EE324: Control Systems Lab
Problem sheet 9

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Contents

1 Nyquist Plots

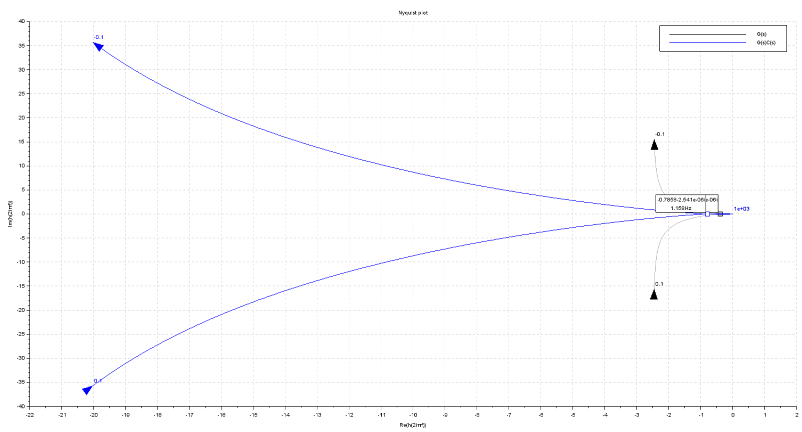
The given transfer function is:

$$G(s) = \frac{10}{s(\frac{s}{5}+1)(\frac{s}{20}+1)}$$

1.1 Lag Compensator

$$C(s) = \frac{s+3}{s+1}$$

Nyquist Plots



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (10)/(s*(s/5 + 1 ) *(s/20 + 1) );
Gs = syslin('c',G);
C = (s+3)/(s+1);
GsCs = syslin('c' , G*C);
nyquist([Gs ; GsCs], 0.1 , 1e3 ,["G(s)" ; "G(s)C(s)"]  );
gm=g_margin(Gs);
phm = p_margin(Gs);
disp(gm,"Gain Margin of G(s)");
disp(phm ,"Phase Margin of G(s)");
gm2 = g_margin(GsCs);
phm2 = p_margin(GsCs);
disp(gm2,"Gain Margin of G(s)C(s)");
disp(phm2 ,"Phase Margin of G(s)C(s)");
```

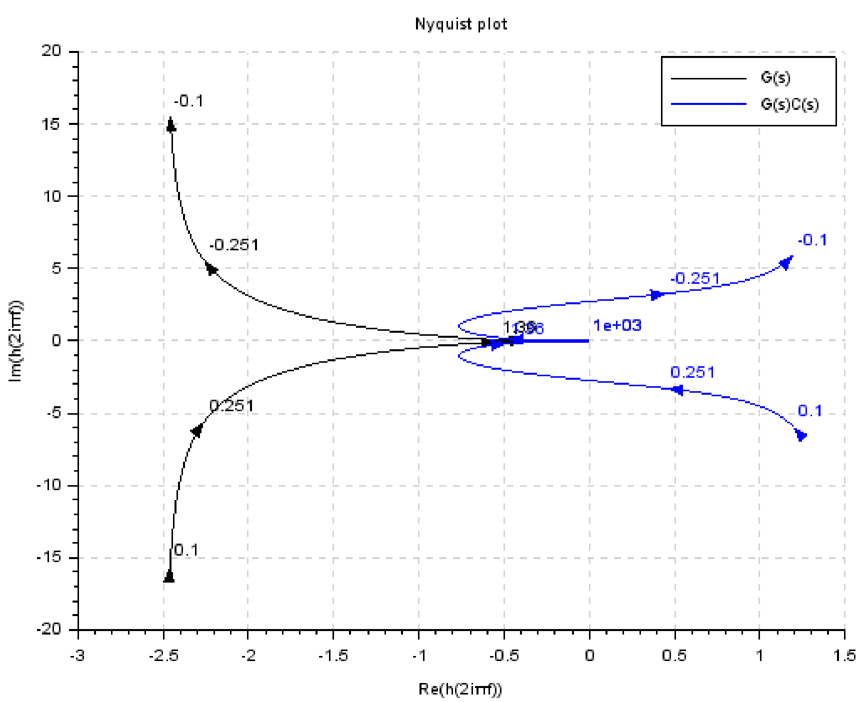
Phase and Gain Margin

```
Gain Margin of G(s)

  7.9588002

Phase Margin of G(s)

  22.535942

Gain Margin of G(s)C(s)

  2.0762546

Phase Margin of G(s)C(s)

  4.0247332
```

1.2 Lead Compensator

$$C(s) = \frac{s+1}{s+3}$$

Nyquist Plots



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (10)/(s*(s/5 + 1 ) *(s/20 + 1) );
Gs = syslin('c',G);
C = (s+1)/(s+3);
GsCs = syslin('c' , G*C);
nyquist([Gs ; GsCs], 0.1 , 1e3 ,["G(s)" ; "G(s)C(s)"]  );
gm=g_margin(Gs);
phm = p_margin(Gs);
disp(gm,"Gain Margin of G(s)");
disp(phm ,"Phase Margin of G(s)");
gm2 = g_margin(GsCs);
phm2 = p_margin(GsCs);
disp(gm2,"Gain Margin of G(s)C(s)");
disp(phm2 ,"Phase Margin of G(s)C(s)");
```

Phase and Gain Margin

```
Gain Margin of G(s)

  7.9588002

Phase Margin of G(s)

  22.535942

Gain Margin of G(s)C(s)

  11.759539

Phase Margin of G(s)C(s)

  43.173118
```

1.3 Observation

On applying a lag compensator, both phase and gain margin reduce, whereas on applying a lead compensator, both phase and gain margin increase

2 Notch Filter

We have that the transfer function of the notch filter is given by-

$$G(s) = \frac{s^2+w_z^2}{s^2+\frac{w_p}{Q}s+w_p^2}$$

Here we want bandstop filter at f=50Hz.

