ON THE EPISTEMOLOGICAL APPLICATION OF THE MATHEMATICAL CONCEPTS OF COMPLETENESS, CONSISTENCY, AND DECIDABILITY TO SYSTEMS OF NATURAL LAW

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Abstract

In the early twentieth century, mathematics was shown to be incomplete and not necessarily consistent by Kurt Gödel in his historic paper, "On Formally Undecidable Propositions in Principia Mathematica and Related Systems," as well as undecidable by Alan Turing in his comparably notable paper, "On Computable Numbers, With an Application to The Entscheidungsproblem." That is, true statements arise in formal systems of arithmetic which cannot be proven, it is possible for a formal system of arithmetic to prove contradictions, and there is no algorithm to determine if a mathematical statement is derivable from a set of given axioms. However, math is not the only field in which formal systems can arise. Established truths about the universe can be organized into formal systems in which these natural laws infer other truths. Here, we summarize the mathematical proofs of incompleteness, inconsistency, and undecidability and apply them to systems of natural laws which describe our universe. With the resulting conclusions, we explore epistemological implications such as the possibility of a recursive inability to obtain certain truth including that of said inability, a proposed innate distinction between logic and nature, and a fundamental exiguity in our ability to extract knowledge from our universe.

Introduction

Mathematics is a system highly regarded as the purest form of logic, capable of providing objective, absolute answers to all questions contained in the system. This idea was epitomized by David Hilbert's Program, a series of hypothesized mathematical postulates which sought to prove that math was complete, consistent, and decidable. However, to Hilbert's dismay, despite its apparent logical rigidity, it was proven by Kurt Gödel that mathematics is not complete. That is, the truth of a statement does not imply the existence of a proof of its truth. He also proved that a consistent set of axioms, one where the axioms never prove a statement and its negation to be true (a system free from contradictions), could not prove its own consistency. In addition, Alan Turing proposed the halting problem in which he established that no algorithm exists that can determine if a statement can be inferred from a set of given axioms, hence disproving the decidability of mathematics. What makes this particularly interesting outside the realm of math is that a system of mathematics is not the only system that can describe aspects of the world we apprehend. When generalized, the laws of nature that describe reliably uniform phenomena we observe, can be considered axioms which admit the existence of a system of natural laws through which we perceive the world around us. This system, however comprehensive it may be, inherently has no less than a possibility of being subject to the same lack of completeness, certain consistency, and decidability that Gödel and Turing proved in mathematics, and at most a definite lack of these properties. Given the fidelity of formal systems in connecting statements to premises, it is evident that the lack of these properties exists in systems of natural laws used to describe natural phenomena, possibly providing the basis to prove fascinating completeness, consistency, and decidability related theories in fields such as theology, epistemology, ontology, etc.

Gödel's First Incompleteness Theorem

Before considering the application of mathematical theorems to the way we understand the environments that surround us, an adequate understanding of these mathematical theorems is necessary. In 1931, Kurt Friedrich Gödel published his landmark paper, "On Formally Undecidable Propositions in Principia Mathematica and Related Systems," which consisted of many propositions regarding the properties of axiomatizable formal systems, most famous of which are the incompleteness and uncertainty of consistency demonstrated by such systems in which arithmetic could be performed[1]. Regarding incompleteness, proposition VI states that, "any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F."[2] By formal system, we mean a set of axiomatic, well-formed formulas, expressed in a language of a finite amount of symbols which is used to describe all formulas in the system, equipped with a set of grammar rules to constitute a well-formed formula, and decidable rules of inference which can be used to prove other formulas, or theorems. The set of axioms must be finite or at least axiomatizable, i.e., admit an effective method of determining whether a given statement is an axiom.[3] Gödel introduces his paper with popular such systems including that of Principia Mathematica and Zermelo-Fraenkel set theory. To prove such systems are incomplete, he begins with a sketch of the rigorously formalized proof allotted to the rest of the paper. For our purposes, the sketch will suffice with the aid of certain considerations we will examine shortly.

