MATH 1013 Mock: Answers and Reminders

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This document is prepared for people who obtained a MATH1013 Mock.

Summary of Testing Areas

Testing Areas

1. **Function and Limits**: (12 Points)
   Q1-4
2. **Differentiation**: (22 Points)
   Q5-9, 16
3. **Appli. of Differentiation**: (42 Points)
   Q10-11, 17, 19, 20(a)-(d)(ii)
4. **Integration**: (24 Points)
   Q12-15, 18, 20(d)(iii)

Section 1

MC Questions (45 Points)

MC Answers

Item	1@	2*	3*	4*	5@
Ans	C	A	C	E	A
Item	6*	7	8*	9	10
Ans	D	B	B	D	B
Item	11	12	13	14*	15
Ans	A	C	B	A	E

Note:

1. Questions with @ tag has easier variants appeared in other streams.
2. Questions with * tag appeared in other streams.

MCQ 1C@/ Function and Limits/ Inverse Domain

Question 1

- Note that $e^{2x} - \frac{1}{2} > -\frac{1}{2}$ for all x. Also, $\cos x \leq 1$ for all x.
- So, the range of $f(x)$ is $(\cos^{-1}(-\frac{1}{2}), \cos^{-1}(1)]$.
- Domain of the inverse of f: $[\cos^{-1}(1), \cos^{-1}(-\frac{1}{2})) = [0, \frac{2\pi}{3})$

Remarks: Closed/ Open Intervals

Pay attention to closed/ open intervals.
It is a usual area of trap in the final exam.

Reference: MATH 1013 2021-22 Fall Midterm #2

Question 2

Apply rationalization to transform the expression to
$\lim_{h\to 3}\frac{\sqrt{1+h}-2}{|h+3|}=\lim_{h\to 3}\frac{\sqrt{1+h}-2}{|h+3|}\cdot\frac{\sqrt{1+h}+2}{\sqrt{1+h}+2}=\lim_{h\to 3}\frac{h-3}{|h-3|(\sqrt{1+h}+2)}$

- Left limit
$$=\lim_{h\to 3^-}\frac{h-3}{-(h-3)(\sqrt{1+h}+2)}=\lim_{h\to 3^-}\frac{1}{-(\sqrt{1+h}+2)}=-\frac{1}{4}$$
- Right limit
$$=\lim_{h\to 3^+}\frac{h-3}{(h-3)(\sqrt{1+h}+2)}=\lim_{h\to 3^+}\frac{1}{\sqrt{1+h}+2}=\frac{1}{4}$$
- As $\lim\limits_{h\to 3^-}\dfrac{\sqrt{1+h}-2}{|h+3|}\neq\lim\limits_{h\to 3^+}\dfrac{\sqrt{1+h}-2}{|h+3|}$, the limit does not exist.

Trap: Q2

Limit Trap for Absolute Values

- Be careful! $|h-3| = -(h-3)$ for $h < 3$.
- Recall that limits only exist when left hand limit $(-)$ equals to the right hand limit $(+)$.
- Absolute values are often culprits of wrong answers in limit questions.

Reference: MATH 1012 Fall 2024-25 Fall Midterm #7

Question 3

- Vertical asymptotes may occur when denominator equals zero. When $(x^2-x)(x^2-5x-6)(x^2+4) = (x)(x-1)(x+1)(x-6)(x^2+4) = 0$, $x = -1$ or 0 or 1 or 6.
- However, the numerator is $(x-1)^2$. The factor $x-1$ cancels out, meaning there is no vertical asymptote at $x = 1$, as the limit at that point is finite.
- Thus, there are only three asymptotes ($x = -1$, $x = 0$, $x = 6$).

Trap: Q3

Factoring Trap of Vertical Asymptotes

Candidates may forget the definition of vertical asymptotes:

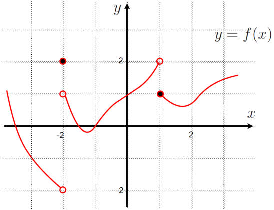
- The limit should tends to $\pm\infty$ when $x \to c$!
- However, when the factor gets cancelled out, the limit when $x \to c$ would be finite.
- Then $x = c$ is NOT a vertical asymptote!

