Ex. 3

5) a. $\lim_{x\to 0} \dfrac{\arcsin 3x}{\arcsin 5x}$

$$= \lim_{x\to 0} \frac{\arcsin 3x}{3x} \cdot \frac{5x}{\arcsin 5x} \cdot \frac{3x}{5x}$$

$$= \left(\lim_{x\to 0} \frac{\arcsin 3x}{3x}\right) \cdot \left(\lim_{x\to 0} \frac{5x}{\arcsin 5x}\right) \cdot \left(\lim_{x\to 0} \frac{3x}{5x}\right)$$

Recall by defn. of arcsin:

$$\sin(\arcsin x) = x$$

So substitute in first limit $t = \arcsin 3x$, so $\sin t = 3x$. And in 2nd limit: $t = \arcsin 5x$, so $\sin t = 5x$.

Since $\sin 0 = 0$ (and sin is continuous), $t \to 0$ as $x \to 0$

$$= \left(\lim_{t\to 0} \frac{t}{\sin t}\right) \cdot \left(\lim_{t\to 0} \frac{\sin t}{t}\right) \cdot \left(\frac{3}{5}\right) = \frac{3}{5}$$

(b) can be solved in a similar manner.

c. $$\lim_{x \to -2} \frac{x^2-4}{\arctan(x+2)}$$ $\to = (x-2)(x+2)$

Substitute $t = x+2$, so $t-4 = x-2$

$$\lim_{t \to 0} \frac{t(t-4)}{\arctan t}$$

Now substitute $y = \arctan t$, so that $t = \tan y$. Since $\tan 0 = 0$, and $\tan$ is continuous at 0, $y \to 0$ as $t \to 0$.

$$\lim_{y \to 0} \frac{(\tan y)(\tan y - 4)}{y}$$

$$= \underbrace{\left(\lim_{y \to 0} \frac{\tan y}{y}\right)}_{=1} \cdot \underbrace{\left(\lim_{y \to 0} (\tan y - 4)\right)}_{=-4} = -4$$

(we did this in class)

From (c) we see that

$$\lim_{x\to 0} \frac{\arctan x}{x} = 1,$$ similar to

$\frac{\arcsin x}{x}$ from part a.

Use this to solve part (d), after a substitution $t = \frac{3}{x}$ (so that $x = \frac{3}{t}$). Note: As $x \to \infty$, $t \to 0^+$.

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graph of $\arctan x$: