CS 663 - Fundamentals of Digital Image Processing
Assignment 05

Gagan Jain - 180100043
Hitesh Kandala - 180070023

November 16, 2020

Solution 1

We have the following, with h_1 and h_2 being blur kernels applied on f_1 and f_2 respectively.

$$g_1 = f_1 + h_2 * f_2 \tag{1}$$
$$g_2 = h_1 * f_1 + f_2 \tag{2}$$

Taking the direct Fourier Transform of the Eq. (1) and (2), we have:

$$G_1 = F_1 + H_2F_2 \tag{3}$$
$$G_2 = H_1F_1 + F_2 \tag{4}$$

Solving for F_1 amd F_2, we get:

$$F_1 = \frac{G_1 - H_2G_2}{1 - H_1H_2} \tag{5}$$
$$F_2 = \frac{G_2 - H_1G_1}{1 - H_1H_2} \tag{6}$$

Now, in order to get back the spatial-domain representation f_1 and f_2, we can take the Inverse Fourier Transform, so that:

$$f_1 = F^{-1}\left(\frac{G_1 - H_2G_2}{1 - H_1H_2}\right) \tag{7}$$
$$f_2 = F^{-1}\left(\frac{G_2 - H_1G_1}{1 - H_1H_2}\right) \tag{8}$$

where F^{-1} represents the Inverse Fourier Transform. Now, it is obvious to see that if we have the product H_1H_2 tending to 1, the expression will blow up and we will not be able to recover the corresponding components of the signal in the way we described. This is clearly something that is not unlikely as for the blur filters h_1 and h_2, we would have:

$$\int_{-\infty}^{\infty} h_1(x)dx = 1 \tag{9}$$
$$\int_{-\infty}^{\infty} h_2(x)dx = 1 \tag{10}$$

In general, we have:

$$H(\omega) = \int_{-\infty}^{\infty} h(x)e^{-2\pi j\omega x}dx \tag{11}$$

It should not be very difficult to see that Eq. (9) and (10) represent the zero frequency (or the static) component of the signal in frequency domain. So, we will necessarily have $H_1(0)H_2(0) = 1$ and as a result, it is not possible to recover the static component of the signal, $F_1(0)$ and $F_2(0)$ using this technique without any modifications.