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Homework 2

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1a. $\lim_{x\to 1}(4x-1) = \lim(4\cdot 1-1) = \lim(3) = 3$

c. $\lim_{x\to 0^-} e^{\frac{1}{x}} = \lim_{x\to\infty} a = e$ $\quad = \lim_{x\to 0^-}(e^{\frac{1}{x}}) = 0$

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$ $\quad$ where $a^{-\infty}, a>1$

f. $\lim_{x\to 0^+}(\ln(x)) = -\infty$

h. $\lim_{x\to -\infty}\left(\frac{1}{\ln(x^2)}\right) = 0$ $\quad \lim_{x\to-\infty}(1) = 1$

$\lim_{x\to-\infty}(\ln(x^2)) = +\infty$

$= \lim_{x\to-\infty}\frac{1}{+\infty} = 0$

n. $\lim_{x\to 0^+}\left(\frac{1}{1+2^{\frac{1}{x}}}\right) =$ $\lim(1) = 1$

$\lim(1) + \lim(2^0) = 2$ $\quad = \lim\left(\frac{1}{2}\right) = \frac{1}{2}$

$\lim(\frac{1}{x}) = 0$ (pre Error)

2e. ① $\lim_{x\to 1}\frac{7x^2-2x-5}{x^2-3x+2} = \lim_{x\to 1}\frac{7\cdot 1^2-2-5}{1-3+2} = \frac{0}{-4} = 0$

$\lim_{x\to 1}\left(\frac{7x^2+5x-7x-5}{x^2-x-2x+2}\right) = \lim_{x\to 1}\left(\frac{x(7x+5)-(7x+5)}{x(x-1)-2(x-1)}\right)$

$= \lim_{x\to 1}\left(\frac{(7x+5)\cdot\cancel{(x-1)}}{\cancel{(x-1)}(x-2)}\right) = \lim_{x\to 1}\left(\frac{7x+5}{x-2}\right) = \lim_{x\to 1}\left(\frac{7+5}{1-2}\right)$

$= \lim_{x\to 1}\left(\frac{12}{-1}\right) = -12$

② $\lim_{x\to 2^+}\left(\frac{7x^2-2x-5}{x^2-3x+2}\right) = \lim_{x\to 2^+}\left((7x^2-2x-5)\cdot\frac{1}{x^2-3x+2}\right)$

$= \lim_{x\to 2^+}(7x^2-2x-5) = (7\cdot 2^2-2\cdot 2-5) = 19$

$\lim_{x\to 2^+}\left(\frac{1}{x^2-3x+2}\right) = \left(\frac{1}{x}\text{ where } +0 \text{ goes to } 2 = \right) = +\infty$ $\quad = 19 \times (+\infty)$ where $a>0$

$= +\infty$

(II) $\lim\limits_{x \to 2} \left(\dfrac{7x^2 - 2x - 5}{x^2 - 3x + 2} \right) = \lim \left(\dfrac{7 \cdot 2^2 - 4 - 5}{2^2 - 6 + 2} \right) = \dfrac{19}{0}$

does not exist

(IV) $\lim\limits_{x \to -1} \left(\dfrac{7x^2 - 2x - 5}{x^2 - 3x + 2} \right) = \lim \left(\dfrac{7 \cdot (-1)^2 - 2 \cdot (-1) - 5}{(-1)^2 - 3 \cdot (-1) + 2} \right) = 1$

g. (I) $\lim\limits_{x \to 2} \left(\dfrac{x^4 - 7x^3 + 2x^2 + 5x - 1}{x^3 + 2x^2 - x - 2} \right) = \left(\dfrac{2^4 - 7 \cdot 2^3 + 2 \cdot 2^2 + 10 - 1}{2^3 + 2 \cdot 4 - 2 - 2} \right) = \dfrac{-23}{12}$

(II) $\lim\limits_{x \to 1} (-//-) = \left(\dfrac{1 - 7 + 2 + 5 - 1}{1 + 2 - 1 - 2} \right) = \dfrac{0}{0}$ ✗

$= \left(\dfrac{x^4 - x^3 - 6x^3 + 6x^2 - 4x^2 + 4x + x - 1}{x^2(x+2) - (x+2)} \right)$

$= \lim\limits_{x \to 1} \left(\dfrac{x^3(x-1) - 6x^2(x-1) - 4x(x-1) + 1(x-1)}{(x+2)(x^2-1)} \right)$

