שנה"ל **התש"ף**, סמסטר **ב'**, מועד **ב**
שאלון בחינה בקורס: מבנה נתונים ותוכניות א'
מספר קורס: **150015**

Student ID#:_______________

- Instructors names: Adina Milston, Meir Komar
- Exam Date: 20 Tammuz 5780 (12/7/2020)
- Length of exam (in minutes): 180
- Materials allowed during exam: Only printed material
- You are not allowed to use a calculator
- The exam has 6 question each worth 20 points. Answer any 5 of them.

1. **It is your responsibility to understand the institutional regulations regarding exams. Any deviation from these rules may lead to the exam being disallowed in addition to other actions against the student.**
2. **If for any reason you are not certain if you understand the intent of the instructor with regard to a question, you are to indicate at the beginning of your answer how you understand the question and answer accordingly. The instructor has the right to determine how many points your understanding and answer are worth.**
3. **Please note that points will be taken off not only for mistakes but for irrelevant material in the answer, lack of adequate explanation for an answer or an unclear and/or ambiguous answer.**

Place an X on the question you **do not** wish to be marked:

1	2	3	4	5	6	Total

שנה"ל **התש"ף**, סמסטר **ב'**, מועד **ב**
שאלון בחינה בקורס: מבנה נתונים ותוכניות א'
מספר קורס: **150015**

General Comments:

1. **When the exam mentions the term run-time without additional qualifications, the worst-case run time is meant (in order of magnitude).**
2. **When the exam stipulates that additional memory cannot be used, you are allowed to use a fixed amount of additional memory.**
3. **In questions where you are required to write an algorithm, you can use algorithms taught in the lecture without rewriting them.**

Question 1

A. (10 pts) For each pair of functions select the appropriate notation: o, ω, O, Ω or θ

If there are several correct answers, select the most accurate,
for example if f (n) = O (g (n)) and also f (n) = o (g (n)) select o.
The answers should be explained briefly.

A1 $f(n) = log\left(\sqrt{n}\right)$, $g(n) = log(n)$

A2 $f(n) = 2n^3 - 4\,log^2 n$, $g(n) = 8n^2 + 10log(n^3) - 4\,log^2 n$

A3 $f(n) = n^{2n}$, $g(n) = n^n$

B. (10 points) Is the following statement true:
$n^3 + 2n - logn = O(0.5n^3 - 4n + log^2 n)$

If so, give values to c and n_0 so that the definition of O holds. If not, explain why.

Question 2

We will define the weight of a node in a binary tree: **1** if it is a leaf, **2** if it has **an only** son and **3** if it **has two sons**.

Let the weight of the tree be the sum of the weights of all its nodes.

In order to calculate the weight of the tree after addition or deletion - recalculate the weight of each of the nodes, and then sum all the weights.

A. Suppose the weight of a binary search tree T is exactly W.

A 1. (5 points) What will be the weight of the tree at most after deleting a node? Explain.

A 2. (5 points) What will be the weight of the tree at least after deleting a node? Explain.

B. Suppose tree T has n nodes.

B1. (5 points) What is the weight of the tree T at most? Explain.

B2. (5 points) What is the weight of the tree T at least? Explain.

<u>Question 3</u>

A. (10 points) Prove or disprove:

A minimum heap of size n can be turned into a binary search tree in the worst case in θ (n) time.

B. (10 points) The following recursive function is given for reversing a single-linked list:

The function receives a pointer p to the first member in the list.

```
RecListReverse(p)

{

  if p==null or next(p)==null

        return p

  q ← RecListReverse(next(p))

  ????????? ←p

  next(p) ← null

  return  q

}
```

What should be written instead of the question marks for the function to perform its task correctly?

The answer should be explained in one or two sentences.

<u>Question 4</u>

A. (10 points) n sections $[a_i, b_i]$ are given. (For every i : $a_i < b_i$).

It is known that each for each segment b_i is an integer in the range 1.. n^2 and a_i is a real number.

We want to decide whether all segments are distinct (foreign) from each other.

Describe in words a linear algorithm (θ (n)) for solving the above problem.

For example:

For n = 5:

[0.48,4], [13.31,14], [3.12,5], [8.412,12], [14.27,16]

Here the algorithm will answer no, because the two sections: [3.12,5], [0.48,4] overlap.

B. (10 points) Given an array A [1… n] of integers (positive, negative or 0) all different from each other.

Describe an algorithm in words that runs in time o (n) [small o], which finds an index i for which A [i] = i exists, if such an index exists.

If there are several such indexes, the algorithm will return one of them. In case it does not exist, the algorithm will return "non-existent".

Explain why the algorithm is correct and analyze the runtime at worst case in terms of θ (theta).

For example, in the following array where n = 10:

1	2	3	4	5	6	7	8	9	10
-5	**-1**	**0**	**2**	**3**	**6**	**7**	**10**	**12**	**13**

There are two indices that meet the requirement: indices 6 and 7.

Question 5

A. (5 points) What is the maximum possible number of rotations while inputting into an AVL tree the values 1,2,…, 10 in any order?

Note: RL or LR rotation is considered one rotation and not two.

B. (5 points) In which indices (indexes) may we find the second largest element (i.e. the previous element to the maximum value) in a minimum heap of size n?

C1. (7 points) Build a binary search tree from the following values according to the algorithm for building a binary search tree (left to right):

18,42,28,56,32,36,28

C2. (3 points) Draw the resulting tree after deleting the value 42 from the tree you built in the previous section.

Question 6

A. (10 points) What is the runtime of the following function? Run time should be expressed as a function of n using θ, and explain your answer.

f(n)

$i = 17$

while ($i < 2.7^n$)

  $i \leftarrow 3i$

B. (10 points) What is the runtime of the following function? Run time should be expressed as a function of n using θ, and explain the answer.

g(n)

$x \leftarrow 1$

for i = 1 to $\sqrt{n}$

  for j = 1 to $\sqrt{n}$

    for k = 1 to $\sqrt{n} - j + 1$

      $x \leftarrow x + 1$