HKUST

MATH1003 Calculus and Linear Algebra

Mock Final Examination (Fall 2024)

Name: ____________________

1 Dec 2024 (Updated)

Student ID: ____________________

Time limit: 3 hours

Lecture Section: ____________________

Directions:

- This is a closed book examination. You may use an ordinary scientific calculator, but calculators with graphical functions are NOT allowed.
- **DO NOT open the exam until instructed to do so.**
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above.
- When instructed to open the exam, please check that you have **14** pages of questions including the cover page. This document is updated, with amendments highlighted in red.
- Answer all questions. Show an appropriate amount of work for each short or long problem. If you do not show enough work, you will get only partial credit.
- **Cheating is a serious violation of the HKUST Academic Code.** This is only a mock exam, with no benefit of cheating here: by cheating, you are not lying to anyone but yourself.
- For answer checking/ marking/ feedback of this mock paper set, please either email me via theskillfulnoob2002@gmail.com or Whatsapp/ Signal via (+852) 9035 4789.

Please read the following statement and sign your signature:

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature:

____

Question No.	Points	Out of
Q. 1-18		45
Q. 19		10
Q. 20		8
Q. 21		8
Q. 22		12
Q.23		17
Total Points		**100**

Part I: Multiple Choice Questions (45 Points)

Answer all of the following multiple choice questions.

- Mark your answers clearly in the Multiple Choice Item Answer Boxes below.
- Mark only one answer for each MC question. Multiple answers will be treated as incorrect answer.

Question	1	2	3	4	5	6	7	8	9
Answer									
Question	10	11	12	13	14	15	16	17	18
Answer									

References

- Qs. 1, 4, 6, 7, 9, 12, 15 are borrowed/ scaled-down from 1012/13 streams of the mock series.
- All MCs except 2, 6, 13, 17, 18 are direct PP references.
- LQ 19, 22, 23 are direct PP references.

1. Suppose $f(x) = \dfrac{x+3}{3x+4}$ and $g(x) = \dfrac{2x}{4x-1}$. The domain of $f\left(g(x)\right)$ is all real numbers except.

   A. $-\dfrac{4}{3}$. B. $\dfrac{1}{4}$. C. $\dfrac{1}{4}$ and $-\dfrac{4}{3}$. D. $\dfrac{2}{11}$ and $\dfrac{1}{4}$. E. $\dfrac{3}{4}$ and $\dfrac{1}{14}$.

2. Find the limit $\lim\limits_{x\to 4} \dfrac{x^2-3x-4}{x^2-5x+4}$ if it exists.

   A. $-\dfrac{5}{3}$ B. $\dfrac{5}{3}$ C. $\dfrac{3}{5}$ D. $-\dfrac{3}{5}$ E. Does not exist

3. Let $f(x)$ and $g(x)$ be functions of x. How many of the following statements must be true?

   I. If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both exist, $\lim_{x\to c} f(x)g(x)$ exists.

   II. If $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ both don't exist, $\lim_{x\to c} f(x)g(x)$ doesn't exist.

   III. If $\lim_{x\to c} f(x)+g(x)$ exists, then both $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist.

   IV. If $\lim_{x\to c} f(x)+g(x)$ doesn't exist, then both $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ don't exist.

