Control Systems Lab: Problem sheet 1

Vinit Awale, 18D070067

January 17, 2021

Contents

1 Analysis in Laplace domain

$$G_1(s) = \frac{10}{s^2 + 2s + 10}$$
$$G_2(s) = \frac{5}{s+5}$$

1.1 Problem a

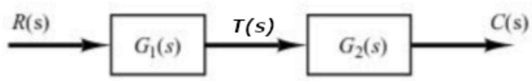


Figure 1: Cascaded system

From the definition of transfer function, we say that the transfer function of the Cascaded systems is defined as $G(s) = \frac{C(s)}{R(s)}$. Also we can easily see that $G_1(s)$ and $G_2(s)$ are defined as follows:

$$G_1(s) = \frac{T(s)}{R(s)} \tag{1.1.1}$$
$$G_2(s) = \frac{C(s)}{T(s)} \tag{1.1.2}$$

From equations 1.1.1 and 1.1.2 and the definition of G(s), we can easily see that,

$$\mathbf{G(s) = G_1(s).G_2(s)}$$

The scilab code for finding the transfer function is as follows

```
clear
clc


s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

n2 = 5;
```

```
d2 = s+5;
g2 = n2/d2

g_cascaded = g1*g2;
disp( g_cascaded,"The transfer function of the cascaded
   system is")
```

For the given systems $G_1(s)$ and $G_2(s)$, the final transfer function is as follows

```
The transfer function of the cascaded
system is

       50
  ------------------
              2   3
  50 + 20s + 7s + s
```

1.2 Problem b

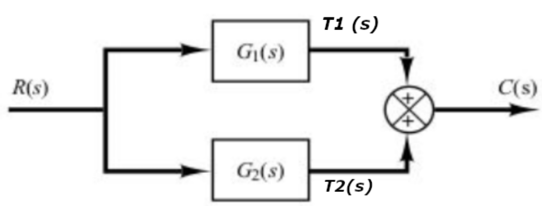


Figure 2: Parallel system

From the definition of transfer function, we say that the transfer function of the Cascaded systems is defined as $G(s) = \dfrac{C(s)}{R(s)}$. Also we can easily see that $G_1(s)$ and $G_2(s)$ are defined as follows:

$$G_1(s) = \frac{T1(s)}{R(s)} \tag{1.2.1}$$

$$G_2(s) = \frac{T2(s)}{R(s)} \tag{1.2.2}$$

From equations 1.2.1 and 1.2.2 and the definition of G(s), we can easily see that,

$$C(s) = T1(s) + T2(s)$$
$$G(s) = \frac{C(s)}{R(s)}$$
$$\implies G(s) = \frac{T1(s)}{R(s)} + \frac{T2(s)}{R(s)}$$
$$\implies \mathbf{G(s) = G_1(s) + G_2(s)}$$

The scilab code for finding the transfer function is as follows

```
clear
clc


s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

n2 = 5;
d2 = s+5;
g2 = n2/d2

g_parallel = g1+g2;
disp( g_parallel,"The transfer function of the parallel
    system is")
```

For the given systems $G_1(s)$ and $G_2(s)$, the final transfer function is as follows

```
 The transfer function of the parallel
 system is
                  2
    100 + 20s + 5s
  ------------------
                2   3
   50 + 20s + 7s + s
```

1.3 Problem c

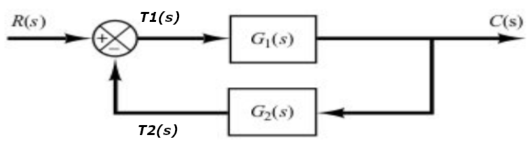


Figure 3: Feed-back system

From the definition of transfer function, we say that the transfer function of the Cascaded systems is defined as $G(s) = \dfrac{C(s)}{R(s)}$. Also we can easily see that $G_1(s)$ and $G_2(s)$ are defined as follows:

$$G_1(s) = \frac{C(s)}{T1(s)} \tag{1.3.1}$$

$$G_2(s) = \frac{T2(s)}{C(s)} \tag{1.3.2}$$

From equations 1.3.1 and 1.3.2 and the definition of G(s), we can easily see that,

