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Homework 3

1a. $\frac{x^3-3x^2+2}{x^2-4x+3} = \frac{x^2-2x-2}{x-3}$ $\quad \cancel{x \neq 1}$

$x \neq 3$

$$\lim_{x\to 3^-} \frac{x^2-2x-2}{x-3} = -\infty$$

$-\infty$

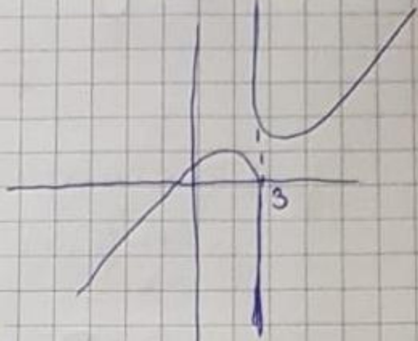
-0

$$\lim_{x\to 3^+} \frac{x^2-2x-2}{x-3} = +\infty$$

$+$

$+$

Is discontinious because x=3 doesn't exist

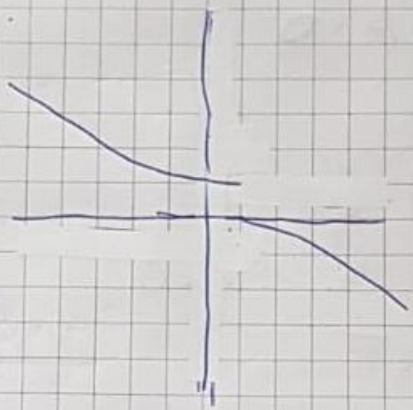


c. $\frac{1}{1+5^{\frac{1}{x-1}}}$

$x \neq 1$

$$\lim_{x\to 1^-} \left(\frac{1}{1+5^{\frac{1}{x-1}}}\right) = -\infty$$

$$\lim_{x\to 1^+} \left(\frac{1}{1+5^{\frac{1}{x-1}}}\right) = 0$$

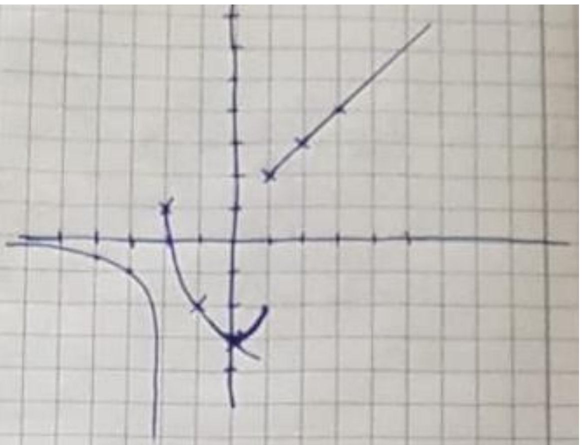


discontinious

bec. $x \neq 1$

(P2)

f. $\begin{cases} \frac{1}{x+2} & : x < -2 \\ x^2 - 3 & : -2 \leq x < 1 \\ x+1 & : x \geq 1 \end{cases}$



