⑥ Ondřej Ondryáš (xondry02)

$$\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n+3^n} = \sum_{n=0}^{\infty} \frac{1}{2^n+3^n} \cdot (x+1)^n \qquad c_n = \frac{1}{2^n+3^n} \qquad x_0 = -1$$

$$r = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| = \lim_{n\to\infty} \frac{\frac{1}{2^n+3^n}}{\frac{1}{2^{n+1}+3^{n+1}}} = \lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{3^n} + \frac{3^{n+1}}{3^n}}{\frac{2^n}{3^n} + \frac{3^n}{3^n}}$$

$$= \lim_{n\to\infty} \frac{2\left(\frac{2}{3}\right)^n + 3}{\left(\frac{2}{3}\right)^n + 1} = \underline{\underline{3}}$$

$\left(\frac{2}{3}\right)^n \to 0$

Oborem konvergence je $(-1-3; -1+3) = (-4; 2)$.

Krajní body:

$$\sum_{n=0}^{\infty} \frac{(-3)^n}{2^n+3^n} \to \lim_{n\to\infty} \frac{(-3)^n}{2^n+3^n} = \lim_{n\to\infty} \frac{\frac{(-1)^n \cancel{3^n}}{\cancel{3^n}}}{\frac{2^n}{3^n} + \frac{\cancel{3^n}}{\cancel{3^n}}} = \lim_{n\to\infty} \frac{(-1)^n}{\frac{2^n}{3^n}+1} = \lim_{n\to\infty} (-1)^n \frac{1}{\left(\frac{2}{3}\right)^n + 1}$$

$$= \underline{\underline{\lim_{n\to\infty} (-1)^n}}$$

tato limita neexistuje
⇒ není splněna nutná podmínka konvergence řady

$$\sum_{n=0}^{\infty} \frac{3^n}{2^n+3^n} \to \lim_{n\to\infty} \frac{3^n}{2^n+3^n} = \lim_{n\to\infty} \frac{\frac{3^n}{3^n}}{\frac{2^n}{3^n} + \frac{3^n}{3^n}} = \lim_{n\to\infty} \frac{1}{\left(\frac{2}{3}\right)^n + 1} = \underline{\underline{1}}$$

limita existuje, ale není 0 ⇒ není splněna nutná podmínka konvergence

Oborem konvergence řady $\sum_{n=0}^{\infty} \frac{(x+1)^n}{2^n+3^n}$ je $(-4; 2)$ (řada konverguje pro $x \in (-4; 2)$.)