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Data Structures and Abstractions

Hash Tables

Lecture 31

Important Advice for LAB/Assignment

- You must complete Lab 10.
  - Submission needed for the last assessed lab
  - Submission needed for the assignment
- Your BST needs to be usable beyond the purposes of the lab/assignment.
- Follow the assignment specifications carefully. Read the QandA file (when available) regularly to see any clarification or advice.
  - If the answer to your question is not there, ask early.
- Be mindful of summing small floating point numbers. Errors accumulate.
  - See the following for some advice:
    - https://en.wikipedia.org/wiki/Kahan_summation_algorithm
    - For a more detailed answer see: "What Every Computer Scientist Should Know About Floating-Point Arithmetic" at http://docs.oracle.com/cd/E19957-01/806-3568/ncg_goldberg.html

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Maps

- Previously we have looked at the STL map class, where any type can be used as a key into a container of paired values, giving direct access to the second part of the data.
- So we can have:

```
map<string,string> DictionaryType;

Dictionary dictionary; // not really a good way to name

dictionary["aardvark"] = "A nocturnal mammal of southern
   Africa"

cout << "arardvark:" << dictionary["aardvark"];

//Work out how may string object constructions occurred in the lines
   //above
```

Hash Tables

- One way to achieve this kind of direct access for the map class is to use what is known as a *hash table.*
  - If keys are unique a balanced binary search tree can be used.
  - If keys are not unique and key are unordered as in *std::unordered_multimap* or *std::unordered_multiset* , then hashing is used.
- When storing the data, the key—in this case “aardvark” —is passed through a *hash function,* to give an index into an ordinary array. [1] [2]
- The quality of the hash function determines how many different keys hash to the same index value. (technically known as “collisions”)
- No hash function is perfect under all conditions, therefore there will always be clashes (“collisions”).
- Therefore there must also be a *collision resolution* defined.
- Hash tables will always have empty space. To work most efficiently they are generally required to be no more than half full.

Dealing with Strings

- The key used in the above example is a string.
- Obviously you cannot pass a string through a mathematical function.
- Therefore strings must be mapped to integers before hashing.
- There are many ways to do this, however it is important to make sure that the method chosen does not promote collisions.

Insertion into a Hash Table

```
• Insert (pair)
•    index = HashFunction (pair.key)
    index = index mod arraySize // in hash table i.e %


    IF array[index] is empty
        array[index] = pair
    ELSE
        HandleCollision (index, pair)
    ENDIF


• End Insert
```

Searching a Hash Table

```
Search (key, found)
        index = HashFunction (key)
        index = index mod arraySize
        IF array[index] is empty
             found = false
        ELSE
             IF array[index].key = key
                 found = true
             ELSE // another key was hashed to the same index
                 found = CollisionSearch (index, pair)
             ENDIF
        ENDIF
End Search
```

Hash Functions

- The ideal function would: [1]
  - be easy to calculate;
  - never produce the same index from two different keys;
  - spread the records evenly throughout the array;
  - deal with ‘bad’ keys better than others.
- Of course no function has all of these attributes under all conditions.
  - It may be possible under restricted conditions where all keys are known in advance.
- Common Hash Functions
  - Truncation
  - Extraction
  - Folding
  - Modular arithmetic
  - Prime number division
  - Mid-square hashing
  - Radix conversion

Radix Conversion

- The best known of these is Radix Conversion.
- Choose a low prime [1] number such as 7, 11 or 13 to use as the base of a polynomial. Then use the digits of the key as the factors of the polynomial.
- Finally modulate by the array size, which should be a prime number.
- For example, if
  key = 32934648
  array size = 997
  Base = 7

  Then
  index = $(3 * 7^7 + 2 * 7^6 + 9 * 7^5 + 3 * 7^4 + 4 * 7^3 + 6 * 7^2 + 4 * 7 + 8)$ MOD 997
  = 2866095 MOD 997
  = 717

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Collision Resolution

- Collision resolution needs to:
  - avoid clustering of records;
  - be as simple to code as possible;
  - only fail when the array is actually full;
  - be ‘reversible’ to allow for deletion/search.
- As before, no method fulfils all these requirements under every possible condition.
- Common collision resolutions are:
  - Linear probing
  - Quadratic probing
  - Random probing
  - Linked collisions
  - Overflow containers

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Probing

- Linear probing simply looks for the next empty space in the array. So if index is full, index+1 is checked, then index+2, index+3 etc. This has the disadvantage of increasing clustering and therefore collisions.