For a standard notch filter we need to keep $w_p = w_z$.

After adjusting the the parameters we get the below transfer function which gives 50Hz bandstop filter.

$G(s) = \frac{s^2+314.16^2}{s^2+\frac{314.16}{10}*s+314.16^2}$

To adjust the steepness we need to vary the quality factor Q. As can be seen below are two different bode plots for specified Q.

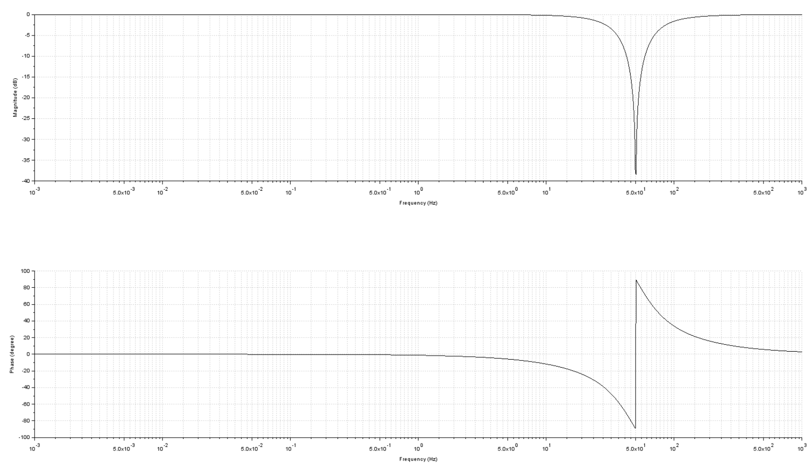


Figure 1: Bode plot for Q=1

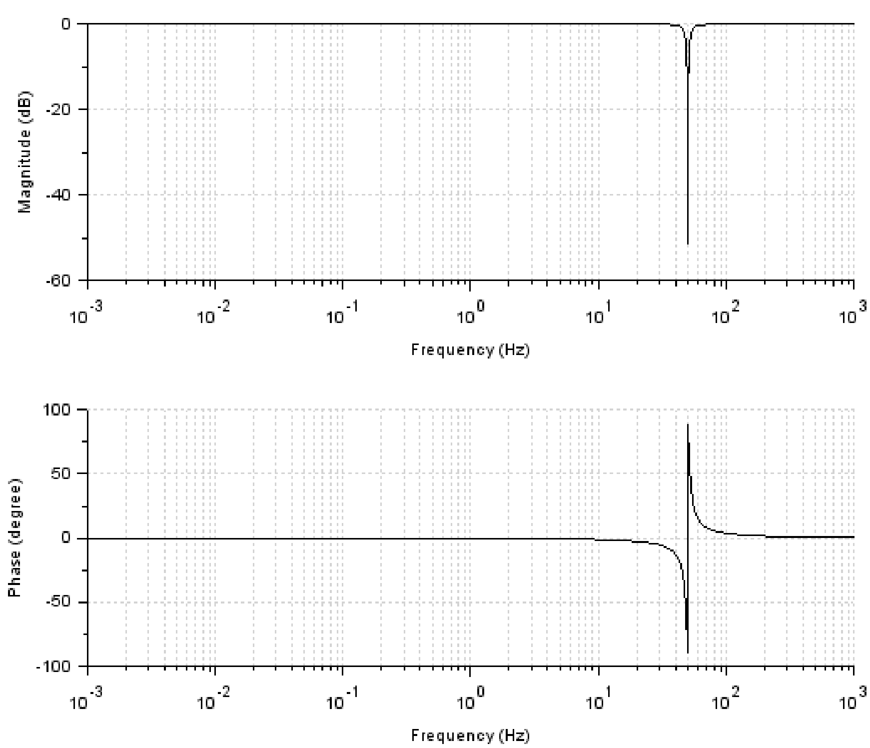


Figure 2: Bode plot for Q=10

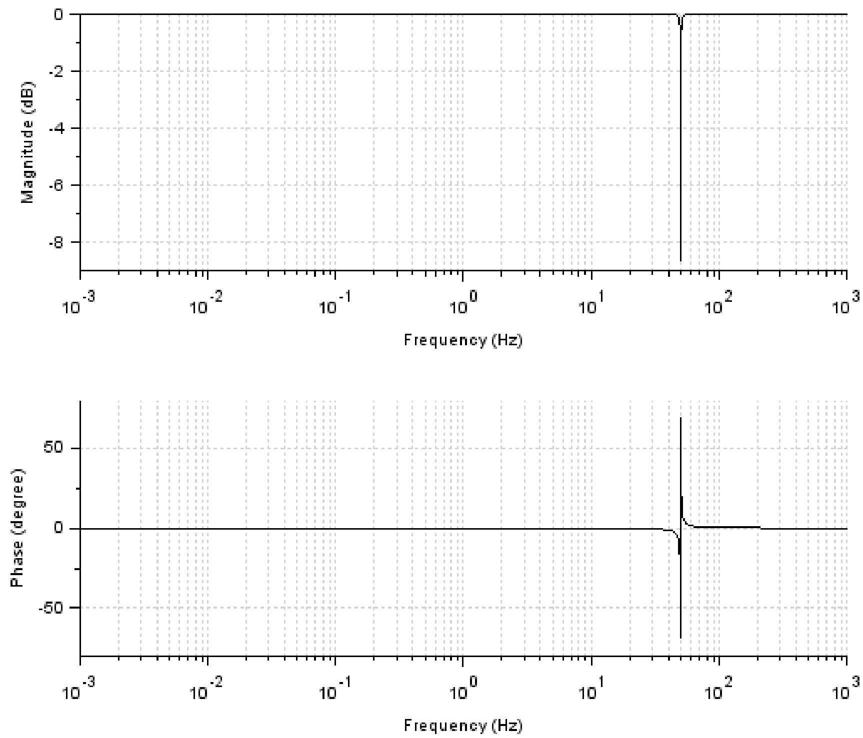


