Assignment 1: CS 663, Fall 2021

Question 4

- Read in the images T1.jpg and T2.jpg from the homework folder using the MATLAB function `imread` and cast them as a double array. Let us call these images as J1 and J2. These are magnetic resonance images of a portion of the human brain, acquired with different settings of the MRI machine. They both represent the same anatomical structures and are perfectly aligned (i.e. any pixel at location (x, y) in both images represents the exact same physical entity). We are going to perform a simulation experiment for image alignment in a setting where the image intensities of physically corresponding pixels are different. To this end, do as follows:

  (a) Write a piece of MATLAB code to rotate the second image by $\theta = 28.5$ degrees anti-clockwise. You can use the `imrotate` function in MATLAB to implement the rotation using any interpolation method. Note that the rotation is performed implicitly about the centroid of the image. While doing so, assign a value of 0 to unoccupied pixels. Let us denote the rotated version of J2 as J3.

  (b) Our job will now be to align J3 with J1 keeping J1 fixed. To this end, we will do a brute-force search over θ ranging from -45 to +45 degrees in steps of 1 degree. For each θ, apply the rotation to J3 to create an intermediate image J4, and compute the following measures of dependence between J1 and J4:

    - the normalized cross-correlation (NCC)
    - the joint entropy (JE)
    - a measure of dependence called quadratic mutual information (QMI) defined as $\sum_{i_1} \sum_{i_2} (p_{I_1 I_2}(i_1, i_2) - p_{I_1}(i_1) p_{I_2}(i_2))^2$, where $p_{I_1 I_2}(i_1, i_2)$ represents the normalized joint histogram (*i.e.*, joint pmf) of I_1 and I_2 ('normalized' means that the entries sum up to one). Here, the random variables I_1, I_2 denote the pixel intensities from the two images respectively. For computing the joint histogram, use a bin-width of 10 in both I_1 and I_2. For computing the marginal histograms p_{I_1} and p_{I_2}, you need to integrate the joint histogram along one of the two directions respectively. You should write your own joint histogram routine in MATLAB - do not use any inbuilt functions for it.

  (c) Plot separate graphs of the values of NCC, JE, QMI versus θ and include them in the report PDF.

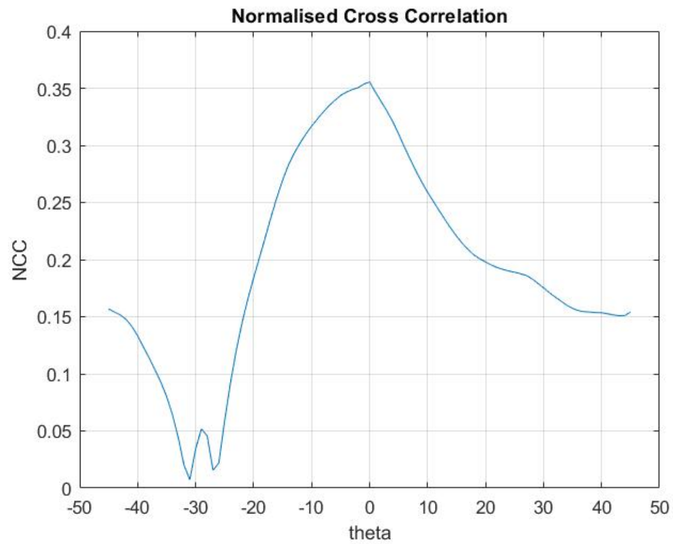
  (d) Determine the optimal rotation between J3 and J1 using each of these three measures. What do you observe from the plots with regard to estimating the rotation? Explain in the report PDF.

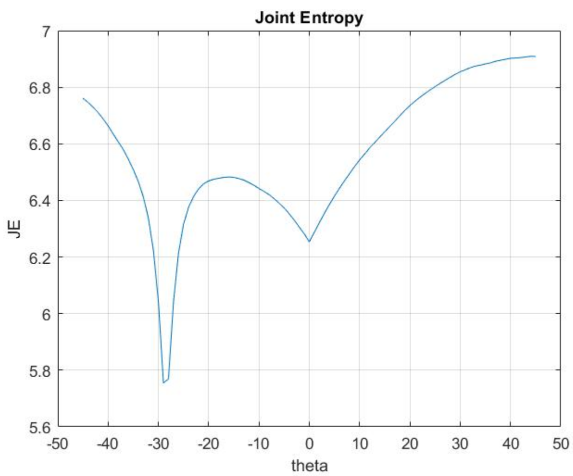
  (e) For the optimal rotation using JE, plot the joint histogram between J1 and J4 using the `imagesc` function in MATLAB along with `colorbar`. Include it in the report PDF.

