

ELEC6089 High Voltage Insulation Systems Assignment 1

HV AC 275kV Bushing Design

Thomas J. Smith, David Mahmoodi, Brendan Hickman, Patrick P. L. Fong

University of Southampton

March 8, 2014

Abstract

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Hello

1 Introduction

The design of electrical equipment always involves an aspect of insulation design. For the safe and efficient operation of electrical equipment it is necessary to have an electrical circuit and a means of isolating this circuit from the surrounding environment [1]. Power systems contain a complex structure of generators, transmission lines, transformers, switchgear and more. All of these different devices require an appropriately selected insulation material in order to isolate the mechanical casings and support structures from the high voltage components [2].

The purpose of this report is to describe the design and simulation of a high voltage bushing. Bushings are an integral part of power system insulation. IEEE standard C57.19.00 describes a bushing as “an insulating structure, including a through conductor or providing a central passage for such a conductor, with provision for mounting on a barrier, conducting or otherwise, for the purpose of insulating the conductor from the barrier and conducting current from one side of the barrier to the other.” [3]. Bushings are required for situations such as connecting the external conductor to the internal windings of a transformer through the walls of the metal oil tank. The walls of the transformer housing will be grounded, but need to be shielded from the incoming high voltage conductor, hence the use of an insulating bushing [1]. An example of this application can be seen in figure 1, as 400kV grid conductors enter an oil filled transformer casing. The shedding on the outer cylinder can be seen in figure 1 which helps increase electrical strength in wet conditions [1].



Figure 1: High Voltage Bushings on the 400kV Transformers at Staythorpe CCGT Power Station, Newark, UK (Taken by TJS)

2 Overview of Grading Methods

Electric field stress control is important in the design of many power system elements, especially cable terminations and bushings [2]. Failure of a bushing can damage the power transformer it is protecting, which can be an expensive mistake [1]. Bushings are required to withstand Electrical, Mechanical and Thermal stresses as defined in the IEEE standard C57.19.00 [3]. The design of the bushing is largely determined by the insulation material chosen and the resolution of these conflicting sources of stress. A good bushing design has insulation that can withstand the applied voltage and thermal characteristics appropriate for the current carried by the conductor [4].

The problem grading methods attempt to resolve is laid out in figure 2. The grounded transformer casing is shown in light grey which is perpendicular to the the bushing insulation shown in dark grey and the high voltage conductor in white. The top of the bushing is exposed to air, while the other side is exposed to transformer oil. Conducting a numerical analysis or simulation would show that the conductor surface within the plane of the transformer casing and at the points marked by red crosses would experience high electric field stress. The bushing insulation is designed to withstand the high electric field between the conductor and the transformer casing, however at the points marked with crosses the interface between the solid insulation and the air/transformer oil would cause surface discharge leading to relatively low flashover voltages [5]. It is therefore necessary to develop methods of reducing electric field stress to a more uniform distribution for both functional purposes and the economic use of space and materials [2].

2.1 Low Voltage and DC Solutions

There are several methods that can be used dependant upon the application. Low voltage solutions include internal and external screening electrodes, while resistive stress control can be used for DC applications.

External screening electrodes are parts outside the conductor and are not electrically connected to the conductor. Some are intended to reduce the electric field strength around the bushing terminal, hence reducing the chance of corona or partial discharge and others are intended to reduce the potential

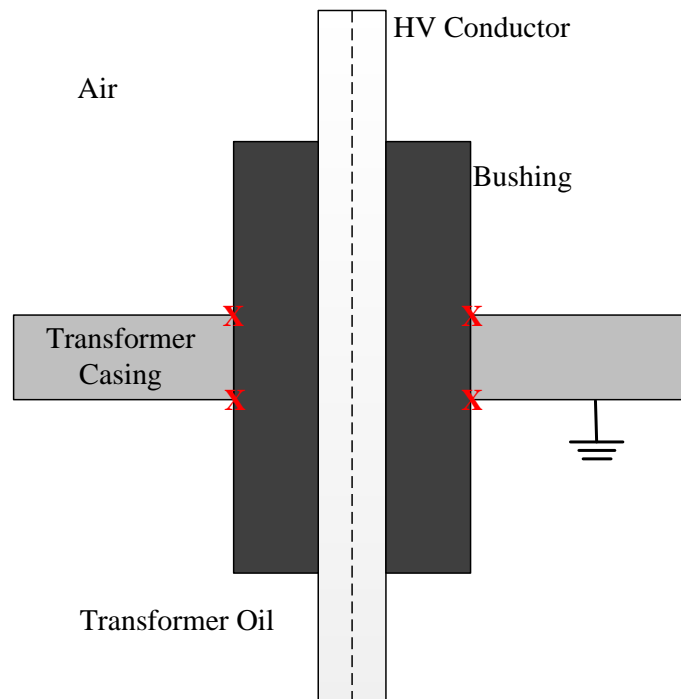


Figure 2: The Bushing Problem

gradient of insulator, hence reducing the chance of electrical breakdown. These screening electrodes come with various shapes according to the different designs. The main types of design take the shape of sphere, toroid or ring, these are shapes which prevent regions of intense electric field and help contain electric field as much as possible. The reduction in corona discharge not only reduces the power loss, it also suppresses the speed of ageing of insulator.

Internal screening electrodes are also used to

<http://www.docstoc.com/docs/144150876/Gas-insulated-high-voltage-bushing-with-shield-electrode-embedded-in-an-annular-insulating-body>

TS - I can't find any good references for these types, only his notes and the hst.tu notes seem to cover it and then it is just lecture slides, not a credible resource like a book or something

2.2 Capacitive Grading

Capacitive grading was first proposed by R.Nagel of Siemens in a German paper published in 1906 [4]. The value of this type of arrangement was quickly recognised, and is now industry standard practice for AC bushing designs for 25kV - 1500kV applications [2]. The general concept of the design is illustrated in figure 3, showing the isolated foils inserted inside the solid bushing insulation. Shown in red in figure 3 is the potential field with no grading, and in blue with the isolated conductive foils

inserted. It shows that the whole dielectric is much more evenly stressed with the capacitive grading method.