Reference: MATH 1013 2020-21 Fall Final #3/ MATH 1003 Sample Final #2

Question 4

"Limit graph" involves these steps:

- Putting the limit inside:
  $\lim\limits_{x\to 1^+} f(3f(x)-5) = f(\lim\limits_{x\to 1^+} 3f(x)-5)$
- Substitution:
  $f(\lim\limits_{x\to 1^+} 3f(x)-5) = f(\lim\limits_{y\to 1^-} 3y-5)$
- The limit evaluates to
  $f(\lim\limits_{u\to -2^-} u) = \lim\limits_{u\to -2^-} f(u) = -2$



Reference: MATH 1012 Fall 2023 Midterm Q7

Question 5

1. Group the terms to reduce the limit to manageable parts:
$$\lim_{h\to 0}\frac{hf(2+h)}{h}+\lim_{h\to 0}\frac{2(f(2+h)-f(2+h+h^2))}{h}$$
2. The first term simplifies to
$\lim_{h\to 0}\frac{hf(2+h)}{h}=f(2+h)=\lim_{h\to 0}f(2+h)=f(2)=4$.
3. The second term is a "0/0" limit. Applying L'Hôpital's Rule yields
$2\lim_{h\to 0}\frac{f'(2+h)-(1+2h)f'(2+h+h^2)}{1}$
$=2\left(\lim_{h\to 0}f'(2+h)-f'(2+h+h^2)+2hf'(2+h+h^2)\right)$

The second term evaluates to $2(f'(2)-f'(2)-0f'(2))=0$.
Final answer : 4.

MCQ 6D*/ Differentiation/ Differentiability

Question 6

- L.H. derivative: $4e^x + m$ (the function is clearly continuous).
- R.H. derivative: $\frac{d(\ln(\ln(x+e)))}{dx} = \frac{d(\ln(\ln(x+e)))}{d(\ln(x+e))} \cdot \frac{d(\ln(x+e))}{d(x+e)} \cdot \frac{d(x+e)}{dx}$
- At $x = 0$, differentiability requires
  $4e^0 + m = \frac{1}{\ln(0+e)} \cdot \frac{1}{0+e} \implies m = \frac{1}{e} - 4$

Reference: MATH 1013 2019-20 Fall Final #6

Question 7

- Chain rule: $\dfrac{d}{dx}\left(f \circ g^{-1}(x)\right)\Big|_{x=1} = f'(g^{-1}(x))\big|_{x=1} \cdot (g^{-1})'(x)\big|_{x=1}$
- The first term evaluates to $f'(1) = 4$.
- By inverse function theorem, the second term evaluates to $\dfrac{1}{g'(g^{-1}(x))} = \dfrac{1}{g(2)} = \dfrac{1}{3}$.
- Combining, the answer is $\frac{4}{3}$.

Reference: MATH1012 Fall 2023-24 Midterm #15

Reminder/ Trap: Q7

Potential Trap on Derivative

In fact, this question could contain a hidden trap!

- If I didn't amend the question, the answer is actually 0 (the expression to differentiate was a constant!)
- Be careful if this happens in the finals...

Question 8

$h(x)$ is an obvious candidate for logarithmic differentiation.

- Employing the method, $\ln h(x) = 2e^{3x} \ln x$.
- Differentiating,
$$\frac{1}{h(x)} h'(x) = \frac{h'(x)}{h(x)} = \frac{2e^{3x}}{x} + 3(2e^{3x})(\ln x) = e^{3x} \left(\frac{2}{x} + 6 \ln x \right)$$

Reference: MATH1013 Fall 2018 Midterm #12

Question 9

Recall the Newton's Method has iteration formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

- The objective is $f(x_n) = x_n^3 - 1 + 3\cos x_n$
- Its derivative $f'(x_n) = 3x_n^2 - 3\sin x_n$
- So, the iteration formula is

$$x_{n+1} = x_n - \frac{x_n^3 - 1 + 3\cos x_n}{3x_n^2 - 3\sin x_n}$$

References: MATH 1013 2018-19 Fall Final #15

MCQ 10B/ Appli. of Differentiation/ Box Optimization

Question 10

The box has width and length both being $(6-2x)$, and height x.

