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Linear Algebra HW 5

1a. does not make an basis

$$\begin{bmatrix} 4 & -2 & 1 \\ 1 & 2 & 4 \\ 5 & 0 & 5 \end{bmatrix} \xrightarrow{R_1 \div 4} = \begin{bmatrix} 1 & -\frac{2}{4} & \frac{1}{4} \\ 1 & 2 & 4 \\ 5 & 0 & 5 \end{bmatrix} \xrightarrow{R_1 \cdot \text{coeff} -1 + R_2} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{4} \\ 0 & 2+\frac{1}{2} & 4-\frac{1}{4} \\ 5 & 0 & 5 \end{bmatrix}$$

$$\xrightarrow[R_1 \cdot \text{coeff} -5 + R_3]{R_2 : \frac{5}{2}} \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{4} \\ 0 & \frac{5}{2} & \frac{15}{4} \\ 0 & \frac{5}{2} & \frac{15}{4} \end{bmatrix} \xrightarrow{R_2 \cdot \text{coeff} \frac{1}{2} + R_1} \begin{bmatrix} 1 & 0 & \frac{1}{4}+\frac{3}{2\cdot 2} \\ 0 & 1 & \frac{3}{2} \\ 0 & \frac{5}{2} & \frac{15}{4} \end{bmatrix} \xrightarrow{R_2(-\frac{5}{2})R_3} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{15}{4}-\frac{5\cdot 3}{2\cdot 2} \end{bmatrix}$$

= number of lin. independent rows or cols of matrix is equal to 2. Therefore the rank is less than number of initial vectors. Therefore they don't form an independent base.

c.

$$\begin{pmatrix} -4 & 11 & 12 & 8 \\ 2 & 5 & 6 & 4 \\ 1 & 2 & 3 & 2 \\ 1 & 1 & -1 & 1 \end{pmatrix} \xrightarrow{R_1 : -4} \begin{matrix} R_2+(-2)R_1 \\ R_3+(-1)R_1 \\ R_4+(-1)R_1 \end{matrix} \begin{pmatrix} 1 & -\frac{11}{4} & -3 & -2 \\ 0 & \frac{21}{2} & 12 & 8 \\ 0 & \frac{19}{4} & 6 & 4 \\ 0 & 1+\frac{11}{4} & -2 & 3 \end{pmatrix} \xrightarrow{R_1 + \frac{11}{4}R_2} \begin{matrix} R_2 : \frac{21}{2} \\ R_3 \cdot (-\frac{19}{4})R_2 \\ R_4 \cdot (-\frac{15}{4})R_2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & \frac{1}{7} & \frac{2}{21} \\ 0 & 1 & \frac{8}{7} & \frac{16}{21} \\ 0 & 0 & \frac{4}{7} & \frac{8}{21} \\ 0 & 0 & 2-\frac{15\cdot 8}{7\cdot 7} & 3-\frac{15\cdot 16}{8\cdot 4} \end{pmatrix} \begin{matrix} R_3 : \frac{4}{7} \\ R_1 \cdot (-\frac{1}{7})R_3 \\ R_2 \cdot -\frac{8}{7}R_3 \\ R_4 \cdot (-\frac{16}{7})R_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{7}+\frac{32}{21} \end{pmatrix} \begin{matrix} R_4 : \frac{2}{3} \\ R_3 + (-\frac{2}{3})R_4 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

number of linear independent is 4. That is equal to n. of vectors.

3a.

$$\begin{pmatrix} 3 & 6 & -3 \\ 7 & 1 & 2 \\ 10 & 4 & 5 \end{pmatrix} \xrightarrow{R_1 \cdot \frac{1}{3}} \begin{pmatrix} 1 & 2 & -1 \\ 7 & 1 & 2 \\ 10 & 4 & 5 \end{pmatrix} \xrightarrow[R_3+(-10)R_1]{R_2+(-7)R_1} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -13 & 9 \\ 0 & -16 & 15 \end{pmatrix}$$

$$\xrightarrow{R_2 \cdot (-\frac{1}{13})} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -\frac{9}{13} \\ 0 & -16 & 15 \end{pmatrix} \xrightarrow{R_3+16R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -\frac{9}{13} \\ 0 & 0 & \frac{51}{13} \end{pmatrix} \xrightarrow{R_3 \cdot \frac{13}{51}} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -\frac{9}{13} \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow[R_1+R_3]{R_2 + \frac{9}{13}R_3} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1+(-2)R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \text{linear independent}$$

1.

$$\begin{vmatrix} 0 & 1 & 0 & 2 & 1 \\ -2 & 1 & 1 & 1 & 0 \\ 5 & -3 & -1 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \end{vmatrix} \xrightarrow[R_2 \cdot (-\frac{1}{2})]{R_1 \leftrightarrow R_2} \begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 5 & -3 & -1 & 0 & -1 \\ 1 & 0 & 0 & -1 & 0 \end{vmatrix} \xrightarrow[R_4+(-1)R_1]{R_3+(-5)R_1}$$

$$\begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & -\frac{1}{2} & \frac{3}{2} & \frac{5}{2} & -1 \\ 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \end{vmatrix} \xrightarrow[R_4+(-\frac{1}{2})R_2]{R_3+\frac{1}{2}R_2} \begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & \frac{3}{2} & \frac{7}{2} & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \end{vmatrix} \xrightarrow{R_3 \cdot \frac{2}{3}}$$