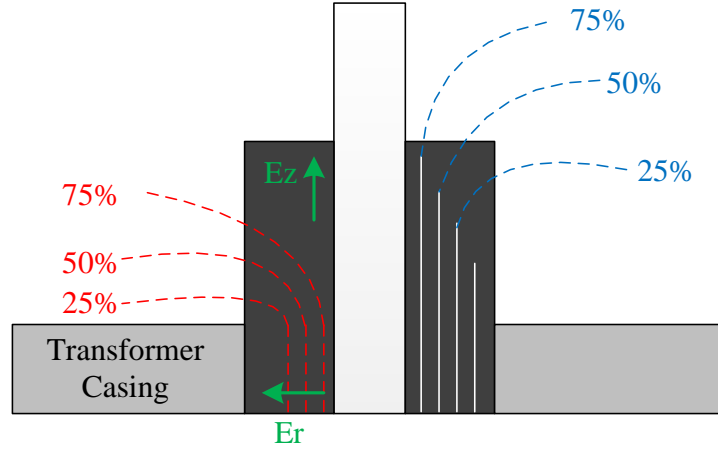


Figure 3: Field Distribution both without capacitive grading (shown in red) and with capacitive grading (shown in blue), modified from [2]

The insulation is stressed in both a radial and axial direction, which sum to give the tangential field. The radial component E_r can cause breakdown of the insulating material, while the axial component E_z can cause surface discharge along the boundary [6]. Attention must be paid to the design and shape of the boundary, so that the critical value for inception voltage for surface discharge is not exceeded []. These can be seen in green in figure 3. These sum up to give the tangential field E_t .

Before proceeding, it is first necessary to introduce some terms. Firstly, the radius of the foil is

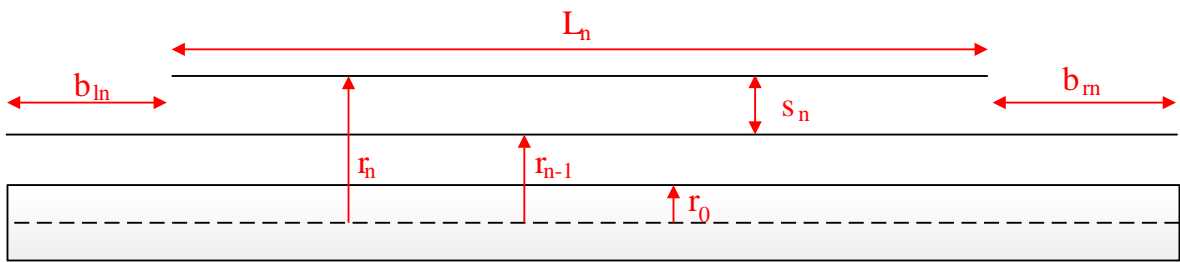


Figure 4: Symbols for calculating capacitive grading, modified from [6]

referenced from the centre of the conductor, and termed r_n . The spacing between each foil is defined in equation 1.

$$S_n = r_n - r_{n-1} \quad (1)$$

Additionally, the length of each foil is referred to as L_n and the difference in length on the right and left side between each foil is termed b_{ln} and b_{rn} . Symmetric double sided capacitive grading is achieved when $b_{ln} = b_{rn}$ [6]. The total number of foils in the system is N . Also note that subscript n denotes the outermost foil.

Inserting isolated conducting foils forms a set of coaxial capacitor units [5]. The equation for the capacitance of one of these capacitors depends on the radial displacement r_n and length of each foil L_n , as in equation 2.

$$C_n = \frac{2\pi\epsilon L_n}{\ln\left(\frac{r_n}{r_{n-1}}\right)} \quad (2)$$

The most widely used method to choose the dimensions and locations of the foils is double sided capacitive grading, of which there are two variants; radial grading and axial grading [6]. The aim of capacitive grading is to evenly distribute the electric field between the foils. To achieve this, an even voltage difference between foils is required as in equation 3, where V is the total voltage difference between the conductor and the casing, N is the number of foils required and ΔV is the voltage between each foil [].

$$\Delta V = \frac{V}{N} \quad (3)$$

For the voltage between each foil to be constant, as in equation 3, the capacitance between each consecutive pair of foils must also be constant. This is expressed as $C_n = C_{n-1} = \dots = C_0$

2.2.1 Radial Grading

The radial spacing and dimension of each foil is determined in the following derivation, which has been verified and modified from [5]. In radial grading, the radial component of the electric field E_r is kept constant between all the foils. The radial electric field is related to the voltage difference and the spacing between each foil, as in equation 4. ΔV is already defined as a constant from equation 3, and so to have equal field the foil spacing S_n should also be constant.

$$E_r = \frac{\Delta V}{S_n} = \text{Constant} \quad (4)$$

Given this condition and equation 2 for coaxial capacitance, the length of each foil is required to change from foil to foil. The lengths and radii of consecutive foils can be calculated from the relationship in equation 5.

$$C_n = \frac{2\pi\epsilon L_n}{\ln\left(\frac{r_n}{r_{n-1}}\right)} = C_{n-1} = \frac{2\pi\epsilon L_{n-1}}{\ln\left(\frac{r_{n-1}}{r_{n-2}}\right)} = \dots = C_1 = \frac{2\pi\epsilon L_1}{\ln\left(\frac{r_1}{r_0}\right)} \quad (5)$$

The common factor of $2\pi\epsilon$ cancels from equation 5 giving a simple equation linking the lengths and radial displacements of consecutive foils, as in equation 6.

$$\frac{L_n}{\ln\left(\frac{r_n}{r_{n-1}}\right)} = \frac{L_{n-1}}{\ln\left(\frac{r_{n-1}}{r_{n-2}}\right)} = \dots = \frac{L_1}{\ln\left(\frac{r_1}{r_0}\right)} \quad (6)$$

An approximate solution for thin foils can then be found. Under the thin foil assumption, $r_n = r_{n-1} + S_n$ and $\frac{S_n}{r_n} \ll 1$ even for the smallest radii of the inner foil. This is shown in equation 8.

$$\ln\left(\frac{r_n}{r_{n-1}}\right) = \ln\frac{1}{1 - \left(\frac{S_n}{r_n}\right)} \approx \frac{S_n}{r_n} \quad (7)$$

$$L_n r_n \approx L_{n-1} r_{n-1} \approx \dots \approx L_0 r_0 \quad (8)$$

Equation 6 can then be used to determine an exact solution while equation 8 can be used to find an approximate solution in conjunction with initial data regarding the length and radial displacement of the first foil and the spacing of the foils to calculate the parameters of all the other foils in the bushing.

