ELEC6089 High Voltage Insulation Systems Assignment 1
HV AC 275kV Bushing Design

Thomas J. Smith, David Mahmoodi, Brendan Hickman, Patrick P. L. Fong

University of Southampton

March 17, 2014

Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Hello aa

1 Introduction

The design of electrical equipment always involves an aspect of insulation design. For the safe and efficient operation of electrical equipment it is necessary to have an electrical circuit and a means of isolating this circuit from the surrounding environment [1]. Power systems contain a complex structure of generators, transmission lines, transformers, switchgear and more. All of these different devices require an appropriately selected insulation material in order to isolate the mechanical casings and support structures from the high voltage components [2].

The purpose of this report is to describe the design and simulation of a high voltage bushing. Bushings are an integral part of power system insulation. IEEE standard C57.19.00 describes a bushing as "an insulating structure, including a through conductor or providing a central passage for such a conductor, with provision for mounting on a barrier, conducting or otherwise, for the purpose of insulating the conductor from the barrier and conducting current from one side of the barrier to the other."[3]. Bushings are required for situations such as connecting the external conductor to the internal windings of a transformer through the walls of the metal oil tank. The walls of the transformer housing will be grounded, but need to be shielded from the incoming high voltage conductor, hence the use of an insulating bushing [1]. An example of this application can be seen in figure 1.1, as 400kV grid conductors enter an oil filled transformer casing. The shedding on the outer cylinder can be seen in figure 1.1 which helps increase electrical strength in wet conditions [1].
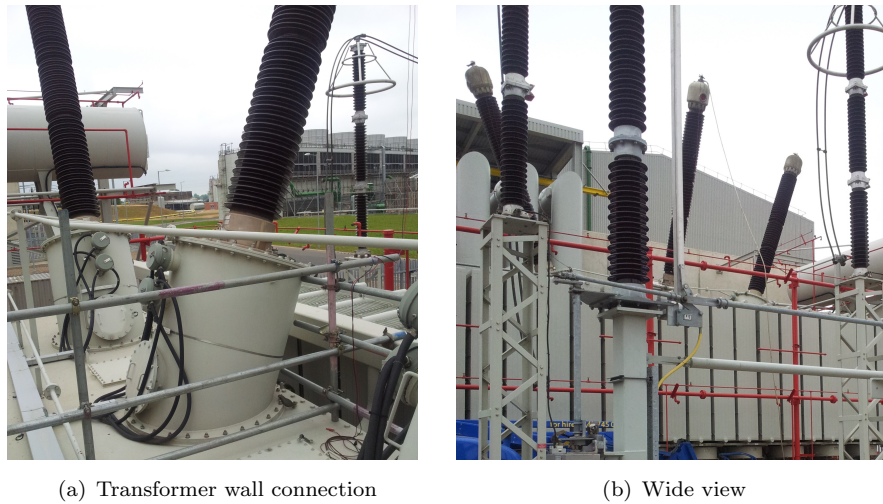
(a) Transformer wall connection   (b) Wide view

Figure 1.1: High Voltage Bushings on the 400kV Transformers at Staythorpe CCGT Power Station, Newark, UK (Taken by TJS)

2 Overview of Grading Methods

Electric field stress control is important in the design of many power system elements, especially cable terminations and bushings [2]. Failure of a bushing can damage the power transformer it is protecting, which can be an expensive mistake [1]. Bushings are required to withstand Electrical, Mechanical and Thermal stresses as defined in the IEEE standard C57.19.00 [3]. The design of the bushing is largely determined by the insulation material chosen and the resolution of these conflicting sources of stress. A good bushing design has insulation that can withstand the applied voltage and thermal characteristics appropriate for the current carried by the conductor [4].
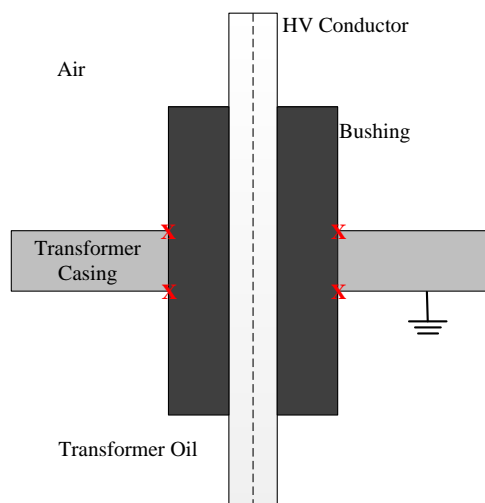

Figure 2.1: The Bushing Problem

The problem grading methods attempt to resolve is laid out in figure 2.1. The grounded transformer casing is shown in light grey which is perpendicular to the the bushing insulation shown in dark grey

and the high voltage conductor in white. The top of the bushing is exposed to air, while the other side is exposed to transformer oil. Conducting a numerical analysis or simulation would show that the conductor surface within the plane of the transformer casing and at the points marked by red crosses would experience high electric field stress. The bushing insulation is designed to withstand the high electric field between the conductor and the transformer casing, however at the points marked with crosses the interface between the solid insulation and the air/transformer oil would cause surface discharge leading to relatively low flashover voltages [5]. It is therefore necessary to develop methods of reducing electric field stress to a more uniform distribution for both functional purposes and the economic use of space and materials [2].

2.1 Low Voltage and DC Solutions

There are several methods that can be used dependant upon the application. Low voltage solutions include internal and external screening electrodes, while resistive stress control can be used for DC applications. Sometimes these solutions are used in combine to achieve the acceptable result.

2.1.1 External Screening Electrode

External screening electrodes are parts outside the conductor and are not electrically connected to the conductor, they are made of metal conductors and some designs use aluminium as the material. Corona ring designs are intended to reduce the electric field strength around the bushing terminal, hence reducing the chance of corona or partial discharge and grading ring designs are intended to reduce the potential gradient of insulator, hence reducing the chance of electrical breakdown. These screening electrodes come with various shapes according to the different designs. The main types of design take the shape of sphere, toroid or ring, these are shapes which prevent regions of intense electric field strength by varying the electric potential distribution and help contain electric field as much as possible. The reduction in corona discharge not only reduces the power loss, it also suppresses the ageing speed of the insulator. Example of these can be seen on figure 1(a) and figure 1(b), where they are placed at the top of the bushings. The diameters of these design are closely related to the electric field strength around the electrode, so the diameters of these designs must be carefully considered in order to avoid electrical breakdown.

2.1.2 Internal Screening Electrode

Internal screening electrodes are also used to control the electric potential distribution so the electric field strength is within acceptable level to avoid chance of breakdown. They are placed inside the insulator and usually in a pressurised gas. Some existing designs have the shape of internal electrode in a 3D cone like shape and maybe referred as a deflector and there are also designs which are disk shape. An example of the field distribution with the cone shape deflector inserted is demonstrated in figure 2.2.

2.1.3 Resistive Stress Control

This is a grading method of bushing involving the use of resistive component to control the distribution of electric field around a conductor
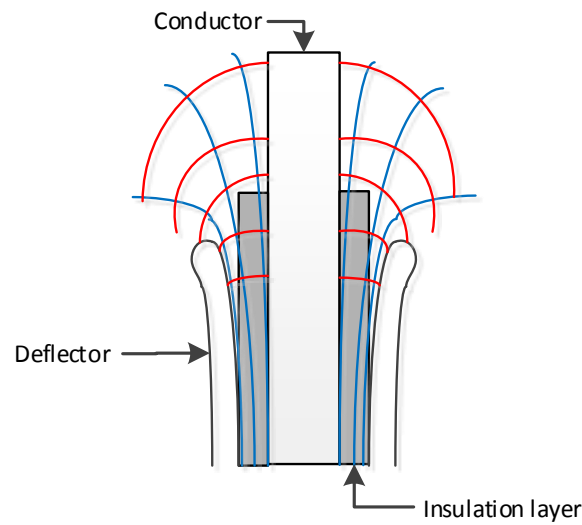
Figure 2.2: Field distribution with deflector

2.2 Capacitive Grading

Capacitive grading was first proposed by R.Nagel of Siemens in a German paper published in 1906 [4]. The value of this type of arrangement was quickly recognised, and is now industry standard practice for AC bushing designs for 25kV - 1500kV applications [2]. The general concept of the design is illustrated in figure 2.3, showing the isolated foils inserted inside the solid bushing insulation. Shown in red in figure 2.3 is the potential field with no grading, and in blue with the isolated conductive foils inserted. It shows that the whole dielectric is much more evenly stressed with the capacitive grading method.
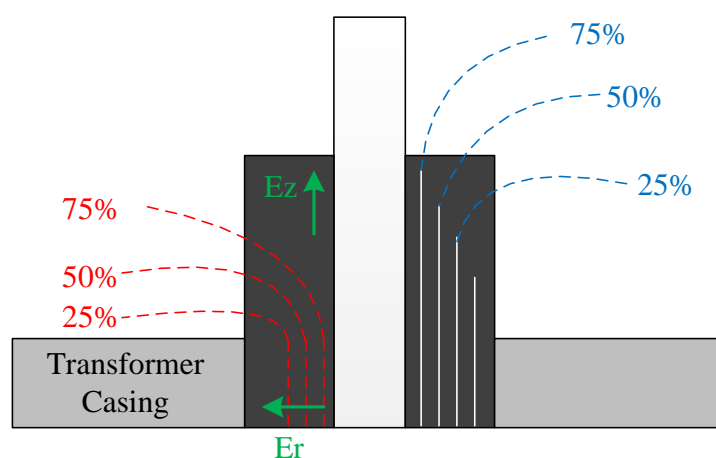

Figure 2.3: Field Distribution both without capacitive grading (shown in red) and with capacitive grading (shown in blue), modified from [2]

The insulation is stressed in both a radial and axial direction, which sum to give the tangential field. The radial component E_r can cause breakdown of the insulating material, while the axial component E_z can cause surface discharge along the boundary [6]. Attention must be paid to the design and shape of the boundary, so that the critical value for inception voltage for surface discharge is not exceeded []. These can be seen in green in figure 2.3. These sum up to give the tangential field E_t.

