A_1

September 2, 2021

EE 679: Computing Assignment 1

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0.1 Question 1:

Given the following specification for a single-formant resonator, obtain the transfer function of the filter H(z) from the relation between resonance frequency / bandwidth, and the pole angle / radius. Plot filter magnitude response (dB magnitude versus frequency) and impulse response.

F1 (formant) = 900 Hz
B1(bandwidth) = 200 Hz
Fs (sampling freq) = 16 kHz

0.2 Solution:

We can calculate the transfer function using the following equation:

$$H(z) = \frac{1}{1 - 2rcos\theta z^{-1} + r^2 z^{-2}}$$

Where r is the magnitude of the pole and $theta$ is the angle of the pole.

$$r = e^{-\pi BT}\theta = 2\pi FT$$

where, T is the sampling period and F is the frequency of the formant.

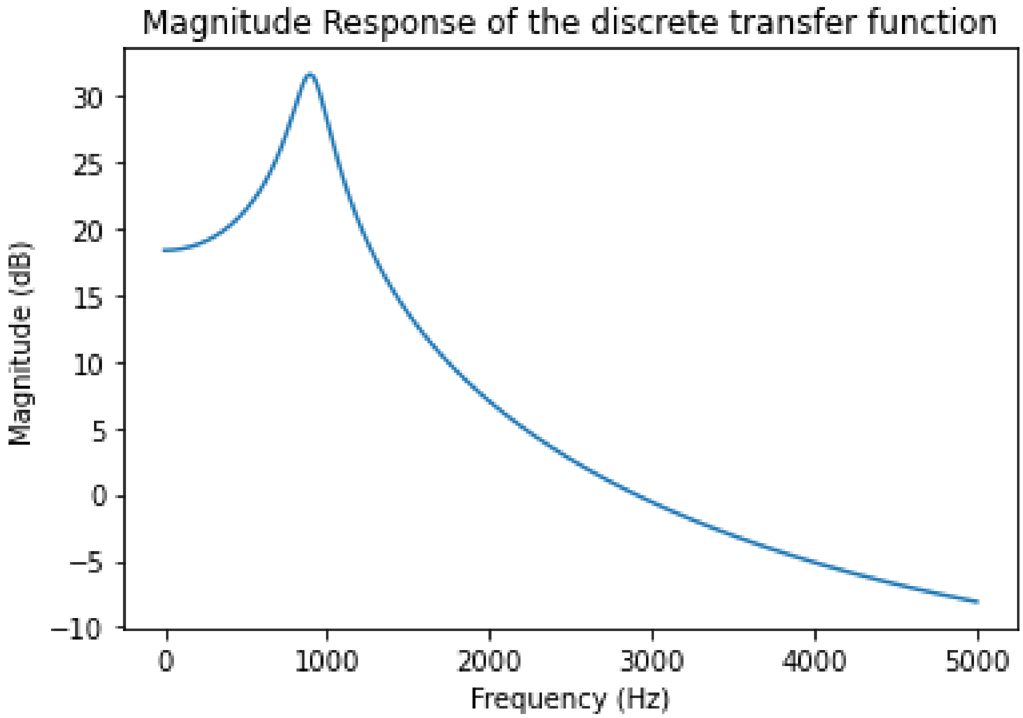
Further, to get the transfer function as a function of the frequency, we can substitute z as:

$$z = e^{j\omega T} = e^{\frac{j2\pi f}{Fs}}$$

$$\implies H(f) = \frac{1}{1 - 2rcos\theta z^{-1} + r^2 z^{-2}}\Bigg|_{z=e^{\frac{j2\pi f}{Fs}}}$$

[8]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.io.wavfile import write

# Input Parameters
```

```
F1 = 900
B1 = 200
Fs = 16000

freq_range = np.arange(0,5000,1)

# Function to calculate the transfer function magnitude from poles parameters
def H_discrete_transfer_mag(F1, B1, Fs):
    """ Function to get the magnitude of thediscrete transfer function from␣
 ↪Formant freq and bandwidth

