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Calculus 1

Homework 4

1 b $\frac{3}{2x-5}$

$$f'(x) = \frac{3'(2x-5) - 3\cdot(2x-5)'}{(2x-5)^2}$$

$$= \frac{0 - 3\cdot((2x)' - 5')}{(2x-5)^2}$$

$$= \frac{-3(2-0)}{(2x-5)^2} = \frac{-6}{(2x-5)^2}$$

1 d. $= \sqrt{(3-4x)}$ $\qquad g(c(x))$

$= (3-4x)^{\frac{1}{2}}$ $\qquad g(c) = c^{\frac{1}{2}}$

$c(x) = 3-4x$

$$f'(x) = c'^{\,\frac{1}{2}} \cdot (3-4x)'$$

$$= \frac{1}{2} c^{-\frac{1}{2}} \cdot (0-4)$$

$$= \frac{1}{2} (3-4x)^{-\frac{1}{2}} \cdot (-4) = \frac{-2}{\sqrt{3-4x}}$$

2. 14. $\sin^5 3x$

$f(g) = g^5$ $\quad f'(x) = (g^5)' \cdot (\sin c)' \cdot (3z)'$

$g(c) = \sin c$

$c(z) = 3z$ $\quad f'(x) = 5g^4 \cdot \cos c \cdot 3$

$f'(x) = 5\sin^4 3x \cdot \cos 3x \cdot 3$

$= 15 \sin^4 3x \cdot \cos 3x$

16. $f(x) = \tan(\sin(x))$

$\cos(x) \cdot \frac{1}{\cos^2(\sin(x))}$

17. $f'(x) = \cos(\cot\sqrt{x})$

$\frac{1}{2\sqrt{x}} \cdot \frac{-1}{\sin^2(\sqrt{x})} \cdot (-\sin(\cot(\sqrt{x}))$

$$= \frac{\sin(\cot(\sqrt{x}))}{2\sqrt{x} \cdot \sin^2(\sqrt{x})}$$

19. $f(x) = \sqrt{\cos(\cot(x))}$

$\frac{1}{2\sqrt{-\sin(\cot(x))}} \cdot -\frac{1}{\sin^2(x)}$

$$= -\frac{1}{2\sqrt{-\sin(\cot(x))}\sin(x)^2}$$

24. $\arcsin \frac{\sqrt{x^2-1}}{x}$ $\quad \frac{(\sqrt{x^2-1})' \cdot x - (\sqrt{x^2-1}) \cdot (x)'}{x^2}$ $\quad \frac{1}{\sqrt{x^2-1}} \cdot 2x \cdot x$

$$\frac{1}{\sqrt{1-\left(\frac{\sqrt{x^2-1}}{x}\right)^2}} \cdot \frac{\frac{1}{\sqrt{x^2-1}} \cdot 2x \cdot x - \sqrt{x^2-1}}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{x^2-1}} - \frac{\sqrt{x^2-1}}{1}}{x^2} = \frac{\frac{x^2}{\sqrt{x^2-1}} - \frac{x^2-1}{\sqrt{x^2-1}}}{x^2}$$

$$= \frac{\frac{1}{\sqrt{x^2-1}}}{x^2} = \frac{1}{x^2\sqrt{x^2-1}}$$

$$\frac{1}{\sqrt{1-\frac{x^2-1}{x^2}}} \cdot \frac{1}{x^2\sqrt{x^2-1}} = \frac{|x|}{x^2\sqrt{x^2-1}}$$

25. $f(x)\, x^2 - 3x$

$2x - 3$ where $x \leq 1$

$x^3 - 2x^2 - 1$

$3x^2 - 2 \cdot 2x - 0$

$= 3x^2 - 4x$ where $-2 < x < 1$

$-2x^2 + 12x + 9$

$-2 \cdot 2x + 12 + 0$

$= -4x + 12$ where $x \leq -2$

P4

27. $f(x) = \frac{|x^2-1|}{x-1}$

$$\frac{(|x^2-1|)'(x-1)-(|x^2-1|)\cdot(x-1)'}{(x-1)^2}$$

$$= \frac{|2x|\cdot(x-1)-|x^2-1|\cdot 1}{x^2-1}$$

$$= \frac{2\cdot|x|\cdot(x-1)-|x^2-1|}{x^2-1}$$

27, 30, 33, 35, 37

30. $\ln(1+|x|)$ $\quad g(x)=1+|x|$

$= |1|$

$$\frac{1}{1+|x|}\cdot|1|$$

33. $\frac{\sqrt{x}\cdot\cos(x)}{\sqrt[5]{x^2-3x}\cdot\ln x}$

$$\frac{(\sqrt{x}\cdot\cos(x))'\cdot(\sqrt[5]{x^2-3x}\cdot\ln(x))-(\sqrt{x}\cdot\cos(x))\cdot(\sqrt[5]{x^2-3x}\cdot\ln(x))'}{(\sqrt[5]{x^2-3x}\cdot\ln(x))^2}$$

