Assignment 1: CS 663, Fall 2021

Question 5

- Read in the images 'goi1.jpg' and 'goi2.jpg' from the homework folder using the MATLAB `imread` function and cast them as double. These are images of the Gateway of India acquired from two different viewpoints. As such, no motion model we have studied in class is really adequate for representing the motion between these images, but it turns out that an affine model is a reasonably good approximation, and you will see this. We will estimate the affine transformation between these two images in the following manner:

  (a) Display both images using `imshow(im1)` and `imshow(im2)` in MATLAB. Use the `ginput` function of MATLAB to manually select (via an easy graphical user interface) and store $n = 12$ pairs of physically corresponding salient feature points from both the images. For this, you can do the following:
  ```
  for i=1:12, figure(1); imshow(im1/255); [x1(i), y1(i)] = ginput(1);
  figure(2); imshow(im2/255); [x2(i), y2(i)] = ginput(1);
  ```
  Tips: Avoid selecting points which are visible in only one image. Try to select them as accurately as possible, but our procedure is robust to small sub-pixel errors. Make sure `x1(i),y1(i)` and `x2(i),y2(i)` are actually physically corresponding points. Salient feature points are typically points that represent corners of various structures.

  (b) Write MATLAB code to determine the affine transformation which converts the first image ('goi1') into the second one ('goi2').

  (c) Using nearest neighbor interpolation that you should implement <u>yourself</u>, warp the first image with the affine transformation matrix determined in the previous step, so that it is now better aligned with the second image. You are not allowed to use any implementation for this already available in MATLAB. Display all three images side by side in the report PDF.

  (d) Repeat the previous step with bilinear interpolation that you should implement <u>yourself</u>. You are not allowed to use any implementation for this already available in MATLAB. Display all three images side by side in the report PDF.

  (e) In the first step, suppose that the n points you chose in the first image happened to be collinear. Explain (in the report PDF) the effect on the estimation of the affine transformation matrix. [5+5+5+5+5=25 points]

Answer:

(c) Result of Nearest Neighbor Interpolation

Figure 1: Image 1

Figure 2: Image 2

Figure 3: Warped Image using Nearest Neighbor Interpolation

(d) Result of Bilinear Interpolation

Figure 4: Image 1

Figure 5: Image 2

Figure 6: Warped Image using Bilinear Interpolation

(e) Let the k control points chosen from the first image (I_1) be the set $S = \{(x_{1,i}, y_{1,i})\}$ where i goes from 1 to k. It has also been give that the control points are perfectly co-linear, i.e., $y_{1,i} = ax_{1,i} + b$. Also, the physically corresponding points in the second image I_2 are from $\{(x_{2,i}, y_{2,i})\}$. From control point method, we know that:

$$\begin{aligned} P_2 &= \begin{pmatrix} x_{2,1} & \dots & x_{2,k} \\ y_{2,1} & \dots & y_{2,k} \\ 1 & \dots & 1 \end{pmatrix} \\ P_1 &= \begin{pmatrix} x_{1,1} & \dots & x_{1,k} \\ y_{1,1} & \dots & y_{1,k} \\ 1 & \dots & 1 \end{pmatrix} \end{aligned} \tag{1}$$

and

$$P_2 = \tilde{A}P_1 \implies P_2P_1^T = \tilde{A}P_1P_1^T \tag{2}$$

We can find $\tilde{A}$ if $P = P_1P_1^T$ is invertible. If the control points are co-linear, then P is given by:

$$P = \begin{pmatrix} \sum_{i=1}^k x_{1,i}^2 & a\sum_{i=1}^k x_{1,i}^2 + b\sum_{i=1}^k x_{1,i} & \sum_{i=1}^k x_{1,i} \\ a\sum_{i=1}^k x_{1,i}^2 + b\sum_{i=1}^k x_{1,i} & a^2\sum_{i=1}^k x_{1,i}^2 + 2ab\sum_{i=1}^k x_{1,i} + kb^2 & a\sum_{i=1}^k x_{1,i} + kb \\ \sum_{i=1}^k x_{1,i} & a\sum_{i=1}^k x_{1,i} + kb & k \end{pmatrix} \tag{3}$$

Clearly, for the above matrix P, $C_2 - aC_1 - bC_3 = 0$, where C_i is th i^{th} column of P. Hence, the matrix P is non-invertible. Therefore, we will not be able to find $\tilde{A}$ using this method. Hence, affine transformation matrix cannot be estimated using the control point method if we choose co-linear control points.