APS502 Computation Project

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Problem 1

In this problem, the goal is to meet obligation with least total price by buying bonds, thus short selling is assumed to be prohibited.

p_i: price of bond i (given)
x_i: amount of bond i to purchase (decision variable)
C_i: coupon payment of bond i (given)
F_i: face value of bond i (given)

$f_{i,j}$: forward rate between period i and period j (derived from spot rate $\sqrt[j-i]{\frac{(1+s_j)^j}{(1+s_i)^i}} - 1$)

z_i: amount of money to be reinvest at period i (decision variable)

Part 1 Formulation

$$Min. \sum_{i=1}^{13} p_i x_i$$

$$s.t. \sum_{i=1}^{12} C_i x_i + F_{13} x_{13} - z_1 \geq y_1$$

$$\sum_{i=1}^{10} C_i x_i + \sum_{i=11}^{12} (C_i + F_i) x_i + (1 + f_{1,2}) z_1 - z_2 \geq y_2$$

$$\sum_{i=1}^{7} C_i x_i + \sum_{i=8}^{10} (C_i + F_i) x_i + (1 + f_{2,3}) z_2 - z_3 \geq y_3$$

$$\sum_{i=1}^{5} C_i x_i + \sum_{i=6}^{7} (C_i + F_i) x_i + (1 + f_{3,4}) z_3 - z_4 \geq y_4$$

$$\sum_{i=1}^{3} C_i x_i + \sum_{i=4}^{5} (C_i + F_i) x_i + (1 + f_{4,5}) z_4 - z_5 \geq y_5$$

$$\sum_{i=1}^{3} (C_i + F_i) x_i + (1 + f_{5,6}) z_5 \geq y_6$$

$$x_i \geq 0 \; for \; i = 1,2, \dots, 13$$

$$z_i \geq 0 \; for \; i = 1,2, \dots, 5$$

Result:

Optimal position:

```
[8.1818, 0, 0, 0, 5.7774, 2.6202, 0, 0, 6.1298, 0, 0.1180, 0, 3.1180]
```

Optimal function value:

```
2.6400e+03
```

Part 2 Formulation

Same formulation as part 1, except for 1 additional constrain:

$$-\sum_{i=1}^{6} p_i x_i + \sum_{i=7}^{13} p_i x_i \geq 0$$

Result:

Optimal position:

```
[0, 8.4112, 0, 0, 5.5027, 0, 3.3565, 0, 6.3502, 0, 0.3184, 0, 3.3184]
```

Optimal function value:

```
2.6444e+03
```

Part 3 Formulation

Same formulation as part 1, except for 1 additional constrain:

$$-\sum_{i=1}^{6} 3p_i x_i + \sum_{i=7}^{13} p_i x_i \geq 0$$

Result:

Optimal position:

```
[0, 7.1267, 0, 0, 0, 0, 10.4052, 0, 6.4638, 0, 0.4216, 0, 3.4216]
```

Optimal function value:

```
2.6796e+03
```

```
Rank 3 portfolios according to cost (> means better):
                  portfolio 3 > portfolio 2 > portfolio 1
The first portfolio costs the least, and the third portfolio costs the
most.
```

Problem 2 Formulation

Q: covariance matrix (given)

μ: expected return vector (given)

w_i: weight of asset i in portfolio (decision variable)

ε: return lower bound (given)

Part 1 Formulation

$$Min. \frac{1}{2} w^T Q w$$
$$s.t. \quad \mu^T w = \varepsilon$$
$$\sum_{i=1}^{3} w_i = 1$$

Results:

Note: the goal return range is set to be from minimum positive return to maximum positive return, as mentioned in the project handout.

a)

expected return (in order of SPY, GOVT, EEMV), adjusted for percentage return

1.0378	0.2501	0.3826

Standard deviation of return (in order of SPY, GOVT, EEMV), adjusted for percentage return

