2) 33. $f(x) = x^5 e^x \log_2 x \cdot \tan x$

$$f(x) = e^{\ln(f(x))} = e^{[\underbrace{5\ln x + x + \ln(\log_2 x) + \ln(\tan x)}]}$$

Hence: $$f'(x) = e^{[\ldots\ldots]} \cdot \left(\frac{5}{x} + 1 + \frac{1}{\log_2 x} \cdot \frac{1}{x \ln 2} + \overset{= \frac{\cos x}{\sin x}}{\frac{1}{\tan x}} \cdot \frac{1}{\cos^2 x} \right]$$

$$= \left(x^5 e^x \log_2 x \cdot \tan x \right) \left(\frac{5}{x} + 1 + \frac{1}{x \log_2 x \ln 2} + \frac{1}{\sin x \cos x} \right)$$

If you now find f' the "long way" - using the product rule many times, and then factor out whatever can be factored out - you get the same answer, just with a lot more work. The point of this question is to teach a quicker method for differentiating a long product / quotient.

4) c. $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

a. f is continuous at 0 by the Sandwich Theorem (we showed this, or something very similar, in class).

b. $f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{h\sin\left(\frac{1}{h}\right) - 0}{h}$

$= \lim_{h \to 0} \sin\left(\frac{1}{h}\right)$ This limit doesn't exist, since $\sin\left(\frac{1}{h}\right)$ oscillates endlessly between 1 and -1 as $h \to 0$.

So f <u>isn't</u> differentiable at 0.

c. $f'_+(0)$ and $f'_-(0)$ involve the same limit as in part b, just from one side; the limit still doesn't exist.

6) c. 2) $f(x) = x + e^x$

f is the sum of continuous functions on all $\mathbb{R}$, hence f is continuous on all $\mathbb{R}$.

f is the sum of monotonic increasing functions on all $\mathbb{R}$, hence f is monotonic increasing on all $\mathbb{R}$.

If f is continuous and monotonic it must be <u>one-to-one</u>, and thus <u>invertible</u>.

The image of f is all of $\mathbb{R}$ (try to understand why) so f^{-1} is defined on all $\mathbb{R}$.

We learned: If $y = f(x)$ then $(f^{-1})'(y) = \dfrac{1}{f'(x)}$

Note that $e+1 = f(1)$, and so:

$$(f^{-1})'(e+1) = \frac{1}{f'(1)}$$

Since $f(x) = x + e^x$, we have $f'(x) = 1 + e^x$, and so $f'(1) = 1 + e$. Hence: $(f^{-1})'(e+1) = \dfrac{1}{1+e}$

$$f\left(f^{-1}(x)\right) = x$$

$$f'\left(f^{-1}(x)\right) \cdot \left(f^{-1}\right)'(x) = 1$$

$$\left(f^{-1}\right)'(x) = \frac{1}{f'\left(f^{-1}(x)\right)}$$