$$\begin{vmatrix} 1 & -//- & -//- & & \\ & -//- & & -//- & \\ 0 & 0 & 1 & \frac{7}{3} & -\frac{1}{3} \\ & -//- & & -//- & \end{vmatrix} \xrightarrow{R_4+(-\frac{1}{2})R_3} \begin{vmatrix} & -//- & \\ & -//- & \\ 0 \; 0 & 0 & -\frac{8}{3} \; \frac{1}{3} \end{vmatrix} \xrightarrow{R_4 \cdot (-\frac{3}{8})} \begin{vmatrix} & -//- & \\ & -//- & \\ & -//- & \\ 0 \; 0 & 0 & 1 \; \frac{1}{8} \end{vmatrix}$$

$$\xrightarrow[\substack{R_2+(-2)R_4 \\ R_1+(\frac{1}{2})R_4}]{R_3+(-\frac{7}{3})R_4} \begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{16} \\ 0 & 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & 0 & -\frac{5}{8} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{vmatrix} \xrightarrow{R_1 + \frac{1}{2}R_3 + \frac{1}{2}R_2} \begin{vmatrix} 1 & 0 & 0 & 0 & \frac{1}{8} \\ 0 & 1 & 0 & 0 & \frac{5}{6} \\ 0 & 0 & 1 & 0 & -\frac{5}{8} \\ 0 & 0 & 0 & 1 & \frac{1}{8} \end{vmatrix}$$

= linear combination

4a. 1. a ~~for~~ makes all $a \in \mathbb{R}$ a linear combination

2.

6b.

$$w \rightarrow \begin{matrix} 1 & -2 & -5 & 3 \\ 2 & 3 & -1 & -4 \\ 3 & 6 & -3 & -5 \\ 4 & 8 & -2 & -7 \end{matrix} \xrightarrow[\substack{R_3+(-3)R_1 \\ R_4+(-4)R_1}]{R_2+(-2)R_1} \begin{matrix} 1 & -2 & -5 & 3 \\ 0 & 7 & 9 & -10 \\ 0 & 12 & 12 & -14 \\ 0 & 16 & 18 & -19 \end{matrix} \xrightarrow{R_2 : 7}$$

$$\begin{matrix} 1 & -2 & -5 & 3 \\ 0 & 1 & \frac{9}{7} & -\frac{10}{7} \\ & -//- & & \end{matrix} \xrightarrow[\substack{R_3+(-12)R_2 \\ R_4+(-16)R_2}]{R_1+2R_2} \begin{matrix} 1 & 0 & -\frac{17}{7} & \frac{1}{7} \\ 0 & 1 & \frac{9}{7} & -\frac{10}{7} \\ 0 & 0 & -\frac{24}{7} & \frac{22}{7} \\ 0 & 0 & -\frac{18}{7} & \frac{23}{7} \end{matrix} \xrightarrow{R_3 : \left(-\frac{24}{7}\right)}$$

$$\begin{matrix} & -//- & & \\ & -//- & & \\ 0 & 0 & 1 & -\frac{11}{12} \\ 0 & 0 & -\frac{18}{7} & \frac{23}{7} \end{matrix} \xrightarrow[\substack{R_2+\left(-\frac{9}{7}\right)R_3 \\ R_4+\left(\frac{18}{7}\right)R_3}]{R_1+\left(\frac{17}{7}\right)R_3} \begin{matrix} 1 & 0 & 0 & -\frac{25}{12} \\ 0 & 1 & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{11}{12} \\ 0 & 0 & 0 & \frac{23}{7} - \frac{18 \cdot 11}{12 \cdot 7} \end{matrix} \xrightarrow{R_4 : \frac{3}{2}}$$

$$\begin{matrix} & -//- & & \\ & -//- & & \\ & -//- & & \\ 0 & 0 & 0 & 1 \end{matrix} \xrightarrow[\substack{R_2+\frac{1}{4}R_4 \\ R_3+\frac{11}{12}R_4}]{R_1+\frac{25}{12}R_4} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \Rightarrow$$

Therefore makes a basis

$w = (4 \quad 8 \quad -2 \quad -7)$

7b.

$$\begin{matrix} -2 & -3 \\ -1 & 12 \end{matrix} \xrightarrow{R_1 : -2} \begin{matrix} 1 & \frac{3}{2} \\ -1 & 12 \end{matrix} \xrightarrow{R_2+R_1} \begin{matrix} 1 & \frac{3}{2} \\ 0 & \frac{27}{2} \end{matrix} \xrightarrow{R_2 : \frac{27}{2}} \begin{matrix} 1 & \frac{3}{2} \\ 0 & 1 \end{matrix}$$

$$\xrightarrow{R_1+\left(-\frac{3}{2}\right)R_2} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \rightarrow \text{base}$$

c.

$$\begin{matrix} 5 & -1 \\ -1 & 12 \end{matrix} \xrightarrow{R_1 : 5} \begin{matrix} 1 & -\frac{1}{5} \\ -1 & 12 \end{matrix} \xrightarrow{R_2+R_1} \begin{matrix} 1 & -\frac{1}{5} \\ 0 & 12-\frac{1}{5} \end{matrix} \xrightarrow{R_2 : \left(12-\frac{1}{5}\right)} \begin{matrix} 1 & -\frac{1}{5} \\ 0 & 1 \end{matrix}$$

$$\xrightarrow{R_1+\left(-\frac{1}{5}\right)R_2} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} = \text{basis}$$