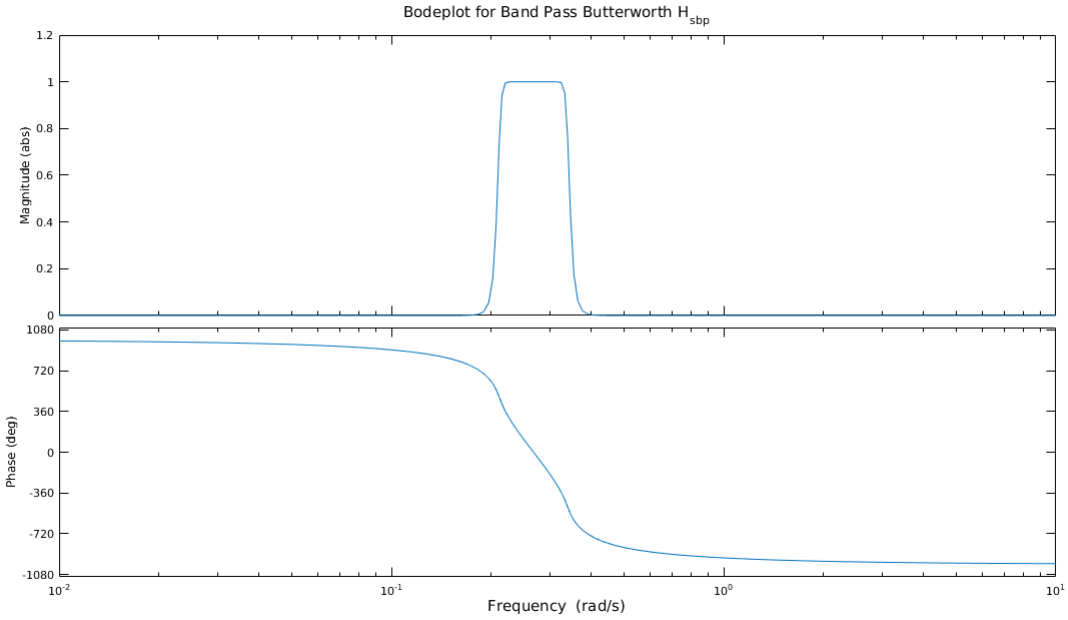

Figure 4: Analog Band Pass Bode Plot

1.8 The Discrete Time Filter Transfer Function

```
H_z_bp_actual =

  3.506e-11 + 2.132e-14 z^-1 - 3.86e-10 z^-2 + 2.501e-12 z^-3 + 1.916e-09 z^-4 + 4.729e-11 z^-5 - 5.919e-09 z^-6 + 2.983e-10 z^
          -7 + 1.099e-08 z^-8 + 9.459e-10 z^-9 - 1.746e-08 z^-10 + 1.368e-09 z^-11 + 1.487e-08 z^-12 + 1.062e-09 z^-13 - 1.23e-08 z^-14
          + 4.184e-10 z^-15 + 5.588e-09 z^-16 + 7.458e-11 z^-17 - 1.951e-09 z^-18 + 5.372e-12 z^-19 + 3.847e-10 z^-20 + 9.726e-14 z^-21
                                                                                                                  - 3.506e-11 z^-22

  ----------------------------------------------------------------------------------------------------------------------------------

  1 - 17.59 z^-1 + 150 z^-2 - 824.9 z^-3 + 3279 z^-4 - 1.002e04 z^-5 + 2.444e04 z^-6 - 4.874e04 z^-7 + 8.081e04 z^-8 - 1.126e05 z^-9
          + 1.329e05 z^-10 - 1.333e05 z^-11 + 1.139e05 z^-12 - 8.269e04 z^-13 + 5.083e04 z^-14 - 2.627e04 z^-15 + 1.129e04 z^-16
                                                  - 3966 z^-17 + 1112 z^-18 - 239.7 z^-19 + 37.36 z^-20 - 3.754 z^-21 + 0.1829 z^-22


Sample time: 7.1429e-06 seconds
Discrete-time transfer function.
```

