

Colorimetric and Photometric Properties of a 2° Fundamental Observer

The availability of tables with standardized data on the properties of a normal visual observer has been a great help in scientific communication. For daily practice these data, going back to the 1920s, still suffice. For basic science they show shortcomings, however. The present paper gives new and extended tables which are intended to be complementary to, rather than to compete with, the old CIE tables.

Introduction

In 1924 the CIE published collected data on the average luminosity function V_λ of the normal 2° observer. These data formed the basis for the present "standard 2° observer." In 1931 this was followed by the definition of his colorimetric properties in the form of the \bar{x}_λ , \bar{y}_λ , \bar{z}_λ colour-matching functions. Since then the CIE standard observer has been generally accepted, in particular because of the easy availability of tabulated data on \bar{x}_λ , \bar{y}_λ , and \bar{z}_λ , and of their derivatives, the chromaticity coordinates x_λ , y_λ .

Of course, measurements done since have shown that the standardized data were not completely correct, but to update the definition was never seriously considered because the corrections concerned only the very ends of the visible spectrum and were therefore insignificant for the daily practice of colorimetry and photometry.

They are significant, however, for visual scientists who work with monochromatic stimuli. Mainly to serve them, Judd¹ calculated colour-mixture functions for a revised standard observer with the same colorimetric properties as the old one, but with a definitely higher sensitivity in the blue part of the spectrum, in accordance with the then known new data. Judd's tables are only available with 10-nm intervals, however, and that may be the reason that his data have never found general acceptance in visual science.

Ourselves having experienced the absence as a handicap we decided to redo and complete Judd's work on this point, and to calculate colour-matching functions, tabulated at

1-nm steps and adapted to our present state of knowledge. The table presented here, for reasons of space, has to be a 5-nm version only. The full 1-nm version is available, however, on request from the author.

The Data Used

To calculate the tables several data sets were used.

(a) Since Judd used the same colorimetric data to form the basis of the CIE 1931 tables, his chromaticity diagram should be a projective transformation of the CIE 1931 diagram. Judd never gave the transformation formulas and obtained his tables by a method of successive approximations. With present-day computer facilities it was easy to obtain the exact formulas by demanding an exact imaging at four arbitrary, but amply spaced wavelengths. In fact we took 450, 500, 550, and 650 nm as reference points, and thus obtained as projective transformation between $(x_\lambda, y_\lambda)_{\text{CIE '31}}$ and $(x'_\lambda, y'_\lambda)_{\text{Judd '51}}$:

$$x'_\lambda = \frac{1.0271 x_\lambda - 0.00008 y_\lambda - 0.00009}{0.03845 x_\lambda + 0.01496 y_\lambda + 1},$$
$$y'_\lambda = \frac{0.00376 x_\lambda + 1.0072 y_\lambda + 0.00764}{0.03845 x_\lambda + 0.01496 y_\lambda + 1}.$$

The (x'_λ, y'_λ) chromaticity diagram thus computed shows only minor last-decimal deviations from Judd's data, these probably being due to Judd's approximation technique. Because the CIE 1931 data are available in seven decimals and with 1-nm intervals, the same degree of definition of the revised chromaticity data could be obtained. In fact we did not go further than five chemicals in floating-point notation.

(b) The CIE 1931 data assumed constant colorimetric properties beyond 700 nm, but since then an infrared colour reversal has been discovered² that makes a revision of the old data necessary. The effect seems to be independent of field size (ref. 3, p. 561), and we could therefore use the well-defined data on the infrared colour reversal of the CIE 1964 10° tables. Therefore we imaged the CIE 10° chromaticity diagram on the Judd diagram by a transformation

TABLE I. Table of chromaticities, colour-matching functions, and receptor system primaries for a 2° fundamental observer

