Linear Algebra 1—Exercise 3.5 (optional): Distances and Angles in $\mathbb{R}^3$

1) Given the points: $A=(2,1,3), B=(-4,1,-3), C=(-16,4,1), D=(0,0,-4)$, the plane $P:\ x-4y+8z-1=0$, and the line $l=\{(5,1,-2)+t(-1,-3,3): t\in\mathbb{R}\}$. What is the distance…
   1. … from A to the line BC ?
   2. … from the line AB to the line l ?
   3. … from the line AB to the line BC ?
   4. … from A to the plane P ?
   5. … from the line AC to the plane P ?
   6. … from the line BD to the plane P ?
   7. … from the plane ABC to the plane P ?
   8. … from the plane ABC to the plane whose equation is $3x+8y+9z=5$?
   9. … between the planes whose equations are $2x-3y+4z=5$ and $4x-2z=1$?
   10. … between the plane containing both A and the line $\{(0,0,2)+t(0,-3,3): t\in\mathbb{R}\}$, and the plane $x-y-z=4$?
   11. … between the lines l and $l_1=\{(0,1,-2)+t(2,0,1): t\in\mathbb{R}\}$?
   12. What is the area of triangle ΔABC ?
   13. What is the volume of the tetrahedron (pyramid with triangular base) $ABCO$ (where O is the origin)?

   [Reminder (for q. 1.13, q. 2): a. The volume of a pyramid is $\frac{1}{3}$ the product of the area of the base and the height. b. $\|\underline{u}\times\underline{v}\|=\|\underline{u}\|\cdot\|\underline{v}\|\cdot\sin\theta$, where θ is the angle between $\underline{u}$ and $\underline{v}$.]

2) Given a tetrahedron $PQRS$, where $P=(3,1,5), Q=(7,2,-1), R=(4,3,-2), D=(-4,1,2)$.
   a. What is the angle between the faces PQR and PRS ?
   b. What is the angle between edge PQ and the face QRS ?
   c. What is the distance between edges PQ and RS ?
   d. What is the volume of the tetrahedron?

3) Given a pyramid (with a quadrilateral base) $ABCDS$, where S is the apex and: $A=(-1,2,4), B=(5,5,-2), C=(7,3,-1), D=(1,0,5), S=(6,8,7)$.
   a. Is the base of the pyramid a square, rhombus, rectangle, parallelogram, or trapezoid? Prove!
   b. What is the angle between the faces ABS and BCS ?
   c. What is the angle between the faces ABS and CDS ?
   d. What is the angle between the face ABS and the base $ABCD$?
   e. What is the volume of the pyramid?
   f. What is the angle between edge AS and the base $ABCD$?

4) Given tetrahedron $ABCS$ with $A=(1,2,3), B=(-3,5,6), C=(8,-3,-1), S=(11,8,t)$ where $t<0$. SM is an altitude of $ABCS$ (M is on the plane ABC) of length $2\sqrt{35}$.

a. Find the coordinates of M.
b. What is the length of the perpendicular segment from S to the edge AC ?
c. What is the volume of the tetrahedron?

5) What are the equations of the planes perpendicular to the plane $3x-7y+4z=0$ and whose distance from the line $\{(8,6,5)+t(2,3,-5):t\in\mathbb{R}\}$ is $3\sqrt{3}$?