Homework 1 Linear Algebra

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P.1

HW1 Leonard Blam

1b.

row eshelon matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 8 \end{array}\right] \xrightarrow[R_3 \to R_3 + (-R_2)]{R_2 \to R_2 + (-R_1)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 7 \end{array}\right] \xrightarrow{R_2 \to R_2 \cdot \frac{1}{2}}$$

$$\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 7 \end{array} \xrightarrow[R_2 \to R_2 + \frac{1}{2}R_3]{R_1 \to R_1 + R_2} \begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{array} \xrightarrow{R_1 \to R_1 + (-\frac{1}{2}R_3)} \begin{array}{ccc|c} 1 & 0 & 0 & -4,5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{array}$$

$$(-4{,}5,\ 3,\ 7)$$

e.

$$\begin{aligned} w + 2x + y - z &= 1 \\ 4w - x + 2y + z &= 0 \end{aligned} \qquad \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 4 & -1 & 2 & 1 & 0 \end{array}$$

$$\xrightarrow{R_2 \to R_2 + (-4R_1)} \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & -9 & -2 & 5 & -4 \end{array} \xrightarrow{R_2 \to -\frac{1}{9}R_2} \begin{array}{cccc|c} 1 & 2 & 1 & -1 & 1 \\ 0 & 1 & \frac{2}{9} & -\frac{5}{9} & \frac{4}{9} \end{array}$$

$$\xrightarrow{R_1 \to R_1 + (-2R_2)} \begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & -\frac{4}{9} & \frac{8}{9} \\ 0 & 1 & \frac{2}{9} & -\frac{5}{9} & \frac{4}{9} \end{array} \longrightarrow \begin{array}{cccc|c} 1 & 0 & \frac{1}{3} & -\frac{4}{9} & \frac{8}{9} \\ 0 & 1 & \frac{2}{9} & -\frac{5}{9} & \frac{4}{9} \end{array} \qquad \begin{aligned} s &= y \\ t &= z \end{aligned}$$

$$w + \tfrac{1}{3}s - \tfrac{4}{9}t = \tfrac{8}{9} \quad \Big| + \tfrac{4}{9}t - \tfrac{1}{3}s$$

$$w = \tfrac{8}{9} + \tfrac{4}{9}t - \tfrac{1}{3}s$$

$$x + \tfrac{2}{9}s - \tfrac{5}{9}t = \tfrac{4}{9} \quad \Big| - \tfrac{2}{9}s + \tfrac{5}{9}t$$

$$x = \tfrac{4}{9} - \tfrac{2}{9}s + \tfrac{5}{9}t$$

$$= \left(\tfrac{8}{9} + \tfrac{4}{9}t - \tfrac{1}{3}s,\ \tfrac{4}{9} - \tfrac{2}{9}s + \tfrac{5}{9}t,\ s,\ t\right)$$

R2

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2b. No, it can happen that it has no solution

ex. as we did on class

2d.

2i. yes, if one of the equation will get to $0,0\ldots0|n$

2j. No, ~~since an equation of~~ there are just three ways that could happen: no solution, one unique sol. or many solution like we learned in class

3c.
$$\left(\begin{array}{ccc|c} 3 & 2 & -3 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right) \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 1R_1}]{R_1 \to R_1/3} \left(\begin{array}{ccc|c} 1 & \frac{2}{3} & -1 & \frac{1}{3} \\ 0 & -\frac{7}{3} & 3 & -\frac{2}{3} \\ 0 & \frac{1}{3} & 2 & \frac{5}{3} \end{array}\right) \xrightarrow[R_3 \to R_3 - \frac{1}{3}R_2]{R_1 \to R_1 - \frac{2}{3}R_2} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{7} & \frac{1}{7} \\ 0 & 1 & -\frac{9}{7} & \frac{2}{7} \\ 0 & 0 & 1 & \frac{11}{17} \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 / \frac{17}{7}} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{7} & \frac{1}{7} \\ 0 & 1 & -\frac{9}{7} & \frac{2}{7} \\ 0 & 0 & 1 & \frac{11}{17} \end{array}\right) \xrightarrow[R_2 \to R_2 - (\frac{-9}{7})R_3]{R_1 \to R_1 - (-\frac{1}{7}R_3)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{4}{17} \\ 0 & 1 & 0 & \frac{19}{17} \\ 0 & 0 & 1 & \frac{11}{17} \end{array}\right)$$