2.2.2 Axial Grading

In axial grading, the axial component of the electric field E_z is kept constant between all of the foils. The following equations prove that the length of each foil must decay by a constant value for each consecutive foil, and the radius at which it is placed is determined by a simple iterative formula.

The axial electric field is related to the voltage difference and the length change between each consecutive foil as in equation 9. Under symmetric capacitive grading, $b_n = b_{ln} = b_{rn}$ with reference to figure 4. ΔV is already defined as a constant from equation 3, and so to have equal field, the change in foil length b_n should also be constant.

$$E_z = \frac{\Delta V}{b_n} = Constant \quad (9)$$

The relationship between L_n and b_n is defined in figure 4, as explained in equation 10.

$$L_n = L_{n-1} - 2b_n \quad (10)$$

Since equation 9 requires the change in foil length b_n to be constant, equation 2 for coaxial capacitance requires the radius of each foil to change from foil to foil. This can be simplified to a similar form as equation 6, except that the initial information required is the radius and length of the first foil, and the constant difference in length between each of the foils. All other lengths and radii can then be calculated.

3 Design Details

The reference model for this project is shown in figure 5. The reference design is a paper impregnated with oil bushing with 21 aluminium foils of $100\mu m$. One side of the bushing is exposed to air, the other to oil, similar to a transformer bushing. The diameter of the conductor is 100mm, the bushing diameter is 300mm. The length of the first foil is 5000mm long, and fixed 2mm into the bushing at the conductor voltage. The outer foil is also set 2mm inside the bushing and is directly connected to the earthed flange. The conductor is used at 275kV AC voltage, and the design was taken from a bushing that was in operation for around 30 years.

3.1 Design Issues

In section 2.2 the initial information required for both radial and axial grading includes the length and radial displacement of the innermost foil. In the reference design the following initial information is given.

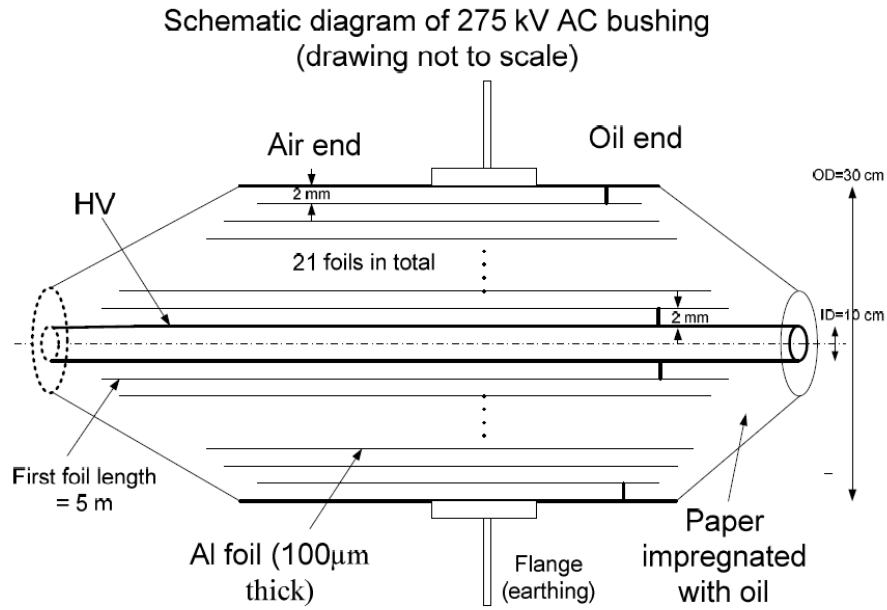


Figure 5: The reference problem taken from [7]

Table 1: Initial Information for Reference Design

Initial Information	Value)
Conductor Diameter (ID)	100mm
First Foil Length L_1	5000mm
First Foil Radius r_1	52mm
Outer Bushing Diameter (OD)	300mm
Outer Foil Radius r_{21}	148mm

This information intuitively fits radial grading best, since there is no requirement to assume the length of the outermost foil. However, there is a discrepancy between the standard literature problem and the reference design. The first foil is connected to the high voltage conductor, and the last foil is connected to the earthed flange. This is to eliminate the electric field on the boundary interface as far as possible on both sides of the bushing, so that the voltage drop occurs exclusively inside the bushing insulation.

This has an impact on the calculations described in section 2.2. Since the innermost foil is at the same voltage as the conductor, there is no capacitance between them, as shown in figure 6, hence the first foil shown on the diagram is not the first index for the iterative calculation. The derivation of the iterative equation assumes a capacitance between each foil. The first foil in figure 5 is therefore indexed as 0 and not 1.

This means that there is not sufficient initial information to proceed, since the first non-connected foil length L_1 is not given. The iterative equations require the first non-connected foil length L_1 to be known as shown in equation 11. All radii variables are known due to the even spacings under radial grading.

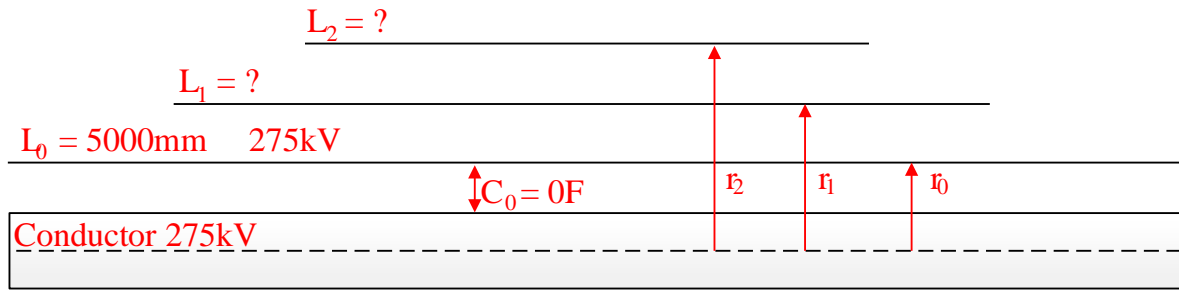


Figure 6: Diagram to explain the assumptions required

$$\frac{L_1}{\ln\left(\frac{r_1}{r_0}\right)} = \frac{L_2}{\ln\left(\frac{r_2}{r_1}\right)} \quad (11)$$

$$L_2 = L_1 \frac{\ln\left(\frac{r_2}{r_1}\right)}{\ln\left(\frac{r_1}{r_0}\right)} \quad (12)$$

If this is not taken into account then a flawed design will be produced. Equation 13 shows a wrongly described first iteration of the radial grading formula which results in the design in figure 7. This is clearly wrong, and does not give the hyperbolic shape from the beginning of the foils.

$$L_1 = 5000mm \frac{\ln\left(\frac{56.8mm}{52mm}\right)}{\ln\left(\frac{52mm}{50mm}\right)} = 11256mm \quad (13)$$

In order to proceed with the calculations there must be an assumption of the length of the first unconnected foil. A reasonable assumption is that this follows the hyperbolic shape of the other foils, which helps to evenly distribute the electric field. To achieve this design mathematically, two assumptions are made.