    Args:
        F1 (int): Formant frequency
        B1 (int): Formant bandwidth
        Fs (int): Sampling frequency

    Returns:
        [ndarray]: Magnitude of the discrete transfer function
    """
    T = 1/Fs                                    # Sampling period
    H_discrete = []                             # Empty list to store the discrete␣
 ↪transfer function
    r1 = np.exp(-np.pi*B1*T)                    # Magnitude of the pole␣
 ↪corresponding to the formant 1
    theta1 = 2*np.pi*F1*T                       # Angle of the pole corresponding␣
 ↪to the formant 1
    freq_range = np.arange(0,5000,1)            # define the range of frequency␣
 ↪values

    for f in freq_range:
        z = np.exp(2j*np.pi*f/Fs)               # complex exponential in discrete␣
 ↪domain
        H = 1/(1-2*r1*np.cos(theta1)*z**-1 + r1**2*z**-2)  # discrete transfer␣
 ↪function calculation
        H_discrete.append(np.abs(H))

    return H_discrete

H = H_discrete_transfer_mag(F1, B1, Fs)

plt.plot(freq_range, 20* np.log10(H))
plt.xlabel('Frequency (Hz)')
plt.ylabel('Magnitude (dB)')
plt.title('Magnitude Response of the discrete transfer function')
plt.show()
```



0.3 Calculation of the Impulse Response

To calculate the Impulse Response we construct the difference equation from the transfer function.

$$H(z) = \frac{1}{1 - 2rcos\theta z^{-1} + r^2 z^{-2}}$$

Comparing H(z) with standard form of transfer function

$$H(z) = \frac{b_0}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

we get:

$$a_0 = 1 a_1 = -2rcos\theta a_2 = r^2 b_0 = 1$$

Hence, we can form the difference equation from the transfer function as follows:

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] \implies a_0 y[n] = b_0 x[n] - a_1 y[n-1] - a_2 y[n-2]$$

Subsituting the values of a_0, a_1, a_2 and b_0 in the above equation we get:

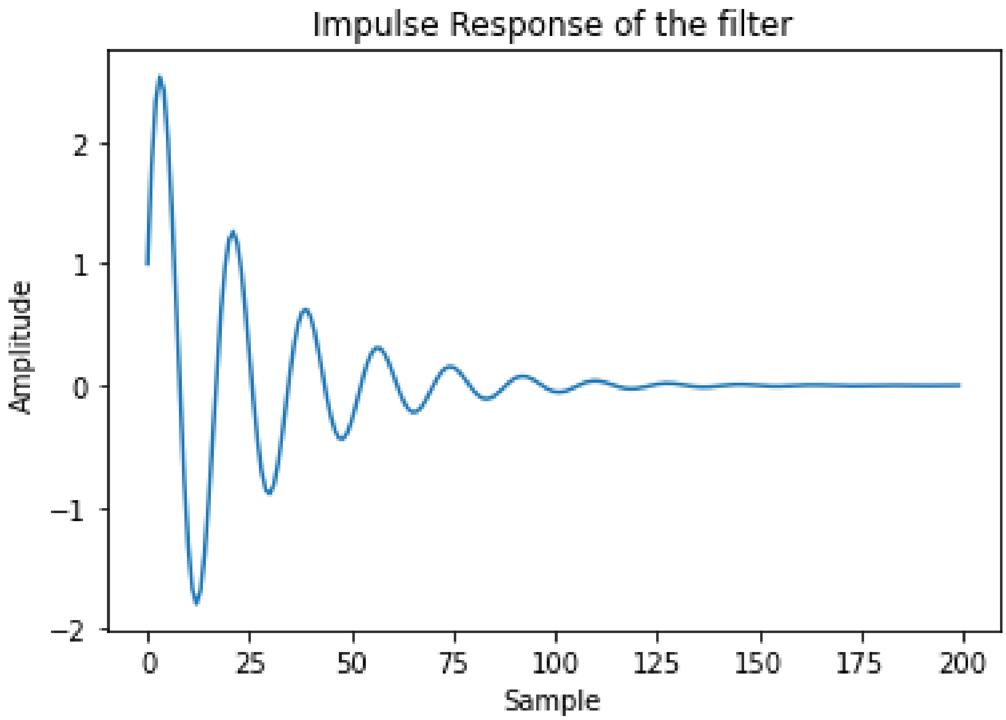
$$y[n] = x[n] + 2rcos\theta x[n-1] - r^2 x[n-2]$$

[9]:
```
# Implement the given filter as a difference equation