4.0237	1.0891	3.7072

covariance matrix (in order of SPY, GOVT, EEMV), adjusted for percentage return

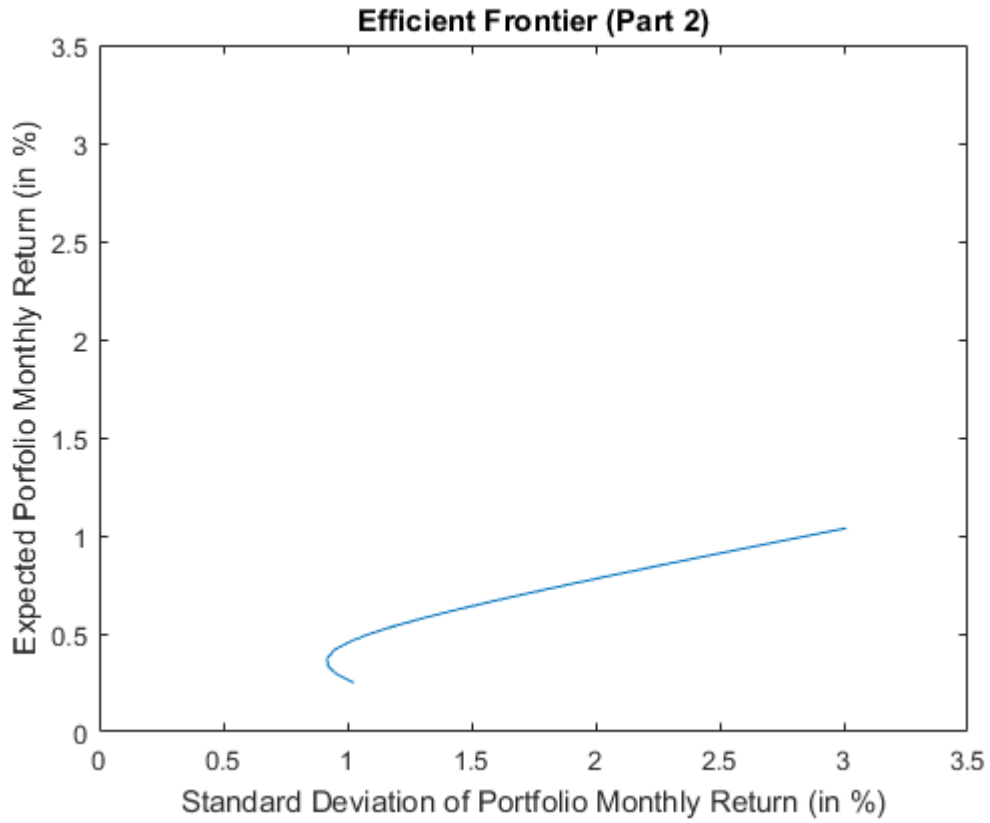
16.1905	-1.4615	10.7078
-1.4615	1.1861	-0.5158
10.7078	-0.5158	13.7437

b)

Table Generated

	1 goal_R	2 w_SPY	3 w_GOVT	4 w_EEMV	5 portf_Variance
1	0.2501	-0.0178	0.9120	0.1059	1.0531
2	0.2915	0.0426	0.8979	0.0595	0.9216
3	0.3330	0.1031	0.8838	0.0131	0.8516
4	0.3744	0.1635	0.8697	-0.0332	0.8430
5	0.4159	0.2239	0.8557	-0.0796	0.8958
6	0.4574	0.2844	0.8416	-0.1260	1.0101
7	0.4988	0.3448	0.8275	-0.1723	1.1858
8	0.5403	0.4052	0.8135	-0.2187	1.4229
9	0.5817	0.4657	0.7994	-0.2651	1.7215
10	0.6232	0.5261	0.7853	-0.3114	2.0815
11	0.6647	0.5865	0.7713	-0.3578	2.5029
12	0.7061	0.6470	0.7572	-0.4042	2.9858
13	0.7476	0.7074	0.7431	-0.4505	3.5301
14	0.7890	0.7678	0.7291	-0.4969	4.1358
15	0.8305	0.8283	0.7150	-0.5433	4.8030
16	0.8720	0.8887	0.7009	-0.5896	5.5316
17	0.9134	0.9491	0.6869	-0.6360	6.3216
18	0.9549	1.0096	0.6728	-0.6824	7.1731
19	0.9963	1.0700	0.6587	-0.7287	8.0860
20	1.0378	1.1304	0.6447	-0.7751	9.0603

Efficient Frontier



c)

Monthly return for only Feb 2021, adjusted for percentage return

2.7805	-1.8414	0.9890

w_portf_1 = [-0.0178 0.9120 0.1059]

return_portf_1 = -1.6241

w_portf_2 = [1/3 1/3 1/3];

return_portf_2 = 0.6427

w_portf_3 = [0.6 0.3 0.1];

return_portf_3 = 1.2148

rank based on return: portfolio 3 > portfolio 2 > portfolio 1

The asset with highest monthly return is SPY, and the asset with lowest monthly return. Portfolio 3 has more weighting in SPY and less weighting in GOVT, compared to porfolio 2. Porfolio 2 has more weighting in SPY and less weighting in GOVT, compared to porfolio 3. Thus porfolio 1 has the highest return.