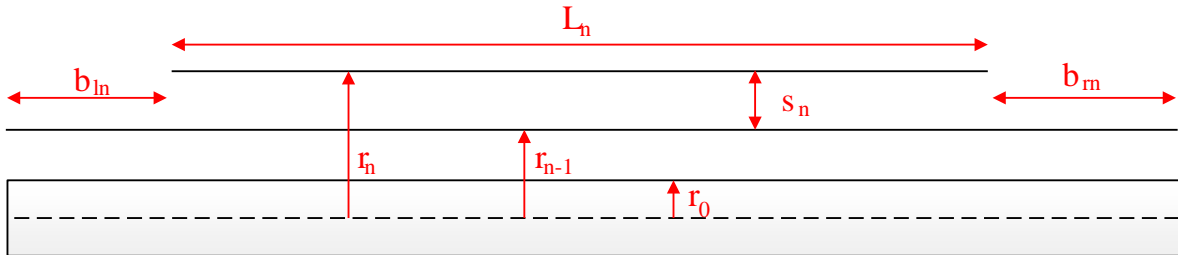Before proceeding, it is first necessary to introduce some terms. Firstly, the radius of the foil is

Figure 2.4: Symbols for calculating capacitive grading, modified from [6]

referenced from the centre of the conductor, and termed r_n. The spacing between each foil is defined in equation 1.

$$S_n = r_n - r_{n-1} \tag{1}$$

Additionally, the length of each foil is referred to as L_n and the difference in length on the right and left side between each foil is termed b_{ln} and b_{rn}. Symmetric double sided capacitive grading is achieved when $b_{ln} = b_{rn}$ [6]. The total number of foils in the system is N. Also note that subscript n denotes the outermost foil.

Inserting isolated conducting foils forms a set of coaxial capacitor units [5]. The equation for the capacitance of one of these capacitors depends on the radial displacement r_n and length of each foil L_n, as in equation 2.

$$C_n = \frac{2\pi\epsilon L_n}{ln(\frac{r_n}{r_{n-1}})} \tag{2}$$

The most widely used method to choose the dimensions and locations of the foils is double sided capacitive grading, of which there are two variants; radial grading and axial grading [6]. The aim of capacitive grading is to evenly distribute the electric field between the foils. To achieve this, an even voltage difference between foils is required as in equation 3, where V is the total voltage difference between the conductor and the casing, N is the number of foils required and ΔV is the voltage between each foil [].

$$\Delta V = \frac{V}{N} \tag{3}$$

For the voltage between each foil to be constant, as in equation 3, the capacitance between each consecutive pair of foils must also be constant. This is expressed as $C_n = C_{n-1} = \cdots = C_0$

2.2.1 Radial Grading

The radial spacing and dimension of each foil is determined in the following derivation, which has been verified and modified from [5]. In radial grading, the radial component of the electric field E_r

is kept constant between all the foils. The radial electric field is related to the voltage difference and the spacing between each foil, as in equation 4. ΔV is already defined as a constant from equation 3, and so to have equal field the foil spacing S_n should also be constant.

$$E_r = \frac{\Delta V}{S_n} = Constant \tag{4}$$

Given this condition and equation 2 for coaxial capacitance, the length of each foil is required to change from foil to foil. The lengths and radii of consecutive foils can be calculated from the relationship in equation 5.

$$C_n = \frac{2\pi\epsilon L_n}{ln(\frac{r_n}{r_{n-1}})} = C_{n-1} = \frac{2\pi\epsilon L_{n-1}}{ln(\frac{r_{n-1}}{r_{n-2}})} = \cdots = C_1 = \frac{2\pi\epsilon L_1}{ln(\frac{r_1}{r_0})} \tag{5}$$

The common factor of $2\pi\epsilon$ cancels from equation 5 giving a simple equation linking the lengths and radial displacements of consecutive foils, as in equation 6.

$$\frac{L_n}{ln(\frac{r_n}{r_{n-1}})} = \frac{L_{n-1}}{ln(\frac{r_{n-1}}{r_{n-2}})} = \cdots = \frac{L_1}{ln(\frac{r_1}{r_0})} \tag{6}$$

An approximate solution for thin foils can then be found. Under the thin foil assumption, $r_n = r_{n-1} + S_n$ and $\frac{S_n}{r_n} << 1$ even for the smallest radii of the inner foil. This is shown in equation 8.

$$ln(\frac{r_n}{r_{n-1}}) = ln\frac{1}{1 - (\frac{S_n}{r_n})} \approx \frac{S_n}{r_n} \tag{7}$$

$$L_n r_n \approx L_{n-1} r_{n-1} \approx \cdots \approx L_1 r_1 \tag{8}$$

Equation 6 can then be used to determine an exact solution while equation 8 can be used to find an approximate solution in conjunction with initial data regarding the length and radial displacement of the first foil and the spacing of the foils to calculate the parameters of all the other foils in the bushing. Nevertheless, it should be noted, at this stage, that r_0 refers to the surface of the conductor.

2.2.2 Axial Grading

In axial grading, the axial component of the electric field E_z is kept constant between all of the foils. The following equations prove that the length of each foil must decay by a constant value for each consecutive foil, and the radius at which it is placed is determined by a simple iterative formula.

The axial electric field is related to the voltage difference and the length change between each consecutive foil as in equation 9. Under symmetric capacitive grading, $b_n = b_{ln} = b_{rn}$ with reference to figure 2.4. ΔV is already defined as a constant from equation 3, and so to have equal field, the change in foil length b_n should also be constant.

$$E_z = \frac{\Delta V}{b_n} = Constant \tag{9}$$

The relationship between L_n and b_n is defined in figure 2.4, as explained in equation 10.

$$L_n = L_{n-1} - 2b_n \tag{10}$$

Since equation 9 requires the change in foil length b_n to be constant, equation 2 for coaxial capacitance requires the radius of each foil to change from foil to foil. This can be simplified to a similar form as equation 6, except that the initial information required, in this case, are different. The necessary parameters are: the length of the first foil(L_1), the radius of conductor and first foil(r_0, r_1). However, for initial calculation of $L_n(n = 1, 2, ...N)$ the size of the constant difference in length between each of the foils should be known. All other lengths and radii can then be calculated.

In case of axial grading where different material is used on each side of the bushing ($b_{ln} \neq b_{rn}$) similar calculation is carried out for each side. The total length of each foil is found by adding L_{ln} and L_{rn}. Furthermore, position of each foil could be calculated using the recursive formulae at equation 11

$$r_n = r_{(n-1)} \exp\left(\frac{L_n}{L_1} ln(\frac{r_1}{r_0})\right) \tag{11}$$

3 Design Details

The reference model for this project is shown in figure 3.1. The reference design is a paper impregnated with oil bushing with 21 aluminium foils of $100\mu m$. One side of the bushing is exposed to air, the other to oil, similar to a transformer bushing. The diameter of the conductor is 100mm, the bushing diameter is 300mm. The length of the first foil is 5000mm long, and fixed 2mm into the bushing at the conductor voltage. The outer foil is also set 2mm inside the bushing and is directly connected to the earthed flange. The conductor is used at 275kV AC voltage, and the design was taken from a bushing that was in operation for around 30 years.
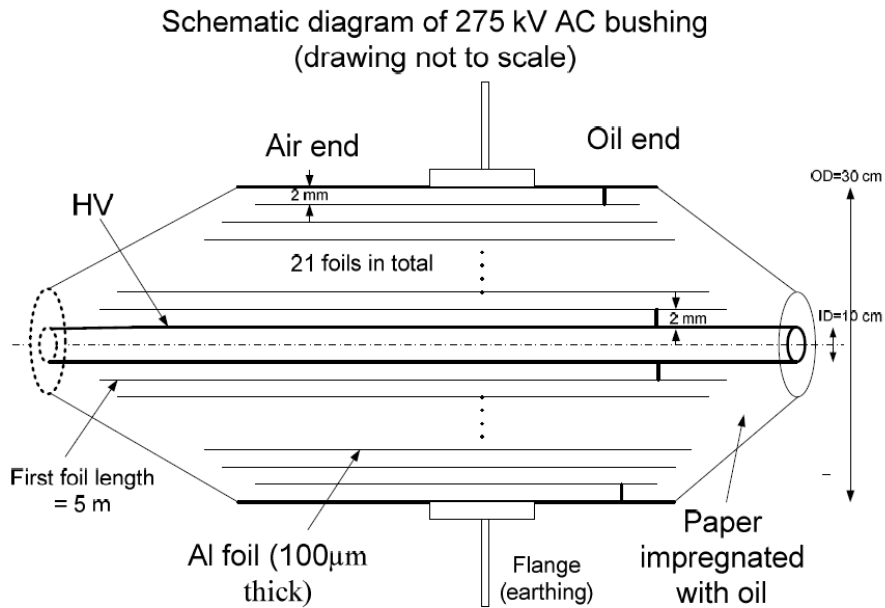

Figure 3.1: The reference problem taken from [7]

3.1 Design Issues

In section 2.2 the initial information required for both radial and axial grading includes the length and radial displacement of the innermost foil. In the reference design the following initial information is given.