def filter(x, F1, B1):
```

```
    """ Function to get the output signal when input signal x is passed through␣
↪filter with Formant Freq F1 and
    Formant Bandwidth B1

    Args:
        x (ndarray): Input signal
        F1 (int): Formant Frequency
        B1 (int): Formant Bandwidth

    Returns:
        [ndarray]: Output signal
    """
    T = 1/Fs                                    # Sampling period
    r1 = np.exp(-np.pi*B1*T)                    # Magnitude of the pole␣
↪corresponding to the formant 1
    theta1 = 2*np.pi*F1*T                       # Angle of the pole corresponding␣
↪to the formant 1
    y = np.zeros(x.size)                        # Initialize output array

    # Assuming initial reset we get the first two terms of the output as
    y[0] = x[0]
    y[1] = x[1] + 2*r1*np.cos(theta1)*y[0]

    # The rest of the output is calculated using the difference equation
    for i in range(2, x.size):
        y[i] = x[i] + 2*r1*np.cos(theta1)*y[i-1] - r1**2*y[i-2]

    return y
```

Now, to calculate the impulse response for the filter with given formant, we pass an impulse through the filter and observe the output.

[10]:
```
impulse = np.zeros(200)
impulse[0] = 1                                  # Impulse signal

impulse_response = filter(impulse,F1,B1)        # Impulse response of the filter

plt.plot(impulse_response)
plt.xlabel('Sample')
plt.ylabel('Amplitude')