LAMBDA	X'	Y'	X̄	Ȳ	Z̄	R	G	B	LOG R	LOG G	LOG B
380	0.17755	0.01320	0.26899E-02	0.20000E-03	0.12260E-01	0.71592E-04	0.38621E-04	0.89786E-04	-4.14514	-4.41318	-4.04679
385	0.17744	0.01322	0.53195E-02	0.35566E-03	0.24222E-01	0.14107E-03	0.77106E-04	0.17739E-03	-3.85057	-4.11291	-3.75108
390	0.17723	0.01315	0.10781E-01	0.80000E-03	0.49250E-01	0.28195E-03	0.15737E-03	0.36067E-03	-3.54983	-3.80307	-3.44288
395	0.17699	0.01316	0.20792E-01	0.15457E-02	0.95135E-01	0.53954E-03	0.30947E-03	0.69671E-03	-3.26798	-3.50938	-3.15695
400	0.17676	0.01303	0.37981E-01	0.28000E-02	0.17409E-00	0.96154E-03	0.56353E-03	0.12749E-02	-3.01703	-3.24908	-2.89452
405	0.17644	0.01301	0.63157E-01	0.46562E-02	0.29013E-00	0.15750E-02	0.95644E-03	0.21247E-02	-2.80272	-3.01934	-2.67260
410	0.17599	0.01303	0.99941E-01	0.74000E-02	0.46053E-00	0.24572E-02	0.15702E-02	0.33726E-02	-2.60957	-2.80405	-2.47203
415	0.17549	0.01306	0.15824E-00	0.11779E-01	0.73166E-00	0.38316E-02	0.25889E-02	0.53582E-02	-2.41661	-2.58689	-2.27098
420	0.17480	0.01333	0.22948E-00	0.17500E-01	0.10658E-00	0.56079E-02	0.40865E-02	0.78053E-02	-2.25120	-2.38864	-2.10671
425	0.17368	0.01401	0.28108E-00	0.22670E-01	0.13146E-00	0.72059E-02	0.58449E-02	0.96272E-02	-2.14231	-2.23323	-2.01850
430	0.17223	0.01512	0.31895E-00	0.27300E-01	0.14672E-00	0.66961E-02	0.78592E-02	0.10744E-01	-2.06067	-2.10462	-1.96882
435	0.17022	0.01677	0.33072E-00	0.32584E-01	0.15796E-00	0.10467E-01	0.10549E-01	0.11538E-01	-1.98016	-1.97678	-1.95675
440	0.16770	0.01907	0.33336E-00	0.37900E-01	0.16166E-00	0.12393E-01	0.13668E-01	0.11839E-01	-1.90682	-1.86429	-1.92670
445	0.16433	0.02199	0.31672E-00	0.42391E-01	0.15682E-00	0.14042E-01	0.16864E-01	0.11485E-01	-1.85259	-1.77303	-1.93088
450	0.15980	0.02589	0.28882E-00	0.46800E-01	0.14717E-00	0.15683E-01	0.20329E-01	0.10778E-01	-1.80458	-1.69167	-1.96746
455	0.15404	0.03092	0.25969E-00	0.52122E-01	0.13740E-00	0.17674E-01	0.24385E-01	0.10063E-01	-1.75265	-1.61282	-1.99729
460	0.14690	0.03787	0.23276E-00	0.60000E-01	0.12917E-00	0.20826E-01	0.29715E-01	0.94595E-02	-1.68140	-1.52703	-2.02413
465	0.13828	0.04803	0.20999E-00	0.72942E-01	0.12356E-00	0.26399E-01	0.37494E-01	0.90488E-02	-1.57841	-1.42604	-2.04341
470	0.12668	0.06505	0.17476E-00	0.90900E-01	0.11138E-00	0.35375E-01	0.47578E-01	0.81567E-02	-1.45291	-1.22293	-2.08049
475	0.11185	0.09499	0.13287E-00	0.11284E-00	0.94220E-00	0.46839E-01	0.58966E-01	0.69001E-02	-1.32810	-1.22940	-2.16115
480	0.09316	0.14086	0.91944E-01	0.13902E-00	0.75596E-00	0.61746E-01	0.71738E-01	0.55362E-02	-1.20939	-1.14425	-2.25679
485	0.07007	0.20838	0.56985E-01	0.16937E-00	0.58640E-00	0.79363E-01	0.86217E-01	0.42944E-02	-1.10038	-1.06441	-2.36710
490	0.04623	0.30304	0.31731E-01	0.20802E-00	0.44669E-00	0.10134E-00	0.10341E-00	0.32713E-02	-0.99423	-0.98544	-2.45529
495	0.02580	0.42042	0.14613E-01	0.25800E-00	0.34116E-00	0.12977E-00	0.12580E-00	0.24985E-02	-0.88679	-0.90031	-2.60233