- Objective function is $V(x) = x(6-2x)(6-2x) = 4x^3 - 24x^2 + 36x$.
- $\dfrac{dV}{dx} = 12x^2 - 48x + 36 = 12(x-3)(x-1)$,
  which attains zero at $x^* = 1$ or $x^* = 3$.
- Note that $\dfrac{d^2V}{dx^2} = 24x - 48$ which is < 0 for $x^* = 1$, and > 0 for $x^* = 3$. Therefore $V(1) = 16$ is the maximum volume attainable.

Reference: Libretexts// MATH 1012 2020-21 Fall Final #15

MCQ 11A/ Appli. of Differentiation/ Critical Points

Question 11

Note that $g(x) = \dfrac{(x-3)^{2024}}{2024} + C$:

- For A, g' changes sign ($-$ to $+$) through $x = 3$.
  So g attains a minimum.
- For B, $g''(x) = 2023(x-3)^{2022}$, which does not change sign through $x = 3$.
  So g does not have an inflection point at $x = 3$.
- For C and D, $\dfrac{d}{dx}(xg(x)) = g(x) + xg'(x)$, which may not be zero as $g(3) = C$ is arbitrary. So they are both not necessarily true.

Beware of the arbitrary constant!

Question 12

Differentiate all answers:

A. $\frac{d}{dx}\left(-\frac{\sin x}{1+\cos x}+C\right)=-\frac{\cos x}{1+\cos x}-\frac{\sin^2 x}{(1+\cos x)^2}$

which simplies to $-\frac{1+\cos x}{(1+\cos x)^2}=-\frac{1}{1+\cos x}$

So, C is the answer as $\frac{d}{dx}\left(-\frac{\sin x}{1+\cos x}+C\right)=\frac{1}{1+\cos x}$.

B. $\frac{d}{dx}\left(\frac{\cos x}{1+\cos x}+C\right)=-\frac{\sin x}{1+\cos x}-\frac{\sin x\cos x}{(1+\cos x)^2}$

which simplifies to $\frac{-\sin x(1+\cos x)}{(1+\cos x)^2}=-\frac{\sin x}{1+\cos x}$.

E. Similarly, $\frac{d}{dx}(\frac{\sin(x)\cos(x)}{1+\cos(x)})=\frac{-\sin^2(x)+\cos^3(x)+\cos^2(x)}{(1+\cos(x))^2}$

Reminder: Q12

Reminder: Antiderivatives

Sometimes you don't know when to start for "elegant integration by substitution".

- If a list of candidates are given, just do it (brute force) by testing with differentiation.
- In this question, spotting the negative sign can help reduce unnecessary calculations across choices.

Reference: MATH 1013 2017-18 Fall Final #12

Question 13

Applying Fundamental Theorem of Calculus with Chain rule,

- Differentiating w.r.t. x yields $f(x) + x^3 f(x^3) \cdot 3x^2 = 4x^3$
- Substituting $x = 1$ yields $f(1) + (1)f(1) \cdot 3(1) = 4(1)$
- $f(1) = 1$

Reference: MATH 1013 2017-18 Fall Final #17

Question 14

Note that $\int_0^1 \frac{1+2e^{3x}}{1+e^{3x}} dx = \int_0^1 \frac{1+e^{3x}}{1+e^{3x}} dx + \int_0^1 \frac{e^{3x}}{1+e^{3x}} dx.$

- The first integral is simply $\int_0^1 dx = [x]_0^1 = 1$
- For the second integral, apply substitution $u = 1 + e^{3x}$: $du = 3e^{3x} dx$, when $x = 1$, $u = 1 + e^3$; when $x = 0$, $u = 2$.
- Then, the second integral $\int_0^1 \frac{e^{3x}}{1+e^{3x}} dx = \int_2^{1+e^3} \frac{1}{3} \frac{du}{u} = \frac{1}{3}[\ln u]_2^{1+e^3} = \frac{1}{3} \ln(\frac{1+e^3}{2})$
- Summing up, the integral evaluates to $1 + \frac{1}{3} \ln(\frac{1+e^3}{2})$.

