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Calculus 1 HW 6

P1

$\int x^n dx = \frac{1}{n+1} x^{n+1}$

$\left(\frac{1}{n+1} x^{n+1}\right)' = x^n$

$x^{-1} = \ln(x)$

not integral form $\left(\sqrt[n]{x} = x^{\frac{1}{n}} \right.$, $\left. \sqrt{x} = x^{\frac{1}{2}} \right)$

1\. 1. $\int \left(2x^3 + \frac{3}{x}\right) \quad \int 2x^3 dx + \int \frac{3}{x} dx$

$= 2\int x^3 dx + 3 \int x^{-1} dx$

$= 2 \cdot \frac{1}{4} x^4 + 3 \ln(x) + c$

2\. $\int \frac{x^2 + 4x - 1}{x} dx = \int \left(x + 4 - \frac{1}{x}\right) dx$

$= \int x \, dx + \int (4) dx - \int \frac{1}{x} dx$

$= \frac{x^2}{2} + 4x - \ln(x) + c$

3\. $\int \frac{(x+3)^2}{x} dx = \int \frac{(x+3)(x+3)}{x} dx = \int \frac{x^2 + 3x + 3x + 9}{x}$

$= \int x \, dx + \int 6 \, dx + \int \frac{9}{x} dx$

$= \frac{x^2}{2} + 6x + 9\ln(x) + c$

5\. $\int \frac{x^2}{x^2+1} dx = \int \frac{x^2+1-1}{x^2+1} dx = \int \frac{x^2+1}{x^2+1} dx \int \frac{1}{x^2+1} dx$

$= \int 1 \, dx - \int \frac{1}{x^2+1} dx = x + \arctan(x) + c$

6\. $\int \sqrt[3]{x} \left(\sqrt{x} + 2\right) dx = \int \sqrt[3]{x} \left(\sqrt{x}\right) dx + \int \sqrt[3]{x} (2) dx$

$= \int \left(x^{\frac{1}{3}} \cdot x^{\frac{1}{2}} + x^{\frac{1}{3}} \cdot 2\right) dx$

$= \int x^{\frac{5}{6}} dx + 2\int x^{\frac{1}{3}} dx$

$= \frac{1}{\frac{5}{6}+1} \cdot x^{1+\frac{5}{6}} + \frac{2}{1+\frac{1}{3}} x^{1+\frac{1}{3}} + C$

$= \frac{6}{11} x^{\frac{11}{6}} + \frac{3}{2} x^{\frac{4}{3}} + C$

8. $\int \cos^2\left(\frac{x}{2}\right) dx$

$\int \frac{\cos x + 1}{2} \, dx = \frac{1}{2}\int \cos x \, dx + \int \frac{1}{2} \, dx$

$= \frac{1}{2}\sin x + \frac{1}{2}x + C$

10. $\int \tan^2 x \, dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx$

$= \int \left(\frac{1}{\cos^2 x}\right) - \int (1) \, dx$

$= \tan x - x + C$

2. 3. $\int \frac{3}{x^2 + 16} \, dx = \frac{1}{4}\arctan\frac{x}{4} + C$

$\underset{4^2}{16}$

$= -\frac{1}{4}\arctan\frac{x}{4} + C$

4. $\int \frac{x^4}{x^2 + 4} \, dx = \int x^2 - 4 + \frac{16}{x^2+4} \, dx$

$\underset{2^2}{4}$

$= \int x^2 \, dx - \int 4 \, dx + \int \frac{16}{x^2+4} \, dx + C$

$= \frac{x^3}{3} - 4x + 8\arctan\frac{x}{2} + C$

8. $\int \sin 2x \cos 3x \, dx$

$= \int \frac{1}{2}\sin(2x - 3x) + \sin(2x + 3x)) \, dx$

$= \int \frac{1}{2}\sin(-x) + \frac{1}{2}\sin(5x) \, dx$

$= \int \frac{1}{2}(-\sin x) \, dx + \int \frac{1}{2}\sin 5x \, dx$

$= \frac{1}{2}\cos(x) + \int \frac{1}{2}\sin 5x \, \frac{1}{5} \, d5x = \frac{1}{2}\cos x + \frac{1}{10}\int \sin 5x \, ds$