500	0.00819	0.54540	0.48491E-02	0.32300E-00	0.26437E-00	0.16637E-00	0.15470E-00	0.19361E-02	-0.77893	-0.81052	-2.71308
505	0.00378	0.66062	0.23215E-02	0.40540E-00	0.20594E-00	0.12189E-00	0.19100E-00	0.15082E-02	-0.67185	-0.71897	-2.82155
510	0.01393	0.75440	0.92899E-02	0.50300E-00	0.15445E-00	0.26888E-00	0.23299E-00	0.11311E-02	-0.57040	-0.63267	-2.94649
515	0.03922	0.81453	0.29276E-01	0.60811E-00	0.10910E-00	0.33075E-00	0.27366E-00	0.79956E-03	-0.48051	-0.55820	-3.09715
520	0.07502	0.83492	0.63791E-01	0.71000E-00	0.76585E-00	0.39264E-00	0.31680E-00	0.56086E-03	-0.40600	-0.49923	-3.25115
525	0.11517	0.82638	0.11081E-00	0.79510E-00	0.56227E-00	0.44691E-00	0.34778E-00	0.41177E-03	-0.34978	-0.45870	-3.38535
530	0.15596	0.80538	0.16632E-00	0.86200E-00	0.41366E-00	0.49250E-00	0.36920E-00	0.30294E-03	-0.30760	-0.43274	-3.51865
535	0.19425	0.76070	0.22769E-00	0.91505E-00	0.29353E-00	0.53117E-00	0.38365E-00	0.21496E-03	-0.27476	-0.41606	-3.66764
540	0.23106	0.75311	0.29269E-00	0.95400E-00	0.20042E-00	0.56277E-00	0.39109E-00	0.14678E-03	-0.24907	-0.40773	-3.83334
545	0.26722	0.72295	0.36225E-00	0.98004E-00	0.13312E-00	0.58795E-00	0.39199E-00	0.97490E-04	-0.23066	-0.40672	-4.01104
550	0.30300	0.69089	0.43635E-00	0.99495E-00	0.87823E-02	0.60771E-00	0.38717E-00	0.64316E-04	-0.21630	-0.41210	-4.19168
555	0.33865	0.65749	0.51513E-00	0.10001E-00	0.58573E-02	0.62286E-00	0.37723E-00	0.42895E-04	-0.20561	-0.42340	-4.36760
560	0.37423	0.62322	0.59748E-00	0.99500E-00	0.40493E-02	0.63292E-00	0.36205E-00	0.29654E-04	-0.19865	-0.44123	-4.52791
565	0.40965	0.58858	0.68121E-00	0.97875E-00	0.29217E-02	0.63713E-00	0.34160E-00	0.21397E-04	-0.19578	-0.46648	-4.68565
570	0.44471	0.55396	0.76425E-00	0.95200E-00	0.22771E-02	0.63551E-00	0.31648E-00	0.16676E-04	-0.19088	-0.49966	-4.77792
575	0.47910	0.51977	0.84294E-00	0.91558E-00	0.19706E-02	0.62810E-00	0.28746E-00	0.14431E-04	-0.20197	-0.54142	-4.84069
580	0.51245	0.48653	0.91635E-00	0.87000E-00	0.18066E-02	0.61459E-00	0.25539E-00	0.13230E-04	-0.21141	-0.59279	-4.87943
585	0.54436	0.45477	0.97970E-00	0.81623E-00	0.15449E-02	0.59482E-00	0.22140E-00	0.11314E-04	-0.22562	-0.65482	-4.94638
590	0.57431	0.42439	0.10230E-00	0.75700E-00	0.12348E-02	0.56979E-00	0.18720E-00	0.90432E-05	-0.24428	-0.72769	-5.04368
595	0.60168	0.39768	0.10513E-00	0.69483E-00	0.11177E-02	0.54042E-00	0.15440E-00	0.81850E-05	-0.26272	-0.81136	-5.08698
600	0.62539	0.37406	0.10550E-00	0.63100E-00	0.90564E-03	0.50634E-00	0.12465E-00	0.66323E-05	-0.29556	-0.90430	-5.17833
605	0.64632	0.35333	0.10362E-00	0.56654E-00	0.69467E-03	0.46843E-00	0.98111E-00	0.50873E-05	-0.32976	-1.00828	-5.29351
610	0.66344	0.33627	0.99239E-00	0.50300E-00	0.42885E-03	0.42713E-00	0.75862E-00	0.31406E-05	-0.36844	-1.11897	-5.42098
615	0.67749	0.32237	0.92861E-00	0.44172E-00	0.31817E-03	0.38396E-00	0.57757E-00	0.23301E-05	-0.41571	-1.23840	-5.63263