1. Foil 0 is not connected to the HV conductor.
2. The conductor surface is spaced a distance of S_n from foil 0.

Assumption 1 is required to be able to use the capacitor derived iterative formula on foil 0. Assumption 2 is required so that the radial spacing is kept constant. The first iteration has been calculated under these assumptions, giving the result in equation 14 which is an expected value. The remainder the foil parameters can then be calculated using the iterative formula.

$$L_1 = 5000mm \frac{\ln\left(\frac{56.8mm}{52mm}\right)}{\ln\left(\frac{52mm}{47.2mm}\right)} = 4558mm \quad (14)$$

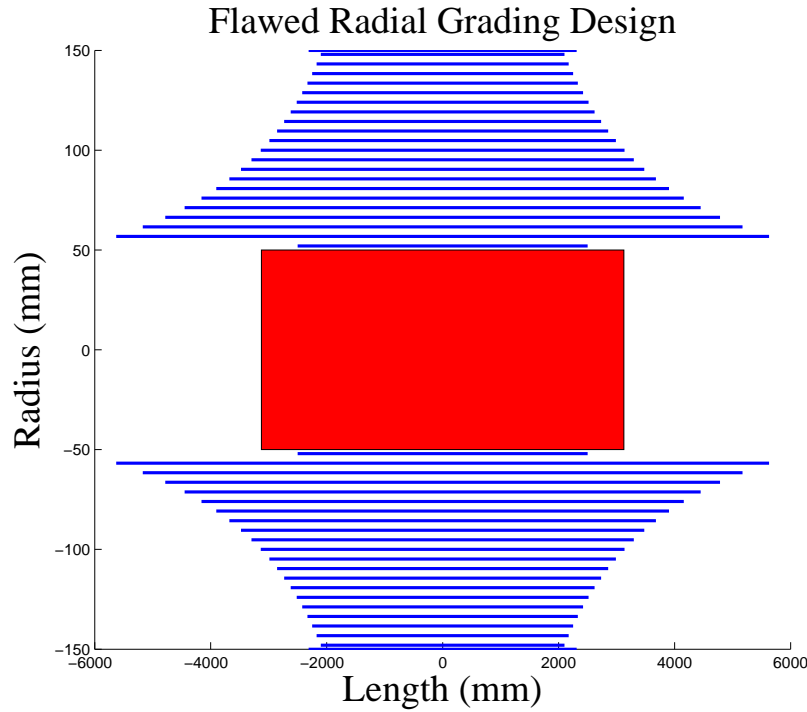


Figure 7: Flawed Radial Grading Design

3.2 Matlab Calculations

A Matlab script was developed to take a required number of foils, and the inner and outer dimensions of the bushing, to calculate the radial location and length of each foil using the radial grading method as described in section 2.2. This script was built to be easily customisable for any number of foils and any initial values, to cater for the calculation of improved designs. It automatically outputs data in a form for direct input into the COMSOL model, and auto-updates a \LaTeX file containing the data in table 2. Finally, the script plots the calculated foil positions in a graph shown in figure 8. Particularly in figure 8(a) the hyperbolic shape can be observed as expected. This allows a quick verification of the scripts accuracy before proceeding to simulation.

DM - To change the figures for the nice ones, remember .eps, times titles that are big enough.

The final information required to be able to proceed to the simulation phase is the relative permittivity of each material. This was gathered from [6] and is shown in table 3.

4 Modelling Results

The following simulations were completed using the COMSOL multiphysics software package. COMSOL uses finite element methods to solve the the laplacian electric field partial differential equation.

Table 2: Radial Grading Calculations Results

Radius(mm)	Length(mm)
52.00	5000.00
56.80	4558.22
61.60	4188.21
66.40	3873.79
71.20	3603.30
76.00	3368.13
80.80	3161.78
85.60	2979.27
90.40	2816.68
95.20	2670.92
100.00	2539.51
104.80	2420.43
109.60	2312.01
114.40	2212.90
119.20	2121.93
124.00	2038.15
128.80	1960.73
133.60	1888.98
138.40	1822.29
143.20	1760.16
148.00	1702.12
150.00	851.06

Table 3: Relative Permittivity of Materials

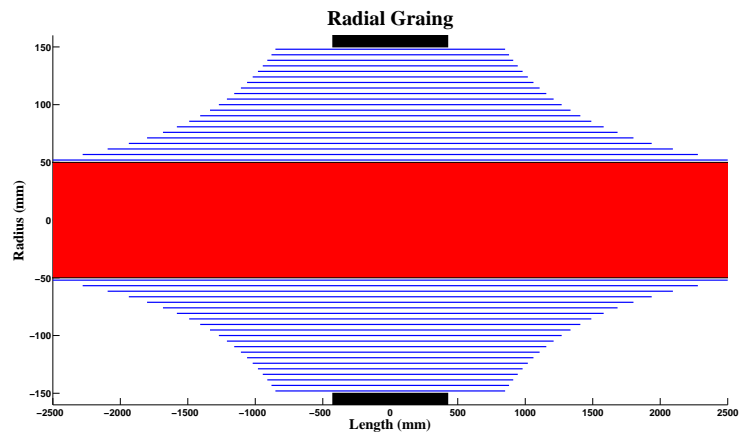
Material	Relative Permittivity (ϵ_r)
Air	1
Oil	2.2
Paper Impregnated with Oil	4
Aluminium	10^8

4.1 No Grading

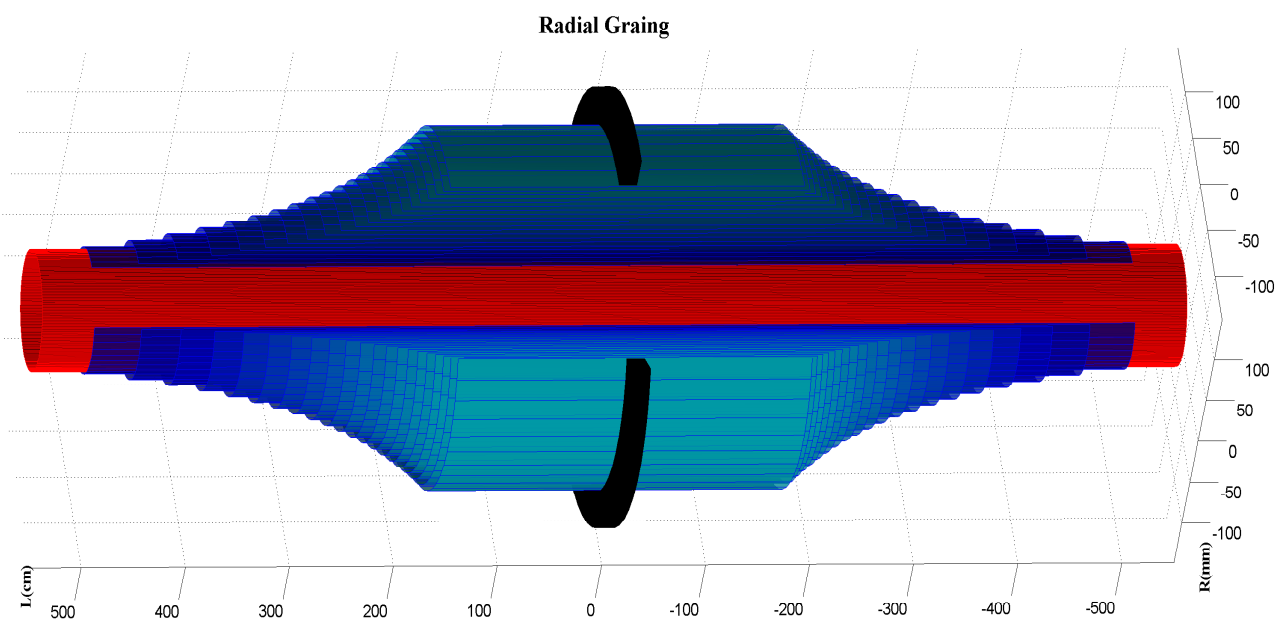
As a baseline for comparison, a bushing with no foils has been constructed and simulated. The geometry of the model was built as in figure 9. The system is an axialsymmetric 2D model, which takes the central vertical point $r = 0$ as the centre of a cylinder.

Once the geometry of the model is defined, a finite element mesh can be created as shown in figure 10. This model is fairly simple, hence a very fine graded mesh was used improving the accuracy of results.

The next stage is to define the relative permittivity of each of the materials used for each sub section of the geometry. The initial conditions must then be set, with the conductor set to 275kV, and the



(a) Wide View



(b) Real Perspective

Figure 8: Representation of foil radial position and length

transformer wall and all outer boundaries earthed. All other boundaries are assumed to be continuity boundaries.

The model can then be solved to give the electric field distribution

TS - Report done up to here 03/03/2014

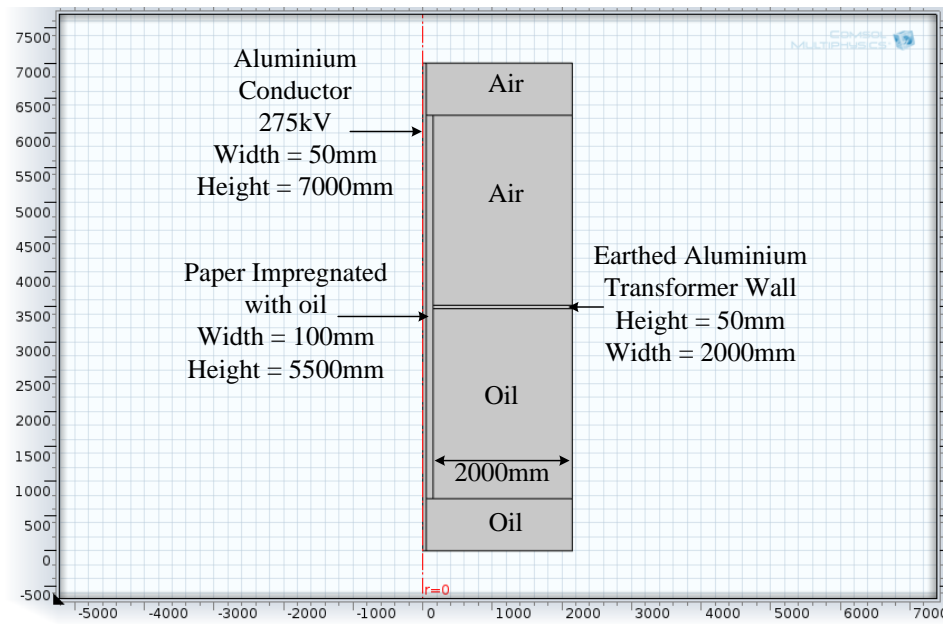


Figure 9: COMSOL Geometry Annotated with Materials - No Grading

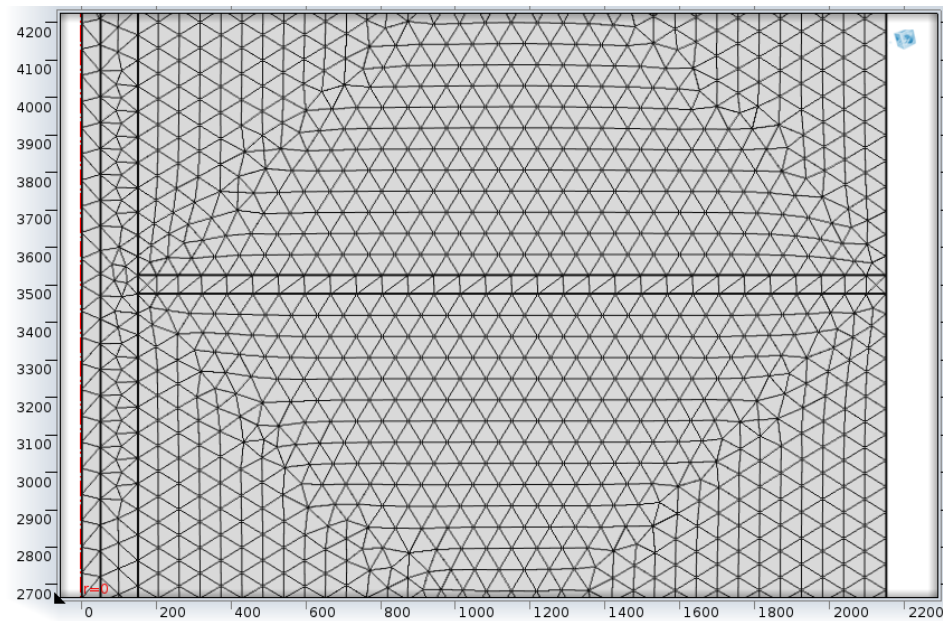


Figure 10: COMSOL Mesh - No Grading

5 Discussion of Results

Comparison and discussion (Suggestions on improvement).