$$\begin{aligned}
T1(s) &= R(s) - T2(s) \\
R(s) &= T1(s) + T2(s) \\
\implies \frac{C(s)}{R(s)} &= \frac{C(s)}{T1(s) + T2(s)} \\
\implies G(s) &= \frac{1}{\frac{1}{G_1(s)} + G_2(s)} \\
\implies \mathbf{G(s)} &= \frac{\mathbf{G_1(s)}}{\mathbf{1 + G_1(s).G_2(s)}}
\end{aligned}$$

The scilab code for finding the transfer function is as follows

```
clear
clc


s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

```

```
n2 = 5;
d2 = s+5;
g2 = n2/d2

g_feedback = g1/(1+ (g1*g2));
disp( g_feedback,"The transfer function of the feedback
    system is")
```

For the given systems $G_1(s)$ and $G_2(s)$, the final transfer function is as follows

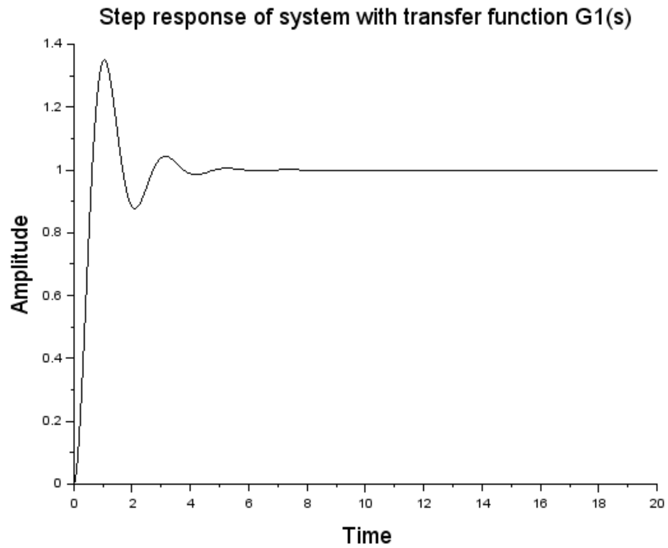
```
 The transfer function of the feedback system is


       50 + 10s
   -------------------
                2   3
   100 + 20s + 7s + s
```

1.4 Problem d

Step response of the system with transfer function G1(s).
The scilab code for plotting the step response is as follows

```
clear
clc

s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

G1 = syslin('c', g1);
t = 0:0.05:20;

gs1 = csim('step' , t , G1);

plot2d(t,gs1)
xlabel('Time','fontsize',4)
ylabel('Amplitude','fontsize',4)

title('Step response of system with transfer function G1(s)',
     'fontsize',4)
```

The output plot obtained is as follows



2 Finding Poles and Zeros

For this part the following functions/attributes are used in Scilab:

1. sys.num : The numerator of the system 'sys' can be extracted using this attribute
2. sys.den : The denominator of a system 'sys' can be extracted using this attribute
3. roots(p) : This function gives the roots of the polynomial 'p'
4. plzr(sys) : The pole-zero plot of a system is obtained using this function

2.1 Cascaded system

Scilab code for finding the poles and zeros of the system and plotting pole-zero plot is as follows