620	0.68869	0.31109	0.84346E-00	0.38100E-00	0.25598E-03	0.33779E-00	0.43222E-00	0.18746E-05	-0.47137	-1.36430	-5.72709
625	0.69761	0.30223	0.73983E-00	0.32952E-00	0.15679E-03	0.28885E-00	0.31663E-00	0.11482E-05	-0.53932	-1.49944	-5.93998
630	0.70478	0.29510	0.63209E-00	0.26500E-00	0.97694E-04	0.24211E-00	0.22886E-00	0.71544E-06	-0.61598	-1.64044	-6.14543
635	0.71077	0.28913	0.53351E-00	0.21702E-00	0.68944E-04	0.20064E-00	0.16384E-00	0.50490E-06	-0.68758	-1.78559	-6.29678
640	0.71566	0.28424	0.44062E-00	0.17500E-00	0.51165E-04	0.16340E-00	0.11596E-00	0.37470E-06	-0.78674	-1.93570	-6.42632
645	0.71958	0.28034	0.35453E-00	0.13912E-00	0.36016E-04	0.13002E-00	0.81015E-02	0.26376E-06	-0.88599	-2.09143	-6.57880
650	0.72348	0.27746	0.27662E-00	0.10700E-00	0.24230E-04	0.10134E-00	0.56600E-02	0.17751E-06	-0.98432	-2.24719	-6.75079
655	0.72457	0.27537	0.21485E-00	0.81652E-01	0.16915E-04	0.77679E-01	0.39726E-02	0.12387E-06	-1.10970	-2.40893	-6.90702
660	0.72593	0.27408	0.16161E-00	0.61000E-01	0.11506E-04	0.58203E-01	0.27964E-02	0.87192E-07	-1.23505	-2.55340	-7.05952
665	0.72721	0.27273	0.11820E-00	0.44327E-01	0.81489E-05	0.42413E-01	0.19149E-02	0.59677E-07	-1.37251	-2.71706	-7.23419
670	0.72820	0.27174	0.85753E-01	0.32000E-01	0.56006E-05	0.30684E-01	0.13160E-02	0.41015E-07	-1.51389	-2.88073	-7.38705
675	0.72891	0.27103	0.63077E-01	0.23454E-01	0.39544E-05	0.22524E-01	0.82949E-03	0.28959E-07	-1.64735	-3.03176	-7.53822
680	0.72940	0.27054	0.45834E-01	0.17000E-01	0.27912E-05	0.16344E-01	0.65609E-03	0.20441E-07	-1.78665	-3.18304	-7.68951
685	0.72970	0.27025	0.32057E-01	0.11872E-01	0.19176E-05	0.11422E-01	0.45081E-03	0.14004E-07	-1.94227	-3.34600	-7.85254
690	0.72987	0.27008	0.22187E-01	0.82100E-02	0.13135E-05	0.79012E-02	0.30878E-03	0.96193E-08	-2.10231	-3.51035	-8.01686
695	0.73003	0.26992	0.15612E-01	0.57723E-02	0.91519E-06	0.55572E-02	0.21513E-03	0.67022E-08	-2.25514	-3.66730	-8.17378
700	0.73010	0.26985	0.11098E-01	0.41020E-02	0.64767E-06	0.39497E-02	0.15227E-03	0.47431E-08	-2.40343	-3.81739	-8.32324
705	0.73006	0.26989	0.79233E-02	0.29291E-02	0.46352E-06	0.28201E-02	0.10898E-03	0.33945E-08	-2.54873	-3.96266	-8.46922
710	0.72995	0.27000	0.56531E-02	0.20910E-02	0.33304E-06	0.20127E-02	0.78287E-04	0.24390E-08	-2.69622	-4.10631	-8.61279
715	0.72979	0.27016	0.40039E-02	0.14822E-02	0.23823E-06	0.14262E-02	0.55936E-04	0.17446E-08	-2.84582	-4.25183	-8.75830
720	0.72958	0.27037	0.28253E-02	0.10470E-02	0.17026E-06	0.10070E-02	0.40023E-04	0.12469E-08	-2.99698	-4.39769	-8.90417
725	0.72932	0.27062	0.19947E-02	0.74015E-03	0.12207E-06	0.71146E-03	0.28691E-04	0.89394E-09	-3.14785	-4.54226	-9.04869
730	0.72904	0.27091	0.13994E-02	0.52000E-03	0.87107E-07	0.49953E-03	0.20472E-04	0.63791E-09	-3.30144	-4.68883	-9.19524
735	0.72873	0.27121	0.96980E-03	0.36093E-03	0.61455E-07	0.34645E-03	0.14441E-04	0.45006E-09	-3.46031	-4.84041	-9.34673
740	0.72840	0.27154	0.66847E-03</								