6 Conclusions

Conclusions.

References

- [1] D.F. Warne. *Newnes Electrical Power Engineer's Handbook*. Elsevier Science, 2005.
- [2] R.E. James, Q. Su, and Institution of Engineering and Technology. *Condition Assessment of High Voltage Insulation in Power System Equipment*. IET power and energy series. Institution of Engineering and Technology, 2008.
- [3] Ieee standard general requirements and test procedure for power apparatus bushings. *IEEE Std C57.19.00-2004 (Revision of IEEE Std C57.19.00-1991)*, pages 1–17, 2005.
- [4] J.H. Harlow. *Electric Power Transformer Engineering*. The Electric Power Engineering Hbk, Second Edition. Taylor & Francis, 2004.
- [5] J. Kuffel, E. Kuffel, and W.S. Zaengl. *High Voltage Engineering Fundamentals*. Elsevier Science, 2000.
- [6] Zeeshan Ahmed. Analysis of partial discharge in OIP bushing models. Master's thesis, Royal Institute of Technology (KTH), September 2011.
- [7] George Chen. ELEC6089 High Voltage Insulation Systems Assignment 1 HV AC 275kV Bushing Design Briefing Notes, February 2014. The University of Southampton.

A Individual Contributions

Team Member	Contribution
Thomas J. Smith 23914254	-
David Mahmoodi 99999999	-
Brendan Hickman 99999999	-
Patrick P. L. Fong 99999999	-

B Meeting Minutes

B.1 Meeting 1 - Kick-off Meeting

Purpose	ELEC6089 Bushing Design Kick Off Meeting
Date and Time	Thursday 20th February 13:30
Venue	GDP Lab Zepler Building, Highfield Campus
Participants	TS (Thomas Smith), DM (David Mahmoodi), BH (Brendan Hickman), PF (Patrick Fong)
Apologies	None
Agenda	Review what we understand of the project so far. Understand the tasks required. Agree expectations of work and schedule. Agree date and agenda of next meeting.

B.1.1 Minutes of the Meeting

ID	Subject	Notes and Discussion	Action
1.0	Research prior to the meeting	BH uploaded the course text to the Facebook working group which has a section on stress control by floating screens. TS uploaded a project from KTH university that had similar guidelines and had a useful description to compound the lecture notes for the module. All agreed to research the topic further and read these sections by the next meeting	ALL A1.0
2.0	Current understanding of task	The group discussed the task at hand. We need to design the bushing using the iterative formulas from the lectures and then build a COMSOL model. The design must be either radial or axial in grading method.	-
3.0	Work Breakdown	The group tried to identify the work to complete. This includes research into field design and grading methods, calculating the bushing design, simulating and report writing. None of these tasks can be completed in parallel, and all need the previous in order to complete the task. Hence each member needs to research, and have knowledge of the design and simulation process. It will become clearer who will be assigned responsibility for what shortly. Currently, remain with all needing to complete research	-
4.0	Next Meeting	First meeting with G. Chen in 2 weeks, Tuesday 4th March. Before then have a first model and have begun verification. Have group Latex template for collaboration, good layout and presentation marks. Use Github. Next meeting on Wednesday 26th.	-

B.1.2 Action List

ID	Action	Comments	Status
A1.0	Research	All to start research. Make notes of all sources. At least reviewed the lecture notes and Kuffel.	Open 20th Feb

Next Meeting: 26th Feb 2014, Location & Time TBA

B.2 Meeting 2 - Progress Meeting

Purpose	ELEC6089 Bushing Design Progress Meeting
Date and Time	Wednesday 26th February 11:30
Venue	GDP Lab Zepler Building, Highfield Campus
Participants	TS (Thomas Smith), DM (David Mahmoodi), BH (Brendan Hickman)
Apologies	PF (Patrick Fong)
Agenda	Review research progress. Clarify project understanding. Start design task. Identify further work.

B.2.1 Minutes of the Meeting

ID	Subject	Notes and Discussion	Action
1.0	Research update	The present team members discussed the task in the context of Kuffel and KTH research. Agreed on bushing definitions and the theory behind capacitive grading. Also took time to verify that the lecture notes matched the explanation in Kuffel. Kuffel pages are 235-241. Also discussed why the capacitors were added, and established the iterative formula to use. All should continue to gain a firmer grounding of the required theory	ALL A1.0
2.0	Github and \LaTeX	TS ran the present through the report template, what was required and how to use the distributed revision control system Git as hosted on GitHub. This should make collaboration much easier than using just our facebook group page.	-

ID	Subject	Notes and Discussion	Action
3.0	Grading Methods	DM left the meeting at this point to read the lecture notes. DA will also perform the grading and we can then use this to independently verify the design. TS and BH started on axial grading method. Both wrote matlab code to calculate spacings. The results were the same, hence reasonable level of confidence of validity.	PF & DM A2.0
4.0	Remaining work	BH and TS identified the remaining work for actioning. The report has an introduction which requires review. Sections on Grading methods (why grade? LV solutions using electrodes, DC solution using resistivity, AC capacitive grading), AC grading types (discussion of axial and radial components of tangential fields, radial and axial derivation) and section on the design details (iterative formula, Matlab calculations, visio diagrams). The design must be built in COMSOL which represents significant work to understand COMSOL. Probably want to simulate a non-graded bushing as a baseline for discussion. Aiming to do both radial and axial grading simulations. Then discuss.	-
5.0	Assignment of work	BH and PF have a key deadline on tuesday 4th March hence largely unavailable until then. TS and DM to get started on tasks. Try and get simulations done before meeting with GC.	TS & DA A3.0 A4.0
4.0	Next Meeting	First meeting with G. Chen Tuesday 4th March. Before then have a first model and have begun verification. Next meeting on Prior to this meeting.	-