Figure 5: Discrete Time Band Pass Transfer Function

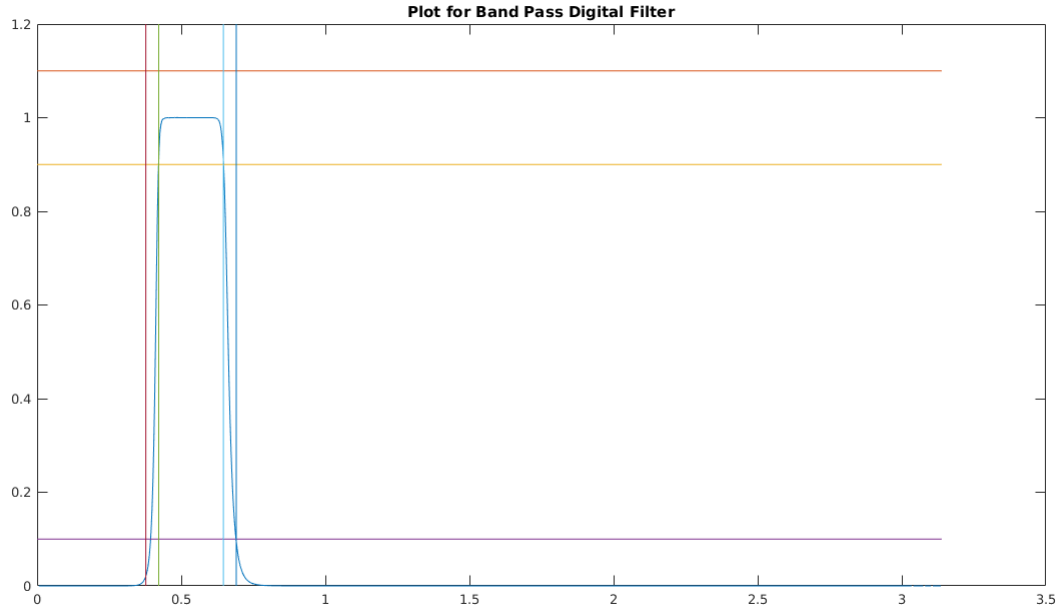

Figure 6: Frequency Magnitude Response of Discrete Time Band Pass Filter

1.9 Direct Form 2 Realization of the Discrete Filter

1.10 FIR Filter Transfer Function to get the same specifications

Given that $\delta = 0.1$, we get

$$A = -20log_{10}(\delta) = -20log_{10}(0.1) = 20$$

For $A < 21$ we get $\alpha = 0$ and hence $\beta = 0$. Also we have $\Delta\omega = w_s - w_p$ We then compute

$$N \geq \frac{A - 8}{2 * 2.285 * \Delta\omega} = 58.507$$

Therefore we choose

$$N = 59$$

.

But with $N = 59$ the resulting FIR doesn't meet all the specifications. Here is the plot for the same