Part 2 Formulation

$$Min.\frac{1}{2}\, w^T Q w$$
$$s.t.\ \ \mu^T w = \varepsilon$$
$$\sum_{i=1}^{8} w_i = 1$$

Results:

a)

expected return (in order of SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN), adjusted for percentage return

1.0378	0.2501	0.3826	1.3984	1.7924	0.7554	1.2699	1.4927

Standard deviation of return (SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN), adjusted for percentage return

4.0237	1.0891	3.7072	5.3498	5.7911	6.6692	5.2377	5.4869

covariance matrix (in order of SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN), adjusted for percentage

16.1905	-1.4615	10.7078	7.6403	13.7581	8.7350	11.1931	17.4814
-1.4615	1.1861	-0.5158	-1.1065	-0.0266	-0.4340	-1.2496	-1.1570
10.7078	-0.5158	13.7437	1.1394	8.8126	2.9226	3.2744	9.4588
7.6403	-1.1065	1.1394	28.6200	10.3192	20.2593	17.5087	9.6883
13.7581	-0.0266	8.8126	10.3192	33.5367	9.7687	11.3675	19.0056
8.7350	-0.4340	2.9226	20.2593	9.7687	44.4787	16.2277	11.2037
11.1931	-1.2496	3.2744	17.5087	11.3675	16.2277	27.4332	15.0881
17.4814	-1.1570	9.4588	9.6883	19.0056	11.2037	15.0881	30.1063

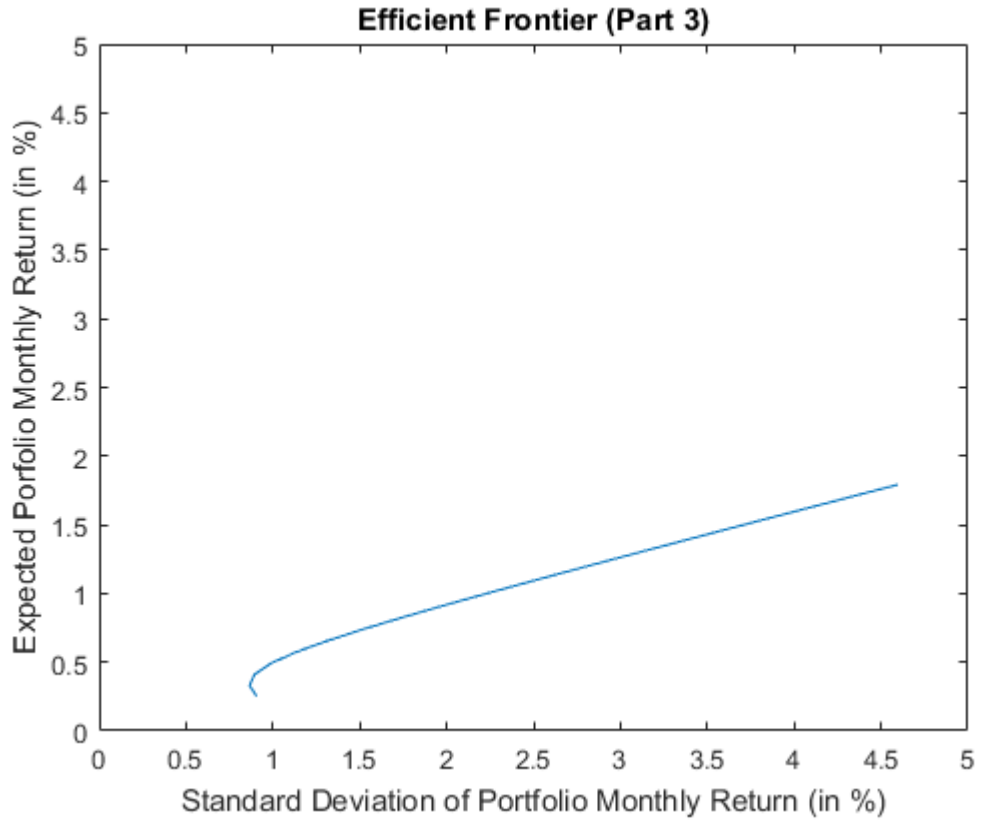
b)

Results:

Table Generated

	1 goal_R	2 w_SPY	3 w_GOVT	4 w_EEMV	5 w_CME	6 w_BR	7 w_CBOE	8 w_ICE	9 w_ACN	10 portf_Variance
1	0.2501	0.1588	0.8929	0.0329	0.0211	-0.0738	-0.0033	0.0121	-0.0406	0.8220
2	0.3312	0.1760	0.8657	-0.0034	0.0407	-0.0463	-0.0147	0.0114	-0.0293	0.7489
3	0.4124	0.1932	0.8386	-0.0397	0.0602	-0.0188	-0.0262	0.0107	-0.0180	0.8027
4	0.4936	0.2104	0.8115	-0.0760	0.0798	0.0087	-0.0377	0.0100	-0.0067	0.9834
5	0.5748	0.2276	0.7843	-0.1123	0.0994	0.0362	-0.0491	0.0094	0.0046	1.2909
6	0.6559	0.2448	0.7572	-0.1485	0.1189	0.0637	-0.0606	0.0087	0.0159	1.7253
7	0.7371	0.2619	0.7301	-0.1848	0.1385	0.0912	-0.0721	0.0080	0.0272	2.2865
8	0.8183	0.2791	0.7029	-0.2211	0.1580	0.1187	-0.0835	0.0074	0.0385	2.9746
9	0.8995	0.2963	0.6758	-0.2574	0.1776	0.1462	-0.0950	0.0067	0.0498	3.7895
10	0.9806	0.3135	0.6486	-0.2936	0.1971	0.1737	-0.1065	0.0060	0.0611	4.7313
11	1.0618	0.3307	0.6215	-0.3299	0.2167	0.2012	-0.1179	0.0053	0.0724	5.7999
12	1.1430	0.3479	0.5944	-0.3662	0.2362	0.2287	-0.1294	0.0047	0.0837	6.9954
13	1.2242	0.3651	0.5672	-0.4025	0.2558	0.2562	-0.1408	0.0040	0.0950	8.3177
14	1.3053	0.3823	0.5401	-0.4388	0.2754	0.2837	-0.1523	0.0033	0.1063	9.7669
15	1.3865	0.3995	0.5129	-0.4750	0.2949	0.3112	-0.1638	0.0027	0.1176	11.3429
16	1.4677	0.4166	0.4858	-0.5113	0.3145	0.3387	-0.1752	0.0020	0.1290	13.0458
17	1.5489	0.4338	0.4587	-0.5476	0.3340	0.3662	-0.1867	0.0013	0.1403	14.8755
18	1.6300	0.4510	0.4315	-0.5839	0.3536	0.3937	-0.1982	6.3514e-04	0.1516	16.8321
19	1.7112	0.4682	0.4044	-0.6201	0.3731	0.4212	-0.2096	-3.6908e-05	0.1629	18.9156
20	1.7924	0.4854	0.3773	-0.6564	0.3927	0.4487	-0.2211	-7.0896e-04	0.1742	21.1259