B.2.2 Action List

ID	Action	Comments	Status
A1.0	Research	All to start research. Make notes of all sources. At least reviewed the lecture notes and Kuffel.	Open 20th Feb
A2.0	Grading	Other members to perform axial grading calculations seperately so that the results can be verified independently	Open 26th Feb
A3.0	COMSOL	Gain an understanding of COMSOL and attempt some simulations.	Open 26th Feb
A4.0	Reporting	Continue to document progress in the report.	Open 26th Feb

Next Meeting: 4th March 2014, Preceeding meeting with GC

C Code Listings

Radial Grading Matlab Code

```

clc
close all
clear all

%%
% Declareing Given Variables
Voltage = 275000;
Inner_diameter = 100;
Outer_diameter = 300;
First_foil_length = 5000;
N = 21;
Foil_Thickness = 0.1;
First_Gap = 2;
Last_Gap = 2;

%%
% Defining new variables
Del_Voltage = Voltage/N-1; %Voltage between each foil
Del_Radius = ((Outer_diameter- Inner_diameter)-2*(First_Gap+Last_Gap))/(2*(N-1)); % Spacing between each f
%Initialise vectors (22 for 3D plotting)
L = zeros(1,N+1);
Radius = zeros(1,N+1);

%%
% Calculation
L(1)=First_foil_length; %The forst foil is 5000mm and connected to the conductor, no capacitance between c
r0 = Inner_diameter/2; %Radius of the conductor
Radius(1)= Inner_diameter/2 + First_Gap; % Radial position of first foil = 52mm

%Calculate the radial positions of all foils
for i=2:N
    Radius(i)=Radius(i-1) + Del_Radius;
end

% Refer to Section 2.2 for an explanation of this assumption
L(2)= log(Radius(2)/Radius(1))* L(1) / log(Radius(1)/(Radius(1)-Del_Radius)) ;

%Follow the iterative formula
for i=3:N
    L(i)= log(Radius(i-1)/Radius(i))* L(i-1) / log(Radius(i-2)/Radius(i-1));
end

%For plotting - add the outer shell
L(N+1)=L(N)-.5*L(N);
Radius(N+1)=Radius(N)+ Last_Gap;

%%
% Ploting
x=zeros(1,2*(N+1));
y=x;
j=1;
for i=1:2:2*(N+1)
    x(i)=L(j)/2;
    x(i+1)=-L(j)/2;
    y(i)=Radius(j);
    y(i+1)=Radius(j);
    j=j+1;
end
y2=-y;

% 2D Plot
figure
axes('FontSize',16,'fontWeight','bold')
rect_H = rectangle('Position', [-1.1.*x(1),- r0, 2.2*x(1), 2*r0]);
set(rect_H, 'FaceColor', 'r')

for i=1:2:2*(N)

```

```

    hold on
    line(x(i:i+1), y(i:i+1), 'LineWidth',2)
    line(x(i:i+1), y2(i:i+1), 'LineWidth',2)
    %axis equal
end
rect_H = rectangle('Position', [-L(N+1)/2,Outer_diameter/2, L(N+1), 10]);
set(rect_H, 'FaceColor',[0, 0, 0])
rect_H = rectangle('Position', [-L(N+1)/2,-10-Outer_diameter/2, L(N+1), 10]);
set(rect_H, 'FaceColor',[0, 0, 0]);
title('Radial Graing','FontName','Times New Roman','FontSize',34,'fontWeight','bold');
xlabel('Length (mm)','FontName','Times New Roman','FontSize',24,'fontWeight','bold')
ylabel('Radius (mm)','FontName','Times New Roman','FontSize',24,'fontWeight','bold')
xlim([x(2) x(1)]);ylim([y2(end)-10 y(end)+10])

% 3D Plot
K=50; scl=.1; % Z direction scalling value for plotting
p= 6*N/3+1; %adjusting the Cut in the 3D shap
figure
axes('FontSize',16,'fontWeight','bold');
R=[r0 r0];
[X,Y,Z] = cylinder(R,5*K);
Z(2,:) = ( L(1)+ .1*L(1))*scl;
Z(1,:) = - Z(2,:);
surf(X,Y,Z, 'FaceColor',[1,0,0],'EdgeColor',[1,0,0]);
for i=1:N
    hold on
    R=[Radius(i) Radius(i)];
    [X,Y,Z] = cylinder(R,K);
    Z(2,:)= L(i);
    Z(1,:)= -L(i);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    testsubject = surf(X,Y,Z);
    set(testsubject, 'FaceAlpha',0.8, 'EdgeColor','b')
    axis equal
end
Ground=Radius(N+1)-1;
for i=1:30
    R=[Ground+i Ground+i];
    [X,Y,Z] = cylinder(R,K);
    Z(2,:) = L(N+1)*scl;
    Z(1,:) = - Z(2,:);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    surf(X,Y,Z, 'FaceColor',[0,0,0],'EdgeColor',[0, 0, 0]);
end
camlight
lighting gouraud
title('Radial Graing','FontName','Times New Roman','FontSize',24,'fontWeight','bold');
xlabel('R(mm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','Rotation',90,'HorizontalAlign','right');
ylabel('L(cm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','HorizontalAlign','right');

% Saving results to file
FID = fopen('RadialVals21.tex', 'w');
fprintf(FID, '\\begin{table}[!htb]\n');
fprintf(FID, '\\caption{Radial Grading Calculations Results}\n');
fprintf(FID, '\\label{table:radialvals}\n');
fprintf(FID, '\\begin{center}\n');
fprintf(FID, '\\begin{tabular}{cc}\n');
fprintf(FID, '\\toprule\n');
fprintf(FID, '\\textbf{Radius(mm)} & \\textbf{Length(mm)} \\\\ \\toprule\n');
for i=1:N+1
    fprintf(FID, '%4.2f & %4.2f \\\\\\n', Radius(i), L(i));
end
fprintf(FID, '\\bottomrule\n');
fprintf(FID, '\\end{tabular}\n');
fprintf(FID, '\\end{center}\n');
fprintf(FID, '\\end{table}\n');