plt.title('Impulse Response of the filter')
plt.show()
```



0.4 Question 2:

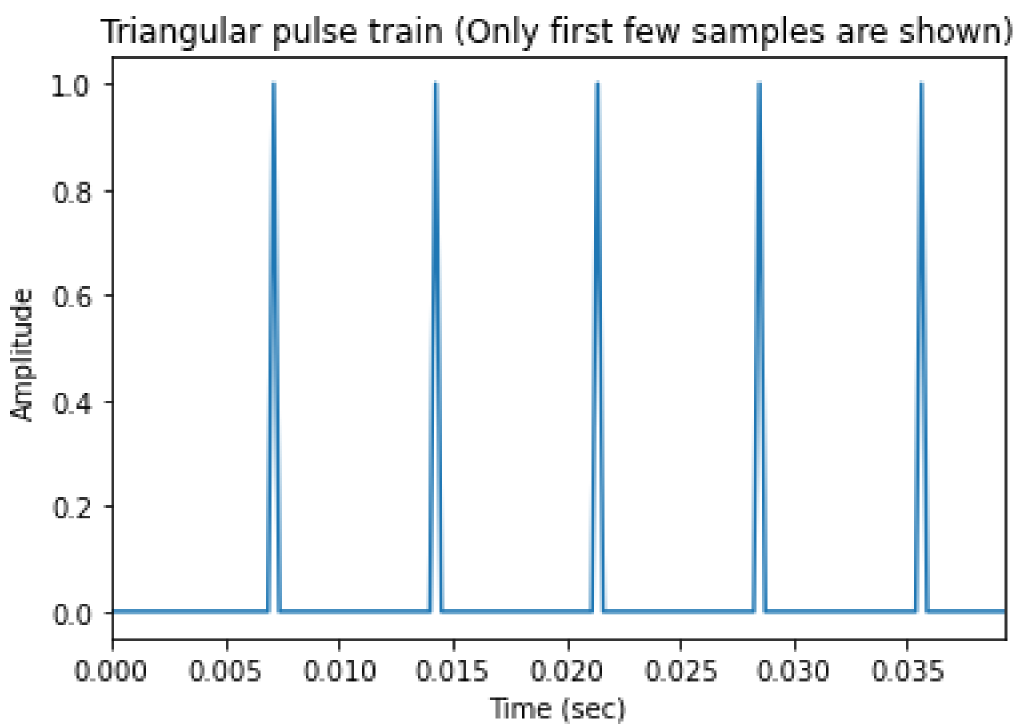
Excite the above resonator ("filter") with a periodic source excitation of F0 = 140 Hz. You can approximate the source signal by a narrow-triangular pulse train. Compute the output of the source-filter system over the duration of 0.5 second using the difference equation implementation of the LTI system. Plot the time domain waveform over a few pitch periods so that you can observe waveform characteristics. Play out the 0.5 sec duration sound and comment on the sound quality.

0.5 Solution:

First we make the required periodic source excitation. For this we assume triangular pulse train with a period of 0.5 sec with triangular pulse of frequency 140 Hz.

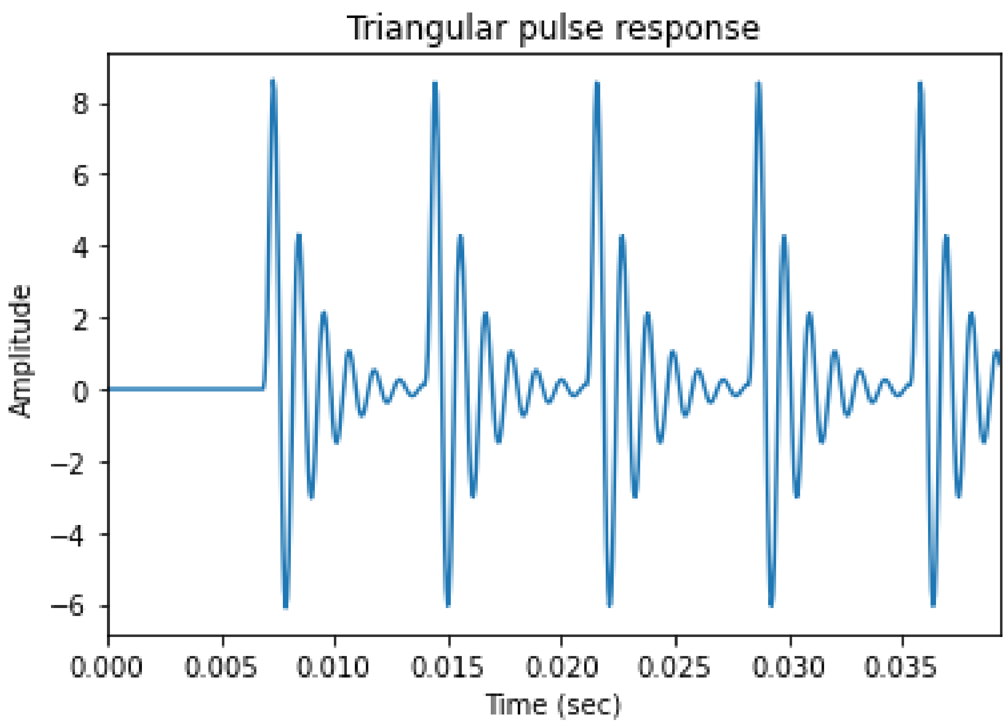
[11]:
```
duration = 0.5                                                    # Duration of the␣
↪signal

F0 = 140                                                          # Impulse frequency
time = np.arange(0, duration, 1/Fs)                               # Time vector
# Function to generate the triangular pulse train with given impulse frequency␣
↪F0
def triangular_pulse_train_gen(F0, duration, Fs):
    triangular_pulse_train = np.zeros(time.size)                  # Empty vector to␣
↪store the output
    # Generating the triangular pulse train
```