Reminder: Q14

Reminder: When to substitute? When to break?

- If you see similar terms in numerator/ denominator, factor/ divide them out. You will likely benefit from it!
- Recall differentiating a constant returns 0. For substitution:
  1. It is common to write $d(e^{ax} + k) = ae^{ax}dx$.
  2. Another common substitution is using $d(x^a + b) = ax^{a-1}dx$.
- Obtain the constant a for substitution by dividing it elsewhere.

It is also a common (careless) error that $e^0 = 1$, NOT zero.

Reference: MATH 1012 2015-16 Fall Final #9

MCQ 15E/ Integration/ Graph

Question 15

With the absolute sign in the integrand, we must split f into increasing and decreasing parts:

- f is increasing in $(0,3)$, and $(\frac{13}{2},8)$; it is decreasing in $(3,\frac{13}{2})$.
- Thus, $\displaystyle\int_0^8 |f'(x)|dx = \int_0^3 f'(x)dx - \int_3^{\frac{13}{2}} f'(x)dx + \int_{\frac{13}{2}}^8 f'(x)dx$
- The answer yields $(4-(-5))-(-3-4)+(0-(-3))=19$.

In recent years, more complex variants like $\displaystyle\int_0^8 f(x)\cdot|f'(x)|dx$ have appeared, but the method remains similar.
References: MATH 1013 2017-18 Fall Final #15/ 2019-20 Final #21

Question 16(a) - Constant A [4 pts]

The given limit is of $0/0$ form.

- [1 pt: First application]
  Applying L'Hôpital once, the limit becomes
  $$\lim_{x\to 0} \frac{2\sin x \cos x + 2Ax}{4x^3} = \lim_{x\to 0} \frac{\sin 2x + 2Ax}{4x^3}$$
- [1 pt: Second Application]
  Applying the second time, $\displaystyle\lim_{x\to 0} \frac{2\cos 2x + 2A}{12x^2}$
- [2 pts: Deduce condition for limit to exist + Answer]
  The limit may exist only if it is of $0/0$ form (the denominator $\to 0$).
  So, $2\cos(2(0)) + 2A = 0 \implies A = -1$

LQ 16 (b)/ Differentiation/ L'Hôpital's Rule

Question 16(b) - Limit L [3 pts]

- [1 pt: Third Application]
  The limit $\lim_{x \to 0} \dfrac{2\cos 2x - 2}{12x^2}$ is of "0/0" form
  Applying L'Hôpital again reduces to $\lim_{x \to 0} \dfrac{-4\sin 2x}{24x}$
- [2 pt: Final Application + Answer]
  Applying L'Hôpital for a final time, the limit evaluates to
  $\lim_{x \to 0} \dfrac{-8\cos 2x}{24} = \dfrac{-8(1)}{24} = -\dfrac{1}{3}$.

Deduct 1 pt in (a) or (b) if does not state "0/0" condition.
Sometimes, you need to deduce the form of limit from its existence: common technique in HKALE (that has appeared in Calc I finals).

LQ 17 (a)/ Appli. of Differentiation/ Similar Solids

Question 17(a) - Similar Solids [4 pts]

- [1 pt: Similar 2D for original cone]
  Suppose the height of the cone being "cut" to form the frustum is h' cm. Applying similar ratios, $\frac{h'}{h'+24} = \frac{40}{50} \implies h' = 96$.
- [1 pt: Original Quantities of Volume/ Surface Area]
  Then, the volume of the cut cone is $\frac{\pi}{3}(96)(40^2) = 51200\pi$ cm^3, and its curved surface area $\pi(\sqrt{96^2+40^2})(40) = 4160\pi$ cm^2.
- [2 pts: Similar 2D/ 3D for desired quantity + Followthrough]
  Applying similar solids, $\frac{V+51200\pi}{51200\pi} = \left(\frac{h+96}{96}\right)^3$ and $\frac{A+4160\pi}{4160\pi} = \left(\frac{h+96}{96}\right)^2$
  So, $V = \frac{25\pi}{432}\left[(96+h)^3 - 96^3\right]$ and $A = \frac{65\pi}{144}[(96+h)^2 - 96^2]$.