```

```
fclose(FID);
```

Axial Grading Matlab Code

```
clc
close all
clear all
%%
% Declareing Given Variables
Voltage = 275000;           %Applied voltage (volt)
Inner_diameter = 100;       %mm
Outer_diameter = 300;       %mm
First_foil_length = 5000;   %mm
N = 21;
Foil_Thickness = 0.1;       %mm
First_Gap = 2;              %mm
Last_Gap = 2;               %mm

E_boundary_surface_Air = 300; %volt/mm
E_boundary_surface_Oil = 3*300; %volt/mm

%%
% Defining new variables
Del_Voltage = Voltage/N-1; %Voltage between each foil
b_Air = Del_Voltage/E_boundary_surface_Air;
b_Oil = Del_Voltage/E_boundary_surface_Oil;
L_Air = b_Air * (N-1);
L_Oil = b_Oil * (N-1);

% Del_Radius = ((Outer_diameter- Inner_diameter)-2*(First_Gap+Last_Gap))/(2*(N-1)); % Spacing between each
%Initialise vectors (22 for 3D plotting)
L = zeros(1,N+1);
L_Air = zeros(1,N+1);
L_Oil = zeros(1,N+1);
Radius = zeros(1,N+1);
R_parameter=1.007; % Parameter for adjesting assumption value of r0

%%
% Calculation
L(1)=First_foil_length; %The forst foil is 5000mm and connected to the conductor, no capacitance between c
r0 = Inner_diameter/2 ; %Radius of the conductor
Radius(1)= Inner_diameter/2 + First_Gap; % Radial position of first foil = 52mm
L_Air(1)= L(1)/2;
L_Oil(1)= L(1)/2;
%Calculate the radial positions of all foils
for i=2:N
    L_Air(i)=L_Air(i-1)- b_Air;
    L_Oil(i)=L_Oil(i-1)- b_Oil;
    L(i)= L_Air(i)+L_Oil(i);
end

% Refer to Section 2.2 for an explanation of this assumption
Radius(2)= Radius(1)* exp( (L(2)/L(1)) * log(Radius(1)/(r0-R_parameter)));

%Follow the iterative formula
for i=3:N
    Radius(i)= Radius(i-1)* exp( (L(i)/L(i-1)) * log(Radius(i-1)/Radius(i-2)));
end

%For plotting - add the outer shell
L(N+1)=L(N)- .5*L(N);
Radius(N+1)=Radius(N)+ Last_Gap;

%%
% Ploting
x=zeros(1,2*(N+1));
y=x;
j=1;
for i=1:2:2*(N+1)
    x(i)=L_Air(j);
    x(i+1)=-L_Oil(j);
```

```

        y(i)=Radius(j);
        y(i+1)=Radius(j);
        j=j+1;
    end
    y2=-y;

% 2D Plot
figure
axes('FontSize',16,'fontWeight','bold')
rect_H = rectangle('Position', [-1.1.*x(1),- r0, 2.2*x(1), 2*r0]);
set(rect_H, 'FaceColor', 'r')

for i=1:2:2*(N)
    hold on
    line(x(i:i+1), y(i:i+1), 'LineWidth',2)
    line(x(i:i+1), y2(i:i+1), 'LineWidth',2)
    %axis equal
end
rect_H = rectangle('Position', [-L(N+1)/2,Outer_diameter/2, L(N+1)/2, 1000]);
set(rect_H, 'FaceColor',[0, 0, 0])
rect_H = rectangle('Position', [-L(N+1)/2,-10-Outer_diameter/2, L(N+1)/2, 10]);
set(rect_H, 'FaceColor',[0, 0, 0])
title('Axial Graing','FontName', 'Times New Roman','FontSize',34,'fontWeight','bold');
xlabel('Length (mm)','FontName', 'Times New Roman','FontSize',24,'fontWeight','bold')
ylabel('Radius (mm)','FontName', 'Times New Roman','FontSize',24,'fontWeight','bold')
xlim([x(2) x(1)]);ylim([y2(end)-10 y(end)+10])

% 3D Plot
K=50; scl=.1; % Z direction scalling value for plotting
p= 6*N/3+1; %adjusting the Cut in the 3D shap
figure
axes('FontSize',16,'fontWeight','bold')
R=[r0 r0];
[X,Y,Z] = cylinder(R,5*K);
Z(2, :) = (L_Air(1)+ .1*L_Air(1))*scl;
Z(1, :) = - Z(2, :);
surf(X,Y,Z, 'FaceColor',[1,0,0],'EdgeColor',[1,0,0]);
for i=1:N
    hold on
    R=[Radius(i) Radius(i)];
    [X,Y,Z] = cylinder(R,K);
    Z(2,:)= L_Air(i);
    Z(1,:)= -L_Oil(i);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    testsubject = surf(X,Y,Z);
    set(testsubject,'FaceAlpha',0.8,'EdgeColor','b')
    axis equal
end
Ground=Radius(N+1)-1;
for i=1:30
    R=[Ground+i Ground+i];
    [X,Y,Z] = cylinder(R,K);
    Z(2, :) = L(N+1) *scl;
    Z(1, :) = - Z(2, :);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    surf(X,Y,Z, 'FaceColor',[0,0,0],'EdgeColor',[0, 0, 0]);
end
camlight
lighting gouraud
title('Radial Graing','FontName','Times New Roman','FontSize',24,'fontWeight','bold');
xlabel('R(mm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','Rotation',90,'HorizontalAlign',
zlabel('L(cm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','HorizontalAlignment','right')

% Saving results to file
FID = fopen('AxialVals21.tex','w');
fprintf(FID, '\\begin{table}[!htb]\n');
fprintf(FID, '\\caption{Radial Grading Calculations Results}\n');

```

```
fprintf(FID, '\\label{table:radialvals}\\n');
fprintf(FID, '\\begin{center}\\n');
fprintf(FID, '\\begin{tabular}{cc}\\n');
fprintf(FID, '\\toprule\\n');
fprintf(FID, '\\textbf{Radius(mm)} & \\textbf{Length(mm)} \\ \\toprule\\n');
for i=1:N+1
    fprintf(FID, '%4.2f & %4.2f \\ \\n', Radius(i), L(i));
end
fprintf(FID, '\\bottomrule\\n');
fprintf(FID, '\\end{tabular}\\n');
fprintf(FID, '\\end{center}\\n');
fprintf(FID, '\\end{table}\\n');
fclose(FID);
```