$= \lim\limits_{x \to 1} \left(\dfrac{\cancel{(x-1)}(x^3 - 6x^2 - 4x + 1)}{(x+2)\cancel{(x-1)}(x+1)} \right)$

$= \lim\limits_{x \to 1} \left(\dfrac{x^3 - 6x^2 - 4x + 1}{x^2 + x + 2x + 2} \right) = \lim\limits_{x \to 1} \left(\dfrac{x^3 - 6x^2 - 4x + 1}{x^2 + 3x + 2} \right)$

$= \lim\limits_{x \to 1} \left(\dfrac{1 - 6 - 4 + 1}{1 + 3 + 2} \right) = -\dfrac{4}{3}$

(III) $\lim\limits_{x \to -1} (-//-) = \left(\dfrac{-1^4 - 7 \cdot (-1)^3 + 2 \cdot (-1)^2 + 5 \cdot (-1) - 1}{(-1)^3 + 2 \cdot (-1)^2 - (-1) - 2} \right) = \dfrac{-2}{0}$ ✗

from (II) $= \lim\limits_{x \to -1} \left(\dfrac{x^3 - 6x^2 - 4x + 1}{x^2 + 3x + 2} \right) = \dfrac{(-1)^3 - 6 \cdot (-1)^2 - 4 \cdot (-1) + 1}{(-1)^2 + 3 \cdot (-1) + 2} = \dfrac{-6 + 4}{0} = \dfrac{-2}{0}$

doesn't exist ✗

(iv) $\lim_{x \to -2} (-\text{"}-) = \left(\dfrac{(-2)^4 - 7\cdot(-2)^3 + 2\cdot(-2)^2 + 5(-2) - 1}{(-2)^3 + 2\cdot(-2)^2 - (-2) - 2} \right) = \dfrac{37}{0}$

from (ii)

$$= \lim_{x \to -2} \left(\frac{x^3 - 6x^2 - 4x + 1}{x^2 + 3x + 2} \right) = \frac{(-2)^3 - 6\cdot(-2)^2 - 4\cdot(-2) + 1}{(-2)^2 + 3\cdot(-2) + 2}$$

$$= \frac{23}{8}$$

(v) $\lim_{x \to -\infty} (-\text{"}-) = \lim_{x \to \infty} (x^4 - 7x^3 + 2x^2 + 5x - 1) = +\infty$ X

$$= \lim_{x \to -\infty} (x^3 + 2x^2 - x - 2) = -\infty$$

$$\lim_{x \to -\infty} \left(\frac{\cancel{x^3}\left(x - 7 + \frac{2}{x} + \frac{5}{x^2} - \frac{1}{x^3}\right)}{\cancel{x^3}\left(1 + \frac{2}{x} - \frac{1}{x^2} - \frac{2}{x^3}\right)} \right)$$

$$= \lim_{-\text{"}-} \left(x - 7 + \frac{2}{x} + \frac{5}{x^2} - \frac{1}{x^3} \right) = -\infty$$

$$= \lim_{-\text{"}-} \Big(\underset{1}{1} + \underset{0}{\frac{2}{x}} - \underset{0}{\frac{1}{x^2}} - \underset{0}{\frac{2}{x^3}} \Big) = 1$$

$$= \frac{-\infty}{1} = -\infty$$

j. $\lim_{x \to 0} (\sqrt{x^2 + 3x + 1} - 1) = 0$ X

$\lim_{x \to 0} (x) = 0$

multiply the func with $\dfrac{\sqrt{x^2 + 3x + 1} + 1}{\sqrt{x^2 + 3x + 1} + 1}$

$$= \frac{\sqrt{x^2 + 3x + 1} - 1}{x} \cdot \frac{\sqrt{x^2 + 3x + 1} + 1}{\sqrt{x^2 + 3x + 1} + 1}$$

$$= \frac{(\sqrt{x^2 + 3x + 1} - 1)(\sqrt{x^2 + 3x + 1} + 1)}{x(\sqrt{x^2 + 3x + 1} + 1)} = \frac{x^2 + 3x + 1 - 1}{x(\sqrt{x^2 + 3x + 1} + 1)}$$