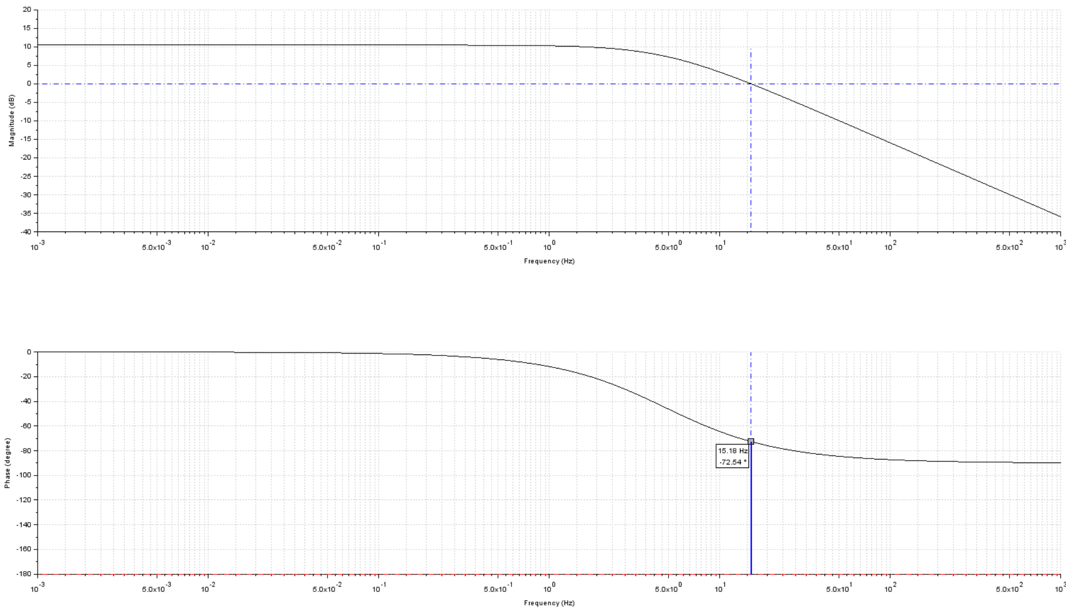
Figure 3: Bode plot for Q=100

Hence if we want steeper plots we must keep the quality factor high.

3 Minimum Delay to destabilize the closed loop system

First we plot the bode plot of the given transfer function:

3.1 Obtained Bode Plot



From the Bode plot the phase at a gain of 0 dB is $-72.54°$ and at 15.18 Hz. Adding a delay of T seconds gives an additional phase of $2\pi f T$. Hence to make the phase reach -2π rad at this frequency we need a delay as follows:

$$-\pi = -\frac{72.54}{180}\pi - 2\pi(15.18)T$$
$$\implies T = 0.01966 sec$$

Now, we plot the Bode Plot

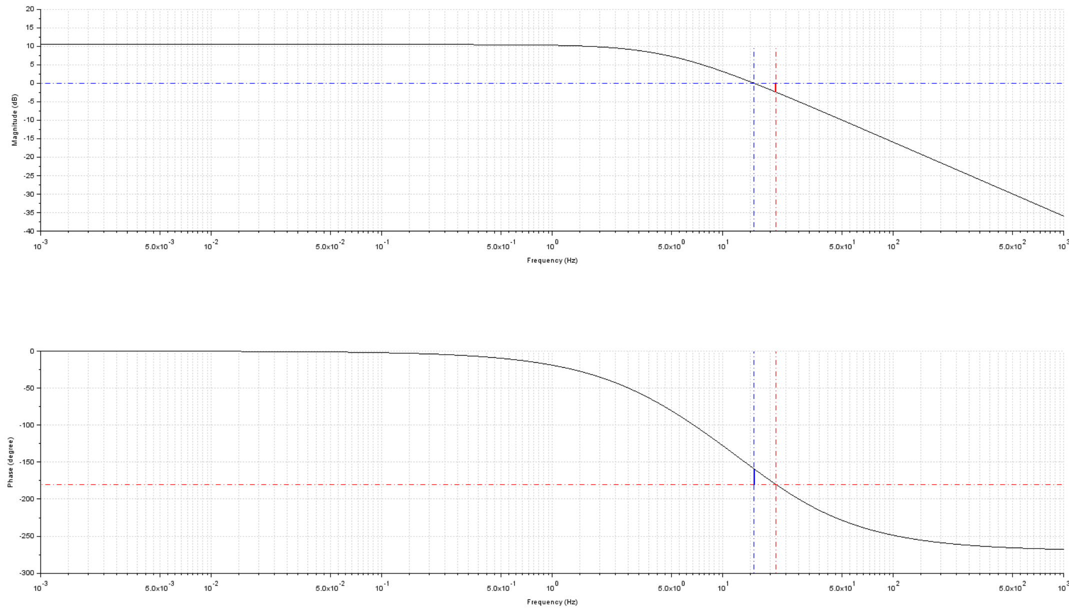
For a delay of T secs we need a transfer function $G(s) = e^{-sT}$

The above G is of the form- We will consider 1/1 pade approximation of exponential function.

$G(s) = \frac{2+sT}{2-sT} = \frac{2/T+s}{2/T-s}$

$C(s) = \frac{100}{s+30}$

Using these transfer functions we plot the Bode Plots.



Hence, we can see that the closed loop system is very close to being unstable as the phase margin is roughly zero.

3.2 Observations

Phase margin without delay is 107.46°
Phase margin with the delay is roughly 0°

4 Difference in Gain Margin

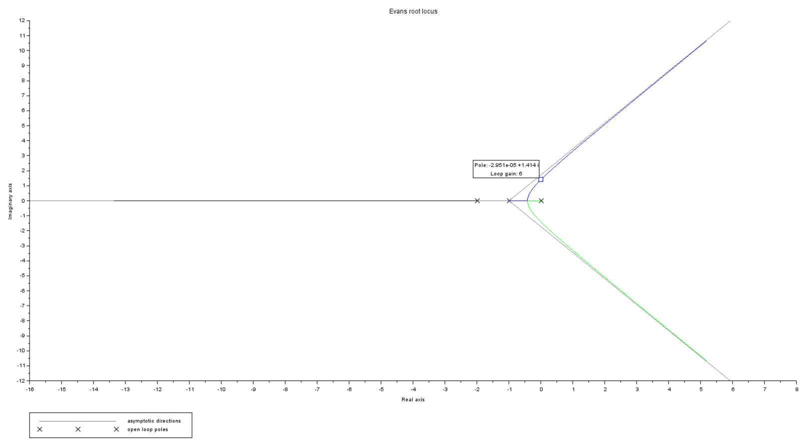
The given open loop transfer function is:

$$G(s) = \frac{1}{s^3 + 3s^2 + 2s}$$

4.1 Root-Locus

To find the gain margin using this method we plot the Root- Locus of the given transfer function.

Obtained Root Locus



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (1)/(s^3+ 3*s^2 + 2*s);
Glin = syslin('c',G);
clf();
evans(Glin);
```

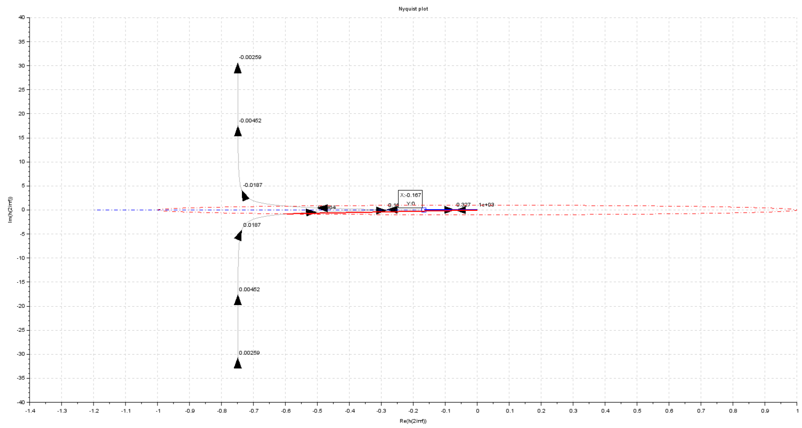
Observation

From the plot we can see that the imaginary axis crossing occurs at the gain of 6. Hence, using Root Locus we get the gain margin to be equal to 6.

4.2 Nyquist Plot

To find the gain margin using this method we plot the Nyquist Plot of the given transfer function.

Obtained Nyquist Plot



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (1)/(s^3+ 3*s^2 + 2*s );
Gs = syslin('c',G);
nyquist([Gs], 0.0187,1e3, ,["G(s)"]  );
//show_margins(Gs , 'nyquist'  );
//disp(g_margin(Gs))
show_margins(Gs,  'nyquist')
```

