



**KTH Electrical Engineering**

# **Electrotechnical Design (EI2440)**

## **PROJECT TASK:**

### **High Voltage Bushing Design**

*January 2010*



INTRODUCTION .....	1
Background .....	1
What is a Bushing? .....	1
Project task .....	1
BUSHINGS .....	2
Solid Bushings .....	2
Capacitance-Graded Bushings .....	4
Useful theory .....	6
The tangential component of E .....	7
PROBLEMS .....	9
Part 1: Voltage and field distributions around bushings .....	9
Part 2: Voltage and field distributions analysis of the transformer tank side of a transformer bushing. ....	11
Questions .....	12
COMSOL Multiphysics .....	13
Getting started .....	13
Draw mode .....	13
Boundary mode .....	13
Subdomain mode .....	13
Mesh mode .....	14
Post mode .....	14
REPORTING .....	16
General notes .....	16
Part 1 .....	16
Part 2 .....	17
APPENDIX A .....	19



# INTRODUCTION

The task of this project is to analyze and optimize the voltage distributions along the surface of the insulating parts of a transformer bushing. The model will only include the electrical properties of the bushings. All calculations will be carried out using MATLAB<sup>®</sup> and a special software for finite element calculations called COMSOL Multiphysics<sup>®</sup> (CMPH). This material is best used in the following way:

1. Read these notes through carefully.
2. Solve the problems and find the answers to the questions in the Problems section with the help from the instructions in the COMSOL Multiphysics section.
3. Write a report on your work using the instructions given in the Reporting section. The report should include the answers to the questions and illustrative plots with explanatory figures where appropriate.

## Background

### *What is a Bushing?*

An electrical bushing can be explained as an apparatus for transmitting power in or out of enclosures, i.e., barriers, of an electrical apparatus such as transformers, circuit breakers, shunt reactors and power capacitors. ANSI/IEEE Std. C57.19.00 (1997) defines an electrical bushing as “an insulating structure, including a through conductor or providing a central passage for such conductor, with provision for mounting a barrier, conducting or otherwise, for the purpose of insulating the conductor from the barrier and of an electrical current from one side of the barrier to the other.”

Since electrical power is the product of voltage and current, the insulation in a bushing must be capable of withstanding the voltage at which it is applied, and its current carrying conductor must be capable of carrying rated current without overheating the adjacent insulation. The bushing must also be able to withstand the various mechanical forces applied to it and in some cases it must also provide sealing (e.g. for transformer oil). However, in this project task we are only concentrating on the electrical properties.

When designing a bushing, the task is basically to lead a high voltage conductor through an, often grounded, barrier. This leads to high electrical stresses that can lead to an electrical breakdown in the bushing insulation due to high electrical fields. In this project we will focus on reducing these fields by means of rational design.

### *Project task*

The task in this project is to analyze and optimize the voltage distributions around transformer bushings by looking at simplified geometries of the different types of bushings.

The project task only involves the electrical properties of the bushing, ignoring the mechanical, chemical and heat transfer properties.

## BUSHINGS

Bushings come in many shapes and sizes and utilize different techniques to reduce electrical stress. However, two main bushing types can be identified, the solid bushings and the capacitance graded bushings. These two bushing types will be described below.

### Solid Bushings

In its simplest form, a bushing is a cylinder of insulating material, such as porcelain, glass, resin-bounded paper, cast resin, polyethylene or hard rubber. The middle of the cylinder is attached to the grounded barrier and covers the high-voltage conductor as shown in Figure 1.

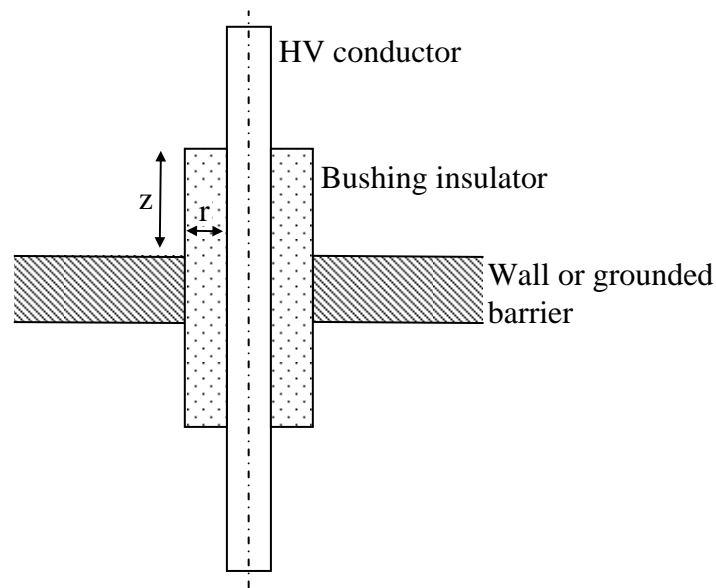


Figure 1. Solid type bushing.

The insulation of a bushing is put under stress both radially and axially, as shown in Figure 2. In this respect the boundary surface between the insulating material and the surrounding medium constitutes a critical area.

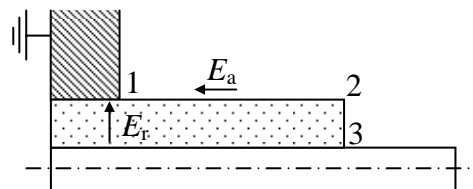


Figure 2. Axial and radial stresses on the bushing insulation.

The radial component,  $E_r$ , of the electric field strength can cause a breakdown in the insulating material. The axial component,  $E_a$ , can under certain circumstances lead to surface discharges along the boundary surface. Since the electrical breakdown

strength of the insulating material itself is much higher than the flashover limit on its surface, the axial stress is generally far more critical. Therefore, considerable attention must be paid to a proper shaping of the boundary so that the critical value of the inception voltage for surface partial discharges is not exceeded.

