Q6)

→ Let $f(x,y) = \delta(x,y)$

$$R_\theta(f) = g(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy.$$

Now, we know, $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x,y)\, h(x,y)\, dx\, dy = h(0,0)$

Here we take $\delta(x\cos\theta + y\sin\theta - \rho)$ as $h(x,y)$

$$\therefore \boxed{R_\theta(f) = \delta(-\rho) = \delta(\rho)}$$

Now, by shifting property of Radon transform,

$$R(f(x-x_0, y-y_0))(\rho,\theta) = R(\cancel{g}f(x,y))(\rho - x_0\cos\theta - y_0\sin\theta)$$

To prove this, see ~~that~~ following,

LHS =

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x-x_0, y-y_0)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

Put $u = x - x_0$, $v = y - y_0$,

$$= \iint f(u,v)\,\delta(u\cos\theta + v\sin\theta - (\rho - x_0\cos\theta - y_0\sin\theta))\,dx\,dy$$

$$= R(f(x,y))(\rho - x_0\cos\theta - y_0\sin\theta) = RHS.$$

Using this,

$$R(\delta(x-x_0, y-y_0))(\rho,\theta) = R(\delta(x,y))(\rho - x_0\cos\theta - y_0\sin\theta, \theta)$$

$$= \delta(\rho - x_0\cos\theta - y_0\sin\theta)$$

Radon transform of unit impulse is $\delta(\rho)$

& of shifted unit impulse is $\delta(\rho - x_0\cos\theta - y_0\sin\theta)$