Linear Algebra HW 6

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1b. $\begin{matrix} 1 & 2 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{matrix} \cdot \begin{matrix} 0 \\ -3 \\ 0 \\ 2 \end{matrix} = \begin{matrix} 1\cdot 0 + 2\cdot(-3) + (-1)\cdot 0 + 5\cdot 2 = -16 \\ 0\cdot 0 + 0\cdot(-3) + 0\cdot 0 + 0\cdot 2 = \underline{\underline{0}} \end{matrix}$

e. $\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix} \cdot \begin{matrix} -1 \\ 1 \\ 1 \end{matrix} = \begin{matrix} a\cdot(-1) + b\cdot 1 + c\cdot 1 = -a+b+c \\ d\cdot(-1) + e\cdot 1 + f\cdot 1 = -d+e+f \\ g\cdot(-1) + h\cdot 1 + i\cdot 1 = -g+h+i \end{matrix}$

2b. $\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}^{-1} = \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \cdot (-\frac{1}{3})} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & -\frac{1}{3} \end{array}\right]$

$\xrightarrow{R_1 + (-2)R_2} \left[\begin{array}{cc|cc} 1 & 0 & 1 & \frac{2}{3} \\ 0 & 1 & 0 & -\frac{1}{3} \end{array}\right] = S = \begin{pmatrix} 1 & \frac{2}{3} \\ 0 & -\frac{1}{3} \end{pmatrix}$

f. $\left.\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array}\right. \xrightarrow[R_3 + (-5)R_1]{R_2 + (-1)R_1} \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array}$

$\xrightarrow[R_3 \cdot (-\frac{1}{4})]{R_1 - R_2} \begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -\frac{1}{4} \end{array} \xrightarrow[R_1 + R_3]{R_2 + (-2)R_3} \begin{array}{ccc|ccc} 1 & 0 & 0 & 3\frac{1}{4} & -1 & -\frac{1}{4} \\ 0 & 1 & 0 & -3\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{5}{4} & 0 & -\frac{1}{4} \end{array}$

$S = \begin{pmatrix} 3\frac{1}{4} & -1 & -\frac{1}{4} \\ -3\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{5}{4} & 0 & -\frac{1}{4} \end{pmatrix}$

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3a.

$$\left[\begin{array}{ccc|c} 2 & 6 & -4 & 0 \\ 2 & 5 & -3 & -2 \\ -3 & 2 & -4 & 4 \end{array}\right] \xrightarrow[R_2+\frac{1}{2}R_3 \quad R_3+2R_2]{\text{find }=0} \begin{array}{cccc} 2 & 6 & -4 & =0 \\ \frac{1}{2} & 6 & -5 & =0 \\ 1 & 12 & -10 & =0 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 6 & -4 & 1 & 0 & 0 \\ 1/2 & 6 & -5 & 0 & 1 & 0 \\ 1 & 12 & -10 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_2-\left(\frac{1}{2}\right)R_3]{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 12 & -10 & 0 & 0 & 1 \\ 0 & 0 & -10 & 0 & 1 & -\frac{1}{2} \\ 2 & 6 & -4 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 12 & -10 & 0 & 0 & 1 \\ 2 & 6 & -4 & 1 & 0 & 0 \\ 0 & 0 & -10 & 0 & 1 & 1/2 \end{array}\right] \xrightarrow[R_3 \cdot \left(-\frac{1}{10}\right)]{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 12 & -10 & 0 & 0 & 1 \\ 0 & -18 & 16 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & -\frac{1}{10} & -\frac{1}{100} \end{array}\right]$$

$$\xrightarrow{R_2 \cdot \left(-\frac{1}{18}\right)} \left[\begin{array}{ccc|ccc} 1 & 12 & -10 & 0 & 0 & 1 \\ 0 & 1 & -\frac{8}{9} & -\frac{1}{18} & 0 & \frac{1}{9} \\ 0 & 0 & 1 & 0 & -\frac{1}{10} & -\frac{1}{100} \end{array}\right] \xrightarrow[R_1+10R_3]{R_2+\frac{8}{9}R_3} \left[\begin{array}{ccc|ccc} 1 & 12 & 0 & 0 & -1 & 0{,}9 \\ 0 & 1 & 0 & -\frac{1}{18} & -0{,}0\overline{8} & 0{,}10\overline{2} \\ 0 & 0 & 1 & 0 & -\frac{1}{10} & -\frac{1}{100} \end{array}\right]$$

$$\xrightarrow{R_1+(-12)R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{6}{9} & -2{,}0\overline{6} & -0{,}32\overline{64} \\ 0 & 1 & 0 & -\frac{1}{18} & -0{,}0\overline{8} & 0{,}10\overline{2} \\ 0 & 0 & 1 & 0 & -\frac{1}{10} & -\frac{1}{100} \end{array}\right]$$

4a. if X. Y are two matrices

$XY = YX = I$

then $A = B^{-1}$ given $B^{-1}A^{-1} = (AB)^{-1}$

multiply with AB to $B^{-1}A^{-1}$

$ABB^{-1}A^{-1} = A(BB^{-1})A^{-1} = AA^{-1} = 1$

multiply with $B^{-1}A^{-1}$ to AB yes

$B^{-1}A^{-1}AB = B^{-1}IB = B^{-1}B = 1$

$AB(B^{-1}A^{-1}) = B^{-1}A^{-1})AB = 1$

$$\underline{(AB)^{-1} = (B)^{-1}(A)^{-1}}$$

6a $A^{-1} = \left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 6 & -1 & 1 \end{array}\right] \xrightarrow{R_2 \cdot \frac{1}{6}} \left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{array}\right]$

$\xrightarrow{R_1 + R_2} \left[\begin{array}{cc|cc} 3 & 0 & \frac{5}{6} & \frac{1}{6} \\ 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{array}\right] \xrightarrow{R_1 : 3} \left[\begin{array}{cc|cc} 1 & 0 & 0{,}2\overline{7} & 0{,}0\overline{5} \\ 0 & 1 & -\frac{1}{6} & \frac{1}{6} \end{array}\right]$

7a. If A and B are invertible than (AB)C is invertible and hence AB is invertible A, B and C is invertible

11a $\left[\begin{array}{ccc|ccc} 1 & -m & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -m & 0 & 0 & 1 \end{array}\right] \longrightarrow$ all numbers w/0 $m \neq 0$

b. $\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - R_2]{R_2 - R_1} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 1 & 0 \\ 0 & 0 & -3 & 0 & -1 & 1 \end{array}\right]$

$\xrightarrow[R_3 \cdot (-\frac{1}{3})]{R_2 \cdot (-\frac{1}{2})} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & -\frac{1}{3} \end{array}\right] \xrightarrow[R_1 + (-R_3)]{R_1 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -\frac{7}{6} & -\frac{2}{3} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{3} & -\frac{1}{3} \end{array}\right]$