Question 17(b) - Rate of Change [4 pts]

- [1 pt: Differentiating w.r.t. t]
  $\dfrac{dV}{dt} = \dfrac{25\pi}{144}(96+h)^2\dfrac{dh}{dt}$ and $\dfrac{dA}{dt} = \dfrac{65\pi}{72}(96+h)\dfrac{dh}{dt}$
- [2 pts: Obtain trend/ proportion on dA/dt]
  1. With $\dfrac{dV}{dt} = 60\pi$ cm^3 s^{-1} fixed, $\dfrac{dh}{dt} \propto \dfrac{1}{(96+h)^2}$
  2. Consequently, $\dfrac{dA}{dt} \propto \dfrac{1}{(96+h)}$, which is decreasing as h increases from 0 to 24. [Exact answer: $\dfrac{dA}{dt} = \dfrac{26}{5(96+h)}\dfrac{dV}{dt}$]
- [1 pt: Answers with end points]
  So, $\frac{dA}{dt}$ decreases from $\frac{dA}{dt}\Big|_{h=0} = \frac{13}{4}\pi$ to $\frac{dA}{dt}\Big|_{h=24} = \frac{13}{5}\pi$ throughout.

Reminder: Q17

Use of Similarity

- Similar triangles/ solids are necessary for such mensuration rate of change questions.
- The key is to visualize the cut/ original solids to clarify what solids/ triangles to apply similarity.
- Beware of the power (2 or 3?) of similarity.
- There were many examples in HKDSE M2.

References:
HKDSE 2012 M2 #6; MATH 1013 2019-2020 Fall Final #12

Question 18(a) - Substitution [5 pts]

Note that $x^3 = x(x^2) = x[(9+x^2)-9]$.

- [1 pt: State substitution]
  Consider the substitution $u = 9 + x^2$. Then $du = 2xdx$.
- [2 pts: Split $x^2 = (x^2+9) - 9$ and complete transformation]
  Rewrite $\dfrac{x^3}{9+x^2}dx = \dfrac{x[(9+x^2)-9]}{9+x^2}dx$
  $= \dfrac{1}{2}(2xdx)[1 - \dfrac{9}{9+x^2}] = \dfrac{1}{2}(1-\dfrac{9}{u})du$
- [2 pts: Evaluate u and back-substitute x]
  So, the integral becomes $\dfrac{1}{2}\displaystyle\int(1-\dfrac{9}{u})du = \dfrac{u}{2} - \dfrac{9}{2}\ln|u| + C$.
  Finally, the answer is $\dfrac{9+x^2}{2} - \dfrac{9}{2}\ln(9+x^2) + C$.

Alternative to LQ 18(a)

- [1 pt: State substitution]
  Consider the substitution $x = 3\tan\theta$. Then $dx = 3\sec^2\theta d\theta$.
- [1 pt: Complete transformation]
  Then $\int \frac{x^3}{9+x^2}dx = \int \frac{27\tan^3\theta(3\sec^2\theta)}{9+9\tan^2\theta}d\theta = \int 9\tan^3\theta d\theta$
- [2 pts: Evaluate u and back-substitute x]
  Note that $\int 9\tan^3\theta d\theta = \int 9\tan\theta\sec^2\theta d\theta - \int 9\tan\theta d\theta$
  $= 9\int \tan\theta d(\tan\theta) + 9\int \frac{d(\cos\theta)}{\cos\theta} = \frac{9}{2}\tan^2\theta + 9\ln|\cos\theta| + C$
- [1 pt: Back-substitute x]
  Finally, the answer is $\frac{x^2}{2} - \frac{9}{2}\ln(9+x^2) + C$.