$= \frac{1}{2}\cos x + \frac{1}{10}(-\cos 5x) +$

25, 26, 27, 30 ... 17, 19, 21

(23) $t = \sqrt[3]{x^2+7}$

11. $\int \frac{3t}{2}\,dt = \frac{3}{2}\int t\,dt = \frac{3}{2}\cdot\frac{t^2}{2} + C$

$= \frac{3}{2}\cdot\frac{\sqrt[3]{x^2+7}^{\,2}}{2} + C = \frac{3\sqrt[3]{(x^2+7)^2}}{4} + C$

13. $\int \frac{e^x\,dx}{1+e^x} = \cancel{\int \frac{de^x}{1+e^x} = \int \frac{d\alpha}{1+\alpha} = \int \frac{1}{1+\alpha}\,d\alpha}$

$= \frac{d(e^x+1)}{e^x+1} = \ln(|e^x+1|) + C$

$t = 3 + \cos(2x)$

15. $\int -\frac{1}{2t}\,dt = -\frac{1}{2}\cdot\int \frac{1}{t}\,dt = -\frac{1}{2}\ln(|t|)$

$= -\frac{1}{2}\ln(|3+\cos(2x)|) + C$

19. $\int \frac{1}{\sqrt{x}(\sqrt{x}+3)}\,dx = \int \frac{2}{t}\,dt \qquad t = \sqrt{x}+3$

$= 2\ln(|t|) = 2\ln(|\sqrt{x}+3|)$

$= 2\ln(\sqrt{x}+3) + C$

21. $\int \frac{2x+3}{(x^2+3x+1)^{10}}\,dx = \int \frac{1}{t^{10}}\,dt \qquad t = x^2+3x+1$

$= \boxed{\int \frac{1}{x^n}\,dx = -\frac{1}{(n-1)\cdot x^{n-1}}} = -\frac{1}{9t^9} = \frac{1}{9(x^2+3x+1)^9} + C$

$t = \cos(x) \quad t = -\sin(x)$

25. $= \int \sin(x)\cos(x)^5 \cdot \frac{1}{-\sin(x)}\,dt$

$= \int -\cos(x)^5\,dt = \int -t^5\,dt = -\int t^5\,dt = -\frac{t^6}{6}$

$= \frac{\cos(x)^6}{6} + C$

26 $\int \tan(x)\,dx$ $\quad t = -\frac{1}{t}\,dt$

$$= \int \frac{\sin(x)}{\cos(x)}\,dx = \int -\frac{1}{t}\,dt = -\int \frac{1}{t}\,dt$$

$$= -\ln(|t|) = -\ln(|\cos(x)|) + C$$

28. $t = \cos(x)$

$$\int -t^2 + t^4\,dt = -\int t^2\,dt + \int t^4\,dt$$

$$= -\frac{t^3}{3} + \frac{t^5}{5}$$

$$= \frac{\cos(x)^3}{3} + \frac{\cos(x)^5}{5} + C$$

30. $t = x + 3$

$$\int t^2\sqrt{t} - 6t\sqrt{t} + 9\sqrt{t}\,dt$$

$$= \int t^2\, t^{\frac{1}{2}} - 6t \cdot t^{\frac{1}{2}} + 9t^{\frac{1}{2}}\,dt$$

$$= \int t^{\frac{5}{2}} - 6t^{\frac{3}{2}} + 9t^{\frac{1}{2}}\,dt$$

$$= \int t^{\frac{5}{2}}\,dt - \int 6t^{\frac{3}{2}}\,dt + \int 9t^{\frac{1}{2}}\,dt$$

$$= \frac{2t^3\sqrt{t}}{7} - \frac{12t^2\sqrt{t}}{5} + 6t\sqrt{t}$$

$$= \frac{2(x+3)^3\sqrt{x+3}}{7} - \frac{12(x+3)^2\sqrt{x+3}}{5} + 6(x+3)\sqrt{x+3}$$

$$= \frac{2\sqrt{x+3}(x^3 + 9x^2 + 27x + 27)}{7} - \frac{12\sqrt{x+3}(x^2 + 6x + 9)}{5} + 6(x+3)\sqrt{x+3} + C$$