Table 3.1: Initial Information for Reference Design

Initial Information	Value
Conductor Diameter (ID)	$100mm$
First Foil Length L_1	$5000mm$
First Foil Radius r_1	$52mm$
Outer Bushing Diameter (OD)	$300mm$
Outer Foil Radius $r_2 1$	$148mm$

This information intuitively fits radial grading best, since there is no requirement to assume the length of the outermost foil. However, there is a discrepancy between the standard literature problem and the reference design. The first foil is connected to the high voltage conductor, and the last foil is connected to the earthed flange. This is to eliminate the electric field on the boundary interface as far as possible on both sides of the bushing, so that the voltage drop occurs exclusively inside the bushing insulation.

This has an impact on the calculations described in section 2.2. Since the innermost foil is at the same voltage as the conductor, there is no capacitance between them, as shown in figure 3.2, hence the first foil shown on the diagram is not index 0 for the iterative calculation. The derivation of the iterative equations assumes a capacitance between the current foil and the previous foil (or conductor) (r_n, r_{n-1}). The first foil in figure 3.1 is therefore indexed as 0 and not 1.
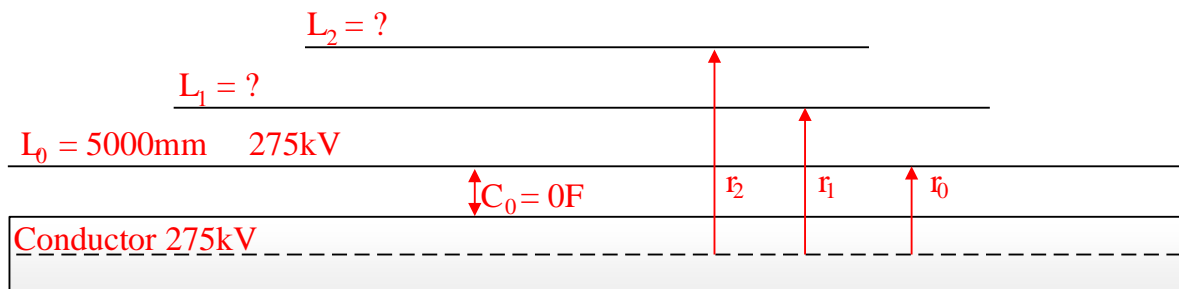
Figure 3.2: Diagram to explain the assumptions required

This means that there is not sufficient initial information to proceed with ether radial or axial grading, since the first non-connected foil length L_1 is not given. The iterative equations require the first non-connected foil length L_1 for radial (L_1 & r_1 for axial) grading to be known as shown in equation 12 and 13. In axial grading all radii variables are known due to the even spacings under radial grading. In the axial grading case, the length of foils are known due to known parameters b_{ln} and b_{rn}.

$$L_2 = L_1 \frac{ln(\frac{r_2}{r_1})}{ln(\frac{r_1}{r_0})} \qquad (Radial\ grading) \tag{12}$$

$$r_2 = r_1 \exp\left(\frac{L_2}{L_1} ln(\frac{r_1}{r_0})\right) \qquad (Axial\ grading) \tag{13}$$

If this is not taken into account then a flawed design will be produced in both cases. Equations 14 and 15 show a wrongly described first iteration of the radial and axial grading formula. For radial grading the resulted design is shown in figure 3.3. This shows that the length of second foil is much bigger than the first foil. This is clearly wrong, and does not give the hyperbolic shape from the beginning of the foils.

$$L_1 = 5000 \frac{ln(\frac{56.8}{52})}{ln(\frac{52}{50})} = 11256mm \qquad (Radial\ grading) \tag{14}$$

$$r_2 = 52 \exp\left(\frac{49418}{5000} ln(\frac{52}{50})\right) = 54.05mm \implies r_{21} = 103.52mm \qquad (Axial\ grading) \tag{15}$$
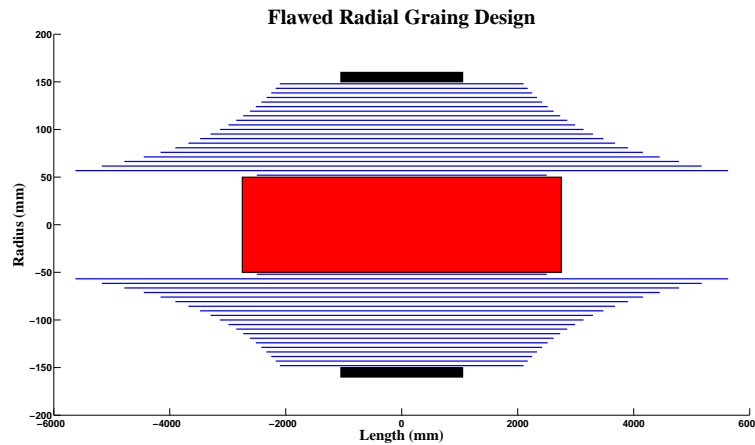
Figure 3.3: Flawed Radial Grading Design

In order to proceed with the calculations there must be an assumption of the length of the first unconnected foil. A reasonable assumption is that this follows the hyperbolic shape of the other foils in radial grading and also in axial grading it is assumed to be the first unconnected foil. These assumptions in both cases help to evenly distribute the electric field radially or axially accordingly. To achieve this design mathematically, the following assumptions are made:

1. Foil 0 is not connected to the HV conductor for both cases.

2. The conductor surface is spaced a distance of S_n from foil 0 in **radial grading**.

3. The conductor surface is spaced an adjustable distance from foil 0 in **axial grading**.

The reason of each assumption according to the design constrains are explained as following:

- Assumption 1 is required to be able to use the capacitor derived iterative formula on foil 0.
- Assumption 2 is required so that the radial spacing is kept constant.

- Assumption 3 is required so that axial grading could be calculated with a varying parameter value for r_0. This makes it possible to adjust the initial gap so that the last foil will be placed exactly at 148mm.

The first iteration has been calculated under these assumptions, giving the result in equations 16 and 17 which are expected values. The remainder of foil parameters can then be calculated using the iterative formulas in each grading.

$$L_1 = 5000 \frac{ln(\frac{56.8}{52})}{ln(\frac{52}{47.2})} = 4558mm \qquad (Radial\ grading) \qquad (16)$$

$$r_2 = 52 \exp\left(\frac{49418}{5000} \ln(\frac{52}{50 - 1.007})\right) = 55.15mm \Longrightarrow r_{21} \simeq 148mm \qquad (Axial\ grading) \qquad (17)$$

4   Matlab Calculations

Two Matlab scripts were developed for computation of radial and axial grading. These scripts were built to be easily customisable for any number of foils and any initial values, to cater for the calculation of improved designs. They also automatically outputs data in a form for direct input into the COMSOL model, auto-updates a LaTeXfile containing the data to form results table and displays the results in both 2D and 3D plots for quick design verification.

4.1   Matlab Radial Grading

In case of radial grading the code takes a required number of foils, and the inner and outer dimensions of the bushing, to calculate the radial location and length of each foil using the radial grading method as described in section 2.2 and also using the assumptions made for redial grading design on 3.1.

For current design with specified parameters, the script plots the calculated foil positions in a 3D graph shown in figure 4.1. Also the 2D plot of this design is shown in figure 1(b). This figure illustrates the hyperbolic shape which was expected for radial grading. These figures allows a quick verification of the scripts accuracy before proceeding to simulation.
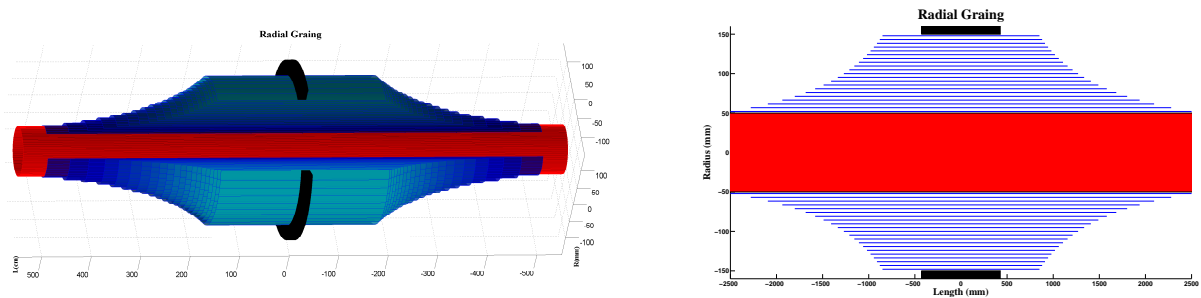
(a) 3D Representation of foil radial position and length

(b) 2D Representation of foil radial position and length

Figure 4.1: Matlab Generated Plots of Geometric Radial Design

Table **??** shows the values obtained for radial grading. The final information required to be able to proceed to the axial simulation phase is the relative permittivity of each material. This was gathered from [6] and is shown in table **??**.

