EE324: CONTROL SYSTEMS LAB
PROBLEM SHEET 7

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Contents

1 Phase and Gain Margin

The given open loop transfer function is,

$$G(s) = \frac{1}{s(s^2+4s+8)}$$

1.1 Part a

For the phase and gain margin to be equal to 0, we find the value of gain (K), such that $G(j\omega) = -1$ where

$$G(j\omega) = \frac{K}{j\omega(-\omega^2+4j\omega+8)}$$

Hence, we can find the value of K by solving the following equation,

$$\frac{K}{j\omega(-\omega^2+4j\omega+8)} = -1$$
$$\implies jK = -\omega^3 + 4j\omega^2 + 8$$

Now, we equate the real and imaginary parts of LHS and RHS to get the following equations as follows,

$$K = 4\omega^2 and - \omega^3 = 8$$

Hence we get $K = 4 \times (2)^2 = 16$

1.2 Part b

The transfer function of the closed loop system will have only only poles.
Thus the bode plot of the system will be monotonic.
Hence we can say that the system will be having only one crossover frequency. So, at the frequency where gain is 1 will be the frequency at which we measure the phase margin. Given the phase margin is zero, i.e. at that phase will be 180. Now we need to measure the gain margin at the same frequency since there will not be any other w for which the phase is 180 (since monotonic). So, we get that the gain margin will also be zero.
Thus it is not possible that the phase margin will be zero and the gain margin as non-zero, since both are measured at the same freq.
Similarly in the vice versa case, both frequencies should be the same, hence here also gain margin as zero and phase margin as non-zero is not possible.

1.3 Part c

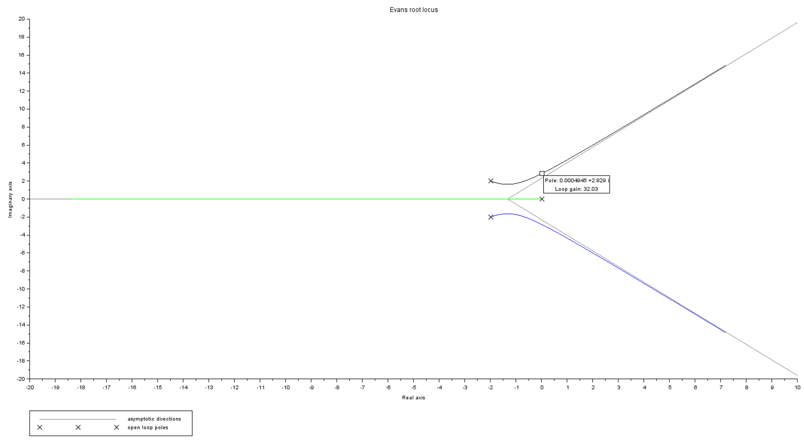
To check the stability we plot the root locus of the given system

Scilab Code

```
clear
close
clc

s = poly(0,'s');
```

```
G = (1)/(s*(s^2 + 4*s + 8));
Glin = syslin('c',G);
clf();
evans(Glin,5000);
```

Obtained Plot



From the root locus we can see that the system becomes stable for gains higher than 32. Hence, for a gain of 16, the system is stable.

2 Effect of Pole-Zero Location in Lag Compensator

The system transfer function in this part is given by:

$$G(s) = \frac{s + K_1}{s + K_2}$$

The ratio of zero magnitude to pole magnitude in the lag compensator is 5 , i.e. $\frac{K_1}{K_2} = 5$.