- Quadratic probing looks for the next empty space using 'square' jumps. So if index is full, index+1 is checked, then index+4, index+9, index+16 etc. This avoids the clustering of linear probing, but can fail when the array is not full.

- Random probing uses a random number generator—from a set starting point—for the increments in index. This avoids clustering, but is harder to reverse when a record is to be deleted (search?). Use pseudo -random number generator.

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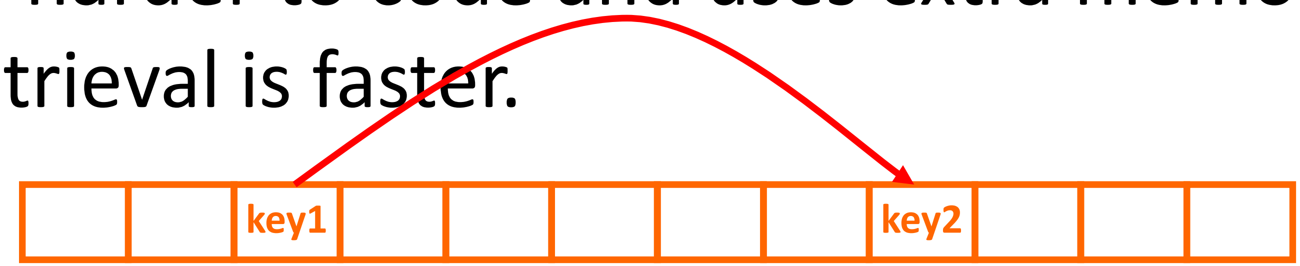
Linking Collisions

- After the first collision, a second hash function is used to generate an alternate position and the two are linked.
- A third collision would then have a link from the second collision and so on.
- This is harder to code and uses extra memory, but retrieval is faster.



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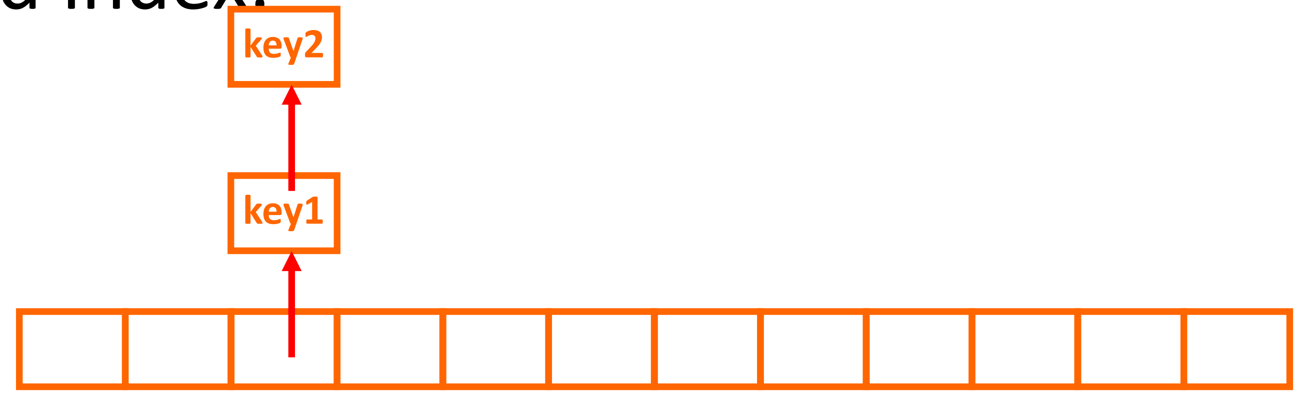
An Overflow Container

- Instead of using a one dimensional array to store data, a two dimensional structure is used.
- The records are placed in a linked list from the hashed index.



Readings

- Reference book, Introduction to Algorithms. Chapter on Hash Tables.