   A. 0 B. 1 C. 2 D. 3 E. 4

4. How many vertical asymptotes does the following function have?

$$y = \frac{x^2-2x+1}{(x^2-x)(x^2-5x-6)(x^2+4)}$$

   A. 1 B. 2 C. 3 D. 4 E. 5

5. For the function $u(x) = \dfrac{3x^5+4x^3+5x}{x^4}$ $(x \neq 0)$, find $u'(x)$.

   A. $u'(x) = 3 + \dfrac{4}{x^2} + \dfrac{15}{x^4}$

   B. $u'(x) = \dfrac{15x^4+12x^2-15}{x^5}$

   C. $u'(x) = 3 - \dfrac{4}{x^2} - \dfrac{15}{x^4}$

   D. $u'(x) = \dfrac{15x^4-12x^2+15}{x^5}$

   E. $u'(x) = 3 - \dfrac{4}{x} - \dfrac{15}{x^3}$

6. Let $p(x) = (x^2+5)^{2024} \cdot (6-x)^{1203}$. Find the value of $p'(0)$.

   A. $1203 \cdot 5^{2023} \cdot 6^{1203}$

   B. $-2024 \cdot 5^{2023} \cdot 6^{1203}$

   C. $2024 \cdot 5^{2024} \cdot 6^{1202}$

   D. $-1203 \cdot 5^{2024} \cdot 6^{1202}$

   E. $1203 \cdot 5^{2024} \cdot 6^{1202}$

7. Define $h(x) = x^{2x}$ for $x > 0$. Find $\dfrac{h'(x)}{h(x)}$.

   A. $2\ln x + 2$ B. $2x\ln x + 2x$ C. $2x\ln x + \ln x$ D. $\ln x + 2$ E. $2\ln x$

8. Let $\dfrac{x^3}{y^3} + x\log_3 y = 10$. Which of the following is correct?

   A. $\dfrac{3x^2}{y^3} + \ln y + \left(\dfrac{x}{\ln 3} - \dfrac{3x^3}{y^4}\right)\dfrac{dy}{dx} = 0$

   B. $\dfrac{3x^2}{y^3} + \dfrac{\ln y}{\ln 3} + \left(\dfrac{x}{y\ln 3} - \dfrac{3x^2}{y^4}\right)\dfrac{dy}{dx} = 10$

   C. $\dfrac{3x^2}{y^3} + \dfrac{\ln y}{\ln 3} + \left(\dfrac{x}{y\ln 3} - \dfrac{3x^3}{y^4}\right)\dfrac{dy}{dx} = 0$

   D. $\dfrac{3x^2}{y^3} + \dfrac{1}{\ln 3} + \left(\dfrac{x}{y\ln 3} - \dfrac{x^3}{y^4}\right)\dfrac{dy}{dx} = 10$

   E. $\dfrac{3x^2}{y^3} + \dfrac{\ln y}{\ln 3} + \left(\dfrac{x\ln y}{y\ln 3} - \dfrac{3x^3}{y^4}\right)\dfrac{dy}{dx} = 0$

9. Find the tangent to the curve $C : y = (x-1)^4 - 5x$ at $x = 2$.

   A. $y = -x - 7$ B. $y = -x + 11$ C. $y = x - 11$ D. $y = x + 7$ E. $y = 4x - 17$

10. Let $q(x) = \dfrac{3x}{x+1}$. Find $q''(1)$.

    A. $-\dfrac{3}{4}$ B. $-\dfrac{3}{2}$ C. $\dfrac{3}{4}$ D. $\dfrac{3}{2}$ E. 3

11. Suppose the graph of $y = a(x)$ has a critical point at $x = c$, and is continuous at $x = c$. Which of the following must be true?

    A. $a(c) = 0$

    B. $a'(c) = 0$

    C. If $a''(c)$ exists, $a''(x)$ changes sign through $x = c$.

    D. $(c, a(c))$ is either a maximum or minimum point of the graph of $y = a(x)$.

    E. None of the above

12. An open-top box is to be made from a square cardboard of side 6 inches by removing a square from each of its corner and folding up the flaps on each side. To maximize the volume of the box $V(x)$ (in cubic inches) where x inches is the side of the squares to remove, what is the objective function $V(x)$, and the conditions on $V'(x)$ and $V''(x)$?

    A. $V(x) = x(6-2x)^2$; $V'(x) = 0$ and $V''(x) < 0$

    B. $V(x) = x(6-2x)^2$; $V'(x) = 0$ and $V''(x) > 0$

    C. $V(x) = x(6-x)^2$; $V'(x) = 0$ and $V''(x) = 0$

    D. $V(x) = x(6-x)^2$; $V'(x) = 0$ and $V''(x) < 0$

    E. $V(x) = x(6-x)^2$; $V'(x) = 0$ and $V''(x) > 0$

13. A company manufactures and sells x backpacks per week. The weekly price-demand equation is $P(x) = 500 - 0.5x$ and the cost of production per backpack is given by $C(x) = 100 + 0.5x$. Which of the following concerning the revenue function $R(x)$ and profit function $F(x)$ is true? *[Hint: Profit = Revenue − Cost]*

    A. $R(x)$ keeps decreasing as x increases.

    B. $F(x) = 400 - x$.

    C. $R(x)$ is maximized at $x = 500$.

    D. $F(x)$ is maximized at $x = 500$.

    E. None of the above

14. If the sides of a cube are growing at a constant rate of $\frac{1}{6}$ cm/ min, how fast is its volume increasing (in cm^3) when the volume is 512 cm^3?

A. 16 B. 32 C. 48 D. 64 E. 96

15. A particle is traveling along the s-axis with velocity function

$$v(t) = -\frac{24}{t+3} + 4$$

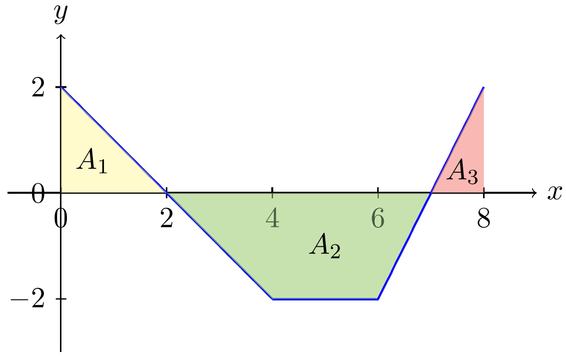
for $t \geq 0$ (in seconds). It is at displacement $s(0) = 0$ initially. Find its displacement $s(t)$ when it is at rest at time $t = T$.