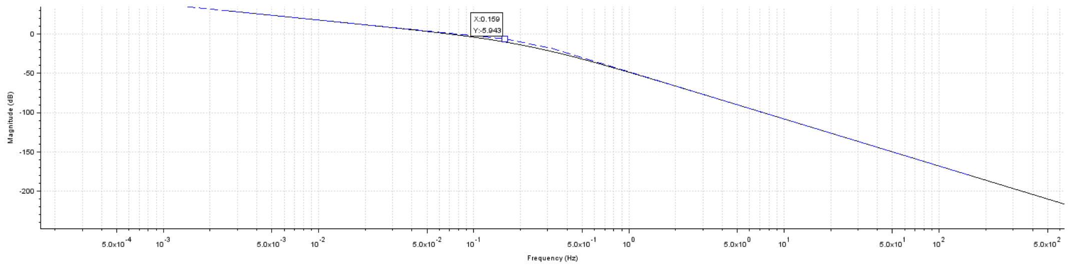
Observation

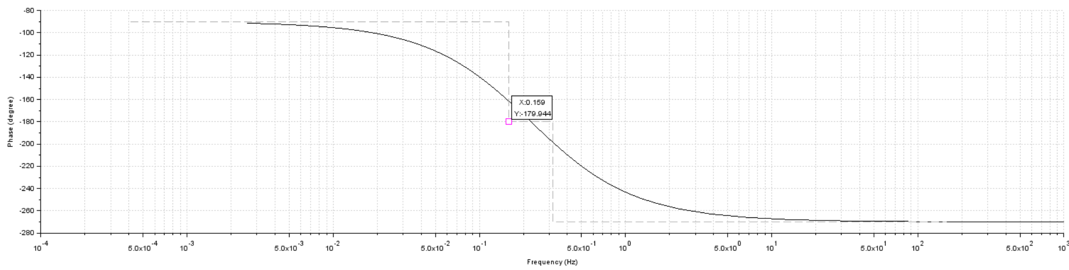
From the plot we can see that the real axis crossing occurs at the gain of 0.167 + 0i. Hence, we can multiply upto a gain of $\frac{1}{0.167} \approx 6$ till the system remains stable. Hence, using nyquist plot as well we get the gain margin to be equal to 6.

4.3 Asymptotic Bode Plot

To find the gain margin using this method we plot the asymptotic Bode plot of the given transfer function.

Obtained Bode Plot





Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (1)/(s^3+ 3*s^2 + 2*s);
Gs = syslin('c',G);
clf();
bode(Gs);
bode_asymp(Gs);
```

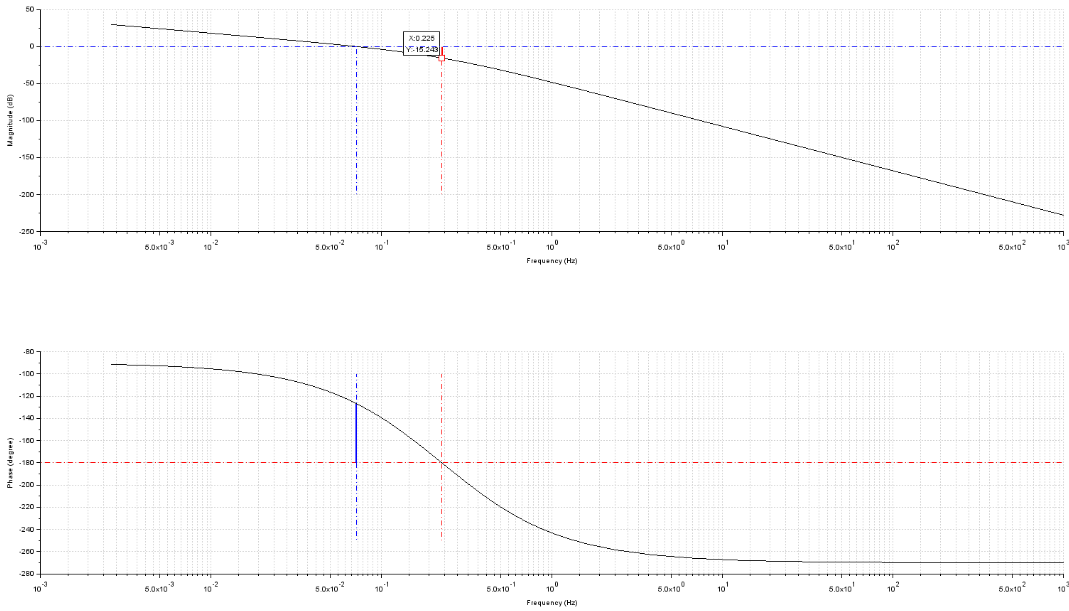
Observation

From the plot we can see that the gain at a phase of $-180°$ is equal to 5.943 (at $\omega = 0.159 rad/s$. Hence we can see the using asymptotic bode plot we get the gain margin again close to 6.

4.4 Bode Plot

To find the gain margin using this method we plot the Bode plot of the given transfer function.

Obtained Bode Plot



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (1)/(s^3+ 3*s^2 + 2*s);
Gs = syslin('c',G);
clf();
bode(Gs);
show_margins(Gs , 'bode')
```

Observation

From the plot we can see that the gain at a phase of $-180°$ is equal to 15.563 dB (= 6). Hence we can see the using asymptotic bode plot we get the gain margin again close to 6.