Question 18(b) - Identifying Riemann Integral [4 pts]

- [2 pts: Summation with interval + Division]
  Rewrite S in summation:
  $$\lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{i^3}{9n^3+ni^2}\right)=\lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}\left(\frac{(\frac{i}{n})^3}{9+(\frac{i}{n})^2}\right)=\lim_{n\to\infty}\frac{4}{n}\sum_{i=1}^{n}f(\frac{i}{n})$$
- [1 pt: End points and total interval]
  So, the Riemann sum uses (right) end-points $x_i=\frac{i}{n}$, meaning the sample points cover $[0,1]$, not $[0,4]$. So, $\Delta x=\frac{1}{n}$.
- [1 pt: Correct Integral]
  The factor 4 in $\frac{4}{n}$ should be factored out, resulting in the integral $4\int_0^1 f(x)dx$. Thus, the student is NOT correct.

Reminder: Q18

In identifying the Riemann integral, always follow the steps:

1. Rewrite the sum as a summation.
2. Divide the fraction to spot any term relevant to $\frac{k \cdot i}{n}$ (k constant).
3. Determine the width $b - a$ with the sample points x_i, not the multiple of $\frac{1}{n}$. Correspondingly, regenerate the term $\Delta x = \frac{b-a}{n}$
4. Finally, convert the sum into the integral with bounds a and b.

Reference: 2018 HKDSE M2 #5(a)

LQ 19 (a)/ Appli. of Differentiation/ Implicit Diff.

Question 19(a) - Implicit Differentiation [4 pts]

- [2 pts: Implicitly differentiate each sides w.r.t. x]
  $y + x\dfrac{dy}{dx} = -y\sin(xy) - x\sin(xy)\dfrac{dy}{dx}$
  So, $y(1+\sin(xy)) + x\dfrac{dy}{dx}(1+\sin(xy)) = 0$. The result follows.
- [2 pts: Considering counterargument]
  For $1+\sin(xy)=0$, $\sin(xy)=-1$, $xy \neq 0$
  But, $\cos(xy) = \sqrt{1-(-1)^2} = 0 = xy$, contradicting with the requirement $xy \neq 0$.
- So, $y + x\dfrac{dy}{dx} = 0 \implies \dfrac{dy}{dx} = -\dfrac{y}{x}$

LQ 19 (b)(i)/ Appli. of Differentiation/ IVT

Question 19(b)(i) - Intermediate Value Theorem [3 pts]

- [1 pt: mention $\cos u$ is decreasing]
  Note that $\dfrac{d}{du}(\cos u - u) = -\sin u - 1 < 0$ for all $0 < u < \dfrac{\pi}{2}$.
  So, $\cos u$ is decreasing throughout $0 < u < \dfrac{\pi}{2}$.
- [2 pts: Applying IVT]
  Noting that $0 - \cos(0) = -1 < 0$ and $\frac{\pi}{2} - \cos(\frac{\pi}{2}) = \frac{\pi}{2} > 0$,
  The equation (*) has exactly one root in $\left[0, \dfrac{\pi}{2}\right]$.

Question 19(b)(ii) - Tangent Slope [4 pts]

- [1 pt: Matching Slope]

  The line L has slope -1. So, $\dfrac{dy}{dx} = -1 = -\dfrac{y}{x} \implies x = y$.

- [3 pts: Applying IVT]

  Substituting $x = y$ into the curve C, we have $x^2 = \cos(x^2)$.

  1. By (i), we know for $0 < x^2 < \frac{\pi}{2}$, there is exactly one value of x satisfying $x^2 = \cos(x^2)$.
     Denoting this value by $x = u$. Then, $u^2 = \cos(u^2)$.
  2. Also note that $x = -u$ is the only value $\in (-\dfrac{\pi}{2}, 0)$ satisfying $x^2 = \cos(x^2)$.

  Thus, there exists two tangents to C parallel to the straight line $L : x + y = 0$.