$$\frac{1}{2\sqrt{x}}\cdot -\sin(x)$$

$$\frac{\left(\frac{1}{2\sqrt{x}}\cdot-\sin(x)\right)\cdot\sqrt[5]{x^2-3x}\cdot\ln(x)-(\sqrt{x}\cdot\cos(x))\cdot\left(\frac{5}{2\sqrt{x^2-3x}}\cdot\frac{1}{x}\right)}{(\sqrt[5]{x^2-3x}\cdot\ln(x))^2}$$

35. $(3x^2-2x)^{5x+3}$

$g(h)= h^{(5x+3)} = h^5$

$h(x)= 3x^2-2x$

$3\cdot 2x-2$

$$(3x^2-2x)^5\cdot 6x-2$$

37. $(\ln(x))^{\arctan(x)}$

$g(h) = h^{\arctan(x)} = h^{\frac{1}{1+x^2}} =$

$h(x) = \ln(x) = \frac{1}{x}$

$$(\ln(x))^{\frac{1}{1+x^2}}\cdot\frac{1}{x}$$

3b. $\frac{(3x^2-5x+2)^5}{(4-3x)^7} = \frac{((3x^2-5x+2)^5)' \cdot (4-3x)^7 - (3x^2-5x+2)^5 \cdot ((4-3x)^7)'}{((4-3x)^7)^2}$

$= \frac{5(3x^2-5x+2)^4 \cdot (4-3x)^7 - (3x-5x+2)^5 \cdot 7(4-3x)^6}{(4-3x)^{14}}$

$= \frac{(3x^2-5x+2)^4(-27x^2+90x-58)}{(4-3x)^8}$

d. $(2x-3)\sqrt{4x-x^2} = \frac{2x-3}{2\sqrt{4x-x^2}} \cdot 2\sqrt{4x-x^2}$

$= \frac{2x-3}{2\sqrt{4x-x^2}} \cdot \frac{2\sqrt{4x-x^2} \cdot 2\sqrt{4x-x^2}}{2\sqrt{4x-x^2}}$

$= \frac{\cancel{2}(-4x^2+15x-6)}{\cancel{2}\sqrt{4x-x^2}}$

6.
4c. a. is continious on 0

$\lim_{x \to 0^-} x\sin\left(\frac{1}{x}\right) = 0$

$\lim_{x \to 0^+} x\sin\left(\frac{1}{x}\right) = 0$

$x\sin\left(\frac{1}{x}\right)$
$= \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

d. a. is discontinious on 0^- & 0^+
is $x \neq 0$

$x^2 \sin\left(\frac{1}{x}\right)$
$= x\cos\left(\frac{1}{x}\right) \cdot \frac{1}{x^2}$

$\frac{(1)'x-(1)\cdot(x)'}{x^2} = \frac{-1}{x^2}$

$\lim_{x \to 0} = x^2 \sin\left(\frac{1}{x}\right) = 0$

$\lim_{x \to 0^-} = x^2 \sin\left(\frac{1}{x}\right) = -\infty$

$\lim_{x \to 0^+} = x^2 \sin\left(\frac{1}{x}\right) = +\infty$

P6

a. $x^3 \sin\left(\frac{1}{x}\right)$

$\lim_{x \to 0^-} = x^3 \sin\left(\frac{1}{x}\right) = 0$

$\lim_{x \to 0^+} = x^3 \sin\left(\frac{1}{x}\right) = \pm\infty$

is discontinuous on 0^+

if $x \neq 0$

$f'(x) = 3x^2 \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$= -3\cos\left(\frac{1}{x}\right)$

5a. $x^2 - 3x$ $\quad f'(x) = 2x - 3$

$\frac{dy}{dx} = \frac{d(x^2-3x)}{d(2x-3)}$

c. $x^3 - 6x^2 + 9x$ $\quad x(x^2 - 6x + 9)$

$\frac{dy}{dx} = \cancel{d(x^3 - 6x^2} \; \frac{d\left(x(x^2-6x+9)\right)}{d(2x-6)}$