4.5 Observation

We can easily see that the gain margin using all the 4 methods is roughly same i.e. 6.

5 Question 5

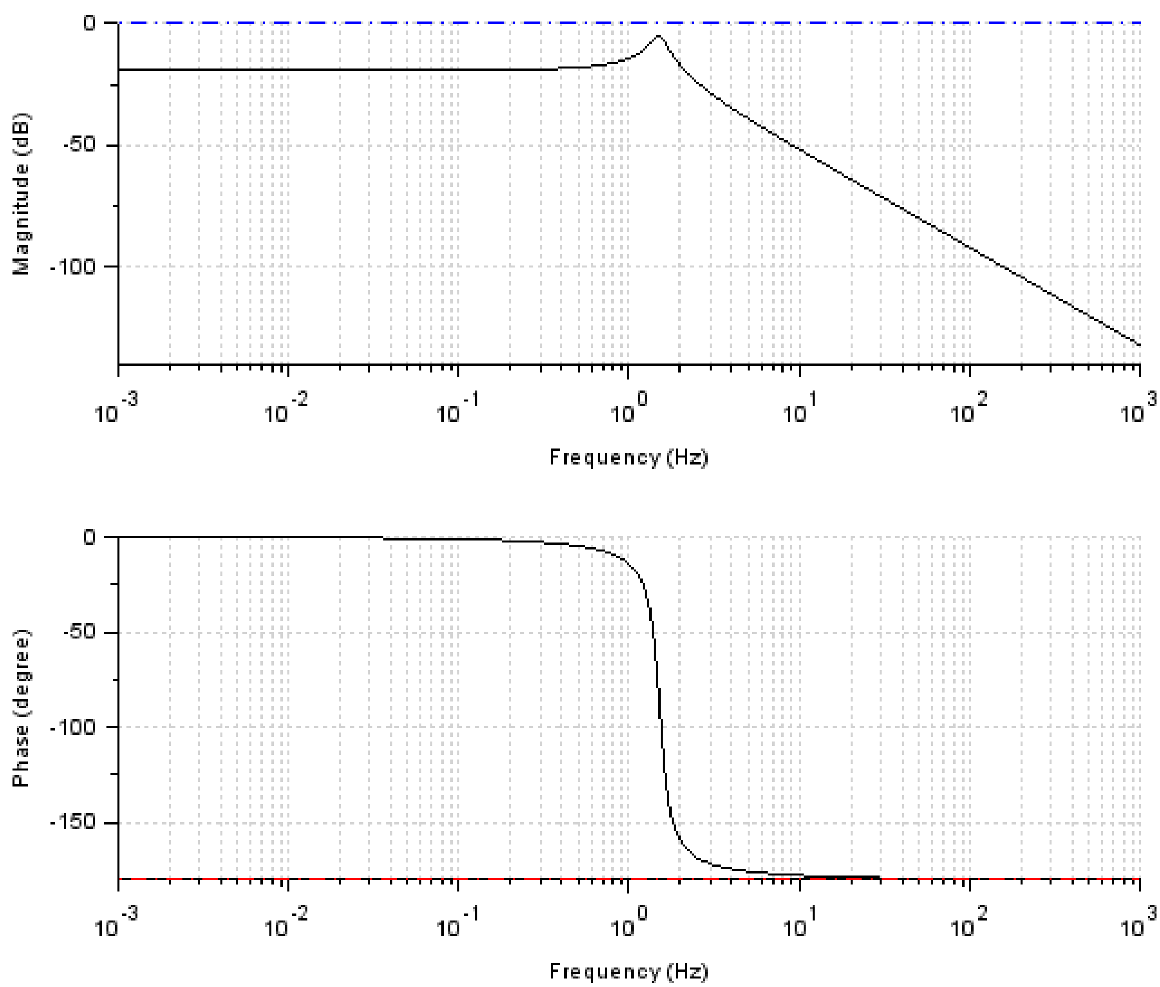
The given open loop transfer function is:

$$G(s) = \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

5.1 Part 1

We plot the Bode plot of the system as follows:

5.2 Obtained Bode Plot



Scilab Code

```
clear
close
clc

s = poly(0,'s');
G = (10*s + 2000)/(s^3 + 202*s^2 + 490*s + 18001);
Gs = syslin('c',G);
clf();
bode(Gs);
show_margins(Gs , 'bode')
[gm,fpcf]=g_margin(Gs);
[phm,fgcf] = p_margin(Gs);
disp(gm,"Gain Margin ");
disp(phm ,"Phase Margin ");
```

```
disp(fgcf ,"Gain Cross-over Frequency ")
disp(fpcf ,"Phase Cross-over Frequency ")
```

Obtained Gain and Phase Margins

```
Gain Margin

  Inf

Phase Margin

   []

Gain Cross-over Frequency

   []

Phase Cross-over Frequency

   []
```

Hence, as the phase plot never crosses the $-180°$ line and the magnitude plot never crosses the 0 dB line for a finite frequency, the gain and the phase margins are infinite.