Foil N.O	Radius(mm)	Length(mm)	Foil N.O	Radius(mm)	Length(mm)
1	52.00	5000.00	12	104.80	2420.43
2	56.80	4558.22	13	109.60	2312.01
3	61.60	4188.21	14	114.40	2212.90
4	66.40	3873.79	15	119.20	2121.93
5	71.20	3603.30	16	124.00	2038.15
6	76.00	3368.13	17	128.80	1960.73
7	80.80	3161.78	18	133.60	1888.98
8	85.60	2979.27	19	138.40	1822.29
9	90.40	2816.68	20	143.20	1760.16
10	95.20	2670.92	21	148.00	1702.12
11	100.00	2539.51	22	150.00	0.00

Table 4.1: Axial Grading Calculations Results

Material	Relative Permittivity (ϵ_r)
Air	1
Oil	2.2
Paper Impregnated with Oil	4
Aluminium	10^8

Table 4.2: Relative Permittivity of Materials

4.2   Matlab Axial Grading

The axial grading script is similar to the radial grading script in design. However, as it is explain in reference [8], the constant difference of foil length in oil and air side of bushing (b_{air}, b_{oil}) is calculated by considering the value of flash over distance L_{air} and L_{oil}. These values are calculated by using the fact that average electric field strength along the boundary surface in oil side should be 3 to 4 Kv/cm and nearly 9 to 12 Kv/cm in air side of the bushing. These values are calculated using equation 18.

$$b_{air} = \frac{\Delta V}{900 \, ^v/_{mm}} \qquad b_{oil} = \frac{\Delta V}{300 \, ^v/_{mm}} \tag{18}$$

Additionally, as shown in equation 19, in the calculation of r_2 a parameter called $R_{parameter}$ is subtracted form r_0 to change the position of assumed conductor surface. The radius of the last foil (r_{21}) could be correctly adjusted by making changes in this parameter and executing the script. $R_{parameter} = 1.007mm$ was the best value found for this design.

$$r_2 = r_1 \exp\left(\frac{L_2}{L_1} ln\left(\frac{r_1}{r_0 - R_{parameter}}\right)\right) \tag{19}$$

The script plots 2D and 3D plots of axial grading to the user. Figure 2(a) illustrates the 3D configuration of design's axial bushing. The 2D plot of this design, which is shown in figure 2(b), shows linear reduction on foil length on both sides. It also illustrates that $b_{air} > b_{oil}$ as it was expected due to the relative permittivity of each material. Finally the calculated values for axial grading, using this design method, are shown in **??**. These values along the given parameters in table **??** were used for COMSOL simulations of this axial grading design.

(a) 3D Representation of foil radial position and length

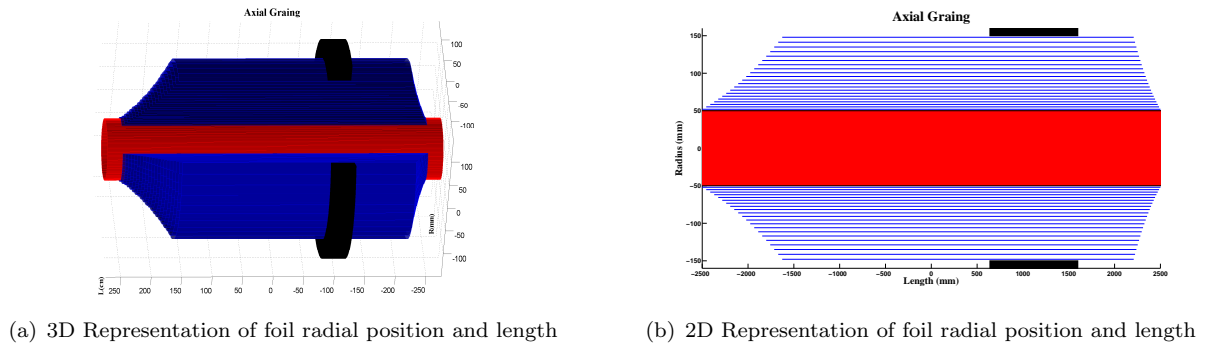(b) 2D Representation of foil radial position and length

Figure 4.2: Matlab Generated Plots of Geometric Axial Design

5 Modelling Results

The following simulations were completed using the COMSOL multiphysics software package. COMSOL is a professional finite element simulation package able to model a variety of physical features. The following models are created using the AC/DC module, which is used to simulate electric and magnetic fields [9]. Specifically, the electrostatics interface is used. This solves a charge conservation equation for a given voltage and spacial distribution of charge [9].

5.1 Finite Element Methods (FEM)

There are inherent difficulties in solving the partial differential equations that govern many practical engineering problems [5]. Despite knowing the equations and appropriate boundary conditions that govern a problem, many are complicated by irregular geometries or other discontinuities. Numerical methods allow approximate solutions to be obtained for problems intractable by analytic methods [10]. In an analytic solution, the whole system is governed by a mathematical equation valid for the entire region of interest. Although these differential equations are often mathematically compact, it is difficult to obtain an answer unless the system is unreasonably simplified [10]. In FEMs, the complex geometry is broken into a series of much smaller and simpler geometries [5]. These geometries can be squares, rectangles or triangles in 2D or the 3D equivalent shapes. These simpler shapes form interconnected subregions for which an approximate function, usually a high order polynomial, can be used to represent the actual function. If the complex is split into an adequate number of simple shapes, these approximate functions closely matches the exact solution [10].

By default COMSOL uses a triangular discretisation to split up a complex geometry in a process called meshing. This forms an unstructured grid of triangles, allowing the mapping of complex or curved geometries. Other numerical methods such as Finite Difference Methods require a structured grid, hence FEMs are more flexible with regards to geometry [5]. Meshing requires an initial understanding of the expected outcomes of the problem, so that the mesh can be refined in areas of interest. Each

triangular element is approximated by a linear interpolation of the potential at the vertices of the triangle. A set of linear algebraic equations are formed by minimising the error between the actual solution and a set of approximate linear trial functions [10].

5.2 Equation Derivation and Boundary Conditions

The electrostatics interface of the AC/DC COMSOL module uses the electric potential V to calculate static electric fields. A Poisson type partial differential equation is derived using classical electrostatics and Gauss' Law [11].

By taking Gauss's Law:
$$\nabla.\mathbf{D} = \rho v \tag{20}$$

the equation for electric flux density $\mathbf{D}$:
$$\mathbf{D} = \epsilon_0\epsilon_r\mathbf{E} \tag{21}$$

this can be combined with the equation for a static electric field:

$$\mathbf{E} = -\nabla V \tag{22}$$

to give by substitution:
$$\nabla.\mathbf{D} = \nabla.(\epsilon_0\epsilon_r\mathbf{E}) = -\nabla.(\epsilon_0\epsilon_r\nabla V) = \rho v \tag{23}$$

which is more usually written:
$$\nabla^2 V = -\frac{\rho v}{\epsilon_0\epsilon_r} \tag{24}$$

where ϵ_0 is the permittivity of free space, ϵ_r is the relative permittivity of the material, $\mathbf{E}$ is the electric field strength and ρv is the volume charge density.

In the special case where there is zero volume charge density, that is $\rho v = 0$ then the equation simplifies to Laplace's Equation:
$$\nabla^2 V = 0 \tag{25}$$

The models used in this paper are 2D axisymmetric, meaning that a 2D model is used to describe a 3D object that can be rotated 360° about a central point $r = 0$ to give a 3D geometry. This assumes that not only is the geometry the same in the φ direction, but also that the electric potential is constant. In this case, Poissons equation can be rewritten in cylindrical coordinates for a 2D axisymmetric model, it is multiplied by r to ensure there are no singularities at $r = 0$ [10].

$$\begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial z} \end{bmatrix}^T .(r \begin{bmatrix} \frac{\partial V}{\partial r} \\ \frac{\partial V}{\partial z} \end{bmatrix}) = -\frac{r\rho v}{\epsilon_0\epsilon_r} \tag{26}$$

The boundary conditions are defined as the following:

1. All boundaries with the conductor and with foil 0 are set to $V = V_0 = 275kV$

2. All boundaries with the transformer wall and with the outermost foil are set to $V = 0$

3. The interface between two insulator sub-domains is defined by $n.(\mathbf{D}_1 - \mathbf{D}_2) = \rho v$ and $n \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$. These equations specify that the normal component of the electric flux density is discontinuous at the interface between two dielectrics, and that the tangential component of

the electric field is continuous across the dielectric interface. n is the normal outward vector pointing from dielectric 2 to dielectric 1.

5.3   Workflow

In order to simulate the electric field distribution within our bushing design, 2D axisymmetric models were created. The general workflow to achieve this is:

1. Build a geometry representing the physical structure of the bushing.

2. Assign each geometric domain a material. The material selection determines the relative permittivity ϵ_r of each domain.

3. Define the charge conservation equation and all initial conditions. This includes setting which boundaries are at ground and conductor potential and setting boundary conditions.

4. Design a mesh. The geometry is split into smaller elements in order to compute the charge conservation equation. For designs with foils, special meshing parameters are required to speed up the process.

5. Carry out the study. This stage is the actual computation of the solution.

6. Post-processing - Display the results in a number of formats including 3D, 2D and 1D plots, or export the data for post-processing in Matlab.