similar to that mentioned under (a). Of course this transformation could not be expected to give a perfect match since the CIE 10° and the CIE 2° data are based on completely different colorimetric data, but the differences are rather small, so that a smooth transition in the 650-nm region from the converted 2° data to the converted 10° data could easily be obtained.

As reference wavelengths we used 450, 500, 550, and 652 nm, the latter seemingly odd choice being determined by trial and error finding that with that choice the rate of change of x and y with λ allowed a smooth transition as well. The projective transformation obtained between $(x_\lambda, y_\lambda)_{\text{CIE } 10^\circ '64}$ and $(x'_\lambda, y'_\lambda)_{\text{Judd '51}}$ reads:

$$x'_\lambda = \frac{0.39564 x_\lambda - 0.12666 y_\lambda + 0.08853}{-0.54397 x_\lambda - 0.51866 y_\lambda + 1},$$

$$y'_\lambda = \frac{-0.03833 x_\lambda + 0.50787 y_\lambda + 0.01056}{-0.54397 x_\lambda - 0.51866 y_\lambda + 1}.$$

(c) Judd postulated a luminosity function that was higher in the blue region than the CIE 1924 data. However, there are indications (ref. 3, p. 435) that his corrections were slight overcorrections below 410 nm. For our V_λ values, we did follow the revisions suggested by Stiles⁴ taking 0.2-log-units-lower values in the extreme-short-wavelength part.

(d) On the above-mentioned basic data, two "aesthetic" corrections were applied. In the first place the data below 380 nm were omitted. The CIE data rest almost completely on unfounded extrapolation, and there seemed to be no reason to incorporate them in the definition of a fundamental observer. In the future we hope to be able to extend the data farther into the ultraviolet on the basis of Tan's⁵ colorimetric data of aphakes.

In the second place the CIE 2° data show some completely unnecessary irregularities in the 380–400-nm region and a colorimetrically insignificant last-decimal concavity of the spectral locus between 435 and 445 nm. Corrections for these inequities were applied.

All the above-mentioned corrections apply only to the ends of the visible spectrum and can be considered as only second-order corrections to Judd's modification of the CIE 1931 data. For that reason no new transformation of the chromaticity diagram was sought to locate the equal-energy point E at $x' = y' = 1/3$ exactly. From the totals of \bar{x}'_λ , \bar{y}'_λ , and \bar{z}'_λ (for the 1-nm interval version) it appears that E is located now at $x' = 0.334$, $y' = 0.336$, not more than 1 jnd off the ideal location.

The resulting "fundamental chromaticity diagram" is shown in Fig. 1. The points marked E, R, G, and B indicate the equal-energy point and the three receptor primary points (see following section).

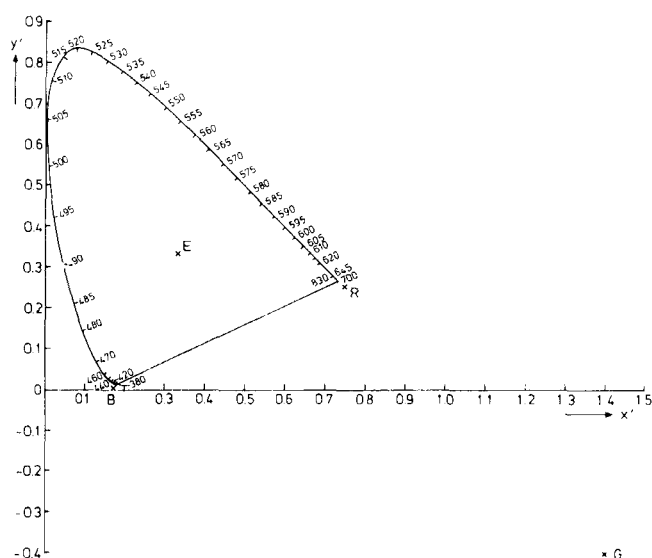


FIG. 1. Fundamental chromaticity diagram with equal energy point E and the three receptor primary points.