5.3 Part 2

Now, we multiply by a gain K such that the steady-state error becomes 0.1.
We know that the steady state error for a unit step is given by:

$$S.S.E. = \lim_{x \to 0} \frac{1}{1 + KG(s)}$$

From the given open loop transfer function, G(s), we can find the value of G(0) = 0.1111 . Hence, the value of K for a steady state error of 0.1 is calculated as:

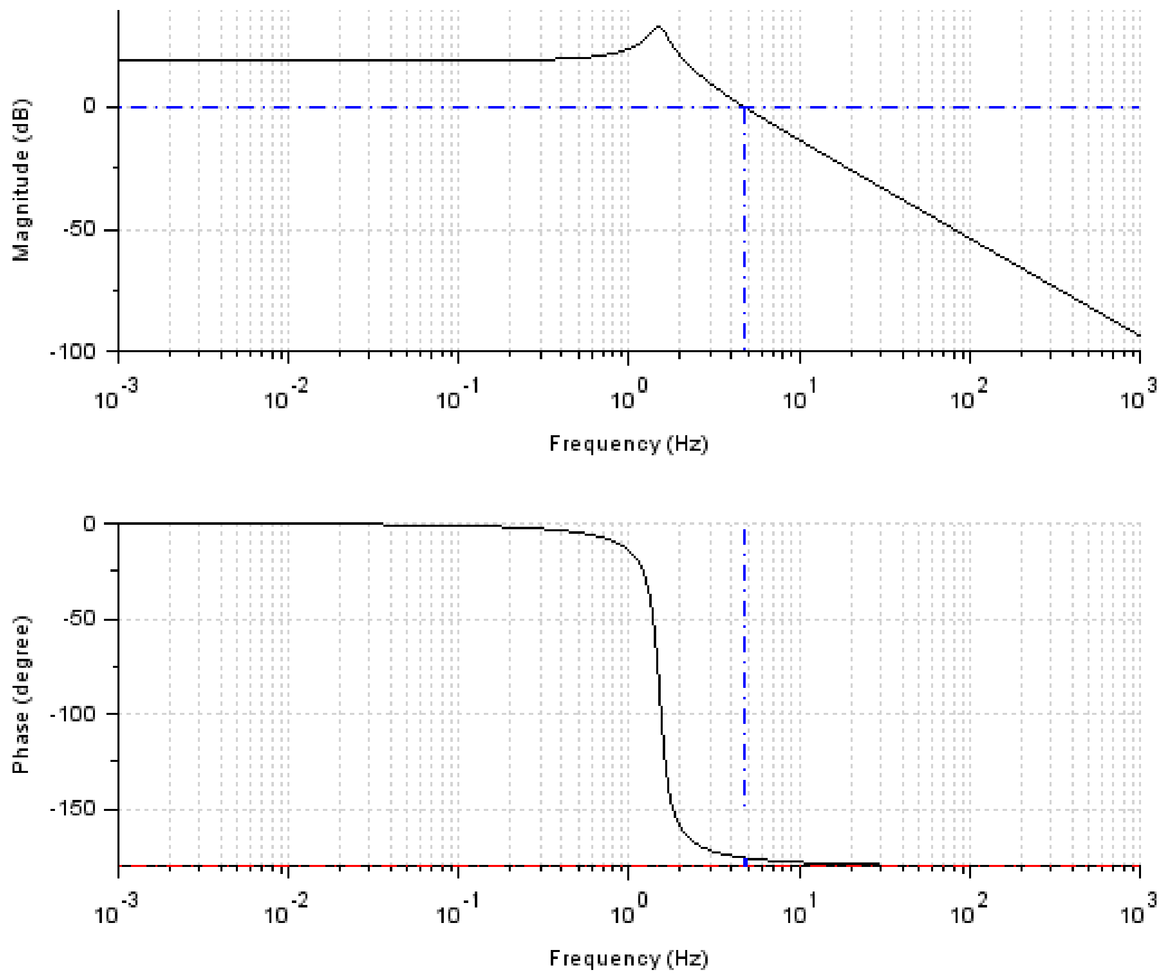
$$0.1 = \frac{1}{1 + K \times 0.1111}$$
$$\implies K = 81.008$$

5.4 Part 3

$$G_1(s) == 81.008 \times \frac{10s + 2000}{s^3 + 202s^2 + 490s + 18001}$$

We get the phase margin, gain margin and the crossover frequecnies as:

5.5 Obtained Bode Plot



Scilab Code

```
clear
close
clc

s = poly(0,'s');
K = 81.008;
G = K*(10*s + 2000)/(s^3 + 202*s^2 + 490*s + 18001);
Gs = syslin('c',G);
clf();
bode(Gs , "rad");
show_margins(Gs , 'bode')
[gm,fpcf]=g_margin(Gs);
[phm,fgcf] = p_margin(Gs);
disp(gm,"Gain Margin ");
disp(phm ,"Phase Margin ");
disp(fgcf ,"Gain Cross-over Frequency ")
disp(fpcf ,"Phase Cross-over Frequency ")
```