3) 3. $u = x \Rightarrow du = dx$
$dv = e^{-x}dx \quad v = -e^{-x}$

$= x(-e^{-x}) - \int -e^{-x}\,dx$

$= x(-e^{-x}) + \int e^{-x}\,dx$

$= x(-e^{-x}) - e^{-x}$

$= -\frac{x+1}{e^{x}} + C$

6. $\int \ln(x)\frac{1}{x^2}\,dx$

$u = \ln(x) \Rightarrow du = \frac{1}{x}dx$

$dv = \frac{1}{x^2}\,dx \quad v = -\frac{1}{x}$

$= \ln(x)\left(-\frac{1}{x}\right) - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx$

$= \ln(x)\left(-\frac{1}{x}\right) + \int \frac{1}{x^2}\,dx$

$= \ln(x)\left(-\frac{1}{x}\right) - \frac{1}{x}$

$= -\frac{\ln(x)}{x} - \frac{1}{x} = -\frac{\ln(x)+1}{x} + C$

$\int x \cdot \frac{1}{\cos(x)^2}\,dx$

$u = x$
$dv = \frac{1}{\cos(x)^2}\,dx$

$= x\tan(x) - \int \tan(x)\,dx \qquad t = \cos(x)$

$= x\tan(x) - \int \frac{\sin(x)}{\cos(x)}\,dx$

$= x\tan(x) - \int -\frac{1}{t}\,dt$

$= x\tan(x) + \ln(|t|)$

$= x\tan(x) + \ln(|\cos(x)|) + C$

14. $u = 0{,}5t$

$= \int 8 u^2 \cdot \sin(u)\,du$

$= 8 \cdot \int u^2 \cdot \sin(u)\,du$

$= 8\left(u^2(-\cos(u)) - \int -\cos(u)\, 2u\,du\right)$

$= 8\left(u^2(-\cos(u)) + 2\int u\cos(u)\,du\right)$

$= 8\left(u^2(\cos(u)) + 2(u\sin(u) - (-\cos(u))\right)$

$= 8\left((0{,}5t)^2(-\cos(0{,}5t)) + 2(0{,}5t\sin(0{,}5t) - (\cos(0{,}5t))\right) + C$

17. $\int e^{\sqrt{x}}\,dx \qquad t = \sqrt{x}$

$= \int 2te^t\,dt$

$= 2\int te^t\,dt$

$= 2\left(te^t - \int e^t\,dt\right)$

$= 2(te^t - e^t)$

$= 2(\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}})$

$= 2e^{\sqrt{x}}\sqrt{x} - 2e^{\sqrt{x}} + C$

20. $\int e^x \sin(2x)\,dx$

$= \int \sin(2x)\, e^x\,dx$

$= \sin(2x)\,e^x - \int e^x \cos(2x)\cdot 2\,dx$

$= \sin(2x)\,e^x - 2\cdot\int e^x \cos(2x)\,dx$

$= \sin(2x)\,e^x - 2\int \cos(2x)\,e^x\,dx$

$= \sin(2x)\,e^x - 2\left(\cos(2x)e^x - \int e^x(-\sin(2x))\,2\,dx\right)$

$= \int e^x \sin(2x)\,dx = \sin(2x)e^x - 2\cos(2x)e^x - 4\int e^x \sin(2x)\,dx$

$= 5\int e^x \sin(2x)\,dx = \sin(2x)e^x - 2\cos(2x)e^x$

$= \int e^x \sin(2x)\,dx = \frac{\sin(2x)e^x}{5} - \frac{2\cos(2x)e^x}{5}$

$= \frac{\sin(2x)e^x - 2e^x\cos(2x)}{5} + C$

Herlitz

(P7)