1g.
$$\left(\begin{array}{cccc|c} 1 & 2 & 6 & -1 & -3 \\ -1 & 1 & -1 & 1 & 3 \\ 2 & -1 & -3 & 3 & 9 \end{array}\right) \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 + R_1} \left(\begin{array}{cccc|c} 1 & 2 & 6 & -1 & -3 \\ 0 & 3 & 5 & 0 & 0 \\ 0 & 1 & -5 & 5 & 15 \end{array}\right) \leftrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 6 & -1 & -3 \\ 0 & 1 & -5 & 5 & 15 \\ 0 & 3 & 5 & 0 & 0 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + (-3R_2)} \left(\begin{array}{cccc|c} 1 & 2 & 6 & -1 & -3 \\ 0 & 1 & -5 & 5 & 15 \\ 0 & 0 & 20 & -15 & -45 \end{array}\right) \xrightarrow[\substack{R_2 \to R_2 + (\frac{1}{4}R_3) \\ R_3 \to R_3 + 3\frac{6}{5}R_2}]{R_1 \to R_1 + (-2R_2)} \left(\begin{array}{cccc|c} 1 & 0 & 16 & -11 & -33 \\ 0 & 1 & 0 & -9\frac{3}{4} & -29\frac{3}{4} \\ 0 & 0 & 1 & 4 & 12 \end{array}\right) \xrightarrow{R_1 \to R_1 + 16R_3}$$

$y + 4 = 12 \quad | -4$

$y = 8$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 53 & 159 \\ 0 & 1 & 0 & -9\frac{3}{4} & -29\frac{3}{4} \\ 0 & 0 & 1 & 4 & 12 \end{array}\right)$$

$z = t$

$w + 53 = 159 \quad | -53$

$w = 106$

$x + (-9\frac{3}{4}) = -29\frac{3}{4}$

$x - 9\frac{3}{4} = -29\frac{3}{4} \quad | + 9\frac{3}{4}$

$x = -20$

$= \underline{(106, -20, 8, t)}$

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3.1. $\begin{matrix} 0 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{matrix} \xrightarrow{R_1 \to R_1 \cdot \frac{1}{2}} \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{matrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{matrix}$

$\xrightarrow{R_3 \to R_3 - R_2} \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

5 $\begin{matrix} 1 & k & k^2 \\ k & k^2 & 1 \\ k^2 & 1 & k \end{matrix}$ if $k=0$ $\begin{matrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}$ ×

if $k=1$ $\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$ ×

if $k=2$ $\begin{matrix} 1 & 2 & 4 \\ 2 & 4 & 1 \\ 4 & 1 & 2 \end{matrix} \xrightarrow[R_3 \to R_3 - 4R_1]{R_1 \to R_1 + \frac{1}{4}R_3,\ R_2 \to R_2 - 2R_1} \begin{matrix} 0 & 1\frac{3}{4} & 3\frac{1}{2} \\ 0 & 0 & -7 \\ 0 & -7 & -14 \end{matrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{matrix} 0 & 1\frac{3}{4} & 3\frac{1}{2} \\ 0 & 1\frac{3}{4} & 0 \\ 0 & -7 & -14 \end{matrix}$

$\xrightarrow[R_3 \to R_3 + 2R_2]{R_1 \to R_1 - R_2} \begin{matrix} 0 & 0 & 3\frac{1}{2} \\ 0 & 1\frac{3}{4} & 0 \\ 0 & 0 & -14 \end{matrix}$ ×

$\begin{matrix} 1 & k & k^2 \\ k & k^2 & 1 \\ k^2 & 1 & k \end{matrix} \Rightarrow 2x^3 - x^6 - 1$

8a. $\begin{matrix} 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 1 \\ a & 1-a & 2-a & 1-a \end{matrix}$

