MIE1622 Assignment 4

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1. Implement Pricing Functions in Python

Please see option_pricing.py for coding.

Note: the currency type for the outputs follows the currency type of the given S0, K, and Sb.

2. Analysis of Results

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Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455
One-step MC price of an European call option is 8.01218579826472
One-step MC price of an European put option is 7.900938483557013
Multi-step MC price of an European call option is 8.024686210402164
Multi-step MC price of an European put option is 7.893013872574593
One-step MC price of an Barrier call option is 7.809997796765999
One-step MC price of an Barrier put option is 0.0
Multi-step MC price of an Barrier call option is 7.974651221278676
Multi-step MC price of an Barrier put option is 1.2719255333600317
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Figure 1: Price Outputs for Different Options with Different Time Step

2.1 Black-Scholes Call and Put Price for European Option

Black-Scholes price of a European call option is 8.02, and 7.90 for a European put option.

2.2 One-Step MC Call and Put Price for European Option and Justification of Number of Paths

The one-step MC for the given European option has call option price of 8.01 and put option price of 7.90.

The call and put prices for European option with one-step MC approach call and put prices for Black-Choles, as the number of paths increase. Following testing has been done:

1k paths: (7.190, 7.600)
10k paths: (7.953, 7.774)
100k paths: (7.974, 7.916)
1 Million paths: (8.012, 7.900)

The prices from 1 million paths are close to the prices for one-step MC, thus 1 million paths are generated. The runtime gets considerably large when the number of paths grows to 10 million.

2.3 Multi-Step MC Call and Put Price for European Option with Justification of Numbers of Steps and Paths:

The multi-step MC for the given European option has call option price of 8.03 and put option price of 7.89.

Number of steps are preferred, from my perspective, to have real-world meaning. Numbers of steps for semiannual and quarter setting are too small. Number of steps for weekly and daily setting are kind of large and can cause heavy computation when the number of paths is large. Thus, monthly setting is chosen, which means the number of steps is 12.

Selection of number of paths follows similar process as one-step MC European option:

10k paths: (8.195, 7.788)
100k paths: (8.078, 7.897)
1 Million paths: (8.025, 7.893)

The prices from 1 million paths are close to the prices for one-step MC, thus 1 million paths are generated. The runtime gets considerably large when the number of paths grows to 10 million.

2.4 One-Step MC Call and Put Price for Barrier Option and Justification of Number of Paths

The one-step MC for the given Barrier option has call option price of 7.81 and put option price of 0.

The difference between European option and knock-in Barrier option is simply that knock-in Barrier option turns the value to 0 for the paths that never hit the barrier value. Since 1 million paths can provide call and put prices for MC European option that are close to Black-Scholes prices, 1 million paths can describe most of the price dynamics and give mostly complete picture of value distribution for MC European option, which means it can also do so for MC Barrier option. Therefore, 1 million paths are generated for MC Barrier option.

2.5 Multi-Step MC Call and Put Price for Barrier Option and Justification of Number of Steps and Paths

The multi-step MC for the given Barrier option has call option price of 7.79 and put option price of 1.27.

Follow same logic mentioned in section 2.4 (one-step MC Barrier Options), the number of steps and paths are exhibited from multi-step MC European Options. Thus 12 steps and 1 million paths are used.

2.6 Chart Illustrating Monte Carlo Pricing Procedure

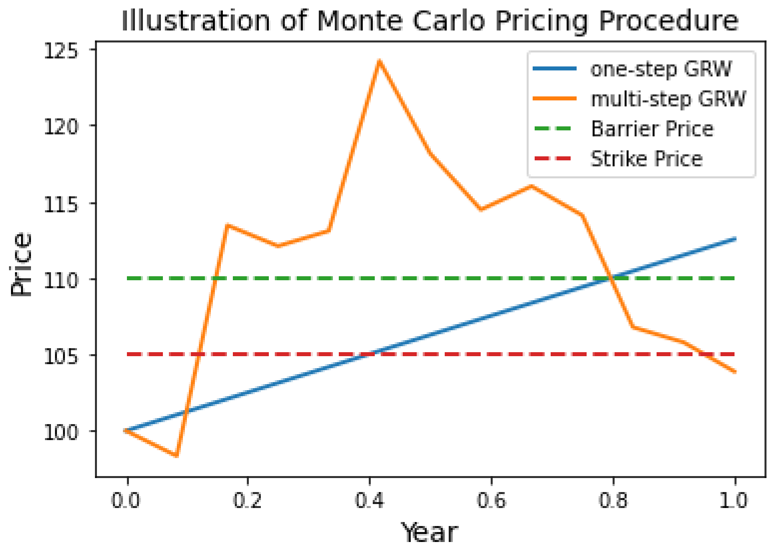

Figure 2: Illustration of Monte Carlo Pricing Procedure

An example path from one-step GRW and an example path from multi-step GRW are plotted in Figure 2. The price of a path depends on its final price at t = 1, as compared to strike price K, and whether it has crossed Barrier price. The prices of all the paths are then averaged and discounted to t = 0.