The lower limit for the required size of the bushing in Figure 2 is calculated in the following way:

$$U = \int_{\text{conductor}}^{\text{ground}} E_t ds = \int_1^2 E_a ds + \int_2^3 E_r ds \quad (1)$$

where,

$U$  = conductor voltage [V].

$E_t$  = the tangential component of the electric field  $\mathbf{E}$  [V/m].

$E_a$  = the axial component of the electric field [V/m].

$E_r$  = the radial component of the electric field [V/m].

The average value of the tangential component of  $\mathbf{E}$  is therefore:

$$\langle E_t \rangle = \frac{U}{L} \quad (2)$$

where,

$L$  = length of the surface of the bushing from the ground surface to the conductor [m],  
i.e.  $L = r + z$ , in Figure 1.

For a given critical value for  $E_{\text{critical}}$  and a given voltage  $U$ ,  $L = U / E_{\text{critical}}$  gives a lower limit for the length of the surface of the bushing. In order not to increase the dimension of the bushing too much, the surface is usually curved as illustrated in Figure 3.

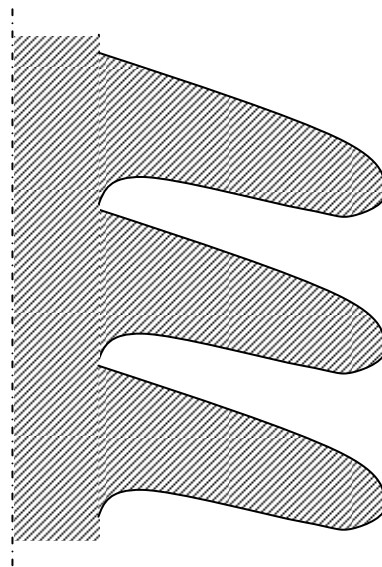
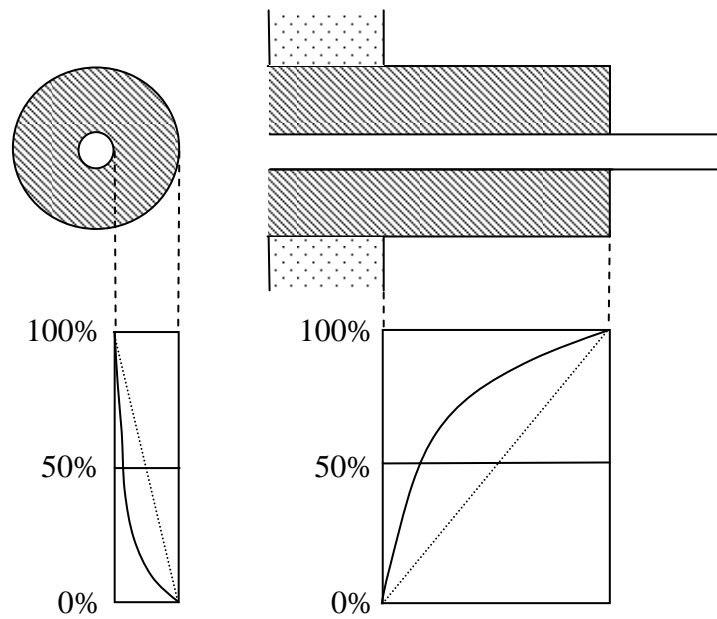


Figure 3. The curved surface of an insulator.

One aim when designing a high-voltage bushing is to obtain an optimal field distribution along the bushing surface, this is done by getting the tangential field component constant along the surface. The reason for this is that a lower field at some point will inevitably lead to a higher field somewhere else along the surface. So, clearly the best design is obtained when the tangential component is constant along the surface, with the value  $E_{optimal} = U / L$ .

However, for a simple cylindrical bushing,  $E_t$  is far from being constant as indicated in Figure 4.



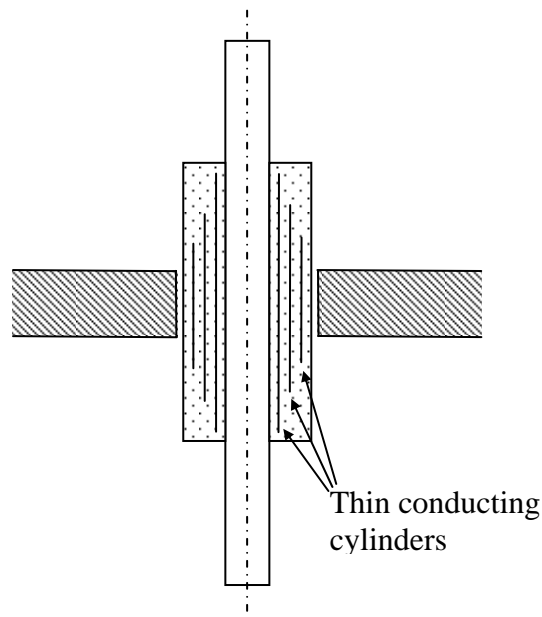
*Figure 4. The voltage distribution across the insulation.*

The voltage is neither evenly distributed across the thickness,  $r$ , of the insulation wall nor along the length of the insulation,  $z$ . Furthermore, as the voltage increases, the dimensions required for this type of bushing become so large that higher voltage applications are not feasible. A better solution is the so-called capacitance-graded or field stress controlled bushing principle.

