EE324: CONTROL SYSTEMS LAB
PROBLEM SHEET 10

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Contents

1 State Space

We consider the following matrices:

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 4 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 4 & 3 & -2 \\ 0 & 5 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 & 4 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 \end{bmatrix}$$

1.1 Part A

The transfer G(s) we obtain is

$$\frac{465 + 49s + 3s^2 + 3s^3}{21 + s - 3s^2 + s^3}$$

Now after using the transformation: $A_t = T^{-1}AT$, $B_t = T^{-1}B$ and $C_t = CT$, we again calculate the transfer function as:

$$\frac{465 + 49s + 3s^2 + 3s^3}{21 + s - 3s^2 + s^3}$$

Hence, we can see that the transfer function remains same before and after using the transformation.

1.2 Part B

We calculate the poles of the transfer function to be:

```
roots are-

  2.4797048 + 2.1374234i
  2.4797048 - 2.1374234i
 -1.9594095
```

And the eigenvalues of A are:

```
eigenvalues of A are

 -1.9594095    0.                          0.
  0.           2.4797048 + 2.1374234i      0.
  0.           0.                          2.4797048 - 2.1374234i
```

Hence, we can see that the poles of the transfer function are the eigenvalues of A.

1.3 Part C

The value of D for a strictly proper transfer function is 0, while that of a bi proper transfer is 1.

Scilab Code

```
clear;
close;
clc;

s = poly(0, 's');
T = [1 0 2; 4 -1 0; 0 2 1];
disp(det(T));

A = [-1 2 0; 4 3 -2; 0 5 1];
B = [8; 3; 5];
C = [-1 0 4];
D = [3];
// part a
I = eye(A);
Gs = D + C*inv(s*I - A)*B;
disp(Gs);

A_t = inv(T)*A*T;
B_t = inv(T)*B;
C_t = C*T;

Gt = D + C_t*inv(s*I - A_t)*B_t;
disp(Gt);

//part b
root = roots(Gs.den);
disp(root,'roots are-');
[vecs, vals] = spec(A);
disp(vals,'eigenvalues of A are');
```

```
//part c
G1 = 1/(s^2 + 3*s + 2);
G2 = (s^2 -s + 3)/(s^2 + 3*s + 2);
sys1 = syslin('c', G1);
sys2 = syslin('c', G2);

ss1 = tf2ss(G1);
ss2 = tf2ss(G2);
disp(ss1(5));
disp(ss2(5));
```

2 Obtain State Space from Transfer Function

We have that the transfer function of given as:

$$G(s) = \frac{s+3}{s^2+5s+4}$$

2.1 Part a

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+3}{S^2+5s+4}$$
$$\implies \frac{Y(s)}{X(s)} \times \frac{X(s)}{U(s)} = \frac{s+3}{s^2+5s+4}$$

Hence, we split the given transfer function as:

$$\frac{X(s)}{U(s)} = \frac{1}{s^2+5s+4} \tag{2.1.1}$$
$$\frac{Y(s)}{X(s)} = \frac{s+3}{1} \tag{2.1.2}$$

Hence, we can simplify equation 2.1.1 as

$$(s^2+5s+4)X(s) = U(s) \tag{2.1.3}$$
$$\implies s^2X(s) + 5sX(s) + 4X(s) = U(s) \tag{2.1.4}$$

Now, we define the following state variables

$$X_1 = X(s) \tag{2.1.5}$$
$$X_2 = \dot{X}_1 = sX(s) \tag{2.1.6}$$

Hence, from equation 2.1.4 we get

$$\dot{X}_2 = -5X_2 - 4X_1 + U(s) \tag{2.1.7}$$

Hence, using equations 2.1.6 and 2.1.7 we can write as follows:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(s) \tag{2.1.8}$$

Also from equation 2.1.2 we get

$$Y(s) = sX(s) + 3X(s)$$
$$\implies Y(s) = X_2 + 3X_1$$

$$\implies Y(s) = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{2.1.9}$$

Finally, using equations 2.1.8 and 2.1.9, we the state space model as:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U(s)$$
$$Y(s) = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

2.2 Part b

$$G(s) = \frac{s+1}{s^2+5s+4}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+1}{S^2+5s+4}$$
$$\implies \frac{Y(s)}{X(s)} \times \frac{X(s)}{U(s)} = \frac{s+1}{s^2+5s+4}$$

