

Assignment 1: CS 663, Fall 2021

Question 2

- Suppose an image is subjected to histogram equalization. Prove that another round of histogram equalization will produce the exact same result as the first round. [15 points]

Answer:

Let L be the number of intensity levels in the input image I_1 . Let r_k be the intensity values of the pixels in the image. Let R be the random variable with PDF $P_R(r)$ from which the values of r_k are taken. We know that, from histogram equalization, the equalized image has intensity values s_k given by:

$$s_k = T(r_k) = (L - 1) \int_0^{r_k} P_R(r) dr \quad (1)$$

Now, let S be the random variable from which s_k is taken. From above:

$$\begin{aligned} P_S(s_k) &= \frac{P_R(r_k)}{|T'(r_k)|} = \frac{P_R(r_k)}{(L - 1)P_R(r_k)} \\ &= \frac{1}{(L - 1)} \end{aligned} \quad (2)$$

(When $x \rightarrow 0$, $\lim_{x \rightarrow 0} \frac{x}{(L-1)x} = \frac{1}{L-1}$.)

Clearly the random variable S is $U(0, L - 1)$. Let the equalized image be I_2 . If we equalize I_2 using the above procedure, assuming m_k to be the new intensity values, we get:

$$\begin{aligned} m_k &= T(s_k) = (L - 1) \int_0^{s_k} P_S(s) ds \\ &= (L - 1) \int_0^{s_k} \frac{1}{(L - 1)} ds \\ &= (L - 1) \frac{s_k}{(L - 1)} \\ &= s_k \end{aligned} \quad (3)$$

Therefore, we get the intensity values after equalization, of the equalized image (m_k), equal to that of the equalized image, i.e., $m_k = s_k$. Therefore another round of histogram equalization will produce the exact same result as the first round.

Hence Proved.