$\frac{2}{3} + \frac{4}{6} = \frac{4}{6} + \frac{4}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

1a. The sum of two rational numbers will always be ~~getting~~ a rational number. Since a rational number has 2 intergers m and n. And by adding it together with m_2 and n_2, it becomes the sum of two intergers written in a rational number.

$\frac{n}{m} + \frac{n_2}{m_2} = \frac{sn}{sm}$ $\quad sn \& sm \in \{\mathbb{Z}\}$

1e. The sum of a rational number and an irrational number will always be irrational

As example: $1/2 + \sqrt{2}$ = irrational

$\{\mathbb{Q}\} + \{\mathbb{R}\} = \{\mathbb{R}\}$

whole number

irrational

$\frac{a}{b} + c = \frac{a + c \cdot b}{b}$

if $\sqrt{3} \cdot 2 = 2\sqrt{3}$

if $a + c =$

like $1 + \sqrt{3} = 1,\overline{333}$ ← irrational claim $\sqrt{3} \notin \mathbb{Q}$

2. $\sqrt{3}$ proof: assume $\sqrt{3}$ is $\in \mathbb{Q}$

$\sqrt{3} = \frac{a}{b}$ in reduced fraction $\quad 3 = \frac{a^2}{b^2}$

$3b^2 = a^2 \rightarrow$ is multiply of 3

$a \rightarrow$ must be multiply for 2

$a^2 = 3b^2$

$a = \frac{3b^2}{a}$

$b^2 = \frac{a^2}{3} \Rightarrow b^2 = \frac{a \cdot \frac{b^4}{a^2}}{3}$

$= 3 \cdot \frac{b^4}{a^2}$

$b^2 = \frac{a^2}{3} = \frac{a}{3} \cdot a$ if a^2 is a multiply of 3 then b^2 is m. of 3

$b \Rightarrow$ is a m. of 3

to sum up: a and b are multiply of 3 and $\Rightarrow \frac{a}{b}$ is not a reduced fraction

and also by 3 and 9

b^2 can be divided by 3

b can be divided by 3

$\sqrt{6}$ claim $\sqrt{6} \notin \mathbb{Q}$

proof: assume $\sqrt{6}$ is $\in \mathbb{Q}$

$\sqrt{6} = \frac{a}{b}$, ($\frac{a}{b}$ is reduced fraction)

$6 = \frac{a^2}{b^2}$

$6b^2 = a^2 \rightarrow$ is a multiple of 6 ($= 2 \cdot 3$)

$a \rightarrow$ must be a multiply of 2 and same for 3

$b^2 = \frac{a^2}{6} = \frac{a}{6} \cdot a$ if a^2 is a multiply of 2 then b^2 is m. of 2 and same to m. of 3!

$b \Rightarrow$ is a m. of 2 and 3

to sum up: a and b are multiply of 6

and $\Rightarrow \frac{a}{b}$ is not a reduced fraction

c. $1+\sqrt{2}$ As we did in class: A rational number added to a irrational number will always irrational

ex. $1 + 1,\dots = 2,\dots$ still irrational

d. $\sqrt{2}+\sqrt{3}$ ~~an irrational number~~

If added it would be rational

then $\sqrt{2} = \frac{a}{b}$ (in reduced form)

$+\ \sqrt{3} = \frac{c}{d}$ (in reduced form)

$\sqrt{R} = \frac{ad+cb}{bd}$ (all in reduced form)

than result should be $R = \left(\frac{ad+cb}{bd}\right)^2$

than all can be devided with 2

a and b and c and d $\rightarrow$ multiply of 2

sums not reduced form

what tells us it's irrational

3d. $\lim\limits_{n\to\infty} \frac{1}{\sqrt{2n^2-5}} = 0$ $\quad \lim\limits_{n\to\infty} \sqrt{2n-5} = \infty \rightarrow \frac{1}{\infty} = 0$

e. $\lim\limits_{n\to\infty} \frac{2n+1}{3n+2} = \lim \frac{\frac{2n+1}{n}}{\frac{3n+2}{n}} \Rightarrow \lim \frac{\frac{2n}{n}+\frac{1}{n}}{\frac{3n}{n}+\frac{2}{n}} \Rightarrow \lim \frac{2+0}{3+0} \Rightarrow \frac{2}{3}$

g. $\lim\limits_{n\to\infty} 3^{-n} = 0$ $\quad \lim\limits_{n\to\infty} 3 = 3$ $\quad \lim\limits_{n\to\infty} -n = -n$

$a^{-\infty}$, $a>1$ is sign as 0

means that $\lim\limits_{n\to\infty} 3^{-n} = 0$

h. $\lim\limits_{n\to\infty} \frac{2}{4^{-n}+1} = 2$ $\quad \frac{\lim 2}{\lim(4^{-n}+1)} = \frac{2}{\lim 4^{-n} + \lim 1} = \frac{2}{0+1} = \frac{2}{1} = 2$

4a. $\left|\frac{2n+1}{3n+2} - \frac{2}{3}\right| < \frac{1}{1000}$ $\left(= n \in \left(-\infty, -\frac{1006}{9}\right) \cup \left(\frac{994}{9}, +\infty\right)\right)$

$\left|\frac{1}{9n+6}\right| < \frac{1}{1000}$

$9n+6 > 1000$

$n > \frac{994}{9}$

$N = \frac{-1006}{9}$ not $n>N$ but right

$n > N$

$n > \frac{994}{9}$

$N = \frac{994}{9}$

6a. $a_n = (-1)^n$

$n \ngeq N$

$\lim_{n\to\infty} = (-1)^n = -1$ $\quad |-1^n - l| \geq \varepsilon$

$-1^n - l \geq \varepsilon = -1^n - \varepsilon \geq l$

$1^n + l \geq \varepsilon = 1^n + \varepsilon \geq l$ ← always diverging

d. $a_n = 2n-1$

$\lim_{n\to\infty} (2n-1) = \lim 2n - \lim 1 = \infty - 1 = \infty$?

$|(2n-1) - l| \geq \varepsilon$

only than diverges → $2n-1-l \geq \varepsilon = 2n-1-\varepsilon \geq l$

$-2n+1+l \geq \varepsilon = -2n+l-\varepsilon \geq -l = 2n-l+\varepsilon \leq l$

7b. $\lim_{n\to\infty} 2^n = +\infty$ $\quad |2^n - l| < \varepsilon$

only if $n > 0$ → $2^n - l < \varepsilon = 2^n - \varepsilon < l$

$-2^n + l < \varepsilon = -2^n - \varepsilon < -l = 2n + \varepsilon > l$

d. $\lim_{n\to\infty} \log_2 n = +\infty$ $\quad |\log_2 n - l| < \varepsilon$

only if $n > 0$ → $\log_2 n - l < \varepsilon = \log_2 n - \varepsilon < l$

$-\log_2 n + l < \varepsilon = -\log_2 n - \varepsilon < -l = \log_2 n + \varepsilon > l$