Additional Data

For fundamental work in vision, a conversion to more basic primaries in terms of cone system sensitivity is wanted. This can be done by using the locations of the dichromatic confusion centres on the premise that dichromats can be considered as normal trichromats missing one receptor system.^{6,7}

The protanopic confusion centre was located by Thomson and Wright⁸ at $x'_p = 0.7465$, $y'_p = 0.2535$, a location never challenged to our knowledge. The location of the deuteranopic confusion centre is less certain,⁹ but recent psychophysical arguments have considerably narrowed the range of possibilities, so that $x'_d = 1.40$, $y'_d = -0.40$ seems to be the most accurate location for the moment.⁷

The data on the tritanopic confusion centre were recently reevaluated by Walraven.¹⁰ His study led to a best choice at $x'_t = 0.1747$, $y'_t = 0.0060$. On the basis of these confusion loci and the above-mentioned explanation of dichromatism, the following conversion formulas of \bar{x}'_λ , \bar{y}'_λ , and \bar{z}'_λ to the cone system fundamentals R_λ , G_λ , and B_λ can be defined:

$$R_\lambda = 0.1551646 \bar{x}'_\lambda + 0.5430763 \bar{y}'_\lambda - 0.0370161 \bar{z}'_\lambda$$

$$G_\lambda = -0.1551646 \bar{x}'_\lambda + 0.4569237 \bar{y}'_\lambda + 0.0296946 \bar{z}'_\lambda$$

$$B_\lambda = 0.0073215 \bar{z}'_\lambda$$

The values of R_λ , G_λ , and B_λ , calculated with these conversion formulas, and the corresponding values of $\log R_\lambda$, $\log G_\lambda$, and $\log B_\lambda$ are listed as the last six columns of Table I. Figure 2 gives a graphical illustration of these spectral sensitivities.

Notes to Table I

LAMBDA is the wavelength in nanometers.

x' , y' are the chromaticity coordinates on the basis of an E-centred chromaticity diagram.

\bar{x}' , \bar{y}' , \bar{z}' are the colour-matching functions on the same basis, and on the basis of a revised course of the CIE luminosity function in the extreme blue.

R , G , B are the spectral sensitivities of, respectively, the long-wavelength (R), middle-wavelength (G), and short-wavelength (B) cone systems.

$\log R$, $\log G$, $\log B$ are the corresponding logarithmic values.

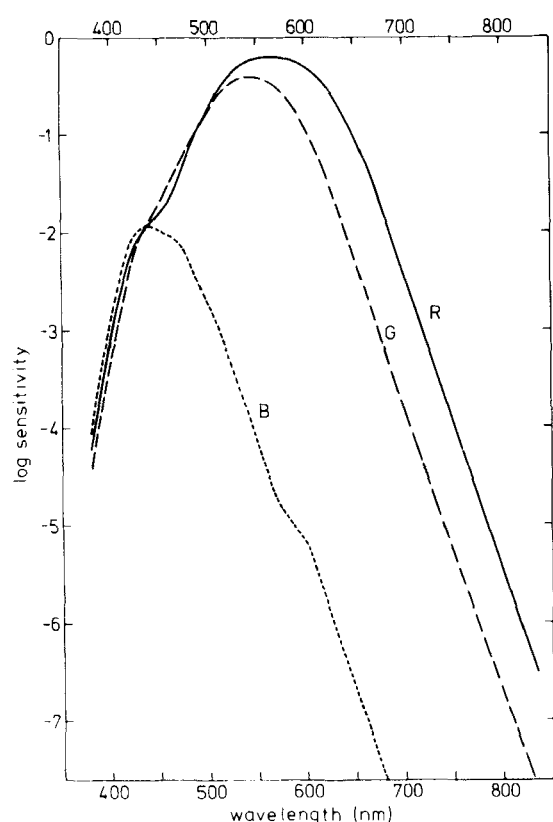


FIG. 2. Spectral sensitivity of the three receptor primary systems.

Table of Data on the Proposed 2° Fundamental Observer

In Table I, x' and y' mean chromaticity coordinates on the basis of an E-centred chromaticity diagram, and \bar{x}' , \bar{y}' , and \bar{z}' mean the colour-matching functions on the same basis, and on a revised course of the luminosity function in the extreme blue.

We have considered several ways to distinguish these "fundamental x, y, z values" from the CIE standard values. It did not seem wise to proceed further in the direction from x to x' , and to introduce x'' or x^* , etc. Since we consider our table a logical continuation of the fundamental updating started by Judd we have opted for a continuation of his indication, so use x', y', z' .

We have the opinion that the values tabulated here reflect the present state of knowledge. We can be sure, though, that new experimental data will sooner or later lead to better tables. Therefore, and to underline its temporary character by definition, it may be suggested to refer to them as the "1978 2° fundamental observer data."

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