Further Exploration

- Khan Academy Video one particular example of the use Hash functions [“Bitcoin: Cryptographic hash functions”]
- Tutorial on Hash functions
  http://research.cs.vt.edu/AVresearch/hashing/

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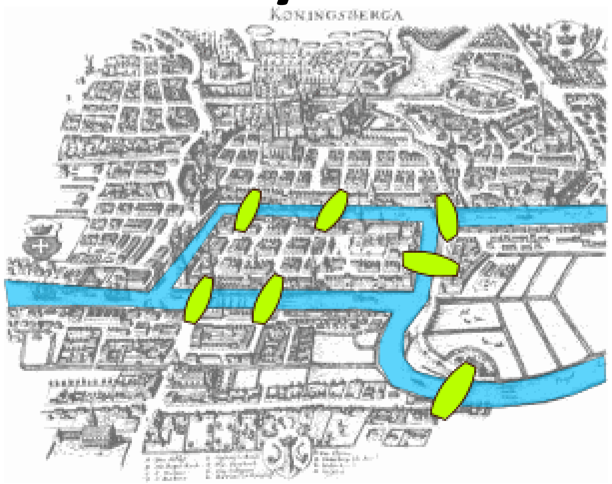
Data Structures and Abstractions

Graph Theory

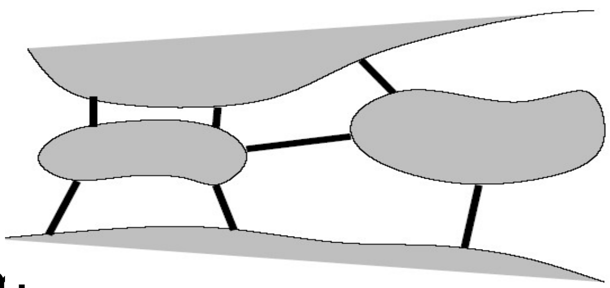
Lecture 32

The Origins of Graph Theory

- Graph Theory (unlike a lot of what we do) dates back to before 1736.
- In Konisberg there were two islands in the middle of a river, connected by 7 bridges. [1]
  - Textbook has the abstract version.



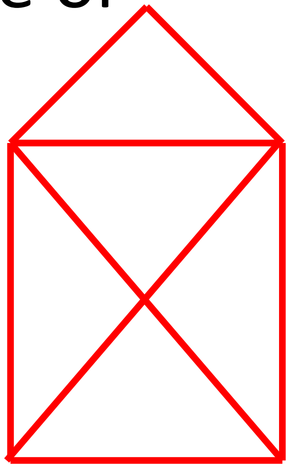
- The question was: “is it possible to cross each bridge exactly once?”
  – Abstract representation is used to investigate solutions.
  – Any solution obtained can then be used for similar problems.



- In 1736, Euler answered this problem by establishing “Graph Theory” as a discipline. (The answer is “no”)

Another Common Problem

- As a child you may have met something similar: Draw the shape below without taking your pen off the page and without going over any line or node more than once.



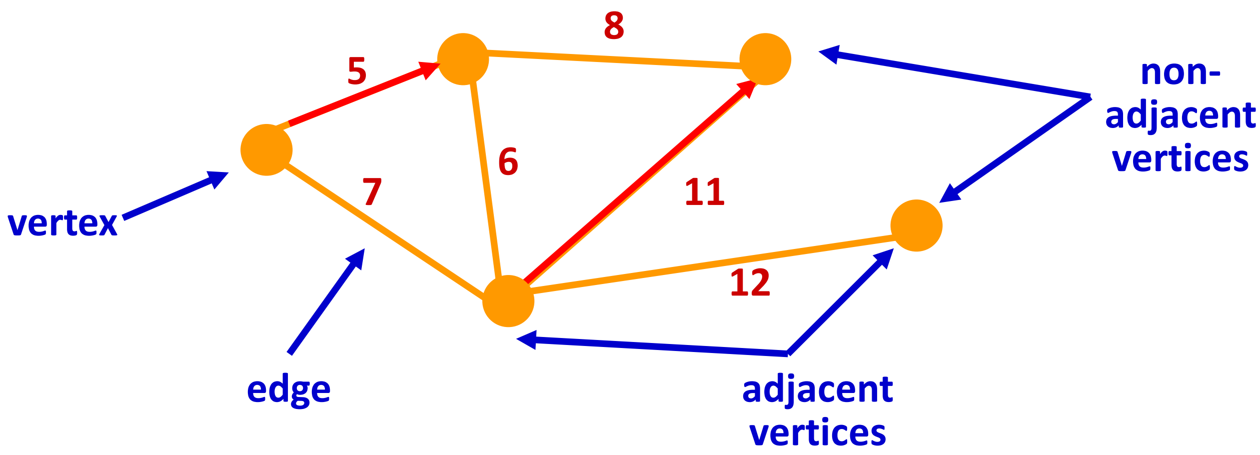
- It is a graph problem, just as the Konisberg problem is a graph problem.