```python
    for i in range(time.size):
        if i%(Fs//F0) == 0 and i != 0:
            triangular_pulse_train[i] = 1
            triangular_pulse_train[i+1] = 0.75
            triangular_pulse_train[i-1] = 0.75
            triangular_pulse_train[i+2] = 0.5
            triangular_pulse_train[i-2] = 0.5
            triangular_pulse_train[i+3] = 0.25
            triangular_pulse_train[i-3] = 0.25

    return triangular_pulse_train


triangular_pulse_train = triangular_pulse_train_gen(F0, duration, Fs)  #␣
↪Generating the triangular pulse train for given parameters

plt.plot(time, triangular_pulse_train)  # Plot the triangular pulse train
plt.xlim(0,5.5/F0)       # Limit the x-axis to show 5 impulse periods
plt.xlabel('Time (sec)')
plt.ylabel('Amplitude')
plt.title('Triangular pulse train (Only first few samples are shown)')
plt.show()
```



Now, we pass this triangular_pulse_train through the filter and observe the output.

[12]:
```
triangular_pulse_response = filter(triangular_pulse_train,F1,B1)  # Pass the␣
 ↪triangular pulse train through the filter

plt.plot(time, triangular_pulse_response)                      # Plot the output␣
 ↪of the filter
plt.xlim(0,5.5/F0)                               # Limit the x-axis to show 5␣
 ↪impulse periods
plt.xlabel('Time (sec)')
plt.ylabel('Amplitude')
plt.title('Triangular pulse response')
plt.show()
```



Now, we play this triangular_pulse_response to observe the sound.

[13]:
```
# Write the input sound to input.wav
input_sound = np.int16(triangular_pulse_train/np.max(np.
 ↪abs(triangular_pulse_train))*32767)
write('input.wav', Fs, input_sound)

