Assignment 1: CS 663, Fall 2021

Question 3

August 15, 2021

- Suppose an image is subjected to histogram equalization. Prove that another round of histogram equalization will produce the exact same result as the first round. [15 points]

Answer:

Let L be the number of intensity levels in the input image I_1. Let r_k be the intensity values of the pixels in the image. Let R be the random variable with PDF $P_R(r)$ from which the values of r_k are taken. We know that, from histogram equalization, the equalized image has intensity values s_k given by:

$$s_k = T(r_k) = (L-1) \int_0^{r_k} P_R(r)dr \tag{1}$$

Now, let S be the random variable from which s_k is taken. From above:

$$P_S(s_k) = \frac{P_R(r_k)}{|T'(r_k)|} = \frac{P_R(r_k)}{(L-1)P_R(r_k)}$$
$$= \frac{1}{(L-1)} \tag{2}$$

(When $x \to 0$, $\lim_{x \to 0} \frac{x}{(L-1)x} = \frac{1}{L-1}$.)

Clearly the random variable S is $U(0, L-1)$. Let the equalized image be I_2. If we equalize I_2 using the above procedure, assuming m_k to be the new intensity values, we get:

$$m_k = T(s_k) = (L-1) \int_0^{s_k} P_S(s)dr$$
$$= (L-1) \int_0^{s_k} \frac{1}{(L-1)}dr \tag{3}$$
$$= (L-1)\frac{s_k}{(L-1)}$$
$$= s_k$$

Therefore, we get the intensity values after equalization, of the equalized image (m_k), equal to that of the equalized image, i.e., $m_k = s_k$. Therefore another round of histogram equalization will produce the exact same result as the first round.

Hence Proved.