Question 19(c) - Mean Value Theorem [2 pts]

- [1 pt: point out discontinuity]
  Note that $f(x)$ is NOT defined at $x = 0$. Thus, $f(x)$ is NOT continuous over $[a, b]$.
  Alternative answer:
  Note that $f'(x) = \dfrac{dy}{dx} = -\dfrac{y}{x}$ is NOT defined at $x = 0$. Thus, $f(x)$ is NOT differentiable over $[a, b]$.
- [1 pt: followthrough]
  As the prerequisites of MVT are not satisfied, the student's application of MVT is incorrect.

Question 22(a) - Asymptotes [2 pts]
Question 22(b) - Derivative [2 pts]

- [2 pts: Asymptotes (Answers only)]
  Horizontal asymptote: $y = 3$; Vertical asymptote: $x = 1$
- [2 pts: Derivatives (Answers only)]
  $$\frac{dy}{dx} = -\frac{4}{(x-1)^2} + \frac{2(k-1)}{(x-1)^3}; \qquad \frac{d^2y}{dx^2} = \frac{8}{(x-1)^3} - \frac{6(k-1)}{(x-1)^4}$$

You may find this unexpectedly short!

- This question highlights the use case of partial fractions/ long div.
- HKDSE features asymptotes in the first part after long div. Uses:
  (1) Easy spotting of asymptotes and (2) Easy differentiation

Question 20(c) - Minimum for all k? [5 pts]

- [1 pt: Set first derivative zero]
  Setting $\frac{dy}{dx} = 0 \implies -4(x-1) + 2(k+1) = 0 \implies x^* = \frac{k+1}{2}$
- [1 pt: Case Undefined]
  For $k = 1$, $x^* = 1$ where Γ is undefined. This cannot be minimum.
- [2 pts: Case Minimum]
  For $k \neq 1$, $\left.\frac{d^2y}{dx^2}\right|_{x=\frac{k+1}{2}} = \left[\frac{8}{(x-1)^3} - \frac{6(k-1)}{(x-1)^4}\right]_{x=\frac{k+1}{2}} = -\frac{32}{(k-1)^3}$.
  For $k < 1$, $k - 1 < 0 \implies -\frac{32}{(k+1)^3} = \left.\frac{d^2y}{dx^2}\right|_{x=\frac{-k+1}{2}} > 0$,
  i.e. Γ has a minimum point.
- [1 pt: Answer]
  Thus, the claim is incorrect: Γ only has minimum for $k \in (-\infty, 1)$.

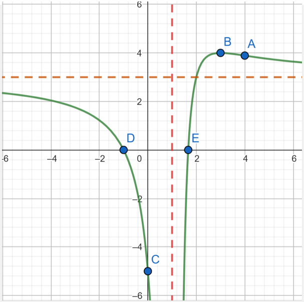
Question 22(d)(i) - Inflection Points to Solve k [2 pts]
Question 22(d)(ii) - Sketch Graph [4 pts]

[1 pt: Set second d. zero + 1 pt: Followthrough]

As $\left.\dfrac{d^2y}{dx^2}\right|_{x=4} = 0$, $\frac{8}{(4-1)^3} - \frac{6(k-1)}{(4-1)^4} = 0 \implies k = 5$

Each characteristic is worth 1 pt:

1. Two asymptotes
2. x-, y- intercepts (points C, D, E)
3. Maximum and Inflection points (points B, A)
4. Overall shape



Question 22(d)(iii) - Area Bounded [3 pts]

- [1 pt: Identify Area Integral]
  The area $A = \int_{\frac{5}{3}}^{4} \frac{3x^2 - 2x - 5}{(x-1)^2}\, dx = \int_{\frac{5}{3}}^{4} 3 + \frac{4}{x-1} - \frac{4}{(x-1)^2}\, dx$
- [1 pt: Evaluate the integral]
  $A = \left[3x + 4\ln|x-1| + \frac{4}{x-1} \right]_{\frac{5}{3}}^{4}$
- [1 pt: Answer]
  The area is $\frac{7}{3} + 4\ln\frac{9}{2} \approx 8.3496.$