5.4   Baseline Model

In order to minimise the computation time required for each model, it was necessary to determine the areas of interest in the model. A bushing geometry was built with no foils inserted, a high quality mesh was produced and the system was solved to find the electric field distribution throughout the bushing and the surrounding area.
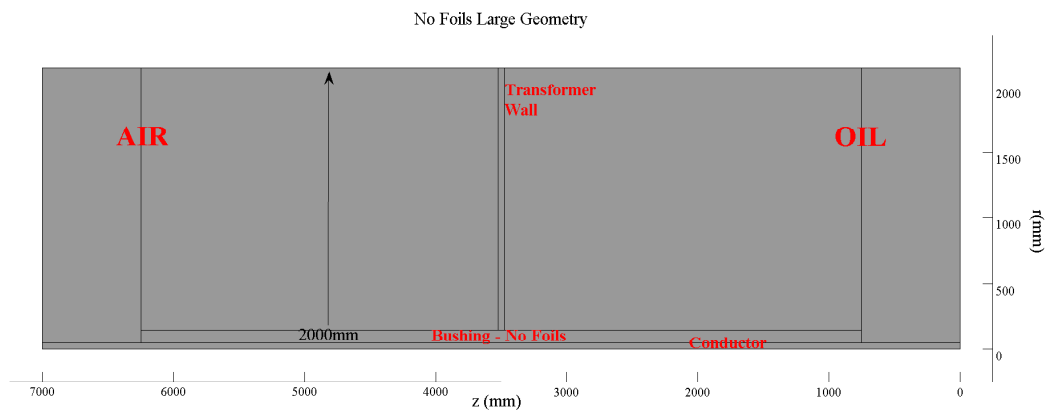

Figure 5.1: Baseline Model Geometry

The model is made up with a very large geometry. The air and oil extends radially 2m from the end of the bushing and 1m in the axial direction. This is to understand the anticipated area of interest in the model. By considering figure 5.3 it is clear that there is very little happening further than 500mm

Extra Fine Mesh for Baseline
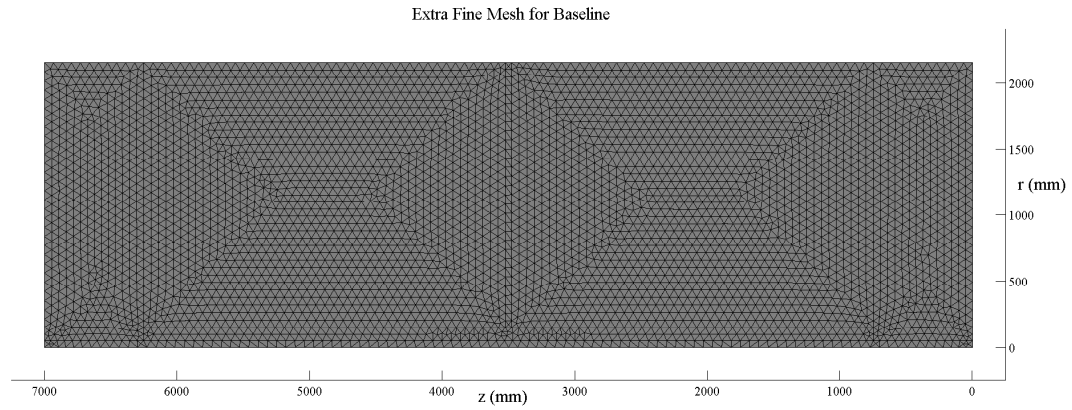
Figure 5.2: Extra Fine Meshing for Baseline Model

Normal Electric Field Strength (V/m)
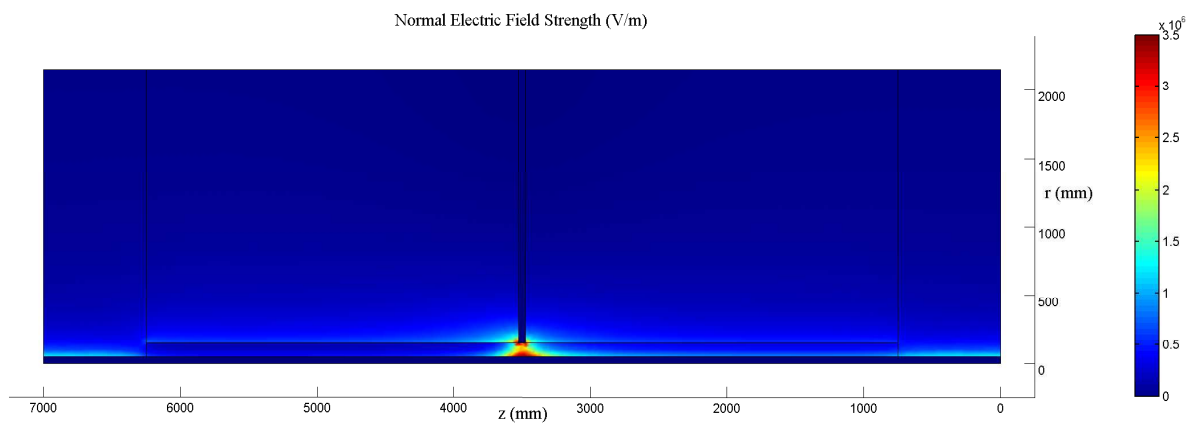
Figure 5.3: Normal Electric Field (V/m) for Baseline Model

radially from the bushing surface and there is very little of interest further than 200mm in the radial direction. Therefore all further models will adhere to this geometry, ensuring that the area of interest is captured, while decreasing simulation times to a minimum.

5.5   No Foils

In order to illustrate the requirement for capacitive grading within AC bushings, a simulation of a bushing with no grading was conducted. Figure 5.4 shows the geometry and materials used for each section. The conductor length is $6000mm$ and has a width of half the inner diameter, $50mm$. The paper impregnated with oil insulation is $5500mm$ long in order to accommodate the first foil length of $5000mm$ with sufficient clearance. The transformer wall is modelled as a $50mm$ high aluminium block that is $500mm$ wide, and is placed vertically in the middle of the design. The surrounding oil and air are $500mm$ wide at the centre of the bushing, and extend to the length of the conductor. The paper impregnated with oil insulation is sloped to avoid sharp corners and high field strengths.

A very fine mesh was used in this model as shown in figure 5.5. This allows for the maximum level of accuracy in the results produced, and is possible since the geometry of this model is not over-complex.
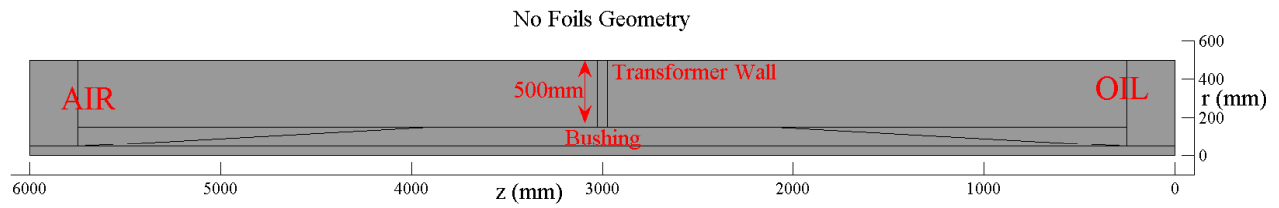
Figure 5.4: Geometry and Materials for No Foils Bushing
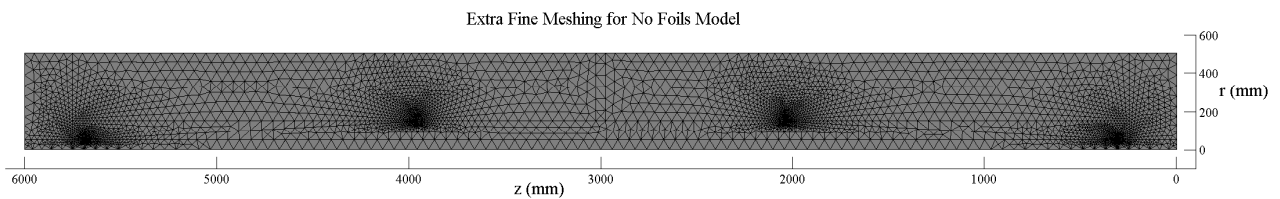
Figure 5.5: Extra Fine Mesh for No Foils Bushing
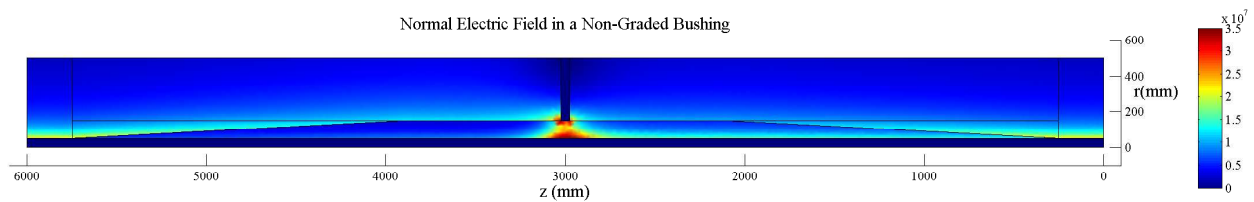
Figure 5.6: Normal Electric Field (V/m) for No Foils Bushing

The motivation for capacitive grading is clear in figure 5.6. The central area of stress is shown zoomed in figure 5.7. The highest stress is shown at the corners of the transformer wall and in the plane between the transformer wall and the conductor. This reaches field strengths greater than $3.5kV/mm$. This simulation bears close resemblance to the theory explained in many texts and explained in figure 2.1 in section 2 of this report. In order for this bushing to function within the design constraints, capacitive grading must be used.
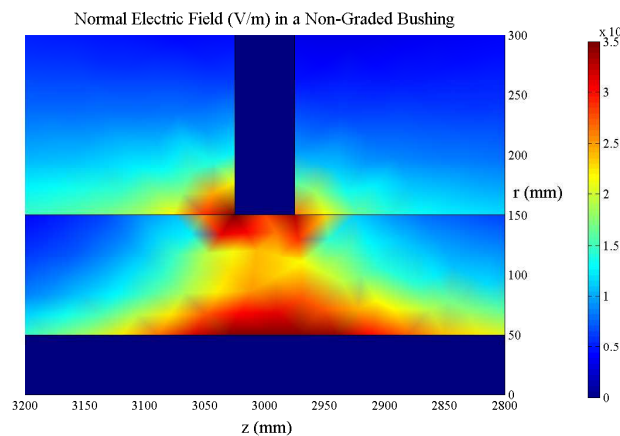
Figure 5.7: Area of very high electric stress

