MA2101: Analysis I Lecture Notes

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1 Algebra of the Real Number System

1.1 Properties of Addition

The properties of addition(+) in the real number system are:

(A1) $x + y = y + x \; \forall \; x, y \in \mathbb{R}$

(A2) $(x + y) + z = x + (y + z) \; \forall \; x, y, z \in \mathbb{R}$

(A3) $\exists! 0 \in \mathbb{R} \; s.t. \; x + 0 = 0 + x = x \; \forall \; x \in \mathbb{R}$

(A4) $\forall x \in \mathbb{R} \exists! y \in \mathbb{R}$ s.t. $x + y = y + x = 0$

1.2 Properties of Multiplication

The properties of multiplication($\cdot$) in the real number system are:

(M1) $x \cdot y = y \cdot x \forall x, y \in \mathbb{R}$

(M2) $(x \cdot y) \cdot z = x \cdot (y \cdot z) \forall x, y, z \in \mathbb{R}$

(M3) $\exists! 1 \in \mathbb{R}$ s.t. $x \cdot 1 = x \forall x \in \mathbb{R}$

(M4) $\forall x \in \mathbb{R} \setminus \{0\} \exists! y \in \mathbb{R}$ s.t. $x \cdot y = y \cdot x = 0$

1.3 Distributive Property

The multiplication operator distributes over addition inn real numbers.

$$x \cdot (y + z) = x \cdot y + x \cdot z$$

Since addition and multiplication have these properties in real numbers, $(\mathbb{R}, +, \cdot)$ is a Field.

1.4 Order in Reals

1.4.1 Law of Trichotomy

Given two $x, y \in \mathbb{R}$, exaclt one of the following statements is true :

(i) $x = y$

(ii) $x > y$

(iii) $x < y$

1.4.2 Properties of "$<$"

(i) If $x < y$ and $y < z$ then $x < z$

(ii) If $x > 0$, $y > 0$ then, $xy > 0$

(iii) If $x < y$ then, $x + z < y + z$ $\forall z \in \mathbb{R}$

(iv) $x < y \Rightarrow -x > -y$

(v) If $x < y$ and $z > 0$ then $xz < yz$

(vi) If $0 < x < y$, then $0 < \frac{1}{y} < \frac{1}{x}$

(vii) $x^2 \geq 0 \forall x \in \mathbb{R}$

Remark 1. *Let $x, y \in \mathbb{R}$ such that, $x \leq y$ and $y \leq x$. Then, $x = y$.*

Proof. Let's assume to teh contrary that $x \neq y$. Then, by the law of trichotomy, either $x < y$ or $y < x$. Let $y < x$. From $x \leq y$ we have either $x < y$ or $x = y$. By the law of trichotomy, neither of them can be true. Hence, $y \not< x$ Now, let $x < y$. From $y \leq x$ we have either $y < x$ or $y = x$. Again, by the law of trichotomy, neither of them can be true. Hence, $x \not< y$ This is a contradiction. Hence, $x = y$ □