

CSE 4309 – Assignment 2

Task-1

$P(\text{sensor} = \text{Maine}) = 0.05$, $P(\text{sensor} = \text{Sahara}) = 0.95$,

$P(\text{temp} \geq 80 \mid \text{Maine}) = 0.2$, $P(\text{temp} < 80 \mid \text{Maine}) = 0.8$

$P(\text{temp} \geq 80 \mid \text{Sahara}) = 0.9$, $P(\text{temp} < 80 \mid \text{Sahara}) = 0.1$

- a) $P(\text{Maine} \mid \text{temp} < 80) = P(\text{Maine}) P(\text{temp} < 80 \mid \text{Maine}) / P(\text{temp} < 80)$
 $= P(\text{Maine}) P(\text{temp} < 80 \mid \text{Maine}) / [P(\text{Maine}) P(\text{temp} < 80 \mid \text{Maine}) + P(\text{Sahara}) P(\text{temp} < 80 \mid \text{Sahara})]$
 $= 0.05 * 0.8 / [0.05 * 0.8 + 0.95 * 0.1]$
 $= 0.04 / [0.04 + 0.095]$
 $= 0.04 / 0.135$
 $= 0.2963$
- b) $P(\text{temp} < 80) = P(\text{Maine} \mid \text{temp} < 80) * P(\text{temp} < 80 \mid \text{Maine}) + P(\text{Sahara} \mid \text{temp} < 80) * P(\text{temp} < 80 \mid \text{Sahara})$
 $= 0.2963 * 0.8 + (1 - 0.2963) * 0.1$
 $= 0.23704 + 0.07037$
 $= 0.30741$
- c) From part b, we get the probability of getting two emails indicating a daily high under 80 degrees for 2 consecutive days. Let's call this event $t_{1,2}$
 The probability of first three emails indicating daily highs under 80 could be represented with the product rule like- $P(t_1, t_2, t_3) = P(t_1 \mid t_2, t_3) * P(t_2 \mid t_3) * P(t_3)$.
 However, knowing the probability for the first 2 days, we could write it as:
 $P(t_{1,2}, t_3) = P(t_{1,2} \mid t_3) * P(t_3) = P(t_3 \mid t_{1,2}) * P(t_{1,2})$ from the bayes rule.
 And $P(t_3 \mid t_{1,2}) = P(t_3 \text{ and } t_{1,2}) / P(t_{1,2})$

$$P(\text{temp} < 80 \text{ at either location}) = 0.2963 * 0.05 + 0.7031 * 0.95 = 0.014815 + 0.667945 = 0.68276$$

Since each day is conditionally independent of the other day, $P(t_3) = 0.68276$

$$\text{Therefore, } P(t_{1,2}, t_3) = P(t_3 \mid t_{1,2}) * P(t_{1,2}) = P(t_3 \text{ and } t_{1,2}) = P(t_3) * P(t_{1,2}) = 0.68276 * 0.30741 = 0.20988725$$

Task-2

P is possibly a probability function. Probability cannot be great than 1. With $P(A) = 0.3$ and $P(B) = 0.6$, the $P(A) + P(B) = 0.9$. If $P(C) + P(D)$ is equal to 0.1, then P is a probability function, otherwise it is not.

Task-3

P is definitely not a probability function because integrating $P(x) = 0.3$ from the limits 0 to 10 would give us 3. This is not less than or equal to 1, so P cannot be a probability density function.

Task-4

- $p(B = r) = 0.4$
- $p(B = b) = 0.6$
- $p(F = a | B = r) = 0.25$
- $p(F = o | B = r) = 0.75$
- $p(F = a | B = b) = 0.75$
- $p(F = o | B = b) = 0.25$

$$P(F = a) = p(F = a, B = r) + p(F = a, B = b)$$

$$= p(F = a | B = r) * p(B = r) + p(F = a | B = b) * p(B = b)$$

$$= 0.25 * 0.4 + 0.75 * 0.6$$

$$= 0.1 + 0.45$$

$$= 0.55$$

$$P(F = o) = 1 - P(F = a) = 0.45$$

$$P(B = r | F = a) = p(F = a | B = r) * p(B = r) / p(F = a) = 0.25 * 0.4 / 0.55 = 0.1818$$

$$P(B = b | F = a) = p(F = a | B = b) * p(B = b) / p(F = a) = 0.75 * 0.6 / 0.55 = 0.8181$$

$$P(B = r | F = o) = p(F = o | B = r) * p(B = r) / p(F = o) = 0.75 * 0.4 / 0.45 = 0.6667$$

$$P(B = b | F = o) = p(F = o | B = b) * p(B = b) / p(F = o) = 0.25 * 0.6 / 0.45 = 0.3333$$

If $F = a$, if the prediction is $B = r$, then the classifier will be 18.18% correct.

If $F = a$, if the prediction is $B = b$, then the classifier will be 81.81% correct.

If $F = o$, if the prediction is $B = r$, then the classifier will be 66.67% correct.

If $F = o$, if the prediction is $B = b$, then the classifier will be 33.33% correct.

Task-5

Run command for your terminal:

```
python3 naive_bayes.py [path of training file] [path of test file]
```

classification accuracy is 0.4483