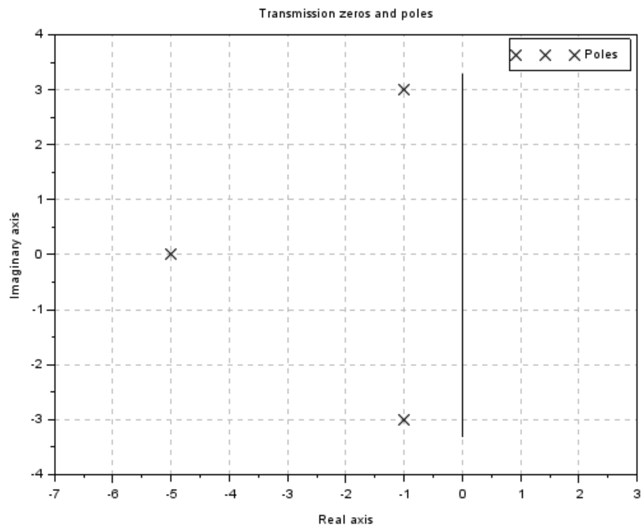
```
clear
close
clc

```

```
s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

n2 = 5;
d2 = s+5;
g2 = n2/d2;

g_cascaded = g1*g2;
G = syslin('c', g_cascaded);

z = roots(G.num);
p = roots(G.den);

disp(z, 'The zeros of the system are : ');
disp(p, 'The poles of the system are : ');

plzr(G);  // Pole-zero plot of the system
```

The output obtained is as follows

```
The zeros of the system are :

    []

The poles of the system are :

  -5.
  -1. + 3.i
  -1. - 3.i
```

The pole-zero plot of the cascaded system is as follows The output obtained is as follows



2.2 Parallel system

Scilab code for finding the poles and zeros of the system and plotting pole-zero plot is as follows

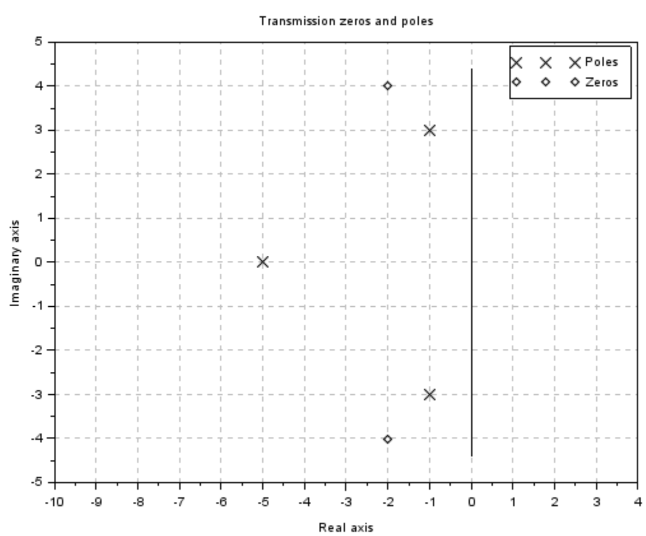
```
clear
close
clc

s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

n2 = 5;
d2 = s+5;
g2 = n2/d2;

g_parallel = g1+g2;
G = syslin('c', g_parallel);

z = roots(G.num);
p = roots(G.den);

disp(z, 'The zeros of the system are : ');
disp(p, 'The poles of the system are : ');
```

```
plzr(G);   // Pole-zero plot of the system
```

The output obtained is as follows

```
 The zeros of the system are :

  -2. + 4.i
  -2. - 4.i

 The poles of the system are :

  -5.
  -1. + 3.i
  -1. - 3.i
```

The pole-zero plot of the parallel system is as follows The output obtained is as follows



2.3 Feedback system

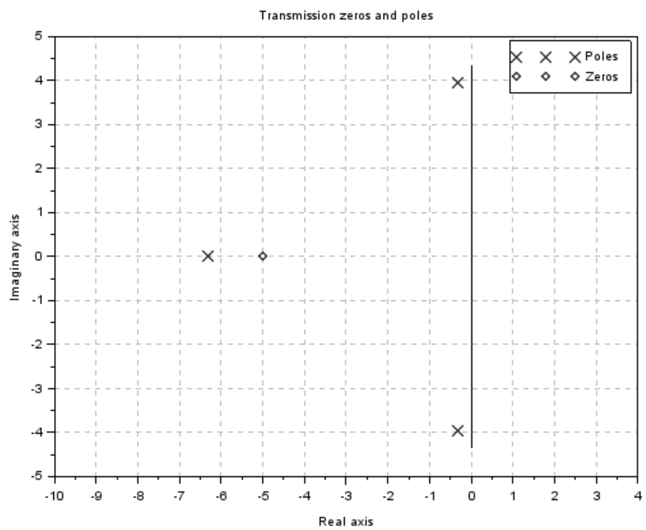
Scilab code for finding the poles and zeros of the system and plotting pole-zero plot is as follows

```
clear
close
clc

s = poly(0,'s');
n1 = 10;
d1 = s^2 + 2*s + 10;
g1 = n1/d1;

n2 = 5;
d2 = s+5;
g2 = n2/d2;

g_feedback = g1/(1+ (g1*g2));
G = syslin('c', g_feedback);

z = roots(G.num);
p = roots(G.den);

disp(z, 'The zeros of the system are : ');
disp(p, 'The poles of the system are : ');

plzr(G);  // Pole-zero plot of the system
```

