Q5)

→ The Radon transform of $f(x,y)$ is,

$$R_\theta(f) = g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy \quad \text{—— ①}$$

Now, $f'(x,y) = f(ax, ay)$; $a \neq 0$.

Then,

$$R_\theta(f') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(ax,ay)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

Let $v = ax$, $w = ay$

$$R_\theta(f') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v,w)\,\delta\left(\frac{v\cos\theta}{a} + \frac{w\sin\theta}{a} - \rho\right)\frac{1}{a^2}\,dv\,dw$$

$$= \frac{1}{a^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v,w)\,\delta(v\cos\theta + w\sin\theta - a\rho)\,dv\,dw$$

$$= \frac{1}{a^2}\,g(a\rho,\theta) \quad \text{— from ①}$$

$$\therefore R_\theta(f') = \frac{1}{a^2}\,g(a\rho,\theta), \quad a \neq 0$$