Efficient Frontier



```
%% problem 1 part 1
f12 = (1.015 ^ 2 / 1.01) - 1;
f23 = (1.02 ^ 3 / 1.05 ^ 2) - 1;
f34 = (1.025 ^ 4 / 1.02 ^ 3) - 1;
f45 = (1.03 ^ 5 / 1.025 ^ 4) - 1;
f56 = (1.035 ^ 6 / 1.03 ^ 5) - 1;
F = 100;
p1 = 108;
p2 = 94;
p3 = 99;
p4 = 92.7;
p5 = 96.6;
p6 = 95.9;
p7 = 92.9;
p8 = 110;
p9 = 104;
p10 = 101;
p11 = 107;
p12 = 102;
p13 = 95.2;
C1 = 10;
C2 = 7;
C3 = 8;
C4 = 6;
C5 = 7;
C6 = 6;
C7 = 5;
C8 = 10;
C9 = 8;
C10 = 6;
C11 = 10;
C12 = 7;
y1 = 500;
y2 = 200;
y3 = 800;
y4 = 400;
y5 = 700;
y6 = 900;
f = [p1 p2 p3 p4 p5 p6 p7 p8 p9 p10 p11 p12 p13 0 0 0 0 0 ]';
A1 = -[C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11 C12 F -1 0 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11+F C12+F 0 1+f12 -1 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8+F C9+F C10+F 0 0 0 0 1+f23 -1 0 0;
    C1 C2 C3 C4 C5 C6+F C7+F 0 0 0 0 0 0 0 0 1+f34 -1 0;
    C1 C2 C3 C4+F C5+F 0 0 0 0 0 0 0 0 0 0 1+f45 -1;
    C1+F C2+F C3+F 0 0 0 0 0 0 0 0 0 0 0 0 0 1+f56];
b1 = -[y1 y2 y3 y4 y5 y6]';
Aeq = [];
beq = [];
lb = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]';
[x1,fval1] = linprog(f, A1, b1, Aeq, beq, lb);
% optimal positions
```

```
% [8.1818, 0, 0, 0, 5.7774, 2.6202, 0, 0, 6.1298, 0, 0.1180, 0, 3.1180]
disp(x1([1:13])');
% optimal function value
% 2.6400e+03
disp(fval1);
%% problem 1 part 2
A2 = -[C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11 C12 F -1 0 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11+F C12+F 0 1+f12 -1 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8+F C9+F C10+F 0 0 0 0 1+f23 -1 0 0;
    C1 C2 C3 C4 C5 C6+F C7+F 0 0 0 0 0 0 0 0 1+f34 -1 0;
    C1 C2 C3 C4+F C5+F 0 0 0 0 0 0 0 0 0 0 0 1+f45 -1;
    C1+F C2+F C3+F 0 0 0 0 0 0 0 0 0 0 0 0 0 1+f56;
    -p1 -p2 -p3 -p4 -p5 -p6 p7 p8 p9 p10 p11 p12 p13 0 0 0 0 0];
b2 = -[y1 y2 y3 y4 y5 y6 0]';
[x2,fval2] = linprog(f, A2, b2, Aeq, beq, lb);
% optimal positions
% [0, 8.4112, 0, 0, 5.5027, 0, 3.3565, 0, 6.3502, 0, 0.3184, 0, 3.3184]
disp(x2([1:13])');
% optimal function value
% 2.6444e+03
disp(fval2);
%% problem 1 part 3
A3 = -[C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11 C12 F -1 0 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8 C9 C10 C11+F C12+F 0 1+f12 -1 0 0 0;
    C1 C2 C3 C4 C5 C6 C7 C8+F C9+F C10+F 0 0 0 0 1+f23 -1 0 0;
    C1 C2 C3 C4 C5 C6+F C7+F 0 0 0 0 0 0 0 0 1+f34 -1 0;
    C1 C2 C3 C4+F C5+F 0 0 0 0 0 0 0 0 0 0 0 1+f45 -1;
    C1+F C2+F C3+F 0 0 0 0 0 0 0 0 0 0 0 0 0 1+f56;
    -3*p1 -3*p2 -3*p3 -3*p4 -3*p5 -3*p6 p7 p8 p9 p10 p11 p12 p13 0 0 0 0 0];
b3 = -[y1 y2 y3 y4 y5 y6 0]';
[x3,fval3] = linprog(f, A3, b3, Aeq, beq, lb);
% optimal positions
% [0, 7.1267, 0, 0, 0, 0, 10.4052, 0, 6.4638, 0, 0.4216, 0, 3.4216]
disp(x3([1:13])');
% optimal function value
% 2.6796e+03
disp(fval3);

%% problem 1 Discussion
% Rank 3 portfolios according to cost: portfolio 3 > portfolio 2 > portfolio 1
% The first portfolio costs the least, and the third portfolio costs the most.

%% problem 2 part 1a)
% expected return (in order of SPY, GOVT, EEMV), adjusted for percentage
mu_1 = 100 * [0.01037794, 0.00250065, 0.00382618];
% covariance matrix (in order of SPY, GOVT, EEMV), adjusted for percentage
cov_1 = 10000 * [0.00161905, -0.00014615,  0.00107078;
       -0.00014615,  0.00011861, -0.00005158;
       0.00107078, -0.00005158,  0.00137437];

% standard deviation
```

```
% [4.0237, 1.0891, 3.7072]
std_1 = sqrt(diag(cov_1))';

% list of 20 return requirements
epsilon_list_1 = linspace(min(mu_1),max(mu_1),20);

%% problem 2 part 1b)
table_data_1 = zeros(20,5);
eff_front_data_1 = zeros(20,2);
table_data_1(1:20,1) = epsilon_list_1;
for i = 1:1:20
    [w,fval] = quadprog(cov_1,zeros(1,3),[],[],[mu_1 ; ones(1,3)],[epsilon_list_1(i);1]);
