Assignment 9

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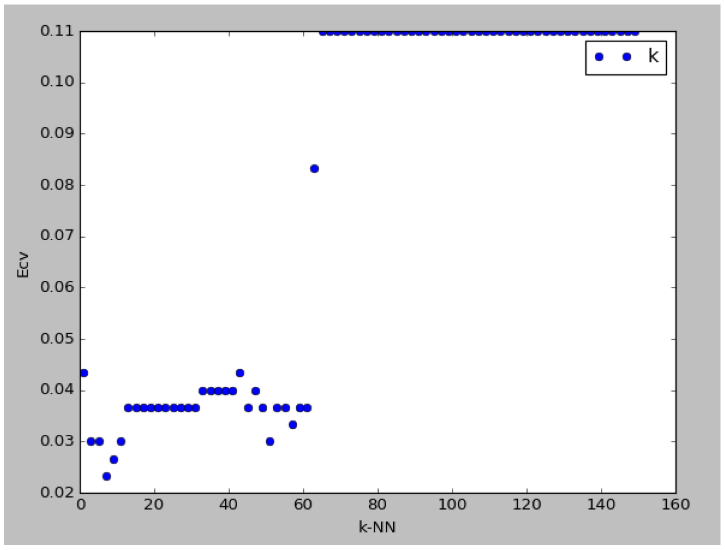
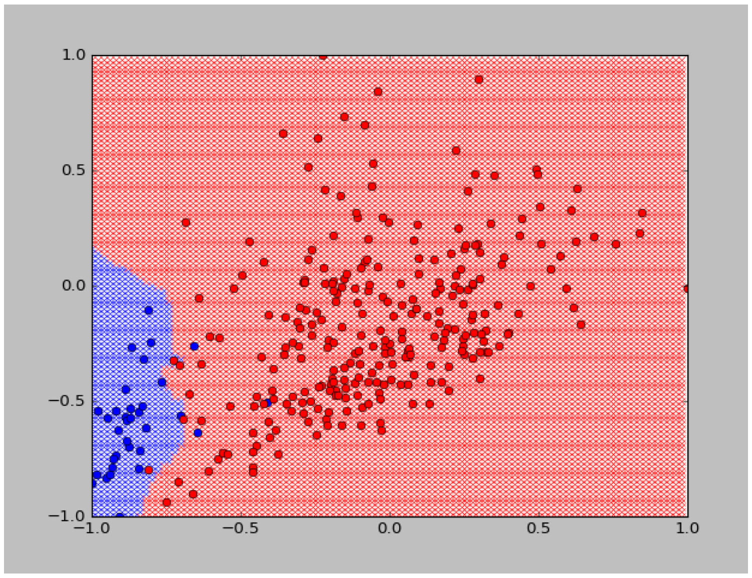
1. k-NN Rule

(a) The following graph was created after doing cross-validation on the training set of size 300. For each odd k from 1 to 150, each point was removed and put back into the k-NN graph trained from the remaining 299 points. From here, error values were calculated and averaged to create the following graph.



The E_{cv} valleys at k = **7**, so 7-NN was selected.

(b) The E_{cv} for 7-NN from the training set is **0.023**. E_{in} was calculated by running each data point on the fully trained 7-NN graph. Of course, this resulted in each point being on top of its own point, but that's okay. $E_{in} = \mathbf{0.2}$. The decision boundary for 7-NN is displayed in the graph below:



(c) E_{test} was calculated to be **0.027**. This was done by running the same 7-NN network on the remaining test points.

2. Overfitting

2-2.png

This is the original plot of all the data points. It is pretty clear where you can draw a line in order to have a pretty decent E_{in}. It's not linearly separable data, but you can't always have that in the real world.

2-1.png

This is the plot with the model without any regularization. As you can see, there is quite a bit of overfitting here. In order to conform with the shape of the 1s, the algorithm outputs a bound that flows with the data points a little bit too much for our comfort. The red dots represent the outputted hypothesis. Also note the large curve on the bottom that's only possible due to a gap in the "not 1" points. We would probably not expect points in this region to actually be considered 1s in real life.

3. Regularization

3-1.png

There is a tinge of underfitting here. Notice the blue points that are to the right of the red curve that could be added to the left side of the curve without any loss in E_{in}. This is typically a sign that we're probably doing a little too much regularization.

4. Cross Validation

4-1.png

Note that I had to start the regularization constant from 0.001 instead of from 0, because the E_{test} at 0 was returning an extremely high value that was skewing the entire graph. On the graph above, imagine there is a red point all the way above the graph at $(0, 1.88)$.

At first, E_{cv} starts at a value when $\lambda = 0$ and goes down initially as you increase regularization. This is because some amount of regularization is very important to make sure that you aren't totally overfitting the data with some crazy curves. However, as you continue to increase regularization, you underfit the data by overconstraining your hypotheses, and your E_{test} and E_{cv} will increase.

5. Pick λ

5-1.png

The minimum E_{cv} was found at $\lambda = 0.014$, and the resulting weights created the above separation. This is a pretty good boundary, as it's quite accurate and does not have complicated curves.

6. Estimate E_{out}

Running the weights generated from a regularization of 0.014 resulted in an E_{out} of **0.0388**. This is a very decent estimate!

7. Is E_{cv} biased?

No it is not biased! We did not touch the test set, so our $E_{cv}(\lambda^*)$ estimates will estimate out E_{test} estimates quite well.

8. Data snooping

However, we are guilty of unconscious data snooping. When we originally scaled and normalized, we did so before removing data for the test set. As a result, our results were unintentionally dependent on the test set, and we have a biased estimate of E_{out}. Fortunately, if we remove for the test set then run our learning algorithm, we can circumvent this.