# Write the output sound to output.wav
output_sound = np.int16(triangular_pulse_response/np.max(np.
 ↪abs(triangular_pulse_response))*32767)
write("output.wav",Fs,output_sound)
```

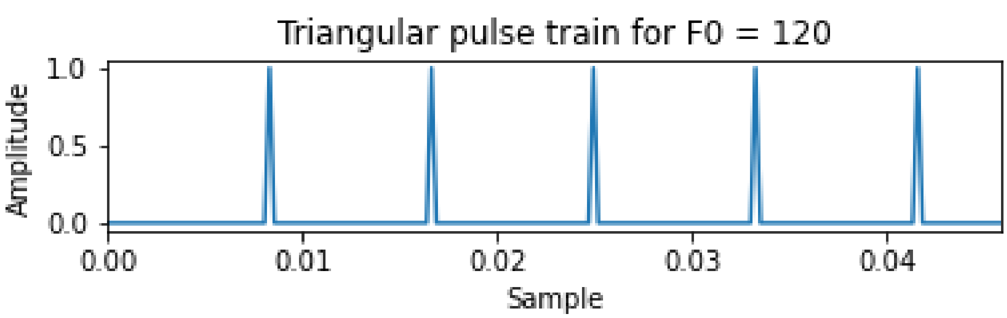
0.6 Question 3:

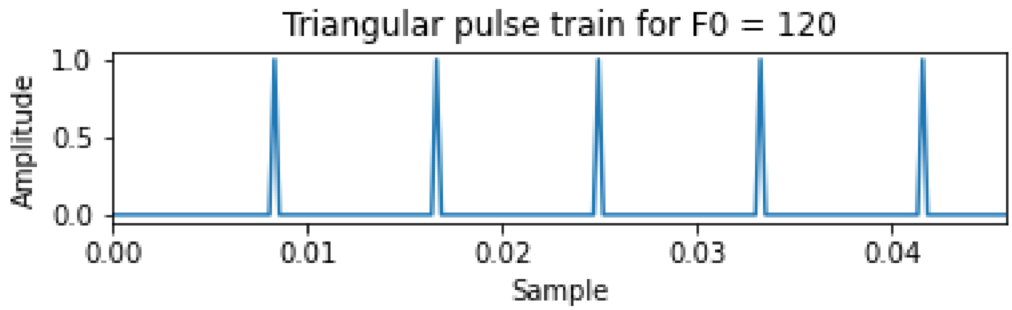
Vary the parameters as indicated below; plot and comment on the differences in waveform and in sound quality for the different parameter combinations. - F0 = 120 Hz, F1 = 300 Hz, B1 = 100 Hz
- F0 = 120 Hz, F1= 1200 Hz, B1 = 200 Hz
- F0 = 180 Hz, F1 = 300 Hz, B1 = 100 Hz

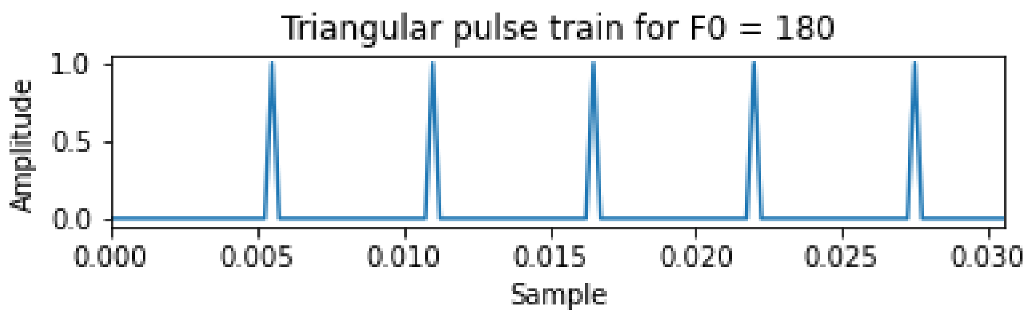
0.7 Solution:

0.7.1 Generate triangular pulse train with period of 0.5 sec and frequencies mentioned above.

[14]:
```
F0s = np.array([120,120,180])                    # Array of F0 values

# Generate triangular pulse train with given impulse frequencies F0s
triangular_pulse_trains = [triangular_pulse_train_gen(F0s[i], duration, Fs) for␣
 ↪i in range(len(F0s))]