1. Panu Raatikainen, "Gödel's Incompleteness Theorems," The Stanford Encyclopedia of Philosophy, Metaphysics Research Lab Stanford University, March 8, 2021, https://plato.stanford.edu/archives/spr2021/entries/goedel-incompleteness/.

2. See note 1 above.

3. See note 1 above.

Consider the formulas of a formal system F capable of arithmetic which have one free variable.[4] The numbers for which a formula is true can be organized into a class (for a class c with element n, c[n] is true). Classes are then arranged into an ordered sequence where R(n) refers to the n^{th} class. We now define the class K as the numbers, n, for which R(n)[n] is not provable. As a class, K is referred to in the sequence, R. Therefore, there is some number q, for which R(q) = K. Consider the statement, R(q)[q]. If R(q)[q] is assumed to be provable, then it states that q belongs to K, which by the definition of K, means R(q)[q] is not provable, a contradiction. If ~R(q)[q] is assumed to be provable, then q must not belong to K. By the definition of K, this occurs if and only if R(q)[q] is provable. Once again, contradiction. Therefore, a statement exists for which neither itself nor is negation can be proven in F.[5]

Now, while such a proof may seem trivial for any system, several specifications were overlooked. First, the process of arithmetization. To be able to operate not only on numbers, but on the properties of these numbers, such as formulas, and include the infrastructure required for self-referential statements involving said properties, it was necessary for Gödel to create a resolute method of translating the formulae of a formal system into numbers which the system could operate on. Since, formulas are just a finite sequence of symbols contained in the language of the system, and proofs are just finite sequences of formulas, effectively mapping the symbols of the language to distinct numbers achieves this goal. The numbers acquired from this process are often called Gödel numbers. Arithmetization lays the groundwork for the next requirement, which Gödel's proof heavily depends on, that is, the system must support diagonalization, the property

4. It may be worth mentioning a formula in this context refers to a logical predicate of a statement where the free variable is the subject of said statement.

5. Gödel, Kurt. "On formally undecidable propositions of principia mathematica and related systems." The undecidable. Hew (1964).

which allows self-reference. Essentially, it must able to be shown that the system entails that a sentence can be constructed for any formula such that the sentence is true if and only if the formula is true when its subject is the Gödel number of the sentence. The proof of diagonalization simply requires a substitution function which maps the Gödel number of a formula with a free variable and some number, n, to the Gödel number of the formula evaluated at n. Some clever substitutions are made with this function and the Gödel number of a statement including this function to arrive at the conclusion above.[6] Finally, the proof relies on the ability to define within the considered system, the concepts used in the proof. Most notably of the proof sketch discussed here, the concept of provability must be definable.

Gödel's Second Incompleteness Theorem

We will now examine Gödel's Second Incompleteness Theorem. Now, Gödel's proof for this theorem relies on a set of conditions known as Löb's Derivability Conditions, however, adapting such restrictions may be too ambitious for epistemological applications. For this reason, we instead look at a model theoretic variation of the proof. The theorem formally states that "no sufficiently strong consistent mathematical theory can prove its own consistency."[7] This is equivalent to the statement, "It is unprovable in set theory (unless it is inconsistent) that there exists a model of set theory." To this end, we start by assuming the opposite, that set theory is consistent and does prove that a model of set theory exists. For context, a set theory is a set of sentences about the properties of sets. Now, let E be a finite set of axioms versatile enough be considered a theory

6. See note 1 above.

7. Thomas Jech, "On Gödel's Second Incompleteness Theorem," *Proceedings of the American Mathematical Society* 121, no. 1 (1994): 311–313, https://doi.org/10.2307/2160398.