$\lim_{x \to -2^-} \left(\frac{1}{x+2} \right) = -\infty$

$\lim_{x \to -2^+} \left(x^2 - 3 \right) = 1$

$\lim_{x \to 1^-} \left(x^2 - 3 \right) = -2$

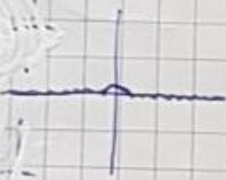
$\lim_{x \to \infty^+} (x+1) = +\infty$

g. $\frac{2x^3}{x-1}$; $x \leq 0$

$\lim_{x \to 1^-} \left(\frac{2x^3}{x-1} \right) = -\infty$

$\lim_{x \to 1^+} \left(+/- \right) = +\infty$

$\frac{\sin(x)}{x}$; $0 < x < \pi$

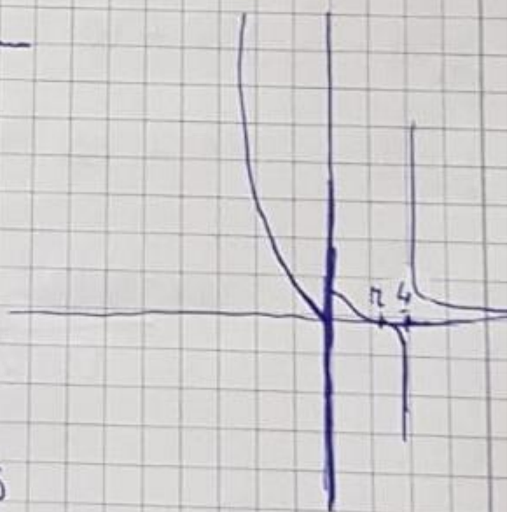


bounded

$\lim_{x \to 0^+} \left(\frac{\sin(x)}{x} \right) = 1$

$\lim_{x \to \pi^-} \left(\frac{\sin(x)}{x} \right) = 0$

$\frac{1}{x-4}$; $x > \pi$



$\lim_{x \to \pi^+} \left(\frac{1}{x-4} \right) = \frac{1}{\pi - 4} = \sim 1,16495$

$x \neq 4$

discontinuous by $x = 4$ and $x = 0$

P3

2b. $\lim_{x\to 0} \sin\frac{1}{x}$ + doesn't exist $\quad A \in x \in \{-1, 1\}$

c. $\lim_{x\to 0} x \sin\frac{1}{x} = \lim_{x\to 0}(x) = 0$

$\lim_{x\to 0} \sin\frac{1}{x} = x \in \{-1, 1\}$

$= 0 \cdot \{-1, 1\} = 0$

$A = 0$

d. $f(x) = \sin(x) \cdot \sin(\frac{1}{x})$

$= \lim_{x\to 0} \sin(x) \cdot \lim_{x\to 0} \frac{1}{x} = 0 \cdot x \in \{-1, 1\} = 0$

$A = 0$

$\exp\left(\frac{-1}{x^2}\right)$

e. $f(x) = \begin{cases} e^{\frac{-1}{x^2}} & : x \neq 0 \\ A & : x = 0 \end{cases}$