2.1 Problem a

We plot the step response of the systems for the following values:

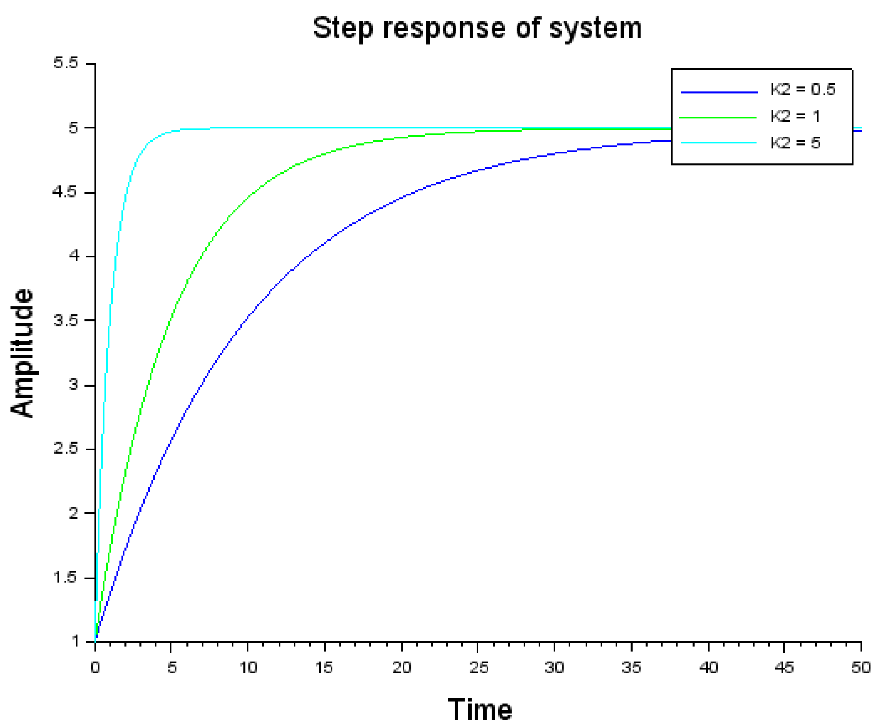
- K1 = 0.5 and K2 = 2.5
- K1 = 1 and K2 = 5
- K1 = 5 and K2 = 25

Scilab Code

```
clear
clc

s = poly(0,'s');

```

```
K1 = [0.5 1 5];
K2 = K1/5;
t = 0:0.001:50;

for i = 1:length(K1)
    G = (s+K1(i) )/(s + K2(i));

    Gs = syslin('c',G);
    ts =  csim('step' ,t, Gs);
    plot2d(t , ts, style = i+1);
end
xlabel('Time','fontsize',4)
ylabel('Amplitude','fontsize',4)

legend("K2 = 0.5" , "K2 = 1" , "K2 = 5")
title('Step response of system', 'fontsize',4)
```

Obtained Plot



From the obtained plot we can easily that further away the pole is from the origin the faster is the response of the system.

2.2 Problem b

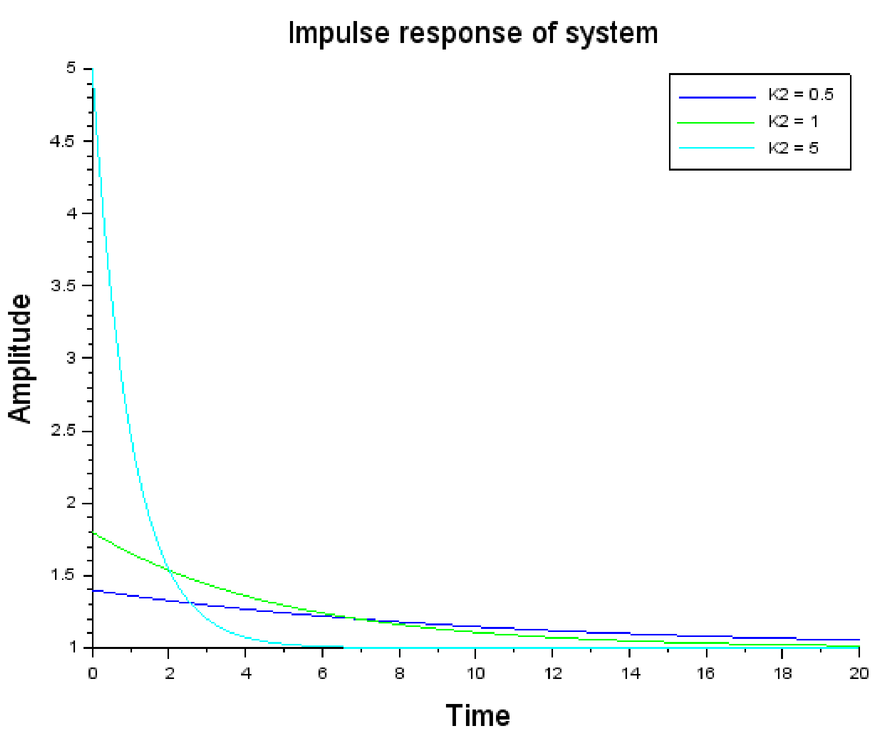
Similar to previous parts we plot the impulse responses of the system for the following values:

- K1 = 0.5 and K2 = 2.5
- K1 = 1 and K2 = 5
- K1 = 5 and K2 = 25

Scilab Code

```
clear
clc

s = poly(0,'s');

K1 = [0.5 1 5];
K2 = K1/5;
t = 0:0.001:20;

for i = 1:length(K1)
    G = (s+K1(i) )/(s + K2(i));

    Gs = syslin('c',G);
    ts =  csim('impulse' ,t, Gs);
    plot2d(t , ts, style = i+1);
end
xlabel('Time','fontsize',4)
ylabel('Amplitude','fontsize',4)

legend("K2 = 0.5" , "K2 = 1" , "K2 = 5")
title('Impulse response of system', 'fontsize',4)
```

Obtained Plot



From the obtained plot we can easily that further away the pole is from the origin the faster is the response of the system.

3 Two phase-crossover frequencies

After part a, the transfer function is,

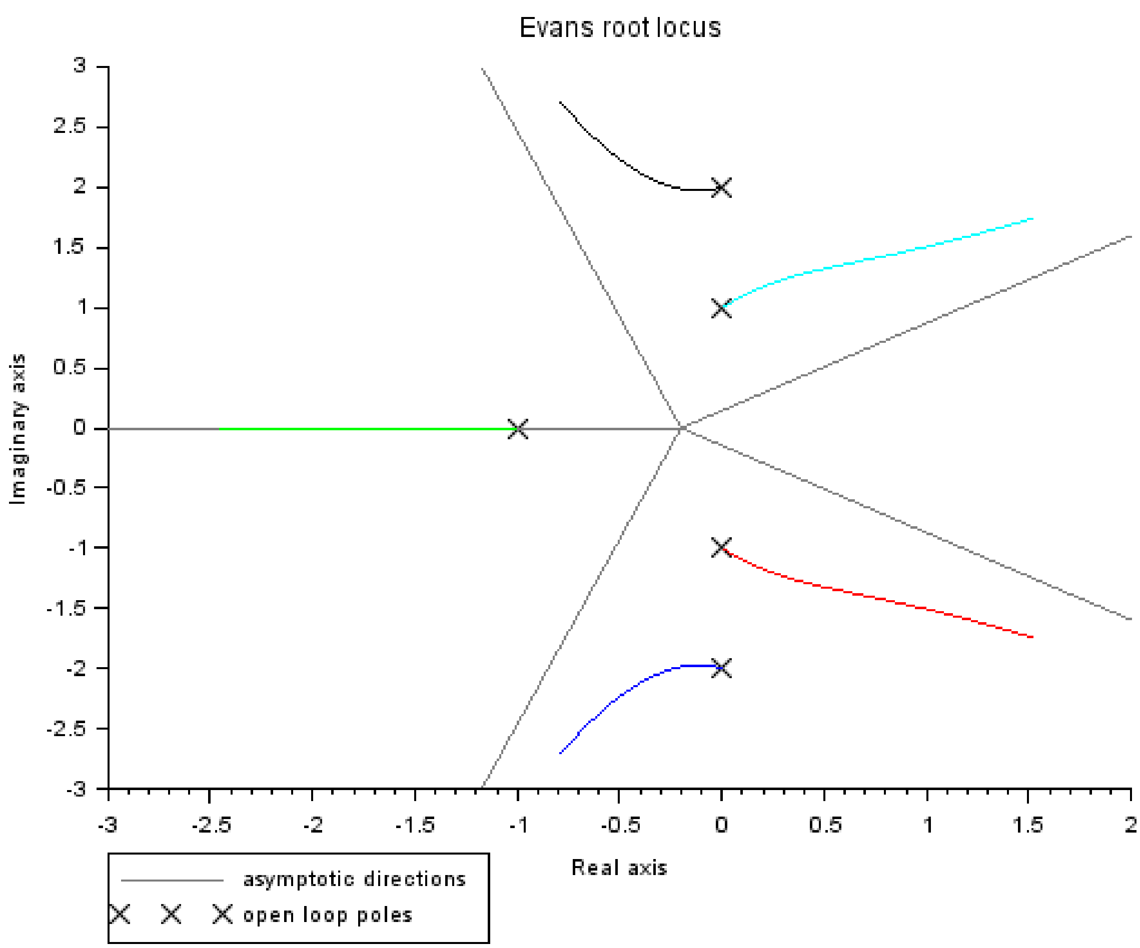
$$G_1(s) = \frac{1}{(s+1)(s^2+4)(s^2+1)}$$

