Linear Algebra 1—Exercise 3: Lines and Planes in $\mathbb{R}^n$

To submit: 1 c, e 2 a 3 b 4 b 5 a, c, e, g 6 a 7 b 11 a 13 c, f 14 a 15 c, e 16 d 18 19

1) Find a parametric representation of each line:
   a. $y=20x$ b. $y=-3x$ c. $y=3x-7$ d. $3x-2y-9=0$ e. $y=0$ f. $x=-2$

2) Find the Cartesian representation of each line:
   a. $\{(2t,t):t\in\mathbb{R}\}$ b. $\{(1,4)+t(-2,-5):t\in\mathbb{R}\}$

3) Find a parametric representation of the line in $\mathbb{R}^3$ which goes through each pair of points:
   a. (1,2,4), (0,-2,3) b. (0,0,0), (2,7,6) c. (1,4,2), (5,-2,1)

4) Find parametric and Cartesian representations of each line in $\mathbb{R}^2$:
   a. The line through (0,-2) and in the direction of the vector (2,-3).
   b. The line through (0,5) and parallel to the line $\{(-5,1)+t(1,-2):t\in\mathbb{R})\}$.
   c. The line through (3,2) and parallel to the x-axis.

5) Find a parametric representation of each line in $\mathbb{R}^3$:
   a. The line through (-1,0,2) and parallel to the vector (0,-2,1)
   b. The line through the origin and parallel to the line $\{(1,4,6)+t(2,-1,2)\mid t\in\mathbb{R})\}$.
   c. The line through (3,7,0) and parallel to the x-axis.
   d. The line through (0,1,2) and parallel to the z-axis.
   e. The line through (9,3,2) and parallel to the y-axis.
   f. The z-axis
   g. The y-axis.

6) Determine the relative position of each pair of lines (identical, intersecting, parallel, or skew). If they intersect, find the point of intersection.
   a. $\{(1,3,5)+s(7,1,-3)\mid s\in\mathbb{R}\}$, $\{(4,6,7)+t(-1,0,2):t\in\mathbb{R}\}$
   b. $\{(3,5,6)+t(8,-3,1):t\in\mathbb{R}\}$, $\{(2,6,0)+s(8,-8,10)\mid s\in\mathbb{R}\}$
   c. $\{(-13,1,2)+t(12,6,3):t\in\mathbb{R}\}$, $\{(-1,3,1)+s(4,1,0):s\in\mathbb{R}\}$

7) Do the points P_1,P_2,P_3 lie on the same line? Explain!
   a. $P_1=(6,9,7),P_2=(9,2,0),P_3=(0,-5,-3)$
   b. $P_1=(1,0,1),P_2=(3,-4,-3),P_3=(4,-6,-5)$

8) Find a parametric representation of the line which goes through the point of intersection of the lines $l_1=\{t(1,4,2):t\in\mathbb{R}\}$ and $l_2=\{(2,4,-8)+s(0,1,1):s\in\mathbb{R}\}$ and is parallel to

the line through the points $(2,-1,4)$ and $(3,-6,0)$. If these lines do not intersect, show that they don't.

9) Given the two lines $L_1=\{(3,4,-2)+s(1,1,0):s\in\mathbb{R}\}$ and $L_2=\{t(-6,3,-6):t\in\mathbb{R}\}\ldots$
   a. Show that these lines do not intersect.
   b. Find the angle between the lines (i.e., the angle formed by their direction vectors).

10) Given the two lines $l_1=\{(1+2s,2-s,4-2s):s\in\mathbb{R}\}$ and $l_2=\{(9+t,5+3t,-4-t):t\in\mathbb{R}\}\ldots$
   a. Show that these lines intersect.
   b. Find the acute angle between the lines.
   c. Find a parametric representation of the line perpendicular to l_1 and l_2 which goes through their point of intersection.

11) Find where the line $L=\{(1,2,3)+t(0,-1,-2):t\in\mathbb{R}\}$ intersects…
   a. the *x-y* plane. b. the *x-z* plane. c. the *y-z* plane.

12) Where does the line $L=\{(2,0,3)+t(0,1,-2):t\in\mathbb{R}\}$ intersect the plane whose equation is $x+y-2z=0$?