and have models that can satisfy its statements. A model M of E is a set with an appropriate signature (a set of non-logical symbols, predicates, used to form statements through interpretations of the symbols as relations on M) where the sentences of E are true in M with respect to the applied signature.[8] When the set of sentences E is true in a model M with respect to its signature like this, we say that M satisfies E. If M and N are models for E, we say M < N if the binary relation (a relation made up of ordered pairs relating one element of a set to one other) on M is the same as the binary relation on a set m in N that N satisfies is true. This becomes, if M < N, then M satisfies a sentence o if and only if N satisfies that one of its elements, m, satisfies o, making m a model of E. We call this statement (1). It follows from (1) that for models X, Y, and Z of E, if X < Y, and Y < Z, then X < Z. We call this statement (2). Now, let S be the set of all Gödel numbers n, such that there is a model, N, of E which satisfies that n is not an element of the set of Gödel numbers, S_n, needed to define n. Let k be the Gödel number representing the set S and let A be the sentence "k is an element of S." It is worth noting that S is the set S_k since k is the Gödel number for S, thereby making S the set of Gödel numbers needed to define k. Therefore, by the definition of S, A says that k is an element of the set with the property that it is true in a model, N, of E that k is not in that set (essentially saying that "I am not provable by a model, N, of the axioms, E"). Thus, statement (3) says it is provable in our set of axioms E, that A is true if and only if there is a model, N, of E such that N satisfies the negation of A. From this, N satisfies that A is true if and only if there exists a model M that satisfies the negation of A. We call this statement (4). Since N is satisfying that k is in S, the set with the property that there is a model, m, of E that satisfies k is not in S, it is satisfying that a model m satisfies a sentence o (o being the sentence "k is not in S,"

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8. Hodges, Wilfrid, en Thomas Scanlon. "First-order Model Theory". The Stanford Encyclopedia of Philosophy. Metaphysics Research Lab, Stanford University, 2018. https://plato.stanford.edu/archives/win2018/entries/modeltheory-fo/.

or the negation of A), a sentence we already established was satisfied by M. Therefore, by (1), M < N, a property we include in (4). In addition, we will say that N is "positive" if it satisfies A, and "negative" otherwise. Taking the negation of (4), we have statement (4*) which says that if N is negative, then all M such that M < N are positive. Now, since we assumed E is consistent and does prove that a model of E exists, we have statement (5), "there exists a model." Also, any model of E, such as N, satisfies E, which means the axioms in E are true on N, which means N satisfies some statement m in E, which as an axiom, satisfies other statements, o, which it infers. By (1), this happens if and only if there is another model of E, M, such that M < N. So, we get that every model N admits a model M < N, our statement (6). Finally, let N_1 be a model of E. If N_1 is positive, (4) shows there must be a negative model, N_2, such that $N_2 < N_1$. There now exists a model $N_3 < N_2$ by (6), which must be positive since N_2 is negative. N_3 then admits a negative N_4 such that $N_4 < N_3$ by (4), which by (2), infers $N_4 < N_2$. By (4*), Since N_2 is negative, all models $N_n < N_2$ must be positive, but N_4 is also negative, so we reach a contradiction which can only be corrected by retracting our original assumption that a consistent set theory does prove that a model of set theory exists, a statement which must be false due to its inferred contradiction. Therefore, the original statement that no consistent system can prove its own consistency holds.[9]

Natural Laws and Systems Thereof

Now that we understand the way in which the lack of completeness and the undecidability of consistency is proven in a mathematical system, it is prudent to realize what this system really is and that it is not the only system bound by logic and theory. As humans, we tend to seek out knowledge, yet "even if we knew everything, we should still want to systematize our

9. See note 7 above

knowledge."[10] This way we have a method of organizing and effectively putting into use the knowledge we so ardently pursue. One means of achieving this is by organizing the laws we discover into deductive systems, which at their most basic, are sets of general axioms. What is special about this kind of system is that the axioms they are made up of can be used to deduce other statements. One may note that the first order theories of math we dealt with in the Gödel proofs were deductive systems consisting of axioms which could be used to deduce, or as we said, infer other theorems. However, the axioms that make up deductive systems do not have to be numeric statements or even statements regarding any structure related to numbers or math. Afterall, an axiom is just a statement that is regarded as inherently true by means of an established truth. Therefore, any natural law–a law involving properties which support relations to other properties[11]–who's truth can be reasonably established through careful, repeated observation and investigative analysis, can qualify as axioms capable of admitting a system. The difficulty comes in expressing these natural laws through a formal system as opposed to a semantically ambiguous deductive system. This will be more useful in supporting a convincing application of the incompleteness theorems.

