Assignment 3: CS 663, Fall 2021

Question 7

- Provide an explanation for the presence of strong spikes in the center of the filters in the second sub-figure Of Fig. 1. Note that the fourier transform magnitudes of these filters are plotted in the first figure. [10 points]

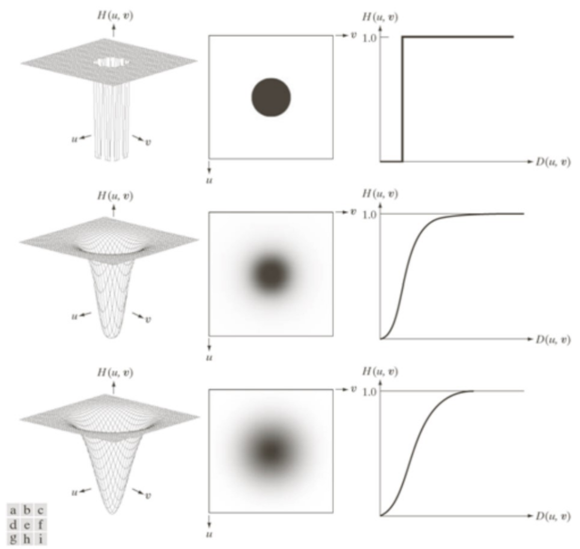

FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

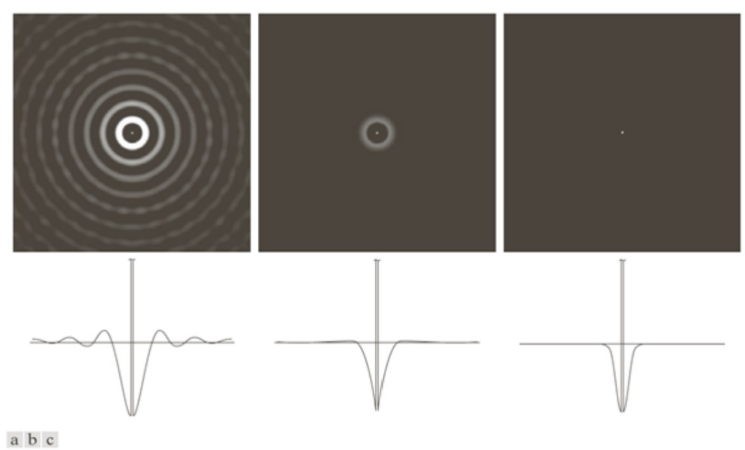

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Figure 1: Figures required for the last question. Fourier domain (first figure) and spatial domain (second figure) representations of various filters.

Answer: From $IDFT$, we know that,

$$h(x,y) = \frac{1}{W_1 W_2} \sum_{u=0}^{W_1-1} \sum_{v=0}^{W_2-1} H(u,v) e^{j2\pi\left(\frac{ux}{W_1} + \frac{vy}{W_2}\right)}$$

Where H(u,v) is the DFT of our chosen filter. In the spatial domain, the centre is at $(x,y) = (0,0)$. Therefore, the intensity at the center is given by $h(0,0)$.

$$h(0,0) = \frac{1}{W_1 W_2} \sum_{u=0}^{W_1-1} \sum_{v=0}^{W_2-1} H(u,v)$$

Now, let $\Delta u = 1$ and $\Delta v = 1$. Clearly,

$$h(0,0) = \frac{1}{W_1 W_2} \sum_{u=0}^{W_1-1} \sum_{v=0}^{W_2-1} H(u,v) \Delta u \Delta v$$

If $W_1, W_2 >> 1$,

$$\begin{aligned} h(0,0) &= \frac{1}{W_1 W_2} \int_0^{W_1-1} \int_0^{W_2-1} H(u,v) du dv \\ &= \frac{1}{W_1 W_2} \int_{\mathcal{X} \in D(u,v)} H(\mathcal{X}) d\mathcal{X} \end{aligned}$$

Where $D(u,v)$ is the Fourier domain through which the $H(u,v)$ is defined. Clearly, the area under $H(\mathcal{X})$, as seen from the first sub-figure is positive. Therefore $h(0,0) > 0$ and hence there is a strong spike at the origin.