## Capacitance-Graded Bushings

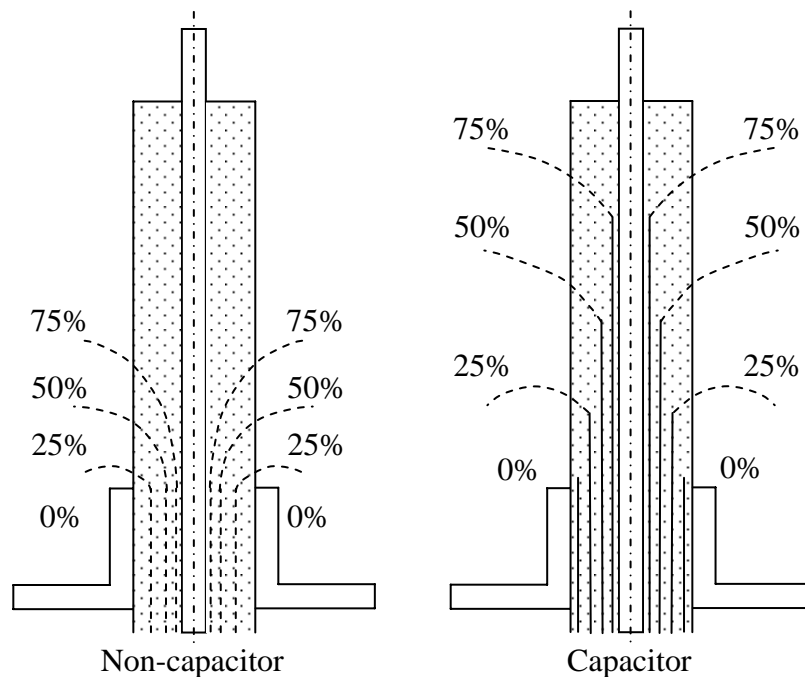
In the capacitance-graded bushings a number of capacitors, in the form of concentric conducting cylinders, are inserted into the bushing, see Figure 5.





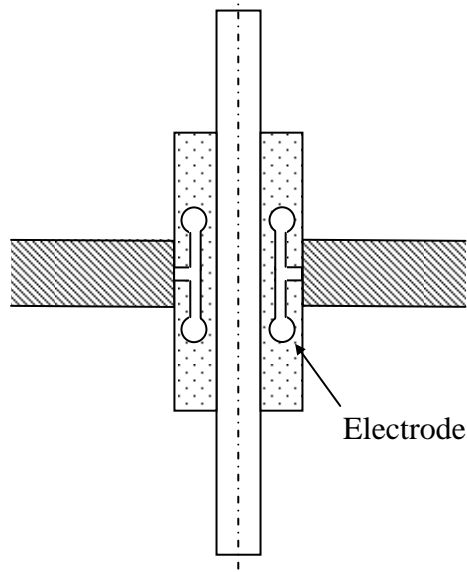
*Figure 5. Capacitance-graded bushing.*

The conducting cylinders have the effect of “spreading out” the voltage gradient more evenly. The comparative voltage distribution in a capacitor and a non-capacitor is demonstrated in Figure 6.



*Figure 6. Voltage distributions of a non-capacitor bushing (left) and of a capacitor bushing (right).*

The capacitive voltage grading is usually used for paper-oil insulated bushings for rated voltages over 60kV. For lower rated voltages, field control carried out by special electrodes is preferred. Such a solution is illustrated in Figure 7.



*Figure 7. Field control by special electrodes.*

In practical cases, external conditions on both sides of the barrier are different, and the external insulation materials of the bushing may therefore also be different. For transformers, mineral oil insulation is typically used inside the tank and atmospheric air is used outside. In such cases the bushing also provides sealing.

Theoretical considerations concerning capacitive voltage grading of a bushing are presented in Chapter 4.3.3 of the textbook “High Voltage Engineering” by E. Kuffel and W.S. Zaengl. This chapter is re-printed in Appendix A.

## Useful theory

This project assignment will only focus on the axisymmetric electrostatic (DC) case. In electrostatic problems, everything is constant with respect to time, and magnetic fields are not studied. The relevant equations are:

$$\mathbf{E} = -\nabla U \quad (3)$$

$$\nabla \cdot \mathbf{D} = \rho_f \quad (4)$$

$$\mathbf{D} = \epsilon \mathbf{E} \quad (5)$$

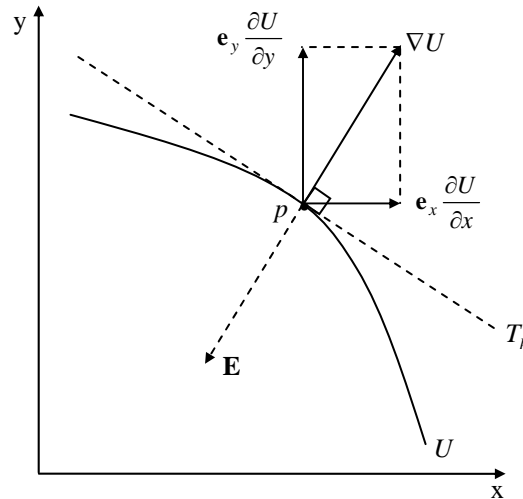
Moreover, since there is no charge density, the equations can then be combined into the Laplace equation:

$$\nabla \cdot \epsilon \nabla U = 0 \quad (6)$$

In order to find the electric field distribution around a bushing, the electrostatic Laplace equation has to be solved.

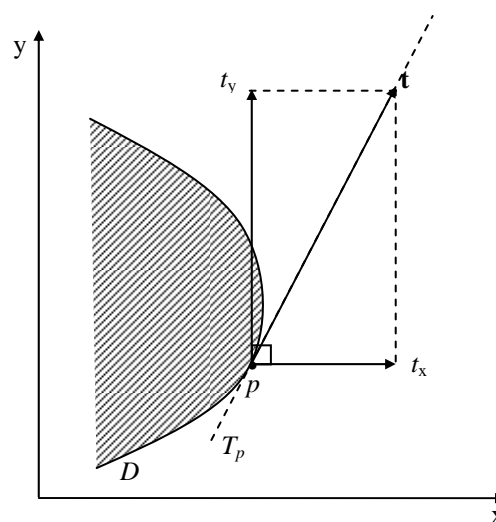
### ***The tangential component of $\mathbf{E}$***

In the problem section you will be asked to investigate and plot the tangential component,  $E_t$ , of  $\mathbf{E}$  along different surfaces of the bushing. Before getting into this it is important to understand what is meant by the tangential component.