Hence, we split the given transfer function as:

$$\frac{X(s)}{U(s)} = \frac{1}{s^2+5s+4} \tag{2.2.1}$$
$$\frac{Y(s)}{X(s)} = \frac{s+1}{1} \tag{2.2.2}$$

Hence, we can simplify equation 2.2.1 as

$$(s^2+5s+4)X(s) = U(s) \tag{2.2.3}$$
$$\implies s^2X(s) + 5sX(s) + 4X(s) = U(s) \tag{2.2.4}$$

Now, we define the following state variables

$$X_1 = X(s) \tag{2.2.5}$$
$$X_2 = \dot{X}_1 = sX(s) \tag{2.2.6}$$

Hence, from equation 2.2.4 we get

$$\dot{X}_2 = -5X_2 - 4X_1 + U(s) \tag{2.2.7}$$

Hence, using equations 2.2.6 and 2.2.7 we can write as follows:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}U(s) \tag{2.2.8}$$

Also from equation 2.2.2 we get

$$Y(s) = sX(s) + X(s)$$
$$\implies Y(s) = X_2 + X_1$$

$$\implies Y(s) = \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} \tag{2.2.9}$$

Finally, using equations 2.2.8 and 2.2.9, we the state space model as:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -4 & -5\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}U(s)$$
$$Y(s) = \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

3 Pole-Zero Cancellation- Part 1

Consider the following matrices:

$$A = \begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 5\end{bmatrix}$$
$$B = \begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$
$$C = \begin{bmatrix}1 & 2 & 3\end{bmatrix}$$

Now, we find the transfer function corresponding to this State Space Model. We know that

$$G(s) = C(sI - A)^{-1}B$$

Now, we calculate $(sI - A)^{-1}$ as

$$sI - A = \begin{bmatrix}s-3 & 0 & 0\\ 0 & s-4 & 0\\ 0 & 0 & s-5\end{bmatrix}$$
$$\implies (sI - A)^{-1} = \begin{bmatrix}\frac{1}{s-3} & 0 & 0\\ 0 & \frac{1}{s-4} & 0\\ 0 & 0 & \frac{1}{s-5}\end{bmatrix}$$

Hence, we can calculate G(s) as

$$
\begin{aligned}
G(s) &= \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{s-3} & 0 & 0 \\ 0 & \frac{1}{s-4} & 0 \\ 0 & 0 & \frac{1}{s-5} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{s-3} & \frac{2}{s-4} & 0 & \frac{3}{s-5} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \\
&= \frac{1}{(s-3)} + \frac{3}{(s-5)} \\
&= \frac{4s-14}{(s-3)(s-5)}
\end{aligned}
$$

Hence, we can see from the above example that the pole at 4, for which the corresponding entry in B was 0, does not appear in the transfer function.

Similarly, if we had chosen C as

$$C = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$$

We would get the final transfer function as

$$G(s) = \frac{1}{s-3}$$

Hence, the pole at 5, for which the corresponding entry of C is 0, does not appear in the transfer function.

4 Pole Zero Cancellation- Part 2

Consider the following matrices:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Now, we find the transfer function corresponding to this State Space Model. We know that

$$G(s) = C(sI - A)^{-1}B$$

Now, we calculate $(sI - A)^{-1}$ as

$$sI - A = \begin{bmatrix} s-3 & 0 & 0 \\ 0 & s-4 & 0 \\ 0 & 0 & s-4 \end{bmatrix}$$

$$\implies (sI - A)^{-1} = \begin{bmatrix} \frac{1}{s-3} & 0 & 0 \\ 0 & \frac{1}{s-4} & 0 \\ 0 & 0 & \frac{1}{s-4} \end{bmatrix}$$

Hence, we can calculate G(s) as

$$
\begin{aligned}
G(s) &= \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{s-3} & 0 & 0 \\ 0 & \frac{1}{s-4} & 0 \\ 0 & 0 & \frac{1}{s-4} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{s-3} & \frac{2}{s-4} & 0 & \frac{3}{s-4} \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \\
&= \frac{1}{(s-3)} + \frac{6}{(s-4)} - \frac{6}{(s-4)} \\
&= \frac{1}{(s-3)}
\end{aligned}
$$

Hence, we can see from the above example that the pole at 4 which was repeated, is cancelled even when the entries in B and C are non zero.