$\frac{5}{2}$ $\frac{8}{2}$

$\xrightarrow{R_1 \leftrightarrow R_2} \begin{matrix} -1 & 2 & 1 & 1 \\ 2 & 1 & 0 & -1 \\ a & 1-a & 2-a & 1-a \end{matrix} \xrightarrow[R_2 + 2R_1]{R_1 + R_2} \begin{matrix} 1 & 3 & 1 & 0 \\ 0 & 5 & 2 & 1 \\ a & 1-a & 2-a & 1-a \end{matrix} \xrightarrow[R_2 \cdot \frac{1}{5},\ R_3 + (-a)R_1]{R_1 - \frac{3}{5}R_2} \begin{matrix} 1 & 0 & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 1 & \frac{2}{5} & \frac{4}{5} \\ 0 & 1-4a & 2-2a & 1-a \end{matrix}$

$\xrightarrow{R_3 \to R_3 + (-1+4a)R_2} \begin{matrix} 1 & 0 & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 1 & \frac{2}{5} & \frac{4}{5} \\ 0 & 0 & \frac{2}{5}(4-x) & \frac{1}{5} - \frac{11}{5}a \end{matrix}$

$9a\ a = 2$

9b.

$$\begin{matrix} 2 & 1 & 0 & -1 \\ -1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{matrix} \xrightarrow{R_1 \Leftrightarrow R_3} \begin{matrix} 1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 1 \\ 2 & 1 & 0 & -1 \end{matrix}$$

$$\xrightarrow[R_3 \Rightarrow R_3 - 2R_1]{R_2 \Rightarrow R_2 + R_1} \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & -2 & -1 \end{matrix} \xrightarrow{R_2 \to R_2 - R_3} \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & -2 & -1 \end{matrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -2 & -3 \end{matrix}$$

$$\xrightarrow{R_3 \to R_3 \div 3R_1} \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{matrix} \quad R_1 \to R_1 - R_3 \quad \left.\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right| \begin{matrix} -3 \\ 2 \\ -3 \end{matrix}$$

$x = -3$
$y = 2$
$z = -3$

$x = (-3, 2, -3)$

1a.

$$\left.\begin{matrix} 1 & 1 & m \\ 1 & m & 1 \\ m & 1 & 1 \end{matrix}\right| \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 \cdot \frac{1}{m} \end{matrix} \quad \left.\begin{matrix} 1 & 1 & m \\ 0 & m-1 & 1-m \\ 1 & \frac{1}{m} & \frac{1}{m} \end{matrix}\right| \begin{matrix} 1 \\ 0 \\ \frac{1}{m} \end{matrix}$$

$$\xrightarrow[R_2 - \frac{1}{m} \cdot R_1]{R_1 \Leftrightarrow R_3} \left.\begin{matrix} m & 1 & 1 \\ 0 & \frac{m^2-1}{m} & \frac{m-1}{m} \\ 1 & 1 & m \end{matrix}\right| \begin{matrix} 1 \\ \frac{m-1}{m} \\ 1 \end{matrix} \xrightarrow{R_3 - \frac{1}{m} \cdot R_1} \left.\begin{matrix} m & 1 & 1 \\ 0 & \frac{m^2-1}{m} & \frac{m-1}{m} \\ 0 & \frac{m-1}{m} & \frac{m^2-1}{m} \end{matrix}\right| \begin{matrix} 1 \\ \frac{m-1}{m} \\ \frac{m-1}{m} \end{matrix} \xrightarrow{R_2 \Leftrightarrow R_3} \left.\begin{matrix} m & 1 & 1 \\ 0 & \frac{m^2-1}{m} & \frac{m-1}{m} \\ 0 & \frac{m-1}{m} & \frac{m^2-1}{m} \end{matrix}\right| \begin{matrix} 1 \\ \frac{m-1}{m} \\ \frac{m-1}{m} \end{matrix}$$

$$\xrightarrow{R_3 \Rightarrow R_3 - (m+1)R_2} \left.\begin{matrix} m & 1 & 1 \\ 0 & \frac{m-1}{m} & \frac{m^2-1}{m} \\ 0 & 0 & (m-1)(m+2) \end{matrix}\right| \begin{matrix} 0 \\ \frac{m-1}{m} \\ -m+1 \end{matrix} \xrightarrow{i} \left.\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right| \begin{matrix} \frac{1}{m+2} \\ \frac{1}{m+2} \\ \frac{1}{m+2} \end{matrix}$$