$\lim_{x\to 0} e^{\frac{-1}{x^2}}$

$\lim_{x\to 0} (e)^{\wedge} \lim_{x\to 0} -\frac{1}{x^2}$

$= e^{\wedge} -\infty = \frac{1}{e^{\infty}} = 0 \qquad A = 0$

3a. $9 = a \cdot 2 + b$

$11 = a \cdot 3 + b$

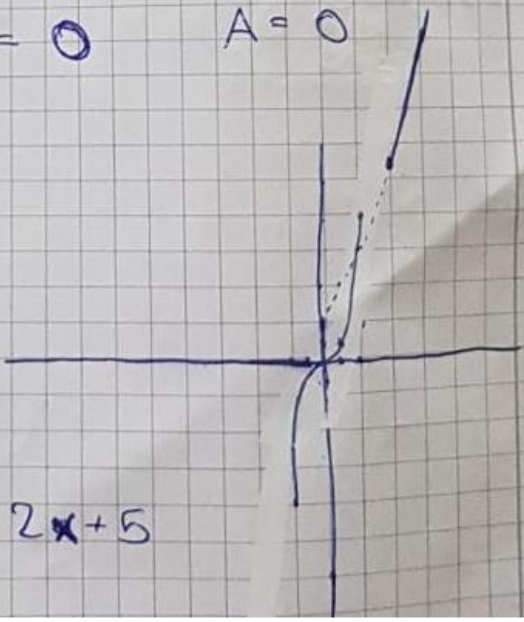
$-$

$2 = 3a + b - 2a - b$

$2 = a$

$b = 5$

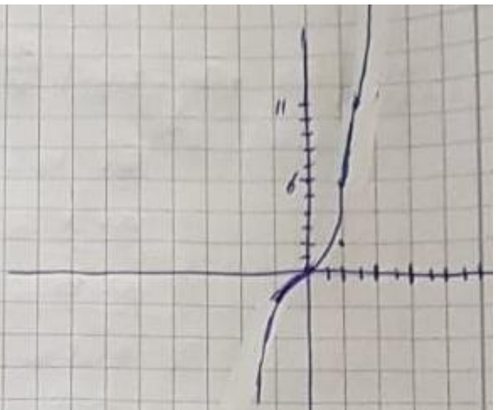
$f(x) = 2x + 5$



b. $ax^3 \; : x < 2$

$ax^2 + bx \; : 2 \leq x < 3$

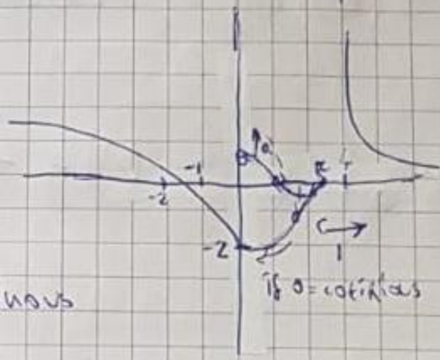
$x^2 + ax \; : x \geq 3$



$a = 0{,}75$ (2, 6)

$b = 1{,}48$ (3; 11,..)

4 a. 1



b. 1 by point x=0
that is a jump discontinuous

1 between ℝ and over
that is a mixed discountinuns

5b $$\lim_{x \to 0} \left(\frac{\frac{d}{dx}(\arcsin(3x))}{\frac{d}{dx}(\tan(5x))} \right) = \lim_{x \to 0} \left(\frac{\frac{3}{\sqrt{1-9x^2}}}{5\sec(5x)^2} \right)$$

$$= \lim_{x \to 0} \left(\frac{3}{5\sqrt{1-9x^2}\sec(5x)^2} \right)$$

$$= \frac{3}{5\sqrt{1-9 \times 0^2}\sec(5 \times 0)^2} = \frac{3}{5}$$

d. $\lim_{x \to \infty} \left(5x \cdot \arctan\left(\frac{3}{x}\right)\right)$

$\arctan\left(\lim_{x \to \infty}\left(\frac{3}{x}\right)\right)$

$\arctan\left(3 \lim_{x \to \infty}\left(\frac{1}{x}\right)\right)$

$\arctan(3 \cdot 0)$

$\arctan(0)$

$\lim_{x \to \infty} (5x \cdot 0) = 0$

$= (\infty \cdot 0) \leftarrow$?Problem!

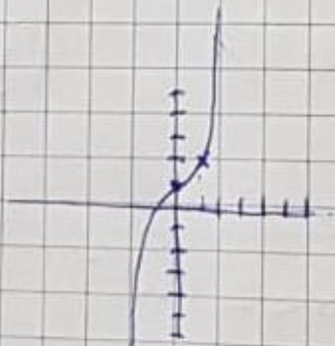
f. $\lim_{x \to -\infty} \left(\arctan(x)\right) = -\frac{\pi}{2}$

7a. $\frac{x^5 - 4x^3 + 5x^2 + 3x + 7}{a} = 0$

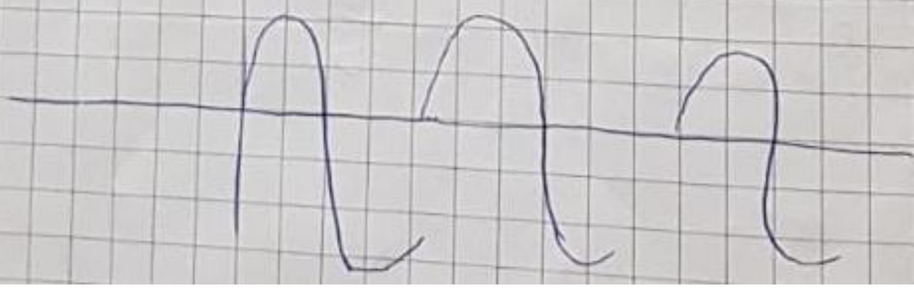
$a \neq 0$

$a \neq -2{,}49245\ldots$

b. $x^2 + 1$

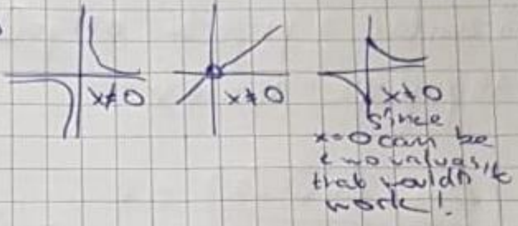


d.