But...

- But the theory itself remained a kind of mathematician-only esoteric field until
  1. Computers became available that could handle graph processing algorithms in reasonable time.
  2. Many of the complex problems of society were recognised to be graph problems.
  3. It was realised that Network traffic and the WWW were graphs.
  4. Some AI applications (simulations, neural networks etc) were discovered to use graph theory.
  5. Computer game playing required graph theory.

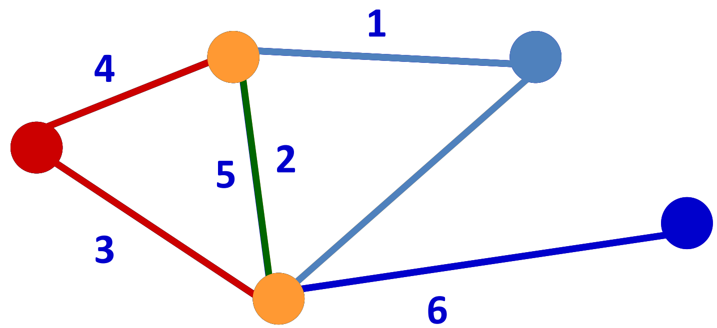
So What is a Graph?

- A graph is a set of *vertices* connected by *edges*.



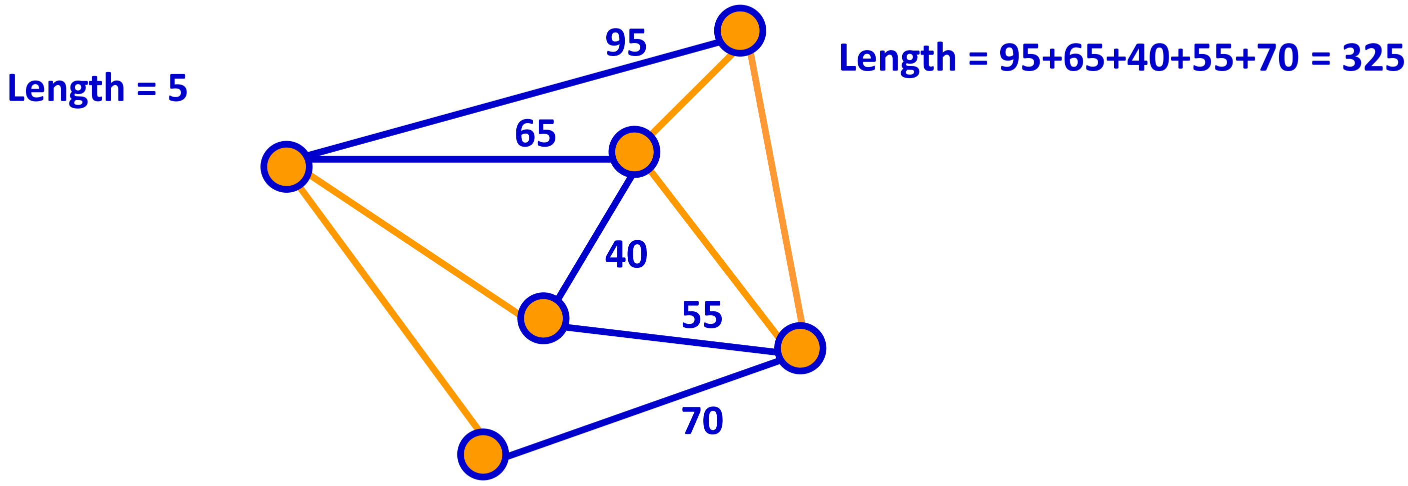
- Two vertices are *adjacent* if they are connected by a single edge.
- A graph is *weighted* if there is a number associated with each edge. (can be cost, distance, ..etc)
- A graph is *directed* if any of the edges are one-way.

