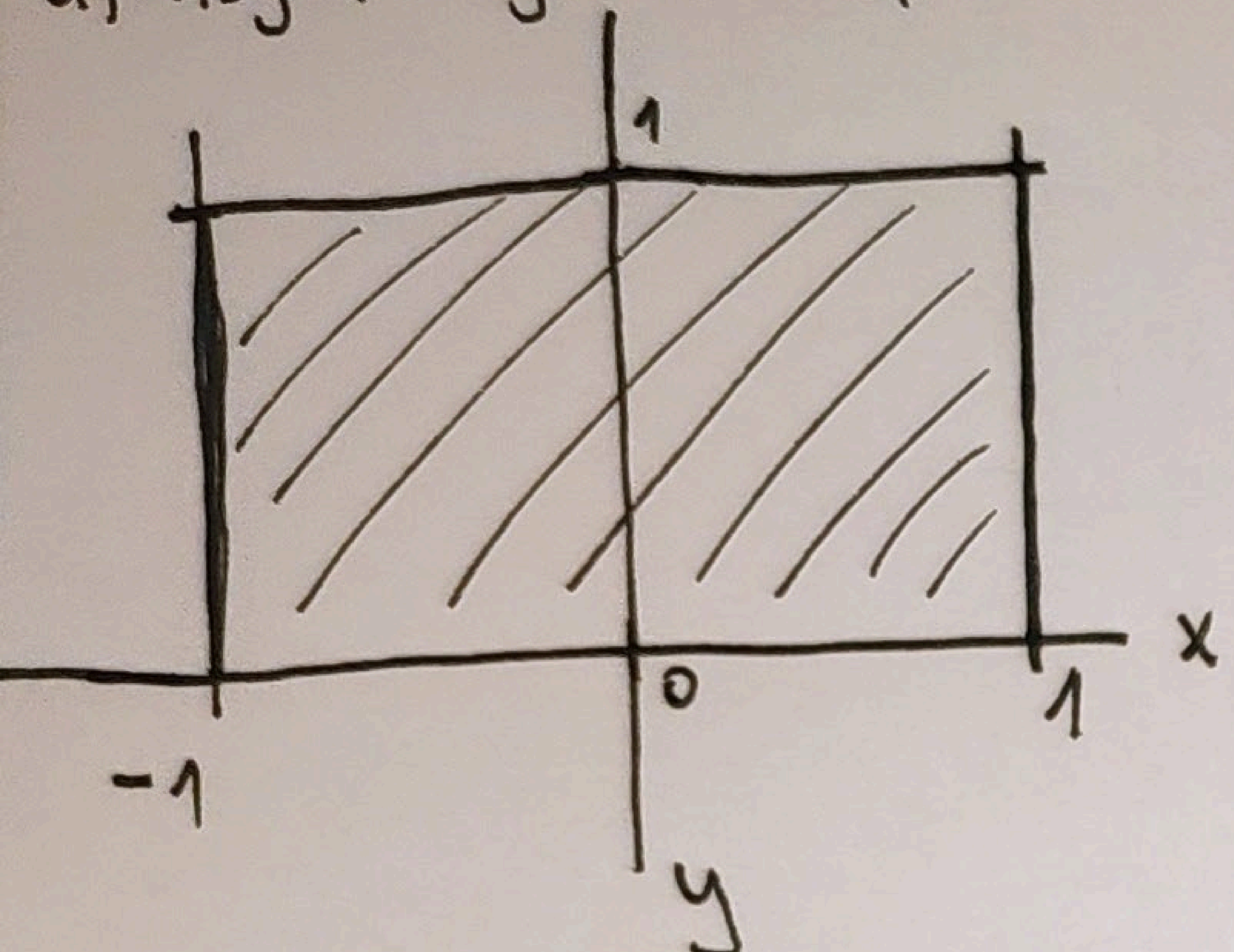
Ondřej Ondryáš, úkol 7, cv. 8

$$f(x,y) = \begin{cases} c(6x^2+2y) & x,y \in \langle -1,1\rangle \times \langle 0,1\rangle \\ 0 & \text{jinak} \end{cases}$$

a) Aby to byla hustota, musí být objem pod grafem $= 1$.



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^1 \left(\int_{-1}^{1} c(6x^2+2y)\,dx \right) dy = c \cdot \int_0^1 \left(\left[\tfrac{6}{3}x^3 + 2xy \right]_{-1}^{1} \right) dy$$

$$= c\int_0^1 (2+2y+2+2y)\,dy = 4c\int_0^1 (y+1)\,dy = 4c\left[\tfrac{1}{2}y^2+y\right]_0^1 = 4c\left(\tfrac{1}{2}+1\right)$$

$$= \underline{6c} \Rightarrow 6c = 1 \Rightarrow \underline{\underline{c = \tfrac{1}{6}}}$$

b) $$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{-1}^{1} c(6x^2+2y)\,dx = c\left[\tfrac{6}{3}x^3+2xy\right]_{-1}^{1} = 4c(y+1) \qquad y \in \langle 0,1\rangle$$

$$F_Y(y) = \cancel{\int_0^y f_Y(y)\,dy} = \int_0^y f_Y(u)\,du = \int_0^y 4c(u+1)\,du = 4c\int_0^y (u+1)\,du = 4c\left[\tfrac{1}{2}u^2+u\right]_0^y$$

$$= 4c\left(\tfrac{1}{2}y^2+y\right), \text{ pro } c=\tfrac{1}{6} \text{ tedy } F_Y(y) = \begin{cases} \underline{\underline{\tfrac{1}{3}y^2+\tfrac{2}{3}y}} & \text{pro } y \in \langle 0;1\rangle \\ 1 & \text{pro } y > 1 \\ 0 & \text{pro } y < 0 \end{cases}$$