    table_data_1(i,2:4) = w';
    % adjust for the 1/2 convention
    table_data_1(i,5) = fval * 2;
    eff_front_data_1(i,1) = sqrt(fval * 2);
    eff_front_data_1(i,2) = dot(mu_1,w);
end
plot(eff_front_data_1(:,1),eff_front_data_1(:,2));
title('Efficient Frontier (Part 2)' );
xlabel('Standard Deviation of Portfolio Monthly Return (in %)' );
ylabel('Expected Porfolio Monthly Return (in %)' );
xlim([0 3.5]);
ylim([0 3.5]);

% generate table
T_1 = array2table(table_data_1, 'VariableNames',{'goal_R','w_SPY','w_GOVT', 'w_EEMV', 'portf_Variance'});

%% problem 2 part 1c)
% w_portf_1 = [-0.0178    0.9120    0.1059]
w_portf_1 = table_data_1(1,2:4);
w_portf_2 = [1/3 1/3 1/3];
w_portf_3 = [0.6 0.3 0.1];

% adjusted for percentage
mu_3 = 100 * [0.02780547, -0.01841394,  0.00988976];

% return_portf_1 = -1.6241
return_portf_1 = dot(mu_3,w_portf_1);
% return_portf_2 = 0.6427
return_portf_2 = dot(mu_3,w_portf_2);
% return_portf_3 = 1.2148
return_portf_3 = dot(mu_3,w_portf_3);

% rank based on return: portfolio 3 > portfolio 2 > portfolio 1
% The asset with highest monthly return is SPY, and the asset with lowest
% monthly return. Portfolio 3 has more weighting in SPY and less weighting
% in GOVT, compared to porfolio 2. Porfolio 2 has more weighting in SPY
% and less weighting in GOVT, compared to porfolio 3. Thus porfolio 1 has
% the highest return.
%% problem 2 part 2a)
```

```
% expected return (in order of SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN), adjusted for percentage
mu_2 = 100 * [0.01037794, 0.00250065, 0.00382618, 0.0139841 , 0.01792375,0.00755397, 0.01269948,
0.01492658];
% covariance matrix (in order of SPY, GOVT, EEMV, CME, BR, CBOE, ICE, ACN), adjusted for percentage
cov_2 = 10000 * [ 0.00161905, -0.00014615,  0.00107078,  0.00076403,  0.00137581, 0.0008735 ,  0.00111931,
0.00174814;
          -0.00014615,  0.00011861, -0.00005158, -0.00011065, -0.00000266, -0.0000434 , -0.00012496,
-0.0001157 ;
          0.00107078, -0.00005158,  0.00137437,  0.00011394,  0.00088126, 0.00029226,  0.00032744,
0.00094588;
          0.00076403, -0.00011065,  0.00011394,  0.002862  ,  0.00103192, 0.00202593,  0.00175087,
0.00096883;
          0.00137581, -0.00000266,  0.00088126,  0.00103192,  0.00335367, 0.00097687,  0.00113675,
0.00190056;
          0.0008735 , -0.0000434 ,  0.00029226,  0.00202593,  0.00097687, 0.00444787,  0.00162277,
0.00112037;
          0.00111931, -0.00012496,  0.00032744,  0.00175087,  0.00113675, 0.00162277,  0.00274332,
0.00150881;
          0.00174814, -0.0001157 ,  0.00094588,  0.00096883,  0.00190056, 0.00112037,  0.00150881,
0.00301063];

% standard deviation
% [4.0237, 1.0891, 3.7072, 5.3498, 5.7911, 6.6692, 5.2377, 5.4869]
std_2 = sqrt(diag(cov_2))';

% list of 20 return requirements
epsilon_list_2 = linspace(min(mu_2),max(mu_2),20);

%% problem 2 part 2b)
table_data_2 = zeros(20,10);
eff_front_data_2 = zeros(20,2);
table_data_2(1:20,1) = epsilon_list_2;
for i = 1:1:20
    [w,fval] = quadprog(cov_2,zeros(1,8),[],[],[mu_2 ; ones(1,8)],[epsilon_list_2(i);1]);
    table_data_2(i,2:9) = w';
    % adjust for the 1/2 convention
    table_data_2(i,10) = fval * 2;
    eff_front_data_2(i,1) = sqrt(fval * 2);
    eff_front_data_2(i,2) = dot(mu_2,w);
end
plot(eff_front_data_2(:,1),eff_front_data_2(:,2));
title('Efficient Frontier (Part 3)' );
xlabel('Standard Deviation of Portfolio Monthly Return (in %)' );
ylabel('Expected Porfolio Monthly Return (in %)' );
xlim([0 5]);
ylim([0 5]);

% generate table
T_2 = array2table(table_data_2, 'VariableNames',{'goal_R','w_SPY','w_GOVT', 'w_EEMV', 'w_CME', 'w_BR',
'w_CBOE', 'w_ICE', 'w_ACN','portf_Variance'});
```

