⑥ Ondřej Ondryáš (xondry02)

$f(t) = 3 - \frac{1}{4}t$ pro $t \in \langle 0; 8)$, $f(t)$ je periodická s $T = 8$

$T = 8 \qquad \omega = \frac{2\pi}{T} = \frac{1}{4}\pi$

$$a_n = \frac{2}{8}\int_0^8 (3 - \tfrac{1}{4}t)\cdot\cos\left(\frac{\pi n t}{4}\right)dt = \frac{2}{8}\left(\int_0^8 3\cos\tfrac{\pi n t}{4}\,dt + \int_0^8 -\tfrac{1}{4}t\cdot\cos\tfrac{\pi n t}{4}\,dt\right) = \frac{2}{8}\left(3\int_0^8 \cos\tfrac{\pi n t}{4}\,dt - \frac{1}{4}\int_0^8 t\cdot\cos\tfrac{\pi n t}{4}\,dt\right)$$

$$\int \cos\tfrac{\pi n t}{4}\,dt = \frac{\sin\frac{\pi n t}{4}}{\frac{\pi n}{4}} + c = \frac{4}{\pi n}\cdot\sin\tfrac{\pi n t}{4} + c$$

$$\int \sin\tfrac{\pi n t}{4}\,dt = -\frac{4}{\pi n}\cos\tfrac{\pi n t}{4} + c$$

$$\int t\cdot\cos\tfrac{\pi n t}{4}\,dt \quad \left|\begin{array}{l} u = t \quad u' = 1 \\ v = \frac{4}{\pi n}\sin\frac{\pi n t}{4} \quad v' = \cos\frac{\pi n t}{4} \end{array}\right| = \frac{4t}{\pi n}\sin\tfrac{\pi n t}{4} - \int \frac{4}{\pi n}\sin\tfrac{\pi n t}{4}\,dt = \frac{4t}{\pi n}\sin\tfrac{\pi n t}{4} - \frac{4}{\pi n}\cdot\left(-\frac{4}{\pi n}\cos\tfrac{\pi n t}{4}\right) + c$$

$$= \frac{4t}{\pi n}\sin\tfrac{\pi n t}{4} + \left(\frac{4}{\pi n}\right)^2\cdot\cos\tfrac{\pi n t}{4} + c$$

$$a_n = \frac{2}{8}\left(3\left[\frac{4}{\pi n}\cdot\sin\tfrac{\pi n t}{4}\right]_0^8 - \frac{1}{4}\left[\frac{4t}{\pi n}\sin\tfrac{\pi n t}{4} + \left(\frac{4}{\pi n}\right)^2\cos\tfrac{\pi n t}{4}\right]_0^8\right) = \frac{2}{8}\left[-\frac{1}{4}\left(\frac{4^2}{\pi^2 n^2}\cdot 1 - \frac{4^2}{\pi^2 n^2}\cdot 1\right)\right] = \frac{2}{8}\cdot\left(-\tfrac{1}{4}\right)\cdot 0 = 0$$

$0 - 0$

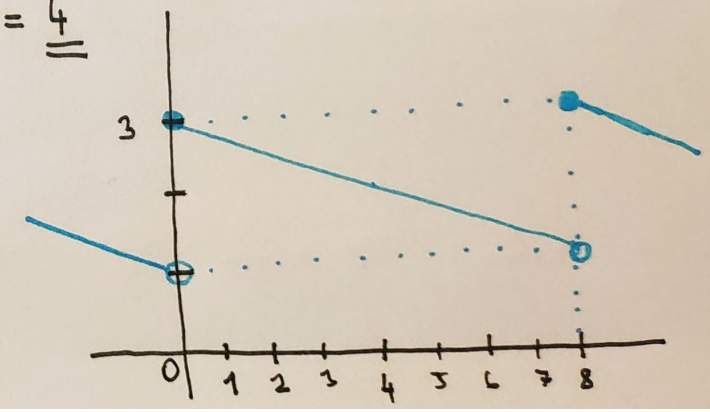
$\sin\frac{8\pi n}{4} = \sin 2\pi n$, pro $n \in \mathbb{N}$ vždy $= 0$