We plot the root-locus plot as,

Scilab Code

```
clear
close
clc

s = poly(0,'s');

G = 1/((s+1)*((s)^2+4)*((s)^2+1));
Glin = syslin('c',G);
clf();
evans(Glin,100);
```

Obtained Root Locus



After following the steps mentioned in the question, without adding the zeros the transfer function is

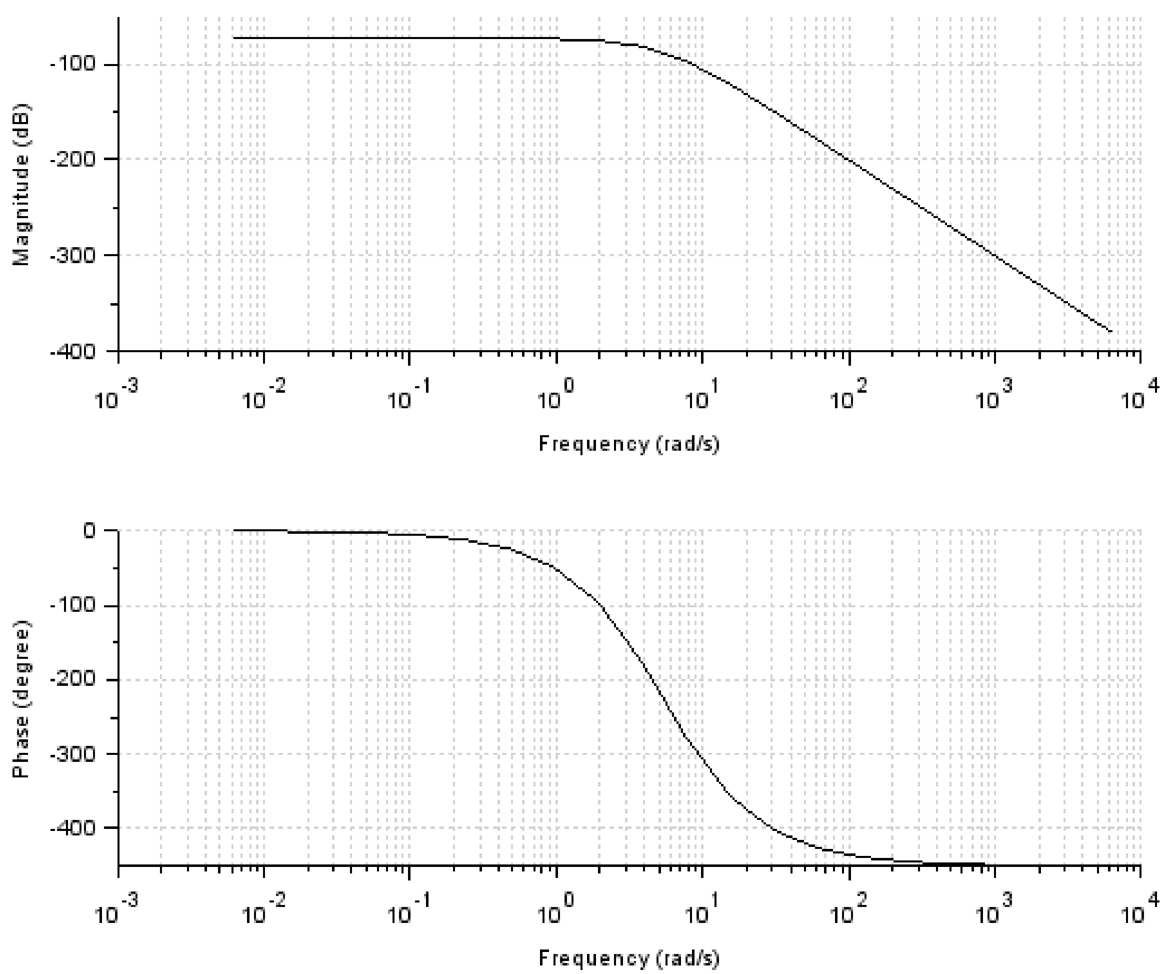
$$G_2(s) = \frac{1}{(s+6)((s+5)^2+4)((s+5)^2+1)}$$