4) 2. $\int \frac{4x^2-3x-4}{x(x-1)(x+2)}\,dx$

$= \int \frac{2}{x} - \frac{1}{x-1} + \frac{3}{x+2}\,dx$

$= \int \frac{2}{x}\,dx - \int \frac{1}{x-1}\,dx + \int \frac{3}{x+2}\,dx$

$= 2\ln(|x|) - \ln(|x-1|) + 3\ln(|x+2|) + C$

3. $= \int -\frac{1}{5x} + \frac{6x+25}{5(x^2-25)}\,dx$

$= \int \frac{1}{-5x}\,dx + \int \frac{6x+25}{5(x^2-25)}\,dx$

$= -\frac{1}{5}\ln(|x|) + \frac{3}{5}\ln(|x^2-25|) + \frac{1}{2}\ln\left(\left|\frac{x-5}{x+5}\right|\right) + C$

5. $\int 2x + \frac{-6x+3}{x^2+1}\,dx = \int 2x\,dx + \int \frac{-6+3}{x^2+1}\,dx$

$= x^2 - 3\ln(x^2+1) + 3\arctan(x) + C$

7. $\int \frac{x+3}{x^2+6x+13}\,dx = \int \frac{1}{2t}\,dx$ $\quad t = x^2+6x+13$

$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln(|t|) = \frac{1}{2}\ln(|x^2+6x+13|) + C$

(P8)

13. $\int \frac{3x+2}{x^2+4x+5} - \frac{2}{x+1}\,dx = \int \frac{3x+2}{x^2+4x+5} - \int \frac{2}{x+1}\,dx$

$= \frac{3}{2}\ln(|x^2+4x+5|) - 4\arctan(x+2) - 2\ln(|x+1|) + C$

14. $\int -\frac{1}{12(x-1)} + \frac{13}{12(x+5)} - \frac{7}{2(x+5)^2}\,dx$

$= -\int \frac{1}{12(x-1)}\,dx + \int \frac{13}{12(x+5)}\,dx - \int \frac{7}{2(x+5)^2}\,dx$

$= -\frac{1}{12}\ln(|x-1|) + \frac{13}{12}\ln(|x+5|) + \frac{7}{2x+10} + C$

18. $\int 1 + \frac{-8x-2}{x^2+4x+5}\,dx = \int 1\,dx + \int \frac{-8x-2}{x^2+4x+5}\,dx$

$= x - 4\ln(|x^2+4x+5|) + 14\arctan(x+2) + C$

5,2. $\int \frac{\sin(2x - \cos(x)}{\sin(x)^2 - 2\sin(x)}\,dx = \int \frac{\sin(2x)}{\sin(x)^2 - 2\sin(x)} - \frac{\cos(x)}{\sin(x)^2 - 2\sin(x)}\,dx$

$= \int \frac{2\cancel{\sin(x)}\cos(x)}{\cancel{\sin(x)}(\sin(x)-2)} - \frac{\cos(x)}{\sin(x)^2 - 2\sin(x)}\,dx$

$= \int \frac{2\cos(x)}{\sin(x)-2}\,dx - \int \frac{\cos(x)}{\sin(x)^2 - 2\sin(x)}\,dx$

$= 2\ln(|\sin(x)-2|) + \frac{1}{2}\ln(|\tan(\frac{x}{2})|) - \frac{1}{2}\ln(|\tan(\frac{x}{2})^2 - \tan(\frac{x}{2}) + 1|)$
$+ C$

(B)

$t = e^x$

4. $3\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = 3\int \frac{1}{\sqrt{1-t^2}}\,dt$

$3\arctan(t) = 3\arctan(e^x) + C$