The first requirement for a formal system is a finite set of symbols which can be used to form statements by concatenating them. Towards a system which mimics our universe (we assume a finite universe with finite subjects[12]), we first realize what sentences can be formed regarding

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9. Frank Plumpton Ramsey, *Foundations: Essays in Philosophy, Logic, Mathematics, and Economics* (Atlantic Highlands, N.J: Humanities Press, 1978), 131.

10. Chris Swoyer, "The nature of natural laws," *Australasian Journal of Philosophy* 60, no. 3 (1982): 203, https://doi.org/10.1080/0004840821234.

12. Ginzburg, Benjamin. "The Finite Universe and Scientific Extrapolation." *The Journal of Philosophy* 32, no. 4 (1935): 85–92. https://doi.org/10.2307/2016605.

the way our universe works. First, we qualify that the truth of a statement is indicative of an event being able to occur. Since an event can never occur out of true spontaneity (there must be a cause, or at least a catalyzing environment which is caused), we adopt basic predicate logic where the structure of any statement in the system claims that a certain property of a certain object can occur. To form predicates, we grant a symbol for property, "p", and symbols to denote the subject of p as an argument of p, "(" and ")"[13]. For our subjects, we use a variable symbol "x" where a subject is anything which can have the property "to exist" within our universe. We also permit iterative symbols "*" for different types (for instance, p* denotes a different property that p**), and "," for different instances of the same type through time, (for instance, x***, and x***,, are of the same type but exist at different times, or as distinct entities, as do x*,** and x**,*). Lastly, we give the traditional logical symbols, "∃" and "∀" for existential and universal quantifier, as well as "∧", "∨", and "¬" for logical conjunction, disjunction, and negation. The sentences formable from this alphabet span not only all properties of all individual objects (and existing things more broadly), but general statements of universal properties such as the laws of motion (through universal quantifiers and logical conjunction). The next requirement is a set of grammar rules to decide if a given statement is a well-formed formula of the formal system (a formula which makes sense given the definition of the symbols). Such rules are quite trivial (clearly "x)p(" wouldn't make sense), so we skip explicitly listing them. We do however note that an infinite combination of these symbols would fail to state anything meaningful about the universe (given a finite universe, infinite

13. Note that these properties are dependent on the structure of the subjects. Also, it may be countered that the use of a single symbol to denote properties, even with type and instance iterators does not effectively define said properties in a formal language. However, the use of p as opposed to a complete set of symbols for all possible properties does not compromise the validity of the formal system. This is because since the we take the universe to be finite, its parts are finite, the parts of those parts' structures are finite, and the resultant properties are finite. Therefore, both sets of symbols are finite and consequently usable in a formal system.

uses of any symbol would refer to universal elements which do not exist). Therefore, we omit such combinations from our grammar rules. Now, a set of axioms can then be formed using these symbols with the proper grammar rules where the statements created are equivalent to those we qualify as natural laws. The final characteristic is a set of inference rules to infer new formulas from the axioms. As with grammar, instead of explicitly stating such rules, we understand that since true spontaneity does not exist, an event in our universe will only occur, and thus a statement in our system will only be true, due to an identifiable cause. Taking these causes as previous statements admits rules of inference from cause to effect. We now have a formal system which competently represents the nature of our universe, the Universal System of Natural Laws if you will, hence forth referred to as U.

Application of Gödel's First Incompleteness Theorem to U

Now that we understand that our universe can be explained by natural laws which can be organized in U, we can prove concepts regarding our ability to know things about the universe by proving parallel concepts about U. First, looking back to the proof for Gödel's First Incompleteness Theorem, we recall that for the proof to be applied to a system, its axioms must either be finite or axiomatizable. If we take the axioms of U to be finite, we can move forward. However even if they were infinite, since U has finite symbols in its language and our grammar rules dictate only finite combinations of symbols can be made, an algorithm could be made to start from the first axiom in U, and list in complete that full statement and every consequent axiom in the set. This makes the set of axioms in U recursively enumerable, which by Craig's theorem makes it axiomatizable.[14] Now, the application of the proof sketch previously discussed to U is trivial. However, confirmation of the necessary conditions of arithmetization, diagonalization, and definability are not so trivial.