$\sin 0 = 0$

$\cos\frac{8}{4}\pi n = \cos 2\pi n$, pro $n \in \mathbb{N}$ vždy $= 1$

$\cos 0 = 1$

$$a_0 = \frac{2}{8}\int_0^8 (3 - \tfrac{1}{4}t)\cdot\cos 0\,dt = \frac{1}{4}\int_0^8 (3 - \tfrac{1}{4}t)\,dt = \frac{1}{4}\left(\left[3t\right]_0^8 - \left[\frac{\frac{1}{4}t^2}{2}\right]_0^8\right) = \frac{1}{4}(24 - 8) = 4$$



$$b_n = \frac{2}{8}\int_0^8 \left(3-\frac{1}{4}t\right)\sin\frac{\pi n t}{4}\,dt = \frac{2}{8}\left(3\int_0^8 \sin\frac{\pi n t}{4}\,dt - \frac{1}{4}\int_0^8 t\cdot\sin\frac{\pi n t}{4}\,dt\right)$$

$$\rightarrow \int \sin\frac{\pi n t}{4}\,dt = -\frac{4}{\pi n}\cos\frac{\pi n t}{4} + c$$

(6) Ondřej Ondrgás

$$\rightarrow \int t\cdot\sin\frac{\pi n t}{4}\,dt \quad \left|\begin{array}{ll} u = t & u' = 1 \\ v = -\frac{4}{\pi n}\cos\frac{\pi n t}{4} & v' = \sin\frac{\pi n t}{4} \end{array}\right| = -\frac{4t}{\pi n}\cos\frac{\pi n t}{4} - \int -\frac{4}{\pi n}\cos\frac{\pi n t}{4} = -\frac{4t}{\pi n}\cos\frac{\pi n t}{4} + \frac{4}{\pi n}\int\cos\frac{\pi n t}{4} =$$

$$= -\frac{4t}{\pi n}\cos\frac{\pi n t}{4} + \left(\frac{4}{\pi n}\right)^2\cdot\sin\frac{\pi n t}{4} + c$$

$$b_n = \frac{2}{8}\left(3\left[-\frac{4}{\pi n}\cos\frac{\pi n t}{4}\right]_0^8 - \frac{1}{4}\left[-\frac{4t}{\pi n}\cos\frac{\pi n t}{4} + \left(\frac{4}{\pi n}\right)^2\cdot\sin\frac{\pi n t}{4}\right]_0^8\right)$$

$\cos 2\pi n = 1 \quad n\in\mathbb{N}$ $\quad \sin 2\pi n = 0 \quad n\in\mathbb{N}$

$\cos 0 = 1$ $\quad \sin 0 = 0$

$$= \frac{2}{8}\left[3\cdot\left(-\frac{4}{n\pi}+\frac{4}{n\pi}\right) - \frac{1}{4}\left(-\frac{4\cdot 8}{\pi n}\cdot 1 + \frac{4\cdot 0}{\pi n}\cdot 1\right)\right] = \frac{2}{8}\cdot\left(-\frac{1}{4}\right)\cdot\left(-\frac{32}{\pi n}\right) = \underline{\underline{\frac{2}{\pi n}}}$$

$$f(t) = \frac{4}{2} + \sum_{n=1}^{\infty}\left(\frac{2}{\pi n}\cdot\sin\left(n\frac{\pi}{4}t\right)\right)$$

$$f(t) = 2 + \sum_{n=1}^{\infty}\left(\frac{2}{\pi n}\cdot\sin\frac{\pi n t}{4}\right) \quad \text{~~(pro } t\in\langle 0;8)\ \wedge f(t) = 3-\frac{1}{4}t\text{~~)} \quad (\text{pro } t\in\mathbb{R} \wedge f(t) = 3-\frac{1}{4}t \text{ na } t\in\langle 0;8) \text{ a } f(t) \text{ je periodická s } T=8)$$

Některé kroky na tomto listu nejsou podrobně rozepsány, protože jsou vcelku zjevné a prakticky shodné s kroky na prvním listu.

Přeji pěkný den!