Assignment 1: CS 663, Fall 2021

Question 3

- Consider two images I and J whose intensity values (in each location) are randomly drawn from the known probability mass functions (PMFs) $p_I(i)$ and $p_J(j)$ respectively. Derive an expression for the PMF of the image $I + J$. The expression resembles which operation that we are currently studying in class (and will discuss on 13th August during the interaction session)? [15+5=20 points]

  Answer: Assuming the given probability mass fucntions take only integer values,

  $$p_{I+J}(k) = \mathbb{P}(I + J = k) = \sum_{i=-\infty}^{\infty} \mathbb{P}(I = i, J = k - i)$$

  given that the intensities of images I and J are randomly drawn, which implies the images are independent w.r.t their intensities:

  $$\mathbb{P}(I = i, J = k - i) = \mathbb{P}(I = i) \cdot \mathbb{P}(J = k - i) = p_I(i) \cdot p_J(k - i)$$

  which gives

  $$p_{I+J}(k) = \sum_{i=-\infty}^{\infty} p_I(i) \cdot p_J(k - i)$$

  The above expression resembles **1D convolution** operating between $p_I(i)$ and $p_J(j)$.