First is arithmetization. With the goal of giving a formal system the ability of self-reference, arithmetization in Gödel's proof took the symbols used to form formulas and convert them to the operatable element of the system, numbers. For U, we need to convert formulas to existing subjects denoted by x, as they are the only objects which can be operated on to determine truth of a sentence. We can do this quite simply by assigning a fixed, distinct, number for each symbol in the language of U. Then, for any formula f, create a subject, x, with a string of type iterators, "*", equal to the number which codes for the first symbol in f, and separate each subsequent string of type iterators (for each subsequent symbol in f) with an instance iterator, ",". If the number of type iterators required to define a formula exceeds the number of finite subjects in the universe (due to an assumed finite universe), another subject can be made repeating the same arithmetization process where to operate on the formula in coded form, the operating formula must be true for both subjects. This coding creates distinct subjects for each distinct formula allowing effective translations in both directions as required for Gödel's proof.

As for diagonalization, the lemma largely depends on mapping an arithmetized formula with a free variable and a subject to the arithmetization of the formula evaluated at the subject. This suggests no immediate concern, as creating relations between elements of our system does not change or demand anything of the makeup of the system. However, the way this relation, R, is used is by forming a formula, S(x, y, z), within our system, that is true when R relates x and y to

14. Craig, William. "On Axiomatizability within a System." *Journal of Symbolic Logic* 18, no. 1 (1953): 30–32. doi:10.2307/2266324.

z. In the context of our system, S would have to be a property of subjects x, y, and z, where, given the nature of R, x is an arithmetized formula, y is an arbitrary subject, and z is the arithmetization of x evaluated at y. However, such an S is not necessarily definable within our system. This issue is mirrored by the inability to confidently define a formula for probability after self-reference is already established. While it is certainly possible for such a property as S to exist in which it is only true of an x and y related to z through R, or a property of provability in which it is only true of an object who represents an arithmetized proof-schema, without an understanding of the machinery which predicates occurrences in the universe the way we have of arithmetic, a proof of existence such a formula's definition within our system is nearly inconceivable. This is where Gödel's ultimate perquisite, "for formal systems in which a certain amount of arithmetic can be performed," comes into play. Gödel can provide a definition of provable by chaining definitions building back to basic arithmetic definitions such as that of divisibility and prime numbers. So, in applying a purely Gödelian proof of the First Incompleteness Theorem to U, we can be optimistic with successful arithmetization and a possible incompleteness, but alas, are forced to accept a lack of adequate machinery to confidently produce self-reference.[15] We now explore a less strictly Gödelian proof for applying the second incompleteness theorem to U.

Application of Gödel's Second Incompleteness Theorem to U

Fortunately, through the model theoretic approach, the proof can be applied to any system as long as the set of axioms is finite, and it is "sufficiently strong enough to formulate the

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15. There are of course other arguments for the incompleteness of U, such as an application of Cantors theorem to demonstrate that the number of elements in a set of the universal truths observed in U will always be less than the number of subsets of U, all of which state only truth, implying truth outside of U. However, we digress, as the scope of this paper is limited to Gödel related proofs.

concepts 'model' and 'satisfies'."[16] From there, assuming the system can prove that a model of itself exists leads to a contradiction which is the same as saying the system cannot prove its own consistency. To apply this to U, we must first sacrifice the possibility of an infinite set of axioms. Luckily, the set of axioms we wish to build for U is meant to represent the natural laws, those truths obtained through careful observation and analysis which we consider to be established truths. Since the qualification of a statement as a natural law, and therefore as an axiom for our U, is that humans can establish it as true, and humans can only observe a finite number of phenomena at a given time, there can only be finite axioms in U. In addition to possessing finite axioms, U easily supports the concept "model" as any set taken with a signature which "satisfies" U when all the sentences of U are true in the model given its signature. Therefore, inability to prove consistency holds in our U, i.e., it is impossible to be sure that no natural laws, axiomatically truthful statements which describe our universe, will not end up inferring directly contradictory statements.