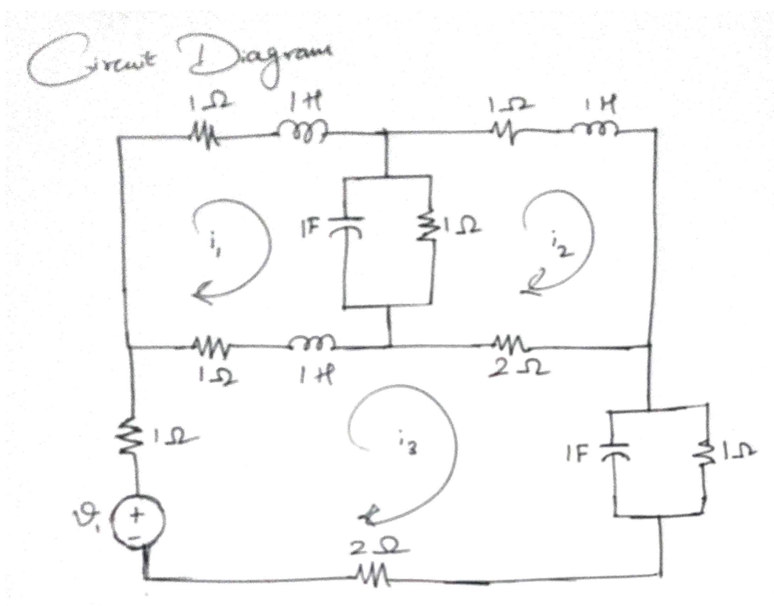
The output obtained is as follows

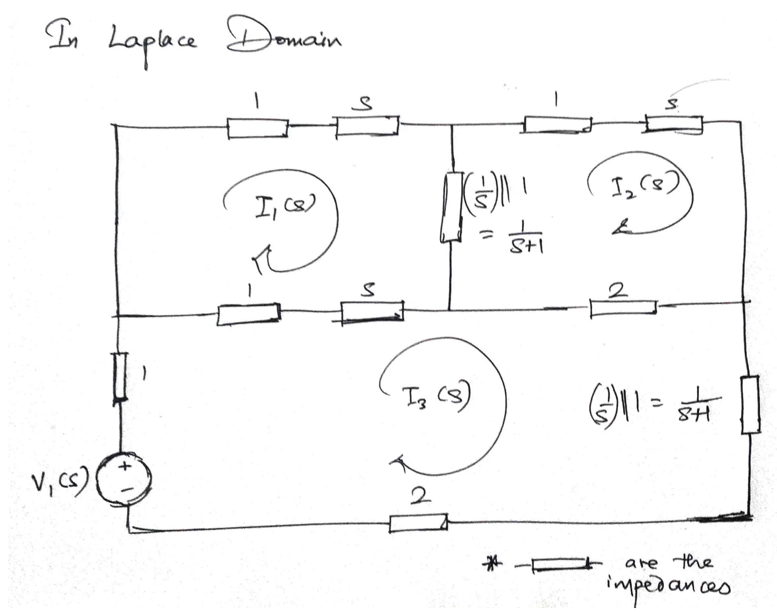
```
The zeros of the system are :

 -5.

The poles of the system are :

 -6.3347665
 -0.3326167 + 3.9592004i
 -0.3326167 - 3.9592004i
```

The pole-zero plot of the feedback system is as follows The output obtained is as follows