# Plot the triangular pulse trains for the three cases
for i in range(len(F0s)):
    plt.subplot(len(F0s),1,i+1)
    plt.plot(time, triangular_pulse_trains[i])
    plt.xlim(0,5.5/F0s[i])
    plt.xlabel('Sample')
    plt.ylabel('Amplitude')
    plt.title('Triangular pulse train for F0 = ' + str(F0s[i]))
    plt.show()
```

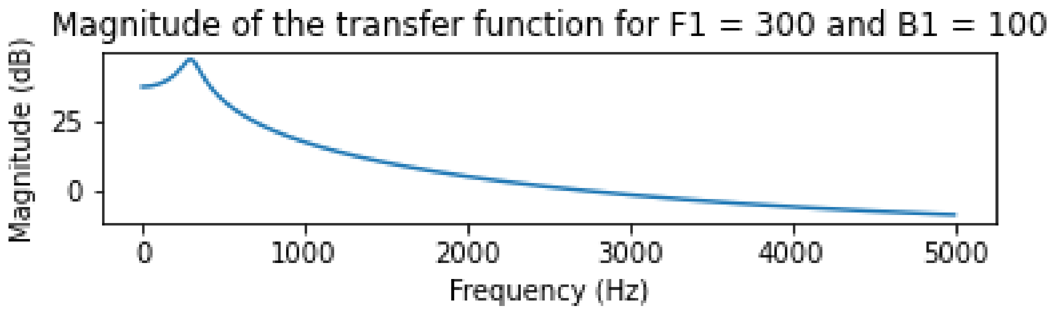


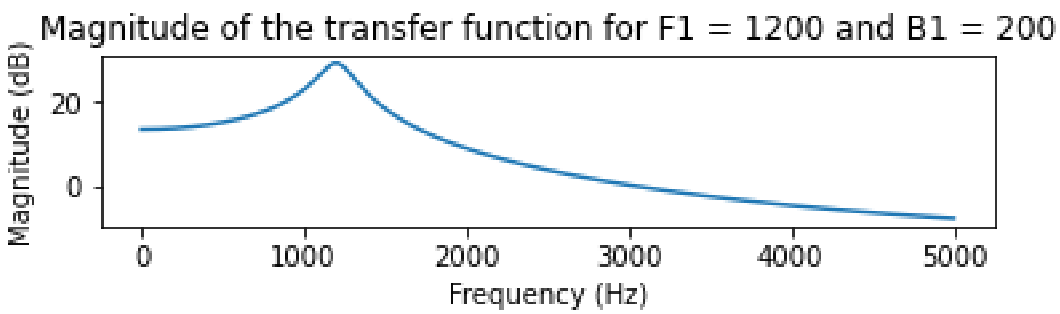


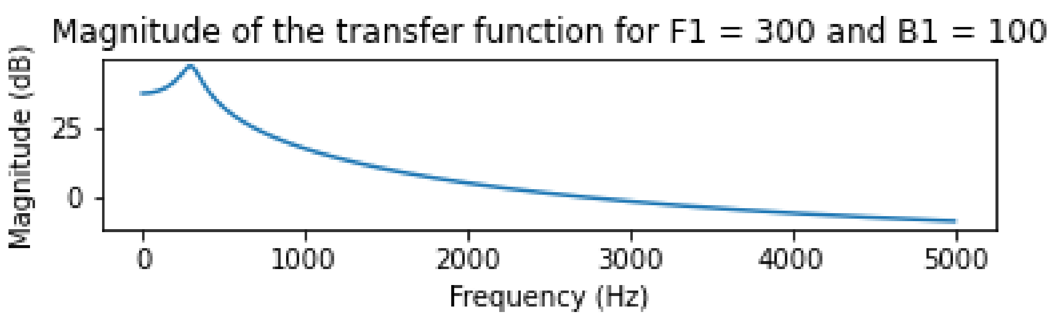


0.7.2 Now, lets have a look at the Magnitude of the transfer function for the given three cases

[15]:
```
F1s = np.array([300,1200,300])                      # Array of F1 values
B1s = np.array([100,200,100])                       # Array of B1 values

H_mags = [H_discrete_transfer_mag(F1s[i], B1s[i], Fs) for i in range(len(F1s))]␣
↪ # Array of H magnitudes

