Calculus I: Exercise 6 (The Indefinite Integral)

Submit: 1 3, 6, 8, 10 2 3, 4, 8, 11, 13, 15, 17, 19, 21, 25, 26, 29, 30 3 3, 6, 8, 10, 14, 17, 20 4 2, 3, 5, 7, 13, 14, 18 5 2, 4, 6

1) Use basic integrals and linear combinations of them to compute the following integrals:

1. $\int\left(2x^3+\frac{3}{x}\right)dx$ 2. $\int\frac{x^2+4x-1}{x}dx$ 3. $\int\frac{(x+3)^2}{x}dx$

4. $\int\frac{(x-2)^3}{x^2}dx$ 5. $\int\frac{x^2}{x^2+1}dx$ 6. $\int\sqrt[3]{x}(\sqrt{x}+2)dx$

7. $\int(x^e+e^x)dx$ 8. $\int\cos^2\frac{x}{2}dx$ 9. $\int(\cos\frac{x}{2}+\sin\frac{x}{2})^2dx$

10. $\int\tan^2 xdx$ 11. $\int\frac{\cos 2x}{\sin^2 x\cos^2 x}dx$ 12. $\int\frac{1}{\sin^2 2x}dx$

2) Use also the method of substitution to compute the following integrals:

1. $\int\sin 5xdx$ 2. $\int e^{-3x}dx$ 3. $\int\frac{3}{x^2+16}dx$ 4. $\int\frac{x^4}{x^2+4}dx$

5. $\int\frac{dx}{\sqrt{16-x^2}}$ 6. $\int\frac{dx}{\sqrt{x^2-16}}$ 7. $\int\frac{dx}{\sqrt{x^2+16}}$ 8. $\int\sin 2x\cos 3xdx$

9. $\int\sin 2x\sin 5xdx$ 10. $\int\frac{x}{x^2+7}dx$ 11. $\int\frac{x}{\sqrt[3]{x^2+7}}dx$ 12. $\int\frac{x^2}{x^3+4}dx$

13. $\int\frac{e^x dx}{1+e^x}$ 14. $\int\frac{(2x+3)dx}{x^2+1}$ 15. $\int\frac{\sin 2xdx}{3+\cos 2x}$ 16. $\int\frac{e^x dx}{1+e^{2x}}$

17. $\int\frac{x+e^{2x}}{x^2+e^{2x}}dx$ 18. $\int\frac{\sin\sqrt{x}}{\sqrt{x}}dx$ 19. $\int\frac{dx}{x+3\sqrt{x}}$ 20. $\int\frac{\sin x}{\cos^7 x}dx$

21. $\int\frac{(2x+3)}{(x^2+3x+1)^{10}}dx$ 22. $\int\frac{e^{\arctan x}dx}{1+x^2}$ 23. $\int\frac{dx}{x\ln x}$ 24. $\int\frac{\tan\ln xdx}{x}$

25. $\int\sin x\cdot\cos^5 x\,dx$ 26. $\int\tan x\,dx$ 27. $\int\tan x\cdot\frac{1}{\cos^2 x}dx$

28. $\int\sin^3 x\cdot\cos^2 x\,dx$ 29. $\int\cos^3 x\,dx$ 30. $\int x^2\sqrt{x+3}\,dx$

3) Use also the method of integration by parts to compute the following integrals:

1. $\int x\sin 2x\,dx$ 2. $\int x\cos 3x\,dx$ 3. $\int xe^{-x}dx$ 4. $\int x\ln x\,dx$

5. $\int x^2\ln x\,dx$ 6. $\int \frac{\ln x}{x^2}dx$ 7. $\int \frac{x}{\sin^2 x}dx$ 8. $\int \frac{x}{\cos^2 x}dx$

9. $\int \frac{x}{\sinh^2 x}dx$ 10. $\int x\sinh 2x\,dx$ 11. $\int \arctan x\,dx$ 12. $\int z\arctan z\,dz$

13. $\int \arcsin x\,dx$ 14. $\int t^2\sin 0.5t\,dt$ 15. $\int \ln^2 x\,dx$ 16. $\int x^3e^{2x}\,dx$ 17. $\int e^{\sqrt{x}}dx$

18. $\int \sin\ln x\,dx$ 19. $\int \cos\ln x\,dx$ 20. $\int e^x\sin 2x\,dx$ 21. $\int e^{3x}\cos 4x\,dx$

4) Compute the following integrals:

1. $\int \frac{20}{(2x+1)(x-2)}dx$ 2. $\int \frac{4x^2-3x-4}{x(x-1)(x+2)}dx$ 3. $\int \frac{x^2+5x+5}{x(x^2-25)}dx$

4. $\int \frac{3x-4}{x^2+5}dx$ 5. $\int \frac{2x^3-4x+3}{x^2+1}dx$ 6. $\int \frac{3x-4}{x^2+2x+5}dx$

7. $\int \frac{x+3}{x^2+6x+13}dx$ 8. $\int \frac{x^2+5x+2}{(1+x^2)(1+x)}dx$ 9. $\int \frac{2x-1}{x^3+1}dx$

10. $\int \frac{4x+6}{x^3-8}dx$ 11. $\int \frac{2x-3}{(x^2+2x+2)(x+1)}dx$ 12. $\int \frac{3}{(2x-3)(x+1)^2}dx$

13. $\int \frac{(x^2-3x-8)dx}{(x^2+4x+5)(x+1)}$ 14. $\int \frac{x^2-4}{(x-1)(x+5)^2}dx$ 15. $\int \frac{5x^3-3x^2+2x-1}{x^2(x^2+1)}dx$

16. $\int \frac{6x^4+29x^3+48x^2-10x-80}{(3x-2)(x^2+4x+8)}dx$ 17. $\int \frac{3x-4}{x+1}dx$ 18. $\int \frac{x^2-4x+3}{x^2+4x+5}dx$

5) Calculate the following integrals:

1. $\int \frac{2e^{2x}+3e^x}{2e^{2x}-5e^x+6}dx$ 2. $\int \frac{\sin 2x-\cos x}{\sin^2 x-2\sin x}dx$ 3. $\int \frac{1+\tan^2 x}{\tan^2 x-4}dx$

4. $\int \frac{3e^x}{\sqrt{1-e^{2x}}}dx$ 5. $\int \frac{2}{x\ln^2 x+9x}dx$ 6. $\int \frac{3+2\ln x}{x\ln^2 x-9x}dx$

Indefinite Integrals:

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \quad , \quad (a \neq -1) \qquad \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x dx = \sin x + C$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C \qquad \int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \qquad \int e^x dx = e^x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C \qquad \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x+\sqrt{x^2+a^2}| + C \qquad \int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x+\sqrt{x^2-a^2}| + C$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C \qquad \int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \sinh x dx = \cosh x + C \qquad \int \cosh x dx = \sinh x + C$$

Trigonometric Identities:

$$\sin(x+y) = \sin x \cos y + \sin y \cos x \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\sin 2x = 2\sin x \cos x \qquad \begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned} \qquad \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \qquad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$