5.6   No Grading

In order to rectify the high electric stress identified in the no foils model, isolated foils are introduced to capacitively grade the electric field. To prove the requirement for grading the lengths or radial displacement of the foils, a simulation where the foils are maintained at the same length was performed. The bushing has 21 foils spaced at an even radial spacing, but with no change in length of each foil. Each foil is $1756mm$ in length, and is centred in the within the bushing.
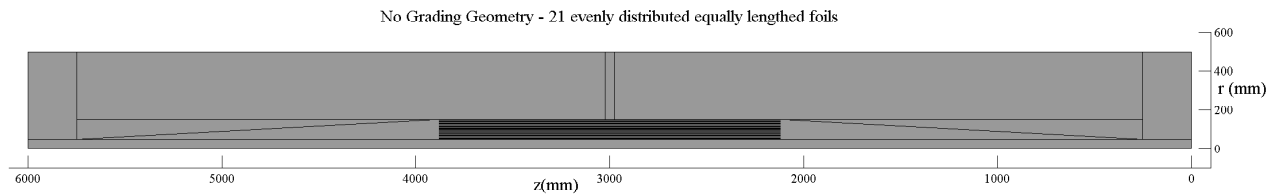
Figure 5.8: Geometry of the No Grading Model

Producing a finite element mesh for this model caused issues with long computation time. Even using the coarsest default setting, the meshing time was of the order of days. This is due to the difference in size of the aspects of the geometry. The foils are just $0.1mm$ thick, which requires a very small set of triangles in order to mesh this area. The other domains are up to $6000mm$ long, which is considerably larger and requires a different size of triangles.

In order to solves the issue, a set of meshing rules were produced. Firstly, the mesh within the conductor and within the foils can be very coarse. Within a conducting material, no electric field is expected, hence there is no reason to finely mesh that area. Areas of interest or areas where the field changes rapidly need to have a very fine mesh to ensure the accuracy of the results is sufficient. The meshing rules dictated a minimum number of points at the tip of each foil, and along the surface of the bushing.
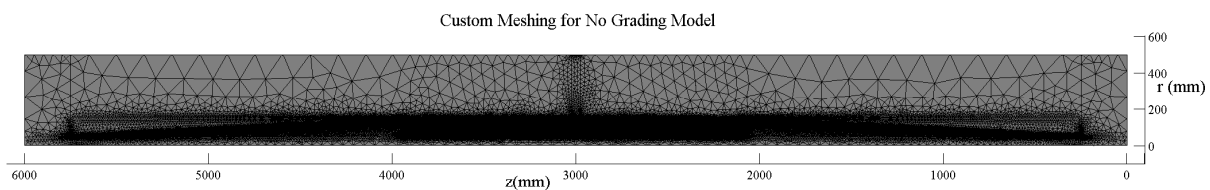
Figure 5.9: User defined meshing for no grading simulation - whole view

5.7   Radial Grading

. . .

5.8   Axial Grading

. . .
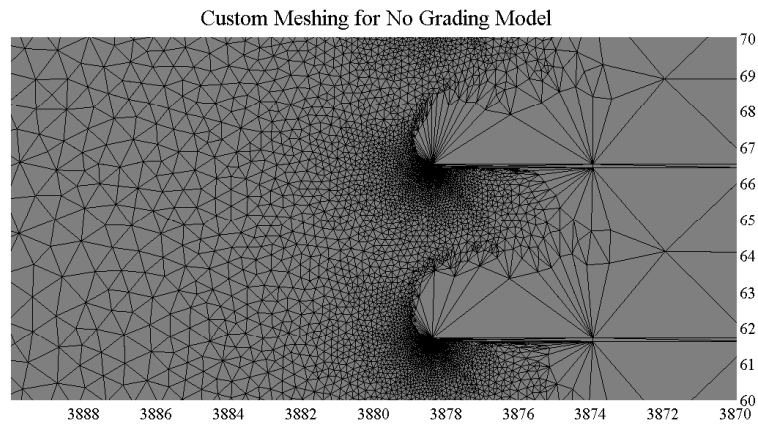
Figure 5.10: User defined meshing for no grading simulation - foil tip view
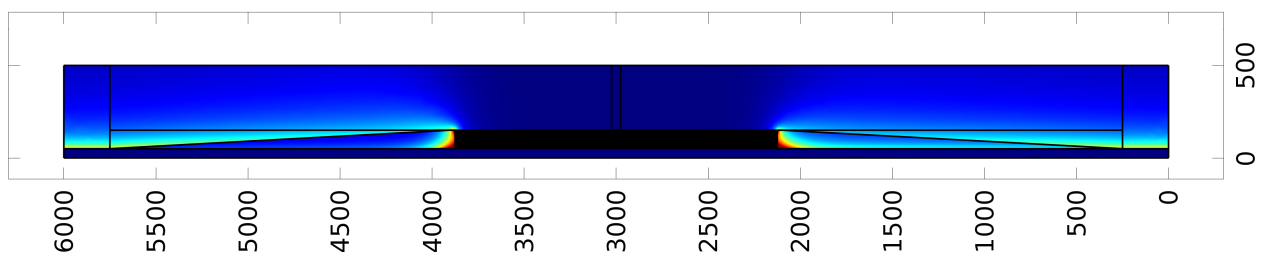
Figure 5.11: Normal Electric Field (V/m)

.

6 Discussion of Results

Comparison and discussion (Suggestions on improvement).

7 Conclusions

Conclusions.

References

[1] D.F. Warne. *Newnes Electrical Power Engineer's Handbook*. Elsevier Science, 2005.

[2] R.E. James, Q. Su, and Institution of Engineering and Technology. *Condition Assessment of High Voltage Insulation in Power System Equipment*. IET power and energy series. Institution of Engineering and Technology, 2008.

[3] Ieee standard general requirements and test procedure for power apparatus bushings. *IEEE Std C57.19.00-2004 (Revision of IEEE Std C57.19.00-1991)*, pages 1–17, 2005.

[4] J.H. Harlow. *Electric Power Transformer Engineering*. The Electric Power Engineering Hbk, Second Edition. Taylor & Francis, 2004.

[5] J. Kuffel, E. Kuffel, and W.S. Zaengl. *High Voltage Engineering Fundamentals*. Elsevier Science, 2000.

[6] Zeeshan Ahmed. Analysis of partial discharge in OIP bushing models. Master's thesis, Royal Institute of Technology (KTH), September 2011.

[7] George Chen. ELEC6089 High Voltage Insulation Systems Assignment 1 HV AC 275kV Bushing Design Briefing Notes, February 2014. The University of Southampton.

[8] Fachgebiet Hochspannungstechnik Technische Universitt Darmstadt. Measures for field and potential control. [Online] Available: `http://www.hst.tu-darmstadt.de/uploads/media/hvt2_v_02.pdf`, last viewed on 23.12.2014.

[9] COMSOL. *Introduction to AC/DC Module - COMSOL Version 4.3*. Part No. CM020104, 2012.

[10] Mohammad R Meshkatoddini. Study of the electric field intensity in bushing integrated zno surge arresters by means of finite element analysis. In *COSMOL Users Conference*, 2006.

[11] W.H. Hayt and J.A. Buck. *Engineering Electromagnetics*. MCGRAW-HILL Higher Education, 2012.

A Individual Contributions

Team Member	Contribution
Thomas J. Smith 23914254	-
David Mahmoodi 99999999	-
Brendan Hickman 99999999	-
Patrick P. L. Fong 99999999	-

B   Meeting Minutes

B.1   Meeting 1 - Kick-off Meeting

Purpose	ELEC6089 Bushing Design Kick Off Meeting
Date and Time	Thursday 20th February 13:30
Venue	GDP Lab Zepler Building, Highfield Campus
Participants	TS (Thomas Smith), DM (David Mahmoodi), BH (Brendan Hickman), PF (Patrick Fong)
Apologies	None
Agenda	Review what we understand of the project so far. Understand the tasks required. Agree expectations of work and schedule. Agree date and agenda of next meeting.

B.1.1   Minutes of the Meeting

ID	Subject	Notes and Discussion	Action
1.0	Research prior to the meeting	BH uploaded the course text to the Facebook working group which has a section on stress control by floating screens. TS uploaded a project from KTH university that had similar guidelines and had a useful description to compound the lecturenotes for the module. All agreed to research the topic further and read these sections by the next meeting	**ALL A1.0**
2.0	Current understanding of task	The group discussed the task at hand. We need to design the bushing using the iterative formulas from the lectures and then build a COMSOL model. The design must be either radial or axial in grading method.	-
3.0	Work Breakdown	The group tried to identify the work to complete. This includes research into field design and grading methods, calculating the bushing design, simulating and report writing. None of these tasks can be completed in parallel, and all need the previous in order to complete the task. Hence each member needs to research, and have knowledge of the design and simulation process. It will become clearer who will be assigned responsibility for what shortly. Currently, remain with all needing to complete research	-
4.0	Next Meeting	First meeting with G. Chen in 2 weeks, Tuesday 4th March. Before then have a first model and have begun verification. Have group Latex template for collaboration, good layout and presentation marks. Use Github. Next meeting on Wednesday 26th.	-

B.1.2   Action List

ID	Action	Comments	Status
A1.0	Research	All to start research. Make notes of all sources. At least reviewed the lecture notes and Kuffel.	Open 20th Feb

Next Meeting: 26th Feb 2014, Location & Time TBA

B.2   Meeting 2 - Progress Meeting

Purpose	ELEC6089 Bushing Design Progress Meeting
Date and Time	Wednesday 26th February 11:30
Venue	GDP Lab Zepler Building, Highfield Campus
Participants	TS (Thomas Smith), DM (David Mahmoodi), BH (Brendan Hickman)
Apologies	PF (Patrick Fong)
Agenda	Review research progress. Clarify project understanding. Start design task. Identify further work.

