Assignment 2: CS 663, Fall 2021

Question 3

- Consider a clean image $I(x, y)$ which gets corrupted by additive noise randomly and independently from a zero mean Gaussian distribution with standard deviation σ. Derive an expression for the PDF of the resulting noisy image. Assume continuous-valued intensities. [10 points]

  Answer: Given:

  1. The clean image is $I(x, y)$.
  2. Additive White Gaussian Noise: $W(x, y) \sim N(0, \sigma^2)$.

  Let the noisy image be $I_n(x, y)$. Also, let $(x, y) \in S$, where S is the set of pixel locations in I. We know that:

  $$I_n(x, y) = I(x, y) + W(x, y) \tag{1}$$

  Now, since the image and the noise are independent, their joint PDF is product of the marginal PDFs i.e.

  $$\mathbb{P}_{I,W}(k) = \mathbb{P}_I(k).\mathbb{P}_W(k)$$

  We can find the Cumulative Distribution Function of the Noisy Image as:

  $$\mathbb{P}(I_n \leq k) = \mathbb{P}(I + W \leq k) = \int_{-\infty}^{\infty} \int_{-\infty}^{k-w} \mathbb{P}_I(i).\mathbb{P}_W(w) di dw$$

  Hence, we can find the PDF, $\mathbb{P}_{I_n}(k)$, by differenctiating the CDF as follows:

  $$\mathbb{P}_{I_n}(k) = \frac{\partial}{\partial k}\mathbb{P}(I_n \leq k)$$
  $$\implies \mathbb{P}_{I_n}(k) = \frac{\partial}{\partial k} \int_{-\infty}^{\infty} \int_{-\infty}^{k-w} \mathbb{P}_I(i).\mathbb{P}_W(w) di dw$$

  Using Leibniz rule we can find the derivative as:

  $$\mathbb{P}_{I_n}(k) = \int_{-\infty}^{\infty} \mathbb{P}_I(k - w).\mathbb{P}_W(w) dw$$

  Hence, we can eaily see that the PDF of the noisy image is the convolution of PDF of original image and the PDF of noise.