Graph Definitions

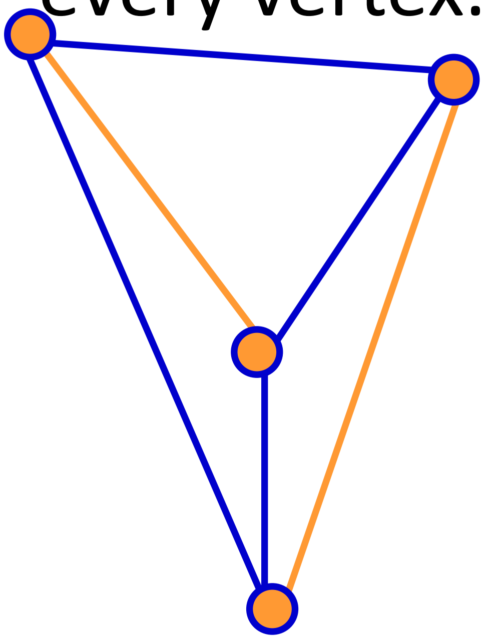


- A *path* is a sequence of adjacent vertices
- A *simple path* is one that has distinct edges: no vertex is visited twice.
- A *cyclic path* is one where the start and finish are the same vertex.
- Two paths are *disjoint* if they have no vertices in common, other than, possibly, their endpoints. [see the red and blue paths]

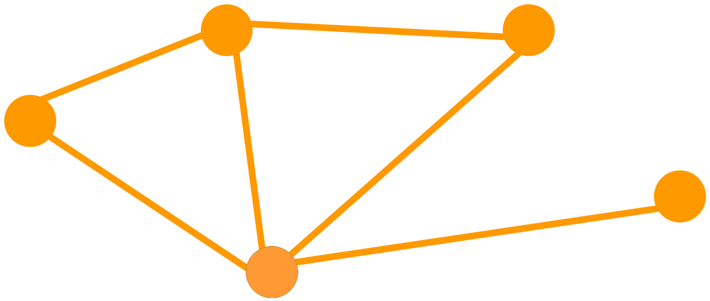
- In an unweighted graph, the length of a path is the number of traversed edges.
- In a weighted graph, the length of a path is the sum of the weights of the traversed edges.



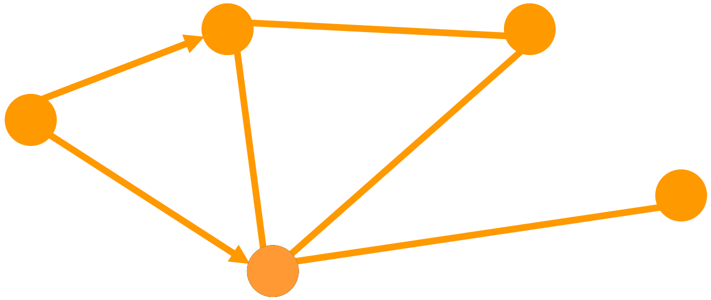
- A *tour* is a cyclic path that touches every vertex.



- A connected graph is one where every vertex is reachable from every other vertex

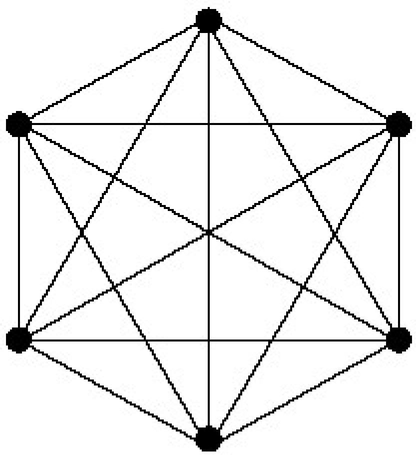


Connected

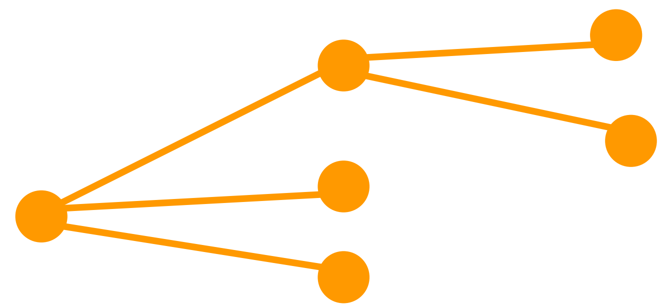


Not connected

- A *complete* graph is one where every vertex is adjacent to every other vertex.



- A graph with no cycles (an *acyclic graph)* is a tree.