	1	2	3	4	5	6
1	-10	-7	-8	-6	-7	-6
2	-10	-7	-8	-6	-7	-6
3	-10	-7	-8	-6	-7	-6
4	-10	-7	-8	-6	-7	-106
5	-10	-7	-8	-106	-107	0
6	-110	-107	-108	0	0	0

	7	8	9	10	11	12
1	-5	-10	-8	-6	-10	-7
2	-5	-10	-8	-6	-110	-107
3	-5	-110	-108	-106	0	0
4	-105	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0

	13	14	15	16	17	18
1	-100	1	0	0	0	0
2	0	-1.0200	1	0	0	0
3	0	0	-0.9625	1	0	0
4	0	0	0	-1.0401	1	0
5	0	0	0	0	-1.0502	1
6	0	0	0	0	0	-1.0604

	1	2	3	4	5	6
1	-10	-7	-8	-6	-7	-6
2	-10	-7	-8	-6	-7	-6
3	-10	-7	-8	-6	-7	-6
4	-10	-7	-8	-6	-7	-106
5	-10	-7	-8	-106	-107	0
6	-110	-107	-108	0	0	0
7	108	94	99	92.7000	96.6000	95.9000

	7	8	9	10	11	12
1	-5	-10	-8	-6	-10	-7
2	-5	-10	-8	-6	-110	-107
3	-5	-110	-108	-106	0	0
4	-105	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	-92.9000	-110	-104	-101	-107	-102

	13	14	15	16	17	18
1	-100	1	0	0	0	0
2	0	-1.0200	1	0	0	0
3	0	0	-0.9625	1	0	0
4	0	0	0	-1.0401	1	0
5	0	0	0	0	-1.0502	1
6	0	0	0	0	0	-1.0604
7	-95.2000	0	0	0	0	0

	1	2	3	4	5	6
1	-10	-7	-8	-6	-7	-6
2	-10	-7	-8	-6	-7	-6
3	-10	-7	-8	-6	-7	-6
4	-10	-7	-8	-6	-7	-106
5	-10	-7	-8	-106	-107	0
6	-110	-107	-108	0	0	0
7	324	282	297	278.1000	289.8000	287.7000

	7	8	9	10	11	12
1	-5	-10	-8	-6	-10	-7
2	-5	-10	-8	-6	-110	-107
3	-5	-110	-108	-106	0	0
4	-105	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	-92.9000	-110	-104	-101	-107	-102

	13	14	15	16	17	18
1	-100	1	0	0	0	0
2	0	-1.0200	1	0	0	0
3	0	0	-0.9625	1	0	0
4	0	0	0	-1.0401	1	0
5	0	0	0	0	-1.0502	1
6	0	0	0	0	0	-1.0604
7	-95.2000	0	0	0	0	0

	1
1	-500
2	-200
3	-800
4	-400
5	-700
6	-900

	1
1	-500
2	-200
3	-800
4	-400
5	-700
6	-900
7	0

	1
1	-500
2	-200
3	-800
4	-400
5	-700
6	-900
7	0

	1	2	3
1	16.1905	-1.4615	10.7078
2	-1.4615	1.1861	-0.5158
3	10.7078	-0.5158	13.7437

	1	2	3	4	5	6
1	16.1905	-1.4615	10.7078	7.6403	13.7581	8.7350
2	-1.4615	1.1861	-0.5158	-1.1065	-0.0266	-0.4340
3	10.7078	-0.5158	13.7437	1.1394	8.8126	2.9226
4	7.6403	-1.1065	1.1394	28.6200	10.3192	20.2593
5	13.7581	-0.0266	8.8126	10.3192	33.5367	9.7687
6	8.7350	-0.4340	2.9226	20.2593	9.7687	44.4787
7	11.1931	-1.2496	3.2744	17.5087	11.3675	16.2277
8	17.4814	-1.1570	9.4588	9.6883	19.0056	11.2037

	7	8
1	11.1931	17.4814
2	-1.2496	-1.1570
3	3.2744	9.4588
4	17.5087	9.6883
5	11.3675	19.0056
6	16.2277	11.2037
7	27.4332	15.0881
8	15.0881	30.1063

	1	2	3
1	1.0378	0.2501	0.3826

	1	2	3	4	5	6
1	1.0378	0.2501	0.3826	1.3984	1.7924	0.7554

	7	8
1	1.2699	1.4927

	1	2	3
1	2.7805	-1.8414	0.9890

	1 goal_R	2 w_SPY	3 w_GOVT	4 w_EEMV	5 portf_Variance
1	0.2501	-0.0178	0.9120	0.1059	1.0531
2	0.2915	0.0426	0.8979	0.0595	0.9216
3	0.3330	0.1031	0.8838	0.0131	0.8516
4	0.3744	0.1635	0.8697	-0.0332	0.8430
5	0.4159	0.2239	0.8557	-0.0796	0.8958
6	0.4574	0.2844	0.8416	-0.1260	1.0101
7	0.4988	0.3448	0.8275	-0.1723	1.1858
8	0.5403	0.4052	0.8135	-0.2187	1.4229
9	0.5817	0.4657	0.7994	-0.2651	1.7215
10	0.6232	0.5261	0.7853	-0.3114	2.0815
11	0.6647	0.5865	0.7713	-0.3578	2.5029
12	0.7061	0.6470	0.7572	-0.4042	2.9858
13	0.7476	0.7074	0.7431	-0.4505	3.5301
14	0.7890	0.7678	0.7291	-0.4969	4.1358
15	0.8305	0.8283	0.7150	-0.5433	4.8030
16	0.8720	0.8887	0.7009	-0.5896	5.5316
17	0.9134	0.9491	0.6869	-0.6360	6.3216
18	0.9549	1.0096	0.6728	-0.6824	7.1731
19	0.9963	1.0700	0.6587	-0.7287	8.0860
20	1.0378	1.1304	0.6447	-0.7751	9.0603