*Figure 8. The tangent to an equipotential line.*

Consider the equipotential line,  $U$ , plotted in the orthogonal coordinate system  $(x,y)$  in Figure 8. Here the line  $T_p$  is tangential to the potential line  $U$  in point  $p$ . From the figure it is clear that the gradient of the scalar potential  $\nabla U$  is perpendicular to the tangential line and that also holds for  $\mathbf{E} = -\nabla U$ .  $\nabla U$  is made up of the two components  $\mathbf{e}_x \frac{\partial U}{\partial x}$  and  $\mathbf{e}_y \frac{\partial U}{\partial y}$ , where  $\mathbf{e}_x$  and  $\mathbf{e}_y$  are unity vectors in the  $x$  and  $y$  directions. This situation is quite different from when plotting the tangential component,  $E_t$ , of  $\mathbf{E}$  along a surface of the geometry.



*Figure 9. The tangential component along a geometrical surface.*

In Figure 9 the line  $D$  represents an arbitrary surface of a bushing geometry. Here,  $\mathbf{t}$  is a vector of unit length along the tangent,  $T_p$ , in the point  $p$  on the line  $D$ . In this case  $p$  is on the surface of the bushing geometry. From Figure 9 it is clear that

$$\mathbf{t} = \mathbf{e}_x t_x + \mathbf{e}_y t_y = (t_x, t_y) \quad (7)$$

By inspecting Figure 8 it can be observed that

$$\mathbf{E} = -\nabla U = -\left(\mathbf{e}_x \frac{\partial U}{\partial x} + \mathbf{e}_y \frac{\partial U}{\partial y}\right) = -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}\right) \quad (8)$$

Hence, the tangential component of  $\mathbf{E}$  can be written as

$$E_t = \mathbf{E} \cdot \mathbf{t} = -(t_x, t_y) \cdot \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}\right) \quad (9)$$

which means that  $E_t$  can be re-written as

$$E_t = -\left(t_x \frac{\partial U}{\partial x} + t_y \frac{\partial U}{\partial y}\right) \quad (10)$$

## PROBLEMS

The project is divided into two parts. In Part 1 you will analyze and improve previously prepared geometries of simple bushings. In Part 2, you will carry out an analysis of a real and more complicated bushing.

### Part 1: Voltage and field distributions around bushings

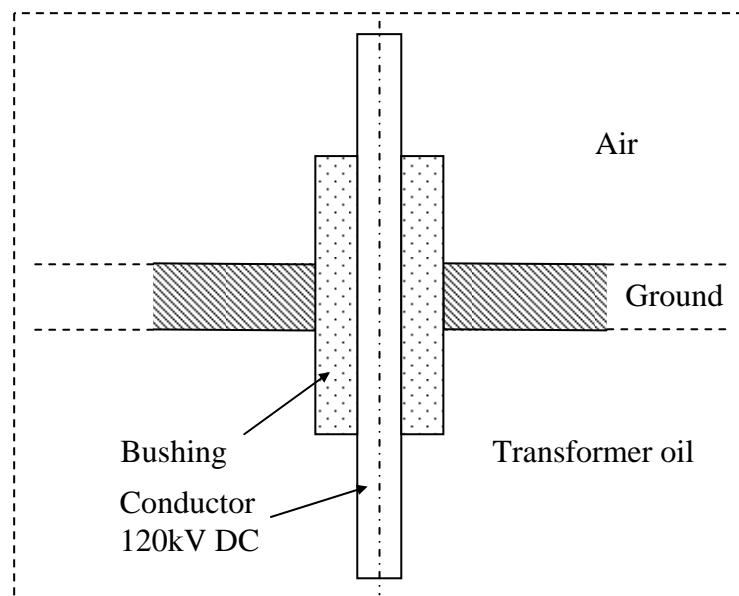
In this part you will analyze and improve previously prepared geometries of simple bushings. The bushing geometries are stored in `\\prosit\public\EI2440\bushing` under the file names: `bushing1.mph`, `bushing2.mph` and `bushing3.mph`.

You will be required to solve a number of problems and answer a number of questions in this part. The solutions to the problems and answers to the questions will be found by investigating the prepared bushing types using COMSOL Multiphysics (CMPH).

Answers to all four questions below are required for each of the three bushing types:

- |                               |                |
|-------------------------------|----------------|
| 1) no voltage grading         | (bushing1.mph) |
| 2) capacitive voltage grading | (bushing2.mph) |
| 3) electrode voltage grading  | (bushing3.mph) |

In each of the cases the environment for the bushing is as illustrated in Figure 10.



*Figure 10. Environment for the bushings in Part 1.*

The simple constructions provided are far from optimal. Therefore, in question 4, you should try to improve them by modifying the geometries. A good construction is characterized by:

- Fulfillment of the electrical field demand.
- Sufficient mechanical strength. (not part of this project)

- Sufficient thermal strength. (not part of this project)
- A simple design, such that manufacturing cost is kept down.
- Low material consumption, such that the material cost is kept low.

For all bushings, you can for instance change the angle of the surface, round off corners, etc. In addition, for the bushings with the capacitive grading, you can try to change the length, number and location of the gradings. For the bushing equipped with the electrode, you can alter the shape of the electrode. Useful design hints are given in section 4.2.4 in the main course compendium “Electrotechnical modeling and design”. Fig 4.42 and Fig 4.43 will facilitate your understanding.