12a. Not true, since $\frac{\sin(x)}{x}$ is not defined by 0 but continuous on both sides

b. yes. Since we say. the function is discontinuous on $x=0$ and all types of discontinuous, that none of them will have a none exist number. That particular lies here on $x=0$

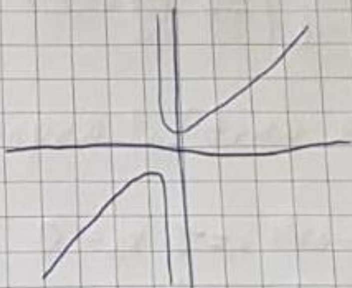


16 c. no touch with x axis

on y-axis: $f(0)=\frac{9}{4}$

$x \in \mathbb{R} \setminus \{-\frac{4}{3}, 2\}$

$x \neq -\frac{4}{3} \quad x \neq 2$



if

$\lim_{x \to -\frac{4}{3}} \left(\frac{2x^3+x-18}{3x^2-2x-8}\right) \to$ doesn't exist $\to x = -\frac{4}{3}$

$\lim_{x \to 2} \left(\frac{2x^3+x-18}{3x^2-2x-8}\right) \to \frac{5}{2} \to$ no existing vertical asymptote

vertical asymptote: $-\frac{4}{3}$

horizontal asymptote: not any

tilt asymptote: $y = \frac{2}{3}x + \frac{4}{9}$

$\lim_{x \to \infty} \left(\frac{\frac{2x^3+x-18}{3x^2-2x-8}}{x}\right) = \frac{2}{3} \quad \lim \left(\frac{2x^3+x-18}{3x^2-2x-8} - \frac{2}{3}x\right) = \frac{4}{9}$

$= y = \frac{2}{3}x + \frac{4}{9}$

e. $x =$ doesn't touches x-axis
$y = f(0)\ \sqrt{7}$

$\lim_{x \to \infty} \left(2x + \sqrt{4x^2+7}\right) = \infty$

$\lim_{x \to -\infty} \left(2x + \sqrt{4x^2+7}\right) = 0$ $= 0$

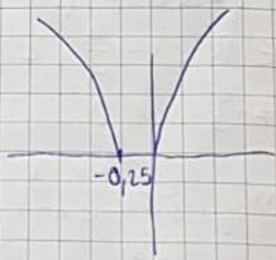
horizontal asymptote on $y = 0$

tilt asymptote: $y = 4x$

$\lim_{x \to \infty} \left(\frac{2x + \sqrt{4x^2+7}}{x}\right) = 4$ $= y = 4x + 0$

$\lim_{x \to -\infty} \left(\frac{2x + \sqrt{4x^2+7}}{x}\right) = 0$ $= y = 4x$

no vertical asymptote



g. x-axis: $x_1 = -\frac{1}{4}$ $x_2 = 0$

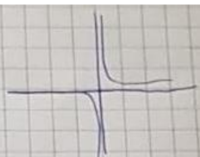
y-axis: $f(0) = 0$

no horizontal asymptote

tilt asymptote by:

$y = 2x + \frac{1}{4}$

$y = -2x - \frac{1}{4}$



i. no x-axis

no y-axis

vertical asymptote by

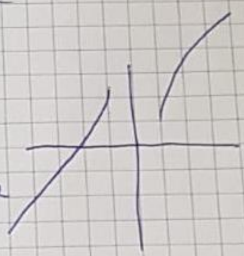
$y = 1$

$y = 0$

$\lim_{x \to \infty} \left(\frac{e^x}{e^x - 1}\right) = 1$

$\lim_{x \to -\infty} \left(\frac{e^x}{e^x - 1}\right) = 0$

no tilt asymptote



j. no y-axis

no horizontal asymptote