13) Find a Cartesian equation and a parametric representation of the plane which…
   a. … goes through the point $(0,2,-1)$ and has normal $3\vec{i}-2\vec{j}-\vec{k}$.
   b. … goes through the point $(2,2,1)$ and is perpendicular to the vector $(1,3,2)$.
   c. … goes through the point $(0,-1,3)$ and is parallel to the plane with equation $3x+y+z=7$.
   d. … goes through the point $(-2,4,-2)$ and is parallel to the plane with equation $2x-3y+5z=2$.
   e. … goes through the points $(1,1,-1),(1,0,2),(3,-2,1)$.
   f. … goes through the points $(-2,0,1),(0,2,3),(1,0,-1)$.
   g. … goes through the point $(2,4,0)$ and is perpendicular to the line $L=\{(5,1,0)+t(1,2,4):t\in\mathbb{R}\}$.
   h. … goes through the point $(2,4,0)$ and is parallel to the *x-y* plane.
   i. … goes through the point $(2,4,5)$ and is parallel to the *y-z* plane.
   j. … goes through the origin and in parallel to the plane with equation $3x-2y+2z+12=0$.

k. … contains the line $L_1 = \{(5,1,0) + t(1,1,4) : t \in \mathbb{R}\}$ and is parallel to the line $L_2 = \{(3,2,2) + t(-1,2,5) : t \in \mathbb{R}\}$.

l. … contains the line $L = \{(-2,4,3) + t(3,1,-1) : t \in \mathbb{R}\}$ and is perpendicular to the plane with equation $x - 2y + z = 6$.

14) Find a parametric representation of the line of intersection of each pair of planes:

a. $-2x + 3y + 7z = -2$, $x + 2y - 3z = -5$

b. $3x + -5y + 2z = 0$, $x + z = 0$

c. $2x - y + 2z = 0$, $y + z = 0$.

15) For each plane in $\mathbb{R}^3$ given in parametric form, find a Cartesian representation; for each given in Cartesian form, find a parametric representation.

a. $\pi = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + 2y + z = 0\}$ b. $\pi = \{(x, y, z) \in \mathbb{R}^3 \mid 3x - 2y + z = 0\}$

c. $\pi = \{(x, y, z) \in \mathbb{R}^3 \mid 6x + z = 0\}$ d. $\pi = \{t(1,2,3) + s(1,4,3) + (2,1,1) \mid t, s \in \Re\}$

e. $\pi = \{t(3,6,4) + s(5,1,7) \mid t, s \in \Re\}$ f. $\pi = \{(x, y, z) = 3x + 2y - 4z + 4 = 0\}$

16) For each line in $\mathbb{R}^3$ given in parametric form, find a Cartesian representation (i.e., the set of solutions to a system of two equations); for each given in Cartesian form, find a parametric representation.

a. $l = \{(t-1, 1-t, 3+t) \mid t \in R\}$

b. $l = \{(1,1,2) + t(1,-4,0) \mid t \in R\}$

c. $l = \{x, y, z) \in \mathbb{R}^3 \mid x - 2y + z = 1,\ 2x + 2y + 2z = 0\}$.

d. $l = \{x, y, z) \in \mathbb{R}^3 \mid x = 0, 2y + 2z = 0\}$

17) (Exam question 5774)

Prove or disprove:

a. The points $A = (0,0,0), B = (2,3,1), C = (1,1,0), D = (1,2,1)$ are all contained in one plane.

b. If $u, v \in \mathbb{R}^3$ and u, v are perpendicular to each other then $u \times v = 0$.

c. If $u, v, w \in \mathbb{R}^3$, $v \times w = 0$, and $w \times u = 0$, then $v \times u = 0$.

18) (Exam question 5774)

Given the following lines:

$l_1 = \{(1 + 2s, -2 + 6s, -1 + s) \mid s \in R\}$, $l_2 = \{(-t, 8 + 2t, 5 + 2t) \mid t \in R\}$

a. Find the point of intersection between l_1 and l_2.

b. Find the acute angle between l_1 and l_2.

c. Find a parametric representation of the plane containing l_1 and l_2.

d. Find a Cartesian representation of the plane containing l_1 and l_2.

19) (Exam question 5775)

Given the following lines:

$L_1 = \{(1+2s, 2-s, 4-2s) \mid s \in \Re\}, \quad L_2 = \{(9+t, 5+3t, -4-t) \mid t \in \Re\}$

a. Show that these lines meet and find the point of intersection.

b. Find the acute angle between the two lines.

c. Find a parametric representation of the line perpendicular to both L_1 and L_2 and passing through the point $(1,1,1)$.

d. Find a Cartesian representation of the plane containing L_1 and L_2.

20) Given the points: $A = (k, 2, 3), B = (2, 2k, 5), C = (1, 1, 4)$.

a. For which values of *k* do all three points lie on one line?

b. For which values of *k* is the line through *A* and *C* parallel to the plane with equation $-x+3y+z+1=0$? In this case find the distance from the line to the plane.

c. For which values of *k* is the line through *A* and *B* perpendicular to the plane with equation $-x+3y+z+1=0$?

(Formula for distance from a point $(\boldsymbol{x_0}, \boldsymbol{y_0}, \boldsymbol{z_0})$ to the plane $\boldsymbol{ax+by+cz+d=0}$:

$$\frac{|\boldsymbol{a x_0 + b y_0 + c z_0 + d}|}{\sqrt{\boldsymbol{a^2+b^2+c^2}}}$$