2.7 Comparison of Three Pricing Strategies for European Option and Discussion of Performance

Black-Scholes derived the call and put prices analytically by plugging in number into the formula, thus it has the highest precision and lowest runtime (no need to generate large amount random samples) among the three strategies.

From section 2.2 and 2.3, it is showed that precision of one-step and multi-step (12-steps) MC are similar (both are high) with number of paths of 100 thousand and 1 million. However, multi-step MC has higher runtime than one-step MC when generating same number of paths, because it generates random number and computes prices for multiple times for each path, while the one-step MC does it once per path. Therefore, one-step MC is preferred over multi-step MC for European option for the sake of runtime.

2.8 Explanation of the Difference between Call and Put Prices obtained for European and Barrier Options

Value of a path for European option is nonzero (always positive if nonzero) if and only if its price at expiration date is higher than strike price for call and lower for put. However, the barrier turns values of some of the paths that has hit barrier price from non-zero to zeros. Therefore, the mean of all prices of different paths would decrease (some positive prices become zero). Thus, both the call and put prices for Barrier option would be lower than the call and put prices for European option, respectively. Particularly for one-step MC, in order to hit barrier price, which is higher than strike price, the path must be a single line segment that ends up with price higher than strike price at expiration date. Thus, the price of put option for one-step MC of Barrier option is 0, meaning that multi-step must be used in this setting.

2.9 Prices of Barrier Option with Modified Volatility

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Multi-step MC price of an Barrier call option with 10% decreased volatility is 7.165897783249576
Multi-step MC price of an Barrier put option with 10% decreased volatility is 0.9696862425404498
Multi-step MC price of an Barrier call option with 10% increased volatility is 8.763452659837156
Multi-step MC price of an Barrier put option with 10% increased volatility is 1.5935218539784357
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Figure 3: Price Outputs of Barrier Option with Modified Volatility

As mentioned in section 2.8, when the barrier price is higher than the strike price, which is higher than the initial price, the put option price for one-step MC is 0. Therefore, multi-step MC is used for Barrier option.

Both call and put prices increase when the volatility is increased by 10% and decreases when the volatility is decreased by 10%. With 10% volatility decreased and multi-step Barrier option, call option price is 7.17 and put option is 0.97. With 10% volatility increased and multi-step Barrier option, call option price is 8.76 and put option is 1.59.

For a normal distribution, when standard deviation increases, the probability of extreme values increases. With volatility increases, there would be more paths that hit barrier price and then reach prices higher than strike prices for call option. Also, there would be more paths that hit barrier price and then reach prices lower than strike prices for put option. Moreover, the positive values of both put and call options tend to be more positive after the increase in volatility (deviate more from expected value). Therefore, both the put and call option prices increase when volatility increases.

Following similar logic, both the put and call option prices decrease when volatility decreases.

3. Strategies to Obtain Same Prices from 2 Procedures

3.1 Design Procedure for Choosing Numbers of Time Steps and Scenarios in MC to Get Same Price (up to cent) as Given by the Black-Scholes Formula

As mentioned in section 2.2 and 2.3, prices from MC approaches prices from Black-Scholes Formula as number of scenarios increases. In addition, the precisions of one-step and multi-step (12 steps) are close for European option with 100 thousand and 1 million, but multi-step MC has longer runtime. Therefore, one-step is used, and the number of scenarios is tuned to reach the goal prices. Each time, if the prices are not close enough, number of scenarios is increased by 10 times, starting from 1 thousand (as in section 2.2). Eventually, with one-step and 10 million scenarios, the goal prices are achieved.

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One-step MC price of an European call option with 10 Million Scenarios is 8.02185044149982
One-step MC price of an European put option with 10 Million Scenarios is 7.896100566598778
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Figure 4: Price Outputs of European Option with One-Step and 10 Million Scenarios

The prices given by Black-Scholes Formula are 8.02135 (call) and 7.90044 (put). The prices given by European option with one-step and 10 million scenarios MC are 8.02185 (call) and 7.89610 (put). The prices are same up to the cent.