$$= \frac{\cancel{x}(x + 3)}{\cancel{x}(\sqrt{x^2 + 3x + 1} + 1)} = \lim_{x \to 0} \frac{(0 + 3)}{\sqrt{0^2 + 3\cdot 0 + 1} + 1} = \frac{3}{2}$$

l. $\lim_{x \to -\infty} \left(\sqrt{x^2+3x}\right) = +\infty$

$\lim_{x \to -\infty} (x) \qquad = -\infty \qquad \times$

$= \left(\left(\sqrt{x^2+3x}+x\right) \frac{\sqrt{x^2+3x}-x}{\sqrt{x^2+3x}-x} \right) = \left(\frac{\left(\sqrt{x^2+3x}+x\right)\left(\sqrt{x^2+3x}-x\right)}{\sqrt{x^2+3x}\quad -x} \right)$

$= \left(\frac{x^2+3x-x^2}{\sqrt{x^2+3x}\quad -x} \right) = \left(\frac{3x}{x\left(-\sqrt{1+\frac{3}{x}}-1\right)} \right) = \frac{3}{-\sqrt{1+\frac{3}{x}}-1}$

$= \lim_{x \to -\infty} \frac{3}{-\sqrt{1+3\cdot 0}-1} = \lim_{x \to -\infty} \frac{3}{-\sqrt{1+0}-1} = \frac{3}{-\sqrt{1}-1} = \frac{3}{-1-1} = -\frac{3}{2}$

m.

p. $= \left(\frac{-x\sqrt{1+\frac{3}{x}+\frac{8}{x^2}}}{x\left(2+\frac{3}{x}\right)} \right) = \frac{\lim_{x \to -\infty}\left(-\sqrt{1+\frac{3}{x}+\frac{8}{x^2}}\right)}{\lim_{x \to -\infty}\left(2+\frac{3}{x}\right)}$

$= \frac{\sqrt{\lim_{x \to -\infty}\left(1+\frac{3}{x}+\frac{8}{x^2}\right)}}{\lim_{-''-}(2)+\lim_{-''-}\left(\frac{3}{x}\right)} = \frac{-\sqrt{\lim_{-''-}(1)+\lim_{-''-}\left(\frac{3}{x}\right)+\lim_{-''-}\left(\frac{8}{x^2}\right)}}{\lim(2)+\lim\left(\frac{3}{x}\right)}$

$= \frac{-\sqrt{1+3\cdot\lim\left(\frac{1}{x}\right)+8\cdot\lim\left(\frac{1}{x^2}\right)}}{2+3\cdot\lim\frac{1}{x}} = \frac{-\sqrt{1+3\cdot 0+8\cdot 0}}{2+3\cdot 0}$

$= \cancel{} = \frac{-\sqrt{1}}{2} = \frac{1}{2}$

3a. $= \lim_{x \to 0} \left(\frac{\sin(3x)}{\sin(5x)} \times \frac{3 \cdot 5x}{3 \cdot 5x} \right)$

$= \lim_{x \to 0} \left(\frac{3\sin(3x) \cdot 5x}{5 \times 3x \sin(5x)} \right) = \lim_{x \to 0} \left(\frac{3}{5} \cdot \frac{\sin(3x)}{3x} \cdot \frac{5x}{\sin(5x)} \right)$

$= \lim_{x \to 0} \left(\frac{3}{5}\right) \cdot \lim_{x \to 0} (1) \cdot \lim_{x \to 0} (1^{-1})$

$= \lim_{x \to 0} \left(\frac{3}{5}\right) = \frac{3}{5}$

P. $\lim_{x \to 0} \frac{2x + 5\sin(x)}{5x - 2} = \frac{2 \times 0 + 5\sin(0)}{5 \times 0 - 2} = \frac{0}{-2} = 0$

t. $\lim_{x \to 0} \left(\frac{\frac{\sin(x)}{\cos(x)} - \sin(x)}{\sin(x)^3} \right) = \left(\frac{\frac{\sin(x) - \cos(x)\sin(x)}{\cos(x)}}{\sin(x)^3} \right)$

$= \left(\frac{\sin(x) - \cos(x)\sin(x)}{\cos(x)\sin(x)^3} \right) = \left(\frac{\sin(x)(1 - \cos(x))}{\cos(x)\sin(x)^3} \right)$