We plot the bode plot as,

Scilab Code

```
clear
close
clc

s = poly(0,'s');

G = 1/((s+6)*((s+5)^2+4)*((s+5)^2+1));
Glin = syslin('c',G);
clf();
bode(Glin,'rad');
```

Obtained Bode Plot



Now, we have to place zeros such that we get two phase-crossover frequencies. The zeros are added at -50,-60,-70,-80. Hence, the transfer function is

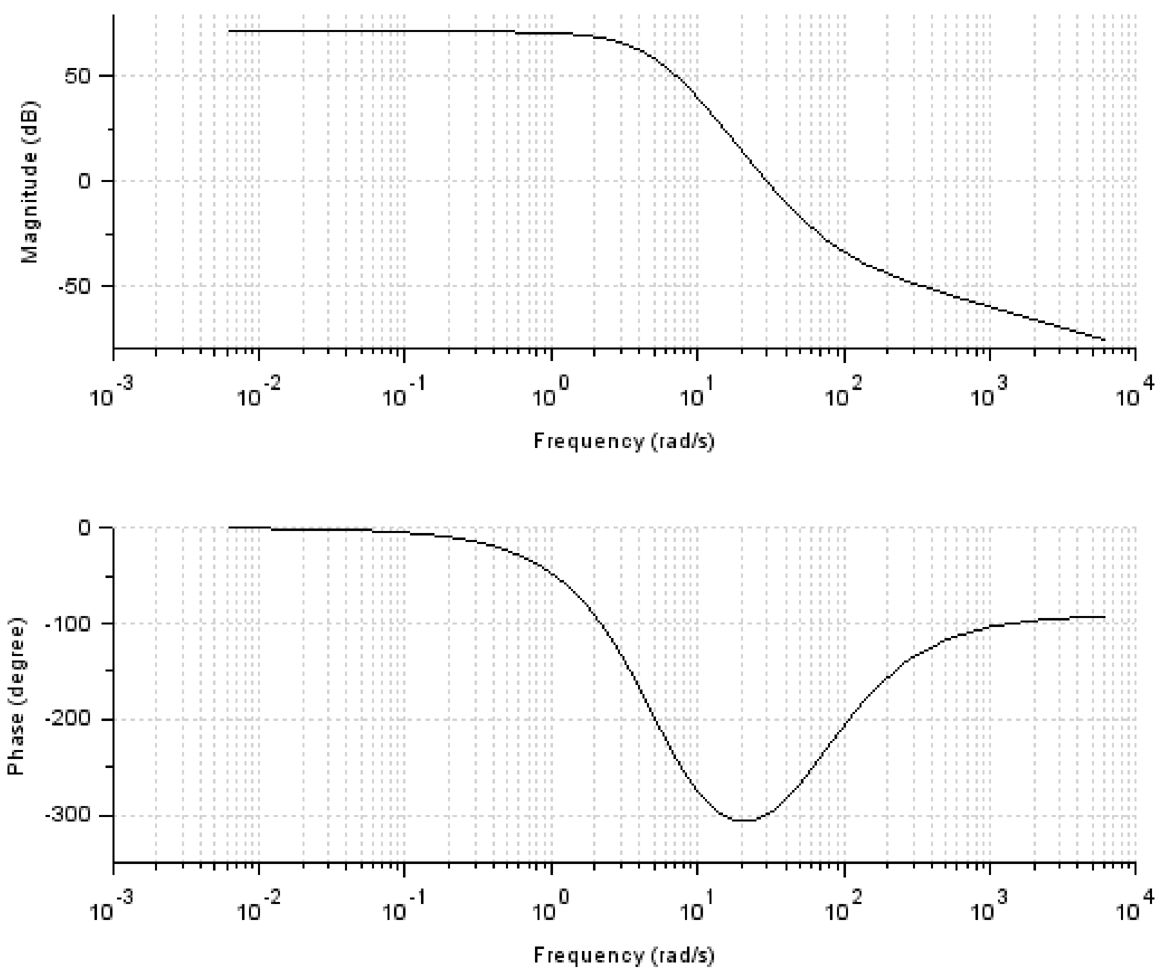
$$G_3(s) = \frac{(s+50)(s+60)(s+70)(s+80)}{(s+6)((s+5)^2+4)((s+5)^2+1)}$$