# Plot the Magnitude of the transfer function for the three cases
for i in range(len(F1s)):
    plt.subplot(len(F1s),1,i+1)
    plt.plot(freq_range, 20*np.log10(H_mags[i]))
    plt.xlabel('Frequency (Hz)')
    plt.ylabel('Magnitude (dB)')
    plt.title('Magnitude of the transfer function for F1 = ' + str(F1s[i]) + '␣
↪and B1 = ' + str(B1s[i]))
    plt.show()
```



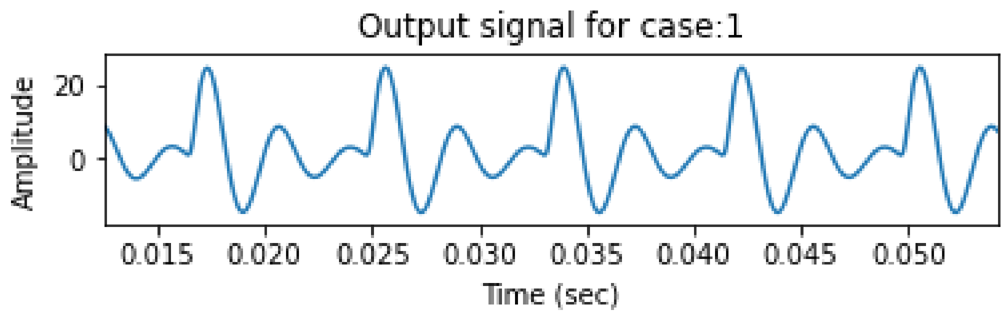


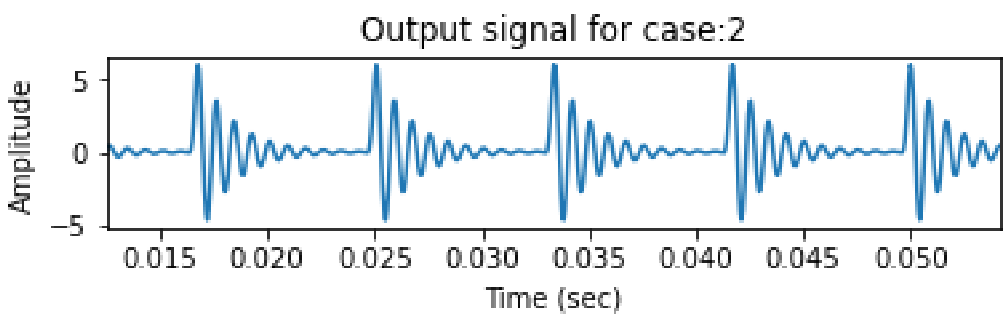


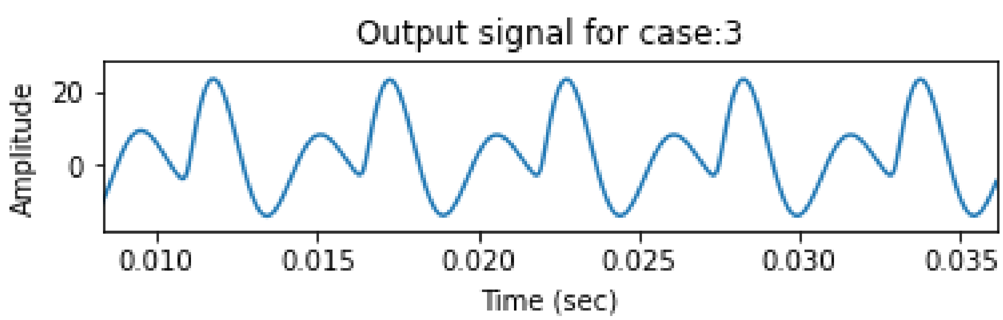
0.7.3 Now let us see the output of the filter for the given three cases.

[18]:
```
outputs = [filter(triangular_pulse_trains[i],F1s[i],B1s[i]) for i in␣
↪range(len(F1s))]  # Array of output signals

# Plot the output signals for the three cases
for i in range(len(F1s)):
    plt.subplot(len(F1s),1,i+1)
    plt.plot(time, outputs[i])
    plt.xlim(1.5/F0s[i],6.5/F0s[i])
```

```
    plt.xlabel('Time (sec)')
    plt.ylabel('Amplitude')
    plt.title('Output signal for case:'+str(i+1))
    plt.show()
```







0.7.4 Now we save the input and output of the filter for the given three cases.

[17]:
```
# Write the input sound to input.wav
input_sounds = [np.int16(triangular_pulse_trains[i])*32767 for i in␣
 ↪range(len(F1s))] # No need to divide by max since it is 1
for i in range(len(F1s)):
    write('input_'+str(i+1)+'.wav', Fs, input_sounds[i])

# Write the output sound to output.wav
output_sounds = [np.int16(outputs[i]/np.max(np.abs([outputs[i]]))*32767) for i␣
 ↪in range(len(F1s))]
for i in range(len(F1s)):
    write("output_"+str(i+1)+".wav",Fs,output_sounds[i])
```

[ ]:
```
```