## Questions

***Q1:***

**Determine the appropriate boundary conditions. Explain and motivate your answer.**

***Q2:***

**Determine the appropriate materials for the different regions. Use Bakelite for the bushing material.**

***Q3:***

**Plot the equipotential lines. Plot and investigate the voltage and the tangential electric field distributions along the surface of the bushing.**

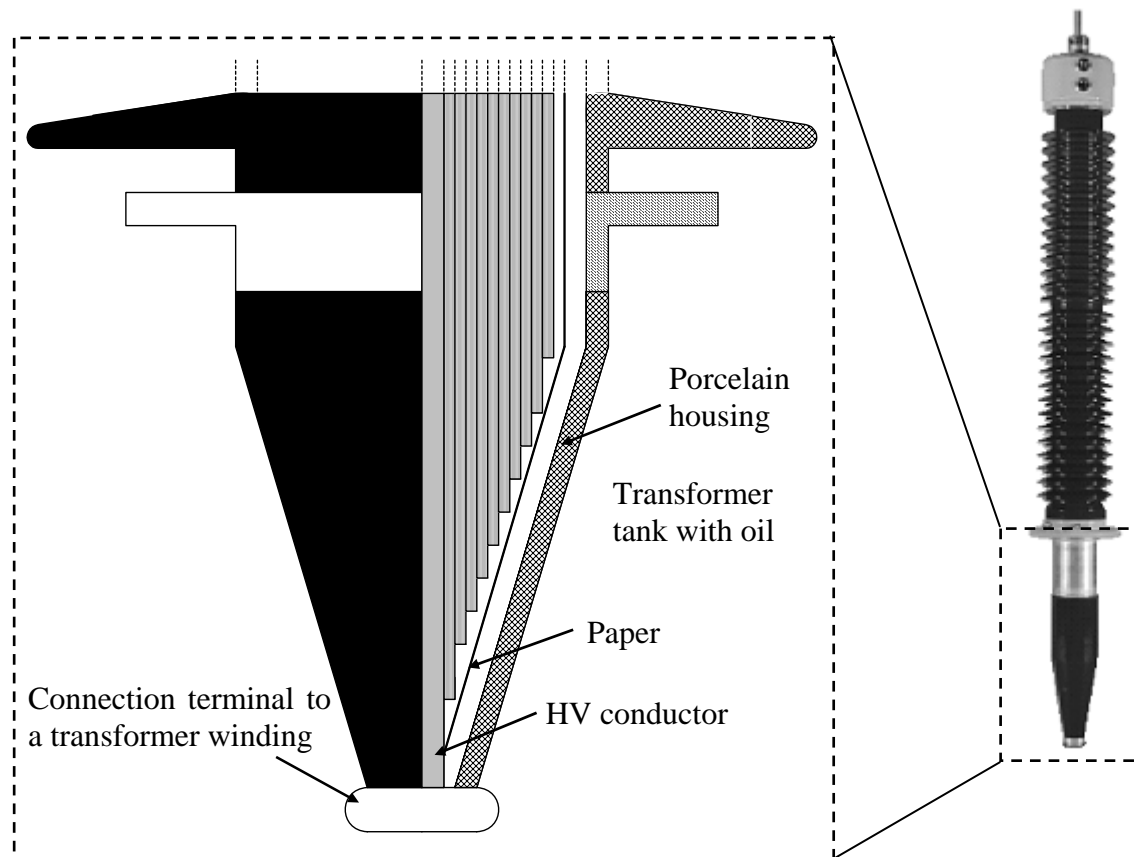
**Check whether any of the analyzed constructions fulfills the requirement that the tangential component of the electric field should not exceed 230 kV/m at any point of the bushing surface.**

***Q4:***

**Modify the geometries to improve the voltage and field distributions. Illustrate by using plots and explanatory figures. Try to fulfill the 230 kV/m requirement from Q3, at least for the bushing types with grading. Be creative. However, try not to increase the overall size of the bushings too much; this is especially important when it comes to the diameter of the bushing.**

## Part 2: Voltage and field distributions analysis of the transformer tank side of a transformer bushing.

The geometry for this problem represents a part of a real transformer bushing (courtesy of ABB Components, Ludvika) and is stored under the file name “bushing4.mph”. The details of the geometry are explained in Figure 11.



*Figure 11. A real transformer bushing.*

The boundary conditions and the materials have already been set. Your task is to generate and analyze the solution. Answer the four questions below and illustrate with plots and explanatory figures where appropriate.

Study the geometry; take note of the pre-selected materials and boundaries. Spend some time on choosing a good method of creating the mesh. Generate a solution.

### *Questions*

**Q5:**

Make plots and observations of the equipotential lines. Check for areas in the construction which are highly stressed by the electric field, and refer to plots showing their geometrical locations.

**Q6:**

Plot and comment on the field strength in the area where the HV conductor comes close to porcelain and oil insulation, make comments. Observe and interpret field values at the gap between the porcelain insulator and the metal flange.

**Q7:**

Plot and comment on the voltage and tangential electric field distributions along the paper and porcelain surfaces of the bushing.

**Q8:**

Assume E-field safety limits of:

- a) 8500 kV/m for the electric strength of the oil.
- b) 4200 kV/m (for the tangential component of the electric field) for the interface between the oil and the bushing solids.

Is the construction meeting these two criteria?



## COMSOL Multiphysics

This section is meant to be supplementary to the COMSOL Multiphysics (CMPH) introduction and will only focus on points worth considering when solving the problems presented in this project.

For further help on CMPH please consult the introduction given in section 3.6.4 in the main course compendium “Electro technical modeling and design”.

### *Getting started*

Before doing anything else you should copy all the files in the folder \\prosit\public\EI2440\Bushing and paste them into your own personal folder under \\prosit\Users\”username”. All together, it should be six (6) files for this project task.

To start CMPH click the **COMSOL 3.5 with Matlab** shortcut on the desktop. A dialog box **Model navigator** appears. In the page **open** find and select the model that you want to open (e.g. bushing1.mph) and click **Ok**.