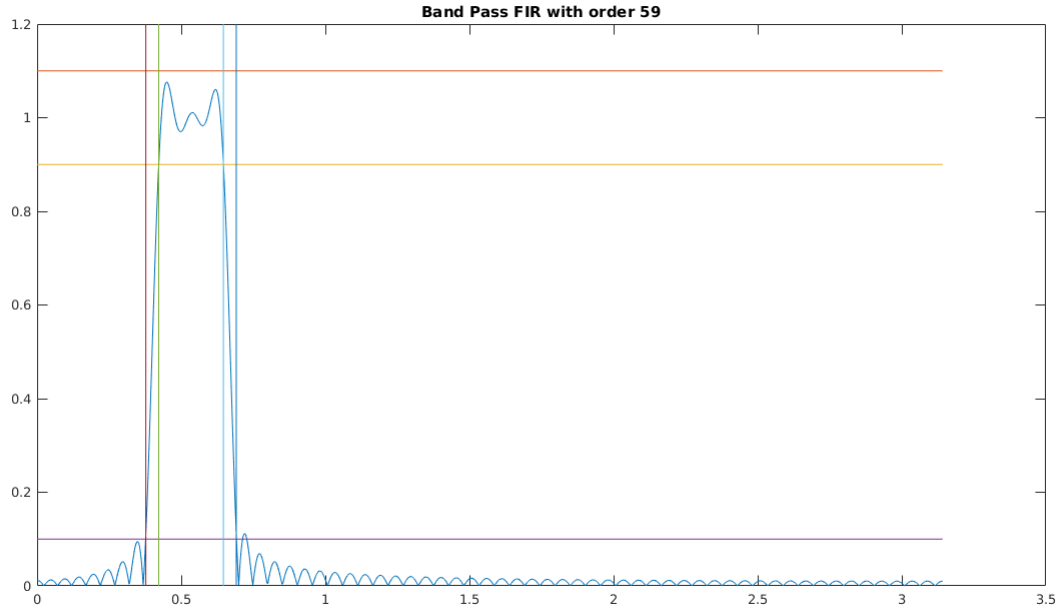

Figure 7: Order 59 FIR Band Pass

On increasing N to $N = 63$, we get an FIR that meets the specifications.

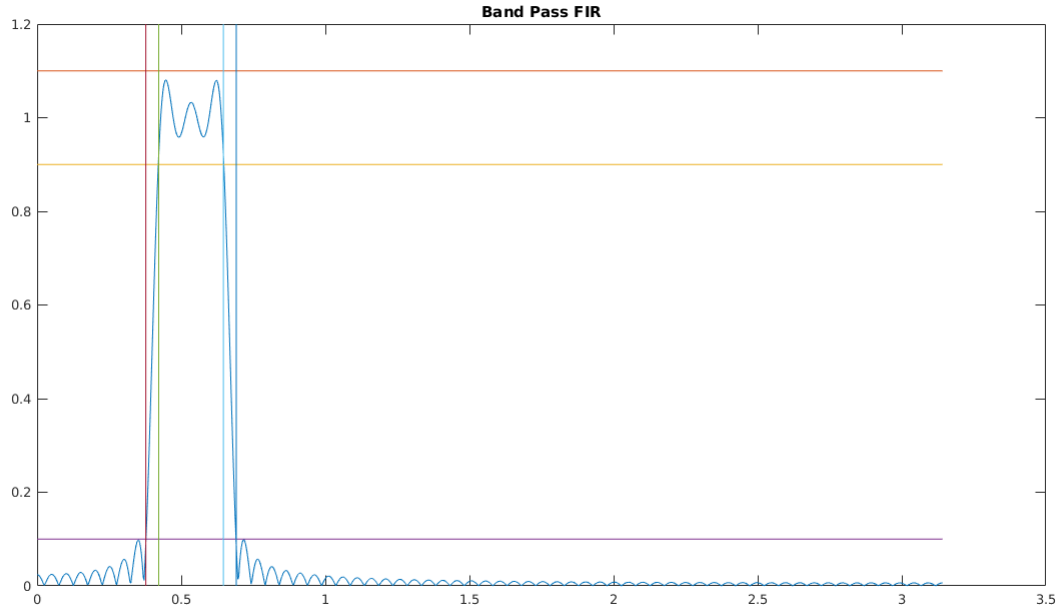

Figure 8: Order 63 FIR Band Pass

$H_{fir\_bp} = 0.010699z^{-59} + 0.0099216z^{-58} + 0.0061769z^{-57} + 0.00067888z^{-56} + -0.0047345z^{-55} + -0.0083073z^{-54} + -0.0090412z^{-53} + -0.0070586z^{-52} + -0.0035005z^{-51} + 1.5549e-17z^{-50} + 0.0020437z^{-49} + 0.0020753z^{-48} + 0.0006074z^{-47} + -0.0010474z^{-46} + -0.0014292z^{-45} + 0.00031922z^{-44} + 0.0039434z^{-43} + 0.0080615z^{-42} + 0.010683z^{-41} + 0.010067z^{-40} + 0.0055652z^{-39} + -0.0019308z^{-38} + -0.010166z^{-37} + -0.016293z^{-36} + -0.017965z^{-35} + -0.014297z^{-34} + -0.0062993z^{-33} + 0.0034278z^{-32} + 0.011658z^{-31} + 0.015779z^{-30} + 0.014805z^{-29} + 0.009742z^{-28} + 0.0031172z^{-27} + -0.0021396z^{-26} +$

