CS 663 - Fundamentals of Digital Image Processing
Assignment 05

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Solution 2

We have, the image f and the gradient image g related by the convolution filter h such that:

$$g = h * f \tag{1}$$

Now, we know that, for a 1D image, by the forward difference method:

$$g(x) = f(x+1) - f(x) \tag{2}$$

This is equivalent to having a filter h(x) = [-1,1] in Eq. (1). Now, taking the Discrete Fourier Transform of Eq. (2) and using the shift property, we have:

$$G(\omega) = F(\omega)e^{2\pi j\omega/N} - F(\omega)$$
$$G(\omega) = F(\omega)(e^{2\pi j\omega/N} - 1) \tag{3}$$

So, the Fourier Transform of the original image is given by:

$$F(\omega) = \frac{G(\omega)}{(e^{2\pi j\omega/N} - 1)} \tag{4}$$

Now, making an argument similar to what we did for Question 1, we can say that when ω is close to zero, the denominator vanishes and thus, the Fourier Transform blows up. This means that we are incapable of recovering the static component of the image using this technique. As a result, we need to use the Boundary Condition assumptions in order to be able to get the Static component of the original image.

Now, for a 2D image, we repeat the same technique, assuming the two partial derivatives to be $g_x(x,y)$ and $g_y(x,y)$. So, we will again have:

$$G_x(u,v) = F(u,v)(e^{2\pi ju/N} - 1) \tag{5}$$
$$G_y(u,v) = F(u,v)(e^{2\pi jv/N} - 1) \tag{6}$$

As a result, we will have the following expressions for the image:

$$F(x,y) = \frac{G_x(u,v)}{(e^{2\pi ju/N} - 1)} = \frac{G_y(u,v)}{(e^{2\pi jy/N} - 1)} \tag{7}$$

Now, again estimating the original image using the partial derivatives is difficult when u = 0 or v = 0 and we will need to look for other methods to estimate the static (zero frequency) contents of the image as different boundary conditions (for u and v) might lead to different estimates for the image