Implications of First and Second Incompleteness Theorems

As we previously observed, we could not effectively demonstrate a method of self-reference to our formal system of natural laws without moving past our abstract representation of U and into a language containing all workings of the universe. However, in our application of the Second Incompleteness Theorem to U (when U's axioms are limited to obtainable natural laws), we found that either U is inconsistent, equivalent to saying no satisfying model for U exists, or U does not guarantee the existence of a model to satisfy its statements. Since its statements are natural laws and models of U are sets paired with a set of relations, this is really stating that U cannot itself

16. See note 7 above.

support the fact that anything can be related in a way to form plausible statements confirming the truth of any statements in U. As opposed to a system of arithmetic where axioms can be chosen and discarded at will, and inconsistency just means $(x = 1) \wedge \sim(x = 1)$, U's axioms represent the most basic truths about our universe, and inconsistency in U means "Gravity pulls A toward B" ∧ "Gravity pushes A away from B". Therefore, either this absurdity is to be believed within our universe, or the inability for U to produce a model to verify the truth of its statements implies that the truths of U are not self-evident. Furthermore, since the truth of any statements inferred from natural laws depends on the veracity of said laws, this implies that nothing U infers is necessarily[17] true either. If we are not careful though, we can find ourselves in an indefinite paradox of repetition where we have inferred that nothing, innate or inferred, is necessarily true, including that which we have inferred, and that statement we have just formed regarding the necessity of truth of the concept of the necessity of truth, and so on and so forth. It is tempting to call this out as an absurdity; however, we recall that the statement at the heart of the paradox is our conclusion we found from the Second Incompleteness Theorem, that natural laws are not necessary, or inherent truths. This only falls apart when we take this conclusion to be a natural law, therefore inferring it might not be true, in which case natural laws are inherently true. However, we previously defined a natural law as those truths established through careful observation and investigative analysis. Where did we observe our conclusion in our universe? We did not. We instead inferred them. But then it must have been inferred from axioms. If those statements which we inferred our conclusion from were natural laws, the laws would therefore be axiomatic (as in necessary), contradicting our conclusion. Yet, looking back at our proofs, one recognizes that the only axioms used to infer our conclusion were statements regarding the function of logical structures such as models, systems,

17. Necessarily true as in true because it cannot not be so.

theories, etc. These tools of reasoning are not physically perceivable in our universe and would have no effect on it if we did not make use of them. In this way, they are not natural laws and therefore, taking them as axioms does not contradict our conclusion regarding natural laws. Not only have we solved a frightening logical dilemma and hence proved that no knowledge extractable from the universe is necessarily true, but one can interpret this as a demonstration that there is something fundamental about reason and logic that separates it from natural law. Reason is not an inherent characteristic of the universe. It is rather our ability to access logic, a web of statements, axiomatic and inferred, connected through implication, whose very existence in our universe relies solely on humans' use of it.