6 a. $x_0 = 3$

$f(3) = 5$

$f'(3) = -2$

b. $f(x) = \frac{3}{2x-4}$ $\quad f^{-1}(x) = \frac{3+4x}{2x}$

$f'(x) = \frac{6}{(2x-4)^2}$

7b. $y = x + \arctan(xy^2)$

$h = 2xy$

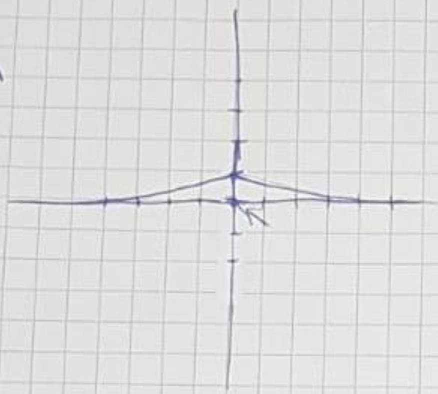
$f' = 1 + \frac{1}{1+(xy)^2} \cdot 2xy$

$g = \arctan(h) = \frac{1}{1+h^2}$

~~f 1 arctan~~

d.

9



14 c. $\left(\dfrac{\frac{d}{dx}(\ln(x)^2)}{\frac{d}{dx}(\sqrt[3]{x})}\right) = \lim_{x\to\infty}\left(\dfrac{\frac{2\ln(x)}{x}}{\frac{1}{3\sqrt[3]{x^2}}}\right) = \left(\dfrac{6\ln(x)\cdot\sqrt[3]{x^2}}{x}\right)$

$= \left(\dfrac{\frac{d}{dx}(6\ln(x)\sqrt[3]{y^2})}{\frac{d}{dx}(x)}\right) = \left(\dfrac{\frac{4\ln(x)+6}{\sqrt[3]{x}}}{1}\right) = \left(\dfrac{\frac{d}{dx}(4\ln(x)+6)}{\frac{d}{dx}(\sqrt[3]{x})}\right)$

$= \left(\dfrac{\frac{4}{x}}{\frac{1}{3\sqrt[3]{x^2}}}\right) = \left(\dfrac{12\sqrt[3]{x^2}}{x}\right) = \left(\dfrac{\frac{d}{dx}(12\sqrt[3]{x^2})}{\frac{d}{dx}(x)}\right) = \left(\dfrac{\frac{8}{\sqrt[3]{x}}}{1}\right)$

$= \left(\dfrac{8}{\sqrt[3]{x}}\right) = \lim_{x\to\infty}(8) = 8$

$\lim_{x\to\infty}(\sqrt[3]{x}) = +\infty$ $\quad \frac{8}{\infty} = \underline{\underline{0}}$

h. $= \left(\dfrac{x \cdot \left(1 + \frac{\sin(x)}{x}\right)}{x \cdot \left(2 - \frac{3\sin(x)}{x}\right)} \right) = \left(\dfrac{1 + \frac{\sin(x)}{x}}{2 - \frac{3\sin(x)}{x}} \right)$

$= \dfrac{1+1}{2-3 \cdot 1} = \dfrac{2}{-1} = \underline{\underline{-2}}$

i. $\int x + \sin(x) \left(\dfrac{1 + \frac{\sin(x)}{x}}{2 - \frac{3\sin(x)}{x}} \right)$

$= \dfrac{\lim\limits_{x \to \infty} (1) + \lim\limits_{x \to \infty} \left(\frac{\sin(x)}{x}\right)}{\lim\limits_{x \to \infty} (2) - \lim\limits_{x \to \infty} \left(\frac{3\sin(x)}{x}\right)}$

$= \dfrac{1 + 0}{2 - 3 \cdot \lim\limits_{x \to \infty} \left(\frac{\sin(x)}{x}\right)} = \dfrac{1}{2 - 3 \cdot 0} = \underline{\underline{\dfrac{1}{2}}}$

15a. $f'(x) = 2x + 1 = 2$ true

$f'(x) = 5 = 0$

b. $f'(x) = 2x + 1 = 2$ true

$\quad \ldots = 7 = 0$

c. $f(x) = \dfrac{2x^2 - 6x}{x - 3}$ $\quad \dfrac{(2x^2 - 6x)'(x-3) - (2x^2 - 6x)(x-3)'}{(x-3)^2}$

$= \dfrac{(4x - 6)(x - 3) - (2x^2 - 6x)(1)}{x^2 - 9}$

$= 4x^2 - 12x - 6x + 18 - 2x^2 - 6x$

$$\lim_{x \to \infty} \left(\frac{2x^2 - 24x + 18}{x^2 - 9} \right) = 2$$

above one is true but bottom one is wrong

$f'(x)\ 4 = 0$

d. $f'(x) = x^2 - 4x \qquad = 2x - 4$ true

$f'(x) = 6 - 3x \qquad = -3$

e. $x^2 - 6x \qquad = 2x - 6$ true

$6 - 2x \qquad = -2$

f. $x^2 - 6x = 2x - 6$ true

$-2x - 4 = -2$