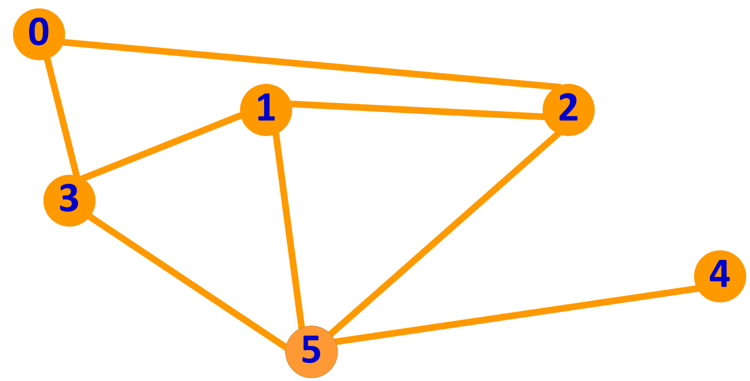


Data Structures to Represent Graphs

- Representing a graph as vertices and edges is fine in the abstract (physical) sense but makes processing too difficult.
- Two alternatives are therefore used within programming:
  - Adjacency matrices
    - Constant access time
    - Slow search time
  - Adjacency list
    - Fast search time
    - Slow access time
- For both of these, the vertices are arbitrarily numbered.

Adjacency Matrix Representation

- The graph is represented as a two dimensional array of boolean.
- A vertex is not considered to connect to itself.

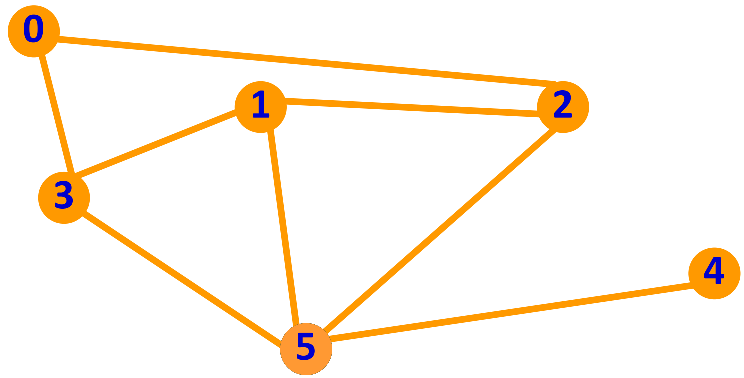


	0	1	2	3	4	5
0	false	false	true	true	false	false
1	false	false	true	true	false	true
2	true	true	false	false	false	true
3	true	true	false	false	false	true
4	false	false	false	false	false	true
5	false	true	true	true	true	false

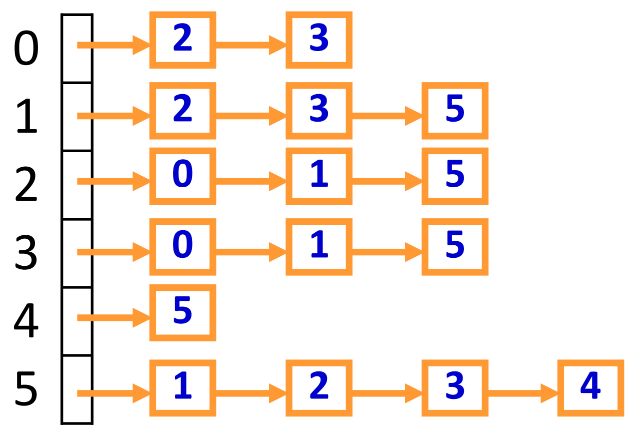
Drawing an Adjacency Matrix

- Make sure you can draw an adjacency matrix for a graph. Use the Graph program to check your answers.

Adjacency List Representation



- The graph is represented as a one dimensional sorted list of connected vertices:



Drawing an Adjacency List

- Make sure you can draw an adjacency list for a graph. Use the Graph program to check your answers.