A. $-12\ln 2$ B. $12(\ln 2 - 1)$ C. $-24\ln 2$ D. $12(1 - 2\ln 6)$ E. $12(1 - 2\ln 2)$

16. Suppose g is a continuous function such that $g''(x) = e^x + 6$, $g'(0) = 2$ and $g(0) = 5$. What is $g(1)$?

A. $e + 7$ B. $e + 8$ C. $e + 9$ D. $e + 10$ E. $2e + 11$

17. Refer to the diagram concerning the plot of $y = f(x)$ below, where the area bounded by $y = f(x)$ and the x-axis are split into regions of respective areas A_1, A_2 and A_3:



The value of the definite integral $\int_8^0 f(x)dx$ of the function given below is given by

A. $A_1 + A_2 + A_3$.

B. $-A_1 + A_2 + A_3$.

C. $A_1 + -A_2 + A_3$.

D. $A_1 + A_2 - A_3$.

E. $-A_1 + A_2 - A_3$.

18. It is given that $F(x)$ is an antiderivative of the continuous function $f(x)$. Which of the following statements is NOT true?

A. $\int_0^1 f(x)dx = F(1) - F(0)$

B. $F(x)$ is continuous.

C. $\frac{d}{dx}F(x) = f(x)$

D. $\int f(x)dx = F(x)$

E. None of the above

Part II: Long Questions (55 Points)

Answer each of the following 5 long questions. Unless otherwise specified, numerical answers should be either exact or correct to 2 decimal places.

19. *(10 pts)* Evaluate the following limits if they exist.

(a) $\displaystyle\lim_{x\to 3^+} \frac{x|3-x|}{x-3}$ [3pts]

(b) $\displaystyle\lim_{h\to 0} \frac{e^8(e^{4h}-1)}{h}$ [3pts]

(c) $\displaystyle\lim_{x\to\infty} \left(2x - \sqrt{4x^2-5x+3}\right)$ [4pts]

20. *(8 pts)* Consider the curve $C : x^3y^2 - y = 6$.

(a) Find x when $y = 3$. [1pts]

(b) Find the tangent equation to C when $y = 3$. [4pts]

(c) A point is moving on C. When its y-coordinate is 3, the x-coordinate is decreasing at a rate of 2 units/ second. Find the rate at which its y-coordinate is increasing/ decreasing. [3pts]

21. *(8 pts)* Suppose at time $t = 0$ hours, Car A is located 520km south of Car B. Car A heads north, while Car B heads east, both at a constant speed. Two conditions are given:

1. the distance between the two cars attains a minimum at $t = 4$ hours.
2. at $t = 13$ hours, the distance between the two cars is increasing at a rate of 60 km per hour.

Let the speeds of Car A and Car B be a and b km/h respectively.

(a) Express S, the distance between the two cars at time t, in terms of t. [2pts]

(b) Using condition 1, show that $a^2 + b^2 - 130a = 0$. [2pts]

(c) Find the speeds of the two cars by constructing another equation on a and b. [4pts]

22. *(12 pts)* Find/Evaluate the following integrals.

(a) Find $\int x^5 - \frac{3}{x} + e^x dx$. [3pts]

(b) Evaluate $\int_1^2 \frac{x^2}{5x^3 - 1} dx$ [Hint: use substitution.] [4pts]

(c) Express the area of the finite region bounded by the graphs of $f(x) = x^3 + 5x^2$ and $g(x) = 6x$ as one/ multiple definite integral(s).
You need not compute the integral(s), just set it/ them up. [5pts]

23. *(17 pts)* Given a function $f(x) = \dfrac{x^3 + 5x^2}{(x-1)^2}$, with first and second derivatives below:

$$f'(x) = \frac{x(x^2 - 3x - 10)}{(x-1)^3}, f''(x) = \frac{2(13x+5)}{(x-1)^4}$$

It is further given that $\dfrac{x^3 + 6x^2}{(x-1)^2} = x + 7 + \dfrac{13}{x-1} + \dfrac{6}{(x-1)^2}$. Denote the graph of $y = f(x)$ by Γ.

(a) Find the x-intercept(s) of Γ. [1pts]

(b) State all asymptotes of Γ. [2pts]

(c) Determine the interval(s) of increase and interval of decrease of $f(x)$.
Also find the minimum/ maximum point(s) of Γ. [5pts]

(d) Determine the the concave upward and concave downward interval(s) of $f(x)$.
Also find the inflection point(s) of Γ. [5pts]

(e) Sketch the graph of

$$\Gamma : y = f(x) = \frac{x^3 + 6x^2}{(x-1)^2}$$

below, including its intercepts, asymptotes, extreme point(s) and inflection point(s).
Note: Grid scales have changed. [4pts]