$= \left(\frac{1 - \cos(x)}{\cos(x)\sin(x)^2} \right) = \left(\frac{1 - \cos(x)}{\cos(x)(1 - \cos(x)^2)} \right)$

$= \left(\frac{1 - \cos(x)}{\cos(x)(1 - \cos(x))(1 + \cos(x))} \right) = \left(\frac{1}{\cos(x)(1 + \cos(x))} \right)$

$= \left(\frac{1}{\cos(x)(\cos(x))^2} \right) = \lim_{x \to 0} \left(\frac{1}{\cos(0)(\cos(0))^2} \right)$

$= \lim_{x \to 0} \left(\frac{1}{1 + 1} \right) = \frac{1}{2}$

(P6) $n, 9, 5, t, v, w$

$$\lim_{x\to 0}\left(x\cdot\left[\frac{7}{x}\right]\right)$$

4b $\lim_{x\to 0}\left(x\cdot\frac{7}{x}\right) \le \qquad\qquad \le \lim_{x\to 0}\left(x\cdot\frac{7}{x+1}\right)$

$7\lim_{x\to 0}\left(x\cdot\frac{1}{x}\right) \le \qquad\qquad \le \lim_{x\to 0}\left(0\cdot\frac{7}{1}\right)$

$7\cdot\lim_{x\to 0}\left(\frac{\frac{d}{dx}f(x)}{\frac{d}{dx}(x)}\right) \le \qquad\qquad \le 7$

$7\cdot\lim(1) \le$

$7 \le \lim_{x\to 0}\left(x\cdot\left[\frac{7}{x}\right]\right) \le 7$

$= 7$

d. $\lim_{x\to 0}\left(x\sqrt{3-2\cos\left(\frac{1}{x-1}\right)}\right) \le \lim_{x\to 0}\left(x\sqrt{3-2\cos\left(\frac{1}{x}\right)}\right) \le \lim_{x\to 0}\left(x\sqrt{3-2\cos\left(\frac{1}{x+1}\right)}\right)$

$\hookrightarrow = 0 \qquad \hookrightarrow 0 \le f(x) \le 0 \qquad \hookrightarrow = 0$

$= 0$

5b. $\lim_{x\to 0}(1-2x)^{\frac{3}{4}x} = \lim_{x\to 0}(1-0)^0 = 1$

i. $\lim_{t\to +\infty}\left(\left(\frac{t-5+4}{t}\right)^{t-5}\right) = \left(\left(\frac{t-1}{t}\right)^{t-5}\right) = \left(\left(\frac{t}{t}+\frac{-1}{t}\right)^{t-5}\right)$ $\boxed{a = b^{\frac{a}{b}}}$

$= \left(\left(1+\frac{-1}{t}\right)^{t\,\frac{t-5}{t}}\right) = \left(\lim_{t\to +\infty}\left(\left(1+\frac{-1}{t}\right)^t\right)\right)^{\lim_{t\to+\infty}\left(\frac{t-5}{t}\right)}$

$= \left(e^{-1}\right)^1 = e^{-1}$ or $\frac{1}{e}$

m. $\lim_{x\to +\infty}\left(\left(2+\frac{1}{x}\right)^x\right) = 2^x\left(1+\frac{1}{2x}\right)^x = \lim_{x\to+\infty}\left(2^x\right)\cdot\lim_{x\to+\infty}\left(1+\frac{1}{2x}\right)^{\lim_{x\to+\infty}(x)}$

$= \infty\cdot 1^\infty$

$= +\infty$

5q. $\lim\limits_{x\to+\infty}\left(\frac{2x+3}{x-4}\right)^{\lim\limits_{x\to+\infty}(3x-5)}$

$= 2^{+\infty}$

$= +\infty$

r. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{3x-2}\right)^{\lim\limits_{x\to\infty}(4x-3)}$

$= \frac{2}{3}^{+\infty} = a^{+\infty}, 0<a<1$ is 0

$= 0$

t. $\lim\limits_{x\to-\infty}\left(\frac{2x+3}{x+4}\right)^{\lim\limits_{x\to-\infty}\left(\frac{3}{x}\right)}$

$= \frac{3}{4}^{-\infty}$

$= +\infty$

u. $\lim\limits_{x\to1^-}(2)^{\lim\limits_{x\to1^-}\left(\frac{1}{1-x}\right)}$

$= 2^{+\infty}$

$= +\infty$