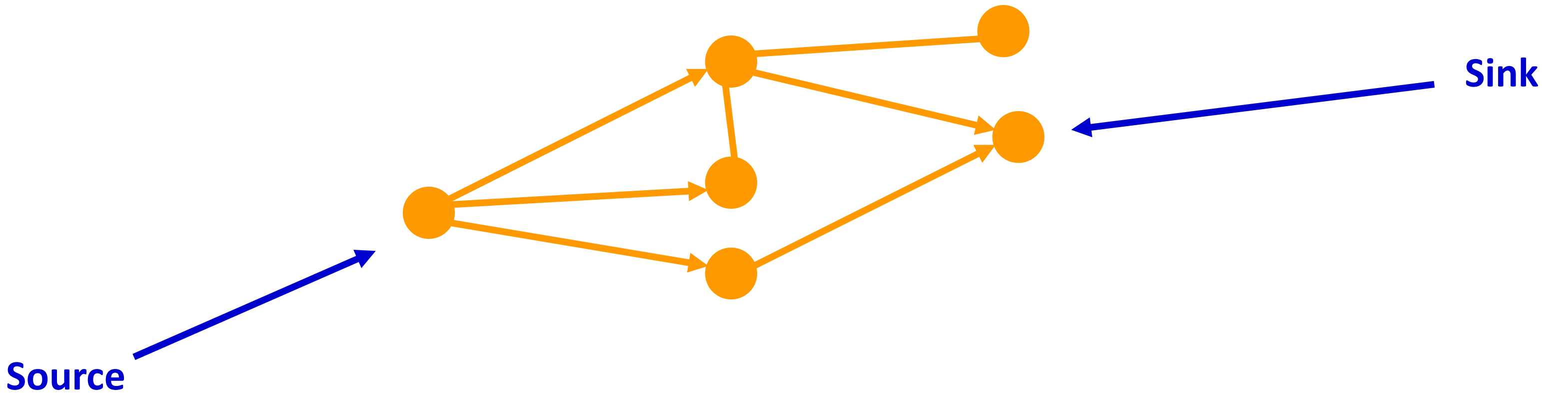
Matrix and List Comparison

- Advantages of Lists
  - More flexible as the size is not fixed
  - Less space used: $O(V+E)$ rather than $O(V^2)$ for a matrix.
  - Faster processing (searching) at each vertex
- Advantages of Matrices
  - Easier to program
  - Access time to find out if a pair of vertices are connected is constant time as opposed to $O(V)$ for lists.

Directed Graph Definitions

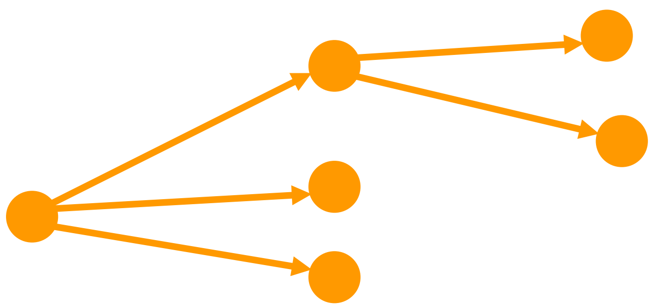
- Directed graphs are also known as di-graphs.
- A vertex is *reachable* from another vertex if there is a path between them.
- It is assumed that each vertex can reach itself.
- The *in-degree* of a vertex is the number of edges leading into a vertex.
- The *out-degree* of a vertex is the number of edges leading out of a vertex.

- A *sink* is a vertex with out-degree of zero.



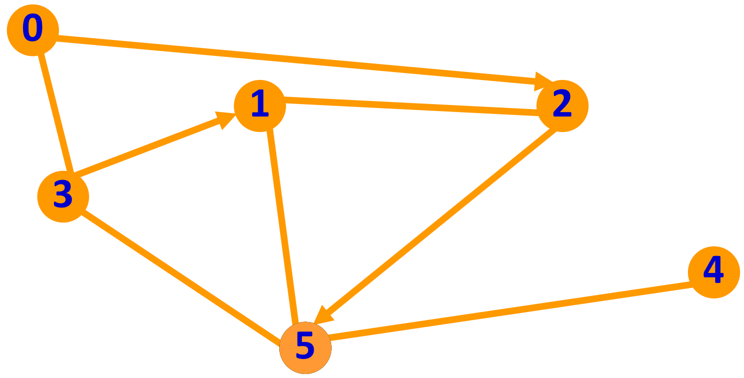
- A *source* is a vertex of in-degree 1: it is reachable only from itself.

- A *map* is a di-graph where every vertex has out-degree 1.
- A di-graph is *strongly connected* if every vertex is reachable from every other vertex.
- A di-graph with no cycles is an Acyclic Directed Graph, or DAG.