Turing Machines, the Halting Problem, and Application to U

One may have noticed that the last concept to examine, Alan Turing's Halting Problem has not been mentioned so far; it shall now be addressed. The problem, which he proposed in his 1936 paper, "On Computable Numbers, With an Application to The Entscheidungsproblem," simply put, is the issue of determining whether a Turing Machine, a rudimentary computer, halts on a given input or continues computing the input indefinitely. It was shown by Turing that there is no machine that can determine this characteristic. If there was such a machine m which determined whether a machine n would halt, then a machine m+ could be made which halts when m determines n does not halt, and does not halt when m determines n does halt. When fed its own program code as program code and input, m+ forces m to determine if m+ halts. By the nature of m+, if m determines that m+ halts, m+ consequently does not halt and vice versa. This contradiction proves that a machine akin to m cannot exist.[18] Fortunately, unlike the incompleteness theorems, an in-depth proof of this is not necessary for our purposes since it is conceivable that for any set of axioms S, the set can be coded into an input as a list of all axioms in S. Consider a Turing Machine T which computes all possible statements inferred in one step from the axioms of S, the set of which is called R_1, then computes all possible statements inferred in one step from the statements of R_1, the set of which can be denoted R_2, and continues to compute sets of statements, R_n, until it forms the statement s, upon which T halts. Turing's proof provides evidence that no machine M could determine if T halts. In other words, there is no algorithm to know if a statement s will be inferred from the original set of input axioms. Since the natural laws in U can be taken as axiomatic statements to form the original set S, this proves that our system is undecidable. That is, there is no way to determine if a statement regarding the nature of our universe can be inferred by its natural laws.

Implications of the Halting Problem

Now, we must realize this explanation of the Halting problem, which while sufficient to claim U's undecidability, it only does so when U is incomplete. Since the proof is not taken from the first principles of U, it does not account for a complete U, which breaks its undecidability since a complete system necessarily has a proof for every statement, which as a finite sequence of formulae, entails a point of halting for any machine T iterating towards a statement s. So, taking into consideration that we have not explicitly proven the incompleteness of U, but have explored its possibility, this new conclusion exhibits the interesting implication mirrored by arithmetic

18. Rybalov, Alexander. "On the strongly generic undecidability of the Halting Problem." *Theoretical Computer Science* 377, (2007): 268-270. https://doi.org/10.1016/j.tcs.2007.02.010.

systems that if U is incomplete, it is therefore undecidable, and so the statements of U, the events which can occur in our universe can not necessarily be shown to follow from natural laws, those which we have access to. This paired with the assumed incompleteness of U would mean that all the knowledge we could extract from our universe would never represent the totality of understanding. To better understand this concept, we can look to the thought experiment, "Mary's Room." In this experiment, Mary is restricted to a room in which all visually sensible attributes of her surroundings are black-and-white. Even though there is nothing in color for her to see, she learns every aspect of how color is perceived by eyes by means of books and videos. Once she has learned every natural law regarding how people can view color, she is finally exposed to it. The creator of the thought-experiment, Frank Jackson, argued that there was something more to learn from actually experiencing color, that she could not gain from the entire collection of natural laws describing the process of viewing color.[19] Just as Mary cannot capture all there is to know about color from understanding every aspect of it, we cannot capture all knowledge about our universe even if we understood every aspect of it.

Conclusion

In the application of Gödel's Incompleteness Theorems and Turing's Halting problem to the Universal System of Natural Laws which describe and explain the universe, we have been able to demonstrate the possibility to develop a formal system to represent the truths of our universe, establish the lack of certainty as to whether it is consistent, and discuss the implications of such a system when it is incomplete and undecidable. First, while our application of the First

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14. Frank Jackson, "What Mary Didn't Know," *The Journal of Philosophy* 83, no. 5 (1986): 291 https://doi.org/10.2307/2026143.

Incompleteness Theorem failed to prove the incompleteness of natural laws, our disproof of certain consistency demonstrated that one cannot be sure that a natural law directly implies the inexistence of a diametrically opposing statements. We cannot know that the Universal System of Natural Laws does not prove both a statement and is opposite true. From this, we explored the interpretation of natural laws and the truth thereof as inherently unnecessary, and the corollary of logic's inherent separation from the universe. Finally, our proof of undecidability demonstrated that in an incomplete universe, it is impossible to know for certain that a statement can be proven true by natural laws. In other words, there is no way to know that our understanding of the universe can be supported by anything we know. Overall, pairing the implications of the Incompleteness Theorems with that of the Halting Problem, we have shown that no discoverable truths, particularly those considered to be the most established we have access to, are innately, necessarily, certainly true, and even if they were, it is perfectly reasonable to question whether they could elucidate all there is to know about our universe.

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