B.2.1   Minutes of the Meeting

ID	Subject	Notes and Discussion	Action
1.0	Research update	The present team members discussed the task in the context of Kuffel and KTH research. Agreed on bushing definitions and the theory behind capacitive grading. Also took time to verify that the lecture notes matched the explanation in Kuffel. Kuffel pages are 235-241. Also discussed why the capacitors were added, and established the iterative formula to use. All should continue to gain a firmer grounding of the required theory	**ALL A1.0**
2.0	Github and LaTeX	TS ran the present through the report template, what was required and how to use the distributed revision control system Git as hosted on GitHub. This should make collaboration much easier than using just our facebook group page.	-

ID	Subject	Notes and Discussion	Action
3.0	Grading Methods	DM left the meeting at this point to read the lecture notes. DA will also perform the grading and we can then use this to idependently verify the design. TS and BH started on axial grading method. Both wrote matlab code to calculate spacings. The results were the same, hence reasonable level of confidence of validity.	**PF & DM A2.0**
4.0	Remaining work	BH and TS identified the remaining work for actioning. The report has an introduction which requires review. Sections on Grading methods (why grade? LV solutions using electrodes, DC solution using resistivity, AC capacitive grading), AC grading types (discussion of axial and radial components of tangential fields, radial and axial derivation) and section on the design details (iterative formula, Matlab calculations, visio diagrams). The design must be built in COMSOL which represents significant work to understand COMSOL. Probably want to simulate a non-graded bushing as a baseline for discussion. Aiming to do both radial and axial grading simulations. Then discuss.	-
5.0	Assignment of work	BH and PF have a key deadline on tuesday 4th March hence largely unavailable until then. TS and DM to get started on tasks. Try and get simulations done before meeting with GC.	**TS & DA A3.0 A4.0**
6.0	Next Meeting	First meeting with G. Chen Tuesday 4th March. Before then have a first model and have begun verification. Next meeting on Prior to this meeting.	-

B.2.2 Action List

ID	Action	Comments	Status
A1.0	Research	All to start research. Make notes of all sources. At least reviewed the lecture notes and Kuffel.	Open 20th Feb
A2.0	Grading	Other members to perform axial grading calculations seperately so that the results can be verified independently	Open 26th Feb
A3.0	COMSOL	Gain an understanding of COMSOL and attempt some simulations.	Open 26th Feb
A4.0	Reporting	Continue to document progress in the report.	Open 26th Feb

Next Meeting: 4th March 2014, Preceeding meeting with GC

B.3 Meeting 3 - Progress Meeting

Purpose	ELEC6089 Bushing Design Progress Meeting
Date and Time	Tuesday 4th March Before Meeting with GC
Venue	Building 58, Highfield Campus
Participants	TS (Thomas Smith), DM (David Mahmoodi), BH (Brendan Hickman), PF (Patrick Fong)
Apologies	-
Agenda	Review progress. Prepare Questions for Meeting. Identify further work required. Allocate work.

B.3.1 Minutes of the Meeting

ID	Subject	Notes and Discussion	Action
1.0	Project Update	DM updated the group with his work in Matlab using the natural log derivation of the grading formula. TS updated the group with report progress and progress using COMSOL. Significant work completed by TS, including Introduction, an overview of grading methods, an attempt to derive the equations and initial COMSOL models. BH and PF have nothing to update due to another deadline just prior to the meeting.	-
2.0	Questions to ask in GC meeting	Meaning of dimensions for 'failed' 275kV bushing? Will it work, must we improve it? Will it fail without foils but work with?	-
2.1		Can we change the length or width of the bushing? The material/insulation of the bushing or the number of foils?	-
2.2		Check that TS output data/graphs are correct. Check conductor, meshing, and length of grounded aluminium transformer.	-
2.3		Should we have radial and axial grading to compare?	-
2.4		Can we design shedding in comsol for the air side as extra to bushing?	-
2.5		Should we count last boundary of oil-end interface as a capacitor, even take it into account?	-
2.6		Check formatting of paper	-
3.0	Meeting with GC	The Group attended a meeting with GC to gain some feedback on progress so far.	-
3.1		Answer to 2.1, 2.3, 2.4 - Later on, you can change dimensions but there are also other (better?) improvements	-

ID	Subject	Notes and Discussion	Action
3.2		Find/mesh tightly the areas of interest, the bushing and not the air to make the meshing more efficient. Look at the interfaces, at end of Al foil there is more chance of discharge. In the middle the field is symmetrical.	-
3.3		Graphs should be a clear demonstration of info, potential distribution and field strength 2/3. Check where the ground/transformer is, is it half in or more or less? Different flashover lengths, partial discharges etc.	-
3.4		The first and last foils are connected to the conductor and ground respectively. Flashover can go through interface of paper&oil	-
3.5		45 page maximum including appendices	-
4.0	Work Allocation	low voltage, DC solutions, electrode grading and external shaping investigation for the report	**PF A5.0**
4.1		Partial discharge, flashover (& voltages) and related criteria so we can judge how good a solution is	**BH A6.0**
4.2		Comsol, further learning + shaping of field	**TS and DM A7.0**
5.0	Next Meeting	Meet on 11th March - 1 week time	-

B.3.2   Action List

ID	Action	Comments	Status
A1.0	Research	All to start research. Make notes of all sources. At least reviewed the lecture notes and Kuffel.	Open 20th Feb Closed 4th March
A2.0	Grading	Other members to perform axial grading calculations seperately so that the results can be verified independently	Open 26th Feb Closed 4th March
A3.0	COMSOL	Gain an understanding of COMSOL and attempt some simulations.	Open 26th Feb Closed 4th March
A4.0	Reporting	Continue to document progress in the report.	Open 26th Feb
A5.0	LV and DC	Research and document LV solutions and DC alternatives.	Open 4th March
A6.0	Criteria	Define a set of assessment criteria.	Open 4th March
A7.0	Simulations	Continue to refine Matlab script and develop COMSOL simulations accurately portraying what is happening so that the design can be assessed critically.	Open 4th March

Next Meeting: 11th March 2014

C   Code Listings

Radial Grading Matlab Code

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ELEC6089 High Volatage Insulation Bushing Design
% Radial grading calculation
% Author - David Mahmoodi
% Date - 14/02/2014
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; close all; clear all
%% Declareing Given Variables
Voltage = 275000;
Inner_diameter = 100;
Outer_diameter = 300;
First_foil_length = 5000;
N = 21;
Foil_Thickness = 0.1;
First_Gap = 2;
Last_Gap  = 2;
%% Defining new variables
Del_Voltage = Voltage/N-1; %Voltage between each foil
Del_Radius = ((Outer_diameter- Inner_diameter)-2*(First_Gap+Last_Gap))/(2*(N-1)); % Spacing between each f
%Initialise vectors (22 for 3D plotting)
L = zeros(1,N+1);
Radius = zeros(1,N+1);
%% Calculation
L(1)=First_foil_length; %The first foil is 5000mm and connected to the conductor, no capacitance between c
r0 = Inner_diameter/2; %Radius of the conductor
Radius(1)= Inner_diameter/2  + First_Gap; % Radial position of first foil = 52mm
%Calculate the radial positions of all foils
for i=2:N
    Radius(i)=Radius(i-1) + Del_Radius;
end
% Refer to Section 2.2 for an explanation of this assumption
L(2)= log(Radius(2)/Radius(1))* L(1) / log(Radius(1)/(Radius(1)-Del_Radius)) ;
%Follow the iterative formula
for i=3:N
    L(i)= log(Radius(i-1)/Radius(i))* L(i-1) / log(Radius(i-2)/Radius(i-1));
end
%For plotting - add the outer shell
L(N+1)=L(N)-.5*L(N);
Radius(N+1)=Radius(N)+ Last_Gap;
%% Ploting
x=zeros(1,2*(N+1));
y=x;  j=1;
for i=1:2:2*(N+1)
    x(i)=L(j)/2;
    x(i+1)=-L(j)/2;
    y(i)=Radius(j);
    y(i+1)=Radius(j);
    j=j+1;
end
y2=-y;
% 2D Plot
figure
axes('FontSize',16,'fontWeight','bold')
rect_H = rectangle('Position', [-1.1.*x(1),- r0, 2.2*x(1), 2*r0]);
set(rect_H, 'FaceColor', 'r')

for i=1:2:2*(N)
    hold on
    line(x(i:i+1), y(i:i+1), 'LineWidth',2)
    line(x(i:i+1), y2(i:i+1),'LineWidth',2)
    %axis equal
end
rect_H = rectangle('Position', [-L(N+1)/2,Outer_diameter/2, L(N+1), 10]);
set(rect_H, 'FaceColor',[0, 0, 0])
rect_H = rectangle('Position', [-L(N+1)/2,-10-Outer_diameter/2, L(N+1), 10]);
set(rect_H, 'FaceColor',[0, 0, 0]);
```