To see if it matches the problem statement, we plot the bode plot

Scilab Code

```
clear
close
clc

s = poly(0,'s');
G =(s+50)*(s+60)*(s+70)*(s+80)/((s+6)*((s+5)^2+4)*((s+5)^2+1));
Glin = syslin('c',G);
clf();
bode(Glin,'rad');
```

Obtained Bode Plot

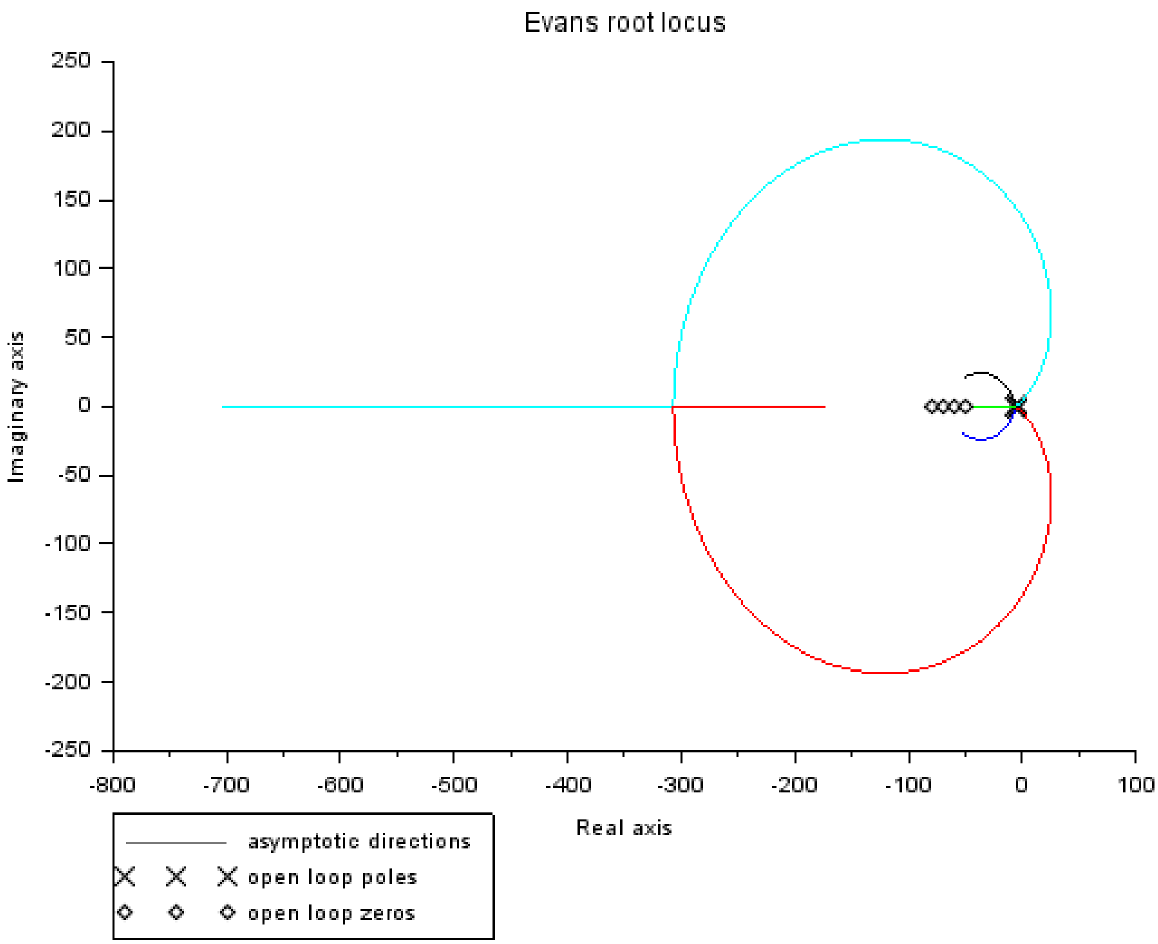


We plot the root-locus plot as,

Scilab Code

```
clear
close
clc

s = poly(0,'s');

G = (s+50)*(s+60)*(s+70)*(s+80)/((s+6)*((s+5)^2+4)*((s+5)^2+1));
Glin = syslin('c',G);
clf();
evans(Glin,1000);
```

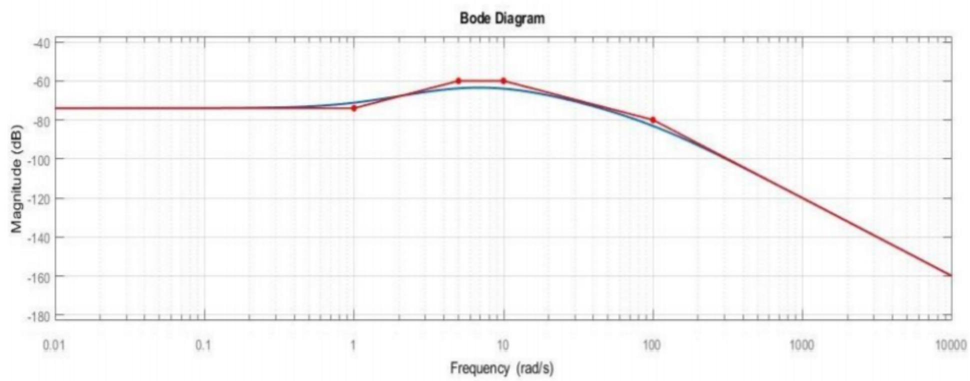
Obtained Root Locus



Hence, we can see that the root locus crosses the imaginary axis two times at different values of gain.

4 Bode Plot

The given magnitude bode plot is:



To design one of the transfer we consider the following pole-zero locations:

- Poles: -4, -10, -100
- Zero: -1

Hence, the total transfer function is,

$$G(s) = \frac{s+1}{(s+5)(s+10)(s+100)}$$

We draw the Bode plot of G(s)

4.1 Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (s+1)/((s+5)*(s+10)*(s+100)) ;
Gs = syslin('c' ,G);
bode(Gs,'rad');
```

4.2 Obtained Bode Plot