$-0.0040068z^{-25}+-0.0023031z^{-24}+0.0011925z^{-23}+0.003537z^{-22}+0.0020435z^{-21}+-0.0042678z^{-20}+$
$-0.013846z^{-19}+-0.022858z^{-18}+-0.026534z^{-17}+-0.021167z^{-16}+-0.0059818z^{-15}+0.01592z^{-14}+$
$0.038073z^{-13}+0.05258z^{-12}+0.052901z^{-11}+0.036481z^{-10}+0.0062323z^{-9}+-0.029837z^{-8}+$
$-0.060854z^{-7}+-0.07654z^{-6}+-0.070733z^{-5}+-0.04374z^{-4}+-0.0026197z^{-3}+0.040796z^{-2}+$
$0.073555z^{-1}+0.085714z^{0}+0.073555z^{1}+0.040796z^{2}+-0.0026197z^{3}+-0.04374z^{4}+-0.070733z^{5}+$
$-0.07654z^{6}+-0.060854z^{7}+-0.029837z^{8}+0.0062323z^{9}+0.036481z^{10}+0.052901z^{11}+0.05258z^{12}+$
$0.038073z^{13}+0.01592z^{14}+-0.0059818z^{15}+-0.021167z^{16}+-0.026534z^{17}+-0.022858z^{18}+$
$-0.013846z^{19}+-0.0042678z^{20}+0.0020435z^{21}+0.003537z^{22}+0.0011925z^{23}+-0.0023031z^{24}+$
$-0.0040068z^{25}+-0.0021396z^{26}+0.0031172z^{27}+0.009742z^{28}+0.014805z^{29}+0.015779z^{30}+$
$0.011658z^{31}+0.0034278z^{32}+-0.0062993z^{33}+-0.014297z^{34}+-0.017965z^{35}+-0.016293z^{36}+$
$-0.010166z^{37}+-0.0019308z^{38}+0.0055652z^{39}+0.010067z^{40}+0.010683z^{41}+0.0080615z^{42}+$
$0.0039434z^{43}+0.00031922z^{44}+-0.0014292z^{45}+-0.0010474z^{46}+0.0006074z^{47}+0.0020753z^{48}+$
$0.0020437z^{49}+1.5549e{-}17z^{50}+-0.0035005z^{51}+-0.0070586z^{52}+-0.0090412z^{53}+-0.0083073z^{54}+$
$-0.0047345z^{55}+0.00067888z^{56}+0.0061769z^{57}+0.0099216z^{58}+0.010699z^{59}$

2 Second Filter Specifications

2.1 Un-normalized Discrete Time Filter Specifications

- Sampling Frequency (f_s) = 140kHz
- $B_L = 7.7kHz$
- $B_H = 10.7kHz$
- Monotonic Passband (Butterworth)

2.2 Normalized Digital Filter Specifications

- $\omega_{p1} = \frac{7.7}{90}2\pi = 0.3007$
- $\omega_{p2} = \frac{10.7}{90}2\pi = 0.5251$
- $\omega_{s1} = \frac{6.7}{90}2\pi = 0.3456$
- $\omega_{s2} = \frac{11.7}{90}2\pi = 0.4802$

2.3 Analog Filter Specifications

- $\Omega_{p1} = tan(\frac{\omega_{p1}}{2}) = 0.1515$
- $\Omega_{p2} = tan(\frac{\omega_{p2}}{2}) = 0.2688$
- $\Omega_{s1} = tan(\frac{\omega_{s1}}{2}) = 0.1745$
- $\Omega_{s2} = tan(\frac{\omega_{s2}}{2}) = 0.2448$

2.4 Frequency transformation to be employed

$$s_L = \frac{s^2 + \Omega_0^2}{Bs} \tag{3}$$

with $B = \Omega_{p2} - \Omega_{p1} = 0.1208$ and $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.2678$
This translates to

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \tag{4}$$

2.5 Frequency transformed lowpass analog filter specifications

From 4

- $\Omega_{Lp} = 1$
- $\Omega_{Ls} = min(\Omega_{Ls1}, \Omega_{Ls2}) = 1.3321$

2.6 The analog Low pass Transfer Function

$$H_{analog}(s_L) = \frac{1}{0.4533s^{11} + 3.423s^{10} + 12.92s^9 + 32.07s^8 + 57.98s^7 + 80.04s^6 + 86.01s^5 + 71.95s^4 + 45.95s^3 + 21.38s^2}$$