	1 goal_R	2 w_SPY	3 w_GOVT	4 w_EEMV	5 w_CME	6 w_BR
1	0.2501	0.1588	0.8929	0.0329	0.0211	-0.0738
2	0.3312	0.1760	0.8657	-0.0034	0.0407	-0.0463
3	0.4124	0.1932	0.8386	-0.0397	0.0602	-0.0188
4	0.4936	0.2104	0.8115	-0.0760	0.0798	0.0087
5	0.5748	0.2276	0.7843	-0.1123	0.0994	0.0362
6	0.6559	0.2448	0.7572	-0.1485	0.1189	0.0637
7	0.7371	0.2619	0.7301	-0.1848	0.1385	0.0912
8	0.8183	0.2791	0.7029	-0.2211	0.1580	0.1187
9	0.8995	0.2963	0.6758	-0.2574	0.1776	0.1462
10	0.9806	0.3135	0.6486	-0.2936	0.1971	0.1737
11	1.0618	0.3307	0.6215	-0.3299	0.2167	0.2012
12	1.1430	0.3479	0.5944	-0.3662	0.2362	0.2287
13	1.2242	0.3651	0.5672	-0.4025	0.2558	0.2562
14	1.3053	0.3823	0.5401	-0.4388	0.2754	0.2837
15	1.3865	0.3995	0.5129	-0.4750	0.2949	0.3112
16	1.4677	0.4166	0.4858	-0.5113	0.3145	0.3387
17	1.5489	0.4338	0.4587	-0.5476	0.3340	0.3662
18	1.6300	0.4510	0.4315	-0.5839	0.3536	0.3937
19	1.7112	0.4682	0.4044	-0.6201	0.3731	0.4212
20	1.7924	0.4854	0.3773	-0.6564	0.3927	0.4487

	7 w_CBOE	8 w_ICE	9 w_ACN	10 portf_Variance
1	-0.0033	0.0121	-0.0406	0.8220
2	-0.0147	0.0114	-0.0293	0.7489
3	-0.0262	0.0107	-0.0180	0.8027
4	-0.0377	0.0100	-0.0067	0.9834
5	-0.0491	0.0094	0.0046	1.2909
6	-0.0606	0.0087	0.0159	1.7253
7	-0.0721	0.0080	0.0272	2.2865
8	-0.0835	0.0074	0.0385	2.9746
9	-0.0950	0.0067	0.0498	3.7895
10	-0.1065	0.0060	0.0611	4.7313
11	-0.1179	0.0053	0.0724	5.7999
12	-0.1294	0.0047	0.0837	6.9954
13	-0.1408	0.0040	0.0950	8.3177
14	-0.1523	0.0033	0.1063	9.7669
15	-0.1638	0.0027	0.1176	11.3429
16	-0.1752	0.0020	0.1290	13.0458
17	-0.1867	0.0013	0.1403	14.8755
18	-0.1982	6.3514e-04	0.1516	16.8321
19	-0.2096	-3.6908e-05	0.1629	18.9156
20	-0.2211	-7.0896e-04	0.1742	21.1259

	1	2
1	1.0262	0.2501
2	0.9600	0.2915
3	0.9228	0.3330
4	0.9181	0.3744
5	0.9465	0.4159
6	1.0050	0.4574
7	1.0889	0.4988
8	1.1929	0.5403
9	1.3121	0.5817
10	1.4427	0.6232
11	1.5821	0.6647
12	1.7279	0.7061
13	1.8788	0.7476
14	2.0337	0.7890
15	2.1916	0.8305
16	2.3519	0.8720
17	2.5143	0.9134
18	2.6783	0.9549
19	2.8436	0.9963
20	3.0100	1.0378

	1	2
1	0.9066	0.2501
2	0.8654	0.3312
3	0.8959	0.4124
4	0.9917	0.4936
5	1.1362	0.5748
6	1.3135	0.6559
7	1.5121	0.7371
8	1.7247	0.8183
9	1.9467	0.8995
10	2.1751	0.9806
11	2.4083	1.0618
12	2.6449	1.1430
13	2.8840	1.2242
14	3.1252	1.3053
15	3.3679	1.3865
16	3.6119	1.4677
17	3.8569	1.5489
18	4.1027	1.6300
19	4.3492	1.7112
20	4.5963	1.7924