Obtained Gain and Phase Margins

```
Gain Margin

  Inf

Phase Margin

  4.2425005

Gain Cross-over Frequency

  4.7689821

Phase Cross-over Frequency

   []
```

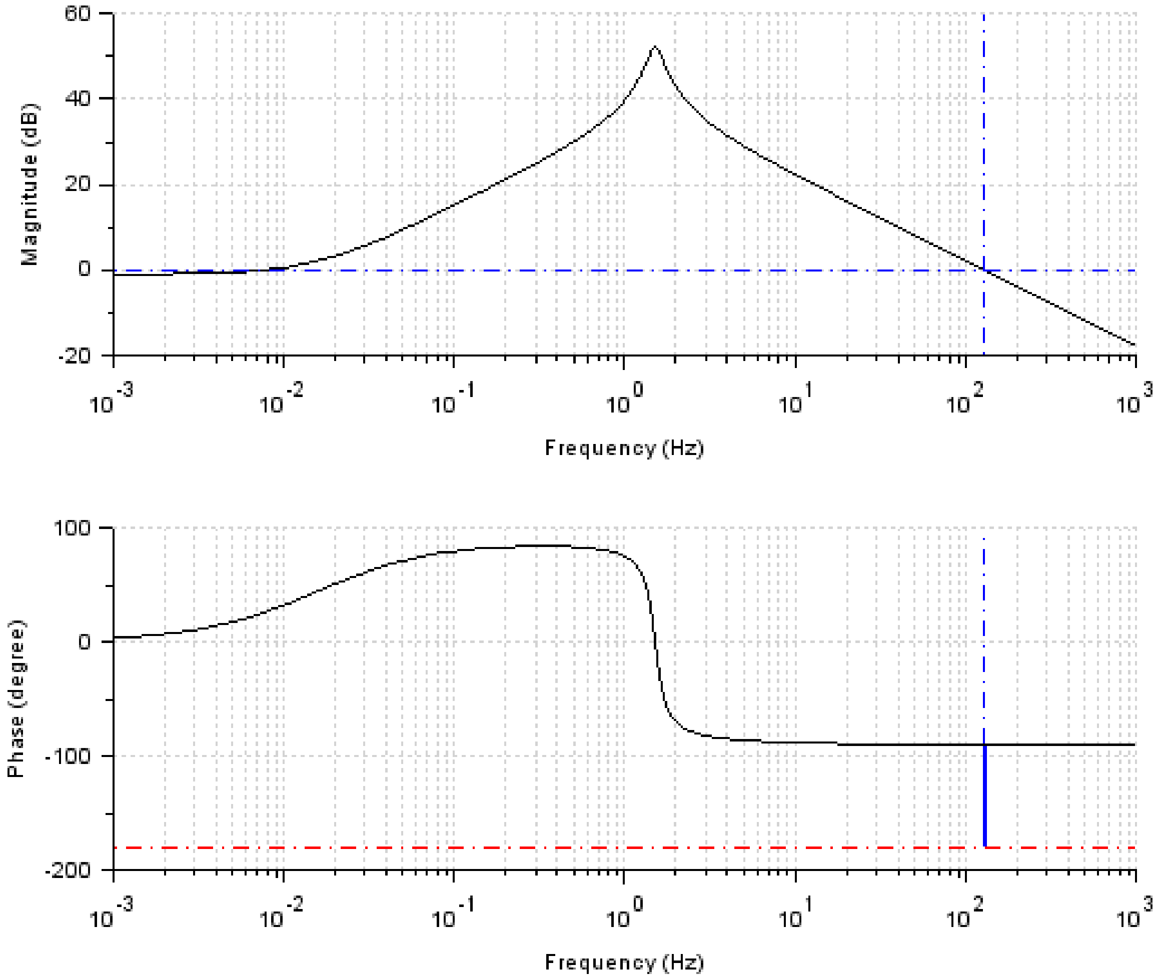
5.6 Part 4

Now, we improve the phase margin by cascading a zero. From the precious Bode plots we can see that the phase plot starts falling near 0Hz. Hence we place a zero very close to the origin and in the LHP at -0.1.

$$C(s) = s + 0.1$$

Hence, the obtained gain and phase margin are as follows:

5.7 Obtained Bode Plot



Scilab Code

```
clear
close
clc

s = poly(0,'s');
K = 81.008;
G = K*(10*s + 2000) * (s+0.1)/(s^3 + 202*s^2 + 490*s + 18001);
Gs = syslin('c',G);
clf();
bode(Gs , "rad");
show_margins(Gs , 'bode')
[gm,fpcf]=g_margin(Gs);
[phm,fgcf] = p_margin(Gs);
disp(gm,"Gain Margin ");
disp(phm ,"Phase Margin ");
disp(fgcf ,"Gain Cross-over Frequency ")
disp(fpcf ,"Phase Cross-over Frequency ")
```

Obtained Gain and Phase Margins

```
Gain Margin

  Inf

Phase Margin

  90.134385

Gain Cross-over Frequency

  128.94552

Phase Cross-over Frequency

    []
```

5.8 Part 5

From the bode plot of the transfer function obtained in part 4, we can easily see that the closed loop system will be stable.