In order to get access to all materials you must change the search path to the “Material data file”. Start CMPH and open any of the four geometry files that you have copied. Choose **Materials/Coefficients Library** under **Options**. In the new dialog box **Materials/Coefficients Library** click the button **Add library**. Enter the search path to where you have stored your copy of the file “mtrl\_lib\_bushing.txt” and click open, if you have copied the file into your personal directory the search path will be \\prosit\Users\”username”\ mtrl lib bushing.txt. The added library (library 8) can now be found at the bottom of the materials list in the dialog box **Materials/Coefficients Library**.

In CMPH there are a number of modes that can be selected either by clicking on of the buttons named by their mode or by **Selection mode** under **Physics**. These modes are described below.

### *Draw mode*

When in draw mode you can study the geometrical configuration of the different bushings and change their geometry.

### *Boundary mode*

In this mode you can apply the boundary conditions that you have determined. Please note that it is not necessary to use **Interior boundaries** in this project.

### *Subdomain mode*

The subdomain mode is where you set the appropriate materials. In Part 1 you are asked to select the different materials for the bushings. The materials are predefined and can be selected by double-clicking the subdomain of interest in the geometry, when in subdomain mode. In the dialog box **Subdomain settings**, found under **Physics**, click the **Load** button for library materials. A new dialog box with a list of pre-defined materials will appear.

Select the material in the list on the left hand side by marking it and then click **Apply**. Repeat until all the appropriate materials are added, and then click **Ok**. All the

different materials you have chosen are now available in the dropdown list **Library material**. Assign the appropriate materials to the different domains.

If you do not wish to use a pre-defined material, you can assign your own values for the different coefficients. Values for the most commonly used materials are easily obtained from the literature.

In electrostatic problems, one often wishes to include conductors. This raises the question of what value should be used for the permittivity. For such materials,

$$\sigma \mathbf{E} = \mathbf{j} = \nabla \times \mathbf{H} + \frac{\partial \mathbf{D}}{\partial t} \quad (11)$$

which means that they simply do not obey any law of the type  $\mathbf{D} = \epsilon \mathbf{E}$ . However, one can use a fictitious permittivity and get the correct results as follows:

In a conductor there is (virtually) no drop in voltage so  $\nabla U = 0$ . By setting  $\epsilon = \infty$  for such regions in the Laplace equation, hence  $\nabla U$  is forced to zero. In CMPH, this can be simulated by choosing a very large value for  $\epsilon_r$  in conductors, for instance  $10^8$ . *It is necessary, **for you**, to perform this assignment to the conducting materials in the capacitive and the electrode graded bushings.*

(A rigorous proof of the correctness of this technique is somewhat complicated and is outside the scope of this course.)

### **Mesh mode**

Select **Mesh mode** under **Mesh** and triangulate the geometry mesh and refine it around the points of interest. If you are uncertain whether the mesh is good enough, you should, after solving and viewing the solution, go back and refine the mesh and generate a new solution and spot the difference. This procedure should be repeated until there is no significant difference between the solutions.

### **Post mode**

When involved in power system component design it is common to plot voltage or field distributions along a surface. In CMPH, this is achieved by selecting **Domain Plot Parameters** under **Postprocessing**.

In order to plot the voltage distribution along a surface, select **Line plot** under the **Line/Extrusion** tab. Select the boundaries that you are interested in and choose the predefined quantity **Electric potential** before pressing **Apply**.

Plotting the tangential component of the **E**-field along a surface is slightly more complicated. Follow the same procedure as for plotting the voltage distribution but instead of using the predefined line expression **Electric potential** you should put your own expression (compare this to equation 10):

$$-t_x * V_x - t_y * V_y$$

where  $t_x$  and  $t_y$  are components of the tangential vector on the boundaries and  $V_x$  and  $V_y$  are the derivatives of the potential. The minus signs are present because

$$\mathbf{E} = -\nabla U.$$

However, since we are only interested in the absolute value of  $E$ , the following expression should be used instead:

$$\text{abs}(t_x * V_x + t_y * V_y)$$

When plotting along surfaces it is very important that you pay attention to the direction along which you are plotting. If you find it difficult to interpret the results it may be useful to plot one boundary at a time.

Furthermore, it is often of interest to find the maximum field in a subdomain. This cannot be done via the GUI (even though it is possible to see the global max and min values in a surface plot by ticking the **Max/Min marker** box in the dialog box **Plot Parameters**). All the necessary information is stored in the CMPH data structure and it is just a matter of extracting the data. In order to do this you can use a script prepared for this purpose. However, first you need to export your FEM-structure by pressing CTRL+f in CMPH. The next step is to open the script called “max\_in\_subdomain.m” stored under `\\prosit\public\EI2440\bushing`. In the second line of the script file you set which subdomain to be studied. After deciding which subdomain you want to study, save the m-file and switch to the MATLAB window.

Run the m-file “max\_in\_subdomain.m” by typing:

```
>>max_in_subdomain
```

The maximum and minimum values of the **E**-field will be displayed. For more information on how this works, study the m-file and note the comments included.

## REPORTING

### *General notes*

For this project you need to hand in a report describing your work, to be accepted the report must fulfill the requirements stated in the course information material regarding report writing. The report shall include answers to all the questions in the problem section. In Part 1 you need to answer the questions for all three different bushing types.

The report should be short and does not need to include any theoretical background; however, a short introduction of two or three lines is required.

All figures must be properly explained and be relevant. There should not be any uncommented figures. Furthermore, it is very important that you explain what the relationship is between the plot and the geometry, see Figure 12 for an example.