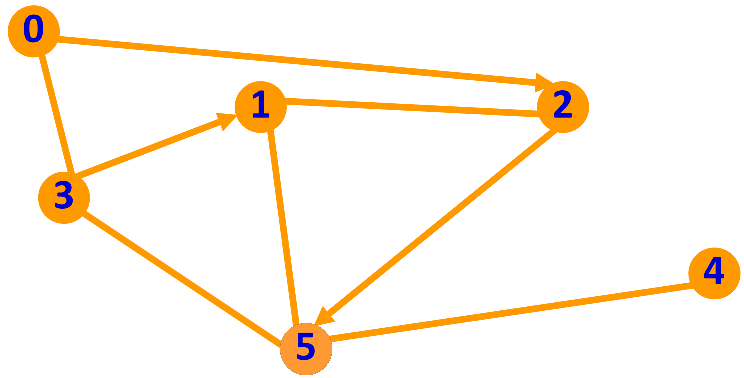
Adjacency Matrix Representation of a Di-graph

- The di-graph is represented as a two dimensional array of boolean.
- Note that in a di-graph vertices are considered to be connected to themselves.

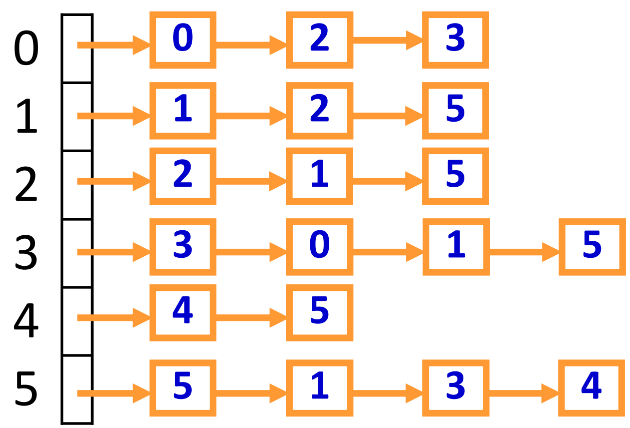


	0	1	2	3	4	5
0	true	false	true	true	false	false
1	false	true	true	false	false	true
2	false	true	true	false	false	true
3	true	true	false	true	false	true
4	false	false	false	false	true	true
5	false	true	false	true	true	true

Adjacency List Representation



- The di-graph is represented as a one dimensional sorted list of connected vertices:



Graph Domains

- Social media – friendship networks
- Interconnections in ecosystems
- Genetics and ancestry
- Chemical structures
- Traversal problems
- Travel itineraries
- Neural networks
- The WWW (the biggest graph of them all)
- Electric circuits
- Scheduling
- Financial transactions
- Compilers use graphs to represent call structures
- Within games software
- UML diagrams, data flow diagrams, E-R diagrams etc
- Automatic diagram generation

etc.

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Some Graph, Di-Graph and DAG Processing Problems

- Searching: how do we get from a particular vertex to another.
- Connectivity: is a given graph connected.
- Find the minimum length set of edges that connects all vertices (the Minimum Spanning Tree or MST).
- Find the shortest path between two vertices.
- Find the shortest path from a specific vertex to all other vertices (the Shortest Path Tree or SPT).
- Planarity: can a specific graph be drawn without any intersecting lines?
- Matching: what is the largest subset of edges with the property that no two are connected to the same vertex?
- Find the tour with the shortest path (mail carrier problem).
- Topological Sort: sort the vertices of a DAG in order of the number of dependencies.

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Complex problems

- Graphs are a powerful tool for modelling complex problems.

- *"The great unexplored frontier is complexity*
  - *I am convinced that nations and people that master the new science of complexity will become the economic, cultural, and political superpowers of the next century."* Heinz Pagels

In Fact

- These problems are NP-Hard.
- There is no solution for any of them that is guaranteed to be solvable in a reasonable amount of time.
  - Restricted case solutions are possible but not in the general case.
- There are only solutions that work quite well in some circumstances.
- This, combined with the large number of domains, makes this field one that is rich in research possibilities.

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Readings

- Textbook Chapter on Graphs.
- The lecture notes and textbook is sufficient for this unit.
- Further exploration:
  - Graphs and Networks has interesting applications of graphs, including a wall chart.
  - How graph problems gave the biggest finite number known that no one can write but ends in 7.
  - Interactive explanations of various algorithms [1]
  - Complex systems: Network thinking, Melanie Mitchell, Artificial Intelligence, vol. 170(18), Science Direct, Elsevier.
  - Reference book, Introduction to Algorithms. Part on Graph Algorithms contains a number of chapters on graph algorithms. (for further study)
  - Consider doing the unit MAS225 where Graphs are covered in detail. https://handbook.murdoch.edu.au/units/12/MAS225