```matlab
title('Radial Graing','FontName','Times New Roman','FontSize',34,'fontWeight','bold');
xlabel('Length (mm)','FontName','Times New Roman','FontSize',24,'fontWeight','bold')
ylabel('Radius (mm)','FontName','Times New Roman','FontSize',24,'fontWeight','bold')
xlim([x(2) x(1)]);ylim([y2(end)-10 y(end)+10])
% 3D Plot
K=50; scl=.1; % Z direction scalling value for plotting
p= 6*N/3+1; %adjusting the Cut in the 3D shap
figure
axes('FontSize',16,'fontWeight','bold');
R=[r0 r0];
[X,Y,Z] = cylinder(R,5*K);
Z(2, :) =( L(1)+ .1*L(1))*scl;
Z(1, :) = - Z(2, :);
surf(X,Y,Z, 'FaceColor',[1,0,0],'EdgeColor', [1,0,0]);
for i=1:N
    hold on
    R=[Radius(i) Radius(i)];
    [X,Y,Z] = cylinder(R,K);
    Z(2,:)= L(i);
    Z(1,:)= -L(i);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    testsubject = surf(X,Y,Z);
    set(testsubject,'FaceAlpha',0.8,'EdgeColor','b')
    axis equal
end
Ground=Radius(N+1)-1;
for i=1:30
    R=[Ground+i Ground+i];
    [X,Y,Z] = cylinder(R,K);
    Z(2, :) = L(N+1)*scl;
    Z(1, :) = - Z(2, :);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    surf(X,Y,Z, 'FaceColor', [0,0,0],'EdgeColor',[0, 0, 0]);
end
camlight
lighting gouraud
title('Radial Graing','FontName','Times New Roman','FontSize',24,'fontWeight','bold');
xlabel('R(mm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','Rotation',90,'HorizontalAli
zlabel('L(cm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','HorizontalAlignment','right
%% Saving results to file
FID = fopen('RadialVals21.tex', 'w');
fprintf(FID, '\\begin{table}[!htb]\n');
fprintf(FID, '\\label{table:radialvals}\n');
fprintf(FID, '\\begin{center}\n');
fprintf(FID, '\\begin{tabular}{|c||cc|c||cc|}\n');
fprintf(FID, '\\toprule\n');
fprintf(FID, '\\textbf{Foil N.O} & \\textbf{Radius(mm)} & \\textbf{Length(mm)} & \\textbf{Foil N.O} & \\te
Max = floor((N+1)/2); L(N+1)= 00;
for i=1:Max
    fprintf(FID, '%.f & %4.2f & %4.2f & %.f & %4.2f & %4.2f \\\\\n',i,Radius(i),L(i),(i+Max),Radius(i+Max
end
fprintf(FID, '\\bottomrule\n');
fprintf(FID, '\\end{tabular}\n');
fprintf(FID, '\\end{center}\n');
fprintf(FID, '\\caption{Axial Grading Calculations Results}\n');
fprintf(FID, '\\end{table}\n');
fclose(FID);
```

Axial Grading Matlab Code

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ELEC6089 High Volatage Insulation Bushing Design
% Axial grading calculation
% Author - David Mahmoodi
% Date - 14/02/2014
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc; close all; clear all
```

```matlab
%% Declareing Given Variables
Voltage = 275000;          %Applied voltage (volt)
Inner_diameter = 100;      %mm
Outer_diameter = 300;      %mm
First_foil_length = 5000; %mm
N = 21;
Foil_Thickness = 0.1;      %mm
First_Gap = 2;             %mm
Last_Gap  = 2;             %mm
E_boundary_surface_Air = 300;   %volt/mm
E_boundary_surface_Oil = 3*300; %volt/mm
%% Defining new variables
Del_Voltage = Voltage/N-1; %Voltage between each foil
b_Air = Del_Voltage/E_boundary_surface_Air;
b_Oil = Del_Voltage/E_boundary_surface_Oil;
%Initialise vectors (22 for 3D plotting)
L = zeros(1,N+1);
L_Air = zeros(1,N+1);
L_Oil = zeros(1,N+1);
Radius = zeros(1,N+1);
R_parameter=1.007; % Parameter for adjesting assumption value of r0
%% Calculation
L(1)=First_foil_length; %The first foil is 5000mm and connected to the conductor, no capacitance between c
r0 = Inner_diameter/2 ; %Radius of the conductor
Radius(1)= Inner_diameter/2  + First_Gap; % Radial position of first foil = 52mm
L_Air(1)= L(1)/2;
L_Oil(1)= L(1)/2;
%Calculate the radial positions of all foils
for i=2:N
    L_Air(i)=L_Air(i-1)- b_Air;
    L_Oil(i)=L_Oil(i-1)- b_Oil;
    L(i)= L_Air(i)+L_Oil(i);
end
% Refer to Section 2.2 for an explanation of this assumption
Radius(2)=  Radius(1)* exp( (L(2)/L(1)) * log(Radius(1)/(r0-R_parameter)));
%Follow the iterative formula
for i=3:N
    Radius(i)= Radius(i-1)* exp( (L(i)/L(i-1)) * log(Radius(i-1)/Radius(i-2)));
end
%For plotting - add the outer shell
L(N+1)=L(N)- .5*L(N);
Radius(N+1)=Radius(N)+ Last_Gap;

%% Ploting
x=zeros(1,2*(N+1));
y=x;  j=1;
for i=1:2:2*(N+1)
    x(i)=L_Air(j);
    x(i+1)=-L_Oil(j);
    y(i)=Radius(j);
    y(i+1)=Radius(j);
    j=j+1;
end
y2=-y;
% 2D Plot
figure
axes('FontSize',16,'fontWeight','bold')
rect_H = rectangle('Position', [-1.1.*x(1),- r0, 2.2*x(1), 2*r0]);
set(rect_H, 'FaceColor', 'r')
for i=1:2:2*(N)
    hold on
    line(-x(i:i+1), -y(i:i+1), 'LineWidth',2)
    line(-x(i:i+1), -y2(i:i+1),'LineWidth',2)
    %axis equal
end
rect_H = rectangle('Position', [L(N+1)/3,Outer_diameter/2, L(N+1)/2, 1000]);
set(rect_H, 'FaceColor',[0, 0, 0])
rect_H = rectangle('Position', [L(N+1)/3,-10-Outer_diameter/2, L(N+1)/2, 10]);
set(rect_H, 'FaceColor',[0, 0, 0])
title('Axial Graing','FontName', 'Times New Roman','FontSize',34,'fontWeight','bold');
xlabel('Length (mm)','FontName', 'Times New Roman','FontSize',24,'fontWeight','bold')
```

```matlab
ylabel('Radius (mm)','FontName', 'Times New Roman','FontSize',24,'fontWeight','bold')
xlim([x(2) x(1)]);ylim([y2(end)-10 y(end)+10])
% 3D Plot
K=50; scl=.1; % Z direction scalling value for plotting
p= 6*N/3+1; %adjusting the Cut in the 3D shap
figure
axes('FontSize',16,'fontWeight','bold')
R=[r0 r0];
[X,Y,Z] = cylinder(R,5*K);
Z(2, :) = (L_Air(1)+ .1*L_Air(1))*scl;
Z(1, :) = - Z(2, :);
surf(X,Y,Z, 'FaceColor',[1,0,0],'EdgeColor', [1,0,0]);
for i=1:N
    hold on
    R=[Radius(i) Radius(i)];
    [X,Y,Z] = cylinder(R,K);
    Z(2,:)= L_Air(i);
    Z(1,:)= -L_Oil(i);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl;
    testsubject = surf(X,Y,Z);
    set(testsubject,'FaceAlpha',0.8,'EdgeColor','b')
    axis equal
end
Ground=Radius(N+1)-1;
for i=1:30
    R=[Ground+i Ground+i];
    [X,Y,Z] = cylinder(R,K);
    Z(2, :) = L(N+1) *scl;
    Z(1, :) = -  Z(2, :);
    X = X(:,1:p);
    Y = Y(:,1:p);
    Z = Z(:,1:p)*scl-100;
    surf(X,Y,Z, 'FaceColor', [0,0,0],'EdgeColor',[0, 0, 0]);
end
camlight
lighting gouraud
title('Axial Graing','FontName','Times New Roman','FontSize',24,'fontWeight','bold');
xlabel('R(mm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','Rotation',90,'HorizontalAli
zlabel('L(cm)','FontName','Times New Roman','FontSize',16,'fontWeight','bold','HorizontalAlignment','right
%% Saving results to file
FID = fopen('AxialVals21.tex', 'w');
fprintf(FID, '\\begin{table}[!htb]\n');
fprintf(FID, '\\label{table:radialvals}\n');
fprintf(FID, '\\begin{center}\n');
fprintf(FID, '\\begin{tabular}{|c||cc|c||cc|}\n');
fprintf(FID, '\\toprule\n');
fprintf(FID, '\\textbf{Foil N.O} & \\textbf{Radius(mm)} & \\textbf{Length(mm)} & \\textbf{Foil N.O} & \\te
Max = floor((N+1)/2); L(N+1)= 00;
for i=1:Max
    fprintf(FID, '%.f & %4.2f & %4.2f & %.f & %4.2f & %4.2f \\\\\n',i,Radius(i),L(i),(i+Max),Radius(i+Max
end
fprintf(FID, '\\bottomrule\n');
fprintf(FID, '\\end{tabular}\n');
fprintf(FID, '\\end{center}\n');
fprintf(FID, '\\caption{Radial Grading Calculations Results}\n');
fprintf(FID, '\\end{table}\n');
fclose(FID);
```