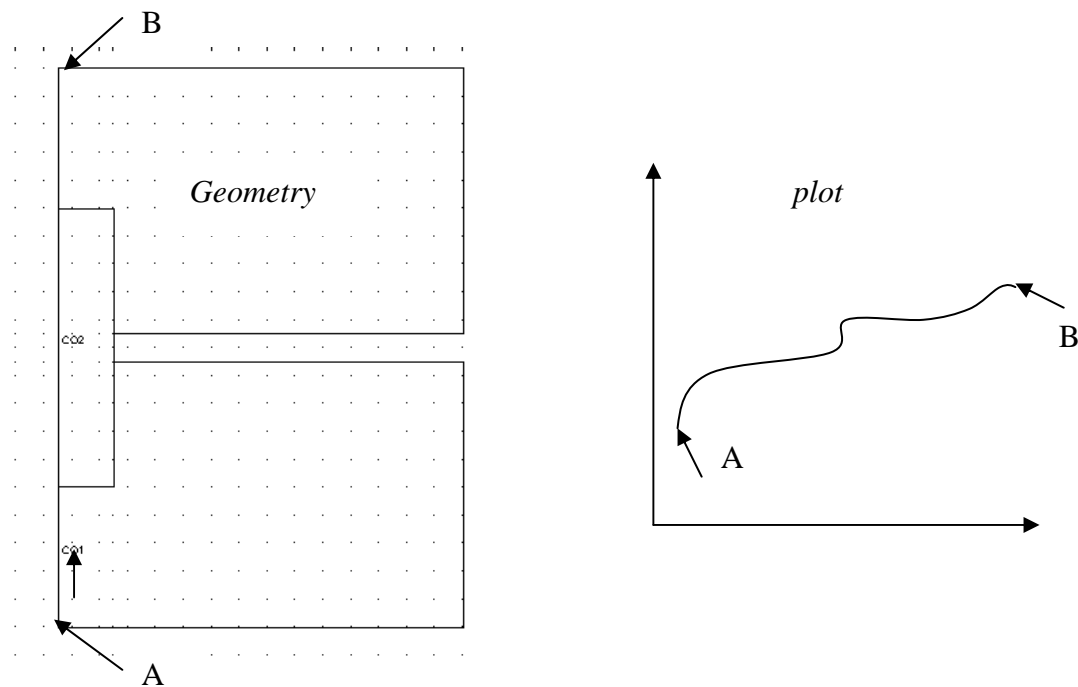


Figure 12. Example of how to explain relationship between the plot and the geometry.

The **Export image** button in the CMPH figure window is very useful for exporting CMPH images to your report.

### *Part 1*

- Write a very brief introduction, including the purpose of the project.
- Explain and motivate your selections of boundary conditions and the materials for the different regions.

- Include for **each original** bushing design:
  - plot of the equipotential lines
  - plot of voltage distribution along the bushing surface
  - plot of tangential electric field distribution along the bushing surface
- Refer to the plots of the field distribution lines along the bushing surface and indicate the geometric locations of the ‘hot spots’ in a separate figure. Why is the field so high in these spots? Do any of the analyzed constructions fulfill the requirements for the tangential electric field?
- Include for **each improved** bushing design:
  - plot of the equipotential lines
  - plot of voltage distribution along the bushing surface
  - plot of tangential electric field distribution along the bushing surface
- Explain the underlying principle behind your modifications. Also, explain any unsuccessful modifications that you tried and why they did not work.
- Present numeric values corresponding to the given and modified design regarding the maximal tangential electric field along air/oil and solid insulation materials. Summarize your results in a table like the one below:

	Before modification	After improvement	Acceptance level	Requirement fulfilled?
Bushing 1				
Bushing 2				
Bushing 3				

## **Part 2**

- Make plots and observations of the equipotential lines. Identify areas in the construction which are highly stressed by the electric field, and refer to plots showing their geometric locations.
- Make plots of the field strength where the HV conductor comes close to the porcelain and oil insulation. Why is it like this? Make comments. Also, comment on the field values at the gap between the porcelain insulator and the metal flange.
- Plot and comment on the voltage and tangential electric field distributions along the paper and porcelain surfaces of the bushing.
- Identify where the bulk oil is stressed maximally, or where it is stressed above the safety limit of 8500 kV/m. Refer to appropriate plots indicating the geometrical location of these hot spots.

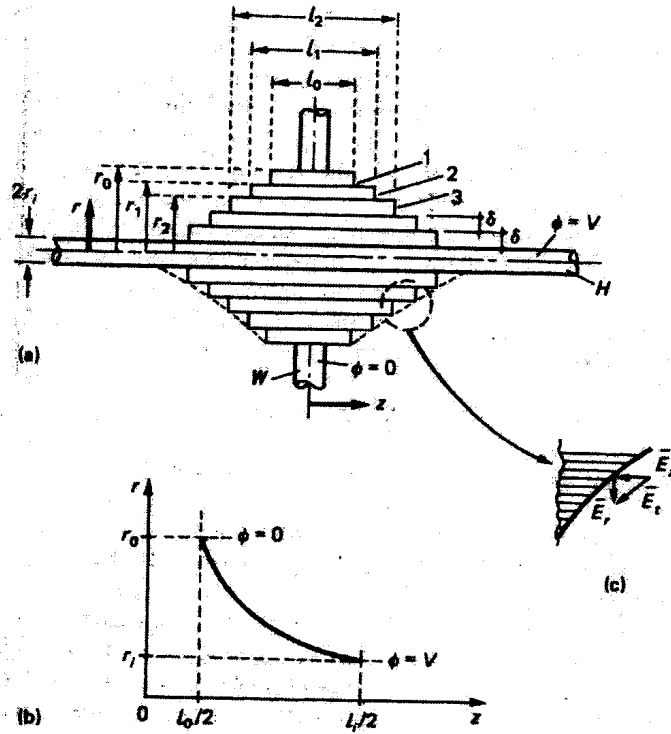
- Identify where in the structure the tangential electric field along oil and bushing solid interface is maximal or higher than the safety limit of 4200 kV/m, and indicate the geometric location of these 'hot spots' in a separate figure. Is the construction meeting the criteria's for the tangential electric field safety limit?
- Present your analyses in a table with references to appropriate plots.