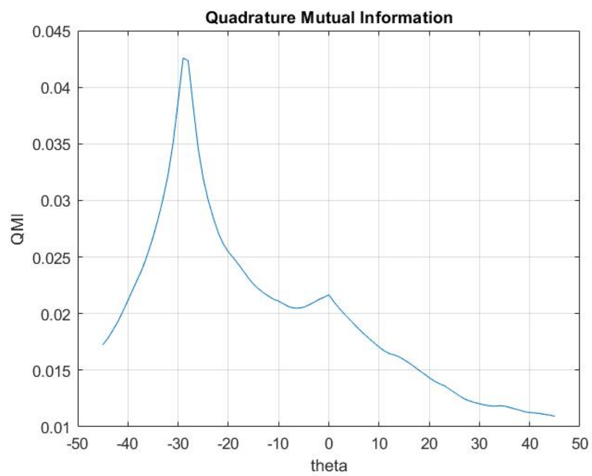
  (f) We have studied NCC and JE in class. What is the intuition regarding QMI? Explain in the report PDF. (Hint: When would random variables I_1 and I_2 be considered statistically independent?) [2+10+2+3+3+5=25 points]

Answer:

(c)





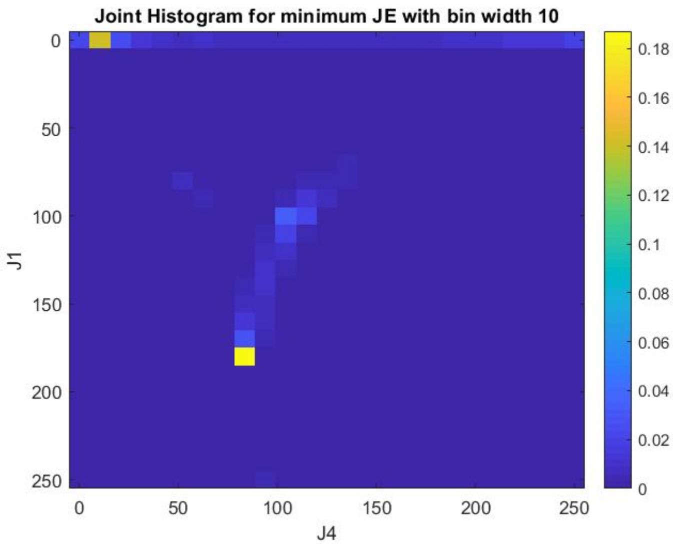


(d) **NCC:** We pick theta for which NCC is the maximum. From the above plot we observe NCC is maximum at theta $= 0^o$, which is clearly not the correct rotation. We also observe a local maxima at $-29^o, -28^o$, but it is not a global maxima. NCC is not a good measure for all kinds of image alignment.

JE: We pick theta for which JE is the minimum. The above plot shows the minima of JE at theta=$\boldsymbol{-29^o}$, close to the expected value. The initial rotation was 28.5^o(anti-clockwise is +ve) and as the step size of theta is 1, we got the answer with precision upto 1 degree.

QMI: We pick theta for which QMI is the maximum. The above plot shows the maxima of QMI at theta=$\boldsymbol{-29^o}$, close to the expected value. The initial rotation was 28.5^o(anti-clockwise is +ve) and as the step size of theta is 1, we got the answer with precision upto 1 degree.

(e) Optimal rotation is -29^o for aligning second image w.r.t first image according to JE.



(f) We know that, when two random variables, I_1 and I_2 are independent, the $P_{I_1,I_2}(i_1,i_2) = P_{I_1}(i_1)P_{I_2}(i_2)$. Therefore, the more the magnitude of difference between $P_{I_1,I_2}(1_1,i_2)$ and $P_{I_1}(i_1)P_{I_2}(i_2)$, the more dependent they are. Hence the images are more dependent/correlated when the QMI, $\sum_i \sum_j (P_{I_1,I_2}(1_1,i_2) - P_{I_1}(i_1)P_{I_2}(i_2))^2$ has larger a value. Therefore, the images are aligned when their QMI is maximum.