3 Mesh Analysis





Now, from the above circuit diagrams we can write the following KVL equations in Laplace domain:

1. For loop 1:

$$
\begin{gathered}
(s+1)I_1 + \frac{1}{s+1}(I_1 - I_2) + (s+1)(I_1 - I_3) = 0 \\
\implies (2s+2+\frac{1}{s+1})I_1 - \frac{1}{s+1}I_2 - (s+1)I_3 = 0
\end{gathered}
$$

2. For loop 2:

$$
\begin{gathered}
\frac{1}{s+1}(I_2 - I_1) + (s+1)I_2 + 2(I_2 - I_3) = 0 \\
\implies -\frac{1}{s+1}I_1 + (3+s+\frac{1}{s+1})I_2 - 2I_3 = 0
\end{gathered}
$$

3. For loop 3:

$$
\begin{gathered}
I_3 + s + 1(I_3 - I_1) + 2(I_3 - I_2) + (2+\frac{1}{s+1})I_3 = V_1 \\
\implies -(s+1)I_1 - 2I_2 + (s+6+\frac{1}{s+1_3} = 0
\end{gathered}
$$

These equations can be represented in a matrix form as follows:

$$\begin{bmatrix} 2s+2+\frac{1}{s+1} & -\frac{1}{s+1} & -s-1 \\ -\frac{1}{s+1} & 3+s+\frac{1}{s+1} & -2 \\ -s-1 & -2 & s+6+\frac{1}{s+1} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}$$

Hence we have,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2s+2+\frac{1}{s+1} & -\frac{1}{s+1} & -s-1 \\ -\frac{1}{s+1} & 3+s+\frac{1}{s+1} & -2 \\ -s-1 & -2 & s+6+\frac{1}{s+1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}$$

Hence, we can find the transfer functions as follows:

$$\begin{bmatrix} \frac{I_1}{V_1} \\ \frac{I_2}{V_1} \\ \frac{I_3}{V_1} \end{bmatrix} = \frac{1}{V_1} \begin{bmatrix} 2s+2+\frac{1}{s+1} & -\frac{1}{s+1} & -s-1 \\ -\frac{1}{s+1} & 3+s+\frac{1}{s+1} & -2 \\ -s-1 & -2 & s+6+\frac{1}{s+1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ V_1 \end{bmatrix}$$

$$\implies \begin{bmatrix} \frac{I_1}{V_1} \\ \frac{I_2}{V_1} \\ \frac{I_3}{V_1} \end{bmatrix} = \begin{bmatrix} 2s+2+\frac{1}{s+1} & -\frac{1}{s+1} & -s-1 \\ -\frac{1}{s+1} & 3+s+\frac{1}{s+1} & -2 \\ -s-1 & -2 & s+6+\frac{1}{s+1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The Scilab code for finding out these transfer function is as follows:

```
clear
close
clc

s = poly(0,'s');

Z = [2*s+2+(1/(s+1)) , -1/(s+1) , (-s-1); -1/(s+1) , s+3+(1/(
    s+1)) , -2; -s-1 , -2 , 6+s+(1/(s+1))];// Defining the
    matrix
Vvec = [0;0;1]; // Defining the vector

I = (Z\Vvec); // Solving the matrix equation

disp(I(1,1),"The transfer function I1(s)/V(s) is")
disp(I(2,1),"The transfer function I2(s)/V(s) is")
disp(I(3,1),"The transfer function I3(s)/V(s) is")
```

The result obtained is as follows

```
The transfer function I1(s)/V(s) is

                    2    3   4
       6 + 14s + 13s + 6s + s
  ---------------------------------
                  2     3     4   5
  57 + 144s + 147s + 74s + 17s + s

The transfer function I2(s)/V(s) is

                      2    3
         7 + 16s + 13s + 4s
  ---------------------------------
                  2     3     4   5
  57 + 144s + 147s + 74s + 17s + s

The transfer function I3(s)/V(s) is

                   2     3    4
     11 + 28s + 27s + 12s + 2s
  ---------------------------------
                  2     3     4   5
  57 + 144s + 147s + 74s + 17s + s
```