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uk. 10, cv. 11

Regresní rovina, hledám odhad $\hat{\beta}$ vektem $(\beta_0, \beta_1, \beta_2)'$ pro model $\vec{y} = X\beta + e$

$$X'X\hat{\beta} = X'y$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 25 & 26 & 25 & 22 & 28 & 27 \\ 7{,}5 & 4{,}3 & 6{,}4 & 5{,}2 & 7{,}9 & 6{,}8 \end{pmatrix} \begin{pmatrix} 1 & 25 & 7{,}5 \\ 1 & & \\ 1 & \vdots & \vdots \\ 1 & & \\ 1 & & \\ 1 & 27 & 6{,}8 \end{pmatrix} \cdot \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 25 & & & \cdots & 27 \\ 7{,}5 & & & \cdots & 6{,}8 \end{pmatrix} \cdot \begin{pmatrix} 1{,}34 \\ 1{,}16 \\ 1{,}25 \\ 1{,}32 \\ 1{,}45 \\ 1{,}32 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 158 & 38{,}1 \\ 158 & 4168 & 1004{,}5 \\ 38{,}1 & 1004{,}5 & 251{,}35 \end{pmatrix} \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 7{,}89 \\ 208{,}14 \\ 50{,}673 \end{pmatrix}$$

$\hat{\beta}_0 = -0{,}126268$

$\hat{\beta}_1 = 0{,}0414240$

$\hat{\beta}_2 = 0{,}0551868$

Modelem tedy je:

$$\hat{y}_i = -0{,}126268 + 0{,}041424\, x_i + 0{,}0551868\, z_i$$

kde x_i je teplota a z_i koncentrace hnojiva

Pro hodnoty 26 °C a 7 ml/l tedy můžeme očekávat výšku 1,337 m.

$$\hat{y} = -0{,}126268 + 0{,}041424 \cdot 26 + 0{,}0551868 \cdot 7 \doteq 1{,}337 \text{ m}$$

Výpočty jsem provedl pomocí webové aplikace https://matrixcalc.org/en/:

X'X

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 25 & 26 & 25 & 27 & 28 & 27 \\ \frac{15}{2} & \frac{43}{10} & \frac{32}{5} & \frac{26}{5} & \frac{79}{10} & \frac{34}{5} \end{pmatrix} \cdot \begin{pmatrix} 1 & 25 & \frac{15}{2} \\ 1 & 26 & \frac{43}{10} \\ 1 & 25 & \frac{32}{5} \\ 1 & 27 & \frac{26}{5} \\ 1 & 28 & \frac{79}{10} \\ 1 & 27 & \frac{34}{5} \end{pmatrix} = \begin{pmatrix} 6 & 158 & \frac{381}{10} \\ 158 & 4168 & \frac{2009}{2} \\ \frac{381}{10} & \frac{2009}{2} & \frac{25139}{100} \end{pmatrix}$$

X'Y

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 25 & 26 & 25 & 27 & 28 & 27 \\ 7.5 & 4.3 & 6.4 & 5.2 & 7.9 & 6.8 \end{pmatrix} \cdot \begin{pmatrix} 1.34 \\ 1.16 \\ 1.25 \\ 1.32 \\ 1.45 \\ 1.37 \end{pmatrix} = \begin{pmatrix} 7.89 \\ 208.14 \\ 50.673 \end{pmatrix}$$

▶ Details (Matrix multiplication)

Solution by Cramer's rule

$$\begin{cases} 6 \cdot x_1 + 158 \cdot x_2 + 38.1 \cdot x_3 = 7.89 \\ 158 \cdot x_1 + 4168 \cdot x_2 + 1004.5 \cdot x_3 = 208.14 \\ 38.1 \cdot x_1 + 1004.5 \cdot x_2 + 251.39 \cdot x_3 = 50.673 \end{cases}$$

$$\Delta = \begin{vmatrix} 6 & 158 & 38.1 \\ 158 & 4168 & 1004.5 \\ 38.1 & 1004.5 & 251.39 \end{vmatrix} = 407.38$$

▶ Details (Triangle's rule)

...

$$\Delta_1 = \begin{vmatrix} 7.89 & 158 & 38.1 \\ 208.14 & 4168 & 1004.5 \\ 50.673 & 1004.5 & 251.39 \end{vmatrix} = -51.4389;$$

▶ Details (Triangle's rule)

...

$$\Delta_2 = \begin{vmatrix} 6 & 7.89 & 38.1 \\ 158 & 208.14 & 1004.5 \\ 38.1 & 50.673 & 251.39 \end{vmatrix} = 16.8753;$$

▶ Details (Triangle's rule)

...

$$\Delta_3 = \begin{vmatrix} 6 & 158 & 7.89 \\ 158 & 4168 & 208.14 \\ 38.1 & 1004.5 & 50.673 \end{vmatrix} = 22.482;$$

▶ Details (Triangle's rule)

...

$$x_1 = \Delta_1 / \Delta = \frac{-51.4389}{407.38} = -0.126268$$

$$x_2 = \Delta_2 / \Delta = \frac{16.8753}{407.38} = 0.0414240$$

$$x_3 = \Delta_3 / \Delta = \frac{22.482}{407.38} = 0.0551868$$

Answer:

$x_1 = -0.126268$

$x_2 = 0.0414240$

$x_3 = 0.0551868$