$i: = \left(\frac{1}{m+2}, \frac{1}{m+2}, \frac{1}{m+2}\right)$

iii $\downarrow$ $R_3 \cdot m$ if $m = 0$ or $m = 1$ or $m = -2$

$$\left.\begin{matrix} m & 1 & 1 \\ 0 & \frac{m-1}{m} & \frac{m^2-1}{m} \\ 0 & 0, & 0 \end{matrix}\right| \begin{matrix} 0 \\ \frac{m-1}{m} \\ (1) \text{ or } (0) \text{ or } (3) \end{matrix}$$

iii = no solution if $m = 1$ or -2

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10d.
$$\left[\begin{array}{ccc|c} 1 & m+1 & 1 & 2m \\ 1 & -1 & m & -1 \\ 1 & m & -1 & 2m+1 \end{array}\right] \xrightarrow{\substack{R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2}} \left[\begin{array}{ccc|c} 0 & m & 1-m & 2m-1 \\ 1 & -1 & m & -1 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -1 & m & -1 \\ 0 & m & 1-m & 2m-1 \\ 0 & 1 & -2 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 \cdot \frac{1}{m}} \left[\begin{array}{ccc|c} 1 & -1 & m & -1 \\ 0 & 1 & \frac{1}{m}\cdot 1-m & 2m-1\cdot\frac{1}{m} \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -1 & m & -1 \\ 0 & 1 & \frac{1}{m}-m & 2m-\frac{1}{m} \\ 0 & 0 & -2-\frac{1}{m}-m & 1-2m-\frac{1}{m} \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & m+1 & 1 & 2m \\ 0 & -m-2 & m-1 & -1-2m \\ 0 & 0 & \frac{3m+3}{-m-2} & \frac{-3m-3}{-m-2} \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & m \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

ii $\Rightarrow$ many solution $(m, 1, -1)$

iii

no solution if

$m = -2$

$m = -1$

11. 1.

$$\left[\begin{array}{ccc|c} 4k & 2k^2 & -1 & 4 \\ 0 & 4k & -1 & k \\ 0 & 16k & -(3k^2+k) & k^2-5 \end{array}\right]$$

if $k=0$
$$\left[\begin{array}{ccc|c} 0 & 0 & -1 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

X

a = wrong

reduced echelon matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-3k^4+22k^2+11k-22}{8k(k-1)(3k+4)} \\ 0 & 1 & 0 & \frac{3k^3+5}{4k(3k^2+k-4)} \\ 0 & 0 & 1 & \frac{4k-k^2+5}{-4+3k^2+k} \end{array}\right]$$

if $k=1$ $\quad \frac{-3+22+11-22}{8(0)\,0}$

X

b = wrong since if k=1 the equation would be an error

and c = wrong since if it's k=5 it has solution

the result is

d

(P. 6)

11.2. is $k = -\frac{4}{3}$ $\quad \frac{-3(-\frac{4}{3})^4 + 22(-\frac{4}{3})^2 + 11(\frac{4}{3}) - 22}{8(-\frac{4}{3})(-\frac{4}{3}-1)(3(-\frac{4}{3})+4)} = \text{Error}$

can't devide with 0

× if $k = 5$ $\quad \frac{-3(5)^4 + 22(5)^2 + 11 \cdot 5 - 22}{8(5)(5-1)(3(5)+4} = \frac{-1292}{3040}$

has a solution

$$\frac{3(5)^3 + 5}{4(5)(3(5)^2 + 5 - 4)} = \frac{1}{4}$$

$$\frac{4(5) - 5^2 + 5}{-4 + 3(5)^2 + 5} = 0$$

result: a. is wrong, since $k=0$ and $k=1$ has no solution

b. is wrong, since $k=5$ has a solution

d. is wrong, since $k=1$ has no solution

must be b.!