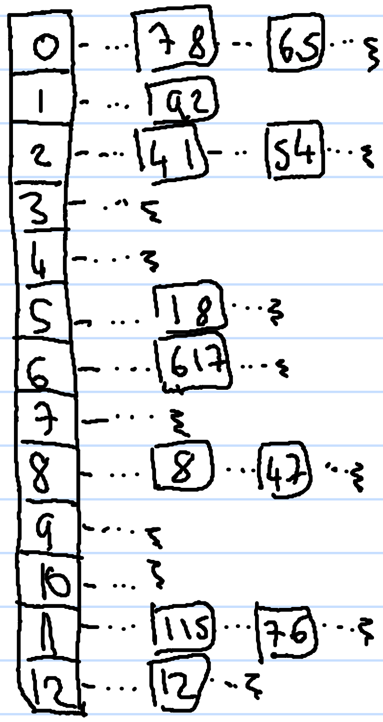
Homework 2 DS2 Leonard Blaum 900086

1.



$$a = n/m = 12/13$$

$$O(1+a) = O(1 + 12/13) = O(1\tfrac{12}{13}) = O(1)$$

2. No, because if we have a table with m space and we want to insert k elements and the size is more then m. Therefore, there must be a collision in one or more of the keys. This is simillar from the pigeonhole principle

3.

	best	worst	average
insertion	$O(1)$	$O(n)$	$O(1)/O(\log n)$
deletion	$O(1)$	$O(n)$	$O(\log n)$
search	$O(1)$	$O(n)$	$O(\log n)$

best case is going to be $O(1)$ because if we insert and this is the first element then it is $O(1)$

worst case in a binary tree is $O(n)$ because we need to go all the whole tree to find it.

4. It is not feasible, because if we shorten the key by q(k) we will have more collision and the runtime will be worse

5a.

0	78
1	53
2	15
3	13
4	4
5	18
6	31
7	72
8	14
9	
10	
11	11
12	

avg time $= O(1 + 10/13) = O(1)$

$h_1(k) = k \bmod 13$

$h_2(k) = 1 + (k \bmod 7)$

$h(k) = 1 + ((k \bmod 13) \bmod 7)$

c.

0	
1	72
2	53
3	15
4	0
5	11
6	18
7	31
8	78
9	4
10	14
11	
12	

avg $= O(n)$

b.

0	78
1	53
2	15
3	13
4	4
5	18
6	14
7	72
8	31
9	
10	
11	11
12	

6. $h_{a,b}(k) = ((ak+b) \bmod p) \bmod m$

$h_{3,4}(k) = ((3k+4) \bmod 691) \bmod 7$

$h_{3,4}(315) = ((3(315)+4) \bmod 691) \bmod 7$

$= (949 \bmod 691) \bmod 7$

$= 258 \bmod 7$

$= \underline{\underline{6}}$

7. We have learned in class the opposite, that the equation is an universal set

Proof from the class:

Let $k, l \in U$ be two keys that satisfy $k \neq l$.
To prove that H is uni set we must show that $P\{h_{a,b}(k) = h_{a,b}(l)\} = \frac{1}{m}$. We designate $r = (ak+b) \bmod p$ and $s = (al+b) \bmod p$. Since $k \neq l$ and $0 \leq a, b, k, l \leq p$, it can be shown that $r \neq s$ *
Since mod p has exactly p dif. possible values, s has exactly $p-1$ possible values because $r \neq s$ and r is a result of a mod p computation.
Since mod m has exactly m different possible values, the # of pos. values for s that satisfy $r \bmod m = s \bmod m$ is the quotient of the # of possible values for s & the # of possible values for mod m: $\frac{p-1}{m}$

The overall probability that $r \bmod m = s \bmod m$ is the quotient of the # of pos. for s where the equation holds and the total # of pos. for s.

Thus: $P(h_{a,b}(k) = h_{a,b}(l)) = P[((ak+b) \bmod p) \bmod m = ((al+b) \bmod p) \bmod m]$

$= P[r \bmod m = s \bmod m] = \frac{p-1}{m} / p-1 = \frac{1}{m}$

We see that the prob. of a collision is $\frac{1}{m}$ so the set is uni set as required.

QED.