



1) $P[S_0 \rightarrow 5] = \frac{1}{5} ?$ $T = \{5\}$ $S_0 = \{S_1, S_3, S_4, 1, 2, 3, 4\}$

$$\begin{aligned} r(5) &= 1 \\ r(S_1) &= 0 \\ r(S_5) &= \frac{1}{2} r(5) + \frac{1}{2} r(S_0) \\ r(S_2) &= \frac{1}{2} r(S_5) + \frac{1}{2} r(S_0) \\ r(S_0) &= \frac{1}{2} r(S_2) + \frac{1}{2} r(S_1) \end{aligned}$$

from this system, we get
 $r(S_0) = \frac{1}{5}$, $r(S_2) = \frac{2}{5}$, $r(S_5) = \frac{3}{5}$
 \hookrightarrow so $P[S_0 \rightarrow 5] = \frac{1}{5} \checkmark$

$P[S_0 \rightarrow 1] = \frac{1}{5} ?$ $T = \{1\}$ $S_0 = \{2, 3, 4, S_4, 5\}$

$$\begin{aligned} r(1) &= 1 \\ r(S_3) &= \frac{1}{2} r(1) + \frac{1}{2} r(2) \\ r(S_4) &= 0 \\ r(S_1) &= \frac{1}{2} r(S_3) + \frac{1}{2} r(S_4) \\ r(S_0) &= \frac{1}{2} r(S_1) + \frac{1}{2} r(S_2) \\ r(S_2) &= \frac{1}{2} r(S_5) + \frac{1}{2} r(S_0) \\ r(S_5) &= \frac{1}{2} r(S_0) \end{aligned}$$

$$\begin{aligned} r(1) &= 1 \quad r(S_3) = \frac{1}{2} \quad r(S_1) = \frac{1}{4} \\ r(S_0) &= \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} r(S_2) \\ r(S_2) &= \frac{1}{2} r(S_5) + \frac{1}{2} r(S_0) \\ r(S_5) &= \frac{1}{2} r(S_0) + \frac{1}{2} r(5) \end{aligned}$$

1, 2, 3, 4 are symmetric,
 so this result
 applies to all of them

$$\begin{aligned} r(S_0) &= \frac{1}{5}, \quad r(S_2) = \frac{3}{5}, \quad r(S_5) = \frac{1}{5} \\ \hookrightarrow \text{so } P[S_0 \rightarrow 1] &= \frac{1}{5} \checkmark \end{aligned}$$

	s_0	s_1	s_2	s_3	s_4	s_5	1	2	3	4	5
$t_0 =$	1	0	0	0	0	0	0	0	0	0	0
$t_1 =$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	0
$t_2 =$	$\frac{1}{4}$	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	0	0
$t_3 =$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	0	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$t_4 =$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$t_5 =$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

Diagram illustrating the evolution of a probability distribution over time steps t_0 to t_5 . The distribution is represented by a vector of probabilities for states s_0 through s_5 and outcomes 1 through 5. Arrows indicate transitions between states over time.

$$P[s_0 \xrightarrow{\leq 5} \{1, 2, 3, 4, 5\}] = 4 \cdot \frac{1}{8} + \frac{3}{16} = \underline{\underline{\frac{11}{16}}}$$

$$P[s_0 \xrightarrow{\leq 5} \{1, 5\}] = \frac{1}{8} + \frac{3}{16} = \underline{\underline{\frac{5}{16}}}$$