	Result from CMPH	Safety limit	Requirement fulfilled?
Maximal <b>E</b> -field in the oil			
Maximal tangential field along oil/bushing solid interfaces			

## APPENDIX A

*Taken from "High Voltage Engineering" by E. Kuffel and W.S. Zaengl.*

Let the cylindrical high-voltage conductor H be surrounded by many layers of thin dielectric sheets of permittivity  $\epsilon$  ( $\epsilon > \epsilon_0$ ). Fig. 1a shows a simplified cross-sectional view of such an arrangement, in which the dielectric sheets of different lengths are interleaved with thin conducting foils providing the floating electrodes. We may treat this system as an arrangement of coaxial cylindrical capacitor units which are series connected. As indicated, the length  $l_0, l_1, \dots, l_n$  of the sheets is increasing from the barrier W to the centre conductor H, and the conditions for the different lengths can be provided by boundary conditions.



*Figure 1. Capacitor bushing. (a) Coaxial capacitor arrangement. (b) Profile of conductive foils for constant radial field intensity  $E_r$  (mean value). (c) Definition of field intensity components*

Let us assume the boundary condition for which the mean value of the field intensity  $E_r$  acting within the sheet remains constant. If every sheet is of equal thickness  $\delta$ , each of the coaxial units is stressed by equal voltages

$\Delta V = E_r \delta$ , if all capacitances are equal. Then  $C_1 = C_2 = \dots = C_n$  with

$$C_1 = \frac{2\pi\epsilon l_0}{\ln\left(\frac{r_0}{r_1}\right)}$$

$$C_2 = \frac{2\pi\epsilon l_1}{\ln\left(\frac{r_1}{r_2}\right)}$$

or

$$\frac{l_0}{\ln\left(\frac{r_0}{r_1}\right)} = \frac{l_1}{\ln\left(\frac{r_1}{r_2}\right)} = \dots = \frac{l_n}{\ln\left(\frac{r_n}{r_{n+1}}\right)} \quad (1)$$

Apart from this exact solution, an approximation is possible for thin sheets. Then  $r_{n+1} = r_n - \delta$  and  $(\delta/r_n) \ll 1$  even for the smallest radius  $r_i$  of the inner conductor, yielding

$$\ln\left(\frac{r_n}{r_{n+1}}\right) = \ln\left(\frac{1}{1 - \frac{\delta}{r_n}}\right) \approx \frac{\delta}{r_n}$$

With this approximation, eqn. (1) becomes

$$l_0 r_0 \approx l_1 r_1 \approx \dots \approx l_n r_n \quad (2)$$

where  $0 \ll n \ll N$ , with  $N$  equal to the total number of sheets. As  $N$  is quite high, we may replace the discrete numbers  $l_n$  and  $r_n$  by the variables  $z = l/2$  and  $r$ . Equation (2) then defines a two-dimensional profile or contour of the conducting foil edges as sketched in Fig. 1b. The given boundary condition provides a hyperbolic profile, along which the potential  $\Phi$  increases steadily between  $r_0$  and  $r_i$ . Neglecting the actual discontinuities of the field intensities produced by the conducting foils, we can now recognize the constant value of the mean radial field strength  $E_r$ .

Whereas  $E_r$  stresses the insulation material of the sheets only, an even more significant component of a field intensity is introduced by the conducting foil edges, as sketched in Fig. 1c. The solid material from the active part of the capacitor bushing also shares a boundary with the surrounding dielectric material, in general atmospheric air or mineral oil. This interface is heavily stressed by a tangential field intensity  $E_t$ , which has the components of  $E_r$  and  $E_z$ , the latter defined as a mean value of the potential difference  $\Delta\Phi$  between each adjacent foil and the increase  $\Delta l = 2\Delta z$  in sheet length, i.e.  $E_z = \Delta\Phi/\Delta l$ . For the small values of  $\Delta\Phi$ ,  $\Delta l$  and  $\Delta r$  provided by the large amount of sections, we may write in differential terms

$$d\Phi = -E_r dr = 2E_z dz$$



$$E_z = -\frac{1}{2}E_r \frac{dr}{dz} \quad (3)$$

with the boundary conditions given by eqn. (2)

$$E_r = V/(r_0 - r_i) \quad \text{and} \quad dr/dz = -(2/l_0 r_0)r^2$$

and thus

$$E_z = \frac{V}{(r_0 - r_i)l_0 r_0} r^2$$

This dependency shows the strong increase of the axial field strength with increasing diameter of the dielectric sheets. It contributes to a nonhomogeneous potential distribution at the surface of the laminated unit, as the mean value of the tangential field intensity is

$$E_T = \sqrt{E_r^2 + E_z^2}$$

if the surface is very close to the foil edges.  $E_t$  is therefore, still highest in the vicinity of the grounded barrier, and the optimum value of the flashover voltage will not be reached.

In practice, such a dimensioning of a capacitor bushing due to constant mean values of the radial field intensity is not adequate and the calculations performed only indicate the problems. One could readily see however, that the conducting foils can be used to control the internal fields  $E_r$  as well as the field strength distribution along the boundaries  $E_t$  and that it will not be possible to keep both these values constant. The dimensioning of bushing thus becomes a very difficult task, if also other important factors are taken into account. First, the surrounding insulation materials cannot be neglected. Secondly, high-voltage bushings, in general are not provided by oil- or resin-impregnated kraft paper. Protection of the active part is provided by porcelain or other solid insulation material housing, having different permittivities and introducing additional field refraction at the interfaces of the differing materials. Due to the heat generated within the high-voltage conductor H, the permissible radial field intensity may be lower in that region than within the outer regions. Finally, careful attention must be paid to the edges of the conduction foils which form regions of locally high fields. Analytical computations of bushing design are, therefore, supplemented by numerical computations which take into account the very different boundary conditions.